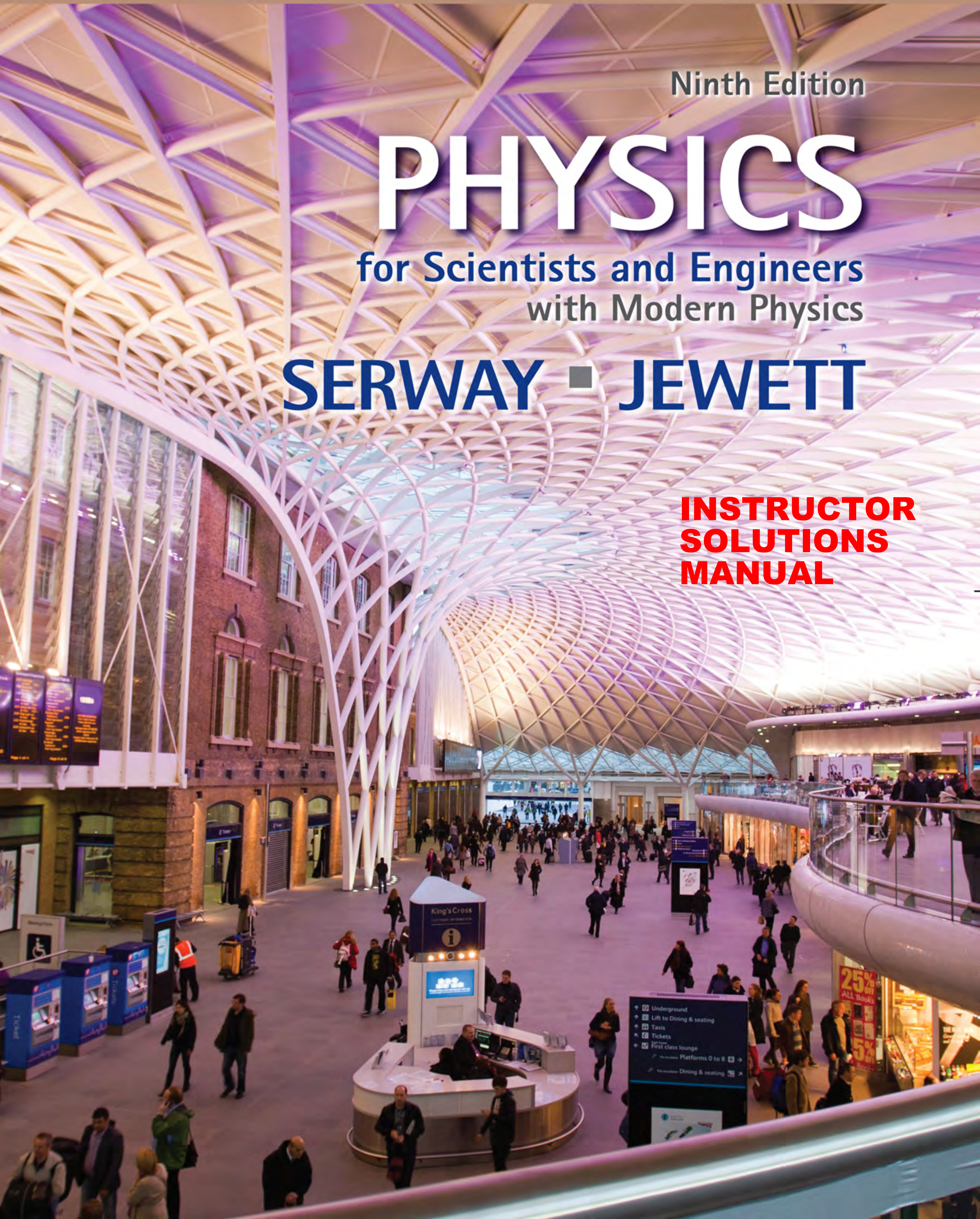
Ninth Edition

PHYSICS

for Scientists and Engineers with Modern Physics

SERWAY ■ JEWETT

INSTRUCTOR SOLUTIONS MANUAL



1

Physics and Measurement

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ1.1 The meterstick measurement, (a), and (b) can all be 4.31 cm. The meterstick measurement and (c) can both be 4.24 cm. Only (d) does not overlap. Thus (a), (b), and (c) all agree with the meterstick measurement.

OQ1.2 Answer (d). Using the relation

$$1\text{ ft} = 12\text{ in}\left(\frac{2.54\text{ cm}}{1\text{ in}}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right) = 0.304\ 8\text{ m}$$

we find that

$$1\,420\text{ ft}^2\left(\frac{0.304\ 8\text{ m}}{1\text{ ft}}\right)^2 = 132\text{ m}^2$$

OQ1.3 The answer is yes for (a), (c), and (e). You cannot add or subtract a number of apples and a number of jokes. The answer is no for (b) and (d). Consider the gauge of a sausage, 4 kg/2 m, or the volume of a cube, $(2\text{ m})^3$. Thus we have (a) yes; (b) no; (c) yes; (d) no; and (e) yes.

OQ1.4 41 € ≈ 41 € (1 L/1.3 €)(1 qt/1 L)(1 gal/4 qt) ≈ (10/1.3) gal ≈ 8 gallons, answer (c).

OQ1.6 The number of decimal places in a sum of numbers should be the same as the smallest number of decimal places in the numbers summed.

$$\begin{array}{r l} 21.4 & \text{s} \\ 15 & \text{s} \\ 17.17 & \text{s} \\ 4.003 & \text{s} \\ \hline 57.573 & \text{s} = 58 \text{ s, answer (d).} \end{array}$$

OQ1.7 The population is about 6 billion = 6×10^9. Assuming about 100 lb per person = about 50 kg per person (1 kg has the weight of about 2.2 lb), the total mass is about $(6 \times 10^9)(50 \text{ kg}) = 3 \times 10^{11}$ kg, answer (d).

OQ1.8 No: A dimensionally correct equation need not be true. Example: 1 chimpanzee = 2 chimpanzee is dimensionally correct.

Yes: If an equation is not dimensionally correct, it cannot be correct.

OQ1.9 Mass is measured in kg; acceleration is measured in m/s^2. Force = mass × acceleration, so the units of force are answer (a) $\text{kg}\cdot\text{m/s}^2$.

OQ1.10 0.02(1.365) = 0.03. The result is $(1.37 \pm 0.03) \times 10^7$ kg. So (d) 3 digits are significant.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ1.1 Density varies with temperature and pressure. It would be necessary to measure both mass and volume very accurately in order to use the density of water as a standard.

CQ1.2 The metric system is considered superior because units larger and smaller than the basic units are simply related by multiples of 10. Examples: 1 km = 10^3 m, 1 mg = 10^{-3} g = 10^{-6} kg, 1 ns = 10^{-9} s.

CQ1.3 A unit of time should be based on a reproducible standard so it can be used everywhere. The more accuracy required of the standard, the less the standard should change with time. The current, very accurate standard is the period of vibration of light emitted by a cesium atom. Depending on the accuracy required, other standards could be: the period of light emitted by a different atom, the period of the swing of a pendulum at a certain place on Earth, the period of vibration of a sound wave produced by a string of a specific length, density, and tension, and the time interval from full Moon to full Moon.

CQ1.4 (a) 0.3 millimeters; (b) 50 microseconds; (c) 7.2 kilograms

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 1.1 Standards of Length, Mass, and Time

P1.1 (a) Modeling the Earth as a sphere, we find its volume as

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(6.37\times10^6\text{ m}\right)^3 = 1.08\times10^{21}\text{ m}^3$$

Its density is then

$$\rho = \frac{m}{V} = \frac{5.98\times10^{24}\text{ kg}}{1.08\times10^{21}\text{ m}^3} = \boxed{5.52\times10^3\text{ kg/m}^3}$$

(b) This value is intermediate $\boxed{\text{between the tabulated densities of aluminum and iron.}}$ Typical rocks have densities around 2000 to 3000 kg/m^3. The average density of the Earth is significantly higher, so higher-density material must be down below the surface.

P1.2 With V = (base area)(height), $V = \left(\pi r^2\right)h$ and $\rho = \frac{m}{V}$, we have

$$\rho = \frac{m}{\pi r^2 h} = \frac{1\text{ kg}}{\pi(19.5\text{ mm})^2(39.0\text{ mm})}\left(\frac{10^9\text{ mm}^3}{1\text{ m}^3}\right)$$

$$\rho = \boxed{2.15\times10^4\text{ kg/m}^3}$$

P1.3 Let V represent the volume of the model, the same in $\rho = \frac{m}{V}$, for both.

Then $\rho_{\text{iron}} = 9.35\text{ kg/V}$ and $\rho_{\text{gold}} = \frac{m_{\text{gold}}}{V}$.

Next, $$\frac{\rho_{\text{gold}}}{\rho_{\text{iron}}} = \frac{m_{\text{gold}}}{9.35\text{ kg}}$$

and $$m_{\text{gold}} = (9.35\text{ kg})\left(\frac{19.3\times10^3\text{ kg/m}^3}{7.87\times10^3\text{ kg/m}^3}\right) = \boxed{22.9\text{ kg}}$$

P1.4 (a) $\rho = m/V$ and $V = (4/3)\pi r^3 = (4/3)\pi(d/2)^3 = \pi d^3/6$, where d is the diameter.

Then $$\rho = 6m/\pi d^3 = \frac{6\left(1.67\times10^{-27}\text{kg}\right)}{\pi\left(2.4\times10^{-15}\text{m}\right)^3} = \boxed{2.3\times10^{17}\text{kg/m}^3}$$

(b) $$\frac{2.3\times10^{17}\text{ kg/m}^3}{22.6\times10^3\text{ kg/m}^3} = \boxed{1.0\times10^{13}\text{ times the density of osmium}}$$

P1.5 For either sphere the volume is $V = \frac{4}{3}\pi r^3$ and the mass is $m = \rho V = \rho\frac{4}{3}\pi r^3$. We divide this equation for the larger sphere by the same equation for the smaller:

$$\frac{m_\ell}{m_s} = \frac{\rho(4/3)\pi r_\ell^3}{\rho(4/3)\pi r_s^3} = \frac{r_\ell^3}{r_s^3} = 5$$

Then $r_\ell = r_s\sqrt[3]{5} = (4.50\text{ cm})\sqrt[3]{5} = \boxed{7.69\text{ cm}}$

***P1.6** The volume of a spherical shell can be calculated from

$$V = V_o - V_i = \frac{4}{3}\pi\left(r_2^3 - r_1^3\right)$$

From the definition of density, $\rho = \frac{m}{V}$, so

$$m = \rho V = \rho\left(\frac{4}{3}\pi\right)\left(r_2^3 - r_1^3\right) = \boxed{\frac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}}$$

Section 1.2 Matter and Model Building

P1.7 From the figure, we may see that the spacing between diagonal planes is half the distance between diagonally adjacent atoms on a flat plane. This diagonal distance may be obtained from the Pythagorean theorem, $L_{\text{diag}} = \sqrt{L^2 + L^2}$. Thus, since the atoms are separated by a distance *L* = 0.200 nm, the diagonal planes are separated by $\frac{1}{2}\sqrt{L^2 + L^2} = \boxed{0.141\text{ nm}}$.

P1.8 (a) Treat this as a conversion of units using 1 Cu-atom = 1.06×10^{-25} kg, and 1 cm = 10^{-2} m:

$$\text{density} = \left(8\ 920\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{10^{-2}\text{ m}}{1\text{ cm}}\right)^3\left(\frac{\text{Cu-atom}}{1.06\times 10^{-25}\text{ kg}}\right)$$

$$= \boxed{8.42\times 10^{22}\ \frac{\text{Cu-atom}}{\text{cm}^3}}$$

(b) Thinking in terms of units, invert answer (a):

$$(\text{density})^{-1} = \left(\frac{1\ \text{cm}^3}{8.42\times10^{22}\ \text{Cu-atoms}}\right)$$
$$= \boxed{1.19\times10^{-23}\ \text{cm}^3/\text{Cu-atom}}$$

(c) For a cube of side *L*,

$$L^3 = 1.19\times10^{-23}\ \text{cm}^3 \rightarrow L = \boxed{2.28\times10^{-8}\text{cm}}$$

Section 1.3 Dimensional Analysis

P1.9 (a) Write out dimensions for each quantity in the equation

$$v_f = v_i + ax$$

The variables v_f and v_i are expressed in units of m/s, so

$$[v_f] = [v_i] = \text{LT}^{-1}$$

The variable a is expressed in units of m/s^2; $[a] = \text{LT}^{-2}$

The variable x is expressed in meters. Therefore, $[ax] = \text{L}^2\text{T}^{-2}$

Consider the right-hand member (RHM) of equation (a):

$$[\text{RHM}] = \text{LT}^{-1} + \text{L}^2\text{T}^{-2}$$

Quantities to be added must have the same dimensions. Therefore, $\boxed{\text{equation (a) } \textbf{is not} \text{ dimensionally correct.}}$

(b) Write out dimensions for each quantity in the equation

$$y = (2\text{ m})\cos(kx)$$

For y, $[y] = \text{L}$

for 2 m, $[2\text{ m}] = \text{L}$

and for (kx), $[kx] = \left[\left(2\text{ m}^{-1}\right)x\right] = \text{L}^{-1}\text{L}$

Therefore we can think of the quantity kx as an angle in radians, and we can take its cosine. The cosine itself will be a pure number with no dimensions. For the left-hand member (LHM) and the right-hand member (RHM) of the equation we have

$$[\text{LHM}] = [y] = \text{L} \qquad [\text{RHM}] = [2\text{ m}][\cos(kx)] = \text{L}$$

These are the same, so $\boxed{\text{equation (b) } \textbf{is} \text{ dimensionally correct.}}$

P1.10 Circumference has dimensions L, area has dimensions L^2, and volume has dimensions L^3. Expression (a) has dimensions $L(L^2)^{1/2} = L^2$, expression (b) has dimensions L, and expression (c) has dimensions $L(L^2) = L^3$. $\boxed{\text{The matches are: (a) and (f), (b) and (d), and (c) and (e).}}$

P1.11 (a) Consider dimensions in terms of their mks units. For kinetic energy *K*:

$$[K] = \left[\left(\frac{p^2}{2m}\right)\right] = \frac{[p]^2}{\text{kg}} = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}$$

Solving for $[p^2]$ and $[p]$ then gives

$$[p]^2 = \frac{\text{kg}^2\cdot\text{m}^2}{\text{s}^2} \quad \rightarrow \quad [p] = \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$\boxed{\text{The units of momentum are kg}\cdot\text{m/s.}}$

(b) Momentum is to be expressed as the product of force (in N) and some other quantity *X*. Considering dimensions in terms of their mks units,

$$[\text{N}]\cdot[X] = [p]$$

$$\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\cdot[X] = \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

$$[X] = \text{s}$$

Therefore, the units of momentum are $\boxed{\text{N}\cdot\text{s}}$.

P1.12 We substitute $[\text{kg}] = [M]$, $[\text{m}] = [L]$, and $[F] = \left[\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\right] = \frac{[M][L]}{[T]^2}$ into Newton's law of universal gravitation to obtain

$$\frac{[M][L]}{[T]^2} = \frac{[G][M]^2}{[L]^2}$$

Solving for $[G]$ then gives

$$[G] = \frac{[L]^3}{[M][T]^2} = \boxed{\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$$

***P1.13** The term *x* has dimensions of L, *a* has dimensions of LT^{-2}, and *t* has dimensions of T. Therefore, the equation $x = ka^m t^n$ has dimensions of

$$L = \left(LT^{-2}\right)^m (T)^n \quad \text{or} \quad L^1T^0 = L^mT^{n-2m}$$

The powers of L and T must be the same on each side of the equation.

Therefore,

$$\mathrm{L}^1 = \mathrm{L}^m \text{ and } \boxed{m = 1}$$

Likewise, equating terms in T, we see that $n - 2m$ must equal 0. Thus, $\boxed{n = 2}$. The value of k, a dimensionless constant, $\boxed{\text{cannot be obtained by dimensional analysis}}$.

P1.14 Summed terms must have the same dimensions.

(a) $[X] = [At^3] + [Bt]$

$$\mathrm{L} = [A]\mathrm{T}^3 + [B]\mathrm{T} \rightarrow \boxed{[A] = \mathrm{L}/\mathrm{T}^3, \text{ and } [B] = \mathrm{L}/\mathrm{T}}.$$

(b) $[dx/dt] = [3At^2] + [B] = \boxed{\mathrm{L}/\mathrm{T}}.$

Section 1.4 Conversion of Units

P1.15 From Table 14.1, the density of lead is 1.13×10^4 kg/m^3, so we should expect our calculated value to be close to this value. The density of water is 1.00×10^3 kg/m^3, so we see that lead is about 11 times denser than water, which agrees with our experience that lead sinks.

Density is defined as $\rho = m/V$. We must convert to SI units in the calculation.

$$\begin{aligned}\rho &= \left(\frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\right)\left(\frac{1 \text{ kg}}{1\,000 \text{ g}}\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 \\ &= \left(\frac{23.94 \text{ g}}{2.10 \text{ cm}^3}\right)\left(\frac{1 \text{ kg}}{1\,000 \text{ g}}\right)\left(\frac{1\,000\,000 \text{ cm}^3}{1 \text{ m}^3}\right) \\ &= \boxed{1.14 \times 10^4 \text{ kg/m}^3}\end{aligned}$$

Observe how we set up the unit conversion fractions to divide out the units of grams and cubic centimeters, and to make the answer come out in kilograms per cubic meter. At one step in the calculation, we note that **one million** cubic centimeters make one cubic meter. Our result is indeed close to the expected value. Since the last reported significant digit is not certain, the difference from the tabulated values is possibly due to measurement uncertainty and does not indicate a discrepancy.

P1.16 The weight flow rate is

$$\left(1\ 200\frac{\text{ton}}{\text{h}}\right)\left(\frac{2000\ \text{lb}}{\text{ton}}\right)\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)=\boxed{667\ \text{lb/s}}$$

P1.17 For a rectangle, Area = Length × Width. We use the conversion 1 m = 3.281 ft. The area of the lot is then

$$A=LW=(75.0\ \text{ft})\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)(125\ \text{ft})\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)=\boxed{871\ \text{m}^2}$$

P1.18 Apply the following conversion factors: 1 in = 2.54 cm, 1 d = 86 400 s, 100 cm = 1m, and 10^9 nm = 1 m. Then, the rate of hair growth per second is

$$\text{rate}=\left(\frac{1}{32}\ \text{in/day}\right)\frac{(2.54\ \text{cm/in})(10^{-2}\ \text{m/cm})(10^{9}\ \text{nm/m})}{86\ 400\ \text{s/day}}$$
$$=\boxed{9.19\ \text{nm/s}}$$

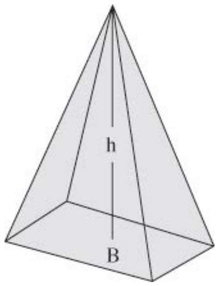
This means the proteins are assembled at a rate of many layers of atoms each second!

P1.19 The area of the four walls is (3.6 + 3.8 + 3.6 + 3.8) m × (2.5 m) = 37 m^2. Each sheet in the book has area (0.21 m)(0.28 m) = 0.059 m^2. The number of sheets required for wallpaper is 37 m^2/0.059 m^2 = 629 sheets = 629 sheets(2 pages/1 sheet) = 1260 pages.

$\boxed{\text{The number of pages in Volume 1 are insufficient.}}$

P1.20 We use the formula for the volume of a pyramid given in the problem and the conversion 43 560 ft^2 = 1 acre. Then,

$$\begin{aligned}V&=Bh\\&=\frac{1}{3}\left[(13.0\ \text{acres})(43\ 560\ \text{ft}^2/\text{acre})\right]\\&\qquad\times(481\ \text{ft})\\&=9.08\times10^{7}\ \text{ft}^3\end{aligned}$$



ANS FIG. P1.20

or

$$\begin{aligned}V&=(9.08\times10^{7}\ \text{ft}^3)\left(\frac{2.83\times10^{-2}\ \text{m}^3}{1\ \text{ft}^3}\right)\\&=\boxed{2.57\times10^{6}\ \text{m}^3}\end{aligned}$$

P1.21 To find the weight of the pyramid, we use the conversion 1 ton = 2 000 lbs:

$$F_g = (2.50 \text{ tons/block})(2.00\times10^6 \text{ blocks})(2\ 000 \text{ lb/ton})$$

$$= \boxed{1.00\times10^{10} \text{ lbs}}$$

P1.22 (a) $\text{rate} = \left(\dfrac{30.0 \text{ gal}}{7.00 \text{ min}}\right)\left(\dfrac{1 \text{ mi}}{60 \text{ s}}\right) = \boxed{7.14\times10^{-2}\dfrac{\text{gal}}{\text{s}}}$

(b) $\text{rate} = 7.14\times10^{-2}\dfrac{\text{gal}}{\text{s}}\left(\dfrac{231 \text{ in}^3}{1 \text{ gal}}\right)\left(\dfrac{2.54 \text{ cm}}{1 \text{ in}}\right)^3\left(\dfrac{1 \text{ m}}{100 \text{ cm}}\right)^3$

$$= \boxed{2.70\times10^{-4}\frac{\text{m}^3}{\text{s}}}$$

(c) To find the time to fill a 1.00-m^3 tank, find the rate time/volume:

$$2.70\times10^{-4}\frac{\text{m}^3}{\text{s}} = \left(\frac{2.70\times10^{-4} \text{ m}^3}{1 \text{ s}}\right)$$

or $$\left(\frac{2.70\times10^{-4} \text{ m}^3}{1 \text{ s}}\right)^{-1} = \left(\frac{1 \text{ s}}{2.70\times10^{-4} \text{ m}^3}\right) = 3.70\times10^3\frac{\text{s}}{\text{m}^3}$$

and so: $$3.70\times10^3 \text{ s}\left(\frac{1 \text{ h}}{3\ 600 \text{ s}}\right) = \boxed{1.03 \text{ h}}$$

***P1.23** It is often useful to remember that the 1 600-m race at track and field events is approximately 1 mile in length. To be precise, there are 1 609 meters in a mile. Thus, 1 acre is equal in area to

$$(1 \text{ acre})\left(\frac{1 \text{ mi}^2}{640 \text{ acres}}\right)\left(\frac{1\ 609 \text{ m}}{\text{mi}}\right)^2 = \boxed{4.05\times10^3 \text{ m}^2}$$

***P1.24** The volume of the interior of the house is the product of its length, width, and height. We use the conversion 1 ft = 0.304 8 m and 100 cm = 1 m.

$$V = LWH$$

$$= (50.0 \text{ ft})\left(\frac{0.304\ 8 \text{ m}}{1 \text{ ft}}\right)\times(26 \text{ ft})\left(\frac{0.304\ 8 \text{ m}}{1 \text{ ft}}\right)$$

$$\times(8.0 \text{ ft})\left(\frac{0.304\ 8 \text{ m}}{1 \text{ ft}}\right)$$

$$= 294.5 \text{ m}^3 = \boxed{290 \text{ m}^3}$$

$$= (294.5 \text{ m}^3)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = \boxed{2.9\times10^8 \text{ cm}^3}$$

Both the 26-ft width and 8.0-ft height of the house have two significant figures, which is why our answer was rounded to 290 m^3.

P1.25 The aluminum sphere must be larger in volume to compensate for its lower density. We require equal masses:

$$m_{A1} = m_{Fe} \qquad \text{or} \qquad \rho_{A1}V_{A1} = \rho_{Fe}V_{Fe}$$

then use the volume of a sphere. By substitution,

$$\rho_{A1}\left(\frac{4}{3}\pi r_{A1}{}^3\right) = \rho_{Fe}\left(\frac{4}{3}\pi(2.00\text{ cm})^3\right)$$

Now solving for the unknown,

$$\begin{aligned} r_{A1}^3 &= \left(\frac{\rho_{Fe}}{\rho_{A1}}\right)(2.00\text{ cm})^3 = \left(\frac{7.86\times10^3\text{ kg/m}^3}{2.70\times10^3\text{ kg/m}^3}\right)(2.00\text{ cm})^3 \\ &= 23.3\text{ cm}^3 \end{aligned}$$

Taking the cube root, $\boxed{r_{Al} = 2.86\text{ cm}}$.

The aluminum sphere is 43% larger than the iron one in radius, diameter, and circumference. Volume is proportional to the cube of the linear dimension, so this excess in linear size gives it the (1.43)(1.43)(1.43) = 2.92 times larger volume it needs for equal mass.

P1.26 The mass of each sphere is $m_{Al} = \rho_{Al}V_{Al} = \dfrac{4\pi\rho_{Al}r_{Al}^3}{3}$

and $m_{Fe} = \rho_{Fe}V_{Fe} = \dfrac{4\pi\rho_{Fe}r_{Fe}^3}{3}$. Setting these masses equal,

$$\frac{4}{3}\pi\rho_{Al}r_{Al}^3 = \frac{4}{3}\pi\rho_{Fe}r_{Fe}^3 \rightarrow r_{Al} = r_{Fe}\sqrt[3]{\frac{\rho_{Fe}}{\rho_{Al}}}$$

$$r_{Al} = r_{Fe}\sqrt[3]{\frac{7.86}{2.70}} = \boxed{r_{Fe}(1.43)}$$

The resulting expression shows that the radius of the aluminum sphere is directly proportional to the radius of the balancing iron sphere. The aluminum sphere is 43% larger than the iron one in radius, diameter, and circumference. Volume is proportional to the cube of the linear dimension, so this excess in linear size gives it the $(1.43)^3 = 2.92$ times larger volume it needs for equal mass.

P1.27 We assume the paint keeps the same volume in the can and on the wall, and model the film on the wall as a rectangular solid, with its volume given by its "footprint" area, which is the area of the wall, multiplied by its thickness *t* perpendicular to this area and assumed to be uniform. Then,

$$V = At \quad \text{gives} \quad t = \frac{V}{A} = \frac{3.78 \times 10^{-3}\ \text{m}^3}{25.0\ \text{m}^2} = \boxed{1.51 \times 10^{-4}\ \text{m}}$$

The thickness of 1.5 tenths of a millimeter is comparable to the thickness of a sheet of paper, so this answer is reasonable. The film is many molecules thick.

P1.28 (a) To obtain the volume, we multiply the length, width, and height of the room, and use the conversion 1 m = 3.281 ft.

$$\begin{aligned} V &= (40.0\ \text{m})(20.0\ \text{m})(12.0\ \text{m}) \\ &= \left(9.60\times 10^3\ \text{m}^3\right)\left(\frac{3.281\ \text{ft}}{1\ \text{m}}\right)^3 \\ &= \boxed{3.39\times 10^5\ \text{ft}^3} \end{aligned}$$

(b) The mass of the air is

$$m = \rho_{\text{air}} V = \left(1.20\ \text{kg/m}^3\right)\left(9.60\times 10^3\ \text{m}^3\right) = 1.15\times 10^4\ \text{kg}$$

The student must look up the definition of weight in the index to find

$$F_g = mg = \left(1.15\times 10^4\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = 1.13\times 10^5\ \text{N}$$

where the unit of N of force (weight) is newtons.

Converting newtons to pounds,

$$F_g = (1.13\times 10^5\ \text{N})\left(\frac{1\ \text{lb}}{4.448\ \text{N}}\right) = \boxed{2.54\times 10^4\ \text{lb}}$$

P1.29 (a) The time interval required to repay the debt will be calculated by dividing the total debt by the rate at which it is repaid.

$$T = \frac{\$16\ \text{trillion}}{\$1000\,/\,\text{s}} = \frac{\$16 \times 10^{12}}{(\$1000\,/\,\text{s})\left(3.156 \times 10^7\ \text{s/yr}\right)} = \boxed{507\ \text{yr}}$$

(b) The number of bills is the distance to the Moon divided by the length of a dollar.

$$N = \frac{D}{\ell} = \frac{3.84\times 10^8\ \text{m}}{0.155\ \text{m}} = \boxed{2.48\times 10^9\ \text{bills}}$$

Sixteen trillion dollars is larger than this two-and-a-half billion dollars by more than six thousand times. The ribbon of bills comprising the debt reaches across the cosmic gulf thousands of times. Similar calculations show that the bills could span the distance between the Earth and the Sun sixteen times. The strip could encircle the Earth's equator nearly 62 000 times. With successive turns wound edge to edge without overlapping, the dollars would cover a zone centered on the equator and about 4.2 km wide.

P1.30 (a) To find the scale size of the nucleus, we multiply by the scaling factor

$$\begin{aligned} d_{\text{nucleus, scale}} &= d_{\text{nucleus, real}}\left(\frac{d_{\text{atom, scale}}}{d_{\text{atom, real}}}\right) \\ &= \left(2.40\times10^{-15}\text{ m}\right)\left(\frac{300\text{ ft}}{1.06\times10^{-10}\text{ m}}\right) \\ &= 6.79\times10^{-3}\text{ ft} \end{aligned}$$

or

$$d_{\text{nucleus, scale}} = \left(6.79\times10^{-3}\text{ ft}\right)\left(\frac{304.8\text{ mm}}{1\text{ ft}}\right) = \boxed{2.07\text{ mm}}$$

(b) The ratio of volumes is simply the ratio of the cubes of the radii:

$$\begin{aligned} \frac{V_{\text{atom}}}{V_{\text{nucleus}}} &= \frac{4\pi r_{\text{atom}}^3/3}{4\pi r_{\text{nucleus}}^3/3} = \left(\frac{r_{\text{atom}}}{r_{\text{nucleus}}}\right)^3 = \left(\frac{d_{\text{atom}}}{d_{\text{nucleus}}}\right)^3 \\ &= \left(\frac{1.06\times10^{-10}\text{ m}}{2.40\times10^{-15}\text{ m}}\right)^3 = \boxed{8.62\times10^{13}\text{ times as large}} \end{aligned}$$

Section 1.5 Estimates and Order-of-Magnitude Calculations

P1.31 Since we are only asked to find an estimate, we do not need to be too concerned about how the balls are arranged. Therefore, to find the number of balls we can simply divide the volume of an average-size living room (perhaps 15 ft × 20 ft × 8 ft) by the volume of an individual Ping-Pong ball. Using the approximate conversion 1 ft = 30 cm, we find

$$V_{\text{Room}} = (15\text{ ft})(20\text{ ft})(8\text{ ft})(30\text{ cm/ft})^3 \approx 6\times10^7\text{ cm}^3$$

A Ping-Pong ball has a diameter of about 3 cm, so we can estimate its volume as a cube:

$$V_{\text{ball}} = (3\text{ cm})(3\text{ cm})(3\text{ cm}) \approx 30\text{ cm}^3$$

The number of Ping-Pong balls that can fill the room is

$$N \approx \frac{V_{\text{Room}}}{V_{\text{ball}}} \approx 2 \times 10^6 \text{ balls} \sim \boxed{10^6 \text{ balls}}$$

So a typical room can hold on the order of a million Ping-Pong balls. As an aside, the actual number is smaller than this because there will be a lot of space in the room that cannot be covered by balls. In fact, even in the best arrangement, the so-called "best packing fraction" is $\frac{1}{6}\pi\sqrt{2} = 0.74$, so that at least 26% of the space will be empty.

P1.32 (a) We estimate the mass of the water in the bathtub. Assume the tub measures 1.3 m by 0.5 m by 0.3 m. One-half of its volume is then

$$V = (0.5)(1.3)(0.5)(0.3) = 0.10 \text{ m}^3$$

The mass of this volume of water is

$$m_{\text{water}} = \rho_{\text{water}} V = \left(1\ 000 \text{ kg/m}^3\right)\left(0.10 \text{ m}^3\right) = 100 \text{ kg} \boxed{\sim 10^2 \text{ kg}}$$

(b) Pennies are now mostly zinc, but consider copper pennies filling 50% of the volume of the tub. The mass of copper required is

$$m_{\text{copper}} = \rho_{\text{copper}} V = \left(8\ 920 \text{ kg/m}^3\right)\left(0.10 \text{ m}^3\right) = 892 \text{ kg} \boxed{\sim 10^3 \text{ kg}}$$

P1.33 Don't reach for the telephone book or do a Google search! Think. Each full-time piano tuner must keep busy enough to earn a living. Assume a total population of 10^7 people. Also, let us estimate that one person in one hundred owns a piano. Assume that in one year a single piano tuner can service about 1 000 pianos (about 4 per day for 250 weekdays), and that each piano is tuned once per year.

Therefore, the number of tuners

$$= \left(\frac{1 \text{ tuner}}{1\ 000 \text{ pianos}}\right)\left(\frac{1 \text{ piano}}{100 \text{ people}}\right)\left(10^7 \text{ people}\right) \sim \boxed{100 \text{ tuners}}$$

If you did reach for an Internet directory, you would have to count. Instead, have faith in your estimate. Fermi's own ability in making an order-of-magnitude estimate is exemplified by his measurement of the energy output of the first nuclear bomb (the Trinity test at Alamogordo, New Mexico) by observing the fall of bits of paper as the blast wave swept past his station, 14 km away from ground zero.

P1.34 A reasonable guess for the diameter of a tire might be 2.5 ft, with a circumference of about 8 ft. Thus, the tire would make

$$\left(50\ 000 \text{ mi}\right)\left(5\ 280 \text{ ft/mi}\right)\left(1 \text{ rev/8 ft}\right) = 3 \times 10^7 \text{ rev} \sim \boxed{10^7 \text{ rev}}$$

Section 1.6 Significant Figures

P1.35 We will use two different methods to determine the area of the plate and the uncertainty in our answer.

METHOD ONE: We treat the best value with its uncertainty as a binomial, (21.3 ± 0.2) cm × (9.8 ± 0.1) cm, and obtain the area by expanding:

$$A = \left[21.3(9.8) \pm 21.3(0.1) \pm 0.2(9.8) \pm (0.2)(0.1)\right]\text{cm}^2$$

The first term gives the best value of the area. The cross terms add together to give the uncertainty and the fourth term is negligible.

$$A = \boxed{209\ \text{cm}^2 \pm 4\ \text{cm}^2}$$

METHOD TWO: We add the fractional uncertainties in the data.

$$A = (21.3\ \text{cm})(9.8\ \text{cm}) \pm \left(\frac{0.2}{21.3} + \frac{0.1}{9.8}\right)$$

$$= 209\ \text{cm}^2 \pm 2\% = 209\ \text{cm}^2 \pm 4\ \text{cm}^2$$

P1.36 (a) The ± 0.2 following the 78.9 expresses uncertainty in the last digit. Therefore, there are $\boxed{\textbf{three}}$ significant figures in 78.9 ± 0.2.

(b) Scientific notation is often used to remove the ambiguity of the number of significant figures in a number. Therefore, all the digits in 3.788 are significant, and 3.788×10^9 has $\boxed{\textbf{four}}$ significant figures.

(c) Similarly, 2.46 has three significant figures, therefore 2.46×10^{-6} has $\boxed{\textbf{three}}$ significant figures.

(d) Zeros used to position the decimal point are not significant. Therefore 0.005 3 has $\boxed{\textbf{two}}$ significant figures.

Uncertainty in a measurement can be the result of a number of factors, including the skill of the person doing the measurements, the precision and the quality of the instrument used, and the number of measurements made.

P1.37 We work to nine significant digits:

$$1\ \text{yr} = 1\ \text{yr}\left(\frac{365.\ 242\ 199\ \text{d}}{1\ \text{yr}}\right)\left(\frac{24\ \text{h}}{1\ \text{d}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right)\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

$$= \boxed{315\ 569\ 26.0\ \text{s}}$$

P1.38 (a) $756 + 37.2 + 0.83 + 2 = 796.03 \rightarrow \boxed{796}$, since the number with the fewest decimal places is 2.

(b) $(0.003\ 2)\{2\text{ s.f.}\}\times(356.3)\{4\text{ s.f.}\}=1.140\ 16=\{2\text{ s.f.}\}\ \boxed{1.1}$

(c) $5.620\{4\text{ s.f.}\}\times\pi\{>4\text{ s.f.}\}=17.656=\{4\text{ s.f.}\}\ \boxed{17.66}$

P1.39 Let o represent the number of ordinary cars and s the number of sport utility vehicles. We know $o = s + 0.947s = 1.947s$, and $o = s + 18$.

We eliminate o by substitution:

$$s+18=1.947s\rightarrow 0.947s=18\rightarrow s=18/0.947=\boxed{19}$$

P1.40 "One and one-third months" = 4/3 months. Treat this problem as a conversion:

$$\left(\frac{1\text{ bar}}{4/3\text{ months}}\right)\left(\frac{12\text{ months}}{1\text{ year}}\right)=\boxed{9\text{ bars/year}}$$

P1.41 The tax amount is \$1.36 – \$1.25 = \$0.11. The tax rate is

$$\$0.11/\$1.25=0.0880=\boxed{8.80\%}$$

P1.42 We are given the ratio of the masses and radii of the planets Uranus and Neptune:

$$\frac{M_N}{M_U}=1.19,\text{ and }\frac{r_N}{r_U}=0.969$$

The definition of density is $\rho=\dfrac{\text{mass}}{\text{volume}}=\dfrac{M}{V}$, where $V=\dfrac{4}{3}\pi r^3$ for a sphere, and we assume the planets have a spherical shape.

We know $\rho_U=1.27\times10^3\text{ kg/m}^3$. Compare densities:

$$\frac{\rho_N}{\rho_U}=\frac{M_N/V_N}{M_U/V_U}=\left(\frac{M_N}{M_U}\right)\left(\frac{V_U}{V_N}\right)=\left(\frac{M_N}{M_U}\right)\left(\frac{r_U}{r_N}\right)^3$$

$$=(1.19)\left(\frac{1}{0.969}\right)^3=1.307\ 9$$

which gives

$$\rho_N=(1.3079)\left(1.27\times10^3\text{ kg/m}^3\right)=\boxed{1.66\times10^3\text{ kg/m}^3}$$

P1.43 Let s represent the number of sparrows and m the number of more interesting birds. We know $s/m = 2.25$ and $s + m = 91$.

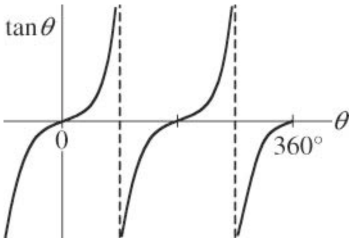
We eliminate m by substitution:

$$m=s/2.25\rightarrow s+s/2.25=91\rightarrow 1.444s=91$$

$$\rightarrow s=91/1.444=\boxed{63}$$

P1.44 We require

$$\sin\theta = -3\cos\theta,\ \text{or}\ \frac{\sin\theta}{\cos\theta} = \tan\theta = -3$$

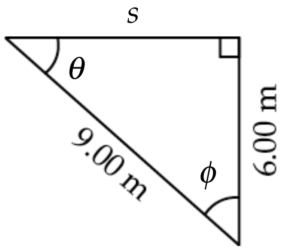


ANS. FIG. P1.44

For $\tan^{-1}(-3) = \arctan(-3)$, your calculator may return –71.6°, but this angle is not between 0° and 360° as the problem requires. The tangent function is negative in the second quadrant (between 90° and 180°) and in the fourth quadrant (from 270° to 360°). The solutions to the equation are then

$$360° - 71.6° = \boxed{288°}\ \text{and}\ 180° - 71.6 = \boxed{108°}$$

***P1.45** (a) ANS. FIG. P1.45 shows that the hypotenuse of the right triangle has a length of 9.00 m and the unknown side is opposite the angle ϕ. Since the two angles in the triangle are not known, we can obtain the length of the unknown side, which we will represent as s, using the Pythagorean Theorem:



ANS. FIG. P1.45

$$s^2 + (6.00\ \text{m})^2 = (9.00\ \text{m})^2$$
$$s^2 = (9.00\ \text{m})^2 - (6.00\ \text{m})^2 = 45$$

which gives $s = \boxed{6.71\ \text{m}}$. We express all of our answers in three significant figures since the lengths of the two known sides of the triangle are given with three significant figures.

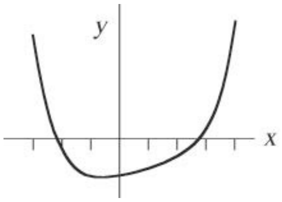
(b) From ANS. FIG. P1.45, the tangent of θ is equal to ratio of the side opposite the angle, 6.00 m in length, and the side adjacent to the angle, s = 6.71 m, and is given by

$$\tan\theta = \frac{6.00\ \text{m}}{s} = \frac{6.00\ \text{m}}{6.71\ \text{m}} = \boxed{0.894}$$

(c) From ANS. FIG. P1.45, the sine of ϕ is equal to ratio of the side opposite the angle, s = 6.71 m, and the hypotenuse of the triangle, 9.00 m in length, and is given by

$$\sin\phi = \frac{s}{9.00\ \text{m}} = \frac{6.71\ \text{m}}{9.00\ \text{m}} = \boxed{0.745}$$

P1.46 For those who are not familiar with solving equations numerically, we provide a detailed solution. It goes beyond proving that the suggested answer works.



ANS. FIG. P1.46

The equation $2x^4 - 3x^3 + 5x - 70 = 0$ is quartic, so we do not attempt to solve it with algebra. To find how many real solutions the equation has and to estimate them, we graph the expression:

x	–3	–2	–1	0	1	2	3	4
$y = 2x^4 - 3x^3 + 5x - 70$	158	–24	–70	–70	–66	–52	26	270

We see that the equation $y = 0$ has two roots, one around $x = -2.2$ and the other near $x = +2.7$. To home in on the first of these solutions we compute in sequence:

When $x = -2.2$, $y = -2.20$. The root must be between $x = -2.2$ and $x = -3$.
When $x = -2.3$, $y = 11.0$. The root is between $x = -2.2$ and $x = -2.3$.
When $x = -2.23$, $y = 1.58$. The root is between $x = -2.20$ and $x = -2.23$.
When $x = -2.22$, $y = 0.301$. The root is between $x = -2.20$ and –2.22.
When $x = -2.215$, $y = -0.331$. The root is between $x = -2.215$ and –2.22.
We could next try $x = -2.218$, but we already know to three-digit precision that the root is $x = -2.22$.

P1.47 When the length changes by 15.8%, the mass changes by a much larger percentage. We will write each of the sentences in the problem as a mathematical equation.

Mass is proportional to length cubed: $m = k\ell^3$, where k is a constant. This model of growth is reasonable because the lamb gets thicker as it gets longer, growing in three-dimensional space.

At the initial and final points, $\mathrm{m_i = k\ell_i^3}$ and $\mathrm{m_f = k\ell_f^3}$

Length changes by 15.8%: 15.8% **of** ℓ means 0.158 **times** ℓ.

Thus $\ell_i + 0.158\,\ell_i = \ell_f$ and $\ell_f = 1.158\,\ell_i$

Mass increases by 17.3 kg: $m_i + 17.3\text{ kg} = m_f$

Now we combine the equations using algebra, eliminating the unknowns ℓ_i, ℓ_f, k, and m_i by substitution:

From $\ell_f = 1.158\,\ell_i$, we have $\ell_f^3 = 1.158^3\ell_i^3 = 1.553\,\ell_i^3$

Then

$$m_f = k\ell_f^3 = k\left(1.553\right)\ell_i^3 = 1.553k\ell_i^3 = 1.553m_i \quad \text{and} \quad m_i = m_f/1.553$$

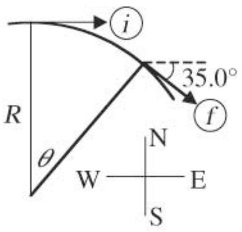
Next,

$$m_i + 17.3\text{ kg} = m_f \quad \text{becomes} \quad m_f/1.553 + 17.3\text{ kg} = m_f$$

Solving, $17.3\text{ kg} = m_f - m_f/1.553 = m_f(1 - 1/1.553) = 0.356\,m_f$

and $$m_f = \frac{17.3\text{ kg}}{0.356} = \boxed{48.6\text{ kg}}.$$

P1.48 We draw the radius to the initial point and the radius to the final point. The angle θ between these two radii has its sides perpendicular, right side to right side and left side to left side, to the 35° angle between the original and final tangential directions of travel. A most useful theorem from geometry then identifies these angles as equal: $\theta = 35°$. The whole circumference of a 360° circle of the same radius is $2\pi R$. By proportion, then



ANS. FIG. P1.48

$$\frac{2\pi R}{360°} = \frac{840\text{ m}}{35°}$$

$$R = \left(\frac{360°}{2\pi}\right)\left(\frac{840\text{ m}}{35°}\right) = \frac{840\text{ m}}{0.611} = \boxed{1.38\times10^3\text{ m}}$$

We could equally well say that the measure of the angle in radians is

$$\theta = 35° = 35°\left(\frac{2\pi\text{ radians}}{360°}\right) = 0.611\text{ rad} = \frac{840\text{ m}}{R}$$

Solving yields $R = 1.38$ km.

P1.49 Use substitution to solve simultaneous equations. We substitute $p = 3q$ into each of the other two equations to eliminate p:

$$\begin{cases} 3qr = qs \\ \frac{1}{2}3qr^2 + \frac{1}{2}qs^2 = \frac{1}{2}qt^2 \end{cases}$$

These simplify to $\begin{cases} 3r = s \\ 3r^2 + s^2 = t^2 \end{cases}$, assuming $q \neq 0$.

We substitute the upper relation into the lower equation to eliminate s:

$$3r^2 + (3r)^2 = t^2 \rightarrow 12r^2 = t^2 \rightarrow \frac{t^2}{r^2} = 12$$

We now have the ratio of t to r: $\boxed{\frac{t}{r} = \pm\sqrt{12} = \pm3.46}$

P1.50 First, solve the given equation for Δt:

$$\Delta t = \frac{4QL}{k\pi d^2(T_h - T_c)} = \left[\frac{4QL}{k\pi(T_h - T_c)}\right]\left[\frac{1}{d^2}\right]$$

(a) Making *d* three times larger with d^2 in the bottom of the fraction makes Δt $\boxed{\text{nine times smaller}}$.

(b) $\boxed{\Delta t \text{ is inversely proportional to the square of } d.}$

(c) $\boxed{\text{Plot } \Delta t \text{ on the vertical axis and } 1/d^2 \text{ on the horizontal axis.}}$

(d) From the last version of the equation, the slope is $\boxed{4QL / k\pi \left(T_h - T_c\right)}$. Note that this quantity is constant as both Δt and *d* vary.

P1.51 (a) The fourth experimental point from the top is a circle: this point lies just above the best-fit curve that passes through the point (400 cm^2, 0.20 g). The interval between horizontal grid lines is 1 space = 0.05 g. We estimate from the graph that the circle has a vertical separation of 0.3 spaces = $\boxed{0.015 \text{ g}}$ above the best-fit curve.

(b) The best-fit curve passes through 0.20 g:

$$\left(\frac{0.015 \text{ g}}{0.20 \text{ g}}\right) \times 100 = \boxed{8\%}$$

(c) The best-fit curve passes through the origin and the point (600 cm^3, 3.1 g). Therefore, the slope of the best-fit curve is

$$\text{slope} = \left(\frac{3.1 \text{ g}}{600 \text{ cm}^3}\right) = \boxed{5.2 \times 10^{-3} \frac{\text{g}}{\text{cm}^3}}$$

(d) $\boxed{\begin{array}{l}\text{For shapes cut from this copy paper, the mass of the cutout} \\ \text{is proportional to its area. The proportionality constant is} \\ 5.2 \text{ g/m}^2 \pm 8\%\text{, where the uncertainty is estimated.}\end{array}}$

(e) $\boxed{\begin{array}{l}\text{This result is to be expected if the paper has thickness and} \\ \text{density that are uniform within the experimental uncertainty.}\end{array}}$

(f) $\boxed{\text{The slope is the areal density of the paper, its mass per unit area.}}$

P1.52 $r = (6.50 \pm 0.20) \text{ cm} = (6.50 \pm 0.20) \times 10^{-2} \text{ m}$

$$m = (1.85 + 0.02) \text{kg}$$

$$\rho = \frac{m}{\left(\frac{4}{3}\right)\pi r^3}$$

also, $\dfrac{\delta\rho}{\rho} = \dfrac{\delta \mathrm{m}}{\mathrm{m}} + \dfrac{3\delta \mathrm{r}}{\mathrm{r}}$

In other words, the percentages of uncertainty are cumulative. Therefore,

$$\frac{\delta\rho}{\rho} = \frac{0.02}{1.85} + \frac{3(0.20)}{6.50} = 0.103,$$

$$\rho = \frac{1.85}{\left(\frac{4}{3}\right)\pi\left(6.5\times10^{-2}\ \mathrm{m}\right)^3} = \boxed{1.61\times10^3\ \mathrm{kg/m^3}}$$

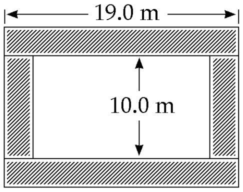
then $\delta\rho = 0.103\rho = \boxed{0.166\times10^3\ \mathrm{kg/m^3}}$

and $\rho \pm \delta\rho = \boxed{(1.61 \pm 0.17)\times10^3\ \mathrm{kg/m^3}} = (1.6 \pm 0.2)\times10^3\ \mathrm{kg/m^3}.$

***P1.53** The volume of concrete needed is the sum of the four sides of sidewalk, or

$$\mathrm{V} = 2\mathrm{V}_1 + 2\mathrm{V}_2 = 2(\mathrm{V}_1 + \mathrm{V}_2)$$

The figure on the right gives the dimensions needed to determine the volume of each portion of sidewalk:



ANS. FIG. P1.53

$$\mathrm{V}_1 = (17.0\ \mathrm{m} + 1.0\ \mathrm{m} + 1.0\ \mathrm{m})(1.0\ \mathrm{m})(0.09\ \mathrm{m}) = 1.70\ \mathrm{m}^3$$

$$\mathrm{V}_2 = (10.0\ \mathrm{m})(1.0\ \mathrm{m})(0.090\ \mathrm{m}) = 0.900\ \mathrm{m}^3$$

$$\mathrm{V} = 2\left(1.70\ \mathrm{m}^3 + 0.900\ \mathrm{m}^3\right) = \boxed{5.2\ \mathrm{m}^3}$$

The uncertainty in the volume is the sum of the uncertainties in each dimension:

$$\left.\begin{array}{l} \dfrac{\delta \ell_1}{\ell_1} = \dfrac{0.12\ \mathrm{m}}{19.0\ \mathrm{m}} = 0.0063 \\[2ex] \dfrac{\delta \mathrm{w}_1}{\mathrm{w}_1} = \dfrac{0.01\ \mathrm{m}}{1.0\ \mathrm{m}} = 0.010 \\[2ex] \dfrac{\delta \mathrm{t}_1}{\mathrm{t}_1} = \dfrac{0.1\ \mathrm{cm}}{9.0\ \mathrm{cm}} = 0.011 \end{array}\right\} \frac{\delta \mathrm{V}}{\mathrm{V}} = 0.006 + 0.010 + 0.011 = 0.027 = \boxed{3\%}$$

Additional Problems

P1.54 (a) Let *d* represent the diameter of the coin and *h* its thickness. The gold plating is a layer of thickness *t* on the surface of the coin; so, the mass of the gold is

$$\begin{aligned} m &= \rho V = \rho\left[2\pi\frac{d^2}{4} + \pi dh\right]t \\ &= \left(19.3\frac{\text{g}}{\text{cm}^3}\right)\left[2\pi\frac{(2.41\text{ cm})^2}{4} + \pi(2.41\text{ cm})(0.178\text{ cm})\right] \\ &\qquad \times\left(1.8\times10^{-7}\text{ m}\right)\left(\frac{10^2\text{ cm}}{1\text{ m}}\right) \\ &= 0.003\ 64\text{ g} \end{aligned}$$

and the cost of the gold added to the coin is

$$\text{cost} = (0.003\ 64\text{ g})\left(\frac{\$10}{1\text{ g}}\right) = \$0.036\ 4 = \boxed{3.64\text{ cents}}$$

(b) $\boxed{\text{The cost is negligible compared to \$4.98.}}$

P1.55 It is desired to find the distance *x* such that

$$\frac{x}{100\text{ m}} = \frac{1\ 000\text{ m}}{x}$$

(i.e., such that *x* is the same multiple of 100 m as the multiple that 1 000 m is of *x*). Thus, it is seen that

$$x^2 = (100\text{ m})(1\ 000\text{ m}) = 1.00\times10^5\text{ m}^2$$

and therefore

$$x = \sqrt{1.00\times10^5\text{ m}^2} = \boxed{316\text{ m}}$$

P1.56 (a) A Google search yields the following dimensions of the intestinal tract:

small intestines: length ≅ 20 ft ≅ 6 m, diameter ≅ 1.5 in ≅ 4 cm

large intestines: length ≅ 5 ft ≅ 1.5 m, diameter ≅ 2.5 in ≅ 6 cm

Treat the intestines as two cylinders: the volume of a cylinder of diameter *d* and length *L* is $V = \frac{\pi}{4}d^2L$.

The volume of the intestinal tract is

$$V = V_{\text{small}} + V_{\text{large}}$$

$$V = \frac{\pi}{4}(0.04\,\text{m})^2(6\,\text{m}) + \frac{\pi}{4}(0.06\,\text{m})^2(1.5\,\text{m})$$

$$= 0.0117\,\text{m}^3 \cong 10^{-2}\,\text{m}^3$$

Assuming 1% of this volume is occupied by bacteria, the volume of bacteria is

$$V_{\text{bac}} = \left(10^{-2}\ \text{m}^3\right)(0.01) = 10^{-4}\ \text{m}^3$$

Treating a bacterium as a cube of side $L = 10^{-6}$ m, the volume of one bacterium is about $L^3 = 10^{-18}$ m^3. The number of bacteria in the intestinal tract is about

$$\left(10^{-4}\ \text{m}^3\right)\left(\frac{1\ \text{bacterium}}{10^{-18}\ \text{m}^3}\right) = \boxed{10^{14}\ \text{bacteria!}}$$

(b) The large number of bacteria suggests they must be $\boxed{\text{beneficial}}$, otherwise the body would have developed methods a long time ago to reduce their number. It is well known that certain types of bacteria in the intestinal tract are beneficial: they aid digestion, as well as prevent dangerous bacteria from flourishing in the intestines.

P1.57 We simply multiply the distance between the two galaxies by the scale factor used for the dinner plates. The scale factor used in the "dinner plate" model is

$$S = \left(\frac{0.25\ \text{m}}{1.0\times 10^5\ \text{light-years}}\right) = 2.5\times 10^{-6}\ \text{m/ly}$$

The distance to Andromeda in the scale model will be

$$D_{\text{scale}} = D_{\text{actual}}S = \left(2.0\times 10^6\ \text{ly}\right)\left(2.5\times 10^{-6}\ \text{m/ly}\right) = \boxed{5.0\ \text{m}}$$

P1.58 Assume the winner counts one dollar per second, and the winner tries to maintain the count without stopping. The time interval required for the task would be

$$\$10^6\left(\frac{1\ \text{s}}{\$1}\right)\left(\frac{1\ \text{hour}}{3600\ \text{s}}\right)\left(\frac{1\ \text{work week}}{40\ \text{hours}}\right) = 6.9\ \text{work weeks.}$$

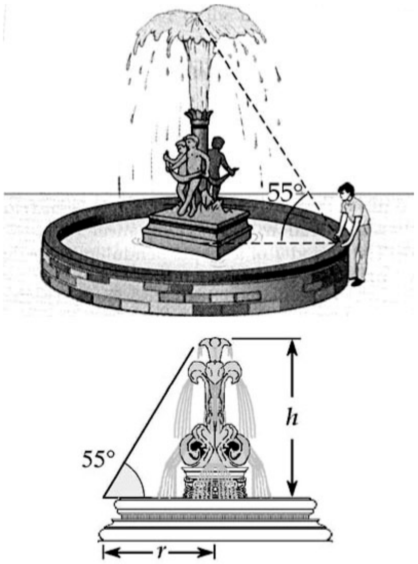
$$\boxed{\begin{array}{l}\text{The scenario has the contestants succeeding on the whole. But the}\\ \text{calculation shows that is impossible. It just takes too long!}\end{array}}$$

P1.59 We imagine a top view to figure the radius of the pool from its circumference. We imagine a straight-on side view to use trigonometry to find the height.

Define a right triangle whose legs represent the height and radius of the fountain. From the dimensions of the fountain and the triangle, the circumference is $C = 2\pi r$ and the angle satisfies $\tan\phi = h/r$.

Then by substitution

$$h = r\tan\phi = \left(\frac{C}{2\pi}\right)\tan\phi$$



ANS. FIG. P1.59

Evaluating,

$$h = \left(\frac{15.0\text{ m}}{2\pi}\right)\tan 55.0° = \boxed{3.41\text{ m}}$$

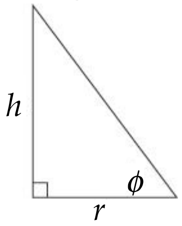
When we look at a three-dimensional system from a particular direction, we may discover a view to which simple mathematics applies.

P1.60 The fountain has height *h*; the pool has circumference *C* with radius *r*. The figure shows the geometry of the problem: a right triangle has base *r*, height *h*, and angle *ϕ*. From the triangle,

$$\tan\phi = h/r$$

We can find the radius of the circle from its circumference, $C = 2\pi r$, and then solve for the height using

$$\boxed{h = r\tan\phi = (\tan\phi)C/2\pi}$$



ANS. FIG. P1.60

P1.61 The density of each material is $\rho = \dfrac{m}{V} = \dfrac{m}{\pi r^2 h} = \dfrac{4m}{\pi D^2 h}$.

$$\text{Al: } \rho = \frac{4(51.5\text{ g})}{\pi(2.52\text{ cm})^2(3.75\text{ cm})} = \boxed{2.75\frac{\text{g}}{\text{cm}^3}\text{; this is 2\% larger}}$$

than the tabulated value, 2.70 g/cm^3.

$$\text{Cu: } \rho = \frac{4(56.3\text{ g})}{\pi(1.23\text{ cm})^2(5.06\text{ cm})} = \boxed{9.36\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 8.92 g/cm^3.

$$\text{brass: } \rho = \frac{4(94.4\text{ g})}{\pi(1.54\text{ cm})^2(5.69\text{ cm})} = \boxed{8.91\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 8.47 g/cm^3.

$$\text{Sn: } \rho = \frac{4(69.1\text{ g})}{\pi(1.75\text{ cm})^2(3.74\text{ cm})} = \boxed{7.68\frac{\text{g}}{\text{cm}^3}\text{; this is 5\% larger}}$$

than the tabulated value, 7.31 g/cm^3.

$$\text{Fe: } \rho = \frac{4(216.1\text{ g})}{\pi(1.89\text{ cm})^2(9.77\text{ cm})} = \boxed{7.88\frac{\text{g}}{\text{cm}^3}\text{; this is 0.3\% larger}}$$

than the tabulated value, 7.86 g/cm^3.

P1.62 The volume of the galaxy is

$$\pi r^2 t = \pi\left(10^{21}\text{ m}\right)^2\left(10^{19}\text{ m}\right) \sim 10^{61}\text{ m}^3$$

If the distance between stars is 4×10^{16}, then there is one star in a volume on the order of

$$\left(4\times10^{16}\text{ m}\right)^3 \sim 10^{50}\text{ m}^3$$

The number of stars is about $\dfrac{10^{61}\text{ m}^3}{10^{50}\text{ m}^3/\text{star}} \sim \boxed{10^{11}\text{ stars}}$.

P1.63 We define an average national fuel consumption rate based upon the total miles driven by all cars combined. In symbols,

$$\text{fuel consumed} = \frac{\text{total miles driven}}{\text{average fuel consumption rate}}$$

or

$$f = \frac{s}{c}$$

For the current rate of 20 mi/gallon we have

$$f = \frac{\left(100 \times 10^6\text{ cars}\right)\left(10^4\text{ (mi/yr)/car}\right)}{20\text{ mi/gal}} = 5 \times 10^{10}\text{ gal/yr}$$

Since we consider the same total number of miles driven in each case, at 25 mi/gal we have

$$f = \frac{\left(100 \times 10^6\text{ cars}\right)\left(10^4\text{ (mi/yr)/car}\right)}{25\text{ mi/gal}} = 4 \times 10^{10}\text{ gal/yr}$$

Thus we estimate a change in fuel consumption of

$$\Delta f = 4\times10^{10}\text{ gal/yr} - 5\times10^{10}\text{ gal/yr} = \boxed{-1\times10^{10}\text{ gal/yr}}$$

The negative sign indicates that the change is a reduction. It is a fuel savings of ten billion gallons each year.

P1.64 (a) The mass is equal to the mass of a sphere of radius 2.6 cm and density 4.7 g/cm^3, minus the mass of a sphere of radius *a* and density 4.7 g/cm^3, plus the mass of a sphere of radius *a* and density 1.23 g/cm^3.

$$m = \rho_1\left(\frac{4}{3}\pi r^3\right) - \rho_1\left(\frac{4}{3}\pi a^3\right) + \rho_2\left(\frac{4}{3}\pi a^3\right)$$

$$= \left(\frac{4}{3}\pi\right)\Big[\left(4.7\ \text{g/cm}^3\right)\left(2.6\ \text{cm}\right)^3 - \left(4.7\ \text{g/cm}^3\right)a^3 + \left(1.23\ \text{g/cm}^3\right)a^3\Big]$$

$$m = \boxed{346\ \text{g} - \left(14.5\ \text{g/cm}^3\right)a^3}$$

(b) The mass is maximum for $\boxed{a = 0}$.

(c) $\boxed{346\ \text{g}}$.

(d) $\boxed{\text{Yes}}$. This is the mass of the uniform sphere we considered in the first term of the calculation.

(e) $\boxed{\text{No change, so long as the wall of the shell is unbroken.}}$

P1.65 Answers may vary depending on assumptions:

typical length of bacterium: $L = 10^{-6}$ m

typical volume of bacterium: $L^3 = 10^{-18}\ \text{m}^3$

surface area of Earth: $A = 4\pi r^2 = 4\pi\left(6.38 \times 10^6\ \text{m}\right)^2 = 5.12 \times 10^{14}\ \text{m}^2$

(a) If we assume the bacteria are found to a depth $d = 1000$ m below Earth's surface, the volume of Earth containing bacteria is about

$$V = \left(4\pi r^2\right)d = 5.12 \times 10^{17}\ \text{m}^3$$

If we assume an average of 1000 bacteria in every 1 mm^3 of volume, then the number of bacteria is

$$\left(\frac{1000\ \text{bacteria}}{1\ \text{mm}^3}\right)\left(\frac{10^3\ \text{mm}}{1\ \text{m}}\right)^3 (5.12 \times 10^{17}\ \text{m}^3) \approx \boxed{5.12 \times 10^{29}\ \text{bacteria}}$$

(b) Assuming a bacterium is basically composed of water, the total mass is

$$\left(10^{29}\ \text{bacteria}\right)\left(\frac{10^{-18}\ \text{m}^3}{1\ \text{bacterium}}\right)\left(\frac{10^3\ \text{kg}}{1\ \text{m}^3}\right) = \boxed{10^{14}\ \text{kg}}$$

P1.66 The rate of volume increase is

$$\frac{dV}{dt}=\frac{d}{dt}\left(\frac{4}{3}\pi r^3\right)=\frac{4}{3}\pi\left(3r^2\right)\frac{dr}{dt}=\left(4\pi r^2\right)\frac{dr}{dt}$$

(a) $\frac{dV}{dt}=4\pi(6.5\text{ cm})^2(0.9\text{ cm/s})=\boxed{478\text{ cm}^3/\text{s}}$

(b) The rate of increase of the balloon's radius is

$$\frac{dr}{dt}=\frac{dV/dt}{4\pi r^2}=\frac{478\text{ cm}^3/\text{s}}{4\pi(13\text{ cm})^2}=\boxed{0.225\text{ cm/s}}$$

(c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one second in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.67 (a) We have $B + C(0) = 2.70\text{ g/cm}^3$ and $B + C(14\text{ cm}) = 19.3\text{ g/cm}^3$.

We know $\boxed{B = 2.70\text{ g/cm}^3}$, and we solve for C by subtracting:

$C(14\text{ cm}) = 19.3\text{ g/cm}^3 - B = 16.6\text{ g/cm}^3$, so $\boxed{C = 1.19\text{ g/cm}^4}$

(b) The integral is

$$m=(9.00\text{ cm}^2)\int_0^{14\text{ cm}}(B+Cx)\,dx$$

$$=(9.00\text{ cm}^2)\left(Bx+\frac{C}{2}x^2\right)\Bigg|_0^{14\text{ cm}}$$

$$m=(9.00\text{ cm}^2)\Big\{\left(2.70\text{ g/cm}^3\right)(14\text{ cm}-0)$$

$$+\left(1.19\text{ g/cm}^4/2\right)\left[(14\text{ cm})^2-0\right]\Big\}$$

$$=340\text{ g}+1046\text{ g}=1390\text{ g}=\boxed{1.39\text{ kg}}$$

P1.68 The table below shows α in degrees, α in radians, tan(α), and sin(α) for angles from 15.0° to 31.1°:

α' (deg)	α (rad)	tan(α)	sin(α)	difference between α and tan α
15.0	0.262	0.268	0.259	2.30%
20.0	0.349	0.364	0.342	4.09%
30.0	0.524	0.577	0.500	9.32%
33.0	0.576	0.649	0.545	11.3%
31.0	0.541	0.601	0.515	9.95%
31.1	0.543	0.603	0.516	10.02%

We see that α in radians, tan(α), and sin(α) start out together from zero and diverge only slightly in value for small angles. Thus $\boxed{31.0°}$ is the largest angle for which $\dfrac{\tan\alpha-\alpha}{\tan\alpha} < 0.1$.

P1.69 We write "millions of cubic feet" as 10^6 ft^3, and use the given units of time and volume to assign units to the equation.

$$V = (1.50\times10^6\,\text{ft}^3/\text{mo})t+(0.008\,00\times10^6\,\text{ft}^3/\text{mo}^2)t^2$$

To convert the units to seconds, use

$$1\text{ month} = (30.0\text{ d})\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 2.59\times10^6\text{ s}$$

to obtain

$$\begin{aligned} V &= \left(1.50\times10^6\ \frac{\text{ft}^3}{\text{mo}}\right)\left(\frac{1\text{ mo}}{2.59\times10^6\text{ s}}\right)t \\ &\quad+\left(0.008\ 00\times10^6\ \frac{\text{ft}^3}{\text{mo}^2}\right)\left(\frac{1\text{ mo}}{2.59\times10^6\text{ s}}\right)^2 t^2 \\ &= (0.579\ \text{ft}^3/\text{s})t+(1.19\times10^{-9}\ \text{ft}^3/\text{s}^2)t^2 \end{aligned}$$

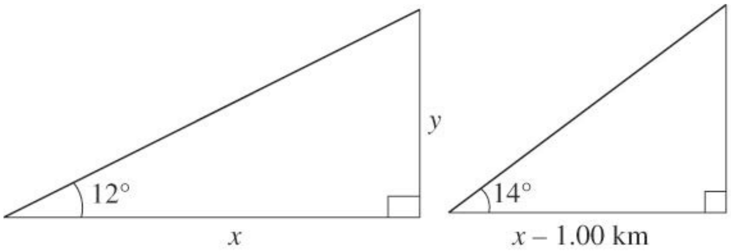
or

$$V = \boxed{0.579t+1.19\times10^{-9}t^2}$$

where *V* is in cubic feet and *t* is in seconds. The coefficient of the first term is the volume rate of flow of gas at the beginning of the month.

The second term's coefficient is related to how much the rate of flow increases every second.

P1.70 (a) and (b), the two triangles are shown.



ANS. FIG. P1.70(a) **ANS. FIG. P1.70(b)**

(c) From the triangles,

$$\tan 12.0° = \frac{y}{x} \to \boxed{y = x\tan 12.0°}$$

and $\tan 14.0° = \dfrac{y}{(x - 1.00\text{ km})} \to \boxed{y = (x - 1.00\text{ km})\tan 14.0°}$.

(d) Equating the two expressions for *y*, we solve to find $\boxed{y = 1.44\text{ km.}}$

P1.71 Observe in Fig. 1.71 that the radius of the horizontal cross section of the bottle is a relative maximum or minimum at the two radii cited in the problem; thus, we recognize that as the liquid level rises, the time rate of change of the diameter of the cross section will be zero at these positions.

The volume of a particular thin cross section of the shampoo of thickness h and area A is $V = Ah$, where $A = \pi r^2 = \pi D^2/4$. Differentiate the volume with respect to time:

$$\frac{dV}{dt} = A\frac{dh}{dt} + h\frac{dA}{dt} = A\frac{dh}{dt} + h\frac{d}{dt}(\pi r^2) = A\frac{dh}{dt} + 2\pi hr\frac{dr}{dt}$$

Because the radii given are a maximum and a minimum value, $dr/dt = 0$, so

$$\frac{dV}{dt} + A\frac{dh}{dt} = Av \to v = \frac{1}{A}\frac{dV}{dt} = \frac{1}{\pi D^2/4}\frac{dV}{dt} = \frac{4}{\pi D^2}\frac{dV}{dt}$$

where $v = dh/dt$ is the speed with which the level of the fluid rises.

(a) For $D = 6.30$ cm,

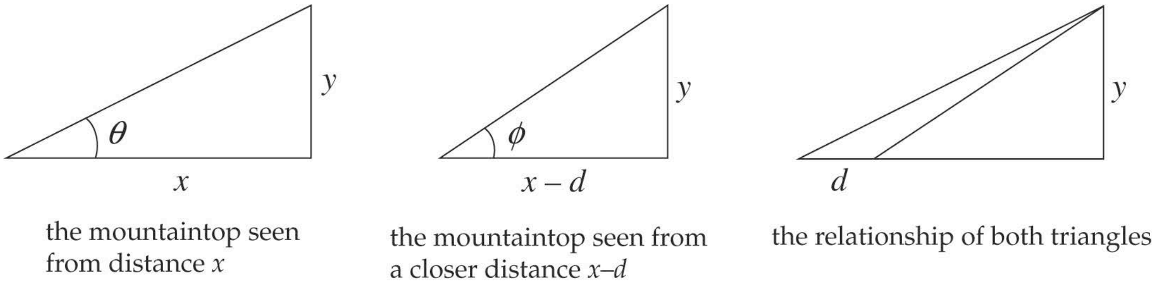
$$v = \frac{4}{\pi(6.30\text{ cm})^2}(16.5\text{ cm}^3/\text{s}) = \boxed{0.529\text{ cm/s}}$$

(b) For $D = 1.35$ cm,

$$v = \frac{4}{\pi(1.35\text{ cm})^2}(16.5\text{ cm}^3/\text{s}) = \boxed{11.5\text{ cm/s}}$$

Challenge Problems

P1.72 The geometry of the problem is shown below.



ANS. FIG. P1.72

From the triangles in ANS. FIG. P1.72,

$$\tan\theta = \frac{y}{x} \rightarrow y = x\tan\theta$$

and

$$\tan\phi = \frac{y}{x-d} \rightarrow y = (x-d)\tan\phi$$

Equate these two expressions for *y* and solve for *x*:

$$x\tan\theta = (x-d)\tan\phi \rightarrow d\tan\phi = x(\tan\phi - \tan\theta)$$

$$\rightarrow x = \frac{d\tan\phi}{\tan\phi - \tan\theta}$$

Take the expression for *x* and substitute it into either expression for *y*:

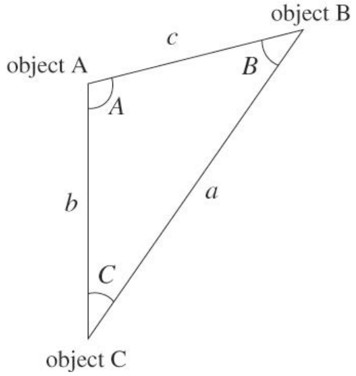
$$y = x\tan\theta = \boxed{\frac{d\tan\phi\tan\theta}{\tan\phi - \tan\theta}}$$

P1.73 The geometry of the problem suggests we use the law of cosines to relate known sides and angles of a triangle to the unknown sides and angles. Recall that the sides *a*, *b*, and *c* with opposite angles *A*, *B*, and *C* have the following relationships:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = c^2 + a^2 - 2ca\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$



ANS. FIG. P1.73

For the cows in the meadow, the triangle has sides $a = 25.0$ m and $b = 15.0$ m, and angle $C = 20.0°$, where object A = cow A, object B = cow B, and object C = you.

(a) Find side *c*:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$c^2 = (25.0\text{ m})^2 + (15.0\text{ m})^2 - 2(25.0\text{ m})(15.0\text{ m})\cos(20.0°)$$

$$c = \boxed{12.1\text{ m}}$$

(b) Find angle *A*:

$$a^2 = b^2 + c^2 - 2bc\cos A \rightarrow$$

$$\cos A = \frac{a^2 - b^2 - c^2}{2bc} = \frac{(25.0\text{ m})^2 - (15.0\text{ m})^2 - (12.1\text{ m})^2}{2(15.0\text{ m})(12.1\text{ m})}$$

$$\rightarrow A = 134.8° = \boxed{135°}$$

(c) Find angle *B*:

$$b^2 = c^2 + a^2 - 2ca\cos B \rightarrow$$

$$\cos B = \frac{b^2 - c^2 - a^2}{2ca} = \frac{(15.0\text{ m})^2 - (25.0\text{ m})^2 - (12.1\text{ m})^2}{2(25.0\text{ m})(12.1\text{ m})}$$

$$\rightarrow B = \boxed{25.2°}$$

(d) For the situation, object A = star A, object B = star B, and object C = our Sun (or Earth); so, the triangle has sides $a = 25.0$ ly, $b = 15.0$ ly, and angle $C = 20.0°$. The numbers are the same, except for units, as in part (b); thus, $\boxed{\text{angle } A = 135°.}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P1.2	$2.15 \times 10^4\ \text{kg/m}^3$
P1.4	(a) $2.3 \times 10^{17}\ \text{kg/m}^3$; (b) 1.0×10^{13} times the density of osmium
P1.6	$\dfrac{4\pi\rho\left(r_2^3 - r_1^3\right)}{3}$
P1.8	(a) $8.42 \times 10^{22}\ \dfrac{\text{Cu-atom}}{\text{cm}^3}$; (b) $1.19 \times 10^{-23}\ \text{cm}^3/\text{Cu-atom}$; (c) 2.28×10^{-8} cm
P1.10	(a) and (f); (b) and (d); (c) and (e)
P1.12	$\dfrac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
P1.14	(a) $[A] = L/T^3$ and $[B] = L/T$; (b) L/T
P1.16	667 lb/s
P1.18	9.19 nm/s
P1.20	$2.57 \times 10^6\ \text{m}^3$
P1.22	(a) $7.14 \times 10^{-2}\ \dfrac{\text{gal}}{\text{s}}$; (b) $2.70 \times 10^{-4}\ \dfrac{\text{m}^3}{\text{s}}$; (c) 1.03 h
P1.24	$290\ \text{m}^3$, $2.9 \times 10^8\ \text{cm}^3$
P1.26	$r_{\text{Fe}}(1.43)$
P1.28	(a) $3.39 \times 10^5\ \text{ft}^3$; (b) 2.54×10^4 lb
P1.30	(a) 2.07 mm; (b) 8.62×10^{13} times as large
P1.32	(a) $\sim 10^2$ kg; (b) $\sim 10^3$ kg
P1.34	10^7 rev
P1.36	(a) 3; (b) 4; (c) 3; (d) 2
P1.38	(a) 796; (b) 1.1; (c) 17.66
P1.40	9 bars / year
P1.42	$1.66 \times 10^3\ \text{kg/m}^3$
P1.44	288°; 108°
P1.46	See P1.46 for complete description.
P1.48	1.38×10^3 m

P1.50 (a) nine times smaller; (b) Δt is inversely proportional to the square of d; (c) Plot Δt on the vertical axis and $1/d^2$ on the horizontal axis; (d) $4QL/k\pi\left(T_h - T_c\right)$

P1.52 $1.61\times10^3\ \mathrm{kg/m^3},\ 0.166\times10^3\ \mathrm{kg/m^3},\ (1.61\pm0.17)\times10^3\ \mathrm{kg/m^3}$

P1.54 3.64 cents; the cost is negligible compared to $4.98.

P1.56 (a) 10^{14} bacteria; (b) beneficial

P1.58 The scenario has the contestants succeeding on the whole. But the calculation shows that is impossible. It just takes too long!

P1.60 $h = r\tan\phi = (\tan\theta)C/2\pi$

P1.62 10^{11} stars

P1.64 (a) $m = 346\ \mathrm{g} - (14.5\ \mathrm{g/cm^3})a^3$; (b) $a = 0$; (c) 346 g; (d) yes; (e) no change

P1.66 (a) 478 $\mathrm{cm^3/s}$; (b) 0.225 cm/s; (c) When the balloon radius is twice as large, its surface area is four times larger. The new volume added in one second in the inflation process is equal to this larger area times an extra radial thickness that is one-fourth as large as it was when the balloon was smaller.

P1.68 31.0°

P1.70 (a-b) see ANS. FIG. P1.70(a) and P1.70(b); (c) $y = x\tan 12.0°$ and $y = (x - 1.00\ \mathrm{km})\tan 14.0°$; (d) $y = 1.44$ km

P1.72 $\dfrac{d\tan\phi\tan\theta}{\tan\phi - \tan\theta}$

2

Motion in One Dimension

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ2.1 Count spaces (intervals), not dots. Count 5, not 6. The first drop falls at time zero and the last drop at 5 × 5 s = 25 s. The average speed is 600 m/25 s = 24 m/s, answer (b).

OQ2.2 The initial velocity of the car is $v_0 = 0$ and the velocity at time t is v. The constant acceleration is therefore given by

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - 0} = \frac{v - 0}{t} = \frac{v}{t}$$

and the average velocity of the car is

$$\bar{v} = \frac{(v + v_0)}{2} = \frac{(v + 0)}{2} = \frac{v}{2}$$

The distance traveled in time t is $\Delta x = \bar{v}t = vt/2$. In the special case where $a = 0$ (and hence $v = v_0 = 0$), we see that statements (a), (b), (c), and (d) are all correct. However, in the general case ($a \neq 0$, and hence

$v \neq 0$) only statements (b) and (c) are true. Statement (e) is not true in either case.

OQ2.3 The bowling pin has a constant downward acceleration while in flight. The velocity of the pin is directed upward on the ascending part of its flight and is directed downward on the descending part of its flight. Thus, only (d) is a true statement.

OQ2.4 The derivation of the equations of kinematics for an object moving in one dimension was based on the assumption that the object had a constant acceleration. Thus, (b) is the correct answer. An object would have constant velocity if its acceleration were zero, so (a) applies to cases of zero acceleration only. The speed (magnitude of the velocity) will increase in time only in cases when the velocity is in the same direction as the constant acceleration, so (c) is not a correct response. An object projected straight upward into the air has a constant downward acceleration, yet its position (altitude) does not always increase in time (it eventually starts to fall back downward) nor is its velocity always directed downward (the direction of the constant acceleration). Thus, neither (d) nor (e) can be correct.

OQ2.5 The maximum height (where $v = 0$) reached by a freely falling object shot upward with an initial velocity $v_0 = +225$ m/s is found from $v_f^2 = v_i^2 + 2a(y_f - y_i) = v_i^2 + 2a\Delta y$, where we replace a with $-g$, the downward acceleration due to gravity. Solving for Δy then gives

$$\Delta y = \frac{\left(v_f^2 - v_i^2\right)}{2a} = \frac{-v_0^2}{2(-g)} = \frac{-(225\text{ m/s})^2}{2\left(-9.80\text{ m/s}^2\right)} = 2.58 \times 10^3\text{ m}$$

Thus, the projectile will be at the $\Delta y = 6.20 \times 10^2$ m level twice, once on the way upward and once coming back down.

The elapsed time when it passes this level coming downward can be found by using $v_f^2 = v_i^2 + 2a\Delta y$ again by substituting $a = -g$ and solving for the velocity of the object at height (displacement from original position) $\Delta y = +6.20 \times 10^2$ m.

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v^2 = (225\text{ m/s})^2 + 2\left(-9.80\text{ m/s}^2\right)\left(6.20 \times 10^2\text{ m}\right)$$
$$v = \pm 196\text{ m/s}$$

The velocity coming down is −196 m/s. Using $v_f = v_i + at$, we can solve for the time the velocity takes to change from +225 m/s to −196 m/s:

$$t = \frac{\left(v_f - v_i\right)}{a} = \frac{\left(-196 \text{ m/s} - 225 \text{ m/s}\right)}{\left(-9.80 \text{ m/s}^2\right)} = 43.0 \text{ s.}$$

The correct choice is (e).

OQ2.6 Once the arrow has left the bow, it has a constant downward acceleration equal to the free-fall acceleration, g. Taking upward as the positive direction, the elapsed time required for the velocity to change from an initial value of 15.0 m/s upward (v_0 = +15.0 m/s) to a value of 8.00 m/s downward (v_f = −8.00 m/s) is given by

$$\Delta t = \frac{\Delta v}{a} = \frac{v_f - v_0}{-g} = \frac{-8.00 \text{ m/s } - \left(+15.0 \text{ m/s}\right)}{-9.80 \text{ m/s}^2} = 2.35 \text{ s}$$

Thus, the correct choice is (d).

OQ2.7 (c) The object has an initial positive (northward) velocity and a negative (southward) acceleration; so, a graph of velocity versus time slopes down steadily from an original positive velocity. Eventually, the graph cuts through zero and goes through increasing-magnitude-negative values.

OQ2.8 (b) Using $v_f^2 = v_i^2 + 2a\Delta y$, with v_i = −12 m/s and Δy = −40 m:

$$v_f^2 = v_i^2 + 2a\Delta y$$
$$v^2 = \left(-12 \text{ m/s}\right)^2 + 2\left(-9.80 \text{ m/s}^2\right)\left(-40 \text{ m}\right)$$
$$v = -30 \text{ m/s}$$

OQ2.9 With original velocity zero, displacement is proportional to the square of time in (1/2)at^2. Making the time one-third as large makes the displacement one-ninth as large, answer (c).

OQ2.10 We take downward as the positive direction with $y = 0$ and $t = 0$ at the top of the cliff. The freely falling marble then has $v_0 = 0$ and its displacement at t = 1.00 s is Δy = 4.00 m. To find its acceleration, we use

$$y = y_0 + v_0 t + \frac{1}{2}at^2 \rightarrow \left(y - y_0\right) = \Delta y = \frac{1}{2}at^2 \rightarrow a = \frac{2\Delta y}{t^2}$$
$$a = \frac{2\left(4.00 \text{ m}\right)}{\left(1.00 \text{ s}\right)^2} = 8.00 \text{ m/s}^2$$

The displacement of the marble (from its initial position) at $t = 2.00$ s is found from

$$\Delta y = \frac{1}{2}at^2$$

$$\Delta y = \frac{1}{2}\left(8.00\ \text{m/s}^2\right)\left(2.00\ \text{s}\right)^2 = 16.0\ \text{m}.$$

The distance the marble has fallen in the 1.00 s interval from $t = 1.00$ s to $t = 2.00$ s is then

$$\Delta y = 16.0\ \text{m} - 4.0\ \text{m} = 12.0\ \text{m}.$$

and the answer is (c).

OQ2.11 In a position vs. time graph, the velocity of the object at any point in time is the slope of the line tangent to the graph at that instant in time. The speed of the particle at this point in time is simply the magnitude (or absolute value) of the velocity at this instant in time. The displacement occurring during a time interval is equal to the difference in x coordinates at the final and initial times of the interval, $\Delta x = x_f - x_i$.

The average velocity during a time interval is the slope of the straight line connecting the points on the curve corresponding to the initial and final times of the interval,

$$\bar{v} = \Delta x/\Delta t$$

Thus, we see how the quantities in choices (a), (e), (c), and (d) can all be obtained from the graph. Only the acceleration, choice (b), *cannot be obtained* from the position vs. time graph.

OQ2.12 We take downward as the positive direction with $y = 0$ and $t = 0$ at the top of the cliff. The freely falling pebble then has $v_0 = 0$ and $a = g = +9.8\ \text{m/s}^2$. The displacement of the pebble at $t = 1.0$ s is given: $y_1 = 4.9$ m. The displacement of the pebble at $t = 3.0$ s is found from

$$y_3 = v_0t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(9.8\ \text{m/s}^2\right)\left(3.0\ \text{s}\right)^2 = 44\ \text{m}$$

The distance fallen in the 2.0-s interval from $t = 1.0$ s to $t = 3.0$ s is then

$$\Delta y = y_3 - y_1 = 44\ \text{m} - 4.9\ \text{m} = 39\ \text{m}$$

and choice (c) is seen to be the correct answer.

OQ2.13 (c) They are the same. After the first ball reaches its apex and falls back downward past the student, it will have a downward velocity of magnitude v_i. This velocity is the same as the velocity of the second ball, so after they fall through equal heights their impact speeds will also be the same.

OQ2.14 (b) Above. Your ball has zero initial speed and smaller average speed during the time of flight to the passing point. So your ball must travel a smaller distance to the passing point than the ball your friend throws.

OQ2.15 Take down as the positive direction. Since the pebble is released from rest, $v_f^2 = v_i^2 + 2a\Delta y$ becomes

$$v_f^2 = (4\ \mathrm{m/s})^2 = 0^2 + 2gh.$$

Next, when the pebble is thrown with speed 3.0 m/s from the same height *h*, we have

$$v_f^2 = (3\ \mathrm{m/s})^2 + 2gh = (3\ \mathrm{m/s})^2 + (4\ \mathrm{m/s})^2 \rightarrow v_f = 5\ \mathrm{m/s}$$

and the answer is (b). Note that we have used the result from the first equation above and replaced $2gh$ with $(4\ \mathrm{m/s})^2$ in the second equation.

OQ2.16 Once the ball has left the thrower's hand, it is a freely falling body with a constant, nonzero, acceleration of $a = -g$. Since the acceleration of the ball is not zero at any point on its trajectory, choices (a) through (d) are all false and the correct response is (e).

OQ2.17 (a) Its speed is zero at points B and D where the ball is reversing its direction of motion. Its speed is the same at A, C, and E because these points are at the same height. The assembled answer is A = C = E > B = D.

(b) The acceleration has a very large positive (upward) value at D. At all the other points it is $-9.8\ \mathrm{m/s^2}$. The answer is D > A = B = C = E.

OQ2.18 (i) (b) shows equal spacing, meaning constant nonzero velocity and constant zero acceleration. (ii) (c) shows positive acceleration throughout. (iii) (a) shows negative (leftward) acceleration in the first four images.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ2.1 The net displacement must be zero. The object could have moved away from its starting point and back again, but it is at its initial position again at the end of the time interval.

CQ2.2 Tramping hard on the brake at zero speed on a level road, you do not feel pushed around inside the car. The forces of rolling resistance and air resistance have dropped to zero as the car coasted to a stop, so the car's acceleration is zero at this moment and afterward.

Tramping hard on the brake at zero speed on an uphill slope, you feel thrown backward against your seat. Before, during, and after the zero-speed moment, the car is moving with a downhill acceleration if you do not tramp on the brake.

CQ2.3 Yes. If a car is travelling eastward and slowing down, its acceleration is opposite to the direction of travel: its acceleration is westward.

CQ2.4 Yes. Acceleration is the time rate of change of the velocity of a particle. If the velocity of a particle is zero at a given moment, and if the particle is not accelerating, the velocity will remain zero; if the particle is accelerating, the velocity will change from zero—the particle will begin to move. Velocity and acceleration are independent of each other.

CQ2.5 Yes. Acceleration is the time rate of change of the velocity of a particle. If the velocity of a particle is nonzero at a given moment, and the particle is not accelerating, the velocity will remain the same; if the particle is accelerating, the velocity will change. The velocity of a particle at a given moment and how the velocity is changing at that moment are independent of each other.

CQ2.6 Assuming no air resistance: (a) The ball reverses direction at its maximum altitude. For an object traveling along a straight line, its velocity is zero at the point of reversal. (b) Its acceleration is that of gravity: $-9.80\ \text{m/s}^2$ ($9.80\ \text{m/s}^2$, downward). (c) The velocity is $-5.00\ \text{m/s}^2$. (d) The acceleration of the ball remains $-9.80\ \text{m/s}^2$ as long as it does not touch anything. Its acceleration changes when the ball encounters the ground.

CQ2.7 (a) No. Constant acceleration only: the derivation of the equations assumes that d^2x/dt^2 is constant. (b) Yes. Zero is a constant.

CQ2.8 Yes. If the speed of the object varies at all over the interval, the instantaneous velocity will sometimes be greater than the average velocity and will sometimes be less.

CQ2.9 No: Car A might have greater acceleration than B, but they might both have zero acceleration, or otherwise equal accelerations; or the driver of B might have tramped hard on the gas pedal in the recent past to give car B greater acceleration just then.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 2.1 Position, Velocity, and Speed

P2.1 The average velocity is the slope, not necessarily of the graph line itself, but of a secant line cutting across the graph between specified points. The slope of the graph line itself is the instantaneous velocity, found, for example, in Problem 6 part (b). On this graph, we can tell positions to two significant figures:

(a) $x = 0$ at $t = 0$ and $x = 10$ m at $t = 2$ s:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{10\text{ m} - 0}{2\text{ s} - 0} = \boxed{5.0\text{ m/s}}$$

(b) $x = 5.0$ m at $t = 4$ s:

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ m} - 0}{4\text{ s} - 0} = \boxed{1.2\text{ m/s}}$$

(c) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{5.0\text{ m} - 10\text{ m}}{4\text{ s} - 2\text{ s}} = \boxed{-2.5\text{ m/s}}$$

(d) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{-5.0\text{ m} - 5.0\text{ m}}{7\text{ s} - 4\text{ s}} = \boxed{-3.3\text{ m/s}}$$

(e) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{0.0\text{ m} - 0.0\text{ m}}{8\text{ s} - 0\text{ s}} = \boxed{0\text{ m/s}}$$

P2.2 We assume that you are approximately 2 m tall and that the nerve impulse travels at uniform speed. The elapsed time is then

$$\Delta t = \frac{\Delta x}{v} = \frac{2\text{ m}}{100\text{ m/s}} = 2 \times 10^{-2}\text{ s} = \boxed{0.02\text{ s}}$$

P2.3 Speed is positive whenever motion occurs, so the average speed must be positive. For the velocity, we take as positive for motion to the right and negative for motion to the left, so its average value can be positive, negative, or zero.

(a) The average speed during any time interval is equal to the total distance of travel divided by the total time:

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{d_{AB} + d_{BA}}{t_{AB} + t_{BA}}$$

But $d_{AB} = d_{BA}$, $t_{AB} = d/v_{AB}$, and $t_{BA} = d/v_{BA}$

so $$\text{average speed} = \frac{d + d}{(d/v_{AB}) + (d/v_{BA})} = \frac{2(v_{AB})(v_{BA})}{v_{AB} + v_{BA}}$$

and

$$\text{average speed} = 2\left[\frac{(5.00\ \text{m/s})(3.00\ \text{m/s})}{5.00\ \text{m/s} + 3.00\ \text{m/s}}\right] = \boxed{3.75\ \text{m/s}}$$

(b) The average velocity during any time interval equals total displacement divided by elapsed time.

$$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t}$$

Since the walker returns to the starting point, $\Delta x = 0$ and $\boxed{v_{x,\text{avg}} = 0}$.

P2.4 We substitute for t in $x = 10t^2$, then use the definition of average velocity:

t (s)	2.00	2.10	3.00
x (m)	40.0	44.1	90.0

(a) $$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{90.0\ \text{m} - 40.0\ \text{m}}{1.00\ \text{s}} = \frac{50.0\ \text{m}}{1.00\ \text{s}} = \boxed{50.0\ \text{m/s}}$$

(b) $$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{44.1\ \text{m} - 40.0\ \text{m}}{0.100\ \text{s}} = \frac{4.10\ \text{m}}{0.100\ \text{s}} = \boxed{41.0\ \text{m/s}}$$

***P2.5** We read the data from the table provided, assume three significant figures of precision for all the numbers, and use Equation 2.2 for the definition of average velocity.

(a) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{2.30\ \text{m} - 0\ \text{m}}{1.00\ \text{s}} = \boxed{2.30\ \text{m/s}}$$

(b) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{57.5\ \text{m} - 9.20\ \text{m}}{3.00\ \text{s}} = \boxed{16.1\ \text{m/s}}$$

(c) $$v_{x,\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{57.5\ \text{m} - 0\ \text{m}}{5.00\ \text{s}} = \boxed{11.5\ \text{m/s}}$$

Section 2.2 Instantaneous Velocity and Speed

P2.6 (a) At any time, t, the position is given by $x = (3.00\ \text{m/s}^2)t^2$.

Thus, at $t_i = 3.00$ s: $x_i = (3.00\ \text{m/s}^2)(3.00\ \text{s})^2 = \boxed{27.0\ \text{m}}$.

(b) At $t_f = 3.00\ \text{s} + \Delta t$: : $x_f = (3.00\ \text{m/s}^2)(3.00\ \text{s} + \Delta t)^2$, or

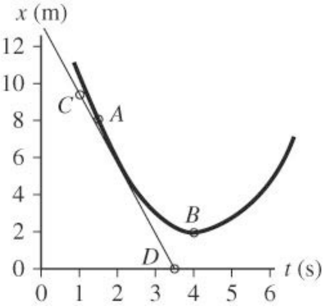
$$x_f = \boxed{27.0\ \text{m} + (18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}$$

(c) The instantaneous velocity at $t = 3.00$ s is:

$$\boxed{\begin{aligned}\lim_{\Delta t\to 0}\frac{\Delta x}{\Delta t} &= \lim_{\Delta t\to 0}\frac{(18.0\ \text{m/s})\Delta t + (3.00\ \text{m/s}^2)(\Delta t)^2}{\Delta t}\\ &= \lim_{\Delta t\to 0}(18.0\ \text{m/s}) + (3.00\ \text{m/s}^2)(\Delta t) = \boxed{18.0\ \text{m/s}}\end{aligned}}$$

P2.7 For average velocity, we find the slope of a secant line running across the graph between the 1.5-s and 4-s points. Then for instantaneous velocities we think of slopes of tangent lines, which means the slope of the graph itself at a point.

We place two points on the curve: Point A, at $t = 1.5$ s, and Point B, at $t = 4.0$ s, and read the corresponding values of x.



ANS. FIG. P2.7

(a) At $t_i = 1.5$ s, $x_i = 8.0$ m (Point A)

At $t_f = 4.0$ s, $x_f = 2.0$ m (Point B)

$$v_{\text{avg}} = \frac{x_f - x_i}{t_f - t_i} = \frac{(2.0 - 8.0)\ \text{m}}{(4.0 - 1.5)\ \text{s}}$$

$$= -\frac{6.0\ \text{m}}{2.5\ \text{s}} = \boxed{-2.4\ \text{m/s}}$$

(b) The slope of the tangent line can be found from points C and D. ($t_C = 1.0$ s, $x_C = 9.5$ m) and ($t_D = 3.5$ s, $x_D = 0$),

$$v \approx \boxed{-3.8\ \text{m/s}}$$

The negative sign shows that the **direction** of v_x is along the negative x direction.

(c) The velocity will be zero when the slope of the tangent line is zero. This occurs for the point on the graph where x has its minimum value. This is at $t \approx \boxed{4.0\ \text{s}}$.

P2.8 We use the definition of average velocity.

(a) $$v_{1,x,\text{ave}} = \frac{(\Delta x)_1}{(\Delta t)_1} = \frac{L-0}{t_1} = \boxed{+L/t_1}$$

(b) $$v_{2,x,\text{ave}} = \frac{(\Delta x)_2}{(\Delta t)_2} = \frac{0-L}{t_2} = \boxed{-L/t_2}$$

(c) To find the average velocity for the round trip, we add the displacement and time for each of the two halves of the swim:

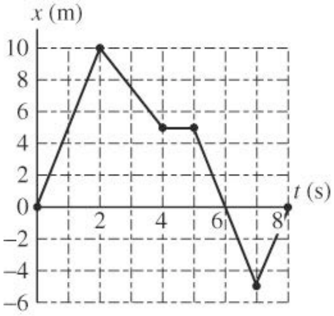
$$v_{x,\text{ave,total}} = \frac{(\Delta x)_{\text{total}}}{(\Delta t)_{\text{total}}} = \frac{(\Delta x)_1 + (\Delta x)_2}{t_1 + t_2} = \frac{+L-L}{t_1+t_2} = \frac{0}{t_1+t_2} = \boxed{0}$$

(d) The average speed of the round trip is the total distance the athlete travels divided by the total time for the trip:

$$v_{\text{ave,trip}} = \frac{\text{total distance traveled}}{(\Delta t)_{\text{total}}} = \frac{|(\Delta x)_1| + |(\Delta x)_2|}{t_1+t_2}$$

$$= \frac{|+L| + |-L|}{t_1+t_2} = \boxed{\frac{2L}{t_1+t_2}}$$

P2.9 The instantaneous velocity is found by evaluating the slope of the $x - t$ curve at the indicated time. To find the slope, we choose two points for each of the times below.



ANS. FIG. P2.9

(a) $$v = \frac{(5-0)\text{ m}}{(1-0)\text{ s}} = \boxed{5\text{ m/s}}$$

(b) $$v = \frac{(5-10)\text{ m}}{(4-2)\text{ s}} = \boxed{-2.5\text{ m/s}}$$

(c) $$v = \frac{(5-5)\text{ m}}{(5\text{ s}-4\text{ s})} = \boxed{0}$$

(d) $$v = \frac{0-(-5\text{ m})}{(8\text{ s}-7\text{ s})} = \boxed{+5\text{ m/s}}$$

Section 2.3 Analysis Model: Particle Under Constant Velocity

P2.10 The plates spread apart distance d of 2.9×10^3 mi in the time interval Δt at the rate of 25 mm/year. Converting units:

$$\left(2.9\times 10^3 \text{ mi}\right)\left(\frac{1609 \text{ m}}{1 \text{ mi}}\right)\left(\frac{10^3 \text{ mm}}{1 \text{ m}}\right) = 4.7\times 10^9 \text{ mm}$$

Use $d = v\Delta t$, and solve for Δt:

$$d = v\Delta t \rightarrow \Delta t = \frac{d}{v}$$

$$\Delta t = \frac{4.7\times 10^9 \text{ mm}}{25 \text{ mm/year}} = \boxed{1.9\times 10^8 \text{ years}}$$

P2.11 (a) The tortoise crawls through a distance D before the rabbit resumes the race. When the rabbit resumes the race, the rabbit must run through 200 m at 8.00 m/s while the tortoise crawls through the distance (1 000 m – D) at 0.200 m/s. Each takes the same time interval to finish the race:

$$\Delta t = \left(\frac{200 \text{ m}}{8.00 \text{ m/s}}\right) = \left(\frac{1\,000 \text{ m} - D}{0.200 \text{ m/s}}\right)$$

Solving,

$$\rightarrow (0.200 \text{ m/s})(200 \text{ m}) = (8.00 \text{ m/s})(1\,000 \text{ m} - D)$$

$$1\,000 \text{ m} - D = \frac{(0.200 \text{ m/s})(200 \text{ m})}{8.00 \text{ m/s}}$$

$$\rightarrow D = 995 \text{ m}$$

So, the tortoise is 1 000 m – D = $\boxed{5.00 \text{ m}}$ from the finish line when the rabbit resumes running.

(b) Both begin the race at the same time: $t = 0$. The rabbit reaches the 800-m position at time t = 800 m/(8.00 m/s) = 100 s. The tortoise has crawled through 995 m when t = 995 m/(0.200 m/s) = 4 975 s. The rabbit has waited for the time interval Δt = 4 975 s – 100 s = $\boxed{4\,875 \text{ s}}$.

P2.12 The trip has two parts: first the car travels at constant speed v_1 for distance d, then it travels at constant speed v_2 for distance d. The first part takes the time interval $\Delta t_1 = d/v_1$, and the second part takes the time interval $\Delta t_2 = d/v_2$.

(a) By definition, the average velocity for the entire trip is $v_{\text{avg}} = \Delta x / \Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = 2d$, and

$\Delta t = \Delta t_1 + \Delta t_2 = d/v_1 + d/v_2$. Putting these together, we have

$$v_{avg} = \left(\frac{\Delta d}{\Delta t}\right) = \left(\frac{\Delta x_1 + \Delta x_2}{\Delta t_1 + \Delta t_2}\right) = \left(\frac{2d}{d/v_1 + d/v_2}\right) = \left(\frac{2v_1 v_2}{v_1 + v_2}\right)$$

We know $v_{avg} = 30$ mi/h and $v_1 = 60$ mi/h.

Solving for v_2 gives

$$(v_1 + v_2)v_{avg} = 2v_1 v_2 \rightarrow v_2 = \left(\frac{v_1 v_{avg}}{2v_1 - v_{avg}}\right).$$

$$v_2 = \left[\frac{(30\text{ mi/h})(60\text{ mi/h})}{2(60\text{ mi/h}) - (30\text{ mi/h})}\right] = \boxed{20\text{ mi/h}}$$

(b) The average velocity for this trip is $v_{avg} = \Delta x/\Delta t$, where $\Delta x = \Delta x_1 + \Delta x_2 = d + (-d) = 0$; so, $v_{avg} = \boxed{0}$.

(c) The average speed for this trip is $v_{avg} = d/\Delta t$, where $d = d_1 + d_2 = d + d = 2d$ and $\Delta t = \Delta t_1 + \Delta t_2 = d/v_1 + d/v_2$; so, the average speed is the same as in part (a): $v_{avg} = \boxed{30\text{ mi/h.}}$

***2.13** (a) The total time for the trip is $t_{total} = t_1 + 22.0$ min $= t_1 + 0.367$ h, where t_1 is the time spent traveling at $v_1 = 89.5$ km/h. Thus, the distance traveled is $\Delta x = v_1 t_1 = v_{avg} t_{total}$, which gives

$$\begin{aligned}(89.5\text{ km/h})t_1 &= (77.8\text{ km/h})(t_1 + 0.367\text{ h}) \\ &= (77.8\text{ km/h})t_1 + 28.5\text{ km}\end{aligned}$$

or $(89.5\text{ km/h} - 77.8\text{ km/h})t_1 = 28.5\text{ km}$

from which, $t_1 = 2.44$ h, for a total time of

$$t_{total} = t_1 + 0.367\text{ h} = \boxed{2.81\text{ h}}$$

(b) The distance traveled during the trip is $\Delta x = v_1 t_1 = v_{avg} t_{total}$, giving

$$\Delta x = v_{avg} t_{total} = (77.8\text{ km/h})(2.81\text{ h}) = \boxed{219\text{ km}}$$

Section 2.4 Acceleration

P2.14 The ball's motion is entirely in the horizontal direction. We choose the positive direction to be the outward direction, perpendicular to the wall. With outward positive, $v_i = -25.0$ m/s and $v_f = 22.0$ m/s. We use Equation 2.13 for one-dimensional motion with constant acceleration, $v_f = v_i + at$, and solve for the acceleration to obtain

$$a = \frac{\Delta v}{\Delta t} = \frac{22.0\ \text{m/s} - (-25.0\ \text{m/s})}{3.50 \times 10^{-3}\ \text{s}} = \boxed{1.34 \times 10^{4}\ \text{m/s}^2}$$

P2.15 (a) Acceleration is the slope of the graph of *v* versus *t*.

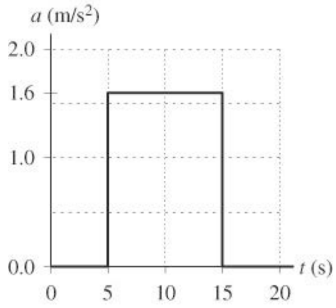
For $0 < t < 5.00$ s, $a = 0$.

For 15.0 s $< t <$ 20.0 s, $a = 0$.

For 5.0 s $< t <$ 15.0 s, $a = \frac{v_f - v_i}{t_f - t_i}$.

$$a = \frac{8.00\ \text{m/s} - (-8.00\ \text{m/s})}{15.0\ \text{s} - 5.00\ \text{s}} = \boxed{1.60\ \text{m/s}^2}$$

We can plot *a*(*t*) as shown in ANS. FIG. P2.15 below.



ANS. FIG. P2.15

For (b) and (c) we use $a = \frac{v_f - v_i}{t_f - t_i}$.

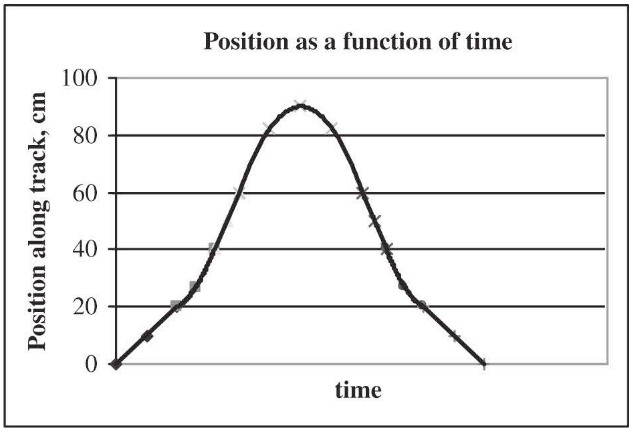
(b) For 5.00 s $< t <$ 15.0 s, $t_i = 5.00$ s, $v_i = -8.00$ m/s, $t_f = 15.0$ s, and $v_f = 8.00$ m/s:

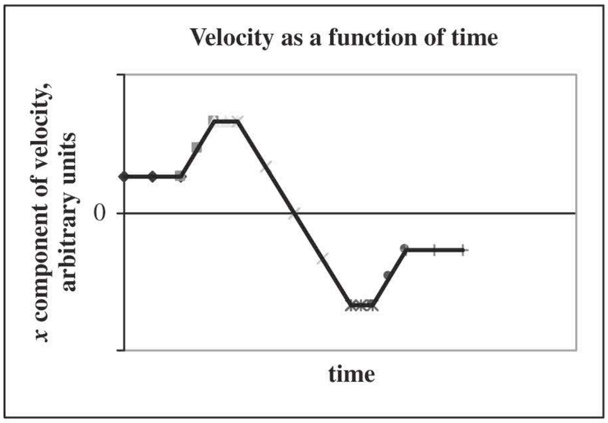
$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00\ \text{m/s} - (-8.00\ \text{m/s})}{15.0\ \text{s} - 5.00\ \text{s}} = \boxed{1.60\ \text{m/s}^2}$$

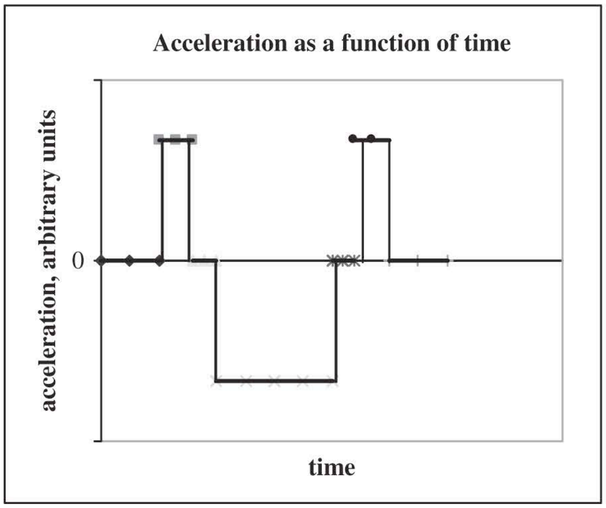
(c) We use $t_i = 0$, $v_i = -8.00$ m/s, $t_f = 20.0$ s, and $v_f = 8.00$ m/s:

$$a = \frac{v_f - v_i}{t_f - t_i} = \frac{8.00\ \text{m/s} - (-8.00\ \text{m/s})}{20.0\ \text{s} - 0} = \boxed{0.800\ \text{m/s}^2}$$

P2.16 The acceleration is zero whenever the marble is on a horizontal section. The acceleration has a constant positive value when the marble is rolling on the 20-to-40-cm section and has a constant negative value when it is rolling on the second sloping section. The position graph is a straight sloping line whenever the speed is constant and a section of a parabola when the speed changes.







P2.17 (a) In the interval $t_i = 0$ s and $t_f = 6.00$ s, the motorcyclist's velocity changes from $v_i = 0$ to $v_f = 8.00$ m/s. Then,

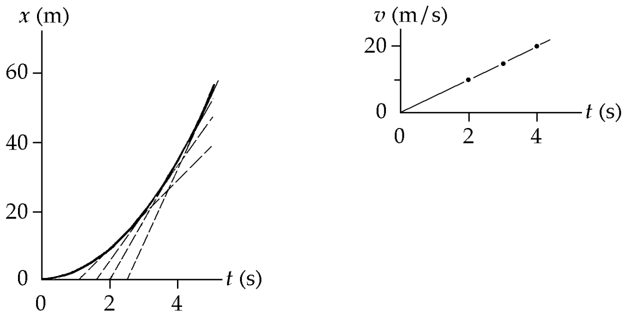
$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} = \frac{8.0\text{ m/s} - 0}{6.0\text{ s} - 0} = \boxed{1.3\text{ m/s}^2}$$

(b) Maximum positive acceleration occurs when the slope of the velocity-time curve is greatest, at $t = 3$ s, and is equal to the slope of the graph, approximately (6 m/s – 2 m/s)/(4 s – 2 s) = $\boxed{2\text{ m/s}^2}$.

(c) The acceleration $a = 0$ when the slope of the velocity-time graph is zero, which occurs at $\boxed{t = 6\text{ s}}$, and also for $\boxed{t > 10\text{ s}}$.

(d) Maximum negative acceleration occurs when the velocity-time graph has its maximum negative slope, at $t = 8$ s, and is equal to the slope of the graph, approximately $\boxed{-1.5\text{ m/s}^2}$.

***P2.18** (a) The graph is shown in ANS. FIG. P2.18 below.



ANS. FIG. P2.18

(b) At $t = 5.0$ s, the slope is $v \approx \dfrac{58\text{ m}}{2.5\text{ s}} \approx \boxed{23\text{ m/s}}$.

At $t = 4.0$ s, the slope is $v \approx \dfrac{54\text{ m}}{3\text{ s}} \approx \boxed{18\text{ m/s}}$.

At $t = 3.0$ s, the slope is $v \approx \dfrac{49\text{ m}}{3.4\text{ s}} \approx \boxed{14\text{ m/s}}$.

At $t = 2.0$ s, the slope is $v \approx \dfrac{36\text{ m}}{4.0\text{ s}} \approx \boxed{9.0\text{ m/s}}$.

(c) $\bar{a} = \dfrac{\Delta v}{\Delta t} \approx \dfrac{23\text{ m/s}}{5.0\text{ s}} \approx \boxed{4.6\text{ m/s}^2}$

(d) The initial velocity of the car was $\boxed{\text{zero}}$.

P2.19 (a) The area under a graph of *a* vs. *t* is equal to the change in velocity, Δv. We can use Figure P2.19 to find the change in velocity during specific time intervals.

The area under the curve for the time interval 0 to 10 s has the shape of a rectangle. Its area is

$$\Delta v = (2\text{ m/s}^2)(10\text{ s}) = 20\text{ m/s}$$

The particle starts from rest, $v_0 = 0$, so its velocity at the end of the 10-s time interval is

$$v = v_0 + \Delta v = 0 + 20\text{ m/s} = \boxed{20\text{ m/s}}$$

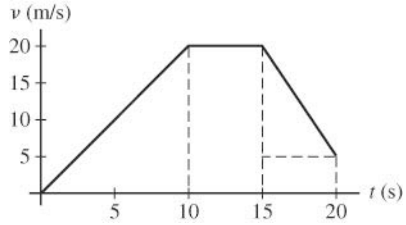
Between $t = 10$ s and $t = 15$ s, the area is zero: $\Delta v = 0$ m/s.

Between $t = 15$ s and $t = 20$ s, the area is a rectangle: $\Delta v = (-3\text{ m/s}^2)(5\text{ s}) = -15\text{ m/s}$.

So, between $t = 0$ s and $t = 20$ s, the total area is $\Delta v = (20\text{ m/s}) + (0\text{ m/s}) + (-15\text{ m/s}) = 5\text{ m/s}$, and the velocity at $t = 20$ s is $\boxed{5\text{ m/s.}}$

(b) We can use the information we derived in part (a) to construct a graph of *x* vs. *t*; the area under such a graph is equal to the displacement, Δx, of the particle.

From (a), we have these points (t, v) = (0 s, 0 m/s), (10 s, 20 m/s), (15 s, 20 m/s), and (20 s, 5 m/s). The graph appears below.



The displacements are:

0 to 10 s (area of triangle): $\Delta x = (1/2)(20\text{ m/s})(10\text{ s}) = 100\text{ m}$

10 to 15 s (area of rectangle): $\Delta x = (20\text{ m/s})(5\text{ s}) = 100\text{ m}$

15 to 20 s (area of triangle and rectangle):

$$\Delta x = (1/2)[(20-5)\text{ m/s}](5\text{ s}) + (5\text{ m/s})(5\text{ s})$$
$$= 37.5\text{ m} + 25\text{ m} = 62.5\text{ m}$$

Total displacement over the first 20.0 s:

$$\Delta x = 100\text{ m} + 100\text{ m} + 62.5\text{ m} = 262.5\text{ m} = \boxed{263\text{ m}}$$

P2.20 (a) The average velocity is the change in position divided by the length of the time interval. We plug in to the given equation.

At $t = 2.00$ s, $x = [3.00(2.00)^2 - 2.00(2.00) + 3.00]$ m = 11.0 m.

At $t = 3.00$ s, $x = [3.00(3.00)^2 - 2.00(3.00) + 3.00]$ m = 24.0 m

so

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{24.0\text{ m} - 11.0\text{ m}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{13.0\text{ m/s}}$$

(b) At all times the instantaneous velocity is

$$v = \frac{d}{dt}\left(3.00t^2 - 2.00t + 3.00\right) = \left(6.00t - 2.00\right)\text{ m/s}$$

At $t = 2.00$ s, $v = [6.00(2.00) - 2.00]$ m/s = $\boxed{10.0\text{ m/s}}$.

At $t = 3.00$ s, $v = [6.00(3.00) - 2.00]$ m/s = $\boxed{16.0\text{ m/s}}$.

(c) $a_{\text{avg}} = \dfrac{\Delta v}{\Delta t} = \dfrac{16.0\text{ m/s} - 10.0\text{ m/s}}{3.00\text{ s} - 2.00\text{ s}} = \boxed{6.00\text{ m/s}^2}$

(d) At all times $a = \dfrac{d}{dt}\left(6.00t - 2.00\right) = \boxed{6.00\text{ m/s}^2}$. This includes both $t = 2.00$ s and $t = 3.00$ s.

(e) From (b), $v = (6.00t - 2.00) = 0 \rightarrow t = (2.00)/(6.00) = \boxed{0.333\text{ s.}}$

P2.21 To find position we simply evaluate the given expression. To find velocity we differentiate it. To find acceleration we take a second derivative.

With the position given by $x = 2.00 + 3.00t - t^2$, we can use the rules for differentiation to write expressions for the velocity and acceleration as functions of time:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(2 + 3t - t^2\right) = 3 - 2t \text{ and } a_x = \frac{dv}{dt} = \frac{d}{dt}(3 - 2t) = -2$$

Now we can evaluate x, v, and a at $t = 3.00$ s.

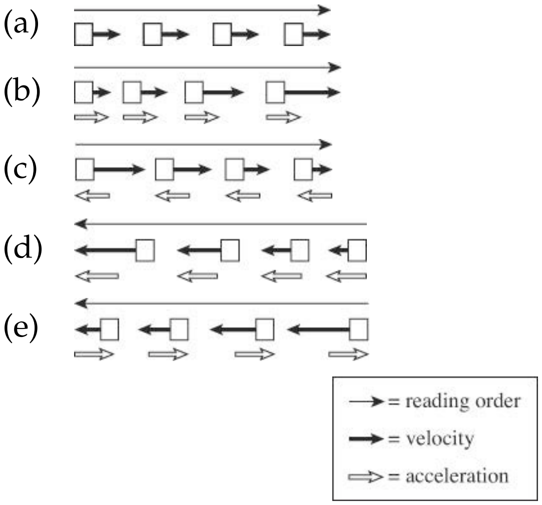
(a) $x = (2.00 + 9.00 - 9.00)$ m = $\boxed{2.00\text{ m}}$

(b) $v = (3.00 - 6.00)$ m/s = $\boxed{-3.00\text{ m/s}}$

(c) $a = \boxed{-2.00\text{ m/s}^2}$

Section 2.5 Motion Diagrams

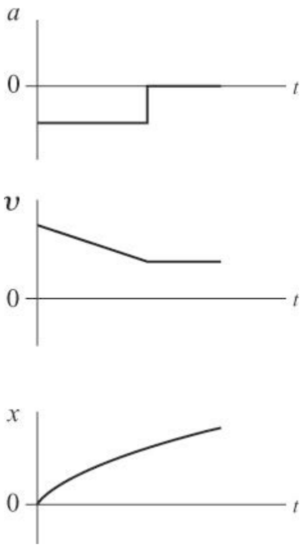
P2.22



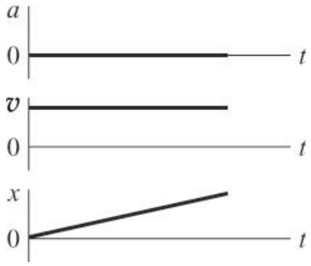
(f) One way of phrasing the answer: The spacing of the successive positions would change with $\boxed{\text{less regularity.}}$

Another way: The object would move with some combination of the kinds of motion shown in (a) through (e). Within one drawing, the acceleration vectors would vary in magnitude and direction.

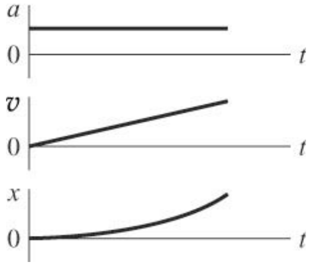
P2.23 (a) The motion is fast at first but slowing until the speed is constant. We assume the acceleration is constant as the object slows.



(b) The motion is constant in speed.



(c) The motion is speeding up, and we suppose the acceleration is constant.



Section 2.6 Analysis Model: Particle Under Constant Acceleration

***P2.24** Method One

Suppose the unknown acceleration is constant as a car moving at $v_{i1} = 35.0 \text{ mi/h}$ comes to a stop, $v_f = 0$ in $x_{f1} - x_i = 40.0$ ft. We find its acceleration from $v_{f1}^2 = v_{i1}^2 + 2a\left(x_{f1} - x_i\right)$:

$$a = \frac{v_{f1}^2 - v_i^2}{2\left(x_{f1} - x_i\right)} = \frac{0 - (35.0 \text{ mi/h})^2}{2(40.0 \text{ ft})}\left(\frac{5\,280 \text{ ft}}{\text{mi}}\right)^2\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)^2 = -32.9 \text{ ft/s}^2$$

Now consider a car moving at $v_{i2} = 70.0 \text{ mi/h}$ and stopping, $v_f = 0$, with $a = -32.9 \text{ ft/s}^2$. From the same equation, its stopping distance is

$$x_{f2} - x_i = \frac{v_{f2}^2 - v_i^2}{2a} = \frac{0 - (70.0 \text{ mi/h})^2}{2\left(-32.9 \text{ ft/s}^2\right)}\left(\frac{5\,280 \text{ ft}}{1 \text{ mi}}\right)^2\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)^2$$

$$= \boxed{160 \text{ ft}}$$

Method Two

For the process of stopping from the lower speed v_{i1} we have $v_f^2 = v_{i1}^2 + 2a\left(x_{f1} - x_i\right)$, $0 = v_{i1}^2 + 2ax_{f1}$, and $v_{i1}^2 = -2ax_{f1}$. For stopping from $v_{i2} = 2v_{i1}$, similarly $0 = v_{i2}^2 + 2ax_{f2}$, and $v_{i2}^2 = -2ax_{f2}$. Dividing gives

$$\frac{v_{i2}^2}{v_{i1}^2} = \frac{x_{f2}}{x_{f1}};\ x_{f2} = 40\text{ ft} \times 2^2 = \boxed{160\text{ ft}}$$

***P2.25** We have $v_i = 2.00 \times 10^4\text{ m/s}$, $v_f = 6.00 \times 10^6\text{ m/s}$, and $x_f - x_i = 1.50 \times 10^{-2}\text{ m}$.

(a) $x_f - x_i = \frac{1}{2}(v_i + v_f)t$:

$$t = \frac{2(x_f - x_i)}{v_i + v_f} = \frac{2(1.50 \times 10^{-2}\text{ m})}{2.00 \times 10^4\text{ m/s} + 6.00 \times 10^6\text{ m/s}}$$

$$= \boxed{4.98 \times 10^{-9}\text{ s}}$$

(b) $v_f^2 = v_i^2 + 2a_x(x_f - x_i)$:

$$a_x = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(6.00 \times 10^6\text{ m/s})^2 - (2.00 \times 10^4\text{ m/s})^2}{2(1.50 \times 10^{-2}\text{ m})}$$

$$= \boxed{1.20 \times 10^{15}\text{ m/s}^2}$$

***P2.26** (a) Choose the initial point where the pilot reduces the throttle and the final point where the boat passes the buoy: $x_i = 0$, $x_f = 100\text{ m}$, $v_{xi} = 30\text{ m/s}$, $v_{xf} = ?$, $a_x = -3.5\text{ m/s}^2$, and $t = ?$

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2:$$

$$100\text{ m} = 0 + (30\text{ m/s})t + \frac{1}{2}(-3.5\text{ m/s}^2)t^2$$

$$(1.75\text{ m/s}^2)t^2 - (30\text{ m/s})t + 100\text{ m} = 0$$

We use the quadratic formula:

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{30\text{ m/s} \pm \sqrt{900\text{ m}^2/\text{s}^2 - 4(1.75\text{ m/s}^2)(100\text{ m})}}{2(1.75\text{ m/s}^2)}$$

$$= \frac{30\text{ m/s} \pm 14.1\text{ m/s}}{3.5\text{ m/s}^2} = 12.6\text{ s} \quad\text{or}\quad \boxed{4.53\text{ s}}$$

The smaller value is the physical answer. If the boat kept moving with the same acceleration, it would stop and move backward, then gain speed, and pass the buoy again at 12.6 s.

(b) $v_{xf} = v_{xi} + a_x t = 30\ \text{m/s} - \left(3.5\ \text{m/s}^2\right)4.53\ \text{s} = \boxed{14.1\ \text{m/s}}$

P2.27 In parts (a) – (c), we use Equation 2.13 to determine the velocity at the times indicated.

(a) The time given is 1.00 s after 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(1.00\ \text{s}) = \boxed{9.00\ \text{m/s}}$$

(b) The time given is 4.00 s after 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(4.00\ \text{s}) = \boxed{-3.00\ \text{m/s}}$$

(c) The time given is 1.00 s before 10:05:00 a.m., so

$$v_f = v_i + at = 13.0\ \text{m/s} + (-4.00\ \text{m/s}^2)(-1.00\ \text{s}) = \boxed{17.0\ \text{m/s}}$$

(d) The graph of velocity versus time is a slanting straight line, having the value 13.0 m/s at 10:05:00 a.m. on the certain date, and sloping down by 4.00 m/s for every second thereafter.

(e) If we also know the velocity at any one instant, then knowing the value of the constant acceleration tells us the velocity at all other instants

P2.28 (a) We use Equation 2.15:

$$x_f - x_i = \frac{1}{2}\left(v_i + v_f\right)t \text{ becomes } 40.0\ \text{m} = \frac{1}{2}\left(v_i + 2.80\ \text{m/s}\right)\left(8.50\ \text{s}\right),$$

which yields $v_i = \boxed{6.61\ \text{m/s}}$.

(b) From Equation 2.13,

$$a = \frac{v_f - v_i}{t} = \frac{2.80\ \text{m/s} - 6.61\ \text{m/s}}{8.50\ \text{s}} = \boxed{-0.448\ \text{m/s}^2}$$

P2.29 The velocity is always changing; there is always nonzero acceleration and the problem says it is constant. So we can use one of the set of equations describing constant-acceleration motion. Take the initial point to be the moment when $x_i = 3.00$ cm and $v_{xi} = 12.0$ cm/s. Also, at $t = 2.00$ s, $x_f = -5.00$ cm.

Once you have classified the object as a particle moving with constant acceleration and have the standard set of four equations in front of you, how do you choose which equation to use? Make a list of all of the six symbols in the equations: x_i, x_f, v_{xi}, v_{xf}, a_x, and t. On the list fill in values as above, showing that x_i, x_f, v_{xi}, and t are known. Identify a_x as the unknown. Choose an equation involving only one unknown and the knowns. That is, choose an equation *not* involving v_{xf}. Thus we choose the kinematic equation

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

and solve for a_x:

$$a_x = \frac{2\left[x_f - x_i - v_{xi}t\right]}{t^2}$$

We substitute:

$$a_x = \frac{2[-5.00\text{ cm} - 3.00\text{ cm} - (12.0\text{ cm/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= \boxed{-16.0\text{ cm/s}^2}$$

P2.30 We think of the plane moving with maximum-size backward acceleration throughout the landing, so the acceleration is constant, the stopping time a minimum, and the stopping distance as short as it can be. The negative acceleration of the plane as it lands can be called deceleration, but it is simpler to use the single general term *acceleration* for all rates of velocity change.

(a) The plane can be modeled as a particle under constant acceleration, with $a_x = -5.00\text{ m/s}^2$. Given $v_{xi} = 100\text{ m/s}$ and $v_{xf} = 0$, we use the equation $v_{xf} = v_{xi} + a_x t$ and solve for t:

$$t = \frac{v_{xf} - v_{xi}}{a_x} = \frac{0 - 100\text{ m/s}}{-5.00\text{ m/s}^2} = \boxed{20.0\text{ s}}$$

(b) Find the required stopping distance and compare this to the length of the runway. Taking x_i to be zero, we get

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

or $$\Delta x = x_f - x_i = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{0 - \left(100\text{ m/s}\right)^2}{2\left(-5.00\text{ m/s}^2\right)} = \boxed{1\,000\text{ m}}$$

(c) The stopping distance is greater than the length of the runway; $\boxed{\text{the plane cannot land}}$.

P2.31 We assume the acceleration is constant. We choose the initial and final points 1.40 s apart, bracketing the slowing-down process. Then we have a straightforward problem about a particle under constant acceleration. The initial velocity is

$$v_{xi} = 632\text{ mi/h} = 632\text{ mi/h}\left(\frac{1\,609\text{ m}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 282\text{ m/s}$$

(a) Taking $v_{xf} = v_{xi} + a_x t$ with $v_{xf} = 0$,

$$a_x = \frac{v_{xf} - v_{xi}}{t} = \frac{0 - 282\text{ m/s}}{1.40\text{ s}} = \boxed{-202\text{ m/s}^2}$$

This has a magnitude of approximately 20*g*.

(b) From Equation 2.15,

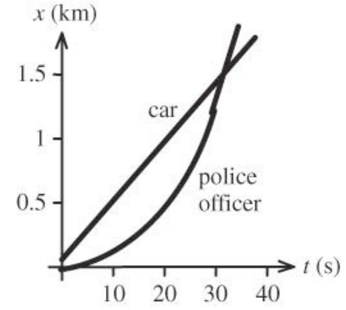
$$x_f - x_i = \frac{1}{2}(v_{xi} + v_{xf})t = \frac{1}{2}(282\text{ m/s} + 0)(1.40\text{ s}) = \boxed{198\text{ m}}$$

P2.32 As in the algebraic solution to Example 2.8, we let *t* represent the time the trooper has been moving. We graph

$$x_{\text{car}} = 45 + 45t$$

and $x_{\text{trooper}} = 1.5t^2$

They intersect at $t = \boxed{31\text{ s}}$.



ANS. FIG. P2.32

***P2.33** (a) The time it takes the truck to reach 20.0 m/s is found from $v_f = v_i + at$. Solving for *t* yields

$$t = \frac{v_f - v_i}{a} = \frac{20.0\text{ m/s} - 0\text{ m/s}}{2.00\text{ m/s}^2} = 10.0\text{ s}$$

The total time is thus $10.0\text{ s} + 20.0\text{ s} + 5.00\text{ s} = \boxed{35.0\text{ s}}$.

(b) The average velocity is the total distance traveled divided by the total time taken. The distance traveled during the first 10.0 s is

$$x_1 = \bar{v}t = \left(\frac{0 + 20.0}{2}\right)(10.0) = 100 \text{ m}$$

With $a = 0$ for this interval, the distance traveled during the next 20.0 s is

$$x_2 = v_i t + \frac{1}{2}at^2 = (20.0)(20.0) + 0 = 400 \text{ m}$$

The distance traveled in the last 5.00 s is

$$x_3 = \bar{v}t = \left(\frac{20.0 + 0}{2}\right)(5.00) = 50.0 \text{ m}$$

The total distance $x = x_1 + x_2 + x_3 = 100 + 400 + 50 = 550$ m, and the average velocity is given by $\bar{v} = \frac{x}{t} = \frac{550}{35.0} = \boxed{15.7 \text{ m/s}}$.

P2.34 We ask whether the constant acceleration of the rhinoceros from rest over a period of 10.0 s can result in a final velocity of 8.00 m/s and a displacement of 50.0 m? To check, we solve for the acceleration in two ways.

1) $t_i = 0$, $v_i = 0$; $t = 10.0$ s, $v_f = 8.00$ m/s:

$$v_f = v_i + at \rightarrow a = \frac{v_f}{t}$$

$$a = \frac{8.00 \text{ m/s}}{10.0 \text{ s}} = 0.800 \text{ m/s}^2$$

2) $t_i = 0$, $x_i = 0$, $v_i = 0$; $t = 10.0$ s, $x_f = 50.0$ m:

$$x_f = x_i + v_i t + \frac{1}{2}at^2 \rightarrow x_f = \frac{1}{2}at^2$$

$$a = \frac{2x_f}{t^2} = \frac{2(50.0 \text{ m})}{(10.0 \text{ s})^2} = 1.00 \text{ m/s}^2$$

$\boxed{\text{The accelerations do not match, therefore the situation is impossible.}}$

P2.35 Since we don't know the initial and final velocities of the car, we will need to use two equations simultaneously to find the speed with which the car strikes the tree. From Equation 2.13, we have

$$v_{xf} = v_{xi} + a_x t = v_{xi} + (-5.60 \text{ m/s}^2)(4.20 \text{ s})$$

$$v_{xi} = v_{xf} + (5.60 \text{ m/s}^2)(4.20 \text{ s}) \qquad \textbf{[1]}$$

and from Equation 2.15,

$$x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

$$62.4\text{ m} = \frac{1}{2}\left(v_{xi} + v_{xf}\right)(4.20\text{ s}) \qquad \textbf{[2]}$$

Substituting for v_{xi} in [2] from [1] gives

$$62.4\text{ m} = \frac{1}{2}\left[v_{xf} + \left(5.60\text{ m/s}^2\right)(4.20\text{ s}) + v_{xf}\right](4.20\text{ s})$$

$$14.9\text{ m/s} = v_{xf} + \frac{1}{2}\left(5.60\text{ m/s}^2\right)(4.20\text{ s})$$

Thus, $v_{xf} = \boxed{3.10\text{ m/s}}$

P2.36 (a) Take any two of the standard four equations, such as

$$v_{xf} = v_{xi} + a_x t$$

$$x_f - x_i = \frac{1}{2}\left(v_{xi} + v_{xf}\right)t$$

Solve one for v_{xi}, and substitute into the other:

$$v_{xi} = v_{xf} - a_x t$$

$$x_f - x_i = \frac{1}{2}\left(v_{xf} - a_x t + v_{xf}\right)t$$

Thus

$$\boxed{x_f - x_i = v_{xf}t - \frac{1}{2}a_x t^2}$$

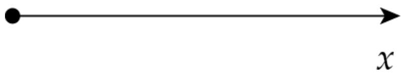
We note that the equation is dimensionally correct. The units are units of length in each term. Like the standard equation $x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$, this equation represents that displacement is a quadratic function of time.

(b) Our newly derived equation gives us for the situation back in problem 35,

$$62.4\text{ m} = v_{xf}(4.20\text{ s}) - \frac{1}{2}\left(-5.60\text{ m/s}^2\right)(4.20\text{ s})^2$$

$$v_{xf} = \frac{62.4\text{ m} - 49.4\text{ m}}{4.20\text{ s}} = \boxed{3.10\text{ m/s}}$$

P2.37 (a) We choose a coordinate system with the x axis positive to the right, in the direction of motion of the speedboat, as shown on the right.



ANS. FIG. P2.37

(b) Since the speedboat is increasing its speed, the particle under constant acceleration model should be used here.

(c) Since the initial and final velocities are given along with the displacement of the speedboat, we use

$$v_{xf}^2 = v_{xi}^2 + 2a\Delta x$$

(d) Solving for the acceleration of the speedboat gives

$$a = \frac{v_{xf}^2 - v_{xi}^2}{2\Delta x}$$

(e) We have $v_i = 20.0\text{ m/s}$, $v_f = 30.0\text{ m/s}$, and $x_f - x_i = \Delta x = 200\text{ m}$:

$$a = \frac{v_{xf}^2 - v_{xi}^2}{2\Delta x} = \frac{(30.0\text{ m/s})^2 - (20.0\text{ m/s})^2}{2(200\text{ m})} = \boxed{1.25\text{ m/s}^2}$$

(f) To find the time interval, we use $v_f = v_i + at$, which gives

$$t = \frac{v_f - v_i}{a} = \frac{30.0\text{ m/s} - 20.0\text{ m/s}}{1.25\text{ m/s}^2} = \boxed{8.00\text{ s}}$$

P2.38 (a) Compare the position equation $x = 2.00 + 3.00t - 4.00t^2$ to the general form

$$x_f = x_i + v_i t + \frac{1}{2}at^2$$

to recognize that $x_i = 2.00\text{ m}$, $v_i = 3.00\text{ m/s}$, and $a = -8.00\text{ m/s}^2$. The velocity equation, $v_f = v_i + at$, is then

$$v_f = 3.00\text{ m/s} - (8.00\text{ m/s}^2)t$$

The particle changes direction when $v_f = 0$, which occurs at $t = \frac{3}{8}\text{ s}$. The position at this time is

$$x = 2.00\text{ m} + (3.00\text{ m/s})\left(\frac{3}{8}\text{ s}\right) - (4.00\text{ m/s}^2)\left(\frac{3}{8}\text{ s}\right)^2$$

$$= \boxed{2.56\text{ m}}$$

(b) From $x_f = x_i + v_i t + \frac{1}{2}at^2$, observe that when $x_f = x_i$, the time is given by $t = -\frac{2v_i}{a}$. Thus, when the particle returns to its initial position, the time is

$$t = \frac{-2(3.00\text{ m/s})}{-8.00\text{ m/s}^2} = \frac{3}{4}\text{ s}$$

and the velocity is

$$v_f = 3.00\text{ m/s} - (8.00\text{ m/s}^2)\left(\frac{3}{4}\text{ s}\right) = \boxed{-3.00\text{ m/s}}$$

P2.39 Let the glider enter the photogate with velocity v_i and move with constant acceleration a. For its motion from entry to exit,

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$\ell = 0 + v_i\Delta t_d + \frac{1}{2}a\Delta t_d^2 = v_d\Delta t_d$$

$$v_d = v_i + \frac{1}{2}a\Delta t_d$$

(a) The speed halfway through the photogate in space is given by

$$v_{hs}^2 = v_i^2 + 2a\left(\frac{\ell}{2}\right) = v_i^2 + av_d\Delta t_d$$

$v_{hs} = \sqrt{v_i^2 + av_d\Delta t_d}$ and this is $\boxed{\text{not equal to } v_d \text{ unless } a = 0}$.

(b) The speed halfway through the photogate in time is given by

$v_{ht} = v_i + a\left(\frac{\Delta t_d}{2}\right)$ and this is $\boxed{\text{equal to } v_d}$ as determined above.

P2.40 (a) Let a stopwatch start from $t = 0$ as the front end of the glider passes point A. The average speed of the glider over the interval between $t = 0$ and $t = 0.628$ s is 12.4 cm/(0.628 s) = $\boxed{19.7\text{ cm/s}}$, and this is the instantaneous speed halfway through the time interval, at $t = 0.314$ s.

(b) The average speed of the glider over the time interval between 0.628 + 1.39 = 2.02 s and 0.628 + 1.39 + 0.431 = 2.45 s is 12.4 cm/(0.431 s) = 28.8 cm/s and this is the instantaneous speed at the instant $t = (2.02 + 2.45)/2 = 2.23$ s.

Now we know the velocities at two instants, so the acceleration is found from

$$[(28.8 - 19.7)\text{ cm/s}]/[(2.23 - 0.314)\text{ s}] = \boxed{4.70\text{ cm/s}^2}$$

(c) $\boxed{\text{The distance between A and B is not used, but the length of the glider is used to find the average velocity during a known time interval.}}$

P2.41 (a) What we know about the motion of an object is as follows: $a = 4.00\ \text{m/s}^2$, $v_i = 6.00\ \text{m/s}$, and $v_f = 12.0\ \text{m/s}$.

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = v_i^2 + 2a\Delta x$$

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[\left(12.0\ \text{m/s}\right)^2 - \left(6.00\ \text{m/s}\right)^2\right]}{2\left(4.00\ \text{m/s}^2\right)} = \boxed{13.5\ \text{m}}$$

(b) From (a), the acceleration and velocity of the object are in the same (positive) direction, so the object speeds up. The distance is $\boxed{13.5\ \text{m}}$ because the object always travels in the same direction.

(c) Given $a = 4.00\ \text{m/s}^2$, $v_i = -6.00\ \text{m/s}$, and $v_f = 12.0\ \text{m/s}$. Following steps similar to those in (a) above, we will find the displacement to be the same: $\boxed{\Delta x = 13.5\ \text{m.}}$ In this case, the object initially is moving in the negative direction but its acceleration is in the positive direction, so the object slows down, reverses direction, and then speeds up as it travels in the positive direction.

(d) We consider the motion in two parts.

(1) Calculate the displacement of the object as it slows down: $a = 4.00\ \text{m/s}^2$, $v_i = -6.00\ \text{m/s}$, and $v_f = 0\ \text{m/s}$.

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[\left(0\ \text{m/s}\right)^2 - \left(-6.00\ \text{m/s}\right)^2\right]}{2\left(4.00\ \text{m/s}^2\right)} = -4.50\ \text{m}$$

The object travels 4.50 m in the negative direction.

(2) Calculate the displacement of the object after it has reversed direction: $a = 4.00\ \text{m/s}^2$, $v_i = 0\ \text{m/s}$, $v_f = 12.0\ \text{m/s}$.

$$\Delta x = \frac{\left(v_f^2 - v_i^2\right)}{2a}$$

$$\Delta x = \frac{\left[(12.0\text{ m/s})^2 - (0\text{ m/s})^2\right]}{2\left(4.00\text{ m/s}^2\right)} = 18.0\text{ m}$$

The object travels 18.0 m in the positive direction.

Total distance traveled: 4.5 m + 18.0 m = $\boxed{22.5\text{ m.}}$

P2.42 (a) For the first car, the speed as a function of time is

$$v_1 = v_{1i} + a_1 t = -3.50\text{ cm/s} + \left(2.40\text{ cm/s}^2\right)t$$

For the second car, the speed is

$$v_2 = v_{2i} + a_2 t = +5.5\text{ cm/s} + 0$$

Setting the two expressions equal gives

$$-3.50\text{ cm/s} + \left(2.40\text{ cm/s}^2\right)t = 5.5\text{ cm/s}$$

Solving for *t* gives

$$t = \frac{9.00\text{ cm/s}}{2.40\text{ cm/s}^2} = \boxed{3.75\text{ s}}$$

(b) The first car then has speed

$$v_1 = v_{1i} + a_1 t = -3.50\text{ cm/s} + \left(2.40\text{ cm/s}^2\right)(3.75\text{ s}) = \boxed{5.50\text{ cm/s}}$$

and this is also the constant speed of the second car.

(c) For the first car, the position as a function of time is

$$\begin{aligned} x_1 &= x_{1i} + v_{1i}t + \frac{1}{2}a_1 t^2 \\ &= 15.0\text{ cm} - (3.50\text{ cm/s})t + \frac{1}{2}\left(2.40\text{ cm/s}^2\right)t^2 \end{aligned}$$

For the second car, the position is

$$x_2 = 10.0\text{ cm} + (5.50\text{ cm/s})t$$

At the point where the cars pass one another, their positions are equal:

$$15.0\text{ cm} - (3.50\text{ cm/s})t + \frac{1}{2}\left(2.40\text{ cm/s}^2\right)t^2 = 10.0\text{ cm} + (5.50\text{ cm/s})t$$

rearranging gives

$$\left(1.20\text{ cm/s}^2\right)t^2 - (9.00\text{ cm/s})t + 5.00\text{ cm} = 0$$

We solve this with the quadratic formula. Suppressing units,

$$t = \frac{9 \pm \sqrt{(9)^2 - 4(1.2)(5)}}{2(1.2)} = \frac{9 \pm \sqrt{57}}{2.4} = 6.90 \text{ s, or } \boxed{0.604 \text{ s}}$$

(d) At $t = 0.604$ s, the second and also the first car’s position is

$$x_{1,2} = 10.0 \text{ cm} + (5.50 \text{ cm/s})(0.604 \text{ s}) = \boxed{13.3 \text{ cm}}$$

At $t = 6.90$ s, both are at position

$$x_{1,2} = 10.0 \text{ cm} + (5.50 \text{ cm/s})(6.90 \text{ s}) = \boxed{47.9 \text{ cm}}$$

(e) The cars are initially moving toward each other, so they soon arrive at the same position x when their speeds are quite different, giving one answer to (c) that is not an answer to (a). The first car slows down in its motion to the left, turns around, and starts to move toward the right, slowly at first and gaining speed steadily. At a particular moment its speed will be equal to the constant rightward speed of the second car, but at this time the accelerating car is far behind the steadily moving car; thus, the answer to (a) is not an answer to (c). Eventually the accelerating car will catch up to the steadily-coasting car, but passing it at higher speed, and giving another answer to (c) that is not an answer to (a).

P2.43 (a) Total displacement = area under the (v, t) curve from $t = 0$ to 50 s. Here, distance is the same as displacement because the motion is in one direction.

$$\Delta x = \frac{1}{2}(50 \text{ m/s})(15 \text{ s}) + (50 \text{ m/s})(40 - 15) \text{ s} + \frac{1}{2}(50 \text{ m/s})(10 \text{ s})$$

$$\Delta x = 1875 \text{ m} = \boxed{1.88 \text{ km}}$$

(b) From $t = 10$ s to $t = 40$ s, displacement is

$$\Delta x = \frac{1}{2}(50 \text{ m/s} + 33 \text{ m/s})(5 \text{ s}) + (50 \text{ m/s})(25 \text{ s}) = \boxed{1.46 \text{ km}}$$

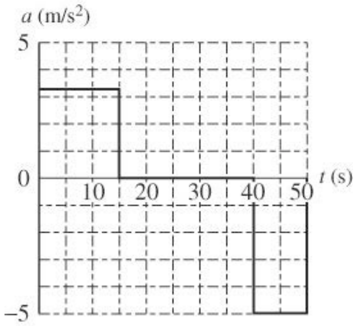
(c) We compute the acceleration for each of the three segments of the car's motion:

$$0 \le t \le 15 \text{ s:} \qquad a_1 = \frac{\Delta v}{\Delta t} = \frac{(50-0)\text{ m/s}}{15\text{ s}-0} = \boxed{3.3\text{ m/s}^2}$$

$$15\text{ s} < t < 40\text{ s:} \qquad \boxed{a_2 = 0}$$

$$40\text{ s} \le t \le 50\text{ s:} \qquad a_3 = \frac{\Delta v}{\Delta t} = \frac{(0-50)\text{ m/s}}{50\text{ s}-40\text{ s}} = \boxed{-5.0\text{ m/s}^2}$$

ANS. FIG. P2.43 shows the graph of the acceleration during this interval.



ANS FIG. P2.43

(d) For segment *0a*,

$$x_1 = 0 + \frac{1}{2}a_1t^2 = \frac{1}{2}\left(3.3\text{ m/s}^2\right)t^2 \text{ or } \boxed{x_1 = \left(1.67\text{ m/s}^2\right)t^2}$$

For segment *ab*,

$$x_2 = \frac{1}{2}(15\text{s})[50\text{ m/s}-0] + (50\text{ m/s})(t-15\text{ s})$$

or $\boxed{x_2 = (50\text{ m/s})t - 375\text{ m}}$

For segment *bc*,

$$x_3 = \begin{pmatrix}\text{area under } v \text{ vs. } t\\ \text{from } t = 0 \text{ to } 40\text{ s}\end{pmatrix} + \frac{1}{2}a_3(t-40\text{ s})^2 + (50\text{ m/s})(t-40\text{ s})$$

or

$$x_3 = 375\text{ m} + 1250\text{ m} + \frac{1}{2}(-5.0\text{ m/s}^2)(t-40\text{ s})^2 + (50\text{ m/s})(t-40\text{ s})$$

which reduces to

$$\boxed{x_3 = (250\text{ m/s})t - \left(2.5\text{ m/s}^2\right)t^2 - 4\,375\text{ m}}$$

(e) $\bar{v} = \frac{\text{total displacement}}{\text{total elapsed time}} = \frac{1\ 875\ \text{m}}{50\ \text{s}} = \boxed{37.5\ \text{m/s}}$

2.44 (a) Take $t = 0$ at the time when the player starts to chase his opponent. At this time, the opponent is a distance $d = (12.0\ \text{m/s})(3.00\ \text{s}) = 36.0\ \text{m}$ in front of the player. At time $t > 0$, the displacements of the players from their initial positions are

$$\Delta x_{\text{player}} = v_{i,\text{player}}t + \frac{1}{2}a_{\text{player}}t^2 = 0 + \frac{1}{2}(4.00\ \text{m/s}^2)t^2 \qquad \textbf{[1]}$$

and

$$\Delta x_{\text{opponent}} = v_{i,\text{opponent}}t + \frac{1}{2}a_{\text{opponent}}t^2 = (12.0\ \text{m/s})t + 0 \qquad \textbf{[2]}$$

When the players are side-by-side, $\Delta x_{\text{player}} = \Delta x_{\text{opponent}} + 36.0\ \text{m}$. **[3]**

Substituting equations [1] and [2] into equation [3] gives

$$\frac{1}{2}(4.00\ \text{m/s}^2)t^2 = (12.0\ \text{m/s})t + 36.0\ \text{m}$$

or $\quad t^2 + (-6.00\ \text{s})t + (-18.0\ \text{s}^2) = 0$

Applying the quadratic formula to this equation gives

$$t = \frac{-(-6.00\ \text{s}) \pm \sqrt{(-6.00\ \text{s})^2 - 4(1)(-18.0\ \text{s}^2)}}{2(1)}$$

which has solutions of $t = -2.20$ s and $t = +8.20$ s. Since the time must be greater than zero, we must choose $t = \boxed{8.20\ \text{s}}$ as the proper answer.

(b) $\Delta x_{\text{player}} = v_{i,\text{player}}t + \frac{1}{2}a_{\text{player}}t^2 = 0 + \frac{1}{2}(4.00\ \text{m/s}^2)(8.20\ \text{s})^2 = \boxed{134\ \text{m}}$

Section 2.7 Freely Falling Objects

P2.45 This is motion with constant acceleration, in this case the acceleration of gravity. The equation of position as a function of time is

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$

Taking the positve y direction as up, the acceleration is $a = (9.80\ \text{m/s}^2$, downward) $= -g$; we also know that $y_i = 0$ and $v_i = 2.80$ m/s. The above equation becomes

$$y_f = v_i t - \frac{1}{2}gt^2$$

$$y_f = (2.80\text{ m/s})t - \frac{1}{2}(9.80\text{ m/s}^2)t^2$$

(a) At $t = 0.100$ s, $y_f = \boxed{0.231\text{ m}}$

(b) At $t = 0.200$ s, $y_f = \boxed{0.364\text{ m}}$

(c) At $t = 0.300$ s, $y_f = \boxed{0.399\text{ m}}$

(d) At $t = 0.500$ s, $y_f = \boxed{0.175\text{ m}}$

P2.46 We can solve (a) and (b) at the same time by assuming the rock passes the top of the wall and finding its speed there. If the speed comes out imaginary, the rock will not reach this elevation.

$$\begin{aligned} v_f^2 &= v_i^2 + 2a(y_f - y_i) \\ &= (7.40\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(3.65\text{ m} - 1.55\text{ m}) \\ &= 13.6\text{ m}^2/\text{s}^2 \end{aligned}$$

which gives $v_f = 3.69$ m/s.

$\boxed{\text{So the rock does reach the top of the wall with } v_f = 3.69\text{ m/s}}$.

(c) The rock travels from $y_i = 3.65$ m to $y_f = 1.55$ m. We find the final speed of the rock thrown down:

$$\begin{aligned} v_f^2 &= v_i^2 + 2a(y_f - y_i) \\ &= (-7.40\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(1.55\text{m} - 3.65\text{ m}) \\ &= 95.9\text{ m}^2/\text{s}^2 \end{aligned}$$

which gives $v_f = -9.79$ m/s.

The change in speed of the rock thrown down is

$$|9.79\text{ m/s} - 7.40\text{ m/s}| = \boxed{2.39\text{ m/s}}$$

(d) The magnitude of the speed change of the rock thrown up is $|7.40\text{ m/s} - 3.69\text{ m/s}| = 3.71$ m/s. This $\boxed{\text{does not agree}}$ with 2.39 m/s.

(e) $\boxed{\text{The upward-moving rock spends more time in flight because its average speed is smaller than the downward-moving rock, so the rock has more time to change its speed.}}$

P2.47 The bill starts from rest, $v_i = 0$, and falls with a downward acceleration of $9.80\ \text{m/s}^2$ (due to gravity). For an average human reaction time of about 0.20 s, we can find the distance the bill will fall:

$$y_f = y_i + v_i t + \frac{1}{2}at^2 \rightarrow \Delta y = v_i t - \frac{1}{2}gt^2$$

$$\Delta y = 0 - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)(0.20\ \text{s})^2 = -0.20\ \text{m}$$

The bill falls about 20 cm—this distance is about twice the distance between the center of the bill and its top edge, about 8 cm. Thus $\boxed{\text{David could not respond fast enough to catch the bill.}}$

P2.48 Since the ball's motion is entirely vertical, we can use the equations for free fall to find the initial velocity and maximum height from the elapsed time. After leaving the bat, the ball is in free fall for $t = 3.00$ s and has constant acceleration $a_y = -g = -9.80\ \text{m/s}^2$.

(a) The initial speed of the ball can be found from

$$v_f = v_i + at$$

$$0 = v_i - gt \rightarrow v_i = gt$$

$$v_i = \left(9.80\ \text{m/s}^2\right)\left(3.00\ \text{s}\right) = \boxed{29.4\ \text{m/s}}$$

(b) Find the vertical displacement Δy:

$$\Delta y = y_f - y_i = \frac{1}{2}\left(v_i + v_f\right)t$$

$$\Delta y = \frac{1}{2}\left(29.4\ \text{m/s} + 0\right)\left(3.00\ \text{s}\right)$$

$$\Delta y = \boxed{44.1\ \text{m}}$$

***P2.49** (a) Consider the upward flight of the arrow.

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = \left(100\ \text{m/s}\right)^2 + 2\left(-9.80\ \text{m/s}^2\right)\Delta y$$

$$\Delta y = \frac{10\,000\ \text{m}^2/\text{s}^2}{19.6\ \text{m/s}^2} = \boxed{510\ \text{m}}$$

(b) Consider the whole flight of the arrow.

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = 0 + (100\ \text{m/s})t + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2$$

The root $t = 0$ refers to the starting point. The time of flight is given by

$$t = \frac{100\ \text{m/s}}{4.90\ \text{m/s}^2} = \boxed{20.4\ \text{s}}$$

P2.50 We are given the height of the helicopter: $y = h = 3.00t^3$.

At $t = 2.00$ s, $y = 3.00(2.00\ \text{s})^3 = 24.0$ m and

$$v_y = \frac{dy}{dt} = 9.00t^2 = 36.0\ \text{m/s}\uparrow$$

If the helicopter releases a small mailbag at this time, the mailbag starts its free fall with velocity 36.0 m/s upward. The equation of motion of the mailbag is

$$y_f = y_i + v_i t + \frac{1}{2}at^2$$

$$y_f = (24.0\ \text{m}) + (36.0\ \text{m/s})t - \left(4.90\ \text{m/s}^2\right)t^2$$

Setting $y_f = 0$, dropping units, and rearranging the equation, we have

$$4.90t^2 - 36.0t - 24.0 = 0$$

We solve for t using the quadratic formula:

$$t = \frac{36.0 \pm \sqrt{(-36.0)^2 - 4(4.90)(-24.0)}}{2(4.90)}$$

Since only positive values of t count, we find $t = \boxed{7.96\ \text{s}}$.

P2.51 The equation for the height of the ball as a function of time is

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

$$0 = 30\ \text{m} + (-8.00\ \text{m/s})t - \left(4.90\ \text{m/s}^2\right)t^2$$

Solving for *t*,

$$t = \frac{+8.00 \pm \sqrt{(-8.00)^2 - 4(-4.90)(30)}}{2(-4.90)} = \frac{+8.00 \pm \sqrt{64 + 588}}{-9.80}$$

$$t = \boxed{1.79\ \text{s}}$$

***P2.52** The falling ball moves a distance of (15 m – *h*) before they meet, where *h* is the height above the ground where they meet. We apply

$$y_f = y_i + v_i t - \frac{1}{2}gt^2$$

to the falling ball to obtain

$$-(15.0\ \text{m} - h) = -\frac{1}{2}gt^2$$

or $$h = 15.0\ \text{m} - \frac{1}{2}gt^2 \qquad \textbf{[1]}$$

Applying $y_f = y_i + v_i t - \frac{1}{2}gt^2$ to the rising ball gives

$$h = (25\ \text{m/s})t - \frac{1}{2}gt^2 \qquad \textbf{[2]}$$

Combining equations [1] and [2] gives

$$(25\ \text{m/s})t - \frac{1}{2}gt^2 = 15.0\ \text{m} - \frac{1}{2}gt^2$$

or $$t = \frac{15\ \text{m}}{25\ \text{m/s}} = \boxed{0.60\ \text{s}}$$

P2.53 We model the keys as a particle under the constant free-fall acceleration. Take the first student's position to be $y_i = 0$ and the second student's position to be $y_f = 4.00\ \text{m}$. We are given that the time of flight of the keys is *t* = 1.50 s, and $a_y = -9.80\ \text{m/s}^2$.

(a) We choose the equation $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$ to connect the data and the unknown.

We solve:

$$v_{yi} = \frac{y_f - y_i - \frac{1}{2}a_y t^2}{t}$$

and substitute:

$$v_{yi} = \frac{4.00\ \text{m} - \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)(1.50\ \text{s})^2}{1.50\ \text{s}} = \boxed{10.0\ \text{m/s}}$$

(b) The velocity at any time $t > 0$ is given by $v_{yf} = v_{yi} + a_y t$.

Therefore, at $t = 1.50$ s,

$$v_{yf} = 10.0\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)\left(1.50\ \text{s}\right) = \boxed{-4.68\ \text{m/s}}$$

The negative sign means that the keys are moving **downward** just before they are caught.

P2.54 (a) The keys, moving freely under the influence of gravity ($a = -g$), undergo a vertical displacement of $\Delta y = +h$ in time t. We use $\Delta y = v_i t + \frac{1}{2}at^2$ to find the initial velocity as

$$\Delta y = v_i t + \frac{1}{2}at^2 = h$$

$$\rightarrow h = v_i t - \frac{1}{2}gt^2$$

$$v_i = \frac{h + \frac{1}{2}gt^2}{t} = \boxed{\frac{h}{t} + \frac{gt}{2}}$$

(b) We find the velocity of the keys just before they were caught (at time t) using $v = v_i + at$:

$$v = v_i + at$$

$$v = \left(\frac{h}{t} + \frac{gt}{2}\right) - gt$$

$$v = \boxed{\frac{h}{t} - \frac{gt}{2}}$$

P2.55 Both horse and man have constant accelerations: they are g downward for the man and 0 for the horse. We choose to do part (b) first.

(b) Consider the vertical motion of the man after leaving the limb (with $v_i = 0$ at $y_i = 3.00$ m) until reaching the saddle (at $y_f = 0$).

Modeling the man as a particle under constant acceleration, we find his time of fall from $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$.

When $v_i = 0$,

$$t = \sqrt{\frac{2\left(y_f - y_i\right)}{a_y}} = \sqrt{\frac{2(0 - 3.00\ \text{m})}{-9.80\ \text{m/s}^2}} = \boxed{0.782\ \text{s}}$$

(a) During this time interval, the horse is modeled as a particle under constant velocity in the horizontal direction.

$$v_{xi} = v_{xf} = 10.0 \text{ m/s}$$

$$x_f - x_i = v_{xi}t = (10.0 \text{ m/s})(0.782 \text{ s}) = \boxed{7.82 \text{ m}}$$

and the ranch hand must let go when the horse is 7.82 m from the tree.

P2.56 (a) Let $t = 0$ be the instant the package leaves the helicopter. The package and the helicopter have a common initial velocity of $-v_i$ (choosing upward as positive). The helicopter has zero acceleration, and the package (in free-fall) has constant acceleration $a_y = -g$.

At times $t > 0$, the velocity of the package is

$$v_p = v_{yi} + a_y t \rightarrow v_p = -v_i - gt = -(v_i + gt)$$

so its speed is $|v_p| = \boxed{v_i + gt}$.

(b) Assume the helicopter is at height H when the package is released. Setting our clock to $t = 0$ at the moment the package is released, the position of the helicopter is

$$y_{hel} = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_{hel} = H + (-v_i)t$$

and the position of the package is

$$y_p = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$y_p = H + (-v_i)t - \frac{1}{2}gt^2$$

The vertical distance, d, between the helicopter and the package is

$$y_{hel} - y_p = [H + (-v_i)t] - \left[H + (-v_i)t - \frac{1}{2}gt^2\right]$$
$$d = \boxed{\frac{1}{2}gt^2}$$

The distance is independent of their common initial speed.

(c) Now, the package and the helicopter have a common initial velocity of $+v_i$ (choosing upward as positive). The helicopter has zero acceleration, and the package (in free-fall) has constant acceleration $a_y = -g$.

At times $t > 0$, the velocity of the package is

$$v_P = v_{yi} + a_y t \rightarrow v_p = +v_i - gt$$

Therefore, the speed of the package at time t is $v_p = \boxed{|v_i - gt|}$.

The position of the helicopter is

$$y_{hel} = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_{hel} = H + (+v_i)t$$

and the position of the package is

$$y_p = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_p = H + (+v_i)t - \frac{1}{2}gt^2$$

The vertical distance, d, between the helicopter and the package is

$$y_{hel} - y_p = [H + (+v_i)t] - \left[H + (+v_i)t - \frac{1}{2}gt^2\right]$$

$$d = \boxed{\frac{1}{2}gt^2}$$

As above, the distance is independent of their common initial speed.

Section 2.8 Kinematic Equations Derived from Calculus

P2.57 This is a derivation problem. We start from basic definitions. We are given $J = da_x/dt$ = constant, so we know that $da_x = Jdt$.

(a) Integrating from the 'initial' moment when we know the acceleration to any later moment,

$$\int_{a_{ix}}^{a_x} da = \int_0^t J\,dt \quad \rightarrow \quad a_x - a_{ix} = J(t - 0)$$

Therefore, $\boxed{a_x = Jt + a_{xi}}$.

From $a_x = dv_x/dt$, $dv_x = a_x\,dt$.

Integration between the same two points tells us the velocity as a function of time:

$$\int_{v_{xi}}^{v_x} dv_x = \int_0^t a_x\,dt = \int_0^t (a_{xi} + Jt)\,dt$$

$$v_x - v_{xi} = a_{xi}t + \frac{1}{2}Jt^2 \quad \text{or} \quad \boxed{v_x = v_{xi} + a_{xi}t + \frac{1}{2}Jt^2}$$

From $v_x = dx/dt$, $dx = v_x dt$. Integrating a third time gives us $x(t)$:

$$\int_{x_i}^{x} dx = \int_0^t v_x\,dt = \int_0^t \left(v_{xi} + a_{xi}t + \frac{1}{2}Jt^2\right)\,dt$$

$$x - x_i = v_{xi}t + \frac{1}{2}a_{xi}t^2 + \frac{1}{6}Jt^3$$

and $\boxed{x = \frac{1}{6}Jt^3 + \frac{1}{2}a_{xi}t^2 + v_{xi}t + x_i}$.

(b) Squaring the acceleration,

$$a_x^2 = (Jt + a_{xi})^2 = J^2t^2 + a_{xi}^2 + 2Ja_{xi}t$$

Rearranging,

$$a_x^2 = a_{xi}^2 + 2J\left(\frac{1}{2}Jt^2 + a_{xi}t\right)$$

The expression for v_x was

$$v_x = \frac{1}{2}Jt^2 + a_{xi}t + v_{xi}$$

So $(v_x - v_{xi}) = \frac{1}{2}Jt^2 + a_{xi}t$

and by substitution

$$\boxed{a_x^2 = a_{xi}^2 + 2J(v_x - v_{xi})}$$

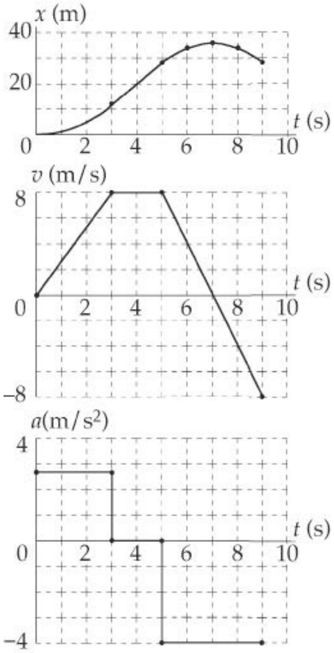
P2.58 (a) See the x vs. t graph on the top panel of ANS. FIG. P2.58, on the next page. Choose $x = 0$ at $t = 0$.

At $t = 3$ s, $x = \frac{1}{2}(8\text{ m/s})(3\text{ s}) = 12$ m.

At $t = 5$ s, $x = 12\text{ m} + (8\text{ m/s})(2\text{ s}) = 28$ m.

At $t = 7$ s, $x = 28\text{ m} + \frac{1}{2}(8\text{ m/s})(2\text{ s})$

$= 36$ m



ANS. FIG. P2.58

(b) See the *a* vs. *t* graph at the bottom right.

$$\text{For } 0 < t < 3\text{ s, } a = \frac{8\text{ m/s}}{3\text{ s}} = 2.67\text{ m/s}^2.$$

For $3 < t < 5$ s, $a = 0$.

At the points of inflection, $t = 3$ and 5 s, the slope of the velocity curve changes abruptly, so the acceleration is not defined.

(c) For $5\text{ s} < t < 9\text{ s}$,

$$a = -\frac{16\text{ m/s}}{4\text{ s}} = \boxed{-4\text{ m/s}^2}$$

(d) The average velocity between $t = 5$ and 7 s is

$$v_{\text{avg}} = (8\text{ m/s} + 0)/2 = 4\text{ m/s}$$

$$\text{At } t = 6\text{ s, } x = 28\text{ m} + (4\text{ m/s})(1\text{ s}) = \boxed{32\text{ m}}$$

(e) The average velocity between $t = 5$ and 9 s is

$$v_{\text{avg}} = [(8\text{ m/s}) + (-8\text{ m/s})]/2 = 0\text{ m/s}$$

$$\text{At } t = 9\text{ s, } x = 28\text{ m} + (0\text{ m/s})(1\text{ s}) = \boxed{28\text{ m}}$$

P2.59 (a) To find the acceleration, we differentiate the velocity equation with respect to time:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left[\left(-5.00\times10^7\right)t^2 + \left(3.00\times10^5\right)t\right]$$

$$\boxed{a = -\left(10.0\times10^7\right)t + 3.00\times10^5}$$

where *a* is in m/s² and *t* is in seconds.

To find the position, take $x_i = 0$ at $t = 0$. Then, from $v = \frac{dx}{dt}$,

$$x - 0 = \int_0^t v\,dt = \int_0^t \left(-5.00\times10^7 t^2 + 3.00\times10^5 t\right)dt$$

$$x = -5.00\times10^7\frac{t^3}{3} + 3.00\times10^5\frac{t^2}{2}$$

which gives

$$\boxed{x = -\left(1.67\times10^7\right)t^3 + \left(1.50\times10^5\right)t^2}$$

where x is in meters and t is in seconds.

(b) The bullet escapes when $a = 0$:

$$a = -\left(10.0\times10^{7}\right)t + 3.00\times10^{5} = 0$$

$$t = \frac{3.00\times10^{5}\text{ s}}{10.0\times10^{7}} = 3.00\times10^{-3}\text{ s} = \boxed{3.00\text{ ms}}$$

(c) Evaluate v when $t = 3.00\times10^{-3}$ s:

$$v = \left(-5.00\times10^{7}\right)\left(3.00\times10^{-3}\right)^{2} + \left(3.00\times10^{5}\right)\left(3.00\times10^{-3}\right)$$

$$v = -450 + 900 = \boxed{450\text{ m/s}}$$

(d) Evaluate x when $t = 3.00\times10^{-3}$ s:

$$x = -\left(1.67\times10^{7}\right)\left(3.00\times10^{-3}\right)^{3} + \left(1.50\times10^{5}\right)\left(3.00\times10^{-3}\right)^{2}$$

$$x = -0.450 + 1.35 = \boxed{0.900\text{ m}}$$

Additional Problems

***P2.60** (a) Assuming a constant acceleration:

$$a = \frac{v_f - v_i}{t} = \frac{42.0\text{ m/s}}{8.00\text{ s}} = \boxed{5.25\text{ m/s}^2}$$

(b) Taking the origin at the original position of the car,

$$x_f = \frac{1}{2}\left(v_i + v_f\right)t = \frac{1}{2}(42.0\text{ m/s})(8.00\text{ s}) = \boxed{168\text{ m}}$$

(c) From $v_f = v_i + at$, the velocity 10.0 s after the car starts from rest is:

$$v_f = 0 + \left(5.25\text{ m/s}^2\right)(10.0\text{ s}) = \boxed{52.5\text{ m/s}}$$

P2.61 (a) From $v^2 = v_i^2 + 2a\Delta y$, the insect's velocity after straightening its legs is

$$v = \sqrt{v_0^2 + 2a\left(\Delta y\right)}$$

$$= \sqrt{0 + 2\left(4\ 000\text{ m/s}^2\right)\left(2.00\times10^{-3}\text{ m}\right)} = \boxed{4.00\text{ m/s}}$$

(b) The time to reach this velocity is

$$t = \frac{v - v_0}{a} = \frac{4.00\text{ m/s} - 0}{4\ 000\text{ m/s}^2} = 1.00\times10^{-3}\text{ s} = \boxed{1.00\text{ ms}}$$

(c) The upward displacement of the insect between when its feet leave the ground and its speed is momentarily zero is

$$\Delta y = \frac{v_f^2 - v_i^2}{2a}$$

$$\Delta y = \frac{0-(4.00\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = \boxed{0.816\text{ m}}$$

P2.62 (a) The velocity is constant between $t_i = 0$ and $t = 4$ s. Its acceleration is $\boxed{0}$.

(b) $a = (v_9 - v_4)/(9\text{ s} - 4\text{ s}) = (18 - [-12])\text{ (m/s)}/5\text{ s} = \boxed{6.0\text{ m/s}^2}$

(c) $a = (v_{18} - v_{13})/(18\text{ s} - 13\text{ s}) = (0 - 18)\text{ (m/s)}/5\text{ s} = \boxed{-3.6\text{ m/s}^2}$

(d) We read from the graph that the speed is zero $\boxed{\text{at t = 6 s and at 18s}}$.

(e) and (f) The object moves away from $x = 0$ into negative coordinates from $t = 0$ to $t = 6$ s, but then comes back again, crosses the origin and moves farther into positive coordinates until $\boxed{t = 18\text{ s}}$, then attaining its maximum distance, which is the cumulative distance under the graph line:

$$\Delta x = (-12\text{ m/s})(4\text{ s}) + \frac{1}{2}(-12\text{ m/s})(2\text{ s}) + \frac{1}{2}(18\text{ m/s})(3\text{ s})$$
$$+(18\text{ m/s})(4\text{ s})\frac{1}{2}(18\text{ m/s})(5\text{ s})$$
$$= \boxed{84\text{ m}}$$

(g) We consider the total distance, rather than the resultant displacement, by counting the contributions computed in part (f) as all positive:

$$d = +60\text{ m} + 144\text{ m} = \boxed{204\text{ m}}$$

P2.63 We set $y_i = 0$ at the top of the cliff, and find the time interval required for the first stone to reach the water using the particle under constant acceleration model:

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

or in quadratic form,

$$-\frac{1}{2}a_y t^2 - v_{yi}t + y_f - y_i = 0$$

(a) If we take the direction downward to be negative,

$$y_f = -50.0\text{ m},\quad v_{yi} = -2.00\text{ m/s},\quad \text{and}\quad a_y = -9.80\text{ m/s}^2$$

Substituting these values into the equation, we find

$$(4.90\text{ m/s}^2)t^2 + (2.00\text{ m/s})t - 50.0\text{ m} = 0$$

We now use the quadratic formula. The stone reaches the pool after it is thrown, so time must be positive and only the positive root describes the physical situation:

$$t = \frac{-2.00\text{ m/s} \pm \sqrt{(2.00\text{ m/s})^2 - 4\left(4.90\text{ m/s}^2\right)(-50.0\text{ m})}}{2\left(4.90\text{ m/s}^2\right)}$$

$$= \boxed{3.00\text{ s}}$$

where we have taken the positive root.

(b) For the second stone, the time of travel is

$$t = 3.00\text{ s} - 1.00\text{ s} = 2.00\text{ s}$$

Since $y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2$,

$$v_{yi} = \frac{\left(y_f - y_i\right) - \frac{1}{2}a_yt^2}{t}$$

$$= \frac{-50.0\text{ m} - \frac{1}{2}\left(-9.80\text{ m/s}^2\right)(2.00\text{ s})^2}{2.00\text{ s}}$$

$$= \boxed{-15.3\text{ m/s}}$$

The negative value indicates the downward direction of the initial velocity of the second stone.

(c) For the first stone,

$$v_{1f} = v_{1i} + a_1t_1 = -2.00\text{ m/s} + (-9.80\text{ m/s}^2)(3.00\text{ s})$$

$$v_{1f} = \boxed{-31.4\text{ m/s}}$$

For the second stone,

$$v_{2f} = v_{2i} + a_2t_2 = -15.3\text{ m/s} + (-9.80\text{ m/s}^2)(2.00\text{ s})$$

$$v_{2f} = \boxed{-34.8\text{ m/s}}$$

P2.64 (a) Area A_1 is a rectangle. Thus, $A_1 = hw = v_{xi}t$.

Area A_2 is triangular. Therefore, $A_2 = \frac{1}{2}bh = \frac{1}{2}t(v_x - v_{xi})$.

The total area under the curve is

$$A = A_1 + A_2 = v_{xi}t + \frac{(v_x - v_{xi})t}{2}$$

and since $v_x - v_{xi} = a_x t$,

$$\boxed{A = v_{xi}t + \frac{1}{2}a_x t^2}$$

(b) $\boxed{\text{The displacement given by the equation is: } x = v_{xi}t + \frac{1}{2}a_x t^2\text{, the same result as above for the total area.}}$

***P2.65** (a) Take initial and final points at top and bottom of the first incline, respectively. If the ball starts from rest, $v_i = 0$, $a = 0.500\ \text{m/s}^2$, and $x_f - x_i = 9.00$ m. Then

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 0^2 + 2(0.500\ \text{m/s}^2)(9.00\ \text{m})$$

$$v_f = \boxed{3.00\ \text{m/s}}$$

(b) To find the time interval, we use

$$x_f - x_i = v_i t + \frac{1}{2}at^2$$

Plugging in,

$$9.00 = 0 + \frac{1}{2}(0.500\ \text{m/s}^2)t^2$$

$$t = \boxed{6.00\ \text{s}}$$

(c) Take initial and final points at the bottom of the first plane and the top of the second plane, respectively: $v_i = 3.00$ m/s, $v_f = 0$, and $x_f - x_i = 15.0$ m. We use

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

which gives

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{0 - (3.00\ \text{m/s})^2}{2(15.0\ \text{m})} = \boxed{-0.300\ \text{m/s}^2}$$

(d) Take the initial point at the bottom of the first plane and the final point 8.00 m along the second plane:

$$v_i = 3.00\ \text{m/s},\ x_f - x_i = 8.00\ \text{m},\ a = -0.300\ \text{m/s}^2$$

$$v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = (3.00\ \text{m/s})^2 + 2\left(-0.300\ \text{m/s}^2\right)(8.00\ \text{m})$$

$$= 4.20\ \text{m}^2/\text{s}^2$$

$$v_f = \boxed{2.05\ \text{m/s}}$$

***P2.66** Take downward as the positive *y* direction.

(a) While the woman was in free fall, $\Delta y = 144$ ft, $v_i = 0$, and we take $a = g = 32.0\ \text{ft/s}^2$. Thus,

$$\Delta y = v_i t + \frac{1}{2}at^2 \rightarrow 144\ \text{ft} = 0 + \left(16.0\ \text{ft/s}^2\right)t^2$$

giving $t_{\text{fall}} = 3.00$ s. Her velocity just before impact is:

$$v_f = v_i + gt = 0 + \left(32.0\ \text{ft/s}^2\right)(3.00\ \text{s}) = \boxed{96.0\ \text{ft/s}}$$

(b) While crushing the box, $v_i = 96.0\ \text{ft/s}$, $v_f = 0$, and $\Delta y = 18.0\ \text{in.} = 1.50\ \text{ft}$. Therefore,

$$a = \frac{v_f^2 - v_i^2}{2(\Delta y)} = \frac{0 - (96.0\ \text{ft/s})^2}{2(1.50\ \text{ft})} = -3.07 \times 10^3\ \text{ft/s}^2$$

or $\boxed{a = 3.07 \times 10^3\ \text{ft/s}^2\ \text{upward}} = 96.0g$.

(c) Time to crush box:

$$\Delta t = \frac{\Delta y}{\bar{v}} = \frac{\Delta y}{\frac{v_f + v_i}{2}} = \frac{2(1.50\ \text{ft})}{0 + 96.0\ \text{ft/s}}$$

or $\boxed{\Delta t = 3.13 \times 10^{-2}\ \text{s}}$

P2.67 (a) The elevator, moving downward at the constant speed of 5.00 m/s has moved $d = v\Delta t = (5.00\ \text{m/s})(5.00\ \text{s}) = 25.0\ \text{m}$ below the position from which the bolt drops. Taking the positive direction to be downward, the initial position of the bolt to be x_B = 0, and setting $t = 0$ when the bolt drops, the position of the top of the elevator is

$$y_E = y_{Ei} + v_{Ei}t + \frac{1}{2}a_E t^2$$

$$y_E = 25.0\ \text{m} + (5.00\ \text{m/s})t$$

and the position of the bolt is

$$y_B = y_{Bi} + v_{Bi}t + \frac{1}{2}a_B t^2$$

$$y_B = \frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

Setting these expressions equal to each other gives

$$y_E = y_B$$

$$25.0\ \text{m} + \left(5.00\ \text{m/s}\right)t = \frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

$$4.90t^2 - 5.00t - 25.0 = 0$$

The (positive) solution to this is $t = \boxed{2.83\ \text{s}}$.

(b) $\boxed{\text{Both problems have an object traveling at constant velocity being overtaken by an object starting from rest traveling in the same direction at a constant acceleration.}}$

(c) $\boxed{\begin{array}{l}\text{The top of the elevator travels a total distance} \\ \qquad d = (5.00\ \text{m/s})(5.00\ \text{s} + 2.83\ \text{s}) = 39.1\ \text{m} \\ \text{from where the bolt drops to where the bolt strikes the top of} \\ \text{the elevator. Assuming 1 floor} \cong 3\ \text{m, this distance is about} \\ \qquad (39.1\ \text{m})(1\ \text{floor}/3\ \text{m}) \cong 13\ \text{floors.}\end{array}}$

P2.68 For the collision not to occur, the front of the passenger train must not have a position that is equal to or greater than the position of the back of the freight train at any time. We can write expressions of position to see whether the front of the passenger car (P) meets the back of the freight car (F) at some time.

Assume at $t = 0$, the coordinate of the front of the passenger car is $x_{Pi} = 0$; and the coordinate of the back of the freight car is $x_{Fi} = 58.5$ m. At later time t, the coordinate of the front of the passenger car is

$$x_P = x_{Pi} + v_{Pi}t + \frac{1}{2}a_P t^2$$

$$x_P = \left(40.0\ \text{m/s}\right)t + \frac{1}{2}\left(-3.00\ \text{m/s}^2\right)t^2$$

and the coordinate of the back of the freight car is

$$x_F = x_{Fi} + v_{Fi}t + \frac{1}{2}a_F t^2$$

$$x_F = 58.5\ \text{m} + \left(16.0\ \text{m/s}\right)t$$

Setting these expression equal to each other gives

$$x_P = x_F$$

$$(40.0\text{ m/s})t + \frac{1}{2}(-3.00\text{ m/s}^2)t^2 = 58.5\text{ m} + (16.0\text{ m/s})t$$

or $$(1.50)t^2 + (-24.0)t + 58.5 = 0$$

after simplifying and suppressing units.

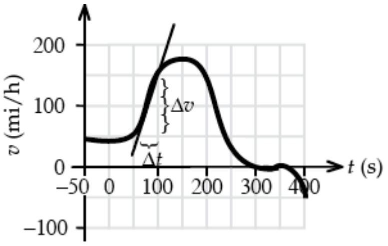
We do not have to solve this equation, we just want to check if a solution exists; if a solution does exist, then the trains collide. A solution does exist:

$$t = \frac{-(-24.0) \pm \sqrt{(-24.0)^2 - 4(1.50)(58.5)}}{2(1.50)}$$

$$t = \frac{24.0 \pm \sqrt{576 - 351}}{3.00} \rightarrow t = \frac{24.0 \pm \sqrt{225}}{3.00} = \frac{24.0 \pm 15}{3.00}$$

$$\boxed{\text{The situation is impossible since there is a finite time for which the front of the passenger train and the back of the freight train are at the same location.}}$$

P2.69 (a) As we see from the graph, from about –50 s to 50 s Acela is cruising at a constant positive velocity in the $+x$ direction. From 50 s to 200 s, Acela accelerates in the $+x$ direction reaching a top speed of about 170 mi/h. Around 200 s, the engineer applies the brakes, and the train, still traveling in the $+x$ direction, slows down and then stops at 350 s. Just after 350 s, Acela reverses direction (v becomes negative) and steadily gains speed in the $-x$ direction.



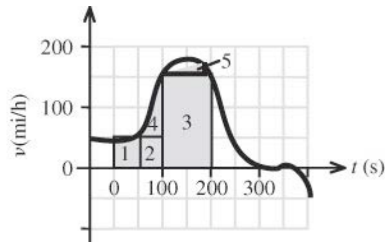
ANS. FIG. P2.69(a)

(b) The peak acceleration between 45 and 170 mi/h is given by the slope of the steepest tangent to the v versus t curve in this interval. From the tangent line shown, we find

$$a = \text{slope} = \frac{\Delta v}{\Delta t} = \frac{(155 - 45)\text{ mi/h}}{(100 - 50)\text{ s}}$$

$$= \boxed{2.2\ (\text{mi/h})/\text{s}} = 0.98\text{ m/s}^2$$

(c) Let us use the fact that the area under the v versus t curve equals the displacement. The train's displacement between 0 and 200 s is equal to the area of the gray shaded region, which we have approximated with a series of triangles and rectangles.



ANS. FIG. P2.69(c)

$$\begin{aligned}\Delta x_{0\to 200\text{ s}} &= \text{area}_1 + \text{area}_2 + \text{area}_3 + \text{area}_4 + \text{area}_5\\ &\approx (50\text{ mi/h})(50\text{ s}) + (50\text{ mi/h})(50\text{ s})\\ &\quad + (160\text{ mi/h})(100\text{ s})\\ &\quad + \frac{1}{2}(50\text{ s})(100\text{ mi/h})\\ &\quad + \frac{1}{2}(100\text{ s})(170\text{ mi/h} - 160\text{ mi/h})\\ &= 24\ 000(\text{mi/h})(\text{s})\end{aligned}$$

Now, at the end of our calculation, we can find the displacement in miles by converting hours to seconds. As 1 h = 3 600 s,

$$\Delta x_{0\to 200\text{ s}} = \left(\frac{24\ 000\text{ mi}}{3\ 600\text{ s}}\right)(\text{s}) = \boxed{6.7\text{ mi}}$$

P2.70 We use the relation $v_f^2 = v_i^2 + 2a(x_f - x_i)$, where $v_i = -8.00$ m/s and $v_f = 16.0$ m/s.

(a) The displacement of the first object is $\Delta x = +20.0$ m. Solving the above equation for the acceleration a, we obtain

$$a = \frac{v_f^2 - v_i^2}{2\Delta x}$$

$$a = \frac{(16.0\text{ m/s})^2 - (-8.00\text{ m/s})^2}{2(20.0\text{ m})}$$

$$a = \boxed{+4.80\text{ m/s}^2}$$

(b) Here, the total distance $d = 22.0$ m. The initial negative velocity and final positive velocity indicate that first the object travels through a negative displacement, slowing down until it reverses direction (where $v = 0$), then it returns to, and passes, its starting point, continuing to speed up until it reaches a speed of 16.0 m/s. We must consider the motion as comprising three displacements; the total distance d is the sum of the lengths of these displacements.

We split the motion into three displacements in which the acceleration remains constant throughout. We can find each displacement using

$$\Delta x = \frac{v_f^2 - v_i^2}{2a}$$

Displacement $\Delta x_1 = -d_1$ for velocity change $-8.00 \to 0$ m/s:

$$\Delta x_1 = \frac{v_f^2 - v_i^2}{2a} = \frac{0-(-8.00\text{ m/s})^2}{2a} = \frac{(-8)^2}{2a} \quad \to \quad d_1 = \frac{8^2}{2a}$$

Displacement $\Delta x_2 = +d_1$ for velocity change $0 \to +8.00$ m/s:

$$\Delta x_2 = \frac{v_f^2 - v_i^2}{2a} = \frac{(8.00\text{ m/s})^2 - 0}{2a} = \frac{8^2}{2a} \quad \to \quad d_2 = \frac{8^2}{2a}$$

Displacement $\Delta x_3 = +d_2$ for velocity change $+8.00 \to +16.0$ m/s:

$$\Delta x_3 = \frac{v_f^2 - v_i^2}{2a} = \frac{(16.0\text{ m/s})^2 - (8.00\text{ m/s})^2}{2a} = \frac{16^2 - 8^2}{2a}$$

$$\to \quad d_3 = \frac{16^2 - 8^2}{2a}$$

Suppressing units, the total distance is $d = d_1 + d_2 + d_3$, or

$$d = d_1 + d_2 + d_3 = 2\left(\frac{8^2}{2a}\right) + \frac{16^2 - 8^2}{2a} = \frac{16^2 + 8^2}{2a}$$

Solving for the acceleration gives

$$a = \frac{v_f^2 - v_i^2}{2d} = \frac{(16\text{ m/s})^2 + (8\text{ m/s})^2}{2d} = \frac{(16\text{ m/s})^2 + (8\text{ m/s})^2}{2(22.0\text{ m})}$$

$$a = \boxed{7.27\text{ m/s}^2}$$

P2.71 (a) In order for the trailing athlete to be able to catch the leader, his speed (v_1) must be greater than that of the leading athlete (v_2), and the distance between the leading athlete and the finish line must be great enough to give the trailing athlete sufficient time to make up the deficient distance, d.

(b) During a time interval t the leading athlete will travel a distance $d_2 = v_2 t$ and the trailing athlete will travel a distance $d_1 = v_1 t$. Only when $d_1 = d_2 + d$ (where d is the initial distance the trailing athlete was behind the leader) will the trailing athlete have caught the leader. Requiring that this condition be satisfied gives the elapsed time required for the second athlete to overtake the first:

$$d_1 = d_2 + d \quad \text{or} \quad v_1 t = v_2 t + d$$

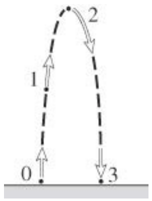
giving

$$v_1t - v_2t = d \quad \text{or} \quad t = \boxed{\frac{d}{(v_1 - v_2)}}$$

(c) In order for the trailing athlete to be able to at least tie for first place, the initial distance *D* between the leader and the finish line must be greater than or equal to the distance the leader can travel in the time *t* calculated above (i.e., the time required to overtake the leader). That is, we must require that

$$D \geq d_2 = v_2t = v_2\left[\frac{d}{(v_1 - v_2)}\right] \quad \text{or} \quad \boxed{d_2 = \frac{v_2d}{v_1 - v_2}}$$

P2.72 Let point 0 be at ground level and point 1 be at the end of the engine burn. Let point 2 be the highest point the rocket reaches and point 3 be just before impact. The data in the table below are found for each phase of the rocket's motion.



ANS. FIG. P2.72

(0 to 1): $v_f^2 - (80.0 \text{ m/s})^2 = 2(4.00 \text{ m/s}^2)(1\,000 \text{ m})$

so $v_f = 120$ m/s. Then, $120 \text{ m/s} = 80.0 \text{ m/s} + (4.00 \text{ m/s}^2)t$

giving $t = 10.0$ s.

(1 to 2) $0 - (120 \text{ m/s})^2 = 2(-9.80 \text{ m/s}^2)(y_f - y_i)$

giving $y_f - y_i = 735$ m,

$0 - 120 \text{ m/s} = (-9.80 \text{ m/s}^2)t$

giving $t = 12.2$ s.

This is the time of maximum height of the rocket.

(2 to 3) $v_f^2 - 0 = 2(-9.80 \text{ m/s}^2)(-1\,735 \text{ m})$ or $v_f = -184$ m/s

Then $v_f = -184 \text{ m/s} = (-9.80 \text{ m/s}^2)t$

giving $t = 18.8$ s.

(a) $t_{\text{total}} = 10 \text{ s} + 12.2 \text{ s} + 18.8 \text{ s} = \boxed{41.0 \text{ s}}$

(b) $(y_f - y_i)_{\text{total}} = \boxed{1.73 \text{ km}}$

(c) $v_{\text{final}} = \boxed{-184 \text{ m/s}}$

		t	x	v	a
0	Launch	0.0	0	80	+4.00
#1	End Thrust	10.0	1 000	120	+4.00
#2	Rise Upwards	22.2	1 735	0	–9.80
#3	Fall to Earth	41.0	0	–184	–9.80

P2.73 We have constant-acceleration equations to apply to the two cars separately.

(a) Let the times of travel for Kathy and Stan be t_K and t_S, where

$$t_S = t_K + 1.00\ \text{s}$$

Both start from rest ($v_{xi,K} = v_{xi,S} = 0$), so the expressions for the distances traveled are

$$x_K = \frac{1}{2}a_{x,K}t_K^2 = \frac{1}{2}(4.90\ \text{m/s}^2)t_K^2$$

and $$x_S = \frac{1}{2}a_{x,S}t_S^2 = \frac{1}{2}(3.50\ \text{m/s}^2)(t_K + 1.00\ \text{s})^2$$

When Kathy overtakes Stan, the two distances will be equal. Setting $x_K = x_S$ gives

$$\frac{1}{2}(4.90\ \text{m/s}^2)t_K^2 = \frac{1}{2}(3.50\ \text{m/s}^2)(t_K + 1.00\ \text{s})^2$$

This we simplify and write in the standard form of a quadratic as

$$t_K^2 - \left(5.00\ t_K\right)\text{s} - 2.50\ \text{s}^2 = 0$$

We solve using the quadratic formula $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, suppressing units, to find

$$t_K = \frac{5 \pm \sqrt{5^2 - 4(1)(-2.5)}}{2(1)} = \frac{5 + \sqrt{35}}{2} = \boxed{5.46\ \text{s}}$$

Only the positive root makes sense physically, because the overtake point must be after the starting point in time.

(b) Use the equation from part (a) for distance of travel,

$$x_K = \frac{1}{2}a_{x,K}t_K^2 = \frac{1}{2}(4.90\ \text{m/s}^2)(5.46\ \text{s})^2 = \boxed{73.0\ \text{m}}$$

(c) Remembering that $v_{xi,K} = v_{xi,S} = 0$, the final velocities will be:

$$v_{xf,K} = a_{x,K}t_K = (4.90\ \text{m/s}^2)(5.46\ \text{s}) = \boxed{26.7\ \text{m/s}}$$

$$v_{xf,S} = a_{x,S}t_S = (3.50\ \text{m/s}^2)(6.46\ \text{s}) = \boxed{22.6\ \text{m/s}}$$

P2.74 (a) While in the air, both balls have acceleration $a_1 = a_2 = -g$ (where upward is taken as positive). Ball 1 (thrown downward) has initial velocity $v_{01} = -v_0$, while ball 2 (thrown upward) has initial velocity $v_{02} = v_0$. Taking $y = 0$ at ground level, the initial y coordinate of each ball is $y_{01} = y_{02} = +h$. Applying $\Delta y = y - y_i = v_i t + \frac{1}{2}at^2$ to each ball gives their y coordinates at time t as

Ball 1: $y_1 - h = -v_0 t + \frac{1}{2}(-g)t^2$ or $\boxed{y_1 = h - v_0 t - \frac{1}{2}gt^2}$

Ball 2: $y_2 - h = +v_0 t + \frac{1}{2}(-g)t^2$ or $\boxed{y_2 = h + v_0 t - \frac{1}{2}gt^2}$

At ground level, $y = 0$. Thus, we equate each of the equations found above to zero and use the quadratic formula to solve for the times when each ball reaches the ground. This gives the following:

Ball 1: $0 = h - v_0 t_1 - \frac{1}{2}gt_1^2 \to gt_1^2 + (2v_0)t_1 + (-2h) = 0$

so $$t_1 = \frac{-2v_0 \pm \sqrt{(2v_0)^2 - 4(g)(-2h)}}{2g} = -\frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Using only the *positive* solution gives

$$t_1 = -\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Ball 2: $0 = h + v_0 t_2 - \frac{1}{2}gt_2^2 \to gt_2^2 + (-2v_0)t_2 + (-2h) = 0$

and $$t_2 = \frac{-(-2v_0) \pm \sqrt{(-2v_0)^2 - 4(g)(-2h)}}{2g} = +\frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Again, using only the *positive* solution,

$$t_2 = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Thus, the difference in the times of flight of the two balls is

$$\Delta t = t_2 - t_1$$

$$= \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}} - \left(-\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}\right) = \boxed{\frac{2v_0}{g}}$$

(b) Realizing that the balls are going *downward* ($v < 0$) as they near the ground, we use $v_f^2 = v_i^2 + 2a(\Delta y)$ with $\Delta y = -h$ to find the velocity of each ball just before it strikes the ground:

Ball 1:

$$v_{1f} = -\sqrt{v_{1i}^2 + 2a_1(-h)} = -\sqrt{(-v_0)^2 + 2(-g)(-h)} = \boxed{-\sqrt{v_0^2 + 2gh}}$$

Ball 2:

$$v_{2f} = -\sqrt{v_{2i}^2 + 2a_2(-h)} = -\sqrt{(+v_0)^2 + 2(-g)(-h)} = \boxed{-\sqrt{v_0^2 + 2gh}}$$

(c) While both balls are still in the air, the distance separating them is

$$d = y_2 - y_1 = \left(h + v_0 t - \frac{1}{2}gt^2\right) - \left(h - v_0 t - \frac{1}{2}gt^2\right) = \boxed{2v_0 t}$$

P2.75 We translate from a pictorial representation through a geometric model to a mathematical representation by observing that the distances x and y are always related by $x^2 + y^2 = L^2$.

(a) Differentiating this equation with respect to time, we have

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

Now the unknown velocity of B is $\frac{dy}{dt} = v_B$ and $\frac{dx}{dt} = -v$,

so the differentiated equation becomes

$$\frac{dy}{dt} = -\frac{x}{y}\left(\frac{dx}{dt}\right) = -\left(\frac{x}{y}\right)(-v) = v_B$$

But $\frac{y}{x} = \tan\theta$, so $v_B = \boxed{\left(\frac{1}{\tan\theta}\right)v}$

(b) We assume that θ starts from zero. At this instant $1/\tan\theta$ is infinite, and the velocity of B is infinitely larger than that of A. As θ increases, the velocity of object B decreases, becoming equal to v when $\theta = 45°$. After that instant, B continues to slow down with non-constant acceleration, coming to rest as θ goes to 90°.

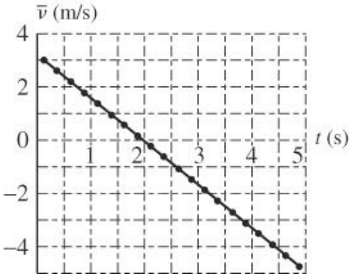
P2.76

Time t **(s)**	**Height** h **(m)**	Δh **(m)**	Δt **(s)**	$\bar{v}$ **(m/s)**	**midpoint time** t **(s)**
0.00	5.00	0.75	0.25	3.00	0.13
0.25	5.75	0.65	0.25	2.60	0.38
0.50	6.40	0.54	0.25	2.16	0.63
0.75	6.94	0.44	0.25	1.76	0.88
1.00	7.38	0.34	0.25	1.36	1.13
1.25	7.72	0.24	0.25	0.96	1.38
1.50	7.96	0.14	0.25	0.56	1.63
1.75	8.10	0.03	0.25	0.12	1.88
2.00	8.13	–0.06	0.25	–0.24	2.13
2.25	8.07	–0.17	0.25	–0.68	2.38
2.50	7.90	–0.28	0.25	–1.12	2.63
2.75	7.62	–0.37	0.25	–1.48	2.88
3.00	7.25	–0.48	0.25	–1.92	3.13
3.25	6.77	–0.57	0.25	–2.28	3.38
3.50	6.20	–0.68	0.25	–2.72	3.63
3.75	5.52	–0.79	0.25	–3.16	3.88
4.00	4.73	–0.88	0.25	–3.52	4.13
4.25	3.85	–0.99	0.25	–3.96	4.38
4.50	2.86	–1.09	0.25	–4.36	4.63
4.75	1.77	–1.19	0.25	–4.76	4.88
5.00	0.58				

TABLE P2.76

The very convincing fit of a single straight line to the points in the graph of velocity versus time indicates that the rock does fall with constant acceleration. The acceleration is the slope of line:

$$a_{\text{avg}} = -1.63\ \text{m/s}^2 = \boxed{1.63\ \text{m/s}^2\ \text{downward}}$$



***P2.77** Distance traveled by motorist = (15.0 m/s)*t*

Distance traveled by policeman $= \frac{1}{2}\left(2.00\ \text{m/s}^2\right)t^2$

(a) Intercept occurs when $15.0t = t^2$, or $t = \boxed{15.0\ \text{s}}$.

(b) $v(\text{officer}) = \left(2.00\ \text{m/s}^2\right)t = \boxed{30.0\ \text{m/s}}$

(c) $x(\text{officer}) = \frac{1}{2}\left(2.00\ \text{m/s}^2\right)t^2 = \boxed{225\ \text{m}}$

***P2.78** The train accelerates with $a_1 = 0.100\ \text{m/s}^2$ then decelerates with $a_2 = -0.500\ \text{m/s}^2$. We can write the 1.00-km displacement of the train as

$$x = 1\,000\ \text{m} = \frac{1}{2}a_1\Delta t_1^2 + v_{1f}\Delta t_2 + \frac{1}{2}a_2\Delta t_2^2$$

with $t = t_1 + t_2$. Now, $v_{1f} = a_1\Delta t_1 = -a_2\Delta t_2$; therefore

$$1\,000\ \text{m} = \frac{1}{2}a_1\Delta t_1^2 + a_1\Delta t_1\left(-\frac{a_1\Delta t_1}{a_2}\right) + \frac{1}{2}a_2\left(\frac{a_1\Delta t_1}{a_2}\right)^2$$

$$1\,000\ \text{m} = \frac{1}{2}a_1\left(1 - \frac{a_1}{a_2}\right)\Delta t_1^2$$

$$1\,000\ \text{m} = \frac{1}{2}\left(0.100\ \text{m/s}^2\right)\left(1 - \frac{0.100\ \text{m/s}^2}{-0.500\ \text{m/s}^2}\right)\Delta t_1^2$$

$$\Delta t_1 = \sqrt{\frac{20\,000}{1.20}}\ \text{s} = 129\ \text{s}$$

$$\Delta t_2 = \frac{a_1\Delta t_1}{-a_2} = \frac{12.9}{0.500}\ \text{s} \approx 26\ \text{s}$$

Total time $= \Delta t = \Delta t_1 + \Delta t_2 = 129\text{ s} + 26\text{ s} = \boxed{155\text{ s}}$

***P2.79** The average speed of every point on the train as the first car passes Liz is given by:

$$\frac{\Delta x}{\Delta t} = \frac{8.60\text{ m}}{1.50\text{ s}} = 5.73\text{ m/s}$$

The train has this as its instantaneous speed halfway through the 1.50-s time. Similarly, halfway through the next 1.10 s, the speed of the train is $\frac{8.60\text{ m}}{1.10\text{ s}} = 7.82\text{ m/s}$. The time required for the speed to change from 5.73 m/s to 7.82 m/s is

$$\frac{1}{2}(1.50\text{ s}) + \frac{1}{2}(1.10\text{ s}) = 1.30\text{ s}$$

so the acceleration is: $a_x = \frac{\Delta v_x}{\Delta t} = \frac{7.82\text{ m/s} - 5.73\text{ m/s}}{1.30\text{ s}} = \boxed{1.60\text{ m/s}^2}$

P2.80 Let the ball fall freely for 1.50 m after starting from rest. It strikes at speed given by

$$v_{xf}^2 = v_{xi}^2 + 2a\left(x_f - x_i\right)$$

$$v_{xf}^2 = 0 + 2\left(-9.80\text{ m/s}^2\right)\left(-1.50\text{ m}\right)$$

$$v_{xf} = -5.42\text{ m/s}$$

If its acceleration were constant, its stopping would be described by

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

$$0 = \left(-5.42\text{ m/s}\right)^2 + 2a_x\left(-10^{-2}\text{ m}\right)$$

$$a_x = \frac{-29.4\text{ m}^2/\text{s}^2}{-2.00\times 10^{-2}\text{ m}} = +1.47\times 10^3\text{ m/s}^2$$

Upward acceleration of this same order of magnitude will continue for some additional time after the dent is at its maximum depth, to give the ball the speed with which it rebounds from the pavement. The ball's maximum acceleration will be larger than the average acceleration we estimate by imagining constant acceleration, but will still be of order of magnitude $\boxed{\sim 10^3\text{m/s}^2}$.

Challenge Problems

P2.81 (a) From the information in the problem, we model the blue car as a particle under constant acceleration. The important "particle" for this part of the problem is the nose of the car. We use the position equation from the particle under constant acceleration model to find the velocity v_0 of the particle as it enters the intersection

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\rightarrow 28.0\text{ m} = 0 + v_0(3.10\text{ s}) + \frac{1}{2}(-2.10\text{ m/s}^2)(3.10\text{ s})^2$$

$$\rightarrow v_0 = 12.3\text{ m/s}$$

Now we use the velocity-position equation in the particle under constant acceleration model to find the displacement of the particle from the first edge of the intersection when the blue car stops:

$$v^2 = v_0^2 + 2a(x - x_0)$$

or $$x - x_0 = \Delta x = \frac{v^2 - v_0^2}{2a} = \frac{0 - (12.3\text{ m/s})^2}{2(-2.10\text{ m/s}^2)} = \boxed{35.9\text{ m}}$$

(b) The time interval during which any part of the blue car is in the intersection is that time interval between the instant at which the nose enters the intersection and the instant when the tail leaves the intersection. Thus, the change in position of the nose of the blue car is 4.52 m + 28.0 m = 32.52 m. We find the time at which the car is at position x = 32.52 m if it is at x = 0 and moving at 12.3 m/s at t = 0:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

$$\rightarrow 32.52\text{ m} = 0 + (12.3\text{ m/s})t + \frac{1}{2}(-2.10\text{ m/s}^2)t^2$$

$$\rightarrow -1.05t^2 + 12.3t - 32.52 = 0$$

The solutions to this quadratic equation are t = 4.04 s and 7.66 s. Our desired solution is the lower of two, so $t = \boxed{4.04\text{s}}$, (The later time corresponds to the blue car stopping and reversing, which it must do if the acceleration truly remains constant, and arriving again at the position x = 32.52 m.)

(c) We again define t = 0 as the time at which the nose of the blue car enters the intersection. Then at time t = 4.04 s, the tail of the blue car leaves the intersection. Therefore, to find the minimum distance from the intersection for the silver car, its nose must enter the intersection at $t = 4.04$ s. We calculate this distance from the position equation:

$$x - x_0 + v_0 t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(5.60\ \text{m/s}^2\right)\left(4.04\ \text{s}\right)^2 = \boxed{45.8\ \text{m}}$$

(d) We use the velocity equation:

$$v = v_0 + at = 0 + \left(5.60\ \text{m/s}^2\right)\left(4.04\ \text{s}\right) = \boxed{22.6\ \text{m/s}}$$

P2.82 (a) Starting from rest and accelerating at $a_b = 13.0\ \text{mi/h}\cdot\text{s}$, the bicycle reaches its maximum speed of $v_{b,\max} = 20.0$ mi/h in a time

$$t_{b,1} = \frac{v_{b,\max} - 0}{a_b} = \frac{20.0\ \text{mi/h}}{13.0\ \text{mi/h}\cdot\text{s}} = 1.54\ \text{s}$$

Since the acceleration a_c of the car is less than that of the bicycle, the car cannot catch the bicycle until some time $t > t_{b,1}$ (that is, until the bicycle is at its maximum speed and coasting). The total displacement of the bicycle at time t is

$$\begin{aligned}\Delta x_b &= \frac{1}{2}a_b t_{b,1}^2 + v_{b,\max}\left(t - t_{b,1}\right)\\ &= \left(\frac{1.47\ \text{ft/s}}{1\ \text{mi/h}}\right)\times\\ &\quad \left[\frac{1}{2}\left(13.0\frac{\text{mi/h}}{\text{s}}\right)\left(1.54\ \text{s}\right)^2 + \left(20.0\ \text{mi/h}\right)\left(t - 1.54\ \text{s}\right)\right]\\ &= \left(29.4\ \text{ft/s}\right)t - 22.6\ \text{ft}\end{aligned}$$

The total displacement of the car at this time is

$$\Delta x_c = \frac{1}{2}a_c t^2 = \left(\frac{1.47\ \text{ft/s}}{1\ \text{mi/h}}\right)\left[\frac{1}{2}\left(9.00\frac{\text{mi/h}}{\text{s}}\right)t^2\right] = \left(6.62\ \text{ft/s}^2\right)t^2$$

At the time the car catches the bicycle, $\Delta x_c = \Delta x_b$. This gives

$$\left(6.62\ \text{ft/s}^2\right)t^2 = \left(29.4\ \text{ft/s}\right)t - 22.6\ \text{ft}$$

or $$t^2 - \left(4.44\ \text{s}\right)t + 3.42\ \text{s}^2 = 0$$

that has only one physically meaningful solution $t > t_{b,1}$. This solution gives the total time the bicycle leads the car and is $t = \boxed{3.45\ \text{s}}$.

(b) The lead the bicycle has over the car continues to increase as long as the bicycle is moving faster than the car. This means until the car attains a speed of $v_c = v_{b,\max} = 20.0$ mi/h. Thus, the elapsed time when the bicycle's lead ceases to increase is

$$t = \frac{v_{b,\max}}{a_c} = \frac{20.0\text{ mi/h}}{9.00\text{ mi/h}\cdot\text{s}} = 2.22\text{ s}$$

At this time, the lead is

$$\begin{aligned}(\Delta x_b - \Delta x_c)_{\max} &= (\Delta x_b - \Delta x_c)\big|_{t=2.22\text{ s}}\\ &= \left[(29.4\text{ ft/s})(2.22\text{ s}) - 22.6\text{ ft}\right]\\ &\quad - \left[(6.62\text{ ft/s}^2)(2.22\text{ s})^2\right]\end{aligned}$$

or $(\Delta x_b - \Delta x_c)_{\max} = \boxed{10.0\text{ ft}}$

P2.83 Consider the runners in general. Each completes the race in a total time interval *T*. Each runs at constant acceleration *a* for a time interval Δt, so each covers a distance (displacement) $\Delta x_a = \frac{1}{2}a\Delta t^2$ where they eventually reach a final speed (velocity) $v = a\Delta t$, after which they run at this constant speed for the remaining time $(T - \Delta t)$ until the end of the race, covering distance $\Delta x_v = v(T - \Delta t) = a\Delta t(T - \Delta t)$. The total distance (displacement) each covers is the same:

$$\begin{aligned}\Delta x &= \Delta x_a + \Delta x_v\\ &= \frac{1}{2}a\Delta t^2 + a\Delta t(T - \Delta t)\\ &= a\left[\frac{1}{2}\Delta t^2 + \Delta t(T - \Delta t)\right]\end{aligned}$$

so $$a = \frac{\Delta x}{\frac{1}{2}\Delta t^2 + \Delta t(T - \Delta t)}$$

where $\Delta x = 100$ m and $T = 10.4$ s.

(a) For Laura (runner 1), $\Delta t_1 = 2.00$ s:

$$a_1 = (100\text{ m})/(18.8\text{ s}^2) = \boxed{5.32\text{ m/s}^2}$$

For Healan (runner 2), $\Delta t_2 = 3.00$ s:

$$a_2 = (100\text{ m})/(26.7\text{ s}^2) = \boxed{3.75\text{ m/s}^2}$$

(b) Laura (runner 1): $v_1 = a_1\Delta t_1 = \boxed{10.6\text{ m/s}}$

Healan (runner 2): $v_2 = a_2\Delta t_2 = \boxed{11.2\text{ m/s}}$

(c) The 6.00-s mark occurs after either time interval Δt. From the reasoning above, each has covered the distance

$$\Delta x = a\left[\frac{1}{2}\Delta t^2 + \Delta t(t - \Delta t)\right]$$

where t = 6.00 s.

Laura (runner 1): Δx_1 = 53.19 m

Healan (runner 2): Δx_2 = 50.56 m

$\boxed{\text{So, Laura is ahead by (53.19 m – 50.56 m) = 2.63 m.}}$

(d) Laura accelerates at the greater rate, so she will be ahead of Healen at, and immediately after, the 2.00-s mark. After the 3.00-s mark, Healan is travelling faster than Laura, so the distance between them will shrink. In the time interval

$\boxed{\text{from the 2.00-s mark to the 3.00-s mark, the distance between them will be the greatest.}}$

During that time interval, the distance between them (the position of Laura relative to Healan) is

$$D = \Delta x_1 - \Delta x_2 = a_1\left[\frac{1}{2}\Delta t_1^2 + \Delta t_1(t - \Delta t_1)\right] - \frac{1}{2}a_2 t^2$$

because Laura has ceased to accelerate but Healan is still accelerating. Differentiating with respect to time, (and doing some simplification), we can solve for the time t when D is an maximum:

$$\frac{dD}{dt} = a_1\Delta t_1 - a_2 t = 0$$

which gives

$$t = \Delta t_1\left(\frac{a_1}{a_2}\right) = (2.00\text{ s})\left(\frac{5.32\text{ m/s}^2}{3.75\text{ m/s}^2}\right) = 2.84\text{ s}$$

Substituting this time back into the expression for D, we find that D = 4.47 m, that is, Laura ahead of Healan by $\boxed{\text{4.47 m.}}$

P2.84 (a) The factors to consider are as follows. The red bead falls through a greater distance with a downward acceleration of g. The blue bead travels a shorter distance, but with acceleration of $g\sin\theta$. A first guess would be that the blue bead "wins," but not by much. We do note, however, that points Ⓐ, Ⓑ, and Ⓒ are the vertices of a right triangle with ⒶⒸ as the hypotenuse.

(b) The red bead is a particle under constant acceleration. Taking downward as the positive direction, we can write

$$\Delta y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

as $\quad D = \frac{1}{2}gt_R^2$

which gives $\boxed{t_R = \sqrt{\frac{2D}{g}}}$.

(c) The blue bead is a particle under constant acceleration, with $a = g\sin\theta$. Taking the direction along L as the positive direction, we can write

$$\Delta y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$$

as $\quad L = \frac{1}{2}(g\sin\theta)t_B^2$

which gives $\boxed{t_B = \sqrt{\frac{2L}{g\sin\theta}}}$.

(d) For the two beads to reach point Ⓒ simultaneously, $t_R = t_B$. Then,

$$\sqrt{\frac{2D}{g}} = \sqrt{\frac{2L}{g\sin\theta}}$$

Squaring both sides and cross-multiplying gives

$$2gD\sin\theta = 2gL$$

or $\quad \sin\theta = \frac{L}{D}$.

We note that the angle between chords ⒶⒸ and ⒷⒸ is $90° - \theta$, so that the angle between chords ⒶⒸ and ⒶⒷ is

θ. Then, $\sin\theta = \frac{L}{D}$, and the beads arrive at point Ⓒ simultaneously.

(e) Once we recognize that the two rods form one side and the hypotenuse of a right triangle with θ as its smallest angle, then the result becomes obvious.

P2.85 The rock falls a distance *d* for a time interval Δt_1 and the sound of the splash travels upward through the same distance *d* for a time interval Δt_2 before the man hears it. The total time interval $\Delta t = \Delta t_1 + \Delta t_2 = 2.40\text{ s}$.

(a) Relationship between distance the rock falls and time interval Δt_1:

$$d = \frac{1}{2} g \Delta t_1^2$$

Relationship between distance the sound travels and time interval Δt_2: $d = v_s \Delta t_2$, where $v_s = 336\text{ m/s}$.

$$d = v_s \Delta t_2 = \frac{1}{2} g \Delta t_1^2$$

Substituting $\Delta t_1 = \Delta t - \Delta t_2$ gives

$$2\frac{v_s \Delta t_2}{g} = \left(\Delta t - \Delta t_2\right)^2$$

$$\left(\Delta t_2\right)^2 - 2\left(\Delta t + \frac{v_s}{g}\right)\Delta t_2 + \Delta t^2 = 0$$

$$\left(\Delta t_2\right)^2 - 2\left(2.40\text{ s} + \frac{336\text{ m/s}}{9.80\text{ m/s}^2}\right)\Delta t_2 + \left(2.40\text{ s}\right)^2 = 0$$

$$\left(\Delta t_2\right)^2 - \left(73.37\right)\Delta t_2 + 5.76 = 0$$

Solving the quadratic equation gives

$$\Delta t_2 = 0.078\,6\text{ s} \quad \rightarrow \quad d = v_s \Delta t_2 = \boxed{26.4\text{ m}}$$

(b) Ignoring the sound travel time,

$d = \frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(2.40\text{ s}\right)^2 = 28.2\text{ m}$, an error of $\boxed{6.82\%}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P2.2	0.02 s
P2.4	(a) 50.0 m/s; (b) 41.0 m/s
P2.6	(a) 27.0 m; (b) 27.0 m + (18.0 m/s) Δt + (3.00 m/s^2)(Δt^2); (c) 18.0 m/s
P2.8	(a) $+L/t_1$; (b) $-L/t_2$; (c) 0; (d) $2L/t_1 + t_2$
P2.10	1.9×10^8 years
P2.12	(a) 20 mi/h; (b) 0; (c) 30 mi/h
P2.14	1.34×10^4 m/s^2
P2.16	See graphs in P2.16.
P2.18	(a) See ANS. FIG. P2.18; (b) 23 m/s, 18 m/s, 14 m/s, and 9.0 m/s; (c) 4.6 m/s^2; (d) zero
P2.20	(a) 13.0 m/s; (b) 10.0 m/s, 16.0 m/s; (c) 6.00 m/s^2; (d) 6.00 m/s^2; (e) 0.333 s
P2.22	(a–e) See graphs in P2.22; (f) with less regularity
P2.24	160 ft.
P2.26	4.53 s
P2.28	(a) 6.61 m/s; (b) −0.448 m/s^2
P2.30	(a) 20.0 s; (b) No; (c) The plane would overshoot the runway.
P2.32	31 s
P2.34	The accelerations do not match.
P2.36	(a) $x_f - x_i = v_{xf} t - \frac{1}{2} a_x t^2$; (b) 3.10 m/s
P2.38	(a) 2.56 m; (b) −3.00 m/s
P2.40	19.7 cm/s; (b) 4.70 cm/s^2; (c) The length of the glider is used to find the average velocity during a known time interval.
P2.42	(a) 3.75 s; (b) 5.50 cm/s; (c) 0.604 s; (d) 13.3 cm, 47.9 cm; (e) See P2.42 part (e) for full explanation.
P2.44	(a) 8.20 s; (b) 134 m
P2.46	(a and b) The rock does not reach the top of the wall with v_f = 3.69 m/s; (c) 2.39 m/s; (d) does not agree; (e) The average speed of the upward-moving rock is smaller than the downward moving rock.
P2.48	(a) 29.4 m/s; (b) 44.1 m

P2.50 7.96 s

P2.52 0.60 s

P2.54 (a) $\frac{h}{t} + \frac{gt}{2}$; (b) $\frac{h}{t} - \frac{gt}{2}$

P2.56 (a) $(v_i + gt)$; (b) $\frac{1}{2}gt^2$; (c) $|v_i - gt|$; (d) $\frac{1}{2}gt^2$

P2.58 (a) See graphs in P2.58; (b) See graph in P2.58; (c) $-4\ \text{m/s}^2$; (d) 32 m; (e) 28 m

P2.60 (a) $5.25\ \text{m/s}^2$; (b) 168 m; (c) 52.5 m/s

P2.62 (a) 0; (b) $6.0\ \text{m/s}^2$; (c) $-3.6\ \text{m/s}^2$; (d) at $t = 6$ s and at 18 s; (e and f) $t = 18$ s; (g) 204 m

P2.64 (a) $A = v_{xi}t + \frac{1}{2}a_x t^2$; (b) The displacement is the same result for the total area.

P2.66 (a) 96.0 ft/s; (b) $3.07 \times 10^3\ \text{ft/s}^2$ upward; (c) 3.13×10^{-2} s

P2.68 The trains do collide.

P2.70 (a) $+4.8\ \text{m/s}^2$; (b) $7.27\ \text{m/s}^2$

P2.72 (a) 41.0 s; (b) 1.73 km; (c) -184 m/s

P2.74 (a) Ball 1: $y_1 = h - v_0 t - \frac{1}{2}gt^2$, Ball 2: $y_2 = h + v_0 t - \frac{1}{2}gt^2$, $\frac{2v_0}{g}$; (b) Ball 1: $-\sqrt{v_0^2 + 2gh}$, Ball 2: $-\sqrt{v_0^2 + 2gh}$; (c) $2v_0 t$

P2.76 (a and b) See TABLE P2.76; (c) $1.63\ \text{m/s}^2$ downward and see graph in P2.76

P2.78 155 s

P2.80 $\sim 10^3\ \text{m/s}^2$

P2.82 (a) 3.45 s; (b) 10.0 ft.

P2.84 (a) The red bead falls through a greater distance with a downward acceleration of g. The blue bead travels a shorter distance, but with acceleration of $g\sin\theta$. A first guess would be that the blue bead "wins," but not by much. (b) $\sqrt{\frac{2D}{g}}$; (c) $\sqrt{\frac{2L}{g\sin\theta}}$; (d) the beads arrive at point Ⓒ simultaneously; (e) Once we recognize that the two rods form one side and the hypotenuse of a right triangle with θ as its smallest angle, then the result becomes obvious.

3

Vectors

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ3.1 Answer (e). The magnitude is $\sqrt{10^2 + 10^2}$ m/s.

OQ3.2 Answer (e). If the quantities x and y are positive, a vector with components $(-x, y)$ or $(x, -y)$ would lie in the second or fourth quadrant, respectively.

***OQ3.3** Answer (a). The vector $-2\vec{\mathbf{D}}_1$ will be twice as long as $\vec{\mathbf{D}}_1$ and in the opposite direction, namely northeast. Adding $\vec{\mathbf{D}}_2$, which is about equally long and southwest, we get a sum that is still longer and due east.

OQ3.4 The ranking is c = e > a > d > b. The magnitudes of the vectors being added are constant, and we are considering the magnitude only—not the direction—of the resultant. So we need look only at the angle between the vectors being added in each case. The smaller this angle, the larger the resultant magnitude.

OQ3.5 Answers (a), (b), and (c). The magnitude can range from the sum of the individual magnitudes, 8 + 6 = 14, to the difference of the individual magnitudes, 8 − 6 = 2. Because magnitude is the “length” of a vector, it is always positive.

OQ3.6 Answer (d). If we write vector $\vec{\mathbf{A}}$ as

$$\left(A_x, A_y\right) = \left(-\left|A_x\right|, \left|A_y\right|\right)$$

and vector $\vec{\mathbf{B}}$ as

$$\left(B_x, B_y\right) = \left(\left|B_x\right|, -\left|B_y\right|\right)$$

then

$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = \left(\left|B_x\right| - \left(-\left|A_x\right|\right), -\left|B_y\right| - \left|A_y\right|\right) = \left(\left|B_x\right| + \left|A_x\right|, -\left|B_y\right| - \left|A_y\right|\right)$$

which would be in the fourth quadrant.

OQ3.7 The answers are (a) yes (b) no (c) no (d) no (e) no (f) yes (g) no. Only force and velocity are vectors. None of the other quantities requires a direction to be described.

OQ3.8 Answer (c). The vector has no y component given. It is therefore 0.

OQ3.9 Answer (d). Take the difference of the x coordinates of the ends of the vector, head minus tail: –4 – 2 = –6 cm.

OQ3.10 Answer (a). Take the difference of the y coordinates of the ends of the vector, head minus tail: 1 – (–2) = 3 cm.

OQ3.11 Answer (c). The signs of the components of a vector are the same as the signs of the points in the quadrant into which it points. If a vector arrow is drawn to scale, the coordinates of the point of the arrow equal the components of the vector. All x and y values in the third quadrant are negative.

OQ3.12 Answer (c). The vertical component is opposite the 30° angle, so sin 30° = (*vertical component*)/50 m.

OQ3.13 Answer (c). A vector in the second quadrant has a negative x component and a positive y component.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ3.1 Addition of a vector to a scalar is not defined. Try adding the speed and velocity, 8.0 m/s + $(15.0 \text{ m/s } \hat{\mathbf{i}})$: Should you consider the sum to be a vector or a scalar? What meaning would it have?

CQ3.2 No, the magnitude of a vector is always positive. A minus sign in a vector only indicates direction, not magnitude.

CQ3.3 (a) The book's displacement is zero, as it ends up at the point from which it started. (b) The distance traveled is 6.0 meters.

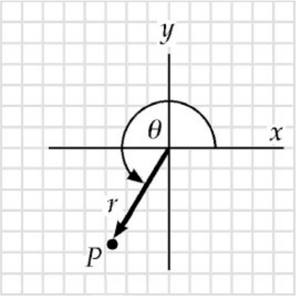
CQ3.4 Vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are perpendicular to each other.

CQ3.5 The inverse tangent function gives the correct angle, relative to the +*x* axis, for vectors in the first or fourth quadrant, and it gives an incorrect answer for vectors in the second or third quadrant. If the *x* and *y* components are both positive, their ratio *y*/*x* is positive and the vector lies in the first quadrant; if the *x* component is positive and the *y* component negative, their ratio *y*/*x* is negative and the vector lies in the fourth quadrant. If the *x* and *y* components are both negative, their ratio *y*/*x* is positive but the vector lies in the third quadrant; if the *x* component is negative and the *y* component positive, their ratio *y*/*x* is negative but the vector lies in the second quadrant.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 3.1 Coordinate Systems

P3.1 ANS. FIG. P3.1 helps to visualize the *x* and *y* coordinates, and trigonometric functions will tell us the coordinates directly. When the polar coordinates (r, θ) of a point P are known, the Cartesian coordinates are found as



$$x = r\cos\theta \quad \text{and} \quad y = r\sin\theta$$

Then,

$$\begin{aligned} x &= r\cos\theta = (5.50\text{ m})\cos 240° \\ &= (5.50\text{ m})(-0.5) = \boxed{-2.75\text{ m}} \\ y &= r\sin\theta = (5.50\text{ m})\sin 240° \\ &= (5.50\text{ m})(-0.866) = \boxed{-4.76\text{ m}} \end{aligned}$$

P3.2 (a) We use $x = r\cos\theta$. Substituting, we have 2.00 = r cos 30.0°, so

$$r = \frac{2.00}{\cos 30.0°} = \boxed{2.31}$$

(b) From $y = r\sin\theta$, we have $y = r\sin 30.0° = 2.31\sin 30.0° = \boxed{1.15}$.

***P3.3** (a) The distance between the points is given by

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(2.00 - [-3.00])^2 + (-4.00 - 3.00)^2} \end{aligned}$$

$$d = \sqrt{25.0 + 49.0} = \boxed{8.60\text{ m}}$$

(b) To find the polar coordinates of each point, we measure the radial distance to that point and the angle it makes with the $+x$ axis:

$$r_1 = \sqrt{(2.00)^2 + (-4.00)^2} = \sqrt{20.0} = \boxed{4.47\text{ m}}$$

$$\theta_1 = \tan^{-1}\left(-\frac{4.00}{2.00}\right) = \boxed{-63.4°}$$

$$r_2 = \sqrt{(-3.00)^2 + (3.00)^2} = \sqrt{18.0} = \boxed{4.24\text{ m}}$$

$$\theta_2 = \boxed{135°} \text{ measured from the } +x \text{ axis.}$$

P3.4 (a) $x = r\cos\theta$ and $y = r\sin\theta$, therefore,

$$x_1 = (2.50\text{ m})\cos 30.0°,\ y_1 = (2.50\text{ m})\sin 30.0°, \text{ and}$$

$$(x_1, y_1) = \boxed{(2.17,\ 1.25)\text{ m}}$$

$$x_2 = (3.80\text{ m})\cos 120°,\ y_2 = (3.80\text{ m})\sin 120°, \text{ and}$$

$$(x_2,\ y_2) = \boxed{(-1.90,\ 3.29)\text{ m}}$$

(b) $d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{4.07^2 + 2.04^2}\text{ m} = \boxed{4.55\text{ m}}$

P3.5 For polar coordinates (r, θ), the Cartesian coordinates are $(x = r\cos\theta, y = r\sin\theta)$, if the angle is measured relative to the $+x$ axis.

(a) $\boxed{(-3.56\text{ cm},\ -2.40\text{ cm})}$

(b) $(+3.56\text{ cm}, -2.40\text{ cm}) \to \boxed{(4.30\text{ cm}, -34.0°)}$

(c) $(7.12\text{ cm},\ 4.80\text{ cm}) \to \boxed{(8.60\text{ cm},\ 34.0°)}$

(d) $(-10.7\text{ cm},\ 7.21\text{ cm}) \to \boxed{(12.9\text{ cm},\ 146°)}$

P3.6 We have $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1}\left(\frac{y}{x}\right)$.

(a) The radius for this new point is

$$\sqrt{(-x)^2 + y^2} = \sqrt{x^2 + y^2} = \boxed{r}$$

and its angle is

$$\tan^{-1}\left(\frac{y}{-x}\right) = \boxed{180° - \theta}$$

(b) $\sqrt{(-2x)^2+(-2y)^2} = \boxed{2r}$. This point is in the third quadrant if (*x*, *y*) is in the first quadrant or in the fourth quadrant if (*x*, *y*) is in the second quadrant. It is at an angle of $\boxed{180° + \theta}$.

(c) $\sqrt{(3x)^2+(-3y)^2} = \boxed{3r}$. This point is in the fourth quadrant if (*x*, *y*) is in the first quadrant or in the third quadrant if (*x*, *y*) is in the second quadrant. It is at an angle of $\boxed{-\theta \text{ or } 360 - \theta}$.

Section 3.2 Vector and Scalar Quantities

Section 3.3 Some Properties of Vectors

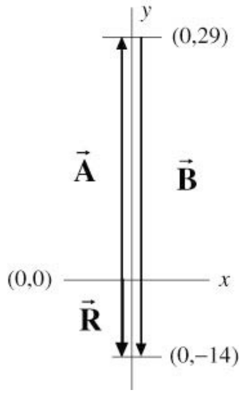
P3.7 Figure P3.7 suggests a right triangle where, relative to angle *θ*, its adjacent side has length *d* and its opposite side is equal to width of the river, *y*; thus,

$$\tan\theta = \frac{y}{d} \rightarrow y = d\tan\theta$$

$$y = (100\text{ m})\tan(35.0°) = 70.0\text{ m}$$

The width of the river is $\boxed{70.0\text{ m}}$.

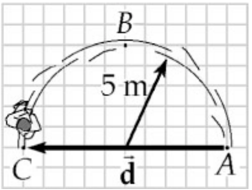
P3.8 We are given $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. When two vectors are added graphically, the second vector is positioned with its tail at the tip of the first vector. The resultant then runs from the tail of the first vector to the tip of the second vector. In this case, vector $\vec{\mathbf{A}}$ will be positioned with its tail at the origin and its tip at the point (0, 29). The resultant is then drawn, starting at the origin (tail of first vector) and going 14 units in the negative *y* direction to the point (0, −14). The second vector, $\vec{\mathbf{B}}$, must then start from the tip of $\vec{\mathbf{A}}$ at point (0, 29) and end on the tip of $\vec{\mathbf{R}}$ at point (0, −14) as shown in the sketch at the right. From this, it is seen that



ANS. FIG. P3.8

$$\boxed{\vec{\mathbf{B}} \text{ is 43 units in the negative } y \text{ direction}}$$

P3.9 In solving this problem we must contrast displacement with distance traveled. We draw a diagram of the skater's path in ANS. FIG. P3.9, which is the view from a hovering helicopter so that we can see the circular path as circular in shape. To start with a concrete example, we have chosen to draw motion *ABC* around one half of a circle of radius 5 m.



ANS. FIG. P3.9

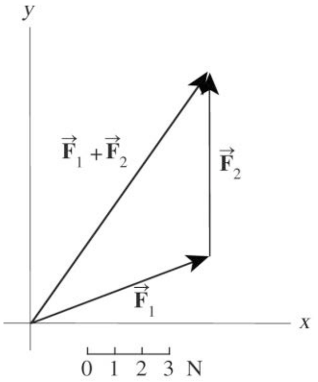
The displacement, shown as $\vec{\mathbf{d}}$ in the diagram, is the straight-line change in position from starting point *A* to finish *C*. In the specific case we have chosen to draw, it lies along a diameter of the circle. Its magnitude is $|\vec{\mathbf{d}}| = |-10.0\hat{\mathbf{i}}| = 10.0$ m.

The distance skated is greater than the straight-line displacement. The distance follows the curved path of the semicircle (*ABC*). Its length is half of the circumference: $s = \frac{1}{2}(2\pi r) = 5.00\pi \text{ m} = 15.7 \text{ m}$.

A straight line is the shortest distance between two points. For any nonzero displacement, less or more than across a semicircle, the distance along the path will be greater than the displacement magnitude. Therefore:

$$\boxed{\text{The situation can never be true because the distance is an arc of a circle between two points, whereas the magnitude of the displacement vector is a straight-line cord of the circle between the same points.}}$$

P3.10 We find the resultant $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ graphically by placing the tail of $\vec{\mathbf{F}}_2$ at the head of $\vec{\mathbf{F}}_1$. The resultant force vector $\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ is of magnitude $\boxed{9.5\text{ N}}$ and at an angle of $\boxed{57° \text{ above the } x \text{ axis}}$.



ANS. FIG. P3.10

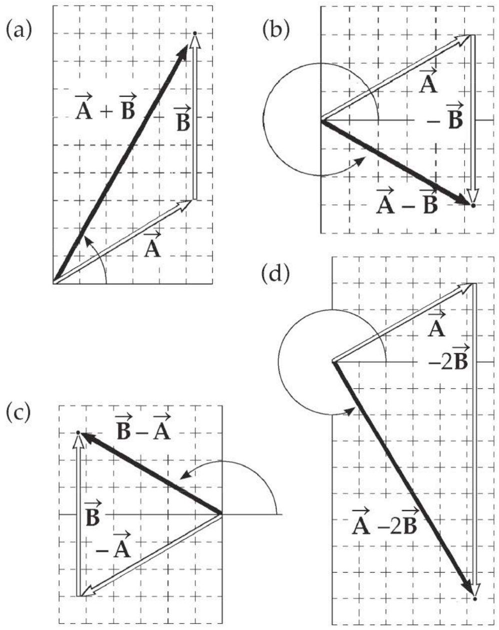
P3.11 To find these vector expressions graphically, we draw each set of vectors. Measurements of the results are taken using a ruler and protractor. (Scale: 1 unit = 0.5 m)

(a) $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{5.2 \text{ m at } 60^\circ}$

(b) $\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{3.0 \text{ m at } 330^\circ}$

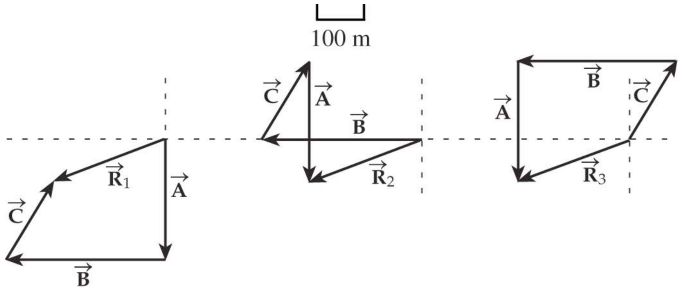
(c) $\vec{\mathbf{B}} - \vec{\mathbf{A}} = \boxed{3.0 \text{ m at } 150^\circ}$

(d) $\vec{\mathbf{A}} - 2\mathbf{B} = \boxed{5.2 \text{ m at } 300^\circ}$



ANS. FIG. P3.11

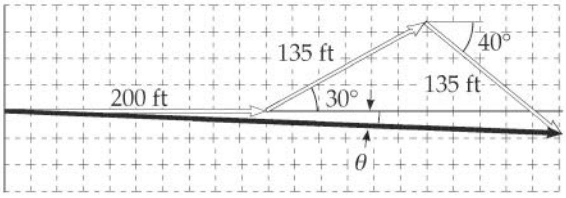
P3.12 (a) The three diagrams are shown in ANS. FIG. P3.12a below.



ANS. FIG. P3.12a

(b) The diagrams in ANS. FIG. P3.12a represent the graphical solutions for the three vector sums: $\vec{\mathbf{R}}_1 = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$, $\vec{\mathbf{R}}_2 = \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{A}}$, and $\vec{\mathbf{R}}_3 = \vec{\mathbf{C}} + \vec{\mathbf{B}} + \vec{\mathbf{A}}$.

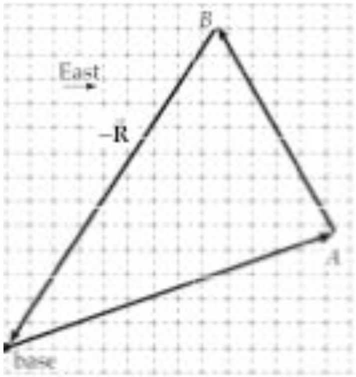
P3.13 The scale drawing for the graphical solution should be similar to the figure to the right. The magnitude and direction of the final displacement from the starting point are obtained by measuring d and θ on the drawing and applying the scale factor used in making the drawing. The results should be $\boxed{d = 420 \text{ ft and } \theta = -3^\circ}$.



(Scale: 1unit = 20 ft)

ANS. FIG. P3.13

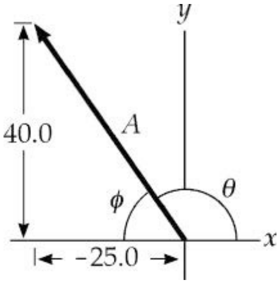
***P3.14** ANS. FIG. P3.14 shows the graphical addition of the vector from the base camp to lake A to the vector connecting lakes A and B, with a scale of 1 unit = 20 km. The distance from lake B to base camp is then the negative of this resultant vector, or $-\vec{\mathbf{R}} = \boxed{\text{310 km at 57° S of W}}$.



ANS. FIG. P3.14

Section 3.4 Components of a Vector and Unit Vectors

P3.15 First we should visualize the vector either in our mind or with a sketch, as shown in ANS. FIG. P3.15. The magnitude of the vector can be found by the Pythagorean theorem:



ANS. FIG. P3.15

$$A_x = -25.0$$

$$A_y = 40.0$$

$$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2}$$

$$= \boxed{47.2 \text{ units}}$$

We observe that

$$\tan\phi = \frac{|A_y|}{|A_x|}$$

so

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan^{-1}\left(\frac{40.0}{25.0}\right) = \tan^{-1}(1.60) = 58.0°$$

The diagram shows that the angle from the $+x$ axis can be found by subtracting from 180°: $\theta = 180° - 58° = \boxed{122°}$

P3.16 We can calculate the components of the vector A using $(A_x, A_y) = (A\cos\theta, A\sin\theta)$ if the angle θ is measured from the $+x$ axis, which is true here. For $A = 35.0$ units and $\theta = 325°$,

$$\boxed{A_x = 28.7 \text{ units}, \ A_y = -20.1 \text{ units}}$$

P3.17 (a) $\boxed{\text{Yes}}$.

(b) Let v represent the speed of the camper. The northward component of its velocity is $v \cos 8.50°$. To avoid crowding the minivan we require $v \cos 8.50° \geq 28$ m/s.

$$\boxed{\begin{array}{l}\text{We can satisfy this requirement simply by taking}\\ v \geq (28.0 \text{ m/s})/\cos 8.50° = 28.3 \text{ m/s.}\end{array}}$$

P3.18 The person would have to walk

$$(3.10 \text{ km})\sin 25.0° = \boxed{1.31 \text{ km north}}$$

and $$(3.10 \text{ km})\cos 25.0° = \boxed{2.81 \text{ km east}}$$

P3.19 Do not think of sin θ = opposite/hypotenuse, but jump right to $y = R \sin \theta$. The angle does not need to fit inside a triangle. We find the x and y components of each vector using $x = r \cos \theta$ and $y = r \sin \theta$. In unit vector notation, $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$.

(a) $x = 12.8 \cos 150°$, $y = 12.8 \sin 150°$, and $\boxed{(x, y) = (-11.1\hat{\mathbf{i}} + 6.40\hat{\mathbf{j}}) \text{ m}}$

(b) $x = 3.30 \cos 60.0°$, $y = 3.30 \sin 60.0°$, and $\boxed{(x, y) = (1.65\hat{\mathbf{i}} + 2.86\hat{\mathbf{j}}) \text{ cm}}$

(c) $x = 22.0 \cos 215°$, $y = 22.0 \sin 215°$, and $\boxed{(x, y) = (-18.0\hat{\mathbf{i}} - 12.6\hat{\mathbf{j}}) \text{ in}}$

P3.20 (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(x_{\text{net}})^2 + (y_{\text{net}})^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00 \text{ blocks}$$

and the angle the resultant makes with the x axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°.$$

The resultant displacement is then $\boxed{5.00 \text{ blocks at } 53.1° \text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.00 \text{ blocks}}$.

P3.21 Let $+x$ be East and $+y$ be North. We can sum the total x and y displacements of the spelunker as

$$\sum x = 250 \text{ m} + (125 \text{ m})\cos 30° = 358 \text{ m}$$
$$\sum y = 75 \text{ m} + (125 \text{ m})\sin 30° - 150 \text{ m} = -12.5 \text{ m}$$

the total displacement is then

$$d = \sqrt{\left(\sum x\right)^2 + \left(\sum y\right)^2} = \sqrt{(358\text{ m})^2 + (-12.5\text{ m})^2} = 358\text{ m}$$

at an angle of

$$\theta = \tan^{-1}\left(\frac{\sum y}{\sum x}\right) = \tan^{-1}\left(-\frac{12.5\text{ m}}{358\text{ m}}\right) = -2.00°$$

or $\boxed{\vec{\mathbf{d}} = 358\text{ m at }2.00°\text{ S of E}}$

P3.22 We use the numbers given in Problem 3.11:

$$\vec{\mathbf{A}} = 3.00\text{ m},\ \theta_A = 30.0°$$

$$A_x = A\cos\theta_A = 3.00\cos 30.0° = 2.60\text{ m},$$

$$A_y = A\sin\theta_A = 3.00\sin 30.0° = 1.50\text{ m}$$

So $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = (2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}})\text{ m}$

$$\vec{\mathbf{B}} = 3.00\text{ m}, \theta_B = 90.0°$$

$$B_x = 0, B_y = 3.00\text{ m} \rightarrow \vec{\mathbf{B}} = 3.00\hat{\mathbf{j}}\text{ m}$$

then $\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(2.60\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right) + 3.00\hat{\mathbf{j}} = \boxed{\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)\text{m}}$

P3.23 We can get answers in unit-vector form just by doing calculations with each term labeled with an $\hat{\mathbf{i}}$ or a $\hat{\mathbf{j}}$. There are, in a sense, only two vectors to calculate, since parts (c), (d), and (e) just ask about the magnitudes and directions of the answers to (a) and (b). Note that the whole numbers appearing in the problem statement are assumed to have three significant figures.

We use the property of vector addition that states that the components of $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ are computed as $R_x = A_x + B_x$ and $R_y = A_y + B_y$.

(a) $\left(\vec{\mathbf{A}} + \vec{\mathbf{B}}\right) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) + \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{2\hat{\mathbf{i}} - 6\hat{\mathbf{j}}}$

(b) $\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right) = \left(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\right) - \left(-\hat{\mathbf{i}} - 4\hat{\mathbf{j}}\right) = \boxed{4\hat{\mathbf{i}} + 2\hat{\mathbf{j}}}$

(c) $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \sqrt{2^2 + 6^2} = \boxed{6.32}$

(d) $\left|\vec{\mathbf{A}} - \vec{\mathbf{B}}\right| = \sqrt{4^2 + 2^2} = \boxed{4.47}$

(e) $\theta_{|\mathbf{A}+\mathbf{B}|} = \tan^{-1}\left(-\dfrac{6}{2}\right) = -71.6° = \boxed{288°}$

$\theta_{|\mathbf{A}-\mathbf{B}|} = \tan^{-1}\left(\dfrac{2}{4}\right) = \boxed{26.6°}$

P3.24 The east and north components of the displacement from Dallas (D) to Chicago (C) are the sums of the east and north components of the displacements from Dallas to Atlanta (A) and from Atlanta to Chicago. In equation form:

$$\begin{aligned} d_{\text{DC east}} &= d_{\text{DA east}} + d_{\text{AC east}} \\ &= (730\text{ mi})\cos 5.00° - (560\text{ mi})\sin 21.0° = 527\text{ miles} \end{aligned}$$

$$\begin{aligned} d_{\text{DC north}} &= d_{\text{DA north}} + d_{\text{AC north}} \\ &= (730\text{ mi})\sin 5.00° + (560\text{ mi})\cos 21.0° = 586\text{ miles} \end{aligned}$$

By the Pythagorean theorem,

$$d = \sqrt{(d_{\text{DC east}})^2 + (d_{\text{DC north}})^2} = 788\text{ mi}$$

Then, $\theta = \tan^{-1}\left(\dfrac{d_{\text{DC north}}}{d_{\text{DC east}}}\right) = 48.0°$

Thus, Chicago is $\boxed{\text{788 miles at 48.0° northeast of Dallas}}$.

P3.25 We use the unit-vector addition method. It is just as easy to add three displacements as to add two. We take the direction east to be along $+\hat{\mathbf{i}}$. The three displacements can be written as:

$$\begin{aligned} \vec{\mathbf{d}}_1 &= (-3.50\text{ m})\hat{\mathbf{j}} \\ \vec{\mathbf{d}}_2 &= (8.20\text{ m})\cos 45.0°\hat{\mathbf{i}} + (8.20\text{ m})\sin 45.0°\hat{\mathbf{j}} \\ &= (5.80\text{ m})\hat{\mathbf{i}} + (5.80\text{ m})\hat{\mathbf{j}} \\ \vec{\mathbf{d}}_3 &= (-15.0\text{ m})\hat{\mathbf{i}} \end{aligned}$$

The resultant is

$$\begin{aligned} \vec{\mathbf{R}} &= \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 = (-15.0\text{ m} + 5.80\text{ m})\hat{\mathbf{i}} + (5.80\text{ m} - 3.50\text{ m})\hat{\mathbf{j}} \\ &= (-9.20\text{ m})\hat{\mathbf{i}} + (2.30\text{ m})\hat{\mathbf{j}} \end{aligned}$$

(or 9.20 m west and 2.30 m north).

The magnitude of the resultant displacement is

$$\left|\vec{\mathbf{R}}\right| = \sqrt{R_x^2 + R_y^2} = \sqrt{(-9.20\text{ m})^2 + (2.30\text{ m})^2} = \boxed{9.48\text{ m}}$$

The direction of the resultant vector is given by

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{2.30\text{ m}}{-9.20\text{ m}}\right) = \boxed{166°}$$

P3.26 (a) See figure to the right.

(b) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$= \boxed{5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}}$

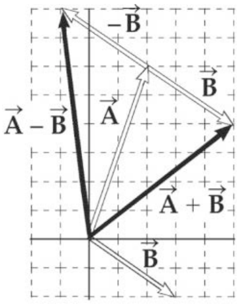
$\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}} = 2.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}$

$= \boxed{-1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}}$

(c) $\vec{\mathbf{C}} = \sqrt{25.0 + 16.0}$ at $\tan^{-1}\left(\frac{4}{5}\right) = \boxed{6.40 \text{ at } 38.7°}$

$\vec{\mathbf{D}} = \sqrt{(-1.00)^2 + (8.00)^2}$ at $\tan^{-1}\left(\frac{8.00}{-1.00}\right)$

$\vec{\mathbf{D}} = 8.06$ at $(180° - 82.9°) = \boxed{8.06 \text{ at } 97.2°}$



ANS. FIG. P3.26

P3.27 We first tabulate the three strokes of the novice golfer, with the x direction corresponding to East and the y direction corresponding to North. The sum of the displacement in each of the directions is shown as the last row of the table.

East	North
x (m)	y (m)
0	4.00
1.41	1.41
–0.500	–0.866
+0.914	4.55

The "hole-in-one" single displacement is then

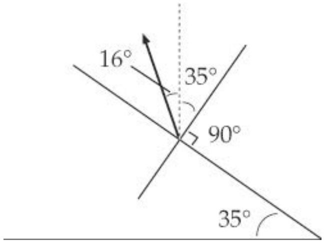
$$|\vec{\mathbf{R}}| = \sqrt{|x|^2 + |y|^2} = \sqrt{(0.914\text{ m})^2 + (4.55\text{ m})^2} = 4.64\text{ m}$$

The angle of the displacement with the horizontal is

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{4.55\text{ m}}{0.914\text{ m}}\right) = 78.6°$$

The expert golfer would accomplish the hole in one with the displacement $\boxed{4.64 \text{ m at } 78.6° \text{ N of E}}$.

P3.28 We take the *x* axis along the slope downhill. (Students, get used to this choice!) The *y* axis is perpendicular to the slope, at 35.0° to the vertical. Then the displacement of the snow makes an angle of 90.0° + 35.0° + 16.0° = 141° with the *x* axis.



ANS. FIG. P3.28

(a) $\boxed{\begin{array}{l}\text{Its component parallel to the surface is} \\ (1.50 \text{ m}) \cos 141° = -1.17 \text{ m}, \\ \text{or 1.17 m toward the top of the hill.}\end{array}}$

(b) $\boxed{\begin{array}{l}\text{Its component perpendicular to the surface is} \\ (1.50 \text{ m}) \sin 141° = 0.944 \text{ m, or 0.944 m away} \\ \text{from the snow.}\end{array}}$

P3.29 (a) The single force is obtained by summing the two forces:

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$$

$$\vec{\mathbf{F}} = 120 \cos(60.0°)\hat{\mathbf{i}} + 120 \sin(60.0°)\hat{\mathbf{j}} - 80.0 \cos(75.0°)\hat{\mathbf{i}} + 80.0 \sin(75.0°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}} = 60.0\hat{\mathbf{i}} + 104\hat{\mathbf{j}} - 20.7\hat{\mathbf{i}} + 77.3\hat{\mathbf{j}} = \left(39.3\hat{\mathbf{i}} + 181\hat{\mathbf{j}}\right) \text{ N}$$

We can also express this force in terms of its magnitude and direction:

$$\left|\vec{\mathbf{F}}\right| = \sqrt{39.3^2 + 181^2} \text{ N} = \boxed{185 \text{ N}}$$

$$\theta = \tan^{-1}\left(\frac{181}{39.3}\right) = \boxed{77.8°}$$

(b) A force equal and opposite the resultant force from part (a) is required for the total force to equal zero:

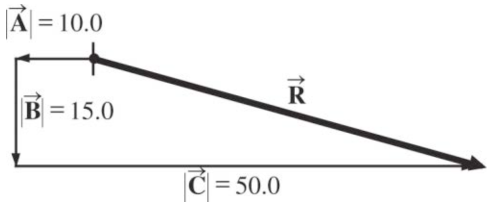
$$\vec{\mathbf{F}}_3 = -\vec{\mathbf{F}} = \boxed{\left(-39.3\hat{\mathbf{i}} - 181\hat{\mathbf{j}}\right) \text{ N}}$$

P3.30 ANS. FIG. P3.30 is a graphical depiction of the three displacements the football undergoes, with $\vec{\mathbf{A}}$ corresponding to the 10.0-yard backward run, $\vec{\mathbf{B}}$ corresponding to the 15.0-yard sideways run, and $\vec{\mathbf{C}}$ corresponding to the 50.0-yard downfield pass. The resultant vector is then

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = -10.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}} + 50.0\hat{\mathbf{i}}$$

$$= 40.0\hat{\mathbf{i}} - 15.0\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{R}}\right| = \left[(40.0)^2 + (-15.0)^2\right]^{1/2} = \boxed{42.7\text{ yards}}$$



ANS. FIG. P3.30

P3.31 (a) We add the components of the three vectors:

$$\vec{\mathbf{D}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} = 6\hat{\mathbf{i}} - 2\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{6^2 + 2^2} = \boxed{6.32\text{ m at }\theta = 342°}$$

(b) Again, using the components of the three vectors,

$$\vec{\mathbf{E}} = -\vec{\mathbf{A}} - \vec{\mathbf{B}} + \vec{\mathbf{C}} = -2\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$$

$$\left|\vec{\mathbf{E}}\right| = \sqrt{2^2 + 12^2} = \boxed{12.2\text{ m at }\theta = 99.5°}$$

P3.32 We are given $\vec{\mathbf{A}} = -8.70\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}$, and $\vec{\mathbf{B}} = 13.2\hat{\mathbf{i}} - 6.60\hat{\mathbf{j}}$, and $\vec{\mathbf{A}} - \vec{\mathbf{B}} + 3\vec{\mathbf{C}} = 0$. Solving for $\vec{\mathbf{C}}$ gives

$$3\vec{\mathbf{C}} = \vec{\mathbf{B}} - \vec{\mathbf{A}} = 21.9\hat{\mathbf{i}} - 21.6\hat{\mathbf{j}}$$

$$\vec{\mathbf{C}} = 7.30\hat{\mathbf{i}} - 7.20\hat{\mathbf{j}} \text{ or } C_x = \boxed{7.30\text{ cm}}; C_y = \boxed{-7.20\text{ cm}}$$

P3.33 Hold your fingertip at the center of the front edge of your study desk, defined as point *O*. Move your finger 8 cm to the right, then 12 cm vertically up, and then 4 cm horizontally away from you. Its location relative to the starting point represents position vector $\vec{\mathbf{A}}$. Move three-fourths of the way straight back toward *O*. Now your fingertip is at the location of $\vec{\mathbf{B}}$. Now move your finger 50 cm straight through *O*, through your left thigh, and down toward the floor. Its position vector now is $\vec{\mathbf{C}}$.

We use unit-vector notation throughout. There is no adding to do here, but just multiplication of a vector by two different scalars.

(a) $\vec{\mathbf{A}} = \boxed{8.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}} - 4.00\hat{\mathbf{k}}}$

(b) $\vec{\mathbf{B}} = \dfrac{\vec{\mathbf{A}}}{4} = \boxed{2.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 1.00\hat{\mathbf{k}}}$

(c) $\vec{\mathbf{C}} = -3\vec{\mathbf{A}} = \boxed{-24.0\hat{\mathbf{i}} - 36.0\hat{\mathbf{j}} + 12.0\hat{\mathbf{k}}}$

P3.34 We are given $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}} = 4.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}} + 3.00\hat{\mathbf{k}}$. The magnitude of the vector is therefore

$$\left|\vec{\mathbf{B}}\right| = \sqrt{4.00^2 + 6.00^2 + 3.00^2} = \boxed{7.81}$$

And the angle of the vector with the three coordinate axes is

$$\alpha = \cos^{-1}\left(\frac{4.00}{7.81}\right) = \boxed{59.2°} \text{ is the angle with the x axis}$$

$$\beta = \cos^{-1}\left(\frac{6.00}{7.81}\right) = \boxed{39.8°} \text{ is the angle with the y axis}$$

$$\gamma = \cos^{-1}\left(\frac{3.00}{7.81}\right) = \boxed{67.4°} \text{ is the angle with the z axis}$$

P3.35 The component description of $\vec{\mathbf{A}}$ is just restated to constitute the answer to part (a): $A_x = -3.00$, $A_y = 2.00$.

(a) $\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} = \boxed{-3.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}}}$

(b) $\left|\vec{\mathbf{A}}\right| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3.00)^2 + (2.00)^2} = \boxed{3.61}$

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{2.00}{-3.00}\right) = -33.7°$$

θ is in the second quadrant, so $\theta = 180° + (-33.7°) = \boxed{146°}$.

(c) $R_x = 0$, $R_y = -4.00$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$, thus $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$ and

$B_x = R_x - A_x = 0 - (-3.00) = 3.00$, $B_y = R_y - A_y = -4.00 - 2.00 = -6.00$.

Therefore, $\vec{\mathbf{B}} = \boxed{3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}$.

P3.36 We carry out the prescribed mathematical operations using unit vectors.

(a) $\vec{\mathbf{C}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \boxed{\left(5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\text{ m}}$

$$\left|\vec{\mathbf{C}}\right| = \sqrt{(5.00\text{ m})^2 + (1.00\text{ m})^2 + (3.00\text{ m})^2} = \boxed{5.92\text{ m}}$$

(b) $\vec{\mathbf{D}} = 2\vec{\mathbf{A}} - \vec{\mathbf{B}} = \boxed{\left(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}}\right)\text{m}}$

$$\left|\vec{\mathbf{D}}\right| = \sqrt{(4.00\text{ m})^2 + (11.0\text{ m})^2 + (15.0\text{ m})^2} = \boxed{19.0\text{ m}}$$

P3.37 (a) Taking components along $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$, we get two equations:

$$6.00a - 8.00b + 26.0 = 0$$

and

$$-8.00a + 3.00b + 19.0 = 0$$

Substituting $a = 1.33b - 4.33$ into the second equation, we find

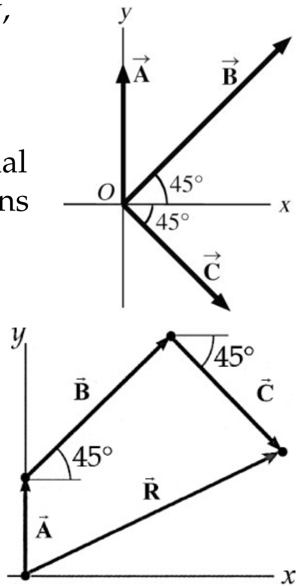
$$-8(1.33b - 4.33) + 3b + 19 = 0 \rightarrow 7.67b = 53.67 \rightarrow b = 7.00$$

and so $a = 1.33(7.00) - 4.33 = 5.00$.

Thus $\boxed{a = 5.00,\ b = 7.00}$. Therefore, $5.00\vec{\mathbf{A}} + 7.00\vec{\mathbf{B}} + \vec{\mathbf{C}} = 0$.

(b) In order for vectors to be equal, all of their components must be equal. A vector equation contains more information than a scalar equation, as each component gives us one equation.

P3.38 The given diagram shows the vectors individually, but not their addition. The second diagram represents a map view of the motion of the ball. According to the definition of a displacement, we ignore any departure from straightness of the actual path of the ball. We model each of the three motions as straight. The simplified problem is solved by straightforward application of the component method of vector addition. It works for adding two, three, or any number of vectors.



ANS. FIG. P3.38

(a) We find the two components of each of the three vectors

$$A_x = (20.0\text{ units})\cos 90° = 0$$

and $A_y = (20.0\text{ units})\sin 90° = 20.0\text{ units}$

$$B_x = (40.0\,\text{units})\cos 45° = 28.3 \text{ units}$$

and $$B_y = (40.0\,\text{units})\sin 45° = 28.3 \text{ units}$$

$$C_x = (30.0\,\text{units})\cos 315° = 21.2 \text{ units}$$

and $$C_y = (30.0\,\text{units})\sin 315° = -21.2 \text{ units}$$

Now adding,

$$R_x = A_x + B_x + C_x = (0 + 28.3 + 21.2) \text{ units} = 49.5 \text{ units}$$

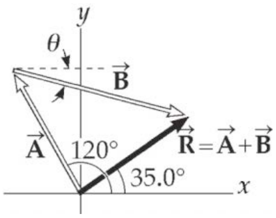
and $$R_y = A_y + B_y + C_y = (20 + 28.3 - 21.2) \text{ units} = 27.1 \text{ units}$$

so $$\vec{\mathbf{R}} = \boxed{49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}}$$

(b) $$\left|\vec{\mathbf{R}}\right| = \sqrt{(49.5)^2 + (27.1)^2} = \boxed{56.4}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{27.1}{49.5}\right) = \boxed{28.7°}$$

P3.39 We will use the component method for a precise answer. We already know the total displacement, so the algebra of solving a vector equation will guide us to do a subtraction.



ANS. FIG. P3.39

We have $\vec{\mathbf{B}} = \vec{\mathbf{R}} - \vec{\mathbf{A}}$:

$$A_x = 150\cos 120° = -75.0 \text{ cm}$$

$$A_y = 150\sin 120° = 130 \text{ cm}$$

$$R_x = 140\cos 35.0° = 115 \text{ cm}$$

$$R_y = 140\sin 35.0° = 80.3 \text{ cm}$$

Therefore,

$$\vec{\mathbf{B}} = [115 - (-75)\hat{\mathbf{i}} + [80.3 - 130]\hat{\mathbf{j}} = \left(190\hat{\mathbf{i}} - 49.7\hat{\mathbf{j}}\right) \text{ cm}$$

$$\left|\vec{\mathbf{B}}\right| = \sqrt{190^2 + 49.7^2} = \boxed{196 \text{ cm}}$$

$$\theta = \tan^{-1}\left(-\frac{49.7}{190}\right) = \boxed{-14.7°}$$

P3.40 First, we sum the components of the two vectors for the male:

$$d_{3mx} = d_{1mx} + d_{2mx} = 0 + (100\text{ cm})\cos 23.0° = 92.1\text{ cm}$$
$$d_{3my} = d_{1my} + d_{2my} = 104\text{ cm} + (100\text{ cm})\sin 23.0° = 143.1\text{ cm}$$

$$\boxed{\begin{array}{l}\text{magnitude: } d_{3m} = \sqrt{(92.1\text{ cm})^2 + (143.1\text{ cm})^2} = 170.1\text{ cm}\\ \text{direction: } \tan^{-1}(143.1/92.1) = 57.2° \text{ above } +x \text{ axis } (\text{first quadrant})\end{array}}$$

followed by the components of the two vectors for the female:

$$d_{3fx} = d_{1fx} + d_{2fx} = 0 + (86.0\text{ cm})\cos 28.0° = 75.9\text{ cm}$$
$$d_{3fy} = d_{1fy} + d_{2fy} = 84.0\text{ cm} + (86.0\text{ cm})\sin 28.0° = 124.4\text{ cm}$$

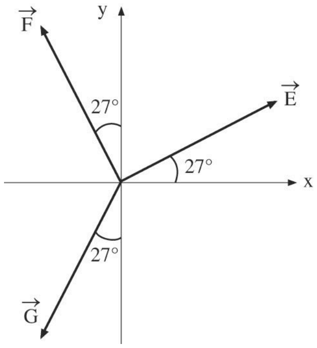
$$\boxed{\begin{array}{l}\text{magnitude: } d_{3f} = \sqrt{(75.9\text{ cm})^2 + (124.4\text{ cm})^2} = 145.7\text{ cm}\\ \text{direction: } \tan^{-1}(124.4/75.9) = 58.6° \text{ above } +x \text{ axis } (\text{first quadrant})\end{array}}$$

P3.41 (a) $\vec{\mathbf{E}} = (17.0\text{ cm})\cos(27.0°)\hat{\mathbf{i}} + (17.0\text{ cm})\sin(27.0°)\hat{\mathbf{j}}$

$$\vec{\mathbf{E}} = \boxed{(15.1\hat{\mathbf{i}} + 7.72\hat{\mathbf{j}})\text{ cm}}$$

(b) $\vec{\mathbf{F}} = (17.0\text{ cm})\cos(117.0°)\hat{\mathbf{i}} + (17.0\text{ cm})\sin(117.0°)\hat{\mathbf{j}}$

$$\vec{\mathbf{F}} = \boxed{\left(-7.72\hat{\mathbf{i}} + 15.1\hat{\mathbf{j}}\right)\text{ cm}}$$



ANS. FIG. P3.41

Note that we did not need to explicitly identify the angle with the positive *x* axis, but by doing so, we don't have to keep track of minus signs for the components.

(c) $\vec{\mathbf{G}} = [(-17.0\text{ cm})\cos(243.0°)]\hat{\mathbf{i}} + [(-17.0\text{ cm})\sin(243.0°)]\hat{\mathbf{j}}$

$$\boxed{\vec{\mathbf{G}} = \left(-7.72\hat{\mathbf{i}} - 15.1\hat{\mathbf{j}}\right)\text{ cm}}$$

P3.42 The position vector from radar station to ship is

$$\vec{\mathbf{S}} = \left(17.3\sin 136°\hat{\mathbf{i}} + 17.3\cos 136°\hat{\mathbf{j}}\right)\text{ km} = \left(12.0\hat{\mathbf{i}} - 12.4\hat{\mathbf{j}}\right)\text{ km}$$

From station to plane, the position vector is

$$\vec{\mathbf{P}} = \left(19.6\sin 153°\hat{\mathbf{i}} + 19.6\cos 153°\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km}$$

or

$$\vec{\mathbf{P}} = \left(8.90\hat{\mathbf{i}} - 17.5\hat{\mathbf{j}} + 2.20\hat{\mathbf{k}}\right)\text{ km}$$

(a) To fly to the ship, the plane must undergo displacement

$$\vec{\mathbf{D}} = \vec{\mathbf{S}} - \vec{\mathbf{P}} = \boxed{\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)\text{ km}}$$

(b) The distance the plane must travel is

$$D = \left|\vec{\mathbf{D}}\right| = \sqrt{(3.12)^2 + (5.02)^2 + (2.20)^2}\text{ km} = \boxed{6.31\text{ km}}$$

P3.43 The hurricane's first displacement is

$$(41.0\text{ km/h})(3.00\text{ h})\text{ at }60.0°\text{ N of W}$$

and its second displacement is

$$(25.0\text{ km/h})(1.50\text{ h})\text{ due North}$$

With $\hat{\mathbf{i}}$ representing east and $\hat{\mathbf{j}}$ representing north, its total displacement is:

$$\begin{aligned}&\left[(41.0\text{ km/h})\cos 60.0°\right](3.00\text{ h})\left(-\hat{\mathbf{i}}\right)\\ &\qquad + \left[(41.0\text{ km/h})\sin 60.0°\right](3.00\text{ h})\hat{\mathbf{j}}\\ &\qquad + (25.0\text{ km/h})(1.50\text{ h})\hat{\mathbf{j}}\\ &= 61.5\text{ km}\left(-\hat{\mathbf{i}}\right) + 144\text{ km }\hat{\mathbf{j}}\end{aligned}$$

with magnitude $\sqrt{(61.5\text{ km})^2 + (144\text{ km})^2} = \boxed{157\text{ km}}$.

P3.44 Note that each shopper must make a choice whether to turn 90° to the left or right, each time he or she makes a turn. One set of such choices, following the rules in the problem, results in the shopper heading in the positive y direction and then again in the positive x direction.

Find the magnitude of the sum of the displacements:

$$\vec{\mathbf{d}} = (8.00\text{ m})\hat{\mathbf{i}} + (3.00\text{ m})\hat{\mathbf{j}} + (4.00\text{ m})\hat{\mathbf{i}} = (12.00\text{ m})\hat{\mathbf{i}} + (3.00\text{ m})\hat{\mathbf{j}}$$

$$\text{magnitude: } d = \sqrt{(12.00\text{ m})^2 + (3.00\text{ m})^2} = 12.4\text{ m}$$

$$\boxed{\text{Impossible because 12.4 m is greater than 5.00 m.}}$$

P3.45 The y coordinate of the airplane is constant and equal to 7.60×10^3 m whereas the x coordinate is given by $x = v_i t$, where v_i is the constant speed in the horizontal direction.

At $t = 30.0$ s we have $x = 8.04 \times 10^3$, so $v_i = 8\,040\text{ m}/30\text{ s} = 268\text{ m/s}$. The position vector as a function of time is

$$\vec{\mathbf{P}} = (268\text{ m/s})t\hat{\mathbf{i}} + (7.60 \times 10^3\text{ m})\hat{\mathbf{j}}$$

At $t = 45.0\text{ s}$, $\vec{\mathbf{P}} = \left[1.21\times10^4\hat{\mathbf{i}} + 7.60\times10^3\hat{\mathbf{j}}\right]\text{ m}$. The magnitude is

$$\vec{\mathbf{P}} = \sqrt{\left(1.21\times10^4\right)^2 + \left(7.60\times10^3\right)^2}\text{ m} = \boxed{1.43\times10^4\text{ m}}$$

and the direction is

$$\theta = \tan^{-1}\left(\frac{7.60\times10^3}{1.21\times10^4}\right) = \boxed{32.2°\text{ above the horizontal}}$$

P3.46 The displacement from the start to the finish is

$$16\hat{\mathbf{i}} + 12\hat{\mathbf{j}} - (5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}) = (11\hat{\mathbf{i}} + 9\hat{\mathbf{j}})$$

The displacement from the starting point to *A* is $f\left(11\hat{\mathbf{i}} + 9\hat{\mathbf{j}}\right)$ meters.

(a) The position vector of point *A* is

$$5\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + f\left(11\hat{\mathbf{i}} + 9\hat{\mathbf{j}}\right) = \boxed{\left[(5+11f)\hat{\mathbf{i}} + (3+9f)\hat{\mathbf{j}}\right]\text{ m}}$$

(b) For $f = 0$ we have the position vector $\boxed{(5+0)\hat{\mathbf{i}} + (3+0)\hat{\mathbf{j}}\text{ meters.}}$

(c) $\boxed{\text{This is reasonable because it is the location of the starting point, } 5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\text{ meters.}}$

(d) For $f = 1 = 100\%$, we have position vector
$(5+11)\hat{\mathbf{i}} + (3+9)\hat{\mathbf{j}}\text{ meters} = \boxed{16\hat{\mathbf{i}} + 12\hat{\mathbf{j}}\text{ meters}}$.

(e) $\boxed{\text{This is reasonable because we have completed the trip, and this is the position vector of the endpoint.}}$

P3.47 Let the positive *x* direction be eastward, the positive *y* direction be vertically upward, and the positive *z* direction be southward. The total displacement is then

$$\begin{aligned}\vec{\mathbf{d}} &= \left(4.80\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}}\right)\text{ cm} + \left(3.70\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm}\\ &= \left(4.80\hat{\mathbf{i}} + 8.50\hat{\mathbf{j}} - 3.70\hat{\mathbf{k}}\right)\text{ cm}\end{aligned}$$

(a) The magnitude is $d = \sqrt{(4.80)^2 + (8.50)^2 + (-3.70)^2}\text{ cm} = \boxed{10.4\text{ cm}}$.

(b) Its angle with the *y* axis follows from
$\cos\theta = \frac{8.50}{10.4}$, giving $\boxed{\theta = 35.5°}$.

Additional Problems

P3.48 The Pythagorean theorem and the definition of the tangent will be the starting points for our calculation.

(a) Take the wall as the xy plane so that the coordinates are $x = 2.00$ m and $y = 1.00$ m; and the fly is located at point P. The distance between two points in the xy plane is

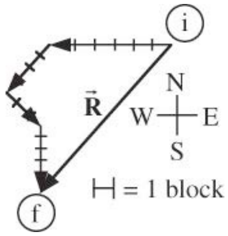
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

so here $d = \sqrt{(2.00\text{ m} - 0)^2 + (1.00\text{ m} - 0)^2} = \boxed{2.24\text{ m}}$

(b) $\theta = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{1.00\text{ m}}{2.00\text{ m}}\right) = 26.6°$, so $\vec{\mathbf{r}} = \boxed{2.24\text{ m}, 26.6°}$

P3.49 We note that $-\hat{\mathbf{i}}$ = west and $-\hat{\mathbf{j}}$ = south. The given mathematical representation of the trip can be written as 6.30 b west + 4.00 b at 40° south of west +3.00 b at 50° south of east +5.00 b south.

(a) The figure on the right shows a map of the successive displacements that the bus undergoes.



ANS. FIG. P3.49

(b) The total odometer distance is the sum of the magnitudes of the four displacements:

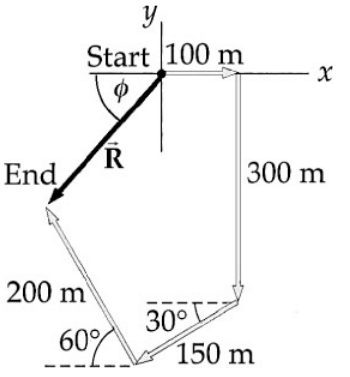
$$6.30\text{ b} + 4.00\text{ b} + 3.00\text{ b} + 5.00\text{ b} = \boxed{18.3\text{ b}}$$

(c)
$$\vec{\mathbf{R}} = (-6.30 - 3.06 + 1.93)\text{ b}\hat{\mathbf{i}} + (-2.57 - 2.30 - 5.00)\text{ b}\hat{\mathbf{j}}$$
$$= -7.44\text{ b}\hat{\mathbf{i}} - 9.87\text{ b}\hat{\mathbf{j}}$$
$$= \sqrt{(7.44\text{ b})^2 + (9.87\text{ b})^2}\text{ at }\tan^{-1}\left(\frac{9.87}{7.44}\right)\text{ south of west}$$
$$= 12.4\text{ b at }53.0°\text{ south of west}$$
$$= \boxed{12.4\text{ b at }233°\text{ counterclockwise from east}}$$

P3.50 To find the new speed and direction of the aircraft, we add the vector components of the wind to the vector velocity of the aircraft:

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (300 + 100\cos 30.0°)\hat{\mathbf{i}} + (100\sin 30.0°)\hat{\mathbf{j}}$$
$$\vec{\mathbf{v}} = (387\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}})\text{ mi/h}$$
$$|\vec{\mathbf{v}}| = \boxed{390\text{ mi/h at }7.37°\text{ N of E}}$$

P3.51 On our version of the diagram we have drawn in the resultant from the tail of the first arrow to the head of the last arrow. The resultant displacement $\vec{\mathbf{R}}$ is equal to the sum of the four individual displacements, $\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4$. We translate from the pictorial representation to a mathematical representation by writing the individual displacements in unit-vector notation:



ANS. FIG. P3.51

$$\vec{\mathbf{d}}_1 = 100\hat{\mathbf{i}} \text{ m}$$
$$\vec{\mathbf{d}}_2 = -300\hat{\mathbf{j}} \text{ m}$$
$$\vec{\mathbf{d}}_3 = (-150 \cos 30°)\hat{\mathbf{i}} \text{ m} + (-150 \sin 30°)\hat{\mathbf{j}} \text{ m} = -130\hat{\mathbf{i}} \text{ m} - 75\hat{\mathbf{j}} \text{ m}$$
$$\vec{\mathbf{d}}_4 = (-200 \cos 60°)\hat{\mathbf{i}} \text{ m} + (200 \sin 60°)\hat{\mathbf{j}} \text{ m} = -100\hat{\mathbf{i}} \text{ m} + 173\hat{\mathbf{j}} \text{ m}$$

Summing the components together, we find

$$R_x = d_{1x} + d_{2x} + d_{3x} + d_{4x} = (100 + 0 - 130 - 100) \text{ m} = -130 \text{ m}$$
$$R_y = d_{1y} + d_{2y} + d_{3y} + d_{4y} = (0 - 300 - 75 + 173) \text{ m} = -202 \text{ m}$$

so altogether

$$\vec{\mathbf{R}} = \vec{\mathbf{d}}_1 + \vec{\mathbf{d}}_2 + \vec{\mathbf{d}}_3 + \vec{\mathbf{d}}_4 = \boxed{\left(-130\hat{\mathbf{i}} - 202\hat{\mathbf{j}}\right)\text{m}}$$

Its magnitude is

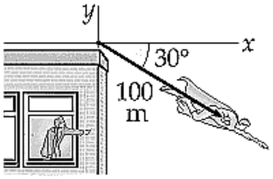
$$\left|\vec{\mathbf{R}}\right| = \sqrt{(-130)^2 + (-202)^2} = \boxed{240 \text{ m}}$$

We calculate the angle $\phi = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{-202}{-130}\right) = 57.2°$.

The resultant points into the third quadrant instead of the first quadrant. The angle counterclockwise from the $+x$ axis is

$$\theta = 180 + \phi = \boxed{237°}$$

***P3.52** The superhero follows a straight-line path at 30.0° below the horizontal. If his displacement is 100 m, then the coordinates of the superhero are:



ANS. FIG. P3.52

$$x = (100 \text{ m})\cos(-30.0°) = \boxed{86.6 \text{ m}}$$
$$y = (100 \text{ m})\sin(-30.0°) = \boxed{-50.0 \text{ m}}$$

P3.53 (a) Take the *x* axis along the tail section of the snake. The displacement from tail to head is

$$\left(240\text{ m}\right)\hat{\mathbf{i}}+\left[(420-240)\text{ m}\right]\cos(180°-105°)\hat{\mathbf{i}}$$
$$-\left(180\text{ m}\right)\sin 75°\hat{\mathbf{j}}=287\text{ m}\hat{\mathbf{i}}-174\text{ m}\hat{\mathbf{j}}$$

Its magnitude is $\sqrt{\left(287\right)^2+\left(174\right)^2}\text{ m}=335\text{ m}$.

From $\mathrm{v}=\dfrac{\text{distance}}{\Delta \mathrm{t}}$, the time for each child's run is

$$\text{Inge: }\Delta\mathrm{t}=\frac{\text{distance}}{\mathrm{v}}=\frac{335\text{ m}\left(\text{h}\right)\left(1\text{ km}\right)\left(3600\text{ s}\right)}{\left(12\text{ km}\right)\left(1000\text{ m}\right)\left(1\text{ h}\right)}=101\text{ s}$$

$$\text{Olaf: }\Delta\mathrm{t}=\frac{420\text{ m}\cdot\text{s}}{3.33\text{ m}}=126\text{ s}$$

Inge wins by $126-101=\boxed{25.4\text{ s}}$.

(b) Olaf must run the race in the same time:

$$\mathrm{v}=\frac{\mathrm{d}}{\Delta\mathrm{t}}=\frac{420\text{ m}}{101\text{ s}}\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{\text{km}}{10^3\text{ m}}\right)=\boxed{15.0\text{ km/h}}$$

P3.54 The position vector from the ground under the controller of the first airplane is

$$\vec{\mathbf{r}}_1=(19.2\text{ km})(\cos 25°)\hat{\mathbf{i}}+(19.2\text{ km})(\sin 25°)\hat{\mathbf{j}}+(0.8\text{ km})\hat{\mathbf{k}}$$
$$=\left(17.4\hat{\mathbf{i}}+8.11\hat{\mathbf{j}}+0.8\hat{\mathbf{k}}\right)\text{ km}$$

The second is at

$$\vec{\mathbf{r}}_2=(17.6\text{ km})(\cos 20°)\hat{\mathbf{i}}+(17.6\text{ km})(\sin 20°)\hat{\mathbf{j}}+(1.1\text{ km})\hat{\mathbf{k}}$$
$$=\left(16.5\hat{\mathbf{i}}+6.02\hat{\mathbf{j}}+1.1\hat{\mathbf{k}}\right)\text{ km}$$

Now the displacement from the first plane to the second is

$$\vec{\mathbf{r}}_2-\vec{\mathbf{r}}_1=\left(-0.863\hat{\mathbf{i}}-2.09\hat{\mathbf{j}}+0.3\hat{\mathbf{k}}\right)\text{ km}$$

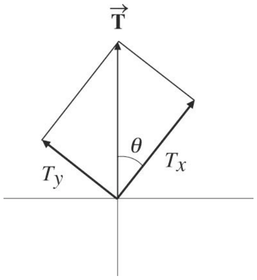
with magnitude

$$\sqrt{\left(0.863\right)^2+\left(2.09\right)^2+\left(0.3\right)^2}\text{ km}=\boxed{2.29\text{ km}}$$

P3.55 (a) The tensions T_x and T_y act as an equivalent tension T (see ANS. FIG. P3.55) which supports the downward weight; thus, the combination is equivalent to 0.150 N, upward. We know that T_x = 0.127 N, and the tensions are perpendicular to each other, so their combined magnitude is

$$T = \sqrt{T_x^2 + T_y^2} = 0.150\text{ N} \rightarrow T_y^2 = (0.150\text{ N})^2 - T_x^2$$

$$T_y^2 = (0.150\text{ N})^2 - (0.127\text{ N})^2 \rightarrow T_y = 0.078\text{ N}$$



ANS. FIG. P3.55

(b) From the figure, $\theta = \tan^{-1}(T_y/T_x) = 32.1°$. The angle the x axis makes with the horizontal axis is $90° - \theta = \boxed{57.9°}$.

(c) From the figure, the angle the y axis makes with the horizontal axis is $\theta = \boxed{32.1°}$.

P3.56 (a) Consider the rectangle in the figure to have height H and width W. The vectors $\vec{A}$ and $\vec{B}$ are related by $\vec{A} + \overrightarrow{ab} + \overrightarrow{bc} = \vec{B}$, where

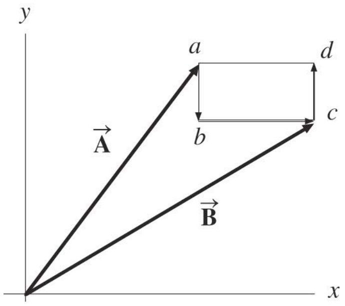
$$\vec{A} = (10.0\text{ m})(\cos 50.0°)\hat{i} + (10.0\text{ m})(\sin 50.0°)\hat{j}$$

$$\vec{A} = (6.42\hat{i} + 7.66\hat{j})\text{ m}$$

$$\vec{B} = (12.0\text{ m})(\cos 30.0°)\hat{i} + (12.0\text{ m})(\sin 30.0°)\hat{j}$$

$$\vec{B} = (10.4\hat{i} + 6.00\hat{j})\text{ m}$$

$$\overrightarrow{ab} = -H\hat{j} \text{ and } \overrightarrow{bc} = W\hat{i}$$



ANS. FIG. P3.56

Therefore,

$$\vec{\mathbf{B}} - \vec{\mathbf{A}} = \overrightarrow{\mathbf{ab}} + \overrightarrow{\mathbf{bc}}$$

$$\left(3.96\hat{\mathbf{i}} - 1.66\hat{\mathbf{j}}\right)\text{ m} = W\hat{\mathbf{i}} - H\hat{\mathbf{j}} \rightarrow W = 3.96\text{ m and } H = 1.66\text{ m}$$

$$\boxed{\text{The perimeter measures } 2(H + W) = 11.24\text{ m.}}$$

(b) The vector from the origin to the upper-right corner of the rectangle (point *d*) is

$$\vec{\mathbf{B}} + H\hat{\mathbf{j}} = 10.4\text{ m}\hat{\mathbf{i}} + (6.00\text{ m} + 1.66\text{ m})\hat{\mathbf{j}} = 10.4\text{ m}\hat{\mathbf{i}} + 7.66\text{ m}\hat{\mathbf{j}}$$

$$\boxed{\begin{array}{l}\text{magnitude: } \sqrt{(10.4\text{ m})^2 + (7.66\text{ m})^2} = 12.9\text{ m} \\ \text{direction: } \tan^{-1}(7.66/10.4) = 36.4° \text{ above } +x \text{ axis } (\text{first quadrant})\end{array}}$$

P3.57 (a) $R_x = \boxed{2.00}$, $R_y = \boxed{1.00}$, $R_z = \boxed{3.00}$

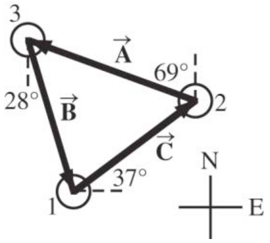
(b) $\left|\vec{\mathbf{R}}\right| = \sqrt{R_x^2 + R_y^2 + R_z^2} = \sqrt{4.00 + 1.00 + 9.00} = \sqrt{14.0} = \boxed{3.74}$

(c)
$$\cos\theta_x = \frac{R_x}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_x = \cos^{-1}\left(\frac{R_x}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{57.7° \text{ from } +x}$$

$$\cos\theta_y = \frac{R_y}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_y = \cos^{-1}\left(\frac{R_y}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{74.5° \text{ from } +y}$$

$$\cos\theta_z = \frac{R_z}{\left|\vec{\mathbf{R}}\right|} \Rightarrow \theta_z = \cos^{-1}\left(\frac{R_z}{\left|\vec{\mathbf{R}}\right|}\right) = \boxed{36.7° \text{ from } +z}$$

P3.58 Let *A* represent the distance from island 2 to island 3. The displacement is $\vec{\mathbf{A}} = A$ at 159°. Represent the displacement from 3 to 1 as $\vec{\mathbf{B}} = B$ at 298°. We have 4.76 km at $37° + \vec{\mathbf{A}} + \vec{\mathbf{B}} = 0$.



ANS. FIG. P3.58

For the *x* components:

$$(4.76\text{ km})\cos 37° + A\cos 159° + B\cos 298° = 0$$

$$3.80\text{ km} - 0.934A + 0.470B = 0$$

$$B = -8.10\text{ km} + 1.99A$$

For the *y* components:

$$(4.76\text{ km})\sin 37° + A\sin 159° + B\sin 298° = 0$$

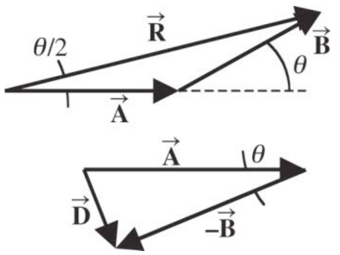
$$2.86\text{ km} + 0.358A - 0.883B = 0$$

(a) We solve by eliminating B by substitution:

$$2.86\text{ km} + 0.358A - 0.883(-8.10\text{ km} + 1.99A) = 0$$
$$2.86\text{ km} + 0.358A + 7.15\text{ km} - 1.76A = 0$$
$$10.0\text{ km} = 1.40A$$
$$A = \boxed{7.17\text{ km}}$$

(b) $B = -8.10\text{ km} + 1.99(7.17\text{ km}) = \boxed{6.15\text{ km}}$

P3.59 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. This can be seen from applying the law of cosines to the isosceles triangle and using the fact that $B = A$.



ANS. FIG. P3.59

Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

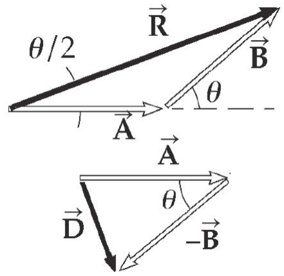
gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = 100D$.

Thus, $2A\cos\left(\frac{\theta}{2}\right) = 200A\sin\left(\frac{\theta}{2}\right)$. This gives

$$\tan\left(\frac{\theta}{2}\right) = 0.010 \text{ and } \boxed{\theta = 1.15°}$$

P3.60 Let θ represent the angle between the directions of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Since $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ have the same magnitudes, $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$ form an isosceles triangle in which the angles are $180° - \theta$, $\frac{\theta}{2}$, and $\frac{\theta}{2}$. The magnitude of $\vec{\mathbf{R}}$ is then $R = 2A\cos\left(\frac{\theta}{2}\right)$. This can be seen by applying the



ANS. FIG. P3.60

law of cosines to the isosceles triangle and using the fact that $B = A$. Again, $\vec{\mathbf{A}}$, $-\vec{\mathbf{B}}$, and $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ form an isosceles triangle with apex angle θ. Applying the law of cosines and the identity

$$1 - \cos\theta = 2\sin^2\left(\frac{\theta}{2}\right)$$

gives the magnitude of $\vec{\mathbf{D}}$ as $D = 2A\sin\left(\frac{\theta}{2}\right)$.

The problem requires that $R = nD$ or

$$\cos\left(\frac{\theta}{2}\right) = n\sin\left(\frac{\theta}{2}\right) \text{ giving } \boxed{\theta = 2\tan^{-1}\left(\frac{1}{n}\right)}.$$

The larger R is to be compared to D, the smaller the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ becomes.

P3.61 (a) We write $\vec{\mathbf{B}}$ in terms of the sine and cosine of the angle θ, and add the two vectors:

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(-60\text{ cm}\hat{\mathbf{j}}\right) + \left(80\text{ cm}\cos\theta\right)\hat{\mathbf{i}} + \left(80\text{ cm}\sin\theta\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = \left(80\text{ cm}\cos\theta\right)\hat{\mathbf{i}} + \left(80\text{ cm}\sin\theta - 60\text{ cm}\right)\hat{\mathbf{j}}$$

Dropping units (cm), the magnitude is

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[\left(80\cos\theta\right)^2 + \left(80\sin\theta - 60\right)^2\right]^{1/2}$$

$$= \left[(80)^2\left(\cos^2\theta + \sin^2\theta\right) - 2(80)(60)\sin\theta + (60)^2\right]^{1/2}$$

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \left[(80)^2 + (60)^2 - 2(80)(60)\sin\theta\right]^{1/2}$$

$$\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{\left[10{,}000 - (9600)\sin\theta\right]^{1/2}\text{ cm}}$$

(b) For $\theta = 270°$, $\sin\theta = -1$, and $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{140\text{ cm}}$.

(c) For $\theta = 90°$, $\sin\theta = 1$, and $\left|\vec{\mathbf{A}} + \vec{\mathbf{B}}\right| = \boxed{20.0\text{ cm}}$.

(d) They do make sense. The maximum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in the same direction, and it is 60 cm + 80 cm. The minimum value is attained when $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ are in opposite directions, and it is 80 cm − 60 cm.

P3.62 We perform the integration:

$$\Delta\vec{\mathbf{r}} = \int_0^{0.380\text{ s}} \vec{\mathbf{v}}\,dt = \int_0^{0.380\text{ s}} \left(1.2\hat{\mathbf{i}}\text{ m/s} - 9.8t\hat{\mathbf{j}}\text{ m/s}^2\right)dt$$

$$= 1.2t\,\hat{\mathbf{i}}\text{ m/s}\Big|_0^{0.380\text{ s}} - \left(9.8\hat{\mathbf{j}}\text{ m/s}^2\right)\frac{t^2}{2}\Bigg|_0^{0.380\text{ s}}$$

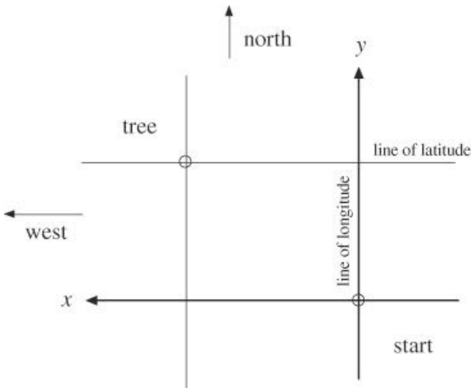
$$= \left(1.2\hat{\mathbf{i}}\text{ m/s}\right)(0.38\text{ s} - 0) - \left(9.8\hat{\mathbf{j}}\text{ m/s}^2\right)\left(\frac{(0.38\text{ s})^2 - 0}{2}\right)$$

$$= \boxed{0.456\hat{\mathbf{i}}\text{ m} - 0.708\hat{\mathbf{j}}\text{ m}}$$

P3.63 (a) $\dfrac{d\vec{\mathbf{r}}}{dt} = \dfrac{d\left(4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2t\hat{\mathbf{k}}\right)}{dt} = -2\hat{\mathbf{k}} = \boxed{-(2.00\text{ m/s})\hat{\mathbf{k}}}$

(b) The position vector at $t = 0$ is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$. At $t = 1$ s, the position is $4\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}}$, and so on. The object is moving straight downward at 2 m/s, so $\dfrac{d\vec{\mathbf{r}}}{dt}$ represents $\boxed{\text{its velocity vector}}$.

P3.64 (a) The very small differences between the angles suggests we may consider this region of Earth to be small enough so that we may consider it to be flat (a plane); therefore, we may consider the lines of latitude and longitude to be parallel and perpendicular, so that we can use them as an *xy* coordinate system. Values of latitude, θ, increase as we travel north, so differences between latitudes can give the *y* coordinate. Values of longitude, ϕ, increase as we travel west, so differences between longitudes can give the *x* coordinate. Therefore, our coordinate system will have +*y* to the north and +*x* to the west.



ANS. FIG. P3.64

Since we are near the equator, each line of latitude and longitude may be considered to form a circle with a radius equal to the radius of Earth, $R = 6.36 \times 10^6$ m. Recall the length *s* of an arc of a circle of radius *R* that subtends an angle (in radians) $\Delta\theta$ (or $\Delta\phi$) is given by $s = R\Delta\theta$ (or $s = R\Delta\phi$). We can use this equation to find the components of the displacement from the starting point to the tree—these are parallel to the *x* and *y* coordinates axes. Therefore, we can regard the origin to be the starting point and the displacements as the x and y coordinates of the tree.

The angular difference $\Delta\phi$ for longitude values is (west being positive)

$$\begin{aligned}\Delta\phi &= [75.64426° - 75.64238°] \\ &= (0.00188°)(\pi \text{ rad}/180°) \\ &= 3.28\times 10^{-5} \text{ rad}\end{aligned}$$

corresponding to the x coordinate (displacement west)

$$x = R\Delta\phi = (6.36\times 10^{6} \text{ m})(3.28\times 10^{-5} \text{ rad}) = 209 \text{ m}$$

The angular difference $\Delta\theta$ for latitude values is (north being positive)

$$\begin{aligned}\Delta\theta &= [0.00162° - (-0.00243°)] \\ &= (0.00405°)(\pi \text{ rad}/180°) \\ &= 7.07\times 10^{-5} \text{ rad}\end{aligned}$$

corresponding to the y coordinate (displacement north)

$$y = R\Delta\theta = (6.36\times 10^{6} \text{ m})(7.07\times 10^{-5} \text{ rad}) = 450 \text{ m}$$

The distance to the tree is

$$d = \sqrt{x^2 + y^2} = \sqrt{(209 \text{ m})^2 + (450 \text{ m})^2} = \boxed{496 \text{ m}}$$

The direction to the tree is

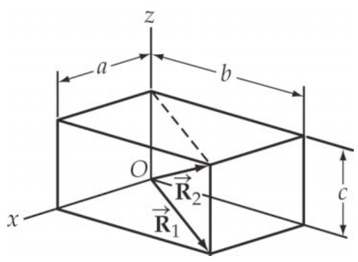
$$\tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{450 \text{ m}}{209 \text{ m}}\right) = 65.1° = \boxed{65.1° \text{ N of W}}$$

(b) $\boxed{\text{Refer to the arguments above. They are justified because the distances involved are small relative to the radius of Earth.}}$

P3.65 (a) From the picture, $\boxed{\vec{\mathbf{R}}_1 = a\hat{\mathbf{i}} + b\hat{\mathbf{j}}.}$

(b) $\boxed{R_1 = \sqrt{a^2 + b^2}}$

(c) $\boxed{\vec{\mathbf{R}}_2 = \vec{\mathbf{R}}_1 + c\hat{\mathbf{k}} = a\hat{\mathbf{i}} + b\hat{\mathbf{j}} + c\hat{\mathbf{k}}}$



ANS. FIG. P3.65

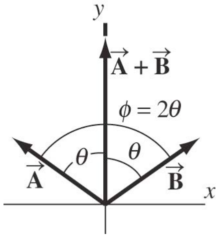
P3.66 Since

$$\vec{\mathbf{A}} + \vec{\mathbf{B}} = 6.00\hat{\mathbf{j}},$$

we have

$$(A_x + B_x)\hat{\mathbf{i}} + (A_y + B_y)\hat{\mathbf{j}} = 0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}}$$

giving $A_x + B_x = 0 \rightarrow A_x = -B_x$.



ANS. FIG. P3.66

Because the vectors have the same magnitude and x components of equal magnitude but of opposite sign, the vectors are reflections of each other in the y axis, as shown in the diagram. Therefore, the two vectors have the same y components:

$$A_y = B_y = (1/2)(6.00) = 3.00$$

Defining θ as the angle between either $\vec{\mathbf{A}}$ or $\vec{\mathbf{B}}$ and the y axis, it is seen that

$$\cos\theta = \frac{A_y}{A} = \frac{B_y}{B} = \frac{3.00}{5.00} = 0.600 \rightarrow \theta = 53.1°$$

The angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is then $\boxed{\phi = 2\theta = 106°}$.

Challenge Problem

P3.67 (a) You start at point A: $\vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_A = (30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}})$ m.

The displacement to B is

$$\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A = 60.0\hat{\mathbf{i}} + 80.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}} = 30.0\hat{\mathbf{i}} + 100\hat{\mathbf{j}}$$

You cover half of this, $(15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}})$, to move to

$$\vec{\mathbf{r}}_2 = 30.0\hat{\mathbf{i}} - 20.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{i}} + 50.0\hat{\mathbf{j}} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}}$$

Now the displacement from your current position to C is

$$\vec{\mathbf{r}}_C - \vec{\mathbf{r}}_2 = -10.0\hat{\mathbf{i}} - 10.0\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} = -55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}$$

You cover one-third, moving to

$$\vec{\mathbf{r}}_3 = \vec{\mathbf{r}}_2 + \Delta\vec{\mathbf{r}}_{23} = 45.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}} + \frac{1}{3}(-55.0\hat{\mathbf{i}} - 40.0\hat{\mathbf{j}}) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}}$$

The displacement from where you are to *D* is

$$\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3 = 40.0\hat{\mathbf{i}} - 30.0\hat{\mathbf{j}} - 26.7\hat{\mathbf{i}} - 16.7\hat{\mathbf{j}} = 13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}$$

You traverse one-quarter of it, moving to

$$\vec{\mathbf{r}}_4 = \vec{\mathbf{r}}_3 + \frac{1}{4}\left(\vec{\mathbf{r}}_D - \vec{\mathbf{r}}_3\right) = 26.7\hat{\mathbf{i}} + 16.7\hat{\mathbf{j}} + \frac{1}{4}\left(13.3\hat{\mathbf{i}} - 46.7\hat{\mathbf{j}}\right)$$
$$= 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}$$

The displacement from your new location to *E* is

$$\vec{\mathbf{r}}_E - \vec{\mathbf{r}}_4 = -70.0\hat{\mathbf{i}} + 60.0\hat{\mathbf{j}} - 30.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}} = -100\hat{\mathbf{i}} + 55.0\hat{\mathbf{j}}$$

of which you cover one-fifth the distance, $-20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}}$, moving to

$$\vec{\mathbf{r}}_4 + \Delta\vec{\mathbf{r}}_{45} = 30.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 20.0\hat{\mathbf{i}} + 11.0\hat{\mathbf{j}} = 10.0\hat{\mathbf{i}} + 16.0\hat{\mathbf{j}}$$

The treasure is at $\boxed{(10.0\text{ m}, 16.0\text{ m})}$.

(b) Following the directions brings you to the average position of the trees. The steps we took numerically in part (a) bring you to

$$\vec{\mathbf{r}}_A + \frac{1}{2}\left(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right) = \left(\frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B}{2}\right)$$

then to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)}{2} + \frac{\vec{\mathbf{r}}_C - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B\right)/2}{3} = \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C}{3}$$

then to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)}{3} + \frac{\vec{\mathbf{r}}_D - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C\right)/3}{4} = \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D}{4}$$

and last to

$$\frac{\left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)}{4} + \frac{\vec{\mathbf{r}}_E - \left(\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D\right)/4}{5}$$
$$= \frac{\vec{\mathbf{r}}_A + \vec{\mathbf{r}}_B + \vec{\mathbf{r}}_C + \vec{\mathbf{r}}_D + \vec{\mathbf{r}}_E}{5}$$

This center of mass of the tree distribution is the same location whatever order we take the trees in.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P3.2 (a) 2.31; (b) 1.15

P3.4 (a) (2.17, 1.25) m, (–1.90, 3.29) m; (b) 4.55m

P3.6 (a) r, 180° – θ; (b) 180° + θ; (c) $-\theta$

P3.8 $\vec{\mathbf{B}}$ is 43 units in the negative y direction

P3.10 9.5 N, 57° above the x axis

P3.12 (a) See ANS. FIG. P3.12; (b) The sum of a set of vectors is not affected by the order in which the vectors are added.

P3.14 310 km at 57° S of W

P3.16 $A_x = 28.7$ units, $A_y = -20.1$ units

P3.18 1.31 km north and 2.81 km east

P3.20 (a) 5.00 blocks at 53.1° N of E; (b) 13.00 blocks

P3.22 $\left(2.60\hat{\mathbf{i}} + 4.50\hat{\mathbf{j}}\right)$ m

P3.24 788 miles at 48.0° northeast of Dallas

P3.26 (a) See ANS. FIG. P3.24; (b) $5.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}, -1.00\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}}$; (c) 6.40 at 38.7°, 8.06 at 97.2°

P3.28 (a) Its component parallel to the surface is (1.50 m) cos 141° = –1.17 m, or 1.17 m toward the top of the hill; (b) Its component perpendicular to the surface is (1.50 m) sin 141° = 0.944 m, or 0.944 m away from the snow.

P3.30 42.7 yards

P3.32 $C_x = 7.30$ cm; $C_y = -7.20$ cm

P3.34 59.2° with the x axis, 39.8° with the y axis, 67.4° with the z axis

P3.36 (a) $5.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}$, 5.92 m; (b) $(4.00\hat{\mathbf{i}} - 11.0\hat{\mathbf{j}} + 15.0\hat{\mathbf{k}})$ m, 19.0) m

P3.38 (a) $49.5\hat{\mathbf{i}} + 27.1\hat{\mathbf{j}}$; (b) 56.4, 28.7°

P3.40 magnitude: 170.1 cm, direction: 57.2° above +x axis (first quadrant); magnitude: 145.7 cm, direction: 58.6° above +x axis (first quadrant)

P3.42 (a) $\left(3.12\hat{\mathbf{i}} + 5.02\hat{\mathbf{j}} - 2.20\hat{\mathbf{k}}\right)$km; (b) 6.31 km

P3.44 Impossible because 12.4 m is greater than 5.00 m

P3.46 (a) $\left(5 = 11\,\mathrm{f}\right)\hat{\mathbf{i}} + \left(3 + 9\,\mathrm{f}\right)\hat{\mathbf{j}}$ meters; (b) $(5+0)\hat{\mathbf{i}} + (3+0)\hat{\mathbf{j}}$ meters; (c) This is reasonable because it is the location of the starting point, $5\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$ meters. (d) $16\hat{\mathbf{i}} + 12\hat{\mathbf{j}}$ meters; (e) This is reasonable because we have completed the trip, and this is the position vector of the endpoint.

P3.48 2.24 m, 26.6°

P3.50 390 mi/h at 7.37° N of E

P3.52 86.6 m, –50.0 m

P3.54 2.29 km

P3.56 (a) The perimeter measures $2(H + W) = 11.24$ m; (b) magnitude: 12.9 m, direction: 36.4° above $+x$ axis (first quadrant)

P3.58 (a) 7.17 km; (b) 6.15 km

P3.60 $\theta = 2\tan^{-1}\left(\dfrac{1}{\mathrm{n}}\right)$

P3.62 $0.456\hat{\mathbf{i}}\ \mathrm{m} - 0.708\hat{\mathbf{j}}\ \mathrm{m}$

P3.64 (a) 496 m, 65.1° N of W; (b) The arguments are justified because the distances involved are small relative to the radius of the Earth.

P3.66 $\phi = 2\theta = 106°$

4

Motion in Two Dimensions

CHAPTER OUTLINE

4.1	The Position, Velocity, and Acceleration Vectors
4.2	Two-Dimensional Motion with Constant Acceleration
4.3	Projectile Motion
4.4	Analysis Model: Particle in Uniform Circular Motion
4.5	Tangential and Radial Acceleration
4.6	Relative Velocity and Relative Acceleration

* An asterisk indicates an item new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ4.1 The car's acceleration must have an inward component and a forward component: answer (e). Another argument: Draw a final velocity vector of two units west. Add to it a vector of one unit south. This represents subtracting the initial velocity from the final velocity, on the way to finding the acceleration. The direction of the resultant is that of vector (e).

OQ4.2 (i) The 45° angle means that at point *A* the horizontal and vertical velocity components are equal. The horizontal velocity component is the same at *A*, *B*, and *C*. The vertical velocity component is zero at *B* and negative at *C*. The assembled answer is $a = b = c > d = 0 > e$.

(ii) The *x component* of acceleration is everywhere zero and the *y component* is everywhere $-9.80\ \text{m/s}^2$. Then we have $a = c = 0 > b = d = e$.

OQ4.3 Because gravity pulls downward, the horizontal and vertical motions of a projectile are independent of each other. Both balls have zero initial vertical components of velocity, and both have the same vertical accelerations, $-g$; therefore, both balls will have identical vertical motions: they will reach the ground at the same time. Answer (b).

OQ4.4 The projectile on the moon is in flight for a time interval six times larger, with the same range of vertical speeds and with the same constant horizontal speed as on Earth. Then its maximum altitude is (d) six times larger.

OQ4.5 The acceleration of a car traveling at constant speed in a circular path is directed toward the center of the circle. Answer (d).

OQ4.6 The acceleration of gravity near the surface of the Moon acts the same way as on Earth, it is constant and it changes only the vertical component of velocity. Answers (b) and (c).

OQ4.7 The projectile on the Moon is in flight for a time interval six times larger, with the same range of vertical speeds and with the same constant horizontal speed as on Earth. Then its range is (d) six times larger.

OQ4.8 Let the positive x direction be that of the girl's motion. The x component of the velocity of the ball relative to the ground is (+5 – 12) m/s = –7 m/s. The x-velocity of the ball relative to the girl is (–7 – 8) m/s = −15 m/s. The relative speed of the ball is +15 m/s, answer (d).

OQ4.9 Both wrench and boat have identical horizontal motions because gravity influences the vertical motion of the wrench only. Assuming neither air resistance nor the wind influences the horizontal motion of the wrench, the wrench will land at the base of the mast. Answer (b).

OQ4.10 While in the air, the baseball is a projectile whose velocity always has a constant horizontal component ($v_x = v_{xi}$) and a vertical component that changes at a constant rate ($\Delta v_y / \Delta t = a_y = -g$). At the highest point on the path, the vertical velocity of the ball is momentarily zero. Thus, at this point, the resultant velocity of the ball is horizontal and its acceleration continues to be directed downward ($a_x = 0$, $a_y = -g$). The only correct choice given for this question is (c).

OQ4.11 The period $T = 2\pi r/v$ changes by a factor of 4/4 = 1. The answer is (a).

OQ4.12 The centripetal acceleration $a = v^2/r$ becomes $(3v)^2/(3r) = 3v^2/r$, so it is 3 times larger. The answer is (b).

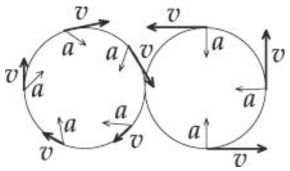
OQ4.13 (a) Yes (b) No: The escaping jet exhaust exerts an extra force on the plane. (c) No (d) Yes (e) No: The stone is only a few times more dense than water, so friction is a significant force on the stone. The answer is (a) and (d).

OQ4.14 With radius half as large, speed should be smaller by a factor of $1/\sqrt{2}$, so that $a = v^2/r$ can be the same. The answer is (d).

ANSWERS TO CONCEPTUAL QUESTIONS

CQ4.1 A parabola results, because the originally forward velocity component stays constant and the rocket motor gives the spacecraft constant acceleration in a perpendicular direction. These are the same conditions for a projectile, for which the velocity is constant in the horizontal direction and there is a constant acceleration in the perpendicular direction. Therefore, a curve of the same shape is the result.

CQ4.2 The skater starts at the center of the eight, goes clockwise around the left circle and then counterclockwise around the right circle.



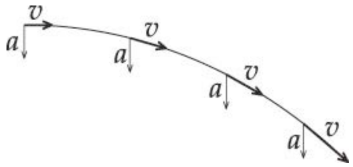
CQ4.3 No, you cannot determine the instantaneous velocity because the points could be separated by a finite displacement, but you can determine the average velocity. Recall the definition of average velocity:

$$\vec{\mathbf{v}}_{\text{avg}} = \frac{\Delta\vec{\mathbf{x}}}{\Delta t}$$

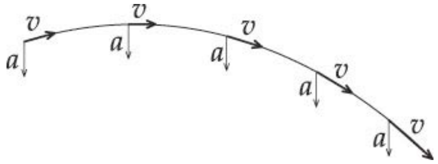
CQ4.4 (a) On a straight and level road that does not curve to left or right.
(b) Either in a circle or straight ahead on a level road. The acceleration magnitude can be constant either with a nonzero or with a zero value.

CQ4.5 (a) Yes, the projectile is in free fall. (b) Its vertical component of acceleration is the downward acceleration of gravity. (c) Its horizontal component of acceleration is zero.

CQ4.6 (a)



(b)

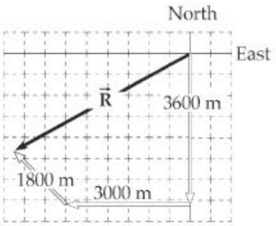


CQ4.7 (a) No. Its velocity is constant in magnitude and direction. (b) Yes. The particle is continuously changing the direction of its velocity vector.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 4.1 The Position, Velocity, and Acceleration Vectors

P4.1 We must use the method of vector addition and the definitions of average velocity and of average speed.



ANS. FIG. P4.1

(a) For each segment of the motion we model the car as a particle under constant velocity. Her displacements are

$$\vec{\mathbf{R}} = (20.0\text{ m/s})(180\text{ s})\ \textbf{south} + (25.0\text{ m/s})(120\text{ s})\ \textbf{west} + (30.0\text{ m/s})(60.0\text{ s})\ \textbf{northwest}$$

Choosing $\hat{\mathbf{i}}$ = east and $\hat{\mathbf{j}}$ = north, we have

$$\vec{\mathbf{R}} = (3.60\text{ km})(-\hat{\mathbf{j}}) + (3.00\text{ km})(-\hat{\mathbf{i}}) + (1.80\text{ km})\cos 45°(-\hat{\mathbf{i}}) + (1.80\text{ km})\sin 45°(\hat{\mathbf{j}})$$

$$\vec{\mathbf{R}} = (3.00 + 1.27)\text{ km}(-\hat{\mathbf{i}}) + (1.27 - 3.60)\text{ km}(\hat{\mathbf{j}}) = (-4.27\hat{\mathbf{i}} - 2.33\hat{\mathbf{j}})\text{ km}$$

The answer can also be written as

$$\vec{\mathbf{R}} = \sqrt{(-4.27\text{ km})^2 + (-2.33\text{ km})^2} = 4.87\text{ km}$$

$$\text{at }\tan^{-1}\left(\frac{2.33}{4.27}\right) = 28.6°$$

or $\boxed{4.87\text{ km at }28.6°\text{ S of W}}$

(b) The total distance or path length traveled is (3.60 + 3.00 + 1.80) km = 8.40 km, so

$$\text{average speed} = \left(\frac{8.40\text{ km}}{6.00\text{ min}}\right)\left(\frac{1.00\text{ min}}{60.0\text{ s}}\right)\left(\frac{1\,000\text{ m}}{\text{km}}\right) = \boxed{23.3\text{ m/s}}$$

(c) $$\text{Average velocity} = \frac{4.87\times 10^3\text{ m}}{360\text{ s}} = \boxed{13.5\text{ m/s along }\vec{\mathbf{R}}}$$

P4.2 The sun projects onto the ground the *x* component of the hawk's velocity:

$$(5.00\text{ m/s})\cos(-60.0°) = \boxed{2.50\text{ m/s}}$$

***P4.3** (a) For the average velocity, we have

$$\vec{\mathbf{v}}_{\text{avg}} = \left(\frac{x(4.00) - x(2.00)}{4.00\text{ s} - 2.00\text{ s}}\right)\hat{\mathbf{i}} + \left(\frac{y(4.00) - y(2.00)}{4.00\text{ s} - 2.00\text{ s}}\right)\hat{\mathbf{j}}$$

$$= \left(\frac{5.00\text{ m} - 3.00\text{ m}}{2.00\text{ s}}\right)\hat{\mathbf{i}} + \left(\frac{3.00\text{ m} - 1.50\text{ m}}{2.00\text{ s}}\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_{\text{avg}} = \boxed{\left(1.00\hat{\mathbf{i}} + 0.750\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) For the velocity components, we have

$$v_x = \frac{dx}{dt} = a = 1.00\text{ m/s}$$

and $$v_y = \frac{dy}{dt} = 2ct = \left(0.250\text{ m/s}^2\right)t$$

Therefore,

$$\vec{\mathbf{v}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = (1.00\text{ m/s})\hat{\mathbf{i}} + \left(0.250\text{ m/s}^2\right)t\,\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{v}}(t = 2.00\text{ s}) = (1.00\text{ m/s})\hat{\mathbf{i}} + (0.500\text{ m/s})\hat{\mathbf{j}}}$$

and the speed is

$$|\vec{\mathbf{v}}(t = 2.00\text{ s})| = \sqrt{(1.00\text{ m/s})^2 + (0.500\text{ m/s})^2} = \boxed{1.12\text{ m/s}}$$

P4.4 (a) From $x = -5.00\sin\omega t$, we determine the components of the velocity by taking the time derivatives of *x* and *y*:

$$v_x = \frac{dx}{dt} = \left(\frac{d}{dt}\right)(-5.00\sin\omega t) = -5.00\omega\cos\omega t$$

and $$v_y = \frac{dy}{dt} = \left(\frac{d}{dt}\right)(4.00 - 5.00\cos\omega t) = 0 + 5.00\omega\sin\omega t$$

At *t* = 0,

$$\vec{\mathbf{v}} = \left(-5.00\omega\cos 0\right)\hat{\mathbf{i}} + \left(5.00\omega\sin 0\right)\hat{\mathbf{j}} = \boxed{-5.00\omega\,\hat{\mathbf{i}}\text{ m/s}}$$

(b) Acceleration is the time derivative of the velocity, so

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(-5.00\omega\cos\omega t\right) = +5.00\omega^2\sin\omega t$$

and $$a_y = \frac{dv_y}{dt} = \left(\frac{d}{dt}\right)(5.00\omega\sin\omega t) = 5.00\omega^2\cos\omega t$$

At t = 0,

$$\vec{\mathbf{a}} = \left(5.00\omega^2\sin 0\right)\hat{\mathbf{i}} + \left(5.00\omega^2\cos 0\right)\hat{\mathbf{j}} = \boxed{5.00\omega^2\hat{\mathbf{j}}\text{ m/s}^2}$$

(c) $\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} = \boxed{(4.00\text{ m})\hat{\mathbf{j}} + (5.00\text{ m})(-\sin\omega t\hat{\mathbf{i}} - \cos\omega t\hat{\mathbf{j}})}$

$$\vec{\mathbf{v}} = \boxed{(5.00\text{ m})\omega\left[-\cos\omega t\,\hat{\mathbf{i}} + \sin\omega t\,\hat{\mathbf{j}}\right]}$$

$$\vec{\mathbf{a}} = \boxed{(5.00\text{ m})\omega^2\left[\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]}$$

(d) $\boxed{\text{the object moves in a circle of radius 5.00 m centered at (0, 4.00 m)}}$

P4.5 (a) The *x* and *y* equations combine to give us the expression for $\vec{\mathbf{r}}$:

$$\boxed{\vec{\mathbf{r}} = 18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}, \text{ where } \vec{\mathbf{r}} \text{ is in meters and } t \text{ is in seconds.}}$$

(b) We differentiate the expression for $\vec{\mathbf{r}}$ with respect to time:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \frac{d}{dt}\left[18.0t\hat{\mathbf{i}} + \left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}\right]$$

$$= \frac{d}{dt}(18.0t)\hat{\mathbf{i}} + \frac{d}{dt}\left(4.00t - 4.90t^2\right)\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{v}} = 18.0\hat{\mathbf{i}} + \left[4.00 - (9.80)t\right]\hat{\mathbf{j}}, \text{ where } \vec{\mathbf{v}} \text{ is in meters per second and } t \text{ is in seconds.}}$$

(c) We differentiate the expression for $\vec{\mathbf{v}}$ with respect to time:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \frac{d}{dt}\left\{18.0\hat{\mathbf{i}} + \left[4.00 - (9.80)t\right]\hat{\mathbf{j}}\right\}$$

$$= \frac{d}{dt}(18.0)\hat{\mathbf{i}} + \frac{d}{dt}\left[4.00 - (9.80)t\right]\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{a}} = -9.80\hat{\mathbf{j}}\text{ m/s}^2}$$

(d) By substitution,

$$\vec{\mathbf{r}}(3.00\text{ s}) = \boxed{(54.0\text{ m})\hat{\mathbf{i}} - (32.1\text{ m})\hat{\mathbf{j}}}$$

$$\vec{\mathbf{v}}(3.00\text{ s}) = \boxed{(18.0\text{ m/s})\hat{\mathbf{i}} - (25.4\text{ m/s})\hat{\mathbf{j}}}$$

$$\vec{\mathbf{a}}(3.00\text{s}) = \boxed{(-9.80\text{ m/s}^2)\hat{\mathbf{j}}}$$

Section 4.2 Two-Dimensional Motion with Constant Acceleration

P4.6 We use the vector versions of the kinematic equations for motion in two dimensions. We write the initial position, initial velocity, and acceleration of the particle in vector form:

$$\vec{\mathbf{a}} = 3.00\hat{\mathbf{j}}\ \text{m/s}^2;\ \vec{\mathbf{v}}_i = 5.00\hat{\mathbf{i}}\ \text{m/s};\ \vec{\mathbf{r}}_i = 0\hat{\mathbf{i}} + 0\hat{\mathbf{j}}$$

(a) The position of the particle is given by Equation 4.9:

$$\begin{aligned}\vec{\mathbf{r}}_f &= \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2 = (5.00\ \text{m/s})t\hat{\mathbf{i}} + \frac{1}{2}(3.00\ \text{m/s}^2)t^2\hat{\mathbf{j}}\\ &= \boxed{5.00t\hat{\mathbf{i}} + 1.50t^2\hat{\mathbf{j}}}\end{aligned}$$

where *r* is in m and *t* in s.

(b) The velocity of the particle is given by Equation 4.8:

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \boxed{5.00\hat{\mathbf{i}} + 3.00t\,\hat{\mathbf{j}}}$$

where *v* is in m/s and *t* in s.

(c) To obtain the particle's position at *t* = 2.00 s, we plug into the equation obtained in part (a):

$$\begin{aligned}\vec{\mathbf{r}}_f &= (5.00\ \text{m/s})(2.00\ \text{s})\hat{\mathbf{i}} + (1.50\ \text{m/s}^2)(2.00\ \text{s})^2\hat{\mathbf{j}}\\ &= (10.0\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}})\ \text{m}\end{aligned}$$

so $x_f = \boxed{10.0\ \text{m}}$, $y_f = \boxed{6.00\ \text{m}}$

(d) To obtain the particle's speed at *t* = 2.00 s, we plug into the equation obtained in part (b):

$$\begin{aligned}\vec{\mathbf{v}}_f &= \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = (5.00\ \text{m/s})\hat{\mathbf{i}} + (3.00\ \text{m/s}^2)(2.00\ \text{s})\hat{\mathbf{j}}\\ &= (5.00\hat{\mathbf{i}} + 6.00\hat{\mathbf{j}})\ \text{m/s}\end{aligned}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(5.00\ \text{m/s})^2 + (6.00\ \text{m/s})^2} = \boxed{7.81\ \text{m/s}}$$

P4.7 (a) We differentiate the equation for the vector position of the particle with respect to time to obtain its velocity:

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = \left(\frac{d}{dt}\right)(3.00\hat{\mathbf{i}} - 6.00t^2\hat{\mathbf{j}}) = \boxed{-12.0t\hat{\mathbf{j}}\ \text{m/s}}$$

(b) Differentiating the expression for velocity with respect to time gives the particle's acceleration:

$$\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} = \left(\frac{d}{dt}\right)\left(-12.0t\hat{\mathbf{j}}\right) = \boxed{-12.0\hat{\mathbf{j}}\ \text{m/s}^2}$$

(c) By substitution, when $t = 1.00$ s,

$$\boxed{\vec{\mathbf{r}} = \left(3.00\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}\right)\text{m};\ \vec{\mathbf{v}} = -12.0\hat{\mathbf{j}}\ \text{m/s}}$$

***P4.8** (a) For the x component of the motion we have $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$.

$$0.01\ \text{m} = 0 + \left(1.80\times10^{7}\ \text{m/s}\right)t + \frac{1}{2}\left(8\times10^{14}\ \text{m/s}^2\right)t^2$$

$$\left(4\times10^{14}\ \text{m/s}^2\right)t^2 + \left(1.80\times10^{7}\ \text{m/s}\right)t - 10^{-2}\ \text{m} = 0$$

$$t = \left(\frac{1}{2\left(4\times10^{14}\,\text{m/s}^2\right)}\right)\left[-1.80\times10^{7}\ \text{m/s} \pm\sqrt{\left(1.8\times10^{7}\ \text{m/s}\right)^2 - 4\left(4\times10^{14}\ \text{m/s}^2\right)\left(-10^{-2}\text{m}\right)}\right]$$

$$= \frac{-1.8\times10^{7} \pm 1.84\times10^{7}\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2}$$

We choose the + sign to represent the physical situation:

$$t = \frac{4.39\times10^{5}\ \text{m/s}}{8\times10^{14}\ \text{m/s}^2} = 5.49\times10^{-10}\ \text{s}$$

Here

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_y t^2 \\ &= 0 + 0 + \frac{1}{2}\left(1.6\times10^{15}\,\text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right)^2 \\ &= 2.41\times10^{-4}\ \text{m} \end{aligned}$$

So, $\boxed{\vec{\mathbf{r}}_f = \left(10.0\,\hat{\mathbf{i}} + 0.241\,\hat{\mathbf{j}}\right)\ \text{mm}}$

(b)
$$\begin{aligned} \vec{\mathbf{v}}_f &= \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t \\ &= 1.80\times10^{7}\,\hat{\mathbf{i}}\ \text{m/s} \\ &\quad + \left(8\times10^{14}\,\hat{\mathbf{i}}\ \text{m/s}^2 + 1.6\times10^{15}\,\hat{\mathbf{j}}\ \text{m/s}^2\right)\left(5.49\times10^{-10}\ \text{s}\right) \\ &= \left(1.80\times10^{7}\ \text{m/s}\right)\hat{\mathbf{i}} + \left(4.39\times10^{5}\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^{5}\ \text{m/s}\right)\hat{\mathbf{j}} \\ &= \boxed{\left(1.84\times10^{7}\ \text{m/s}\right)\hat{\mathbf{i}} + \left(8.78\times10^{5}\ \text{m/s}\right)\hat{\mathbf{j}}} \end{aligned}$$

(c) $$\left|\vec{\mathbf{v}}_f\right| = \sqrt{\left(1.84\times10^7\ \text{m/s}\right)^2 + \left(8.78\times10^5\ \text{m/s}\right)^2} = \boxed{1.85\times10^7\ \text{m/s}}$$

(d) $$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{8.78\times10^5}{1.84\times10^7}\right) = \boxed{2.73°}$$

P4.9 Model the fish as a particle under constant acceleration. We use our old standard equations for constant-acceleration straight-line motion, with *x* and *y* subscripts to make them apply to parts of the whole motion. At $t = 0$,

$$\vec{\mathbf{v}}_i = \left(4.00\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right)\ \text{m/s and } \hat{\mathbf{r}}_i = (10.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})\ \text{m}$$

At the first "final" point we consider, 20.0 s later,

$$\vec{\mathbf{v}}_f = (20.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})\ \text{m/s}$$

(a) $$a_x = \frac{\Delta v_x}{\Delta t} = \frac{20.0\ \text{m/s} - 4.00\ \text{m/s}}{20.0\ \text{s}} = \boxed{0.800\ \text{m/s}^2}$$

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{-5.00\ \text{m/s} - 1.00\ \text{m/s}}{20.0\ \text{s}} = \boxed{-0.300\ \text{m/s}^2}$$

(b) $$\theta = \tan^{-1}\left(\frac{-0.300\ \text{m/s}^2}{0.800\ \text{m/s}^2}\right) = -20.6° = \boxed{339°\ \text{from} + x\ \text{axis}}$$

(c) At $t = 25.0$ s the fish's position is specified by its coordinates and the direction of its motion is specified by the direction angle of its velocity:

$$\begin{aligned} x_f &= x_i + v_{xi}t + \frac{1}{2}a_xt^2 \\ &= 10.0\ \text{m} + \left(4.00\ \text{m/s}\right)(25.0\ \text{s}) + \frac{1}{2}(0.800\ \text{m/s}^2)(25.0\ \text{s})^2 \\ &= \boxed{360\ \text{m}} \end{aligned}$$

$$\begin{aligned} y_f &= y_i + v_{yi}t + \frac{1}{2}a_yt^2 \\ &= -4.00\ \text{m} + \left(1.00\ \text{m/s}\right)(25.0\ \text{s}) + \frac{1}{2}(-0.300\ \text{m/s}^2)(25.0\ \text{s})^2 \\ &= \boxed{-72.7\ \text{m}} \end{aligned}$$

$$v_{xf} = v_{xi} + a_xt = 4.00\ \text{m/s} + \left(0.800\ \text{m/s}^2\right)(25.0\ \text{s}) = 24\ \text{m/s}$$

$$v_{yf} = v_{yi} + a_yt = 1.00\ \text{m/s} - \left(0.300\ \text{m/s}^2\right)(25.0\ \text{s}) = -6.50\ \text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-6.50\ \text{m/s}}{24.0\ \text{m/s}}\right) = \boxed{-15.2°}$$

P4.10 The directions of the position, velocity, and acceleration vectors are given with respect to the x axis, and we know that the components of a vector with magnitude A and direction θ are given by $A_x = A\cos\theta$ and $A_y = A\sin\theta$; thus we have

$$\vec{\mathbf{r}}_i = 29.0\cos 95.0^\circ\,\hat{\mathbf{i}} + 29.0\sin 95.0^\circ\,\hat{\mathbf{j}} = -2.53\,\hat{\mathbf{i}} + 28.9\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}_i = 4.50\cos 40.0^\circ\,\hat{\mathbf{i}} + 4.50\sin 40.0^\circ\,\hat{\mathbf{j}} = 3.45\,\hat{\mathbf{i}} + 2.89\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{a}} = 1.90\cos 200^\circ\,\hat{\mathbf{i}} + 1.90\sin 200^\circ\,\hat{\mathbf{j}} = -1.79\,\hat{\mathbf{i}} + -0.650\,\hat{\mathbf{j}}$$

where $\vec{\mathbf{r}}$ is in m, $\vec{\mathbf{v}}$ in m/s, $\vec{\mathbf{a}}$ in m/s^2, and t in s.

(a) From $\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t$,

$$\boxed{\vec{\mathbf{v}}_f = (3.45 - 1.79t)\hat{\mathbf{i}} + (2.89 - 0.650t)\hat{\mathbf{j}}}$$

where $\vec{\mathbf{v}}$ in m/s and t in s.

(b) The car's position vector is given by

$$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$
$$= (-2.53 + 3.45t + \frac{1}{2}(-1.79)t^2)\hat{\mathbf{i}} + (28.9 + 2.89t + \frac{1}{2}(-0.650)t^2)\hat{\mathbf{j}}$$

$$\boxed{\vec{\mathbf{r}}_f = (-2.53 + 3.45t - 0.893t^2)\hat{\mathbf{i}} + (28.9 + 2.89t - 0.325t^2)\hat{\mathbf{j}}}$$

where $\vec{\mathbf{r}}$ is in m and t in s.

Section 4.3 Projectile Motion

P4.11 At the maximum height $v_y = 0$, and the time to reach this height is found from

$$v_{yf} = v_{yi} + a_y t \text{ as } t = \frac{v_{yf} - v_{yi}}{a_y} = \frac{0 - v_{yi}}{-g} = \frac{v_{yi}}{g}.$$

The vertical displacement that has occurred during this time is

$$(\Delta y)_{\max} = v_{y,\text{avg}}t = \left(\frac{v_{yf} + v_{yi}}{2}\right)t = \left(\frac{0 + v_{yi}}{2}\right)\left(\frac{v_{yi}}{g}\right) = \frac{v_{yi}^2}{2g}$$

Thus, if $(\Delta y)_{\max} = 12\text{ ft}\left(\frac{1\text{ m}}{3.281\text{ ft}}\right) = 3.66\text{ m}$, then

$$v_{yi} = \sqrt{2g(\Delta y)_{\max}} = \sqrt{2(9.80\text{ m/s}^2)(3.66\text{ m})} = 8.47\text{ m/s}$$

and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_i = \frac{v_{yi}}{\sin\theta} = \frac{8.47\text{ m/s}}{\sin 45°} = \boxed{12.0\text{ m/s}}$$

***P4.12** From Equation 4.13 with $R = 15.0$ m, $v_i = 3.00$ m/s, and $\theta_{\max} = 45.0°$:

$$g_{\text{planet}} = \frac{v_i^2 \sin 2\theta}{R} = \frac{v_i^2 \sin 90°}{R} = \frac{9.00\text{ m}^2/\text{s}^2}{15.0\text{ m}} = \boxed{0.600\text{ m/s}^2}$$

P4.13 (a) The mug leaves the counter horizontally with a velocity v_{xi} (say). If t is the time at which it hits the ground, then since there is no horizontal acceleration,

$$x_f = v_{xi}t \to t = x_f/v_{xi} \to t = (1.40\text{ m}/v_{xi})$$

At time t, it has fallen a distance of 1.22 m with a downward acceleration of 9.80 m/s^2. Then

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$
$$0 = 1.22\text{ m} - (4.90\text{ m/s}^2)(1.40\text{ m}/v_{xi})^2$$

Thus,

$$v_{xi} = \sqrt{\frac{\left(4.90\text{ m/s}^2\right)(1.40\text{ m})^2}{1.22\text{ m}}} = \boxed{2.81\text{ m/s}}$$

(b) The vertical velocity component with which it hits the floor is

$$v_{yf} = v_{yi} + a_y t \to v_{yf} = v_{yi} + (-g)(1.40\text{ m}/v_{xi})$$
$$v_{yf} = 0 + (-9.80\text{ m/s}^2)(1.40\text{ m}/2.81\text{ m/s}) = -4.89\text{ m/s}$$

Hence, the angle θ at which the mug strikes the floor is given by

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-4.89\text{ m/s}}{2.81\text{ m/s}}\right) = -60.2°$$

$$\boxed{\text{The mug's velocity is } 60.2° \text{ below the horizontal when it strikes the ground.}}$$

P4.14 The mug is a projectile from just after leaving the counter until just before it reaches the floor. Taking the origin at the point where the mug leaves the bar, the coordinates of the mug at any time t are

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \to x_f = 0 + v_{xi}t \to x_f = v_{xi}t$$

and

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2 \rightarrow y_f = -0 + 0 - \frac{1}{2}gt^2 \rightarrow y_f = -\frac{1}{2}gt^2$$

(a) When the mug reaches the floor, $y_f = h$ and $x_f = d$, so

$$-h = -\frac{1}{2}gt^2 \rightarrow h = \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$$

is the time of impact, and

$$x_f = v_{xi}t \rightarrow d = v_{xi}t \rightarrow v_{xi} = \frac{d}{t}$$

$$\boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$$

(b) Just before impact, the *x* component of velocity is still

$$v_{xf} = v_{xi}$$

while the *y* component is

$$v_{yf} = v_{yi} + at \rightarrow v_{yf} = 0 - gt = -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$$

Then the direction of motion just before impact is below the horizontal at an angle of

$$\theta = \tan^{-1}\frac{v_{yf}}{v_{xf}} = \tan^{-1}\left(\frac{-\sqrt{2gh}}{d\sqrt{\frac{g}{2h}}}\right)$$

$$\theta = \tan^{-1}\left(\frac{-2h}{d}\right) = -\tan^{-1}\left(\frac{2h}{d}\right)$$

because the *x* component of velocity is positive (forward) and the *y* component is negative (downward).

$$\boxed{\text{The direction of the mug's velocity is } \tan^{-1}(2h/d) \text{ below the horizontal.}}$$

P4.15 We ignore the trivial case where the angle of projection equals zero degrees.

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}; \quad R = \frac{v_i^2(\sin 2\theta_i)}{g}; \quad 3h = R$$

so $$\frac{3v_i^2\sin^2\theta_i}{2g} = \frac{v_i^2(\sin 2\theta_i)}{g}$$

or $$\frac{2}{3} = \frac{\sin^2\theta_i}{\sin 2\theta_i} = \frac{\tan\theta_i}{2}$$

thus, $$\theta_i = \tan^{-1}\left(\frac{4}{3}\right) = \boxed{53.1°}$$

P4.16 The horizontal range of the projectile is found from $x = v_{xi}t = v_i\cos\theta_i t$. Plugging in numbers,

$$x = (300 \text{ m/s})(\cos 55.0°)(42.0 \text{ s})$$

$$x = \boxed{7.23\times 10^3 \text{ m}}$$

The vertical position of the projectile is found from

$$y = v_{yi}t - \frac{1}{2}gt^2 = v_i\sin\theta_i t - \frac{1}{2}gt^2$$

Plugging in numbers,

$$y = (300 \text{ m/s})(\sin 55.0°)(42.0 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(42.0 \text{ s})^2$$

$$= \boxed{1.68\times 10^3 \text{ m}}$$

P4.17 (a) The vertical component of the salmon's velocity as it leaves the water is

$$v_{yi} = +v_i\sin\theta = +(6.26 \text{ m/s})\sin 45.0° \simeq +4.43 \text{ m/s}$$

When the salmon returns to water level at the end of the leap, the vertical component of velocity will be

$$v_{yf} = -v_{yi} \simeq -4.43 \text{ m/s}$$

If the salmon jumps out of the water at $t = 0$, the time interval required for it to return to the water is

$$\Delta t_1 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-4.43 \text{ m/s} - 4.43 \text{ m/s}}{-9.80 \text{ m/s}^2} \simeq 0.903 \text{ s}$$

The horizontal distance traveled during the leap is

$$L = v_{xi}\Delta t_i = (v_i\cos\theta)\Delta t_i$$
$$= (6.26 \text{ m/s})\cos 45.0°(0.903 \text{ s}) = 4.00 \text{ m}$$

To travel this same distance underwater, at speed $v = 3.58$ m/s, requires a time interval of

$$\Delta t_2 = \frac{L}{v} = \frac{4.00 \text{ m}}{3.58 \text{ m/s}} \simeq 1.12 \text{ s}$$

The average horizontal speed for the full porpoising maneuver is then

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{2L}{\Delta t_1 + \Delta t_2} = \frac{2(4.00 \text{ m})}{0.903 \text{ s} + 1.12 \text{ s}} = \boxed{3.96 \text{ m/s}}$$

(b) From (a), the total time interval for the porpoising maneuver is

$$\Delta t = 0.903 \text{ s} + 1.12 \text{ s} = 2.02 \text{ s}$$

Without porpoising, the time interval to travel distance $2L$ is

$$\Delta t_2 = \frac{2L}{v} = \frac{8.00 \text{ m}}{3.58 \text{ m/s}} \simeq 2.23 \text{ s}$$

The percentage difference is

$$\frac{\Delta t_1 - \Delta t_2}{\Delta t_2} \times 100\% = -9.6\%$$

$$\boxed{\text{Porpoising reduces the time interval by 9.6\%.}}$$

P4.18 (a) We ignore the trivial case where the angle of projection equals zero degrees. Because the projectile motion takes place over level ground, we can use Equations 4.12 and 4.13:

$$R = h \rightarrow \frac{v_i^2 \sin 2\theta_i}{g} = \frac{v_i^2 \sin^2 \theta_i}{2g}$$

Expanding,

$$2\sin 2\theta_i = \sin^2 \theta_i$$

$$4\sin\theta_i \cos\theta_i = \sin^2 \theta_i$$

$$\tan\theta_i = 4$$

$$\theta_i = \tan^{-1}(4) = \boxed{76.0°}$$

(b) The maximum range is attained for $\theta_i = 45°$:

$$R = \frac{v_i^2 \sin\left[2(76.0°)\right]}{g} \text{ and } R_{\text{max}} = \frac{v_i^2 \sin\left[2(45.0°)\right]}{g} = \frac{v_i^2}{g}$$

then

$$R_{\max} = \frac{v_i^2 \sin[2(76.0°)]}{g\ \sin[2(76.0°)]} = \frac{R}{\sin[2(76.0°)]}$$

$$R_{\max} = \boxed{2.13R}$$

(c) $\boxed{\text{Since } g \text{ divides out, the answer is the same on every planet.}}$

***P4.19** Consider the motion from original zero height to maximum height *h:*

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right) \text{ gives } 0 = v_{yi}^2 - 2g(h-0)$$

or $$v_{yi} = \sqrt{2gh}$$

Now consider the motion from the original point to half the maximum height:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right) \text{ gives } v_{yh}^2 = 2gh + 2(-g)\left(\frac{1}{2}h - 0\right)$$

so $$v_{yh} = \sqrt{gh}$$

At maximum height, the speed is $v_x = \frac{1}{2}\sqrt{v_x^2 + v_{yh}^2} = \frac{1}{2}\sqrt{v_x^2 + gh}$

Solving,

$$v_x = \sqrt{\frac{gh}{3}}$$

Now the projection angle is

$$\theta_i = \tan^{-1}\frac{v_{yi}}{v_x} = \tan^{-1}\frac{\sqrt{2gh}}{\sqrt{gh/3}} = \tan^{-1}\sqrt{6} = \boxed{67.8°}$$

P4.20 (a) $x_f = v_{xi}t = (8.00 \text{ m/s})\cos 20.0° (3.00 \text{ s}) = \boxed{22.6 \text{ m}}$

(b) Taking *y* positive downwards,

$$y_f = v_{yi}t + \frac{1}{2}gt^2$$

$$y_f = (8.00 \text{ m/s})\sin 20.0°(3.00 \text{ s}) + \frac{1}{2}(9.80 \text{ m/s}^2)(3.00 \text{ s})^2$$

$$= \boxed{52.3 \text{ m}}$$

(c) $$10.0 \text{ m} = (8.00 \text{ m/s})(\sin 20.0°)t + \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Suppressing units,

$$4.90t^2 + 2.74t - 10.0 = 0$$

$$t = \frac{-2.74 \pm \sqrt{(2.74)^2 + 196}}{9.80} = \boxed{1.18\text{ s}}$$

P4.21 The horizontal component of displacement is $x_f = v_{xi}t = (v_i\cos\theta_i)t$. Therefore, the time required to reach the building a distance d away is $t = \frac{d}{v_i\cos\theta_i}$. At this time, the altitude of the water is

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2 = v_i\ \sin\theta_i\left(\frac{d}{v_i\cos\theta_i}\right) - \frac{g}{2}\left(\frac{d}{v_i\cos\theta_i}\right)^2$$

Therefore, the water strikes the building at a height h above ground level of

$$h = y_f = \boxed{d\tan\theta_i - \frac{gd^2}{2v_i^2\cos^2\theta_i}}$$

P4.22 (a) The time of flight of a water drop is given by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$0 = y_1 - \frac{1}{2}gt^2$$

For $t_1 > 0$, the root is $t_1 = \sqrt{\frac{2y_1}{g}} = \sqrt{\frac{2(2.35\text{ m})}{9.8\text{ m/s}^2}} = 0.693\text{ s}$.

The horizontal range of a water drop is

$$x_{f1} = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$= 0 + 1.70\text{ m/s}\,(0.693\text{ s}) + 0 = 1.18\text{ m}$$

This is about the width of a town sidewalk, so $\boxed{\text{there is}}$ space for a walkway behind the waterfall. Unless the lip of the channel is well designed, water may drip on the visitors. A tall or inattentive person may get his or her head wet.

(b) Now the flight time t_2 is given by

$$0 = y_2 + 0 - \frac{1}{2}gt_2^2,\ \text{where } y_2 = \frac{y_1}{12}:$$

$$t_2 = \sqrt{\frac{2y_2}{g}} = \sqrt{\frac{2y_1}{g(12)}} = \frac{1}{\sqrt{12}} \times \sqrt{\frac{2y_1}{g}} = \frac{t_1}{\sqrt{12}}$$

From the same equation as in part (a) for horizontal range, $x_2 = v_2 t_2$, where $x_2 = x_1/12$:

$$x_2 = v_2 t_2 \rightarrow \frac{x_1}{12} = v_2 \frac{t_1}{\sqrt{12}}$$

$$v_2 = \frac{x_1}{t_1\sqrt{12}} = \frac{v_1}{\sqrt{12}} = \frac{1.70 \text{ m/s}}{\sqrt{12}} = \boxed{0.491 \text{ m/s}}$$

The rule that the scale factor for speed is the square root of the scale factor for distance is Froude's law, published in 1870.

P4.23 (a) From the particle under constant velocity model in the x direction, find the time at which the ball arrives at the goal:

$$x_f = x_i + v_i t \rightarrow t = \frac{x_f - x_i}{v_{xi}} = \frac{36.0 \text{ m} - 0}{(20 \text{ m/s}) \cos 53.0°} = 2.99 \text{ s}$$

From the particle under constant acceleration model in the y direction, find the height of the ball at this time:

$$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$$

$$y_f = 0 + (20.0 \text{ m/s}) \sin 53.0°(2.99 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(2.99 \text{ s})^2$$

$$y_f = 3.94 \text{ m}$$

Therefore, the ball clears the crossbar by

$$3.94 \text{ m} - 3.05 \text{ m} = \boxed{0.89 \text{ m}}$$

(b) Use the particle under constant acceleration model to find the time at which the ball is at its highest point in its trajectory:

$$v_{yf} = v_{yi} - gt \rightarrow t = \frac{v_{yf} - v_{yi}}{g} = \frac{(20.0 \text{ m/s})\sin 53.0° - 0}{9.80 \text{ m/s}^2} = 1.63 \text{ s}$$

$$\boxed{\text{Because this is earlier than the time at which the ball reaches the goal, the ball clears the goal on its way down.}}$$

P4.24 From the instant he leaves the floor until just before he lands, the basketball star is a projectile. His vertical velocity and vertical displacement are related by the equation $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$. Applying this to the upward part of his flight gives $0 = v_{yi}^2 + 2(-9.80 \text{ m/s}^2)(1.85 - 1.02) \text{ m}$. From this, $v_{yi} = 4.03$ m/s. [Note that this is the answer to part (c) of this problem.]

For the downward part of the flight, the equation gives $v_{yf}^2 = 0 + 2\left(-9.80\,\text{m/s}^2\right)(0.900 - 1.85)$ m. Thus, the vertical velocity just before he lands is $v_{yf} = -4.32$ m/s.

(a) His hang time may then be found from $v_{yf} = v_{yi} + a_y t$:

$$-4.32\text{ m/s} = 4.03\text{ m/s} + (-9.80\text{ m/s}^2)t$$

or $t = \boxed{0.852\text{ s}}$.

(b) Looking at the total horizontal displacement during the leap, $x = v_{xi}t$ becomes

$$2.80\text{ m} = v_{xi}\,(0.852\text{ s})$$

which yields $v_{xi} = \boxed{3.29\text{ m/s}}$.

(c) $v_{yi} = \boxed{4.03\text{ m/s}}$. See above for proof.

(d) The takeoff angle is: $\theta = \tan^{-1}\dfrac{v_{yi}}{v_{xi}} = \tan^{-1}\left(\dfrac{4.03\text{ m/s}}{3.29\text{ m/s}}\right) = \boxed{50.8°}$

(e) Similarly for the deer, the upward part of the flight gives

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = v_{yi}^2 + 2\left(-9.80\ \text{m/s}^2\right)(2.50 - 1.20)\text{ m}$$

so $v_{yi} = 5.04$ m/s

For the downward part, $v_{yf}^2 = v_{yi}^2 + 2a_y(y_f - y_i)$ yields

$$v_{yf}^2 = 0 + 2\left(-9.80\,\text{m/s}^2\right)(0.700 - 2.50)\text{ m and } v_{yf} = -5.94\text{ m/s}.$$

The hang time is then found as $v_{yf} = v_{yi} + a_y t$:

$$-5.94\text{ m/s} = 5.04\text{ m/s} + (-9.80\text{ m/s}^2)t \text{ and}$$

$$\boxed{t = 1.12\text{ s}}$$

P4.25 (a) For the horizontal motion, we have $x_f = d = 24$ m:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$24\text{ m} = 0 + v_i\left(\cos 53°\right)\left(2.2\text{ s}\right) + 0$$

$$v_i = \boxed{18.1\text{ m/s}}$$

(b) As it passes over the wall, the ball is above the street by

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_f = 0 + (18.1 \text{ m/s})(\sin 53°)(2.2 \text{ s}) + \frac{1}{2}(-9.8 \text{ m/s}^2)(2.2 \text{ s})^2 = 8.13 \text{ m}$$

So it clears the parapet by 8.13 m – 7 m = $\boxed{1.13 \text{ m}}$.

(c) Note that the highest point of the ball's trajectory is not directly above the wall. For the whole flight, we have from the trajectory equation:

$$y_f = (\tan\theta_i)x_f - \left(\frac{g}{2v_i^2\cos^2\theta_i}\right)x_f^2$$

or $$6 \text{ m} = (\tan 53°)x_f - \left(\frac{9.8 \text{ m/s}^2}{2(18.1 \text{ m/s})^2\cos^2 53°}\right)x_f^2$$

Solving,

$$(0.041\,2 \text{ m}^{-1})x_f^2 - 1.33x_f + 6 \text{ m} = 0$$

and, suppressing units,

$$x_f = \frac{1.33 \pm \sqrt{1.33^2 - 4(0.041\,2)(6)}}{2(0.041\,2)}$$

This yields two results:

x_f = 26.8 m or 5.44 m

The ball passes twice through the level of the roof.

It hits the roof at distance from the wall

26.8 m – 24 m = $\boxed{2.79 \text{ m}}$

P4.26 We match the given equations:

$$x_f = 0 + (11.2 \text{ m/s})(\cos 18.5°)t$$

$$0.360 \text{ m} = 0.840 \text{ m} + (11.2 \text{ m/s})(\sin 18.5°)t - \frac{1}{2}(9.80 \text{ m/s}^2)t^2$$

to the equations for the coordinates of the final position of a projectile:

$$x_f = x_i + v_{xi}t$$

$$y_f = y_i + v_{yi}t - \frac{1}{2}gt^2$$

For the equations to represent the same functions of time, all coefficients must agree: $x_i = 0$, $y_i = 0.840$ m, $v_{xi} = (11.2 \text{ m/s}) \cos 18.5°$, $v_{yi} = (11.2 \text{ m/s}) \sin 18.5°$, and $g = 9.80 \text{ m/s}^2$.

(a) Then the original position of the athlete's center of mass is the point with coordinates $\left(x_i, y_i\right) = \boxed{(0, 0.840 \text{ m})}$. That is, his original position has position vector $\vec{\mathbf{r}} = 0\hat{\mathbf{i}} + 0.840\hat{\mathbf{j}}$ m.

(b) His original velocity is $\vec{\mathbf{v}}_i = (11.2 \text{ m/s})(\cos 18.5°)\hat{\mathbf{i}} + (11.2 \text{ m/s})(\sin 18.5°)\hat{\mathbf{j}} = \boxed{11.2 \text{ m/s at } 18.5°}$ above the *x* axis.

(c) From $(4.90 \text{ m/s}^2)t^2 - (3.55 \text{ m/s})t - 0.48 \text{ m} = 0$, we find the time of flight, which must be positive. Suppressing units,

$$t = \frac{-(-3.55) + \sqrt{(-3.55)^2 - 4(4.90)(-0.48)}}{2(4.90)} = 0.841 \text{ s}$$

Then $x_f = (11.2 \text{ m/s}) \cos 18.5°(0.841 \text{ s}) = \boxed{8.94 \text{ m}}$.

P4.27 Model the rock as a projectile, moving with constant horizontal velocity, zero initial vertical velocity, and with constant vertical acceleration. Note that the sound waves from the splash travel in a straight line to the soccer player's ears. The time of flight of the rock follows from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$-40.0 \text{ m} = 0 + 0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2$$

$$t = 2.86 \text{ s}$$

The extra time 3.00 s – 2.86 s = 0.140 s is the time required for the sound she hears to travel straight back to the player. It covers distance

$$(343 \text{ m/s})\ 0.143 \text{ s} = 49.0 \text{ m} = \sqrt{x^2 + (40.0 \text{ m})^2}$$

where *x* represents the horizontal distance the rock travels. Solving for *x* gives $x = 28.3$ m. Since the rock moves with constant speed in the *x* direction and travels horizontally during the 2.86 s that it is in flight,

$$x = 28.3 \text{ m} = v_{xi}t + 0t^2$$

$$\therefore v_{xi} = \frac{28.3 \text{ m}}{2.86 \text{ s}} = \boxed{9.91 \text{ m/s}}$$

P4.28 The initial velocity components of the projectile are

$$x_i = 0 \quad \text{and} \quad y_i = h$$

$$v_{xi} = v_i \cos\theta \quad \text{and} \quad v_{yi} = v_i \sin\theta$$

while the constant acceleration components are

$$a_x = 0 \quad \text{and} \quad a_y = -g$$

The coordinates of the projectile are

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 = (v_i \cos\theta)t \quad \text{and}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = h + (v_i \sin\theta)t - \frac{1}{2}gt^2$$

and the components of velocity are

$$v_{xf} = v_{xi} + a_x t = v_i \cos\theta \quad \text{and}$$

$$v_{yf} = v_{yi} + a_y t = v_i \sin\theta - gt$$

(a) We know that when the projectile reaches its maximum height, $v_{yf} = 0$:

$$v_{yf} = v_i \sin\theta - gt = 0 \rightarrow \boxed{t = \frac{v_i \sin\theta}{g}}$$

(b) At the maximum height, $y = h_{\max}$:

$$h_{\max} = h + \left(v_i \sin\theta\right)t - \frac{1}{2}gt^2$$

$$h_{\max} = h + v_i \sin\theta \frac{v_i \sin\theta}{g} - \frac{1}{2}g\left(\frac{v_i \sin\theta}{g}\right)^2$$

$$\boxed{h_{\max} = h + \frac{\left(v_i \sin\theta\right)^2}{2g}}$$

P4.29 (a) Initial coordinates: $\boxed{x_i = 0.00 \text{ m}, y_i = 0.00 \text{ m}}$

(b) Components of initial velocity: $\boxed{v_{xi} = 18.0 \text{ m/s}, v_{yi} = 0}$

(c) $\boxed{\text{Free fall motion, with constant downward acceleration } g = 9.80 \text{ m/s}^2.}$

(d) $\boxed{\text{Constant velocity motion in the horizontal direction.}}$ There is no horizontal acceleration from gravity.

(e) $$v_{xf} = v_{xi} + a_x t \quad \rightarrow \quad \boxed{v_{xf} = v_{xi}}$$

$$v_{yf} = v_{yi} + a_y t \quad \rightarrow \quad \boxed{v_{yf} = -gt}$$

(f) $$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \quad \rightarrow \quad \boxed{x_f = v_{xi}t}$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 \quad \rightarrow \quad \boxed{y_f = -\frac{1}{2}gt^2}$$

(g) We find the time of impact:

$$y_f = -\frac{1}{2}gt^2$$

$$-h = -\frac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(50.0\text{ m})}{9.80\text{ m/s}^2}} = \boxed{3.19\text{ s}}$$

(h) At impact, $v_{xf} = v_{xi} = 18.0$ m/s, and the vertical component is

$$v_{yf} = -gt$$

$$= -g\sqrt{\frac{2h}{g}} = -\sqrt{2gh} = -\sqrt{2(9.80\text{ m/s}^2)(50.0\text{ m})} = -31.3\text{ m/s}$$

Thus,

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(18.0\text{ m/s})^2 + (-31.3\text{ m/s})^2} = \boxed{36.1\text{ m/s}}$$

and

$$\theta_f = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-31.3}{18.0}\right) = \boxed{-60.1^\circ}$$

which in this case means the velocity points into the fourth quadrant because its *y* component is negative.

P4.30 (a) When a projectile is launched with speed v_i at angle θ_i above the horizontal, the initial velocity components are $v_{xi} = v_i \cos\theta_i$ and $v_{yi} = v_i \sin\theta_i$. Neglecting air resistance, the vertical velocity when the projectile returns to the level from which it was launched (in this case, the ground) will be $v_y = -v_{yi}$. From this information, the total time of flight is found from $v_{yf} = v_{yi} + a_y t$ to be

$$t_{\text{total}} = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-v_{yi} - v_{yi}}{-g} = \frac{2v_{yi}}{g} \quad \text{or} \quad t_{\text{total}} = \frac{2v_i \sin\theta_i}{g}$$

Since the horizontal velocity of a projectile with no air resistance is constant, the horizontal distance it will travel in this time (i.e., its range) is given by

$$R = v_{xi} t_{\text{total}} = (v_i \cos\theta_i)\left(\frac{2v_i \sin\theta_i}{g}\right) = \frac{v_i^2}{g}(2\sin\theta_i \cos\theta_i)$$
$$= \frac{v_i^2 \sin(2\theta_i)}{g}$$

Thus, if the projectile is to have a range of $R = 81.1$ m when launched at an angle of $\theta_i = 45.0°$, the required initial speed is

$$v_i = \sqrt{\frac{Rg}{\sin(2\theta_i)}} = \sqrt{\frac{(81.1\text{ m})(9.80\text{ m/s}^2)}{\sin(90.0°)}} = \boxed{28.2\text{ m/s}}$$

(b) With $v_i = 28.2$ m/s and $\theta_i = 45.0°$ the total time of flight (as found above) will be

$$t_{\text{total}} = \frac{2v_i \sin\theta_i}{g} = \frac{2(28.2\text{ m/s})\sin(45.0°)}{9.80\text{ m/s}^2} = \boxed{4.07\text{ s}}$$

(c) Note that at $\theta_i = 45.0°$, and that $\sin(2\theta_i)$ will decrease as θ_i is increased above this optimum launch angle. Thus, if the range is to be kept constant while the launch angle is increased above 45.0°, we see from $v_i = \sqrt{Rg/\sin(2\theta_i)}$ that

$\boxed{\text{the required initial velocity will increase}}$.

Observe that for $\theta_i < 90°$, the function $\sin\theta_i$ increases as θ_i is increased. Thus, increasing the launch angle above 45.0° while keeping the range constant means that both v_i and $\sin\theta_i$ will increase. Considering the expression for t_{total} given above, we see that $\boxed{\text{the total time of flight will increase}}$.

P4.31 We first consider the vertical motion of the stone as it falls toward the water. The initial y velocity component of the stone is

$$v_{yi} = v_i \sin\theta = -(4.00\text{ m/s})\sin 60.0° = -3.46\text{ m/s}$$

and its y coordinate is

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2 = h + (v_i \sin\theta)t - \frac{1}{2}gt^2$$
$$y_f = 2.50 - 3.46t - 4.90t^2$$

where y is in m and t in s. We have taken the water's surface to be at $y = 0$. At the water,

$$4.90t^2 + 3.46t - 2.50 = 0$$

Solving for the positive root of the equation, we get

$$t = \frac{-3.46 + \sqrt{(3.46)^2 - 4(4.90)(-2.50)}}{2(4.90)}$$

$$t = \frac{-3.46 + 7.81}{9.80}$$

$$t = 0.443 \text{ s}$$

The y component of velocity of the stone when it reaches the water at this time t is

$$v_{yf} = v_{yi} + a_y t = -3.46 - gt = -7.81 \text{ m/s}$$

After the stone enters to water, its speed, and therefore the magnitude of each velocity component, is reduced by one-half. Thus, the y component of the velocity of the stone in the water is

$$v_{yi} = (-7.81 \text{ m/s})/2 = -3.91 \text{ m/s},$$

and this component remains constant until the stone reaches the bottom. As the stone moves through the water, its y coordinate is

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$y_f = -3.91t$$

The stone reaches the bottom of the pool when $y_f = -3.00$ m:

$$y_f = -3.91t = -3.00 \to t = 0.767 \text{ s}$$

The total time interval the stone takes to reach the bottom of the pool is

$$\Delta t = 0.443 \text{ s} + 0.767 \text{ s} = \boxed{1.21 \text{ s}}$$

***P4.32** (a) The time for the ball to reach the fence is

$$t = \frac{\Delta x}{v_{xi}} = \frac{130 \text{ m}}{v_i \cos 35.0°} = \frac{159 \text{ m}}{v_i}$$

At this time, the ball must be $\Delta y = 21.0 \text{ m} - 1.00 \text{ m} = 20.0 \text{ m}$ above its launch position, so

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$

gives

$$20.0\text{ m} = \left(v_i \sin 35.0°\right)\left(\frac{159\text{ m}}{v_i}\right) - \frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(\frac{159\text{ m}}{v_i}\right)^2$$

or

$$\left(159\text{ m}\right)\sin 35.0° - 20.0\text{ m} = \frac{\left(4.90\text{ m/s}^2\right)\left(159\text{ m}\right)^2}{v_i^2}$$

from which we obtain

$$v_i = \sqrt{\frac{\left(4.90\text{ m/s}^2\right)\left(159\text{ m}\right)^2}{\left(159\text{ m}\right)\sin 35.0° - 20.0\text{ m}}} = \boxed{41.7\text{ m/s}}$$

(b) From our equation for the time of flight above,

$$t = \frac{159\text{ m}}{v_i} = \frac{159\text{ m}}{41.7\text{ m/s}} = \boxed{3.81\text{ s}}$$

(c) When the ball reaches the wall (at *t* = 3.81 s),

$$v_x = v_i \cos 35.0° = \left(41.7\text{ m/s}\right)\cos 35.0° = \boxed{34.1\text{ m/s}}$$

$$\begin{aligned} v_y &= v_i \sin 35.0° + a_y t \\ &= \left(41.7\text{ m/s}\right)\sin 35.0° - \left(9.80\text{ m/s}^2\right)\left(3.81\text{ s}\right) \\ &= \boxed{-13.4\text{ m/s}} \end{aligned}$$

and $v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(34.1\text{ m/s}\right)^2 + \left(-13.4\text{ m/s}\right)^2} = \boxed{36.7\text{ m/s}}$

Section 4.4 Analysis Model: Particle in Uniform Circular Motion

P4.33 Model the discus as a particle in uniform circular motion. We evaluate its centripetal acceleration from the standard equation proved in the text.

$$a_c = \frac{v^2}{r} = \frac{\left(20.0\text{ m/s}\right)^2}{1.06\text{ m}} = \boxed{377\text{ m/s}^2}$$

The mass is unnecessary information.

P4.34 Centripetal acceleration is given by $a = \frac{v^2}{R}$. To find the velocity of a point at the equator, we note that this point travels through $2\pi R_E$ (where $R_E = 6.37 \times 10^6$ m is Earth's radius) in 24.0 hours. Then,

$$v = \frac{2\pi R_E}{T} = \frac{2\pi\left(6.37\times10^6\ \text{m}\right)}{(24\ \text{h})(3\,600\ \text{s/h})} = 463\ \text{m/s}$$

and,

$$a = \frac{v^2}{R} = \frac{(463\ \text{m/s})^2}{6.37\times10^6\ \text{m}}$$

$$= \boxed{0.033\,7\ \text{m/s}^2\ \text{directed toward the center of Earth}}$$

***P4.35** Centripetal acceleration is given by $a_c = \frac{v^2}{r}$. Let *f* represent the rotation rate. Each revolution carries each bit of metal through distance $2\pi r$, so $v = 2\pi r f$ and

$$a_c = \frac{v^2}{r} = 4\pi^2 r f^2 = 100g$$

A smaller radius implies smaller acceleration. To meet the criterion for each bit of metal we consider the minimum radius:

$$f = \left(\frac{100g}{4\pi^2 r}\right)^{1/2} = \left(\frac{100\cdot 9.8\ \text{m/s}^2}{4\pi^2(0.021\ \text{m})}\right)^{1/2}$$

$$= 34.4\ \frac{1}{\text{s}}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \boxed{2.06\times10^3\ \text{rev/min}}$$

***P4.36** The radius of the tire is $r = 0.500$ m. The speed of the stone on its outer edge is

$$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.500\ \text{m})}{(60.0\ \text{s}/200\ \text{rev})} = \boxed{10.5\ \text{m/s}}$$

and its acceleration is

$$a = \frac{v^2}{R} = \frac{(10.5\ \text{m/s})^2}{0.500\ \text{m}} = \boxed{219\ \text{m/s}^2\ \text{inward}}$$

P4.37 Centripetal acceleration is $a_c = \frac{v^2}{r} \to v = \sqrt{a_c r}$, where $a_c = 20.0g$, and speed v is in meters per second if r is in meters.

We can convert the speed into a rotation rate, in rev/min, by using the relations 1 revolution = $2\pi r$, and 1 min = 60 s:

$$v = \sqrt{a_c r} = \sqrt{a_c r}\left(\frac{1\ \text{rev}}{2\pi r}\right) = \frac{1\ \text{rev}}{2\pi}\sqrt{\frac{a_c}{r}}$$

$$= \frac{1\ \text{rev}}{2\pi}\sqrt{\frac{20.0\left(9.80\ \text{m/s}^2\right)}{29.0\ \text{ft}}\left(\frac{3.281\ \text{ft}}{1\ \text{m}}\right)}\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

$$= \boxed{45.0\ \text{rev/min}}$$

P4.38 (a) Using the definition of speed and noting that the ball travels in a circular path,

$$v = \frac{d}{\Delta t} = \frac{2\pi R}{T}$$

where R is the radius of the circle and T is the period, that is, the time interval required for the ball to go around once. For the periods given in the problem,

$$8.00\ \text{rev/s} \rightarrow T = \frac{1}{8.00\ \text{rev/s}} = 0.125\ \text{s}$$

$$6.00\ \text{rev/s} \rightarrow T = \frac{1}{6.00\ \text{rev/s}} = 0.167\ \text{s}$$

Therefore, the speeds in the two cases are:

$$8.00\ \text{rev/s} \rightarrow v = \frac{2\pi(0.600\ \text{m})}{0.125\ \text{s}} = 30.2\,\text{m/s}$$

$$6.00\ \text{rev/s} \rightarrow v = \frac{2\pi(0.900\ \text{m})}{0.167\ \text{s}} = 33.9\,\text{m/s}$$

Therefore, $\boxed{6.00\ \text{rev/s}}$ gives the greater speed of the ball.

(b) Acceleration $= \frac{v^2}{r} = \frac{(9.60\pi\ \text{m/s})^2}{0.600\ \text{m}} = \boxed{1.52\times 10^3\ \text{m/s}^2}$.

(c) At 6.00 rev/s, acceleration $= \frac{(10.8\pi\ \text{m/s})^2}{0.900\ \text{m}} = \boxed{1.28\times 10^3\ \text{m/s}^2}$. So 8 rev/s gives the higher acceleration.

***P4.39** The satellite is in free fall. Its acceleration is due to gravity and is by effect a centripetal acceleration: $a_c = g$. So

$$\frac{v^2}{r} = g$$

Solving for the velocity,

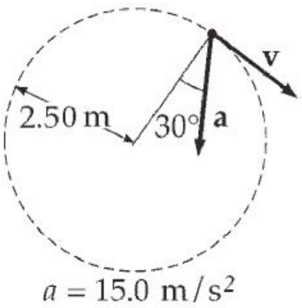
$$v = \sqrt{rg} = \sqrt{(6,400 + 600)(10^3\ \text{m})(8.21\ \text{m/s}^2)}$$
$$= \boxed{7.58 \times 10^3\ \text{m/s}}$$
$$v = \frac{2\pi r}{T}$$

and

$$T = \frac{2\pi r}{v} = \frac{2\pi(7,000 \times 10^3\ \text{m})}{7.58 \times 10^3\ \text{m/s}} = \boxed{5.80 \times 10^3\ \text{s}}$$
$$T = 5.80 \times 10^3\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 96.7\ \text{min}$$

Section 4.5 Tangential and Radial Acceleration

P4.40 From the given magnitude and direction of the acceleration we can find both the centripetal and the tangential components. From the centripetal acceleration and radius we can find the speed in part (b). $r = 2.50$ m, $a = 15.0$ m/s^2.



ANS. FIG. P4.40

(a) The acceleration has an inward radial component:

$$a_c = a\cos 30.0° = (15.0\ \text{m/s}^2)(\cos 30°)$$
$$= \boxed{13.0\ \text{m/s}^2}$$

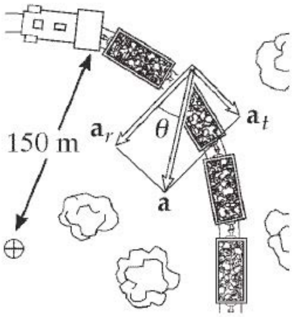
(b) The speed at the instant shown can be found by using

$$a_c = \frac{v^2}{r}$$
$$v^2 = ra_c = 2.50\ \text{m}(13.0\ \text{m/s})$$
$$= 32.5\ \text{m}^2/\text{s}^2$$
$$v = \sqrt{32.5}\ \text{m/s} = \boxed{5.70\ \text{m/s}}$$

(c) $a^2 = a_t^2 + a_r^2$

so $a_t = \sqrt{a^2 - a_r^2} = \sqrt{(15.0\ \text{m/s}^2)^2 - (13.0\ \text{m/s}^2)^2} = \boxed{7.50\ \text{m/s}^2}$

P4.41 Since the train is changing both its speed and direction, the acceleration vector will be the vector sum of the tangential and radial acceleration components. The tangential acceleration can be found from the changing speed and elapsed time, while the radial acceleration can be found from the radius of curvature and the train's speed.



ANS. FIG. P4.41

First, let's convert the speed units from km/h to m/s:

$$v_i = 90.0\text{ km/h} = (90.0\text{ km/h})(10^3\text{ m/km})(1\text{ h}/3600\text{ s})$$
$$= 25.0\text{ m/s}$$
$$v_f = 50.0\text{ km/h} = (50.0\text{ km/h})(10^3\text{ m/km})(1\text{ h}/3600\text{ s})$$
$$= 13.9\text{ m/s}$$

The tangential acceleration and radial acceleration are, respectively,

$$a_t = \frac{\Delta v}{\Delta t} = \frac{13.9\text{ m/s} - 25.0\text{ m/s}}{15.0\text{ s}} = -0.741\text{ m/s}^2 \quad \text{(backward)}$$

and $$a_r = \frac{v^2}{r} = \frac{(13.9\text{ m/s})^2}{150\text{ m}} = 1.29\text{ m/s} \quad \text{(inward)}$$

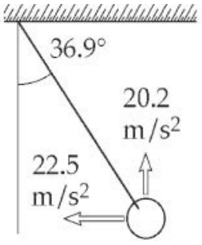
so $$a = \sqrt{a_c^2 + a_t^2} = \sqrt{\left(1.29\text{ m/s}^2\right)^2 + \left(-0.741\text{ m/s}^2\right)^2} = 1.48\text{ m/s}^2$$

at an angle of

$$\tan^{-1}\left(\frac{|a_t|}{a_c}\right) = \tan^{-1}\left(\frac{0.741\text{ m/s}^2}{1.29\text{ m/s}^2}\right) = 29.9°$$

therefore, $\vec{\mathbf{a}} = \boxed{1.48\text{ m/s}^2\text{ inward and }29.9°\text{ backward}}$

P4.42 (a) See ANS. FIG. P4.42.



ANS. FIG. P4.42

(b) The components of the 20.2 m/s^2 and the 22.5 m/s^2 accelerations along the rope together constitute the centripetal acceleration:

$$a_c = \left(22.5\text{ m/s}^2\right)\cos\left(90.0° - 36.9°\right) + \left(20.2\text{ m/s}^2\right)\cos 36.9° = \boxed{29.7\text{ m/s}^2}$$

(c) $a_c = \dfrac{v^2}{r}$ so $v = \sqrt{a_c r} = \sqrt{29.7\text{ m/s}^2(1.50\text{ m})} = 6.67\text{ m/s}$ tangent to the circle.

P4.43 The particle's centripetal acceleration is $v^2/r = (3\text{ m/s})^2/2\text{ m} = 4.50\text{ m/s}^2$. The total acceleration magnitude can be larger than or equal to this, but not smaller.

(a) $\boxed{\text{Yes. The particle can be either speeding up or slowing down, with a tangential component of acceleration of magnitude } \sqrt{6^2 - 4.5^2} = 3.97\text{ m/s.}}$

(b) $\boxed{\text{No. The magnitude of the acceleration cannot be less than } v^2/r = 4.5\text{ m/s}^2.}$

Section 4.6 Relative Velocity and Relative Acceleration

***P4.44** The westward speed of the airplane is the horizontal component of its velocity vector, and the northward speed of the wind is the vertical component of its velocity vector, which has magnitude and direction given by

$$v = \sqrt{(150\text{ km/h})^2 + (30.0\text{ km/h})^2} = \boxed{153\text{ km/h}}$$

$$\theta = \tan^{-1}\left(\frac{30.0\text{ km/h}}{150\text{ km/h}}\right) = \boxed{11.3° \text{ north of west}}$$

P4.45 The airplane (AP) travels through the air (W) that can move relative to the ground (G). The airplane is to make a displacement of 750 km north. Treat north as positive y and west as positive x.

(a) The wind (W) is blowing at 35.0 km/h, south. The northern component of the airplane's velocity relative to the ground is

$$\begin{aligned}(v_{\text{AP,G}})_y &= (v_{\text{AP,W}})_y + (v_{\text{W,G}})_y = 630\text{ km/h} - 35.0\text{ km/h} \\ &= 595\text{ km/h}\end{aligned}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\begin{aligned}&\Delta y = \left(v_{\text{AP,G}}\right)_y \Delta t \rightarrow \\ &\Delta t = \frac{\Delta y}{\left(v_{\text{AP,G}}\right)_y} = \frac{750\text{ km}}{595\text{ km/h}} = 1.26\text{ h}\end{aligned}}$$

(b) The wind (W) is blowing at 35.0 km/h, north. The northern component of the airplane's velocity relative to the ground is

$$\begin{aligned}(v_{\text{AP,G}})_y &= (v_{\text{AP,W}})_y + (v_{\text{W,G}})_y = 630\text{ km/h} + 35.0\text{ km/h}\\ &= 665\text{ km/h}\end{aligned}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\Delta t = \frac{\Delta y}{\left(v_{\text{AP,G}}\right)_y} = \frac{750\text{ km}}{665\text{ km/h}} = 1.13\text{ h}}$$

(c) Now, the wind (W) is blowing at 35.0 km/h, east. The airplane must travel directly north to reach its destination, so it must head somewhat west and north so that the east component of the wind's velocity is cancelled by the airplane's west component of velocity. If the airplane heads at an angle θ measured west of north, then

$$\begin{aligned}(v_{\text{AP,G}})_x &= (v_{\text{AP,W}})_x + (v_{\text{W,G}})_x\\ &= (630\text{ km/h})\sin\theta + (-35.0\text{ km/h}) = 0\end{aligned}$$

$$\sin\theta = 35.0/630 \rightarrow \theta = 3.18°$$

The northern component of the airplane's velocity relative to the ground is

$$\begin{aligned}(v_{\text{AP,G}})_y &= (v_{\text{AP,W}})_y + (v_{\text{W,G}})_y = (630\text{ km/h})\cos 3.18° + 0\\ &= 629\text{ km/h}\end{aligned}$$

We can find the time interval the airplane takes to travel 750 km north:

$$\boxed{\Delta t = \frac{\Delta y}{\left(v_{\text{AP,G}}\right)_y} = \frac{750\text{ km}}{629\text{ km/h}} = 1.19\text{ h}}$$

P4.46 Consider the direction the first beltway (B1) moves to be the positive direction. The first beltway moves relative to the ground (G) with velocity $v_{\text{B1,G}} = v_1$.

(a) The woman's velocity relative to the ground is $v_{\text{WG}} = v_{\text{W,B1}} + v_{\text{B1,G}} = v_1 + 0 = v_1$. The time interval required for the woman to travel distance L relative to the ground is

$$\boxed{\Delta t_{\text{woman}} = \frac{L}{v_1}}$$

(b) The man's (M) velocity relative to the ground is $v_{MG} = v_{M,B1} + v_{B1,G} = v_2 + v_1$. The time interval required for the man to travel distance L relative to the ground is

$$\boxed{\Delta t_{man} = \frac{L}{v_1 + v_2}}$$

(c) The second beltway (B2) moves in the negative direction; its velocity is $v_{B2,G} = -v_1$, and the child (C) rides on the second beltway; his velocity relative to the ground is

$$v_{CG} = v_{C,B2} + v_{B2,G} = 0 - v_1 = -v_1$$

The man's velocity relative to the child is

$$v_{MC} = v_{M,B1} + v_{B1,G} + v_{G,B2} + v_{B2,C}$$
$$v_{MC} = v_{M,B1} + v_{B1,G} - v_{B2,G} - v_{C,B2}$$
$$v_{MC} = v_2 + v_1 - (-v_1) + 0 = v_1 + 2\,v_2$$

so, the time interval required for the man to travel distance L relative to the child is

$$\boxed{\Delta t_{man} = \frac{L}{v_1 + 2v_2}}$$

P4.47 Both police car (P) and motorist (M) move relative to the ground (G). Treating west as the positive direction, the components of their velocities (in km/h) are:

$$v_{PG} = 95.0 \text{ km/h (west)} \qquad v_{PG} = 80 \text{ km/h (west)}$$

(a) $v_{MP} = v_{MG} + v_{GP} = v_{MG} - v_{PG} = 80.0 \text{ km/h} - 95.0 \text{ km/h} = -15.0 = \boxed{15.0 \text{ km/h, east}}$

(b) $v_{PM} = -v_{MP} + \boxed{15.0 \text{ km/h, west}}$

(c) Relative to the motorist, the police car approaches at 15.0 km/h:

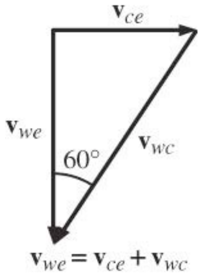
$$d = v\Delta t$$
$$\to \Delta t = \frac{d}{v} = \frac{0.250 \text{ km}}{15.0 \text{ km/h}} = \left(1.67 \times 10^{-2} \text{ h}\right)\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{60.0 \text{ s}}$$

We define the following velocity vectors:

$\vec{v}_{ce}$ = the velocity of the car relative to the Earth

$\vec{v}_{wc}$ = the velocity of the water relative to the car

$\vec{v}_{we}$ = the velocity of the water relative to the Earth



ANS. FIG. P4.48

These velocities are related as shown in ANS. FIG. P4.48

(a) Since $\vec{v}_{we}$ is vertical, $v_{wc}\sin 60.0° = v_{ce} = 50.0$ km/h or

$$\vec{v}_{wc} = \boxed{57.7\text{ km/h at }60.0°\text{ west of vertical}}$$

(b) Since $\vec{v}_{ce}$ has zero vertical component,

$$v_{we} = v_{wc}\cos 60.0° = (57.7\text{ km/h})\cos 60.0°$$
$$= \boxed{28.9\text{ km/h downward}}$$

P4.49 (a) To an observer at rest in the train car, the bolt accelerates downward and toward the rear of the train.

$$a = \sqrt{(2.50\text{ m/s})^2 + (9.80\text{ m/s})^2} = \boxed{10.1\text{ m/s}^2}$$

$$\tan\theta = \frac{2.50\text{ m/s}^2}{9.80\text{ m/s}^2} = 0.255$$

$$\theta = \boxed{14.3°\text{ to the south from the vertical}}$$

To this observer, the bolt moves as if it were in a gravitational field of 9.80 m/s^2 down + 2.50 m/s^2 south.

(b) $a = \boxed{9.80\text{ m/s}^2\text{ vertically downward}}$

(c) $\boxed{\text{If it is at rest relative to the ceiling at release, the bolt moves on a straight line download and southward at 14.3 degrees from the vertical.}}$

(d) $\boxed{\text{The bolt moves on a parabola with a vertical axis.}}$

P4.50 The total time interval in the river is the longer time spent swimming upstream (against the current) plus the shorter time swimming downstream (with the current). For each part, we will use the basic equation $t = d/v$, where v is the speed of the student relative to the shore.

(a) Total time = time upstream plus time downstream:

$$t_{up} = \frac{1\,000 \text{ m}}{1.20 \text{ m/s} - 0.500 \text{ m/s}} = 1.43 \times 10^3 \text{ s}$$

$$t_{down} = \frac{1\,000 \text{ m}}{1.20 \text{ m/s} + 0.500 \text{ m/s}} = 588 \text{ s}$$

Therefore, $t_{total} = 1.43 \times 10^3 \text{ s} + 588 \text{ s} = \boxed{2.02 \times 10^3 \text{ s}}$.

(b) Total time in still water $t = \frac{d}{v} = \frac{2\,000}{1.20} = \boxed{1.67 \times 10^3 \text{ s}}$.

(c) Swimming with the current does not compensate for the time lost swimming against the current.

P4.51 The student must swim faster than the current to travel upstream.

(a) The speed of the student relative to shore is $v_{up} = c - v$ while swimming upstream (against the current), and $v_{down} = c + v$ while swimming downstream (with the current).

Note, The student must swim faster than the current to travel upstream. The time interval required to travel distance *d* upstream is then

$$\Delta t_{up} = \frac{d}{v_{up}} = \frac{d}{c - v}$$

and the time interval required to swim the same distance *d* downstream is

$$\Delta t_{down} = \frac{d}{v_{down}} = \frac{d}{c + v}$$

The time interval for the round trip is therefore

$$\Delta t = \Delta t_{up} + \Delta t_{down} = \frac{d}{c - v} + \frac{d}{c + v} = d\frac{(c + v) + (c - v)}{(c - v)(c + v)}$$

$$\boxed{\Delta t = \frac{2dc}{c^2 - v^2}}$$

(b) In still water, $v = 0$, so $v_{up} = v_{down} = c$; the equation for the time interval for the complete trip reduces to

$$\boxed{\Delta t = \frac{2d}{c}}$$

(c) The equation for the time interval for the complete trip can be written as

$$\Delta t = \frac{2dc}{c^2 - v^2} = \frac{2d}{c\left(1 - \frac{v^2}{c^2}\right)}$$

Because the denominator is always smaller than c, $\boxed{\text{swimming with and against the current is always longer than in still water}}$.

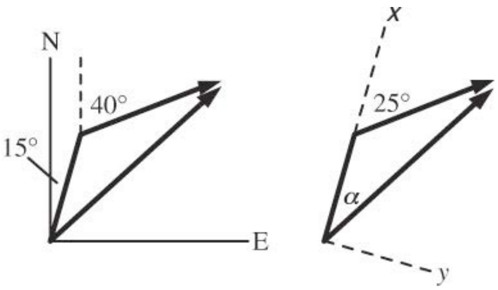
P4.52 Choose the x axis along the 20-km distance. The y components of the displacements of the ship and the speedboat must agree:

$$(26\text{ km/h})t\sin(40.0° - 15.0°) = (50\text{ km/h})t\sin\alpha$$

$$\alpha = \sin^{-1}\left(\frac{11.0\text{ km/h}}{50\text{ km/h}}\right) = 12.7°$$

The speedboat should head

$$15.0° + 12.7° = \boxed{27.7°\text{ E of N}}$$



ANS. FIG. P4.52

P4.53 Identify the student as the S' observer and the professor as the S observer. For the initial motion in S', we have

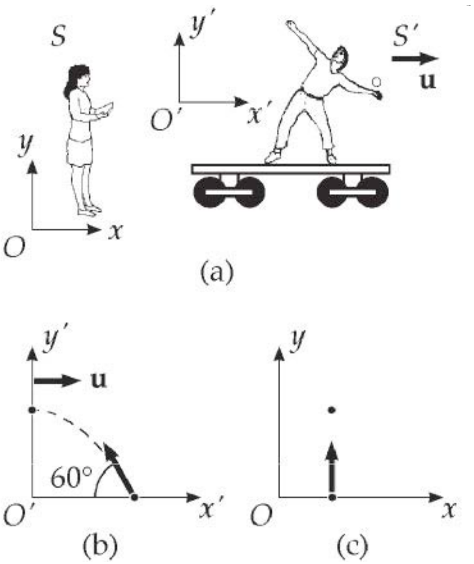
$$\frac{v'_y}{v'_x} = \tan 60.0° = \sqrt{3}$$

Let u represent the speed of S' relative to S. Then because there is no x motion in S, we can write $v_x = v'_x + u = 0$ so that $v'_x = -u = -10.0$ m/s. Hence the ball is thrown backwards in S'. Then,

$$v'_y = v'_y = \sqrt{3}\left|v'_x\right| = 10.0\sqrt{3}\text{ m/s}$$

Using $v_y^2 = 2gh$ we find

$$h = \frac{\left(10.0\sqrt{3}\text{ m/s}\right)^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{15.3\text{ m}}$$



ANS. FIG. P4.53

The motion of the ball as seen by the student in *S′* is shown in ANS. FIG. P4.53(b). The view of the professor in *S* is shown in ANS. FIG. P4.53(c).

P4.54 (a) For the boy to catch the can at the same location on the truck bed, he must throw it $\boxed{\text{straight up, at } 0° \text{ to the vertical}}$.

(b) We find the time of flight of the can by considering its horizontal motion:

$$16.0 \text{ m} = (9.50 \text{ m/s})t + 0 \rightarrow t = 1.68 \text{ s}$$

For the free fall of the can, $y_f = y_i + v_{yi}t - \frac{1}{2}a_y t^2$:

$$0 = 0 + v_{yi}(1.68 \text{ s}) - \frac{1}{2}(9.80 \text{ m/s}^2)(1.68 \text{ s})^2$$

which gives $v_{yi} = \boxed{8.25 \text{ m/s}}$.

(c) The boy sees the can always over his head, traveling in $\boxed{\text{a straight up and down line}}$.

(d) The ground observer sees the can move as a projectile traveling in $\boxed{\text{a symmetric parabola opening downward}}$.

(e) Its initial velocity is

$$\sqrt{(9.50 \text{ m/s})^2 + (8.25 \text{ m/s})^2} = \boxed{12.6 \text{ m/s north}}$$

at an angle of

$$\tan^{-1}\left(\frac{8.25 \text{ m/s}}{9.50 \text{ m/s}}\right) = \boxed{41.0° \text{ above the horizontal}}$$

Additional Problems

***P4.55** After the string breaks the ball is a projectile, and reaches the ground at time *t*:

$$y_f = v_{yi}t + \frac{1}{2}a_y t^2$$

$$-1.20 \text{ m} = 0 + \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$

so t = 0.495 s. Its constant horizontal speed is

$$v_x = \frac{x}{t} = \frac{2.00\text{ m}}{0.495\text{ s}} = 4.04\text{ m/s}$$

so before the string breaks

$$a_c = \frac{v_x^2}{r} = \frac{(4.04\text{ m/s})^2}{0.300\text{ m}} = \boxed{54.4\text{ m/s}^2}$$

***P4.56** The maximum height of the ball is given by Equation 4.12:

$$h = \frac{v_i^2 \sin^2\theta_i}{2g}$$

Equation 4.13 then gives the horizontal range of the ball:

$$R = \frac{v_i^2 \sin 2\theta_i}{g} = \frac{2v_i^2 \sin\theta_i \cos\theta_i}{g}$$

If $h = \frac{R}{6}$, Equation 4.12 yields

$$v_i \sin\theta_i = \sqrt{\frac{gR}{3}} \qquad \textbf{[1]}$$

Substituting equation [1] above into Equation 4.13 gives

$$R = \frac{2\left(\sqrt{gR/3}\right)v_i \cos\theta_i}{g}$$

which reduces to

$$v_i \cos\theta_i = \frac{1}{2}\sqrt{3gR} \qquad \textbf{[2]}$$

(a) From $v_{yf} = v_{yi} + a_y t$, the time to reach the peak of the path $\left(\text{where } v_{yf} = 0\right)$ is found to be

$$t_{\text{peak}} = v_i \sin\theta_i / g$$

Using equation [1], this gives

$$t_{\text{peak}} = \sqrt{\frac{R}{3g}}$$

The total time of the ball's flight is then

$$\boxed{t_{\text{flight}} = 2t_{\text{peak}} = 2\sqrt{\frac{R}{3g}}}$$

(b) At the path's peak, the ball moves horizontally with speed

$$v_{peak} = v_{xi} = v_i \cos\theta_i$$

Using equation [2], this becomes

$$v_{peak} = \boxed{\frac{1}{2}\sqrt{3gR}}$$

(c) The initial vertical component of velocity is $v_{yi} = v_i \sin\theta_i$. From equation [1],

$$v_{yi} = \boxed{\sqrt{\frac{gR}{3}}}$$

(d) Squaring equations [1] and [2] and adding the results,

$$v_i^2\left(\sin^2\theta_i + \cos^2\theta_i\right) = \frac{gR}{3} + \frac{3gR}{4} = \frac{13gR}{12}$$

Thus, the initial speed is

$$v_i = \boxed{\sqrt{\frac{13gR}{12}}}$$

(e) Dividing equation [1] by [2] yields

$$\tan\theta_i = \frac{v_i \sin\theta_i}{v_i \cos\theta_i} = \left[\frac{\left(\sqrt{gR/3}\right)}{\left(\frac{1}{2}\sqrt{3gR}\right)}\right] = \frac{2}{3}$$

Therefore,

$$\theta_i = \tan^{-1}\left(\frac{2}{3}\right) = \boxed{33.7°}$$

(f) For a given initial speed, the projection angle yielding maximum peak height is $\theta_i = 90.0°$. With the speed found in (d), Equation 4.12 then yields

$$h_{max} = \frac{\left(13gR/12\right)\sin^2 90.0°}{2g} = \boxed{\frac{13}{24}R}$$

(g) For a given initial speed, the projection angle yielding maximum range is $\theta_i = 45.0°$. With the speed found in (d), Equation 4.13 then gives

$$R_{max} = \frac{\left(13gR/12\right)\sin 90.0°}{g} = \boxed{\frac{13}{12}R}$$

P4.57 We choose positive y to be in the downward direction. The ball when released has velocity components $v_{xi} = v$ and $v_{yi} = 0$, where v is the speed of the man. We can find the length of the time interval the ball takes to fall the distance h using

$$\Delta y = \frac{1}{2}a_y(\Delta t)^2 = \frac{1}{2}g(\Delta t)^2 \rightarrow \Delta t = \sqrt{\frac{2h}{g}}$$

The horizontal displacement of the ball during this time interval is

$$\Delta x = v_{xi}\ \Delta t = v\sqrt{\frac{2h}{g}} = 7.00h$$

Solve for the speed:

$$v = \sqrt{\frac{49.0gh}{2}} = \sqrt{\frac{49.0(9.80\ \text{m/s}^2)h}{2}} = 15.5\sqrt{h}$$

where h is in m and v in m/s.

If we express the height as a function of speed, we have

$$h = (4.16 \times 10^{-2})v^2$$

where h is in m and v is in m/s.

For a normally proportioned adult, h is about 0.50 m, which would mean that $v = 15.5\sqrt{0.50} = 11$ m/s, which is about 39 km/h; no normal adult could walk "briskly" at that speed. If the speed were a realistic typical speed of 4 km/h, from our equation for h, we find that the height would be about 4 cm, much too low for a normal adult.

P4.58 (a) From $\vec{\mathbf{a}} = d\vec{\mathbf{v}}/dt$, we have

$$\int_i^f d\vec{\mathbf{v}} = \int_i^f \vec{\mathbf{a}}\,dt = \Delta\vec{\mathbf{v}}$$

Then

$$\vec{\mathbf{v}} - 5\hat{\mathbf{i}}\ \text{m/s} = \int_0^t 6\ t^{1/2}dt\ \hat{\mathbf{j}} = 6\left.\frac{t^{3/2}}{3/2}\right|_0^t \hat{\mathbf{j}} = 4\ t^{3/2}\ \hat{\mathbf{j}}\ \text{m/s}$$

so $\vec{\mathbf{v}} = \boxed{\left(5\ \hat{\mathbf{i}} + 4\ t^{3/2}\ \hat{\mathbf{j}}\right)\ \text{m/s}}$.

(b) From $\vec{\mathbf{v}} = d\vec{\mathbf{r}}/dt$, we have

$$\int_i^f d\vec{\mathbf{r}} = \int_i^f \vec{\mathbf{v}}\,dt = \Delta\vec{\mathbf{r}}$$

Then

$$\vec{\mathbf{r}} - 0 = \int_0^t \left(5\,\hat{\mathbf{i}} + 4\,t^{3/2}\,\hat{\mathbf{j}}\right)dt = \left(5t\,\hat{\mathbf{i}} + 4\frac{t^{5/2}}{5/2}\hat{\mathbf{j}}\right)\Bigg|_0^t$$

$$= \boxed{\left(5t\,\hat{\mathbf{i}} + 1.6\,t^{5/2}\,\hat{\mathbf{j}}\right)\ \mathrm{m}}$$

P4.59 (a) The speed at the top is

$$v_x = v_i\cos\theta_i = (143\ \mathrm{m/s})\cos 45° = \boxed{101\ \mathrm{m/s}}$$

(b) In free fall the plane reaches altitude given by

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

$$0 = \left(143\ \mathrm{m/s}\ \sin 45°\right)^2 + 2\left(-9.80\ \mathrm{m/s^2}\right)\left(y_f - 31\ 000\ \mathrm{ft}\right)$$

$$y_f = 31\ 000\ \mathrm{ft} + 522\ \mathrm{m}\left(\frac{3.28\ \mathrm{ft}}{1\ \mathrm{m}}\right) = \boxed{3.27\times 10^4\ \mathrm{ft}}$$

(c) For the whole free-fall motion $v_{yf} = v_{yi} + a_y t$:

$$-101\ \mathrm{m/s} = +101\ \mathrm{m/s} - \left(9.80\ \mathrm{m/s^2}\right)t$$

$$t = \boxed{20.6\ \mathrm{s}}$$

P4.60 (a) The acceleration is that of gravity: $\boxed{9.80\ \mathrm{m/s^2},\ \text{downward.}}$

(b) The horizontal component of the initial velocity is $v_{xi} = v_i\cos 40.0°$ $= 0.766v_i$, and the time required for the ball to move 10.0 m horizontally is

$$t = \frac{\Delta x}{v_{xi}} = \frac{10.0\ \mathrm{m}}{0.766\ v_i} = \frac{13.1\ \mathrm{m}}{v_i}$$

At this time, the vertical displacement of the ball must be

$$\Delta y = y_f - y_i = 3.05\ \mathrm{m} - 2.00\ \mathrm{m} = 1.05\ \mathrm{m}$$

Thus, $\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$ becomes

$$1.05\ \mathrm{m} = \left(\cancel{v_i}\ \sin 40.0°\right)\frac{13.1\ \mathrm{m}}{\cancel{v_i}} + \frac{1}{2}\left(-9.80\ \mathrm{m/s^2}\right)\frac{(13.1\ \mathrm{m})^2}{v_i^2}$$

or $$1.05\ \mathrm{m} = 8.39\ \mathrm{m} - \frac{835\ \mathrm{m^3/s^2}}{v_i^2}$$

which yields

$$v_i = \sqrt{\frac{835\ \text{m}^3/\text{s}^2}{8.39\ \text{m} - 1.05\ \text{m}}} = \boxed{10.7\ \text{m/s}}$$

P4.61 Both Lisa and Jill start from rest. Their accelerations are

$$\vec{\mathbf{a}}_\text{L} = (3.00\,\hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}})\ \text{m/s}^2$$

$$\vec{\mathbf{a}}_\text{J} = (1.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})\ \text{m/s}^2$$

Integrating these, and knowing that they start from rest, we find their velocities:

$$\vec{\mathbf{v}}_\text{L} = (3.00t\,\hat{\mathbf{i}} - 2.00t\,\hat{\mathbf{j}})\ \text{m/s}$$

$$\vec{\mathbf{v}}_\text{J} = (1.00t\,\hat{\mathbf{i}} + 3.00t\,\hat{\mathbf{j}})\ \text{m/s}$$

Integrating again, and knowing that they start from the origin, we find their positions:

$$\vec{\mathbf{r}}_\text{L} = (1.50t^2\,\hat{\mathbf{i}} - 1.00t^2\,\hat{\mathbf{j}})\ \text{m}$$

$$\vec{\mathbf{r}}_\text{J} = (0.50\,t^2\,\hat{\mathbf{i}} + 1.50t^2\,\hat{\mathbf{j}})\ \text{m}$$

All of the above are with respect to the ground (G).

(a) In general, Lisa's velocity with respect to Jill is

$$\vec{\mathbf{v}}_\text{LJ} = \vec{\mathbf{v}}_\text{LG} + \vec{\mathbf{v}}_\text{GJ} = \vec{\mathbf{v}}_\text{LG} - \vec{\mathbf{v}}_\text{JG}$$

$$\vec{\mathbf{v}}_\text{LJ} = \vec{\mathbf{v}}_\text{L} - \vec{\mathbf{v}}_\text{J} = (3.00t\,\hat{\mathbf{i}} - 2.00t\,\hat{\mathbf{j}}) - (1.00t\,\hat{\mathbf{i}} + 3.00t\,\hat{\mathbf{j}})$$

$$\vec{\mathbf{v}}_\text{LJ} = (2.00t\,\hat{\mathbf{i}} - 5.00t\,\hat{\mathbf{j}})$$

When $t = 5.00\ \text{s}, \vec{\mathbf{v}}_\text{LJ} = (10.0\,\hat{\mathbf{i}} - 25.0\,\hat{\mathbf{j}})\ \text{m/s},$ so the speed (magnitude) is

$$\boxed{v = \sqrt{(10.0)^2 + (25.0)^2} = 26.9\ \text{m/s}}$$

(b) In general, Lisa's position with respect to Jill is

$$\vec{\mathbf{r}}_\text{LJ} = \vec{\mathbf{r}}_\text{L} - \vec{\mathbf{r}}_\text{J} = (1.50t^2\,\hat{\mathbf{i}} - 1.00t^2\,\hat{\mathbf{j}}) - (0.50t^2\,\hat{\mathbf{i}} + 1.50t^2\,\hat{\mathbf{j}})$$

$$\vec{\mathbf{r}}_\text{LJ} = (1.00t^2\,\hat{\mathbf{i}} - 2.50t^2\,\hat{\mathbf{j}})$$

When $t = 5.00\ \text{s}, \vec{\mathbf{r}}_\text{LJ} = (25.0\hat{\mathbf{i}} - 62.5\hat{\mathbf{j}})\ \text{m},$ and their distance apart is

$$\boxed{d = \sqrt{(25.0\ \text{m})^2 + (62.5\ \text{m})^2} = 67.3\ \text{m}}$$

(c) In general, Lisa's acceleration with respect to Jill is

$$\vec{\mathbf{a}}_{LJ} = \vec{\mathbf{a}}_L - \vec{\mathbf{a}}_J = (3.00\,\hat{\mathbf{i}} - 2.00\,\hat{\mathbf{j}}) - (1.00\,\hat{\mathbf{i}} + 3.00\,\hat{\mathbf{j}})$$

$$\vec{\mathbf{a}}_{LJ} = \boxed{(2.00\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})\text{ m/s}^2}$$

P4.62 (a) The stone's initial velocity components (at $t = 0$) are v_{xi} and $v_{yi} = 0$, and the stone falls through a vertical displacement $\Delta y = -h$. We find the time t when the stone strikes the ground using

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2 \rightarrow -h = 0 - \frac{1}{2}gt^2 \rightarrow \boxed{t = \sqrt{\frac{2h}{g}}}$$

(b) To find the stone's initial horizontal component of velocity, we know at the above time t, the stone's horizontal displacement is $\Delta x = d$:

$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow d = v_{xi}t \rightarrow v_{0x} = \frac{d}{t} \rightarrow \boxed{v_{xi} = d\sqrt{\frac{g}{2h}}}$$

(c) The vertical component of velocity at time t is

$$v_{yf} = v_{yi} + a_y t = 0 - gt \rightarrow v_{yf} = -g\sqrt{\frac{2h}{g}} \rightarrow v_{yf} = -\sqrt{2gh}$$

and the horizontal component does not change; therefore, the speed of the stone as it reaches the ocean is

$$\boxed{v_f = \sqrt{v_{xf}{}^2 + v_{yf}{}^2} = \sqrt{\left(\frac{d^2 g}{2h}\right) + (2gh)}}$$

(d) From above,

$$\theta_f = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-\sqrt{2gh}}{d\sqrt{\dfrac{g}{2h}}}\right)$$

$$\theta_f = -\tan^{-1}\left(\frac{2h}{d}\right)$$

which means the velocity points below the horizontal by angle

$$\boxed{\theta_f = \tan^{-1}\left(\frac{2h}{d}\right)}$$

P4.63 We use a fixed coordinate system that, viewed from above, has its positive *x* axis passing through point A when the flea jumps, and its positive *y* axis 90° counterclockwise from its *x* axis. Its positive *z* axis is upward. The turntable rotates clockwise. At $t = 0$, the flea jumps straight up relative to the turntable, but the turntable is spinning, so the flea has both horizontal and vertical components of velocity relative to the fixed coordinate axes. Because the turntable is spinning clockwise, the horizontal velocity of the flea is in the negative *y* directon:

$$v_y = \left(-33.3\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi(10.0\text{ cm})}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = -34.9\text{ cm/s}$$

The vertical motion of the flea is independent of its horizontal motion. The time interval the flea takes to rise to a height *h* of 5.00 cm is the same time interval the flea takes to drop back to the turntable. We find the interval to drop using

$$z_f = z_i + v_{zi}t + \frac{1}{2}a_z t^2 \rightarrow 0 = h - \frac{1}{2}gt^2 \rightarrow t = \sqrt{\frac{2h}{g}}$$

where *h* is in m and *t* in s. Substituting, we find

$$t = \sqrt{\frac{2(0.050\ 0\text{ m})}{9.80\text{ m/s}^2}} = 0.101\text{ s}$$

The total time interval for the flea to leave the surface of the turntable and return is twice this: $\Delta t = 0.202$ s.

(a) Find the clockwise angle the turntable rotates through in the time interval Δt:

$$\begin{aligned}\Delta\theta &= \left(\frac{33.3\text{ rev}}{\text{min}}\right)(0.202\text{ s})\\ &= \left[\left(\frac{33.3\text{ rev}}{\text{min}}\right)\left(\frac{360^\circ}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right](0.202\text{ s})\\ &= 40.4^\circ\end{aligned}$$

Point A lies 10.0 cm from the origin. When the flea jumps, the line passing from the origin to point A coincides with the positive *x* axis, but when the flea lands, the line makes an angle of –40.4° with the positive *x* axis:

$$\vec{\mathbf{r}}_A = [10.0\cos(-40.4^\circ)]\hat{\mathbf{i}} + [10.0\sin(-40.4^\circ)]\hat{\mathbf{j}}$$

$$\vec{\mathbf{r}}_A = \boxed{(7.61\hat{\mathbf{i}} - 6.48\hat{\mathbf{j}})\text{cm}}$$

(b) During this time interval, the flea goes through a horizontal *y* displacement

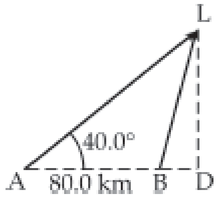
$$\Delta y = v_y \Delta t = (-34.9 \text{ cm/s})(0.202 \text{ s}) = -7.05 \text{ cm}.$$

The flea has no motion parallel to the *x* axis; therefore, the position of point B where the flea lands is

$$\vec{r}_B = \boxed{(10.0\hat{\mathbf{i}} - 7.05\hat{\mathbf{j}}) \text{ cm}}$$

***P4.64** ANS. FIG. P4.64 shows the triangles ALB and ALD. To find the length $\overline{AL}$, we write

$$\overline{AL} = v_1 t = (90.0 \text{ km/h})(2.50 \text{ h}) = 225 \text{ km}$$



ANS. FIG. P4.64

To find the distance travelled by the second couple, we need to determine the length $\overline{BD}$:

$$\begin{aligned}\overline{BD} &= \overline{AD} - \overline{AB} \\ &= \overline{AL}\cos 40.0° - 80.0 \text{ km} = 92.4 \text{ km}\end{aligned}$$

Then, from the triangle BLD in ANS. FIG. P4.64,

$$\begin{aligned}\overline{BL} &= \sqrt{(\overline{BD})^2 + (\overline{DL})^2} \\ &= \sqrt{(92.4 \text{ km})^2 + (\overline{AL}\sin 40.0°)^2} = 172 \text{ km}\end{aligned}$$

Note that the law of cosines can also be used for the triangle ABL to solve for the length BD. Since Car 2 travels this distance in 2.50 h, its constant speed is

$$v_2 = \frac{172 \text{ km}}{2.5 \text{ h}} = \boxed{68.8 \text{ km/h}}$$

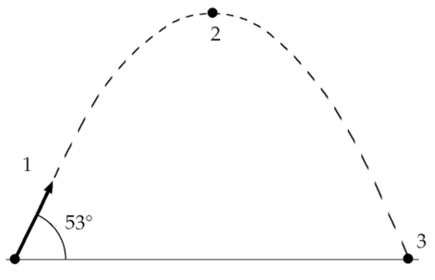
***P4.65** Consider the rocket's trajectory in 3 parts as shown in the diagram on the right. Our initial conditions give:

$$a_y = (30.0 \text{ m/s}^2)\sin 53.0° = 24.0 \text{ m/s}^2$$

$$a_x = (30.0 \text{ m/s}^2)\cos 53.0° = 18.1 \text{ m/s}^2$$

$$v_{yi} = (100 \text{ m/s})\sin 53.0° = 79.9 \text{ m/s}$$

$$v_{xi} = (100 \text{ m/s})\cos 53.0° = 60.2 \text{ m/s}$$



ANS. FIG. P4.65

The distances traveled during each phase of the motion are given in Table P4.65 below.

Path Part #1:

$$\begin{aligned} v_{yf} &= v_{yi} + a_y t \\ &= 79.9\text{ m/s} + \left(24.0\text{ m/s}^2\right)(3.00\text{ s}) \\ &= 152\text{ m/s} \end{aligned}$$

$$\begin{aligned} v_{xf} &= v_{xi} + a_x t \\ &= 60.2\text{ m/s} + \left(18.1\text{ m/s}^2\right)(3.00\text{ s}) \\ &= 114\text{ m/s} \end{aligned}$$

$$\begin{aligned} \Delta y &= v_{yi}t + \frac{1}{2}a_y t^2 \\ &= (79.9\text{ m/s})(3.00\text{ s}) + \frac{1}{2}\left(24.0\text{ m/s}^2\right)(3.00\text{ s})^2 \\ &= 347\text{ m} \end{aligned}$$

$$\begin{aligned} \Delta x &= v_{xi}t + \frac{1}{2}a_x t^2 \\ &= (60.2\text{ m/s})(3.00\text{ s}) + \frac{1}{2}\left(18.1\text{ m/s}^2\right)(3.00\text{ s})^2 \\ &= 262\text{ m} \end{aligned}$$

Path Part #2:

Now $a_x = 0$, $a_y = -9.80\text{ m/s}^2$, $v_{xf} = v_{xi} = 114\text{ m/s}$, $v_{yi} = 152\text{ m/s}$, and $v_{yf} = 0$, so

$$\begin{aligned} v_{yf} &= v_{yi} + a_y t \\ 0 &= 152\text{ m/s} - \left(9.80\text{ m/s}^2\right)t \end{aligned}$$

which gives $t = 15.5\text{ s}$

$$\Delta x = v_{xf}t = (114\text{ m/s})(15.5\text{ s}) = 1.77\times 10^3\text{ m}$$

$$\Delta y = (152\text{ m/s})(15.5\text{ s}) - \frac{1}{2}\left(9.80\text{ m/s}^2\right)(15.5\text{ s})^2 = 1.17\times 10^3\text{ m}$$

Path Path #3:

With $v_{yi} = 0$, $a_x = 0$, $a_y = -9.80\text{ m/s}^2$, and $v_{xf} = v_{xi} = 114\text{ m/s}$, then

$$\begin{aligned} \left(v_{yf}\right)^2 - \left(v_{yi}\right)^2 &= 2a\Delta y \\ \left(v_{yf}\right)^2 - 0 &= 2\left(-9.80\text{ m/s}^2\right)\left(-1.52\times 10^3\text{ m}\right) \end{aligned}$$

which gives $v_{yf} = -173\text{ m/s}$

We find the time from $v_{yf} = v_{yi} - gt$, which gives

$-173\text{ m/s} - 0 = -(9.80\text{ m/s}^2)t$, or $t = 17.6$ s

$\Delta x = v_{xf}t = 114(17.6) = 2.02 \times 10^3\text{ m}$

(a) $\Delta y(\max) = \boxed{1.52 \times 10^3\text{ m}}$

(b) $t(\text{net}) = 3.00\text{ s} + 15.5\text{ s} + 17.6\text{ s} = \boxed{36.1\text{ s}}$

(c) $\Delta x(\text{net}) = 262\text{ m} + 1.77 \times 10^3\text{ m} + 2.02 \times 10^3\text{ m}$

$\Delta x(\text{net}) = \boxed{4.05 \times 10^3\text{ m}}$

	Path Part		
	#1	#2	#3
a_y	24.0	–9.80	–9.80
a_x	18.1	0.0	0.0
v_{yf}	152	0.0	–173
v_{xf}	114	114	114
v_{yi}	79.9	152	0.0
v_{xi}	60.2	114	114
Δy	347	1.17×10^3	-1.52×10^3
Δx	262	1.77×10^3	2.02×10^3
t	3.00	15.5	17.6

Table P4.65

***P4.66** Take the origin at the mouth of the cannon. We have $x_f = v_{xi}t$, which gives

$$2\,000\text{ m} = (1\,000\text{ m/s})\cos\theta_i t$$

Therefore,

$$t = \frac{2.00\ \text{s}}{\cos\theta_i}$$

From $y_f = v_{yi}t + \frac{1}{2}a_y t^2$:

$$800\ \text{m} = (1\,000\ \text{m/s})\sin\theta_i\, t + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2$$

$$800\ \text{m} = (1\,000\ \text{m/s})\sin\theta_i\left(\frac{2.00\ \text{s}}{\cos\theta_i}\right) - \frac{1}{2}(9.80\ \text{m/s}^2)\left(\frac{2.00\ \text{s}}{\cos\theta_i}\right)^2$$

$$800\ \text{m}(\cos^2\theta_i) = 2\,000\ \text{m}(\sin\theta_i\cos\theta_i) - 19.6\ \text{m}$$

$$19.6\ \text{m} + 800\ \text{m}(\cos^2\theta_i) = 2\,000\ \text{m}\sqrt{1-\cos^2\theta_i}\,(\cos\theta_i)$$

$$\begin{aligned}384 + (31\ 360)\cos^2\theta_i + (640\ 000)\cos^4\theta_i \\ = (4\ 000\ 000)\cos^2\theta_i - (4\ 000\ 000)\cos^4\theta_i\end{aligned}$$

$$4\ 640\ 000\cos^4\theta_i - 3\ 968\ 640\cos^2\theta_i + 384 = 0$$

$$\cos^2\theta_i = \frac{3\ 968\ 640 \pm \sqrt{(3\ 968\ 640)^2 - 4(4\ 640\ 000)(384)}}{9\ 280\ 000}$$

$$\cos\theta_i = 0.925 \text{ or } \cos\theta_i = 0.009\ 84$$

$$\theta_i = \boxed{22.4° \text{ or } 89.4°} \qquad \text{(Both solutions are valid.)}$$

P4.67 Given the initial velocity, we can calculate the height change of the ball as it moves 130 m horizontally. So this is what we do, expecting the answer to be inconsistent with grazing the top of the bleachers. We assume the ball field is horizontal. We think of the ball as a particle in free fall (moving with constant acceleration) between the point just after it leaves the bat until it crosses above the cheap seats.

The initial components of velocity are

$$v_{xi} = v_i\cos\theta = 41.7\cos 35.0° = 34.2\ \text{m/s}$$
$$v_{yi} = v_i\sin\theta = 41.7\sin 35.0° = 23.9\ \text{m/s}$$

We find the time when the ball has traveled through a horizontal displacement of 130 m:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow x_f = x_i + v_{xi}t \rightarrow t = (x_f - x_i)/v_{xi}$$

$$t = \frac{130\ \text{m} - 0}{34.2\ \text{m/s}} = 3.80\ \text{s}$$

Now we find the vertical position of the ball at this time:

$$y_f = y_i + v_{yi}t + \frac{1}{2}at^2 = 0 + v_{yi}t - \frac{1}{2}t^2$$
$$y_f = (23.9\text{ m/s})(3.80\text{ s}) - (4.90\text{ m/s}^2)(3.80\text{ s})^2 = 20.1\text{ m}$$

$$\boxed{\text{The ball would not be high enough to have cleared the 24.0-m-high bleachers.}}$$

P4.68 At any time *t*, the two drops have identical *y* coordinates. The distance between the two drops is then just twice the magnitude of the horizontal displacement either drop has undergone. Therefore,

$$d = 2|x(t)| = 2(v_{xi}t) = 2(v_i\ \cos\theta_i)t = \boxed{2v_i t\ \cos\theta_i}$$

P4.69 (a) The Moon's gravitational acceleration is the probe's centripetal acceleration: (For the Moon's radius, see end papers of text.)

$$a = \frac{v^2}{r}$$
$$\frac{1}{6}(9.80\text{ m/s}^2) = \frac{v^2}{1.74\times 10^6\text{ m}}$$
$$v = \sqrt{2.84\times 10^6\text{ m}^2/\text{s}^2} = \boxed{1.69\text{ km/s}}$$

(b) The time interval can be found from

$$v = \frac{2\pi r}{T}$$
$$T = \frac{2\pi r}{v} = \frac{2\pi(1.74\times 10^6\text{ m})}{1.69\times 10^3\text{ m/s}} = 6.47\times 10^3\text{ s} = \boxed{1.80\text{ h}}$$

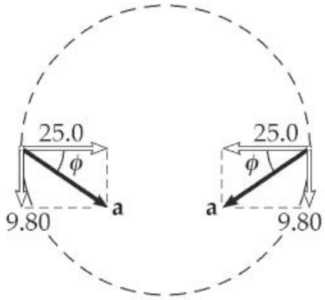
P4.70 (a) The length of the cord is given as $r = 1.00$ m. At the positions with $\theta = 90.0°$ and 270°,

$$a_c = \frac{v^2}{r} = \frac{(5.00\text{ m/s})^2}{1.00\text{ m}} = \boxed{25.0\text{m/s}^2}$$

(b) The tangential acceleration is only the acceleration due to gravity,

$$a_t = g = \boxed{9.80\text{ m/s}^2}$$

(c) See ANS. FIG. P4.70.



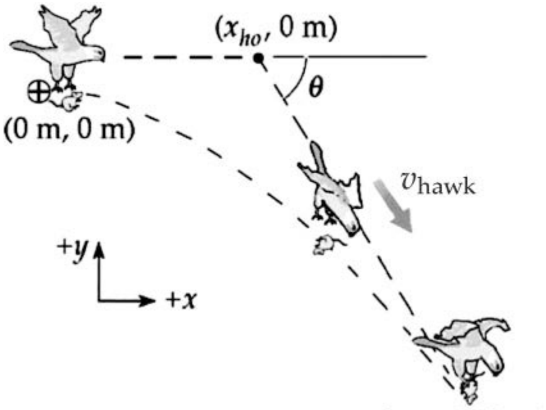
ANS. FIG. P4.70

(d) The magnitude and direction of the total acceleration at these positions is given by

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{(25.0 \text{ m/s}^2)^2 + (9.80 \text{ m/s}^2)^2} = \boxed{26.8 \text{ m/s}^2}$$

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{9.80 \text{ m/s}^2}{25.0 \text{ m/s}^2}\right) = \boxed{21.4°}$$

P4.71 We know the distance that the mouse and hawk move down, but to find the diving speed of the hawk, we must know the time interval of descent, so we will solve part (c) first. If the hawk and mouse both maintain their original horizontal velocity of 10 m/s (as the mouse should without air resistance), then the hawk only needs to think about diving straight down, but to a ground-based observer, the path will appear to be a straight line angled less than 90° below horizontal.



ANS. FIG. P4.71

We begin with the simple calculation of the free-fall time interval for the mouse.

(c) The mouse falls a total vertical distance y = 200 m – 3.00 m = 197 m. The time interval of fall is found from (with $v_{yi} = 0$)

$$y = v_{yi}t - \frac{1}{2}gt^2 \quad \rightarrow \quad t = \sqrt{\frac{2(197 \text{ m})}{9.80 \text{ m/s}^2}} = \boxed{6.34 \text{ s}}$$

(a) To find the diving speed of the hawk, we must first calculate the total distance covered from the vertical and horizontal components. We already know the vertical distance y; we just need the horizontal distance during the same time interval (minus the 2.00-s late start).

$$x = v_{xi}(t - 2.00 \text{ s}) = (10.0 \text{ m/s})(6.34 \text{ s} - 2.00 \text{ s}) = 43.4 \text{ m}$$

The total distance is

$$d = \sqrt{x^2 + y^2} = \sqrt{(43.4 \text{ m})^2 + (197 \text{ m})^2} = 202 \text{ m}$$

So the hawk's diving speed is

$$v = \frac{\Delta d}{\Delta t} = \frac{\sqrt{(197 \text{ m})^2 + (43.4 \text{ m})^2}}{4.34 \text{ s}} = \boxed{46.5 \text{ m/s}}$$

(b) at an angle below the horizontal of

$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{197\text{ m}}{43.4\text{ m}}\right) = \boxed{77.6°}$$

P4.72 (a) We find the x coordinate from $x = 12t$. We find the y coordinate from $49t - 4.9t^2$. Then we find the projectile's distance from the origin as $(x^2 + y^2)^{½}$, with these results:

***t* (s)**	0	1	2	3	4	5	6	7	8	9	10
***r* (m)**	0	45.7	82.0	109	127	136	138	133	124	117	120

(b) From the table, it looks like the magnitude of r is largest at a bit less than 6 s.

The vector $\vec{v}$ tells how $\vec{r}$ is changing. If $\vec{v}$ at a particular point has a component along $\vec{r}$, then $\vec{r}$ will be increasing in magnitude (if $\vec{v}$ is at an angle less than 90° from $\vec{r}$) or decreasing (if the angle between $\vec{v}$ and $\vec{r}$ is more than 90°). To be at a maximum, the distance from the origin must be momentarily staying constant, and the only way this can happen is for the angle between velocity and displacement to be a right angle. Then $\vec{r}$ will be changing in direction at that point, but not in magnitude.

(c) When $t = 5.70$ s, $r = \boxed{138\text{ m.}}$

(d) $\boxed{\text{We can require } dr^2/dt = 0 = (d/dt)[(12t)^2 + (49t - 4.9t^2)^2]\text{, which results in the solution.}}$

P4.73 (a) The time of flight must be positive. It is determined by

$$y_f = y_i + v_{yi}t + (1/2)a_yt^2 \rightarrow \quad 0 = 1.20 + v_i \sin 35.0°t - 4.90t^2$$

From the quadratic formula, and suppressing units, we find

$$t = \frac{0.574v_i + \sqrt{0.329v_i^2 + 23.52}}{9.80}$$

Then the range follows from $x = v_{xi}t + 0 = v_0t$ as

$$\boxed{x(v_i) = v_i\sqrt{0.164\,3 + 0.002\,299v_i^2} + 0.047\,94v_i^2}$$

where x is in meters and v_i is in meters per second.

(b) Substituting $v_i = 0.100$ gives $x(v_i = 0.100) = \boxed{0.041\,0\text{ m}}$

(c) Substituting $v_i = 100$ gives $x(v_i = 100) = \boxed{961\text{ m}}$

(d) When v_i is small, v_i^2 becomes negligible. The expression $x(v_i)$ simplifies to $\mathrm{v_i}\sqrt{0.164\,3+0}+0=\boxed{0.405\ \mathrm{v_i}}$. Note that this gives nearly the answer to part (b).

(e) When v_i is large, v_i is negligible in comparison to v_i^2. Then $x(v_i)$ simplifies to

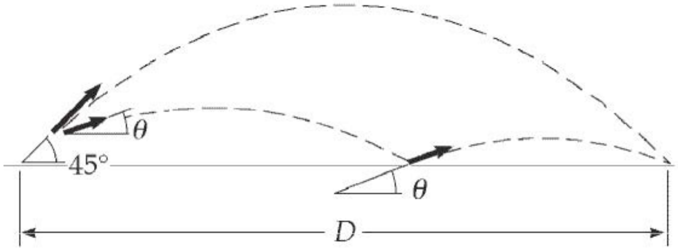
$$\mathrm{x}(\mathrm{v_i})\cong \mathrm{v_i}\sqrt{0+0.002\,299\ \mathrm{v_i^2}}+0.047\,94\ \mathrm{v_i^2}=\boxed{0.0959\ \mathrm{v_i^2}}$$

This nearly gives the answer to part (c).

(f) $\boxed{\text{The graph of x versus v}_\text{i}\text{ starts from the origin as a straight line with slope 0.405 s. Then it curves upward above this tangent line, getting closer and closer to the parabola } \mathrm{x}=(0.095\,9\ \mathrm{s^2/m})\mathrm{v_i}^2.}$

P4.74 The special conditions allowing use of the horizontal range equation applies. For the ball thrown at 45°,

$$\mathrm{D}=\mathrm{R}_{45}=\frac{\mathrm{v_i^2}\sin 90°}{\mathrm{g}}$$



ANS. FIG. P4.74

For the bouncing ball,

$$\mathrm{D}=\mathrm{R_1}+\mathrm{R_2}=\frac{\mathrm{v_i^2}\sin 2\theta}{\mathrm{g}}+\frac{(\mathrm{v_i}/2)^2\sin 2\theta}{\mathrm{g}}$$

where θ is the angle it makes with the ground when thrown and when bouncing.

(a) We require:

$$\frac{\mathrm{v_i^2}}{\mathrm{g}}=\frac{\mathrm{v_i^2}\sin 2\theta}{\mathrm{g}}+\frac{\mathrm{v_i^2}\sin 2\theta}{4\mathrm{g}}$$

$$\sin 2\theta=\frac{4}{5}$$

$$\boxed{\theta=26.6°}$$

(b) The time for any symmetric parabolic flight is given by

$$y_f = v_{yi}t - \frac{1}{2}gt^2$$

$$0 = v_i \sin\theta_i t - \frac{1}{2}gt^2$$

If $t = 0$ is the time the ball is thrown, then $t = \frac{2v_i \sin\theta_i}{g}$ is the time at landing. So for the ball thrown at 45.0°:

$$t_{45} = \frac{2v_i \sin 45.0°}{g}$$

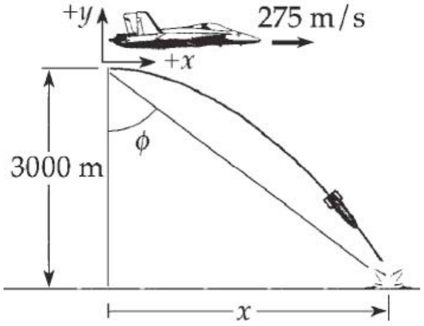
For the bouncing ball,

$$t = t_1 + t_2 = \frac{2v_i \sin 26.6°}{g} + \frac{2(v_i/2)\sin 26.6°}{g} = \frac{3v_i \sin 26.6°}{g}$$

The ratio of this time to that for no bounce is

$$\frac{3v_i \sin 26.6°/g}{2v_i \sin 45.0°/g} = \frac{1.34}{1.41} = \boxed{0.949}$$

P4.75 We model the bomb as a particle with constant acceleration, equal to the downward free-fall acceleration, from the moment after release until the moment before impact. After we find its range it will be a right-triangle problem to find the bombsight angle.



ANS. FIG. P4.75

(a) We take the origin at the point under the plane at bomb release. In its horizontal flight, the bomb has $v_{yi} = 0$ and $v_{xi} = 275$ m/s. We represent the height of the plane as y.

Then, $\Delta y = -\frac{1}{2}gt^2$; $\Delta x = v_i t$

Combining the equations to eliminate t gives:

$$\Delta y = -\frac{1}{2}g\left(\frac{\Delta x}{v_i}\right)^2$$

From this, $(\Delta x)^2 = \left(\frac{-2\Delta y}{g}\right)v_i^2$. Thus

$$\Delta x = v_i\sqrt{\frac{-2\Delta y}{g}} = (275\text{ m/s})\sqrt{\frac{-2(-3\,000\text{ m})}{9.80\text{ m/s}^2}}$$

$$= 6.80\times10^3\text{ m} = \boxed{6.80\text{ km}}$$

(b) The plane has the same velocity as the bomb in the *x* direction. Therefore, the plane will be $\boxed{\text{3 000 m directly above the bomb}}$ when it hits the ground.

(c) When ϕ is measured from the vertical, $\tan\phi = \frac{\Delta x}{\Delta y}$;

therefore, $\phi = \tan^{-1} = \left(\frac{\Delta x}{\Delta y}\right) = \tan^{-1}\left(\frac{6\,800\text{ m}}{3\,000\text{ m}}\right) = \boxed{66.2°}$.

P4.76 Equation of bank: $y^2 = 16x$ **[1]**

Equations of motion: $x = v_i t$ **[2]**

$$y = -\frac{1}{2}gt^2 \quad \textbf{[3]}$$

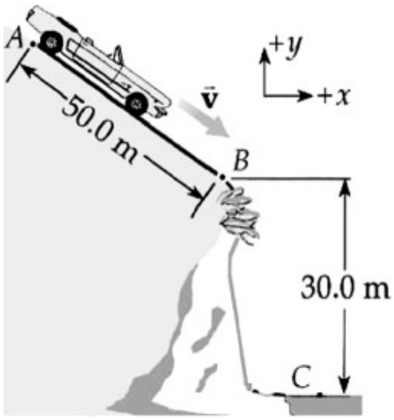
Substitute for *t* from [2] into [3]: $y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)$. Equate *y* from the bank equation to *y* from the equations of motion:

$$16x = \left[-\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right)\right]^2 \Rightarrow \frac{g^2x^4}{4v_i^4} - 16x = x\left(\frac{g^2x^3}{4v_i^4} - 16\right) = 0$$

From this, $x = 0$ or $x^3 = \frac{64v_i^4}{g^2}$ and $x = 4\left(\frac{10^4}{9.80^2}\right)^{1/3}\text{ m} = \boxed{18.8\text{ m}}$.

Also, $y = -\frac{1}{2}g\left(\frac{x^2}{v_i^2}\right) = -\frac{1}{2}\frac{(9.80\text{ m/s}^2)(18.8\text{ m})^2}{(10.0\text{ m/s})^2} = \boxed{-17.3\text{ m}}$

P4.77 The car has one acceleration while it is on the slope and a different acceleration when it is falling, so we must take the motion apart into two different sections. Our standard equations only describe a chunk of motion during which acceleration stays constant. We imagine the acceleration to change instantaneously at the brink of the cliff, but the velocity and the position must be the same just before point *B* and just after point *B*.



ANS. FIG. P4.77

(a) From point *A* to point *B* (along the incline), the car can be modeled as a particle under constant acceleration in one dimension, starting from rest ($v_i = 0$). Therefore, taking Δx to be the position along the incline,

$$v_f^2 - v_i^2 = 2a\Delta x$$

$$v_f^2 - 0 = 2(4.00\text{ m/s}^2)(50.0\text{ m})$$

$$v_f = \boxed{20.0\text{ m/s}}$$

(b) We can find the elapsed time interval from

$$v_f = v_i + at$$

$$20.0\text{ m/s} = 0 + \left(4.00\text{ m/s}^2\right)t$$

$$t = \boxed{5.00\text{ s}}$$

(c) Initial free-fall conditions give us $v_{xi} = 20.0\cos 37.0° = 16.0$ m/s and $v_{yi} = -20.0\sin 37.0° = -12.0$ m/s. Since $a_x = 0$, $v_{xf} = v_{xi}$ and

$$v_{yf} = -\sqrt{2a_y\Delta y + v_{yi}^2}$$

$$= -\sqrt{2\left(-9.80\text{ m/s}^2\right)\left(-30.0\text{ m}\right) + \left(-12.0\text{ m/s}\right)^2}$$

$$= -27.1\text{ m/s}$$

$$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{\left(16.0\text{ m/s}\right)^2 + \left(-27.1\text{ m/s}\right)^2}$$

$$= \boxed{31.5\text{ m/s at } 59.4° \text{ below the horizontal}}$$

(d) From point *B* to *C*, the time is

$$t_1 = 5\text{ s};\ t_2 = \frac{v_{yf} - v_{yi}}{a_y} = \frac{-27.1\text{ m/s} + 12.0\text{ m/s}}{-9.80\text{ m/s}^2} = 1.53\text{ s}$$

The total elapsed time interval is

$$t = t_1 + t_2 = \boxed{6.53 \text{ s}}$$

(e) The horizontal distance covered is

$$\Delta x = v_{xi}t_2 = (16.0 \text{ m/s})(1.53 \text{ s}) = \boxed{24.5 \text{ m}}$$

P4.78 (a) Coyote: $\Delta x = \frac{1}{2}at^2 \rightarrow 70.0 \text{ m} = \frac{1}{2}(15.0 \text{ m/s}^2)\,t^2$

Roadrunner: $\Delta x = v_{xi}t \rightarrow 70.0 \text{ m} = v_{xi}t$

Solving the above, we get

$$v_{xi} = \boxed{22.9 \text{ m/s}} \text{ and } t = \boxed{3.06 \text{ s}}$$

(b) At the edge of the cliff, $v_{xi} = at = (15.0 \text{ m/s}^2)(3.06 \text{ s}) = 45.8 \text{ m/s}$

Substituting $\Delta y = -100$ m into $\Delta y = \frac{1}{2}a_yt^2$, we find

$$-100 \text{ m} = \frac{1}{2}(-9.80 \text{ m/s}^2)t^2$$
$$t = 4.52 \text{ s}$$
$$\Delta x = v_xt + \frac{1}{2}a_xt^2$$
$$= (45.8 \text{ m/s})(4.52 \text{ s}) + \frac{1}{2}(15.0 \text{ m/s}^2)(4.52 \text{ s})^2$$

Solving, $\Delta x = \boxed{360 \text{ m}}$.

(c) For the Coyote's motion through the air,

$$v_{xf} = v_{xi} + a_xt = 45.8 \text{ m/s} + (15 \text{ m/s}^2)(4.52 \text{ s}) = \boxed{114 \text{ m/s}}$$
$$v_{yf} = v_{yi} + a_yt = 0 - (9.80 \text{ m/s}^2)(4.52 \text{ s}) = \boxed{-44.3 \text{ m/s}}$$

P4.79 (a) Reference frame: Earth

The ice chest floats downstream 2 km in time interval Δt, so

$$2 \text{ km} = v_{ow}\Delta t \rightarrow \Delta t = 2 \text{ km}/v_{ow}$$

The upstream motion of the boat is described by

$$d = (v - v_{ow})(15 \text{ min})$$

and the downstream motion is described by

$$d + 2 \text{ km} = (v - v_{ow})(\Delta t - 15 \text{ min})$$

We substitute the above expressions for Δt and d:

$$(\mathrm{v}-\mathrm{v}_{\mathrm{ow}})(15\text{ min})+2\text{ km}=(\mathrm{v}+\mathrm{v}_{\mathrm{ow}})\left(\frac{2\text{ km}}{\mathrm{v}_{\mathrm{ow}}}-15\text{ min}\right)$$

$$\begin{aligned}&\mathrm{v}(15\text{ min})-\mathrm{v}_{\mathrm{ow}}(15\text{ min})+2\text{ km}\\ &\qquad=\frac{\mathrm{v}}{\mathrm{v}_{\mathrm{ow}}}(2\text{ km})+2\text{ km}-\mathrm{v}(15\text{ min})-\mathrm{v}_{\mathrm{ow}}(15\text{ min})\end{aligned}$$

$$\mathrm{v}(30\text{ min})=\frac{\mathrm{v}}{\mathrm{v}_{\mathrm{ow}}}(2\text{ km})$$

$$\mathrm{v}_{\mathrm{ow}}=\boxed{4.00\text{ km/h}}$$

(b) Reference frame: water

After the boat travels so that it and its starting point are 2 km apart, the chest enters the water, where, in the frame of the water, it is motionless. The boat then travels upstream for 15 min at speed v, and then downstream at the same speed, to return to the same point where the chest is at rest in the water. Thus, the boat travels for a total time interval of 30 min. During this same time interval, the starting point approaches the chest at speed v_{ow}, traveling 2 km. Thus,

$$\mathrm{v}_{\mathrm{ow}}=\frac{\Delta\mathrm{x}}{\Delta\mathrm{t}_{\mathrm{total}}}=\frac{2\text{ km}}{30\text{ min}}=\boxed{4.00\text{ km/h}}$$

P4.80 Think of shaking down the mercury in an old fever thermometer. Swing your hand through a circular arc, quickly reversing direction at the bottom end. Suppose your hand moves through one-quarter of a circle of radius 60 cm in 0.1 s. Its speed is

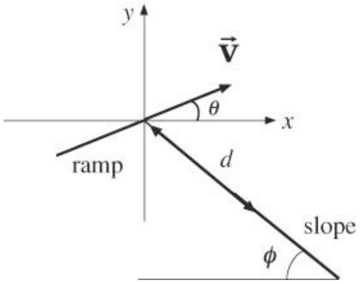
$$\frac{\frac{1}{4}(2\pi)(0.6\text{ m})}{0.1\text{ s}}\approx 9\text{ m/s}$$

and its centripetal acceleration is $\dfrac{\mathrm{v}^2}{\mathrm{r}}\approx\dfrac{(9\text{ m/s})^2}{0.6\text{ m}}\boxed{\sim 10^2\text{ m/s}^2}$.

The tangential acceleration of stopping and reversing the motion will make the total acceleration somewhat larger, but will not affect its order of magnitude.

Challenge Problems

P4.81 ANS. FIG. P4.81 indicates that a line extending along the slope will past through the end of the ramp, so we may take the position of the skier as she leaves the ramp to be the origin of our coordinate system.



ANS. FIG. P4.81

(a) Measured from the end of the ramp, the skier lands a distance *d* down the slope at time *t*:

$$\Delta x = v_{xi}t$$
$$\rightarrow d\cos 50.0° = (10.0\text{ m/s})(\cos 15.0°)t$$

and

$$\Delta y = v_{yi}t + \frac{1}{2}gt^2 \rightarrow$$
$$-d\sin 50.0° = (10.0\text{ m/s})(\sin 15.0°)t + \frac{1}{2}(-9.80\text{ m/s}^2)t^2$$

Solving, $d = \boxed{43.2\text{ m}}$ and $t = 2.88$ s.

(b) Since $a_x = 0$,

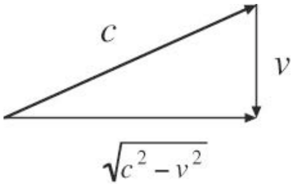
$$v_{xf} = v_{xi} = (10.0\text{ m/s})\cos 15.0° = \boxed{9.66\text{ m/s}}$$
$$v_{yf} = v_{yi} + a_y t = (10.0\text{ m/s})\sin 15.0° - (9.80\text{ m/s}^2)(2.88\text{ s})$$
$$= \boxed{-25.6\text{ m/s}}$$

(c) Air resistance would ordinarily decrease the values of the range and landing speed. As an airfoil, she can deflect air downward so that the air deflects her upward. This means she can get some lift and increase her distance.

P4.82 (a) For Chris, his speed downstream is *c* + *v*, while his speed upstream is *c* – *v*.

Therefore, the total time for Chris is

$$\boxed{\Delta t_1 = \frac{L}{c+v} + \frac{L}{c-v} = \frac{2\ L/c}{1 - v^2/c^2}}$$



ANS. FIG. P4.82

(b) Sarah must swim somewhat upstream to counteract the effect from the current. As is shown in the diagram, the magnitude of her cross-stream velocity is $\sqrt{c^2 - v^2}$.

Thus, the total time for Sarah is

$$\boxed{\Delta t_2 = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2\,L/c}{\sqrt{1 - v^2/c^2}}}$$

(c) Since the term $\left(1 - v^2/c^2\right) < 1, \Delta t_1 > \Delta t_2$, so Sarah, who swims cross-stream, returns first.

***P4.83** Let the river flow in the *x* direction.

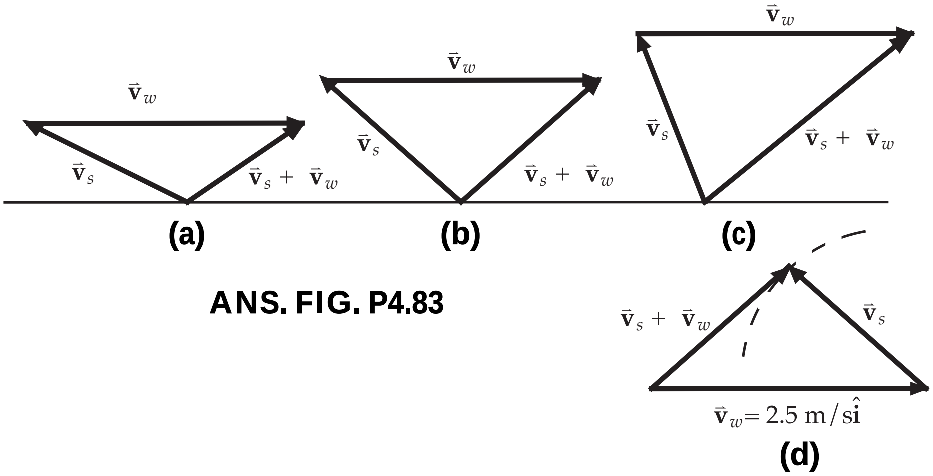
(a) To minimize time, $\boxed{\text{swim perpendicular to the banks}}$ in the *y* direction. You are in the water for time *t* in $\Delta y = v_y t$,

$$t = \frac{80\text{ m}}{1.5\text{ m/s}} = 53.3\text{ s}$$

(b) The water carries you downstream by

$$\Delta x = v_x t = (2.50\text{ m/s})53.3\text{ s} = \boxed{133\text{ m}}$$

(c) To minimize downstream drift, you should swim so that your resultant velocity $\vec{\mathbf{v}}_s + \vec{\mathbf{v}}_w$ is perpendicular to your swimming velocity $\vec{\mathbf{v}}_s$ relative to the water. This is shown graphically in the upper row of ANS. FIG. P4.83. Unlike the situations shown in ANS. FIG. P4.83(a) and ANS. FIG. P4.83(b), this condition (shown in ANS. FIG. P4.83(b)) maximizes the angle between the resultant velocity and the shore. The angle between $\vec{\mathbf{v}}_s$ and the shore is given by $\cos\theta = \frac{1.5\text{ m/s}}{2.5\text{ m/s}}$, $\boxed{\theta = 53.1°}$.



ANS. FIG. P4.83

(d) See ANS. FIG. P4.83(d). Now,

$$v_y = v_s \sin\theta = (1.5 \text{ m/s})\sin 53.1° = 1.20 \text{ m/s}$$

$$t = \frac{\Delta y}{v_y} = \frac{80 \text{ m}}{1.2 \text{ m/s}} = 66.7 \text{ s}$$

$$\Delta x = v_x t = [2.5 \text{ m/s} - (1.5 \text{ m/s})\cos 53.1°](66.7 \text{ s}) = \boxed{107 \text{ m}}$$

P4.84 Measure heights above the level ground. The elevation y_b of the ball follows

$$y_b = R + 0 - \frac{1}{2}gt^2$$

with $x = v_i t$ so $y_b = R - \frac{gx^2}{2v_i^2}$.

(a) The elevation y_r of points on the rock is described by

$$y_r^2 + x^2 = R^2$$

We will have $y_b = y_r$ at $x = 0$, but for all other x we require the ball to be above the rock's surface as in $y_b > y_r$. Then $y_b^2 + x^2 > R^2$:

$$\left(R - \frac{gx^2}{2v_i^2}\right)^2 + x^2 > R^2$$

$$R^2 - \frac{gx^2R}{v_i^2} + \frac{g^2x^4}{4v_i^4} + x^2 > R^2$$

$$\frac{g^2x^4}{4v_i^4} + x^2 > \frac{gx^2R}{v_i^2}$$

If this inequality is satisfied for x approaching zero, it will be true for all x. If the ball's parabolic trajectory has large enough radius of curvature at the start, the ball will clear the whole rock:

$1 > \frac{gR}{v_i^2}$, so

$$\boxed{v_i > \sqrt{gR}}$$

(b) With $v_i = \sqrt{gR}$ and $y_b = 0$, we have $0 = R - \frac{gx^2}{2gR}$

or $x = R\sqrt{2}$. The distance from the rock's base is

$$x - R = \boxed{\left(\sqrt{2} - 1\right)R}$$

P4.85 When the bomb has fallen a vertical distance 2.15 km, it has traveled a horizontal distance x_f given by

$$x_f = \sqrt{(3.25\text{ km})^2 - (2.15\text{ km})^2} = 2.437\text{ km}$$

The vertical displacement of the bomb is

$$y_f = x_f \tan\theta_i - \frac{g x_f^2}{2v_i^2 \cos^2\theta_i}$$

Substituting,

$$-2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - \frac{(9.8\text{ m/s}^2)(2\,437\text{ m})^2}{2(280\text{ m/s})^2\cos^2\theta_i}$$

or

$$-2\,150\text{ m} = (2\,437\text{ m})\tan\theta_i - (371.19\text{ m})(1+\tan^2\theta_i)$$

$$\therefore \tan^2\theta_i - 6.565\tan\theta_i - 4.792 = 0$$

$$\therefore \tan\theta_i = \frac{1}{2}\left(6.565 \pm \sqrt{(6.565)^2 - 4(1)(-4.792)}\right) = 3.283 \pm 3.945$$

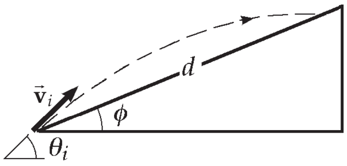
We select the negative solution, since θ_i is below the horizontal.

$$\therefore \tan\theta_i = -0.662,\ \boxed{\theta_i = -33.5°}$$

P4.86 (a) The horizontal distance traveled by the projectile is given by

$$x_f = v_{xi}t = (v_i\cos\theta_i)t$$

$$\rightarrow t = \frac{x_f}{v_i\cos\theta_i}$$



ANS. FIG. P4.86

We substitute this into the equation for the displacement in y:

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = (\tan\theta_i)(x_f) - \frac{g}{2v_i^2\cos^2\theta_i}x_f^2$$

Now setting $x_f = d\cos\phi$ and $y_f = d\sin\phi$, we have

$$d\sin\phi = (\tan\theta_i)(d\cos\phi) - \frac{g}{2v_i^2\cos^2\theta_i}(d\cos\phi)^2$$

Solving for d yields

$$d = \frac{2v_i^2\cos\theta_i\left[\sin\theta_i\cos\phi - \sin\phi\cos\theta_i\right]}{g\cos^2\phi}$$

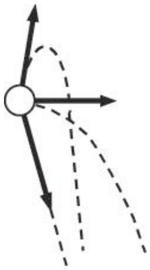
or $$d = \boxed{\frac{2v_i^2 \cos\theta_i \sin(\theta_i - \phi)}{g\cos^2\phi}}$$

(b) Setting $\frac{d}{d\theta_i}(d) = 0$ leads to

$$\boxed{\theta_i = 45° + \frac{\phi}{2}} \text{ and } \boxed{d_{\max} = \frac{v_i^2(1-\sin\phi)}{g\cos^2\phi}}$$

P4.87 For the smallest impact angle

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right)$$



ANS. FIG. P4.87

we want to minimize v_{yf} and maximize $v_{xf} = v_{xi}$. The final y component of velocity is related to v_{yi} by $v_{yf}^2 = v_{yi}^2 + 2gh$, so we want to minimize v_{yi} and maximize v_{xi}. Both are accomplished by making the initial velocity horizontal. Then $v_{xi} = v$, $v_{yi} = 0$, and $v_{yf} = \sqrt{2gh}$. At last, the impact angle is

$$\theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \boxed{\tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)}$$

P4.88 We follow the steps outlined in Example 4.5, eliminating $t = \frac{d\cos\phi}{v_i\cos\theta}$ to find

$$\frac{v_i \sin\theta \, d\cos\phi}{v_i\cos\theta} - \frac{gd^2\cos^2\phi}{2v_i^2\cos^2\theta} = -d\sin\phi$$

Clearing the fractions gives

$$2v_i^2\cos\theta\sin\theta\cos\phi - gd\cos^2\phi = -2v_i^2\cos^2\theta\sin\phi$$

To maximize d as a function of θ, we differentiate through with respect to θ and set $\frac{d}{d\theta}(d) = 0$:

$$2v_i^2\cos\theta\cos\theta\cos\phi + 2v_i^2\sin\theta(-\sin\theta)\cos\phi$$
$$- g\left[\frac{d}{d\theta}(d)\right]\cos^2\phi = -2v_i^2 2\cos\theta(-\sin\theta)\sin\phi$$

We use the trigonometric identities from Appendix B4:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \text{ and } \sin 2\theta = 2\sin\theta\cos\theta$$

to find

$$\cos\phi\cos 2\theta = \sin 2\theta\sin\phi$$

Next, $\dfrac{\sin\phi}{\cos\phi} = \tan\phi$ and $\cot 2\theta = \dfrac{1}{\tan 2\theta}$ give cot 2θ = tan ϕ so

$$\phi = 90° - 2\theta \text{ and } \theta = 45° - \frac{\phi}{2}$$

P4.89 Find the highest firing angle θ_H for which the projectile will clear the mountain peak; this will yield the range of the closest point of bombardment. Next find the lowest firing angle; this will yield the maximum range under these conditions if both θ_H and θ_L are > 45°, x = 2 500 m, y = 1 800 m, and v_i = 250 m/s.

$$y_f = v_{yi}t - \frac{1}{2}gt^2 = v_i\ (\sin\theta)t - \frac{1}{2}gt^2$$
$$x_f = v_{xi}t = v_i\ (\cos\theta)t$$

Thus,

$$t = \frac{x_f}{v_i\cos\theta}$$

Substitute into the expression for y_f:

$$y_f = v_i(\sin\theta)\frac{x_f}{v_i\cos\theta} - \frac{1}{2}g\left(\frac{x_f}{v_i\cos\theta}\right)^2 = x_f\tan\theta - \frac{gx_f^2}{2v_i^2\cos^2\theta}$$

but $\dfrac{1}{\cos^2\theta} = \tan^2\theta + 1$, so $y_f = x_f\tan\theta - \dfrac{gx_f^2}{2v_i^2}\left(\tan^2\theta + 1\right)$ and

$$0 = \frac{gx_f^2}{2v_i^2}\tan^2\theta - x_f\tan\theta + \frac{gx_f^2}{2v_i^2} + y_f$$

Substitute values, use the quadratic formula, and find

$\tan\theta = 3.905$ or 1.197, which gives $\theta_H = 75.6°$ and $\theta_L = 50.1°$.

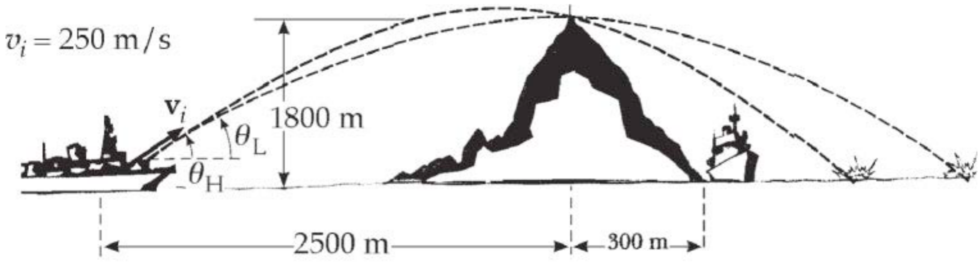
Range $\left(\text{at } \theta_H\right) = \dfrac{v_i^2\sin 2\theta_H}{g} = 3.07\times10^3$ m from enemy ship

$$3.07\times10^3\text{ m} - 2\,500\text{ m} - 300\text{ m} = 270\text{ m from shore}$$

$$\text{Range}\left(\text{at }\theta_L\right) = \frac{v_i^2 \sin 2\theta_L}{g} = 6.28\times10^3\ \text{m from enemy ship}$$

$$6.28\times10^3\ \text{m} - 2\ 500\ \text{m} - 300\ \text{m} = 3.48\times10^3\ \text{m from shore}$$

Therefore, the safe distance is $\boxed{< 270\ \text{m}}$ or $\boxed{> 3.48\times10^3\ \text{m}}$ from the shore.



ANS. FIG. P4.89

ANSWERS TO EVEN-NUMBERED PROBLEMS

P4.2 2.50 m/s

P4.4 (a) $-5.00\omega\,\hat{\mathbf{i}}$ m/s; (b) $-5.00\omega^2\,\hat{\mathbf{j}}$ m/s;
(c) $(4.00\text{ m})\hat{\mathbf{j}}+(5.00\text{ m})\left(-\sin\omega t\hat{\mathbf{i}}-\cos\omega t\hat{\mathbf{j}}\right)$,
$(5.00\text{ m})\omega\left[-\cos\omega\hat{\mathbf{i}}+\sin\omega t\hat{\mathbf{j}}\right]$, $(5.00\text{ m})\omega^2\left[\sin\omega t\hat{\mathbf{i}}+\cos\omega\hat{\mathbf{j}}\right]$; (d) a circle of radius 5.00 m centered at (0, 4.00 m)

P4.6 (a) $5.00t\hat{\mathbf{i}}+1.50t^2\hat{\mathbf{j}}$; (b) $5.00\hat{\mathbf{i}}+3.00t\,\hat{\mathbf{j}}$; (c) 10.0 m, 6.00 m; (d) 7.81 m/s

P4.8 (a) $\left(10.0\,\hat{\mathbf{i}}+0.241\,\hat{\mathbf{j}}\right)$ mm; (b) $\left(1.84\times10^7\text{ m/s}\right)\hat{\mathbf{i}}+\left(8.78\times10^5\text{ m/s}\right)\hat{\mathbf{j}}$;
(c) 1.85×10^7 m/s; (d) 2.73°

P4.10 (a) $\vec{\mathbf{v}}_f=(3.45-1.79t)\hat{\mathbf{i}}+\left(2.89-0.650t\right)\hat{\mathbf{j}}$;
(b) $\vec{\mathbf{r}}_f=(-25.3+3.45t-0.893t^2)\hat{\mathbf{i}}+\left(28.9+2.89t-0.325t^2\right)\hat{\mathbf{j}}$

P4.12 0.600 m/s^2

P4.14 (a) $v_{xi}=d\sqrt{\dfrac{g}{2h}}$, (b) The direction of the mug's velocity is $\tan^{-1}(2h/d)$ below the horizontal.

P4.16 $x=7.23\times10^3$ m, $y=1.68\times10^3$ m

P4.18 (a) 76.0°, (b) $R_{\max}=2.13R$, (c) the same on every planet

P4.20 (a) 22.6 m; (b) 52.3 m; (c) 1.18 s

P4.22 (a) there is; (b) 0.491 m/s

P4.24 (a) 0.852 s; (b) 3.29 m/s; (c) 4.03 m/s; (d) 50.8°; (e) $t=1.12$ s

P4.26 (a) (0, 0.840 m); (b) 11.2 m/s at 18.5°; (c) 8.94 m

P4.28 (a) $t=v_i\sin\theta/g$; (b) $h_{\max}=h+\dfrac{\left(v_i\sin\theta\right)^2}{2g}$

P4.30 (a) 28.2 m/s; (b) 4.07 s; (c) the required initial velocity will increase, the total time of flight will increase

P4.32 (a) 41.7 m/s; (b) 3.81 s; (c) $v_x=34.1$ m/s, $v_y=-13.4$ m/s, $v=36.7$ m/s

P4.24 0.033 7 m/s^2 directed toward the center of Earth

P4.36 10.5 m/s, 219 m/s^2 inward

P4.38 (a) 6.00 rev/s; (b) 1.52×10^3 m/s^2; (c) 1.28×10^3 m/s^2

P4.40 (a) 13.0 m/s^2; (b) 5.70 m/s; (c) 7.50 m/s^2

P4.42 (a) See ANS. FIG. P4.42; (b) 29.7 m/s^2; (c) 6.67 m/s tangent to the circle

P4.44 153 km/h at 11.3° north of west

P4.46 (a) $\Delta t_{\text{woman}} = \dfrac{L}{v_1}$; (b) $\Delta t_{\text{man}} = \dfrac{L}{v_1 + v_2}$; (c) $\Delta t_{\text{man}} = \dfrac{L}{v_1 + 2v_2}$

P4.48 (a) 57.7 km/h at 60.0° west of vertical; (b) 28.9 km/h downward

P4.50 (a) 2.02×10^3 s; (b) 1.67×10^3 s; (c) Swimming with the current does not compensate for the time lost swimming against the current.

P4.52 27.7° E of N

P4.54 (a) straight up, at 0° to the vertical; (b) 8.25 m/s; (c) a straight up and down line; (d) a symmetric parabola opening downward; (e) 12.6 m/s north at $\tan^{-1}(8.25/9.5) = 41.0°$ above the horizontal

P4.56 (a) $2\sqrt{\dfrac{R}{3g}}$; (b) $\dfrac{1}{2}\sqrt{3gR}$; (c) $\sqrt{\dfrac{gR}{3}}$; (d) $\sqrt{\dfrac{13gR}{12}}$; (e) 33.7°; (f) $\dfrac{13}{24}R$; (g) $\dfrac{13}{12}R$

P4.58 (a) $5\hat{\mathbf{i}} + 4t^{3/2}\hat{\mathbf{j}}$; (b) $5t\hat{\mathbf{i}} + 1.6t^{5/2}\hat{\mathbf{j}}$

P4.60 (a) 9.80 m/s^2, downward; (b) 10.7 m/s

P4.62 (a) $t = \sqrt{\dfrac{2h}{g}}$; (b) $v_{xi} = d\sqrt{\dfrac{g}{2h}}$; (c) $v_f = \sqrt{v_{xf}^{\,2} + v_{yf}^{\,2}} = \sqrt{\left(\dfrac{d^2 g}{2h}\right) + (2gh)}$; (d) $\theta_f = \tan^{-1}\left(\dfrac{2h}{d}\right)$

P4.64 68.8 km/h

P4.66 22.4° or 89.4°

P4.68 $2v_i t \cos\theta_i$

P4.70 (a) 25.0 m/s^2; (b) 9.80 m/s^2; (c) See ANS. FIG. P4.70; (d) 26.8 m/s^2, 21.4°

P4.72 (a) See table in P4.72(a); (b) From the table, it looks like the magnitude of *r* is largest at a bit less than 6 s; (c) 138 m; (d) We can require $dr^2/dt = 0 = (d/dt)[(12t)^2 + (49t - 4.9t^2)^2]$, which results in the solution.

P4.74 (a) $\theta = 26.6°$; (b) 0.949

P4.76 18.8 m, −17.3 m

P4.78 (a) 22.9 m/s and 3.06 s; (b) 360 m; (c) 114 m/s, −44.3 m/s

P4.80 $\sim 10^2\ \text{m/s}^2$

P4.82 (a) $\Delta t_1 = \dfrac{L}{c+v} + \dfrac{L}{c-v} = \dfrac{2L/c}{1-v^2/c^2}$; (b) $\Delta t_2 = \dfrac{2L}{\sqrt{c^2-v^2}} = \dfrac{2L/c}{\sqrt{1-v^2/c^2}}$;

(c) Sarah, who swims cross-stream, returns first

P4.84 (a) $v_i > \sqrt{gR}$; (b) $x - R = \left(\sqrt{2}-1\right)R$

P4.86 (a) See P4.86a for derivation; (b) $d_{\text{max}} = 45° + \dfrac{\phi}{2}$, $\theta_i = \dfrac{v_i^2(1-\sin\phi)}{g\cos^2\phi}$

P4.88 See P4.88 for complete derivation.

5

The Laws of Motion

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ5.1 Answer (d). The stopping distance will be the same if the mass of the truck is doubled. The normal force and the friction force both double, so the backward acceleration remains the same as without the load.

OQ5.2 Answer (b). Newton's 3rd law describes all objects, breaking or whole. The force that the locomotive exerted on the wall is the same as that exerted by the wall on the locomotive. The framing around the wall could not exert so strong a force on the section of the wall that broke out.

OQ5.3 Since they are on the order of a thousand times denser than the surrounding air, we assume the snowballs are in free fall. The net force on each is the gravitational force exerted by the Earth, which does not depend on their speed or direction of motion but only on the snowball mass. Thus we can rank the missiles just by mass: d > a = e > b > c.

OQ5.4 Answer (e). The stopping distance will decrease by a factor of four if the initial speed is cut in half.

OQ5.5 Answer (b). An air track or air table is a wonderful thing. It exactly cancels out the force of the Earth's gravity on the gliding object, to display free motion and to imitate the effect of being far away in space.

OQ5.6 Answer (b). 200 N must be greater than the force of friction for the box's acceleration to be forward.

OQ5.7 Answer (a). Assuming that the cord connecting m_1 and m_2 has constant length, the two masses are a fixed distance (measured along the cord) apart. Thus, their speeds must always be the same, which means that their accelerations must have equal magnitudes. The magnitude of the downward acceleration of m_2 is given by Newton's second law as

$$a_2 = \frac{\sum F_y}{m_2} = \frac{m_2 g - T}{m_2} = g - \left(\frac{T}{m_2}\right) < g$$

where T is the tension in the cord, and downward has been chosen as the positive direction.

OQ5.8 Answer (d). Formulas a, b, and e have the wrong units for speed. Formulas a and c would give an imaginary answer.

OQ5.9 Answer (b). As the trailer leaks sand at a constant rate, the total mass of the vehicle (truck, trailer, and remaining sand) decreases at a steady rate. Then, with a constant net force present, Newton's second law states that the magnitude of the vehicle's acceleration ($a = F_{\text{net}}/m$) will *steadily increase*.

OQ5.10 Answer (c). When the truck accelerates forward, the crate has the natural tendency to remain at rest, so the truck tends to slip under the crate, leaving it behind. However, friction between the crate and the bed of the truck acts in such a manner as to oppose this relative motion between truck and crate. Thus, the friction force acting on the crate will be in the forward horizontal direction and tend to accelerate the crate forward. The crate will slide only when the coefficient of static friction is inadequate to prevent slipping.

OQ5.11 Both answers (d) and (e) are *not true*: (d) is not true because the value of the velocity's constant magnitude need not be zero, and (e) is not true because there may be *no force* acting on the object. An object in equilibrium has zero acceleration $(\vec{\mathbf{a}} = 0)$, so both the magnitude and direction of the object's velocity must be *constant*. Also, Newton's second law states that the *net force* acting on an object in equilibrium is zero.

OQ5.12 Answer (d). All the other possibilities would make the total force on the crate be different from zero.

OQ5.13 Answers (a), (c), and (d). A free-body diagram shows the forces exerted on the object by other objects, and the net force is the sum of those forces.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ5.1 A portion of each leaf of grass extends above the metal bar. This portion must accelerate in order for the leaf to bend out of the way. If the bar moves fast enough, the grass will not have time to increase its speed to match the speed of the bar. The leaf's mass is small, but when its acceleration is very large, the force exerted by the bar on the leaf puts the leaf under tension large enough to shear it off.

CQ5.2 When the hands are shaken, there is a large acceleration of the surfaces of the hands. If the water drops were to stay on the hands, they must accelerate along with the hands. The only force that can provide this acceleration is the friction force between the water and the hands. (There are adhesive forces also, but let's not worry about those.) The static friction force is not large enough to keep the water stationary with respect to the skin at this large acceleration. Therefore, the water breaks free and slides along the skin surface. Eventually, the water reaches the end of a finger and then slides off into the air. This is an example of Newton's first law in action in that the drops continue in motion while the hand is stopped.

CQ5.3 When the bus starts moving, the mass of Claudette is accelerated by the force of the back of the seat on her body. Clark is standing, however, and the only force on him is the friction between his shoes and the floor of the bus. Thus, when the bus starts moving, his feet start accelerating forward, but the rest of his body experiences almost no accelerating force (only that due to his being attached to his accelerating feet!). As a consequence, his body tends to stay almost at rest, according to Newton's first law, relative to the ground. Relative to Claudette, however, he is moving toward her and falls into her lap.

CQ5.4 The resultant force is zero, as the acceleration is zero.

CQ5.5 First ask, "Was the bus moving forward or backing up?" If it was moving forward, the passenger is lying. A fast stop would make the suitcase fly toward the front of the bus, not toward the rear. If the bus was backing up at any reasonable speed, a sudden stop could not make a suitcase fly far. Fine her for malicious litigiousness.

CQ5.6 Many individuals have a misconception that throwing a ball in the air gives the ball some kind of a "force of motion" that the ball carries after it leaves the hand. This is the "force of the throw" that is mentioned in the problem. The upward motion of the ball is explained by saying that the "force of the throw" exceeds the gravitational force—of course, this explanation confuses upward velocity with downward acceleration—the hand applies a force on the ball *only* while they are in contact; once the ball leaves the hand, the hand no longer has any influence on the ball's motion. The *only* property of the ball that it carries from its interaction with the hand is the *initial* upward velocity imparted to it by the thrower. Once the ball leaves the hand, the only force on the ball is the gravitational force. (a) If there were a "force of the throw" felt by the ball after it leaves the hand and the force exceeded the gravitational force, the ball would accelerate upward, not downward! (b) If the "force of the throw" equaled the gravitational force, the ball would move upward with a constant velocity, rather than slowing down and coming back down! (c) The magnitude is zero because there is no "force of the throw." (d) The ball moves away from the hand because the hand imparts a velocity to the ball and then the hand stops moving.

CQ5.7 (a) force: The *Earth* attracts the *ball* downward with the force of gravity—reaction force: the *ball* attracts the *Earth* upward with the force of gravity; force: the *hand* pushes up on the *ball*—reaction force: the *ball* pushes down on the *hand*.

(b) force: The *Earth* attracts the *ball* downward with the force of gravity—reaction force: the *ball* attracts the *Earth* upward with the force of gravity.

CQ5.8 (a) The air inside pushes outward on each patch of rubber, exerting a force perpendicular to that section of area. The air outside pushes perpendicularly inward, but not quite so strongly. (b) As the balloon takes off, all of the sections of rubber feel essentially the same outward forces as before, but the now-open hole at the opening on the west side feels no force – except for a small amount of drag to the west from the escaping air. The vector sum of the forces on the rubber is to the east.

The small-mass balloon moves east with a large acceleration. (c) Hot combustion products in the combustion chamber push outward on all the walls of the chamber, but there is nothing for them to push on at the open rocket nozzle. The net force exerted by the gases on the chamber is up if the nozzle is pointing down. This force is larger than the gravitational force on the rocket body, and makes it accelerate upward.

CQ5.9 The molecules of the floor resist the ball on impact and push the ball back, upward. The actual force acting is due to the forces between molecules that allow the floor to keep its integrity and to prevent the ball from passing through. Notice that for a ball passing through a window, the molecular forces weren't strong enough.

CQ5.10 The tension in the rope when pulling the car is twice that in the tug-of-war. One could consider the car as behaving like another team of twenty more people.

CQ5.11 An object cannot exert a force on itself, so as to cause acceleration. If it could, then objects would be able to accelerate themselves, without interacting with the environment. You cannot lift yourself by tugging on your bootstraps.

CQ5.12 Yes. The table bends down more to exert a larger upward force. The deformation is easy to see for a block of foam plastic. The sag of a table can be displayed with, for example, an optical lever.

CQ5.13 As the barbell goes through the bottom of a cycle, the lifter exerts an upward force on it, and the scale reads the larger upward force that the floor exerts on them together. Around the top of the weight's motion, the scale reads less than average. If the weightlifter throws the barbell upward so that it loses contact with his hands, the reading on the scale will return to normal, reading just the weight of the weightlifter, until the barbell lands back in his hands, at which time the reading will jump upward.

CQ5.14 The sack of sand moves up with the athlete, regardless of how quickly the athlete climbs. Since the athlete and the sack of sand have the same weight, the acceleration of the system must be zero.

CQ5.15 If you slam on the brakes, your tires will skid on the road. The force of kinetic friction between the tires and the road is less than the maximum static friction force. Antilock brakes work by "pumping" the brakes (much more rapidly than you can) to minimize skidding of the tires on the road.

CQ5.16 (a) Larger: the tension in A must accelerate two blocks and not just one. (b) Equal. Whenever A moves by 1 cm, B moves by 1 cm. The two blocks have equal speeds at every instant and have equal accelerations. (c) Yes, backward, equal. The force of cord B on block 1 is the tension in the cord.

CQ5.17 As you pull away from a stoplight, friction exerted by the ground on the tires of the car accelerates the car forward. As you begin running forward from rest, friction exerted by the floor on your shoes causes your acceleration.

CQ5.18 It is impossible to string a horizontal cable without its sagging a bit. Since the cable has a mass, gravity pulls it downward. A vertical component of the tension must balance the weight for the cable to be in equilibrium. If the cable were completely horizontal, then there would be no vertical component of the tension to balance the weight. If a physicist would testify in court, the city employees would win.

CQ5.19 (a) Yes, as exerted by a vertical wall on a ladder leaning against it. (b) Yes, as exerted by a hammer driving a tent stake into the ground. (c) Yes, as the ball accelerates upward in bouncing from the floor. (d) No; the two forces describe the same interaction.

CQ5.20 The clever boy bends his knees to lower his body, then starts to straighten his knees to push his body up—that is when the branch breaks. In order to give himself an upward acceleration, he must push down on the branch with a force greater than his weight so that the branch pushes up on him with a force greater than his weight.

CQ5.21 (a) As a man takes a step, the action is the force his foot exerts on the Earth; the reaction is the force of the Earth on his foot. (b) The action is the force exerted on the girl’s back by the snowball; the reaction is the force exerted on the snowball by the girl’s back. (c) The action is the force of the glove on the ball; the reaction is the force of the ball on the glove. (d) The action is the force exerted on the window by the air molecules; the reaction is the force on the air molecules exerted by the window. We could in each case interchange the terms “action” and “reaction.”

CQ5.22 (a) Both students slide toward each other. When student A pulls on the rope, the rope pulls back, causing her to slide toward Student B. The rope also pulls on the pulley, so Student B slides because he is gripping a rope attached to the pulley. (b) Both chairs slide because there is tension in the rope that pulls on both Student A and the pulley connected to Student B. (c) Both chairs slide because when Student B pulls on his rope, he pulls the pulley which puts tension into the rope passing over the pulley to Student A. (d) Both chairs slide because when Student A pulls on the rope, it pulls on her and also pulls on the pulley.

CQ5.23 If you have ever seen a car stuck on an icy road, with its wheels spinning wildly, you know the car has great difficulty moving forward until it "catches" on a rough patch. (a) Friction exerted by the road is the force making the car accelerate forward. Burning gasoline can provide energy for the motion, but only external forces—forces exerted by objects outside—can accelerate the car. (b) If the car moves forward slowly as it speeds up, then its tires do not slip on the surface. The rubber contacting the road moves toward the rear of the car, and static friction opposes relative sliding motion by exerting a force on the rubber toward the front of the car. If the car is under control (and not skidding), the relative speed is zero along the lines where the rubber meets the road, and static friction acts rather than kinetic friction.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 5.1 The Concept of Force
Section 5.2 Newton's First Law and Inertial Frames
Section 5.3 Mass
Section 5.4 Newton's Second Law
Section 5.5 The Gravitational Force and Weight
Section 5.6 Newton's Third Law

***P5.1** (a) The woman's weight is the magnitude of the gravitational force acting on her, given by

$$F_g = mg = 120\text{ lb} = (4.448\text{ N/lb})(120\text{ lb}) = \boxed{534\text{ N}}$$

(b) Her mass is $m = \frac{F_g}{g} = \frac{534\text{ N}}{9.80\text{ m/s}^2} = \boxed{54.5\text{ kg}}$

***P5.2** We are given $F_g = mg = 900\text{ N}$, from which we can find the man's mass,

$$m = \frac{900\text{ N}}{9.80\text{ m/s}^2} = 91.8\text{ kg}$$

Then, his weight on Jupiter is given by

$$(F_g)_{\text{on Jupiter}} = 91.8\text{ kg}(25.9\text{ m/s}^2) = \boxed{2.38\text{ kN}}$$

P5.3 We use Newton's second law to find the force as a vector and then the Pythagorean theorem to find its magnitude. The givens are $m = 3.00$ kg and $\vec{\mathbf{a}} = \left(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$.

(a) The total vector force is

$$\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}} = (3.00\ \text{kg})(2.00\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\ \text{m/s}^2 = \boxed{(6.00\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}})\ \text{N}}$$

(b) Its magnitude is

$$\left|\vec{\mathbf{F}}\right| = \sqrt{\left(F_x\right)^2 + \left(F_y\right)^2} = \sqrt{(6.00\ \text{N})^2 + (15.0\ \text{N})^2} = \boxed{16.2\ \text{N}}$$

P5.4 Using the reference axes shown in Figure P5.4, we see that

$$\sum F_x = T\cos 14.0° - T\cos 14.0° = 0$$

and

$$\sum F_y = -T\sin 14.0° - T\sin 14.0° = -2T\sin 14.0°$$

Thus, the magnitude of the resultant force exerted on the tooth by the wire brace is

$$R = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} = \sqrt{0 + (-2T\sin 14.0°)^2} = 2T\sin 14.0°$$

or

$$R = 2\left(18.0\ \text{N}\right)\sin 14.0° = \boxed{8.71\ \text{N}}$$

P5.5 We use the particle under constant acceleration and particle under a net force models. We first calculate the acceleration of the puck:

$$\vec{\mathbf{a}} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\left(8.00\hat{\mathbf{i}} + 10.0\hat{\mathbf{j}}\right)\text{m/s} - 3.00\hat{\mathbf{i}}\ \text{m/s}}{8.00\ \text{s}}$$
$$= 0.625\hat{\mathbf{i}}\ \text{m/s}^2 + 1.25\hat{\mathbf{j}}\ \text{m/s}^2$$

In $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$, the only horizontal force is the thrust $\vec{\mathbf{F}}$ of the rocket:

(a) $\vec{\mathbf{F}} = (4.00\ \text{kg})\left(0.625\hat{\mathbf{i}}\ \text{m/s}^2 + 1.25\hat{\mathbf{j}}\ \text{m/s}^2\right) = \boxed{\left(2.50\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}\right)\ \text{N}}$

(b) Its magnitude is $|\vec{\mathbf{F}}| = \sqrt{(2.50\ \text{N})^2 + (5.00\ \text{N})^2} = \boxed{5.59\ \text{N}}$

P5.6 (a) Let the *x* axis be in the original direction of the molecule's motion. Then, from $v_f = v_i + at$, we have

$$a = \frac{v_f - v_i}{t} = \frac{-670\text{ m/s} - 670\text{ m/s}}{3.00 \times 10^{-13}\text{ s}} = \boxed{-4.47 \times 10^{15}\text{ m/s}^2}$$

(b) For the molecule, $\sum \vec{F} = m\vec{a}$. Its weight is negligible.

$$\vec{F}_{\text{wall on molecule}} = \left(4.68 \times 10^{-26}\text{ kg}\right)\left(-4.47 \times 10^{15}\text{ m/s}^2\right)$$
$$= -2.09 \times 10^{-10}\text{ N}$$
$$\vec{F}_{\text{molecule on wall}} = \boxed{+2.09 \times 10^{-10}\text{ N}}$$

***P5.7** Imagine a quick trip by jet, on which you do not visit the rest room and your perspiration is just canceled out by a glass of tomato juice. By subtraction, $\left(F_g\right)_p = mg_p$ and $\left(F_g\right)_C = mg_C$ give

$$\Delta F_g = m\left(g_p - g_C\right)$$

For a person whose mass is 90.0 kg, the change in weight is

$$\Delta F_g = 90.0\text{ kg}\left(9.809\,5 - 9.780\,8\right) = \boxed{2.58\text{ N}}$$

A precise balance scale, as in a doctor's office, reads the same in different locations because it compares you with the standard masses on its beams. A typical bathroom scale is not precise enough to reveal this difference.

P5.8 The force on the car is given by $\sum \vec{F} = m\vec{a}$, or, in one dimension, $\sum F = ma$. Whether the car is moving to the left or the right, since it's moving at constant speed, $a = 0$ and therefore $\sum F = \boxed{0}$ for both parts (a) and (b).

P5.9 We find the mass of the baseball from its weight: $w = mg$, so $m = w/g$ = 2.21 N/9.80 m/s^2 = 0.226 kg.

(a) We use $x_f = x_i + \frac{1}{2}(v_i + v_f)t$ and $x_f - x_i = \Delta x$, with $v_i = 0$, $v_f = 18.0$ m/s, and $\Delta t = t = 170\text{ ms} = 0.170\text{ s}$:

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$$
$$\Delta x = \frac{1}{2}(0 + 18.0\text{ m/s})(0.170\text{ s}) = \boxed{1.53\text{ m}}$$

(b) We solve for acceleration using $v_{xf} = v_{xi} + a_x t$, which gives

$$a_x = \frac{v_{xf} - v_{xi}}{t}$$

where a is in m/s^2, v is in m/s, and t in s. Substituting gives

$$a_x = \frac{18.0\text{ m/s} - 0}{0.170\text{ s}} = 106\text{ m/s}^2$$

Call $\vec{\mathbf{F}}_1$ = force of pitcher on ball, and $\vec{\mathbf{F}}_2$ = force of Earth on ball (weight). We know that

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}$$

Writing this equation in terms of its components gives

$$\sum F_x = F_{1x} + F_{2x} = ma_x \qquad \sum F_y = F_{1y} + F_{2y} = ma_y$$

$$\sum F_x = F_{1x} + 0 = ma_x \qquad \sum F_y = F_{1y} - 2.21\text{ N} = 0$$

Solving,

$$F_{1x} = (0.226\text{ kg})(106\text{ m/s}^2) = 23.9\text{ N} \text{ and } F_{1y} = 2.21\text{ N}$$

Then,

$$\begin{aligned} F_1 &= \sqrt{(F_{1x})^2 + (F_{1y})^2} \\ &= \sqrt{(23.9\text{ N})^2 + (2.21\text{ N})^2} = 24.0\text{ N} \end{aligned}$$

and $\theta = \tan^{-1}\left(\dfrac{2.21\text{ N}}{23.9\text{ N}}\right) = 5.29°$

$\boxed{\text{The pitcher exerts a force of 24.0 N forward at 5.29° above the horizontal.}}$

P5.10 (a) Use $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$, where $v_i = 0$, $v_f = v$, and $\Delta t = t$:

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t = \boxed{\frac{1}{2}vt}$$

(b) Use $v_{xf} = v_{xi} + a_x t$:

$$v_{xf} = v_{xi} + a_x t \to a_x = \frac{v_{xf} - v_{xi}}{t} \to a_x = \frac{v - 0}{t} = \frac{v}{t}$$

Call $\vec{\mathbf{F}}_1$ = force of pitcher on ball, and $\vec{\mathbf{F}}_2 = -F_g = -mg =$ gravitational force on ball. We know that

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}$$

writing this equation in terms of its components gives

$$\sum F_x = F_{1x} + F_{2x} = ma_x \qquad \sum F_y = F_{1y} + F_{2y} = ma_y$$

$$\sum F_x = F_{1x} + 0 = ma_x \qquad \sum F_y = F_{1y} - mg = 0$$

Solving and substituting from above,

$$F_{1x} = mv/t \qquad F_{1y} = mg$$

then the magnitude of F_1 is

$$\begin{aligned} F_1 &= \sqrt{(F_{1x})^2 + (F_{1y})^2} \\ &= \sqrt{(mv/t)^2 + (mg)^2} = \boxed{m\sqrt{(v/t)^2 + g^2}} \end{aligned}$$

and its direction is

$$\theta = \tan^{-1}\left(\frac{mg}{mv/t}\right) = \boxed{\tan^{-1}\left(\frac{gt}{v}\right)}$$

P5.11 Since this is a linear acceleration problem, we can use Newton's second law to find the force as long as the electron does not approach relativistic speeds (as long as its speed is much less than 3×10^8 m/s), which is certainly the case for this problem. We know the initial and final velocities, and the distance involved, so from these we can find the acceleration needed to determine the force.

(a) From $v_f^2 = v_i^2 + 2ax$ and $\sum F = ma$, we can solve for the acceleration and then the force: $a = \dfrac{v_f^2 - v_i^2}{2x}$

Substituting to eliminate a, $\sum F = \dfrac{m\left(v_f^2 - v_i^2\right)}{2x}$

Substituting the given information,

$$\sum F = \frac{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left[\left(7.00 \times 10^{5}\ \text{m/s}\right)^2 - \left(3.00 \times 10^{5}\ \text{m/s}\right)^2\right]}{2(0.050\,0\ \text{m})}$$

$$\sum F = \boxed{3.64 \times 10^{-18}\ \text{N}}$$

(b) The Earth exerts on the electron the force called weight,

$$F_g = mg = (9.11 \times 10^{-31}\,\text{kg})(9.80\ \text{m/s}^2) = 8.93 \times 10^{-30}\,\text{N}$$

The accelerating force is

$$\boxed{4.08 \times 10^{11}\ \text{times the weight of the electron.}}$$

P5.12 We first find the acceleration of the object:

$$\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$4.20\ \text{m}\hat{\mathbf{i}} - 3.30\ \text{m}\hat{\mathbf{j}} = 0 + \frac{1}{2}\vec{\mathbf{a}}(1.20\ \text{s})^2 = \left(0.720\ \text{s}^2\right)\vec{\mathbf{a}}$$

$$\vec{\mathbf{a}} = \left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\ \text{m/s}^2$$

Now $\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}$ becomes

$$\vec{\mathbf{F}}_g + \vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}$$

$$\vec{\mathbf{F}}_2 = 2.80\ \text{kg}\left(5.83\hat{\mathbf{i}} - 4.58\hat{\mathbf{j}}\right)\ \text{m/s}^2 + \left(2.80\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}}_2 = \boxed{\left(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}}\right)\ \text{N}}$$

P5.13 (a) $\boxed{\text{Force exerted by spring on hand, to the left; force exerted by spring on wall, to the right.}}$

(b) Force exerted by wagon on handle, downward to the left. Force exerted by wagon on planet, upward. Force exerted by wagon on ground, downward.

(c) Force exerted by football on player, downward to the right. Force exerted by football on planet, upward.

(d) Force exerted by small-mass object on large-mass object, to the left.

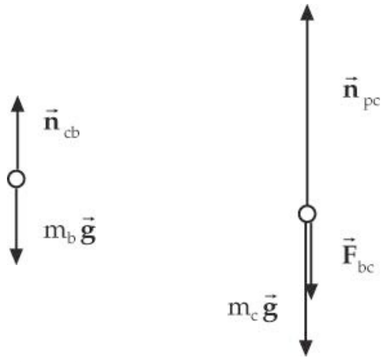
(e) Force exerted by negative charge on positive charge, to the left.

(f) Force exerted by iron on magnet, to the left.

P5.14 The free-body diagrams are shown in ANS. FIG. P5.14 below.

(a) $\vec{\mathbf{n}}_{cb}$ = normal force of cushion on brick
$m_b\vec{\mathbf{g}}$ = gravitational force on brick

(b) $\vec{\mathbf{n}}_{pc}$ = normal force of pavement on cushion
$m_b\vec{\mathbf{g}}$ = gravitational force on cushion
$\vec{\mathbf{F}}_{bc}$ = force of brick on cushion



brick (a) cushion (b)

ANS. FIG.P5.14

(c) force: normal force of cushion on brick ($\vec{\mathbf{n}}_{cb}$) → reaction force: force of brick on cushion ($\vec{\mathbf{F}}_{bc}$)
force: gravitational force of Earth on brick ($m_b\vec{\mathbf{g}}$) → reaction force: gravitational force of brick on Earth
force: normal force of pavement on cushion ($\vec{\mathbf{n}}_{pc}$) → reaction force: force of cushion on pavement
force: gravitational force of Earth on cushion ($m_c\vec{\mathbf{g}}$) → reaction force: gravitational force of cushion on Earth

***P5.15** (a) We start from the sum of the two forces:

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left(-6.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}\right) + \left(-3.00\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)$$
$$= \left(-9.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\text{ N}$$

The acceleration is then:

$$\vec{\mathbf{a}} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{\left(-9.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\text{ N}}{2.00\text{ kg}}$$
$$= \left(-4.50\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right)\text{ m/s}^2$$

and the velocity is found from

$$\vec{\mathbf{v}}_f = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \vec{\mathbf{a}}t$$

$$\vec{\mathbf{v}}_f = \left[\left(-4.50\hat{\mathbf{i}} + 1.50\hat{\mathbf{j}}\right)\text{ m/s}^2\right](10.0\text{ s})$$
$$= \boxed{\left(-45.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) The direction of motion makes angle θ with the x direction.

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(-\frac{15.0\text{ m/s}}{45.0\text{ m/s}}\right)$$

$$\theta = -18.4° + 180° = \boxed{162°\text{ from the }+x\text{ axis}}$$

(c) Displacement:

$$x\text{-displacement} = x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

$$= \frac{1}{2}\left(-4.50\ \text{m/s}^2\right)(10.0\ \text{s})^2 = -225\ \text{m}$$

$$y\text{-displacement} = y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$$

$$= \frac{1}{2}\left(+1.50\ \text{m/s}^2\right)(10.0\ \text{s})^2 = +75.0\ \text{m}$$

$$\Delta\vec{\mathbf{r}} = \boxed{\left(-225\hat{\mathbf{i}} + 75.0\hat{\mathbf{j}}\right)\ \text{m}}$$

(d) Position: $\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \Delta\vec{\mathbf{r}}$

$$\vec{\mathbf{r}}_f = \left(-2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}\right) + \left(-225\hat{\mathbf{i}} + 75.0\hat{\mathbf{j}}\right) = \boxed{\left(-227\hat{\mathbf{i}} + 79.0\hat{\mathbf{j}}\right)\ \text{m}}$$

***P5.16** Since the two forces are perpendicular to each other, their resultant is

$$F_R = \sqrt{(180\ \text{N})^2 + (390\ \text{N})^2} = 430\ \text{N}$$

at an angle of

$$\theta = \tan^{-1}\left(\frac{390\ \text{N}}{180\ \text{N}}\right) = 65.2^\circ\ \text{N of E}$$

From Newton's second law,

$$a = \frac{F_R}{m} = \frac{430\ \text{N}}{270\ \text{kg}} = 1.59\ \text{m/s}^2$$

or

$$\vec{\mathbf{a}} = \boxed{1.59\ \text{m/s}^2\ \text{at}\ 65.2^\circ\ \text{N of E}}$$

P5.17 (a) With the wind force being horizontal, the only vertical force acting on the object is its own weight, *mg*. This gives the object a downward acceleration of

$$a_y = \frac{\sum F_y}{m} = \frac{-mg}{m} = -g$$

The time required to undergo a vertical displacement $\Delta y = -h$, starting with initial vertical velocity $v_{0y} = 0$, is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$-h = 0 - \frac{g}{2}t^2 \qquad \text{or} \qquad \boxed{t = \sqrt{\frac{2h}{g}}}$$

(b) The only horizontal force acting on the object is that due to the wind, so $\sum F_x = F$ and the horizontal acceleration will be

$$a_x = \frac{\sum F_x}{m} = \boxed{\frac{F}{m}}$$

(c) With $v_{0x} = 0$, the horizontal displacement the object undergoes while falling a vertical distance *h* is given by $\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$ as

$$\Delta x = 0 + \frac{1}{2}\left(\frac{F}{m}\right)\left(\sqrt{\frac{2h}{g}}\right)^2 = \boxed{\frac{Fh}{mg}}$$

(d) The total acceleration of this object while it is falling will be

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(F/m)^2 + (-g)^2} = \boxed{\sqrt{(F/m)^2 + g^2}}$$

P5.18 For the same force *F*, acting on different masses $F = m_1a_1$ and $F = m_2a_2$. Setting these expressions for *F* equal to one another gives:

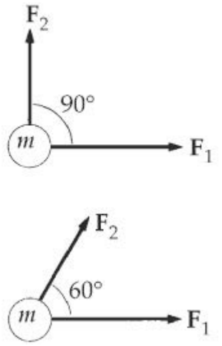
(a) $\frac{m_1}{m_2} = \frac{a_2}{a_1} = \boxed{\frac{1}{3}}$

(b) The acceleration of the combined object is found from

$$F = (m_1 + m_2)a = 4m_1a$$

or $$a = \frac{F}{4m_1} = \frac{1}{4}(3.00\ \text{m/s}^2) = \boxed{0.750\ \text{m/s}^2}$$

P5.19 We use the particle under a net force model and add the forces as vectors. Then Newton's second law tells us the acceleration.



ANS. FIG. P5.19

(a) $\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (20.0\hat{\mathbf{i}} + 15.0\hat{\mathbf{j}})\ \text{N}$

Newton's second law gives, with $m = 5.00$ kg,

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \left(4.00\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$$

or $\boxed{a = 5.00\ \text{m/s}^2 \text{ at } \theta = 36.9°}$

(b) In this configuration,

$$F_{2x} = 15.0 \cos 60.0° = 7.50\ \text{N}$$
$$F_{2y} = 15.0 \sin 60.0° = 13.0\ \text{N}$$
$$\vec{\mathbf{F}}_2 = \left(7.50\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right)\ \text{N}$$

Then,

$$\sum \vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \left[20.0\hat{\mathbf{i}} + \left(7.50\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}}\right)\right]\ \text{N}$$
$$= (27.5\hat{\mathbf{i}} + 13.0\hat{\mathbf{j}})\ \text{N}$$

and $\vec{\mathbf{a}} = \dfrac{\sum \vec{\mathbf{F}}}{m} = \left(5.50\hat{\mathbf{i}} + 2.60\hat{\mathbf{j}}\right)\ \text{m/s}^2 = \boxed{6.08\ \text{m/s}^2 \text{ at } 25.3°}$

P5.20 (a) You and the Earth exert equal forces on each other: $m_y g = M_E a_E$. If your mass is 70.0 kg,

$$a_E = \frac{(70.0\ \text{kg})(9.80\ \text{m/s}^2)}{5.98 \times 10^{24}\ \text{kg}} = \boxed{\sim 10^{-22}\ \text{m/s}^2} \qquad \textbf{[1]}$$

(b) You and the planet move for equal time intervals Δt according to $\Delta x = \frac{1}{2}a(\Delta t)^2$. If the seat is 50.0 cm high,

$$\sqrt{\frac{2\Delta x_y}{a_y}} = \sqrt{\frac{2\Delta x_E}{a_E}}$$

$$\Delta x_E = \frac{a_E}{a_y}\Delta x_y$$

We substitute for $\frac{a_E}{a_y}$ from [1] to obtain

$$\Delta x_E = \frac{70.0\text{ kg}(0.500\text{ m})}{5.98\times 10^{24}\text{ kg}}$$

$$\boxed{\Delta x_E \sim 10^{-23}\text{ m}}$$

P5.21 (a) $\boxed{\text{15.0 lb up,}}$ to counterbalance the Earth's force on the block.

(b) $\boxed{\text{5.00 lb up,}}$ the forces on the block are now the Earth pulling down with 15.0 lb and the rope pulling up with 10.0 lb. The forces from the floor and rope together balance the weight.

(c) $\boxed{0,}$ the block now accelerates up away from the floor.

P5.22 $\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}$ reads

$$\left(-2.00\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{i}} - 3.00\hat{\mathbf{j}} - 45.0\hat{\mathbf{i}}\right)\text{ N} = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

where $\hat{\mathbf{a}}$ represents the direction of $\vec{\mathbf{a}}$:

$$\left(-42.0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\text{ N} = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

$$\sum\vec{\mathbf{F}} = \sqrt{(42.0)^2 + (1.00)^2}\text{ N at }\tan^{-1}\left(\frac{1.00}{42.0}\right)\text{ below the }-x\text{ axis}$$

$$\sum\vec{\mathbf{F}} = 42.0\text{ N at }181° = m\left(3.75\text{ m/s}^2\right)\hat{\mathbf{a}}$$

For the vectors to be equal, their magnitudes and their directions must be equal.

(a) Therefore $\boxed{\hat{\mathbf{a}}\text{ is at }181°}$ counter-clockwise from the x axis

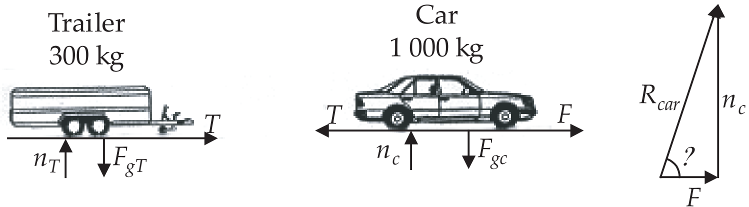
(b) $m = \frac{42.0\text{ N}}{3.75\text{ m/s}^2} = \boxed{11.2\text{ kg}}$

(c) $v = |\vec{\mathbf{v}}| = 0 + |\vec{\mathbf{a}}|t = (3.75\text{ m/s}^2)(10.00\text{ s}) = \boxed{37.5\text{ m/s}}$

(d) $\vec{\mathbf{v}} = \vec{\mathbf{v}}_i + |\vec{\mathbf{a}}|\, t = 0 + \dfrac{\vec{\mathbf{F}}}{m} t$

$$\vec{\mathbf{v}} = \frac{\left(-42.0\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\ \text{N}}{11.2\ \text{kg}}(10.0\ \text{s}) = \left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right)\ \text{m/s}$$

So, $\vec{\mathbf{v}}_f = \boxed{\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right)\ \text{m/s}}$



ANS. FIG. P5.23

* Choose the $+x$ direction to be horizontal and forward with the $+y$ vertical and upward. The common acceleration of the car and trailer then has components of $a_x = +2.15\ \text{m/s}^2$ and $a_y = 0$.

(a) The net force on the car is horizontal and given by

$$\left(\sum F_x\right)_{\text{car}} = F - T = m_{\text{car}} a_x = (1\,000\ \text{kg})\left(2.15\ \text{m/s}^2\right) = \boxed{2.15 \times 10^3\ \text{N forward}}$$

(b) The net force on the trailer is also horizontal and given by

$$\left(\sum F_x\right)_{\text{trailer}} = +T = m_{\text{trailer}} a_x = (300\ \text{kg})\left(2.15\ \text{m/s}^2\right) = \boxed{645\ \text{N forward}}$$

(c) Consider the free-body diagrams of the car and trailer. The only horizontal force acting on the trailer is $T = 645$ N forward, exerted on the trailer by the car. Newton's third law then states that the force the trailer exerts on the car is $\boxed{645\ \text{N toward the rear}}$.

(d) The road exerts two forces on the car. These are F and n_c shown in the free-body diagram of the car. From part (a), $F = T + 2.15 \times 10^3\ \text{N} = +2.80 \times 10^3\ \text{N}$. Also, $\left(\sum F_y\right)_{\text{car}} = n_c - F_{gc} = m_{\text{car}} a_y = 0$, so $n_c = F_{gc} = m_{\text{car}} g = 9.80 \times 10^3\ \text{N}$. The resultant force exerted on the car by the road is then

$$R_{\text{car}} = \sqrt{F^2 + n_c^2} = \sqrt{\left(2.80 \times 10^3\ \text{N}\right)^2 + \left(9.80 \times 10^3\ \text{N}\right)^2} = 1.02 \times 10^4\ \text{N}$$

at $\theta = \tan^{-1}\left(\frac{n_c}{F}\right) = \tan^{-1}(3.51) = 74.1°$ above the horizontal and forward. Newton's third law then states that the resultant force exerted on the road by the car is

$\boxed{1.02 \times 10^4 \text{ N at } 74.1° \text{ below the horizontal and rearward}}$.

P5.24 $v = v_i - kx$ implies the acceleration is given by

$$a = \frac{dv}{dt} = 0 - k\frac{dx}{dt} = -kv$$

Then the total force is

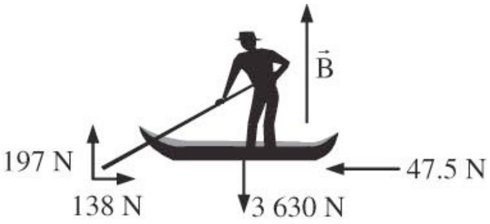
$$\sum F = ma = m(-kv)$$

The resistive force is opposite to the velocity:

$$\boxed{\sum \vec{\mathbf{F}} = -km\vec{\mathbf{v}}}$$

Section 5.7 Analysis Models Using Newton's Second Law

P5.25 As the worker through the pole exerts on the lake bottom a force of 240 N downward at 35° behind the vertical, the lake bottom through the pole exerts a force of 240 N upward at 35° ahead of the vertical. With the *x* axis horizontally forward, the pole force on the boat is



ANS. FIG. P5.25

$$\left(240\cos 35°\hat{\mathbf{j}} + 240\sin 35°\hat{\mathbf{i}}\right) \text{ N} = \left(138\hat{\mathbf{i}} + 197\hat{\mathbf{j}}\right) \text{ N}$$

The gravitational force of the whole Earth on boat and worker is $F_g = mg = 370 \text{ kg } (9.8 \text{ m/s}^2) = 3\,630$ N down. The acceleration of the boat is purely horizontal, so

$$\sum F_y = ma_y \text{ gives } +B + 197 \text{ N} - 3\,630 \text{ N} = 0$$

(a) The buoyant force is $B = \boxed{3.43 \times 10^3 \text{ N}}$.

(b) The acceleration is given by

$$\sum F_x = ma_x: \quad +138\text{ N} - 47.5\text{ N} = (370\text{ kg})a$$

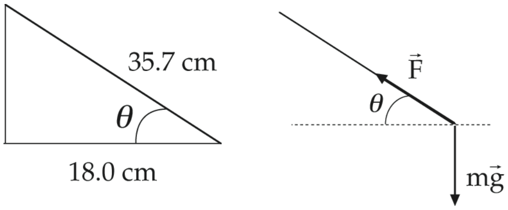
$$a = \frac{90.2\text{ N}}{370\text{ kg}} = 0.244\text{ m/s}^2$$

According to the constant-acceleration model,

$$\begin{aligned} v_{xf} &= v_{xi} + a_x t \\ &= 0.857\text{ m/s} + (0.244\text{ m/s}^2)(0.450\text{ s}) \\ &= 0.967\text{ m/s} \end{aligned}$$

$$\vec{\mathbf{v}}_f = \boxed{0.967\hat{\mathbf{i}}\text{ m/s}}$$

P5.26 (a) The left-hand diagram in ANS. FIG. P5.26(a) shows the geometry of the situation and lets us find the angle of the string with the horizontal:



ANS. FIG. P5.26(a)

$$\cos\theta = 28/35.7 = 0.784$$

or $\theta = 38.3°$

The right-hand diagram in ANS. FIG. P5.26(a) is the free-body diagram. The weight of the bolt is

$$w = mg = (0.065\text{ kg})(9.80\text{ m/s}^2) = 0.637\text{ N}$$

(b) To find the tension in the string, we apply Newton's second law in the x and y directions:

$$\sum F_x = ma_x: -T\cos 38.3° + F_{\text{magnetic}} = 0 \quad \textbf{[1]}$$

$$\sum F_y = ma_y: +T\sin 38.3° - 0.637\text{ N} = 0 \quad \textbf{[2]}$$

from equation [2],

$$T = \frac{0.637\text{ N}}{\sin 38.3°} = \boxed{1.03\text{ N}}$$

(c) Now, from equation [1],

$$F_{\text{magnetic}} = T\cos 38.3° = (1.03\text{ N})\cos 38.3° = \boxed{0.805\text{ N to the right}}$$

P5.27 (a)
$$\boxed{\begin{array}{l} P\cos 40.0° - n = 0 \text{ and } P\sin 40.0° - 220\text{ N} = 0 \\ P = 342\text{ N and } n = 262\text{ N} \end{array}}$$

(b)
$$\boxed{\begin{array}{l} P - n\cos 40.0° - 220\text{ N}\sin 40.0° = 0 \\ \text{and } n\sin 40.0° - 220\text{ N}\cos 40.0° = 0 \\ n = 262\text{ N and } P = 342\text{ N.} \end{array}}$$

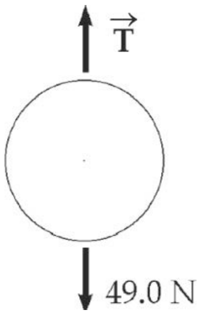
(c) $\boxed{\begin{array}{l}\text{The results agree. The methods are basically of the same level} \\ \text{of difficulty. Each involves one equation in one unknown and} \\ \text{one equation in two unknowns. If we are interested in } n \\ \text{without finding } P\text{, method (b) is simpler.}\end{array}}$

P5.28 (a) Isolate either mass:

$$T + mg = ma = 0$$
$$|T| = |mg|$$

The scale reads the tension T, so

$$T = mg = (5.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{49.0\text{ N}}$$

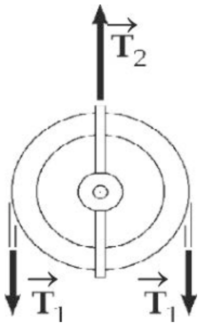


ANS. FIG. P5.28 (a) and (b)

(b) The solution to part (a) is also the solution to (b).

(c) Isolate the pulley:

$$\vec{T}_2 + 2\vec{T}_1 = 0$$
$$T_2 = 2|T_1| = 2mg = \boxed{98.0\text{ N}}$$



ANS. FIG. P5.28(c)

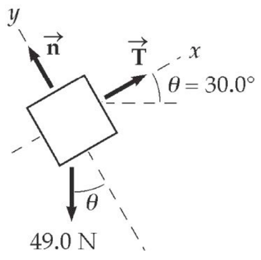
(d) $\sum\vec{F} = \vec{n} + \vec{T} + m\vec{g} = 0$

Take the component along the incline,

$$n_x + T_x + mg_x = 0$$

or $\quad 0 + T - mg\sin 30.0° = 0$

$$T = mg\sin 30.0° = \frac{mg}{2}$$
$$= \frac{(5.00\text{ kg})(9.80\text{ m/s}^2)}{2}$$
$$= \boxed{24.5\text{ N}}$$



ANS. FIG. P5.28(d)

***P5.29** (a) The resultant external force acting on this system, consisting of all three blocks having a total mass of 6.0 kg, is 42 N directed horizontally toward the right. Thus, the acceleration produced is

$$a = \frac{\sum F}{m} = \frac{42\text{ N}}{6.0\text{ kg}} = \boxed{7.0\text{ m/s}^2\text{ horizontally to the right}}$$

(b) Draw a free-body diagram of the 3.0-kg block and apply Newton's second law to the horizontal forces acting on this block:

$$\sum F_x = ma_x:$$

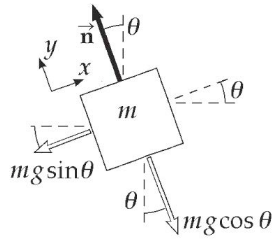
$$42\text{ N} - T = (3.0\text{ kg})(7.0\text{ m/s}^2) \quad \rightarrow \quad T = \boxed{21\text{ N}}$$

(c) The force accelerating the 2.0-kg block is the force exerted on it by the 1.0-kg block. Therefore, this force is given by

$$F = ma = (2.0\text{ kg})(7.0\text{ m/s}^2) = 14\text{ N}$$

or $\vec{\mathbf{F}} = \boxed{14\text{ N horizontally to the right}}$

P5.30 (a) ANS. FIG. P5.30 shows the forces on the object. The two forces acting on the block are the normal force, n, and the weight, mg. If the block is considered to be a point mass and the x axis is chosen to be parallel to the plane, then the free-body diagram will be as shown in the figure to the right. The angle θ is the angle of inclination of the plane. Applying Newton's second law for the accelerating system (and taking the direction up the plane as the positive x direction), we have



ANS. FIG. P5.30(a)

$$\sum F_y = n - mg\cos\theta = 0: \; n = mg\cos\theta$$

$$\sum F_x = -mg\sin\theta = ma: \; a = -g\sin\theta$$

(b) When $\theta = 15.0°$,

$$a = \boxed{-2.54\text{ m/s}^2}$$

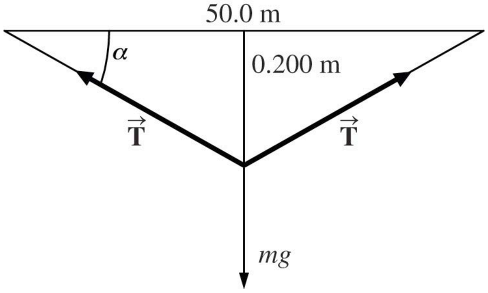
(c) Starting from rest,

$$v_f^2 = v_i^2 + 2a(x_f - x_i) = 2a\Delta x$$

$$|v_f| = \sqrt{2|a|\Delta x} = \sqrt{2|-2.54\text{ m/s}^2|(2.00\text{ m})} = \boxed{3.19\text{ m/s}}$$

P5.31 We use Newton's second law with the forces in the *x* and *y* directions in equilibrium.

(a) At the point where the bird is perched, the wire's midpoint, the forces acting on the wire are the tension forces and the force of gravity acting on the bird. These forces are shown in ANS. FIG. P5.31(a) below.



ANS. FIG. P5.31(a)

(b) The mass of the bird is *m* = 1.00 kg, so the force of gravity on the bird, its weight, is *mg* = (1.00 kg)(9.80 m/s^2) = 9.80 N. To calculate the angle α in the free-body diagram, we note that the base of the triangle is 25.0 m, so that

$$\tan\alpha = \frac{0.200\text{ m}}{25.0\text{ m}} \quad \rightarrow \quad \alpha = 0.458°$$

Each of the tension forces has *x* and *y* components given by

$$T_x = T\cos\alpha \quad \text{and} \quad T_y = T\sin\alpha$$

The *x* components of the two tension forces cancel out. In the *y* direction,

$$\sum F_y = 2T\sin\alpha - mg = 0$$

which gives

$$T = \frac{mg}{2\sin\alpha} = \frac{9.80\text{ N}}{2\sin 0.458°} = \boxed{613\text{ N}}$$

P5.32 To find the net force, we differentiate the equations for the position of the particle once with respect to time to obtain the velocity, and once again to obtain the acceleration:

$$v_x = \frac{dx}{dt} = \frac{d}{dt}\left(5t^2 - 1\right) = 10t, \;\; v_y = \frac{dy}{dt} = \frac{d}{dt}\left(3t^3 + 2\right) = 9t^2$$

$$a_x = \frac{dv_x}{dt} = 10, \;\; a_y = \frac{dv_y}{dt} = 18t$$

Then, at $t = 2.00$ s, $a_x = 10.0\text{ m/s}^2$, $a_y = 36.0\text{ m/s}^2$, and Newton's second law gives us

$$\sum F_x = ma_x\text{:}\;\; 3.00\text{ kg}(10.0\text{ m/s}^2) = 30.0\text{ N}$$

$$\sum F_y = ma_y\text{:}\;\; 3.00\text{ kg}(36.0\text{ m/s}^2) = 108\text{ N}$$

$$\sum F = \sqrt{F_x^2 + F_y^2} = \boxed{112\text{ N}}$$

P5.33 From equilibrium of the sack:

$$T_3 = F_g \qquad \textbf{[1]}$$

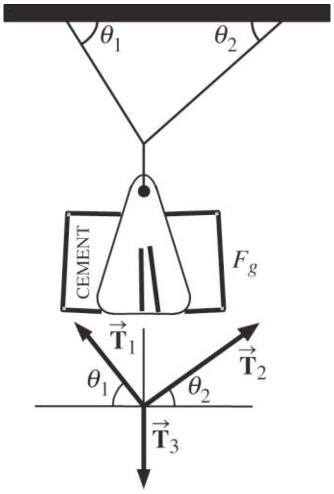
From $\sum F_y = 0$ for the knot:

$$T_1 \sin\theta_1 + T_2 \sin\theta_2 = F_g \qquad \textbf{[2]}$$

From $\sum F_x = 0$ for the knot:

$$T_1 \cos\theta_1 = T_2 \cos\theta_2 \qquad \textbf{[3]}$$

Eliminate T_2 by using $T_2 = T_1 \cos\theta_1 / \cos\theta_2$ and solve for T_1:



ANS. FIG. P5.33

$$T_1 = \frac{F_g \cos\theta_2}{\left(\sin\theta_1 \cos\theta_2 + \cos\theta_1 \sin\theta_2\right)} = \frac{F_g \cos\theta_2}{\sin\left(\theta_1 + \theta_2\right)}$$

$$T_3 = F_g = \boxed{325\text{ N}}$$

$$T_1 = F_g\left(\frac{\cos 40.0^\circ}{\sin 100.0^\circ}\right) = \boxed{253\text{ N}}$$

$$T_2 = T_1\left(\frac{\cos\theta_1}{\cos\theta_2}\right) = (253\text{ N})\left(\frac{\cos 60.0^\circ}{\cos 40.0^\circ}\right) = \boxed{165\text{ N}}$$

P5.34 See the solution for T_1 in Problem 5.33. The equations indicate that the tension is directly proportional to F_g.

***P5.35** Let us call the forces exerted by each person F_1 and F_2. Thus, for pulling in the same direction, Newton's second law becomes

$$F_1 + F_2 = (200\text{ kg})(1.52\text{ m/s}^2)$$

or $$F_1 + F_2 = 304\text{ N} \qquad \textbf{[1]}$$

When pulling in opposite directions,

$$F_1 - F_2 = (200\text{ kg})(-0.518\text{ m/s}^2)$$

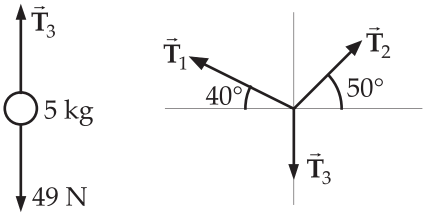
or $$F_1 - F_2 = -104\text{ N} \qquad \textbf{[2]}$$

Solving [1] and [2] simultaneously, we find

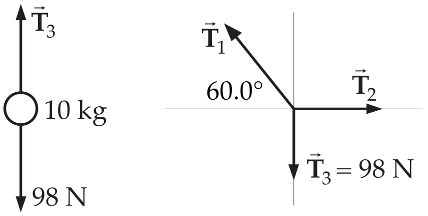
$$F_1 = \boxed{100\text{ N}} \text{ and } F_2 = \boxed{204\text{ N}}$$

***P5.36** (a) First construct a free-body diagram for the 5.00-kg mass as shown in the Figure 5.36a. Since the mass is in equilibrium, we can require $T_3 - 49.0\text{ N} = 0$ or $T_3 = 49.0\text{ N}$. Next, construct a free-body diagram for the knot as shown in ANS. FIG. P5.36(a).

Again, since the system is moving at constant velocity, $a = 0$, and applying Newton's second law in component form gives



ANS. FIG. 5.36(a)



ANS. FIG. 5.36(b)

$$\sum F_x = T_2 \cos 50.0° - T_1 \cos 40.0° = 0$$
$$\sum F_y = T_2 \sin 50.0° + T_1 \sin 40.0° - 49.0\text{ N} = 0$$

Solving the above equations simultaneously for T_1 and T_2 gives $\boxed{T_1 = 31.5\text{ N}}$ and $\boxed{T_2 = 37.5\text{ N}}$ and above we found $\boxed{T_3 = 49.0\text{ N}}$.

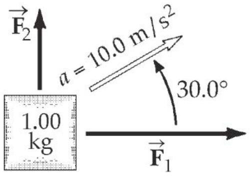
(b) Proceed as in part (a) and construct a free-body diagram for the mass and for the knot as shown in ANS. FIG. P5.36(b). Applying Newton's second law in each case (for a constant-velocity system), we find:

$$T_3 - 98.0\text{ N} = 0$$
$$T_2 - T_1\cos 60.0° = 0$$
$$T_1\sin 60.0° - T_3 = 0$$

Solving this set of equations we find:

$$\boxed{T_1 = 113\text{ N},}\ \boxed{T_2 = 56.6\text{ N},}\ \text{and}\ \boxed{T_3 = 98.0\text{ N}}$$

P5.37 Choose a coordinate system with $\hat{\mathbf{i}}$ East and $\hat{\mathbf{j}}$ North. The acceleration is



ANS. FIG. P5.37

$$\vec{\mathbf{a}} = [(10.0\cos 30.0°)\hat{\mathbf{i}} + (10.0\sin 30.0°)\hat{\mathbf{j}}]\text{ m/s}^2$$
$$= (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\text{ m/s}^2$$

From Newton's second law,

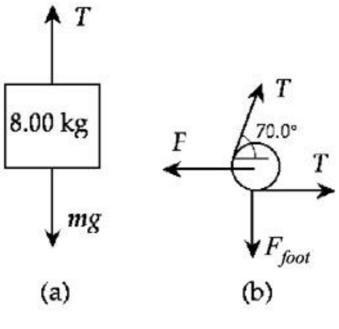
$$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}} = (1.00\text{ kg})(8.66\hat{\mathbf{i}}\text{ m/s}^2 + 5.00\hat{\mathbf{j}}\text{ m/s}^2)$$
$$= (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}})\text{ N}$$

and $\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$

So the force we want is

$$\vec{\mathbf{F}}_1 = \sum\vec{\mathbf{F}} - \vec{\mathbf{F}}_2 = (8.66\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}} - 5.00\hat{\mathbf{j}})\text{ N}$$
$$= 8.66\hat{\mathbf{i}}\text{ N} = \boxed{8.66\text{ N east}}$$

P5.38 (a) Assuming frictionless pulleys, the tension is uniform through the entire length of the rope. Thus, the tension at the point where the rope attaches to the leg is the same as that at the 8.00-kg block. ANS. FIG. P5.38(a) gives a free-body diagram of the suspended block. Recognizing that the block has zero acceleration, Newton's second law gives



ANS. FIG. P5.38

$$\sum F_y = T - mg = 0$$

or

$$T = mg = (8.00\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{78.4\ \text{N}}$$

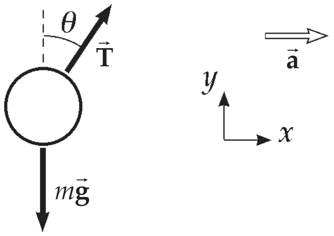
(b) ANS. FIG. P5.38(b) gives a free-body diagram of the pulley near the foot. Here, F is the magnitude of the force the foot exerts on the pulley. By Newton's third law, this is the same as the magnitude of the force the pulley exerts on the foot. Applying the second law gives

$$\sum F_x = T + T\cos 70.0° - F = ma_x = 0$$

or

$$F = T(1 + \cos 70.0°) = (78.4\ \text{N})(1 + \cos 70.0°) = \boxed{105\ \text{N}}$$

***P5.39** (a) Assume the car and mass accelerate horizontally. We consider the forces on the suspended object.



ANS. FIG. P5.39

$$\sum F_y = ma_y:\quad +T\cos\theta - mg = 0$$

$$\sum F_x = ma_x:\quad +T\sin\theta = ma$$

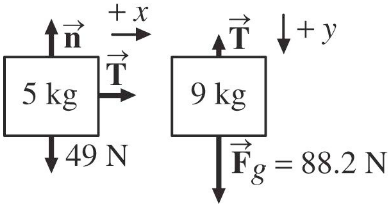
Substitute $T = \dfrac{mg}{\cos\theta}$ from the first equation into the second,

$$\frac{mg\sin\theta}{\cos\theta} = mg\tan\theta = ma$$

$$\boxed{a = g\tan\theta}$$

(b) $a = (9.80\ \text{m/s}^2)\tan 23.0° = \boxed{4.16\ \text{m/s}^2}$

P5.40 (a) The forces on the objects are shown in ANS. FIG. P5.40.



ANS. FIG. P5.40

(b) and (c) First, consider m_1, the block moving along the horizontal. The only force in the direction of movement is T. Thus,

$$\sum F_x = ma$$

or $T = (5.00\text{ kg})a$ **[1]**

Next consider m_2, the block that moves vertically. The forces on it are the tension T and its weight, 88.2 N.

We have $\sum F_y = ma$:

$$88.2\text{ N} - T = (9.00\text{ kg})a \qquad \textbf{[2]}$$

Note that both blocks must have the same magnitude of acceleration. Equations [1] and [2] can be added to give 88.2 N = (14.0 kg)a. Then

$$\boxed{a = 6.30\text{ m/s}^2 \text{ and } T = 31.5\text{ N}}$$

P5.41 (a) and (b) The slope of the graph of upward velocity versus time is the acceleration of the person's body. At both time 0 and time 0.5 s, this slope is (18 cm/s)/0.6 s = 30 cm/s^2.

For the person's body,

$$\sum F_y = ma_y:$$
$$+F_{\text{bar}} - (64.0\text{ kg})(9.80\text{ m/s}^2) = (64.0\text{ kg})(0.3\text{ m/s}^2)$$

Note that there is no floor touching the person to exert a normal force, and that he does not exert any extra force "on himself." Solving, $F_{\text{bar}} = \boxed{646\text{ N up}}$.

(c) a_y = slope of v_y versus t graph = 0 at t = 1.1 s. The person is moving with maximum speed and is momentarily in equilibrium:

$$\sum F_y = ma_y:$$
$$+F_{\text{bar}} - (64.0\text{ kg})(9.80\text{ m/s}^2) = 0$$

$$F_{\text{bar}} = \boxed{627\text{ N up}}$$

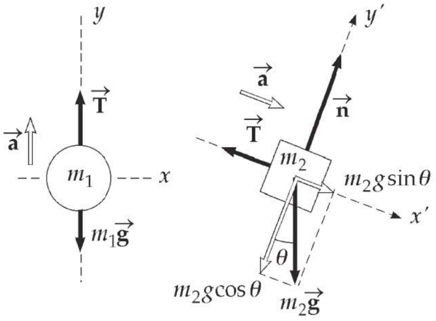
(d) a_y = slope of v_y versus t graph = (0 – 24 cm/s)/(1.7 s – 1.3 s) = –60 cm/s^2

$$\sum F_y = ma_y:$$
$$+F_{\text{bar}} - (64.0\text{ kg})(9.80\text{ m/s}^2) = (64.0\text{ kg})(-0.6\text{ m/s}^2)$$

$$F_{\text{bar}} = \boxed{589\text{ N up}}$$

P5.42 $m_1 = 2.00$ kg, $m_2 = 6.00$ kg, $\theta = 55.0°$

(a) The forces on the objects are shown in ANS. FIG. P5.42.



ANS. FIG. P5.42

(b) $\sum F_x = m_2 g\sin\theta - T = m_2 a$ and

$$T - m_1 g = m_1 a$$

$$a = \frac{m_2 g\sin\theta - m_1 g}{m_1 + m_2}$$

$$= \frac{(6.00\text{ kg})(9.80\text{ m/s}^2)\sin 55.0° - (2.00\text{ kg})(9.80\text{ m/s}^2)}{2.00\text{ kg} + 6.00\text{ kg}}$$

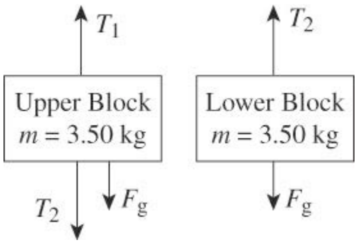
$$= \boxed{3.57\ \text{m/s}^2}$$

(c) $T = m_1(a + g) = (2.00\text{ kg})(3.57\text{ m/s}^2 + 9.80\text{ m/s}^2) = \boxed{26.7\text{ N}}$

(d) Since $v_i = 0$, $v_f = at = (3.57\ \text{m/s}^2)(2.00\text{ s}) = \boxed{7.14\ \text{m/s}}$.

P5.43 (a) Free-body diagrams of the two blocks are shown in ANS. FIG. P5.43. Note that each block experiences a downward gravitational force

$$F_g = (3.50\text{ kg})(9.80\ \text{m/s}^2) = 34.3\text{ N}$$



ANS. FIG. P5.43

Also, each has the same upward acceleration as the elevator, in this case $a_y = +1.60\text{ m/s}^2$.

Applying Newton's second law to the lower block:

$$\sum F_y = ma_y \quad \Rightarrow \quad T_2 - F_g = ma_y$$

or

$$T_2 = F_g + ma_y = 34.3\text{ N} + (3.50\text{ kg})(1.60\ \text{m/s}^2) = \boxed{39.9\text{ N}}$$

Next, applying Newton's second law to the upper block:

$$\sum F_y = ma_y \quad \Rightarrow \quad T_1 - T_2 - F_g = ma_y$$

or

$$T_1 = T_2 + F_g + ma_y = 39.9\text{ N} + 34.3\text{ N} + (3.50\text{ kg})(1.60\ \text{m/s}^2)$$

$$= \boxed{79.8\ \text{N}}$$

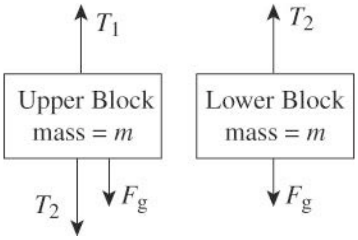
(b) Note that the tension is greater in the upper string, and this string will break first as the acceleration of the system increases. Thus, we wish to find the value of a_y when $T_1 = 85.0$. Making use of the general relationships derived in (a) above gives:

$$T_1 = T_2 + F_g + ma_y = \left(F_g + ma_y\right) + F_g + ma_y = 2F_g + 2ma_y$$

or

$$a_y = \frac{T_1 - 2F_g}{2m} = \frac{85.0\text{ N} - 2(34.3\text{ N})}{2(3.50\text{ kg})} = \boxed{2.34\text{ m/s}^2}$$

P5.44 (a) Free-body diagrams of the two blocks are shown in ANS. FIG. P5.44. Note that each block experiences a downward gravitational force $F_g = mg$.



ANS. FIG. P5.44

Also, each has the same upward acceleration as the elevator, $a_y = +a$.

Applying Newton's second law to the lower block:

$$\sum F_y = ma_y \quad \Rightarrow \quad T_2 - F_g = ma_y$$

or $T_2 = mg + ma = \boxed{m(g + a)}$

Next, applying Newton's second law to the upper block:

$$\sum F_y = ma_y \quad \Rightarrow \quad T_1 - T_2 - F_g = ma_y$$

or

$$T_1 = T_2 + F_g + ma_y = (mg + ma) + mg + ma = 2(mg + ma)$$
$$= \boxed{2m(g + a)} = 2T_2$$

(b) Note that $\boxed{T_1 = 2T_2\text{, so the upper string breaks first}}$ as the acceleration of the system increases.

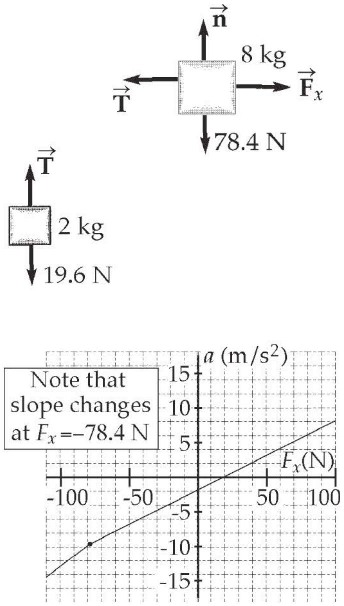
(c) When the upper string breaks, both blocks will be in free fall with $a = -g$. Then, using the results of part (a), $T_2 = m(g + a) = m(g - g) = \boxed{0}$ and $T_1 = 2T_2 = \boxed{0}$.

P5.45 Forces acting on $m_1 = 2.00$-kg block:

$$T - m_1 g = m_1 a \qquad \textbf{[1]}$$

Forces acting on $m_2 = 8.00$-kg block:

$$F_x - T = m_2 a \qquad \textbf{[2]}$$



Note that slope changes at $F_x = -78.4$ N

ANS. FIG. P5.45

(a) Eliminate T and solve for a:

$$a = \frac{F_x - m_1 g}{m_1 + m_2}$$

$$\boxed{a > 0 \text{ for } F_x > m_1 g = 19.6 \text{ N}}$$

(b) Eliminate a and solve for T:

$$T = \frac{m_1}{m_1 + m_2}\left(F_x + m_2 g\right)$$

$$\boxed{T = 0 \text{ for } F_x \le -m_2 g = -78.4 \text{ N}}$$

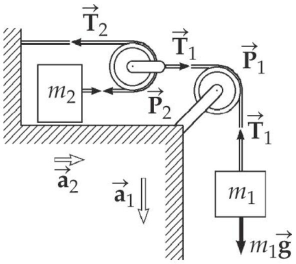
Note that if $F_x < -m_2 g$, the cord is loose, so mass m_2 is in free fall and mass m_1 accelerates under the action of F_x only.

(c) See ANS. FIG. P5.45.

F_x, N	–100	–78.4	–50.0	0	50.0	100
a_x, m/s^2	–12.5	–9.80	–6.96	–1.96	3.04	8.04

P5.46 (a) Pulley P_2 has acceleration a_1.

Since m_2 moves *twice* the distance P_2 moves in the same time, m_2 has twice the acceleration of P_2, i.e., $\boxed{a_2 = 2a_1}$.



ANS. FIG. P5.46

(b) From the figure, and using

$$\sum F = ma: \quad m_1 g - T_1 = m_1 a_1 \qquad \textbf{[1]}$$

$$T_2 = m_2 a_2 = 2m_2 a_1 \qquad \textbf{[2]}$$

$$T_1 - 2T_2 = 0 \qquad \textbf{[3]}$$

Equation [1] becomes $m_1g - 2T_2 = m_1a_1$. This equation combined with equation [2] yields

$$\frac{T_2}{m_2}\left(2m_2 + \frac{m_2}{2}\right) = m_1 g$$

$$\boxed{T_2 = \frac{m_1m_2}{2m_2 + \frac{1}{2}m_1}g} \text{ and } \boxed{T_2 = \frac{m_1m_2}{m_2 + \frac{1}{4}m_1}g}$$

(c) From the values of T_2 and T_1, we find that

$$a_2 = \frac{T_2}{m_2} = \boxed{\frac{m_1 g}{2m_2 + \frac{1}{2}m_1}} \text{ and } a_1 = \frac{1}{2}a_2 = \boxed{\frac{m_1 g}{4m_2 + m_1}}$$

***P5.47** We use the particle under constant acceleration and particle under a net force models. Newton's law applies for each axis. After it leaves your hand, the block's speed changes only because of one component of its weight:

$$\sum F_x = ma_x$$
$$-mg\sin 20.0° = ma$$

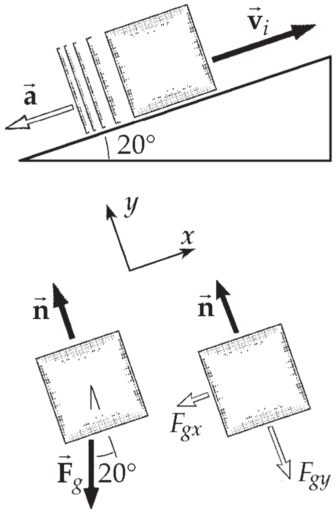
$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$

Taking $v_f = 0$, $v_i = 5.00$ m/s, and $a = -g\sin(20.0°)$ gives, suppressing units,

$$0 = (5.00)^2 - 2(9.80)\sin(20.0°)(x_f - 0)$$

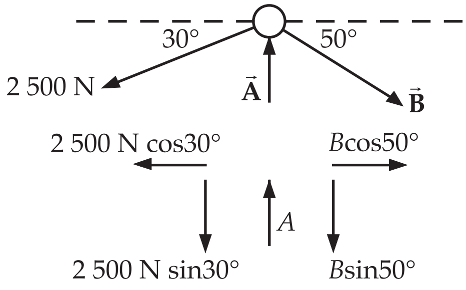
or

$$x_f = \frac{25.0}{2(9.80)\sin(20.0°)} = \boxed{3.73 \text{ m}}$$



ANS. FIG. P5.47

***P5.48** We assume the vertical bar is in compression, pushing up on the pin with force A, and the tilted bar is in tension, exerting force B on the pin at –50.0°.



ANS. FIG. P5.48

$\sum F_x = 0$:

$$-2\,500\text{ N}\cos 30° + B\cos 50° = 0$$

$$\boxed{B = 3.37 \times 10^3\text{ N}}$$

$\sum F_y = 0$:

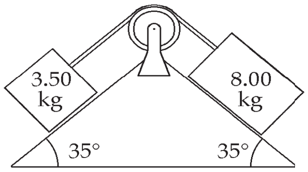
$$-2\,500\text{ N}\sin 30° + A - 3.37 \times 10^3\text{ N }\sin 50° = 0$$

$$\boxed{A = 3.83 \times 10^3\text{ N}}$$

Positive answers confirm that

$$\boxed{B\text{ is in tension and }A\text{ is in compression.}}$$

P5.49 Since it has a larger mass, we expect the 8.00-kg block to move down the plane. The acceleration for both blocks should have the same magnitude since they are joined together by a non-stretching string. Define up the left-hand plane as positive for the 3.50-kg object and down the right-hand plane as positive for the 8.00-kg object.



ANS. FIG. P5.49

$$\sum F_1 = m_1 a_1: \quad -m_1 g\sin 35.0° + T = m_1 a$$
$$\sum F_2 = m_2 a_2: \quad m_2 g\sin 35.0° - T = m_2 a$$

and, suppressing units,

$$-(3.50)(9.80)\sin 35.0° + T = 3.50a$$
$$(8.00)(9.80)\sin 35.0° - T = 8.00a.$$

Adding, we obtain $+45.0\text{ N} - 19.7\text{ N} = (11.5\text{ kg})a$.

(a) Thus the acceleration is $\boxed{a = 2.20\text{ m/s}^2}$. By substitution,

$$-19.7\text{ N} + T = (3.50\text{ kg})(2.20\text{ m/s}^2) = 7.70\text{ N}$$

(b) The tension is $\boxed{T = 27.4\text{ N}}$

P5.50 Both blocks move with acceleration $a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right)g$:

$$a = \left(\frac{7\text{ kg} - 2\text{ kg}}{7\text{ kg} + 2\text{ kg}}\right)(9.80\text{ m/s}^2) = 5.44\text{ m/s}^2$$

(a) Take the upward direction as positive for m_1.

$$v_{yf}^2 = v_{yi}^2 + 2a_x(y_f - y_i)$$

$$0 = (-2.4\text{ m/s})^2 + 2(5.44\text{ m/s}^2)(y_f - 0)$$

$$y_f = -\frac{5.76\text{ m}^2/\text{s}^2}{2(5.44\text{ m/s}^2)} = -0.529\text{ m}$$

$$y_f = \boxed{0.529\text{ m below its initial level}}$$

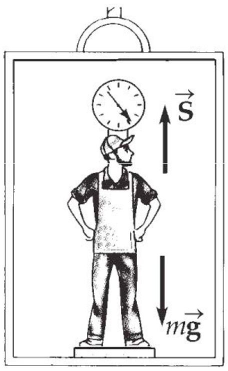
(b) $v_{yf} = v_{yi} + a_y t$: $v_{yf} = -2.40\text{ m/s} + (5.44\text{ m/s}^2)(1.80\text{ s})$

$$v_{yf} = \boxed{7.40\text{ m/s upward}}$$

P5.51 We draw a force diagram and apply Newton's second law for each part of the elevator trip to find the scale force. The acceleration can be found from the change in speed divided by the elapsed time.

Consider the force diagram of the man shown as two arrows. The force F is the upward force exerted on the man by the scale, and his weight is

$$F_g = mg = (72.0\text{ kg})(9.80\text{ m/s}^2) = 706\text{ N}$$



ANS. FIG. P5.51

With $+y$ defined to be upwards, Newton's second law gives

$$\sum F_y = +F_s - F_g = ma$$

Thus, we calculate the upward scale force to be

$$F_s = 706\text{ N} + (72.0\text{ kg})a \qquad \textbf{[1]}$$

where a is the acceleration the man experiences as the elevator changes speed.

(a) Before the elevator starts moving, the elevator's acceleration is zero ($a = 0$). Therefore, equation [1] gives the force exerted by the scale on the man as 706 N upward, and the man exerts a downward force of $\boxed{706\text{ N}}$ on the scale.

(b) During the first 0.800 s of motion, the man accelerates at a rate of

$$a_x = \frac{\Delta v}{\Delta t} = \frac{1.20\text{ m/s} - 0}{0.800\text{ s}} = 1.50\text{ m/s}^2$$

Substituting a into equation [1] then gives

$$F = 706\text{ N} + (72.0\text{ kg})(1.50\text{ m/s}^2) = \boxed{814\text{ N}}$$

(c) While the elevator is traveling upward at constant speed, the acceleration is zero and equation [1] again gives a scale force $F = \boxed{706\text{ N}}$.

(d) During the last 1.50 s, the elevator first has an upward velocity of 1.20 m/s, and then comes to rest with an acceleration of

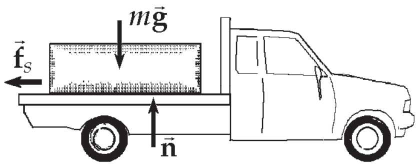
$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 1.20\text{ m/s}}{1.50\text{ s}} = -0.800\text{ m/s}^2$$

Thus, the force of the man on the scale is

$$F = 706\text{ N} + (72.0\text{ kg})(-0.800\text{ m/s}^2) = \boxed{648\text{ N}}$$

Section 5.8 Forces of Friction

***P5.52** If the load is on the point of sliding forward on the bed of the slowing truck, static friction acts backward on the load with its maximum value, to give it the same acceleration as the truck:



ANS. FIG. P5.52

$$\Sigma F_x = ma_x: \quad -f_s = m_{\text{load}}a_x$$

$$\Sigma F_y = ma_y: \quad n - m_{\text{load}}g = 0$$

Solving for the normal force and substituting into the *x* equation gives:

$$-\mu_s m_{\text{load}} g = m_{\text{load}} a_x \qquad \text{or} \qquad a_x = -\mu_s g$$

We can then use

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

Which becomes

$$0 = v_{xi}^2 + 2\left(-\mu_s g\right)\left(x_f - 0\right)$$

(a) $x_f = \dfrac{v_{xi}^2}{2\mu_s g} = \dfrac{(12.0\ \text{m/s})^2}{2(0.500)\left(9.80\ \text{m/s}^2\right)} = \boxed{14.7\ \text{m}}$

(b) From the expression $x_f = \dfrac{v_{xi}^2}{2\mu_s g}$,

$\boxed{\text{neither mass affects the answer}}$

P5.53 Using $m = 12.0 \times 10^{-3}$ kg, $v_i = 260$ m/s, $v_f = 0$, $\Delta x = (x_f - x_i) = 0.230$ m, and $v_f^2 = v_i^2 + 2a(x_f - x_i)$, we find the acceleration of the bullet: $a = -1.47 \times 10^5\ \text{m/s}^2$. Newton's second law then gives

$$\sum F_x = ma_x$$

$$f_k = ma = -1.76 \times 10^5\ \text{N}$$

The (kinetic) friction force is $\boxed{1.76 \times 10^5\ \text{N in the negative } x \text{ direction}}$.

P5.54 We apply Newton's second law to the car to determine the maximum static friction force acting on the car:

$$\sum F_y = ma_y: \quad +n - mg = 0$$

$$f_s \le \mu_s n = \mu_s mg$$

This maximum magnitude of static friction acts so long as the tires roll without skidding.

$$\sum F_x = ma_x \rightarrow -f_s = ma$$

The maximum acceleration is

$$a = -\mu_s g$$

The initial and final conditions are: $x_i = 0$, $v_i = 50.0\text{ mi/h} = 22.4\text{ m/s}$, and $v_f = 0$. Then,

$$v_f^2 = v_i^2 + 2a(x_f - x_i) \rightarrow v_i^2 = 2\mu_s g x_f$$

(a) $$x_f = \frac{v_i^2}{2\mu_s g}$$

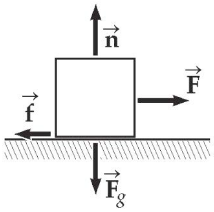
$$x_f = \frac{(22.4\text{ m/s})^2}{2(0.100)(9.80\text{ m/s}^2)} = \boxed{256\text{ m}}$$

(b) $$x_f = \frac{v_i^2}{2\mu_s g}$$

$$x_f = \frac{(22.4\text{ m/s})^2}{2(0.600)(9.80\text{ m/s}^2)} = \boxed{42.7\text{ m}}$$

P5.55 For equilibrium: $f = F$ and $n = F_g$. Also, $f = \mu n$, i.e.,

$$\mu = \frac{f}{n} = \frac{F}{F_g}$$



ANS. FIG. P5.55

In parts (a) and (b), we replace F with the magnitude of the applied force and μ with the appropriate coefficient of friction.

(a) The coefficient of static friction is found from

$$\mu_s = \frac{F}{F_g} = \frac{75.0\text{ N}}{(25.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.306}$$

(b) The coefficient of kinetic friction is found from

$$\mu_k = \frac{F}{F_g} = \frac{60.0\text{ N}}{(25.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.245}$$

P5.56 Find the acceleration of the car, which is the same as the acceleration of the book because the book does not slide.

For the car: $v_i = 72.0$ km/h = 20.0 m/s, $v_f = 0$, $\Delta x = (x_f - x_i) = 30.0$ m. Using $v_f^2 = v_i^2 + 2a(x_f - x_i)$, we find the acceleration of the car:

$$a = -6.67 \text{ m/s}^2$$

Now, find the maximum acceleration that friction can provide. Because the book does not slide, static friction provides the force that slows down the book. We have the coefficient of static friction, $\mu_s = 0.550$, and we know $f_s \le \mu_s n$. The book is on a horizontal seat, so friction acts in the horizontal direction, and the vertical normal force that the seat exerts on the book is equal in magnitude to the force of gravity on the book: $n = F_g = mg$. For maximum acceleration, the static friction force will be a maximum, so $f_s = \mu_s n = \mu_s mg$. Applying Newton's second law, we find the acceleration that friction can provide for the book:

$$\sum F_x = ma_x:$$

$$-f_s = ma$$

$$-\mu_s mg = ma$$

which gives $a = -\mu_s g = -(0.550)(9.80 \text{ m/s}^2) = -5.39 \text{ m/s}^2$, which is too small for the stated conditions.

The situation is impossible because maximum static friction cannot provide the acceleration necessary to keep the book stationary on the seat.

P5.57 The x and y components of Newton's second law as the eraser begins to slip are

$$-f + mg\sin\theta = 0 \text{ and } +n - mg\cos\theta = 0$$

with $f = \mu_s n$ or $\mu_k n$, these equations yield

$$\mu_s = \tan\theta_c = \tan 36.0° = \boxed{0.727}$$

$$\mu_k = \tan\theta_c = \tan 30.0° = \boxed{0.577}$$

P5.58 We assume that all the weight is on the rear wheels of the car.

(a) We find the record time from

$$F = ma: \quad \mu_s mg = ma \quad \text{or} \quad a = \mu_s g$$

But

$$\Delta x = \frac{at^2}{2} = \frac{\mu_s g t^2}{2}$$

so $\mu_s = \dfrac{2\Delta x}{gt^2}$

$$\mu_s = \frac{2(0.250\ \text{mi})(1\,609\ \text{m/mi})}{(9.80\ \text{m/s}^2)(4.43\ \text{s})^2} = \boxed{4.18}$$

(b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

P5.59 Maximum static friction provides the force that produces maximum acceleration, resulting in a minimum time interval to accelerate through $\Delta x = 3.00$ m. We know that the maximum force of static friction is $f_s = \mu_s n$. If the shoe is on a horizontal surface, friction acts in the horizontal direction. Assuming that the vertical normal force is maximal, equal in magnitude to the force of gravity on the person, we have $n = F_g = mg$; therefore, the maximum static friction force is

$$f_s = \mu_s n = \mu_s mg$$

Applying Newton's second law:

$$\sum F_x = ma_x:$$

$$f_s = ma$$

$$\mu_s mg = ma \quad \rightarrow \quad a = \mu_s g$$

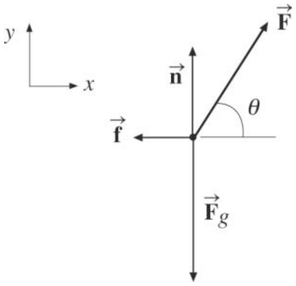
We find the time interval $\Delta t = t$ to accelerate from rest through $\Delta x =$ 3.00 m using $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$:

$$\Delta x = \frac{1}{2}a_x(\Delta t)^2 \rightarrow \Delta t = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{2\Delta x}{\mu_s g}}$$

(a) For $\mu_s = 0.500$, $\Delta t = \boxed{1.11\text{ s}}$

(b) For $\mu_s = 0.800$, $\Delta t = \boxed{0.875\text{ s}}$

P5.60 (a) See the free-body diagram of the suitcase in ANS. FIG. P5.60(a).



ANS. FIG. P5.60(a)

(b) $m_{\text{suitcase}} = 20.0\text{ kg}$, $F = 35.0\text{ N}$

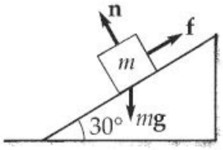
$$\sum F_x = ma_x: \quad -20.0\text{ N} + F\cos\theta = 0$$
$$\sum F_y = ma_y: \quad +n + F\sin\theta - F_g = 0$$

$$F\cos\theta = 20.0\text{ N}$$
$$\cos\theta = \frac{20.0\text{ N}}{35.0\text{ N}} = 0.571$$
$$\boxed{\theta = 55.2°}$$

(c) With $F_g = (20.0\text{ kg})(9.80\text{ m/s}^2)$,

$$n = F_g - F\sin\theta = \left[196\text{ N} - (35.0\text{ N})(0.821)\right]$$
$$\boxed{n = 167\text{ N}}$$

P5.61 We are given: $m = 3.00$ kg, $\theta = 30.0°$, $x = 2.00$ m, $t = 1.50$ s



ANS. FIG. P5.61

(a) At constant acceleration,

$$x_f = v_i t + \tfrac{1}{2}at^2$$

Solving,

$$a = \frac{2(x_f - v_i t)}{t^2} = \frac{2(2.00\text{ m} - 0)}{(1.50\text{ s})^2} = \boxed{1.78\text{ m/s}^2}$$

From the acceleration, we can calculate the friction force, answer (c), next.

(c) Take the positive *x* axis down parallel to the incline, in the direction of the acceleration. We apply Newton's second law:

$$\sum F_x = mg\sin\theta - f = ma$$

Solving, $f = m(g\sin\theta - a)$

Substituting,

$$f = (3.00\text{ kg})[(9.80\text{ m/s}^2)\sin 30.0° - 1.78\text{ m/s}^2] = \boxed{9.37\text{ N}}$$

(b) Applying Newton's law in the *y* direction (perpendicular to the incline), we have no burrowing-in or taking-off motion. Then the *y* component of acceleration is zero:

$$\sum F_y = n - mg\cos\theta = 0$$

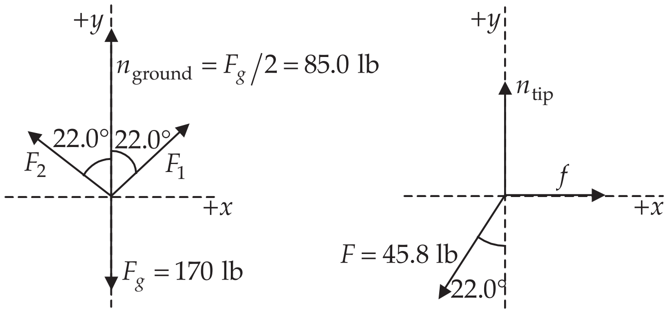
Thus $n = mg\cos\theta$

Because $f = \mu_k n$

we have $\mu_k = \dfrac{f}{mg\cos\theta} = \dfrac{9.37\text{ N}}{(3.00\text{ kg})\left(9.80\text{ m/s}^2\right)\cos 30.0°} = \boxed{0.368}$

(d) $v_f = v_i + at$ so $v_f = 0 + (1.78\text{ m/s}^2)(1.50\text{ s}) = \boxed{2.67\text{ m/s}}$

***P5.62** The free-body diagrams for this problem are shown in ANS. FIG. P5.62.



Free-Body Diagram of Person

Free-Body Diagram of Crutch Tip

ANS. FIG. P5.62

From the free-body diagram for the person,

$$\sum F_x = F_1\sin(22.0°) - F_2\sin(22.0°) = 0$$

which gives $F_1 = F_2 = F$. Then, $\sum F_y = 2F\cos 22.0° + 85.0\text{ lbs} - 170\text{ lbs} = 0$ yields $F = 45.8$ lb.

(a) Now consider the free-body diagram of a crutch tip.

$$\sum F_x = f - (45.8\text{ lb})\sin 22.0° = 0$$

or $f = 17.2$ lb.

$$\sum F_y = n_{\text{tip}} - (45.8\text{ lb})\cos 22.0° = 0$$

which gives $n_{\text{tip}} = 42.5$ lb.

For minimum coefficient of friction, the crutch tip will be on the verge of slipping, so $f = (f_s)_{\max} = \mu_s n_{\text{tip}}$ and

$$\mu_s = \frac{f}{n_{\text{tip}}} = \frac{17.2\text{ lb}}{42.5\text{ lb}} = \boxed{0.404}$$

(b) As found above, the compression force in each crutch is

$$F_1 = F_2 = F = \boxed{45.8\text{ lb}}$$

P5.63 Newton's second law for the 5.00-kg mass gives

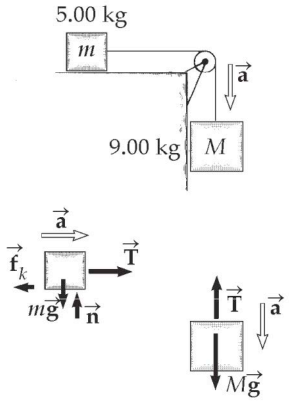
$$T - f_k = (5.00\text{ kg})a$$

Similarly, for the 9.00-kg mass,

$$(9.00\text{ kg})g - T = (9.00\text{ kg})a$$

Adding these two equations gives:

$$\begin{aligned}&(9.00\text{ kg})(9.80\text{ m/s}^2)\\ &\quad - 0.200(5.00\text{ kg})(9.80\text{ m/s}^2)\\ &\qquad = (14.0\text{ kg})a\end{aligned}$$

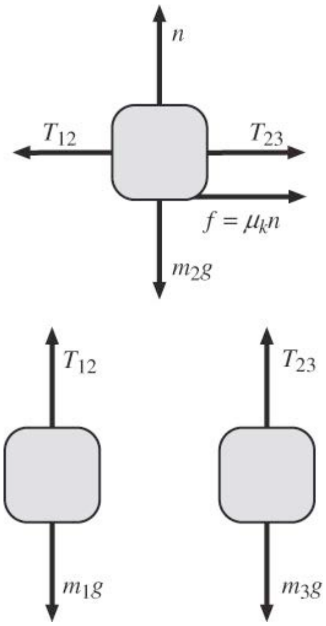


ANS. FIG. P5.63

Which yields $a = 5.60\text{ m/s}^2$. Plugging this into the first equation above gives

$$T = (5.00\text{ kg})(5.60\text{ m/s}^2) + 0.200(5.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{37.8\text{ N}}$$

P5.64 (a) The free-body diagrams for each object appear on the right.



ANS. FIG. P5.64(a)

(b) Let a represent the positive magnitude of the acceleration $-a\hat{\mathbf{j}}$ of m_1, of the acceleration $-a\hat{\mathbf{i}}$ of m_2, and of the acceleration $+a\hat{\mathbf{j}}$ of m_3. Call T_{12} the tension in the left cord and T_{23} the tension in the cord on the right.

For m_1, $\sum F_y = ma_y$:

$$+T_{12} - m_1 g = -m_1 a$$

For m_2, $\sum F_x = ma_x$:

$$-T_{12} + \mu_k n + T_{23} = -m_2 a$$

and $\sum F_y = ma_y$, giving $n - m_2 g = 0$.

For m_3, $\sum F_y = ma_y$, giving $T_{23} - m_3 g = +m_3 a$.

We have three simultaneous equations:

$$\begin{aligned} -T_{12} + 39.2\text{ N} &= (4.00\text{ kg})a \\ +T_{12} - 0.350(9.80\text{ N}) - T_{23} &= (1.00\text{ kg})a \\ +T_{23} - 19.6\text{ N} &= (2.00\text{ kg})a \end{aligned}$$

Add them up (this cancels out the tensions):

$$+39.2\text{ N} - 3.43\text{ N} - 19.6\text{ N} = (7.00\text{ kg})a$$

$$a = \boxed{2.31\text{ m/s}^2\text{, down for } m_1\text{, left for } m_2\text{, and up for } m_3}$$

(c) Now $-T_{12} + 39.2\text{ N} = (4.00\text{ kg})(2.31\text{ m/s}^2)$

$$\boxed{T_{12} = 30.0\text{ N}}$$

and $T_{23} - 19.6\text{ N} = (2.00\text{ kg})(2.31\text{ m/s}^2)$

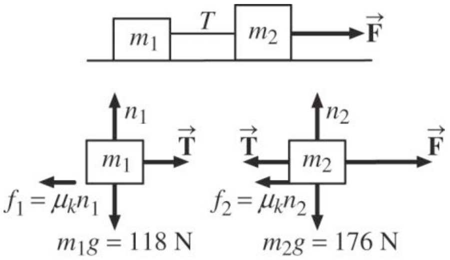
$$\boxed{T_{23} = 24.2\text{ N}}$$

(d) If the tabletop were smooth, friction disappears ($\mu_k = 0$), and so the acceleration would become larger. For a larger acceleration, according to the equations above, the tensions change:

$$T_{12} = m_1 g - m_1 a \rightarrow \boxed{T_{12} \text{ decreases}}$$

$$T_{23} = m_3 g + m_3 a \rightarrow \boxed{T_{23} \text{ increases}}$$

P5.65 Because the cord has constant length, both blocks move the same number of centimeters in each second and so move with the same acceleration. To find just this acceleration, we could model the 30-kg system as a particle under a net force. That method would not help to finding the tension, so we treat the two blocks as separate accelerating particles.



ANS. FIG. P5.65

(a) ANS. FIG. P5.65 shows the free-body diagrams for the two blocks. The tension force exerted by block 1 on block 2 is the same size as the tension force exerted by object 2 on object 1. The tension in a light string is a constant along its length, and tells how strongly the string pulls on objects at both ends.

(b) We use the free-body diagrams to apply Newton's second law.

For m_1: $\sum F_x = T - f_1 = m_1 a$ or $T = m_1 a + f_1$ **[1]**

And also $\sum F_y = n_1 - m_1 g = 0$ or $n_1 = m_1 g$

Also, the definition of the coefficient of friction gives

$$f_1 = \mu n_1 = (0.100)(12.0 \text{ kg})(9.80 \text{ m/s}^2) = 11.8 \text{ N}$$

For m_2: $\sum F_x = F - T - f_2 = ma$ **[2]**

Also from the y component, $n_2 - m_2 g = 0$ or $n_2 = m_2 g$

And again $f_2 = \mu n_2 = (0.100)(18.0 \text{ kg})(9.80 \text{ m/s}^2) = 17.6 \text{ N}$

Substituting T from equation [1] into [2], we get

$$F - m_1 a - f_1 - f_2 = m_2 a \quad \text{or} \quad F - f_1 - f_2 = m_2 a + m_1 a$$

Solving for *a*,

$$a = \frac{F - f_1 - f_2}{m_1 + m_2} = \frac{(68.0\text{ N} - 11.8\text{ N} - 17.6\text{ N})}{(12.0\text{ kg} + 18.0\text{ kg})} = \boxed{1.29\text{ m/s}^2}$$

(c) From equation [1],

$$T = m_1 a + f_1 = (12.0\text{ kg})(1.29\text{ m/s}^2) + 11.8\text{ N} = \boxed{27.2\text{ N}}$$

P5.66 (a) To find the maximum possible value of *P*, imagine impending upward motion as case 1. Setting $\sum F_x = 0$:

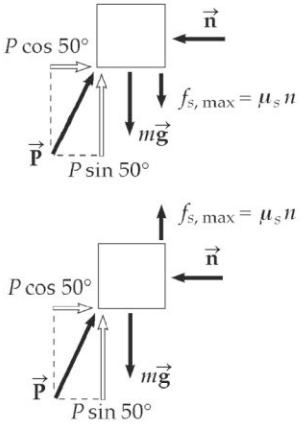
$$P\cos 50.0° - n = 0$$

with $f_{s,\max} = \mu_s n$:

$$f_{s,\max} = \mu_s P\cos 50.0°$$
$$= 0.250(0.643)P = 0.161P$$

Setting $\sum F_y = 0$:

$$P\sin 50.0° - 0.161P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$
$$P_{\max} = \boxed{48.6\text{ N}}$$



ANS. FIG. P5.66

To find the minimum possible value of *P*, consider impending downward motion. As in case 1,

$$f_{s,\max} = 0.161P$$

Setting $\sum F_y = 0$:

$$P\sin 50.0° + 0.161P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$
$$P_{\min} = \boxed{31.7\text{ N}}$$

(b) $\boxed{\text{If } P > 48.6\text{ N, the block slides up the wall. If } P < 31.7\text{ N, the block slides down the wall.}}$

(c) We repeat the calculation as in part (a) with the new angle.

Consider impending upward motion as case 1. Setting

$$\sum F_x = 0: \quad P\cos 13° - n = 0$$
$$f_{s,\,\max} = \mu_s n: \quad f_{s,\,\max} = \mu_s P\cos 13°$$
$$= 0.250(0.974)P = 0.244P$$

Setting

$$\sum F_y = 0: \quad P\sin 13° - 0.244P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$
$$P_{\max} = -1\,580\text{ N}$$

The push cannot really be negative. However large or small it is, it cannot produce upward motion. To find the minimum possible value of *P*, consider impending downward motion. As in case 1,

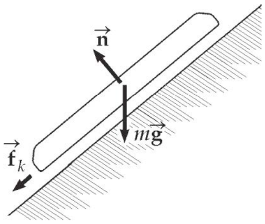
$$f_{s,\,\max} = 0.244P$$

Setting

$$\sum F_y = 0: \quad P\sin 13° + 0.244P - (3.00\text{ kg})(9.80\text{ m/s}^2) = 0$$
$$P_{\min} = \boxed{62.7\text{ N}}$$

$$\boxed{\text{P} \geq 62.7\text{ N. The block cannot slide up the wall. If P} < 62.7\text{ N, the block slides down the wall.}}$$

P5.67 We must consider separately the rock when it is in contact with the roof and when it has gone over the top into free fall. In the first case, we take *x* and *y* as parallel and perpendicular to the surface of the roof:



ANS. FIG. P5.67

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$

then friction is $f_k = \mu_k n = \mu_k mg\cos\theta$

$$\sum F_x = ma_x: \quad -f_k - mg\sin\theta = ma_x$$

$$\begin{aligned} a_x &= -\mu_k g\cos\theta - g\sin\theta = \left(-0.400\cos 37.0° - \sin 37.0°\right)(9.80\ \text{m/s}^2) \\ &= -9.03\ \text{m/s}^2 \end{aligned}$$

The rock goes ballistic with speed given by

$$\begin{aligned} v_{xf}^2 &= v_{xi}^2 + 2a_x\left(x_f - x_i\right) = \left(15\ \text{m/s}\right)^2 + 2\left(-9.03\ \text{m/s}^2\right)\left(10\ \text{m} - 0\right) \\ &= 44.4\ \text{m}^2/\text{s}^2 \\ v_{xf} &= 6.67\ \text{m/s} \end{aligned}$$

For the free fall, we take *x* and *y* horizontal and vertical:

$$\begin{aligned} v_{yf}^2 &= v_{yi}^2 + 2a_y\left(y_f - y_i\right) \\ y_f &= \frac{v_{yf}^2 - v_{yi}^2}{2a_y} \\ &= \frac{0 - \left(6.67\ \text{m/s}\ \sin 37°\right)^2}{2\left(-9.8\ \text{m/s}^2\right)} = 0.822\ \text{m above the top of the roof} \end{aligned}$$

Then $y_{\text{tot}} = 10.0\ \text{m}\ \sin 37.0° + 0.822\ \text{m} = \boxed{6.84\ \text{m}}$.

P5.68 The motion of the salmon as it breaks the surface of the water and eventually leaves must be modeled in two steps. The first is over a distance of 0.750 m, until half of the salmon is above the surface, while a constant force, *P*, is applied upward. In this motion, the initial velocity of the salmon as it nears the surface is 3.58 m/s and ends with the salmon having a velocity, $v_{½}$, when it is half out of the water. This is then the initial velocity for the second motion, where gravity is a second force to be considered acting on the fish. This motion is again over a distance of 0.750 m, and results with the salmon having a final velocity of 6.26 m/s.

The vertical motion equations, in each case, would be

$$a_{1y} = \frac{v_{1yf}^2 - v_{1yi}^2}{2\ \Delta y} = \frac{v_{1/2}^2 - \left(3.58\ \text{m/s}\right)^2}{2\ \left(0.750\ \text{m}\right)} = \frac{v_{1/2}^2 - \left(12.8\ \text{m}^2/\text{s}^2\right)}{1.50\ \text{m}}$$

and

$$a_{2y} = \frac{v_{2yf}^2 - v_{2yi}^2}{2\ \Delta y} = \frac{\left(6.26\ \text{m/s}\right)^2 - v_{1/2}^2}{2\ \left(0.750\ \text{m}\right)} = \frac{\left(39.2\ \text{m}^2/\text{s}^2\right) - v_{1/2}^2}{1.50\ \text{m}}$$

Solving for the square of the velocity in each case and equating the expressions, we find

$$v_{1/2}^2 = (1.50 \text{ m})a_{1y} + (12.8 \text{ m}^2/\text{s}^2)$$

$$v_{1/2}^2 = (39.2 \text{ m}^2/\text{s}^2) - (1.50 \text{ m})a_{2y}$$

$$(1.50 \text{ m})a_{1y} + (12.8 \text{ m}^2/\text{s}^2) = (39.2 \text{ m}^2/\text{s}^2) - (1.50 \text{ m})a_{2y}$$
$$a_{1y} = (17.6 \text{ m/s}^2) - a_{2y}$$

In the first motion, the relationship between the net acceleration and the net force can be written as

$$\sum F_y = P = ma_{1y}$$
$$P = (61.0 \text{ kg})a_{1y}$$

Substituting from above,

$$P = (61.0 \text{ kg})\left[(17.6 \text{ m/s}^2) - a_{2y}\right]$$
$$P = 1\,070 \text{ N} - (61.0 \text{ kg})a_{2y}$$

In the second motion, the relationship between the net acceleration and the net force can be written as

$$\sum F_y = P - mg = ma_{2y}$$
$$P = mg + ma_{2y} = (61.0 \text{ kg})(9.80 \text{ m/s}^2) + (61.0 \text{ kg})a_{2y}$$
$$P = 598 \text{ N} + (61.0 \text{ kg})a_{2y}$$

Equating these two equations for, *P*,

$$1\,070 \text{ N} - (61.0 \text{ kg})a_{2y} = 598 \text{ N} + (61.0 \text{ kg})a_{2y}$$
$$-(122.0 \text{ kg})a_{2y} = -472 \text{ N}$$
$$a_{2y} = 3.87 \text{ m/s}^2$$

Plugging into either of the above,

$$P = 598 \text{ N} + (61.0 \text{ kg})(3.87 \text{ m/s}^2)$$
$$P = \boxed{834 \text{ N}}$$

P5.69 Take $+x$ in the direction of motion of the tablecloth. For the mug:

$$\sum F_x = ma_x: \quad 0.1\text{ N} = 0.2\text{ kg } a_x$$
$$a_x = 0.5\text{ m/s}^2$$

Relative to the tablecloth, the acceleration of the mug is $0.500\text{ m/s}^2 - 3.00\text{ m/s}^2 = -2.50\text{ m/s}^2$. The mug reaches the edge of the tablecloth after time given by

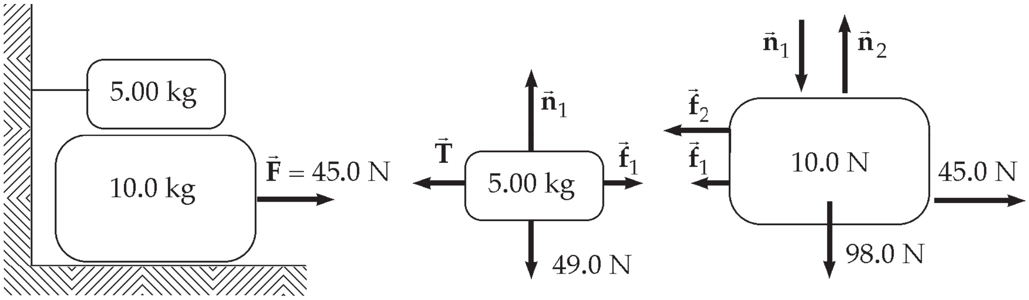
$$\Delta x = v_{xi}t + \frac{1}{2}a_x t^2$$
$$-0.300\text{ m} = 0 + \frac{1}{2}\left(-2.50\text{ m/s}^2\right)t^2$$
$$t = 0.490\text{ s}$$

The motion of the mug relative to tabletop is over distance

$$\frac{1}{2}a_x t^2 = \frac{1}{2}\left(0.500\text{ m/s}^2\right)(0.490\text{ s})^2 = \boxed{0.060\,0\text{ m}}$$

The tablecloth slides 36 cm over the table in this process.

***P5.70** (a) The free-body diagrams are shown in the figure below.



ANS. FIG. P5.70(a)

f_1 and n_1 appear in both diagrams as action-reaction pairs.

(b) For the 5.00-kg mass, Newton's second law in the y direction gives:

$$n_1 = m_1 g = (5.00\text{ kg})\left(9.80\text{ m/s}^2\right) = 49.0\text{ N}$$

In the x direction,

$$f_1 - T = 0$$

$$T = f_1 = \mu mg = 0.200(5.00\text{ kg})(9.80\text{ m/s}^2) = \boxed{9.80\text{ N}}$$

For the 10.0-kg mass, Newton's second law in the *x* direction gives:

$$45.0\text{ N} - f_1 - f_2 = (10.0\text{ kg})a$$

In the *y* direction,

$$n_2 - n_1 - 98.0\text{ N} = 0$$

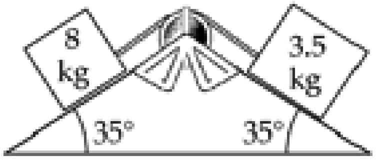
$$f_2 = \mu n_2 = \mu(n_1 + 98.0\text{ N}) = 0.20(49.0\text{ N} + 98.0\text{ N}) = 29.4\text{ N}$$

$$45.0\text{ N} - 9.80\text{ N} - 29.4\text{ N} = (10.0\text{ kg})a$$

$$a = \boxed{0.580\text{ m/s}^2}$$

***P5.71** For the right-hand block (m_1), $\sum F_1 = m_1 a$ gives

$$-m_1 g\sin 35.0° - f_{k,1} + T = m_1 a$$



ANS. FIG. P5.71

or

$$-(3.50\text{ kg})(9.80\text{ m/s}^2)\sin 35.0° - \mu_s(3.50\text{ kg})(9.80\text{ m/s}^2)\cos 35.0° + T = (3.50\text{ kg})(1.50\text{ m/s}^2) \quad \textbf{[1]}$$

For the left-hand block (m_2), $\sum F_2 = m_2 a$ gives

$$+m_2 g\sin 35.0° - f_{k,2} - T = m_2 a$$

$$+(8.00\text{ kg})(9.80\text{ m/s}^2)\sin 35.0° - \mu_s(8.00\text{ kg})(9.80\text{ m/s}^2)\cos 35.0° - T = (8.00\text{ kg})(1.50\text{ m/s}^2) \quad \textbf{[2]}$$

Solving equations [1] and [2] simultaneously gives

(a) $\boxed{\mu_k = 0.087\ 1}$

(b) $\boxed{T = 27.4\text{ N}}$

Additional Problems

P5.72 (a) Choose the black glider plus magnet as the system.

$$\sum F_x = ma_x \to +0.823\ \text{N} = (0.24\ \text{kg})a$$

$$a = \boxed{3.43\ \text{m/s}^2\ \text{toward the scrap iron}}$$

(b) The force of attraction the magnet exerts on the scrap iron is the same as in (a):

$$a_{\text{black}} = \boxed{3.43\ \text{m/s}^2\ \text{toward the scrap iron}}$$

By Newton's third law, the force the black glider exerts on the magnet is equal and opposite to the force exerted on the scrap iron:

$$\sum F_x = ma_x \to -0.823\ \text{N} = -(0.12\ \text{kg})\ a$$

$$a = \boxed{-6.86\ \text{m/s}^2\ \text{toward the magnet}}$$

P5.73 Let situation 1 be the original situation, with $\sum F_1 = m_1 a_1 = m_1(8.40\ \text{mi/h}\cdot\text{s})$. Let situation 2 be the case with larger force $\sum F_2 = (1+0.24)\sum F_1 = m_1 a_2 = 1.24 m_1 a_1$, so $a_2 = 1.24 a_1$. Let situation 3 be the case with the original force but with smaller mass:

$$\sum F_3 = \sum F_1 = m_3 a_3 = (1-0.24)m_1 a_3$$

$$a_3 = \frac{\sum F_1}{0.76 m_1} = 1.32 a_1$$

(a) With $1.32a$ greater than $1.24a_1$, $\boxed{\text{reducing the mass}}$ gives a larger increase in acceleration.

(b) Now with both changes,

$$\sum F_4 = m_4 a_4$$

$$1.24 \sum F_1 = 0.76 m_1 a_4$$

$$a_4 = \frac{1.24}{0.76}\frac{\sum F_1}{m_1} = \frac{1.24}{0.76}(8.40\ \text{mi/h}\cdot\text{s}) = \boxed{13.7\ \text{mi/h}\cdot\text{s}}$$

P5.74 Find the acceleration of the block according to the kinematic equations. The book travels through a displacement of 1.00 m in a time interval of 0.483 s. Use the equation $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$, where $\Delta x = x_f - x_i = 1.00$ m, $\Delta t = t = 0.483$ s, and $v_i = 0$:

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \rightarrow a = \frac{2\Delta x}{t^2} = 8.57\ \text{m/s}^2$$

Now, find the acceleration of the block caused by the forces. See the free-body diagram below. We take the positive *y* axis is perpendicular to the incline; the positive *x* axis is parallel and down the incline.

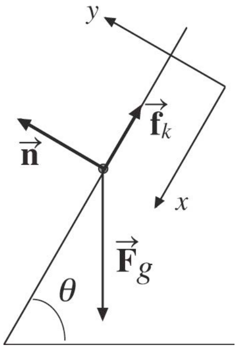
$$\sum F_y = ma_y:$$

$$n - mg\cos\theta = 0 \rightarrow n = mg\cos\theta$$

$$\sum F_x = ma_x:$$

$$mg\sin\theta - f_k = ma$$

where $f_k = \mu_k n = \mu_k mg\cos\theta$



ANS. FIG. P5.74

Substituting the express for kinetic friction into the *x*-component equation gives

$$mg\sin\theta - \mu_k mg\cos\theta = ma \rightarrow a = g(\sin\theta - \mu_k\cos\theta)$$

For $\mu_k = 0.300$, and $\theta = 60.0°$, $a = 7.02\ \text{m/s}^2$.

$\boxed{\text{The situation is impossible because these forces on the book cannot produce the acceleration described.}}$

P5.75 (a) Since the puck is on a horizontal surface, the normal force is vertical. With $a_y = 0$, we see that

$$\sum F_y = ma_y \rightarrow n - mg = 0 \quad \text{or} \quad n = mg$$

Once the puck leaves the stick, the only horizontal force is a friction force in the negative *x* direction (to oppose the motion of the puck). The acceleration of the puck is

$$a_x = \frac{\Sigma F_x}{m} = \frac{-f_k}{m} = \frac{-\mu_k n}{m} = \frac{-\mu_k(mg)}{m} = \boxed{-\mu_k g}$$

(b) Then $v_{xf}^2 = v_{xi}^2 + 2a\Delta x$ gives the horizontal displacement of the puck before coming to rest as

$$\Delta x = \frac{v_{xf}^2 - v_{xi}^2}{2a_x} = \frac{0 - v_i^2}{2(-\mu_k g)} = \boxed{\frac{v_i^2}{2\mu_k g}}$$

***P5.76** (a) Let *x* represent the position of the glider along the air track. Then $z^2 = x^2 + h_0^2$, $x = (z^2 - h_0^2)^{1/2}$, and $v_x = \frac{dx}{dt} = \frac{1}{2}(z^2 - h_0^2)^{-1/2}(2z)\frac{dz}{dt}$.

Now $\frac{dz}{dt}$ is the rate at which the string passes over the pulley, so it is equal to v_y of the counterweight.

$$v_x = z(z^2 - h_0^2)^{-1/2} v_y = uv_y$$

(b) $a_x = \frac{dv_x}{dt} = \frac{d}{dt}uv_y = u\frac{dv_y}{dt} + v_y\frac{du}{dt}$

At release from rest, $v_y = 0$ and $a_x = ua_y$.

(c) $\sin 30.0° = \frac{80.0 \text{ cm}}{z}$, $z = 1.60$ m,

$$u = (z^2 - h_0^2)^{-1/2} z = \left[(1.6 \text{ m})^2 - (0.8 \text{ m})^2\right]^{-1/2}(1.6 \text{ m}) = 1.15 \text{ m}$$

For the counterweight, $\sum F_y = ma_y$:

$$T - (0.5 \text{ kg})(9.80 \text{ m/s}^2) = -(0.5 \text{ kg})a_y$$

$$a_y = (-2 \text{ kg}^{-1})T + (9.80 \text{ m/s}^2)$$

For the glider, $\sum F_x = ma_x$:

$$\begin{aligned} T\cos 30° &= (1.00 \text{ kg})\ a_x = (1.15 \text{ kg})a_y \\ &= (1.15 \text{ kg})\left[(-2 \text{ kg}^{-1})T + 9.80 \text{ m/s}^2\right] \\ &= -2.31T + 11.3 \text{ N} \end{aligned}$$

$$3.18T = 11.3 \text{ N}$$

$$T = \boxed{3.56 \text{ N}}$$

***P5.77** When an object of mass *m* is on this frictionless incline, the only force acting parallel to the incline is the parallel component of weight, $mg\sin\theta$, directed down the incline. The acceleration is then

$$a=\frac{mg\sin\theta}{m}=g\sin\theta=\left(9.80\ \text{m/s}^2\right)\sin 35.0°=5.62\ \text{m/s}^2$$

directed down the incline.

(a) Taking up the incline as positive, the time for the sled projected up the incline to come to rest is given by

$$t=\frac{v_f-v_i}{a}=\frac{0-5.00\ \text{m/s}}{-5.62\ \text{m/s}^2}=0.890\ \text{s}$$

The distance the sled travels up the incline in this time is

$$\Delta x=v_{\text{avg}}t=\left(\frac{v_f+v_i}{2}\right)=\left(\frac{0+5.00\ \text{m/s}}{2}\right)(0.890\ \text{s})=\boxed{2.23\ \text{m}}$$

(b) The time required for the first sled to return to the bottom of the incline is the same as the time needed to go up, that is, *t* = 0.890 s. In this time, the second sled must travel down the entire 10.0-m length of the incline. The needed initial velocity is found from

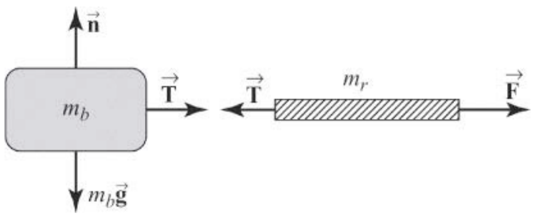
$$\Delta x=v_i t+\frac{1}{2}at^2$$

which gives

$$v_i=\frac{\Delta x}{t}-\frac{at}{2}=\frac{-10.0\ \text{m}}{0.890\ \text{s}}-\frac{\left(-5.62\ \text{m/s}^2\right)(0.890\ \text{s})}{2}=-8.74\ \text{m/s}$$

or $\boxed{8.74\ \text{m/s down the incline}}$

P5.78 (a) free-body diagrams of block and rope are shown in ANS. FIG. P5.78(a):



ANS. FIG. P5.78(a)

(b) Applying Newton's second law to the rope yields

$$\sum F_x = ma_x \quad \Rightarrow \quad F - T = m_r a \quad \text{or} \quad T = F - m_r a \qquad \textbf{[1]}$$

Then, applying Newton's second law to the block, we find

$$\sum F_x = ma_x \quad \Rightarrow T = m_b a \quad \text{or} \quad F - m_r a = m_b a$$

which gives

$$\boxed{a = \frac{F}{m_b + m_r}}$$

(c) Substituting the acceleration found above back into equation [1] gives the tension at the left end of the rope as

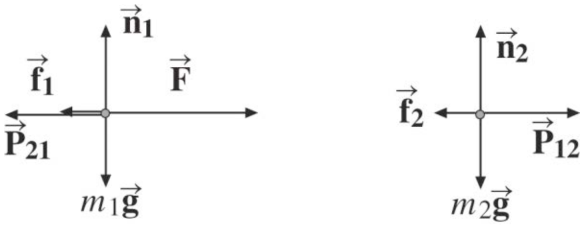
$$T = F - m_r a = F - m_r\left(\frac{F}{m_b + m_r}\right) = F\left(\frac{m_b + \cancel{m_r} - \cancel{m_r}}{m_b + m_r}\right)$$

or $$\boxed{T = \left(\frac{m_b}{m_b + m_r}\right)F}$$

(d) From the result of (c) above, we see that as m_r approaches zero, T approaches F. Thus,

$$\boxed{\text{the tension in a cord of negligible mass is constant along its length.}}$$

P5.79 (a) The free-body diagrams of the two blocks shown in ANS. FIG. P5.79(a):



ANS. FIG. P5.79(a)

Vertical forces sum to zero because the blocks move on a horizontal surface; therefore, $a_y = 0$ for each block.

$\sum F_{1y} = m_1 a_y$:

$-m_1 g + n_1 = 0 \rightarrow n_1 = m_1 g$

Kinetic friction is:

$f_1 = \mu_1 n_1 = \mu_1 m_1 g$

$\sum F_{2y} = m_2 a_y$:

$-m_2 g + n_2 = 0 \rightarrow n_2 = m_2 g$

Kinetic friction is:

$f_2 = \mu_2 n_2 = \mu_2 m_2 g$

(b) The net force on the system of the blocks would be equal to the magnitude of the force, F, minus the friction force on each block. The blocks will have the same acceleration.

(c) The net force on the mass, m_1, would be equal to the force, F, minus the friction force on m_1 and the force P_{21}, as identified in the free-body diagram.

(d) The net force on the mass, m_2, would be equal to the force, P_{12}, minus the friction force on m_2, as identified in the free-body diagram.

(e) The blocks are pushed to the right by force $\vec{\mathbf{F}}$, so kinetic friction $\vec{\mathbf{f}}$ acts on each block to the left. Each block has the same horizontal acceleration, $a_x = a$. Each block exerts an equal and opposite force on the other, so those forces have the same magnitude: $P_{12} = P_{21} = P$.

$\sum F_{1x} = m_1 a_x$:

$F - P - f_1 = m_1 a$

$\boxed{F - P - \mu_1 m_1 g = m_1 a}$

$\sum F_{2x} = m_2 a_x$:

$P - f_2 = m_2 a$

$\boxed{P - \mu_2 m_2 g = m_2 a}$

(f) Adding the above two equations of x components, we find

$$F - P - \mu_1 m_1 g + P - \mu_2 m_2 g = m_1 a + m_2 a$$

$$F - \mu_1 m_1 g - \mu_2 m_2 g = (m_1 + m_2) a \rightarrow$$

$$\boxed{a = \frac{F - \mu_1 m_1 g - \mu_2 m_2 g}{m_1 + m_2}}$$

(g) From the x component equation for block 2, we have

$$P - \mu_2 m_2 g = m_2 a \rightarrow P = \mu_2 m_2 g + m_2 a$$

$$\boxed{P = \left(\frac{m_2}{m_1 + m_2}\right)\left[F + \left(\mu_2 - \mu_1\right)m_1 g\right]}$$

We see that when the coefficients of friction are equal, $\mu_1 = \mu_2$, the magnitude P is independent of friction.

P5.80 (a) The cable does not stretch: Whenever one car moves 1 cm, the other moves 1 cm.

$$\boxed{\text{At any instant they have the same velocity and at all instants they have the same acceleration.}}$$

(b) Consider the BMW as the object.

$$\sum F_y = ma_y:$$
$$+T - mg = ma$$
$$+T - (1\ 461\text{ kg})(9.80\text{ m/s}^2) = \left(1\ 461\text{ kg}\right)(1.25\text{ m/s}^2)$$

$$T = \boxed{1.61 \times 10^4\text{ N}}$$

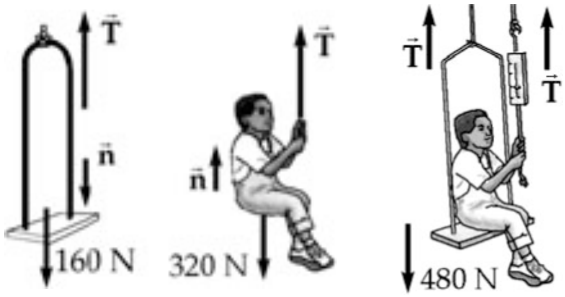
(c) Consider both cars as the object.

$$\sum F_y = ma_y:$$
$$+T - \left(m + M\right)g = \left(m + M\right)a$$
$$+T - (1\ 461\text{ kg} + 1\,207\text{ kg})(9.80\text{ m/s}^2)$$
$$= (1\ 461\text{ kg} + 1\,207\text{ kg})(1.25\text{ m/s}^2)$$

$$T_{\text{above}} = \boxed{2.95 \times 10^4\text{ N}}$$

P5.81 (a) ANS. FIG. P5.81(a) shows the free-body diagrams for this problem.

Note that the same-size force n acts up on Nick and down on chair, and cancels out in the diagram. The same-size force T = 250 N acts up on Nick and up on chair, and appears twice in the diagram.



ANS. FIG. P5.81(a)

(b) First consider Nick and the chair together as the system. Note that **two** ropes support the system, and $T = 250$ N in each rope.

Applying $\sum F = ma$, $\quad 2T - (160\text{ N} + 320\text{ N}) = ma$

where $\quad m = \dfrac{480\text{ N}}{9.80\text{ m/s}^2} = 49.0\text{ kg}$

Solving for *a* gives $\quad a = \dfrac{(500 - 480)\text{ N}}{49.0\text{ kg}} = \boxed{0.408\text{ m/s}^2}$

(c) On Nick, we apply

$$\sum F = ma: \qquad n + T - 320\text{ N} = ma$$

where $\quad m = \dfrac{320\text{ N}}{9.80\text{ m/s}^2} = 32.7\text{ kg}$

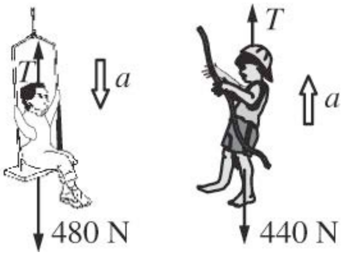
The normal force is the one remaining unknown:

$$n = ma + 320\text{ N} - T$$

Substituting, $\quad n = (32.7\text{ kg})\left(0.408\text{ m/s}^2\right) + 320\text{ N} - 250\text{ N}$

gives $\quad n = \boxed{83.3\text{ N}}$

P5.82 See ANS. FIG. P5.82 showing the free-body diagrams. The rope has tension *T*.



ANS. FIG. P5.82

(a) As soon as Nick passes the rope to the other child,

$$\boxed{\text{Nick and the seat, with total weight 480 N, will accelerate down and the child, with smaller weight 440 N, will accelerate up.}}$$

On Nick and the seat,

$$\sum F_y = +480\text{ N} - T = \frac{480\text{ N}}{9.80\text{ m/s}^2}a$$

On the child,

$$\sum F_y = +T - 440\text{ N} = \frac{440\text{ N}}{9.80\text{ m/s}^2}a$$

Adding,

$$+480\text{ N} - T + T - 440\text{ N} = (49.0\text{ kg} + 44.9\text{ kg})a$$

$$a = \frac{40\text{ N}}{93.9\text{ kg}} = \boxed{0.426\text{ m/s}^2 = a}$$

The rope tension is $T = 440\text{ N} + (44.9\text{ kg})(0.426\text{ m/s}^2) = 459\text{ N}$.

(b) The rope must support Nick and the seat, so the rope tension is 480 N.

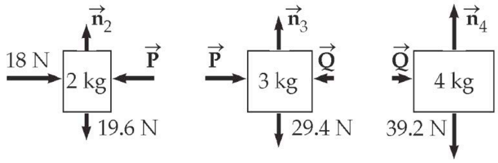
$$\boxed{\text{In problem 81, a rope tension of 250 N does not make the rope break. In part (a), the rope is strong enough to support tension 459 N. But now the tension everywhere in the rope is 480 N, so it can exceed the breaking strength of the rope.}}$$

The tension in the chain supporting the pulley is 480 N + 480 N = 960 N, so the chain may break first.

P5.83 (a) See free-body diagrams in ANS. FIG. P5.83.

(b) We write $\sum F_x = ma_x$ for each object.

$$18\text{ N} - P = (2\text{ kg})a$$
$$P - Q = (3\text{ kg})a$$
$$Q = (4\text{ kg})a$$



ANS. FIG. P5.83

Adding gives

$$18\text{ N} = (9\text{ kg})a \to a = \boxed{2.00\text{ m/s}^2}$$

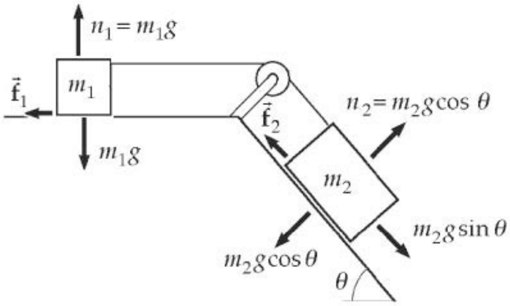
(c) The resultant force on any object is $\sum \vec{F} = m\vec{a}$: All have the same acceleration:

$$\sum \vec{F} = (4\text{ kg})(2\text{ m/s}^2) = \boxed{8.00\text{ N on the 4-kg object}}$$
$$\sum \vec{F} = (3\text{ kg})(2\text{ m/s}^2) = \boxed{6.00\text{ N on the 3-kg object}}$$
$$\sum \vec{F} = (2\text{ kg})(2\text{ m/s}^2) = \boxed{2.00\text{ N on the 2-kg object}}$$

(d) From above, $P = 18\text{ N} - (2\text{ kg})a \to \boxed{P = 14.0\text{ N}}$, and $Q = (4\text{ kg})a \to \boxed{Q = 8.00\text{ N}}$.

(e) Introducing the heavy block reduces the acceleration because the mass of the system (plasterboard-heavy block-you) is greater. The 3-kg block models the heavy block of wood. The contact force on your back is represented by Q, which is much less than the force F. The difference between F and Q is the net force causing acceleration of the 5-kg pair of objects.

P5.84 (a) For the system to start to move when released, the force tending to move m_2 down the incline, $m_2 g \sin\theta$, must exceed the maximum friction force which can retard the motion:



ANS. FIG. P5.84

$$f_{\text{max}} = f_{1,\text{max}} + f_{2,\text{max}} = \mu_{s,1} n_1 + \mu_{s,2} n_2$$
$$f_{\text{max}} = \mu_{s,1} m_1 g + \mu_{s,2} m_2 g \cos\theta$$

From the table of coefficients of friction in the text, we take $\mu_{s,1} = 0.610$ (aluminum on steel) and $\mu_{s,2} = 0.530$ (copper on steel). With

$$m_1 = 2.00 \text{ kg}, m_2 = 6.00 \text{ kg}, \theta = 30.0°$$

the maximum friction force is found to be $f_{\max} = 38.9$ N. This exceeds the force tending to cause the system to move,

$m_2 g \sin\theta = 6.00 \text{ kg}\left(9.80 \text{ m/s}^2\right)\sin 30° = 29.4 \text{ N}$. Hence,

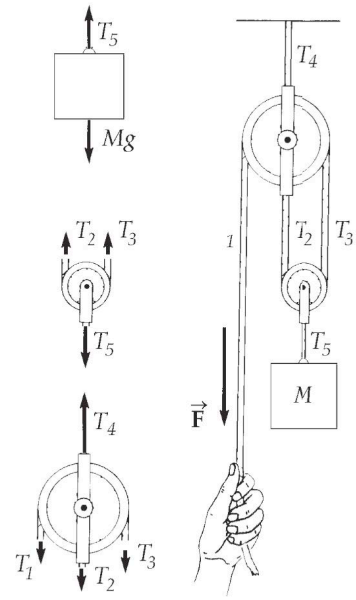
$$\boxed{\text{the system will not start to move when released}}$$

(b) and (c) $\boxed{\text{No answer because the blocks do not move.}}$

(d) The friction forces increase in magnitude until the total friction force retarding the motion, $f = f_1 + f_2$, equals the force tending to set the system in motion. That is, until

$$\boxed{f = m_2 g \sin\theta = 29.4 \text{ N}}$$

P5.85 (a) See ANS. FIG. P5.85 showing the forces. All forces are in the vertical direction. The lifting can be done at constant speed, with zero acceleration and total force zero on each object.



ANS. FIG. 5.85

(b) For M, $\sum F = 0 = T_5 - Mg$

so $T_5 = Mg$

Assume frictionless pulleys. The tension is constant throughout a light, continuous rope. Therefore, $T_1 = T_2 = T_3$.

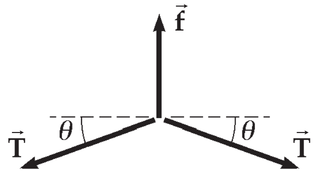
For the bottom pulley,

$$\sum F = 0 = T_2 + T_3 - T_5$$

so $2T_2 = T_5$. Then $\boxed{T_1 = T_2 = T_3 = \frac{Mg}{2}}$, $\boxed{T_4 = \frac{3Mg}{2}}$, and $\boxed{T_5 = Mg}$.

(c) Since $F = T_1$, we have $\boxed{F = \frac{Mg}{2}}$.

***P5.86** (a) Consider forces on the midpoint of the rope. It is nearly in equilibrium just before the car begins to move. Take the *y* axis in the direction of the force you exert:



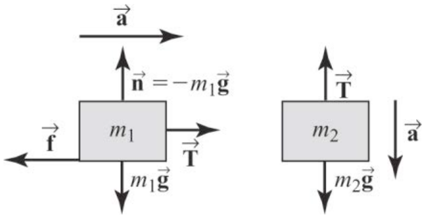
ANS. FIG. P5.86

$$\sum F_y = ma_y: \qquad -T\sin\theta + f - T\sin\theta = 0$$

$$\boxed{T = \frac{f}{2\sin\theta}}$$

(b) $T = \dfrac{100\ \text{N}}{2\sin 7°} = \boxed{410\ \text{N}}$

***P5.87** The acceleration of the system is found from



ANS. FIG. 5.87

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$

since $v_{yi} = 0$, we obtain

$$a = \frac{2\Delta y}{t^2} = \frac{2(1.00\ \text{m})}{(1.20\ \text{s})^2} = 1.39\ \text{m/s}^2$$

Using the free-body diagram for m_2, Newton's second law gives

$$\begin{aligned} &\sum F_{y2} = m_2 a: \\ &\quad m_2 g - T = m_2 a \\ &\quad T = m_2(g - a) \\ &\quad\quad = (5.00\ \text{kg})(9.80\ \text{m/s}^2 - 1.39\ \text{m/s}^2) \\ &\quad\quad = 42.1\ \text{N} \end{aligned}$$

Then, applying Newton's second law to the horizontal motion of m_1,

$$\begin{aligned} &\sum F_{x1} = m_1 a: \\ &\quad T - f = m_1 a \\ &\quad f = T - m_1 a \\ &\quad\quad = 42.1\ \text{N} - (10.0\ \text{kg})(1.39\ \text{m/s}^2) = 28.2\ \text{N} \end{aligned}$$

Since $n = m_1 g = 98.0$ N, we have

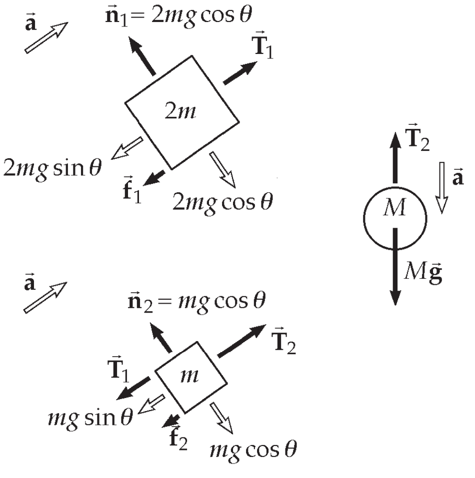
$$\mu_k = \frac{f}{n} = \frac{28.2\text{ N}}{98.0\text{ N}} = \boxed{0.288}$$

***P5.88** Applying Newton's second law to each object gives:

$$T_1 = f_1 + 2m(g\sin\theta + a) \qquad \textbf{[1]}$$

$$T_2 - T_1 = f_2 + m(g\sin\theta + a) \qquad \textbf{[2]}$$

$$T_2 = M(g - a) \qquad \textbf{[3]}$$



ANS. FIG. P5.88

(a), (b) Assuming that the system is in equilibrium ($a = 0$) and that the incline is frictionless, ($f_1 = f_2 = 0$), the equations reduce to

$$\boxed{T_1 = 2mg\sin\theta} \qquad \textbf{[1']}$$

$$T_2 - T_1 = mg\sin\theta \qquad \textbf{[2']}$$

$$T_2 = Mg \qquad \textbf{[3']}$$

Substituting [1′] and [3′] into equation [2′] then gives

$$\boxed{M = 3m\sin\theta}$$

so equation [3′] becomes $\boxed{T_2 = 3mg\sin\theta}$

(c), (d) $M = 6m\sin\theta$ (double the value found above), and $f_1 = f_2 = 0$. With these conditions present, the equations become $T_1 = 2m(g\sin\theta + a)$, $T_2 - T_1 = m(g\sin\theta + a)$ and $T_2 = 6m\sin\theta(g - a)$. Solved simultaneously, these yield

$$\boxed{a = \frac{g\sin\theta}{1 + 2\sin\theta}},\ \boxed{T_1 = 4mg\sin\theta\left(\frac{1 + \sin\theta}{1 + 2\sin\theta}\right)} \text{ and}$$

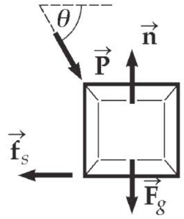
$$\boxed{T_2 = 6mg\sin\theta\left(\frac{1 + \sin\theta}{1 + 2\sin\theta}\right)}$$

(e) Equilibrium ($a = 0$) and impending motion **up** the incline, so $\mathrm{M} = \mathrm{M}_{\max}$ while $f_1 = 2\mu_s mg\cos\theta$ and $f_2 = \mu_s mg\cos\theta$, both directed **down** the incline. Under these conditions, the equations become $T_1 = 2mg(\sin\theta + \mu_s\cos\theta)$, $T_2 - T_1 = mg(\sin\theta + \mu_s\cos\theta)$, and $T_2 = M_{\max}g$, which yield $\boxed{M_{\max} = 3m(\sin\theta + \mu_s\cos\theta)}$.

(f) Equilibrium ($a = 0$) and impending motion **down** the incline, so $\mathrm{M} = \mathrm{M}_{\min}$, while $f_1 = 2\mu_s mg\cos\theta$ and $f_2 = \mu_s mg\cos\theta$, both directed **up** the incline. Under these conditions, the equations are $T_1 = 2mg(\sin\theta - \mu_s\cos\theta)$, $T_2 - T_1 = mg(\sin\theta - \mu_s\cos\theta)$, and $T_2 = M_{\min}g$, which yield $\boxed{M_{\min} = 3m(\sin\theta - \mu_s\cos\theta)}$. When this expression gives a negative value, it corresponds physically to a mass *M* hanging from a cord over a pulley at the bottom end of the incline.

(g) $\boxed{T_{2,\max} - T_{2,\min} = M_{\max}g - M_{\min}g = 6\mu_s mg\cos\theta}$

P5.89 (a) The crate is in equilibrium, just before it starts to move. Let the normal force acting on it be *n* and the friction force, f_s.



ANS. FIG. P5.89

Resolving vertically: $\sum F_y = ma_y$ gives

$$n = F_g + P\sin\theta$$

Horizontally, $\sum F_x = ma_x$ gives

$$P\cos\theta = f$$

But,

$$f_s \le \mu_s n$$

i.e.,

$$P\cos\theta \le \mu_s\left(F_g + P\sin\theta\right)$$

or

$$P\left(\cos\theta - \mu_s\sin\theta\right) \le \mu_s F_g$$

Divide by $\cos\theta$:

$$P\left(1 - \mu_s\tan\theta\right) \le \mu_s F_g\sec\theta$$

Then

$$\boxed{P_{\text{minimum}} = \frac{\mu_s F_g \sec\theta}{1-\mu_s \tan\theta}}$$

(b) To set the crate into motion, the x component (P cos θ) must overcome friction $f_s = \mu_s n$:

$$P\cos\theta \ge \mu_s n = \mu_s\left(F_g + P\sin\theta\right)$$

$$P\left(\cos\theta - \mu_s\sin\theta\right) \ge \mu_s F_g$$

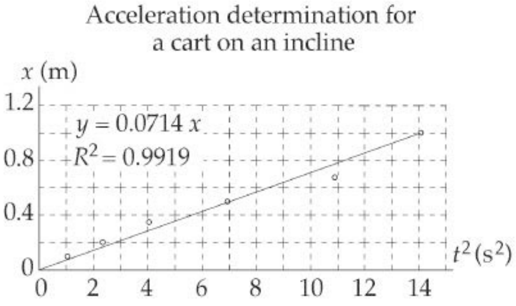
For this condition to be satisfied, it must be true that

$$\left(\cos\theta - \mu_s\sin\theta\right) > 0 \rightarrow \mu_s\tan\theta < 1 \rightarrow \tan\theta < \frac{1}{\mu_s}$$

If this condition is not met, no value of P can move the crate.

P5.90 (a) See table below and graph in ANS. FIG. P5.90(a).

t(s)	t^2(s^2)	x(m)
0	0	0
1.02	1.04 0	0.100
1.53	2.34 1	0.200
2.01	4.04 0	0.350
2.64	6.97 0	0.500
3.30	10.89	0.750
3.75	14.06	1.00



ANS. FIG. P5.90(a)

(b) From $x = \frac{1}{2}at^2$ the slope of a graph of x versus t^2 is $\frac{1}{2}a$, and

$$a = 2 \times \text{slope} = 2\left(0.071\,4\ \text{m/s}^2\right) = \boxed{0.143\ \text{m/s}^2}$$

(c) From $a' = g\sin\theta$,

$$a' = 9.80\ \text{m/s}^2\left(\frac{1.77\,4}{127.1}\right) = 0.137\ \text{m/s}^2\text{, different by 4\%.}$$

The difference is accounted for by the uncertainty in the data, which we may estimate from the third point as

$$\frac{0.350 - (0.071\,4)(4.04)}{0.350} = 18\%$$

$\boxed{\text{Thus the acceleration values agree.}}$

P5.91 (a) The net force on the cushion is in a fixed direction, downward and forward making angle $\theta = \tan^{-1}(F/mg)$ with the vertical.

Because the cushion starts from rest, the direction of its line of motion will be the same as that of the net force.

We show the path is a straight line another way. In terms of a standard coordinate system, the x and y coordinates of the cushion are

$$y = h - \frac{1}{2}gt^2$$

$$x = \frac{1}{2}(F/m)t^2 \rightarrow t^2 = (2m/F)x$$

Substitution of t^2 into the equation for y gives

$$y = h - (mg/f)x$$

which is an equation for a straight line.

(b) Because the cushion starts from rest, it will move in the direction of the net force which is the direction of its acceleration; therefore, it will move with increasing speed and its velocity changes in magnitude.

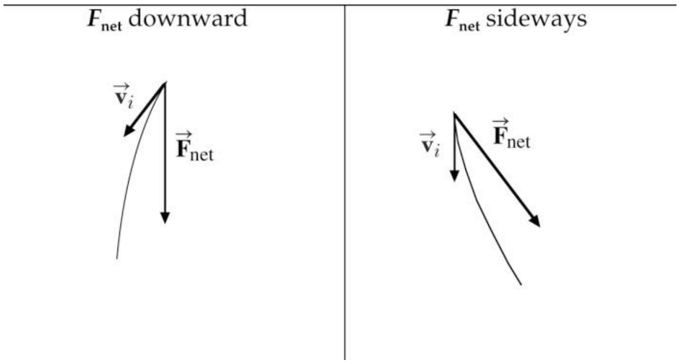
(c) Since the line of motion is in the direction of the net force, they both make the same angle with the vertical. Refer to Figure P5.91 in the textbook: in terms of a right triangle with angle θ, height h, and base x,

$$\tan\theta = x/h = F/mg \rightarrow x = hF/mg$$
$$x = \frac{(8.00\text{ m})(2.40\text{ N})}{(1.20\text{ kg})(9.80\text{ m/s}^2)}$$

and the cushion will land a distance

$x = \boxed{\text{1.63 m from the base of the building}}$.

(d) The cushion will move along a tilted parabola. If the cushion were experiencing a constant net force directed vertically downward (as is normal with gravity), and if its initial velocity were down and somewhat to the left, the trajectory would have the shape of a parabola that we would expect for projectile motion. Because the constant net force is "sideways"—at an angle θ counterclockwise from the vertical—the cushion would travel a similar trajectory as described above, but rotated counterclockwise by the angle θ so that the initial velocity is directed downward. See the figures.



ANS. FIG. P5.91(d)

P5.92 (a) When block 2 moves down 1 cm, block 1 moves 2 cm forward, so block 1 always has twice the speed of block 2, and $\boxed{a_1 = 2a_2}$ relates the magnitudes of the accelerations.

(b) Let T represent the uniform tension in the cord.

For block 1 as object,

$$\sum F_x = m_1a_1: \quad T = m_1a_1 = m_1(2a_2)$$
$$T = 2m_1a_2 \qquad [1]$$

For block 2 as object,

$$\sum F_y = m_2 a_2: \quad T + T - m_2 g = m_2\left(-a_2\right)$$

$$2T - m_2 g = -m_2 a_2 \tag*{[2]}$$

To solve simultaneously we substitute equation [1] into equation [2]:

$$2\left(2m_1 a_2\right) - m_2 g = -m_2 a_2 \rightarrow 4m_1 a_2 + m_2 a_2 = m_2 g$$

$$a_2 = \frac{m_2 g}{4m_1 + m_2}$$

for $m_2 = 1.30$ kg: $\boxed{a_2 = 12.7\ \text{N}\ (1.30\ \text{kg} + 4\ m_1)^{-1}\ \text{down}}$

(c) If m_1 is very much less than 1.30 kg, a_2 approaches

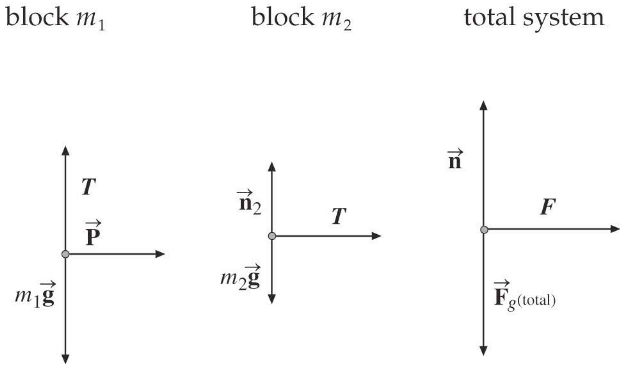
$$12.7\ \text{N}/1.30\ \text{kg} = \boxed{9.80\ \text{m/s}^2\ \text{down}}$$

(d) If m_1 approaches infinity, $\boxed{a_2\ \text{approaches zero}}$.

(e) From equation (2) above, $2T = m_2 g + m_2 a_2 = 12.74\ \text{N} + 0$, $\boxed{T = 6.37\ \text{N}}$

(f) $\boxed{\text{Yes. As } m_1 \text{ approaches zero, block 2 is essentially in free fall. As } m_2 \text{ becomes negligible compared to } m_1,\ m_2 \text{ has very little weight, so the system is nearly in equilibrium.}}$

P5.93 We will use $\sum F = ma$ on each object, so we draw force diagrams for the $M + m_1 + m_2$ system, and also for blocks m_1 and m_2. Remembering that normal forces are always perpendicular to the contacting surface, and always **push** on a body, draw n_1 and n_2 as shown. Note that m_1 is in contact with the cart, and therefore feels a normal force exerted by the cart. Remembering that ropes always **pull** on



ANS. FIG. P5.93

bodies toward the center of the rope, draw the tension force $\vec{\mathbf{T}}$. Finally, draw the gravitational force on each block, which always points downwards.

Applying $\sum F = ma$,

For m_1: $T - m_1 g = 0$

For m_2: $T = m_2 a$

Eliminating T,

$$a = \frac{m_1 g}{m_2}$$

For all three blocks:

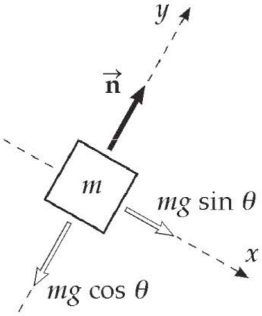
$$\boxed{F = \left(M + m_1 + m_2\right)\frac{m_1 g}{m_2}}$$

P5.94 (a) $\sum F_y = ma_y$:

$$n - mg\cos\theta = 0$$

or $n = (8.40\ \text{kg})(9.80\ \text{m/s}^2)\cos\theta$

$$\boxed{n = (82.3\ \text{N})\cos\theta}$$



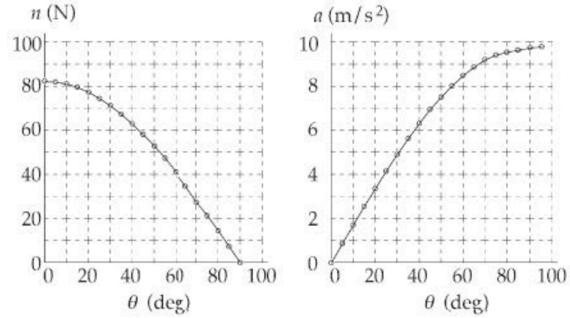
(b) $\sum F_x = ma_x$:

$$mg\sin\theta = ma$$

or

$$a = g\sin\theta$$

$$\boxed{a = (9.80\ \text{m/s}^2)\sin\theta}$$



ANS. FIG. P5.94

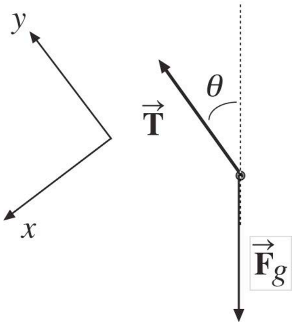
(c)

θ, deg	n, N	a, m/s^2
0.00	82.3	0.00
5.00	82.0	0.854
10.0	81.1	1.70
15.0	79.5	2.54
20.0	77.4	3.35
25.0	74.6	4.14
30.0	71.3	4.90
35.0	67.4	5.62
40.0	63.1	6.30
45.0	58.2	6.93
50.0	52.9	7.51
55.0	47.2	8.03
60.0	41.2	8.49
65.0	34.8	8.88
70.0	28.2	9.21
75.0	21.3	9.47
80.0	14.3	9.65
85.0	7.17	9.76
90.0	0.00	9.80

(d) At 0°, the normal force is the full weight and the acceleration is zero. At 90°, the mass is in free fall next to the vertical incline.

P5.95 Refer to the free-body diagram in ANS. FIG. P5.95. Choose the x axis pointing down the slope so that the string makes the angle θ with the vertical. The acceleration is obtained from $v_f = v_i + at$:

$$a = (v_f - v_i)/t = (30.0\text{ m/s}^2 - 0)/6.00\text{ s}$$
$$a = 5.00\text{ m/s}^2$$



ANS. FIG. P5.95

Because the string stays perpendicular to the ceiling, we know that the toy moves with the same acceleration as the van, 5.00 m/s^2 parallel to the hill. We take the x axis in this direction, so

$$a_x = 5.00\text{ m/s}^2 \quad\text{and}\quad a_y = 0$$

The only forces on the toy are the string tension in the y direction and the planet's gravitational force, as shown in the force diagram. The size of the latter is $mg = (0.100\text{ kg})(9.80\text{ m/s}^2) = 0.980\text{ N}$

(a) Using $\sum F_x = ma_x$ gives $(0.980\text{ N})\sin\theta = (0.100\text{ kg})(5.00\text{ m/s}^2)$

Then $\sin\theta = 0.510$ and $\theta = \boxed{30.7°}$

(b) Using $\sum F_y = ma_y$ gives $+T - (0.980\text{ N})\cos\theta = 0$

$T = (0.980\text{ N})\cos 30.7° = \boxed{0.843\text{ N}}$

Challenge Problems

P5.96 $\sum\vec{F} = m\vec{a}$ gives the object's acceleration:

$$\vec{\mathbf{a}} = \frac{\sum\vec{\mathbf{F}}}{m} = \frac{\left(8.00\hat{\mathbf{i}} - 4.00t\hat{\mathbf{j}}\right)\text{ N}}{2.00\text{ kg}}$$
$$\vec{\mathbf{a}} = \left(4.00\text{ m/s}^2\right)\hat{\mathbf{i}} - \left(2.00\text{ m/s}^3\right)t\hat{\mathbf{j}} = \frac{d\vec{\mathbf{v}}}{dt}$$

(a) To arrive at an equation for the instantaneous velocity of object, we must integrate the above equation.

$$\begin{aligned}
d\vec{\mathbf{v}} &= \left(4.00\ \text{m/s}^2\right)dt\hat{\mathbf{i}} - \left(2.00\ \text{m/s}^3\right)t\,dt\hat{\mathbf{j}} \\
\int d\vec{\mathbf{v}} &= \int\left(4.00\ \text{m/s}^2\right)dt\hat{\mathbf{i}} - \int\left(2.00\ \text{m/s}^3\right)t\,dt\hat{\mathbf{j}} \\
\vec{\mathbf{v}} &= \left[\left(4.00\ \text{m/s}^2\right)t + c_1\right]\hat{\mathbf{i}} - \left[\left(1.00\ \text{m/s}^3\right)t^2 + c_2\right]\hat{\mathbf{j}}
\end{aligned}$$

In order to evaluate the constants of integration, we observe that the object is at rest when $t = 0$ s.

$$\vec{\mathbf{v}}(t=0) = 0 = \left[\left(4.00\ \text{m/s}^2\right)0 + c_1\right]\hat{\mathbf{i}} - \left[\left(1.00\ \text{m/s}^3\right)0^2 + c_2\right]\hat{\mathbf{j}}$$

or $c_1 = c_2 = 0$

and

$$\vec{\mathbf{v}} = \left[\left(4.00\ \text{m/s}^2\right)t\right]\hat{\mathbf{i}} - \left[\left(1.00\ \text{m/s}^3\right)t^2\right]\hat{\mathbf{j}}$$

Thus, when $v = 15.0$ m/s,

$$\begin{aligned}
|\vec{\mathbf{v}}| = 15.0\ \text{m/s} &= \sqrt{\left[\left(4.00\ \text{m/s}^2\right)t\right]^2 + \left[\left(1.00\ \text{m/s}^3\right)t^2\right]^2} \\
15.0\ \text{m/s} &= \sqrt{\left[\left(16.0\ \text{m}^2/\text{s}^4\right)t^2\right] + \left[\left(1.00\ \text{m}^2/\text{s}^6\right)t^4\right]} \\
225\ \text{m}^2/\text{s}^2 &= \left[\left(16.0\ \text{m}^2/\text{s}^4\right)t^2\right] + \left[\left(1.00\ \text{m}^2/\text{s}^6\right)t^4\right] \\
0 &= \left(1.00\ \text{m}^2/\text{s}^6\right)t^4 + \left(16.0\ \text{m}^2/\text{s}^4\right)t^2 - 225\ \text{m}^2/\text{s}^2
\end{aligned}$$

We now need a solution to the above equation, in order to find t. The equation can be factored as,

$$0 = \left(t^2 - 9\right)\left(t^2 + 25\right)$$

The solution for t, here, comes from the first factor:

$$\begin{aligned}
t^2 - 9.00 &= 0 \\
t &= \pm 3.00\ \text{s} = \boxed{3.00\ \text{s}}
\end{aligned}$$

(b) In order to find the object's position at this time, we need to integrate the velocity equation, using the assumption that the objects starts at the origin (the constants of integration will again be equal to 0, as before).

$$d\vec{\mathbf{r}} = \left(4.00\ \text{m/s}^2\right)t\,dt\,\hat{\mathbf{i}} - \left(1.00\ \text{m/s}^3\right)t^2 dt\,\hat{\mathbf{j}}$$
$$\int d\vec{\mathbf{r}} = \int\left(4.00\ \text{m/s}^2\right)t\,dt\,\hat{\mathbf{i}} - \int\left(1.00\ \text{m/s}^3\right)t^2\,dt\,\hat{\mathbf{j}}$$
$$\vec{\mathbf{r}} = \left[\left(2.00\ \text{m/s}^2\right)t^2\right]\hat{\mathbf{i}} - \left[\left(0.333\ \text{m/s}^3\right)t^3\right]\hat{\mathbf{j}}$$

Now, using the time above and finding the magnitude of this displacement vector,

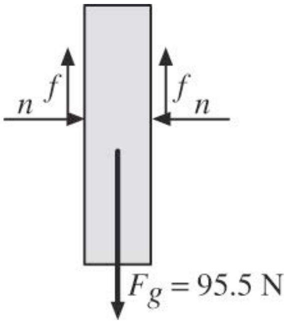
$$|\vec{\mathbf{r}}| = \sqrt{\left[\left(2.00\ \text{m/s}^2\right)(3.00\ \text{s})^2\right]^2 + \left[\left(0.333\ \text{m/s}^3\right)(3.00\ \text{s})^3\right]^2}$$
$$|\vec{\mathbf{r}}| = \boxed{20.1\ \text{m}}$$

(c) Using the displacement vector found in part (b),

$$\vec{\mathbf{r}} = \left[\left(2.00\ \text{m/s}^2\right)t^2\right]\hat{\mathbf{i}} - \left[\left(0.333\ \text{m/s}^3\right)t^3\right]\hat{\mathbf{j}}$$
$$\vec{\mathbf{r}} = \left[\left(2.00\ \text{m/s}^2\right)(3.00\ \text{s})^2\right]\hat{\mathbf{i}} - \left[\left(0.333\ \text{m/s}^3\right)(3.00\ \text{s})^3\right]\hat{\mathbf{j}}$$
$$\vec{\mathbf{r}} = \boxed{(18.0\ \text{m})\hat{\mathbf{i}} - (9.00\ \text{m})\hat{\mathbf{j}}}$$

P5.97 Since the board is in equilibrium, $\sum F_x = 0$ and we see that the normal forces must be the same on both sides of the board. Also, if the minimum normal forces (compression forces) are being applied, the board is on the verge of slipping and the friction force on each side is

$$f = \left(f_s\right)_{\max} = \mu_s n$$



ANS. FIG. P5.97

The board is also in equilibrium in the vertical direction, so

$$\sum F_y = 2f - F_g = 0 \text{, or } f = \frac{F_g}{2}$$

The minimum compression force needed is then

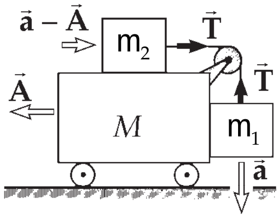
$$n = \frac{f}{\mu_s} = \frac{F_g}{2\mu_s} = \frac{95.5\text{ N}}{2(0.663)} = \boxed{72.0\text{ N}}$$

***P5.98** We apply Newton's second law to each of the three masses, reading the forces from ANS. FIG. P5.98:

$$m_2(a - A) = T \Rightarrow \quad a = \frac{T}{m_2} + A \qquad \textbf{[1]}$$

$$MA = R_x = T \Rightarrow \quad A = \frac{T}{M} \qquad \textbf{[2]}$$

$$m_1 a = m_1 g - T \Rightarrow \quad T = m_1(g - a) \qquad \textbf{[3]}$$



ANS. FIG. P5.98

(a) Substitute the value for *a* from [1] into [3] and solve for *T*:

$$T = m_1\left[g - \left(\frac{T}{m_2} + A\right)\right]$$

Substitute for *A* from [2]:

$$T = m_1\left[g - \left(\frac{T}{m_2} + \frac{T}{M}\right)\right] \Rightarrow T = \boxed{m_2 g\left[\frac{m_1 M}{m_2 M + m_1(m_2 + M)}\right]}$$

(b) Solve [3] for *a* and substitute value of *T*:

$$a = g - \frac{T}{m_1} = g - m_2 g\left[\frac{M}{m_2 M + m_1(m_2 + M)}\right]$$

$$= g\left[1 - \frac{m_2 M}{m_2 M + m_1(m_2 + M)}\right]$$

$$= \boxed{\left[\frac{g m_1(m_2 + M)}{m_2 M + m_1(m_2 + M)}\right]}$$

(c) From [2], $A = \frac{T}{M}$. Substitute the value of *T*:

$$A = \frac{T}{M} = \boxed{\left[\frac{m_1 m_2 g}{m_2 M + m_1(m_2 + M)}\right]}$$

(d) The acceleration of m_1 is given by

$$a - A = \boxed{\left[\frac{m_1 M g}{m_2 M + m_1(m_2 + M)}\right]}$$

P5.99 (a) The cord makes angle θ with the horizontal where

$$\theta = \tan^{-1}\left(\frac{0.100 \text{ m}}{0.400 \text{ m}}\right) = 14.0°$$

Applying Newton's second law in the *y* direction gives

$$\sum F_y = ma_y:$$

$$T\sin\theta - mg + n = 0$$

$$(+10 \text{ N})\sin 14.0° - (2.20 \text{ kg})(9.80 \text{ m/s}^2) + n = 0$$

which gives $n = 19.1$ N. Applying Newton's second law in the *x* direction then gives

$$\sum F_x = ma_x:$$

$$T\cos\theta - f_k = ma$$

$$T\cos\theta - \mu_k n = ma$$

$$(+10 \text{ N})\cos 14.0° - 0.400(19.1 \text{ N}) = (2.20 \text{ kg})\, a$$

which gives

$$a = \boxed{0.931 \text{ m/s}^2}$$

(b) $\boxed{\begin{array}{l}\text{When } x \text{ is large we have } n = 21.6\text{ N, } f_k = 8.62\text{ N, and} \\ a = (10\text{ N} - 8.62\text{ N})/2.2\text{ kg} = 0.625\text{ m/s}^2. \\ \text{As } x \text{ decreases, the acceleration increases gradually, passes} \\ \text{through a maximum, and then drops more rapidly, becoming} \\ \text{negative. At } x = 0 \text{ it reaches the value } a = [0 - 0.4(21.6\text{ N} - 10\text{ N})]/ \\ 2.2\text{ kg} = -2.10\text{ m/s}^2.\end{array}}$

(c) We carry through the same calculations as in part (a) for a variable angle, for which $\cos\theta = x[x^2 + (0.100\text{ m})^2]^{-1/2}$ and $\sin\theta = (0.100\text{ m})[x^2 + (0.100\text{ m})^2]^{-1/2}$. We find

$$a = \left(\frac{1}{2.20\text{ kg}}\right)(10\text{ N})x\left[x^2 + 0.100^2\right]^{-1/2}$$

$$-0.400\left(21.6\text{ N} - (10\text{ N})(0.100)\left[x^2 + 0.100^2\right]^{-1/2}\right)$$

$$a = 4.55x\left[x^2 + 0.100^2\right]^{-1/2} - 3.92 + 0.182\left[x^2 + 0.100^2\right]^{-1/2}$$

Now to maximize a we take its derivative with respect to x and set it equal to zero:

$$\frac{da}{dx} = 4.55\left(x^2 + 0.100^2\right)^{-1/2} + 4.55x\left(-\frac{1}{2}\right)2x\left(x^2 + 0.100^2\right)^{-3/2}$$

$$+0.182\left(-\frac{1}{2}\right)2x\left(x^2 + 0.100^2\right)^{-3/2} = 0$$

Solving,

$$4.55\left(x^2 + 0.1^2\right) - 4.55x^2 - 0.182x = 0$$

or $x = \boxed{0.250\text{ m}}$

At this point, suppressing units,

$$a = (4.55)(0.250)\left[0.250^2 + 0.100^2\right]^{-1/2} - 3.92$$

$$+0.182\left[0.250^2 + 0.100^2\right]^{-1/2}$$

$$= \boxed{0.976\text{ m/s}^2}$$

(d) We solve, suppressing units,

$$0 = 4.55x\left[x^2 + 0.100^2\right]^{-1/2} - 3.92 + 0.182\left[x^2 + 0.100^2\right]^{-1/2}$$

$$3.92\left[x^2 + 0.100^2\right]^{1/2} = 4.55x + 0.182$$

$$15.4\left[x^2 + 0.100^2\right] = 20.7x^2 + 1.65x + 0.033\,1$$

which gives the quadratic equation

$$5.29x^2 + 1.65x - 0.121 = 0$$

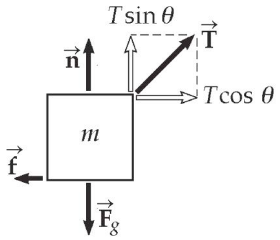
Only the positive root is directly meaningful, so

$$x = \boxed{0.0610\text{ m}}$$

P5.100 The force diagram is shown on the right. With motion impending,

$$n + T\sin\theta - mg = 0$$

$$f = \mu_s\left(mg - T\sin\theta\right)$$



ANS. FIG. P5.100

and

$$T\cos\theta - \mu_s mg + \mu_s T\sin\theta = 0$$

so $$T = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta}$$

To minimize T, we maximize $\cos\theta + \mu_s\sin\theta$:

$$\frac{d}{d\theta}\left(\cos\theta + \mu_s\sin\theta\right) = 0 = -\sin\theta + \mu_s\cos\theta$$

Therefore, the angle where tension T is a minimum is

$$\theta = \tan^{-1}\left(\mu_s\right) = \tan^{-1}\left(0.350\right) = 19.3°$$

What is the tension at this angle? From above,

$$T = \frac{0.350\left(1.30\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\cos 19.3° + 0.350\sin 19.3°} = 4.21\text{ N}$$

$$\boxed{\begin{array}{l}\text{The situation is impossible because at the angle of minimum tension,}\\ \text{the tension exceeds 4.00 N.}\end{array}}$$

P5.101 (a) Following the in-chapter example about a block on a frictionless incline, we have

$$a = g\sin\theta = \left(9.80\ \text{m/s}^2\right)\sin 30.0°$$

$$\boxed{a = 4.90\ \text{m/s}^2}$$

(b) The block slides distance *x* on the incline, with $\sin 30.0° = \dfrac{0.500\ \text{m}}{x}$.

$$x = 1.00\ \text{m: } v_f^2 = v_i^2 + 2a\left(x_f - x_i\right) = 0 + 2\left(4.90\ \text{m/s}^2\right)\left(1.00\ \text{m}\right)$$

$$v_f = \boxed{3.13\ \text{m/s}}\ \text{after time } t_s = \frac{2x_f}{v_f} = \frac{2\left(1.00\ \text{m}\right)}{3.13\ \text{m/s}} = 0.639\ \text{s}$$

(c) To calculate the horizontal range of the block, we need to first determine the time interval during which it is in free fall. We use $y_f - y_i = v_{yi}t + \frac{1}{2}a_y t^2$, and substitute, noting that $v_{yi} = (-3.13\ \text{m/s})\sin 30.0°$.

$$-2.00 = \left(-3.13\ \text{m/s}\right)\sin 30.0° t - \frac{1}{2}\left(9.80\ \text{m/s}^2\right)t^2$$

$$\left(4.90\ \text{m/s}^2\right)t^2 + \left(1.56\ \text{m/s}\right)t - 2.00\ \text{m} = 0$$

Solving for *t* gives

$$t = \frac{-1.56\ \text{m/s} \pm \sqrt{\left(1.56\ \text{m/s}\right)^2 - 4\left(4.90\ \text{m/s}^2\right)\left(-2.00\ \text{m}\right)}}{9.80\ \text{m/s}^2}$$

Only the positive root is physical, with *t* = 0.499 s. The horizontal range of the block is then

$$x_f = v_x t = \left[\left(3.13\ \text{m/s}\right)\cos 30.0°\right]\left(0.499\ \text{s}\right) = \boxed{1.35\ \text{m}}$$

(d) The total time from release to impact is then

$$\text{total time } = t_s + t = 0.639\ \text{s} + 0.499\ \text{s} = \boxed{1.14\ \text{s}}$$

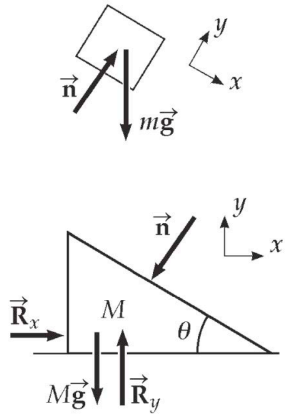
(e) $\boxed{\text{The mass of the block makes no difference}}$, as acceleration due to gravity, whether an object is in free fall or on a frictionless incline, is independent of the mass of the object.

P5.102 Throughout its up and down motion after release the block has

$$\sum F_y = ma_y: \quad +n - mg\cos\theta = 0$$
$$n = mg\cos\theta$$

Let $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$ represent the force of table on incline. We have

$$\sum F_x = ma_x: \quad +R_x - n\sin\theta = 0$$
$$R_x = mg\cos\theta\sin\theta$$
$$\sum F_y = ma_y: \quad -Mg - n\cos\theta + R_y = 0$$
$$R_y = Mg + mg\cos^2\theta$$



ANS. FIG. P5.102

$$\boxed{\vec{\mathbf{R}} = mg\cos\theta\sin\theta \text{ to the right} + \left(M + m\cos^2\theta\right)g \text{ upward}}$$

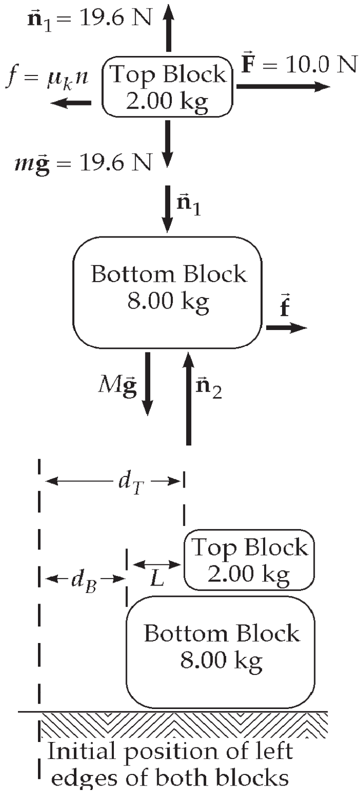
***P5.103** (a) First, draw a free-body diagram of the top block (top panel in ANS. FIG. P5.103). Since $a_y = 0$, $n_1 = 19.6$ N, and

$$f_k = \mu_k n_1 = 0.300(19.6\text{ N})$$
$$= 5.88\text{ N}$$

From $\sum F_x = ma_T$,

$$10.0\text{ N} - 5.88\text{ N} = (2.00\text{ kg})a_T$$

or $a_T = 2.06\text{ m/s}^2$ (for top block). Now draw a free-body diagram (middle figure) of the bottom block and observe that $\sum F_x = Ma_B$ gives $f = 5.88\text{ N} = (8.00\text{ kg})a_B$ or $a_B = 0.735\text{ m/s}^2$ (for the bottom block). In time t, the distance each block moves (starting from rest) is



ANS. FIG. P5.103

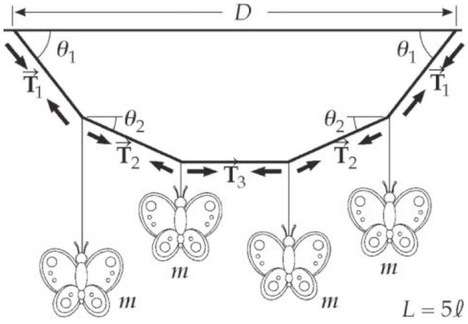
$d_T = \frac{1}{2}a_T t^2$ and $d_B = \frac{1}{2}a_B t^2$. For the top block to reach the right edge of the bottom block (see bottom figure), it is necessary that $d_T = d_B + L$ or

$$\frac{1}{2}\left(2.06\ \text{m/s}^2\right)t^2 = \frac{1}{2}\left(0.735\ \text{m/s}^2\right)t^2 + 3.00\ \text{m}$$

which gives $t = \boxed{2.13\ \text{s}}$.

(b) From above, $d_B = \frac{1}{2}\left(0.735\ \text{m/s}^2\right)(2.13\ \text{s})^2 = \boxed{1.67\ \text{m}}$.

P5.104 (a) Apply Newton's second law to two points where butterflies are attached on either half of mobile (the other half is the same, by symmetry).



ANS. FIG. P5.104

$T_2\cos\theta_2 - T_1\cos\theta_1 = 0$ **[1]**

$T_1\sin\theta_1 - T_2\sin\theta_2 - mg = 0$ **[2]**

$T_2\cos\theta_2 - T_3 = 0$ **[3]**

$T_2\sin\theta_2 - mg = 0$ **[4]**

Substituting [4] into [2] for $T_2\sin\theta_2$,

$$T_1\sin\theta_1 - mg - mg = 0$$

Then

$$\boxed{T_1 = \frac{2mg}{\sin\theta_1}}$$

Substitute [3] into [1] for $T_2\cos\theta_2$:

$$T_3 - T_1\cos\theta_1 = 0,\ T_3 = T_1\cos\theta_1$$

Substitute value of T_1:

$$T_3 = 2mg\frac{\cos\theta_1}{\sin\theta_1} = \boxed{\frac{2mg}{\tan\theta_1} = T_3}$$

From equation [4],

$$T_2 = \frac{mg}{\sin\theta_2}$$

(b) Divide [4] by [3]:

$$\frac{T_2\sin\theta_2}{T_2\cos\theta_2} = \frac{mg}{T_3}$$

Substitute value of T_3:

$$\tan\theta_2 = \frac{mg\tan\theta_1}{2mg}, \boxed{\theta_2 = \tan^{-1}\left(\frac{\tan\theta_1}{2}\right)}$$

Then we can finish answering part (a):

$$\boxed{T_2 = \frac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}}$$

(c) *D* is the horizontal distance between the points at which the two ends of the string are attached to the ceiling.

$$\boxed{\begin{array}{l} D = 2\ell\cos\theta_1 + 2\ell\cos\theta_2 + \ell \text{ and } L = 5\ell \\ D = \frac{L}{5}\left\{2\cos\theta_1 + 2\cos\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right] + 1\right\} \end{array}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P5.2	2.38 kN
P5.4	8.71 N
P5.6	(a) -4.47×10^{15} m/s^2; (b) $+2.09 \times 10^{-10}$ N
P5.8	(a) zero; (b) zero
P5.10	(a) $\frac{1}{2}vt$; (b) magnitude: $m\sqrt{(v/t)^2 + g^2}$, direction: $\tan^{-1}\left(\frac{gt}{v}\right)$
P5.12	$\left(16.3\hat{\mathbf{i}} + 14.6\hat{\mathbf{j}}\right)$ N
P5.14	(a–c) See free-body diagrams and corresponding forces in P5.14.
P5.16	1.59 m/s^2 at 65.2° N of E
P5.18	(a) $\frac{1}{3}$; (b) 0.750 m/s^2
P5.20	(a) ~10^{-22} m/s^2; (b) $\Delta x \sim 10^{-23}$ m
P5.22	(a) $\hat{\mathbf{a}}$ is at 181°; (b) 11.2 kg; (c) 37.5 m/s; (d) $\left(-37.5\hat{\mathbf{i}} - 0.893\hat{\mathbf{j}}\right)$ m/s
P5.24	$\sum\vec{\mathbf{F}} = -km\vec{\mathbf{v}}$
P5.26	(a) See ANS. FIG. P5.26; (b) 1.03 N; (c) 0.805 N to the right
P5.28	(a) 49.0 N; (b) 49.0 N; (c) 98.0 N; (d) 24.5 N
P5.30	(a) See ANS. FIG. P5.30(a); (b) -2.54 m/s^2; (c) 3.19 m/s
P5.32	112 N
P5.34	See P5.33 for complete derivation.
P5.36	(a) $T_1 = 31.5$ N, $T_2 = 37.5$ N, $T_3 = 49.0$ N; (b) $T_1 = 113$ N, $T_2 = 56.6$ N, $T_3 = 98.0$ N
P5.38	(a) 78.4 N; (b) 105 N
P5.40	$a = 6.30$ m/s^2 and $T = 31.5$ N

P5.42 (a) See ANS FIG P5.42; (b) 3.57 m/s^2; (c) 26.7 N; (d) 7.14 m/s

P5.44 (a) $2m(g + a)$; (b) $T_1 = 2T_2$, so the upper string breaks first; (c) 0, 0

P5.46 (a) $a_2 = 2a_1$; (b) $T_2 = \dfrac{m_1m_2}{2m_2 + \frac{1}{2}m_1}g$ and $T_2 = \dfrac{m_1m_2}{m_2 + \frac{1}{4}m_1}g$; (c) $\dfrac{m_1 g}{2m_2 + \frac{1}{2}m_1}$ and $\dfrac{m_1 g}{4m_2 + m_1}$

P5.48 $B = 3.37 \times 10^3$ N, $A = 3.83 \times 10^3$ N, B is in tension and A is in compression.

P5.50 (a) 0.529 m below its initial level; (b) 7.40 m/s upward

P5.52 (a) 14.7 m; (b) neither mass is necessary

P5.54 (a) 256 m; (b) 42.7 m

P5.56 The situation is impossible because maximum static friction cannot provide the acceleration necessary to keep the book stationary on the seat.

P5.58 (a) 4.18; (b) Time would increase, as the wheels would skid and only kinetic friction would act; or perhaps the car would flip over.

P5.60 (a) See ANS. FIG. P5.60; (b) $\theta = 55.2°$; (c) $n = 167$ N

P5.62 (a) 0.404; (b) 45.8 lb

P5.64 (a) See ANS. FIG. P5.64; (b) 2.31 m/s^2, down for m_1, left for m_2, and up for m_3; (c) $T_{12} = 30.0$ N and $T_{23} = 24.2$ N; (d) T_{12} decreases and T_{23} increases

P5.66 (a) 48.6 N, 31. 7 N; (b) If $P > 48.6$ N, the block slides up the wall. If $P < 31.7$ N, the block slides down the wall; (c) 62.7 N, $P \geq 62.7$ N, the block cannot slide up the wall. If $P < 62.7$ N, the block slides down the wall

P5.68 834 N

P5.70 (a) See P5.70 for complete solution; (b) 9.80 N, 0.580 m/s^2

P5.72 (a) 3.43 m/s^2 toward the scrap iron; (b) 3.43 m/s^2 toward the scrap iron; (c) –6.86 m/s^2 toward the magnet

P5.74 The situation is impossible because these forces on the book cannot produce the acceleration described.

P4.76 (a) and (b) See P5.76 for complete derivation; (c) 3.56 N

P5.78 (a) See ANS. FIG. P5.78(a); (b) $a = \dfrac{F}{m_b + m_r}$; (c) $T = \left(\dfrac{m_b}{m_b + m_r}\right)F$; (d) the tension in a cord of negligible mass is constant along its length

P5.80 (a) At any instant they have the same velocity and at all instants they have the same acceleration; (b) 1.61×10^4 N; (c) 2.95×10^4 N

P5.82 (a) Nick and the seat, with total weight 480 N, will accelerate down and the child, with smaller weight 440 N, will accelerate up; (b) In P5.81, a rope tension of 250 N does not make the rope break. In part (a), the rope is strong enough to support tension 459 N. But now the tension everywhere in the rope is 480 N, so it can exceed the breaking strength of the rope.

P5.84 (a) The system will not start to move when released; (b and c) no answer; (d) $f = m_2 g \sin\theta = 29.4$ N

P5.86 (a) $T = \dfrac{f}{2\sin\theta}$; (b) 410 N

P5.88 (a) $M = 3m\sin\theta$; (b) $T_1 = 2mg\sin\theta,\ T_2 = 3mg\sin\theta$; (c) $a = \dfrac{g\sin\theta}{1+2\sin\theta}$;

(d) $T_1 = 4mg\sin\theta\left(\dfrac{1+\sin\theta}{1+2\sin\theta}\right),\ T_2 = 6mg\sin\theta\left(\dfrac{1+\sin\theta}{1+2\sin\theta}\right)$;

(e) $M_{\max} = 3m(\sin\theta + \mu_s\cos\theta)$; (f) $M_{\min} = 3m(\sin\theta - \mu_s\cos\theta)$;

(g) $T_{2,\max} - T_{2,\min} = M_{\max}g - M_{\min}g = 6\mu_s mg\cos\theta$

P5.90 See table in P5.90 and ANS. FIG P5.90; (b) 0.143 m/s^2; (c) The acceleration values agree.

P5.92 (a) $a_1 = 2a_2$; (b) $a_2 = 12.7\text{ N }(1.30\text{ kg} + 4m_1)^{-1}$ down; (c) 9.80 m/s^2 down; (d) a_2 approaches zero; (e) $T = 6.37$ N; (f) yes

P5.94 (a) $n = (8.23\text{ N})\cos\theta$; (b) $a = (9.80\text{ m/s}^2)\sin\theta$; (c) See ANS. FIG P5.94; (d) At 0°, the normal force is the full weight, and the acceleration is zero. At 90° the mass is in free fall next to the vertical incline.

P5.96 (a) 3.00 s; (b) 20.1 m; (c) $(18.0\text{m})\hat{\mathbf{i}} - (9.00\text{m})\hat{\mathbf{j}}$

P5.98 (a) $m_2 g\left[\dfrac{m_1 M}{m_2 M + m_1(m_2 + M)}\right]$; (b) $\left[\dfrac{g m_1(m_2 + M)}{m_2 M + m_1(m_2 + M)}\right]$; (c) $\left[\dfrac{m_1 m_2 g}{m_2 M + m_1(m_2 + M)}\right]$; (d) $\left[\dfrac{m_1 M g}{m_2 M + m_1(m_2 + M)}\right]$

P5.100 The situation is impossible because at the angle of minimum tension, the tension exceeds 4.00 N

P5.102 $\vec{\mathbf{R}} = mg\cos\theta\sin\theta$ to the right $+\left(M + m\cos^2\theta\right)g$ upward

P5.104 (a) $T_1 = \dfrac{2mg}{\sin\theta_1}, \dfrac{2mg}{\tan\theta_1} = T_3$; (b) $\theta_2 = \tan^{-1}\left(\dfrac{\tan\theta_1}{2}\right)$, $T_2 = -\dfrac{mg}{\sin\left[\tan^{-1}\left(\frac{1}{2}\tan\theta_1\right)\right]}$; (c) See P5.104 for complete explanation.

6

Circular Motion and Other Applications of Newton's Laws

CHAPTER OUTLINE

6.1	Extending the Particle in Uniform Circular Motion Model
6.2	Nonuniform Circular Motion
6.3	Motion in Accelerated Frames
6.4	Motion in the Presence of Velocity-Dependent Resistive Forces

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ6.1 (a) A > C = D > B = E = 0. At constant speed, centripetal acceleration is largest when radius is smallest. A straight path has infinite radius of curvature. (b) Velocity is north at *A*, west at *B*, and south at *C*. (c) Acceleration is west at *A*, nonexistent at *B*, east at *C*, to be radially inward.

OQ6.2 Answer (a). Her speed increases, until she reaches terminal speed.

OQ6.3 (a) Yes. Its path is an arc of a circle; the direction of its velocity is changing. (b) No. Its speed is not changing.

OQ6.4 (a) Yes, point *C*. Total acceleration here is centripetal acceleration, straight up. (b) Yes, point *A*. The speed at *A* is zero where the bob is reversing direction. Total acceleration here is tangential acceleration, to the right and downward perpendicular to the cord. (c) No. (d) Yes, point *B*. Total acceleration here is to the right and either downwards or upwards depending on whether the magnitude of the centripetal acceleration is smaller or larger than the magnitude of the tangential acceleration.

OQ6.5 Answer (b). The magnitude of acceleration decreases as the speed increases because the air resistance force increases, counterbalancing more and more of the gravitational force.

OQ6.6 (a) No. When $v = 0$, $v^2/r = 0$.

(b) Yes. Its speed is changing because it is reversing direction.

OQ6.7 (i) Answer (c). The iPod shifts backward relative to the student's hand. The cord then pulls the iPod upward and forward, to make it gain speed horizontally forward along with the airplane. (ii) Answer (b). The angle stays constant while the plane has constant acceleration. This experiment is described in the book *Science from your Airplane Window* by Elizabeth Wood.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ6.1 (a) Friction, either static or kinetic, exerted by the roadway where it meets the rubber tires accelerates the car forward and then maintains its speed by counterbalancing resistance forces. Most of the time static friction is at work. But even kinetic friction (racers starting) will still move the car forward, although not as efficiently. (b) The air around the propeller pushes forward on its blades. Evidence is that the propeller blade pushes the air toward the back of the plane. (c) The water pushes the blade of the oar toward the bow. Evidence is that the blade of the oar pushes the water toward the stern.

CQ6.2 The drag force is proportional to the speed squared and to the effective area of the falling object. At terminal velocity, the drag and gravity forces are in balance. When the parachute opens, its effective area increases greatly, causing the drag force to increase greatly. Because the drag and gravity forces are no longer in balance, the greater drag force causes the speed to decrease, causing the drag force to decrease until it and the force of gravity are in balance again.

CQ6.3 The speed changes. The tangential force component causes tangential acceleration.

CQ6.4 (a) The object will move in a circle at a constant speed.

(b) The object will move in a straight line at a changing speed.

CQ6.5 The person in the elevator is in an accelerating reference frame. The apparent acceleration due to gravity, "g," is changed inside the elevator. "g" $= g \pm a$

CQ6.6 I would not accept that statement for two reasons. First, to be "beyond the pull of gravity," one would have to be infinitely far away from all other matter. Second, astronauts in orbit are moving in a circular path. It is the gravitational pull of Earth on the astronauts that keeps them in orbit. In the space shuttle, just above the atmosphere, gravity is only slightly weaker than at the Earth's surface. Gravity does its job most clearly on an orbiting spacecraft, because the craft feels no other forces and is in free fall.

CQ6.7 This is the same principle as the centrifuge. All the material inside the cylinder tends to move along a straight-line path, but the walls of the cylinder exert an inward force to keep everything moving around in a circular path.

CQ6.8 (a) The larger drop has higher terminal speed. In the case of spheres, the text demonstrates that terminal speed is proportional to the square root of radius. (b) When moving with terminal speed, an object is in equilibrium and has zero acceleration.

CQ6.9 Blood pressure cannot supply the force necessary both to balance the gravitational force and to provide the centripetal acceleration to keep blood flowing up to the pilot's brain.

CQ6.10 The water has inertia. The water tends to move along a straight line, but the bucket pulls it in and around in a circle.

CQ6.11 The current consensus is that the laws of physics are probabilistic in nature on the fundamental level. For example, the Uncertainty Principle (to be discussed later) states that the position and velocity (actually, momentum) of any particle cannot both be known exactly, so the resulting predictions cannot be exact. For another example, the moment of the decay of any given radioactive atomic nucleus cannot be predicted, only the average rate of decay of a large number of nuclei can be predicted—in this sense, quantum mechanics implies that the future is indeterminate. How the laws of physics are related to our sense of free will is open to debate.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 6.1 Extending the Particle in Uniform Circular Motion Model

P6.1 We are given $m = 3.00$ kg, $r = 0.800$ m. The string will break if the tension exceeds the weight corresponding to 25.0 kg, so

$$T_{\max} = Mg = (25.0 \text{ kg})(9.80 \text{ m/s}^2) = 245 \text{ N}$$

When the 3.00-kg mass rotates in a horizontal circle, the tension causes the centripetal acceleration,

so $T = \dfrac{mv^2}{r}$

Then

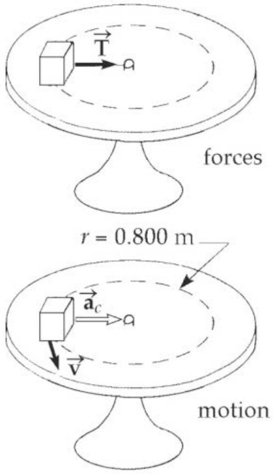
$$v^2 = \frac{rT}{m} = \frac{(0.800 \text{ m})T}{3.00 \text{ kg}} \le \frac{(0.800 \text{ m})T_{\max}}{3.00 \text{ kg}}$$
$$= \frac{(0.800 \text{ m})(245 \text{ N})}{3.00 \text{ kg}} = 65.3 \text{ m}^2/\text{s}^2$$

This represents the maximum value of v^2, or

$$0 \le v \le \sqrt{65.3} \text{ m/s}$$

which gives

$$\boxed{0 \le v \le 8.08 \text{ m/s}}$$



ANS. FIG. P6.1

P6.2 (a) The astronaut's orbital speed is found from Newton's second law, with

$$\sum F_y = ma_y: \; mg_{\text{moon}} \text{ down} = \frac{mv^2}{r} \text{ down}$$

solving for the velocity gives

$$v = \sqrt{g_{\text{moon}} r} = \sqrt{(1.52 \text{ m/s}^2)(1.7 \times 10^6 \text{ m} + 100 \times 10^3 \text{ m})}$$
$$v = \boxed{1.65 \times 10^3 \text{ m/s}}$$

(b) To find the period, we use $v = \dfrac{2\pi r}{T}$ and solve for T:

$$T = \frac{2\pi(1.8 \times 10^6 \text{ m})}{1.65 \times 10^3 \text{ m/s}} = \boxed{6.84 \times 10^3 \text{ s}} = 1.90 \text{ h}$$

P6.3 (a) The force acting on the electron in the Bohr model of the hydrogen atom is directed radially inward and is equal to

$$F = \frac{mv^2}{r} = \frac{(9.11\times10^{-31}\text{ kg})(2.20\times10^{6}\text{ m/s})^2}{0.529\times10^{-10}\text{ m}}$$
$$= \boxed{8.33\times10^{-8}\text{ N inward}}$$

(b) $a = \frac{v^2}{r} = \frac{(2.20\times10^{6}\text{ m/s})^2}{0.529\times10^{-10}\text{ m}} = \boxed{9.15\times10^{22}\text{ m/s}^2\text{ inward}}$

P6.4 In $\sum F = m\frac{v^2}{r}$, both *m* and *r* are unknown but remain constant. Symbolically, write

$$\sum F_{\text{slow}} = \left(\frac{m}{r}\right)(14.0\text{ m/s})^2 \text{ and } \sum F_{\text{fast}} = \left(\frac{m}{r}\right)(18.0\text{ m/s})^2$$

Therefore, $\sum F$ is proportional to v^2 and increases by a factor of $\left(\frac{18.0}{14.0}\right)^2$ as *v* increases from 14.0 m/s to 18.0 m/s. The total force at the higher speed is then

$$\sum F_{\text{fast}} = \left(\frac{18.0}{14.0}\right)^2 \sum F_{\text{slow}} = \left(\frac{18.0}{14.0}\right)^2 (130\text{ N}) = \boxed{215\text{ N}}$$

This force must be $\boxed{\text{horizontally inward}}$ to produce the driver's centripetal acceleration.

P6.5 We neglecting relativistic effects. With 1 u = 1.661 x 10^{-27} kg, and from Newton's second law, we obtain

$$F = ma_c = \frac{mv^2}{r}$$
$$= (2\times1.661\times10^{-27}\text{ kg})\frac{(2.998\times10^{7}\text{ m/s})^2}{(0.480\text{ m})}$$
$$= \boxed{6.22\times10^{-12}\text{ N}}$$

P6.6 (a) The car's speed around the curve is found from

$$v = \frac{235\text{ m}}{36.0\text{ s}} = 6.53\text{ m/s}$$

This is the answer to part (b) of this problem. We calculate the radius of the curve from $\frac{1}{4}(2\pi r) = 235\text{ m},$ which gives *r* = 150 m.

The car's acceleration at point *B* is then

$$\vec{\mathbf{a}}_r = \left(\frac{v^2}{r}\right) \text{ toward the center}$$

$$= \frac{(6.53 \text{ m/s})^2}{150 \text{ m}} \text{ at } 35.0° \text{ north of west}$$

$$= (0.285 \text{ m/s}^2)\left(\cos 35.0°(-\hat{\mathbf{i}}) + \sin 35.0°\hat{\mathbf{j}}\right)$$

$$= \boxed{(-0.233\hat{\mathbf{i}} + 0.163\hat{\mathbf{j}}) \text{ m/s}^2}$$

(b) From part (a), $v = \boxed{6.53 \text{ m/s}}$

(c) We find the average acceleration from

$$\vec{\mathbf{a}}_{\text{avg}} = \frac{(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i)}{\Delta t}$$

$$= \frac{(6.53\hat{\mathbf{j}} - 6.53\hat{\mathbf{i}}) \text{ m/s}}{36.0 \text{ s}}$$

$$= \boxed{(-0.181\hat{\mathbf{i}} + 0.181\hat{\mathbf{j}}) \text{ m/s}^2}$$

P6.7 Standing on the inner surface of the rim, and moving with it, each person will feel a normal force exerted by the rim. This inward force causes the 3.00 m/s^2 centripetal acceleration:

$$a_c = v^2/r \quad \text{so} \quad v = \sqrt{a_c r} = \sqrt{(3.00 \text{ m/s}^2)(60.0 \text{ m})} = 13.4 \text{ m/s}$$

The period of rotation comes from $v = \frac{2\pi r}{T}$:

$$T = \frac{2\pi r}{v} = \frac{2\pi(60.0 \text{ m})}{13.4 \text{ m/s}} = 28.1 \text{ s}$$

so the frequency of rotation is

$$f = \frac{1}{T} = \frac{1}{28.1 \text{ s}} = \left(\frac{1}{28.1 \text{ s}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{2.14 \text{ rev/min}}$$

P6.8 ANS. FIG. P6.8 shows the free-body diagram for this problem.

(a) The forces acting on the pendulum in the vertical direction must be in balance since the acceleration of the bob in this direction is zero. From Newton's second law in the *y* direction,

$$\sum F_y = T\cos\theta - mg = 0$$

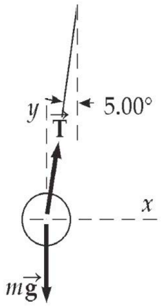
Solving for the tension *T* gives

$$T = \frac{mg}{\cos\theta} = \frac{(80.0\ \text{kg})(9.80\ \text{m/s}^2)}{\cos 5.00°} = 787\ \text{N}$$

In vector form,

$$\vec{\mathbf{T}} = T\sin\theta\hat{\mathbf{i}} + T\cos\theta\hat{\mathbf{j}}$$

$$= \boxed{(68.6\ \text{N})\hat{\mathbf{i}} + (784\ \text{N})\hat{\mathbf{j}}}$$



ANS. FIG. P6.8

(b) From Newton's second law in the *x* direction,

$$\sum F_x = T\sin\theta = ma_c$$

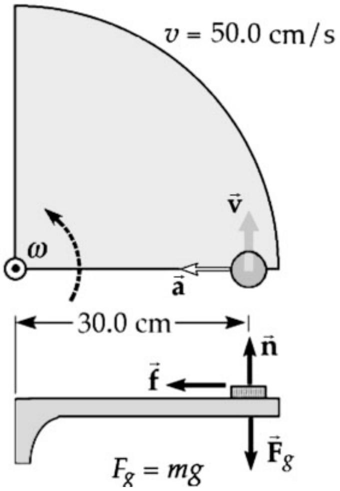
which gives

$$a_c = \frac{T\sin\theta}{m} = \frac{(787\ \text{N})\sin 5.00°}{80.0\ \text{kg}} = \boxed{0.857\ \text{m/s}^2}$$

toward the center of the circle.

The length of the wire is unnecessary information. We could, on the other hand, use it to find the radius of the circle, the speed of the bob, and the period of the motion.

P6.9 ANS. FIG. P6.9 shows the constant maximum speed of the turntable and the centripetal acceleration of the coin.



ANS. FIG. P6.9

(a) The force of $\boxed{\text{static friction}}$ causes the centripetal acceleration.

(b) From ANS. FIG. P6.9,

$$ma\hat{\mathbf{i}} = f\,\hat{\mathbf{i}} + n\hat{\mathbf{j}} + mg\left(-\hat{\mathbf{j}}\right)$$

$$\sum F_y = 0 = n - mg$$

thus, $n = mg$ and

$$\sum F_r = m\frac{v^2}{r} = f = \mu n = \mu mg$$

Then,

$$\mu = \frac{v^2}{rg} = \frac{(50.0\ \text{cm/s})^2}{(30.0\ \text{cm})(980\ \text{cm/s}^2)} = \boxed{0.085\ 0}$$

P6.10 We solve for the tensions in the two strings:

$$F_g = mg = (4.00\ \text{kg})(9.80\ \text{m/s}^2) = 39.2\ \text{N}$$

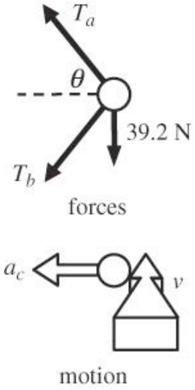
The angle θ is given by

$$\theta = \sin^{-1}\left(\frac{1.50\ \text{m}}{2.00\ \text{m}}\right) = 48.6°$$

The radius of the circle is then

$$r = (2.00\ \text{m})\cos 48.6° = 1.32\ \text{m}$$

Applying Newton's second law,



ANS. FIG. P6.10

$$\sum F_x = ma_x = \frac{mv^2}{r}$$

$$T_a \cos 48.6° + T_b \cos 48.6° = \frac{(4.00\ \text{kg})(3.00\ \text{m/s})^2}{1.32\ \text{m}}$$

$$T_a + T_b = \frac{27.27\ \text{N}}{\cos 48.6°} = 41.2\ \text{N} \qquad \textbf{[1]}$$

$$\sum F_y = ma_y: \quad T_a \sin 48.6° - T_b \sin 48.6° - 39.2\ \text{N} = 0$$

$$T_a - T_b = \frac{39.2\ \text{N}}{\sin 48.6°} = 52.3\ \text{N} \qquad \textbf{[2]}$$

To solve simultaneously, we add the equations in T_a and T_b:

$$(T_a + T_b) + (T_a - T_b) = 41.2\ \text{N} + 52.3\ \text{N}$$

$$T_a = \frac{93.8\ \text{N}}{2} = 46.9\ \text{N}$$

This means that $T_b = 41.2\ \text{N} - T_a = -5.7\ \text{N}$, which we may interpret as meaning the lower string pushes rather than pulls!

$$\boxed{\text{The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.}}$$

To answer the **What if?**, we go back to equation [2] above and substitute *mg* for the weight of the object. Then,

$$\sum F_y = ma_y: \quad T_a \sin 48.6° - T_b \sin 48.6° - mg = 0$$

$$T_a - T_b = \frac{(4.00\ \text{kg})g}{\sin 48.6°} = 5.33g$$

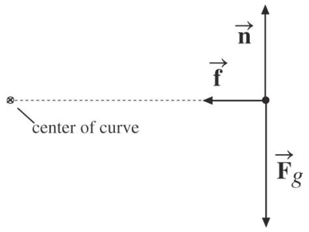
We then add this equation to equation [2] to obtain

$$(T_a + T_b) + (T_a - T_b) = 41.2 \text{ N} + 5.33g$$

or $$T_a = 20.6 \text{ N} + 2.67g \text{ and } T_b = 41.2 \text{ N} - T_a = 41.2 \text{ N} - 2.67g$$

For this situation to be possible, T_b must be > 0, or $g < 7.72 \text{ m/s}^2$. This is certainly the case on the surface of the Moon and on Mars.

P6.11 Call the mass of the egg crate m. The forces on it are its weight $F_g = mg$ vertically down, the normal force n of the truck bed vertically up, and static friction f_s directed to oppose relative sliding motion of the crate on the truck bed. The friction force is directed radially inward. It is the only horizontal force on the crate, so it must provide the centripetal acceleration. When the truck has maximum speed, friction f_s will have its maximum value with $f_s = \mu_s n$.



ANS. FIG. P6.11

Newton's second law in component form becomes

$$\sum F_y = ma_y \quad \text{giving} \quad n - mg = 0 \quad \text{or} \quad n = mg$$

$$\sum F_x = ma_x \quad \text{giving} \quad f_s = ma_r$$

From these three equations,

$$\mu_s n \le \frac{mv^2}{r} \quad \text{and} \quad \mu_s mg \le \frac{mv^2}{r}$$

The mass divides out. The maximum speed is then

$$v \le \sqrt{\mu_s rg} = \sqrt{0.600(35.0 \text{ m})(9.80 \text{ m/s}^2)} \quad \rightarrow \quad v \le \boxed{14.3 \text{ m/s}}$$

Section 6.2 Nonuniform Circular Motion

P6.12 (a) The external forces acting on the water are

$\boxed{\text{the gravitational force}}$

and $\boxed{\text{the contact force exerted on the water by the pail}}$.

(b) The $\boxed{\text{contact force exerted by the pail}}$ is the most important in causing the water to move in a circle. If the gravitational force acted alone, the water would follow the parabolic path of a projectile.

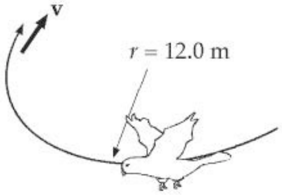
(c) When the pail is inverted at the top of the circular path, it cannot hold the water up to prevent it from falling out. If the water is not to spill, the pail must be moving fast enough that the required centripetal force is at least as large as the gravitational force. That is, we must have

$$m\frac{v^2}{r} \geq mg \quad \text{or} \quad v \geq \sqrt{rg} = \sqrt{(1.00\text{ m})(9.80\text{ m/s}^2)} = \boxed{3.13\text{ m/s}}$$

(d) If the pail were to suddenly disappear when it is at the top of the circle and moving at 3.13 m/s, $\boxed{\text{the water would follow the}}$ $\boxed{\text{parabolic path of a projectile}}$ launched with initial velocity components of $v_{xi} = 3.13$ m/s, $v_{yi} = 0$.

P6.13 (a) The hawk's centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(4.00\text{ m/s})^2}{12.0\text{ m}} = \boxed{1.33\text{ m/s}^2}$$



ANS. FIG. P6.13

(b) The magnitude of the acceleration vector is

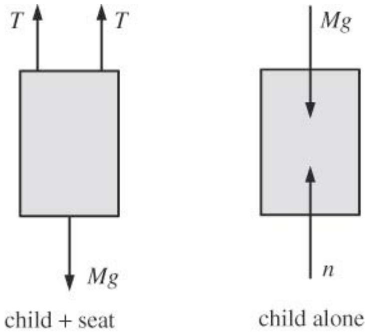
$$a = \sqrt{a_c^2 + a_t^2}$$

$$= \sqrt{(1.33\text{ m/s}^2)^2 + (1.20\text{ m/s}^2)^2} = \boxed{1.79\text{ m/s}^2}$$

at an angle

$$\theta = \tan^{-1}\left(\frac{a_c}{a_t}\right) = \tan^{-1}\left(\frac{1.33\text{ m/s}^2}{1.20\text{ m/s}^2}\right) = \boxed{48.0°\text{ inward}}$$

6.14 We first draw a force diagram that shows the forces acting on the child-seat system and apply Newton's second law to solve the problem. The child's path is an arc of a circle, since the top ends of the chains are fixed. Then at the lowest point the child's motion is changing in direction: He moves with centripetal acceleration even as his speed is not changing and his tangential acceleration is zero.



ANS. FIG. P6.14

(a) ANS. FIG. P6.14 shows that the only forces acting on the system of child + seat are the tensions in the two chains and the weight of the boy:

$$\sum F = F_{\text{net}} = 2T - mg = ma = \frac{mv^2}{r}$$

with

$$F_{\text{net}} = 2T - mg = 2(350\text{ N}) - (40.0\text{ kg})(9.80\text{ m/s}^2) = 308\text{ N}$$

solving for *v* gives

$$v = \sqrt{\frac{F_{\text{net}}r}{m}} = \sqrt{\frac{(308\text{ N})(3.00\text{ m})}{40.0\text{ kg}}} = \boxed{4.81\text{ m/s}}$$

(b) The normal force from the seat on the child accelerates the child in the same way that the total tension in the chain accelerates the child-seat system. Therefore, $n = 2T = \boxed{700\text{ N}}$.

P6.15 See the forces acting on seat (child) in ANS. FIG. P6.14.

(a) $\sum F = 2T - Mg = \dfrac{Mv^2}{R}$

$$v^2 = (2T - Mg)\left(\frac{R}{M}\right)$$

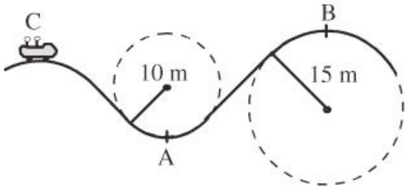
$$\boxed{v = \sqrt{(2T - Mg)\left(\frac{R}{M}\right)}}$$

(b) $n - Mg = F = \dfrac{Mv^2}{R}$

$$\boxed{n = Mg + \frac{Mv^2}{R}}$$

P6.16 (a) We apply Newton's second law at point A, with *v* = 20.0 m/s, *n* = force of track on roller coaster, and *R* = 10.0 m:

$$\sum F = \frac{Mv^2}{R} = n - Mg$$



ANS. FIG. P6.16

From this we find

$$n = Mg + \frac{Mv^2}{R} = (500\text{ kg})(9.80\text{ m/s}^2) + \frac{(500\text{ kg})(20.0\text{ m/s}^2)}{10.0\text{ m}}$$

$$n = 4\,900\text{ N} + 20\,000\text{ N} = \boxed{2.49 \times 10^4\text{ N}}$$

(b) At point B, the centripetal acceleration is now downward, and Newton's second law now gives

$$\sum F = n - Mg = -\frac{Mv^2}{R}$$

The maximum speed at B corresponds to the case where the rollercoaster begins to fly off the track, or when $n = 0$. Then,

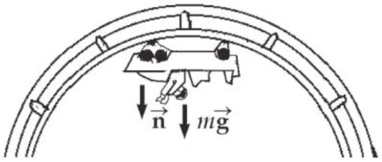
$$-Mg = -\frac{Mv_{\text{max}}^2}{R}$$

which gives

$$v_{\text{max}} = \sqrt{Rg} = \sqrt{(15.0\text{ m})(9.80\text{ m/s}^2)} = \boxed{12.1\text{ m/s}}$$

P6.17 (a) $a_c = \dfrac{v^2}{r}$

$$r = \frac{v^2}{a_c} = \frac{(13.0\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{8.62\text{ m}}$$



ANS. FIG. P6.17

(b) Let n be the force exerted by the rail.

Newton's second law gives

$$Mg + n = \frac{Mv^2}{r}$$

$$n = M\left(\frac{v^2}{r} - g\right) = M(2g - g) = \boxed{Mg\text{, downward}}$$

(c) $a_c = \dfrac{v^2}{r}$, or $a_c = \dfrac{(13.0\text{ m/s})^2}{20.0\text{ m}} = \boxed{8.45\text{ m/s}^2}$

(d) If the force exerted by the rail is n_1,

then $n_1 + Mg = \dfrac{Mv^2}{r} = Ma_c$

$n_1 = M(a_c - g)$ which is < 0 since $a_c = 8.45\text{ m/s}^2$

Thus, the normal force would have to point away from the center of the curve. Unless they have belts, the riders will fall from the cars.

In a teardrop-shaped loop, the radius of curvature r decreases, causing the centripetal acceleration to increase. The speed would decrease as the car rises (because of gravity), but the overall effect is that the required centripetal force increases, meaning the normal force increases--there is less danger if not wearing a seatbelt.

P6.18 (a) Consider radial forces on the object, taking inward as positive.

$$\sum F_r = ma_r: \quad T - mg\cos\theta = \frac{mv^2}{r}$$

Solving for the tension gives

$$\begin{aligned} T &= mg\cos\theta + \frac{mv^2}{r} \\ &= (0.500\text{ kg})(9.80\text{ m/s}^2)\cos 20.0° \\ &\qquad + (0.500\text{ kg})(8.00\text{ m/s})^2/2.00\text{ m} \\ &= 4.60\text{ N} + 16.0\text{ N} = \boxed{20.6\text{ N}} \end{aligned}$$

(b) We already found the radial component of acceleration,

$$a_r = \frac{v^2}{r} = \frac{(8.00\text{ m/s})^2}{2.00\text{ m}} = \boxed{32.0\text{ m/s}^2\text{ inward}}$$

Consider the tangential forces on the object:

$$\sum F_t = ma_t: \quad mg\sin\theta = ma_t$$

Solving for the tangential component of acceleration gives

$$\begin{aligned} a_t &= g\sin\theta = (9.80\text{ m/s}^2)\sin 20.0° \\ &= \boxed{3.35\text{ m/s}^2\text{ downward tangent to the circle}} \end{aligned}$$

(c) The magnitude of the acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(32.0\text{ m/s}^2)^2 + (3.35\text{ m/s}^2)^2} = 32.2\text{ m/s}^2$$

at an angle of

$$\tan^{-1}\left(\frac{3.35\text{ m/s}^2}{32.0\text{ m/s}^2}\right) = 5.98°$$

Thus, the acceleration is

$$\boxed{32.2\text{ m/s}^2\text{ inward and below the cord at }5.98°}$$

(d) $\boxed{\text{No change.}}$

(e) $\boxed{\begin{array}{l}\text{If the object is swinging down it is gaining speed, and if the object} \\ \text{is swinging up it is losing speed, but the forces are the same;} \\ \text{therefore, its acceleration is regardless of the direction of swing.}\end{array}}$

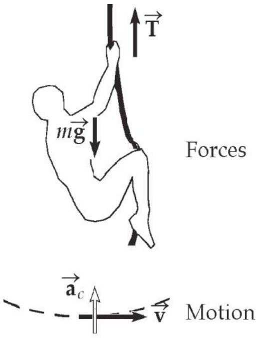
P6.19 Let the tension at the lowest point be *T*. From Newton's second law, $\sum F = ma$ and

$$T - mg = ma_c = \frac{mv^2}{r}$$

$$T = m\left(g + \frac{v^2}{r}\right)$$

$$T = (85.0\ \text{kg})\left[9.80\ \text{m/s}^2 + \frac{(8.00\ \text{m/s})^2}{10.0\ \text{m}}\right]$$

$$= 1.38\ \text{kN} > 1\,000\ \text{N}$$



ANS. FIG. P6.19

$\boxed{\text{He doesn't make it across the river because the vine breaks.}}$

Section 6.3 Motion in Accelerated Frames

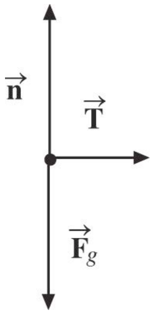
P6.20 (a) From $\sum F_x = Ma$, we obtain

$$a = \frac{T}{M} = \frac{18.0\ \text{N}}{5.00\ \text{kg}} = \boxed{3.60\ \text{m/s}^2} \text{ to the right}$$

(b) If v = const, $a = 0$, so $\boxed{T = 0}$. (This is also an equilibrium situation.)

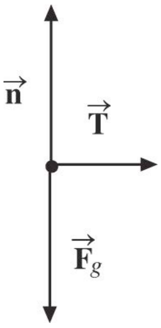
(c) $\boxed{\text{Someone in the car (noninertial observer) claims that the forces on the mass along } x \text{ are } T \text{ and a fictitious force } (-Ma).}$

(d) $\boxed{\text{Someone at rest outside the car (inertial observer) claims that } T \text{ is the only force on } M \text{ in the } x \text{ direction.}}$



ANS. FIG. P6.20

P6.21 The only forces acting on the suspended object are the force of gravity $m\vec{g}$ and the force of tension T forward and upward at angle θ with the vertical, as shown in the free-body diagram in ANS. FIG. P6.21. Applying Newton's second law in the x and y directions,



ANS. FIG. P6.21

$$\sum F_x = T\sin\theta = ma \qquad \textbf{[1]}$$

$$\sum F_y = T\cos\theta - mg = 0$$

or

$$T\cos\theta = mg \qquad \textbf{[2]}$$

(a) Dividing equation [1] by [2] gives

$$\tan\theta = \frac{a}{g} = \frac{3.00\ \text{m/s}^2}{9.80\ \text{m/s}^2} = 0.306$$

Solving for θ, $\theta = \boxed{17.0°}$

(b) From equation [1],

$$T = \frac{ma}{\sin\theta} = \frac{(0.500\ \text{kg})(3.00\ \text{m/s}^2)}{\sin(17.0°)} = \boxed{5.12\ \text{N}}$$

P6.22 In an inertial reference frame, the girl is accelerating horizontally inward at

$$\frac{v^2}{r} = \frac{(5.70\ \text{m/s})^2}{2.40\ \text{m}} = 13.5\ \text{m/s}^2$$

In her own noninertial frame, her head feels a horizontally outward fictitious force equal to its mass times this acceleration. Together this force and the weight of her head add to have a magnitude equal to the mass of her head times an acceleration of

$$\sqrt{g^2 + \left(\frac{v^2}{r}\right)^2} = \sqrt{(9.80\ \text{m/s}^2)^2 + (13.5\ \text{m/s}^2)^2} = 16.7\ \text{m/s}^2$$

This is larger than g by a factor of $\frac{16.7\ \text{m/s}}{9.80\ \text{m/s}} = 1.71$.

Thus, the force required to lift her head is larger by this factor, or the required force is

$$F = 1.71(55.0\ \text{N}) = \boxed{93.8\ \text{N}}$$

P6.23 The scale reads the upward normal force exerted by the floor on the passenger. The maximum force occurs during upward acceleration (when starting an upward trip or ending a downward trip). The minimum normal force occurs with downward acceleration. For each respective situation,

$$\sum F_y = ma_y \quad \text{becomes for starting} \quad +591\text{ N} - mg = +ma$$

$$\text{and for stopping} \quad +391\text{ N} - mg = -ma$$

where a represents the magnitude of the acceleration.

(a) These two simultaneous equations can be added to eliminate a and solve for mg:

$$+\,591\text{ N} - mg + 391\text{ N} - mg = 0$$

or $\quad 982\text{ N} - 2mg = 0$

$$F_g = mg = \frac{982\text{ N}}{2} = \boxed{491\text{ N}}$$

(b) From the definition of weight, $m = \frac{F_g}{g} = \frac{491\text{ N}}{9.80\text{ m/s}^2} = \boxed{50.1\text{ kg}}$

(c) Substituting back gives $+591\text{ N} - 491\text{ N} = (50.1\text{ kg})a$, or

$$a = \frac{100\text{ N}}{50.1\text{ kg}} = \boxed{2.00\text{ m/s}^2}$$

P6.24 Consider forces on the backpack as it slides in the Earth frame of reference.

$$\sum F_y = ma_y:\quad +n - mg = ma,\ n = m(g+a),\ f_k = \mu_k m(g+a)$$

$$\sum F_x = ma_x:\quad -\mu_k m(g+a) = ma_x$$

The motion across the floor is described by

$$L = vt + \frac{1}{2}a_x t^2 = vt - \frac{1}{2}\mu_k(g+a)t^2$$

We solve for μ_k:

$$vt - L = \frac{1}{2}\mu_k(g+a)t^2$$

$$\boxed{\mu_k = \frac{2(vt-L)}{(g+a)t^2}}$$

P6.25 The water moves at speed

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.120\text{ m})}{7.25\text{ s}} = 0.104\text{ m/s}$$

The top layer of water feels a downward force of gravity *mg* and an outward fictitious force in the turntable frame of reference,

$$\frac{mv^2}{r} = \frac{m\left(0.104 \text{ m/s}\right)^2}{0.12 \text{ m}} = m\,9.01\times10^{-2} \text{ m/s}^2$$

It behaves as if it were stationary in a gravity field pointing downward and outward at

$$\tan^{-1}\left(\frac{0.090\,1 \text{ m/s}^2}{9.8 \text{ m/s}^2}\right) = \boxed{0.527°}$$

Its surface slopes upward toward the outside, making this angle with the horizontal.

Section 6.4 Motion in the Presence of Velocity-Dependent Resistive Forces

P6.26 (a) $\rho = \frac{m}{V}$, $A = 0.020\,1 \text{ m}^2$, $R = \frac{1}{2}\rho_{\text{air}} A D v_T^2 = mg$

$$m = \rho_{\text{bead}} V = 0.830 \text{ g/cm}^3\left[\frac{4}{3}\pi\left(8.00 \text{ cm}\right)^3\right] = 1.78 \text{ kg}$$

Assuming a drag coefficient of *D* = 0.500 for this spherical object, and taking the density of air at 20°C from the endpapers, we have

$$v_T = \sqrt{\frac{2\left(1.78 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)}{0.500\left(1.20 \text{ kg/m}^3\right)\left(0.020\,1 \text{ m}^2\right)}} = \boxed{53.8 \text{ m/s}}$$

(b) From $v_f^2 = v_i^2 + 2gh = 0 + 2gh$, we solve for *h*:

$$h = \frac{v_f^2}{2g} = \frac{\left(53.8 \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{148 \text{ m}}$$

P6.27 With 100 km/h = 27.8 m/s, the resistive force is

$$R = \frac{1}{2}D\rho A v^2 = \frac{1}{2}(0.250)\left(1.20 \text{ kg/m}^3\right)\left(2.20 \text{ m}^2\right)\left(27.8 \text{ m/s}\right)^2$$
$$= 255 \text{ N}$$

$$a = -\frac{R}{m} = -\frac{255 \text{ N}}{1\,200 \text{ kg}} = \boxed{-0.212 \text{ m/s}^2}$$

P6.28 Given $m = 80.0$ kg, $v_T = 50.0$ m/s, we write

$$mg = \frac{D\rho A v_T^2}{2}$$

which gives

$$\frac{D\rho A}{2} = \frac{mg}{v_T^2} = 0.314\ \text{kg/m}$$

(a) At $v = 30.0$ m/s,

$$a = g - \frac{D\rho A v^2/2}{m} = 9.80\ \text{m/s}^2 - \frac{(0.314\ \text{kg/m})(30.0\ \text{m/s})^2}{80.0\ \text{kg}}$$

$$= \boxed{6.27\ \text{m/s}^2\ \text{downward}}$$

(b) At $v = 50.0$ m/s, terminal velocity has been reached.

$$\sum F_y = 0 = mg - R$$

$$\Rightarrow R = mg = (80.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{784\ \text{N directed up}}$$

(c) At $v = 30.0$ m/s,

$$\frac{D\rho A v^2}{2} = (0.314\ \text{kg/m})(30.0\ \text{m/s})^2 = \boxed{283\ \text{N upward}}$$

P6.29 Since the upward velocity is constant, the resultant force on the ball is zero. Thus, the upward applied force equals the sum of the gravitational and drag forces (both downward):

$$F = mg + bv$$

The mass of the copper ball is

$$m = \frac{4\pi\rho r^3}{3} = \left(\frac{4}{3}\right)\pi(8.92\times10^3\ \text{kg/m}^3)(2.00\times10^{-2}\ \text{m})^3$$

$$= 0.299\ \text{kg}$$

The applied force is then

$$F = mg + bv = (0.299\ \text{kg})(9.80\ \text{m/s}^2)$$

$$+ (0.950\ \text{kg/s})(9.00\times10^{-2}\ \text{m/s})$$

$$= \boxed{3.01\ \text{N}}$$

P6.30 (a) The acceleration of the Styrofoam is given by

$$a = g - Bv$$

When $v = v_T$, $a = 0$ and $g = Bv_T \rightarrow B = \dfrac{g}{v_T}$

The Styrofoam falls 1.50 m at constant speed v_T in 5.00 s.

Thus,

$$v_T = \frac{h}{\Delta t} = \frac{1.50\text{ m}}{5.00\text{ s}} = 0.300\text{ m/s}$$

Then

$$\boxed{B = \frac{g}{v_T} = \frac{9.80\text{ m/s}^2}{0.300\text{ m/s}} = 32.7\text{ s}^{-1}}$$

(b) At $t = 0$, $v = 0$, and $a = g = \boxed{9.80\text{ m/s}^2\text{ down}}$

(c) When $v = 0.150$ m/s,

$$\begin{aligned} a &= g - Bv \\ &= 9.80\text{ m/s}^2 - (32.7\text{ s}^{-1})(0.150\text{ m/s}) \\ &= \boxed{4.90\text{ m/s}^2\text{ down}} \end{aligned}$$

P6.31 We have a particle under a net force in the special case of a resistive force proportional to speed, and also under the influence of the gravitational force.

(a) The speed v varies with time according to Equation 6.6,

$$v = \frac{mg}{b}\left(1 - e^{-bt/m}\right) = v_T\left(1 - e^{-t/\tau}\right)$$

where $v_T = mg/b$ is the terminal speed. Hence,

$$b = \frac{mg}{v_T} = \frac{(3.00 \times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{2.00 \times 10^{-2}\text{ m/s}} = \boxed{1.47\text{ N}\cdot\text{s/m}}$$

(b) To find the time interval for v to reach $0.632v_T$, we substitute $v = 0.632v_T$ into Equation 6.6, giving

$$0.632v_T = v_T(1 - e^{-bt/m}) \qquad \text{or} \qquad 0.368 = e^{-(1.47t/0.003\,00)}$$

Solve for t by taking the natural logarithm of each side of the equation:

$$\ln(0.368) = -\frac{1.47\text{ t}}{3.00 \times 10^{-3}} \qquad \text{or} \qquad -1 = -\frac{1.47\text{ t}}{3.00 \times 10^{-3}}$$

or $$t = -\left(\frac{m}{b}\right)\ln(0.368) = \boxed{2.04 \times 10^{-3}\ \text{s}}$$

(c) At terminal speed, $R = v_T b = mg$. Therefore,

$$R = v_T b = mg = \left(3.00 \times 10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = \boxed{2.94 \times 10^{-2}\ \text{N}}$$

P6.32 We write

$$-kmv^2 = -\frac{1}{2}D\rho A v^2$$

so

$$k = \frac{D\rho A}{2m} = \frac{0.305\left(1.20\ \text{kg/m}^3\right)\left(4.2 \times 10^{-3}\ \text{m}^2\right)}{2\left(0.145\ \text{kg}\right)} = 5.3 \times 10^{-3}/\text{m}$$

solving for the velocity as the ball crosses home plate gives

$$v = v_i e^{-kx} = \left(40.2\ \text{m/s}\right)e^{-\left(5.3\times10^{-3}/\text{m}\right)\left(18.3\ \text{m}\right)} = \boxed{36.5\ \text{m/s}}$$

P6.33 We start with Newton's second law,

$$\sum F = ma$$

substituting,

$$-kmv^2 = m\frac{dv}{dt}$$

$$-k\,dt = \frac{dv}{v^2}$$

$$-k\int_0^t dt = \int_{v_i}^{v} v^{-2}\,dv$$

integrating both sides gives

$$-k(t-0) = \left.\frac{v^{-1}}{-1}\right|_{v_i}^{v} = -\frac{1}{v} + \frac{1}{v_i}$$

$$\frac{1}{v} = \frac{1}{v_i} + kt = \frac{1 + v_i kt}{v_i}$$

$$\boxed{v = \frac{v_i}{1 + v_i kt}}$$

P6.34 (a) Since the window is vertical, the normal force is horizontal and is given by $n = 4.00$ N. To find the vertical component of the force, we note that the force of kinetic friction is given by

$$f_k = \mu_k n = 0.900(4.00\text{ N}) = 3.60\text{ N upward}$$

to oppose downward motion. Newton's second law then becomes

$$\sum F_y = ma_y: \quad +3.6\text{ N} - (0.16\text{ kg})(9.8\text{ m/s}^2) + P_y = 0$$

$$P_y = -2.03\text{ N} = \boxed{2.03\text{ N down}}$$

(b) Now, with the increased downward force, Newton's second law gives

$$\sum F_y = ma_y:$$

$$+3.60\text{ N} - (0.160\text{ kg})(9.80\text{ m/s}^2) - 1.25(2.03\text{ N}) = 0.160\text{ kg } a_y$$

then

$$a_y = -0.508\text{ N}/0.16\text{ kg} = -3.18\text{ m/s}^2 = \boxed{3.18\text{ m/s}^2\text{ down}}$$

(c) At terminal velocity,

$$\sum F_y = ma_y: \quad +(20.0\text{ N}\cdot\text{s/m})v_T - (0.160\text{ kg})(9.80\text{ m/s}^2) - 1.25(2.03\text{ N}) = 0$$

Solving for the terminal velocity gives

$$v_T = 4.11\text{ N}/(20\text{ N}\cdot\text{s/m}) = \boxed{0.205\text{ m/s down}}$$

P6.35 (a) We must fit the equation $v = v_i e^{-ct}$ to the two data points:

At $t = 0$, $v = 10.0$ m/s, so $v = v_i e^{-ct}$ becomes

$$10.0\text{ m/s} = v_i e^0 = (v_i)(1)$$

which gives $v_i = 10.0$ m/s

At $t = 20.0$ s, $v = 5.00$ m/s so the equation becomes

$$5.00\text{ m/s} = (10.0\text{ m/s})e^{-c(20.0\text{ s})}$$

giving $0.500 = e^{-c(20.0\text{ s})}$

or $-20.0c = \ln\left(\frac{1}{2}\right) \to c = -\frac{\ln\left(\frac{1}{2}\right)}{20.0} = \boxed{3.47\times10^{-2}\text{ s}^{-1}}$

(b) At $t = 40.0$ s

$$v = (10.0\text{ m/s})e^{-40.0c} = (10.0\text{ m/s})(0.250) = \boxed{2.50\text{ m/s}}$$

(c) The acceleration is the rate of change of the velocity:

$$a = \frac{dv}{dt} = \frac{d}{dt} v_i e^{-ct} = v_i\left(e^{-ct}\right)(-c) = -c\left(v_i e^{-ct}\right)$$
$$= \boxed{-cv}$$

Thus, the acceleration is a negative constant times the speed.

P6.36 In $R = \frac{1}{2} D\rho A v^2$, we estimate that the coefficient of drag for an open palm is $D = 1.00$, the density of air is $\rho = 1.20\ \text{kg/m}^3$, the area of an open palm is $A = (0.100\ \text{m})(0.160\ \text{m}) = 1.60 \times 10^{-2}\ \text{m}^2$, and $v = 29.0\ \text{m/s}$ (65 miles per hour). The resistance force is then

$$R = \frac{1}{2}(1.00)\left(1.20\ \text{kg/m}^3\right)\left(1.60 \times 10^{-2}\ \text{m}^2\right)(29.0\ \text{m/s})^2 = 8.07\ \text{N}$$

or $R \sim \boxed{10^1\ \text{N}}$

Additional Problems

P6.37 Because the car travels at a constant speed, it has no tangential acceleration, but it does have centripetal acceleration because it travels along a circular arc. The direction of the centripetal acceleration is toward the center of curvature, and the direction of velocity is tangent to the curve.

Point A

direction of velocity: $\boxed{\text{East}}$

direction of the centripetal acceleration: $\boxed{\text{South}}$

Point B

direction of velocity: $\boxed{\text{South}}$

direction of the centripetal acceleration: $\boxed{\text{West}}$

P6.38 The free-body diagram of the passenger is shown in ANS. FIG. P6.38. From Newton's second law,

$$\sum F_y = ma_y$$

$$n - mg = \frac{mv^2}{r}$$

ANS. FIG. P6.38

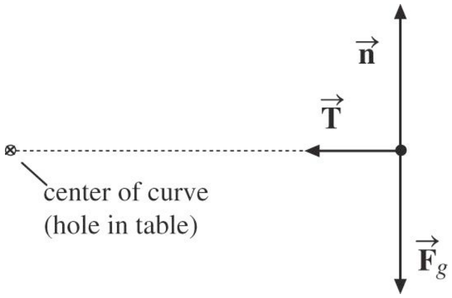
which gives

$$n = mg + \frac{mv^2}{r}$$

$$= (50\ \text{kg})(9.80\ \text{m/s}^2) + \frac{(50.0\ \text{kg})(19\ \text{m/s})^2}{25\ \text{m}}$$

$$= \boxed{1.2 \times 10^3\ \text{N}}$$

P6.39 The free-body diagram of the rock is shown in ANS. FIG. P6.39. Take the *x* direction inward toward the center of the circle. The mass of the rock does not change. We know when $r_1 = 2.50$ m, $v_1 = 20.4$ m/s, and $T_1 = 50.0$ N. To find T_2 when $r_2 = 1.00$ m, and $v_2 = 51.0$ m/s, we use Newton's second law in the horizontal direction:



ANS. FIG. P6.39

$$\sum F_x = ma_x$$

In both cases,

$$T_1 = \frac{mv_1^2}{r_1} \quad \text{and} \quad T_2 = \frac{mv_2^2}{r_2}$$

Taking the ratio of the two tensions gives

$$\frac{T_2}{T_1} = \frac{v_2^2}{v_1^2}\frac{r_1}{r_2} = \left(\frac{51.0\ \text{m/s}}{20.4\ \text{m/s}}\right)^2\left(\frac{2.50\ \text{m}}{1.00\ \text{m}}\right) = 15.6$$

then

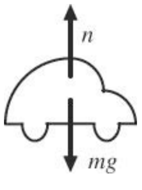
$$T_2 = 15.6T_1 = 15.6(50.0\ \text{N}) = \boxed{781\ \text{N}}$$

We assume the tension in the string is not altered by friction from the hole in the table.

P6.40 (a) We first convert the speed of the car to SI units:

$$v = (30\ \text{km/h})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\,000\ \text{m}}{1\ \text{km}}\right)$$

$$= 8.33\ \text{m/s}$$



ANS. FIG. P6.40

Newton's second law in the vertical direction then gives

$$\sum F_y = ma_y: \qquad +n - mg = -\frac{mv^2}{r}$$

Solving for the normal force,

$$\begin{aligned} n &= m\left(g - \frac{v^2}{r}\right) \\ &= (1\,800\text{ kg})\left[9.80\text{ m/s}^2 - \frac{(8.33\text{ m/s})^2}{20.4\text{ m}}\right] \\ &= \boxed{1.15\times10^4\text{ N up}} \end{aligned}$$

(b) At the maximum speed, the weight of the car is just enough to provide the centripetal force, so $n = 0$. Then $mg = \dfrac{mv^2}{r}$ and

$$v = \sqrt{gr} = \sqrt{(9.80\text{ m/s}^2)(20.4\text{ m})} = \boxed{14.1\text{ m/s}} = 50.9\text{ km/h}$$

P6.41 (a) The free-body diagram in ANS. FIG. P6.40 shows the forces on the car in the vertical direction. Newton's second law then gives

$$\sum F_y = ma_y = \frac{mv^2}{R}$$

$$mg - n = \frac{mv^2}{R} \qquad n = \boxed{mg - \frac{mv^2}{R}}$$

(b) When $n = 0$, $\qquad mg = \dfrac{mv^2}{R}$

Then, $v = \boxed{\sqrt{gR}}$

A more gently curved bump, with larger radius, allows the car to have a higher speed without leaving the road. This speed is proportional to the square root of the radius.

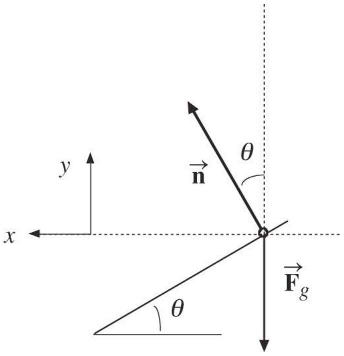
P6.42 The free-body diagram for the object is shown in ANS. FIG. P6.42. The object travels in a circle of radius $r = L\cos\theta$ about the vertical rod.

Taking inward toward the center of the circle as the positive x direction, we have

$$\sum F_x = ma_x: \quad n\sin\theta = \frac{mv^2}{r}$$

$$\sum F_y = ma_y:$$

$$n\cos\theta - mg = 0 \rightarrow n\cos\theta = mg$$



ANS. FIG. P6.42

Dividing, we find

$$\frac{n\sin\theta}{n\cos\theta} = \frac{mv^2/r}{gr} \qquad \to \qquad \tan\theta = \frac{v^2}{gr}$$

Solving for *v* gives

$$v^2 = gr\tan\theta$$

$$v^2 = g(L\cos\theta)\tan\theta$$

$$\boxed{v = (gL\sin\theta)^{1/2}}$$

P6.43 Let v_i represent the speed of the object at time 0. We have

$$\int_{v_i}^{v}\frac{dv}{v} = -\frac{b}{m}\int_i^t dt \qquad \ln v\Big|_{v_i}^{v} = -\frac{b}{m}t\Big|_i^t$$

$$\ln v - \ln v_i = -\frac{b}{m}(t-0) \qquad \ln\left(v/v_i\right) = -\frac{bt}{m}$$

$$v/v_i = e^{-bt/m} \qquad \boxed{v = v_i e^{-bt/m}}$$

From its original value, the speed decreases rapidly at first and then more and more slowly, asymptotically approaching zero.

$$\boxed{\text{In this model the object keeps losing speed forever. It travels a finite distance in stopping.}}$$

The distance it travels is given by

$$\int_0^r dr = v_i\int_0^t e^{-bt/m}dt$$

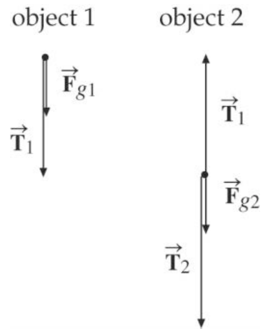
$$r = -\frac{m}{b}v_i\int_0^t e^{-bt/m}\left(-\frac{b}{m}dt\right) = -\frac{m}{b}v_i\,e^{-bt/m}\Big|_0^t$$

$$= -\frac{m}{b}v_i\left(e^{-bt/m}-1\right) = \frac{mv_i}{b}\left(1-e^{-bt/m}\right)$$

As *t* goes to infinity, the distance approaches $\frac{mv_i}{b}(1-0) = mv_i/b$.

P6.44 The radius of the path of object 1 is twice that of object 2. Because the strings are always "collinear," both objects take the same time interval to travel around their respective circles; therefore, the speed of object 1 is twice that of object 2.

The free-body diagrams are shown in ANS. FIG. P6.44. We are given $m_1 = 4.00$ kg, $m_2 = 3.00$ kg, $v = 4.00$ m/s, and $\ell = 0.500$ m.



ANS. FIG. P6.44

Taking down as the positive direction, we have

$$\text{Object 1: } T_1 + m_1 g = \frac{m_1 v_1^2}{r_1}, \quad \text{where } v_1 = 2v,\ r_1 = 2\ell.$$

$$\text{Object 2: } T_2 - T_1 + m_2 g = \frac{m_2 v_2^2}{r_2}, \quad \text{where } v_2 = v,\ r_2 = 2\ell.$$

(a) From above:

$$T_1 = \frac{m_1 v_1^2}{r_1} - m_1 g = m_1\left(\frac{v_1{}^2}{r_1} - g\right)$$

$$T_1 = (4.00\text{ kg})\left[\frac{[2(4.00\text{ m/s})]^2}{2(0.500\text{ m})} - 9.80\text{ m/s}^2\right]$$

$$T_1 = 216.8\text{ N} = \boxed{217\text{ N}}$$

(b) From above:

$$T_2 = T_1 + \frac{m_2 v_2^2}{r_2} - m_2 g$$

$$T_2 = T_1 + m_2\left(\frac{v_2{}^2}{r_2} - g\right)$$

$$T_2 = T_1 + (3.00\text{ kg})\left[\frac{(4.00\text{ m/s})^2}{0.500\text{ m}} - 9.80\text{ m/s}^2\right]$$

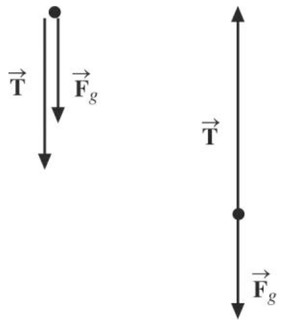
$$T_2 = 216.8\text{ N} + 66.6\text{ N} = 283.4\text{ N} = \boxed{283\text{ N}}$$

(c) $\boxed{\text{From above, } T_2 > T_1 \text{ always, so string 2 will break first.}}$

P6.45 (a) At each point on the vertical circular path, two forces are acting on the ball (see ANS. FIG. P6.45):

(1) $\boxed{\text{The downward gravitational force}}$ with constant magnitude $F_g = mg$

(2) $\boxed{\text{The tension force in the string, always directed toward the center of the path}}$



ANS. FIG. P6.45

(b) ANS. FIG. P6.45 shows the forces acting on the ball when it is at the highest point on the path (left-hand diagram) and when it is at the bottom of the circular path (right-hand diagram). Note that the gravitational force has the same magnitude and direction at each point on the circular path. The tension force varies in magnitude at different points and is always directed toward the center of the path.

(c) At the top of the circle, $F_c = mv^2/r = T + F_g$, or

$$T = \frac{mv^2}{r} - F_g = \frac{mv^2}{r} - mg = m\left(\frac{v^2}{r} - g\right)$$

$$= (0.275\text{ kg})\left[\frac{(5.20\text{ m/s})^2}{0.850\text{ m}} - 9.80\text{ m/s}^2\right] = \boxed{6.05\text{ N}}$$

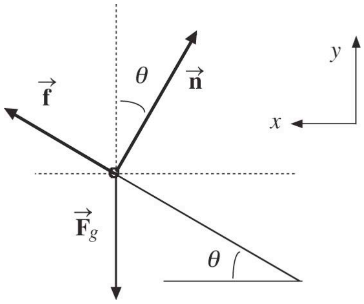
(d) At the bottom of the circle, $F_c = mv^2/r = T - F_g = T - mg$, and solving for the speed gives

$$v^2 = \frac{r}{m}(T - mg) = r\left(\frac{T}{m} - g\right) \quad \text{and} \quad v = \sqrt{r\left(\frac{T}{m} - g\right)}$$

If the string is at the breaking point at the bottom of the circle, then $T = 22.5$ N, and the speed of the object at this point must be

$$v = \sqrt{(0.850\text{ m})\left(\frac{22.5\text{ N}}{0.275\text{ kg}} - 9.80\text{ m/s}^2\right)} = \boxed{7.82\text{ m/s}}$$

P6.46 The free-body diagram is shown on the right, where it is assumed that friction points up the incline, otherwise, the child would slide down the incline. The net force is directed left toward the center of the circular path in which the child travels. The radius of this path is $R = d\cos\theta$.



ANS. FIG. P6.46

Three forces act on the child, a normal force, static friction, and gravity. The relations of their force components are:

$$\sum F_x: \quad f_s\cos\theta - n\sin\theta = mv^2/R \qquad \textbf{[1]}$$

$$\sum F_y: \quad f_s\sin\theta + n\cos\theta - mg = 0 \rightarrow$$

$$f_s\sin\theta + n\cos\theta = mg \qquad \textbf{[2]}$$

Solve for the static friction and normal force.

To solve for static friction, multiply equation [1] by cos θ and equation [2] by sin θ and add:

$$\cos\theta\left[f_s\cos\theta - n\sin\theta\right] + \sin\theta\left[f_s\sin\theta - n\cos\theta\right] = \cos\theta\left(\frac{mv^2}{R}\right) + \sin\theta(mg)$$

$$f_s = mg\sin\theta + \left(\frac{mv^2}{R}\right)\cos\theta$$

To solve for the normal force, multiply equation [1] by –sin θ and equation [2] by cos θ and add:

$$-\sin\theta\left[f_s\cos\theta - n\sin\theta\right] + \cos\theta\left[f_s\sin\theta - n\cos\theta\right] = -\sin\theta\left(\frac{mv^2}{R}\right) + \cos\theta(mg)$$

$$n = mg\cos\theta - \left(\frac{mv^2}{R}\right)\sin\theta$$

In the above, we have used $\sin^2\theta + \cos^2\theta = 1$.

If the above equations are to be consistent, static friction and the normal force must satisfy the condition $f_s \le \mu_s n$; this means

$$(mg)\sin\theta + (mv^2/R)\cos\theta \le \mu_s[(mg)\cos\theta - (mv^2/R)\sin\theta] \rightarrow$$
$$v^2(\cos\theta + \mu_s\sin\theta) \le g\,R(\mu_s\cos\theta - \sin\theta)$$

Using this result, and that $R = d\cos\theta$, we have the requirement that

$$v \le \sqrt{\frac{gd\cos\theta(\mu_s\cos\theta - \sin\theta)}{(\cos\theta + \mu_s\sin\theta)}}$$

If this condition cannot be met, if v is too large, the physical situation cannot exist.

The values given in the problem are $d = 5.32$ m, $\mu_s = 0.700$, $\theta = 20.0°$, and $v = 3.75$ m/s. Check whether the given value of v satisfies the above condition:

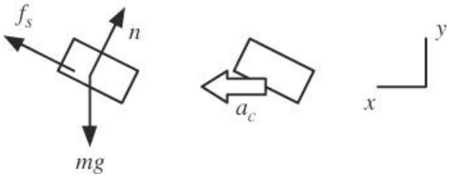
$$\sqrt{\frac{(9.80\text{ m/s}^2)(5.32\text{ m})\cos 20.0°\left[(0.700)\cos 20.0° - \sin 20.0°\right]}{(\cos 20.0° + 0.700\sin 20.0°)}}$$
$$= 3.62\text{ m/s}$$

$$\boxed{\text{The situation is impossible because the speed of the child given in the problem is too large: static friction could not keep the child in place on the incline.}}$$

P6.47 (a) The speed of the bag is

$$\frac{2\pi(7.46\ \text{m})}{38\ \text{s}} = 1.23\ \text{m/s}$$

The total force on it must add to



ANS. FIG. P6.47

$$ma_c = \frac{mv^2}{r}$$

$$= \frac{(30\ \text{kg})(1.23\ \text{m/s})^2}{7.46\ \text{m}} = 6.12\ \text{N}$$

Newton's second law gives

$$\sum F_x = ma_x: \quad f_s\cos 20.0° - n\sin 20.0° = 6.12\ \text{N}$$

$$\sum F_y = ma_y: \quad f_s\sin 20.0° + n\cos 20.0° - (30.0\ \text{kg})(9.80\ \text{m/s}^2) = 0$$

Solving for the normal force gives

$$n = \frac{f_s\cos 20.0° - 6.12\ \text{N}}{\sin 20.0°}$$

Substituting,

$$f_s\sin 20.0° + f_s\frac{\cos^2 20.0°}{\sin 20.0°} - (6.12\ \text{N})\frac{\cos 20.0°}{\sin 20.0°} = 294\ \text{N}$$

$$f_s(2.92) = 294\ \text{N} + 16.8\ \text{N}$$

$$f_s = \boxed{106\ \text{N}}$$

(b) The speed of the bag is now

$$v = \frac{2\pi(7.94\ \text{m})}{34\ \text{s}} = 1.47\ \text{m/s}$$

which corresponds to a total force of

$$ma_c = \frac{mv^2}{r}$$

$$= \frac{(30\ \text{kg})(1.47\ \text{m/s})^2}{7.94\ \text{m}} = 8.13\ \text{N}$$

Newton's second law then gives

$$f_s\cos 20 - n\sin 20 = 8.13\ \text{N}$$

$$f_s\sin 20 + n\cos 20 = 294\ \text{N}$$

Solving for *n*,

$$n = \frac{f_s\cos 20.0° - 8.13\ \text{N}}{\sin 20.0°}$$

Substituting,

$$f_s \sin 20.0° + f_s \frac{\cos^2 20.0°}{\sin 20.0°} - (8.13\ \text{N})\frac{\cos 20.0°}{\sin 20.0°} = 294\ \text{N}$$

$$f_s(2.92) = 294\ \text{N} + 22.4\ \text{N}$$

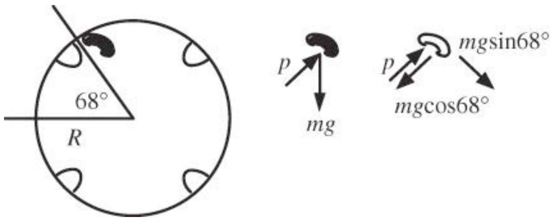
$$f_s = 108\ \text{N}$$

$$n = \frac{(108\ \text{N})\cos 20.0° - 8.13\ \text{N}}{\sin 20.0°} = 273\ \text{N}$$

$$\mu_s = \frac{f_s}{n} = \frac{108\ \text{N}}{273\ \text{N}} = \boxed{0.396}$$

P6.48 When the cloth is at a lower angle θ, the radial component of $\sum F = ma$ reads

$$n + mg\sin\theta = \frac{mv^2}{r}$$



ANS. FIG. P6.48

At $\theta = 68.0°$, the normal force drops to zero and $g\sin 68° = \frac{v^2}{r}$:

$$v = \sqrt{rg\sin 68°} = \sqrt{(0.33\ \text{m})(9.8\ \text{m/s}^2)\sin 68°} = 1.73\ \text{m/s}$$

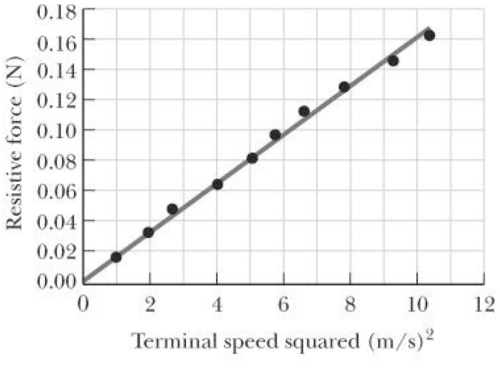
The rate of revolution is

$$\text{angular speed} = (1.73\ \text{m/s})\left(\frac{1\ \text{rev}}{2\pi r}\right)\left(\frac{2\pi r}{2\pi(0.33\ \text{m})}\right)$$

$$= \boxed{0.835\ \text{rev/s}} = 50.1\ \text{rev/min}$$

P6.49 The graph in Figure 6.16b is shown in ANS. FIG. P6.49.

(a) The graph line is straight, so we may use any two points on it to find the slope. It is convenient to take the origin as one point, and we read ($9.9\ \text{m}^2/\text{s}^2$, 0.16 N) as the coordinates of another point. Then the slope is



ANS. FIG. P6.49

$$\text{slope} = \frac{0.160\ \text{N} - 0}{9.9\ \text{m}^2/\text{s}^2} = \boxed{0.016\ 2\ \text{kg/m}}$$

(b) In $R = \frac{1}{2} D\rho A v^2$, we identify the vertical-axis variable as *R* and the horizontal-axis variable as v^2. Then the slope is

$$\text{slope} = \frac{R}{v^2} = \frac{\frac{1}{2} D\rho A v^2}{v^2} = \boxed{\frac{1}{2} D\rho A}$$

(c) We follow the directions in the problem statement:

$$\frac{1}{2} D\rho A = 0.0162\ \text{kg/m}$$

$$D = \frac{2(0.0162\ \text{kg/m})}{(1.20\ \text{kg/m}^3)\pi(0.105\ \text{m})^2} = \boxed{0.778}$$

(d) From the table, the eighth point is at force

$$mg = 8(1.64 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2) = 0.129\ \text{N}$$

and horizontal coordinate $(2.80\ \text{m/s})^2$. The vertical coordinate of the line is here

$$(0.016\,2\ \text{kg/m})(2.80\ \text{m/s})^2 = 0.127\ \text{N}$$

The scatter percentage is

$$\frac{0.129\ \text{N} - 0.127\ \text{N}}{0.127\ \text{N}} = \boxed{1.5\%}$$

(e) The interpretation of the graph can be stated thus:

For stacked coffee filters falling at terminal speed, a graph of air resistance force as a function of squared speed demonstrates that the force is proportional to the speed squared within the experimental uncertainty estimated as 2%. This proportionality agrees with that described by the theoretical equation $R = \frac{1}{2} D\rho A v^2$. The value of the constant slope of the graph implies that the drag coefficient for coffee filters is $D = 0.78 \pm 2\%$.

P6.50 (a) The forces acting on the ice cube are the Earth's gravitational force, straight down, and the basin's normal force, upward and inward at 35.0° with the vertical. We choose the x and y axes to be horizontal and vertical, so that the acceleration is purely in the x direction. Then

$$\sum F_x = ma_x: \quad n\sin 35° = mv^2/R$$

$$\sum F_y = ma_y: \quad n\cos 35° - mg = 0$$

Dividing eliminates the normal force:

$$n\sin 35.0°/n\cos\ 35.0° = mv^2/Rmg$$

$$\tan 35.0° = v^2/Rg$$

$$\boxed{v = \sqrt{Rg\tan 35.0°} = \sqrt{\left(6.86\text{ m/s}^2\right)R}}$$

(b) $\boxed{\text{The mass is unnecessary.}}$

(c) The answer to (a) indicates that the speed is proportional to the square root of the radius, so $\boxed{\text{increasing the radius will make the required speed increase.}}$

(d) The period of revolution is given by

$$T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{Rg\tan 35.0°}} = \left(2.40\text{ s}/\sqrt{\text{m}}\right)\sqrt{R}$$

$\boxed{\text{When the radius increases, the period increases.}}$

(e) $\boxed{\text{On a larger circle, the ice cube's speed is proportional to } \sqrt{R} \text{ but the distance it travels is proportional to } R\text{, so the time interval required is proportional to } R/\sqrt{R} = \sqrt{R}.}$

P6.51 Take the positive x axis up the hill. Newton's second law in the x direction then gives

$$\sum F_x = ma_x: \quad +T\sin\theta - mg\sin\phi = ma$$

from which we obtain

$$a = \frac{T}{m}\sin\theta - g\sin\phi \qquad \textbf{[1]}$$

In the y direction,

$$\sum F_y = ma_y: \quad +T\cos\theta - mg\cos\phi = 0$$

Solving for the tension gives

$$T = \frac{mg\cos\phi}{\cos\theta} \qquad \textbf{[2]}$$

Substituting for *T* from [2] into [1] gives

$$a = \frac{g\cos\phi\sin\theta}{\cos\theta} - g\sin\phi$$

$$a = \boxed{g(\cos\phi\tan\theta - \sin\phi)}$$

P6.52 (a) We first convert miles per hour to feet per second:

$$v = (300\ \text{mi/h})\left(\frac{88.0\ \text{ft/s}}{60.0\ \text{mi/h}}\right) = 440\ \text{ft/s at the top of the loop}$$

and $v = 450\ \text{mi/h} = 660\ \text{ft/s}$ at the bottom of the loop.

At the lowest point, his seat exerts an upward force; therefore, his weight seems to increase. His apparent weight is

$$F_g' = mg + m\frac{v^2}{r} = 160\ \text{lb} + \left(\frac{160\ \text{lb}}{32.0\ \text{ft/s}^2}\right)\frac{(660\ \text{ft/s})^2}{1\,200\ \text{ft}} = \boxed{1\,975\ \text{lb}}$$

(b) At the highest point, the force of the seat on the pilot is directed down and

$$F_g' = mg - m\frac{v^2}{r} = 160\ \text{lb} - \left(\frac{160\ \text{lb}}{32.0\ \text{ft/s}^2}\right)\frac{(440\ \text{ft/s})^2}{1\,200\ \text{ft}} = \boxed{-647\ \text{lb}}$$

Since the plane is upside down, the seat exerts this downward force as a normal force.

(c) When $F_g' = 0$, then $mg = \frac{mv^2}{R}$. If we vary the aircraft's R and v such that this equation is satisfied, then the pilot feels weightless.

P6.53 (a) The only horizontal force on the car is the force of friction, with a maximum value determined by the surface roughness (described by the coefficient of static friction) and the normal force (here equal to the gravitational force on the car).

(b) From Newton's second law in one dimension,

$$\sum F_x = ma_x: \quad -f = ma \quad \rightarrow \quad a = -\frac{f}{m} = \left(v^2 - v_0^2\right)/2\left(x - x_0\right)$$

solving for the stopping distance gives

$$x - x_0 = \frac{m\left(v^2 - v_0^2\right)}{2f} = \frac{(1\ 200\ \text{kg})\left[0^2 - (20.0\ \text{m/s})^2\right]}{2(-7\ 000\ \text{N})} = \boxed{34.3\ \text{m}}$$

(c) Newton's second law now gives

$$f = \frac{mv^2}{r}$$

or $$r = \frac{mv^2}{f} = \frac{(1\ 200\ \text{kg})(20.0\ \text{m/s})^2}{7\ 000\ \text{N}} = \boxed{68.6\ \text{m}}$$

A top view shows that you can avoid running into the wall by turning through a quarter-circle, if you start at least this far away from the wall.

(d) Braking is better. You should not turn the wheel. If you used any of the available friction force to change the direction of the car, it would be unavailable to slow the car, and the stopping distance would be longer.

(e) The conclusion is true in general. The radius of the curve you can barely make is twice your minimum stopping distance.

P6.54 (a) Since the object of mass m_2 is in equilibrium, $\sum F_y = T - m_2 g = 0$

or $T = \boxed{m_2 g}$.

(b) The tension in the string provides the required centripetal acceleration of the puck.

Thus, $F_c = T = \boxed{m_2 g}$.

(c) From $F_c = \dfrac{m_1 v^2}{R}$,

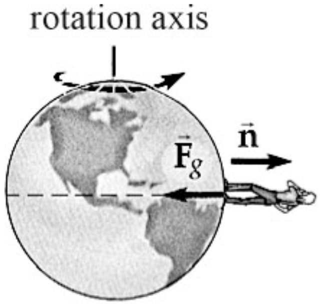
we have $v = \sqrt{\dfrac{RF_c}{m_1}} = \boxed{\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}}$

(d) $\boxed{\text{The puck will spiral inward, gaining speed as it does so.}}$ It gains speed because the extra-large string tension pulls at an angle of less than 90° to the direction of the inward-spiraling velocity, producing forward tangential acceleration as well as inward radial acceleration of the puck.

(e) $\boxed{\text{The puck will spiral outward, slowing down as it does so.}}$

P6.55 (a) The gravitational force exerted by the planet on the person is

$$mg = (75.0\text{ kg})(9.80\text{ m/s}^2)$$
$$= \boxed{735\text{ N}}\text{ down}$$



ANS. FIG. P6.55

Let n represent the force exerted on the person by a scale, which is an upward force whose size is her "apparent weight." The true weight is mg down. For the person at the equator, summing up forces on the object in the direction towards the Earth's center gives $\sum F = ma$:

$$mg - n = ma_c$$

where $a_c = v^2/R_E = 0.033\,7\text{ m/s}^2$

is the centripetal acceleration directed toward the center of the Earth.

Thus, we can solve part (c) before part (b) by noting that

$$n = m(g - a_c) < mg$$

(c) or $mg = n + ma_c > n$.

$\boxed{\text{The gravitational force is greater. The normal force is smaller, just as one experiences at the top of a moving ferris wheel.}}$

(b) If $m = 75.0$ kg and $g = 9.80\text{ m/s}^2$, at the equator we have

$$n = m(g - a_c) = (75.0\text{ kg})(9.800\text{ m/s}^2 - 0.033\,7\text{ m/s}^2) = \boxed{732\text{ N}}$$

P6.56 (a) $v = v_i + kx$ implies the acceleration is

$$\boxed{a = \frac{dv}{dt} = 0 + k\frac{dx}{dt} = +kv}$$

(b) The total force is

$$\sum F = ma = m(+kv)$$

As a vector, the force is parallel or antiparallel to the velocity:

$$\boxed{\sum \vec{\mathbf{F}} = km\vec{\mathbf{v}}}$$

(c) For k positive, some feedback mechanism could be used to impose such a force on an object for a while. The object's speed rises exponentially.

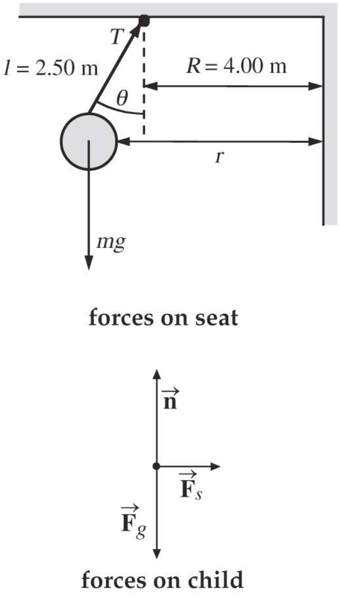
(d) For k negative, think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed.

P6.57 (a) As shown in the free-body diagram on the right, the mass at the end of the chain is in vertical equilibrium. Thus,

$$T\cos\theta = mg \qquad \textbf{[1]}$$

Horizontally, the mass is accelerating toward the center of a circle of radius *r*:

$$T\sin\theta = ma_r = \frac{mv^2}{r} \qquad \textbf{[2]}$$



forces on seat

forces on child

ANS. FIG. P6.57

Here, *r* is the sum of the radius of the circular platform $R = D/2 = 4.00$ m and $2.50\sin\theta$:

$$r = (2.50\sin\theta + 4.00)\text{ m}$$
$$r = (2.50\sin 28.0° + 4.00)\text{ m}$$
$$= 5.17\text{ m}$$

We solve for the tension *T* from [1]:

$$T\cos\theta = mg \rightarrow T = \frac{mg}{\cos\theta}$$

and substitute into [2] to obtain

$$\tan\theta = \frac{a_r}{g} = \frac{v^2}{gr}$$

$$v^2 = gr\tan\theta = (9.80\text{ m/s}^2)(5.17\text{ m})(\tan 28.0°)$$

$$v = \boxed{5.19\text{ m/s}}$$

(b) The free-body diagram for the child is shown in ANS. FIG. P6.57.

(c) $T = \dfrac{mg}{\cos\theta} = \dfrac{(40.0\text{ kg})(9.80\text{ m/s}^2)}{\cos 28.0°} = \boxed{444\text{ N}}$

P6.58 (a) The putty, when dislodged, rises and returns to the original level in time t. To find t, we use $v_f = v_i + at$: i.e., $-v = +v - gt$ or $t = \dfrac{2v}{g}$, where v is the speed of a point on the rim of the wheel.

If R is the radius of the wheel, $v = \dfrac{2\pi R}{t}$, so $t = \dfrac{2v}{g} = \dfrac{2\pi R}{v}$.

Thus, $v^2 = \pi Rg$ and $v = \boxed{\sqrt{\pi Rg}}$.

(b) The putty is dislodged when F, the force holding it to the wheel, is

$$F = \frac{mv^2}{R} = \boxed{m\pi g}$$

P6.59 (a) The wall's normal force pushes inward:

$$\sum F_{\text{inward}} = ma_{\text{inward}}$$

becomes

$$n = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2 = \frac{4\pi^2 Rm}{T^2}$$

The friction and weight balance:

$$\sum F_{\text{upward}} = ma_{\text{upward}}$$

becomes

$$+f - mg = 0$$

so with the person just ready to start sliding down,

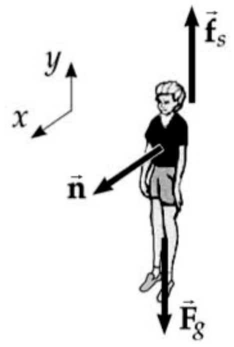
$$f_s = \mu_s n = mg$$

Substituting,

$$\mu_s n = \mu_s \frac{4\pi^2 Rm}{T^2} = mg$$

Solving,

$$T^2 = \frac{4\pi^2 R\mu_s}{g}$$



ANS. FIG. P6.59

gives

$$\boxed{T = \sqrt{\frac{4\pi^2 R\mu_s}{g}}}$$

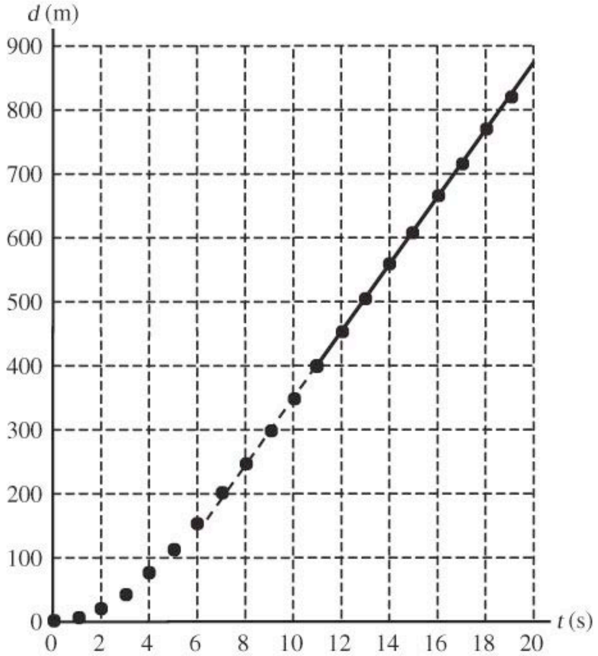
(b) The gravitational and friction forces remain constant. (Static friction adjusts to support the weight.) The normal force increases. The person remains in motion with the wall.

(c) The gravitational force remains constant. The normal and friction forces decrease. The person slides relative to the wall and downward into the pit.

P6.60 (a)

t(s)	*d* (m)	*t*(s)	*d* (m)
1.00	4.88	11.0	399
2.00	18.9	12.0	452
3.00	42.1	13.0	505
4.00	43.8	14.0	558
5.00	112	15.0	611
6.00	154	16.0	664
7.00	199	17.0	717
8.00	246	18.0	770
9.00	296	19.0	823
10.0	347	20.0	876

(b)



(c) A straight line fits the points from $t = 11.0$ s to 20.0 s quite precisely. Its slope is the terminal speed.

$$v_T = \text{slope} = \frac{876\text{ m} - 399\text{ m}}{20.0\text{ s} - 11.0\text{ s}} = \boxed{53.0\text{ m/s}}$$

P6.61 (a) If the car is about to slip *down* the incline, *f* is directed up the incline.

$$\sum F_y = n\cos\theta + f\sin\theta - mg = 0$$

where $f = \mu_s n$. Substituting,

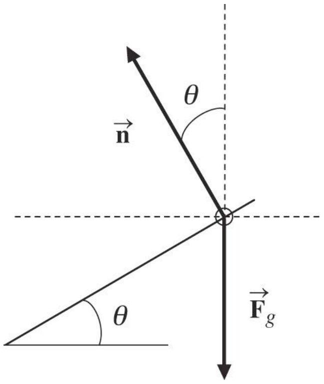
$$n = \frac{mg}{\cos\theta\left(1 + \mu_s\tan\theta\right)}$$

and $f = \dfrac{\mu_s mg}{\cos\theta\left(1 + \mu_s\tan\theta\right)}$

Then, $\sum F_x = n\sin\theta - f\cos\theta = m\dfrac{v_{\min}^2}{R}$

yields

$$\boxed{v_{\min} = \sqrt{\frac{Rg\left(\tan\theta - \mu_s\right)}{1 + \mu_s\tan\theta}}}$$



ANS. FIG. P6.61

When the car is about to slip *up* the incline, *f* is directed down the incline.

Then,

$$\sum F_y = n\cos\theta - f\sin\theta - mg = 0, \text{ with } f = \mu_s n$$

This yields

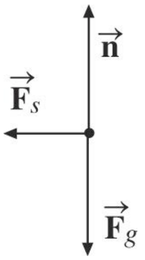
$$n = \frac{mg}{\cos\theta(1-\mu_s\tan\theta)} \text{ and } f = \frac{\mu_s mg}{\cos\theta(1-\mu_s\tan\theta)}$$

In this case, $\sum F_x = n\sin\theta + f\cos\theta = m\frac{v_{\max}^2}{R}$, which gives

$$\boxed{v_{\max} = \sqrt{\frac{Rg(\tan\theta+\mu_s)}{1-\mu_s\tan\theta}}}$$

(b) If $v_{\min} = \sqrt{\frac{Rg(\tan\theta-\mu_s)}{1+\mu_s\tan\theta}} = 0$, then $\boxed{\mu_s = \tan\theta}$.

P6.62 There are three forces on the child, a vertical normal force, a horizontal force (combination of friction and a horizontal force from a seat belt), and gravity.



ANS. FIG. P6.62

$$\sum F_x: \quad F_s = mv^2/R$$

$$\sum F_y: \quad n - mg = 0 \rightarrow n = mg$$

The magnitude of the net force is

$$F_{net} = \sqrt{(mv^2/R)^2 + (mg)^2}$$

with a direction of

$$\theta = \tan^{-1}\left[\frac{mg}{mv^2/R}\right] = \tan^{-1}\left[\frac{gR}{v^2}\right] \text{ above the horizontal}$$

For *m* = 40.0 kg and *R* = 10.0 m:

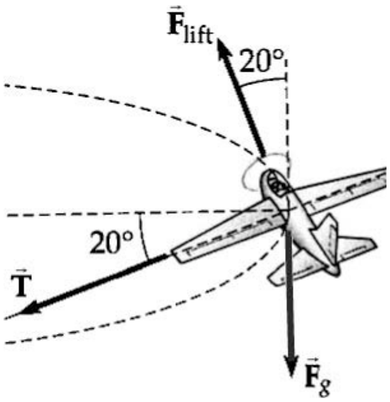
$$F_{net} = \sqrt{\left[\frac{(40.0\text{ kg})(3.00\text{ m/s})^2}{10.0\text{ m}}\right]^2 + \left[(40.0\text{ kg})(9.80\text{ m/s}^2)\right]^2}$$

$$\boxed{F_{net} = 394\text{ N}}$$

direction: $\theta = \tan^{-1}\left[\frac{(9.80\text{ m/s}^2)(10.0\text{ m})}{(3.00\text{ m/s})^2}\right] \rightarrow \boxed{\theta = 84.7°}$

P6.63 The plane's acceleration is toward the center of the circle of motion, so it is horizontal. The radius of the circle of motion is (60.0 m) cos 20.0° = 56.4 m and the acceleration is

$$a_c = \frac{v^2}{r} = \frac{(35\ \text{m/s})^2}{56.4\ \text{m}}$$
$$= 21.7\ \text{m/s}^2$$



ANS. FIG. P6.63

We can also calculate the weight of the airplane:

$$F_g = mg$$
$$= (0.750\ \text{kg})(9.80\ \text{m/s}^2)$$
$$= 7.35\ \text{N}$$

We define our axes for convenience. In this case, two of the forces—one of them our force of interest—are directed along the 20.0° line. We define the x axis to be directed in the $+\vec{T}$ direction, and the y axis to be directed in the direction of lift. With these definitions, the x component of the centripetal acceleration is

$$a_{cx} = a_c \cos 20.0°$$

and $\sum F_x = ma_x$ yields $T + F_g \sin 20.0° = ma_{cx}$

Solving for T,

$$T = ma_{cx} - F_g \sin 20.0°$$

Substituting,

$$T = (0.750\ \text{kg})(21.7\ \text{m/s}^2)\cos 20.0° - (7.35\ \text{N})\sin 20.0°$$

Computing,

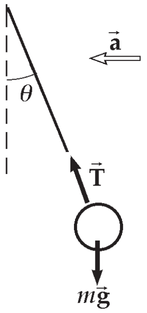
$$T = 15.3\ \text{N} - 2.51\ \text{N} = \boxed{12.8\ \text{N}}$$

***P6.64** (a) While the car negotiates the curve, the accelerometer is at the angle θ.

Horizontally: $T\sin\theta = \frac{mv^2}{r}$

Vertically: $T\cos\theta = mg$

where r is the radius of the curve, and v is the speed of the car.



ANS. FIG. P6.64

By division, $\tan\theta = \dfrac{v^2}{rg}$

Then

$$a_c = \frac{v^2}{r} = g\tan\theta:$$

$$a_c = \left(9.80\ \text{m/s}^2\right)\tan 15.0°$$

$$a_c = \boxed{2.63\ \text{m/s}^2}$$

(b) $r = \dfrac{v^2}{a_c}$ gives $r = \dfrac{(23.0\ \text{m/s})^2}{2.63\ \text{m/s}^2} = \boxed{201\ \text{m}}$

(c) $v^2 = rg\tan\theta = (201\ \text{m})\left(9.80\ \text{m/s}^2\right)\tan 9.00°$

$$v = \boxed{17.7\ \text{m/s}}$$

Challenge Problems

P6.65 We find the terminal speed from

$$v = \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right] \qquad \textbf{[1]}$$

where $\exp(x) = e^x$ is the exponential function.

At $t \to \infty$: $v \to v_T = \dfrac{mg}{b}$

At $t = 5.54$ s: $0.500 v_T = v_T\left[1 - \exp\left(\dfrac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right)\right]$

Solving,

$$\exp\left(\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}}\right) = 0.500$$

$$\frac{-b(5.54\ \text{s})}{9.00\ \text{kg}} = \ln 0.500 = -0.693$$

$$b = \frac{(9.00\ \text{kg})(0.693)}{5.54\ \text{s}} = 1.13\ \text{kg/s}$$

(a) From $v_T = \frac{mg}{b}$, we have

$$v_T = \frac{(9.00\text{ kg})(9.80\text{ m/s}^2)}{1.13\text{ kg/s}} = \boxed{78.3\text{ m/s}}$$

(b) We substitute $0.750v_T$ on the left-hand side of equation [1]:

$$0.750v_T = v_T\left[1 - \exp\left(\frac{-1.13t}{9.00\text{ s}}\right)\right]$$

and solve for *t*:

$$\exp\left(\frac{-1.13t}{9.00\text{ s}}\right) = 0.250$$

$$t = \frac{9.00(\ln 0.250)}{-1.13}\text{ s} = \boxed{11.1\text{ s}}$$

(c) We differentiate equation [1] with respect to time,

$$\frac{dx}{dt} = \left(\frac{mg}{b}\right)\left[1 - \exp\left(-\frac{bt}{m}\right)\right]$$

then, integrate both sides

$$\int_{x_0}^{x} dx = \int_0^t \left(\frac{mg}{b}\right)\left[1 - \exp\left(\frac{-bt}{m}\right)\right]dt$$

$$x - x_0 = \frac{mgt}{b} + \left(\frac{m^2g}{b^2}\right)\exp\left(\frac{-bt}{m}\right)\Bigg|_0^t$$

$$= \frac{mgt}{b} + \left(\frac{m^2g}{b^2}\right)\left[\exp\left(\frac{-bt}{m}\right) - 1\right]$$

At *t* = 5.54 s,

$$x = (9.00\text{ kg})(9.80\text{ m/s}^2)\left(\frac{5.54\text{ s}}{1.13\text{ kg/s}}\right)$$

$$+\left(\frac{(9.00\text{ kg})^2(9.80\text{ m/s}^2)}{(1.13\text{ kg/s})^2}\right)[\exp(-0.693) - 1]$$

$$x = 434\text{ m} + 626\text{ m}(-0.500) = \boxed{121\text{ m}}$$

P6.66 (a) From Problem 6.33,

$$v = \frac{dx}{dt} = \frac{v_i}{1 + v_i kt}$$

$$\int_0^x dx = \int_0^t v_i \frac{dt}{1 + v_i kt} = \frac{1}{k}\int_0^t \frac{v_i k dt}{1 + v_i kt}$$

$$x\Big|_0^x = \frac{1}{k}\ln\left(1 + v_i kt\right)\Big|_0^t$$

$$x - 0 = \frac{1}{k}\left[\ln\left(1 + v_i kt\right) - \ln 1\right]$$

$$\boxed{x = \frac{1}{k}\ln\left(1 + v_i kt\right)}$$

(b) We have $\ln\left(1 + v_i kt\right) = kx$

$$1 + v_i kt = e^{kx} \quad \text{so} \quad v = \frac{v_i}{1 + v_i kt} = \frac{v_i}{e^{kx}} = \boxed{v_i e^{-kx} = v}$$

P6.67 Let the *x* axis point eastward, the *y* axis upward, and the *z* axis point southward.

(a) The range is $Z = \dfrac{v_i^2 \sin 2\theta_i}{g}$

The initial speed of the ball is therefore

$$v_i = \sqrt{\frac{gZ}{\sin 2\theta_i}} = \sqrt{\frac{\left(9.80 \text{ m/s}^2\right)\left(285 \text{ m}\right)}{\sin 96.0°}} = 53.0 \text{ m/s}$$

The time the ball is in the air is found from $\Delta y = v_{iy}t + \frac{1}{2}a_y t^2$ as

$$0 = \left(53.0 \text{ m/s}\right)\left(\sin 48.0°\right)t - \left(4.90 \text{ m/s}^2\right)t^2$$

giving $t = \boxed{8.04 \text{ s}}$.

(b) $$v_{xi} = \frac{2\pi R_e \cos\phi_i}{86\,400 \text{ s}} = \frac{2\pi\left(6.37 \times 10^6 \text{ m}\right)\cos 35.0°}{86\,400 \text{ s}} = \boxed{379 \text{ m/s}}$$

(c) 360° of latitude corresponds to a distance of $2\pi R_e$, so 285 m is a change in latitude of

$$\Delta\phi = \left(\frac{S}{2\pi R_e}\right)\left(360°\right) = \left(\frac{285 \text{ m}}{2\pi\left(6.37 \times 10^6 \text{ m}\right)}\right)\left(360°\right)$$

$$= 2.56 \times 10^{-3} \text{ degrees}$$

The final latitude is then

$$\phi_f = \phi_i - \Delta\phi = 35.0° - 0.002\,56° = 34.997\,4°$$

The cup is moving eastward at a speed

$$v_{xf} = \frac{2\pi R_e \cos\phi_f}{86\,400 \text{ s}}$$

which is larger than the eastward velocity of the tee by

$$\begin{aligned} \Delta v_x = v_{xf} - v_{xi} &= \left(\frac{2\pi R_e}{86\,400 \text{ s}}\right)\left[\cos\phi_f - \cos\phi_i\right] \\ &= \left(\frac{2\pi R_e}{86\,400 \text{ s}}\right)\left[\cos(\phi - \Delta\phi) - \cos\phi_i\right] \\ &= \left(\frac{2\pi R_e}{86\,400 \text{ s}}\right)\left[\cos\phi_i \cos\Delta\phi + \sin\phi_i \sin\Delta\phi - \cos\phi_i\right] \end{aligned}$$

Since $\Delta\phi$ is such a small angle, $\cos\Delta\phi \approx 1$ and

$$\Delta v_x \approx \left(\frac{2\pi R_e}{86\,400 \text{ s}}\right)\sin\phi_i \sin\Delta\phi$$

$$\begin{aligned} \Delta v_x &\approx \left[\frac{2\pi(6.37\times 10^6 \text{ m})}{86\,400 \text{ s}}\right]\sin 35.0° \sin 0.002\,56° \\ &= \boxed{1.19\times 10^{-2} \text{ m/s}} \end{aligned}$$

(d) $\Delta x = (\Delta v_x)t = (1.19\times 10^{-2} \text{ m/s})(8.04 \text{ s}) = 0.095\,5 \text{ m} = \boxed{9.55 \text{ cm}}$

P6.68 (a) We let *R* represent the radius of the hoop and *T* represent the period of its rotation. The bead moves in a circle with radius $r = R\sin\theta$ at a speed of

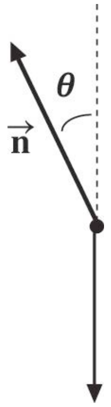
$$v = \frac{2\pi r}{T} = \frac{2\pi R \sin\theta}{T}$$

The normal force has an inward radial component of $n\sin\theta$ and an upward component of $n\cos\theta$.

$$\sum F_y = ma_y: \quad n\cos\theta - mg = 0$$

or

$$n = \frac{mg}{\cos\theta}$$



ANS. FIG. P6.68

Then $\sum F_x = n\sin\theta = m\dfrac{v^2}{r}$ becomes

$$\left(\frac{mg}{\cos\theta}\right)\sin\theta = \frac{m}{R\sin\theta}\left(\frac{2\pi R\sin\theta}{T}\right)^2$$

which reduces to $\dfrac{g\sin\theta}{\cos\theta} = \dfrac{4\pi^2 R\sin\theta}{T^2}$

This has two solutions: $\sin\theta = 0 \quad \Rightarrow \quad \theta = 0°$ **[1]**

and $\cos\theta = \dfrac{gT^2}{4\pi^2 R}$ **[2]**

If *R* = 15.0 cm and *T* = 0.450 s, the second solution yields

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.450\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 0.335 \quad \text{or} \quad \theta = 70.4°$$

Thus, in this case, the bead can ride at two positions: $\boxed{\theta = 70.4°}$ and $\boxed{\theta = 0°}$.

(b) At this slower rotation, solution [2] above becomes

$$\cos\theta = \frac{(9.80\ \text{m/s}^2)(0.850\ \text{s})^2}{4\pi^2(0.150\ \text{m})} = 1.20\text{, which is impossible.}$$

In this case, the bead can ride only at the bottom of the loop, $\boxed{\theta = 0°}$.

(c) $\boxed{\text{There is only one solution for (b) because the period is too large.}}$

(d) $\boxed{\begin{array}{l}\text{The equation that the angle must satisfy has two solutions}\\ \text{whenever } 4\pi^2 R > gT^2 \text{ but only the solution } 0° \text{ otherwise.}\\ \text{The loop's rotation must be faster than a certain threshold}\\ \text{value in order for the bead to move away from the lowest}\\ \text{position. Zero is always a solution for the angle.}\end{array}}$

(e) $\boxed{\begin{array}{l}\text{From the derivation of the solution in (a), there are never more}\\ \text{than two solutions.}\end{array}}$

P6.69 At terminal velocity, the accelerating force of gravity is balanced by friction drag:

$$mg = arv + br^2v^2$$

(a) With $r = 10.0\ \mu\text{m}$, $mg = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$

For water, $m = \rho V = 1\,000\ \text{kg/m}^3\left[\dfrac{4}{3}\pi\left(10^{-5}\ \text{m}\right)^3\right]$

$$mg = 4.11\times10^{-11} = \left(3.10\times10^{-9}\right)v + \left(0.870\times10^{-10}\right)v^2$$

Assuming v is small, ignore the second term on the right hand side: $\boxed{v = 0.013\,2\ \text{m/s}}$

(b) With $r = 100\ \mu\text{m}$, $mg = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$

Here we cannot ignore the second term because the coefficients are of nearly equal magnitude.

$$mg = 4.11\times10^{-8} = \left(3.10\times10^{-8}\right)v + \left(0.870\times10^{-8}\right)v^2$$

Taking the positive root,

$$v = \frac{-3.10 + \sqrt{(3.10)^2 + 4(0.870)(4.11)}}{2(0.870)} = \boxed{1.03\ \text{m/s}}$$

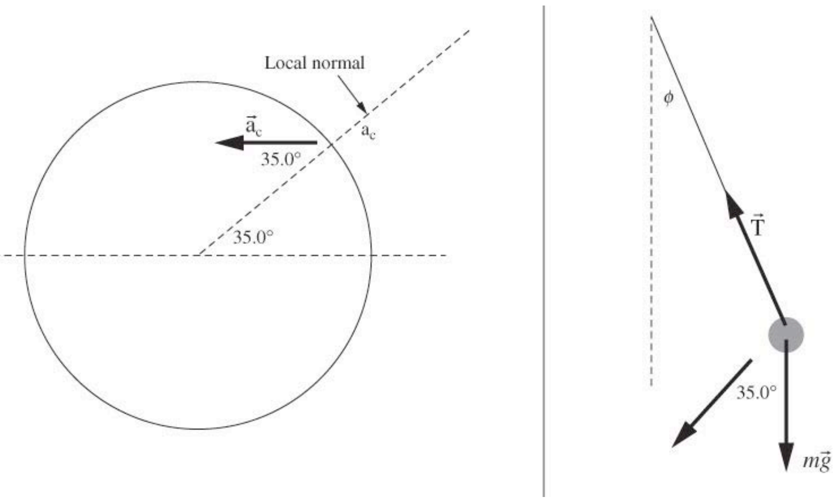
(c) With $r = 1.00\ \text{mm}$, $mg = \left(3.10\times10^{-7}\right)v + \left(0.870\times10^{-6}\right)v^2$

Assuming $v > 1\ \text{m/s}$, and ignoring the first term:

$$4.11\times10^{-5} = \left(0.870\times10^{-6}\right)v^2 \qquad v = \boxed{6.87\ \text{m/s}}$$

P6.70 At a latitude of 35°, the centripetal acceleration of a plumb bob is directed at 35° to the local normal, as can be seen from the following diagram below at left.

Therefore, if we look at a diagram of the forces on the plumb bob and its acceleration with the local normal in a vertical orientation, we see the second diagram in ANS. FIG. P6.70:



ANS. FIG. P6.70

We first find the centripetal acceleration of the plumb bob. The first figure shows that the radius of the circular path of the plumb bob is *R* cos 35.0°, where *R* is the radius of the Earth. The acceleration is

$$a_c = \frac{v^2}{r} = \frac{1}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 R\cos 35.0°}{T^2}$$

$$= \frac{4\pi^2\left(6.37\times 10^6\ \text{m}\right)\cos 35.0°}{\left(86\ 400\ \text{s}\right)^2} = 0.027\ 6\ \text{m/s}^2$$

Apply the particle under a net force model to the plumb bob in both *x* and *y* directions in the second diagram:

$$x\text{:}\ \ T\sin\phi = ma_c\sin 35.0°$$

$$y\text{:}\ \ mg - T\cos\phi = ma_c\cos 35.0°$$

Divide the equations:

$$\tan\phi = \frac{a_c\sin 35.0°}{g - a_c\cos 35.0°}$$

$$\tan\phi = \frac{\left(0.027\ 6\ \text{m/s}^2\right)\sin 35.0°}{9.80\ \text{m/s}^2 - \left(0.027\ 6\ \text{m/s}^2\right)\cos 35.0°} = 1.62\times 10^{-3}$$

$$\phi = \tan^{-1}\left(1.62\times 10^{-3}\right) = \boxed{0.092\ 8°}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P6.2	(a) 1.65×10^3 m/s; (b) 6.84×10^3 s
P6.4	215 N, horizontally inward
P6.6	(a) $\left(-0.233\hat{\mathbf{i}} + 0.163\hat{\mathbf{j}}\right)$ m/s^2; (b) 6.53 m/s, $\left(-0.181\hat{\mathbf{i}} + 0.181\hat{\mathbf{j}}\right)$ m/s^2
P6.8	(a) $(68.6\text{ N})\hat{\mathbf{i}} + (784\text{ N})\hat{\mathbf{j}}$; (b) $a = 0.857$ m/s^2
P6.10	The situation is impossible because the speed of the object is too small, requiring that the lower string act like a rod and push rather than like a string and pull.
P6.12	(a) the gravitational force and the contact force exerted on the water by the pail; (b) contact force exerted by the pail; (c) 3.13 m/s; (d) the water would follow the parabolic path of a projectile
P6.14	(a) 4.81 m/s; (b) 700 N
P6.16	(a) 2.49×10^4 N; (b) 12.1 m/s
P6.18	(a) 20.6 N; (b) 32.0 m/s^2 inward, 3.35 m/s^2 downward tangent to the circle; (c) 32.2 m/s^2 inward and below the cord at 5.98°; (d) no change; (e) acceleration is regardless of the direction of swing
P6.20	(a) 3.60 m/s^2; (b) $T = 0$; (c) noninertial observer in the car claims that the forces on the mass along x are T and a fictitious force $(-Ma)$; (d) inertial observer outside the car claims that T is the only force on M in the x direction
P6.22	93.8 N
P6.24	$\dfrac{2(vt-L)}{(g+a)t^2}$
P6.26	(a) 53.8 m/s; (b) 148 m
P6.28	(a) 6.27 m/s^2 downward; (b) 784 N directed up; (c) 283 N upward
P6.30	(a) 32.7 s^{-1}; (b) 9.80 m/s^2 down; (c) 4.90 m/s^2 down
P6.32	36.5 m/s
P6.34	(a) 2.03 N down; (b) 3.18 m/s^2 down; (c) 0.205 m/s down
P6.36	10^1 N
P6.38	1.2×10^3 N
P6.40	(a) 1.15×10^4 N up; (b) 14.1 m/s
P6.42	See Problem 6.42 for full derivation.

P6.44 (a) 217 N; (b) 283 N; (c) $T_2 > T_1$ always, so string 2 will break first

P6.46 The situation is impossible because the speed of the child given in the problem is too large: static friction could not keep the child in place on the incline

P6.48 0.835 rev/s

P6.50 (a) $v = \sqrt{Rg\tan 35.0°} = \sqrt{\left(6.86\text{ m/s}^2\right)R}$; (b) the mass is unnecessary; (c) increasing the radius will make the required speed increase; (d) when the radius increases, the period increases; (e) the time interval required is proportional to $R/\sqrt{R} = \sqrt{R}$

P6.52 (a) 1 975 lb; (b) −647 lb; (c) When $F_g' = 0$, then $mg = \dfrac{mv^2}{R}$.

P6.54 (a) m_2g; (b) m_2g; (c) $\sqrt{\left(\dfrac{m_2}{m_1}\right)gR}$; (d) The puck will spiral inward, gaining speed as it does so; (e) The puck will spiral outward, slowing down as it does so

P6.56 (a) $a = +kv$; (b) $\sum\vec{\mathbf{F}} = km\vec{\mathbf{v}}$; (c) some feedback mechanism could be used to impose such a force on an object; (d) think of a duck landing on a lake, where the water exerts a resistive force on the duck proportional to its speed

P6.58 (a) $\sqrt{\pi Rg}$; (b) $m\pi g$

P6.60 (a) See table in P6.60 (a); (b) See graph in P6.60 (b); (c) 53.0 m/s

P6.62 84.7°

P6.64 (a) 2.63 m/s^2; (b) 201 m; (c) 17.7 m/s

P6.66 (a) $x = \dfrac{1}{k}\ln\left(1 + v_ikt\right)$; (b) $v = v_ie^{-kx}$

P6.68 (a) $\theta\ = 70.4°$ and $\theta = 0°$; (b) $\theta = 0°$; (c) the period is too large; (d) Zero is always a solution for the angle; (e) there are never more than two solutions

P6.70 0.092 8°

7

Energy of a System

CHAPTER OUTLINE

7.1 Systems and Environments
7.2 Work Done by a Constant Force
7.3 The Scalar Product of Two Vectors
7.4 Work Done by a Varying Force
7.5 Kinetic Energy and the Work-Kinetic Energy Theorem
7.6 Potential Energy of a System
7.7 Conservative and Nonconservative Forces
7.8 Relationship Between Conservative Forces and Potential Energy
7.9 Energy Diagrams and Equilibrium of a System

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ7.1 Answer (c). Assuming that the cabinet has negligible speed during the operation, all of the work Alex does is used in increasing the gravitational potential energy of the cabinet-Earth system. However, in addition to increasing the gravitational potential energy of the cabinet-Earth system by the same amount as Alex did, John must do work overcoming the friction between the cabinet and ramp. This means that the total work done by John is greater than that done by Alex.

OQ7.2 Answer (d). The work–energy theorem states that $W_{net} = \Delta K = K_f - K_i$. Thus, if $W_{net} = 0$, then $K_f - K_i$ or $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$, which leads to the conclusion that the speed is unchanged ($v_f = v_i$). The velocity of the particle involves both magnitude (speed) and direction. The work–energy theorem shows that the magnitude or speed is unchanged when $W_{net} = 0$, but makes no statement about the direction of the velocity.

OQ7.3 Answer (a). The work done on the wheelbarrow by the worker is

$$W = (F\cos\theta)\Delta x = (50\text{ N})(5.0\text{ m}) = +250\text{ J}$$

OQ7.4 Answer (c). The system consisting of the cart's fixed, initial kinetic energy is the mechanical energy that can be transformed due to friction from the surface. Therefore, the loss of mechanical energy is $\Delta E_{mech} = -f_k d = -(6\text{ N})(0.06\text{ m}) = 0.36\text{ J}.$ This product must remain the same in all cases. For the cart rolling through gravel, $-(9\text{ N})(d) = 0.36\text{ J}$ tells us $d = 4$ cm.

OQ7.5 The answer is a > b = e > d > c. Each dot product has magnitude $(1)\cdot(1)\cdot\cos\theta$, where θ is the angle between the two factors. Thus for (a) we have $\cos 0 = 1$. For (b) and (e), $\cos 45^\circ = 0.707$. For (c), $\cos 180^\circ = -1$. For (d), $\cos 90^\circ = 0$.

OQ7.6 Answer (c). The net work needed to accelerate the object from $v = 0$ to v is

$$W_1 = KE_{1f} - KE_{1i} = \frac{1}{2}mv^2 - \frac{1}{2}m(0)^2 = \frac{1}{2}mv^2$$

The work required to accelerate the object from speed v to speed $2v$ is

$$W_2 = KE_{2f} - KE_{2i} = \frac{1}{2}m(2v)^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}m\left(4v^2 - v^2\right) = 3\left(\frac{1}{2}mv^2\right) = 3W_1$$

OQ7.7 Answer (e). As the block falls freely, only the conservative gravitational force acts on it. Therefore, mechanical energy is conserved, or $KE_f + PE_f = KE_i + PE_i$. Assuming that the block is released from rest ($KE_i = 0$), and taking $y = 0$ at ground level ($PE_f = 0$), we have that

$$KE_f = PE_i \quad \text{or} \quad \frac{1}{2}mv_f^2 = mgy \quad \text{and} \quad y_i = \frac{v_f^2}{2g}$$

Thus, to double the final speed, it is necessary to increase the initial height by a factor of four.

OQ7.8 (i) Answer (b). Tension is perpendicular to the motion. (ii) Answer (c). Air resistance is opposite to the motion.

OQ7.9 Answer (e). Kinetic energy is proportional to mass.

OQ7.10 (i) Answers (c) and (e). The force of block on spring is equal in magnitude and opposite to the force of spring on block.

(ii) Answers (c) and (e). The spring tension exerts equal-magnitude forces toward the center of the spring on objects at both ends.

OQ7.11 Answer (a). Kinetic energy is proportional to squared speed. Doubling the speed makes an object's kinetic energy four times larger.

OQ7.12 Answer (b). Since the rollers on the ramp used by David were frictionless, he did not do any work overcoming nonconservative forces as he slid the block up the ramp. Neglecting any change in kinetic energy of the block (either because the speed was constant or was essentially zero during the lifting process), the work done by either Mark or David equals the increase in the gravitational potential energy of the block-Earth system as the block is lifted from the ground to the truck bed. Because they lift identical blocks through the same vertical distance, they do equal amounts of work.

OQ7.13 (i) Answer: a = b = c = d. The gravitational acceleration is quite precisely constant at locations separated by much less than the radius of the planet.

(ii) Answer: c = d > a = b. The mass but not the elevation affects the gravitational force.

(iii) Answer: c > b = d > a. Gravitational potential energy of the object-Earth system is proportional to mass times height.

OQ7.14 Answer (d). $4.00\text{ J} = \frac{1}{2}k(0.100\text{ m})^2$. Therefore, $k = 800\text{ N/m}$ and to stretch the spring to 0.200 m requires extra work

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\text{ J} = 12.0\text{ J}$$

OQ7.15 Answer (a). The system consisting of the cart's fixed, initial kinetic energy is the mechanical energy that can be transformed due to friction from the surface. Therefore, the loss of mechanical energy is $\Delta E_{\text{mech}} = -f_k d = -(6\text{ N})(0.06\text{ m}) = 0.36\text{ J}$. This product must remain the same in all cases. For the cart rolling through gravel, $-(f_k)(0.18\text{ m}) = 0.36\text{ J}$ tells us $f_k = 2\text{ N}$.

OQ7.16 Answer (c). The ice cube is in neutral equilibrium. Its zero acceleration is evidence for equilibrium.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ7.1 Yes. The floor of a rising elevator does work on a passenger. A normal force exerted by a stationary solid surface does no work.

CQ7.2 Yes. Object 1 exerts some forward force on object 2 as they move through the same displacement. By Newton's third law, object 2 exerts an equal-size force in the opposite direction on object 1. In $W = F\Delta r\cos\theta$, the factors F and Δr are the same, and θ differs by 180°, so object 2 does −15.0 J of work on object 1. The energy transfer is 15 J from object 1 to object 2, which can be counted as a change in energy of −15 J for object 1 and a change in energy of +15 J for object 2.

CQ7.3 It is sometimes true. If the object is a particle initially at rest, the net work done on the object is equal to its final kinetic energy. If the object is not a particle, the work could go into (or come out of) some other form of energy. If the object is initially moving, its initial kinetic energy must be added to the total work to find the final kinetic energy.

CQ7.4 The scalar product of two vectors is positive if the angle between them is between 0° and 90°, including 0°. The scalar product is negative when $90° < \theta \leq 180°$.

CQ7.5 No. Kinetic energy is always positive. Mass and squared speed are both positive.

CQ7.6 Work is only done in accelerating the ball from rest. The work is done over the effective length of the pitcher's arm—the distance his hand moves through windup and until release. He extends this distance by taking a step forward.

CQ7.7 (a) Positive work is done by the chicken on the dirt.

(b) The person does no work on anything in the environment. Perhaps some extra chemical energy goes through being energy transmitted electrically and is converted into internal energy in his brain; but it would be very hard to quantify "extra."

(c) Positive work is done on the bucket.

(d) Negative work is done on the bucket.

(e) Negative work is done on the person's torso.

CQ7.8 (a) Not necessarily. It does if it makes the object's speed change, but not if it only makes the direction of the velocity change.

(b) Yes, according to Newton's second law.

CQ7.9 The gravitational energy of the key-Earth system is lowest when the key is on the floor letter-side-down. The average height of particles in the key is lowest in that configuration. As described by $F = -dU/dx$, a force pushes the key downhill in potential energy toward the bottom of a graph of potential energy versus orientation angle. Friction removes mechanical energy from the key-Earth system, tending to leave the key in its minimum-potential energy configuration.

CQ7.10 There is no violation. Choose the book as the system. You did positive work (average force and displacement are in same direction) and the Earth did negative work (average force and displacement are in opposite directions) on the book. The average force you exerted just counterbalanced the weight of the book. The total work on the book is zero, and is equal to its overall change in kinetic energy.

CQ7.11 $k' = 2k$. Think of the original spring as being composed of two half-springs. The same force F that stretches the whole spring by x stretches each of the half-springs by $x/2$; therefore, the spring constant for each of the half-springs is $k' = [F/(x/2)] = 2(F/x) = 2k$.

CQ7.12 A graph of potential energy versus position is a straight horizontal line for a particle in neutral equilibrium. The graph represents a constant function.

CQ7.13 Yes. As you ride an express subway train, a backpack at your feet has no kinetic energy as measured by you since, according to you, the backpack is not moving. In the frame of reference of someone on the side of the tracks as the train rolls by, the backpack is moving and has mass, and thus has kinetic energy.

CQ7.14 Force of tension on a ball moving in a circle on the end of a string. Normal force and gravitational force on an object at rest or moving across a level floor.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 7.2 Work Done by a Constant Force

P7.1 (a) The 35-N force applied by the shopper makes a 25° angle with the displacement of the cart (horizontal). The work done on the cart by the shopper is then

$$W_{\text{shopper}} = (F\cos\theta)\Delta x = (35.0\text{ N})(50.0\text{ m})\cos 25.0°$$

$$= \boxed{1.59\times10^{3}\text{ J}}$$

(b) The force exerted by the shopper is now completely horizontal and will be equal to the friction force, since the cart stays at a constant velocity. In part (a), the shopper's force had a downward vertical component, increasing the normal force on the cart, and thereby the friction force. Because there is no vertical component here, the friction force will be less, and the $\boxed{\text{the force is smaller}}$ than before.

(c) Since the horizontal component of the force is less in part (b), the work performed by the shopper on the cart over the same 50.0-m distance is $\boxed{\text{the same}}$ as in part (b).

P7.2 (a) The work done on the raindrop by the gravitational force is given by

$$W = mgh = \left(3.35\times10^{-5}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(100\text{ m}\right) = \boxed{3.28\times10^{-2}\text{ J}}$$

(b) Since the raindrop is falling at constant velocity, all forces acting on the drop must be in balance, and $R = mg$, so

$$W_{\text{air resistance}} = \boxed{-3.28\times10^{-2}\text{ J}}$$

P7.3 (a) The work done by a constant force is given by

$$W = Fd\cos\theta$$

where θ is the angle between the force and the displacement of the object. In this case, $F = -mg$ and $\theta = 180°$, giving

$$W = (281.5\text{ kg})(9.80\text{ m/s}^2)[(17.1\text{ cm})(1\text{ m}/10^2\text{ cm})] = \boxed{472\text{ J}}$$

(b) If the object moved upward at constant speed, the net force acting on it was zero. Therefore, the magnitude of the upward force applied by the lifter must have been equal to the weight of the object:

$$F = mg = (281.5\text{ kg})(9.80\text{ m/s}^2) = 2.76\times10^3\text{ N} = \boxed{2.76\text{ kN}}$$

P7.4 Assuming the mass is lifted at constant velocity, the total upward force exerted by the two men equals the weight of the mass: $F_{\text{total}} = mg = (653.2\text{ kg})(9.80\text{ m/s}^2) = 6.40\times10^3$ N. They exert this upward force through a total upward displacement of 96 inches (4 inches per lift for each of 24 lifts). The total work would then be

$$W_{\text{total}} = (6.40\times10^3\text{ N})[(96\text{ in})(0.025\,4\text{ m/1 in})] = \boxed{1.56\times10^4\text{ J}}$$

P7.5 We apply the definition of work by a constant force in the first three parts, but then in the fourth part we add up the answers. The total (net) work is the sum of the amounts of work done by the individual forces, and is the work done by the total (net) force. This identification is not represented by an equation in the chapter text, but is something you know by thinking about it, without relying on an equation in a list.

The definition of work by a constant force is $W = F\Delta r\cos\theta$.

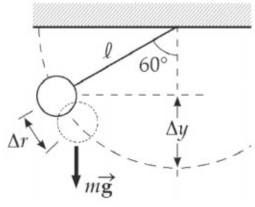
(a) The applied force does work given by

$$W = F\Delta r\cos\theta = (16.0\text{ N})(2.20\text{ m})\cos 25.0° = \boxed{31.9\text{ J}}$$

(b), (c) The normal force and the weight are both at 90° to the displacement in any time interval. Both do $\boxed{0}$ work.

(d) $\sum W = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

P7.6 **METHOD ONE**



ANS. FIG. P7.6

Let ϕ represent the instantaneous angle the rope makes with the vertical as it is swinging up from $\phi_i = 0$ to $\phi_f = 60°$. In an incremental bit of motion from angle ϕ to $\phi + d\phi$, the definition of radian measure implies that $\Delta r = (12.0\text{ m})d\phi$. The angle θ between the incremental displacement and the force of gravity is $\theta = 90° + \phi$. Then

$$\cos\theta = \cos(90° + \phi) = -\sin\phi$$

The work done by the gravitational force on Spiderman is

$$W = \int_i^f F\cos\theta\, dr = \int_{\phi=0}^{\phi=60°} mg(-\sin\phi)(12.0\text{ m})d\phi$$

$$= -mg(12.0\text{ m})\int_0^{60°} \sin\phi\, d\phi$$

$$= (-80.0\text{ kg})(9.80\text{ m/s}^2)(12\text{ m})(-\cos\phi)\Big|_0^{60°}$$

$$= (-784\text{ N})(12.0\text{ m})(-\cos 60° + 1) = \boxed{-4.70\times 10^3\text{ J}}$$

METHOD TWO

The force of gravity on Spiderman is $mg = (80\text{ kg})(9.8\text{ m/s}^2) = 784\text{ N}$ down. Only his vertical displacement contributes to the work gravity does. His original y coordinate below the tree limb is –12 m. His final y coordinate is $(-12.0\text{ m})\cos 60.0° = -6.00\text{ m}$. His change in elevation is $-6.00\text{ m} - (-12.0\text{ m})$. The work done by gravity is

$$W = F\Delta r\cos\theta = (784\text{ N})(6.00\text{ m})\cos 180° = \boxed{-4.70\text{ kJ}}$$

Section 7.3 The Scalar Product of Two Vectors

P7.7

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}\right)\cdot\left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right)$$

$$\begin{aligned}\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = {} & A_xB_x\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}\right) + A_xB_y\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{j}}\right) + A_xB_z\left(\hat{\mathbf{i}}\cdot\hat{\mathbf{k}}\right)\\ & + A_yB_x\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{i}}\right) + A_yB_y\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}\right) + A_yB_z\left(\hat{\mathbf{j}}\cdot\hat{\mathbf{k}}\right)\\ & + A_zB_x\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{i}}\right) + A_zB_y\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{j}}\right) + A_zB_z\left(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}}\right)\end{aligned}$$

And since $\hat{\mathbf{i}}\cdot\hat{\mathbf{i}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{k}}\cdot\hat{\mathbf{k}} = 1$ and $\hat{\mathbf{i}}\cdot\hat{\mathbf{j}} = \hat{\mathbf{i}}\cdot\hat{\mathbf{k}} = \hat{\mathbf{j}}\cdot\hat{\mathbf{k}} = 0$,

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = \boxed{A_xB_x + A_yB_y + A_zB_z}$$

P7.8 $A = 5.00$; $B = 9.00$; $\theta = 50.0°$

$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$$

P7.9

$$\vec{\mathbf{A}} - \vec{\mathbf{B}} = \left(3.00\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}\right) - \left(-\hat{\mathbf{i}} + 2.00\hat{\mathbf{j}} + 5.00\hat{\mathbf{k}}\right) = 4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}$$

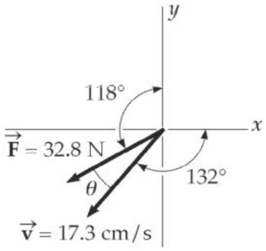
$$\begin{aligned}\vec{\mathbf{C}}\cdot\left(\vec{\mathbf{A}} - \vec{\mathbf{B}}\right) &= \left(2.00\hat{\mathbf{j}} - 3.00\hat{\mathbf{k}}\right)\cdot\left(4.00\hat{\mathbf{i}} - \hat{\mathbf{j}} - 6.00\hat{\mathbf{k}}\right) = 0 + (-2.00) + (+18.0)\\ &= \boxed{16.0}\end{aligned}$$

P7.10 We must first find the angle between the two vectors. It is

$$\begin{aligned}\theta &= (360° - 132°) - (118° + 90.0°)\\ &= 20.0°\end{aligned}$$

Then

$$\begin{aligned}\vec{\mathbf{F}}\cdot\vec{\mathbf{r}} &= Fr\cos\theta\\ &= (32.8\text{ N})(0.173\text{ m})\cos 20.0°\end{aligned}$$



ANS. FIG. P7.10

or $\vec{\mathbf{F}}\cdot\vec{\mathbf{r}} = 5.33\text{ N}\cdot\text{m} = \boxed{5.33\text{ J}}$

P7.11 (a) We use the mathematical representation of the definition of work.

$$\begin{aligned}W &= \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = F_x x + F_y y = (6.00)(3.00)\text{ N}\cdot\text{m} + (-2.00)(1.00)\text{ N}\cdot\text{m}\\ &= \boxed{16.0\text{ J}}\end{aligned}$$

(b)

$$\begin{aligned}\theta &= \cos^{-1}\left(\frac{\vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}}}{F\Delta r}\right)\\ &= \cos^{-1}\frac{16\text{ N}\cdot\text{m}}{\sqrt{(6.00\text{ N})^2 + (-2.00\text{ N})^2}\cdot\sqrt{(3.00\text{ m})^2 + (1.00\text{ m})^2}}\\ &= \boxed{36.9°}\end{aligned}$$

P7.12 (a) $\vec{\mathbf{A}} = 3.00\hat{\mathbf{i}} - 2.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 4.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}}$

$$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\frac{12.0 + 8.00}{\sqrt{13.0}\cdot\sqrt{32.0}}\right) = \boxed{11.3°}$$

(b) $\vec{\mathbf{A}} = -2.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$$\cos\theta = \left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \frac{-6.00 - 16.0}{\sqrt{20.0}\cdot\sqrt{29.0}} \quad\rightarrow\quad \theta = \boxed{156°}$$

(c) $\vec{\mathbf{A}} = \hat{\mathbf{i}} - 2.00\hat{\mathbf{j}} + 2.00\hat{\mathbf{k}}$

$\vec{\mathbf{B}} = 3.00\hat{\mathbf{j}} + 4.00\hat{\mathbf{k}}$

$$\theta = \cos^{-1}\left(\frac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}\left(\frac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$$

P7.13 Let θ represent the angle between $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$. Turning by 25.0° makes the dot product larger, so the angle between $\vec{\mathbf{C}}$ and $\vec{\mathbf{B}}$ must be smaller. We call it $\theta - 25.0°$. Then we have

$$5A\cos\theta = 30 \qquad \text{and} \qquad 5A\cos(\theta - 25.0°) = 35$$

Then

$$A\cos\theta = 6 \qquad \text{and} \qquad A(\cos\theta\cos 25.0° + \sin\theta\sin 25.0°) = 7$$

Dividing,

$$\cos 25.0° + \tan\theta\sin 25.0° = 7/6$$

or $\tan\theta = (7/6 - \cos 25.0°)/\sin 25.0° = 0.616$

Which gives $\theta = 31.6°$. Then the direction angle of A is

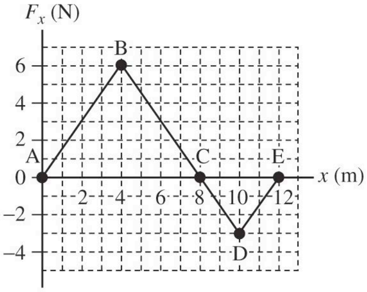
$$60.0° - 31.6° = 28.4°$$

Substituting back,

$$A\cos 31.6° = 6 \quad \text{so} \quad \vec{\mathbf{A}} = \boxed{7.05\text{ m at }28.4°}$$

Section 7.4 Work Done by a Varying Force

P7.14 $W = \int_i^f F dx =$ area under curve from x_i to x_f



ANS. FIG. P7.14

(a) $x_i = 0$ and $x_f = 800$ m

$$W_{0\to 8} = \text{area of triangle ABC}$$

$$= \left(\frac{1}{2}\right)\text{AC}\times\text{height}$$

$$W_{0\to 8} = \left(\frac{1}{2}\right)\times 8.00\text{ m}\times 6.00\text{ N}$$

$$= \boxed{24.0\text{ J}}$$

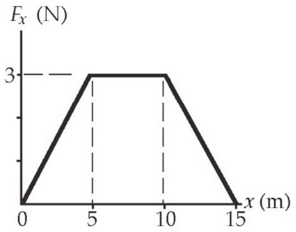
(b) $x_i = 8.00$ m and $x_f = 10.0$ m

$$W_{8\to 10} = \text{area of } \Delta\text{CDE} = \left(\frac{1}{2}\right)\text{CE}\times\text{height},$$

$$W_{8\to 10} = \left(\frac{1}{2}\right)\times(2.00\text{ m})\times(-3.00\text{ N}) = \boxed{-3.00\text{ J}}$$

(c) $W_{0\to 10} = W_{0\to 8} + W_{0\to 10} = 24.0 + (-3.00) = \boxed{21.0\text{ J}}$

P7.15 We use the graphical representation of the definition of work. *W* equals the area under the force-displacement curve. This definition is still written $W = \int F_x dx$ but it is computed geometrically by identifying triangles and rectangles on the graph.



ANS. FIG. P7.15

(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$$

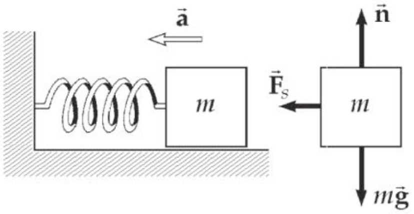
(b) For the region $5.00 \le x \le 10.0$, $W = (3.00\text{ N})(5.00\text{ m}) = \boxed{15.0\text{ J}}$

(c) For the region $10.00 \le x \le 15.0$, $W = \dfrac{(3.00\text{ N})(5.00\text{ m})}{2} = \boxed{7.50\text{ J}}$

(d) For the region $0 \le x \le 15.0$, $W = (7.50 + 7.50 + 15.0)\text{ J} = \boxed{30.0\text{ J}}$

P7.16 $\sum F_x = ma_x$: $kx = ma$

$$k = \frac{ma}{x} = \frac{(4.70\times10^{-3}\text{ kg})(0.800)(9.80\text{ m/s}^2)}{0.500\times10^{-2}\text{m}} = \boxed{7.37\text{ N/m}}$$



ANS. FIG. P7.16

P7.17 When the load of mass M = 4.00 kg is hanging on the spring in equilibrium, the upward force exerted by the spring on the load is equal in magnitude to the downward force that the Earth exerts on the load, given by $w = Mg$. Then we can write Hooke's law as $Mg = +kx$. The spring constant, force constant, stiffness constant, or Hooke's-law constant of the spring is given by

$$k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{2.50\times10^{-2}\text{ m}} = 1.57\times10^3\text{ N/m}$$

(a) For the 1.50-kg mass,

$$y = \frac{mg}{k} = \frac{(1.50\text{ kg})(9.80\text{ m/s}^2)}{1.57\times10^3\text{ N/m}} = 0.009\ 38\text{ m} = \boxed{0.938\text{ cm}}$$

(b) $\text{Work} = \frac{1}{2}ky^2 = \frac{1}{2}\left(1.57\times10^3\text{ N/m}\right)\left(4.00\times10^{-2}\text{ m}\right)^2 = \boxed{1.25\text{ J}}$

P7.18 In $F = -kx$, F refers to the size of the force that the spring exerts on each end. It pulls down on the doorframe in part (a) in just as real a sense as it pulls on the second person in part (b).

(a) Consider the upward force exerted by the bottom end of the spring, which undergoes a downward displacement that we count as negative:

$$k = -F/x = -(7.50\text{ kg})(9.80\text{ m/s}^2)/(-0.415\text{ m} + 0.350\text{ m})$$
$$= -73.5\text{ N}/(-0.065\text{ m}) = \boxed{1.13\text{ kN/m}}$$

(b) Consider the end of the spring on the right, which exerts a force to the left:

$$x = -F/k = -(-190\text{ N})/(1130\text{ N/m}) = 0.168\text{ m}$$

The length of the spring is then

$$0.350\text{ m} + 0.168\text{ m} = \boxed{0.518\text{ m} = 51.8\text{ cm}}$$

P7.19 (a) Spring constant is given by $F = kx$:

$$k = \frac{F}{x} = \frac{230\text{ N}}{0.400\text{ m}} = \boxed{575\text{ N/m}}$$

(b) $\text{Work } = F_{\text{avg}}x = \left(\frac{230\text{ N} - 0}{2}\right)(0.400\text{ m}) = \boxed{46.0\text{ J}}$

P7.20 The same force makes both light springs stretch.

(a) The hanging mass moves down by

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= (1.5\text{ kg})(9.8\text{ m/s}^2)\left(\frac{1}{1\,200\text{ N/m}} + \frac{1}{1\,800\text{ N/m}}\right)$$

$$= \boxed{2.04 \times 10^{-2}\text{ m}}$$

(b) We define the effective spring constant as

$$k = \frac{F}{x} = \frac{mg}{mg(1/k_1 + 1/k_2)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

$$= \left(\frac{1}{1\,200\text{ N/m}} + \frac{1}{1\,800\text{ N/m}}\right)^{-1} = \boxed{720\text{ N/m}}$$

P7.21 (a) The force mg is the tension in each of the springs. The bottom of the upper (first) spring moves down by distance $x_1 = |F|/k_1 = mg/k_1$. The top of the second spring moves down by this distance, and the second spring also stretches by $x_2 = mg/k_2$. The bottom of the lower spring then moves down by distance

$$x_{\text{total}} = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = \boxed{mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

(b) From the last equation we have

$$mg = \frac{x_1 + x_2}{\frac{1}{k_1} + \frac{1}{k_2}}$$

This is of the form

$$|F| = \left(\frac{1}{1/k_1 + 1/k_2}\right)(x_1 + x_2)$$

The downward displacement is opposite in direction to the upward force the springs exert on the load, so we may write $F = -k_{eff}\,x_{\text{total}}$, with the effective spring constant for the pair of springs given by

$$k_{\text{eff}} = \boxed{\frac{1}{1/k_1 + 1/k_2}}$$

P7.22 $$[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg}\cdot\text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$$

P7.23 (a) If the weight of the first tray stretches all four springs by a distance equal to the thickness of the tray, then the proportionality expressed by Hooke's law guarantees that each additional tray will have the same effect, so that the top surface of the top tray can always have the same elevation above the floor if springs with the right spring constant are used.

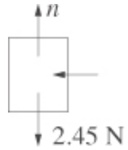
(b) The weight of a tray is (0.580 kg)(9.8 m/s^2) = 5.68 N. The force $\frac{1}{4}(5.68\text{ N}) = 1.42\text{ N}$ should stretch one spring by 0.450 cm, so its spring constant is

$$k = \frac{|F_s|}{x} = \frac{1.42\text{ N}}{0.004\,5\text{ m}} = \boxed{316\text{ N/m}}$$

(c) We did not need to know the length or width of the tray.

P7.24 The spring exerts on each block an outward force of magnitude

$$|F_s| = kx = (3.85\text{ N/m})(0.08\text{ m}) = 0.308\text{ N}$$



ANS. FIG. P7.24

Take the $+x$ direction to the right. For the light block on the left, the vertical forces are given by

$$F_g = mg = (0.250\text{ kg})(9.80\text{ m/s}^2) = 2.45\text{ N}$$

and $\sum F_y = 0$

so $n - 2.45\text{ N} = 0 \quad \rightarrow \quad n = 2.45\text{ N}$

Similarly, for the heavier block,

$$n = F_g = (0.500\text{ kg})(9.80\text{ m/s}^2) = 4.90\text{ N}$$

(a) For the block on the left,

$$\sum F_x = ma_x: \quad -0.308\text{ N} = (0.250\text{ kg})a$$

$$a = \boxed{-1.23\text{ m/s}^2}$$

For the heavier block,

$$+0.308\text{ N} = (0.500\text{ kg})a$$

$$a = \boxed{0.616\text{ m/s}^2}$$

(b) For the block on the left, $f_k = \mu_k n = 0.100(2.45\text{ N}) = 0.245\text{ N}$.

$$\sum F_x = ma_x$$

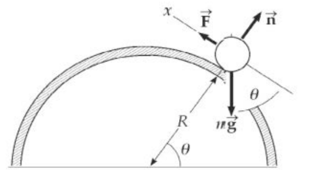
$$-0.308\text{ N} + 0.245\text{ N} = (0.250\text{ kg})a$$

$$a = \boxed{-0.252\text{ m/s}^2 \text{ if the force of static friction is not too large}}.$$

For the block on the right, $f_k = \mu_k n = 0.490$ N. The maximum force of static friction would be larger, so no motion would begin and the acceleration is $\boxed{\text{zero}}$.

(c) Left block: $f_k = 0.462(2.45\text{ N}) = 1.13$ N. The maximum static friction force would be larger, so the spring force would produce no motion of this block or of the right-hand block, which could feel even more friction force. For both, $a = \boxed{0}$.

P7.25 (a) The radius to the object makes angle θ with the horizontal. Taking the x axis in the direction of motion tangent to the cylinder, the object's weight makes an angle θ with the $-x$ axis. Then,



ANS. FIG. P7.25

$$\sum F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

(b) $$W = \int_i^f \vec{F}\cdot d\vec{r}$$

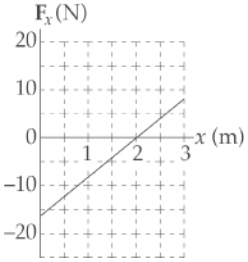
We use radian measure to express the next bit of displacement as $dr = Rd\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg\cos\theta R\,d\theta = mgR\sin\theta\Big|_0^{\pi/2} = mgR(1-0) = \boxed{mgR}$$

P7.26 The force is given by $F_x = (8x - 16)$ N.

(a) See ANS. FIG. P7.26 to the right.

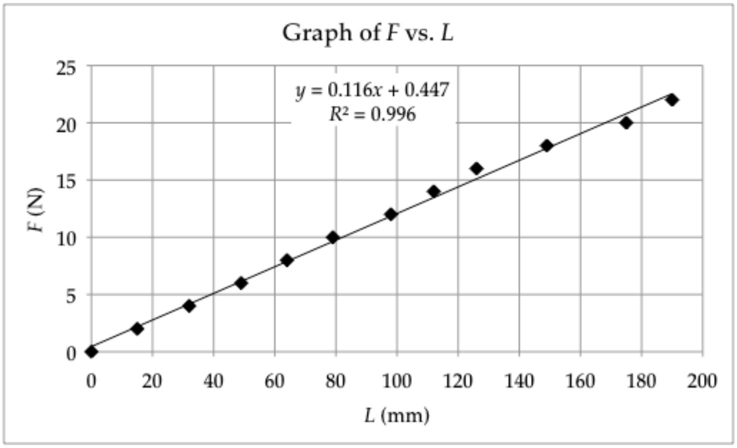
(b) $$W_{\text{net}} = \frac{-(2.00\text{ m})(16.0\text{ N})}{2} + \frac{(1.00\text{ m})(8.00\text{ N})}{2} = \boxed{-12.0\text{ J}}$$



ANS. FIG. P7.26

P7.27 (a)

F (N)	L (mm)	F (N)	L (mm)
0.00	0.00	12.0	98.0
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190



ANS FIG. P7.27(a)

(b) By least-squares fitting, its slope is 0.116 N/mm = $\boxed{116\text{ N/m}}$.

(c) To draw the straight line we use all the points listed and also the origin. If the coils of the spring touched each other, a bend or nonlinearity could show up at the bottom end of the graph. If the spring were stretched "too far," a nonlinearity could show up at the top end. But there is no visible evidence for a bend in the graph near either end.

(d) In the equation $F = kx$, the spring constant k is the slope of the F-versus-x graph.

$$\boxed{k = 116\ \text{N/m}}$$

(e) $F = kx = (116\ \text{N/m})(0.105\ \text{m}) = \boxed{12.2\ \text{N}}$

P7.28 (a) We find the work done by the gas on the bullet by integrating the function given:

$$W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$$

$$W = \int_0^{0.600\ \text{m}} \left(15\,000\ \text{N} + 10\,000x\ \text{N/m} - 25\,000x^2\ \text{N/m}^2\right) dx \cos 0°$$

$$W = 15\,000x + \frac{10\,000x^2}{2} - \frac{25\,000x^3}{3}\Bigg|_0^{0.600\ \text{m}}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

(b) Similarly,

$$W = (15.0\ \text{kN})(1.00\ \text{m}) + \frac{(10.0\ \text{kN/m})(1.00\ \text{m})^2}{2} - \frac{(25.0\ \text{kN/m}^2)(1.00\ \text{m})^3}{3}$$

$$W = 11.67\ \text{kJ} = \boxed{11.7\ \text{kJ}}$$

(c) $$\frac{11.7\ \text{kJ} - 9.00\ \text{kJ}}{9.00\ \text{kJ}} \times 100\% = 29.6\%$$

$$\boxed{\text{The work is greater by 29.6\%.}}$$

P7.29 $$W = \int_i^f \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_0^{5\ \text{m}} \left(4x\hat{\mathbf{i}} + 3y\hat{\mathbf{j}}\right) \text{N} \cdot dx\hat{\mathbf{i}}$$

$$\int_0^{5\ \text{m}} (4\ \text{N/m})x\,dx + 0 = (4\ \text{N/m})\frac{x^2}{2}\Bigg|_0^{5\ \text{m}} = \boxed{50.0\ \text{J}}$$

P7.30 We read the coordinates of the two specified points from the graph as

$$a = (5\text{ cm}, -2\text{ N}) \text{ and } b = (25\text{ cm}, 8\text{ N})$$

We can then write *u* as a function of *v* by first finding the slope of the curve:

$$\text{slope} = \frac{u_b - u_a}{v_b - v_a} = \frac{8\text{ N} - (-2\text{ N})}{25\text{ cm} - 5\text{ cm}} = 0.5\text{ N/cm}$$

The *y* intercept of the curve can be found from $u = mv + b$, where $m = 0.5$ N/cm is the slope of the curve, and *b* is the *y* intercept. Plugging in point *a*, we obtain

$$u = mv + b$$
$$-2\text{ N} = (0.5\text{ N/cm})(5\text{ cm}) + b$$
$$b = -4.5\text{ N}$$

Then,

$$u = mv + b = (0.5\text{ N/cm})v - 4.5\text{ N}$$

(a) Integrating the function above, suppressing units, gives

$$\int_a^b u\,dv = \int_5^{25}(0.5v - 4.5)\,dv = \left[0.5v^2/2 - 4.5v\right]_5^{25}$$
$$= 0.25(625 - 25) - 4.5(25 - 5)$$
$$= 150 - 90 = 60\text{ N}\cdot\text{cm} = \boxed{0.600\text{ J}}$$

(b) Reversing the limits of integration just gives us the negative of the quantity:

$$\int_b^a u\,dv = \boxed{-0.600\text{ J}}$$

(c) This is an entirely different integral. It is larger because all of the area to be counted up is positive (to the right of $v = 0$) instead of partly negative (below $u = 0$).

$$\int_a^b v\,du = \int_{-2}^{8}(2u + 9)\,du = \left[2u^2/2 + 9u\right]_{-2}^{8}$$
$$= 64 - (-2)^2 + 9(8 + 2)$$
$$= 60 + 90 = 150\text{ N}\cdot\text{cm} = \boxed{1.50\text{ J}}$$

Section 7.5 Kinetic Energy and the Work-Kinetic Energy Theorem

P7.31 $\vec{\mathbf{v}}_i = \left(6.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)\ \text{m/s}^2$

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{37.0}\ \text{m/s}$

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(3.00\ \text{kg})\left(37.0\ \text{m}^2/\text{s}^2\right) = \boxed{55.5\ \text{J}}$$

(b) $\vec{\mathbf{v}}_f = 8.00\hat{\mathbf{i}} + 4.00\hat{\mathbf{j}}$

$$v_f^2 = \vec{\mathbf{v}}_f \cdot \vec{\mathbf{v}}_f = 64.0 + 16.0 = 80.0\ \text{m}^2/\text{s}^2$$

$$\Delta K = K_f - K_i = \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{3.00}{2}(80.0) - 55.5 = \boxed{64.5\ \text{J}}$$

P7.32 (a) Since the applied force is horizontal, it is in the direction of the displacement, giving $\theta = 0°$. The work done by this force is then

$$W_{F_0} = \left(F_0 \cos\theta\right)\Delta x = F_0\left(\cos 0\right)\Delta x = F_0\Delta x$$

and

$$F_0 = \frac{W_{F_0}}{\Delta x} = \frac{350\ \text{J}}{12.0\ \text{m}} = \boxed{29.2\ \text{N}}$$

(b) If the applied force is greater than 29.2 N, the crate would accelerate in the direction of the force, so its $\boxed{\text{speed would increase}}$ with time.

(c) If the applied force is less than 29.2 N, the $\boxed{\text{crate would slow down and come to rest.}}$

P7.33 (a) $K_A = \frac{1}{2}(0.600\ \text{kg})\left(2.00\ \text{m/s}\right)^2 = \boxed{1.20\ \text{J}}$

(b) $\frac{1}{2}mv_B^2 = K_B:\ v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50\ \text{J})}{0.600\ \text{kg}}} = \boxed{5.00\ \text{m/s}}$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}m\left(v_B^2 - v_A^2\right) = 7.50\ \text{J} - 1.20\ \text{J} = \boxed{6.30\ \text{J}}$

P7.34 (a) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50\ \text{J})}{4.00\ \text{kg}}} = \boxed{1.94\ \text{m/s}}$$

(b) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5\ \text{J})}{4.00\ \text{kg}}} = \boxed{3.35\ \text{m/s}}$$

(c) $\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0\ \text{J})}{4.00\ \text{kg}}} = \boxed{3.87\ \text{m/s}}$$

P7.35 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall. Let $d = 5.00$ m represent the distance over which the driver falls freely, and $h = 0.12$ the distance it moves the piling.

$$\sum W = \Delta K: \quad W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

so $$(mg)(h + d)\cos 0° + (\bar{F})(d)\cos 180° = 0 - 0$$

Thus,

$$\bar{F} = \frac{(mg)(h + d)}{d} = \frac{(2\,100\ \text{kg})(9.80\ \text{m/s}^2)(5.12\ \text{m})}{0.120\ \text{m}}$$

$$= \boxed{8.78 \times 10^5\ \text{N}}$$

The force on the pile driver is $\boxed{\text{upward.}}$

P7.36 (a) $v_f = 0.096(3.00 \times 10^8\ \text{m/s}) = 2.88 \times 10^7\ \text{m/s}$

$$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(2.88 \times 10^7\ \text{m/s})^2 = \boxed{3.78 \times 10^{-16}\ \text{J}}$$

(b) $K_i + W = K_f : 0 + F\Delta r\cos\theta = K_f$

$$F(0.028\ \text{m})\cos 0° = 3.78 \times 10^{-16}\ \text{J}$$

$$F = \boxed{1.35 \times 10^{-14}\ \text{N}}$$

(c) $\sum F = ma$: $a = \dfrac{\sum F}{m} = \dfrac{1.35 \times 10^{-14}\ \text{N}}{9.11 \times 10^{-31}\ \text{kg}} = \boxed{1.48 \times 10^{+16}\ \text{m/s}^2}$

(d) $v_{xf} = v_{xi} + a_x t$: $2.88 \times 10^7\ \text{m/s} = 0 + \left(1.48 \times 10^{16}\ \text{m/s}^2\right)t$

$t = \boxed{1.94 \times 10^{-9}\ \text{s}}$

P7.37 (a) $K_i + \sum W = K_f = \dfrac{1}{2}mv_f^2$

$0 + \sum W = \dfrac{1}{2}(15.0 \times 10^{-3}\ \text{kg})(780\ \text{m/s})^2 = \boxed{4.56\ \text{kJ}}$

(b) As shown in part (a), the net work performed on the bullet is $\boxed{4.56\ \text{kJ.}}$

(c) $F = \dfrac{W}{\Delta r \cos\theta} = \dfrac{4.56 \times 10^3\ \text{J}}{(0.720\ \text{m})\cos 0°} = \boxed{6.34\ \text{kN}}$

(d) $a = \dfrac{v_f^2 - v_i^2}{2x_f} = \dfrac{(780\ \text{m/s})^2 - 0}{2(0.720\ \text{m})} = \boxed{422\ \text{km/s}^2}$

(e) $\sum F = ma = (15 \times 10^{-3}\ \text{kg})(422 \times 10^3\ \text{m/s}^2) = \boxed{6.34\ \text{kN}}$

(f) $\boxed{\text{The forces are the same. The two theories agree.}}$

P7.38 (a) As the bullet moves the hero's hand, work is done on the bullet to decrease its kinetic energy. The average force is opposite to the displacement of the bullet:

$$W_{\text{net}} = F_{\text{avg}}\Delta x \cos\theta = -F_{\text{avg}}\Delta x = \Delta K$$

$$F_{\text{avg}} = \frac{\Delta K}{-\Delta x} = \frac{0 - \frac{1}{2}\left(7.80 \times 10^{-3}\ \text{kg}\right)\left(575\ \text{m/s}\right)^2}{-0.055\ 0\ \text{m}}$$

$$\boxed{F_{\text{avg}} = 2.34 \times 10^4\ \text{N, opposite to the direction of motion}}$$

(b) If the average force is constant, the bullet will have a constant acceleration and its average velocity while stopping is $\bar{v} = (v_f + v_i)/2$. The time required to stop is then

$$\Delta t = \frac{\Delta x}{\bar{v}} = \frac{2(\Delta x)}{v_f + v_i} = \frac{2(5.50 \times 10^{-2}\text{m})}{0 + 575\ \text{m/s}} = \boxed{1.91 \times 10^{-4}\text{s}}$$

P7.39 (a) $K = \frac{1}{2}mv^2 = \frac{1}{2}m(v_x{}^2 + v_y{}^2)$

$$= \frac{1}{2}(5.75\text{ kg})[(5.00\text{ m/s})^2 + (-3.00\text{ m/s})^2] = \boxed{97.8\text{ J}}$$

(b) We know $F_x = ma_x$ and $F_y = ma_y$. At $t = 0$, $x_i = y_i = 0$, and $v_{xi} = 5.00$ m/s, $v_{yi} = -3.00$ m/s; at $t = 2.00$ s, $x_f = 8.50$ m, $y_f = 5.00$ m.

$$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$$

$$a_x = \frac{2(x_f - x_i - v_{xi}t)}{t^2} = \frac{2[8.50\text{ m} - 0 - (5.00\text{ m/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= -0.75\text{ m/s}^2$$

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

$$a_y = \frac{2(y_f - y_i - v_{yi}t)}{t^2} = \frac{2[5.00\text{ m} - 0 - (-3.00\text{ m/s})(2.00\text{ s})]}{(2.00\text{ s})^2}$$

$$= 5.50\text{ m/s}^2$$

$$F_x = ma_x = (5.75\text{ kg})(-0.75\text{ m/s}^2) = -4.31\text{ N}$$

$$F_y = ma_y = (5.75\text{ kg})(5.50\text{ m/s}^2) = 31.6\text{ N}$$

$$\boxed{\vec{\mathbf{F}} = \left(-4.31\hat{\mathbf{i}} + 31.6\hat{\mathbf{j}}\right)\text{ N}}$$

(c) We can obtain the particle's speed at $t = 2.00$ s from the particle under constant acceleration model, or from the nonisolated system model. From the former,

$$v_{xf} = v_{xi} + a_x t = (5.00\text{ m/s}) + \left(-0.75\text{ m/s}^2\right)(2.00\text{ s}) = 3.50\text{ m/s}$$

$$v_{xf} = v_{yi} + a_y t = (-3.00\text{ m/s}) + \left(5.50\text{ m/s}^2\right)(2.00\text{ s}) = 8.00\text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(3.50\text{ m/s})^2 + (8.00\text{ m/s})^2} = \boxed{8.73\text{ m/s}}$$

From the nonisolated system model,

$$\sum W = \Delta K: \quad W_{\text{ext}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

The work done by the force is given by

$$W_{\text{ext}} = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = F_x\Delta r_x + F_y\Delta r_y$$

$$= (-4.31\text{ N})(8.50\text{ m}) + (31.6\text{ N})(5.00\text{ m}) = 121\text{ J}$$

then,

$$\frac{1}{2}mv_f^2 = W_{ext} + \frac{1}{2}mv_i^2 = 121\text{ J} + 97.8\text{ J} = 219\text{ J}$$

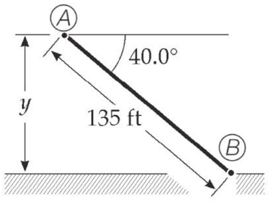
which gives

$$v_f = \sqrt{\frac{2(219\text{ J})}{5.75\text{ kg}}} = \boxed{8.73\text{ m/s}}$$

Section 7.6 Potential Energy of a System

P7.40 (a) With our choice for the zero level for potential energy of the car-Earth system when the car is at point Ⓑ,

$$\boxed{U_B = 0}$$



ANS. FIG. P7.40

When the car is at point Ⓐ, the potential energy of the car-Earth system is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With 135 ft = 41.1 m, this height is found as:

$$y = (41.1\text{ m})\sin 40.0^\circ = 26.4\text{ m}$$

Thus,

$$U_A = (1\,000\text{ kg})\left(9.80\text{ m/s}^2\right)(26.4\text{ m}) = \boxed{2.59\times 10^5\text{ J}}$$

The change in potential energy of the car-Earth system as the car moves from Ⓐ to Ⓑ is

$$U_B - U_A = 0 - 2.59\times 10^5\text{ J} = \boxed{-2.59\times 10^5\text{ J}}$$

(b) With our choice of the zero configuration for the potential energy of the car-Earth system when the car is at point Ⓐ, we have $\boxed{U_A = 0}$. The potential energy of the system when the car is at point Ⓑ is given by $U_B = mgy$, where y is the vertical distance of point Ⓑ below point Ⓐ. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number.

Thus,

$$U_B = (1\,000\text{ kg})\left(9.80\text{ m/s}^2\right)(-26.5\text{ m}) = \boxed{-2.59\times10^5\text{ J}}$$

The change in potential energy when the car moves from Ⓐ to Ⓑ is

$$U_B - U_A = -2.59\times10^5\text{ J} - 0 = \boxed{-2.59\times10^5\text{ J}}$$

P7.41 Use $U = mgy$, where y is measured relative to a reference level. Here, we measure y to be relative to the top edge of the well, where we take $y = 0$.

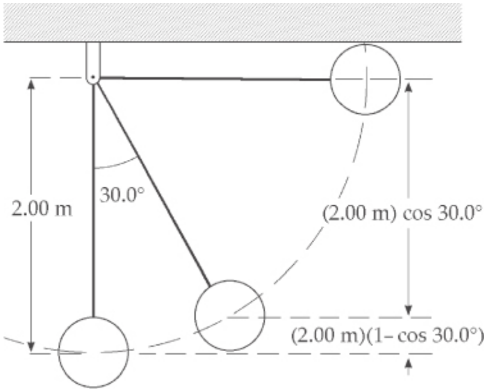
(a) $y = 1.3$ m: $U = mgy = (0.20\text{ kg})(9.80\text{ m/s}^2)(1.3\text{ m}) = \boxed{+2.5\text{ J}}$

(b) $y = -5.0$ m: $U = mgy = (0.20\text{ kg})(9.80\text{ m/s}^2)(-5.0\text{ m}) = \boxed{-9.8\text{ J}}$

(c) $\Delta U = U_f - U_i = (-9.8\text{ J}) - (2.5\text{ J}) = -12.3 = \boxed{-12\text{ J}}$

P7.42 (a) We take the zero configuration of system potential energy with the child at the lowest point of the arc. When the swing is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$\begin{aligned} U_g &= mgy \\ &= (400\text{ N})(2.00\text{ m}) \\ &= \boxed{800\text{ J}} \end{aligned}$$



ANS. FIG. P7.42

(b) From the sketch, we see that at an angle of 30.0° the child is at a vertical height of (2.00 m) (1 – cos 30.0°) above the lowest point of the arc. Thus,

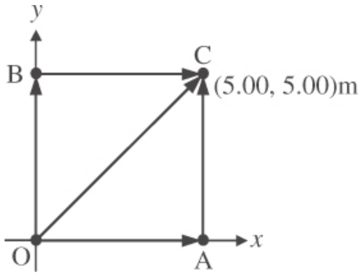
$$U_g = mgy = (400\text{ N})(2.00\text{ m})(1 - \cos 30.0°) = \boxed{107\text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

Section 7.7 Conservative and Nonconservative Forces

P7.43 The gravitational force is downward:

$$F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$$



ANS. FIG. P7.43

(a) Work along OAC = work along OA + work along AC

$$= F_g(\text{OA})\cos 90.0° + F_g(\text{AC})\cos 180°$$

$$= (39.2\text{ N})(5.00\text{ m})(0) + (39.2\text{ N})(5.00\text{ m})(-1)$$

$$= \boxed{-196\text{ J}}$$

(b) W along OBC = W along OB + W along BC

$$= (39.2\text{ N})(5.00\text{ m})\cos 180° + (39.2\text{ N})(5.00\text{ m})\cos 90.0°$$

$$= \boxed{-196\text{ J}}$$

(c) Work along OC = F_g(OC) cos 135°

$$= (39.2\text{ N})\left(5.00\times\sqrt{2}\text{ m}\right)\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196\text{ J}}$$

(d) $\boxed{\text{The results should all be the same, since the gravitational force is conservative.}}$

P7.44 (a) $W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$, and if the force is constant, this can be written as

$W = \vec{\mathbf{F}}\cdot\int d\vec{\mathbf{r}} = \boxed{\vec{\mathbf{F}}\cdot\left(\vec{\mathbf{r}}_f - \vec{\mathbf{r}}_i\right)\text{, which depends only on the end points,}}$

$\boxed{\text{and not on the path}}$.

(b) $W = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}} = \int\left(3\hat{\mathbf{i}} + 4\hat{\mathbf{j}}\right)\cdot\left(dx\hat{\mathbf{i}} + dy\hat{\mathbf{j}}\right)$

$$= (3.00\text{ N})\int_0^{5.00\text{ m}} dx + (4.00\text{ N})\int_0^{5.00\text{ m}} dy$$

$$W = (3.00\text{ N})x\Big|_0^{5.00\text{ m}} + (4.00\text{ N})y\Big|_0^{5.00\text{ m}} = 15.0\text{ J} + 20.0\text{ J} = \boxed{35.0\text{ J}}$$

The same calculation applies for all paths.

P7.45 In the following integrals, remember that

$$\hat{\mathbf{i}}\cdot\hat{\mathbf{i}}=\hat{\mathbf{j}}\cdot\hat{\mathbf{j}}=1 \quad \text{and} \quad \hat{\mathbf{i}}\cdot\hat{\mathbf{j}}=0$$

(a) The work done on the particle in its first section of motion is

$$W_{\text{OA}}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

and since along this path, $y = 0$, that means $W_{\text{OA}} = 0$.

In the next part of its path,

$$W_{\text{AC}}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2\,dy$$

For $x = 5.00$ m, $W_{\text{AC}} = 125$ J

and $W_{\text{OAC}}=0+125=\boxed{125\text{ J}}$.

(b) Following the same steps,

$$W_{\text{OB}}=\int_0^{5.00\text{ m}} dy\hat{\mathbf{j}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} x^2\,dy$$

Since along this path, $x = 0$, that means $W_{\text{OB}} = 0$.

$$W_{\text{BC}}=\int_0^{5.00\text{ m}} dx\hat{\mathbf{i}}\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int_0^{5.00\text{ m}} 2y\,dx$$

Since $y = 5.00$ m, $W_{\text{BC}} = 50.0$ J.

$$W_{\text{OAC}}=0+125=\boxed{125\text{ J}}$$

(c) $W_{\text{OC}}=\int\left(dx\hat{\mathbf{i}}+dy\hat{\mathbf{j}}\right)\cdot\left(2y\hat{\mathbf{i}}+x^2\hat{\mathbf{j}}\right)=\int\left(2y\,dx+x^2dy\right)$

Since $x = y$ along OC, $W_{\text{OC}}=\int_0^{5.00\text{ m}}\left(2x+x^2\right)dx=\boxed{66.7\text{ J}}$

(d) $\boxed{\text{F is nonconservative.}}$

(e) $\boxed{\text{The work done on the particle depends on the path followed by the particle.}}$

P7.46 Along each step of motion, to overcome friction you must push with a force of 3.00 N in the direction of travel along the path, so in the expression for work, $\cos\theta=\cos 0°=1$.

(a) $W=(3.00\text{ N})(5.00\text{ m})(1)+(3.00\text{ N})(5.00\text{ m})(1)=\boxed{30.0\text{ J}}$

(b) The distance CO is $(5.00^2 + 5.00^2)^{1/2}$ m = 7.07 m

$$W = (3.00\text{ N})(5.00\text{ m})(1) + (3.00\text{ N})(5.00\text{ m})(1) + (3.00\text{ N})(7.07\text{ m})(1) = \boxed{51.2\text{ J}}$$

(c) $W = (3.00\text{ N})(7.07\text{ m})(1) + (3.00\text{ N})(7.07\text{ m})(1) = \boxed{42.4\text{ J}}$

(d) $\boxed{\text{Friction is a nonconservative force.}}$

Section 7.8 Relationship Between Conservative Forces and Potential Energy

P7.47 We use the relation of force to potential energy as the force is the negative derivative of the potential energy with respect to distance:

$$U(r) = \frac{A}{r}$$

$$F_r = -\frac{\partial U}{\partial r} = -\frac{d}{dr}\left(\frac{A}{r}\right) = \boxed{\frac{A}{r^2}}$$

If *A* is positive, the positive value of radial force indicates a force of repulsion.

P7.48 We need to be very careful in identifying internal and external work on the book-Earth system. The first 20.0 J, done by the librarian on the system, is external work, so the system now contains an additional 20.0 J compared to the initial configuration. When the book falls and the system returns to the initial configuration, the 20.0 J of work done by the gravitational force from the Earth is *internal* work. This work only transforms the gravitational potential energy of the system to kinetic energy. It does *not* add more energy to the system. Therefore, the book hits the ground with 20.0 J of kinetic energy. The book-Earth system now has zero gravitational potential energy, for a total energy of 20.0 J, which is the energy put into the system by the librarian.

P7.49

$$F_x = -\frac{\partial U}{\partial x} = -\frac{\partial(3x^3y - 7x)}{\partial x} = -(9x^2y - 7) = 7 - 9x^2y$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial(3x^3y - 7x)}{\partial y} = -(3x^3 - 0) = -3x^3$$

Thus, the force acting at the point (*x*, *y*) is

$$\vec{\mathbf{F}} = F_x\hat{\mathbf{i}} + F_y\hat{\mathbf{j}} = \boxed{\left(7 - 9x^2y\right)\hat{\mathbf{i}} - 3x^3\hat{\mathbf{j}}}$$

P7.50 (a) We use Equation 7.27 relating the potential energy of the system to the conservative force acting on the particle, with $U_i = 0$:

$$U = U_f - U_i = U_f - 0$$

$$= -\int_0^x \left(-Ax + Bx^2\right)dx = A\frac{x^2}{2} - B\frac{x^3}{3}\Bigg|_0^x = \boxed{\frac{Ax^2}{2} - \frac{Bx^3}{3}}$$

(b) From (a), $U(2.00 \text{ m}) = 2A - 2.67B$, and $U(3.00 \text{ m}) = 4.5A - 9B$.

$$\boxed{\Delta U = (4.5A - 9B) - (2A - 2.67B) = 2.5A - 6.33B}$$

(c) If we consider the particle alone as a system, the change in its kinetic energy is the work done by the force on the particle: $W = \Delta K$. For the entire system of which this particle is a member, this work is internal work and equal to the negative of the change in potential energy of the system:

$$\boxed{\Delta K = -\Delta U = -2.5A + 6.33B}$$

P7.51 (a) For a particle moving along the *x* axis, the definition of work by a variable force is

$$W_F = \int_{x_i}^{x_f} F_x dx$$

Here $F_x = (2x + 4)$ N, $x_i = 1.00$ m, and $x_f = 5.00$ m.

So

$$W_F = \int_{1.00\text{ m}}^{5.00\text{ m}} (2x + 4)dx \text{ N} \cdot \text{m} = x^2 + 4x\Big]_{1.00\text{ m}}^{5.00\text{ m}} \text{ N} \cdot \text{m}$$

$$= \left(5^2 + 20 - 1 - 4\right) \text{ J} = \boxed{40.0 \text{ J}}$$

(b) The change in potential energy of the system is the negative of the internal work done by the conservative force on the particle: $\Delta U = -W_{\text{int}} = \boxed{-40.0 \text{ J}}$

(c) From $\Delta K = K_f - \dfrac{mv_1^2}{2}$, we obtain

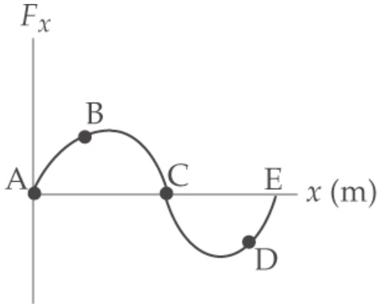
$$K_f = \Delta K + \frac{mv_1^2}{2} = 40.0 \text{ J} + \frac{(5.00 \text{ kg})(3.00 \text{ m/s})^2}{2} = \boxed{62.5 \text{ J}}$$

Section 7.9 Energy Diagrams and Equilibrium of a System

P7.52 (a) $\boxed{\text{F}_x \text{ is zero at points A, C, and E; } \text{F}_x \text{ is positive at point B and negative at point D.}}$

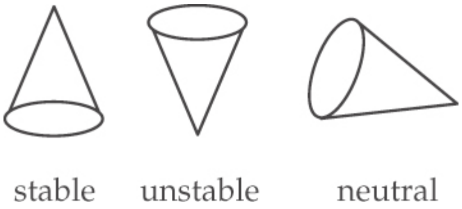
(b) $\boxed{\text{A and E are unstable, and C is stable.}}$

(c) ANS. FIG. P7.52 shows the curve for F_x vs. x for this problem.



ANS. FIG. P7.52

P7.53 The figure below shows the three equilibrium configurations for a right circular cone.



ANS. FIG. P7.53

Additional Problems

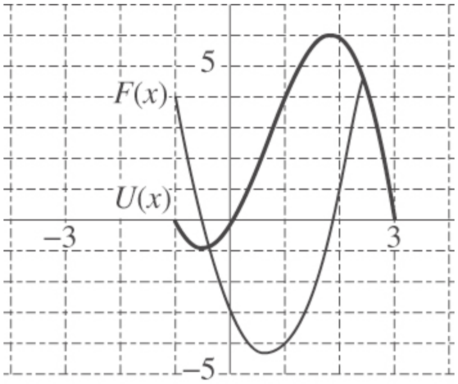
P7.54 (a)
$$\vec{\mathbf{F}} = -\frac{d}{dx}\left(-x^3 + 2x^2 + 3x\right)\hat{\mathbf{i}}$$
$$= \boxed{\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}}$$

(b) $F = 0$ when

$x = \boxed{1.87 \text{ and } -0.535}.$

(c) The stable point is at $x = -0.535$, point of minimum $U(x)$.

The unstable point is at $x = 1.87$, maximum in $U(x)$.



ANS. FIG. P7.54

P7.55 Initially, the ball's velocity is

$$\vec{\mathbf{v}} = (40.0\text{ m/s})\cos 30.0°\hat{\mathbf{i}} + (40.0\text{ m/s})\sin 30.0°\hat{\mathbf{j}}$$

At its apex, the ball's velocity is

$$\vec{\mathbf{v}} = (40.0\text{ m/s})\cos 30.0°\hat{\mathbf{i}} + 0\hat{\mathbf{j}} = (34.6\text{ m/s})\hat{\mathbf{i}}$$

The ball's kinetic energy of the ball at this point is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.150\text{ kg})(34.6\text{ m/s})^2 = \boxed{90.0\text{ J}}$$

P7.56 We evaluate $\int_{12.8}^{23.7} \frac{375dx}{x^3 + 3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3 + 3.75(12.8)} + \frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \cdots \frac{375(0.100)}{(23.6)^3 + 3.75(23.6)} = 0.806$$

and

$$\frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \frac{375(0.100)}{(13.0)^3 + 3.75(13.0)} + \cdots \frac{375(0.100)}{(23.7)^3 + 3.75(23.7)} = 0.791$$

The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799\text{ N}\cdot\text{m}}$.

P7.57 (a) The equivalent spring constant for the stel balls is

$$k = \frac{|F|}{|x|} = \frac{16\,000\text{ N}}{0.000\,2\text{ m}} = \boxed{8\times 10^7\text{ N/m}}$$

(b) $\boxed{\text{A time interval}}$. If the interaction occupied no time, the force exerted by each ball on the other could be infinite, and that cannot happen.

(c) We assume that steel has the density of its main constituent, iron, shown in Table 14.1. Then its mass is

$$\rho V = \rho\left(\frac{4}{3}\right)\pi r^3 = \left(\frac{4\pi}{3}\right)\left(7\ 860\text{ kg/m}^3\right)\left(0.025\ 4\text{ m/2}\right)^3$$
$$= 0.067\ 4\text{ kg}$$

its kinetic energy is then

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(0.067\ 4\ \text{kg})(5\ \text{m/s})^2 = \boxed{0.8\ \text{J}}$$

(d) Imagine one ball running into an infinitely hard wall and bouncing off elastically. The original kinetic energy becomes elastic potential energy

$$0.843\ \text{J} = (1/2)(8\times10^7\ \text{N/m})\,x^2 \qquad x = 0.145\ \text{mm} \approx \boxed{0.15\,\text{mm}}$$

(e) The ball does not really stop with constant acceleration, but imagine it moving 0.145 mm forward with average speed $(5\ \text{m/s} + 0)/2 = 2.5\ \text{m/s}$. The time interval over which it stops is then

$$0.145\ \text{mm}/(2.5\ \text{m/s}) = 6\times10^{-5}\ \text{s} \approx \boxed{10^{-4}\ \text{s}}$$

P7.58 The work done by the applied force is

$$W = \int_i^f F_{\text{applied}}dx = \int_0^{x_{\max}} -\left[-\left(k_1x + k_2x^2\right)\right]dx$$

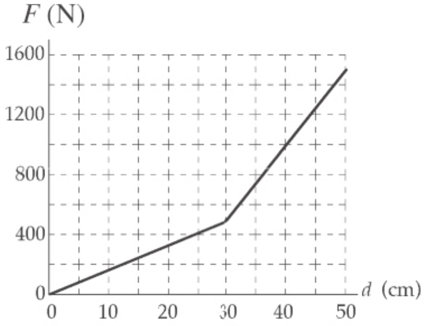
$$= \int_0^{x_{\max}} k_1x\,dx + \int_0^{x_{\max}} k_2x^2dx = k_1\frac{x^2}{2}\bigg|_0^{x_{\max}} + k_2\frac{x^3}{3}\bigg|_0^{x_{\max}}$$

$$= \boxed{k_1\frac{x_{\max}^2}{2} + k_2\frac{x_{\max}^3}{3}}$$

P7.59 Compare an initial picture of the rolling car with a final picture with both springs compressed. From conservation of energy, we have

$$K_i + \sum W = K_f$$

Work by both springs changes the car's kinetic energy.



ANS. FIG. P7.59

$$K_i + \frac{1}{2}k_1\left(x_{1i}^2 - x_{1f}^2\right) + \frac{1}{2}k_2\left(x_{2i}^2 - x_{2f}^2\right) = K_f$$

Substituting,

$$\frac{1}{2}mv_i^2 + 0 - \frac{1}{2}(1\,600\ \text{N/m})(0.500\ \text{m})^2 + 0 - \frac{1}{2}(3\,400\ \text{N/m})(0.200\ \text{m})^2 = 0$$

Which gives

$$\frac{1}{2}(6\,000\text{ kg})v_i^2 - 200\text{ J} - 68.0\text{ J} = 0$$

Solving for v_i,

$$v_i = \sqrt{\frac{2(268\text{ J})}{6\,000\text{ kg}}} = \boxed{0.299\text{ m/s}}$$

P7.60 Apply the work-energy theorem to the ball. The spring is initially compressed by $x_{\text{sp},i} = d = 5.00$ cm. After the ball is released from rest, the spring pushes the ball up the incline the distance d, doing positive work on the ball, and gravity does negative work on the ball as it travels up the incline a distance Δx from its starting point. Solve for Δx.

$$K_i + W_s + W_g = K_f$$

$$\frac{1}{2}mv_i^2 + \left(\frac{1}{2}kx_{\text{sp},i}^2 - \frac{1}{2}kx_{\text{sp, f}}^2\right) - mg\Delta x\sin\theta = \frac{1}{2}mv_f^2$$

$$0 + \left(\frac{1}{2}kd^2 - 0\right) - mg\Delta x\sin 10.0° = 0$$

$$\Delta x = \frac{kd^2}{2mg\sin 10.0°} = \frac{(1.20\text{ N/cm})(5.00\text{ cm})(0.0500\text{ m})}{2(0.100\text{ kg})\left(9.80\text{ m/s}^2\right)\sin 10.0°}$$

$$= 0.881\text{ m}$$

Thus, the ball travels up the incline a distance of 0.881 m after it is released.

Applying the work-kinetic energy theorem to the ball, one finds that it momentarily comes to rest at a distance up the incline of only 0.881 m. This distance is much smaller than the height of a professional basketball player, so the ball will not reach the upper end of the incline to be put into play in the machine. The ball will simply stop momentarily and roll back to the spring; not an exciting entertainment for any casino visitor!

P7.61 (a) $\vec{\mathbf{F}}_1 = (25.0\text{ N})\left(\cos 35.0°\hat{\mathbf{i}} + \sin 35.0°\hat{\mathbf{j}}\right) = \boxed{\left(20.5\hat{\mathbf{i}} + 14.3\hat{\mathbf{j}}\right)\text{ N}}$

$\vec{\mathbf{F}}_2 = (42.0\text{ N})\left(\cos 150°\hat{\mathbf{i}} + \sin 150°\hat{\mathbf{j}}\right) = \boxed{\left(-36.4\hat{\mathbf{i}} + 21.0\hat{\mathbf{j}}\right)\text{ N}}$

(b) $\sum\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \boxed{\left(-15.9\hat{\mathbf{i}} + 35.3\hat{\mathbf{j}}\right)\text{ N}}$

(c) $$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \boxed{\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\ \text{m/s}^2}$$

(d) $$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)\ \text{m/s} + \left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\ \text{s})$$

$$\vec{\mathbf{v}}_f = \boxed{\left(-5.54\hat{\mathbf{i}} + 23.7\hat{\mathbf{j}}\right)\ \text{m/s}}$$

(e) $$\vec{\mathbf{r}}_f = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$\vec{\mathbf{r}}_f = 0 + \left(4.00\hat{\mathbf{i}} + 2.50\hat{\mathbf{j}}\right)(\text{m/s})(3.00\ \text{s}) + \frac{1}{2}\left(-3.18\hat{\mathbf{i}} + 7.07\hat{\mathbf{j}}\right)\left(\text{m/s}^2\right)(3.00\ \text{s})^2$$

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}}_f = \boxed{\left(-2.30\hat{\mathbf{i}} + 39.3\hat{\mathbf{j}}\right)\ \text{m}}$$

(f) $$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(5.00\ \text{kg})\left[(5.54)^2 + (23.7)^2\right]\left(\text{m/s}^2\right) = \boxed{1.48\ \text{kJ}}$$

(g) $$K_f = \frac{1}{2}mv_i^2 + \sum \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}}$$

$$K_f = \frac{1}{2}(5.00\ \text{kg})\left[(4.00)^2 + (2.50)^2\right](\text{m/s})^2 + \left[(-15.9\ \text{N})(-2.30\ \text{m}) + (35.3\ \text{N})(39.3\ \text{m})\right]$$

$$K_f = 55.6\ \text{J} + 1\,426\ \text{J} = \boxed{1.48\ \text{kJ}}$$

(h) $\boxed{\text{The work-kinetic energy theorem is consistent with Newton's second law, used in deriving it.}}$

P7.62 (a) We write

$$F = ax^b$$

$$1\,000\ \text{N} = a(0.129\ \text{m})^b$$

$$5\,000\ \text{N} = a(0.315\ \text{m})^b$$

Dividing the two equations gives

$$5 = \left(\frac{0.315}{0.129}\right)^b = 2.44^b$$

$$\ln 5 = b \ln 2.44$$

$$b = \frac{\ln 5}{\ln 2.44} = \boxed{1.80}$$

$$a = \frac{1\,000\text{ N}}{(0.129\text{ m})^{1.80}} = \boxed{4.01\times 10^{4}\text{ N/m}^{1.8}}$$

(b) $$W = \int_i^f F_{\text{applied}}\,dx = \int_0^x ax^b\,dx = \left.\frac{ax^{b+1}}{b+1}\right|_0^x = \frac{ax^{b+1}}{b+1} - 0 = \frac{ax^{b+1}}{b+1}$$

$$W = \frac{(4.01\times 10^{4}\text{ N/m}^{1.8})x^{2.8}}{2.80}$$

For $x = 0.250$ m,

$$W = \frac{(4.01\times 10^{4}\text{ N/m}^{1.8})(0.250\text{ m})^{2.8}}{2.80}$$

$$= \frac{(4.01\times 10^{4}\text{ N/m}^{1.8})(0.250)^{2.8}(\text{m}^{2.8})}{2.80}$$

$$W = \frac{(4.01\times 10^{4}\text{ N}\cdot\text{m})(0.250)^{2.8}}{2.80} = \boxed{295\text{ J}}$$

P7.63 The component of the weight force parallel to the incline, *mg* sin θ, accelerates the block down the incline through a distance *d* until it encounters the spring, after which the spring force, pushing up the incline, opposes the weight force and slows the block through a distance *x* until the block eventually is brought to a momentary stop. The weight force does positive work on the block as it slides down the incline through total distance ($d + x$), and the spring force does negative work on the block as it slides through distance *x*. The normal force does no work. Applying the work-energy theorem,

$$K_i + W_g + W_s = K_f$$

$$\frac{1}{2}mv_i^2 + mg\sin\theta(d+x) + \left(\frac{1}{2}kx_{\text{sp},i}^2 - \frac{1}{2}kx_{\text{sp,f}}^2\right) = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv^2 + mg\sin\theta(d+x) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

Dividing by *m*, we have

$$\frac{1}{2}v^2 + g\sin\theta(d+x) - \frac{k}{2m}x^2 = 0 \;\rightarrow$$

$$\frac{k}{2m}x^2 - (g\sin\theta)x - \left[\frac{v^2}{2} + (g\sin\theta)d\right] = 0$$

Solving for *x*, we have

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 - 4\left(\frac{k}{2m}\right)\left[-\left(\frac{v^2}{2} + (g\sin\theta)d\right)\right]}}{2\left(\frac{k}{2m}\right)}$$

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

Because distance *x* must be positive,

$$x = \frac{g\sin\theta + \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

For $v = 0.750$ m/s, $k = 500$ N/m, $m = 2.50$ kg, $\theta = 20.0°$, and $g =$ 9.80 m/s^2, we have $g\sin\theta = (9.80\text{ m/s}^2)\sin 20.0° = 3.35\text{ m/s}^2$ and k/m = (500 N/m)/(2.50 kg) = 200 N/m · kg. Suppressing units, we have

$$x = \frac{3.35 + \sqrt{(3.35)^2 + (200)\left[(0.750)^2 + 2(3.35)(0.300)\right]}}{200}$$

$$= \boxed{0.131\text{ m}}$$

P7.64 The component of the weight force parallel to the incline, *mg*sin *θ*, accelerates the block down the incline through a distance *d* until it encounters the spring, after which the spring force, pushing up the incline, opposes the weight force and slows the block through a distance *x* until the block eventually is brought to a momentary stop. The weight force does positive work on the block as it slides down the incline through total distance (*d* + *x*), and the spring force does negative work on the block as it slides through distance *x*. The normal force does no work.

Applying the work-energy theorem,

$$K_i + W_g + W_s = K_f$$

$$\frac{1}{2}mv_i^2 + mg\sin\theta(d + x) + \left(\frac{1}{2}kx_{sp,i}^2 - \frac{1}{2}kx_{sp,f}^2\right) = \frac{1}{2}mv_f^2$$

$$\frac{1}{2}mv^2 + mg\sin\theta(d + x) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

Dividing by *m*, we have

$$\frac{1}{2}v^2 + g\sin\theta(d+x) - \frac{k}{2m}x^2 = 0 \quad\rightarrow$$

$$\frac{k}{2m}x^2 - (g\sin\theta)x - \left[\frac{v^2}{2} + (g\sin\theta)d\right] = 0$$

Solving for *x*, we have

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 - 4\left(\frac{k}{2m}\right)\left[-\left(\frac{v^2}{2} + (g\sin\theta)d\right)\right]}}{2\left(\frac{k}{2m}\right)}$$

$$x = \frac{g\sin\theta \pm \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

Because distance *x* must be positive,

$$\boxed{x = \frac{g\sin\theta + \sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}}$$

P7.65 (a) The potential energy of the system at point *x* is given by 5 plus the negative of the work the force does as a particle feeling the force is carried from *x* = 0 to location *x*.

$$dU = -F\,dx$$

$$\int_5^U dU = -\int_0^x 8e^{-2x}\,dx$$

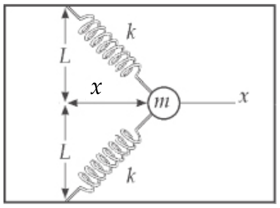
$$U - 5 = -\left(\frac{8}{[-2]}\right)\int_0^x e^{-2x}(-2\,dx)$$

$$U = 5 - \left(\frac{8}{[-2]}\right)e^{-2x}\Big|_0^x = 5 + 4e^{-2x} - 4\cdot 1 = \boxed{1 + 4e^{-2x}}$$

(b) The force must be conservative because the work the force does on the object on which it acts depends only on the original and final positions of the object, not on the path between them. There is a uniquely defined potential energy for the associated force.

Challenge Problems

P7.66 (a) The new length of each spring is $\sqrt{x^2+L^2}$, so its extension is $\sqrt{x^2+L^2}-L$ and the force it exerts is $k\left(\sqrt{x^2+L^2}-L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components (with $\cos\theta = \dfrac{x}{\sqrt{x^2+L^2}}$) add to



ANS. FIG. P7.66

$$\vec{\mathbf{F}} = -2k\left(\sqrt{x^2+L^2}-L\right)\frac{x}{\sqrt{x^2+L^2}}\hat{\mathbf{i}}$$

$$= \boxed{-2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)\hat{\mathbf{i}}}$$

(b) Choose $U = 0$ at $x = 0$. Then at any point the potential energy of the system is

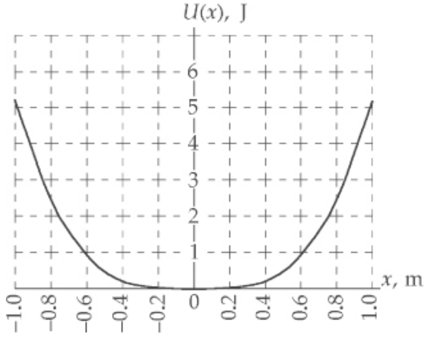
$$U(x) = -\int_0^x F_x\,dx = -\int_0^x\left(-2kx+\frac{2kLx}{\sqrt{x^2+L^2}}\right)dx$$

$$= 2k\int_0^x x\,dx - 2kL\int_0^x \frac{x}{\sqrt{x^2+L^2}}dx$$

$$U(x) = \boxed{kx^2+2kL\left(L-\sqrt{x^2+L^2}\right)}$$

(c) $U(x) = (40.0\text{ N/m})x^2 + (96.0\text{ N})\left(1.20\text{ m}-\sqrt{x^2+1.44\text{ m}^2}\right)$

For negative x, $U(x)$ has the same value as for positive x. The only equilibrium point (i.e., where $F_x = 0$) is $\boxed{x=0}$.



ANS FIG. P7.66(c)

(d) If we consider the particle alone as a system, the change in its kinetic energy is the work done by the force of the springs on the particle: $W = \Delta K$. For the entire system of particle and springs, this work is internal work and equal to the negative of the change in potential energy of the system: $\Delta K = -\Delta U$. From part (c), we evaluate U for $x = 0.500$ m:

$$\begin{aligned} U &= (40.0\text{ N/m})(0.500\text{ m})^2 \\ &\quad + (96.0\text{ N})\left(1.20\text{ m} - \sqrt{(0.500\text{ m})^2 + 1.44\text{ m}^2}\right) \\ &= 0.400\text{ J} \end{aligned}$$

Now find the speed of the particle:

$$\frac{1}{2}mv^2 = -\Delta U$$

$$v = \sqrt{\frac{-2\Delta U}{m}} = \sqrt{\frac{-2}{1.18\text{ kg}}(0 - 0.400\text{ J})} = \boxed{0.823\text{ m/s}}$$

P7.67 (a) We assume the spring lies in the horizontal plane of the motion, then the radius of the puck's motion is $r = L_0 + x$, where $L_0 = 0.155$ m is the unstretched length. The spring force causes the puck's centripetal acceleration:

$$F = mv^2/r \quad \to \quad kx = m(2\pi r/T)^2/r \quad \to \quad kT^2x = 4\pi^2 mr$$

Substituting $r = (L_0 + x)$, we have

$$kT^2x = 4\pi^2 m(L_0 + x)$$

$$kx = \frac{(4\pi^2 mL_0)}{T^2} + \frac{x(4\pi^2 m)}{T^2}$$

$$x\left(k - \frac{4\pi^2 m}{T^2}\right) = \frac{4\pi^2 mL_0}{T^2}$$

$$x = \frac{4\pi^2 mL_0/T^2}{k - 4\pi^2 mL_0/T^2}$$

For $k = 4.30$ N/m, $L_0 = 0.155$ m, and $T = 1.30$ s, we have

$$\begin{aligned} x &= \frac{4\pi^2 m(0.155\text{ m})/(1.30\text{ s})^2}{4.30\text{ N/m} - 4\pi^2 m/(1.30\text{ s})^2} \\ &= \frac{(3.62\text{ m/s}^2)m}{4.30\text{ kg/s}^2 - (23.36/\text{s}^2)m} \\ &= \frac{(3.62\text{ m})m}{[4.30\text{ kg} - (23.36)m]}\frac{1/\text{s}^2}{1/\text{s}^2} \end{aligned}$$

$$\boxed{x = \frac{(3.62\ \text{m})m}{4.30\ \text{kg} - (23.4)m}}$$

(b) For $m = 0.070$ kg,

$$x = \frac{(3.62\ \text{m})[0.070\ \text{kg}]}{4.30\ \text{kg} - 23.36(0.070\ \text{kg})}$$

$$= \boxed{0.095\ 1\ \text{m}}$$

(c) We double the puck mass and find

$$x = \frac{(3.6208\ \text{m})[0.140\ \text{kg}]}{4.30\ \text{kg} - 23.360(0.140\ \text{kg})}$$

$$= \boxed{0.492\ \text{m}}$$

more than twice as big!

(d) For $m = 0.180$ kg,

$$x = \frac{(3.62\ \text{m})[0.180\ \text{kg}]}{4.30\ \text{kg} - 23.36(0.180\ \text{kg})}$$

$$= \frac{0.652}{0.0952}\ \text{m} = \boxed{6.85\ \text{m}}$$

We have to get a bigger table!

(e) When the denominator of the fraction goes to zero, the extension becomes infinite. This happens for 4.3 kg – 23.4 $m = 0$; that is for $m = 0.184$ kg. For any larger mass, the spring cannot constrain the motion. $\boxed{\text{The situation is impossible.}}$

(f) $\boxed{\text{The extension is directly proportional to } m \text{ when } m \text{ is only a few grams. Then it grows faster and faster, diverging to infinity for } m = 0.184\ \text{kg.}}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P7.2	(a) 3.28×10^{-2} J; (b) -3.28×10^{-2} J
P7.4	1.56×10^{4} J
P7.6	method one: -4.70×10^{3} J; method two: -4.70 kJ
P7.8	28.9
P7.10	5.33 J
P7.12	(a) 11.3°; (b) 156°; (c) 82.3°
P7.14	(a) 24.0 J; (b) -3.00 J; (c) 21.0 J
P7.16	7.37 N/m
P7.18	(a) 1.13 kN/m; (b) 0.518 m = 51.8 cm
P7.20	(a) 2.04×10^{-2} m; (b) 720 N/m
P7.22	kg/s^2
P7.24	(a) $-1.23\ \text{m/s}^2$, $0.616\ \text{m/s}^2$; (b) $-0.252\ \text{m/s}^2$ if the force of static friction is not too large, zero; (c) 0
P7.26	(a) See ANS FIG P7.26; (b) -12.0 J
P7.28	(a) 9.00 kJ; (b) 11.7 kJ; (c) The work is greater by 29.6%
P7.30	(a) 0.600 J; (b) -0.600 J; (c) 1.50 J
P7.32	(a) 29.2 N; (b) speed would increase; (c) crate would slow down and come to rest.
P7.34	(a) 1.94 m/s; (b) 3.35 m/s; (c) 3.87 m/s
P7.36	(a) 3.78×10^{-16} J; (b) 1.35×10^{-14} N; (c) $1.48 \times 10^{+16}\ \text{m/s}^2$; (d) 1.94×10^{-9} s
P7.38	(a) $F_{\text{avg}} = 2.34 \times 10^{4}\ \text{N}$, opposite to the direction of motion; (b) 1.91×10^{-4} s
P7.40	(a) $U_B = 0$, 2.59×10^{5} J; (b) $U_A = 0$, -2.59×10^{5} J, -2.59×10^{5} J
P7.42	(a) 800 J; (b) 107 J; (c) $U_g = 0$
P7.44	(a) $\vec{\mathbf{F}} \cdot \left(\vec{\mathbf{r}}_{\text{fi}} - \vec{\mathbf{r}}_{\text{i}}\right)$, which depends only on end points, and not on the path; (b) 35.0 J
P7.46	(a) 30.0 J; (b) 51.2 J; (c) 42.4 J; (d) Friction is a nonconservative force
P7.48	The book hits the ground with 20.0 J of kinetic energy. The book-Earth now has zero gravitational potential energy, for a total energy of 20.0 J, which is the energy put into the system by the librarian.

P7.50 (a) $\frac{Ax^2}{2}-\frac{Bx^3}{3}$; (b) $\Delta U = (4.5A - 9B) - (2A - 2.67B) = 2.5A - 6.33B$;

(c) $\Delta K = -\Delta U = -2.5A + 6.33B$

P7.52 (a) F_x is zero at points A, C, and E; F_x is positive at point B and negative at point D; (b) A and E are unstable, and C is stable; (c) See ANS FIG P7.52

P7.54 (a) $\left(3x^2 - 4x - 3\right)\hat{\mathbf{i}}$; (b) 1.87 and −0.535; (c) See ANS. FIG. P7.54

P7.56 0.799 N · m

P7.58 $k_1\frac{x_{\text{max}}^2}{2} + k_2\frac{x_{\text{max}}^3}{3}$

P7.60 The ball will simply stop momentarily and roll back to the spring.

P7.62 (a) $b = 1.80$, $a = 4.01 \times 10^4\ \text{N/m}^{1.8}$; (b) 295 J

P7.64

$$x = \frac{g\sin\theta\sqrt{(g\sin\theta)^2 + \left(\frac{k}{m}\right)\left[v^2 + 2(g\sin\theta)d\right]}}{k/m}$$

P7.66 (a) $-2kx\left(1-\frac{L}{\sqrt{x^2+L^2}}\right)\hat{\mathbf{i}}$; (b) $kx^2 + 2kL\left(L - \sqrt{x^2+L^2}\right)$; (c) See ANS. FIG. P7.66(c), $x = 0$; (d) $v = 0.823$ m/s

8

Conservation of Energy

CHAPTER OUTLINE

8.1	Analysis Model: Nonisolated System (Energy)
8.2	Analysis Model: Isolated System (Energy)
8.3	Situations Involving Kinetic Friction
8.4	Changes in Mechanical Energy for Nonconservative Forces
8.5	Power

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ8.1 Answer (a). We assume the light band of the slingshot puts equal amounts of kinetic energy into the missiles. With three times more speed, the bean has nine times more squared speed, so it must have one-ninth the mass.

OQ8.2 (i) Answer (b). Kinetic energy is proportional to mass.

(ii) Answer (c). The slide is frictionless, so $v = (2gh)^{1/2}$ in both cases.

(iii) Answer (a). g for the smaller child and $g\sin\theta$ for the larger.

OQ8.3 Answer (d). The static friction force that each glider exerts on the other acts over no distance relative to the surface of the other glider. The air track isolates the gliders from outside forces doing work. The gliders-Earth system keeps constant mechanical energy.

OQ8.4 Answer (c). Once the athlete leaves the surface of the trampoline, only a conservative force (her weight) acts on her. Therefore, the total mechanical energy of the athlete-Earth system is constant during her flight: $K_f + U_f = K_i + U_i$. Taking the $y = 0$ at the surface of the trampoline, $U_i = mgy_i = 0$. Also, her speed when she reaches maximum height is zero, or $K_f = 0$. This leaves us with $U_f = K_i$, or $mgy_{\max} = \frac{1}{2}mv_i^2$, which gives the maximum height as

$$y_{\max} = \frac{v_i^2}{2g} = \frac{(8.5\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 3.7\text{ m}$$

OQ8.5 (a) Yes: a block slides on the floor where we choose $y = 0$.

(b) Yes: a picture on the classroom wall high above the floor.

(c) Yes: an eraser hurtling across the room.

(d) Yes: the block stationary on the floor.

OQ8.6 In order the ranking: c > a = d > b. We have $\frac{1}{2}mv^2 = \mu_k mgd$ so $d = v^2/2\mu_k g$. The quantity v^2/μ_k controls the skidding distance. In the cases quoted respectively, this quantity has the numerical values: (a) 5 (b) 1.25 (c) 20 (d) 5.

OQ8.7 Answer (a). We assume the climber has negligible speed at both the beginning and the end of the climb. Then $K_f = K_i$, and the work done by the muscles is

$$\begin{aligned} W_{nc} &= 0 + (U_f - U_i) = mg(y_f - y_i) \\ &= (70.0\text{ kg})(9.80\text{ m/s}^2)(325\text{ m}) \\ &= 2.23 \times 10^5\text{ J} \end{aligned}$$

The average power delivered is

$$P = \frac{W_{nc}}{\Delta t} = \frac{2.23 \times 10^5\text{ J}}{(95.0\text{ min})(60\text{ s / 1 min})} = 39.1\text{ W}$$

OQ8.8 Answer (d). The energy is internal energy. Energy is never "used up." The ball finally has no elevation and no compression, so the ball-Earth system has no potential energy. There is no stove, so no energy is put in by heat. The amount of energy transferred away by sound is minuscule.

OQ8.9 Answer (c). Gravitational energy is proportional to the mass of the object in the Earth's field.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ8.1 (a) No. They will not agree on the original gravitational energy if they make different $y = 0$ choices. (b) Yes, (c) Yes. They see the same change in elevation and the same speed, so they do agree on the change in gravitational energy and on the kinetic energy.

CQ8.2 The larger engine is unnecessary. Consider a 30-minute commute. If you travel the same speed in each car, it will take the same amount of time, expending the same amount of energy. The extra power available from the larger engine isn't used.

CQ8.3 Unless an object is cooled to absolute zero, then that object will have internal energy, as temperature is a measure of the energy content of matter. Potential energy is not measured for single objects, but for systems. For example, a system comprised of a ball and the Earth will have potential energy, but the ball itself can never be said to have potential energy. An object can have zero kinetic energy, but this measurement is dependent on the reference frame of the observer.

CQ8.4 All the energy is supplied by foodstuffs that gained their energy from the Sun.

CQ8.5 (a) The total energy of the ball-Earth system is conserved. Since the system initially has gravitational energy *mgh* and no kinetic energy, the ball will again have zero kinetic energy when it returns to its original position. Air resistance will cause the ball to come back to a point slightly below its initial position. (b) If she gives a forward push to the ball from its starting position, the ball will have the same kinetic energy, and therefore the same speed, at its return: the demonstrator will have to duck.

CQ8.6 Yes, if it is exerted by an object that is moving in our frame of reference. The flat bed of a truck exerts a static friction force to start a pumpkin moving forward as it slowly starts up.

CQ8.7
(a) original elastic potential energy into final kinetic energy

(b) original chemical energy into final internal energy

(c) original chemical potential energy in the batteries into final internal energy, plus a tiny bit of outgoing energy transmitted by mechanical waves

(d) original kinetic energy into final internal energy in the brakes

(e) energy input by heat from the lower layers of the Sun, into energy transmitted by electromagnetic radiation

(f) original chemical energy into final gravitational energy

CQ8.8 (a) (**i**) A campfire converts chemical energy into internal energy, within the system wood-plus-oxygen, and before energy is transferred by heat and electromagnetic radiation into the surroundings. If all the fuel burns, the process can be 100% efficient.

(**ii**) Chemical-energy-into-internal-energy is also the conversion as iron rusts, and it is the main conversion in mammalian metabolism.

(b) (**i**) An escalator motor converts electrically transmitted energy into gravitational energy. As the system we may choose motor-plus-escalator-and-riders. The efficiency could be, say 90%, but in many escalators a significant amount of internal energy is generated and leaves the system by heat.

(**ii**) A natural process, such as atmospheric electric current in a lightning bolt, which raises the temperature of a particular region of air so that the surrounding air buoys it up, could produce the same electricity-to-gravitational energy conversion with low efficiency.

(c) (**i**) A diver jumps up from a diving board, setting it vibrating temporarily. The material in the board rises in temperature slightly as the visible vibration dies down, and then the board cools off to the constant temperature of the environment. This process for the board-plus-air system can have 100% efficiency in converting the energy of vibration into energy transferred by heat. The energy of vibration is all elastic energy at instants when the board is momentarily at rest at turning points in its motion.

(**ii**) For a natural process, you could think of the branch of a palm tree vibrating for a while after a coconut falls from it.

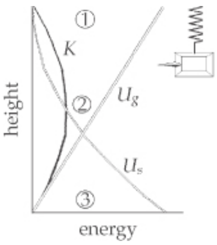
(d) (**i**) Some of the energy transferred by sound in a shout results in kinetic energy of a listener’s eardrum; most of the mechanical-wave energy becomes internal energy as the sound is absorbed by all the surfaces it falls upon.

(**ii**) We would also assign low efficiency to a train of water waves doing work to shift sand back and forth in a region near a beach.

(e) (**i**) A demonstration solar car takes in electromagnetic-wave energy in sunlight and turns some fraction of it temporarily into the car’s kinetic energy. A much larger fraction becomes internal energy in the solar cells, battery, motor, and air pushed aside.

(**ii**) Perhaps with somewhat higher net efficiency, the pressure of light from a newborn star pushes away gas and dust in the nebula surrounding it.

CQ8.9 The figure illustrates the relative amounts of the forms of energy in the cycle of the block, where the vertical axis shows position (height) and the horizontal axis shows energy. Let the gravitational energy (U_g) be zero for the configuration of the system when the block is at the lowest point in the motion, point (3). After the block moves downward through position (2), where its kinetic energy (K) is a maximum, its kinetic energy converts into extra elastic potential energy in the spring (U_s). After the block starts moving up at its lower turning point (3), this energy becomes both kinetic energy and gravitational potential energy, and then just gravitational energy when the block is at its greatest height (1) where its elastic potential energy is the least. The energy then turns back into kinetic and elastic potential energy as the block descends, and the cycle repeats.



ANS. FIG. CQ8.9

CQ8.10 Lift a book from a low shelf to place it on a high shelf. The net change in its kinetic energy is zero, but the book-Earth system increases in gravitational potential energy. Stretch a rubber band to encompass the ends of a ruler. It increases in elastic energy. Rub your hands together or let a pearl drift down at constant speed in a bottle of shampoo. Each system (two hands; pearl and shampoo) increases in internal energy.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 8.1 Analysis Model: Nonisolated system (Energy)

P8.1 (a) The toaster coils take in energy by electrical transmission. They increase in internal energy and put out energy by heat into the air and energy by electromagnetic radiation as they start to glow.

$$\boxed{\Delta E_{\text{int}} = Q + T_{\text{ET}} + T_{\text{ER}}}$$

(b) The car takes in energy by matter transfer. Its fund of chemical potential energy increases. As it moves, its kinetic energy increases and it puts out energy by work on the air, energy by heat in the exhaust, and a tiny bit of energy by mechanical waves in sound.

$$\boxed{\Delta K + \Delta U + \Delta E_{\text{int}} = W + Q + T_{\text{MW}} + T_{\text{MT}}}$$

(c) You take in energy by matter transfer. Your fund of chemical potential energy increases. You are always putting out energy by heat into the surrounding air.

$$\boxed{\Delta U = Q + T_{MT}}$$

(d) Your house is in steady state, keeping constant energy as it takes in energy by electrical transmission to run the clocks and, we assume, an air conditioner. It absorbs sunlight, taking in energy by electromagnetic radiation. Energy enters the house by matter transfer in the form of natural gas being piped into the home for clothes dryers, water heaters, and stoves. Matter transfer also occurs by means of leaks of air through doors and windows.

$$\boxed{0 = Q + T_{MT} + T_{ET} + T_{ER}}$$

P8.2 (a) The system of the ball and the Earth is isolated. The gravitational energy of the system decreases as the kinetic energy increases.

$$\boxed{\Delta K + \Delta U = 0}$$

$$\left(\frac{1}{2}mv^2 - 0\right) + \left(-mgh - 0\right) = 0 \rightarrow \frac{1}{2}mv^2 = mgy$$

$$\boxed{v = \sqrt{2gh}}$$

(b) The gravity force does positive work on the ball as the ball moves downward. The Earth is assumed to remain stationary, so no work is done on it.

$$\Delta K = W$$

$$\left(\frac{1}{2}mv^2 - 0\right) = mgh \rightarrow \frac{1}{2}mv^2 = mgy$$

$$\boxed{v = \sqrt{2gh}}$$

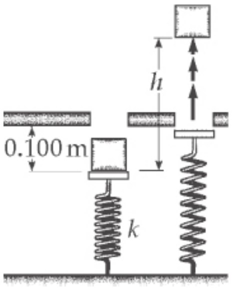
Section 8.2 Analysis Model: Isolated system (Energy)

P8.3 From conservation of energy for the block-spring-Earth system,

$$U_{gf} = U_{si}$$

or

$$(0.250\ \text{kg})(9.80\ \text{m/s}^2)h = \left(\frac{1}{2}\right)(5\,000\ \text{N/m})(0.100\ \text{m})^2$$



ANS. FIG. P8.3

This gives a maximum height, $h = \boxed{10.2\ \text{m}}$.

P8.4 (a) $\Delta K + \Delta U = 0 \rightarrow \Delta K = -\Delta U$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\left(mgy_f - mgy_i\right)$$

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_i^2 + mgy_f$$

We use the Pythagorean theorem to express the original kinetic energy in terms of the velocity components (kinetic energy itself does not have components):

$$\left(\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2\right) = \left(\frac{1}{2}mv_{xf}^2 + 0\right) + mgy_f$$

$$\frac{1}{2}mv_{xi}^2 + \frac{1}{2}mv_{yi}^2 = \frac{1}{2}mv_{xf}^2 + mgy_f$$

Because $v_{xi} = v_{xf}$, we have

$$\frac{1}{2}mv_{yi}^2 = mgy_f \rightarrow y_f = \frac{v_{yi}^2}{2g}$$

so for the first ball:

$$y_f = \frac{v_{yi}^2}{2g} = \frac{\left[(1\,000\ \text{m/s})\sin 37.0°\right]^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{1.85 \times 10^4\ \text{m}}$$

and for the second,

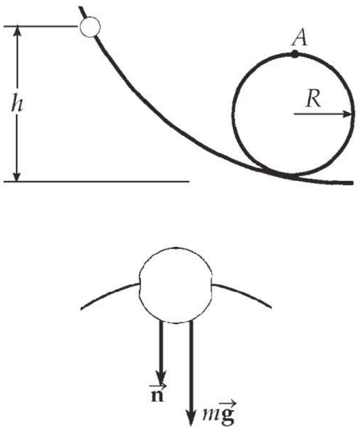
$$y_f = \frac{(1\,000\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{5.10 \times 10^4\ \text{m}}$$

(b) The total energy of each ball-Earth system is constant with value

$$E_{\text{mech}} = K_i + U_i = K_i + 0$$

$$E_{\text{mech}} = \frac{1}{2}(20.0\text{ kg})(1\,000\text{ m/s})^2 = \boxed{1.00\times10^7\text{ J}}$$

P8.5 The speed at the top can be found from the conservation of energy for the bead-track-Earth system, and the normal force can be found from Newton's second law.



ANS. FIG. P8.5

(a) We define the bottom of the loop as the zero level for the gravitational potential energy.

Since $v_i = 0$,

$$E_i = K_i + U_i = 0 + mgh = mg(3.50R)$$

The total energy of the bead at point Ⓐ can be written as

$$E_A = K_A + U_A = \frac{1}{2}mv_A^2 + mg(2R)$$

Since mechanical energy is conserved, $E_i = E_A$, we get

$$mg(3.50R) = \frac{1}{2}mv_A^2 + mg(2R)$$

simplifying,

$$v_A^2 = 3.00\text{ gR}$$

$$\boxed{v_A = \sqrt{3.00gR}}$$

(b) To find the normal force at the top, we construct a force diagram as shown, where we assume that n is downward, like mg. Newton's second law gives $\sum F = ma_c$, where a_c is the centripetal acceleration.

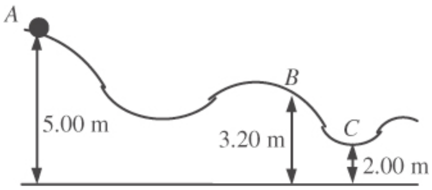
$$\sum F_y = ma_y: \quad n + mg = \frac{mv^2}{r}$$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00gR}{R} - g\right] = 2.00mg$$

$$n = 2.00(5.00\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)$$

$$= \boxed{0.098\,0\text{ N downward}}$$

P8.6 (a) Define the system as the block and the Earth.



ANS. FIG. P8.6

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv_B^2 - 0\right) + \left(mgh_B - mgh_A\right) = 0$$

$$\frac{1}{2}mv_B^2 = mg\left(h_A - h_B\right)$$

$$v_B = \sqrt{2g\left(h_A - h_B\right)}$$

$$v_B = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(5.00\ \text{m} - 3.20\ \text{m}\right)} = \boxed{5.94\ \text{m/s}}$$

Similarly,

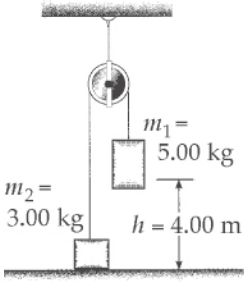
$$v_C = \sqrt{2g\left(h_A - h_C\right)}$$

$$v_C = \sqrt{2g\left(5.00 - 2.00\right)} = \boxed{7.67\ \text{m/s}}$$

(b) Treating the block as the system,

$$W_g\Big|_{A\to C} = \Delta K = \frac{1}{2}mv_C^2 - 0 = \frac{1}{2}\left(5.00\ \text{kg}\right)\left(7.67\ \text{m/s}\right)^2 = \boxed{147\ \text{J}}$$

P8.7 We assign height $y = 0$ to the table top. Using conservation of energy for the system of the Earth and the two objects:



ANS. FIG. P8.7

(a) Choose the initial point before release and the final point, which we code with the subscript *fa*, just before the larger object hits the floor. No external forces do work on the system and no friction acts within the system. Then total mechanical energy of the system remains constant and the energy version of the isolated system model gives

$$(K_A + K_B + U_g)_i = (K_A + K_B + U_g)_{fa}$$

At the initial point, K_{Ai} and K_{Bi} are zero and we define the gravitational potential energy of the system as zero. Thus the total initial energy is zero, and we have

$$0 = \frac{1}{2}(m_1 + m_2)v_{fa}^2 + m_2gh + m_1g(-h)$$

Here we have used the fact that because the cord does not stretch, the two blocks have the same speed. The heavier mass moves down, losing gravitational potential energy, as the lighter mass moves up, gaining gravitational potential energy. Simplifying,

$$(m_1 - m_2)gh = \frac{1}{2}(m_1 + m_2)v_{fa}^2$$

$$v_{fa} = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}} = \sqrt{\frac{2(5.00\text{ kg} - 3.00\text{ kg})g(4.00\text{ m})}{(5.00\text{ kg} + 3.00\text{ kg})}}$$

$$= \sqrt{19.6}\text{ m/s} = \boxed{4.43\text{ m/s}}$$

(b) Now we apply conservation of energy for the system of the 3.00-kg object and the Earth during the time interval between the instant when the string goes slack and the instant at which the 3.00-kg object reaches its highest position in its free fall.

$$\Delta K + \Delta U = 0 \quad \rightarrow \quad \Delta K = -\Delta U$$

$$0 - \frac{1}{2}m_2 v^2 = -m_2 g\Delta y \rightarrow \Delta y = \frac{v^2}{2g}$$

$$\Delta y = 1.00\text{ m}$$

$$y_{\text{max}} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

P8.8 We assume $m_1 > m_2$. We assign height $y = 0$ to the table top.

(a) $\Delta K + \Delta U = 0$

$$\Delta K_1 + \Delta K_2 + \Delta U_1 + \Delta U_2 = 0$$

$$\left[\frac{1}{2}m_1 v^2 - 0\right] + \left[\frac{1}{2}m_2 v^2 - 0\right] + (0 - m_1 gh) + (m_2 gh - 0) = 0$$

$$\frac{1}{2}(m_1 + m_2)v^2 = m_1 gh - m_2 gh = (m_1 - m_2)gh$$

$$v = \boxed{\sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2}}}$$

(b) We apply conservation of energy for the system of mass m_2 and the Earth during the time interval between the instant when the string goes slack and the instant mass m_2 reaches its highest position in its free fall.

$$\Delta K + \Delta U = 0 \quad \rightarrow \quad \Delta K = -\Delta U$$

$$0 - \frac{1}{2}m_2 v^2 = -m_2 g\,\Delta y \rightarrow \Delta y = \frac{v^2}{2g}$$

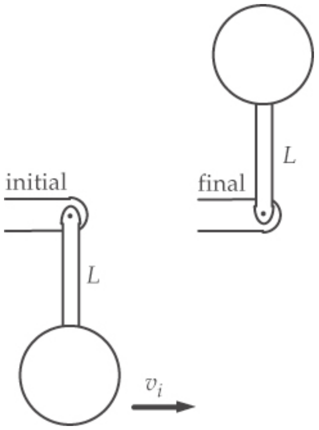
The maximum height of the block is then

$$y_{\max} = h + \Delta y = h + \frac{2(m_1 - m_2)gh}{2g(m_1 + m_2)} = h + \frac{(m_1 - m_2)h}{m_1 + m_2}$$

$$y_{\max} = \frac{(m_1 + m_2)h}{m_1 + m_2} + \frac{(m_1 - m_2)h}{m_1 + m_2}$$

$$y_{\max} = \boxed{\frac{2m_1 h}{m_1 + m_2}}$$

P8.9 The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation of the ball-Earth system between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there. We ignore the mass of the "light" rod.



ANS. FIG. P8.9

$\Delta K + \Delta U = 0$:

$$\left(0 - \frac{1}{2}mv_i^2\right) + \left[mg(2L) - 0\right] = 0$$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80\ \text{m/s}^2)(0.770\ \text{m})}$$

$$v_i = \boxed{5.49\ \text{m/s}}$$

P8.10 (a) One child in one jump converts chemical energy into mechanical energy in the amount that the child-Earth system has as gravitational energy when she is at the top of her jump:

$$mgy = (36\ \text{kg})(9.80\ \text{m/s}^2)(0.25\ \text{m}) = 88.2\ \text{J}$$

For all of the jumps of the children the energy is

$$12(1.05 \times 10^6)(88.2\ \text{J}) = \boxed{1.11 \times 10^9\ \text{J}}$$

(b) The seismic energy is modeled as

$$E = \left(\frac{0.01}{100}\right)(1.11 \times 10^9\ \text{J}) = 1.11 \times 10^5\ \text{J}$$

making the Richter magnitude

$$\frac{\log E - 4.8}{1.5} = \frac{\log(1.11 \times 10^5) - 4.8}{1.5} = \frac{5.05 - 4.8}{1.5} = \boxed{0.2}$$

P8.11 When block B moves up by 1 cm, block A moves down by 2 cm and the separation becomes 3 cm. We then choose the final point to be when B has moved up by $\frac{h}{3}$ and has speed $\frac{v_A}{2}$. Then A has moved down $\frac{2h}{3}$ and has speed v_A:

$$\Delta K + \Delta U = 0$$

$$\left(K_A + K_B + U_g\right)_f - \left(K_A + K_B + U_g\right)_i = 0$$

$$\left(K_A + K_B + U_g\right)_i = \left(K_A + K_B + U_g\right)_f$$

$$0 + 0 + 0 = \frac{1}{2}mv_A^2 + \frac{1}{2}m\left(\frac{v_A}{2}\right)^2 + \frac{mgh}{3} - \frac{mg2h}{3}$$

$$\frac{mgh}{3} = \frac{5}{8}mv_A^2$$

$$\boxed{v_A = \sqrt{\frac{8gh}{15}}}$$

Section 8.3 Situations Involving Kinetic Friction

P8.12 We could solve this problem using Newton's second law, but we will use the nonisolated system energy model, here written as $-f_k d = K_f - K_i$, where the kinetic energy change of the sled after the kick results only from the friction between the sled and ice. The weight and normal force both act at 90° to the motion, and therefore do no work on the sled. The friction force is

$$f_k = \mu_k n = \mu_k mg$$

Since the final kinetic energy is zero, we have

$$-f_k d = -K_i$$

or $$\frac{1}{2}mv_i^2 = \mu_k mgd$$

Thus,

$$d = \frac{mv_i^2}{2f_k} = \frac{mv_i^2}{2\mu_k mg} = \frac{v_i^2}{2\mu_k g} = \frac{(2.00\text{ m/s})^2}{2(0.100)\left(9.80\text{ m/s}^2\right)} = \boxed{2.04\text{ m}}$$

P8.13 We use the nonisolated system energy model, here written as $-f_k d = K_f - K_i$, where the kinetic energy change of the sled after the kick results only from the friction between the sled and ice.

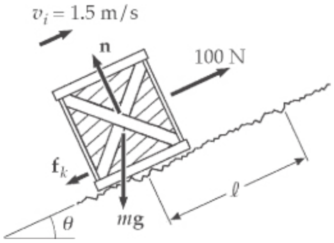
$$\Delta K + \Delta U = -f_k d:$$

$$0 - \frac{1}{2}mv^2 = -f_k d$$

$$\frac{1}{2}mv^2 = \mu_k mgd$$

which gives $\boxed{d = \dfrac{v^2}{2\mu_k g}}$

P8.14 (a) The force of gravitation is



$$(10.0 \text{ kg})(9.80 \text{ m/s}^2) = 98.0 \text{ N}$$

straight down, at an angle of

$$(90.0° + 20.0°) = 110.0°$$

with the motion. The work done by the gravitational force on the crate is

$$W_g = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = mg\ell\cos(90.0° + \theta)$$

$$= (98.0 \text{ N})(5.00 \text{ m})\cos 110.0° = \boxed{-168 \text{ J}}$$

(b) We set the x and y axes parallel and perpendicular to the incline, respectively.

From $\sum F_y = ma_y$, we have

$$n - (98.0 \text{ N})\cos 20.0° = 0$$

so $n = 92.1$ N

and

$$f_k = \mu_k n = 0.400\ (92.1 \text{ N}) = 36.8 \text{ N}$$

Therefore,

$$\Delta E_{\text{int}} = f_k d = (36.8 \text{ N})(5.00 \text{ m}) = \boxed{184 \text{ J}}$$

(c) $W_F = F\ell = (100 \text{ N})(5.00 \text{ m}) = \boxed{500 \text{ J}}$

(d) We use the energy version of the nonisolated system model.

$$\Delta K = -f_k d + \sum W_{\text{other forces}}$$

$$\Delta K = -f_k d + W_g + W_{\text{applied force}} + W_n$$

The normal force does zero work, because it is at 90° to the motion.

$$\Delta K = -184\ \text{J} - 168\ \text{J} + 500\ \text{J} + 0 = \boxed{148\ \text{J}}$$

(e) Again, $K_f - K_i = -f_k d + \sum W_{\text{other forces}}$, so

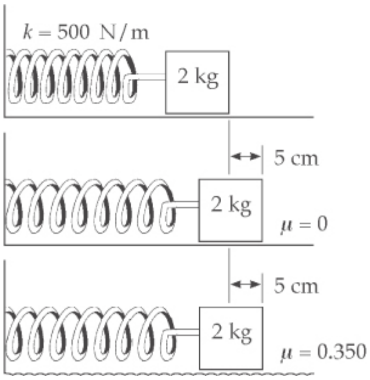
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \sum W_{\text{other forces}} - f_k d$$

$$v_f = \sqrt{\frac{2}{m}\left[\Delta K + \frac{1}{2}mv_i^2\right]}$$

$$= \sqrt{\left(\frac{2}{10.0\ \text{kg}}\right)\left[148\ \text{J} + \frac{1}{2}(10.0\ \text{kg})(1.50\ \text{m/s})^2\right]}$$

$$v_f = \sqrt{\frac{2\left(159\ \text{kg}\cdot\text{m}^2/\text{s}^2\right)}{10.0\ \text{kg}}} = \boxed{5.65\ \text{m/s}}$$

P8.15 (a) The spring does positive work on the block:



ANS. FIG. P8.15

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2$$

$$W_s = \frac{1}{2}(500\ \text{N/m})\left(5.00\times 10^{-2}\ \text{m}\right)^2 - 0$$

$$= 0.625\ \text{J}$$

Applying $\Delta K = W_s$:

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_s \rightarrow \frac{1}{2}mv_f^2 - 0 = W_s$$

so

$$v_f = \sqrt{\frac{2(W_s)}{m}}$$

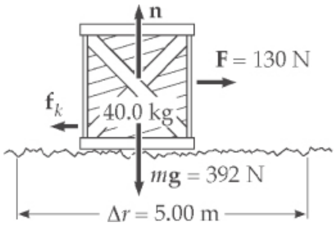
$$= \sqrt{\frac{2(0.625)}{2.00}}\ \text{m/s} = \boxed{0.791\ \text{m/s}}$$

(b) Now friction results in an increase in internal energy $f_k d$ of the block-surface system. From conservation of energy for a nonisolated system,

$$W_s = \Delta K + \Delta E_{\text{int}}$$
$$\Delta K = W_s - f_k d$$
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_s - f_k d = W_s - \mu_s mgd$$
$$\frac{1}{2}mv_f^2 = 0.625\text{ J} - (0.350)(2.00\text{ kg})(9.80\text{ m/s}^2)(0.050\,0\text{ m})$$
$$\frac{1}{2}(2.00\text{ kg})v_f^2 = 0.625\text{ J} - 0.343\text{ J} = 0.282\text{ J}$$
$$v_f = \sqrt{\frac{2(0.282)}{2.00}}\text{ m/s} = \boxed{0.531\text{ m/s}}$$

P8.16 $\sum F_y = ma_y$: $n - 392\text{ N} = 0$

$$n = 392\text{ N}$$
$$f_k = \mu_k n = (0.300)(392\text{ N}) = 118\text{ N}$$



ANS. FIG. P8.16

(a) The applied force and the motion are both horizontal.

$$W_F = Fd\cos\theta$$
$$= (130\text{ N})(5.00\text{ m})\cos 0°$$
$$= \boxed{650\text{ J}}$$

(b) $\Delta E_{\text{int}} = f_k d = (118\text{ N})(5.00\text{ m}) = \boxed{588\text{ J}}$

(c) Since the normal force is perpendicular to the motion,

$$W_n = nd\cos\theta = (392\text{ N})(5.00\text{ m})\cos 90° = \boxed{0}$$

(d) The gravitational force is also perpendicular to the motion, so

$$W_g = mgd\cos\theta = (392\text{ N})(5.00\text{ m})\cos(-90°) = \boxed{0}$$

(e) We write the energy version of the nonisolated system model as

$$\Delta K = K_f - K_i = \sum W_{\text{other}} - \Delta E_{\text{int}}$$
$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

P8.17 (a) $\Delta E_{\text{int}} = -\Delta K = -\dfrac{1}{2}m\left(v_f^2 - v_i^2\right)$:

$$\Delta E_{\text{int}} = -\frac{1}{2}(0.400\text{ kg})\left[(6.00)^2 - (8.00)^2\right](\text{m/s})^2 = \boxed{5.60\text{ J}}$$

(b) After N revolutions, the object comes to rest and $K_f = 0$.

Thus,

$$\Delta E_{\text{int}} = -\Delta K$$

$$f_k d = -(0 - K_i) = \frac{1}{2}mv_i^2$$

or

$$\mu_k mg\left[N(2\pi r)\right] = \frac{1}{2}mv_i^2$$

This gives

$$N = \frac{\frac{1}{2}mv_i^2}{\mu_k mg(2\pi r)} = \frac{\frac{1}{2}(8.00\text{ m/s})^2}{(0.152)\left(9.80\text{ m/s}^2\right)2\pi(1.50\text{ m})}$$

$$= \boxed{2.28\text{ rev}}$$

Section 8.4 Changes in Mechanical Energy for Nonconservative Forces

P8.18 (a) If only conservative forces act, then the total mechanical energy does not change.

$$\Delta K + \Delta U = 0 \quad \text{or} \quad U_f = K_i - K_f + U_i$$

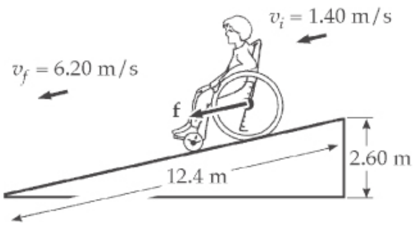
$$U_f = 30.0\text{ J} - 18.0\text{ J} + 10.0\text{ J} = \boxed{22.0\text{ J}}$$

$$E = K + U = 30.0\text{ J} + 10.0\text{ J} = \boxed{40.0\text{ J}}$$

(b) $\boxed{\text{Yes}}$, if the potential energy is less than 22.0 J.

(c) $\boxed{\begin{array}{l}\text{If the potential energy is 5.00 J, the total mechanical energy}\\ \text{is } E = K + U = 18.0\text{ J} + 5.00\text{ J} = 23.0\text{ J, less than the original}\\ \text{40.0 J. The total mechanical energy has decreased, so a non-}\\ \text{conservative force must have acted.}\end{array}}$

P8.19 The boy converts some chemical energy in his body into mechanical energy of the boy-chair-Earth system. During this conversion, the energy can be measured as the work his hands do on the wheels.



ANS. FIG. P8.19

$$\Delta K + \Delta U + \Delta U_{\text{body}} = -f_k d$$

$$\left(K_f - K_i\right) + \left(U_f - U_i\right) + \Delta U_{\text{body}} = -f_k d$$

$$K_i + U_i + W_{\text{hands-on-wheels}} - f_k d = K_f$$

Rearranging and renaming, we have

$$\frac{1}{2}mv_i^2 + mgy_i + W_{\text{by boy}} - f_k d = \frac{1}{2}mv_f^2$$

$$W_{\text{by boy}} = \frac{1}{2}m\left(v_f^2 - v_i^2\right) - mgy_i + f_k d$$

$$W_{\text{by boy}} = \frac{1}{2}(47.0\text{ kg})\left[(6.20\text{ m/s})^2 - (1.40\text{ m/s})^2\right] - (47.0\text{ kg})(9.80\text{ m/s}^2)(2.60\text{ m}) + (41.0\text{ N})(12.4\text{ m})$$

$$W_{\text{by boy}} = \boxed{168\text{ J}}$$

P8.20 (a) Apply conservation of energy to the bead-string-Earth system to find the speed of the bead at Ⓑ. Friction transforms mechanical energy of the system into internal energy $\Delta E_{\text{int}} = f_k d$.

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

$$\left[\frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2\right] + \left(mgy_B - mgy_A\right) + f_k d = 0$$

$$\left[\frac{1}{2}mv_B^2 - 0\right] + \left(0 - mgy_A\right) + f_k d = 0 \rightarrow \frac{1}{2}mv_B^2 = mgy_A - f_k d$$

$$v_B = \sqrt{2gy_A - \frac{2f_k d}{m}}$$

For $y_A = 0.200$ m, $f_k = 0.025$ N, $d = 0.600$ m, and $m = 25.0 \times 10^{-3}$ kg:

$$v_B = \sqrt{2\left(9.80\text{ m/s}^2\right)(0.200\text{ m}) - \frac{2(0.025\text{ N})(0.600\text{ m})}{25.0\times10^{-3}\text{ kg}}}$$

$$= \sqrt{2.72}\text{ m/s}$$

$$\boxed{v_B = 1.65\text{ m/s}}$$

(b) The red bead slides a greater distance along the curved path, so friction transforms more of the mechanical energy of the system into internal energy. There is less of the system's original potential energy in the form of kinetic energy when the bead arrives at point Ⓑ. The result is that the green bead arrives at point Ⓑ first and at higher speed.

P8.21 Use Equation 8.16: $\Delta E_{\text{mech}} = \Delta K + \Delta U = -f_k d$

$$(K_f - K_i) + (U_f - U_i) = -f_k d$$
$$K_i + U_i - f_k d = K_f + U_f$$

(a) $K_i + U_i - f_k d = K_f + U_f$

$$0 + \frac{1}{2}kx^2 - f\Delta x = \frac{1}{2}mv^2 + 0$$

$$\frac{1}{2}(8.00\text{ N/m})(5.00\times10^{-2}\text{ m})^2 - (3.20\times10^{-2}\text{ N})(0.150\text{ m}) = \frac{1}{2}(5.30\times10^{-3}\text{ kg})v^2$$

$$v = \sqrt{\frac{2(5.20\times10^{-3}\text{ J})}{5.30\times10^{-3}\text{ kg}}} = \boxed{1.40\text{ m/s}}$$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $|\vec{F}_s| = kx$; the spring is compressed by

$$\frac{3.20\times10^{-2}\text{ N}}{8.00\text{ N/m}} = 0.400\text{ cm}$$

and the ball has moved

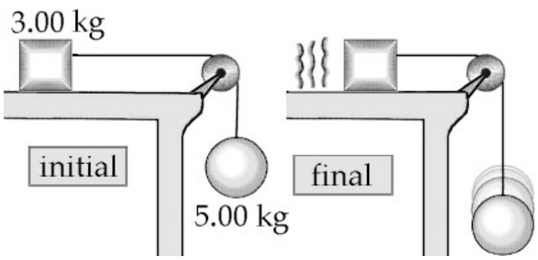
$$5.00\text{ cm} - 0.400\text{ cm} = \boxed{4.60\text{ cm from the start}}$$

(c) Between start and maximum speed points,

$$\frac{1}{2}kx_i^2 - f\Delta x = \frac{1}{2}mv^2 + \frac{1}{2}kx_f^2$$

$$\frac{1}{2}(8.00\text{ N/m})(5.00\times10^{-2}\text{ m})^2 - (3.20\times10^{-2}\text{ N})(4.60\times10^{-2}\text{ m})$$
$$= \frac{1}{2}(5.30\times10^{-3}\text{ kg})v^2 + \frac{1}{2}(8.00\text{ N/m})(4.00\times10^{-3}\text{ m})^2$$

$$v = \boxed{1.79\text{ m/s}}$$



ANS. FIG. P8.22

P8.22 For the Earth plus objects 1 (block) and 2 (ball), we write the energy model equation as

$$(K_1 + K_2 + U_1 + U_2)_f - (K_1 + K_2 + U_1 + U_2)_i = \sum W_{\text{other forces}} - f_k d$$

Choose the initial point before release and the final point after each block has moved 1.50 m. Choose $U = 0$ with the 3.00-kg block on the tabletop and the 5.00-kg block in its final position.

So $K_{1i} = K_{2i} = U_{1i} = U_{1f} = U_{2f} = 0$

We have chosen to include the Earth in our system, so gravitation is an internal force. Because the only external forces are friction and normal forces exerted by the table and the pulley at right angles to the motion,

$$\sum W_{\text{other forces}} = 0$$

We now have

$$\frac{1}{2}m_1 v_f^2 + \frac{1}{2}m_2 v_f^2 + 0 + 0 - 0 - 0 - 0 - m_2 g y_{2i} = 0 - f_k d$$

where the friction force is

$$f_k = \mu_k n = \mu_k m_A g$$

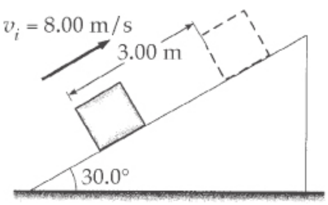
The friction force causes a negative change in mechanical energy because the force opposes the motion. Since all of the variables are known except for v_f, we can substitute and solve for the final speed.

$$\frac{1}{2}m_1 v_f^2 + \frac{1}{2}m_2 v_f^2 - m_2 g y_{2i} = -f_k d$$

$$v^2 = \frac{2gh(m_2 - \mu_k m_1)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\ \text{m/s}^2)(1.50\ \text{m})[5.00\ \text{kg} - 0.400(3.00\ \text{kg})]}{8.00\ \text{kg}}}$$

$$= \boxed{3.74\ \text{m/s}}$$



ANS. FIG. P8.23

P8.23 We consider the block-plane-planet system between an initial point just after the block has been given its shove and a final point when the block comes to rest.

(a) The change in kinetic energy is

$$\Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$= 0 - \frac{1}{2}(5.00\text{ kg})(8.00\text{ m/s})^2 = \boxed{-160\text{ J}}$$

(b) The change in gravitational potential energy is

$$\Delta U = U_f - U_i = mgh$$
$$= (5.00\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$$

(c) The nonisolated system energy model we write as

$$\Delta K + \Delta U = \sum W_{\text{other forces}} - f_k d = 0 - f_k d$$

The force of friction is the only unknown, so we may find it from

$$f_k = \frac{\Delta K - \Delta U}{d} = \frac{+160\text{ J} - 73.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) The forces perpendicular to the incline must add to zero.

$$\sum F_y = 0: \quad +n - mg\cos 30.0° = 0$$

Evaluating,

$$n = mg\cos 30.0° = (5.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0° = 42.4\text{ N}$$

Now $f_k = \mu_k n$ gives

$$\mu_k = \frac{f_k}{n} = \frac{28.8\text{ N}}{42.4\text{ N}} = \boxed{0.679}$$

P8.24 (a) The object drops distance $d = 1.20$ m until it hits the spring, then it continues until the spring is compressed a distance x.

$$\Delta K + \Delta U = 0$$
$$K_f - K_i + U_f - U_i = 0$$
$$0 - 0 + \left(\frac{1}{2}kx^2 - 0\right) + \left[mg(-x) - mgd\right] = 0$$
$$\frac{1}{2}kx^2 - mg(x + d) = 0$$

$$\frac{1}{2}(320\text{ N/m})x^2-(1.50\text{ kg})(9.80\text{ m/s}^2)(x+1.20\text{ m})=0$$

Dropping units, we have

$$160x^2-(14.7)x-17.6=0$$

$$x=\frac{14.7\pm\sqrt{(-14.7)^2-4(160)(-17.6)}}{2(160)}$$

$$x=\frac{14.7\pm 107}{320}$$

The negative root does not apply because *x* is a distance. We have

$$x=\boxed{0.381\text{ m}}$$

(b) This time, friction acts through distance $(x+d)$ in the object-spring-Earth system:

$$\Delta K+\Delta U=-f_k(x+d)$$

$$0-0+\left(\frac{1}{2}kx^2-0\right)+\left[mg(-x)-mgd\right]=-f_k(x+d)$$

$$\frac{1}{2}kx^2-(mg-f_k)x-(mg-f_k)d=0$$

where $mg-f_k=14.0$ N. Suppressing units, we have

$$160x^2-14.0x-16.8=0$$

$$160x^2-14.0x-16.8=0$$

$$x=\frac{14.0\pm\sqrt{(-14.0)^2-4(160)(-16.8)}}{2(160)}$$

$$x=\frac{14.0\pm 105}{320}$$

The positive root is $x=\boxed{0.371\text{ m.}}$

(c) On the Moon, we have

$$\frac{1}{2}kx^2-mg(x+d)=0$$

$$\frac{1}{2}(320\text{ N/m})x^2-(1.50\text{ kg})(1.63\text{ m/s}^2)(x+1.20\text{ m})=0$$

Suppressing units,

$$160x^2-2.45x-2.93=0$$

$$x = \frac{2.45 \pm \sqrt{(-2.45)^2 - 4(160)(-2.93)}}{2(160)}$$

$$x = \frac{2.45 \pm 43.3}{320}$$

$$x = \boxed{0.143 \text{ m}}$$

P8.25 The spring is initially compressed by $x_i = 0.100$ m. The block travels up the ramp distance d.

The spring does work $W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2 = \frac{1}{2}kx_i^2 - 0 = \frac{1}{2}kx_i^2$ on the block.

Gravity does work $W_g = mgd\cos(90° + 60.0°) = mgd\sin(60.0°)$ on the block. There is no friction.

(a) $\sum W = \Delta K$: $\quad W_s + W_g = 0$

$$\frac{1}{2}kx_i^2 - mgd\ \sin(60.0°) = 0$$

$$\frac{1}{2}(1.40 \times 10^3 \text{ N/m})(0.100 \text{ m})^2$$

$$-(0.200 \text{ kg})(9.80 \text{ m/s}^2)d(\sin 60.0°) = 0$$

$$d = \boxed{4.12 \text{ m}}$$

(b) Within the system, friction transforms kinetic energy into internal energy:

$$\Delta E_{\text{int}} = f_k d = (\mu_k n)d = \mu_k(mg\cos 60.0°)d$$

$$\sum W = \Delta K + \Delta E_{\text{int}}: \quad W_s + W_g - \Delta E_{\text{int}} = 0$$

$$\frac{1}{2}kx_i^2 - mgd\sin 60.0° - \mu_k mg\cos 60.0° d = 0$$

$$\frac{1}{2}(1.40 \times 10^3 \text{ N/m})(0.100 \text{ m})^2$$

$$-(0.200 \text{ kg})(9.80 \text{ m/s}^2)d(\sin 60.0°)$$

$$-(0.400)(0.200 \text{ kg})(9.80 \text{ m/s}^2)(\cos 60.0°)d = 0$$

$$d = \boxed{3.35 \text{ m}}$$

P8.26 Air resistance acts like friction. Consider the whole motion:

$$\Delta K + \Delta U = -f_{\text{air}}d \rightarrow K_i + U_i - f_{\text{air}}d = K_f + U_f$$

(a) $0 + mgy_i - f_1d_1 - f_2d_2 = \frac{1}{2}mv_f^2 + 0$

$$(80.0\text{ kg})(9.80\text{ m/s}^2)1\,000\text{ m} - (50.0\text{ N})(800\text{ m}) - (3\,600\text{ N})(200\text{ m}) = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$784\,000\text{ J} - 40\,000\text{ J} - 720\,000\text{ J} = \frac{1}{2}(80.0\text{ kg})v_f^2$$

$$v_f = \sqrt{\frac{2(24\,000\text{ J})}{80.0\text{ kg}}} = \boxed{24.5\text{ m/s}}$$

(b) $\boxed{\text{Yes. This is too fast for safety.}}$

(c) Now in the same energy equation as in part (a), d_2 is unknown, and $d_1 = 1\,000\text{ m} - d_2$:

$$784\,000\text{ J} - (50.0\text{ N})(1\,000\text{ m} - d_2) - (3\,600\text{ N})d_2 = \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2$$

$$784\,000\text{ J} - 50\,000\text{ J} - (3\,550\text{ N})d_2 = 1\,000\text{ J}$$

$$d_2 = \frac{733\,000\text{ J}}{3\,550\text{ N}} = \boxed{206\text{ m}}$$

(d) $\boxed{\text{The air drag is proportional to the square of the skydiver's speed, so it will change quite a bit, It will be larger than her 784-N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down whenever she moves near terminal speed.}}$

P8.27 (a) $\boxed{\text{Yes, the child-Earth system is isolated because the only force that can do work on the child is her weight. The normal force from the slide can do no work because it is always perpendicular to her displacement. The slide is frictionless, and we ignore air resistance.}}$

(b) $\boxed{\text{No, because there is no friction.}}$

(c) At the top of the water slide,

$$U_g = mgh \quad\text{and}\quad K = 0: \quad E = 0 + mgh \rightarrow \boxed{E = mgh}$$

(d) At the launch point, her speed is v_i, and height $h = h/5$:

$$E = K + U_g$$

$$E = \boxed{\frac{1}{2}mv_i^2 + \frac{mgh}{5}}$$

(e) At her maximum airborne height, $h = y_{\max}$:

$$E = \frac{1}{2}mv^2 + mgh = \frac{1}{2}m(v_{xi}{}^2 + v_{yi}{}^2) + mgy_{\max}$$

$$E = \frac{1}{2}m(v_{xi}{}^2 + 0) + mgy_{\max} \to E = \boxed{\frac{1}{2}mv_{xi}^2 + mgy_{\max}}$$

(f) $E = mgh = \dfrac{1}{2}mv_i^2 + mgh/5 \to \boxed{v_i = \sqrt{\dfrac{8gh}{5}}}$

(g) At the launch point, her velocity has components $v_{xi} = v_i \cos\theta$ and $v_{yi} = v_i \sin\theta$:

$$E = \frac{1}{2}mv_i^2 + \frac{mgh}{5} = \frac{1}{2}mv_{xi}^2 + mgy_{\max}$$

$$\to \frac{1}{2}\cancel{m}v_i^2 + \frac{\cancel{m}gh}{5} = \frac{1}{2}\cancel{m}\left(v_i\cos\theta\right)^2 + \cancel{m}gy_{\max}$$

$$\to \frac{1}{2}v_i^2\left(1-\cos^2\theta\right) + \frac{gh}{5} = gh_{\max}$$

$$\to h_{\max} = \frac{1}{2\cancel{g}}\left(\frac{8\cancel{g}h}{5}\right)\left(1-\cos^2\theta\right) + \frac{\cancel{g}h}{5\cancel{g}}$$

$$\to h_{\max} = \left(\frac{4h}{5}\right)\left(1-\cos^2\theta\right) + \frac{h}{5} \to \boxed{h_{\max} = h\left(1-\frac{4}{5}\cos^2\theta\right)}$$

(h) No. If friction is present, mechanical energy of the system would not be conserved, so her kinetic energy at all points after leaving the top of the waterslide would be reduced when compared with the frictionless case. Consequently, her launch speed, maximum height reached, and final speed would be reduced as well.

Section 8.5 Power

P8.28 (a) The moving sewage possesses kinetic energy in the same amount as it enters and leaves the pump. The work of the pump increases the gravitational energy of the sewage-Earth system. We take the equation $K_i + U_{gi} + W_{\text{pump}} = K_f + U_{gf}$, subtract out the K terms, and choose $U_{gi} = 0$ at the bottom of the pump, to obtain $W_{\text{pump}} = mgy_f$. Now we differentiate through with respect to time:

$$\begin{aligned} P_{\text{pump}} &= \frac{\Delta m}{\Delta t} g y_f = \rho \frac{\Delta V}{\Delta t} g y_f \\ &= \left(1\,050 \text{ kg/m}^3\right)\left(1.89 \times 10^6 \text{ L/d}\right) \\ &\quad \times \left(\frac{1 \text{ m}^3}{1\,000 \text{ L}}\right)\left(\frac{1 \text{ d}}{86\,400 \text{ s}}\right)\left(\frac{9.80 \text{ m}}{\text{s}^2}\right)(5.49 \text{ m}) \\ &= \boxed{1.24 \times 10^3 \text{ W}} \end{aligned}$$

(b)
$$\begin{aligned} \text{efficiency} &= \frac{\text{useful output work}}{\text{total input work}} = \frac{\text{useful output work}/\Delta t}{\text{useful input work}/\Delta t} \\ &= \frac{\text{mechanical output power}}{\text{input electric power}} = \frac{1.24 \text{ kW}}{5.90 \text{ kW}} \\ &= \boxed{0.209} = 20.9\% \end{aligned}$$

The remaining power, 5.90 – 1.24 kW = 4.66 kW, is the rate at which internal energy is injected into the sewage and the surroundings of the pump.

P8.29 The Marine must exert an 820-N upward force, opposite the gravitational force, to lift his body at constant speed. The Marine's power output is the work he does divided by the time interval:

$$\text{Power} = \frac{W}{t}$$

$$P = \frac{mgh}{t} = \frac{(820 \text{ N})(12.0 \text{ m})}{8.00 \text{ s}} = 1\,230 \text{ W} = \boxed{1.23 \text{ kW}}$$

P8.30 (a) $$P_{\text{av}} = \frac{W}{\Delta t} = \frac{K_f}{\Delta t} = \frac{mv^2}{2\Delta t} = \frac{(0.875 \text{ kg})(0.620 \text{ m/s})^2}{2\left(21 \times 10^{-3} \text{ s}\right)} = \boxed{8.01 \text{ W}}$$

(b) $\boxed{\text{Some of the energy transferring into the system of the train goes into internal energy in warmer track and moving parts and some leaves the system by sound. To account for this as well as the stated increase in kinetic energy, energy must be transferred at a rate higher than 8.01 W.}}$

P8.31 When the car moves at constant speed on a level roadway, the power used to overcome the total friction force equals the power input from the engine, or $P_{\text{output}} = f_{\text{total}}v = P_{\text{input}}$. This gives

$$\begin{aligned} f_{\text{total}} &= \frac{P_{\text{input}}}{v} = \frac{175\text{ hp}}{29\text{ m/s}}\left(\frac{746\text{ W}}{1\text{ hp}}\right) \\ &= 4.5\times10^5\text{ N or about }\boxed{5\times10^5\text{ N.}} \end{aligned}$$

P8.32 Neglecting any variation of gravity with altitude, the work required to lift a 3.20×10^7 kg load at constant speed to an altitude of $\Delta y = 1.75$ km is

$$\begin{aligned} W = \Delta PE_g &= mg(\Delta y) \\ &= (3.20\times10^7\text{ kg})(9.80\text{ m/s}^2)(1.75\times10^3\text{ m}) \\ &= 5.49\times10^{11}\text{ J} \end{aligned}$$

The time required to do this work using a $P = 2.70$ kW $= 2.70\times10^3$ J/s pump is

$$\begin{aligned} \Delta t = \frac{W}{P} &= \frac{5.49\times10^{11}\text{ J}}{2.70\times10^3\text{ J/s}} = \boxed{2.03\times10^8\text{ s}} \\ &= (2.03\times10^8\text{ s})\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) \\ &= \boxed{5.64\times10^4\text{ h}} = 6.44\text{ yr} \end{aligned}$$

P8.33 energy = power × time

For the 28.0-W bulb:

Energy used = (28.0 W)(1.00×10^4 h) = 280 kWh

total cost = \$4.50 + (280 kWh)(\$0.200/kWh) = \$60.50

For the 100-W bulb:

Energy used = (100 W)(1.00×10^4 h) = 1.00×10^3 kWh

$$\#\text{ of bulbs used } = \frac{1.00\times10^4\text{ h}}{750\text{ h/bulb}} = 13.3 = 13\text{ bulbs}$$

$$\text{total cost} = 13(\$0.420) + (1.00 \times 10^3 \text{ kWh})(\$0.200/\text{kWh}) = \$205.46$$

Savings with energy-efficient bulb:

$$\$205.46 - \$60.50 = \$144.96 = \boxed{\$\ 145}$$

P8.34 The useful output energy is

$$120 \text{ Wh}(1 - 0.60) = mg(y_f - y_i) = F\,\Delta y$$

$$\Delta y = \frac{120 \text{ W}(3\ 600 \text{ s})0.40}{890 \text{ N}}\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right) = \boxed{194 \text{ m}}$$

P8.35 A 1 300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine is equal to its final kinetic energy,

$$\frac{1}{2}(1\ 300 \text{ kg})(24.6 \text{ m/s})^2 = 390 \text{ kJ}$$

with power $P = \dfrac{390\ 000 \text{ J}}{15.0 \text{ s}}\boxed{\sim 10^4 \text{ W},}$ around 30 horsepower.

P8.36 $P = \dfrac{W}{\Delta t}$

older-model: $W = \dfrac{1}{2}mv^2$

newer-model: $W = \dfrac{1}{2}m(2v)^2 = \dfrac{1}{2}(4mv^2) \rightarrow P_{\text{newer}} = \dfrac{4mv^2}{2\Delta t} = 4\dfrac{mv^2}{2\Delta t}$

$$\boxed{\text{The power of the sports car is four times that of the older-model car.}}$$

***P8.37** (a) The fuel economy for walking is

$$\frac{1 \text{ h}}{220 \text{ kcal}}\left(\frac{3 \text{ mi}}{\text{h}}\right)\left(\frac{1 \text{ kcal}}{4\ 186 \text{ J}}\right)\left(\frac{1.30 \times 10^8 \text{ J}}{1 \text{ gal}}\right) = \boxed{423 \text{ mi/gal}}$$

(b) For bicycling:

$$\frac{1 \text{ h}}{400 \text{ kcal}}\left(\frac{10 \text{ mi}}{\text{h}}\right)\left(\frac{1 \text{ kcal}}{4\ 186 \text{ J}}\right)\left(\frac{1.30 \times 10^8 \text{ J}}{1 \text{ gal}}\right) = \boxed{776 \text{ mi/gal}}$$

P8.38 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}\Delta t = \left[\frac{0 + 1.75 \text{ m/s}}{2}\right](3.00 \text{ s}) = 2.63 \text{ m}$$

The motor and the Earth's gravity do work on the elevator car:

$$W_{\text{motor}} + W_{\text{gravity}} = \Delta K$$

$$W_{\text{motor}} + (mg\Delta y)\cos 180° = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{\text{motor}} - (mg\Delta y) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W_{\text{motor}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mg\Delta y$$

$$W_{\text{motor}} = \frac{1}{2}(650\text{ kg})(1.75\text{ m/s})^2 - 0 + (650\text{ kg})g(2.63\text{ m})$$

$$= 1.77 \times 10^4\text{ J}$$

Also, $W = \bar{P}\Delta t$ so $\bar{P} = \dfrac{W}{\Delta t} = \dfrac{1.77 \times 10^4\text{ J}}{3.00\text{ s}} = \boxed{5.91 \times 10^3\text{ W}} = 7.92\text{ hp}$.

(b) When moving upward at constant speed (v = 1.75 m/s), the applied force equals the weight = (650 kg)(9.80 m/s^2) = 6.37 × 10^3 N. Therefore,

$$P = Fv = (6.37 \times 10^3\text{ N})(1.75\text{ m/s}) = \boxed{1.11 \times 10^4\text{ W}} = 14.9\text{ hp}$$

P8.39 As the piano is lifted at constant speed up to the apartment, the total work that must be done on it is

$$W_{nc} = \Delta K + \Delta U_g = 0 + mg(y_f - y_i)$$

$$= (3.50 \times 10^3\text{ N})(25.0\text{ m})$$

$$= 8.75 \times 10^4\text{ J}$$

The three workmen (using a pulley system with an efficiency of 0.750) do work on the piano at a rate of

$$P_{\text{net}} = 0.750\left(3P_{\substack{\text{single}\\\text{worker}}}\right) = 0.750\left[3(165\text{ W})\right] = 371\text{ W} = 371\text{ J/s}$$

so the time required to do the necessary work on the piano is

$$\Delta t = \frac{W_{nc}}{P_{\text{net}}} = \frac{8.75 \times 10^4\text{ J}}{371\text{ J/s}} = \boxed{236\text{ s}} = (236\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{3.93\text{ min}}$$

P8.40 (a) Burning 1 kg of fat releases energy

$$1\text{ kg}\left(\frac{1\,000\text{ g}}{1\text{ kg}}\right)\left(\frac{9\text{ kcal}}{1\text{ g}}\right)\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 3.77 \times 10^7\text{ J}$$

The mechanical energy output is

$$\left(3.77\times10^{7}\ \text{J}\right)(0.20) = nFd\cos\theta$$

where *n* is the number of flights of stairs. Then

$$7.53\times10^{6}\ \text{J} = nmg\Delta y\cos 0°$$

$$7.53\times10^{6}\ \text{J} = n(75\ \text{kg})\left(9.8\ \text{m/s}^2\right)(80\ \text{steps})(0.150\ \text{m})$$

$$7.53\times10^{6}\ \text{J} = n\left(8.82\times10^{3}\ \text{J}\right)$$

where the number of times she must climb the stairs is

$$n = \frac{7.53\times10^{6}\ \text{J}}{8.82\times10^{3}\ \text{J}} = \boxed{854}$$

(b) Her mechanical power output is

$$P = \frac{W}{t} = \frac{8.82\times10^{3}\ \text{J}}{65\ \text{s}} = \boxed{136\ \text{W}} = (136\ \text{W})\left(\frac{1\ \text{hp}}{746\ \text{W}}\right)$$

$$= \boxed{0.182\ \text{hp}}$$

(c) $\boxed{\text{This method is impractical compared to limiting food intake.}}$

P8.41 The energy of the car-Earth system is $E = \frac{1}{2}mv^2 + mgy$:

$$E = \frac{1}{2}mv^2 + mgd\sin\theta$$

where *d* is the distance the car has moved along the track.

$$P = \frac{dE}{dt} = mv\frac{dv}{dt} + mgv\sin\theta$$

(a) When speed is constant,

$$P = mgv\sin\theta = (950\ \text{kg})\left(9.80\ \text{m/s}^2\right)(2.20\ \text{m/s})\sin 30.0°$$

$$= \boxed{1.02\times10^{4}\ \text{W}}$$

(b) $\frac{dv}{dt} = a = \frac{2.20\ \text{m/s} - 0}{12\ \text{s}} = 0.183\ \text{m/s}^2$

Maximum power is injected just before maximum speed is attained:

$$P = mva + mgv\sin\theta$$

$$= (950\ \text{kg})(2.20\ \text{m/s})\left(0.183\ \text{m/s}^2\right) + 1.02\times10^{4}\ \text{W}$$

$$= \boxed{1.06\times10^{4}\ \text{W}}$$

(c) At the top end,

$$\begin{aligned}&\frac{1}{2}\text{m}v^2+\text{mgd}\sin\theta\\&\quad=950\text{ kg}\left(\frac{1}{2}(2.20\text{ m/s})^2+(9.80\text{ m/s}^2)(1\,250\text{ m})\sin 30°\right)\\&\quad=\boxed{5.82\times10^6\text{ J}}\end{aligned}$$

Additional Problems

***P8.42** At a pace I could keep up for a half-hour exercise period, I climb two stories up, traversing forty steps each 18 cm high, in 20 s. My output work becomes the final gravitational energy of the system of the Earth and me,

$$\text{mgy}=(85\text{ kg})(9.80\text{ m/s}^2)(40\times0.18\text{ m})=6\,000\text{ J}$$

making my sustainable power $\frac{6\,000\text{ J}}{20\text{ s}}=\boxed{\sim10^2\text{ W}}$.

P8.43 (a) $U_A=\text{mgR}=(0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m})=\boxed{0.588\text{ J}}$

(b) $K_A+U_A=K_B+U_B$

$$K_B=K_A+U_A-U_B=\text{mgR}=\boxed{0.588\text{ J}}$$

(c) $v_B=\sqrt{\frac{2K_B}{m}}=\sqrt{\frac{2(0.588\text{ J})}{0.200\text{ kg}}}=\boxed{2.42\text{ m/s}}$

(d) $U_C=\text{mgh}_C=(0.200\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})=\boxed{0.392\text{ J}}$

$$K_C=K_A+U_A-U_C=\text{mg}(h_A-h_C)$$
$$K_C=(0.200\text{ kg})(9.80\text{ m/s}^2)(0.300-0.200)\text{ m}=\boxed{0.196\text{ J}}$$

P8.44 (a) Let us take $U=0$ for the particle-bowl-Earth system when the particle is at Ⓑ. Since $v_B=1.50$ m/s and $m=200$ g,

$$K_B=\frac{1}{2}\text{m}v_B^2=\frac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2=\boxed{0.225\text{ J}}$$

(b) At Ⓐ, $v_i=0$, $K_A=0$, and the whole energy at Ⓐ is $U_A=mgR$:

$$\begin{aligned}E_i&=K_A+U_A=0+\text{mgR}=(0.200\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m})\\&=0.588\text{ J}\end{aligned}$$

At Ⓑ,

$$E_f = K_B + U_B = 0.225\text{ J} + 0$$

The decrease in mechanical energy is equal to the increase in internal energy.

$$E_{\text{mech, i}} + \Delta E_{\text{int}} = E_{\text{mech, f}}$$

The energy transformed is

$$\Delta E_{\text{int}} = -\Delta E_{\text{mech}} = E_{\text{mech, i}} - E_{\text{mech, f}} = 0.588\text{ J} - 0.225\text{ J} = \boxed{0.363\text{ J}}$$

(c) $\boxed{\text{No.}}$

(d) $\boxed{\text{It is possible to find an effective coefficient of friction, but not the actual value of } \mu \text{ since } n \text{ and } f \text{ vary with position.}}$

P8.45 Taking $y = 0$ at ground level, and using conservation of energy from when the boy starts from rest ($v_i = 0$) at the top of the slide ($y_i = H$) to the instant he leaves the lower end ($y_f = h$) of the frictionless slide at speed v, where his velocity is horizontal ($v_{xf} = v$, $v_{yf} = 0$), we have

$$E_0 = E_{\text{top}} \;\rightarrow\; \frac{1}{2}mv^2 + mgh = 0 + mgH$$

or $$v^2 = 2g(H - h) \qquad \textbf{[1]}$$

Considering his flight as a projectile after leaving the end of the slide,

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2$$

gives the time to drop distance h to the ground as

$$-h = 0 + \frac{1}{2}(-g)t^2 \qquad \text{or} \qquad t = \sqrt{\frac{2h}{g}}$$

The horizontal distance traveled (at constant horizontal velocity) during this time is d, so

$$d = vt = v\sqrt{\frac{2h}{g}} \qquad \text{and} \qquad v = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting this expression for v into equation [1] above gives

$$\frac{gd^2}{2h} = 2g(H - h) \qquad \text{or} \qquad \boxed{H = h + \frac{d^2}{4h}}$$

P8.46 (a) Mechanical energy is conserved in the two blocks-Earth system:

$$m_2 gy = \frac{1}{2}(m_1 + m_2)v^2$$

$$v = \left[\frac{2m_2 gy}{m_1 + m_2}\right]^{1/2} = \left[\frac{2(1.90\text{ kg})(9.80\text{ m/s}^2)(0.900\text{ m})}{5.40\text{ kg}}\right]^{1/2}$$

$$= \boxed{2.49\text{ m/s}}$$

(b) For the 3.50-kg block from when the string goes slack until just before the block hits the floor, conservation of energy gives

$$\frac{1}{2}(m_2)v^2 + m_2 gy = \frac{1}{2}(m_2)v_d^2$$

$$v_d = \left[2gy + v^2\right]^{1/2} = \left[2(9.80\text{ m/s}^2)(1.20\text{ m}) + (2.49\text{ m/s})^2\right]^{1/2}$$

$$= \boxed{5.45\text{ m/s}}$$

(c) The 3.50-kg block takes this time in flight to the floor: from $y = (1/2)gt^2$ we have $t = [2(1.2)/9.8]^{1/2} = 0.495$ s. Its horizontal component of displacement at impact is then

$$x = v_d t = (2.49\text{ m/s})(0.495\text{ s}) = \boxed{1.23\text{ m}}$$

(d) $\boxed{\text{No.}}$

(e) $\boxed{\text{Some of the kinetic energy of } m_2 \text{ is transferred away as sound and some is transformed to internal energy in } m_1 \text{ and the floor.}}$

P8.47 (a) Given $m = 4.00$ kg and $x = t + 2.0t^3$, we find the velocity by differentiating:

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(t + 2t^3\right) = 1 + 6t^2$$

Then the kinetic energy from its definition is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)\left(1 + 6t^2\right)^2 = \boxed{2 + 24t^2 + 72t^4}$$

where K is in J and t is in s.

(b) Acceleration is the measure of how fast velocity is changing:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(1 + 6t^2\right) = \boxed{12t}$$

where a is in m/s^2 and t is in s.

Newton's second law gives the total force exerted on the particle by the rest of the universe:

$$\sum F = ma = (4.00\text{ kg})(12t) = \boxed{48t}$$

where *F* is in N and *t* is in s.

(c) Power is how fast work is done to increase the object's kinetic energy:

$$P = \frac{dW}{dt} = \frac{dK}{dt} = \frac{d}{dt}\left(2.00 + 24t^2 + 72t^4\right) = \boxed{48t + 288t^3}$$

where *P* is in W [watts] and *t* is in s.

Alternatively, we could use $P = Fv = 48t(1.00 + 6.0t^2)$.

(d) The work-kinetic energy theorem $\Delta K = \sum W$ lets us find the work done on the object between $t_i = 0$ and $t_f = 2.00$ s. At $t_i = 0$ we have $K_i = 2.00$ J. At $t_f = 2.00$ s, suppressing units,

$$K_f = [2 + 24(2.00\text{ s})^2 + 72(2.00\text{ s})^4] = 1250\text{ J}$$

Therefore the work input is

$$W = K_f - K_i = 1\,248\text{ J} = \boxed{1.25 \times 10^3\text{ J}}$$

Alternatively, we could start from

$$W = \int_{t_i}^{t_f} P dt = \int_0^{2\text{ s}} \left(48t + 288t^3\right) dt$$

P8.48 The distance traveled by the ball from the top of the arc to the bottom is πR. The change in internal energy of the system due to the nonconservative force, the force exerted by the pitcher, is

$$\Delta E = Fd\cos 0° = F(\pi R)$$

We shall assign the gravitational energy of the ball-Earth system to be zero with the ball at the bottom of the arc.

Then

$$\Delta E_{\text{mech}} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$$

becomes

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R) = \frac{1}{2}mv_i^2 + mg2R + F(\pi R)$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + (2mg + \pi F)R$$

Solve for *R*, which is the length of her arms.

$$R = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{2mg + \pi F} = m\frac{v_f^2 - v_i^2}{4mg + 2\pi F}$$

$$R = (0.180\ \text{kg})\frac{(25.0\ \text{m/s})^2 - 0}{4(0.180\ \text{kg})g + 2\pi(12.0\ \text{N})} = 1.36\ \text{m}$$

$$\boxed{\text{We find that her arms would need to be 1.36 m long to perform this task. This is significantly longer than the human arm.}}$$

P8.49 (a) $(K + U_g)_A = (K + U_g)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0$$

$$v_B = \sqrt{2gy_A} = \sqrt{2(9.80\ \text{m/s}^2)6.30\ \text{m}} = \boxed{11.1\ \text{m/s}}$$

(b) $(K + U_g + U_{\text{chemical}})_B = (K + U_g)_D$

$$\frac{1}{2}mv_B^2 + U_{\text{chemical}} = \frac{1}{2}mv_D^2 + mg(y_D - y_B)$$

$$U_{\text{chemical}} = \frac{1}{2}mv_D^2 - \frac{1}{2}mv_B^2 + mg(y_D - y_B)$$

$$= \frac{1}{2}m(v_D^2 - v_B^2) + mg(y_D - y_B)$$

$$U_{\text{chemical}} = \frac{1}{2}(76.0\ \text{kg})\left[(5.14\ \text{m/s})^2 - (11.1\ \text{m/s})^2\right] + (76.0\ \text{kg})(9.80\ \text{m/s}^2)(6.30\ \text{m})$$

$$U_{\text{chemical}} = \boxed{1.00 \times 10^3\ \text{J}}$$

(c) $(K + U_g)_D = (K + U_g)_E$ where *E* is the apex of his motion:

$$\frac{1}{2}mv_D^2 + 0 = 0 + mg(y_E - y_D)$$

$$y_E - y_D = \frac{v_D^2}{2g} = \frac{(5.14\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{1.35\ \text{m}}$$

P8.50 (a) Simplified, the equation is

$$0 = (9700\ \text{N/m})x^2 - (450.8\ \text{N})x - 1395\ \text{N}\cdot\text{m}$$

Then

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$=\frac{450.8\ \text{N}\pm\sqrt{(450.8\ \text{N})^2-4(9700\ \text{N/m})(-1395\ \text{N}\cdot\text{m})}}{2(9700\ \text{N/m})}$$

$$=\frac{450.8\ \text{N}\pm 7370\ \text{N}}{19\ 400\ \text{N/m}}=\boxed{0.403\ \text{m or }-0.357\ \text{m}}$$

(b) From a perch at a height of 2.80 m above the top of a pile of mattresses, a 46.0-kg child jumps upward at 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which they are compressed when the child lands on them.

(c) $\boxed{0.023\ 2\ \text{m.}}$

(d) This result is the distance by which the mattresses compress if the child just stands on them. It is the location of the equilibrium position of the oscillator.

P8.51 (a) The total external work done on the system of Jonathan-bicycle is

$$W=\Delta K=\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2$$

$$=\frac{1}{2}(85.0\ \text{kg})\left[(1.00\ \text{m/s})^2-(6.00\ \text{m/s})^2\right]$$

$$=\boxed{-1\ 490\ \text{J}}$$

(b) Gravity does work on the Jonathan-bicycle system, and the potential (chemical) energy stored in Jonathan's body is transformed into kinetic energy:

$$\Delta K+\Delta U_{\text{chem}}=W_g$$

$$\Delta U_{\text{chem}}=W_g-\Delta K=-mgh-\Delta K$$

$$\Delta U_{\text{chem}}=-(85.0\ \text{kg})g(7.30\ \text{m})-\Delta K=-6\ 080-1\ 490$$

$$=\boxed{-7\ 570\ \text{J}}$$

(c) Jonathan does work on the bicycle (and his mass). Treat his work as coming from outside the bicycle-Jonathan's mass system:

$$\Delta K + \Delta U_g = W_j$$

$$W_f = \Delta K + mgh = -1\ 490\text{ J} + 6\ 080\text{ J} = \boxed{4\ 590\text{ J}}$$

P8.52 (a) The total external work done on the system of Jonathan-bicycle is

$$W = \Delta K = \boxed{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$$

(b) Gravity does work on the Jonathan-bicycle system, and the potential (chemical) energy stored in Jonathan's body is transformed into kinetic energy:

$$\Delta K + \Delta U_{\text{chem}} = W_g$$

$$\Delta U_{\text{chem}} = W_g - \Delta K = \boxed{-mgh - \left(\frac{1}{2}mv_f{}^2 - \frac{1}{2}mv_i{}^2\right)}$$

(c) Jonathan does work on the bicycle (and his mass). Treat his work as coming from outside the bicycle-Jonathan's mass system:

$$\Delta K + \Delta U_g = W_j$$

$$W_j = \Delta K + mgh = \boxed{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgh}$$

P8.53 (a) The block-spring-surface system is isolated with a nonconservative force acting. Therefore, Equation 8.2 becomes

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$

$$\left(\tfrac{1}{2}mv^2 - 0\right) + \left(\tfrac{1}{2}kx^2 - \tfrac{1}{2}kx_i{}^2\right) + f_k\left(x_i - x\right) = 0$$

To find the maximum speed, differentiate the equation with respect to *x*:

$$mv\frac{dv}{dx} + kx - f_k = 0$$

Now set $dv/dx = 0$:

$$kx - f_k = 0 \rightarrow \quad x = \frac{f_k}{k} = \frac{4.0\text{ N}}{1.0 \times 10^3\text{ N/m}} = 4.0 \times 10^{-3}\text{ m}$$

This is the compression distance of the spring, so the position of the block relative to $x = 0$ is $\boxed{x = -4.0 \times 10^{-3}\text{ m.}}$

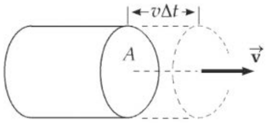
(b) By the same approach,

$$kx - f_k = 0 \rightarrow \quad x = \frac{f_k}{k} = \frac{10.0\ \text{N}}{1.0 \times 10^3\ \text{N/m}} = 1.0 \times 10^{-2}\ \text{m}$$

so the position of the block is $\boxed{x = -1.0 \times 10^{-2}\ \text{m.}}$

P8.54 $P\Delta t = W = \Delta K = \dfrac{(\Delta m)v^2}{2}$

The density is $\rho = \dfrac{\Delta m}{\text{volume}} = \dfrac{\Delta m}{A\Delta x}$



ANS. FIG. P8.54

Substituting this into the first equation and solving for *P*, since $\dfrac{\Delta x}{\Delta t} = v$ for a constant speed, we get

$$P = \boxed{\frac{\rho A v^3}{2}}$$

Also, since *P* = *Fv*,

$$\boxed{F = \frac{\rho A v^2}{2}}$$

Our model predicts the same proportionalities as the empirical equation, and gives $D = 1$ for the drag coefficient. Air actually slips around the moving object, instead of accumulating in front of it. For this reason, the drag coefficient is not necessarily unity. It is typically less than one for a streamlined object and can be greater than one if the airflow around the object is complicated.

P8.55 $P = \dfrac{1}{2} D\rho\pi r^2 v^3$

(a) We use 1.20 kg/m^3 for the density of air, and calculate

$$P_a = \frac{1}{2}(1)\left(1.20\ \text{kg/m}^3\right)\pi(1.50\ \text{m})^2(8.00\ \text{m/s})^3$$
$$= \boxed{2.17 \times 10^3\ \text{W}}$$

(b) We solve part (b) by proportion:

$$\frac{P_b}{P_a} = \frac{v_b^3}{v_a^3} = \left(\frac{24\ \text{m/s}}{8\ \text{m/s}}\right)^3 = 3^3 = 27$$

$$P_b = 27\left(2.17 \times 10^3\ \text{W}\right) = 5.86 \times 10^4\ \text{W} = \boxed{58.6\ \text{kW}}$$

P8.56 (a) In Example 8.3, $m = 35.0\ g$, $y_A = -0.120$ m, $y_B = 0$, and $k = 958$ N/m. Friction $f_k = 2.00$ N acts over distance $d = 0.600$ m. For the ball-spring-Earth system, $K_i = 0$, $U_{gi} = mgy_A$, $U_{si} = \frac{1}{2}kx^2$, where $x = |y_A|$; $K_f = 0$, $U_{gf} = mgy_C$, and $U_{sf} = 0$.

$$\Delta K + \Delta U = -f_k d$$

$$0 + \left(mgy_C - mgy_A\right) + \left(0 - \frac{1}{2}kx^2\right) = -f_k d$$

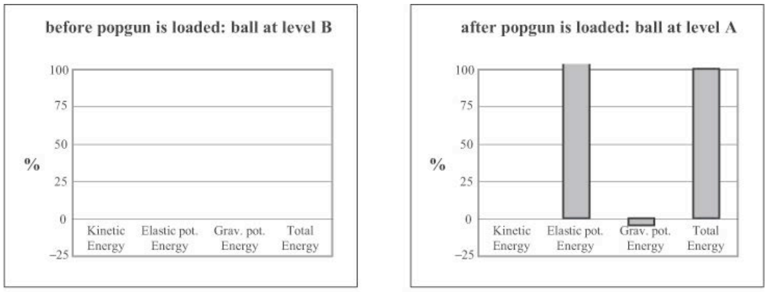
$$mgy_C = mgy_A + \frac{1}{2}kx^2 - f_k d$$

$$y_C = y_A + \frac{\frac{1}{2}kx^2 - f_k d}{mg}$$

$$= -0.120 + \frac{\frac{1}{2}(958\ \text{N/m})(0.120\ \text{m})^2 - (2.00\ \text{N})(0.600\ \text{m})}{(0.035\ \text{kg})g}$$

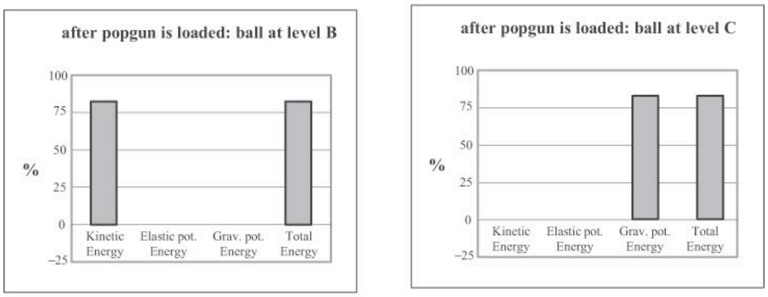
$$= \boxed{16.5\ \text{m}}$$

(b) The ball-spring-Earth system is not isolated as the popgun is loaded. In addition, as the ball travels up the barrel, a nonconservative force acts within the system. The system is isolated after the ball leaves the barrel.



non-isolated system

non-isolated system from A to B



non-isolated system from A to B

isolated system from B to C

ANS. FIG. P8.56

P8.57 (a) To calculate the change in kinetic energy, we integrate the expression for *a* as a function of time to obtain the car's velocity:

$$v = \int_0^t a\,dt = \int_0^t \left(1.16t - 0.210t^2 + 0.240t^3\right)dt$$

$$= 1.16\frac{t^2}{2} - 0.210\frac{t^3}{3} + 0.240\frac{t^4}{4}\bigg|_0^t = 0.580t^2 - 0.070t^3 + 0.060t^4$$

At $t = 0$, $v_i = 0$. At $t = 2.5$ s,

$$v_f = \left(0.580\text{ m/s}^3\right)(2.50\text{ s})^2 - \left(0.070\text{ m/s}^4\right)(2.50\text{ s})^3 + \left(0.060\text{ m/s}^5\right)(2.50\text{ s})^4 = 4.88\text{ m/s}$$

The change in kinetic energy during this interval is then

$$K_i + W = K_f$$

$$0 + W = \frac{1}{2}mv_f^2 = \frac{1}{2}(1\,160\text{ kg})(4.88\text{ m/s})^2 = \boxed{1.38\times10^4\text{ J}}$$

(b) The road does work on the car when the engine turns the wheels and the car moves. The engine and the road together transform chemical potential energy in the gasoline into kinetic energy of the car.

$$P = \frac{W}{\Delta t} = \frac{1.38\times10^4\text{ J}}{2.50\text{ s}}$$

$$P = \boxed{5.52\times10^3\text{ W}}$$

(c) The value in (b) represents only energy that leaves the engine and is transformed to kinetic energy of the car. Additional energy leaves the engine by sound and heat. More energy leaves the engine to do work against friction forces and air resistance.

P8.58 At the bottom of the circle, the initial speed of the coaster is 22.0 m/s. As the coaster travels up the circle, it will slow down. At the top of the track, the centripetal acceleration must be at least that of gravity, *g*, to remain on the track. Apply conservation of energy to the roller coaster-Earth system to find the speed of the coaster at the top of the circle so that we may find the centripetal acceleration of the coaster.

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv_{\text{top}}^2 - \frac{1}{2}mv_{\text{bottom}}^2\right) + \left(mgy_{\text{top}} - mgy_{\text{bottom}}\right) = 0$$

$$\left(\frac{1}{2}mv_{\text{top}}^2-\frac{1}{2}mv_{\text{bottom}}^2\right)+\left(mg2R-0\right)=0\to v_{\text{top}}^2=v_{\text{bottom}}^2-4gR$$

$$v_{\text{top}}^2=(22.0\text{ m/s})^2-4g(12.0\text{ m})=13.6\text{ m}^2/\text{s}^2$$

For this speed, the centripetal acceleration is

$$a_c=\frac{v_{\text{top}}^2}{R}=\frac{13.6\text{ m}^2/\text{s}^2}{12.0\text{ m}}=1.13\text{ m/s}^2$$

The centripetal acceleration of each passenger as the coaster passes over the top of the circle is 1.13 m/s^2. Since this is less than the acceleration due to gravity, the unrestrained passengers will fall out of the cars!

P8.59 (a) The energy stored in the spring is the elastic potential energy, $U=\frac{1}{2}kx^2$, where $k=850\text{ N/m}$. At $x=6.00\text{ cm}$,

$$U=\frac{1}{2}kx^2=\frac{1}{2}(850\text{ N/m})(0.0600\text{ m})^2=\boxed{1.53\text{ J}}$$

At the equilibrium position, $x=0$, $U=\boxed{0\text{ J}}$.

(b) Applying energy conservation to the block-spring system:

$$\Delta K+\Delta U=0$$

$$\left(\frac{1}{2}mv_f^2-\frac{1}{2}mv_i^2\right)+\left(U_f-U_i\right)=0\to\left(\frac{1}{2}mv_f^2-0\right)=-\left(U_f-U_i\right)$$

$$\frac{1}{2}mv_f^2=U_i-U_f$$

because the block is released from rest. For $x_f=0$, $U=0$, and

$$\frac{1}{2}mv_f^2=U_i-U_f\to v_f=\sqrt{\frac{2\left(U_i-U_f\right)}{m}}$$

$$v_f=\sqrt{\frac{2(1.53\text{ J})}{1.00\text{ kg}}}$$

$$\boxed{v_f=1.75\text{ m/s}}$$

(c) From (b) above, for $x_f=x_i/2=3.00\text{ cm}$,

$$U=\frac{1}{2}kx^2=\frac{1}{2}(850\text{ N/m})(0.0300\text{ m})^2=0.383\text{ J}$$

and

$$\frac{1}{2}mv_f^2 = U_i - U_f \rightarrow v_f = \sqrt{\frac{2(U_i - U_f)}{m}}$$

$$v_f = \sqrt{\frac{2(1.53\text{ J} - 0.383\text{ J})}{1.00\text{ kg}}} = \sqrt{\frac{2(1.15\text{ J})}{1.00\text{ kg}}}$$

$$\boxed{v_f = 1.51\text{ m/s}}$$

P8.60 (a) The suggested equation $P\Delta t = bwd$ implies all of the following cases:

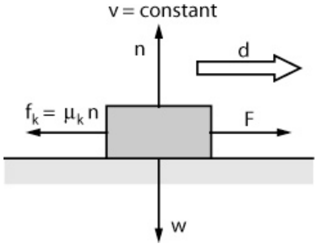
(1) $P\Delta t = b\left(\frac{w}{2}\right)(2d)$

(2) $P\left(\frac{\Delta t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $P\left(\frac{\Delta t}{2}\right) = bw\left(\frac{d}{2}\right)$ and

(4) $\left(\frac{P}{2}\right)\Delta t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.



ANS FIG. P8.60

(b) $\boxed{\text{For one example, consider a horizontal force } F \text{ pushing an object of weight } w \text{ at constant velocity across a horizontal floor with which the object has coefficient of friction } \mu_k.}$

$\sum \vec{F} = m\vec{a}$ implies that

$$+n - w = 0 \text{ and } F - \mu_k n = 0$$

so that $F = \mu_k w$.

As the object moves a distance d, the agent exerting the force does work

$$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd$$

and puts out power $P = \dfrac{W}{\Delta t}$

This yields the equation $P\Delta t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.

P8.61 $k = 2.50 \times 10^4$ N/m, $m = 25.0$ kg

$x_A = -0.100$ m, $U_g\big|_{x=0} = U_s\big|_{x=0} = 0$

(a) At point A, the total energy of the child-pogo-stick-Earth system is given by

$$E_{\text{mech}} = K_A + U_{gA} + U_s \quad \rightarrow \quad E_{\text{mech}} = 0 + mgx_A + \frac{1}{2}kx_A^2$$

$$E_{\text{mech}} = (25.0\ \text{kg})(9.80\ \text{m/s}^2)(-0.100\ \text{m}) + \frac{1}{2}(2.50 \times 10^4\ \text{N/m})(-0.100\ \text{m})^2$$

$$E_{\text{mech}} = -24.5\ \text{J} + 125\ \text{J} = \boxed{100\ \text{J}}$$

(b) Since only conservative forces are involved, the total energy of the child-pogo-stick-Earth system at point *C* is the same as that at point *A*.

$$K_C + U_{gC} + U_{sC} = K_A + U_{gA} + U_{sA}$$

$$0 + (25.0\ \text{kg})(9.80\ \text{m/s}^2)x_C + 0 = 0 - 24.5\ \text{J} + 125\ \text{J}$$

$$x_C = \boxed{0.410\ \text{m}}$$

(c) $K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$

$$\frac{1}{2}(25.0\ \text{kg})v_B^2 + 0 + 0 = 0 + (-24.5\ \text{J}) + 125\ \text{J}$$

$$v_B = \boxed{2.84\ \text{m/s}}$$

(d) The energy of the system for configurations in which the spring is compressed is

$$E = K + \frac{1}{2}kx^2 - mgx$$

where x is the compression distance of the spring.

To find the position *x* for which the kinetic energy is a maximum, solve this expression for *K*, differentiate with respect to *x*, and set the result equal to zero:

$$K = E - \frac{1}{2}kx^2 + mgx$$

$$\frac{dK}{dx} = 0 - kx + mg = 0 \quad \rightarrow \quad x = \frac{mg}{k}$$

Substitute numerical values:

$$x = \frac{(25.0\ \text{kg})(9.80\ \text{m/s}^2)}{2.50 \times 10^4\ \text{N/m}} = 0.009\ 8\ \text{m} = 0.98\ \text{cm}$$

Because this is the value for the compression distance of the spring, this position is 0.98 cm below $x = 0$.

$$K = K_{\max} \text{ at } x = \boxed{-9.80\ \text{mm}}$$

(e) $$K_{\max} = K_A + \left(U_{gA} - U_g\big|_{x=-9.80\ \text{mm}}\right) + \left(U_{sA} - U_s\big|_{x=-9.80\ \text{mm}}\right)$$

or

$$\frac{1}{2}(25.0\ \text{kg})v_{\max}^2$$
$$= (25.0\ \text{kg})(9.80\ \text{m/s}^2)\left[(-0.100\ \text{m}) - (-0.009\ 8\ \text{m})\right]$$
$$+\frac{1}{2}(2.50\times10^4\ \text{N/m})\left[(-0.100\ \text{m})^2 - (-0.009\ 8\ \text{m})^2\right]$$

yielding $v_{\max} = \boxed{2.85\ \text{m/s}}$

P8.62 (a) Between the second and the third picture, $\Delta E_{\text{mech}} = \Delta K + \Delta U$:

$$-\mu mgd = -\frac{1}{2}mv_i^2 + \frac{1}{2}kd^2$$

$$\frac{1}{2}(50.0\ \text{N/m})d^2 + 0.250(1.00\ \text{kg})(9.80\ \text{m/s}^2)d$$
$$-\frac{1}{2}(1.00\ \text{kg})(3.00\ \text{m/s})^2 = 0$$

$$d = \frac{[-2.45 \pm 21.35]\ \text{N}}{50.0\ \text{N/m}} = \boxed{0.378\ \text{m}}$$

(b) Between picture two and picture four, $\Delta E_{\text{mech}} = \Delta K + \Delta U$:

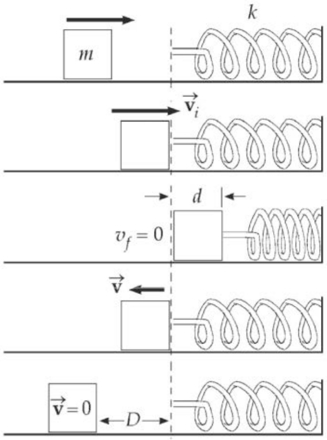
$$-\mu mg(2d) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

which gives

$$v_f = \sqrt{(3.00\text{ m/s})^2 - \frac{2}{(1.00\text{ kg})}(2.45\text{ N})(2)(0.378\text{ m})}$$
$$= \boxed{2.30\text{ m/s}}$$

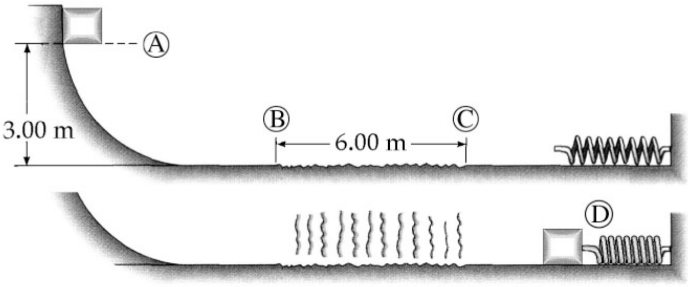
(c) For the motion from picture two to picture five in the figure below, $\Delta E_{\text{mech}} = \Delta K + \Delta U$:

$$-\mu mg(D + 2d) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$D = \frac{(1.00\text{ kg})(3.00\text{ m/s})^2}{2(0.250)(1.00\text{ kg})(9.80\text{ m/s}^2)} - 2(0.378\text{ m}) = \boxed{1.08\text{ m}}$$



ANS. FIG P8.62

P8.63 The easiest way to solve this problem about a chain-reaction process is by considering the energy changes experienced by the block between the point of release (initial) and the point of full compression of the spring (final). Recall that the change in potential energy (gravitational and elastic) plus the change in kinetic energy must equal the work done on the block by non-conservative forces. We choose the gravitational potential energy to be zero along the flat portion of the track.



ANS. FIG. P8.63

There is zero spring potential energy in situation Ⓐ and zero gravitational potential energy in situation Ⓓ. Putting the energy equation into symbols:

$$K_D - K_A - U_{gA} + U_{sD} = -f_k d_{BC}$$

Expanding into specific variables:

$$0 - 0 - mgy_A + \frac{1}{2}kx_s^2 = -f_k d_{BC}$$

The friction force is $f_k = \mu_k mg$, so

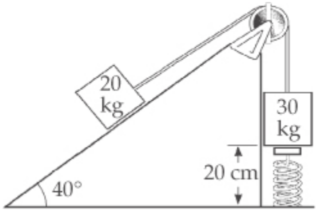
$$mgy_A - \frac{1}{2}kx^2 = \mu_k mgd$$

Solving for the unknown variable μ_k gives

$$\mu_k = \frac{y_A}{d} - \frac{kx^2}{2mgd}$$

$$= \frac{3.00\text{ m}}{6.00\text{ m}} - \frac{(2\,250\text{ N/m})(0.300\text{ m})^2}{2(10.0\text{ kg})\left(9.80\text{ m/s}^2\right)(6.00\text{ m})} = \boxed{0.328}$$

P8.64 We choose the zero configuration of potential energy for the 30.0-kg block to be at the unstretched position of the spring, and for the 20.0-kg block to be at its lowest point on the incline, just before the system is released from rest. From conservation of energy, we have



ANS. FIG. P8.64

$$(K+U)_i = (K+U)_f$$

$$0 + (30.0\text{ kg})\left(9.80\text{ m/s}^2\right)(0.200\text{ m}) + \frac{1}{2}(250\text{ N/m})(0.200\text{ m})^2$$

$$= \frac{1}{2}(20.0\text{ kg} + 30.0\text{ kg})v^2$$

$$+ (20.0\text{ kg})\left(9.80\text{ m/s}^2\right)(0.200\text{ m})\sin 40.0°$$

$$58.8\text{ J} + 5.00\text{ J} = (25.0\text{ kg})v^2 + 25.2\text{ J}$$

$$\boxed{v = 1.24\text{ m/s}}$$

P8.65 (a) For the isolated spring-block system,

$$\Delta K + \Delta U = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + \left(0 - \frac{1}{2}kx^2\right) = 0$$

$$x = \sqrt{\frac{m}{k}}v = \sqrt{\frac{0.500\text{ kg}}{450\text{ N/m}}}(12.0\text{ m/s})$$

$$x = \boxed{0.400\text{ m}}$$

(b) $\Delta K + \Delta U + \Delta E_{\text{int}} = 0$

$$\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) + (2mgR - 0) + f_k(\pi R) = 0$$

$$v_f = \sqrt{v_i^2 - 4gR - \frac{2\pi f_k R}{m}}$$

$$= \sqrt{(12.0\text{ m/s})^2 - 4(9.80\text{ m/s}^2)(1.00\text{ m}) - \frac{2\pi(7.00\text{ N})(1.00\text{ m})}{0.500\text{ kg}}}$$

$$v_f = \boxed{4.10\text{ m/s}}$$

(c) Does the block fall off at or before the top of the track? The block falls if $a_c < g$.

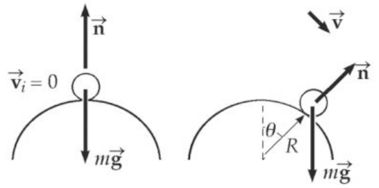
$$a_c = \frac{v_T^2}{R} = \frac{(4.10\text{ m/s})^2}{1.00\text{ m}} = 16.8\text{ m/s}^2$$

Therefore $a_c > g$ and the $\boxed{\text{block stays on the track}}$.

P8.66 m = mass of pumpkin

R = radius of silo top

$$\sum F_r = ma_r \Rightarrow n - mg\cos\theta = -m\frac{v^2}{R}$$



ANS. FIG. P8.66

When the pumpkin first loses contact with the surface, $n = 0$.

Thus, at the point where it leaves the surface: $v^2 = Rg\cos\theta$.

Choose $U_g = 0$ in the $\theta = 90.0°$ plane. Then applying conservation of energy for the pumpkin-Earth system between the starting point and the point where the pumpkin leaves the surface gives

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}mv^2 + mgR\cos\theta = 0 + mgR$$

Using the result from the force analysis, this becomes

$$\frac{1}{2}\mathrm{mRg}\cos\theta + \mathrm{mgR}\cos\theta = \mathrm{mgR}\text{, which reduces to}$$

$$\cos\theta = \frac{2}{3}\text{, and gives } \theta = \cos^{-1}\left(2/3\right) = \boxed{48.2°}$$

as the angle at which the pumpkin will lose contact with the surface.

P8.67 Convert the speed to metric units:

$$\mathrm{v} = \left(100\ \mathrm{km/h}\right)\left(\frac{1\ 000\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\frac{1\ \mathrm{h}}{3\ 600\ \mathrm{s}}\right) = 27.8\ \mathrm{m/s}$$

Write Equation 8.2 for this situation, treating the car and surrounding air as an isolated system with a nonconservative force acting:

$$\Delta K + \Delta U_{\mathrm{grav}} + \Delta U_{\mathrm{fuel}} + \Delta E_{\mathrm{int}} = 0$$

The power of the engine is a measure of how fast it can convert chemical potential energy in the fuel to other forms. The magnitude of the change in energy to other forms is equal to the negative of the change in potential energy in the fuel: $\Delta E_{\text{other forms}} = -\Delta U_{\mathrm{fuel}}$. Therefore, if the car moves a distance *d* along the hill,

$$\begin{aligned} P &= -\frac{\Delta U_{\mathrm{fuel}}}{\Delta t} = -\frac{\left(-\Delta K - \Delta U_{\mathrm{grav}} - \Delta E_{\mathrm{int}}\right)}{\Delta t} \\ &= \frac{0 + \left(\mathrm{mgd}\sin 3.2° - 0\right) + \frac{1}{2}D\rho A v^2 d}{\Delta t} \\ &= \mathrm{mgv}\sin 3.2° + \frac{1}{2}D\rho A v^3 \end{aligned}$$

where we have recognized $d/\Delta t$ as the speed *v* of the car. Substituting numerical values,

$$\begin{aligned} P = &\left(1\ 500\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right)\left(27.8\ \mathrm{m/s}\right)\sin 3.2° \\ &+ \frac{1}{2}\left(0.330\right)\left(1.20\ \mathrm{kg/m^3}\right)\left(2.50\ \mathrm{m^2}\right)\left(27.8\ \mathrm{m/s}\right)^3 \end{aligned}$$

$$\boxed{P = 33.4\ \mathrm{kW} = 44.8\ \mathrm{hp}}$$

The actual power will be larger than this because additional energy coming from the engine is used to do work against internal friction in the moving parts of the car and rolling friction with the road. In addition, some energy from the engine is radiated away by sound. Finally, some of the energy from the fuel raises the internal energy of the engine, and energy leaves the warm engine by heat into the cooler air.

P8.68 (a) Energy is conserved in the swing of the pendulum, and the stationary peg does no work. So the ball's speed does not change when the string hits or leaves the peg, and the ball swings equally high on both sides.

(b) The ball will swing in a circle of radius $R = (L - d)$ about the peg. If the ball is to travel in the circle, the minimum centripetal acceleration at the top of the circle must be that of gravity:

$$\frac{mv^2}{R} = g \rightarrow v^2 = g(L-d)$$

When the ball is released from rest, $U_i = mgL$, and when it is at the top of the circle, $U_i = mg2(L-d)$, where height is measured from the bottom of the swing. By energy conservation,

$$mgL = mg2(L-d) + \frac{1}{2}mv^2$$

From this and the condition on v^2 we find $\boxed{d = \frac{3L}{5}}$.

P8.69 If the spring is just barely able to lift the lower block from the table, the spring lifts it through no noticeable distance, but exerts on the block a force equal to its weight Mg. The extension of the spring, from $|\vec{\mathbf{F}}_s| = kx$, must be Mg/k. Between an initial point at release and a final point when the moving block first comes to rest, we have

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

$$0 + mg\left(-\frac{4mg}{k}\right) + \frac{1}{2}k\left(\frac{4mg}{k}\right)^2 = 0 + mg\left(\frac{Mg}{k}\right) + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2$$

$$-\frac{4m^2g^2}{k} + \frac{8m^2g^2}{k} = \frac{mMg^2}{k} + \frac{M^2g^2}{2k}$$

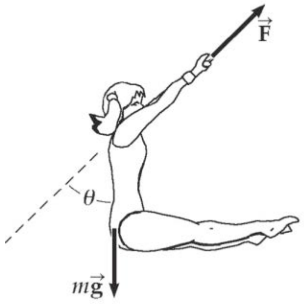
$$4m^2 = mM + \frac{M^2}{2}$$

$$\frac{M^2}{2} + mM - 4m^2 = 0$$

$$M = \frac{-m \pm \sqrt{m^2 - 4\left(\frac{1}{2}\right)\left(-4m^2\right)}}{2\left(\frac{1}{2}\right)} = -m \pm \sqrt{9m^2}$$

Only a positive mass is physical, so we take $M = m(3-1) = \boxed{2m}$.

P8.70 The force needed to hang on is equal to the force F the trapeze bar exerts on the performer. From the free-body diagram for the performer's body, as shown,



ANS. FIG. P8.70

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or $$F = mg\cos\theta + m\frac{v^2}{\ell}$$

At the bottom of the swing, $\theta = 0°$, so

$$F = mg + m\frac{v^2}{\ell}$$

The performer cannot sustain a tension of more than $1.80mg$. What is the force F at the bottom of the swing? To find out, apply conservation of mechanical energy of the performer-Earth system as the performer moves between the starting point and the bottom:

$$mg\ell(1 - \cos 60.0°) = \frac{1}{2}mv^2 \rightarrow \frac{mv^2}{\ell} = 2mg(1 - \cos 60.0°) = mg$$

Hence, $F = mg + m\frac{v^2}{\ell} = mg + mg = 2mg$ at the bottom.

$$\boxed{\text{The tension at the bottom is greater than the performer can withstand; therefore the situation is impossible.}}$$

***P8.71** We first determine the energy output of the runner:

$$= (0.600 \text{ J/kg}\cdot\text{step})(60.0 \text{ kg})\left(\frac{1 \text{ step}}{1.50 \text{ m}}\right) = 24.0 \text{ J/m}$$

From this we calculate the force exerted by the runner per step:

$$F = (24.0 \text{ J/m})(1 \text{ N}\cdot\text{m/J}) = 24.0 \text{ N}$$

Then, from the definition of power, $P = Fv$, we obtain

$$v = \frac{P}{F} = \frac{70.0 \text{ W}}{24.0 \text{ N}} = \boxed{2.92 \text{ m/s}}$$

P8.72 (a) At the top of the loop the car and riders are in free fall:

$$\sum F_y = ma_y:$$

$$mg \text{ down} = \frac{mv^2}{R} \text{ down}$$

$$v = \sqrt{Rg}$$

Energy of the car-riders-Earth system is conserved between release and top of loop:

$$K_i + U_{gi} = K_f + U_{gf}:$$

$$0 + mgh = \frac{1}{2}mv^2 + mg(2R)$$

$$gh = \frac{1}{2}Rg + g(2R)$$

$$\boxed{h = \frac{5R}{2}}$$

(b) Let *h* now represent the height $\geq 2.5\,R$ of the release point. At the bottom of the loop we have

$$mgh = \frac{1}{2}mv_b^2$$

or $v_b^2 = 2gh$

then, from $\sum F_y = ma_y$:

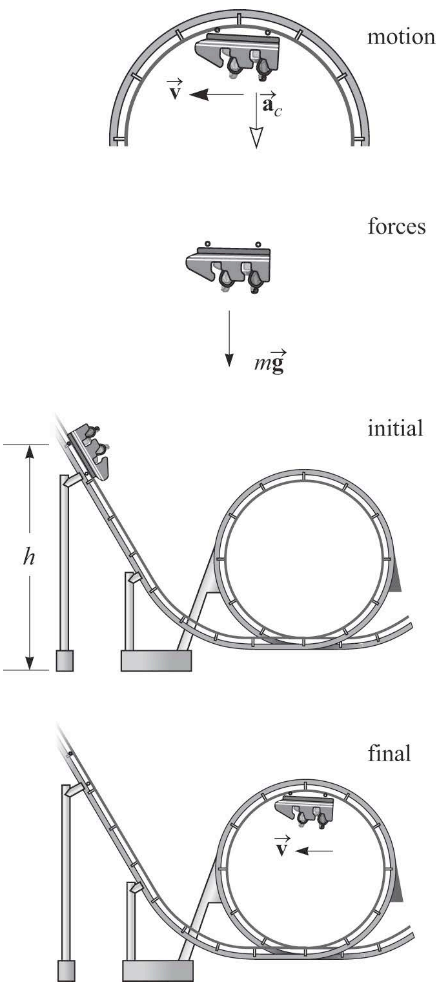
$$n_b - mg = \frac{mv_b^2}{R}(\text{up})$$

$$n_b = mg + \frac{m(2gh)}{R}$$

At the top of the loop,

$$mgh = \frac{1}{2}mv_t^2 + mg(2R)$$

$$v_t^2 = 2gh - 4gR$$



ANS. FIG. P8.72

from $\sum F_y = ma_y$:

$$-n_t - mg = -\frac{mv_t^2}{R}$$

$$n_t = -mg + \frac{m}{R}\left(2gh - 4gR\right)$$

$$n_t = \frac{m\left(2gh\right)}{R} - 5mg$$

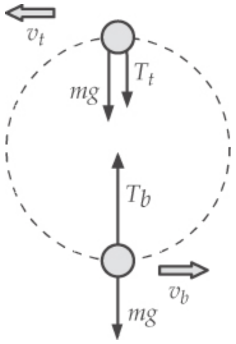
Then the normal force at the bottom is larger by

$$n_b - n_t = mg + \frac{m\left(2gh\right)}{R} - \frac{m\left(2gh\right)}{R} + 5mg = \boxed{6mg}$$

Note that this is the same result we will obtain for the difference in the tension in the string at the top and bottom of a vertical circle in Problem 73.

P8.73 Applying Newton's second law at the bottom (*b*) and top (*t*) of the circle gives

$$T_b - mg = \frac{mv_b^2}{R} \quad \text{and} \quad -T_t - mg = -\frac{mv_t^2}{R}$$



ANS. FIG. P8.73

Adding these gives

$$T_b = T_t + 2mg + \frac{m\left(v_b^2 - v_t^2\right)}{R}$$

Also, energy must be conserved and $\Delta U + \Delta K = 0$.

So, $$\frac{m\left(v_b^2 - v_t^2\right)}{2} + \left(0 - 2mgR\right) = 0 \quad \text{and} \quad \frac{m\left(v_b^2 - v_t^2\right)}{R} = 4mg$$

Substituting into the above equation gives $\boxed{T_b = T_t + 6mg}$.

P8.74 (a) $\boxed{\text{No.}}$ The system of the airplane and the surrounding air is nonisolated. There are two forces acting on the plane that move through displacements, the thrust due to the engine (acting across the boundary of the system) and a resistive force due to the air (acting within the system). Since the air resistance force is nonconservative, some of the energy in the system is transformed to internal energy in the air and the surface of the airplane. Therefore, the change in kinetic energy of the plane is less than the positive work done by the engine thrust. So, $\boxed{\text{mechanical energy is not conserved in this case.}}$

(b) Since the plane is in level flight, $U_{gf} = U_{gi}$ and the conservation of energy for nonisolated systems reduces to

$$\sum W_{\text{other forces}} = W = \Delta K + \Delta U + \Delta E_{\text{int}}$$

or

$$W = W_{\text{thrust}} = K_f - K_i - fs$$

$$F(\cos 0°)s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 - f(\cos 180°)s$$

This gives

$$v_f = \sqrt{v_i^2 + \frac{2(F - f)s}{m}}$$

$$= \sqrt{(60.0\text{ m/s})^2 + \frac{2\left[(7.50 - 4.00)\times 10^4\text{ N}\right](500\text{ m})}{1.50\times 10^4\text{ kg}}}$$

$$v_f = \boxed{77.0\text{ m/s}}$$

P8.75 (a) As at the end of the process analyzed in Example 8.8, we begin with a 0.800-kg block at rest on the end of a spring with stiffness constant 50.0 N/m, compressed 0.092 4 m. The energy in the spring is $(1/2)(50\text{ N/m})(0.092\,4\text{ m})^2 = 0.214\text{ J}$. To push the block back to the unstressed spring position would require work against friction of magnitude 3.92 N (0.092 4 m) = 0.362 J.

$\boxed{\text{Because 0.214 J is less than 0.362 J, the spring cannot push the object back to } x = 0.}$

(b) The block approaches the spring with energy

$$\frac{1}{2}mv^2 = \frac{1}{2}(0.800\text{ kg})(1.20\text{ m/s})^2 = 0.576\text{ J}$$

It travels against friction by equal distances in compressing the spring and in being pushed back out, so half of the initial kinetic energy is transformed to internal energy in its motion to the right and the rest in its motion to the left. The spring must possess one-half of this energy at its maximum compression:

$$\frac{0.576\text{ J}}{2} = \frac{1}{2}(50.0\text{ N/m})x^2$$

so $x = 0.107\text{ m}$

For the compression process we have the conservation of energy equation

$$0.576\text{ J} + \mu_k\, 7.84\text{ N}\left(0.107\text{ m}\right)\cos 180° = 0.288\text{ J}$$

$$\text{so } \mu_k = 0.288\text{ J}/0.841\text{ J} = \boxed{0.342}$$

As a check, the decompression process is described by

$$0.288\text{ J} + \mu_k\, 7.84\text{ N}\left(0.107\text{ m}\right)\cos 180° = 0$$

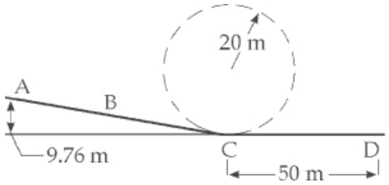
which gives the same answer for the coefficient of friction.

***P8.76** As it moves at constant speed, the bicycle is in equilibrium. The forward friction force is equal in magnitude to the air resistance, which we write as av^2, where a is a proportionality constant. The exercising woman exerts the friction force on the ground; by Newton's third law, it is this same magnitude again. The woman's power output is $P = Fv = av^3 = ch$, where c is another constant and h is her heart rate. We are given $a(22\text{ km/h})^3 = c(90\text{ beats/min})$. For her minimum heart rate we have $av_{\text{min}}^3 = c\left(136\text{ beats/min}\right)$. By division $\left(\dfrac{v_{\text{min}}}{22\text{ km/h}}\right)^3 = \dfrac{136}{90}$.

$$v_{\text{min}} = \left(\frac{136}{90}\right)^{1/3}\left(22\text{ km/h}\right) = \boxed{25.2\text{ km/h}}$$

$$\text{Similarly, } v_{\text{max}} = \left(\frac{166}{90}\right)^{1/3}\left(22\text{ km/h}\right) = \boxed{27.0\text{ km/h}}.$$

P8.77 (a) Conservation of energy for the sled-rider-Earth system, between A and C:



ANS. FIG. P8.77

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}m\left(2.50\text{ m/s}\right)^2 + m\left(9.80\text{ m/s}^2\right)\left(9.76\text{ m}\right) = \frac{1}{2}mv_C^2 + 0$$

$$v_C = \sqrt{\left(2.50\text{ m/s}\right)^2 + 2\left(9.80\text{ m/s}^2\right)\left(9.76\text{ m}\right)} = \boxed{14.1\text{ m/s}}$$

(b) Incorporating the loss of mechanical energy during the portion of the motion in the water, we have, for the entire motion between A and D (the rider's stopping point),

$$K_i + U_{gi} - f_k d = K_f + U_{gf}:$$

$$\frac{1}{2}(80.0\text{ kg})(2.50\text{ m/s})^2$$

$$+(80.0\text{ kg})(9.80\text{ m/s}^2)(9.76\text{ m}) - f_k d = 0 + 0$$

$$-f_k d = 7.90\times 10^3\text{ J}$$

The water exerts a friction force

$$f_k = \frac{7.90\times 10^3\text{ J}}{d} = \frac{7.90\times 10^3\text{ N}\cdot\text{m}}{50.0\text{ m}} = 158\text{ N}$$

and also a normal force of

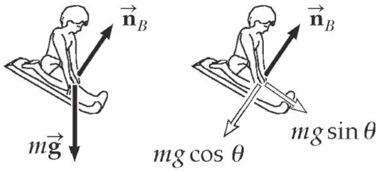
$$n = mg = (80.0\text{ kg})(9.80\text{ m/s}^2) = 784\text{ N}$$

The magnitude of the water force is

$$\sqrt{(158\text{ N})^2 + (784\text{ N})^2} = \boxed{800\text{ N}}$$

(c) The angle of the slide is

$$\theta = \sin^{-1}\left(\frac{9.76\text{ m}}{54.3\text{ m}}\right) = 10.4°$$



ANS. FIG. P8.77(c)

For forces perpendicular to the track at B,

$$\sum F_y = ma_y: \qquad n_B - mg\cos\theta = 0$$

$$n_B = (80.0\text{ kg})(9.80\text{ m/s}^2)\cos 10.4° = \boxed{771\text{ N}}$$

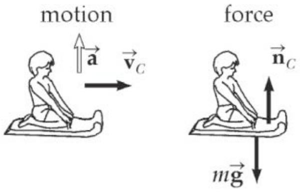
(d) $\sum F_y = ma_y$:

$$+n_C - mg = \frac{mv_C^2}{r}$$

$$n_C = (80.0\text{ kg})(9.80\text{ m/s}^2)$$

$$+\frac{(80.0\text{ kg})(14.1\text{ m/s})^2}{20.0\text{ m}}$$

$$n_C = \boxed{1.57\times 10^3\text{ N up}}$$



ANS. FIG. P8.77(d)

The rider pays for the thrills of a giddy height at A, and a high speed and tremendous splash at C. As a bonus, he gets the quick change in direction and magnitude among the forces we found in parts (d), (b), and (c).

P8.78 (a) Maximum speed occurs after the needle leaves the spring, before it enters the body. We assume the needle is fired horizontally.

ANS. FIG. P8.78(a)

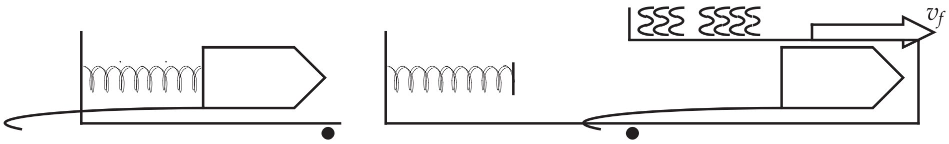
$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + \frac{1}{2}kx^2 - 0 = \frac{1}{2}mv_{max}^2 + 0$$

$$\frac{1}{2}(375\ \text{N/m})(0.081\ \text{m})^2 = \frac{1}{2}(0.005\,6\ \text{kg})v_{max}^2$$

$$\left(\frac{2(1.23\ \text{J})}{0.005\,6\ \text{kg}}\right)^{1/2} = v_{max} = \boxed{21.0\ \text{m/s}}$$

(b) The same energy of 1.23 J as in part (a) now becomes partly internal energy in the soft tissue, partly internal energy in the organ, and partly kinetic energy of the needle just before it runs into the stop. We write a conservation of energy equation to describe this process:



ANS. FIG. P8.78(b)

$$K_i + U_i - f_{k1}d_1 - f_{k2}d_2 = K_f + U_f$$

$$0 + \frac{1}{2}kx^2 - f_{k1}d_1 - f_{k2}d_2 = \frac{1}{2}mv_f^2 + 0$$

$$1.23\ \text{J} - 7.60\ \text{N}(0.024\ \text{m}) - 9.20\ \text{N}(0.035\ \text{m}) = \frac{1}{2}(0.005\,6\ \text{kg})v_f^2$$

$$\left(\frac{2(1.23\ \text{J} - 0.182\ \text{J} - 0.322\ \text{J})}{0.005\,6\ \text{kg}}\right)^{1/2} = v_f = \boxed{16.1\ \text{m/s}}$$

Challenge Problems

P8.79 (a) Let *m* be the mass of the whole board. The portion on the rough surface has mass *mx/L*. The normal force supporting it is $\frac{mxg}{L}$ and the friction force is $\frac{\mu_k mgx}{L} = ma$. Then

$$\boxed{a = \frac{\mu_k gx}{L} \text{ opposite to the motion}}$$

(b) In an incremental bit of forward motion *dx*, the kinetic energy converted into internal energy is $f_k dx = \frac{\mu_k mgx}{L}dx$. The whole energy converted is

$$\frac{1}{2}mv^2 = \int_0^L \frac{\mu_k mgx}{L}dx = \frac{\mu_k mg}{L}\frac{x^2}{2}\Big|_0^L = \frac{\mu_k mgL}{2}$$

$$\boxed{v = \sqrt{\mu_k gL}}$$

P8.80 (a) $U_g = mgy = (64.0\text{ kg})(9.80\text{ m/s}^2)y = \boxed{(627\text{ N})y}$

(b) At the original height and at all heights above 65.0 m – 25.8 m = 39.2 m, the cord is unstretched and $\boxed{U_s = 0}$. Below 39.2 m, the cord extension *x* is given by *x* = 39.2 m – *y*, so the elastic energy is

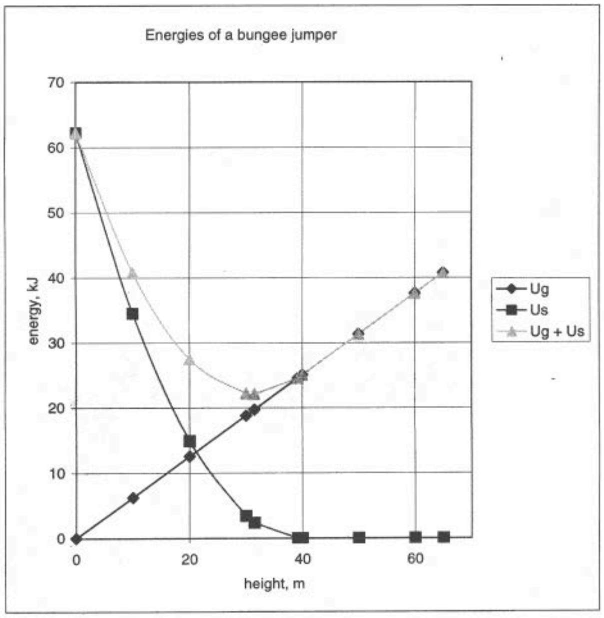
$$U_s = \frac{1}{2}kx^2 = \boxed{\frac{1}{2}(81.0\text{ N/m})(39.2\text{ m} - y)^2}.$$

(c) For *y* > 39.2 m, $U_g + U_s = \boxed{(627\text{ N})y}$

For $y \le 39.2$ m,

$$U_g + U_s = (627\text{ N})y + 40.5\text{ N/m}\left(1\ 537\text{ m}^2 - (78.4\text{ m})y + y^2\right)$$
$$= \boxed{(40.5\text{ N/m})y^2 - (2\ 550\text{ N})y + 62\ 200\text{ J}}$$

(d) See the graph in ANS. FIG. P8.80(d) below.



ANS. FIG. P8.80(d)

(e) At minimum height, the jumper has zero kinetic energy and the system has the same total energy as it had when the jumper was at his starting point. $K_i + U_i = K_f + U_f$ becomes

$$(627\text{ N})(65.0\text{ m}) = (40.5\text{ N/m})y_f^2 - (2\ 550\text{ N})y_f + 62\ 200\text{ J}$$

Suppressing units,

$$0 = 40.5y_f^2 - 2\ 550y_f + 21\ 500$$

$$y_f = \boxed{10.0\text{ m}}\ \left[\text{the solution 52.9 m is unphysical}\right]$$

(f) The total potential energy has a minimum, representing a $\boxed{\text{stable equilibrium}}$ position. To find it, we require $\frac{dU}{dy} = 0.$

Suppressing units, we get

$$\frac{d}{dy}\left(40.5y^2 - 2\ 550y + 62\ 200\right) = 0 = 81y - 2\ 550$$

$$y = \boxed{31.5\text{ m}}$$

(g) Maximum kinetic energy occurs at minimum potential energy. Between the takeoff point and this location, we have

$$K_i + U_i = K_f + U_f$$

Suppressing units,

$$0 + 40\ 800$$
$$= \frac{1}{2}(64.0)\mathsf{v}_{\max}^2 + 40.5(31.5)^2 - 2\ 550(31.5) + 62\ 200$$
$$\mathsf{v}_{\max} = \left(\frac{2(40\ 800 - 22\ 200)}{64.0\ \text{kg}}\right)^{1/2} = \boxed{24.1\ \text{m/s}}$$

P8.81 The geometry reveals $\mathsf{D} = \mathsf{L}\sin\theta + \mathsf{L}\sin\phi$,

$$50.0\ \text{m} = 40.0\ \text{m}(\sin 50° + \sin\phi),\ \phi = 28.9°$$

(a) From takeoff to landing for the Jane-Earth system:

$$\Delta\mathsf{K} + \Delta\mathsf{U} + \Delta\mathsf{E}_{\text{int}} = 0$$
$$\left(0 - \frac{1}{2}\mathsf{mv}_\mathsf{i}^2\right) + \left[\mathsf{mg}(-\mathsf{L}\cos\phi) - \mathsf{mg}(-\mathsf{L}\cos\theta)\right] + \mathsf{FD} = 0$$
$$\frac{1}{2}\mathsf{mv}_\mathsf{i}^2 + \mathsf{mg}(-\mathsf{L}\cos\theta) + \mathsf{FD}(-1) = 0 + \mathsf{mg}(-\mathsf{L}\cos\phi)$$
$$\frac{1}{2}(50.0\ \text{kg})\,\mathsf{v}_\mathsf{i}^2 + (50.0\ \text{kg})(9.80\ \text{m/s}^2)(-40.0\ \text{m})\cos 50°$$
$$-(110\ \text{N})(50.0\ \text{m})$$
$$= (50.0\ \text{kg})(9.80\ \text{m/s}^2)(-40.0\ \text{m})\cos 28.9°$$
$$\frac{1}{2}(50.0\ \text{kg})\mathsf{v}_\mathsf{i}^2 - 1.26\times10^4\ \text{J} - 5.5\times10^3\ \text{J} = -1.72\times10^4\ \text{J}$$
$$\mathsf{v}_\mathsf{i} = \sqrt{\frac{2(947\ \text{J})}{50.0\ \text{kg}}} = \boxed{6.15\ \text{m/s}}$$

(b) For the swing back:

$$\Delta\mathsf{K} + \Delta\mathsf{U} = \Delta\mathsf{E}_{\text{mech}}$$
$$\left(0 - \frac{1}{2}\mathsf{mv}_\mathsf{i}^2\right) + \left[\mathsf{mg}(-\mathsf{L}\cos\theta) - \mathsf{mg}(-\mathsf{L}\cos\phi)\right] = \mathsf{FD}$$
$$\frac{1}{2}\mathsf{mv}_\mathsf{i}^2 + \mathsf{mg}(-\mathsf{L}\cos\phi) + \mathsf{FD}(+1) = 0 + \mathsf{mg}(-\mathsf{L}\cos\theta)$$
$$\frac{1}{2}(130\ \text{kg})\mathsf{v}_\mathsf{i}^2 + (130\ \text{kg})(9.80\ \text{m/s}^2)(-40.0\ \text{m})\cos 28.9°$$
$$+(110\ \text{N})(50.0\ \text{m})$$
$$= (130\ \text{kg})(9.80\ \text{m/s}^2)(-40.0\ \text{m})\cos 50°$$

$$\frac{1}{2}(130\text{ kg})v_i^2 - 4.46\times10^4\text{ J} + 5\,500\text{ J} = -3.28\times10^4\text{ J}$$

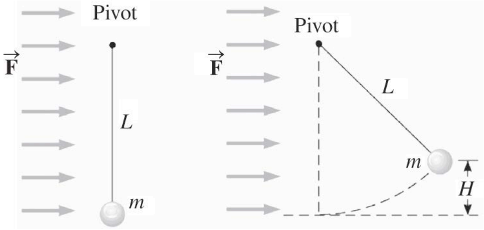
$$v_i = \sqrt{\frac{2(6\,340\text{ J})}{130\text{ kg}}} = \boxed{9.87\text{ m/s}}$$

P8.82 (a) Take the original point where the ball is released and the final point where its upward swing stops at height *H* and horizontal displacement

$$x = \sqrt{L^2 - (L-H)^2} = \sqrt{2LH - H^2}$$

Since the wind force is purely horizontal, it does work

$$W_{\text{wind}} = \int \vec{\mathbf{F}}\cdot d\vec{\mathbf{s}} = F\int dx = F\sqrt{2LH - H^2}$$



ANS FIG. P8.82

The work-energy theorem can be written:

$$K_i + U_{gi} + W_{\text{wind}} = K_f + U_{gf}\text{, or}$$

$$0 + 0 + F\sqrt{2LH - H^2} = 0 + mgH$$

giving

$$F^2 2LH - F^2H^2 = m^2g^2H^2$$

Here the solution $H = 0$ represents the lower turning point of the ball's oscillation, and the upper limit is at $F^2(2L) = (F^2 + m^2g^2)H$. Solving for *H* yields

$$H = \frac{2LF^2}{F^2 + m^2g^2} = \frac{2L}{1 + (mg/F)^2}$$

$$= \frac{2(0.800\text{ m})}{1+(0.300\text{ kg})^2(9.8\text{ m/s}^2)^2/F^2} = \boxed{\frac{1.60\text{ m}}{1 + 8.64\text{ N}^2/F^2}}$$

(b) $H = 1.6\text{ m}[1 + 8.64/1]^{-1} = \boxed{0.166\text{ m}}$

(c) $H = 1.6\text{ m}[1 + 8.64/100]^{-1} = \boxed{1.47\text{ m}}$

(d) As $F \to 0$, $\boxed{H \to 0\text{ as is reasonable.}}$

(e) As $F \to \infty$, $\boxed{H \to 1.60\text{ m}}$, which would be hard to approach experimentally.

(f) Call θ the equilibrium angle with the vertical and *T* the tension in the string.

$$\sum F_x = 0 \Rightarrow T\sin\theta = F, \text{ and}$$
$$\sum F_y = 0 \Rightarrow T\cos\theta = mg$$

Dividing: $\tan\theta = \dfrac{F}{mg}$

Then

$$\cos\theta = \frac{mg}{\sqrt{(mg)^2 + F^2}} = \frac{1}{\sqrt{1 + (F/mg)^2}} = \frac{1}{\sqrt{1 + F^2/8.64\text{ N}^2}}$$

Therefore, $H_{eq} = L(1 - \cos\theta) = \boxed{(0.800\text{ m})\left(1 - \dfrac{1}{\sqrt{1 + F^2/8.64\text{ N}^2}}\right)}$

(g) For *F* = 10 N, $H_{eq} = 0.800\text{ m}[1 - (1 + 100/8.64)^{-1/2}] = \boxed{0.574\text{ m}}$

(h) As $F \to \infty$, $\tan\theta \to \infty$, $\theta \to 90.0°$, $\cos\theta \to 0$, and $H_{eq} \to \boxed{0.800\text{ m}}$.

A very strong wind pulls the string out horizontal, parallel to the ground.

P8.83 The coaster-Earth system is isolated as the coaster travels up the circle. Find how high the coaster travels from the bottom:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv^2 + 0 = 0 + mgh \to h = \frac{v^2}{2g} = \frac{(15.0\text{ m/s})^2}{2g} = 11.5\text{ m}$$

For this situation, the coaster stops at height 11.5 m, which is lower than the height of 24 m at the top of the circular section; in fact, it is close to halfway to the top. The passengers will be supported by the normal force from the backs of their seats. Because of the usual position of a seatback, there may be a slight downhill incline of the seatback that would tend to cause the passengers to slide out. Between the force the passengers can exert by hanging on to a part of the car and the friction between their backs and the back of their seat, the passengers should be able to avoid sliding out of the cars. Therefore, this situation is $\boxed{\text{less dangerous}}$ than that in the original higher-speed situation, where the coaster is upside down.

P8.84 (a) Let mass m_1 of the chain laying on the table and mass m_2 hanging off the edge. For the hanging part of the chain, apply the particle in equilibrium model in the vertical direction:

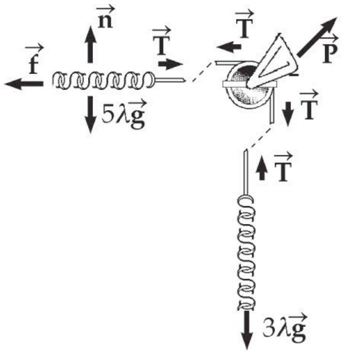
$$m_2 g - T = 0 \qquad \textbf{[1]}$$

For the part of the chain on the table, apply the particle in equilibrium model in both directions:

$$n - m_1 g = 0 \qquad \textbf{[2]}$$

$$T - f_s = 0 \qquad \textbf{[3]}$$

Assume that the length of chain hanging over the edge is such that the chain is on the verge of slipping. Add equations [1] and [3], impose the assumption of impending motion, and substitute equation [2]:



ANS. FIG. P8.84

$$n - m_1 g = 0$$

$$f_s = m_2 g \rightarrow \mu_s n = m_2 g$$

$$\rightarrow \mu_s m_1 g = m_2 g$$

$$\rightarrow m_2 = \mu_s m_1 = 0.600 m_1$$

From the total length of the chain of 8.00 m, we see that

$$m_1 + m_2 = 8.00\lambda$$

where λ is the mass of a one meter length of chain. Substituting for m_2,

$$m_1 + 0.600 m_1 = 8.00\lambda \rightarrow 1.60 m_1 = 8.00\lambda \rightarrow m_1 = 5.00\lambda$$

From this result, we find that $\boxed{m_2 = 3.00\lambda}$ and we see that 3.00 m of chain hangs off the table in the case of impending motion.

(b) Let x represent the variable distance the chain has slipped since the start.

Then length $(5 - x)$ remains on the table, with now

$$\sum F_y = 0: \qquad +n - (5 - x)\lambda g = 0 \rightarrow n = (5 - x)\lambda g$$

$$f_k = \mu_k n = 0.4(5 - x)\lambda g = 2\lambda g - 0.4 x\lambda g$$

Consider energies of the chain-Earth system at the initial moment when the chain starts to slip, and a final moment when $x = 5$, when the last link goes over the brink. Measure heights above the final position of the leading end of the chain. At the moment the final link slips off, the center of the chain is at $y_f = 4$ meters.

Originally, 5 meters of chain is at height 8 m and the middle of the dangling segment is at height $8 - \frac{3}{2} = 6.5$ m.

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f:$$

$$0 + \left(m_1 g y_1 + m_2 g y_2\right)_i - \int_i^f f_k dx = \left(\frac{1}{2}mv^2 + mgy\right)_f$$

$$(5\lambda g)8 + (3\lambda g)6.5 - \int_0^5 (2\lambda g - 0.4x\lambda g)dx = \frac{1}{2}(8\lambda)v^2 + (8\lambda g)4$$

$$40.0g + 19.5g - 2.00g\int_0^5 dx + 0.400g\int_0^5 x\,dx = 4.00v^2 + 32.0g$$

$$27.5g - 2.00gx\Big|_0^5 + 0.400g\frac{x^2}{2}\Big|_0^5 = 4.00v^2$$

$$27.5g - 2.00g(5.00) + 0.400g(12.5) = 4.00v^2$$

$$22.5g = 4.00v^2$$

$$v = \sqrt{\frac{(22.5\text{ m})(9.80\text{ m/s}^2)}{4.00}} = \boxed{7.42\text{ m/s}}$$

P8.85 (a) For a 5.00-m cord the spring constant is described by $F = kx$, $mg = k\,(1.50\text{ m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

$$k = \left(\frac{5.00\text{ m}}{L}\right)\left(\frac{mg}{1.50\text{ m}}\right) = 3.33mg/L$$

From the isolated system model,

$$\left(K + U_g + U_s\right)_i = \left(K + U_g + U_s\right)_f$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$

$$mg\left(y_i - y_f\right) = \frac{1}{2}kx_f^2 = \frac{1}{2}(3.33)\left(\frac{mg}{L}\right)x_f^2$$

here $y_i - y_f = 55\text{ m} = L + x_f$. Substituting,

$$(55.0\text{ m})L = \frac{1}{2}(3.33)(55.0\text{ m} - L)^2$$

$$(55.0\text{ m})L = 5.04\times10^3\text{ m}^2 - (183\text{ m})L + 1.67L^2$$

Suppressing units, we have

$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)(5.04\times10^3)}}{2(1.67)} = \frac{238 \pm 152}{3.33} = \boxed{25.8\text{ m}}$$

Only the value of *L* less than 55 m is physical.

(b) From part (a), $k = 3.33\left(\dfrac{mg}{25.8\text{ m}}\right)$, with

$$x_{\max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$$

From Newton's second law,

$$\sum F = ma: \qquad +kx_{\max} - mg = ma$$

$$3.33\frac{mg}{25.8\text{ m}}(29.2\text{ m}) - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P8.2 (a) $\Delta K + \Delta U = 0$, $v = \sqrt{2gh}$; (b) $v = \sqrt{2gh}$

P8.4 (a) 1.85×10^4 m, 5.10×10^4 m; (b) 1.00×10^7 J

P8.6 (a) 5.94 m/s, 7.67 m/s; (b) 147 J

P8.8 (a) $\sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2}}$; (b) $\frac{2m_1h}{m_1 + m_2}$

P8.10 (a) 1.11×10^9 J; (b) 0.2

P8.12 2.04 m

P8.14 (a) −168 J; (b) 184 J; (c) 500 J; (d) 148 J; (e) 5.65 m/s

P8.16 (a) 650 J; (b) 588 J; (c) 0; (d) 0; (e) 62.0 J; (f) 1.76 m/s

P8.18 (a) 22.0 J, $E = K + U = 30.0\text{ J} + 10.0\text{ J} = 40.0\text{ J}$; (b) Yes; (c) The total mechanical energy has decreased, so a nonconservative force must have acted.

P8.20 (a) $v_B = 1.65\text{ m/s}^2$; (b) green bead, see P8.20 for full explanation

P8.22 3.74 m/s

P8.24 (a) 0.381 m; (b) 0.371 m; (c) 0.143 m

P8.26 (a) 24.5 m/s; (b) Yes. This is too fast for safety; (c) 206 m; (d) see P8.26(d) for full explanation

P8.28 (a) 1.24×10^3 W; (b) 0.209

P8.30 (a) 8.01 W; (b) see P8.30(b) for full explanation

P8.32 2.03×10^8 s, 5.64×10^4 h

P8.34 194 m

P8.36 The power of the sports car is four times that of the older-model car.

P8.38 (a) 5.91×10^3 W; (b) 1.11×10^4 W

P8.40 (a) 854; (b) 0.182 hp; (c) This method is impractical compared to limiting food intake.

P8.42 $\sim 10^2$ W

P8.44 (a) 0.225 J; (b) −0.363 J; (c) no; (d) It is possible to find an effective coefficient of friction but not the actual value of μ since n and f vary with position.

P8.46 (a) 2.49 m/s; (b) 5.45 m/s; (c) 1.23 m; (d) no; (e) Some of the kinetic energy of m_2 is transferred away as sound and to internal energy in m_1 and the floor.

P8.48 We find that her arms would need to be 1.36 m long to perform this task. This is significantly longer than the human arm.

P8.50 (a) 0.403 m or –0.357 m (b) From a perch at a height of 2.80 m above the top of a pile of mattresses, a 46.0-kg child jumps upward at 2.40 m/s. The mattresses behave as a linear spring with force constant 19.4 kN/m. Find the maximum amount by which they are compressed when the child lands on them; (c) 0.023 2 m; (d) This result is the distance by which the mattresses compress if the child just stands on them. It is the location of the equilibrium position of the oscillator.

P8.52 (a) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_f^2$; (b) $-mgh - \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right)$; (c) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_f^2 + mgh$

P8.54 $\frac{\rho A v^3}{2}$; $F = \frac{\rho A v^2}{2}$; see P8.54 for full explanation

P8.56 (a) 16.5 m; (b) See ANS. FIG. P8.56

P8.58 Unrestrained passengers will fall out of the cars

P8.60 (a) See P8.60(a) for full explanation; (b) see P8.60(b) for full explanation

P8.62 (a) 0.378 m; (b) 2.30 m/s; (c) 1.08 m

P8.64 1.24 m/s

P8.66 48.2°

P8.68 $\frac{3L}{5}$

P8.70 The tension at the bottom is greater than the performer can withstand.

P8.72 (a) 5R/2; (b) $6mg$

P8.74 (a) No, mechanical energy is not conserved in this case; (b) 77.0 m/s

P8.76 25.2 km/h and 27.0 km/h

P8.78 (a) 21.0 m/s; (b) 16.1 m/s

P8.80 (a) $(627\text{ N})y$; (b) $U_s = 0$, $\frac{1}{2}(81\text{ N/m})(39.2\text{m} - y)^2$; (c) $(627\text{ N})y$, $(40.5\text{ N/m})\,y^2 - (2\,550\text{ N})y + 62\,200\text{ J}$; (d) See ANS. FIG. P7.78(d); (e) 10.0 m; (f) stable equilibrium, 31.5 m; (g) 24.1 m/s

P8.82 (a) $\frac{1.60\text{ m}}{1 + 8.64\text{ N}^2/F^2}$; (b) 0.166 m; (c) 1.47 m; (d) $H \to 0$ as is reasonable; (e) $H \to 1.60$ m; (f) $(0.800\text{ m})\left(1 - \frac{1}{\sqrt{1 + F^2/8.64\text{ N}^2}}\right)$; (g) 0.574 m; (h) 0.800 m

P8.84 (a) 3.00λ; (b) 7.42 m/s

9

Linear Momentum and Collisions

CHAPTER OUTLINE

9.1	Linear Momentum
9.2	Analysis Model: Isolated System (Momentum)
9.3	Analysis Model: Nonisolated System (Momentum)
9.4	Collisions in One Dimension
9.5	Collisions in Two Dimensions
9.6	The Center of Mass
9.7	Systems of Many Particles
9.8	Deformable Systems
9.9	Rocket Propulsion

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ9.1 Think about how much the vector momentum of the Frisbee changes in a horizontal plane. This will be the same in magnitude as your momentum change. Since you start from rest, this quantity directly controls your final speed. Thus (b) is largest and (c) is smallest. In between them, (e) is larger than (a) and (a) is larger than (c). Also (a) is equal to (d), because the ice can exert a normal force to prevent you from recoiling straight down when you throw the Frisbee up. The assembled answer is b > e > a = d > c.

OQ9.2 (a) No: mechanical energy turns into internal energy in the coupling process.

(b) No: the Earth feeds momentum into the boxcar during the downhill rolling process.

(c) Yes: total energy is constant as it turns from gravitational into kinetic.

(d) Yes: If the boxcar starts moving north, the Earth, very slowly, starts moving south.

(e) No: internal energy appears.

(f) Yes: Only forces internal to the two-car system act.

OQ9.3 (i) Answer (c). During the short time the collision lasts, the total system momentum is constant. Whatever momentum one loses the other gains.

(ii) Answer (a). The problem implies that the tractor's momentum is negligible compared to the car's momentum before the collision. It also implies that the car carries most of the kinetic energy of the system. The collision slows down the car and speeds up the tractor, so that they have the same final speed. The faster-moving car loses more energy than the slower tractor gains because a lot of the car's original kinetic energy is converted into internal energy.

OQ9.4 Answer (a). We have $m_1 = 2$ kg, $v_{1i} = 4$ m/s; $m_2 = 1$ kg, and $v_{1i} = 0$. We find the velocity of the 1-kg mass using the equation derived in Section 9.4 for an elastic collision:

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{4\text{ kg}}{3\text{ kg}}\right)(4\text{ m/s}) + \left(\frac{1\text{ kg}}{3\text{ kg}}\right)(0) = 5.33\text{ m/s}$$

OQ9.5 Answer (c). We choose the original direction of motion of the cart as the positive direction. Then, $v_i = 6$ m/s and $v_f = -2$ m/s. The change in the momentum of the cart is

$$\begin{aligned}\Delta p = mv_f - mv_i = m(v_f - v_i) &= (5\text{ kg})(-2\text{ m/s} - 6\text{ m/s})\\ &= -40\text{ kg}\cdot\text{m/s}.\end{aligned}$$

OQ9.6 Answer (c). The impulse given to the ball is $I = F_{\text{avg}}\Delta t = mv_f - mv_i$. Choosing the direction of the final velocity of the ball as the positive direction, this gives

$$\begin{aligned}F_{\text{avg}} = \frac{m(v_f - v_i)}{\Delta t} &= \frac{(57.0 \times 10^{-3}\text{ kg})[25.0\text{ m/s} - (-21.0\text{ m/s})]}{0.060\text{ s}}\\ &= 43.7\text{ kg}\cdot\text{m/s}^2 = 43.7\text{ N}\end{aligned}$$

OQ9.7 Answer (a). The magnitude of momentum is proportional to speed and the kinetic energy is proportional to speed squared. The speed of the rocket becomes 4 times larger, so the kinetic energy becomes 16 times larger.

OQ9.8 Answer (d). The magnitude of momentum is proportional to speed and the kinetic energy is proportional to speed squared. The speed of the rocket becomes 2 times larger, so the magnitude of the momentum becomes 2 times larger.

OQ9.9 Answer (c). The kinetic energy of a particle may be written as

$$\mathrm{KE}=\frac{mv^2}{2}=\frac{m^2v^2}{2m}=\frac{(mv)^2}{2m}=\frac{p^2}{2m}$$

The ratio of the kinetic energies of two particles is then

$$\frac{(\mathrm{KE})_2}{(\mathrm{KE})_1}=\frac{p_2^2/2m_2}{p_1^2/2m_1}=\left(\frac{p_2}{p_1}\right)^2\left(\frac{m_1}{m_2}\right)$$

We see that, if the magnitudes of the momenta are equal ($p_2 = p_1$), the kinetic energies will be equal only if the masses are also equal. The correct response is then (c).

OQ9.10 Answer (d). Expressing the kinetic energy as $KE = p^2/2m$, we see that the ratio of the magnitudes of the momenta of two particles is

$$\frac{p_2}{p_1}=\frac{\sqrt{2m_2(\mathrm{KE})_2}}{\sqrt{2m_1(\mathrm{KE})_1}}=\sqrt{\left(\frac{m_2}{m_1}\right)\frac{(\mathrm{KE})_2}{(\mathrm{KE})_1}}$$

Thus, we see that if the particles have equal kinetic energies [$(KE)_2 = (KE)_1$], the magnitudes of their momenta are equal only if the masses are also equal. However, momentum is a *vector quantity* and we can say the two particles have equal momenta only it both the magnitudes and directions are equal, making choice (d) the correct answer.

OQ9.11 Answer (b). Before collision, the bullet, mass $m_1 = 10.0$ g, has speed $v_{1i} = v_b$, and the block, mass $m_2 = 200$ g, has speed $v_{2i} = 0$. After collision, the objects have a common speed (velocity) $v_{1f} = v_{2f} = v$. The collision of the bullet with the block is completely inelastic:

$$m_1v_{1i}+m_2v_2=m_1v_{1f}+m_2v_{2f}$$

$$m_1v_b=(m_1+m_2)v\,,\quad \text{so}\qquad v_b=v\left(\frac{m_1+m_2}{m_1}\right)$$

The kinetic friction, $f_k = \mu_k n$, slows down the block with acceleration of magnitude $\mu_k g$. The block slides to a stop through a distance $d = 8.00$ m. Using $v_f^2 = v_i^2 + 2a(x_f - x_i)$, we find the speed of the block just after the collision:

$$v = \sqrt{2(0.400)(9.80\text{ m/s}^2)(8.00\text{ m})} = 7.92\text{ m/s}.$$

Using the results above, the speed of the bullet before collision is

$$v_b = (7.92\,\text{m/s})\left(\frac{10 + 200}{10.0}\right) = 166\text{ m/s}.$$

OQ9.12 Answer (c). The masses move through the same distance under the same force. Equal net work inputs imply equal kinetic energies.

OQ9.13 Answer (a). The same force gives the larger mass a smaller acceleration, so the larger mass takes a longer time interval to move through the same distance; therefore, the impulse given to the larger mass is larger, which means the larger mass will have a greater final momentum.

OQ9.14 Answer (d). Momentum of the ball-Earth system is conserved. Mutual gravitation brings the ball and the Earth together into one system. As the ball moves downward, the Earth moves upward, although with an acceleration on the order of 10^{25} times smaller than that of the ball. The two objects meet, rebound, and separate.

OQ9.15 Answer (d). Momentum is the same before and after the collision. Before the collision the momentum is

$$m_1 v_1 + m_2 v_2 = (3\text{ kg})(+2\text{ m/s}) + (2\text{ kg})(-4\text{ m/s}) = -2\text{ kg}\cdot\text{m/s}$$

OQ9.16 Answer (a). The ball gives more rightward momentum to the block when the ball reverses its momentum.

OQ9.17 Answer (c). Assuming that the collision was head-on so that, after impact, the wreckage moves in the original direction of the car's motion, conservation of momentum during the impact gives

$$(m_c + m_t)v_f = m_c v_{0c} + m_t v_{0t} = m_c v + m_t(0)$$

or

$$v_f = \left(\frac{m_c}{m_c + m_t}\right)v = \left(\frac{m}{m + 2m}\right)v = \frac{v}{3}$$

OQ9.18 Answer (c). Billiard balls all have the same mass and collisions between them may be considered to be elastic. The dual requirements of conservation of kinetic energy and conservation of momentum in a one-dimensional, elastic collision are summarized by the two relations:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad [1]$$

and

$$v_{1i} - v_{2i} = \left(v_{1f} - v_{2f}\right) \qquad [2]$$

In this case, $m_1 = m_2$ and the masses cancel out of the first equation. Call the blue ball #1 and the red ball #2 so that $v_{1i} = -3v$, $v_{2i} = +v$, $v_{1f} = v_{\text{blue}}$, and $v_{2f} = v_{\text{red}}$. Then, the two equations become

$$-3v + v = v_{\text{blue}} + v_{\text{red}} \quad \text{or} \quad v_{\text{blue}} + v_{\text{red}} = v \qquad [1]$$

and

$$-3v - v = -\left(v_{\text{blue}} - v_{\text{red}}\right) \quad \text{or} \quad \left(v_{\text{blue}} - v_{\text{red}}\right) = 4v \qquad [2]$$

Adding the final versions of these equations yields $2v_{\text{blue}} = 2v$, or $v_{\text{blue}} = v$. Substituting this result into either [1] or [2] above then yields $v_{\text{red}} = -3v$.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ9.1 The passenger must undergo a certain momentum change in the collision. This means that a certain impulse must be exerted on the passenger by the steering wheel, the window, an air bag, or something. By increasing the distance over which the momentum change occurs, the time interval during which this change occurs is also increased, resulting in the force on the passenger being decreased.

CQ9.2 If the golfer does not "follow through," the club is slowed down by the golfer before it hits the ball, so the club has less momentum available to transfer to the ball during the collision.

CQ9.3 Its speed decreases as its mass increases. There are no external horizontal forces acting on the box, so its momentum cannot change as it moves along the horizontal surface. As the box slowly fills with water, its mass increases with time. Because the product mv must be constant, and because m is increasing, the speed of the box must decrease. Note that the vertically falling rain has no horizontal momentum of its own, so the box must "share" its momentum with the rain it catches.

CQ9.4 (a) It does not carry force, force requires another object on which to act.

(b) It cannot deliver more kinetic energy than it possesses. This would violate the law of energy conservation.

(c) It can deliver more momentum in a collision than it possesses in its flight, by bouncing from the object it strikes.

CQ9.5 Momentum conservation is not violated if we choose as our system the planet along with you. When you receive an impulse forward, the Earth receives the same size impulse backwards. The resulting acceleration of the Earth due to this impulse is much smaller than your acceleration forward, but the planet's backward momentum is equal in magnitude to your forward momentum. If we choose you as the system, momentum conservation is not violated because you are not an isolated system.

CQ9.6 The rifle has a much lower speed than the bullet and much less kinetic energy. Also, the butt distributes the recoil force over an area much larger than that of the bullet.

CQ9.7 The time interval over which the egg is stopped by the sheet (more for a faster missile) is much longer than the time interval over which the egg is stopped by a wall. For the same change in momentum, the longer the time interval, the smaller the force required to stop the egg. The sheet increases the time interval so that the stopping force is never too large.

CQ9.8 (a) Assuming that both hands are never in contact with a ball, and one hand is in contact with any one ball 20% of the time, the total contact time with the system of three balls is 3(20%) = 60% of the time. The center of mass of the balls is in free fall, moving up and then down with the acceleration due to gravity, during the 40% of the time when the juggler's hands are empty. During the 60% of the time when the juggler is engaged in catching and tossing, the center of mass must accelerate up with a somewhat smaller average acceleration. The center of mass moves around in a little closed loop with a parabolic top and likely a circular bottom, making three revolutions for every one revolution that one ball makes.

(b) On average, in one cycle of the system, the center of mass of the balls does not change position, so its average acceleration is zero (i.e., the average net force on the system is zero). Letting T represent the time for one cycle and F_g the weight of one ball, we have $F_J(0.60T) = 3F_gT$, and $F_J = 5F_g$. The average force exerted by the juggler is five times the weight of one ball.

CQ9.9 (a) In empty space, the center of mass of a rocket-plus-fuel system does not accelerate during a burn, because no outside force acts on this system. The rocket body itself does accelerate as it blows exhaust containing momentum out the back.

(b) According to the text's 'basic expression for rocket propulsion,' the change in speed of the rocket body will be larger than the speed of the exhaust relative to the rocket, if the final mass is less than 37% of the original mass.

CQ9.10 To generalize broadly, around 1740 the English favored position (a), the Germans position (b), and the French position (c). But in France Emilie de Chatelet translated Newton's *Principia* and argued for a more inclusive view. A Frenchman, Jean D'Alembert, is most responsible for showing that each theory is consistent with the others. All the theories are equally correct. Each is useful for giving a mathematically simple and conceptually clear solution for some problems. There is another comprehensive mechanical theory, the angular impulse–angular momentum theorem, which we will glimpse in Chapter 11. It identifies the product of the torque of a force and the time it acts as the cause of a change in motion, and change in angular momentum as the effect.

We have here an example of how scientific theories are different from what people call a theory in everyday life. People who think that different theories are mutually exclusive should bring their thinking up to date to around 1750.

CQ9.11 No. Impulse, $\vec{\mathbf{F}}\Delta t$, depends on the force and the time interval during which it is applied.

CQ9.12 No. Work depends on the force and on the displacement over which it acts.

CQ9.13 (a) Linear momentum is conserved since there are no external forces acting on the system. The fragments go off in different directions and their vector momenta add to zero.

(b) Kinetic energy is not conserved because the chemical potential energy initially in the explosive is converted into kinetic energy of the pieces of the bomb.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 9.1 Linear Momentum

P9.1 (a) The momentum is $p = mv$, so $v = p/m$ and the kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \boxed{\frac{p^2}{2m}}$$

(b) $K = \frac{1}{2}mv^2$ implies $v = \sqrt{\frac{2K}{m}}$ so $p = mv = m\sqrt{\frac{2K}{m}} = \boxed{\sqrt{2mK}}$.

P9.2 $K = p^2/2m$, and hence, $p = \sqrt{2mK}$. Thus,

$$m = \frac{p^2}{2\cdot K} = \frac{(25.0\text{ kg}\cdot\text{m/s})^2}{2(275\text{ J})} = \boxed{1.14\text{ kg}}$$

and

$$v = \frac{p}{m} = \frac{\sqrt{2m(K)}}{m} = \sqrt{\frac{2(K)}{m}} = \sqrt{\frac{2(275\text{ J})}{1.14\text{ kg}}} = \boxed{22.0\text{ m/s}}$$

P9.3 We apply the impulse-momentum theorem to relate the change in the horizontal momentum of the sled to the horizontal force acting on it:

$$\Delta p_x = F_x\Delta t \rightarrow F_x = \frac{\Delta p_x}{\Delta t} = \frac{mv_{xf} - mv_{xi}}{\Delta t}$$

$$F_x = \frac{-(17.5\text{ kg})(3.50\text{ m/s})}{8.75\text{ s}}$$

$$F_x = \boxed{7.00\text{ N}}$$

***P9.4** We are given $m = 3.00$ kg and $\vec{\mathbf{v}} = (3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})$ m/s.

(a) The vector momentum is then

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = (3.00\text{ kg})\left[(3.00\hat{\mathbf{i}} - 4.00\hat{\mathbf{j}})\text{ m/s}\right]$$
$$= (9.00\hat{\mathbf{i}} - 12.0\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}$$

Thus, $\boxed{p_x = 9.00\text{ kg}\cdot\text{m/s}}$ and $\boxed{p_y = -12.0\text{ kg}\cdot\text{m/s}}$.

(b) $p = \sqrt{p_x^2 + p_y^2} = \sqrt{(9.00\text{ kg}\cdot\text{m/s})^2 + (12.0\text{ kg}\cdot\text{m/s})^2}$

$= \boxed{15.0\text{ kg}\cdot\text{m/s}}$

at an angle of

$$\theta = \tan^{-1}\left(\frac{p_y}{p_x}\right) = \tan^{-1}(-1.33) = \boxed{307°}$$

P9.5 We apply the impulse-momentum theorem to find the average force the bat exerts on the baseball:

$$\Delta\vec{\mathbf{p}} = \vec{\mathbf{F}}\Delta t \to \vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = m\left(\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t}\right)$$

Choosing the direction toward home plate as the positive *x* direction, we have $\vec{\mathbf{v}}_i = (45.0 \text{ m/s})\hat{\mathbf{i}}$, $\vec{\mathbf{v}}_f = (55.0 \text{ m/s})\hat{\mathbf{j}}$, and $\Delta t = 2.00$ ms:

$$\vec{\mathbf{F}}_{\text{on ball}} = m\frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{\Delta t} = (0.145 \text{ kg})\frac{(55.0 \text{ m/s})\hat{\mathbf{j}} - (45.0 \text{ m/s})\hat{\mathbf{i}}}{2.00 \times 10^{-3} \text{ s}}$$

$$\vec{\mathbf{F}}_{\text{on ball}} = \left(-3.26\hat{\mathbf{i}} + 3.99\hat{\mathbf{j}}\right) \text{ N}$$

By Newton's third law,

$$\vec{\mathbf{F}}_{\text{on bat}} = -\vec{\mathbf{F}}_{\text{on ball}} \quad \text{so} \quad \boxed{\vec{\mathbf{F}}_{\text{on bat}} = \left(+3.26\hat{\mathbf{i}} - 3.99\hat{\mathbf{j}}\right) \text{ N}}$$

Section 9.2 Analysis Model: Isolated system (Momentum)

P9.6 (a) The girl-plank system is isolated, so horizontal momentum is conserved.

We measure momentum relative to the ice: $\vec{\mathbf{p}}_{gi} + \vec{\mathbf{p}}_{pi} = \vec{\mathbf{p}}_{gf} + \vec{\mathbf{p}}_{pf}$.

The motion is in one dimension, so we can write,

$$v_{gi}\hat{\mathbf{i}} = v_{gp}\hat{\mathbf{i}} + v_{pi}\hat{\mathbf{i}} \to v_{gi} = v_{gp} + v_{pi}$$

where v_{gi} denotes the velocity of the girl relative to the ice, v_{gp} the velocity of the girl relative to the plank, and v_{pi} the velocity of the plank relative to the ice. The momentum equation becomes

$$0 = m_g v_{gi}\hat{\mathbf{i}} + m_p v_{pi}\hat{\mathbf{i}} \to 0 = m_g v_{gi} + m_p v_{pi}$$

$$0 = m_g\left(v_{gp} + v_{pi}\right) + m_p v_{pi}$$

$$0 = m_g v_{gp} + \left(m_g + m_p\right)v_{pi} \to v_{pi} = -\left(\frac{m_g}{m_g + m_p}\right)v_{gp}$$

solving for the velocity of the plank gives

$$\mathrm{v}_{pi} = -\left(\frac{\mathrm{m}_g}{\mathrm{m}_g + \mathrm{m}_p}\right)\mathrm{v}_{gp} = -\left(\frac{45.0\text{ kg}}{45.0\text{ kg} + 150\text{ kg}}\right)(1.50\text{ m/s})$$

$$\boxed{\mathrm{v}_{pi} = -0.346\text{ m/s}}$$

(b) Using our result above, we find that

$$\mathrm{v}_{gi} = \mathrm{v}_{gp} + \mathrm{v}_{pi} = (1.50\text{ m/s}) + (-0.346\text{ m/s})$$

$$\boxed{\mathrm{v}_{gi} = 1.15\text{ m/s}}$$

P9.7 (a) The girl-plank system is isolated, so horizontal momentum is conserved.

We measure momentum relative to the ice: $\vec{\mathbf{p}}_{gi} + \vec{\mathbf{p}}_{pi} = \vec{\mathbf{p}}_{gf} + \vec{\mathbf{p}}_{pf}$.

The motion is in one dimension, so we can write

$$\mathrm{v}_{gi}\hat{\mathbf{i}} = \mathrm{v}_{gp}\hat{\mathbf{i}} + \mathrm{v}_{pi}\hat{\mathbf{i}} \rightarrow \mathrm{v}_{gi} = \mathrm{v}_{gp} + \mathrm{v}_{pi}$$

where v_{gi} denotes the velocity of the girl relative to the ice, v_{gp} the velocity of the girl relative to the plank, and v_{pi} the velocity of the plank relative to the ice. The momentum equation becomes

$$0 = \mathrm{m}_g\mathrm{v}_{gi}\hat{\mathbf{i}} + \mathrm{m}_p\mathrm{v}_{pi}\hat{\mathbf{i}} \rightarrow 0 = \mathrm{m}_g\mathrm{v}_{gi} + \mathrm{m}_p\mathrm{v}_{pi}$$

$$0 = \mathrm{m}_g\left(\mathrm{v}_{gp} + \mathrm{v}_{pi}\right) + \mathrm{m}_p\mathrm{v}_{pi}$$

$$0 = \mathrm{m}_g\mathrm{v}_{gp} + \left(\mathrm{m}_g + \mathrm{m}_p\right)\mathrm{v}_{pi}$$

solving for the velocity of the plank gives

$$\boxed{\mathrm{v}_{pi} = -\left(\frac{\mathrm{m}_g}{\mathrm{m}_g + \mathrm{m}_p}\right)\mathrm{v}_{gp}}$$

(b) Using our result above, we find that

$$\mathrm{v}_{gi} = \mathrm{v}_{gp} + \mathrm{v}_{pi} = \mathrm{v}_{gp}\frac{\left(\mathrm{m}_g + \mathrm{m}_p\right)}{\mathrm{m}_g + \mathrm{m}_p} - \frac{\mathrm{m}_g}{\mathrm{m}_g + \mathrm{m}_p}\mathrm{v}_{gp}$$

$$\mathrm{v}_{gi} = \frac{\left(\mathrm{m}_g + \mathrm{m}_p\right)\mathrm{v}_{gp} - \mathrm{m}_g\mathrm{v}_{gp}}{\mathrm{m}_g + \mathrm{m}_p}$$

$$\mathrm{v}_{gi} = \frac{\mathrm{m}_g\mathrm{v}_{gp} + \mathrm{m}_p\mathrm{v}_{gp} - \mathrm{m}_g\mathrm{v}_{gp}}{\mathrm{m}_g + \mathrm{m}_p}$$

$$\boxed{v_{gi} = \left(\frac{m_p}{m_g + m_p}\right) v_{gp}}$$

P9.8 (a) Brother and sister exert equal-magnitude oppositely-directed forces on each other for the same time interval; therefore, the impulses acting on them are equal and opposite. Taking east as the positive direction, we have

impulse on boy: $I = F\Delta t = \Delta p = (65.0\text{ kg})(-2.90\text{ m/s}) = -189\text{ N}\cdot\text{s}$

impulse on girl: $I = -F\Delta t = -\Delta p = +189\text{ N}\cdot\text{s} = mv_f$

Her speed is then

$$v_f = \frac{I}{m} = \frac{189\text{ N}\cdot\text{s}}{40.0\text{ kg}} = 4.71\text{ m/s}$$

meaning $\boxed{\text{she moves at 4.71 m/s east}}$.

(b) original chemical potential energy in girl's body = total final kinetic energy

$$\begin{aligned} U_{\text{chemical}} &= \frac{1}{2}m_{\text{boy}}v_{\text{boy}}^2 + \frac{1}{2}m_{\text{girl}}v_{\text{girl}}^2 \\ &= \frac{1}{2}(65.0\text{ kg})(2.90\text{ m/s})^2 + \frac{1}{2}(40.0\text{ kg})(4.71\text{ m/s})^2 \\ &= \boxed{717\text{ J}} \end{aligned}$$

(c) $\boxed{\text{Yes.}}$ System momentum is conserved with the value zero.

(d) The forces on the two siblings are internal forces, which cannot change the momentum of the system— $\boxed{\text{the system is isolated}}$.

(e) $\boxed{\text{Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.}}$

***P9.9** We assume that the velocity of the blood is constant over the 0.160 s. Then the patient's body and pallet will have a constant velocity of $\frac{6\times 10^{-5}\ \text{m}}{0.160\ \text{s}} = 3.75\times 10^{-4}\ \text{m/s}$ in the opposite direction. Momentum conservation gives

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}:$$

$$0 = m_{\text{blood}}(0.500\ \text{m/s}) + (54.0\ \text{kg})(-3.75\times 10^{-4}\ \text{m/s})$$

$$m_{\text{blood}} = 0.040\ 5\ \text{kg} = \boxed{40.5\ \text{g}}$$

P9.10 I have mass 72.0 kg and can jump to raise my center of gravity 25.0 cm. I leave the ground with speed given by

$$v_f^2 - v_i^2 = 2a(x_f - x_i): \qquad 0 - v_i^2 = 2(-9.80\ \text{m/s}^2)(0.250\ \text{m})$$

$$v_i = 2.20\ \text{m/s}$$

Total momentum of the system of the Earth and me is conserved as I push the planet down and myself up:

$$0 = (5.98\times 10^{24}\ \text{kg})(-v_e) + (85.0\ \text{kg})(2.20\ \text{m/s})$$

$$v_e \sim \boxed{10^{-23}\ \text{m/s}}$$

P9.11 (a) For the system of two blocks $\Delta p = 0$, or $p_i = p_f$. Therefore,

$$0 = mv_m + (3m)(2.00\ \text{m/s})$$

Solving gives $v_m = \boxed{-6.00\ \text{m/s}}$ (motion toward the left).

(b) $$\frac{1}{2}kx^2 = \frac{1}{2}mv_M^2 + \frac{1}{2}(3m)v_{3M}^2$$

$$= \frac{1}{2}(0.350\ \text{kg})(-6.00\ \text{m/s})^2 + \frac{3}{2}(0.350\ \text{kg})(2.00\ \text{m/s})^2$$

$$= \boxed{8.40\ \text{J}}$$

(c) $\boxed{\text{The original energy is in the spring.}}$

(d) A force had to be exerted over a displacement to compress the spring, transferring energy into it by work.

$\boxed{\text{The cord exerts force, but over no displacement.}}$

(e) $\boxed{\text{System momentum is conserved with the value zero.}}$

(f) The forces on the two blocks are internal forces, which cannot change the momentum of the system— $\boxed{\text{the system is isolated.}}$

(g) $\boxed{\text{Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.}}$

Section 9.3 Analysis Model: Nonisolated system (Momentum)

P9.12 (a) $I = F_{avg}\Delta t$, where *I* is the impulse the man must deliver to the child:

$$I = F_{avg}\Delta t = \Delta p_{child} = m_{child}\left|v_f - v_i\right| \rightarrow F_{avg} = \frac{m_{child}\left|v_f - v_i\right|}{\Delta t}$$

Solving for the average force gives

$$F_{avg} = \frac{m_{child}\left|v_f - v_i\right|}{\Delta t} = \frac{(12.0\text{ kg})\left|0 - 60\text{ mi/h}\right|}{0.10\text{ s}}\left(\frac{0.447\text{ m/s}}{1\text{ mi/h}}\right)$$
$$= \boxed{3.22\times 10^3\text{ N}}$$

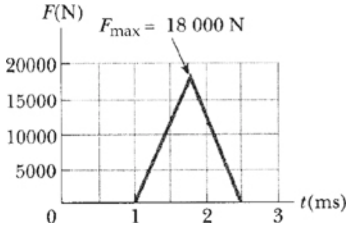
or

$$F_{avg} = \left(3.22\times 10^3\text{ N}\right)\left(\frac{0.224\,8\text{ lb}}{1\text{ N}}\right) \approx \boxed{720\text{ lb}}$$

(b) $\boxed{\text{The man's claim is nonsense.}}$ He would not be able to exert a force of this magnitude on the child. In reality, the violent forces during the collision would tear the child from his arms.

(c) $\boxed{\text{These devices are essential for the safety of small children.}}$

P9.13 (a) The impulse delivered to the ball is equal to the area under the *F*-*t* graph. We have a triangle and so to get its area we multiply half its height times its width:



ANS. FIG. P9.13

$$I = \int F dt = \text{area under curve}$$

$$I = \frac{1}{2}\left(1.50\times 10^{-3}\text{ s}\right)\left(18\,000\text{ N}\right) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $F = \dfrac{13.5\ \text{N}\cdot\text{s}}{1.50\times 10^{-3}\ \text{s}} = \boxed{9.00\ \text{kN}}$

P9.14 (a) The impulse the floor exerts on the ball is equal to the change in momentum of the ball:

$$\begin{aligned}\Delta\vec{\mathbf{p}} &= m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right) = m\left(v_f - v_i\right)\hat{\mathbf{j}}\\ &= (0.300\ \text{kg})\left[(5.42\ \text{m/s}) - (-5.86\ \text{m/s})\right]\hat{\mathbf{j}}\\ &= \boxed{3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}}\end{aligned}$$

(b) Estimating the contact time interval to be 0.05 s, from the impulse-momentum theorem, we find

$$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}}{0.05\ \text{s}} \rightarrow \boxed{\vec{\mathbf{F}} = 7\times 10^2\ \text{N}\ \hat{\mathbf{j}}}$$

P9.15 (a) The mechanical energy of the isolated spring-mass system is conserved:

$$\begin{aligned}&K_i + U_{si} = K_f + U_{sf}\\ &0 + \frac{1}{2}kx^2 = \frac{1}{2}mv^2 + 0\\ &v = x\sqrt{\frac{k}{m}}\end{aligned}$$

(b) $I = \left|\vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i\right| = mv_f - 0 = mx\sqrt{\dfrac{k}{m}} = x\sqrt{km}$

(c) For the glider, $W = K_f - K_i = \dfrac{1}{2}mv^2 - 0 = \dfrac{1}{2}kx^2$

The mass makes $\boxed{\text{no difference}}$ to the work.

***P9.16** We take the *x* axis directed toward the pitcher.

(a) In the *x* direction, $p_{xi} + I_x = p_{xf}$:

$$\begin{aligned}I_x &= p_{xf} - p_{xi}\\ &= (0.200\ \text{kg})(40.0\ \text{m/s})\cos 30.0°\\ &\qquad - (0.200\ \text{kg})(15.0\ \text{m/s})(-\cos 45.0°)\\ &= 9.05\ \text{N}\cdot\text{s}\end{aligned}$$

In the *y* direction, $p_{yi} + I_y = p_{yf}$:

$$\begin{aligned} I_y &= p_{yf} - p_{yi} \\ &= (0.200\text{ kg})(40.0\text{ m/s})\sin 30.0° \\ &\qquad - (0.200\text{ kg})(15.0\text{ m/s})(-\sin 45.0°) \\ &= 6.12\text{ N}\cdot\text{s} \end{aligned}$$

Therefore, $\vec{\mathbf{I}} = \boxed{(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}})\text{ N}\cdot\text{s}}$

(b) $\vec{\mathbf{I}} = \frac{1}{2}(0 + \vec{\mathbf{F}}_m)(4.00\text{ ms}) + \vec{\mathbf{F}}_m(20.0\text{ ms}) + \frac{1}{2}\vec{\mathbf{F}}_m(4.00\text{ ms})$

$\vec{\mathbf{F}}_m \times 24.0 \times 10^{-3}\text{ s} = (9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}})\text{ N}\cdot\text{s}$

$\vec{\mathbf{F}}_m = \boxed{(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}})\text{ N}}$

***P9.17** (a) From the kinematic equations,

$$\Delta t = \frac{\Delta x}{v_{\text{avg}}} = \frac{2\Delta x}{v_f + v_i} = \frac{2(1.20\text{ m})}{0 + 25.0\text{ m/s}} = \boxed{9.60 \times 10^{-2}\text{ s}}$$

(b) We find the average force from the momentum-impulse theorem:

$$F_{\text{avg}} = \frac{\Delta p}{\Delta t} = \frac{m\Delta v}{\Delta t} = \frac{(1\,400\text{ kg})(25.0\text{ m/s} - 0)}{9.60 \times 10^{-2}\text{ s}} = \boxed{3.65 \times 10^5\text{ N}}$$

(c) Using the particle under constant acceleration model,

$$a_{\text{avg}} = \frac{\Delta v}{\Delta t} = \frac{25.0\text{ m/s} - 0}{9.60 \times 10^{-2}\text{ s}} = (260\text{ m/s}^2)\left(\frac{1\text{ g}}{9.80\text{ m/s}^2}\right) = \boxed{26.5g}$$

P9.18 We assume that the initial direction of the ball is in the –*x* direction.

(a) The impulse delivered to the ball is given by

$$\begin{aligned} \vec{\mathbf{I}} &= \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i \\ &= (0.060\,0\text{ kg})(40.0\text{ m/s})\hat{\mathbf{i}} - (0.060\,0\text{ m/s})(20.0\text{ m/s})(-\hat{\mathbf{i}}) \\ &= \boxed{3.60\hat{\mathbf{i}}\text{ N}\cdot\text{s}} \end{aligned}$$

(b) We choose the tennis ball as a nonisolated system for energy. Let the time interval be from just before the ball is hit until just after. Equation 9.2 for conservation of energy becomes

$$\Delta K + \Delta E_{\text{int}} = T_{\text{MW}}$$

Solving for the energy sum $\Delta E_{\text{int}} - T_{\text{MW}}$ and substituting gives

$$\Delta E_{\text{int}} - T_{\text{MW}} = -\Delta K = -\left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = \frac{1}{2}m\left(v_i^2 - v_f^2\right)$$

Substituting numerical values gives,

$$\begin{aligned}\Delta E_{\text{int}} - T_{\text{MW}} &= -\frac{1}{2}(0.060\,0\text{ kg})\left[(20.0\text{ m/s})^2 - (40.0\text{ m/s})^2\right]\\ &= 36.0\text{ J}\end{aligned}$$

There is no way of knowing how the energy splits between ΔE_{int} and T_{MW} without more information.

P9.19 (a) The impulse is in the *x* direction and equal to the area under the *F-t* graph:

$$\begin{aligned}\text{I} &= \left(\frac{0+4\text{ N}}{2}\right)(2\text{ s}-0)+(4\text{ N})(3\text{ s}-2\text{ s})+\left(\frac{4\text{ N}+0}{2}\right)(5\text{ s}-3\text{ s})\\ &= 12.0\text{ N}\cdot\text{s}\end{aligned}$$

$$\boxed{\vec{\mathbf{I}} = 12.0\text{ N}\cdot\text{s }\hat{\mathbf{i}}}$$

(b) From the momentum-impulse theorem,

$$m\vec{\mathbf{v}}_i + \vec{\mathbf{F}}\Delta t = m\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}\Delta t}{m} = 0 + \frac{12.0\,\hat{\mathbf{i}}\text{ N}\cdot\text{s}}{2.50\text{ kg}} = \boxed{4.80\,\hat{\mathbf{i}}\text{ m/s}}$$

(c) From the same equation,

$$\vec{\mathbf{v}}_f = \vec{\mathbf{v}}_i + \frac{\vec{\mathbf{F}}\Delta t}{m} = -2.00\,\hat{\mathbf{i}}\text{ m/s} + \frac{12.0\,\hat{\mathbf{i}}\text{ N}\cdot\text{s}}{2.50\text{ kg}} = \boxed{2.80\,\hat{\mathbf{i}}\text{ m/s}}$$

(d) $\vec{\mathbf{F}}_{\text{avg}}\Delta t = 12.0\hat{\mathbf{i}}\text{ N}\cdot\text{s} = \vec{\mathbf{F}}_{\text{avg}}(5.00\text{ s}) \rightarrow \vec{\mathbf{F}}_{\text{avg}} = \boxed{2.40\hat{\mathbf{i}}\text{ N}}$

P9.20 (a) A graph of the expression for force shows a parabola opening down, with the value zero at the beginning and end of the 0.800-s interval. We integrate the given force to find the impulse:

$$\begin{aligned}\text{I} &= \int_0^{0.800\text{s}} F\,dt\\ &= \int_0^{0.800\text{s}} (9\,200\text{ t N/s} - 11\,500\text{ t}^2\text{ N/s}^2)dt\\ &= \left[\frac{1}{2}(9\,200\text{ N/s})t^2 - \frac{1}{3}(11\,500\text{ N/s}^2)t^3\right]_0^{0.800\text{s}}\\ &= \frac{1}{2}(9\,200\text{ N/s})(0.800\text{ s})^2 - \frac{1}{3}(11\,500\text{ N/s}^2)(0.800\text{ s})^3\\ &= 2\,944\text{ N}\cdot\text{s} - 1\,963\text{ N}\cdot\text{s} = 981\text{ N}\cdot\text{s}\end{aligned}$$

The athlete imparts a downward impulse to the platform, so the platform imparts to her an impulse of $\boxed{981\text{ N}\cdot\text{s, up.}}$

(b) We could find her impact speed as a free-fall calculation, but we choose to write it as a conservation-of-energy calculation:

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{impact}}^2$$

$$v_{\text{impact}} = \sqrt{2gy_{\text{top}}} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(0.600\ \text{m}\right)}$$

$$= \boxed{3.43\ \text{m/s, down}}$$

(c) Gravity, as well as the platform, imparts impulse to her during the interaction with the platform

$$I = \Delta p$$

$$I_{\text{grav}} + I_{\text{platform}} = mv_f - mv_i$$

$$-mg\Delta t + I_{\text{platform}} = mv_f - mv_i$$

solving for the final velocity gives

$$v_f = v_i - mg\Delta t + \frac{I_{\text{platform}}}{m}$$

$$= \left(-3.43\ \text{m/s}\right) - \left(9.80\ \text{m/s}^2\right)\left(0.800\ \text{s}\right) + \frac{981\ \text{N}\cdot\text{s}}{65.0\ \text{kg}}$$

$$= \boxed{3.83\ \text{m/s, up}}$$

Note that the athlete is putting a lot of effort into jumping and does not exert any force "on herself." The usefulness of the force platform is to measure her effort by showing the force she exerts on the floor.

(d) Again energy is conserved in upward flight:

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{takeoff}}^2$$

which gives

$$y_{\text{top}} = \frac{v_{\text{takeoff}}^2}{2g} = \frac{\left(3.83\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{0.748\ \text{m}}$$

P9.21 After 3.00 s of pouring, the bucket contains

(3.00 s)(0.250 L/s) = 0.750 liter

of water, with mass (0.750 L)(1 kg/1 L) = 0.750 kg, and feeling gravitational force (0.750 kg)(9.80 m/s^2) = 7.35 N. The scale through the bucket must exert 7.35 N upward on this stationary water to support its weight. The scale must exert another 7.35 N to support the 0.750-kg bucket itself.

Water is entering the bucket with speed given by

$$mgy_{\text{top}} = \frac{1}{2}mv_{\text{impact}}^2$$

$$v_{\text{impact}} = \sqrt{2gy_{\text{top}}} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(2.60\text{ m}\right)}$$
$$= 7.14\text{ m/s, downward}$$

The scale exerts an extra upward force to stop the downward motion of this additional water, as described by

$$mv_{\text{impact}} + F_{\text{extra}}t = mv_f$$

The rate of change of momentum is the force itself:

$$\left(\frac{dm}{dt}\right)v_{\text{impact}} + F_{\text{extra}} = 0$$

which gives

$$F_{\text{extra}} = -\left(\frac{dm}{dt}\right)v_{\text{impact}} = -\left(0.250\text{ kg/s}\right)\left(-7.14\text{ m/s}\right) = 1.78\text{ N}$$

Altogether the scale must exert 7.35 N + 7.35 N + 1.78 N = $\boxed{16.5\text{ N}}$

Section 9.4 Collisions in One Dimension

P9.22 (a) Conservation of momentum gives

$$m_T v_{Tf} + m_C v_{Cf} = m_T v_{Ti} + m_C v_{Ci}$$

Solving for the final velocity of the truck gives

$$v_{Tf} = \frac{m_T v_{Ti} + m_C\left(v_{Ci} - v_{Cf}\right)}{m_T}$$
$$= \frac{\left(9\ 000\text{ kg}\right)\left(20.0\text{ m/s}\right) + \left(1\ 200\text{ kg}\right)\left[\left(25.0 - 18.0\right)\text{ m/s}\right]}{9\ 000\text{ kg}}$$

$$v_{Tf} = \boxed{20.9\text{ m/s East}}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\Delta KE = KE_f - KE_i = \left[\frac{1}{2}m_C v_{Cf}^2 + \frac{1}{2}m_T v_{Tf}^2\right] - \left[\frac{1}{2}m_C v_{Ci}^2 + \frac{1}{2}m_T v_{Ti}^2\right]$$

$$= \frac{1}{2}\left[m_C\left(v_{Cf}^2 - v_{Ci}^2\right) + m_T\left(v_{Tf}^2 - v_{Ti}^2\right)\right]$$

$$= \frac{1}{2}\left\{\left(1\ 200\text{ kg}\right)\left[(18.0\text{ m/s})^2 - (25.0\text{ m/s})^2\right]\right.$$

$$\left. + \left(9\ 000\text{ kg}\right)\left[(20.9\text{ m/s})^2 - (20.0\text{ m/s})^2\right]\right\}$$

$$\Delta KE = \boxed{-8.68\times 10^3\text{ J}}$$

Note: If 20.9 m/s were used to determine the energy lost instead of 20.9333 as the answer to part (a), the answer would be very different. We have kept extra digits in all intermediate answers until the problem is complete.

(c) The mechanical energy of the car-truck system has decreased. $\boxed{\text{Most of the energy was transformed to internal energy with some being carried away by sound.}}$

P9.23 Momentum is conserved for the bullet-block system:

$$mv + 0 = (m + M)v_f$$

$$v = \left(\frac{m+M}{m}\right)v_f = \left(\frac{10.0\times 10^{-3}\text{ kg} + 5.00\text{ kg}}{10.0\times 10^{-3}\text{ kg}}\right)\left(0.600\text{ m/s}\right)$$

$$= \boxed{301\text{ m/s}}$$

P9.24 The collision is completely inelastic.

(a) Momentum is conserved by the collision:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$mv_1 + (2m)v_2 = mv_f + 2mv_f = 3mv_f$$

$$v_f = \frac{mv_1 + 2mv_2}{3m} \rightarrow \boxed{v_f = \frac{1}{3}\left(v_1 + 2v_2\right)}$$

(b) We compute the change in mechanical energy of the car-truck system from

$$\Delta K = K_f - K_i = \frac{1}{2}(3m)v_f^2 - \left[\frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2\right]$$

$$\Delta K = \frac{3m}{2}\left[\frac{1}{3}(v_1 + 2v_2)\right]^2 - \left[\frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2\right]$$

$$\Delta K = \frac{3m}{2}\left(\frac{v_1^2}{9} + \frac{4v_1v_2}{9} + \frac{4v_2^2}{9}\right) - \frac{mv_1^2}{2} - mv_2^2$$

$$= m\left(\frac{v_1^2}{6} + \frac{2v_1v_2}{3} + \frac{2v_2^2}{3} - \frac{v_1^2}{2} - v_2^2\right)$$

$$\Delta K = m\left(\frac{v_1^2}{6} + \frac{4v_1v_2}{6} + \frac{4v_2^2}{6} - \frac{3v_1^2}{6} - \frac{6v_2^2}{6}\right)$$

$$= m\left(-\frac{2v_1^2}{6} + \frac{4v_1v_2}{6} - \frac{2v_2^2}{6}\right)$$

$$\Delta K = \boxed{-\frac{m}{3}(v_1^2 + v_2^2 - 2v_1v_2)}$$

***P9.25** (a) We write the law of conservation of momentum as

$$mv_{1i} + 3mv_{2i} = 4mv_f$$

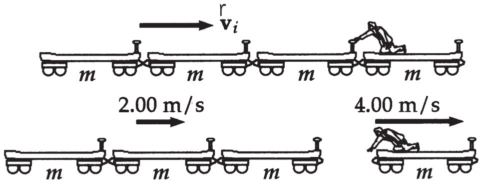
or $$v_f = \frac{4.00\text{ m/s} + 3(2.00\text{ m/s})}{4} = \boxed{2.50\text{ m/s}}$$

(b) $$K_f - K_i = \frac{1}{2}(4m)v_f^2 - \left[\frac{1}{2}mv_{1i}^2 + \frac{1}{2}(3m)v_{2i}^2\right]$$

$$= \frac{1}{2}(2.50\times 10^4\text{ kg})[4(2.50\text{ m/s})^2 - (4.00\text{ m/s})^2 - 3(2.00\text{ m/s})^2]$$

$$= \boxed{-3.75\times 10^4\text{ J}}$$

***P9.26** (a) The internal forces exerted by the actor do not change the total momentum of the system of the four cars and the movie actor. Conservation of momentum gives



ANS. FIG. P9.26

$$(4m)v_i = (3m)(2.00\text{ m/s}) + m(4.00\text{ m/s})$$

$$v_i = \frac{6.00\text{ m/s} + 4.00\text{ m/s}}{4} = \boxed{2.50\text{ m/s}}$$

(b) $W_{\text{actor}} = K_f - K_i$

$$= \frac{1}{2}\left[(3m)(2.00\ \text{m/s})^2 + m(4.00\ \text{m/s})^2\right] - \frac{1}{2}(4m)(2.50\ \text{m/s})^2$$

$$W_{\text{actor}} = \frac{(2.50\times 10^4\ \text{kg})}{2}(12.0 + 16.0 - 25.0)(\text{m/s})^2 = \boxed{37.5\ \text{kJ}}$$

(c) The event considered here is the time reversal of the perfectly inelastic collision in the previous problem. The same momentum conservation equation describes both processes.

P9.27 (a) From the text's analysis of a one-dimensional elastic collision with an originally stationary target, the x component of the neutron's velocity changes from v_i to $v_{1f} = (1 - 12)v_i/13 = -11v_i/13$. The x component of the target nucleus velocity is $v_{2f} = 2v_i/13$.

The neutron started with kinetic energy $\frac{1}{2}m_1v_i^2$.

The target nucleus ends up with kinetic energy $\frac{1}{2}(12m_1)\left(\frac{2v_i}{13}\right)^2$.

Then the fraction transferred is

$$\frac{\frac{1}{2}(12m_1)(2v_i/13)^2}{\frac{1}{2}m_1v_i^2} = \frac{48}{169} = \boxed{0.284}$$

Because the collision is elastic, the other 71.6% of the original energy stays with the neutron. The carbon is functioning as a *moderator* in the reactor, slowing down neutrons to make them more likely to produce reactions in the fuel.

(b) The final kinetic energy of the neutron is

$$K_n = (0.716)(1.60\times 10^{-13}\ \text{J}) = \boxed{1.15\times 10^{-13}\ \text{J}}$$

and the final kinetic energy of the carbon nucleus is

$$K_C = (0.284)(1.60\times 10^{-13}\ \text{J}) = \boxed{4.54\times 10^{-14}\ \text{J}}$$

***P9.28** Let's first analyze the situation in which the wood block, of mass $m_w = 1.00$ kg, is held in a vise. The bullet of mass $m_b = 7.00$ g is initially moving with speed v_b and then comes to rest in the block due to the kinetic friction force f_k between the block and the bullet as the bullet deforms the wood fibers and moves them out of the way. The result is an increase in internal energy in the wood and the bullet. Identify the wood and the bullet as an isolated system for energy during the collision:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substituting for the energies:

$$\left(0 - \frac{1}{2}m_b v_b^2\right) + f_k d = 0 \qquad \textbf{[1]}$$

where d = 8.00 cm is the depth of penetration of the bullet in the wood.

Now consider the second situation, where the block is sitting on a frictionless surface and the bullet is fired into it. Identify the wood and the bullet as an isolated system for energy during the collision:

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substituting for the energies:

$$\left[\frac{1}{2}\left(m_b + m_w\right)v_f^2 - \frac{1}{2}m_b v_b^2\right] + f_k d' = 0 \qquad \textbf{[2]}$$

where v_f is the speed with which the block and imbedded bullet slide across the table after the collision and d' is the depth of penetration of the bullet in this situation. Identify the wood and the bullet as an isolated system for momentum during the collision:

$$\Delta p = 0 \quad\rightarrow\quad p_i = p_f \quad\rightarrow\quad m_b v_b = \left(m_b + m_w\right)v_f \qquad \textbf{[3]}$$

Solving equation [3] for v_b, we obtain

$$v_b = \frac{\left(m_b + m_w\right)v_f}{m_b} \qquad \textbf{[4]}$$

Solving equation [1] for $f_k d$ and substituting for v_b from equation [4] above:

$$f_k d = \frac{1}{2}m_b v_b^2 = \frac{1}{2}m_b\left[\frac{\left(m_b + m_w\right)v_f}{m_b}\right]^2 = \frac{1}{2}\frac{\left(m_b + m_w\right)^2}{m_b}v_f^2 \qquad \textbf{[5]}$$

Solving equation [2] for f_kd' and substituting for v_b from equation [4]:

$$f_kd' = -\left[\frac{1}{2}\left(m_b + m_w\right)v_f^2 - \frac{1}{2}m_bv_b^2\right]$$

$$= -\left[\frac{1}{2}\left(m_b + m_w\right)v_f^2 - \frac{1}{2}m_b\left[\frac{\left(m_b + m_w\right)v_f}{m_b}\right]^2\right]$$

$$f_kd' = \frac{1}{2}\left[\frac{m_w}{m_b}\left(m_b + m_w\right)\right]v_f^2 \qquad \textbf{[6]}$$

Dividing equation [6] by [5] gives

$$\frac{f_kd'}{f_kd} = \frac{d'}{d} = \frac{\frac{1}{2}\left[\frac{m_w}{m_b}\left(m_b + m_w\right)\right]v_f^2}{\frac{1}{2}\left[\frac{\left(m_b + m_w\right)^2}{m_b}\right]v_f^2} = \frac{m_w}{m_b + m_w}$$

Solving for d' and substituting numerical values gives

$$d' = \left(\frac{m_w}{m_b + m_w}\right)d = \left[\frac{1.00\text{ kg}}{0.007\ 00\text{ kg} + 1.00\text{ kg}}\right]\left(8.00\text{ cm}\right) = \boxed{7.94\text{ cm}}$$

***P9.29** (a) The speed v of both balls just before the basketball reaches the ground may be found from $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ as

$$v = \sqrt{v_{yi}^2 + 2a_y\Delta y} = \sqrt{0 + 2\left(-g\right)\left(-h\right)} = \sqrt{2gh}$$

$$= \sqrt{2\left(9.80\text{ m/s}^2\right)\left(1.20\text{ m}\right)} = \boxed{4.85\text{ m/s}}$$

(b) Immediately after the basketball rebounds from the floor, it and the tennis ball meet in an elastic collision. The velocities of the two balls just before collision are

for the tennis ball (subscript t): $v_{ti} = -v$

and for the basketball (subscript b): $v_{bi} = +v$

We determine the velocity of the tennis ball immediately after this elastic collision as follows:

Momentum conservation gives

$$m_tv_{tf} + m_bv_{bf} = m_tv_{ti} + m_bv_{bi}$$

or $$m_tv_{tf} + m_bv_{bf} = \left(m_b - m_t\right)v \qquad \textbf{[1]}$$

From the criteria for a perfectly elastic collision:

$$v_{ti} - v_{bi} = -\left(v_{tf} - v_{bf}\right)$$

or $$v_{bf} = v_{tf} + v_{ti} - v_{bi} = v_{tf} - 2v \qquad \textbf{[2]}$$

Substituting equation [2] into [1] gives

$$m_t v_{tf} + m_b\left(v_{tf} - 2v\right) = \left(m_b - m_t\right)v$$

or the upward speed of the tennis ball immediately after the collision is

$$v_{tf} = \left(\frac{3m_b - m_t}{m_t + m_b}\right)v = \left(\frac{3m_b - m_t}{m_t + m_b}\right)\sqrt{2gh}$$

The vertical displacement of the tennis ball during its rebound following the collision is given by $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$ as

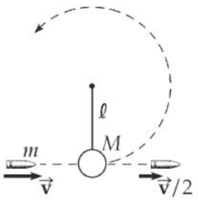
$$\Delta y = \frac{v_{yf}^2 - v_{yi}^2}{2a_y} = \frac{0 - v_{tf}^2}{2(-g)} = \left(\frac{1}{\cancel{2g}}\right)\left(\frac{3m_b - m_t}{m_t + m_b}\right)^2\left(\cancel{2g}h\right)$$

$$= \left(\frac{3m_b - m_t}{m_t + m_b}\right)^2 h$$

Substituting,

$$\Delta y = \left[\frac{3(590\text{ g}) - (57.0\text{ g})}{57.0\text{ g} + 590\text{ g}}\right]^2(1.20\text{ m}) = \boxed{8.41\text{ m}}$$

P9.30 Energy is conserved for the bob-Earth system between bottom and top of the swing. At the top the stiff rod is in compression and the bob nearly at rest.



ANS. FIG. P9.30

$$K_i + U_i = K_f + U_f: \quad \frac{1}{2}Mv_b^2 + 0 = 0 + Mg2\ell$$

$$v_b^2 = 4g\ell \quad \text{so} \quad v_b = 2\sqrt{g\ell}$$

Momentum of the bob-bullet system is conserved in the collision:

$$mv = m\frac{v}{2} + M\left(2\sqrt{g\ell}\right) \rightarrow \boxed{v = \frac{4M}{m}\sqrt{g\ell}}$$

P9.31 The collision between the clay and the wooden block is completely inelastic. Momentum is conserved by the collision. Find the relation between the speed of the clay (C) just before impact and the speed of the clay+block (CB) just after impact:

$$\vec{\mathbf{p}}_{\text{Bi}} + \vec{\mathbf{p}}_{\text{Ci}} = \vec{\mathbf{p}}_{\text{Bf}} + \vec{\mathbf{p}}_{\text{Cf}} \rightarrow m_B v_{\text{Bi}} + m_C v_{\text{Ci}} = m_B v_{\text{Bf}} + m_C v_{\text{Cf}}$$

$$M(0) + mv_C = mv_{CB} + Mv_{CB} = (m + M)v_{CB}$$

$$v_C = \frac{(m + M)}{m} v_{CB}$$

Now use conservation of energy in the presence of friction forces to find the relation between the speed v_{CB} just after impact and the distance the block slides before stopping:

$$\Delta K + \Delta E_{\text{int}} = 0: \quad 0 - \frac{1}{2}(m + M)v_{CB}^2 - fd = 0$$

and $-fd = -\mu nd = -\mu(m + M)gd$

$$\rightarrow \frac{1}{2}(m + M)v_{CB}^2 = \mu(m + M)gd \rightarrow v_{CB} = \sqrt{2\mu gd}$$

Combining our results, we have

$$v_C = \frac{(m + M)}{m}\sqrt{2\mu gd}$$

$$= \frac{(12.0\text{ g} + 100\text{ g})}{12.0\text{ g}}\sqrt{2(0.650)\left(9.80\text{ m/s}^2\right)(7.50\text{ m})}$$

$$\boxed{v_C = 91.2\text{ m/s}}$$

P9.32 The collision between the clay and the wooden block is completely inelastic. Momentum is conserved by the collision. Find the relation between the speed of the clay (C) just before impact and the speed of the clay+block (CB) just after impact:

$$\vec{\mathbf{p}}_{\text{Bi}} + \vec{\mathbf{p}}_{\text{Ci}} = \vec{\mathbf{p}}_{\text{Bf}} + \vec{\mathbf{p}}_{\text{Cf}} \rightarrow m_B v_{\text{Bi}} + m_C v_{\text{Ci}} = m_B v_{\text{Bf}} + m_C v_{\text{Cf}}$$

$$M(0) + mv_C = mv_{CB} + Mv_{CB} = (m + M)v_{CB}$$

$$v_C = \frac{(m + M)}{m} v_{CB}$$

Now use conservation of energy in the presence of friction forces to find the relation between the speed v_{CB} just after impact and the distance the block slides before stopping:

$$\Delta K + \Delta E_{\text{int}} = 0: \quad 0 - \frac{1}{2}(m + M)v_{CB}^2 - fd = 0$$

and $\quad -fd = -\mu nd = -\mu(m + M)gd$

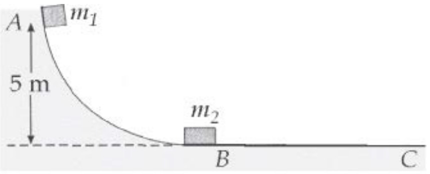
Then,

$$\frac{1}{2}(m+M)v_{CB}^2 = \mu(m+M)gd \rightarrow v_{CB} = \sqrt{2\mu gd}$$

Combining our results, we have

$$\boxed{v_C = \frac{(m+M)}{m}\sqrt{2\mu gd}}$$

P9.33 The mechanical energy of the isolated block-Earth system is conserved as the block of mass m_1 slides down the track. First we find v_1, the speed of m_1 at B before collision:



ANS. FIG. P9.33

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_1v_1^2 + 0 = 0 + m_1gh$$

$$v_1 = \sqrt{2(9.80\text{ m/s}^2)(5.00\text{ m})} = 9.90\text{ m/s}$$

Now we use the text's analysis of one-dimensional elastic collisions to find v_{1f}, the speed of m_1 at B just after collision.

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_1 = -\frac{1}{3}(9.90)\text{ m/s} = -3.30\text{ m/s}$$

Now the 5-kg block bounces back up to its highest point after collision according to

$$m_1gh_{\max} = \frac{1}{2}m_1v_{1f}^2$$

which gives

$$h_{\max} = \frac{v_{1f}^2}{2g} = \frac{(-3.30\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{0.556\text{ m}}$$

P9.34 (a) Using conservation of momentum, $\left(\sum\vec{\mathbf{p}}\right)_{\text{before}} = \left(\sum\vec{\mathbf{p}}\right)_{\text{after}}$, gives

$$(4.00\text{ kg})(5.00\text{ m/s}) + (10.0\text{ kg})(3.00\text{ m/s}) + (3.00\text{ kg})(-4.00\text{ m/s}) = [(4.00+10.0+3.00)\text{ kg}]v$$

Therefore, $v = +2.24$ m/s, or $\boxed{\text{2.24 m/s toward the right}}$.

(b) $\boxed{\text{No.}}$ For example, if the 10.0-kg and 3.00-kg masses were to stick together first, they would move with a speed given by solving

$$(13.0\text{ kg})v_1 = (10.0\text{ kg})(3.00\text{ m/s}) + (3.00\text{ kg})(-4.00\text{ m/s})$$

or $v_1 = +1.38\text{ m/s}$

Then when this 13.0-kg combined mass collides with the 4.00-kg mass, we have

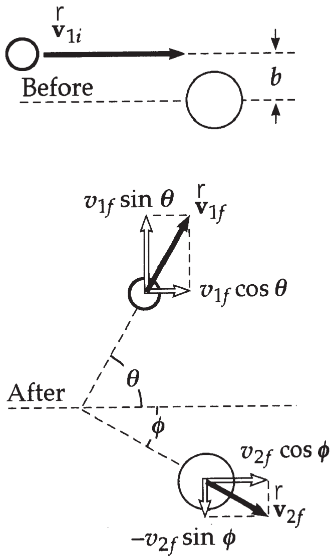
$$(17.0\text{ kg})v = (13.0\text{ kg})(1.38\text{ m/s}) + (4.00\text{ kg})(5.00\text{ m/s})$$

and $v = +2.24$ m/s, just as in part (a).

$\boxed{\text{Coupling order makes no difference to the final velocity.}}$

Section 9.5 Collisions in Two Dimensions

***P9.35** (a) We write equations expressing conservation of the x and y components of momentum, with reference to the figures on the right. Let the puck initially at rest be m_2. In the x direction,



ANS. FIG. P9.35

$$m_1 v_{1i} = m_1 v_{1f}\cos\theta + m_2 v_{2f}\cos\phi$$

which gives

$$v_{2f}\cos\phi = \frac{m_1 v_{1i} - m_1 v_{1f}\cos\theta}{m_2}$$

or

$$v_{2f}\cos\phi = \left(\frac{1}{0.300\text{ kg}}\right)\left[(0.200\text{ kg})(2.00\text{ m/s}) - (0.200\text{ kg})(1.00\text{ m/s})\cos 53.0°\right]$$

In the y direction,

$$0 = m_1 v_{1f}\sin\theta - m_2 v_{2f}\sin\phi$$

which gives

$$v_{2f}\sin\phi = \frac{m_1 v_{1f}\sin\theta}{m_2}$$

or

$$0 = (0.200\text{ kg})(1.00\text{ m/s})\sin 53.0° - (0.300\text{ kg})(\mathrm{v}_{2\mathrm{f}}\sin\phi)$$

From these equations, we find

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{0.532}{0.932} = 0.571 \quad \text{or} \quad \boxed{\phi = 29.7°}$$

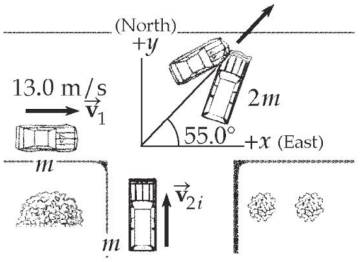
$$\text{Then } \mathrm{v}_{2\mathrm{f}} = \frac{0.160\text{ kg}\cdot\text{m/s}}{(0.300\text{ kg})(\sin 29.7°)} = \boxed{1.07\text{ m/s}}$$

(b) $\mathrm{K}_\mathrm{i} = \frac{1}{2}(0.200\text{ kg})(2.00\text{ m/s})^2 = 0.400\text{ J}$ and

$$\mathrm{K}_\mathrm{f} = \frac{1}{2}(0.200\text{ kg})(1.00\text{ m/s})^2 + \frac{1}{2}(0.300\text{ kg})(1.07\text{ m/s})^2 = 0.273\text{ J}$$

$$\mathrm{f}_{\text{lost}} = \frac{\Delta\mathrm{K}}{\mathrm{K}_\mathrm{i}} = \frac{\mathrm{K}_\mathrm{f} - \mathrm{K}_\mathrm{i}}{\mathrm{K}_\mathrm{i}} = \frac{0.273\text{ J} - 0.400\text{ J}}{0.400\text{ J}} = \boxed{-0.318}$$

P9.36 We use conservation of momentum for the system of two vehicles for both northward and eastward components, to find the original speed of car number 2.



ANS. FIG. P8.26

For the eastward direction:

$$\mathrm{m}(13.0\text{ m/s}) = 2\mathrm{m}\mathrm{V}_\mathrm{f}\cos 55.0°$$

For the northward direction:

$$\mathrm{m}\mathrm{v}_{2\mathrm{i}} = 2\mathrm{m}\mathrm{V}_\mathrm{f}\sin 55.0°$$

Divide the northward equation by the eastward equation to find:

$$\mathrm{v}_{2\mathrm{i}} = (13.0\text{ m/s})\tan 55.0° = 18.6\text{ m/s} = 41.5\text{ mi/h}$$

Thus, the driver of the northbound car was untruthful. His original speed was more than 35 mi/h.

P9.37 We will use conservation of both the *x* component and the *y* component of momentum for the two-puck system, which we can write as a single vector equation.

$$\mathrm{m}_1\vec{\mathbf{v}}_{1\mathrm{i}} + \mathrm{m}_2\vec{\mathbf{v}}_{2\mathrm{i}} = \mathrm{m}_1\vec{\mathbf{v}}_{1\mathrm{f}} + \mathrm{m}_2\vec{\mathbf{v}}_{2\mathrm{f}}$$

Both objects have the same final velocity, which we call $\vec{\mathbf{v}}_f$. Doing the algebra and substituting to solve for the one unknown gives

$$\vec{\mathbf{v}}_f = \frac{m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i}}{m_1 + m_2}$$

$$= \frac{(3.00\text{ kg})(5.00\hat{\mathbf{i}}\text{ m/s}) + (2.00\text{ kg})(-3.00\hat{\mathbf{j}}\text{ m/s})}{3.00\text{ kg} + 2.00\text{ kg}}$$

and calculating gives $\vec{\mathbf{v}}_f = \dfrac{15.0\hat{\mathbf{i}} - 6.00\hat{\mathbf{j}}}{5.00}$ m/s = $\boxed{(3.00\hat{\mathbf{i}} - 1.20\hat{\mathbf{j}})\text{ m/s}}$

P9.38 We write the conservation of momentum in the *x* direction, $p_{xf} = p_{xi}$, as

$$mv_O \cos 37.0° + mv_Y \cos 53.0° = m(5.00\text{ m/s})$$

$$0.799v_O + 0.602v_Y = 5.00\text{ m/s} \qquad \textbf{[1]}$$

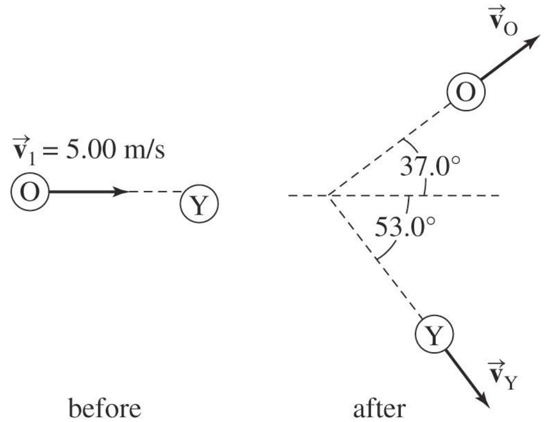
and the conservation of momentum in the *y* direction, $p_{yf} = p_{yi}$, as

$$mv_O \sin 37.0° - mv_Y \sin 53.0° = 0$$

$$0.602v_O = 0.799v_Y \qquad \textbf{[2]}$$

Solving equations [1] and [2] simultaneously gives,

$\boxed{v_O = 3.99\text{ m/s}}$ and $\boxed{v_Y = 3.01\text{ m/s}}$



ANS. FIG. P9.38

P9.39 ANS. FIG. P9.38 illustrates the collision. We write the conservation of momentum in the *x* direction, $p_{xf} = p_{xi}$, as

$$mv_O \cos\theta + mv_Y \cos(90.0° - \theta) = mv_i$$

$$v_O \cos\theta + v_Y \sin\theta = v_i \qquad \textbf{[1]}$$

and the conservation of momentum in the y direction, $p_{yf} = p_{yi}$, as

$$mv_O \sin\theta - mv_Y \cos(90.0° - \theta) = 0$$

$$v_O \sin\theta = v_Y \cos\theta \qquad \textbf{[2]}$$

From equation [2],

$$v_O = v_Y\left(\frac{\cos\theta}{\sin\theta}\right) \qquad \textbf{[3]}$$

Substituting into equation [1],

$$v_Y\left(\frac{\cos^2\theta}{\sin\theta}\right) + v_Y \sin\theta = v_i$$

so

$$v_Y\left(\cos^2\theta + \sin^2\theta\right) = v_i \sin\theta, \text{ and } \boxed{v_Y = v_i \sin\theta}$$

Then, from equation [3], $\boxed{v_O = v_i \cos\theta}$.

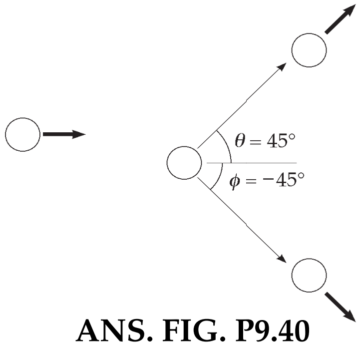
We did not need to write down an equation expressing conservation of mechanical energy. In this situation, the requirement on perpendicular final velocities is equivalent to the condition of elasticity.

***P9.40** (a) The vector expression for conservation of momentum, $\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f$ gives $p_{xi} = p_{xf}$ and $p_{yi} = p_{yf}$.

$$mv_i = mv\cos\theta + mv\cos\phi \qquad \textbf{[1]}$$

$$0 = mv\sin\theta + mv\sin\phi \qquad \textbf{[2]}$$

From [2], $\sin\theta = -\sin\phi$ so $\theta = -\phi$.



ANS. FIG. P9.40

Furthermore, energy conservation for the system of two protons requires

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

so

$$\boxed{v = \frac{v_i}{\sqrt{2}}}$$

(b) Hence, [1] gives

$$v_i = \frac{2v_i \cos\theta}{\sqrt{2}}$$

with $\theta = \boxed{45.0°}$ and $\phi = \boxed{-45.0°}$.

P9.41 By conservation of momentum for the system of the two billiard balls (with all masses equal), in the *x* and *y* directions separately,

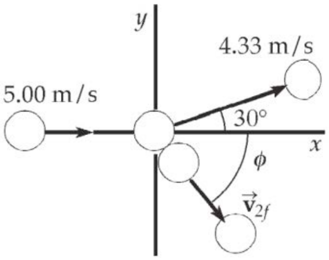
$$5.00\text{ m/s} + 0 = (4.33\text{ m/s})\cos 30.0° + v_{2\,fx}$$

$$v_{2\,fx} = 1.25\text{ m/s}$$

$$0 = (4.33\text{ m/s})\sin 30.0° + v_{2\,fy}$$

$$v_{2\,fy} = -2.16\text{ m/s}$$

$$\vec{\mathbf{v}}_{2\,f} = \boxed{2.50\text{ m/s at } -60.0°}$$



ANS. FIG. P9.41

Note that we did not need to explicitly use the fact that the collision is perfectly elastic.

P9.42 (a) $\boxed{\text{The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction—thus the collision is perfectly inelastic.}}$

(b) First, we conserve momentum for the system of two football players in the *x* direction (the direction of travel of the fullback):

$$(90.0\text{ kg})(5.00\text{ m/s}) + 0 = (185\text{ kg})V\cos\theta$$

where θ is the angle between the direction of the final velocity V and the *x* axis. We find

$$V\cos\theta = 2.43\text{ m/s} \qquad \textbf{[1]}$$

Now consider conservation of momentum of the system in the *y* direction (the direction of travel of the opponent):

$$(95.0\text{ kg})(3.00\text{ m/s}) + 0 = (185\text{ kg})V\sin\theta$$

which gives

$$V\sin\theta = 1.54\text{ m/s} \qquad \textbf{[2]}$$

Divide equation [2] by [1]:

$$\tan\theta = \frac{1.54}{2.43} = 0.633$$

From which, $\boxed{\theta = 32.3°}$.

Then, either [1] or [2] gives $V = \boxed{2.88 \text{ m/s}}$.

(c) $$K_i = \frac{1}{2}(90.0 \text{ kg})(5.00 \text{ m/s})^2 + \frac{1}{2}(95.0 \text{ kg})(3.00 \text{ m/s})^2 = 1.55\times 10^3 \text{ J}$$

$$K_f = \frac{1}{2}(185 \text{ kg})(2.88 \text{ m/s})^2 = 7.67\times 10^2 \text{ J}$$

Thus, the kinetic energy lost is $\boxed{786 \text{ J into internal energy}}$.

P9.43 (a) With three particles, the total final momentum of the system is $m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} + m_3\vec{\mathbf{v}}_{3f}$ and it must be zero to equal the original momentum. The mass of the third particle is

$$m_3 = (17.0 - 5.00 - 8.40) \times 10^{-27} \text{ kg}$$

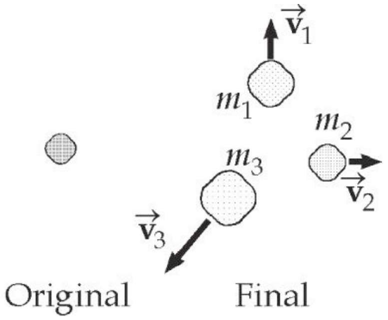
or $m_3 = 3.60 \times 10^{-27}$ kg

Solving $m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f} + m_3\vec{\mathbf{v}}_{3f} = 0$ for $\vec{\mathbf{v}}_{3f}$ gives

$$\vec{\mathbf{v}}_{3f} = -\frac{m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}}{m_3}$$

$$\vec{\mathbf{v}}_{3f} = -\frac{(3.36\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}}) \times 10^{-20} \text{ kg}\cdot\text{m/s}}{3.60 \times 10^{-27} \text{ kg}}$$

$$= \boxed{(-9.33\times 10^6\hat{\mathbf{i}} - 8.33\times 10^6\hat{\mathbf{j}}) \text{ m/s}}$$



ANS. FIG. P9.43

(b) The original kinetic energy of the system is zero.

The final kinetic energy is $K = K_{1f} + K_{2f} + K_{3f}$.

The terms are

$$K_{1f} = \frac{1}{2}(5.00\times 10^{-27} \text{ kg})(6.00\times 10^6 \text{ m/s})^2 = 9.00\times 10^{-14} \text{ J}$$

$$K_{2f} = \frac{1}{2}(8.40\times10^{-27}\ \text{kg})(4.00\times10^{6}\ \text{m/s})^2 = 6.72\times10^{-14}\ \text{J}$$

$$\begin{aligned} K_{3f} &= \frac{1}{2}(3.60\times10^{-27}\ \text{kg}) \\ &\qquad \times\left[(-9.33\times10^{6}\ \text{m/s})^2 + (-8.33\times10^{6}\ \text{m/s})^2\right] \\ &= 28.2\times10^{-14}\ \text{J} \end{aligned}$$

Then the system kinetic energy is

$$\begin{aligned} K &= 9.00\times10^{-14}\ \text{J} + 6.72\times10^{-14}\ \text{J} + 28.2\times10^{-14}\ \text{J} \\ &= \boxed{4.39\times10^{-13}\ \text{J}} \end{aligned}$$

P9.44 The initial momentum of the system is 0. Thus,

$$(1.20m)v_{Bi} = m(10.0\ \text{m/s})$$

and $v_{Bi} = 8.33\ \text{m/s}$

From conservation of energy,

$$K_i = \frac{1}{2}m(10.0\ \text{m/s})^2 + \frac{1}{2}(1.20m)(8.33\ \text{m/s})^2 = \frac{1}{2}m(183\ \text{m}^2/\text{s}^2)$$

$$K_f = \frac{1}{2}m(v_G)^2 + \frac{1}{2}(1.20m)(v_B)^2 = \frac{1}{2}\left(\frac{1}{2}m(183\ \text{m}^2/\text{s}^2)\right)$$

or $v_G^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2$ **[1]**

From conservation of momentum,

$$mv_G = (1.20m)v_B$$

or $v_G = 1.20v_B$ **[2]**

Solving [1] and [2] simultaneously, we find

$$(1.20v_B)^2 + 1.20v_B^2 = 91.7\ \text{m}^2/\text{s}^2$$

$$v_B = (91.7\ \text{m}^2/\text{s}^2/2.64)^{1/2}$$

which gives

$\boxed{v_B = 5.89\ \text{m/s}}$ (speed of blue puck after collision)

and $\boxed{v_G = 7.07\ \text{m/s}}$ (speed of green puck after collision)

Section 9.6 The Center of Mass

P9.45 The x coordinate of the center of mass is

$$x_{CM} = \frac{\sum m_i x_i}{\sum m_i} = \frac{0+0+0+0}{2.00\text{ kg} + 3.00\text{ kg} + 2.50\text{ kg} + 4.00\text{ kg}} = 0$$

and the y coordinate of the center of mass is

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$$

$$= \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg} + 2.50\text{ kg} + 4.00\text{ kg}}\right) \times [(2.00\text{ kg})(3.00\text{ m}) + (3.00\text{ kg})(2.50\text{ m}) + (2.50\text{ kg})(0) + (4.00\text{ kg})(-0.500\text{ m})]$$

$$y_{CM} = 1.00\text{ m}$$

Then $\boxed{\vec{\mathbf{r}}_{CM} = \left(0\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right)\text{ m}}$

P9.46 Let the x axis start at the Earth's center and point toward the Moon.

$$x_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

$$= \frac{\left(5.97\times 10^{24}\text{ kg}\right)(0) + \left(7.35\times 10^{22}\text{ kg}\right)\left(3.84\times 10^{8}\text{ m}\right)}{6.05\times 10^{24}\text{ kg}}$$

$$= \boxed{4.66\times 10^{6}\text{ m from the Earth's center}}$$

The center of mass is within the Earth, which has radius 6.37×10^6 m. It is 1.7 Mm below the point on the Earth's surface where the Moon is straight overhead.

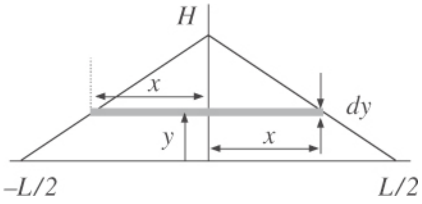
P9.47 The volume of the monument is that of a thick triangle of base $L = 64.8$ m, height $H = 15.7$ m, and width $W = 3.60$ m: $V = ½ LHW = 1.83 \times 10^3\text{ m}^3$. The monument has mass $M = \rho V = (3\,800\text{ kg/m}^3)V = 6.96 \times 10^6$ kg. The height of the center of mass (CM) is $y_{CM} = H/3$ (derived below). The amount of work done on the blocks is

$$U_g = Mgy_{CM}$$

$$= Mg\frac{H}{3} = \left(6.96\times 10^{6}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(\frac{15.7\text{ m}}{3}\right)$$

$$= \boxed{3.57\times 10^{8}\text{ J}}$$

We derive $y_{CM} = H/3$ here:



ANS. FIG. P9.47

We model the monument with the figure shown above. Consider the monument to be composed of slabs of infinitesimal thickness dy stacked on top of each other. A slab at height y has a infinitesimal volume element $dV = 2xWdy$, where W is the width of the monument and x is a function of height y.

The equation of the sloping side of the monument is

$$y = H - \frac{H}{L/2}x \rightarrow y = H - \frac{2H}{L}x \rightarrow y = H\left(1 - \frac{2}{L}x\right)$$

where x ranges from 0 to $+L/2$. Therefore,

$$x = \frac{L}{2}\left(1 - \frac{y}{H}\right)$$

where y ranges from 0 to H. The infinitesimal volume of a slab at height y is then

$$dV = 2xWdy = LW\left(1 - \frac{y}{H}\right)dy.$$

The mass contained in a volume element is $dm = \rho dV$.

Because of the symmetry of the monument, its CM lies above the origin of the coordinate axes at position y_{CM}:

$$y_{CM} = \frac{1}{M}\int_0^M y\,dm = \frac{1}{M}\int_0^V y\rho\,dV = \frac{1}{M}\int_0^H y\rho LW\left(1 - \frac{y}{H}\right)dy$$

$$y_{CM} = \frac{\rho LW}{M}\int_0^H\left(y - \frac{y^2}{H}\right)dy = \frac{\rho LW}{M}\left(\frac{y^2}{2} - \frac{y^3}{3H}\right)\Bigg|_0^H$$

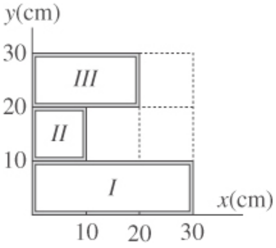
$$= \frac{\rho LW}{M}\left(\frac{H^2}{2} - \frac{H^3}{3H}\right)$$

$$y_{CM} = \frac{\rho LWH^2}{M}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6}\frac{\rho LWH^2}{\left(\frac{1}{2}\rho LWH\right)} = \left(\frac{2}{1}\right)\frac{H}{6}$$

$$y_{CM} = \frac{H}{3}$$

where we have used $M = \rho\left(\frac{1}{2}LHW\right)$.

P9.48 We could analyze the object as nine squares, each represented by an equal-mass particle at its center. But we will have less writing to do if we think of the sheet as composed of three sections, and consider the mass of each section to be at the geometric center of that section. Define the mass per unit area to be σ, and number the rectangles as shown. We can then calculate the mass and identify the center of mass of each section.



ANS. FIG. P9.48

$$m_I = (30.0\text{ cm})(10.0\text{ cm})\sigma \quad \text{with} \quad CM_I = (15.0\text{ cm}, 5.00\text{ cm})$$

$$m_{II} = (10.0\text{ cm})(20.0\text{ cm})\sigma \quad \text{with} \quad CM_{II} = (5.00\text{ cm}, 20.0\text{ cm})$$

$$m_{III} = (10.0\text{ cm})(10.0\text{ cm})\sigma \quad \text{with} \quad CM_{III} = (15.0\text{ cm}, 25.0\text{ cm})$$

The overall center of mass is at a point defined by the vector equation:

$$\vec{\mathbf{r}}_{CM} \equiv \left(\sum m_i \vec{\mathbf{r}}_i\right)/\sum m_i$$

Substituting the appropriate values, $\vec{\mathbf{r}}_{CM}$ is calculated to be:

$$\vec{\mathbf{r}}_{CM} = \left(\frac{1}{\sigma\left(300\text{ cm}^2 + 200\text{ cm}^2 + 100\text{ cm}^2\right)}\right) \times\left\{\sigma[(300)(15.0\hat{\mathbf{i}} + 5.00\hat{\mathbf{j}}) + (200)(5.00\hat{\mathbf{i}} + 20.0\hat{\mathbf{j}}) +(100)(15.0\hat{\mathbf{i}} + 25.0\hat{\mathbf{j}})]\text{ cm}^3\right\}$$

Calculating,

$$\vec{\mathbf{r}}_{CM} = \frac{4\,500\hat{\mathbf{i}} + 1\,500\hat{\mathbf{j}} + 1\,000\hat{\mathbf{i}} + 4\,000\hat{\mathbf{j}} + 1\,500\hat{\mathbf{i}} + 2\,500\hat{\mathbf{j}}}{600}\text{ cm}$$

and evaluating, $\vec{\mathbf{r}}_{CM} = \boxed{(11.7\hat{\mathbf{i}} + 13.3\hat{\mathbf{j}})\text{ cm}}$

P9.49 This object can be made by wrapping tape around a light, stiff, uniform rod.

(a) $M = \int_0^{0.300\text{ m}} \lambda\,dx = \int_0^{0.300\text{ m}} \left[50.0 + 20.0x\right]dx$

$$M = \left[50.0x + 10.0x^2\right]_0^{0.300\text{ m}} = \boxed{15.9\text{ g}}$$

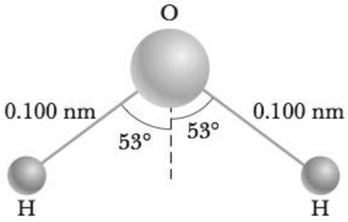
(b) $$x_{CM} = \frac{\int_{\text{all mass}} x\,dm}{M} = \frac{1}{M}\int_0^{0.300\text{ m}} \lambda x\,dx = \frac{1}{M}\int_0^{0.300\text{ m}} \left[50.0x + 20.0x^2\right]dx$$

$$x_{CM} = \frac{1}{15.9\text{ g}}\left[25.0x^2 + \frac{20x^3}{3}\right]_0^{0.300\text{ m}} = \boxed{0.153\text{ m}}$$

***P9.50** We use a coordinate system centered in the oxygen (O) atom, with the *x* axis to the right and the *y* axis upward. Then, from symmetry,

$$\boxed{x_{CM} = 0}$$

and



ANS. FIG. P9.50

$$y_{CM} = \frac{\sum m_i y_i}{\sum m_i}$$

$$= \left(\frac{1}{15.999\text{ u} + 1.008\text{ u} + 1.008\text{ u}}\right) \times [0 - (1.008\text{ u})(0.100\text{ nm})\cos 53.0° - (1.008\text{ u})(0.100\text{ nm})\cos 53.0°]$$

$$\boxed{\text{The center of mass of the molecule lies on the dotted line shown in ANS. FIG. P9.50, 0.006 73 nm below the center of the O atom.}}$$

Section 9.7 Systems of Many Particles

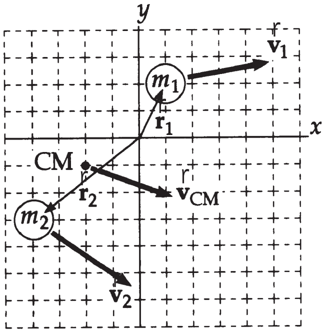
P9.51 (a) $$\vec{\mathbf{v}}_{CM} = \frac{\sum m_i \vec{\mathbf{v}}_i}{M} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{M}$$

$$= \left(\frac{1}{5.00\text{ kg}}\right)[(2.00\text{ kg})(2.00\hat{\mathbf{i}}\text{ m/s} - 3.00\hat{\mathbf{j}}\text{ m/s}) + (3.00\text{ kg})(1.00\hat{\mathbf{i}}\text{ m/s} + 6.00\hat{\mathbf{j}}\text{ m/s})]$$

$$\vec{\mathbf{v}}_{CM} = \boxed{\left(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) $$\vec{\mathbf{p}} = M\vec{\mathbf{v}}_{CM} = (5.00\text{ kg})(1.40\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}})\text{ m/s} = \boxed{(7.00\hat{\mathbf{i}} + 12.0\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}}$$

***P9.52** (a) ANS. FIG. P9.52 shows the position vectors and velocities of the particles.



ANS. FIG. P9.52

(b) Using the definition of the position vector at the center of mass,

$$\vec{r}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2}$$

$$\vec{r}_{CM} = \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\right)[(2.00\text{ kg})(1.00\text{ m}, 2.00\text{ m}) + (3.00\text{ kg})(-4.00\text{ m}, -3.00\text{ m})]$$

$$\vec{r}_{CM} = \boxed{(-2.00\hat{i} - 1.00\hat{j})\text{ m}}$$

(c) The velocity of the center of mass is

$$\vec{\mathbf{v}}_{CM} = \frac{\vec{\mathbf{P}}}{M} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1 + m_2}$$

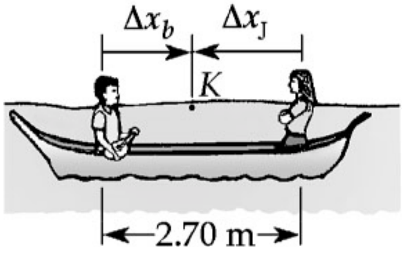
$$= \left(\frac{1}{2.00\text{ kg} + 3.00\text{ kg}}\right)[(2.00\text{ kg})(3.00\text{ m/s}, 0.50\text{ m/s}) + (3.00\text{ kg})(3.00\text{ m/s}, -2.00\text{ m/s})]$$

$$\vec{\mathbf{v}}_{CM} = \boxed{(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}})\text{ m/s}}$$

(d) The total linear momentum of the system can be calculated as $\vec{\mathbf{P}} = M\vec{\mathbf{v}}_{CM}$ or as $\vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2$. Either gives

$$\vec{\mathbf{P}} = \boxed{(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}})\text{ kg}\cdot\text{m/s}}$$

P9.53 No outside forces act on the boat-plus-lovers system, so its momentum is conserved at zero and the center of mass of the boat-passengers system stays fixed:



ANS. FIG. P9.53

$$x_{CM,i} = x_{CM,f}$$

Define K to be the point where they kiss, and Δx_J and Δx_b as shown in the figure. Since Romeo moves with the boat (and thus $\Delta x_{\text{Romeo}} = \Delta x_b$), let m_b be the combined mass of Romeo and the boat. The front of the boat and the shore are to the right in this picture, and we take the positive *x* direction to the right. Then,

$$m_J\Delta x_J + m_b\Delta x_b = 0$$

Choosing the *x* axis to point toward the shore,

$$(55.0\text{ kg})\Delta x_J + (77.0\text{ kg} + 80.0\text{ kg})\Delta x_b = 0$$

and $\Delta x_J = -2.85\Delta x_b$

As Juliet moves away from shore, the boat and Romeo glide toward the shore until the original 2.70-m gap between them is closed. We describe the relative motion with the equation

$$|\Delta x_J| + \Delta x_b = 2.70\text{ m}$$

Here the first term needs absolute value signs because Juliet's change in position is toward the left. An equivalent equation is then

$$-\Delta x_J + \Delta x_b = 2.70\text{ m}$$

Substituting, we find

$$+2.85\Delta x_b + \Delta x_b = 2.70\text{ m}$$

so $\Delta x_b = \dfrac{2.70\text{ m}}{3.85} = \boxed{0.700\text{ m}}$ towards the shore

P9.54 The vector position of the center of mass is (suppressing units)

$$\vec{r}_{CM} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2} = \frac{3.5\left[\left(3\hat{\mathbf{i}} + 3\hat{\mathbf{j}}\right)t + 2\hat{\mathbf{j}}t^2\right] + 5.5\left[3\hat{\mathbf{i}} - 2\hat{\mathbf{i}}t^2 + 6\hat{\mathbf{j}}t\right]}{3.5 + 5.5}$$

$$= \left(1.83 + 1.17t - 1.22t^2\right)\hat{\mathbf{i}} + \left(-2.5t + 0.778t^2\right)\hat{\mathbf{j}}$$

(a) At *t* = 2.50 s,

$$\vec{\mathbf{r}}_{CM} = (1.83 + 1.17\cdot 2.5 - 1.22\cdot 6.25)\hat{\mathbf{i}} + (-2.5\cdot 2.5 + 0.778\cdot 6.25)\hat{\mathbf{j}}$$

$$= \boxed{\left(-2.89\hat{\mathbf{i}} - 1.39\hat{\mathbf{j}}\right)\text{ cm}}$$

(b) The velocity of the center of mass is obtained by differentiating the expression for the vector position of the center of mass with respect to time:

$$\vec{\mathbf{v}}_{CM} = \frac{d\vec{\mathbf{r}}_{CM}}{dt} = (1.17 - 2.44t)\hat{\mathbf{i}} + (-2.5 + 1.56t)\hat{\mathbf{j}}$$

At *t* = 2.50 s,

$$\vec{\mathbf{v}}_{CM} = (1.17 - 2.44\cdot 2.5)\hat{\mathbf{i}} + (-2.5 + 1.56\cdot 2.5)\hat{\mathbf{j}}$$

$$= (-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}})\text{ cm/s}$$

Now, the total linear momentum is the total mass times the velocity of the center of mass.

$$\vec{\mathbf{p}} = (9.00\ \text{g})(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}})\ \text{cm/s}$$

$$= \boxed{(-44.5\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}})\ \text{g}\cdot\text{cm/s}}$$

(c) As was shown in part (b), $\boxed{\left(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}\right)\ \text{cm/s}}$

(d) Differentiating again, $\vec{\mathbf{a}}_{\text{CM}} = \dfrac{d\vec{\mathbf{v}}_{\text{CM}}}{dt} = (-2.44)\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}$

The center of mass acceleration is $\boxed{\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)\ \text{cm/s}^2}$ at t = 2.50 s and at all times.

(e) The net force on the system is equal to the total mass times the acceleration of the center of mass:

$$\vec{\mathbf{F}}_{\text{net}} = (9.00\ \text{g})\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)\ \text{cm/s}^2 = \boxed{\left(-220\hat{\mathbf{i}} + 140\hat{\mathbf{j}}\right)\ \mu\text{N}}$$

P9.55 (a) Conservation of momentum for the two-ball system gives us:

$$(0.200\ \text{kg})(1.50\ \text{m/s}) + (0.300\ \text{kg})(-0.400\ \text{m/s}) = (0.200\ \text{kg})v_{1f} + (0.300\ \text{kg})v_{2f}$$

Relative velocity equation:

$$v_{2f} - v_{1f} = 1.90\ \text{m/s}$$

Then, suppressing units, we have

$$0.300 - 0.120 = 0.200v_{1f} + 0.300(1.90 + v_{1f})$$

$$v_{1f} = -0.780\ \text{m/s} \qquad v_{2f} = 1.12\ \text{m/s}$$

$$\boxed{\vec{\mathbf{v}}_{1f} = -0.780\hat{\mathbf{i}}\ \text{m/s}} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 1.12\hat{\mathbf{i}}\ \text{m/s}}$$

(b) Before, $\vec{\mathbf{v}}_{\text{CM}} = \dfrac{(0.200\ \text{kg})(1.50\ \text{m/s})\hat{\mathbf{i}} + (0.300\ \text{kg})(-0.400\ \text{m/s})\hat{\mathbf{i}}}{0.500\ \text{kg}}$

$$\boxed{\vec{\mathbf{v}}_{\text{CM}} = (0.360\ \text{m/s})\hat{\mathbf{i}}}$$

Afterwards, the center of mass must move at the same velocity, because the momentum of the system is conserved.

Section 9.8 Deformable Systems

P9.56 (a) $\boxed{\text{Yes}}$ The only horizontal force on the vehicle is the frictional force exerted by the floor, so it gives the vehicle all of its final momentum, $(6.00\text{ kg})(3.00\hat{\mathbf{i}}\text{ m/s}) = \boxed{18.0\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}}$.

(b) $\boxed{\text{No. The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work.}}$

(c) $\boxed{\text{Yes, we could say that the final momentum of the cart came from the floor or from the Earth through the floor.}}$

(d) $\boxed{\text{No. The kinetic energy came from the original gravitational potential energy of the Earth-elevated load system, in the amount } KE = \left(\frac{1}{2}\right)(6.00\text{ kg})(3.00\text{ m/s})^2 = 27.0\text{ J.}}$

(e) $\boxed{\text{Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the wheels from slipping backward.}}$

P9.57 (a) When the cart hits the bumper it immediately stops, and the hanging particle keeps moving with its original speed v_i. The particle swings up as a pendulum on a fixed pivot, keeping constant energy. Measure elevations from the pivot:

$$\frac{1}{2}mv_i^2 + mg(-L) = 0 + mg(-L\cos\theta)$$

Then $v_i = \boxed{\sqrt{2gL(1-\cos\theta)}}$

(b) The bumper continues to exert a force to the left until the particle has swung down to $\boxed{\text{its lowest point}}$. This leftward force is necessary to reverse the rightward motion of the particle and accelerate it to the left.

P9.58 (a) $\boxed{\text{Yes}}$ The floor exerts a force, larger than the person's weight over time as he is taking off.

(b) $\boxed{\text{No}}$ The work by the floor on the person is zero because the force exerted by the floor acts over zero distance.

(c) He leaves the floor with a speed given by $\frac{1}{2}mv^2 = mgy_f$, or

$$v = \sqrt{2gy_f} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(0.150\ \text{m}\right)} = 1.71\ \text{m/s}$$

so his momentum immediately after he leaves the floor is

$$p = mv = \left(60.0\ \text{kg}\right)\left(1.71\ \text{m/s up}\right) = \boxed{103\ \text{kg}\cdot\text{m/s up}}$$

(d) Yes. You could say that it came from the planet, that gained momentum 103 kg · m/s down, but it came through the force exerted by the floor over a time interval on the person, so it came through the floor or from the floor through direct contact.

(e) His kinetic energy is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(60.0\ \text{kg}\right)\left(1.71\ \text{m/s}\right)^2 = \boxed{88.2\ \text{J}}$$

(f) No. The energy came from chemical energy in the person's leg muscles. The floor did no work on the person.

P9.59 Consider the motion of the center of mass (CM) of the system of the two pucks. Because the pucks have equal mass *m*, the CM lies at the midpoint of the line connecting the pucks.

(a) The force *F* accelerates the CM to the right at the rate

$$a_{CM} = \frac{F}{2m}$$

According to Figure P9.59, when the force has moved through distance *d*, the CM has moves through distance $D_{CM} = d - \frac{1}{2}\ell$. We can find the speed of the CM, which is the same as the speed *v* of the pucks when they meet and stick together:

$$v_f^2 = v_i^2 + 2a_{CM}\left(x_f - x_i\right)$$

$$v_{CM}^2 = 0 + 2\left(\frac{F}{2m}\right)\left(d - \frac{1}{2}\ell\right) \rightarrow \qquad v = v_{CM} = \boxed{\sqrt{\frac{F\left(2d - \ell\right)}{2m}}}$$

(b) The force F does work on the system through distance d, the work done is $W = Fd$. Relate this work to the change in kinetic energy and internal energy:

$$\Delta K + \Delta E_{\text{int}} = W$$

$$\text{where } \Delta K = \frac{1}{2}(2m)v_{\text{CM}}^2 = m\left[\frac{F(2d-\ell)}{2m}\right] = \frac{F(2d-\ell)}{2}$$

$$\left[\frac{F(2d-\ell)}{2}\right] + \Delta E_{\text{int}} = Fd \rightarrow \Delta E_{\text{int}} = Fd - \left[\frac{F(2d-\ell)}{2}\right]$$

$$\Delta E_{\text{int}} = Fd - Fd + \frac{F\ell}{2}$$

$$\Delta E_{\text{int}} = \boxed{\frac{F\ell}{2}}$$

Section 9.9 Rocket Propulsion

P9.60 (a) The fuel burns at a rate given by

$$\frac{dM}{dt} = \frac{12.7\text{ g}}{1.90\text{ s}} = 6.68 \times 10^{-3}\text{ kg/s}$$

From the rocket thrust equation,

$$\text{Thrust} = v_e\frac{dM}{dt}: \; 5.26\text{ N} = v_e\left(6.68 \times 10^{-3}\text{ kg/s}\right)$$

$$v_e = \boxed{787\text{ m/s}}$$

(b) $v_f - v_i = v_e \ln\left(\dfrac{M_i}{M_f}\right)$:

$$v_f - 0 = (787\text{ m/s})\ln\left(\frac{53.5\text{ g} + 25.5\text{ g}}{53.5\text{ g} + 25.5\text{ g} - 12.7\text{ g}}\right)$$

$$v_f = \boxed{138\text{ m/s}}$$

***P9.61** The force exerted on the water by the hose is

$$F = \frac{\Delta p_{\text{water}}}{\Delta t} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.600\text{ kg})(25.0\text{ m/s}) - 0}{1.00\text{ s}}$$
$$= \boxed{15.0\text{ N}}$$

According to Newton's third law, the water exerts a force of equal magnitude back on the hose. Thus, the gardener must apply a 15.0-N force (in the direction of the velocity of the exiting water stream) to hold the hose stationary.

P9.62 (a) The thrust, *F*, is equal to the time rate of change of momentum as fuel is exhausted from the rocket.

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv_e)$$

Since the exhaust velocity v_e is a constant,

$$F = v_e(dm/dt),\text{ where } dm/dt = 1.50\times10^4\text{ kg/s}$$

and $v_e = 2.60\times10^3\text{ m/s}$.

Then $F = (2.60\times10^3\text{m/s})(1.50\times10^4\text{kg/s}) = \boxed{3.90\times10^7\text{ N}}$

(b) Applying $\sum F = ma$ gives

$$\sum F_y = \text{Thrust} - Mg = Ma:$$
$$3.90\times10^7\text{ N} - (3.00\times10^6\text{ kg})(9.80\text{ m/s}^2) = (3.00\times10^6\text{ kg})a$$

$$a = \boxed{3.20\text{ m/s}^2}$$

P9.63 In $v = v_e \ln\frac{M_i}{M_f}$ we solve for M_i.

(a) $M_i = e^{v/v_e}M_f \quad\rightarrow\quad M_i = e^5(3.00\times10^3\text{ kg}) = 4.45\times10^5\text{ kg}$

The mass of fuel and oxidizer is

$$\Delta M = M_i - M_f = (445 - 3.00)\times10^3\text{kg}$$
$$= \boxed{442\text{ metric tons}}$$

(b) $\Delta M = e^2(3.00\text{ metric tons}) - 3.00\text{ metric tons} = \boxed{19.2\text{ metric tons}}$

(c) This is much less than the suggested value of 442/2.5. Mathematically, the logarithm in the rocket propulsion equation is not a linear function. Physically, a higher exhaust speed has an extra-large cumulative effect on the rocket body's final speed, by counting again and again in the speed the body attains second after second during its burn.

P9.64 (a) From the equation for rocket propulsion in the text,

$$v - 0 = v_e \ln\left(\frac{M_i}{M_f}\right) = -v_e \ln\left(\frac{M_f}{M_i}\right)$$

Now, $M_f = M_i - kt$, so $v = -v_e \ln\left(\frac{M_i - kt}{M_i}\right) = -v_e \ln\left(1 - \frac{k}{M_i}t\right)$

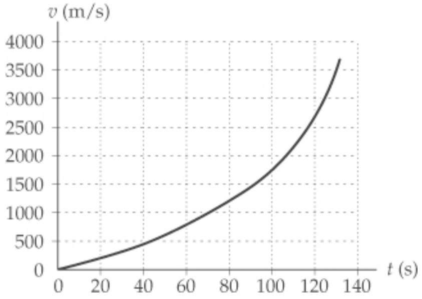
With the definition, $T_p \equiv \frac{M_i}{k}$, this becomes

$$v(t) = \boxed{-v_e \ln\left(1 - \frac{t}{T_p}\right)}$$

(b) With, $v_e = 1\,500$ m/s, and $T_p = 144$ s,

$$v = -(1\,500\text{ m/s})\ln\left(1 - \frac{t}{144\text{ s}}\right)$$

t (s)	*v* (m/s)
0	0
20	224
40	488
60	808
80	1 220
100	1 780
120	2 690
132	3 730



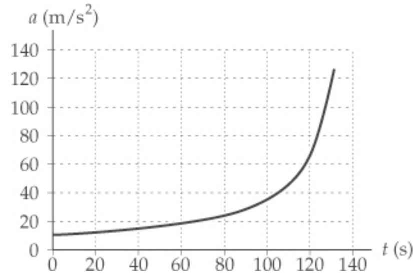
ANS. FIG. P9.64(b)

(c) $$a(t)=\frac{dv}{dt}=\frac{d\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]}{dt}=-v_e\left(\frac{1}{1-\frac{t}{T_p}}\right)\left(-\frac{1}{T_p}\right)=\left(\frac{v_e}{T_p}\right)\left(\frac{1}{1-\frac{t}{T_p}}\right),$$

or $$a(t)=\boxed{\frac{v_e}{T_p-t}}$$

(d) With, $v_e = 1\,500$ m/s, and $T_p = 144$ s, $a=\dfrac{1\,500\text{ m/s}}{144\text{ s}-t}$.

t (s)	a (m/s^2)
0	10.4
20	12.1
40	14.4
60	17.9
80	23.4
100	34.1
120	62.5
132	125



ANS. FIG. P9.64(d)

(e) $$x(t)=0+\int_0^t v\,dt=\int_0^t\left[-v_e\ln\left(1-\frac{t}{T_p}\right)\right]dt=v_eT_p\int_0^t\ln\left[1-\frac{t}{T_p}\right]\left(-\frac{dt}{T_p}\right)$$

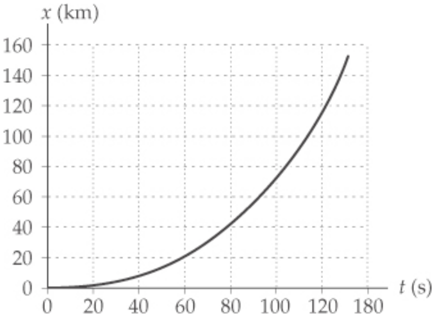
$$x(t)=v_eT_p\left[\left(1-\frac{t}{T_p}\right)\ln\left(1-\frac{t}{T_p}\right)-\left(1-\frac{t}{T_p}\right)\right]_0^t$$

$$x(t)=\boxed{v_e\left(T_p-t\right)\ln\left(1-\frac{t}{T_p}\right)+v_et}$$

(f) With, $v_e = 1.500$ m/s = 1.50 km/s, and $T_p = 144$ s,

$$x=1.50(144-t)\ln\left(1-\frac{t}{144}\right)+1.50t$$

t (s)	*x* (m)
0	0
20	2.19
40	9.23
60	22.1
80	42.2
100	71.7
120	115
132	153



ANS. FIG. P9.64(f)

Additional Problems

P9.65 (a) At the highest point, the velocity of the ball is zero, so momentum is also $\boxed{\text{zero}}$.

(b) Use $v_{yf}^2 = v_{yi}^2 + 2a(y_f - y_i)$ to find the maximum height H_{max}:

$$0 = v_i + 2(-g)H_{\text{max}}$$

or $$H_{\text{max}} = \frac{v_i^2}{2g}$$

Now, find the speed of the ball for $(y_f - y_i) = \frac{1}{2}H_{\text{max}}$:

$$v_f^2 = v_i^2 = 2(-g)\left(\frac{1}{2}H_{\text{max}}\right)$$

$$= v_i^2 - 2g\left(\frac{1}{2}\right)\left(\frac{v_i^2}{2g}\right) = v_i^2 - \frac{1}{2}v_i^2 = \frac{1}{2}v_i^2$$

which gives $v_f = \frac{v_i}{\sqrt{2}}$

Then, $p_f = mv_f = \boxed{\frac{mv_i}{\sqrt{2}}\text{, upward}}$

P9.66 (a) The system is isolated because the skater is on frictionless ice — if it were otherwise, she would be able to move. Initially, the horizontal momentum of the system is zero, and this quantity is conserved; so when she throws the gloves in one direction, she will move in the opposite direction because the total momentum will remain zero. The system has total mass M. After the skater throws the gloves, the mass of the gloves, m, is moving with velocity $\vec{\mathbf{v}}_{\text{gloves}}$ and the mass of the skater less the gloves, $M - m$, is moving with velocity $\vec{\mathbf{v}}_{\text{girl}}$:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f}$$

$$0 = (M - m)\vec{\mathbf{v}}_{\text{girl}} + m\vec{\mathbf{v}}_{\text{gloves}} \rightarrow \vec{\mathbf{v}}_{\text{girl}} = \boxed{-\left(\frac{m}{M-m}\right)\vec{\mathbf{v}}_{\text{gloves}}}$$

The term $M - m$ is the total mass less the mass of the gloves.

(b) $\boxed{\text{As she throws the gloves and exerts a force on them, the gloves exert an equal and opposite force on her (Newton's third law) that causes her to accelerate from rest to reach the velocity } \vec{\mathbf{v}}_{\text{girl}}.}$

P9.67 In $\vec{\mathbf{F}}\Delta t = \Delta(m\vec{\mathbf{v}})$, one component gives

$$\Delta p_y = m(v_{yf} - v_{yi}) = m(v\cos 60.0° - v\cos 60.0°) = 0$$

So the wall does not exert a force on the ball in the y direction. The other component gives

$$\begin{aligned}\Delta p_x &= m(v_{xf} - v_{xi}) = m(-v\sin 60.0° - v\sin 60.0°)\\ &= -2mv\sin 60.0° = -2(3.00\text{ kg})(10.0\text{ m/s})\sin 60.0°\\ &= -52.0\text{ kg}\cdot\text{m/s}\end{aligned}$$

So $$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t} = \frac{\Delta p_x\hat{\mathbf{i}}}{\Delta t} = \frac{-52.0\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s}}{0.200\text{ s}} = -260\hat{\mathbf{i}}\text{ N}$$

P9.68 (a) In the same symbols as in the text's Example, the original kinetic energy is

$$K_A = \frac{1}{2}m_1v_{1A}^2$$

The example shows that the kinetic energy immediately after latching together is

$$K_B = \frac{1}{2}\left(\frac{m_1v_{1A}^2}{m_1 + m_2}\right)$$

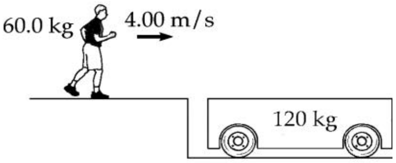
so the fraction of kinetic energy remaining as kinetic energy is

$$K_B/K_A = \boxed{m_1/(m_1 + m_2)}$$

(b) Momentum is conserved in the collision so momentum after divided by momentum before is $\boxed{1.00}$.

(c) Energy is an entirely different thing from momentum. A comparison: When a photographer's single-use flashbulb flashes, a magnesium filament oxidizes. Chemical energy disappears. (Internal energy appears and light carries some energy away.) The measured mass of the flashbulb is the same before and after. It can be the same in spite of the 100% energy conversion, because energy and mass are totally different things in classical physics. In the ballistic pendulum, conversion of energy from mechanical into internal does not upset conservation of mass or conservation of momentum.

***P9.69** (a) Conservation of momentum for this totally inelastic collision gives



ANS. FIG. P9.69

$$m_p v_i = (m_p + m_c) v_f$$

$$(60.0 \text{ kg})(4.00 \text{ m/s}) = (120 \text{ kg} + 60.0 \text{ kg}) v_f$$

$$\vec{\mathbf{v}}_f = \boxed{1.33\hat{\mathbf{i}} \text{ m/s}}$$

(b) To obtain the force of friction, we first consider Newton's second law in the *y* direction, $\sum F_y = 0$, which gives

$$n - (60.0 \text{ kg})(9.80 \text{ m/s}) = 0$$

or n = 588 N. The force of friction is then

$$f_k = \mu_k n = (0.400)(588 \text{ N}) = 235 \text{ N}$$

$$\vec{\mathbf{f}}_k = \boxed{-235\hat{\mathbf{i}} \text{ N}}$$

(c) The change in the person's momentum equals the impulse, or

$$p_i + I = p_f$$

$$mv_i + Ft = mv_f$$

$$(60.0\text{ kg})(4.00\text{ m/s}) - (235\text{ N})t = (60.0\text{ kg})(1.33\text{ m/s})$$

$$t = \boxed{0.680\text{ s}}$$

(d) The change in momentum of the person is

$$m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i = (60.0\text{ kg})(1.33 - 4.00)\hat{\mathbf{i}}\text{ m/s} = \boxed{-160\hat{\mathbf{i}}\text{ N}\cdot\text{s}}$$

The change in momentum of the cart is

$$(120\text{ kg})(1.33\text{ m/s}) - 0 = \boxed{+160\hat{\mathbf{i}}\text{ N}\cdot\text{s}}$$

(e) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}[(4.00 + 1.33)\text{ m/s}](0.680\text{ s}) = \boxed{1.81\text{ m}}$

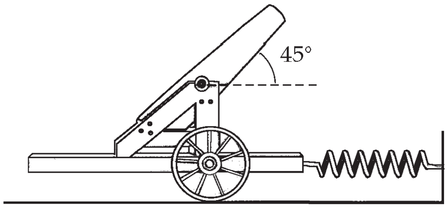
(f) $x_f - x_i = \frac{1}{2}(v_i + v_f)t = \frac{1}{2}(0 + 1.33\text{ m/s})(0.680\text{ s}) = \boxed{0.454\text{ m}}$

(g)
$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(60.0\text{ kg})(1.33\text{ m/s})^2 - \frac{1}{2}(60.0\text{ kg})(4.00\text{ m/s})^2 = \boxed{-427\text{ J}}$$

(h) $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}(120\text{ kg})(1.33\text{ m/s})^2 - 0 = \boxed{107\text{ J}}$

(i) The force exerted by the person on the cart must be equal in magnitude and opposite in direction to the force exerted by the cart on the person. The changes in momentum of the two objects must be equal in magnitude and must add to zero. Their changes in kinetic energy are different in magnitude and do not add to zero. The following represent two ways of thinking about why. The distance moved by the cart is different from the distance moved by the point of application of the friction force to the cart. The total change in mechanical energy for both objects together, –320 J, becomes +320 J of additional internal energy in this perfectly inelastic collision.

***P9.70** (a) Use conservation of the horizontal component of momentum for the system of the shell, the cannon, and the carriage, from just before to just after the cannon firing:



ANS. FIG. P9.70

$$p_{xf} = p_{xi}$$

$$m_{\text{shell}}v_{\text{shell}}\cos 45.0° + m_{\text{cannon}}v_{\text{recoil}} = 0$$

$$(200\text{ kg})(125\text{ m/s})\cos 45.0° + (5\,000\text{ kg})v_{\text{recoil}} = 0$$

or $v_{\text{recoil}} = \boxed{-3.54\text{ m/s}}$

(b) Use conservation of energy for the system of the cannon, the carriage, and the spring from right after the cannon is fired to the instant when the cannon comes to rest.

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$$

$$0 + 0 + \frac{1}{2}kx_{\text{max}}^2 = \frac{1}{2}mv_{\text{recoil}}^2 + 0 + 0$$

$$x_{\text{max}} = \sqrt{\frac{mv_{\text{recoil}}^2}{k}} = \sqrt{\frac{(5\,000\text{ kg})(-3.54\text{ m/s})^2}{2.00\times 10^4\text{ N/m}}} = \boxed{1.77\text{ m}}$$

(c) $|F_{s,\text{ max}}| = kx_{\text{max}}$

$$|F_{s,\text{ max}}| = (2.00\times 10^4\text{ N/m})(1.77\text{ m}) = \boxed{3.54\times 10^4\text{ N}}$$

(d) $\boxed{\text{No.}}$ The spring exerts a force on the system during the firing. The force represents an impulse, so the momentum of the system is not conserved in the horizontal direction. Consider the vertical direction. There are two vertical forces on the system: the normal force from the ground and the gravitational force. During the firing, the normal force is larger than the gravitational force. Therefore, there is a net impulse on the system in the upward direction. The impulse accounts for the initial vertical momentum component of the projectile.

P9.71 (a) Momentum of the bullet-block system is conserved in the collision, so you can relate the speed of the block and bullet right after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet-block-Earth system to relate the speed after the collision to the maximum height.

(b) Momentum is conserved by the collision. Find the relation between the speed of the bullet v_i just before impact and the speed of the bullet + block v just after impact:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \rightarrow m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2i}$$

$$mv_i + M(0) = mv + Mv = (m + M)v$$

$$\rightarrow \qquad v_i = \frac{(m + M)}{m}v$$

For the bullet-block-Earth system, total energy is conserved. Find the relation between the speed of the bullet-block v and the height h the block climbs to:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m + M)v^2 + 0 = (m + M)gh \to v = \sqrt{2gh}$$

Combining our results, we find

$$v_i = \frac{m + M}{m}\sqrt{2gh} = \left(\frac{1.255\ \text{kg}}{0.005\ 00\ \text{kg}}\right)\sqrt{2\left(9.80\ \text{m/s}^2\right)\left(0.220\ \text{m}\right)}$$

$$v_i = \boxed{521\ \text{m/s}}$$

P9.72 (a) Momentum of the bullet-block system is conserved in the collision, so you can relate the speed of the block and bullet right after the collision to the initial speed of the bullet. Then, you can use conservation of mechanical energy for the bullet-block-Earth system to relate the speed after the collision to the maximum height.

(b) Momentum is conserved by the collision. Find the relation between the speed of the bullet v_i just before impact and the speed of the bullet + block v just after impact:

$$\vec{\mathbf{p}}_{1i} + \vec{\mathbf{p}}_{2i} = \vec{\mathbf{p}}_{1f} + \vec{\mathbf{p}}_{2f} \to m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2i}$$

$$mv_i + M(0) = mv + Mv = (m + M)v$$

$$\to \qquad v_i = \frac{(m + M)}{m}v$$

For the bullet-block-Earth system, total energy is conserved. Find the relation between the speed of the bullet-block v and the height h the block climbs to:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}(m + M)v^2 + 0 = (m + M)gh \to v = \sqrt{2gh}$$

Combining our results, we find $\boxed{v_i = \frac{m + M}{m}\sqrt{2gh}}$.

***P9.73** Momentum conservation for the system of the two objects can be written as

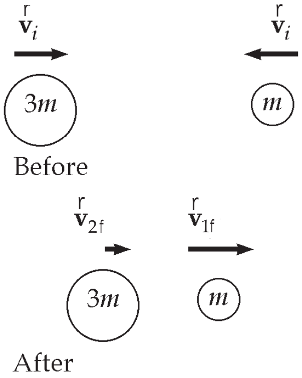
$$3mv_i - mv_i = mv_{1f} + 3mv_{2f}$$

The relative velocity equation then gives

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f}$$

or

$$-v_i - v_i = -v_{1f} + v_{2f}$$
$$2v_i = v_{1f} + 3v_{2f}$$



ANS. FIG. P9.73

Which gives

$$0 = 4v_{2f}$$

or $v_{1f} = \boxed{2v_i}$ and $v_{2f} = \boxed{0}$.

P9.74 (a) The mass of the sleigh plus you is 270 kg. Your velocity is 7.50 m/s in the x direction. You unbolt a 15.0-kg seat and throw it back at the ravening wolves, giving it a speed of 8.00 m/s relative to you. Find the velocity of the sleigh afterward, and the velocity of the seat relative to the ground.

(b) We substitute $v_{1f} = 8.00\text{ m/s} - v_{2f}$:

$$(270\text{ kg})(7.50\text{ m/s}) = (15.0\text{ kg})(-8.00\text{ m/s} + v_{2f}) + (255\text{ kg})v_{2f}$$
$$2\ 025\text{ kg}\cdot\text{m/s} = -120\text{ kg}\cdot\text{m/s} + (270\text{ kg})v_{2f}$$
$$v_{2f} = \frac{2\ 145\text{ m/s}}{270} = 7.94\text{ m/s}$$
$$v_{1f} = 8.00\text{ m/s} - 7.94\text{ m/s} = 0.055\ 6\text{ m/s}$$

The final velocity of the seat is $-0.055\,6\hat{\mathbf{i}}$ m/s. That of the sleigh is $7.94\hat{\mathbf{i}}$ m/s.

(c) You transform potential energy stored in your body into kinetic energy of the system:

$$\Delta K + \Delta U_{\text{body}} = 0$$
$$\Delta U_{\text{body}} = -\Delta K = K_i - K_f$$

$$\Delta U_{\text{body}} = \frac{1}{2}(270\text{ kg})(7.50\text{ m/s})^2$$
$$-\left[\frac{1}{2}(15.0\text{ kg})(0.0556\text{ m/s})^2\right.$$
$$\left.+\frac{1}{2}(255\text{ kg})(7.94\text{ m/s})^2\right]$$
$$\Delta U_{\text{body}} = 7\ 594\text{ J} - [0.023\ 1\text{ J} + 8\ 047\text{ J}]$$
$$\Delta U_{\text{body}} = \boxed{-453\text{ J}}$$

P9.75 (a) When the spring is fully compressed, each cart moves with same velocity *v*. Apply conservation of momentum for the system of two gliders

$$p_i = p_f: \qquad m_1v_1 + m_2v_2 = (m_1 + m_2)v \ \rightarrow \ \boxed{v = \frac{m_1v_1 + m_2v_2}{m_1 + m_2}}$$

(b) Only conservative forces act; therefore, $\Delta E = 0$.

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_2^2 = \frac{1}{2}(m_1 + m_2)v^2 + \frac{1}{2}kx_m^2$$

Substitute for *v* from (a) and solve for x_m.

$$x_m^2 = \left(\frac{1}{k(m_1 + m_2)}\right)[(m_1 + m_2)m_1v_1^2 + (m_1 + m_2)m_2v_2^2$$
$$-(m_1v_1)^2 - (m_2v_2)^2 - 2m_1m_2v_1v_2]$$
$$x_m = \sqrt{\frac{m_1m_2(v_1^2 + v_2^2 - 2v_1v_2)}{k(m_1 + m_2)}} = \boxed{(v_1 - v_2)\sqrt{\frac{m_1m_2}{k(m_1 + m_2)}}}$$

(c) $m_1v_1 + m_2v_2 = m_1v_{1f} + m_2v_{2f}$

Conservation of momentum: $m_1(v_1 - v_{1f}) = m_2(v_{2f} - v_2)$ **[1]**

Conservation of energy: $\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$

which simplifies to: $m_1(v_1^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_2^2)$

Factoring gives

$$m_1(v_1 - v_{1f})(v_1 + v_{1f}) = m_2(v_{2f} - v_2)\cdot(v_{2f} + v_2)$$

and with the use of the momentum equation (equation [1]),

this reduces to $v_1 + v_{1f} = v_{2f} + v_2$

or $v_{1f} = v_{2f} + v_2 - v_1$ **[2]**

Substituting equation [2] into equation [1] and simplifying yields

$$\boxed{v_{2f} = \frac{2m_1 v_1 + (m_2 - m_1)v_2}{m_1 + m_2}}$$

Upon substitution of this expression for into equation [2], one finds

$$\boxed{v_{1f} = \frac{(m_1 - m_2)v_1 + 2m_2 v_2}{m_1 + m_2}}$$

Observe that these results are the same as two equations given in the chapter text for the situation of a perfectly elastic collision in one dimension. Whatever the details of how the spring behaves, this collision ends up being just such a perfectly elastic collision in one dimension.

P9.76 We hope the momentum of the equipment provides enough recoil so that the astronaut can reach the ship before he loses life support! But can he do it?

Relative to the spacecraft, the astronaut has a momentum $p = (150\text{ kg})(20\text{ m/s}) = 3\,000\text{ kg}\cdot\text{m/s}$ away from the spacecraft. He must throw enough equipment away so that his momentum is reduced to at least zero relative to the spacecraft, so the equipment must have momentum of at least 3 000 kg · m/s relative to the spacecraft. If he throws the equipment at 5.00 m/s relative to himself in a direction away from the spacecraft, the velocity of the equipment will be 25.0 m/s away from the spacecraft. How much mass travelling at 25.0 m/s is necessary to equate to a momentum of 3 000 kg · m/s?

$$p = 3\,000\text{ kg}\cdot\text{m/s} = m(25.0\text{ m/s})$$

which gives

$$m = \frac{3\,000\text{ kg}\cdot\text{m/s}}{25.0\text{ m/s}} = 120\text{ kg}$$

$$\boxed{\text{In order for his motion to reverse under these condition, the final mass of the astronaut and space suit is 30 kg, much less than is reasonable.}}$$

P9.77 Use conservation of mechanical energy for a block-Earth system in which the block slides down a frictionless surface from a height *h*:

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f \to \frac{1}{2}mv^2+0=0+mgh \to v=\sqrt{2gh}$$

Note this also applies in reverse, a mass travelling at speed *v* will climb to a height *h* on a frictionless surface: $h=\frac{v^2}{2g}$.

From above, we see that because each block starts from the same height *h*, each block has the same speed *v* when it meets the other block:

$$v_1=v_2=v=\sqrt{2\left(9.80\text{ m/s}^2\right)\left(5.00\text{ m}\right)}=9.90\text{ m/s}$$

Apply conservation of momentum to the two-block system:

$$m_1v_{1f}+m_2v_{2f}=m_1v+m_2(-v)$$

$$m_1v_{1f}+m_2v_{2f}=\left(m_1-m_2\right)v \qquad \textbf{[1]}$$

For an elastic, head-on collision:

$$v_{1i}-v_{2i}=v_{1f}-v_{2f}$$

$$v-(-v)=v_{2f}-v_{1f}$$

$$v_{2f}=v_{1f}+2v \qquad \textbf{[2]}$$

Substituting equation [2] into [1] gives

$$m_1v_{1f}+m_2\left(v_{1f}+2v\right)=\left(m_1-m_2\right)v$$

$$\left(m_1+m_2\right)v_{1f}=\left(m_1-m_2\right)v-2m_2v$$

$$v_{1f}=\left(\frac{m_1-3m_2}{m_1+m_2}\right)v=\left[\frac{2.00\text{ kg}-3(4.00\text{ kg})}{2.00\text{ kg}+4.00\text{ kg}}\right](9.90\text{ m/s})$$

$$=-16.5\text{ m/s}$$

Using this result and equation [2], we have

$$v_{2f}=v_{1f}+2v=\left(\frac{m_1-3m_2}{m_1+m_2}\right)v+2v$$

$$v_{2f}=\left(\frac{3m_1-m_2}{m_1+m_2}\right)v=\left[\frac{3(2.00\text{ kg})-4.00\text{ kg}}{2.00\text{ kg}+4.00\text{ kg}}\right](9.90\text{ m/s})$$

$$=3.30\text{ m/s}$$

Using our result above, we find the height that each block rises to:

$$h_1 = \frac{v_{1f}^2}{2g} = \frac{(-16.5\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{13.9\ \text{m}}$$

and $$h_2 = \frac{v_{2f}^2}{2g} = \frac{(3.30\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{0.556\ \text{m}}$$

P9.78 (a) Proceeding step by step, we find the stone's speed just before collision, using energy conservation for the stone-Earth system:

$$m_a g y_i = \frac{1}{2} m_a v_i^2$$

which gives

$$v_i = \sqrt{2gh} = [2(9.80\ \text{m/s}^2)(1.80\ \text{m})]^{1/2} = 5.94\ \text{m/s}$$

Now for the elastic collision with the stationary cannonball, we use the specialized Equation 9.22 from the chapter text, with $m_1 = 80.0$ kg and $m_2 = m$:

$$v_{\text{cannonball}} = v_{2f} = \frac{2m_1 v_{1i}}{m_1 + m_2} = \frac{2(80.0\ \text{kg})(5.94\ \text{m/s})}{80.0\ \text{kg} + m}$$
$$= \frac{950\ \text{kg}\cdot\text{m/s}}{80.0\ \text{kg} + m}$$

The time for the cannonball's fall into the ocean is given by

$$\Delta y = v_{yi}t + \frac{1}{2}a_y t^2 \rightarrow -36.0 = \frac{1}{2}(-9.80)t^2 \rightarrow t = 2.71\ \text{s}$$

so its horizontal range is

$$R = v_{2f}t = (2.71\ \text{s})\left(\frac{950\ \text{kg}\cdot\text{m/s}}{80.0\ \text{kg} + m}\right)$$
$$= \boxed{\frac{2.58\times 10^3\ \text{kg}\cdot\text{m}}{80.0\ \text{kg} + m}}$$

(b) The maximum value for *R* occurs for $m \rightarrow 0$, and is

$$R = \frac{2.58\times 10^3\ \text{kg}\cdot\text{m}}{80.0\ \text{kg} + m} = \frac{2.58\times 10^3\ \text{kg}\cdot\text{m}}{80.0\ \text{kg} + 0} = \boxed{32.2\ \text{m}}$$

(c) As indicated in part (b), the maximum range corresponds to $\boxed{m \rightarrow 0}$

(d) Yes, until the cannonball splashes down. No; the kinetic energy of the system is split between the stone and the cannonball after the collision and we don't know how it is split without using the conservation of momentum principle.

(e) The range is equal to the product of $v_{\text{cannonball}}$, the speed of the cannonball after the collision, and t, the time at which the cannonball reaches the ocean. But $v_{\text{cannonball}}$ is proportional to v_i, the speed of the stone just before striking the cannonball, which is, in turn, proportional to the square root of g. The time t at which the cannonball strikes the ocean is inversely proportional to the square root of g. Therefore, the product $R = (v_{\text{cannonball}})t$ is *independent* of g. At a location with weaker gravity, the stone would be moving more slowly before the collision, but the cannonball would follow the same trajectory, moving more slowly over a longer time interval.

P9.79 We will use the subscript 1 for the blue bead and the subscript 2 for the green bead. Conservation of mechanical energy for the blue bead-Earth system, $K_i + U_i = K_f + U_f$, can be written as

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh$$

where v_1 is the speed of the blue bead at point B just before it collides with the green bead. Solving for v_1 gives

$$v_1 = \sqrt{2gh} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(1.50 \text{ m}\right)} = 5.42 \text{ m/s}$$

Now recall Equations 9.21 and 9.22 for an elastic collision:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right)v_{2i}$$

$$v_{2f} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i}$$

For this collision, the green bead is at rest, so $v_{2i} = 0$, and Equation 9.22 simplifies to

$$v_{2f} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{2i} = \left(\frac{2m_2}{m_1 + m_2}\right)v_{1i}$$

Plugging in gives

$$v_{2f} = \left(\frac{2(0.400\ \text{kg})}{0.400\ \text{kg} + 0.600\ \text{kg}}\right)(5.42\ \text{m/s}) = 4.34\ \text{m/s}$$

Now, we use conservation of the mechanical energy of the green bead after collision to find the maximum height the ball will reach. This gives

$$0 + m_2 g\, y_{\max} = \frac{1}{2} m_2 v_{2f}^2 + 0$$

Solving for $y_{\max}$ gives

$$y_{\max} = \frac{v_{2f}^2}{2g} = \frac{(4.34\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{0.960\ \text{m}}$$

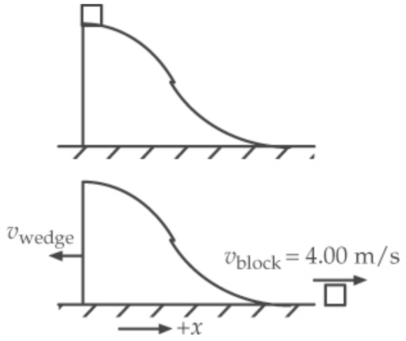
P9.80 (a) The initial momentum of the system is zero, which remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have

$$m_2 v_{\text{wedge}} + m_1 v_{\text{block}} = 0$$

or

$$(3.00\ \text{kg}) v_{\text{wedge}} + (0.500\ \text{kg})(+4.00\ \text{m/s}) = 0$$

so $v_{\text{wedge}} = \boxed{-0.667\ \text{m/s}}$



ANS. FIG. P9.80

(b) Using conservation of energy for the block-wedge-Earth system as the block slides down the smooth (frictionless) wedge, we have

$$\left[K_{\text{block}} + U_{\text{system}}\right]_i + \left[K_{\text{wedge}}\right]_i = \left[K_{\text{block}} + U_{\text{system}}\right]_f + \left[K_{\text{wedge}}\right]_f$$

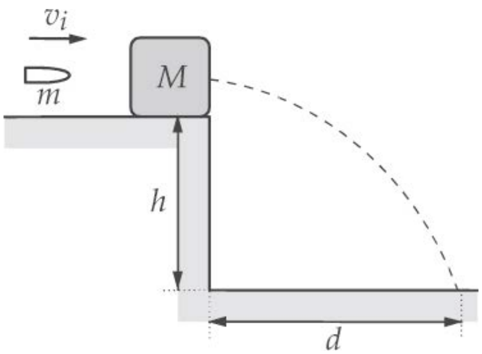
or $$\left[0 + m_1 g h\right] + 0 = \left[\frac{1}{2} m_1 (4.00\ \text{m/s})^2 + 0\right] + \frac{1}{2} m_2 (-0.667\ \text{m/s})^2$$

which gives $\boxed{h = 0.952\ \text{m}}$

***P9.81** Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M+m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad \textbf{[1]}$$



ANS. FIG. P9.81

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus, $$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into [1] from above gives

$$v_i = \left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}} = \left(\frac{250\text{ g} + 8.00\text{ g}}{8.00\text{ g}}\right)\sqrt{\frac{(9.80\text{ m/s}^2)(2.00\text{ m})^2}{2(1.00\text{ m})}}$$
$$= \boxed{143\text{ m/s}}$$

P9.82 Refer to ANS. FIG. P9.81. Using conservation of momentum from just before to just after the impact of the bullet with the block:

$$mv_i = (M+m)v_f$$

or $$v_i = \left(\frac{M+m}{m}\right)v_f \qquad \textbf{[1]}$$

The speed of the block and embedded bullet just after impact may be found using kinematic equations:

$$d = v_f t \quad \text{and} \quad h = \frac{1}{2}gt^2$$

Thus, $$t = \sqrt{\frac{2h}{g}} \quad \text{and} \quad v_f = \frac{d}{t} = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting into [1] from above gives $v_i = \boxed{\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}}$.

P9.83 (a) From conservation of momentum,

$$\vec{\mathbf{p}}_{1i}+\vec{\mathbf{p}}_{2i}=\vec{\mathbf{p}}_{1f}+\vec{\mathbf{p}}_{2f}\to m_1v_{1i}+m_2v_{2i}=m_1v_{1f}+m_2v_{2i}$$

$$\begin{aligned}&(0.500\text{ kg})\left(2.00\hat{\mathbf{i}}-3.00\hat{\mathbf{j}}+1.00\hat{\mathbf{k}}\right)\text{ m/s}\\ &\quad+(1.50\text{ kg})\left(-1.00\hat{\mathbf{i}}+2.00\hat{\mathbf{j}}-3.00\hat{\mathbf{k}}\right)\text{ m/s}\\ &\quad=(0.500\text{ kg})\left(-1.00\hat{\mathbf{i}}+3.00\hat{\mathbf{j}}-8.00\hat{\mathbf{k}}\right)\text{ m/s}\\ &\quad+(1.50\text{ kg})\vec{\mathbf{v}}_{2f}\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{v}}_{2f}&=\left(\frac{1}{1.50\text{ kg}}\right)\Big[\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\\ &\quad+\left(0.500\hat{\mathbf{i}}-1.50\hat{\mathbf{j}}+4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\Big]\\ &=\boxed{0}\end{aligned}$$

The original kinetic energy is

$$\begin{aligned}&\frac{1}{2}(0.500\text{ kg})\left(2^2+3^2+1^2\right)\text{ m}^2/\text{s}^2\\ &\quad+\frac{1}{2}(1.50\text{ kg})\left(1^2+2^2+3^2\right)\text{m}^2/\text{s}^2=14.0\text{ J}\end{aligned}$$

The final kinetic energy is

$$\frac{1}{2}(0.500\text{ kg})\left(1^2+3^2+8^2\right)\text{ m}^2/\text{s}^2+0=18.5\text{ J}$$

different from the original energy so the collision is $\boxed{\text{inelastic}}$.

(b) We follow the same steps as in part (a):

$$\begin{aligned}&\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\text{ kg}\cdot\text{m/s}\\ &\quad=(0.500\text{ kg})\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{ m/s}\\ &\quad+(1.50\text{ kg})\vec{\mathbf{v}}_{2f}\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{v}}_{2f}&=\left(\frac{1}{1.50\text{ kg}}\right)\left(-0.5\hat{\mathbf{i}}+1.5\hat{\mathbf{j}}-4\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m/s}\\ &\quad+\left(0.125\hat{\mathbf{i}}-0.375\hat{\mathbf{j}}+1\hat{\mathbf{k}}\right)\text{kg}\cdot\text{m/s}\\ &=\boxed{\left(-0.250\hat{\mathbf{i}}+0.750\hat{\mathbf{j}}-2.00\hat{\mathbf{k}}\right)\text{m/s}}\end{aligned}$$

We see $\vec{\mathbf{v}}_{2f}=\vec{\mathbf{v}}_{1f}$ so the collision is $\boxed{\text{perfectly inelastic}}$.

(c) Again, from conservation of momentum,

$$\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}$$

$$=\left(0.500\ \text{kg}\right)\left(-1\hat{\mathbf{i}}+3\hat{\mathbf{j}}+a\hat{\mathbf{k}}\right)\text{m/s}+\left(1.50\ \text{kg}\right)\vec{\mathbf{v}}_{2\text{f}}$$

$$\vec{\mathbf{v}}_{2\text{f}}=\left(\frac{1}{1.50\ \text{kg}}\right)\left[\left(-0.500\hat{\mathbf{i}}+1.50\hat{\mathbf{j}}-4.00\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}\right.$$

$$\left.+\left(0.500\hat{\mathbf{i}}-1.50\hat{\mathbf{j}}-0.500a\hat{\mathbf{k}}\right)\ \text{kg}\cdot\text{m/s}\right]$$

$$=\boxed{(-2.67-0.333a)\hat{\mathbf{k}}\ \text{m/s}}$$

Then, from conservation of energy:

$$14.0\ \text{J}=\frac{1}{2}\left(0.500\ \text{kg}\right)\left(1^2+3^2+a^2\right)\ \text{m}^2/\text{s}^2$$

$$+\frac{1}{2}\left(1.50\ \text{kg}\right)(2.67+0.333a)^2\ \text{m}^2/\text{s}^2$$

$$=2.50\ \text{J}+0.250a^2+5.33\ \text{J}+1.33a+0.0833a^2$$

This gives, suppressing units, a quadratic equation in a,

$$0=0.333a^2+1.33a-6.167=0$$

which solves to give

$$a=\frac{-1.33\pm\sqrt{1.33^2-4(0.333)(-6.167)}}{0.667}$$

With $\boxed{a=2.74}$,

$$\vec{\mathbf{v}}_{2\text{f}}=\left(-2.67-0.333(2.74)\right)\hat{\mathbf{k}}\ \text{m/s}=\boxed{-3.58\hat{\mathbf{k}}\ \text{m/s}}$$

With $\boxed{a=-6.74}$,

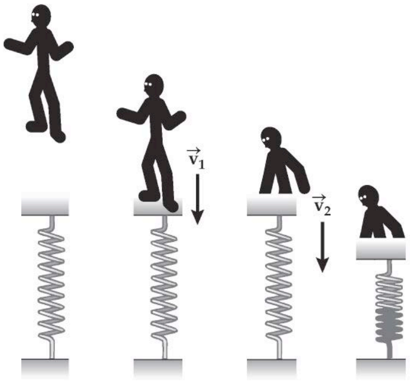
$$\vec{\mathbf{v}}_{2\text{f}}=\left(-2.67-0.333(-6.74)\right)\hat{\mathbf{k}}\ \text{m/s}=\boxed{-0.419\hat{\mathbf{k}}\ \text{m/s}}$$

P9.84 Consider the motion of the firefighter during the three intervals: (1) before, (2) during, and (3) after collision with the platform.

(a) While falling a height of 4.00 m, her speed changes from $v_i=0$ to v_1 as found from

$$\Delta E=\left(K_f+U_f\right)-\left(K_i-U_i\right)$$

$$K_f=\Delta E-U_f+K_i+U_i$$



ANS FIG. P9.84

When the initial position of the platform is taken as the zero level of gravitational potential, we have

$$\frac{1}{2}mv_1^2 = fh\cos(180°) - 0 + 0 + mgh$$

Solving for v_1 gives

$$v_1 = \sqrt{\frac{2(-fh + mgh)}{m}}$$

$$= \sqrt{\frac{2\left[-(300\text{ N})(4.00\text{ m}) + (75.0\text{ kg})(9.80\text{ m/s}^2)(4.00\text{ m})\right]}{75.0\text{ kg}}}$$

$$= \boxed{6.81\text{ m/s}}$$

(b) During the inelastic collision, momentum of the firefighter-platform system is conserved; and if v_2 is the speed of the firefighter and platform just after collision, we have $mv_1 = (m + M)v_2$, or

$$v_2 = \frac{m_1 v_1}{m + M} = \frac{(75.0\text{ kg})(6.81\text{ m/s})}{75.0\text{ kg} + 20.0\text{ kg}} = 5.38\text{ m/s}$$

Following the collision and again solving for the work done by nonconservative forces, using the distances as labeled in the figure, we have (with the zero level of gravitational potential at the initial position of the platform)

$$\Delta E = K_f + U_{fg} + U_{fs} - K_i - U_{ig} - U_{is}$$

or $$-fs = 0 + (m + M)g(-s) + \frac{1}{2}ks^2 - \frac{1}{2}(m + M)v^2 - 0 - 0$$

This results in a quadratic equation in s:

$$2\,000s^2 - (931)s + 300s - 1\,375 = 0$$

with solution $\boxed{s = 1.00 \text{ m}}$

P9.85 Each primate swings down according to

$$mgR = \frac{1}{2}mv_1^2 \text{ and } MgR = \frac{1}{2}Mv_1^2 \quad \rightarrow \quad v_1 = \sqrt{2gR}$$

For the collision,

$$-mv_1 + Mv_1 = +(m + M)v_2$$

$$v_2 = \frac{M - m}{M + m}v_1$$

While the primates are swinging up,

$$\frac{1}{2}(M + m)v_2^2 = (M + m)gR(1 - \cos 35°)$$

$$v_2 = \sqrt{2gR(1 - \cos 35.0°)}$$

$$\sqrt{2gR(1 - \cos 35.0°)}(M + m) = (M - m)\sqrt{2gR}$$

$$0.425M + 0.425m = M - m$$

$$1.425m = 0.575M$$

which gives

$$\boxed{\frac{m}{M} = 0.403}$$

P9.86 (a) We can obtain the initial speed of the projectile by utilizing conservation of momentum:

$$m_1v_{1A} + 0 = (m_1 + m_2)v_B$$

Solving for v_{1A} gives

$$v_{1A} = \frac{m_1 + m_2}{m_1}\sqrt{2gh}$$

$$v_{1A} \cong \boxed{6.29 \text{ m/s}}$$

(b) We begin with the kinematic equations in the x and y direction:

$$x = x_0 + v_{x0}t + \frac{1}{2}a_xt^2$$

$$y = y_0 + v_{y0}t + \frac{1}{2}a_yt^2$$

And simplify by plugging in $x_0 = y_0 = 0$, $v_{y0} = 0$, $v_{x0} = v_{1A}$, $a_x = 0$, and $a_y = g$:

$$\frac{1}{2}gt^2 = y \text{ and } x = v_{1At}$$

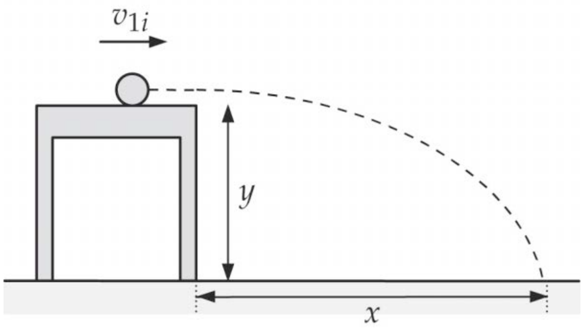
Combining them gives

$$v_{1A} = \frac{x}{\sqrt{2y/g}} = x\sqrt{\frac{g}{2y}}$$

Substituting the numerical values from the problem statement gives

$$v_{1A} = x\sqrt{\frac{g}{2y}} = (2.57\text{ m})\sqrt{\frac{9.80\text{ m/s}^2}{2(0.853\text{ m})}} = \boxed{6.16\text{ m/s}}$$

(c) $\boxed{\text{Most of the 2\% difference between the values for speed could be accounted for by air resistance.}}$



ANS. FIG. P9.86

P9.87 The force exerted by the spring on each block is in magnitude.

$$|F_s| = kx = (3.85\text{ N/m})(0.08\text{ m}) = 0.308\text{ N}$$

(a) With no friction, the elastic energy in the spring becomes kinetic energy of the blocks, which have momenta of equal magnitude in opposite directions. The blocks move with constant speed after they leave the spring. From conservation of energy,

$$(K+U)_i = (K+U)_f$$

$$\frac{1}{2}kx^2 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2$$

$$\frac{1}{2}(3.85\text{ N/m})(0.080\ 0\text{ m})^2$$
$$= \frac{1}{2}(0.250\text{ kg})v_{1f}^2 + \frac{1}{2}(0.500\text{ kg})v_{2f}^2 \qquad \textbf{[1]}$$

And from conservation of linear momentum,

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 = (0.250\text{ kg})v_{1f}(-\hat{\mathbf{i}}) + (0.500\text{ kg})v_{2f}\hat{\mathbf{i}}$$

$$v_{1f} = 2v_{2f}$$

Substituting this into [1] gives

$$0.012\ 3\text{ J} = \frac{1}{2}(0.250\text{ kg})\left(2v_{2f}\right)^2 + \frac{1}{2}(0.500\text{ kg})v_{2f}^2$$
$$= \frac{1}{2}(1.50\text{ kg})v_{2f}^2$$

Solving,

$$v_{2f} = \left(\frac{0.012\ 3\text{ J}}{0.750\text{ kg}}\right)^{1/2} = 0.128\text{ m/s} \qquad \boxed{\vec{\mathbf{v}}_{2f} = 0.128\hat{\mathbf{i}}\text{ m/s}}$$

$$v_{1f} = 2(0.128\text{ m/s}) = 0.256\text{ m/s} \qquad \boxed{\vec{\mathbf{v}}_{1f} = -0.256\hat{\mathbf{i}}\text{ m/s}}$$

(b) For the lighter block,

$$\sum F_y = ma_y, \quad n - 0.250\text{ kg}\left(9.80\text{ m/s}^2\right) = 0, \quad n = 2.45\text{ N},$$

$$f_k = \mu_k n = 0.1\left(2.45\text{ N}\right) = 0.245\text{ N}.$$

We assume that the maximum force of static friction is a similar size. Since 0.308 N is larger than 0.245 N, this block moves. For the heavier block, the normal force and the frictional force are twice as large: f_k = 0.490 N. Since 0.308 N is less than this, the heavier block stands still. In this case, the frictional forces exerted by the floor change the momentum of the two-block system. The lighter block will gain speed as long as the spring force is larger than the friction force: that is until the spring compression becomes x_f given by

$$\left|F_s\right| = kx, \ 0.245\text{ N} = (3.85\text{ N}\cdot\text{m})x_f, \ 0.063\ 6\text{ m} = x_f$$

Now for the energy of the lighter block as it moves to this maximum-speed point, we have

$$K_i + U_i - f_k d = K_f + U_f$$

$$0 + 0.012\,3\text{ J} - (0.245\text{ N})(0.08 - 0.063\,6\text{ m}) = \frac{1}{2}(0.250\text{ kg})v_f^2 + \frac{1}{2}(3.85\text{ N/m})(0.063\,6\text{ m})^2$$

$$0.012\,3\text{ J} - 0.004\,01\text{ J} = \frac{1}{2}(0.250\text{ kg})v_f^2 + 0.007\,80\text{ J}$$

$$\left(\frac{2(0.000\,515\text{ J})}{0.250\text{ kg}}\right)^{1/2} = v_f = 0.064\,2\text{ m/s}$$

Thus for the heavier block the maximum velocity is $\boxed{0}$ and for the lighter block, $\boxed{-0.064\,2\hat{\mathbf{i}}\text{ m/s}}$.

(c) For the lighter block, $f_k = 0.462(2.45\text{ N}) = 1.13\text{ N}$. The force of static friction must be at least as large. The 0.308-N spring force is too small to produce motion of either block. Each has $\boxed{0}$ maximum speed.

P9.88 The orbital speed of the Earth is

$$v_E = \frac{2\pi r}{T} = \frac{2\pi(1.496\times10^{11}\text{ m})}{3.156\times10^{7}\text{ s}} = 2.98\times10^{4}\text{ m/s}$$

In six months the Earth reverses its direction, to undergo momentum change

$$m_E|\Delta\vec{\mathbf{v}}_E| = 2m_Ev_E = 2(5.98\times10^{24}\text{ kg})(2.98\times10^{4}\text{ m/s}) = 3.56\times10^{29}\text{ kg}\cdot\text{m/s}$$

Relative to the center of mass, the Sun always has momentum of the same magnitude in the opposite direction. Its 6-month momentum change is the same size, $m_S|\Delta\vec{\mathbf{v}}_S| = 3.56\times10^{29}\text{ kg}\cdot\text{m/s}$

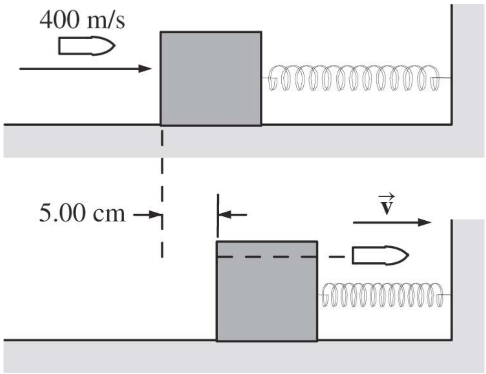
Then $$|\Delta\vec{\mathbf{v}}_S| = \frac{3.56\times10^{29}\text{ kg}\cdot\text{m/s}}{1.991\times10^{30}\text{ kg}} = \boxed{0.179\text{ m/s}}$$

ANS. FIG. P9.88

P9.89 (a) We find the speed when the bullet emerges from the block by using momentum conservation:

$$mv_i = MV_i + mv$$

The block moves a distance of 5.00 cm. Assume for an approximation that the block quickly reaches its maximum velocity, V_i, and the bullet kept going with a constant velocity, v. The block then compresses the spring and stops. After the collision, the mechanical energy is conserved in the block-spring system:



ANS. FIG. P9.89

$$\frac{1}{2}MV_i^2 = \frac{1}{2}kx^2$$

$$V_i = \sqrt{\frac{(900\text{ N/m})(5.00\times10^{-2}\text{ m})^2}{1.00\text{ kg}}} = 1.50\text{ m/s}$$

$$v = \frac{mv_i - MV_i}{m}$$

$$= \frac{(5.00\times10^{-3}\text{ kg})(400\text{ m/s}) - (1.00\text{ kg})(1.50\text{ m/s})}{5.00\times10^{-3}\text{ kg}}$$

$$v = \boxed{100\text{ m/s}}$$

(b) Identifying the system as the block and the bullet and the time interval from just before the collision to just after the collision,

$$\Delta K + \Delta E_{\text{int}} = 0 \quad \text{gives}$$

$$\Delta E_{\text{int}} = -\Delta K = -\left(\frac{1}{2}mv^2 + \frac{1}{2}MV_i^2 - \frac{1}{2}mv_i^2\right)$$

Then

$$\Delta E_{\text{int}} = -\left[\frac{1}{2}(0.005\,00\text{ kg})(100\text{ m/s})^2 + \frac{1}{2}(1.00\text{ kg})(1.50\text{ m/s})^2 - \frac{1}{2}(0.005\,00\text{ kg})(400\text{ m/s})^2\right]$$

$$= \boxed{374\text{ J}}$$

P9.90 (a) We have, from the impulse-momentum theorem, $\vec{\mathbf{p}}_i + \vec{\mathbf{F}}t = \vec{\mathbf{p}}_f$:

$$(3.00\text{ kg})(7.00\text{ m/s})\hat{\mathbf{j}} + \left(12.0\hat{\mathbf{i}}\text{ N}\right)(5.00\text{ s}) = (3.00\text{ kg})\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \boxed{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m/s}}$$

(b) The particle's acceleration is

$$\vec{\mathbf{a}} = \frac{\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i}{t} = \frac{\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}} - 7.00\hat{\mathbf{j}}\right)\text{ m/s}}{5.00\text{ s}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$$

(c) From Newton's second law,

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m} = \frac{12.0\hat{\mathbf{i}}\text{ N}}{3.00\text{ kg}} = \boxed{4.00\hat{\mathbf{i}}\text{ m/s}^2}$$

(d) The vector displacement of the particle is

$$\Delta\vec{\mathbf{r}} = \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$= \left(7.00\text{ m/s}\,\hat{\mathbf{j}}\right)(5.00\text{ s}) + \frac{1}{2}\left(4.00\text{ m/s}^2\hat{\mathbf{i}}\right)(5.00\text{ s})^2$$

$$\Delta\vec{\mathbf{r}} = \boxed{\left(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}}\right)\text{ m}}$$

(e) Now, from the work-kinetic energy theorem, the work done on the particle is

$$W = \vec{\mathbf{F}}\cdot\Delta\vec{\mathbf{r}} = \left(12.0\hat{\mathbf{i}}\text{ N}\right)\left(50.0\hat{\mathbf{i}}\text{ m} + 35.0\hat{\mathbf{j}}\text{ m}\right) = \boxed{600\text{ J}}$$

(f) The final kinetic energy of the particle is

$$\frac{1}{2}mv_f^2 = \frac{1}{2}(3.00\text{ kg})\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\cdot\left(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}}\right)\text{ m}^2/\text{s}^2$$

$$\frac{1}{2}mv_f^2 = (1.50\text{ kg})\left(449\text{ m}^2/\text{s}^2\right) = \boxed{674\text{ J}}$$

(g) The final kinetic energy of the particle is

$$\frac{1}{2}mv_i^2 + W = \frac{1}{2}(3.00\text{ kg})(7.00\text{ m/s})^2 + 600\text{ J} = \boxed{674\text{ J}}$$

(h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

P9.91 We note that the initial velocity of the target particle is zero (that is, $v_{2i} = 0$). Then, from conservation of momentum,

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0 \qquad \textbf{[1]}$$

For head-on elastic collisions, $v_{1i} - v_{2i} = (v_{1f} - v_{2f})$, and with $v_{2i} = 0$, this gives

$$v_{2f} = v_{1i} + v_{1f} \qquad \textbf{[2]}$$

Substituting equation [2] into [1] yields

$$m_1 v_{1f} + m_2 (v_{1i} + v_{1f}) = m_1 v_{1i}$$

or

$$(m_1 + m_2) v_{1f} = (m_1 - m_2) v_{1i}$$

which gives

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} \qquad \textbf{[3]}$$

Now, we substitute equation [3] into [2] to obtain

$$v_{2f} = v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} \qquad \textbf{[4]}$$

Equations [3] and [4] can now be used to answer both parts (a) and (b).

(a) If $m_1 = 2.00\text{ g}, m_2 = 1.00\text{ g}$, and $v_{1i} = 8.00\text{ m/s}$, then

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} = \left(\frac{2.00\text{ g} - 1.00\text{ g}}{2.00\text{ g} + 1.00\text{ g}} \right) (8.00\text{ m/s}) = \boxed{2.67\text{ m/s}}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} = \left[\frac{2(2.00\text{ g})}{2.00\text{ g} + 1.00\text{ g}} \right] (8.00\text{ m/s}) = \boxed{10.7\text{ m/s}}$$

(b) If $m_1 = 2.00\text{ g}, m_2 = 10.0\text{ g}$, and $v_{1i} = 8.00\text{ m/s}$, we find

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2} \right) v_{1i} = \left(\frac{2.00\text{ g} - 10.0\text{ g}}{2.00\text{ g} + 10.0\text{ g}} \right) (8.00\text{ m/s}) = \boxed{5.33\text{ m/s}}$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{1i} = \left[\frac{2(2.00\text{ g})}{2.00\text{ g} + 10.0\text{ g}} \right] (8.00\text{ m/s}) = \boxed{2.67\text{ m/s}}$$

(c) The final kinetic energy of the 2.00-g particle in each case is:

Case (a):

$$\mathrm{KE}_{1\,\mathrm{f}} = \frac{1}{2}m_1 v_{1\,\mathrm{f}}^2 = \frac{1}{2}\left(2.00\times10^{-3}\ \mathrm{kg}\right)\left(2.67\ \mathrm{m/s}\right)^2 = \boxed{7.11\times10^{-3}\ \mathrm{J}}$$

Case (b):

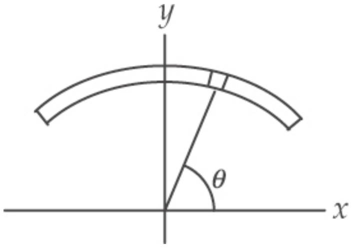
$$\mathrm{KE}_{1\,\mathrm{f}} = \frac{1}{2}m_1 v_{1\,\mathrm{f}}^2 = \frac{1}{2}\left(2.00\times10^{-3}\ \mathrm{kg}\right)\left(5.33\ \mathrm{m/s}\right)^2 = \boxed{2.84\times10^{-2}\ \mathrm{J}}$$

Since the incident kinetic energy is the same in cases (a) and (b), we observe that

$$\boxed{\text{the incident particle loses more kinetic energy in case (a), in which the target mass is 1.00 g.}}$$

Challenge Problems

P9.92 Take the origin at the center of curvature. We have $L = \frac{1}{4}2\pi r,\ r = \frac{2L}{\pi}$. An incremental bit of the rod at angle θ from the x axis has mass given by $\frac{dm}{r d\theta} = \frac{m}{L},\ dm = \frac{mr}{L}d\theta,$ where we have used the definition of radian measure. Now



ANS. FIG. P9.92

$$y_{\mathrm{CM}} = \frac{1}{M}\int_{\text{all mass}} y\,dm = \frac{1}{M}\int_{\theta=45^\circ}^{135^\circ} r\sin\theta\frac{Mr}{L}d\theta = \frac{r^2}{L}\int_{45^\circ}^{135^\circ}\sin\theta\,d\theta$$

$$= \left(\frac{2L}{\pi}\right)^2\frac{1}{L}(-\cos\theta)\Big|_{45^\circ}^{135^\circ} = \frac{4L}{\pi^2}\left(\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right) = \frac{4\sqrt{2}L}{\pi^2}$$

The top of the bar is above the origin by $r = \frac{2L}{\pi}$, so the center of mass is below the middle of the bar by

$$\frac{2L}{\pi} - \frac{4\sqrt{2}L}{\pi^2} = \frac{2}{\pi}\left(1-\frac{2\sqrt{2}}{\pi}\right)L = \boxed{0.063\ 5\ L}$$

P9.93 The x component of momentum for the system of the two objects is

$$p_{1ix} + p_{2ix} = p_{1fx} + p_{2fx}$$

$$-mv_i + 3mv_i = 0 + 3mv_{2x}$$

The y component of momentum of the system is

$$0 + 0 = -mv_{1y} + 3mv_{2y}$$

By conservation of energy of the system,

$$+\frac{1}{2}mv_i^2 + \frac{1}{2}3mv_i^2 = \frac{1}{2}mv_{1y}^2 + \frac{1}{2}3m\left(v_{2x}^2 + v_{2y}^2\right)$$

we have $v_{2x} = \dfrac{2v_i}{3}$

also $v_{1y} = 3v_{2y}$

So the energy equation becomes

$$4v_i^2 = 9v_{2y}^2 + \frac{4v_i^2}{3} + 3v_{2y}^2$$

$$\frac{8v_i^2}{3} = 12v_{2y}^2$$

or $v_{2y} = \dfrac{\sqrt{2}v_i}{3}$

(a) The object of mass m has final speed

$$v_{1y} = 3v_{2y} = \boxed{\sqrt{2}v_i}$$

and the object of mass $3m$ moves at

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4v_i^2}{9} + \frac{2v_i^2}{9}}$$

$$\sqrt{v_{2x}^2 + v_{2y}^2} = \boxed{\sqrt{\frac{2}{3}}v_i}$$

(b) $\theta = \tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right)$

$$\theta = \tan^{-1}\left(\frac{\sqrt{2}v_i}{3}\frac{3}{2v_i}\right) = \boxed{35.3°}$$

P9.94 A picture one second later differs by showing five extra kilograms of sand moving on the belt.

(a) $\frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} = (0.750\ \text{m/s})(5.00\ \text{kg/s}) = \boxed{3.75\ \text{N}}$

(b) The only horizontal force on the sand is belt friction, which causes the momentum of the sand to change: $F = \frac{dp}{dt} = \boxed{3.75\ \text{N}}$ as above.

(c) The belt is in equilibrium:

$$\sum F_x = ma_x: \quad +F_{\text{ext}} - f = 0 \quad \text{and} \quad F_{\text{ext}} = \boxed{3.75\ \text{N}}$$

(d) $W = F\Delta r\cos\theta = (3.75\ \text{N})(0.750\ \text{m})\cos 0° = \boxed{2.81\ \text{J}}$

(e) $\frac{dK}{dt} = \frac{d\left(\frac{1}{2}mv^2\right)}{dt} = \frac{1}{2}v^2\frac{dm}{dt} = \frac{1}{2}(0.750\ \text{m/s})^2(5.00\ \text{kg/s}) = \boxed{1.41\ \text{J/s}}$

(f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

P9.95 Depending on the length of the cord and the time interval Δt for which the force is applied, the sphere may have moved very little when the force is removed, or we may have x_1 and x_2 nearly equal, or the sphere may have swung back, or it may have swung back and forth several times. Our solution applies equally to all of these cases.

(a) The applied force is constant, so the center of mass of the glider-sphere system moves with constant acceleration. It starts, we define, from $x = 0$ and moves to $(x_1 + x_2)/2$. Let v_1 and v_2 represent the horizontal components of velocity of glider and sphere at the moment the force stops. Then the velocity of the center of mass is $v_{CM} = (v_1 + v_2)/2$, and because the acceleration is constant we have

$$\frac{x_1 + x_2}{2} = \left(\frac{v_1 + v_2}{2}\right)\left(\frac{\Delta t}{2}\right)$$

which gives

$$\Delta t = 2\left(\frac{x_1 + x_2}{v_1 + v_2}\right)$$

The impulse-momentum theorem for the glider-sphere system is

$$F\Delta t = mv_1 + mv_2$$

or

$$2F\left(\frac{x_1 + x_2}{v_1 + v_2}\right) = m(v_1 + v_2)$$

$$2F(x_1 + x_2) = m(v_1 + v_2)^2$$

Dividing both sides by 4m and rearranging gives

$$\frac{2F(x_1 + x_2)}{4m} = \frac{m(v_1 + v_2)^2}{4m}$$

$$\frac{F(x_1 + x_2)}{2m} = \frac{(v_1 + v_2)^2}{4} = v_{\text{CM}}^2$$

or

$$v_{\text{CM}} = \boxed{\sqrt{\frac{F(x_1 + x_2)}{2m}}}$$

(b) The applied force does work that becomes, after the force is removed, kinetic energy of the constant-velocity center-of-mass motion plus kinetic energy of the vibration of the glider and sphere relative to their center of mass. The applied force acts only on the glider, so the work-energy theorem for the pushing process is

$$Fx_1 = \frac{1}{2}(2m)v_{\text{CM}}^2 + E_{\text{vib}}$$

Substitution gives

$$Fx_1 = \frac{1}{2}(2m)\left[\frac{F(x_1 + x_2)}{2m}\right] + E_{\text{vib}} = \frac{1}{2}Fx_1 + \frac{1}{2}Fx_2 + E_{\text{vib}}$$

Then,

$$E_{\text{vib}} = \frac{1}{2}Fx_1 - \frac{1}{2}Fx_2$$

When the cord makes its largest angle with the vertical, the vibrational motion is turning around. No kinetic energy is associated with the vibration at this moment, but only gravitational energy:

$$mgL(1 - \cos\theta) = F(x_1 - x_2)/2$$

Solving gives

$$\boxed{\theta = \cos^{-1}[1 - F(x_1 - x_2)/2mgL]}$$

P9.96 The force exerted by the table is equal to the change in momentum of each of the links in the chain. By the calculus chain rule of derivatives,

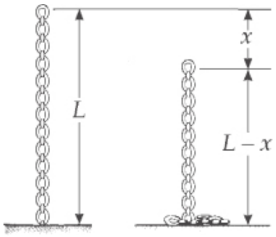
$$F_1 = \frac{dp}{dt} = \frac{d(mv)}{dt} = v\frac{dm}{dt} + m\frac{dv}{dt}$$

We choose to account for the change in momentum of each link by having it pass from our area of interest just before it hits the table, so that

$$v\frac{dm}{dt} \neq 0 \text{ and } m\frac{dv}{dt} = 0$$

Since the mass per unit length is uniform, we can express each link of length *dx* as having a mass *dm*:

$$dm = \frac{M}{L}dx$$



ANS. FIG. P9.96

The magnitude of the force on the falling chain is the force that will be necessary to stop each of the elements *dm*.

$$F_1 = v\frac{dm}{dt} = v\left(\frac{M}{L}\right)\frac{dx}{dt} = \left(\frac{M}{L}\right)v^2$$

After falling a distance x, the square of the velocity of each link $v^2 = 2gx$ (from kinematics), hence

$$F_1 = \frac{2Mgx}{L}$$

The links already on the table have a total length x, and their weight is supported by a force F_2:

$$F_2 = \frac{Mgx}{L}$$

Hence, the *total* force on the chain is

$$F_{\text{total}} = F_1 + F_2 = \boxed{\frac{3Mgx}{L}}$$

That is, *the total force is three times the weight of the chain on the table at that instant.*

ANSWERS TO EVEN-NUMBERED PROBLEMS

P9.2	1.14 kg; 22.0 m/s
P9.4	(a) $p_x = 9.00\ \text{kg}\cdot\text{m/s}$, $p_y = -12.0\ \text{kg}\cdot\text{m/s}$; (b) 15.0 kg · m/s
P9.6	(a) $v_{pi} = -0.346\ \text{m/s}$; (b) $v_{gi} = 1.15\ \text{m/s}$
P9.8	(a) 4.71 m/s East; (b) 717 J
P9.10	10^{-23} m/s
P9.12	(a) 3.22×10^3 N, 720 lb; (b) not valid; (c) These devices are essential for the safety of small children.
P9.14	(a) $\Delta\vec{\mathbf{p}} = 3.38\ \text{kg}\cdot\text{m/s}\ \hat{\mathbf{j}}$; (b) $\vec{\mathbf{F}} = 7 \times 10^2\ \text{N}\hat{\mathbf{j}}$
P9.16	(a) $\left(9.05\hat{\mathbf{i}} + 6.12\hat{\mathbf{j}}\right)\ \text{N}\cdot\text{s}$; (b) $\left(377\hat{\mathbf{i}} + 255\hat{\mathbf{j}}\right)\ \text{N}$
P9.18	(a) $3.60\hat{\mathbf{i}}\ \text{N}\cdot\text{s}$ away from the racket; (b) −36.0 **J**
P9.20	(a) $981\ \text{N}\cdot\text{s}$, up; (b) 3.43 m/s, down; (c) 3.83 m/s, up; (d) 0.748 m
P9.22	(a) 20.9 m/s East; (b) −8.68 × 103 J; (c) Most of the energy was transformed to internal energy with some being carried away by sound.
P9.24	(a) $v_f = \dfrac{1}{3}\left(v_1 + 2v_2\right)$; (b) $\Delta K = -\dfrac{m}{3}\left(v_1^2 + v_2^2 - 2v_1v_2\right)$
P9.26	(a) 2.50 m/s; (b) 37.5 kJ; (c) The event considered in this problem is the time reversal of the perfectly inelastic collision in Problem 9.25. The same momentum conservation equation describes both processes.
P9.28	7.94 cm
P9.30	$v = \dfrac{4M}{m}\sqrt{g\ell}$
P9.32	$v_c = \dfrac{\left(m + M\right)}{m}\sqrt{2\mu gd}$
P9.34	(a) 2.24 m/s toward the right; (b) No. Coupling order makes no difference to the final velocity.
P9.36	The driver of the northbound car was untruthful. His original speed was more than 35 mi/h.
P9.38	$v_O = 3.99\ \text{m/s}$ and $v_Y = 3.01\ \text{m/s}$

P9.40 $v = \frac{v_i}{\sqrt{2}}$, 45.0°, –45.0°

P9.42 The opponent grabs the fullback and does not let go, so the two players move together at the end of their interaction; (b) $\theta = 32.3°$, 2.88 m/s; (c) 786 J into internal energy

P9.44 $v_B = 5.89$ m/s; $v_G = 7.07$ m/s

P9.46 4.67×10^6 m from the Earth's center

P9.48 11.7 cm; 13.3 cm

P9.50 The center of mass of the molecule lies on the dotted line shown in ANS. FIG. P9.50, 0.006 73 nm below the center of the O atom.

P9.52 (a) See ANS. FIG. P8.42; (b) $\left(-2.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)$ m; (c) $\left(3.00\hat{\mathbf{i}} - 1.00\hat{\mathbf{j}}\right)$ m/s; (d) $\left(15.0\hat{\mathbf{i}} - 5.00\hat{\mathbf{j}}\right)$ kg · m/s

P9.54 (a) $\left(-2.89\hat{\mathbf{i}} - 1.39\hat{\mathbf{j}}\right)$cm; (b) $\left(-44.5\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}}\right)$g · cm/s; (c) $\left(-4.94\hat{\mathbf{i}} + 1.39\hat{\mathbf{j}}\right)$cm/s; (d) $\left(-2.44\hat{\mathbf{i}} + 1.56\hat{\mathbf{j}}\right)$cm/s^2; (e) $\left(-220\hat{\mathbf{i}} + 140\hat{\mathbf{j}}\right)\mu$N

P9.56 (a) Yes. $18.0\hat{\mathbf{i}}$ kg · m/s; (b) No. The friction force exerted by the floor on each stationary bit of caterpillar tread acts over no distance, so it does zero work; (c) Yes, we could say that the final momentum of the card came from the floor or from the Earth through the floor; (d) No. The kinetic energy came from the original gravitational potential energy of the Earth-elevated load system, in the amount 27.0 J; (e) Yes. The acceleration is caused by the static friction force exerted by the floor that prevents the wheels from slipping backward.

P9.58 (a) yes; (b) no; (c) 103 kg·m/s, up; (d) yes; (e) 88.2 J; (f) no, the energy came from chemical energy in the person's leg muscles

P9.60 (a) 787 m/s; (b) 138 m/s

P9.62 (a) 3.90×10^7 N; (b) 3.20 m/s^2

P9.64 (a) $-v_e \ln\left(1 - \frac{t}{T_p}\right)$; (b) See ANS. FIG. P9.64(b); (c) $\frac{v_e}{T_p - t}$; (d) See ANS. FIG. P9.64(d); (e) $v_e\left(T_p - t\right)\ln\left(1 - \frac{t}{T_p}\right) + v_e t$; (f) See ANS. FIG. P9.64(f)

P9.66 (a) $-\left(\frac{m}{M-m}\right)\vec{\mathbf{v}}_{\text{gloves}}$; (b) As she throws the gloves and exerts a force on them, the gloves exert an equal and opposite force on her that causes her to accelerate from rest to reach the velocity $\vec{\mathbf{v}}_{\text{girl}}$.

P9.68 (a) $K_E/K_A = m_1/(m_1 + m_2)$; (b) 1.00; (c) See P9.68(c) for argument.

P9.70 (a) –3.54 m/s; (b) 1.77 m; (c) 3.54×10^4 N; (d) No

P9.72 (a) See P9.72(a) for description; (b) $v_i = \frac{m+M}{m}\sqrt{2gh}$

P9.74 (a) See P9.74 for complete statement; (b) The final velocity of the seat is $-0.055\,6\hat{\mathbf{i}}$ m/s. That of the sleigh is $7.94\hat{\mathbf{i}}$ m/s; (c) −453 J

P9.76 In order for his motion to reverse under these conditions, the final mass of the astronaut and space suit is 30 kg, much less than is reasonable.

P9.78 (a) $2.58 \times 10^3\ \text{kg} \cdot \text{m}/(80\ \text{kg} + m)$; (b) 32.2 m; (c) $m \to 0$; (d) See P9.78(d) for complete answer; (e) See P9.78(e) for complete answer.

P9.80 (a) –0.667 m/s; (b) h = 0.952 m

P9.82 $\left(\frac{M+m}{m}\right)\sqrt{\frac{gd^2}{2h}}$

P9.84 (a) 6.81 m/s; (b) s = 1.00 m

P9.86 (a) 6.29 m/s; (b) 6.16 m/s; (c) Most of the 2% difference between the values for speed could be accounted for by air resistance.

P9.88 0.179 m/s

P9.90 (a) $(20.0\hat{\mathbf{i}} + 7.00\hat{\mathbf{j}})\ \text{m/s}$; (b) $4.00\hat{\mathbf{i}}\ \text{m/s}^2$; (c) $4.00\hat{\mathbf{i}}\ \text{m/s}^2$;

(d) $(50.0\hat{\mathbf{i}} + 35.0\hat{\mathbf{j}})\ \text{m}$; (e) 600 J; (f) 674 J; (g) 674 J; (h) The accelerations computed in different ways agree. The kinetic energies computed in different ways agree. The three theories are consistent.

P9.92 0.063 5L

P9.94 (a) 3.75 N; (b) 3.75 N; (c) 3.75 N; (d) 2.81 J; (e) 1.41 J/s; (f) One-half of the work input becomes kinetic energy of the moving sand and the other half becomes additional internal energy. The internal energy appears when the sand does not elastically bounce under the hopper, but has friction eliminate its horizontal motion relative to the belt. By contrast, all of the impulse input becomes momentum of the moving sand.

P9.96 $\frac{3Mgx}{L}$

10

Rotation of a Rigid Object About a Fixed Axis

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ10.1 Answer (c). The wheel has a radius of 0.500 m and made 320 revolutions. The distance traveled is

$$s = r\theta = (0.500\text{ m})(320\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 1.00\times10^{3}\text{ m} = 1.00\text{ km}$$

OQ10.2 Answer (b). Any object moving in a circular path undergoes a constant change in the direction of its velocity. This change in the direction of velocity is an acceleration, always directed toward the center of the path, called the centripetal acceleration, $a_c = v^2/r = r\omega^2$. The tangential speed of the object is $v_t = r\omega$, where ω is the angular velocity. If ω is not constant, the object will have both an angular acceleration, $\alpha_{\text{avg}} = \Delta\omega / \Delta t$, and a tangential acceleration, $a_t = r\alpha$. The only untrue statement among the listed choices is (b). Even when ω is constant, the object still has centripetal acceleration.

OQ10.3 Answer: b = e > a = d > c = 0. The tangential acceleration has magnitude $(3/\text{s}^2)r$, where r is the radius. It is constant in time. The radial acceleration has magnitude $\omega^2 r$, so it is $(4/\text{s}^2)r$ at the first and last moments mentioned and it is zero at the moment the wheel reverses.

OQ10.4 Answer (d). The angular displacement will be

$$\Delta\theta = \omega_{\text{avg}}\Delta t = \left(\frac{\omega_f + \omega_i}{2}\right)\Delta t$$
$$= \left(\frac{12.00\text{ rad/s} + 4.00\text{ rad/s}}{2}\right)(4.00\text{ s}) = 32.0\text{ rad}$$

OQ10.5 **(i)** Answer (d). The speedometer measures the number of revolutions per second of the tires. A larger tire will rotate fewer times to cover the same distance. The speedometer reading is assumed proportional to the rotation rate of the tires, $\omega = v/R$, for a standard tire radius R, but the actual reading is $\omega = v/(1.3)R$, or 1.3 times smaller. Example: When the car travels at 13 km/h, the speedometer reads 10 km/h.

(ii) Answer (d). If the driver uses the odometer reading to calculate fuel economy, this reading is a factor of 1.3 too small because the odometer assumes 1 rev = $2\pi R$ for a standard tire radius R, whereas the actual distance traveled is $1.3(2\pi R)$, so the fuel economy in miles per gallon will appear to be lower by a factor of 1.3. Example: If the car travels 13 km, the odometer will read 10 km. If the car actually makes 13 km/gal, the calculation will give 10 km/gal.

OQ10.6 **(i)** Answer (a). Smallest I is about the x axis, along which the larger-mass balls lie.

(ii) Answer (c). The balls all lie at a distance from the z axis, which is perpendicular to both the x and y axes and passes through the origin.

OQ10.7 Answer (a). The accelerations are not equal, but greater in case (a). The string tension above the 50-N object is less than its weight while the object is accelerating downward because it does not fall with the acceleration of gravity.

OQ10.8 Answers (a), (b), (e). The object must rotate with a nonzero and constant angular acceleration. Its moment of inertia would not change unless there were a rearrangement of mass within the object.

OQ10.9 (**i**) Answer (a). The basketball has rotational as well as translational kinetic energy.

(**ii**) Answer (c). The motions of their centers of mass are identical.

(**iii**) Answer (a). The basketball-Earth system has more kinetic energy than the ice-Earth system due to the rotational kinetic energy of the basketball. Therefore, when the kinetic energy of both systems has transformed to gravitational potential energy when the objects momentarily come to rest at their highest point on the ramp, the basketball will be at a higher location, corresponding to the larger gravitational potential energy.

OQ10.10 **(i)** Answer (c). The airplane momentarily has zero torque acting on it. It was speeding up in its angular rotation before this instant of time and begins slowing down just after this instant.

(**ii**) Answer (b). Although the angular speed is zero at this instant, there is still an angular acceleration because the wound-up string applies a torque to the airplane. This is similar to a ball thrown upward, which we studied earlier: at the top of its flight, it momentarily comes to rest, but is still accelerating because the gravitational force is acting on it.

OQ10.11 Answer (e). The sphere of twice the radius has eight times the volume and eight times the mass, and the r^2 term in $I = \frac{2}{5}mr^2$ also becomes four times larger.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ10.1 Yes. For any object on which a net force acts but no net torque, the translational kinetic energy will change but the rotational kinetic energy will not. For example, if you drop an object, it will gain translational kinetic energy due to work done on the object by the gravitational force. Any rotational kinetic energy the object has is unaffected by dropping it.

CQ10.2 No, just as an object need not be moving to have mass.

CQ10.3 If the object is free to rotate about any axis, the object will start to rotate if the two forces act along <u>different</u> lines of action. Then the torques of the forces will not be equal in magnitude and opposite in direction.

CQ10.4 Attach an object, of known mass m, to the cord. You could measure the time that it takes the object to fall a measured distance after being released from rest. Using this information, the linear acceleration of the mass can be calculated, and then the torque on the rotating object and its angular acceleration. It is assumed the mass of the cord has negligible effect on the motion as the cord unwinds.

CQ10.5 We have from Example 10.6 the means to calculate a and α. You could use $\omega = \alpha t$ and $v = at$.

CQ10.6 The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is at a different distance from the axis than before. Compare the moments of inertia of a uniform rigid rod about axes perpendicular to the rod, first passing through its center of mass, then passing through an end. For example, if you wiggle repeatedly a meterstick back and forth about an axis passing through its center of mass, you will find it does not take much effort to reverse the direction of rotation. However, if you move the axis to an end, you will find it more difficult to wiggle the stick back and forth. The moment of inertia about the end is much larger, because much of the mass of the stick is farther from the axis.

CQ10.11 No, only if its angular velocity changes.

CQ10.12 Adding a small sphere of mass m to the end will increase the moment of inertia of the system from $(1/3)ML^2$ to $(1/3)ML^2 + mL^2$, and the initial potential energy would be $(1/2)MgL + mgL$. Following Example 10.11, the final angular speed ω would be

$$\omega = \sqrt{\frac{3g}{L}}\sqrt{\frac{M+2m}{M+3m}}$$

If $m = M$, $\omega = \sqrt{\frac{3g}{L}}\sqrt{\frac{M+2m}{M+3m}} = \sqrt{\frac{3g}{L}}\sqrt{\frac{3M}{4M}} = \sqrt{\frac{9g}{4L}}$

Therefore, ω would increase.

CQ10.13 (a) The sphere would reach the bottom first. (b) The hollow cylinder would reach the bottom last. First imagine that each object has the same mass and the same radius. Then they all have the same torque due to gravity acting on them. The one with the smallest moment of inertia will thus have the largest angular acceleration and reach the bottom of the plane first. Equation 10.52 describes the speed of an object rolling down an inclined plane. In the denominator, I_{CM} will be a numerical factor (e.g., 2/5 for the sphere) multiplied by MR^2. Therefore, the mass and radius will cancel in the equation and the center-of-mass speed will be independent of mass and radius.

CQ10.14 (a) Sewer pipe: $I_{CM} = MR^2$. (b) Embroidery hoop: $I_{CM} = MR^2$. (c) Door: $I = \frac{1}{3}MR^2$. (d) Coin: $I_{CM} = \frac{1}{2}MR^2$. The distribution of mass along lines parallel to the axis makes no difference to the moment of inertia.

CQ10.15 (a) The tricycle rolls forward. (b) The tricycle rolls forward. (c) The tricycle rolls backward. (d) The tricycle does not roll, but may skid forward. (e) The tricycle rolls backward. (f) To answer these questions, think about the torque of the string tension about an axis at the bottom of the wheel, where the rubber meets the road. This is the instantaneous axis of rotation in rolling. Cords A and B produce clockwise torques about this axis. Cords C and E produce counterclockwise torques. Cord D has zero lever arm.

CQ10.16 As one finger slides towards the center, the normal force exerted by the sliding finger on the ruler increases. At some point, this normal force will increase enough so that static friction between the sliding finger and the ruler will stop their relative motion. At this moment the other finger starts sliding along the ruler towards the center. This process repeats until the fingers meet at the center of the ruler.

Next step: Try a rod with a nonuniform mass distribution.

Next step: Wear a piece of sandpaper as a ring on one finger to change its coefficient of friction.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 10.1 Angular Position, Velocity, and Acceleration

P10.1 (a) The Earth rotates 2π radians (360°) on its axis in 1 day. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\ \text{rad}}{1\ \cancel{\text{day}}}\left(\frac{1\ \cancel{\text{day}}}{8.64\times 10^4\ \text{s}}\right) = \boxed{7.27\times 10^{-5}\ \text{rad/s}}$$

(b) Because of its angular speed, $\boxed{\text{the Earth bulges at the equator}}$.

P10.2 (a) $\alpha = \frac{\Delta\omega}{\Delta t} = \frac{1.00\ \text{rev/s} - 0}{30.0\ \text{s}} = \left(3.33\times 10^{-2}\ \frac{\cancel{\text{rev}}}{\text{s}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right)$

$= \boxed{0.209\ \text{rad/s}^2}$

(b) $\boxed{\text{Yes.}}$ When an object starts from rest, its angular speed is related to the angular acceleration and time by the equation $\omega = \alpha(\Delta t)$.

Thus, the angular speed is directly proportional to both the angular acceleration and the time interval. If the time interval is held constant, doubling the angular acceleration will double the angular speed attained during the interval.

P10.3 (a) $\theta|_{t=0} = \boxed{5.00\text{ rad}}$

$$\omega|_{t=0} = \left.\frac{d\theta}{dt}\right|_{t=0} = 10.0 + 4.00t|_{t=0} = \boxed{10.0\text{ rad/s}}$$

$$\alpha|_{t=0} = \left.\frac{d\omega}{dt}\right|_{t=0} = \boxed{4.00\text{ rad/s}^2}$$

(b) $\theta|_{t=3.00\text{ s}} = 5.00 + 30.0 + 18.0 = \boxed{53.0\text{ rad}}$

$$\omega|_{t=3.00\text{s}} = \left.\frac{d\theta}{dt}\right|_{t=3.00\text{s}} = 10.0 + 4.00t|_{t=3.00\text{s}} = \boxed{22.0\text{rad/s}}$$

$$\alpha|_{t=3.00\text{s}} = \left.\frac{d\omega}{dt}\right|_{t=3.00\text{s}} = \boxed{4.00\text{ rad/s}^2}$$

P10.4 $\alpha = \dfrac{d\omega}{dt} = 10 + 6t \quad \rightarrow \quad \int_0^{\omega} d\omega = \int_0^t (10 + 6t)dt \quad \rightarrow \quad \omega - 0 = 10t + \dfrac{6}{2}t^2$

$\omega = \dfrac{d\theta}{dt} = 10t + 3t^2 \quad \rightarrow \quad \int_0^{\theta} d\theta = \int_0^t (10t + 3t^2)dt \quad \rightarrow \quad \theta - 0 = \dfrac{10t^2}{2} + \dfrac{3t^3}{3}$

$\theta = 5t^2 + t^3$. At $t = 4.00$ s, $\theta = 5(4.00\text{ s})^2 + (4.00\text{ s})^3 = \boxed{144\text{ rad}}$

Section 10.2 Analysis Model: Rigid Object Under Constant Angular Acceleration

P10.5 (a) We start with $\omega_f = \omega_i + \alpha t$ and solve for the angular acceleration α:

$$\alpha = \frac{\omega - \omega_i}{t} = \frac{12.0\text{ rad/s}}{3.00\text{ s}} = \boxed{4.00\text{ rad/s}^2}$$

(b) The angular position of a rigid object under constant angular acceleration is given by Equation 10.7:

$$\theta = \omega_i t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(4.00\text{ rad/s}^2)(3.00\text{ s})^2 = \boxed{18.0\text{ rad}}$$

P10.6 $\omega_i = 3\,600 \text{ rev/min} = 3.77\times10^2 \text{ rad/s}$

$\theta = 50.0 \text{ rev} = 3.14\times10^2 \text{ rad and } \omega_f = 0$

$\omega_f^2 = \omega_i^2 + 2\alpha\theta$

$0 = \left(3.77\times10^2 \text{ rad/s}\right)^2 + 2\alpha\left(3.14\times10^2 \text{ rad}\right)$

$\alpha = \boxed{-2.26\times10^2 \text{ rad/s}^2}$

P10.7 We are given $\alpha = -2.00 \text{ rad/s}^2$, $\omega_f = 0$, and

$$\omega_i = \frac{100 \text{ rev}}{1.00 \text{ min}}\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1.00 \text{ rev}}\right) = \frac{10\pi}{3} \text{ rad/s}$$

(a) From $\omega_f - \omega_i = \alpha t$, we have

$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - (10\pi/3)}{-2.00} \text{ s} = \boxed{5.24 \text{ s}}$$

(b) Since the motion occurs with constant angular acceleration, we write

$$\theta_f = \bar{\omega}t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left(\frac{10\pi}{6} \text{ rad/s}\right)\left(\frac{10\pi}{6} \text{ s}\right) = \boxed{27.4 \text{ rad}}$$

P10.8 (a) From $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta)$, the angular displacement is

$$\Delta\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{(2.2 \text{ rad/s})^2 - (0.06 \text{ rad/s})^2}{2(0.70 \text{ rad/s}^2)} = \boxed{3.5 \text{ rad}}$$

(b) From the equation given above for $\Delta\theta$, observe that when the angular acceleration is constant, the displacement is proportional to the difference in the *squares* of the final and initial angular speeds. Thus, the angular displacement would $\boxed{\text{increase by a factor of 4}}$ if both of these speeds were doubled.

***P10.9** We are given $\omega_f = 2.51\times10^4 \text{ rev/min} = 2.63\times10^3 \text{ rad/s}$

(a) $\alpha = \dfrac{\omega_f - \omega_i}{t} = \dfrac{2.63\times10^3 \text{ rad/s} - 0}{3.20 \text{ s}} = \boxed{8.21\times10^2 \text{ rad/s}^2}$

(b) $\theta_f = \omega_i t + \dfrac{1}{2}\alpha t^2 = 0 + \dfrac{1}{2}\left(8.21\times10^2 \text{ rad/s}^2\right)(3.20 \text{ s})^2 = \boxed{4.21\times10^3 \text{ rad}}$

P10.10 According to the definition of average angular speed (Eq. 10.2), the disk's average angular speed is 50.0 rad/10.0 s = 5.00 rad/s. According to the average angular speed expressed as $(\omega_i + \omega_f)/2$ in the model of a rigid object under constant angular acceleration, the average angular speed of the disk is (0 + 8.00 rad/s)/2 = 4.00 rad/s. Because these two numbers do not match, the angular acceleration of the disk cannot be constant.

P10.11 $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ are two equations in two unknowns, ω_i and α.

$$\omega_i = \omega_f - \alpha t\text{:}\ \ \theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega t - \frac{1}{2}\alpha t^2$$

$$(37.0\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = (98.0\text{ rad/s})(3.00\text{ s}) - \frac{1}{2}\alpha(3.00\text{ s})^2$$

$$232\text{ rad} = 294\text{ rad} - (4.50\text{ s}^2)\alpha\text{:}\quad \alpha = \frac{61.5\text{ rad}}{4.50\text{ s}^2} = \boxed{13.7\text{ rad/s}^2}$$

P10.12 $\omega = 5.00\text{ rev/s} = 10.0\pi\text{ rad/s}$. We will break the motion into two stages: (1) a period during which the tub speeds up and (2) a period during which it slows down.

While speeding up, $\theta_1 = \bar{\omega}t = \frac{0 + 10.0\pi\text{ rad/s}}{2}(8.00\text{ s}) = 40.0\pi\text{ rad}$.

While slowing down, $\theta_2 = \bar{\omega}t = \frac{10.0\pi\text{ rad/s} + 0}{2}(12.0\text{ s}) = 60.0\pi\text{ rad}$.

So, $\theta_{\text{total}} = \theta_1 + \theta_2 = 100\pi\text{ rad} = \boxed{50.0\text{ rev}}$.

***P10.13** We use $\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$ and $\omega_f = \omega_i + \alpha t$ to obtain

$$\omega_i = \omega_f - \alpha t \quad\text{and}\quad \theta_f - \theta_i = (\omega_f - \alpha t)t + \frac{1}{2}\alpha t^2 = \omega_f t - \frac{1}{2}\alpha t^2$$

Solving for the final angular speed gives

$$\omega_f = \frac{\theta_f - \theta_i}{t} + \frac{1}{2}\alpha t = \frac{62.4\text{ rad}}{4.20\text{ s}} + \frac{1}{2}(-5.60\text{ rad/s}^2)(4.20\text{ s})$$
$$= \boxed{3.10\text{ rad/s}^2}$$

P10.14 (a) Let R_E represent the radius of the Earth. The base of the building moves east at $v_1 = \omega R_E$, where ω is one revolution per day. The top of the building moves east at $v_2 = \omega(R_E + h)$. Its eastward speed relative to the ground is $v_2 - v_1 = \omega h$. The object's time of fall is given by $\Delta y = 0 + \frac{1}{2}gt^2$, $t = \sqrt{\frac{2h}{g}}$. During its fall the object's eastward motion is unimpeded so its deflection distance is

$$\Delta x = (v_2 - v_1)t = \omega h\sqrt{\frac{2h}{g}} = \boxed{\omega h^{3/2}\left(\frac{2}{g}\right)^{1/2}}$$

(b) $$\left(\frac{2\pi \text{ rad}}{86400 \text{ s}}\right)(50.0 \text{ m})^{3/2}\left(\frac{2}{9.80 \text{ m/s}^2}\right)^{1/2} = \boxed{1.16 \text{ cm}}$$

(c) $\boxed{\text{The deflection is only 0.02\% of the original height, so it is negligible in many practical cases.}}$

(d) $\boxed{\text{Decrease.}}$ Because the displacement is proportional to angular speed and the angular acceleration is constant, the displacement decreases linearly in time.

Section 10.3 Angular and Translational Quantities

P10.15 (a) From $v = r\omega$, we have

$$\omega = \frac{v}{r} = \frac{45.0 \text{ m/s}}{250 \text{ m}} = \boxed{0.180 \text{ rad/s}}$$

(b) Traveling at constant speed along a circular track, the car will experience a centripetal acceleration given by

$$a_r = \frac{v^2}{r} = \frac{(45.0 \text{ m/s})^2}{250 \text{ m}} = \boxed{8.10 \text{ m/s}^2 \text{ toward the center of track}}$$

P10.16 Estimate the tire's radius at 0.250 m and miles driven as 10 000 per year. Then,

$$\theta = \frac{s}{r} = \left(\frac{1.00 \times 10^4 \text{ mi}}{0.250 \text{ m}}\right)\left(\frac{1609 \text{ m}}{1 \text{ mi}}\right) = 6.44 \times 10^7 \text{ rad/yr}$$

$$\theta = \left(6.44\times 10^{7}\ \text{rad/yr}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = 1.02\times 10^{7}\ \text{rev/yr or } \boxed{\sim 10^{7}\ \text{rev/yr}}$$

P10.17 (a) The final angular speed is

$$\omega = \frac{v}{r} = \frac{25.0\ \text{m/s}}{1.00\ \text{m}} = \boxed{25.0\ \text{rad/s}}$$

(b) We solve for the angular acceleration from $\omega_f^2 = \omega_i^2 + 2\alpha\left(\Delta\theta\right)$:

$$\alpha = \frac{\omega_f^2 - \omega_i^2}{2\left(\Delta\theta\right)} = \frac{(25.0\ \text{rad/s})^2 - 0}{2\left[\left(1.25\ \text{rev}\right)(2\pi\ \text{rad/rev})\right]} = \boxed{39.8\ \text{rad/s}^2}$$

(c) From the definition of angular acceleration,

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{25.0\ \text{rad/s}}{39.8\ \text{rad/s}^2} = \boxed{0.628\ \text{s}}$$

P10.18 (a) Consider a tooth on the front sprocket. It gives this speed, relative to the frame, to the link of the chain it engages:

$$v = r\omega = \left(\frac{0.152\ \text{m}}{2}\right)\left(76\ \text{rev/min}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= \boxed{0.605\ \text{m/s}}$$

(b) Consider the chain link engaging a tooth on the rear sprocket:

$$\omega = \frac{v}{r} = \frac{0.605\ \text{m/s}}{\left(0.070\ \text{m}\right)/2} = \boxed{17.3\ \text{rad/s}}$$

(c) Consider the wheel tread and the road. A thread could be unwinding from the tire with this speed relative to the frame:

$$v = r\omega = \left(\frac{0.673\ \text{m}}{2}\right)\left(17.3\ \text{rad/s}\right) = \boxed{5.82\ \text{m/s}}$$

(d) $\boxed{\text{We did not need to know the length of the pedal cranks}}$, but we could use that information to find the linear speed of the pedals:

$$v = r\omega = \left(0.175\ \text{m}\right)\left(7.96\ \text{rad/s}\right)\left(\frac{1}{1\ \text{rad}}\right) = 1.39\ \text{m/s}$$

P10.19 Given $r = 1.00$ m, $\alpha = 4.00\ \text{rad/s}^2$, $\omega_i = 0$, and $\theta_i = 57.3° = 1.00$ rad:

(a) $\omega_f = \omega_i + \alpha t = 0 + \alpha t$

At $t = 2.00$ s, $\omega_f = 4.00\ \text{rad/s}^2\left(2.00\ \text{s}\right) = \boxed{8.00\ \text{rad/s}}$

(b) $v = r\omega = (1.00\text{ m})(8.00\text{ rad/s}) = \boxed{8.00\text{ m/s}}$

(c) $|a_r| = a_c = r\omega^2 = (1.00\text{ m})(8.00\text{ rad/s})^2 = 64.0\text{ m/s}^2$

$a_t = r\alpha = (1.00\text{ m})(4.00\text{ rad/s}^2) = 4.00\text{ m/s}^2$

The magnitude of the total acceleration is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(64.0\text{ m/s}^2)^2 + (4.00\text{ m/s}^2)^2} = \boxed{64.1\text{ m/s}^2}$$

The direction the total acceleration vector makes with respect to the radius to point *P* is

$$\phi = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{4.00}{64.0}\right) = \boxed{3.58°}$$

(d) $\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2 = (1.00\text{ rad}) + \frac{1}{2}(4.00\text{ rad/s}^2)(2.00\text{ s})^2 = \boxed{9.00\text{ rad}}$

P10.20 (a) We first determine the distance travelled by the car during the 9.00-s interval:

$$s = \bar{v}t = \frac{v_i + v_f}{2}t = (11.0\text{ m/s})(9.00\text{ s}) = 99.0\text{ m}$$

the number of revolutions completed by the tire is then

$$\theta = \frac{s}{r} = \frac{99.0\text{ m}}{0.290\text{ m}} = 341\text{ rad} = \boxed{54.3\text{ rev}}$$

(b) $\omega_f = \dfrac{v_f}{r} = \dfrac{22.0\text{ m/s}}{0.290\text{ m}} = 75.9\text{ rad/s} = \boxed{12.1\text{ rev/s}}$

P10.21 Every part of this problem is about using radian measure to relate rotation of the whole object to the linear motion of a point on the object.

(a) $\omega = 2\pi f = \dfrac{2\pi\text{ rad}}{1\text{ rev}}\left(\dfrac{1\,200\text{ rev}}{60.0\text{ s}}\right) = \boxed{126\text{ rad/s}}$

(b) $v = \omega r = (126\text{ rad/s})(3.00\times10^{-2}\text{ m}) = \boxed{3.77\text{ m/s}}$

(c) $a_c = \omega^2 r = (126\text{ rad/s})^2(8.00\times10^{-2}\text{ m}) = 1\,260\text{ m/s}^2$ so

$\vec{\mathbf{a}}_r = \boxed{1.26\text{ km/s}^2\text{ toward the center}}$

(d) $s = r\theta = \omega r t = (126\text{ rad/s})(8.00\times10^{-2}\text{ m})(2.00\text{ s}) = \boxed{20.1\text{ m}}$

P10.22 (a) $\boxed{5.77\text{ cm}}$

(b) Yes. The top of the ladder is displaced

$$\theta = s/r = 0.690\text{ m}/4.90\text{ m} \cong 0.141\text{ rad}$$

from vertical about its right foot. The left foot of the ladder is displaced by the same angle below the horizontal; therefore,

$$\theta = 0.690\text{ m}/4.90\text{ m} = t/0.410\text{ m} \to t = 5.77\text{ cm}$$

Note that we are approximating the straight-line distance of 0.690 m as an arc length because it is much smaller than the length of the ladder. The thickness of the rock is a cruder approximation of an arc length because the rung of the ladder is much shorter than the length of the ladder.

P10.23 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $m(1.70\text{ m/s}^2)$. Its radially inward component is $ma_c = \frac{mv^2}{r} = m\omega^2 r$, which increases with time: this takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a_t$$
$$= m\pi(1.70\text{ m/s}^2)$$

With skidding impending we have $\sum F_y = ma_y,\ +n - mg = 0,\ n = mg$:

$$f_s = \mu_s n = \mu_s mg = \sqrt{m^2(1.70\text{ m/s}^2)^2 + m^2\pi^2(1.70\text{ m/s}^2)^2}$$
$$\mu_s = \frac{1.70\text{ m/s}^2}{g}\sqrt{1+\pi^2} = \boxed{0.572}$$

P10.24 The force of static friction must act forward and then more and more inward on the tires, to produce both tangential and centripetal acceleration. Its tangential component is $ma = mr\alpha$. Its radially inward component is $ma_c = \frac{mv^2}{r} = m\omega^2 r$ which increases with time; this takes the maximum value

$$m\omega_f^2 r = mr(\omega_i^2 + 2\alpha\Delta\theta) = mr\left(0 + 2\alpha\frac{\pi}{2}\right) = m\pi r\alpha = m\pi a$$

With skidding impending we have

$$\sum F_y = ma_y: \quad +n - mg = 0 \quad \rightarrow \quad n = mg$$

$$f_s = \mu_s n = \mu_s mg = \sqrt{(ma_t)^2 + (ma_c)^2} = \sqrt{m^2a^2 + m^2\pi^2a^2}$$

$$\mu_s = \boxed{\frac{a}{g}\sqrt{1+\pi^2}}$$

P10.25 (a) The general expression for angular velocity is

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}\left(2.50t^2 - 0.600t^3\right) = 5.00t - 1.80t^2$$

where ω is in radians/second and *t* is in seconds.

The angular velocity will be a maximum when

$$\frac{d\omega}{dt} = \frac{d}{dt}\left(5.00t - 1.80t^2\right) = 5.00 - 3.60t = 0$$

Solving for the time *t*, we find

$$t = \frac{5.00}{3.60} = 1.39\,\text{s}$$

Placing this value for *t* into the equation for angular velocity, we find

$$\omega_{\text{max}} = 5.00t - 1.80t^2 = 5.00(1.39) - 1.80(1.39)^2 = \boxed{3.47\,\text{rad/s}}$$

(b) $v_{\text{max}} = \omega_{\text{max}} r = (3.47\,\text{rad/s})(0.500\text{ m}) = \boxed{1.74\,\text{m/s}}$

(c) The roller reverses its direction when the angular velocity is zero—recall an object moving vertically upward against gravity reverses its motion when its velocity reaches zero at the maximum height.

$$\omega = 5.00t - 1.80t^2 = t(5.00 - 1.80t) = 0$$

$$\rightarrow 5.00 - 1.80t = 0 \rightarrow t = \frac{5.00}{1.80} = 2.78\,\text{s}$$

The driving force should be removed from the roller at $t = \boxed{2.78\,\text{s}}$.

(d) Set $t = 2.78$ s in the expression for angular position:

$$\theta = 2.50t^2 - 0.600t^3 = 2.50(2.78)^2 - 0.600(2.78)^3 = 6.43\,\text{rad}$$

or $$(6.43\text{ rad})\left(\frac{1\text{ rotation}}{2\pi\,\text{rad}}\right) = \boxed{1.02\text{ rotations}}$$

P10.26 The object starts with $\theta_i = 0$. The location of its final position on the circle is found from $9\,\text{rad} - 2\pi = 2.72\,\text{rad} = 156°$.

(a) Its position vector is

$$3.00\text{ m at }156° = (3.00\text{ m})\cos 156°\hat{\mathbf{i}} + (3.00\text{ m})\sin 156°\hat{\mathbf{j}}$$

$$= \boxed{\left(-2.73\hat{\mathbf{i}} + 1.24\hat{\mathbf{j}}\right)\text{ m}}$$

(b) $\boxed{\text{It is in the second quadrant, at }156°}$

(c) The object's velocity is $v = \omega r = (1.50\text{ rad/s})(3.00\text{ m}) = 4.50\text{ m/s}$ at 90°. After the displacement, its velocity is

$$4.50\text{ m/s at }90°+156° \quad \text{or}$$

$$4.50\text{ m/s at }246° = (4.50\text{ m/s})\cos 246°\hat{\mathbf{i}} + (4.50\text{ m/s})\sin 246°\hat{\mathbf{j}}$$

$$= \boxed{\left(-1.85\hat{\mathbf{i}} - 4.10\hat{\mathbf{j}}\right)\text{ m/s}}$$

(d) $\boxed{\text{It is moving toward the third quadrant, at }246°}$.

(e) Its acceleration is v^2/r, opposite in direction to its position vector. This is

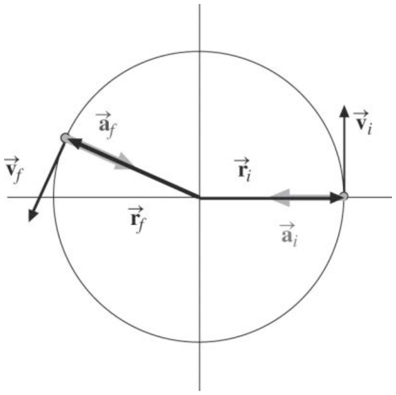
$$\frac{(4.50\text{ m/s})^2}{3.00\text{ m}}\text{ at }\;180°+156° \quad \text{or}$$

$$6.75\text{ m/s}^2\text{ at }336° = \left(6.75\text{ m/s}^2\right)\cos 336°\hat{\mathbf{i}}$$

$$+\left(6.75\text{ m/s}^2\right)\sin 336°\hat{\mathbf{j}}$$

$$= \boxed{\left(6.15\hat{\mathbf{i}} - 2.78\hat{\mathbf{j}}\right)\text{ m/s}^2}$$

(f) ANS. FIG. P10.26 shows the initial and final position, velocity, and acceleration vectors.



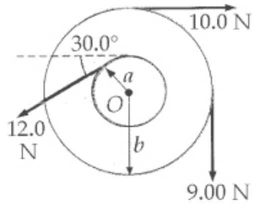
ANS. FIG. P10.26

(g) The total force is given by

$$\mathsf{F} = m\mathsf{a} = (4.00\ \text{kg})(6.15\hat{\mathbf{i}} - 2.78\,\hat{\mathbf{j}})\ \text{m/s}^2 = \boxed{(24.6\,\hat{\mathbf{i}} - 11.1\,\hat{\mathbf{j}})\ \text{N}}$$

Section 10.4 Torque

P10.27 To find the net torque, we add the individual torques, remembering to apply the convention that a torque producing clockwise rotation is negative and a counterclockwise rotation is positive.

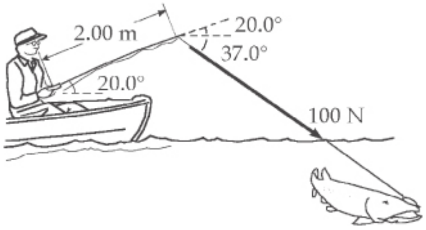


ANS. FIG. P10.27

$$\begin{aligned}\sum \tau &= (0.100\ \text{m})(12.0\ \text{N}) \\ &\quad - (0.250\ \text{m})(9.00\ \text{N}) \\ &\quad - (0.250\ \text{m})(10.0\ \text{N}) \\ &= \boxed{-3.55\ \text{N}\cdot\text{m}}\end{aligned}$$

The thirty-degree angle is unnecessary information.

P10.28 We resolve the 100-N force into components perpendicular to and parallel to the rod, as



ANS. FIG. P10.28

$$F_{\text{par}} = (100\ \text{N})\cos 57.0° = 54.5\ \text{N}$$

and

$$F_{\text{perp}} = (100\ \text{N})\sin 57.0° = 83.9\ \text{N}$$

The torque of F_{par} is zero since its line of action passes through the pivot point.

The torque of F_{perp} is

$$\tau = (83.9\ \text{N})(2.00\ \text{m}) = \boxed{168\ \text{N}\cdot\text{m}}\ \text{(clockwise)}$$

Section 10.5 Analysis Model: Rigid Object Under a Net Torque

P10.29 The flywheel is a solid disk of mass *M* and radius *R* with axis through its center.

$$\left.\begin{aligned}&\sum\tau = I\alpha\\ &I = \frac{1}{2}MR^2\end{aligned}\right\} -T_u r + T_b r = \frac{1}{2}MR^2\alpha \to T_b = T_u + \frac{MR^2\alpha}{2r}$$

$$T_b = 135\text{ N} + \frac{(80.0\text{ kg})(0.625\text{ m})^2(-1.67\text{ rad/s}^2)}{2(0.230\text{ m})} = \boxed{21.5\text{ N}}$$

P10.30 (a) The moment of inertia of the wheel, modeled as a disk, is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.00\text{ kg})(7.00\times10^{-2}\text{ m})^2 = 4.90\times10^{-3}\text{ kg}\cdot\text{m}^2$$

From Newton's second law for rotational motion,

$$\alpha = \frac{\sum\tau}{I} = \frac{0.600}{4.90\times10^{-3}} = 122\text{ rad/s}^2$$

then, from $\alpha = \dfrac{\Delta\omega}{\Delta t}$, we obtain

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{1200(2\pi/60)}{122} = \boxed{1.03\text{ s}}$$

(b) The number of revolutions is determined from

$$\Delta\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2}(122\text{ rad/s})(1.03\text{ s})^2 = 64.7\text{ rad} = \boxed{10.3\text{ rev}}$$

***P10.31** (a) We first determine the moment of inertia of the merry-go-round:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(150\text{ kg})(1.50\text{ m})^2 = 169\text{ kg}\cdot\text{m}^2$$

To find the angular acceleration, we use

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \left(\frac{0.500\text{ rev/s} - 0}{2.00\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{\pi}{2}\text{ rad/s}^2$$

From the definition of torque, $\tau = F\cdot r = I\alpha$, we obtain

$$F = \frac{I\alpha}{r} = \frac{(169\text{ kg}\cdot\text{m}^2)\left(\dfrac{\pi}{2}\text{ rad/s}^2\right)}{1.50\text{ m}} = \boxed{177\text{ N}}$$

P10.32 (a) See ANS. FIG. P10.32 below for the force diagrams. For m_1, $\sum F_y = ma_y$ gives

$$+n - m_1 g = 0$$

$$n_1 = m_1 g$$

with $f_{k1} = \mu_k n_1$.

$\sum F_x = ma_x$ gives

$$-f_{k1} + T_1 = m_1 a \qquad [1]$$

For the pulley, $\sum \tau = I\alpha$ gives

$$-T_1 R + T_2 R = \frac{1}{2} MR^2 \left(\frac{a}{R}\right)$$

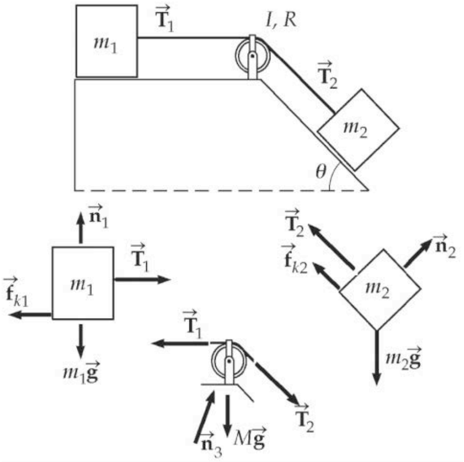
or $$-T_1 + T_2 = \frac{1}{2} MR\left(\frac{a}{R}\right) \to -T_1 + T_2 = \frac{1}{2} Ma \qquad [2]$$

For m_2,

$$+n_2 - m_2 g \cos\theta = 0 \to n_2 = m_2 g \cos\theta$$

$$f_{k2} = \mu_k n_2$$

$$-f_{k2} - T_2 + m_2 g \sin\theta = m_2 a \qquad [3]$$



ANS. FIG. P10.32

(b) Add equations [1], [2], and [3] and substitute the expressions for f_{k1} and n_1, and $-f_{k2}$ and n_2:

$$-f_{k1} + T_1 + \left(-T_1 + T_2\right) - f_{k2} - T_2 + m_2 g\sin\theta = m_1 a + \frac{1}{2}Ma + m_2 a$$

$$-f_{k1} - f_{k2} + m_2 g\sin\theta = \left(m_1 + m_2 + \frac{1}{2}M\right)a$$

$$-\mu_k m_1 g - \mu_k m_2 g\cos\theta + m_2 g\sin\theta = \left(m_1 + m_2 + \frac{1}{2}M\right)a$$

$$a = \frac{m_2\left(\sin\theta - \mu_k\cos\theta\right) - \mu_k m_1}{m_1 + m_2 + \frac{1}{2}M}g$$

$$a = \frac{(6.00\text{ kg})\left(\sin 30.0^\circ - 0.360\cos 30.0^\circ\right) - 0.360(2.00\text{ kg})}{(2.00\text{ kg}) + (6.00\text{ kg}) + \frac{1}{2}(10.0\text{ kg})}g$$

$$a = \boxed{0.309\text{ m/s}^2}$$

(c) From equation [1]:

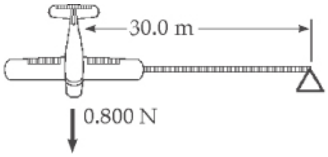
$$-f_{k1} + T_1 = m_1 a \rightarrow T_1 = 2.00\text{ kg}\left(0.309\text{ m/s}^2\right) + 7.06\text{ N} = \boxed{7.67\text{ N}}$$

From equation [2]:

$$-T_1 + T_2 = \frac{1}{2}Ma \rightarrow T_2 = 7.67\text{ N} + 5.00\text{ kg}\left(0.309\text{ m/s}^2\right)$$

$$= \boxed{9.22\text{ N}}$$

P10.33 We use the definition of torque and the relationship between angular and translational acceleration, with $m = 0.750$ kg and $F = 0.800$ N:



ANS. FIG. P10.33

(a) $\tau = rF = (30.0\text{ m})(0.800\text{ N}) = \boxed{24.0\text{ N}\cdot\text{m}}$

(b) $$\alpha = \frac{\tau}{I} = \frac{rF}{mr^2} = \frac{24.0\text{ N}\cdot\text{m}}{(0.750\text{ kg})(30.0\text{ m})^2}$$

$$= \boxed{0.0356\text{ rad/s}^2}$$

(c) $a_t = \alpha r = \left(0.0356\text{ rad/s}^2\right)(30.0\text{ m}) = \boxed{1.07\text{ m/s}^2}$

P10.34 (a) The chosen tangential force produces constant torque and therefore constant angular acceleration. Since the disk starts from rest, we write

$$\theta_f - \theta_i = \omega_i t + \frac{1}{2}\alpha t^2$$

$$\theta_f - 0 = 0 + \frac{1}{2}\alpha t^2$$

$$\theta_f = \frac{1}{2}\alpha t^2$$

Solving for the angular acceleration gives

$$\alpha = \frac{2\theta_f}{t^2} = \frac{2(2.00\text{ rev})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)}{(10.0\text{ s})^2} = 0.251\text{ rad/s}^2$$

We then obtain the required combination of *F* and *R* from the rigid object under a net torque model:

$$\sum\tau = I\alpha: \qquad FR = (100\text{ kg}\cdot\text{m}^2)(0.251\text{ rad/s}^2) = 25.1\text{ N}\cdot\text{m}$$

$$\boxed{\text{For } F = 25.1\text{ N, } R = 1.00\text{ m. For } F = 10.0\text{ N, } R = 2.51\text{ m.}}$$

(b) $\boxed{\text{No. Infinitely many pairs of values that satisfy this requirement exist: for any } F \le 50.0\text{ N, } R = 25.1\text{ N}\cdot\text{m}/F\text{, as long as } R \le 3.00\text{ m.}}$

P10.35 (a) From the rigid object under a net torque model, $\sum\tau = I\alpha$ gives

$$I = \frac{\sum\tau}{\alpha} = \frac{\sum\tau}{\Delta\omega}\Delta t = \frac{36.0\text{ N}\cdot\text{m}}{10.0\text{ rad/s}}(6.00\text{ s}) = \boxed{21.6\text{ kg}\cdot\text{m}^2}$$

(b) For the portion of the motion during which the wheel slows down,

$$\left|\sum\tau\right| = |I\alpha| = \left|I\frac{\Delta\omega}{\Delta t}\right| = \left|(21.6\text{ kg}\cdot\text{m}^2)\left(\frac{-10.0\text{ rad/s}}{60.0\text{ s}}\right)\right|$$

$$= \boxed{3.60\text{ N}\cdot\text{m}}$$

(c) During the first portion of the motion,

$$\Delta\theta = \omega_{\text{avg}}\Delta t = \left(\frac{\omega_i + \omega_f}{2}\right)\Delta t = \left(\frac{0 + 10.0\text{ rad/s}}{2}\right)(6.00\text{ s})$$

$$= 30\text{ rad}$$

During the second portion,

$$\Delta\theta = \omega_{\text{avg}}\Delta t = \left(\frac{\omega_i + \omega_f}{2}\right)\Delta t = \left(\frac{10.0\ \text{rad/s} + 0}{2}\right)(60.0\ \text{s})$$
$$= 300\ \text{rad}$$

Therefore, the total angle is 330 rad or $\boxed{52.5\ \text{revolutions}}$.

P10.36 (a) Let T_1 represent the tension in the cord above m_1 and T_2 the tension in the cord above the lighter mass. The two blocks move with the same acceleration because the cord does not stretch, and the angular acceleration of the pulley is a/R. For the heavier mass we have

$$\sum F = m_1 a \rightarrow T_1 - m_1 g = m_1(-a) \quad \text{or} \quad -T_1 + m_1 g = m_1 a$$

For the lighter mass,

$$\sum F = m_2 a \quad \rightarrow \quad T_2 - m_2 g = m_2 a$$

We assume the pulley is a uniform disk: $I = (1/2)MR^2$

$$\sum \tau = I\alpha \quad \rightarrow \quad +T_1 R - T_2 R = \frac{1}{2}MR^2(a/R)$$

or $\quad T_1 - T_2 = \dfrac{1}{2}Ma$

Add up the three equations in a:

$$-T_1 + m_1 g + T_2 - m_2 g + T_1 - T_2 = m_1 a + m_2 a + \frac{1}{2}Ma$$

$$a = \frac{m_1 - m_2}{m_1 + m_2 + \frac{1}{2}M}g$$
$$= \frac{20.0\ \text{kg} - 12.5\ \text{kg}}{20.0\ \text{kg} + 12.5\ \text{kg} + \frac{1}{2}(5.00\ \text{kg})}(9.80\ \text{m/s}^2)$$
$$= 2.10\ \text{m/s}^2$$

Next, $x = 0 + 0 + \dfrac{1}{2}at^2 \rightarrow t = \sqrt{\dfrac{2x}{a}} = \sqrt{\dfrac{2(4.00\ \text{m})}{2.10\ \text{m/s}^2}} = \boxed{1.95\ \text{s}}$

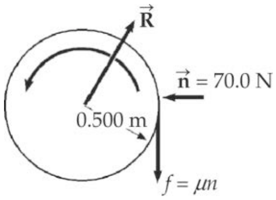
(b) If the pulley were massless, the acceleration would be larger by a factor 35/32.5 and the time shorter by the square root of the factor 32.5/35. That is, the time would be reduced by 3.64%.

P10.37 From the rigid object under a net torque model,

$$\sum \tau = I\alpha$$

$$-f_k R = \mu_k F R = \left(\frac{1}{2}MR^2\right)\frac{\Delta\omega}{\Delta t}$$

$$\mu_k = -\frac{MR\Delta\omega}{2F\Delta t}$$



ANS. FIG. P10.37

Substitute numerical values:

$$\mu_k = -\frac{(100\text{ kg})(0.500\text{ m})(-50.0\text{ rev/min})}{2(70.0\text{ N})(6.00\text{ s})}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)$$

$$= \boxed{0.312}$$

Section 10.6 Calculation of Moments of Inertia

P10.38 Model your body as a cylinder of mass 60.0 kg and a radius of 12.0 cm. Then its moment of inertia is

$$\frac{1}{2}MR^2 = \frac{1}{2}(60.0\text{ kg})(0.120\text{ m})^2 = 0.432\text{ kg}\cdot\text{m}^2$$

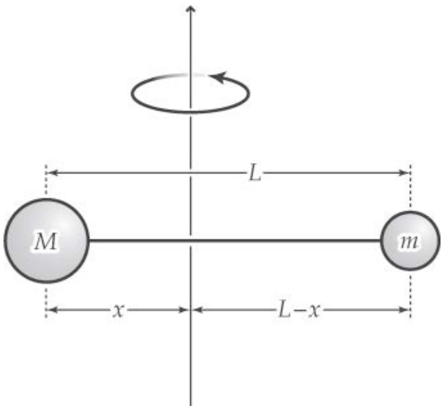
$$\sim \boxed{10^0\text{ kg}\cdot\text{m}^2 = 1\text{ kg}\cdot\text{m}^2}$$

P10.39 (a) Every particle in the door could be slid straight down into a high-density rod across its bottom, without changing the particle's distance from the rotation axis of the door. Thus, a rod 0.870 m long with mass 23.0 kg, pivoted about one end, has the same rotational inertia as the door:

$$I = \frac{1}{3}ML^2 = \frac{1}{3}(23.0\text{ kg})(0.870\text{ m})^2 = \boxed{5.80\text{ kg}\cdot\text{m}^2}$$

(b) The $\boxed{\text{height of the door is unnecessary}}$ data.

P10.40 (a) We take a coordinate system with mass M at the origin. The distance from the axis to the origin is also x. The moment of ineria about the axis is



ANS. FIG. P10.40

$$I = Mx^2 + m(L-x)^2$$

To find the extrema in the moment of inertia, we differentiate I with respect to x:

$$\frac{dI}{dx} = 2Mx - 2m(L-x) = 0$$

Solving for x then gives

$$x = \frac{mL}{M+m}$$

Differentiating again gives $\frac{d^2I}{dx^2} = 2m + 2M$; therefore, I is at a minimum when the axis of rotation passes through $x = \frac{mL}{M+m}$, which is also the position of the center of mass of the system if we take mass M to lie at the origin of a coordinate system.

(b) The moment of inertia about an axis passing through x is

$$I_{CM} = M\left[\frac{mL}{M+m}\right]^2 + m\left[1 - \frac{m}{M+m}\right]^2 L^2 = \frac{Mm}{M+m}L^2$$

$$\rightarrow \boxed{I_{CM} = \mu L^2, \text{ where } \mu = \frac{Mm}{M+m}}$$

P10.41 Treat the tire as consisting of three hollow cylinders: two sidewalls and a tread region. The moment of inertia of a hollow cylinder, where $R_2 > R_1$, is $I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$, and the mass of a hollow cylinder of height (or thickness) t is $M = \rho\pi\left(R_2^2 - R_1^2\right)t$. Substituting the expression for mass M into the expression for I, we get

$$I = \frac{1}{2}\rho\pi\left(R_2^2 - R_1^2\right)t\left(R_1^2 + R_2^2\right) = \frac{1}{2}\rho\pi t\left(R_2^4 - R_1^4\right)$$

The two sidewalls have inner radius $r_1 = 16.5$ cm, outer radius $r_2 = 30.5$ cm, and height $t_{\text{side}} = 0.635$ cm. The tread region has inner radius $r_2 = 30.5$ cm, outer radius $r_3 = 33.0$ cm, and height $t_{\text{tread}} = 20.0$ cm. The density of the rubber is $1.10 \times 10^3\ \text{kg/m}^3$.

For the tire (two sidewalls: $R_1 = r_1$, $R_2 = r_2$; tread region: $R_1 = r_2$, $R_2 = r_3$)

$$\begin{aligned} I_{\text{total}} &= 2\left[\frac{1}{2}\rho\pi t_{\text{side}}\left(R_2^4 - R_1^4\right)\right] + \frac{1}{2}\rho\pi t_{\text{tread}}\left(R_2^4 - R_1^4\right) \\ &= 2\left[\frac{1}{2}\rho\pi t_{\text{side}}\left(r_2^4 - r_1^4\right)\right] + \frac{1}{2}\rho\pi t_{\text{tread}}\left(r_3^4 - r_2^4\right) \end{aligned}$$

Substituting,

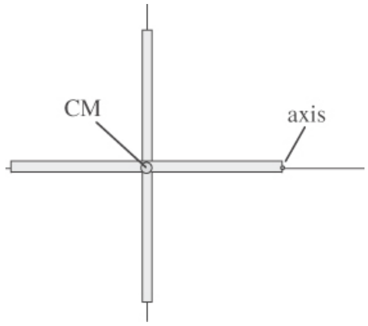
$$\begin{aligned} I_{\text{total}} = 2\Big\{&\frac{1}{2}\left(1.10\times10^3\ \text{kg/m}^3\right)\pi\left(6.35\times10^{-3}\ \text{m}\right) \\ &\times\left[(0.305\ \text{m})^4 - (0.165\ \text{m})^4\right]\Big\} \\ &+\frac{1}{2}\left(1.10\times10^3\ \text{kg/m}^3\right)\pi(0.200\ \text{m})\left[(0.330\ \text{m})^4 - (0.305\ \text{m})^4\right] \\ &= 2\left(8.68\times10^{-2}\ \text{kg}\cdot\text{m}^2\right) + 1.11\ \text{kg}\cdot\text{m}^2 = \boxed{1.28\ \text{kg}\cdot\text{m}^2} \end{aligned}$$

P10.42 We use x as a measure of the distance of each mass element dm in the rod from the y' axis:

$$\boxed{I_{y'} = \int_{\text{all mass}} r^2 dm = \int_0^L x^2 \frac{M}{L}dx = \frac{M}{L}\frac{x^3}{3}\bigg|_0^L = \frac{1}{3}ML^2}$$

P10.43 We assume the rods are thin, with radius much less than L. Note that the center of mass (CM) of the rod combination lies at the origin of the coordinate system. Because the axis of rotation is parallel to the y axis, we can first calculate the moment of inertia of the rods about the y axis, then use the parallel-axis theorem to find the moment about the axis of rotation.

The moment of the rod on the y axis about the y axis itself is essentially zero (axis through center, parallel to rod) because the rod is thin. The moments of the rods on the x and z axes are each $I = \frac{1}{12}mL^2$ (axis through center, perpendicular to rod) from the table in the chapter.



ANS. FIG. P10.43

The total moment of the three rods about the *y* axis (and about the CM) is

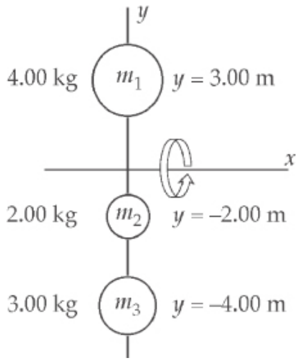
$$\begin{aligned} I_{\text{CM}} &= I_{\text{on x axis}} + I_{\text{on y axis}} + I_{\text{on z axis}} \\ &= \frac{1}{12}mL^2 + 0 + \frac{1}{12}mL^2 = \frac{1}{6}mL^2 \end{aligned}$$

For the moment of the rod-combination about the axis of rotation, the parallel-axis theorem gives

$$I = I_{\text{CM}} + 3m\left(\frac{L}{2}\right)^2 = \left[\frac{1}{6} + \frac{3}{4}\right]mL^2 = \left[\frac{2}{12} + \frac{9}{12}\right]mL^2 = \boxed{\frac{11}{12}mL^2}$$

Section 10.7 Rotational Kinetic Energy

P10.44 The masses and distances from the rotation axis for the three particles are:



ANS. FIG. P10.44

$$m_1 = 4.00 \text{ kg}, \; r_1 = |y_1| = 3.00 \text{ m}$$

$$m_2 = 2.00 \text{ kg}, \; r_2 = |y_2| = 2.00 \text{ m}$$

$$m_3 = 3.00 \text{ kg}, \; r_3 = |y_3| = 4.00 \text{ m}$$

and $\omega = 2.00 \text{ rad/s}$ about the *x* axis.

(a) $I_x = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$

$$\begin{aligned} I_x &= (4.00 \text{ kg})(3.00 \text{ m})^2 + (2.00 \text{ kg})(2.00 \text{ m})^2 \\ &\quad + (3.00 \text{ kg})(4.00 \text{ m})^2 \\ &= \boxed{92.0 \text{ kg}\cdot\text{m}^2} \end{aligned}$$

(b) $K_R = \frac{1}{2}I_x\omega^2 = \frac{1}{2}(92.0 \text{ kg}\cdot\text{m}^2)(2.00 \text{ m})^2 = \boxed{184 \text{ J}}$

(c) $v_1 = r_1\omega = (3.00 \text{ m})(2.00 \text{ rad/s}) = \boxed{6.00 \text{ m/s}}$

$v_2 = r_2\omega = (2.00 \text{ m})(2.00 \text{ rad/s}) = \boxed{4.00 \text{ m/s}}$

$v_3 = r_3\omega = (4.00 \text{ m})(2.00 \text{ rad/s}) = \boxed{8.00 \text{ m/s}}$

(d) $K_1 = \frac{1}{2}m_1v_1^2 = \frac{1}{2}(4.00 \text{ kg})(6.00 \text{ m/s})^2 = 72.0 \text{ J}$

$$K_2 = \frac{1}{2}m_2v_2^2 = \frac{1}{2}(2.00\text{ kg})(4.00\text{ m/s})^2 = 16.0\text{ J}$$

$$K_3 = \frac{1}{2}m_3v_3^2 = \frac{1}{2}(3.00\text{ kg})(8.00\text{ m/s})^2 = 96.0\text{ J}$$

$$K = K_1 + K_2 + K_3 = 72.0\text{ J} + 16.0\text{ J} + 96.0\text{ J} = \boxed{184\text{ J}} = \frac{1}{2}I_x\omega^2$$

(e) $\boxed{\text{The kinetic energies computed in parts (b) and (d) are the same.}}$

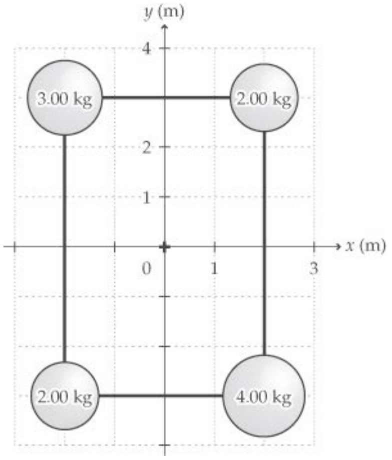
Rotational kinetic energy of an object rotating about a fixed axis can be viewed as the total translational kinetic energy of the particles moving in circular paths.

P10.45 (a) All four particles are at a distance r from the z axis, with

$$r^2 = (3.00\text{ m})^2 + (2.00\text{ m})^2 = 13.0\text{ m}^2$$

Thus the moment of inertia is

$$\begin{aligned} I_z &= \sum m_i r_i^2 \\ &= (3.00\text{ kg})(13.0\text{ m}^2) \\ &\quad + (2.00\text{ kg})(13.0\text{ m}^2) \\ &\quad + (4.00\text{ kg})(13.0\text{ m}^2) \\ &\quad + (2.00\text{ kg})(13.0\text{ m}^2) \\ &= \boxed{143\text{ kg}\cdot\text{m}^2} \end{aligned}$$



ANS. FIG. P10.45

(b) The rotational kinetic energy of the four-particle system is

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}(143\text{ kg}\cdot\text{m}^2)(6.00\text{ rad/s})^2 = \boxed{2.57\times 10^3\text{ J}}$$

P10.46 The cam is a solid disk of radius R that has had a small disk of radius $R/2$ cut from it. To find the moment of inertia of the cam, we use the parallel-axis theorem to find the moment of inertia of the solid disk about an axis at distance $R/2$ from its CM, then subtract off the moment of inertia of the small disk of radius $R/2$ with axis through its center.

By the parallel-axis theorem, the moment of inertia of the solid disk about an axis $R/2$ from its CM is

$$I_{\text{disk}} = I_{\text{CM}} + M_{\text{disk}}\left(\frac{R}{2}\right)^2 = \frac{1}{2}M_{\text{disk}}R^2 + \frac{1}{4}M_{\text{disk}}R^2 = \frac{3}{4}M_{\text{disk}}R^2$$

With half the radius, the cut-away small disk has one-quarter the face area and one-quarter the volume and one-quarter the mass M_{disk} of the original solid disk:

$$\frac{M_{\text{small disk}}}{M_{\text{disk}}} = \frac{(R/2)^2}{R^2} = \frac{1}{4}$$

The moment of inertia of the small disk of radius $R/2$ about an axis through its CM is

$$I_{\text{small disk}} = \frac{1}{2}M_{\text{small disk}}\left(\frac{R}{2}\right)^2 = \frac{1}{2}\left[\frac{1}{4}M_{\text{disk}}\right]\frac{R^2}{4} = \frac{1}{32}M_{\text{disk}}R^2$$

Subtracting the moment of the small disk from the solid disk, we find for the cam

$$I_{\text{cam}} = I_{\text{disk}} - I_{\text{small disk}} = \frac{3}{4}M_{\text{disk}}R^2 - \frac{1}{32}M_{\text{disk}}R^2$$

$$I_{\text{cam}} = M_{\text{disk}}R^2\left[\frac{24}{32} - \frac{1}{32}\right] = \frac{23}{32}M_{\text{disk}}R^2$$

The mass of the cam is $M = M_{\text{disk}} - M_{\text{small disk}} = M_{\text{disk}} - \frac{1}{4}M_{\text{disk}} = \frac{3}{4}M_{\text{disk}}$, therefore

$$I_{\text{cam}} = \frac{23}{32}M_{\text{disk}}R^2\left(\frac{M}{\frac{3}{4}M_{\text{disk}}}\right) = MR^2\left(\frac{23}{32}\right)\left(\frac{4}{3}\right) = \frac{23}{24}MR^2$$

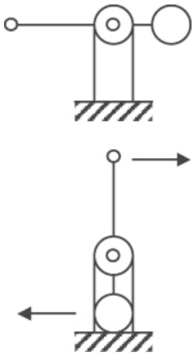
The moment of inertia of the cam-shaft is the sum of the moments of the cam and the shaft:

$$\begin{aligned} I_{\text{cam-shaft}} &= I_{\text{cam}} + I_{\text{shaft}} = \frac{23}{24}MR^2 + \frac{1}{2}M\left(\frac{R}{2}\right)^2 \\ &= MR^2\left[\frac{23}{24} + \frac{1}{8}\right] = MR^2\left[\frac{23}{24} + \frac{3}{24}\right] \\ I_{\text{cam-shaft}} &= \frac{26}{24}MR^2 = \frac{13}{12}MR^2 \end{aligned}$$

The kinetic energy of the cam-shaft combination rotating with angular speed ω is

$$K = \frac{1}{2}I_{\text{can-shaft}}\omega^2 = \frac{1}{2}\left(\frac{13}{12}MR^2\right)\omega^2 = \boxed{\frac{13}{24}MR^2\omega^2}$$

P10.47 (a) Identify the two objects and the Earth as an isolated system. The maximum speed of the lighter object will occur when the rod is in the vertical position so let's define the time interval as from when the system is released from rest to when the rod reaches a vertical orientation. So, for the isolated system,



ANS. FIG. P10.47

$$\Delta K + \Delta U = 0$$

$$\left[\left(\frac{1}{2}I_1\omega^2 + \frac{1}{2}I_2\omega^2\right) - 0\right] + \left[m_1 g y_1 + m_2 g y_2 - 0\right] = 0$$

$$\omega = \sqrt{\frac{-2g\left(m_1 y_1 + m_2 y_2\right)}{I_1 + I_2}} = \sqrt{\frac{-2g\left(m_1 y_1 + m_2 y_2\right)}{m_1 r_1^2 + m_2 r_2^2}}$$

$$= \sqrt{\frac{-2\left(9.80 \text{ m/s}^2\right)\left[\left(0.120 \text{ kg}\right)\left(2.86 \text{ m}\right) + \left(60.0 \text{ kg}\right)\left(-0.140 \text{ m}\right)\right]}{\left(0.120 \text{ kg}\right)\left(2.86 \text{ m}\right)^2 + \left(60.0 \text{ kg}\right)\left(0.140 \text{ m}\right)^2}}$$

$$= 8.55 \text{ rad/s}$$

Then, the tangential speed of the lighter object is,

$$v = r\omega = \left(2.86 \text{ m}\right)(8.55 \text{ rad/s}) = \boxed{24.5 \text{ m/s}}$$

(b) $\boxed{\text{No.}}$ The overall acceleration is not constant. It has to move either in a straight line or parabolic path to have a chance of being under constant acceleration. The circular path presented here rules out that possibility.

(c) $\boxed{\text{No.}}$ It does not move with constant tangential acceleration, since the angular acceleration is not constant. See explanation in part (d).

(d) $\boxed{\text{No.}}$ The lever arm of the gravitational force acting on the 60-kg mass changes during the motion. As a result, the torque changes, and so does the angular acceleration.

(e) $\boxed{\text{No.}}$ The angular velocity changes, therefore the angular momentum of the trebuchet changes.

(f) $\boxed{\text{Yes.}}$ The mechanical energy stays constant because the system is isolated—that is how we solved the problem in (a).

Section 10.8 Energy Considerations in Rotational Motion

P10.48 From the rigid object under a net torque model,

$$\sum\tau = I\alpha \quad\rightarrow\quad \alpha = \frac{\sum\tau}{I} = \frac{FR}{\frac{1}{2}MR^2} = \frac{2F}{MR}$$

From the definition of rotational kinetic energy and the rigid object under constant angular acceleration model,

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left(\omega_i + \alpha t\right)^2 = \frac{1}{2}I\alpha^2t^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{2F}{MR}\right)^2 t^2$$
$$= \frac{F^2t^2}{M}$$

Substituting,

$$K = \frac{\left(50.0\text{ N}\right)^2\left(3.00\text{ s}\right)^2}{800\text{ N}/9.80\text{ m/s}^2} = \boxed{276\text{ J}}$$

P10.49 The moment of inertia of a thin rod about an axis through one end is $I = \frac{1}{3}ML^2$. The total rotational kinetic energy is given as

$$K_R = \frac{1}{2}I_h\omega_h^2 + \frac{1}{2}I_m\omega_m^2$$

with $$I_h = \frac{m_hL_h^2}{3} = \frac{60.0\text{ kg}\left(2.70\text{ m}\right)^2}{3} = 146\text{ kg}\cdot\text{m}^2$$

and $$I_m = \frac{m_mL_m^2}{3} = \frac{100\text{ kg}\left(4.50\text{ m}\right)^2}{3} = 675\text{ kg}\cdot\text{m}^2$$

In addition,

$$\omega_h = \frac{2\pi\text{ rad}}{12\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$$

while $$\omega_m = \frac{2\pi\text{ rad}}{1\text{ h}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$$

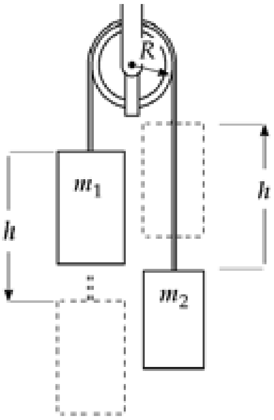
Therefore,

$$K_R = \frac{1}{2}\left(146\text{ kg}\cdot\text{m}^2\right)\left(1.45\times10^{-4}\text{ rad/s}\right)^2$$
$$+\frac{1}{2}\left(675\text{ kg}\cdot\text{m}^2\right)\left(1.75\times10^{-3}\text{ rad/s}\right)^2$$
$$= \boxed{1.04\times10^{-3}\text{ J}}$$

***P10.50** Take the two objects, pulley, and Earth as the system. If we neglect friction in the system, then mechanical energy is conserved and we can state that the increase in kinetic energy of the system equals the decrease in potential energy. Since $K_i = 0$ (the system is initially at rest), we have

$$\Delta K = K_f - K_i$$
$$= \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2 + \frac{1}{2}I\omega^2$$

where m_1 and m_2 have a common speed. But $v = R\omega$ so that $\Delta K = \frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v^2$.



ANS. FIG. P10.50

From ANS. FIG. P10.50, we see that the system loses potential energy because of the motion of m_1 and gains potential energy because of the motion of m_2. Applying the law of conservation of energy, $\Delta K + \Delta U = 0$, gives

$$\frac{1}{2}\left(m_1 + m_2 + \frac{I}{R^2}\right)v^2 + m_2gh - m_1gh = 0$$

$$\boxed{v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{R^2}}}}$$

Since $v = R\omega$, the angular speed of the pulley at this instant is given by

$$\boxed{\omega = \frac{v}{R} = \sqrt{\frac{2(m_1 - m_2)gh}{m_1R^2 + m_2R^2 + I}}}$$

P10.51 For the nonisolated system of the top,

$$W = \Delta K \quad \to \quad F\Delta x = \left(\frac{1}{2}I\omega^2 - 0\right)$$
$$\to \quad \omega = \sqrt{\frac{2F\Delta x}{I}} = \sqrt{\frac{2(5.57\text{ N})(0.800\text{ m})}{4 \times 10^{-4}\text{ kg}\cdot\text{m}^2}} = \boxed{149\text{ rad/s}}$$

P10.52 The power output of the bus is $P = \frac{E}{\Delta t}$, where

$$E = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}MR^2\omega^2\right) = \frac{1}{4}MR^2\omega^2$$

is the stored energy and $\Delta t = \frac{d}{v}$ is the time it can roll. Then $\frac{1}{4}MR^2\omega^2 = P\Delta t = \frac{Pd}{v}$. The maximum range of the bus is then $d = \frac{MR^2\omega^2 v}{4P}$.

For average $P = (25.0\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 18\ 650\text{ W}$ and average v =35.0 km/h = 9.72 m/s, the maximum range is

$$\begin{aligned} d &= \frac{MR^2\omega^2 v}{4P} \\ &= \frac{(1\,200\text{ kg})(0.500\text{ m})^2(3\ 000\cdot 2\pi/60\text{ s})^2(9.72\text{ m/s})}{4(18\,650\text{ W})} \\ &= 3.86\text{ km} \end{aligned}$$

The situation is impossible because the range is only 3.86 km, not city-wide.

P10.53 (a) Apply $\Delta K + \Delta U + \Delta E_{\text{int}} = 0$, where $\Delta E_{\text{int}} = f_k d$, $\mu = 0.250$, and $f_k = \mu n_2 = \mu m_2 g$. Both translational and rotational kinetic energy are present in the system. $v_i = 0.820$ m/s. Find v. The angular speed of the pulley is $\omega_i = v_i/R_2$, and $\omega = v/R_2$. Mass m_1 drops by $h = d$ when mass m_2 moves distance $d = 0.700$ m.

$I = \frac{1}{2}M\left(R_1^2 + R_2^2\right)$, where $R_1 = 0.020\ 0$ m, $R_2 = 0.030\ 0$ m, and $M = 0.350$ kg.

$$(K_f - K_i) + (U_f - U_f) + \Delta E_{\text{int}} = 0$$

$$\left(\frac{1}{2}m_2 v^2 - \frac{1}{2}m_2 v_i^2\right) + \left(\frac{1}{2}m_1 v^2 - \frac{1}{2}m_1 v_i^2\right) + \left(\frac{1}{2}I\omega^2 - \frac{1}{2}I\omega_i^2\right) + (m_1 gy - m_1 gy_i) + f_k d = 0$$

$$\frac{1}{2}(m_1 + m_2)(v^2 - v_i^2) + \frac{1}{2}I\left[\left(\frac{v}{R_2}\right)^2 - \left(\frac{v_i}{R_2}\right)^2\right] + m_1 g(y - y_i) + \mu m_2 gd = 0$$

$$\frac{1}{2}(m_1+m_2)(v^2-v_i^2) + \frac{1}{2}\left[\frac{1}{2}M\left(R_1^2+R_2^2\right)\right]\left(\frac{1}{R_2}\right)^2\left[v^2-v_i^2\right] + m_1 g(-d) + \mu m_2 gd = 0$$

$$\frac{1}{2}(m_1+m_2)(v^2-v_i^2) + \frac{1}{2}\left[\frac{1}{2}M\left(1+\frac{R_1^2}{R_2^2}\right)\right]\left[v^2-v_i^2\right] = gd(m_1-\mu m_2)$$

$$\frac{1}{2}\left[(m_1+m_2)+\frac{1}{2}M\left(1+\frac{R_1^2}{R_2^2}\right)\right](v^2-v_i^2) = gd(m_1-\mu m_2)$$

$$v = \left\{v_i^2 + \frac{4gd(m_1-\mu_k m_2)}{2(m_1+m_2)+M\left(1+\frac{R_1^2}{R_2^2}\right)}\right\}^{1/2}$$

Suppressing units,

$$v = \left\{(0.820)^2 + \frac{4(9.80)(0.700)[0.420-(0.250)(0.850)]}{\left[2(0.420+0.850)+0.350\left(1+\frac{(0.020\ 0)^2}{(0.030\ 0)^2}\right)\right]}\right\}^{1/2}$$

$$= \boxed{1.59\ \text{m/s}}$$

(b) $\omega = \frac{v}{r} = \frac{1.59\ \text{m/s}}{0.030\ 0\ \text{m}} = \boxed{53.1\ \text{rad/s}}$

P10.54 (a) For the isolated rod-ball-Earth system,

$$\Delta K + \Delta U = 0 \rightarrow (K_f - 0) + (0 - U_i) = 0 \rightarrow K_f = U_i$$

$$K_f = m_{\text{rod}} g y_{\text{CM, rod}} + m_{\text{ball}} g y_{\text{CM, ball}}$$

$$= (m_{\text{rod}} y_{\text{CM, rod}} + m_{\text{ball}} y_{\text{CM, ball}})g$$

$$= [(1.20\ \text{kg})(0.120\ \text{m}) + (2.00\ \text{kg})(0.280\ \text{m})](9.80\ \text{m/s}^2)$$

$$= \boxed{6.90\ \text{J}}$$

(b) We assume the rod is thin. For the compound object

$$\begin{aligned} I &= \frac{1}{3}M_{\text{rod}}L^2 + \left[\frac{2}{5}m_{\text{ball}}R^2 + M_{\text{ball}}D^2\right] \\ &= \frac{1}{3}(1.20\ \text{kg})(0.240\ \text{m})^2 \\ &\qquad + \frac{2}{5}(2.00\ \text{kg})(4.00\times10^{-2}\ \text{m})^2 + (2.00\ \text{kg})(0.280\ \text{m})^2 \end{aligned}$$

$$I = 0.181\ \text{kg}\cdot\text{m}^2$$

$$K_f = \frac{1}{2}\omega^2 \rightarrow \omega = \sqrt{\frac{2K_f}{I}} = \sqrt{\frac{2(6.90\ \text{J})}{0.181\ \text{kg}\cdot\text{m}^2}} = \boxed{8.73\ \text{rad/s}}$$

(c) $v = r\omega = (0.280\ \text{m})(8.73\ \text{rad/s}) = \boxed{2.44\ \text{m/s}}$

(d) $v_f^2 = v_i^2 + 2a(y_f - y_i)$

$$v_f = \sqrt{0 + 2(9.80\ \text{m/s}^2)(0.280\ \text{m})} = 2.34\ \text{m/s}$$

$\boxed{\text{The speed it attains in swinging is greater by}}$

$$\frac{2.44}{2.34} = \boxed{1.0432\ \text{times}}$$

P10.55 The gravitational force exerted on the reel is

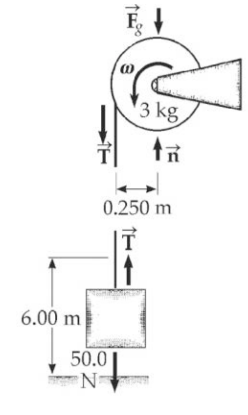
$$mg = (5.10\ \text{kg})(9.80\ \text{m/s}^2) = 50.0\ \text{N down}$$

We use $\sum\tau = I\alpha$ to find T and a.

First find I for the reel, which we know is a uniform disk.

$$\begin{aligned} I &= \frac{1}{2}MR^2 = \frac{1}{2}(3.00\ \text{kg})(0.250\ \text{m})^2 \\ &= 0.093\ 8\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

The forces on the reel are shown in ANS. FIG. P10.55, including a normal force exerted by its axle. From the diagram, we can see that the tension is the only force that produces a torque causing the reel to rotate.



ANS. FIG. P10.55

$\sum\tau = I\alpha$ becomes

$$n(0) + F_{gp}(0) + T(0.250\ \text{m}) = (0.093\ 8\ \text{kg}\cdot\text{m}^2)(a/0.250\ \text{m}) \qquad \textbf{[1]}$$

where we have applied $a_t = r\alpha$ to the point of contact between string and reel. For the object that moves down,

$$\sum F_y = ma_y \qquad \text{becomes} \qquad 50.0\text{ N} - T = (5.10\text{ kg})a \qquad [2]$$

Note that we have defined downwards to be positive, so that positive linear acceleration of the object corresponds to positive angular acceleration of the reel. We now have our two equations in the unknowns T and a for the two connected objects. Substituting T from equation [2] into equation [1], we have

$$\left[50.0\text{ N} - (5.10\text{ kg})a\right](0.250\text{ m}) = (0.093\ 8\text{ kg}\cdot\text{m}^2)\left(\frac{a}{0.250\text{ m}}\right)$$

(b) Solving for a from above gives

$$50.0\text{ N} - (5.10\text{ kg})a = (1.50\text{ kg})a$$

$$a = \frac{50.0\text{ N}}{6.60\text{ kg}} = \boxed{7.57\text{ m/s}^2}$$

Because we eliminated T in solving the simultaneous equations, the answer for a, required for part (b), emerged first. No matter—we can now substitute back to get the answer to part (a).

(a) $T = 50.0\text{ N} - 5.10\text{ kg }(7.57\text{ m/s}^2) = \boxed{11.4\text{ N}}$

(c) For the motion of the hanging weight,

$$v_f^2 = v_i^2 + 2a(y_f - y_i) = 0^2 + 2(7.57\text{ m/s}^2)(6.00\text{ m})$$

$$v_f = 9.53\text{ m/s (down)}$$

(d) The isolated-system energy model can take account of multiple objects more easily than Newton's second law. Like your bratty cousins, the equation for conservation of energy grows between visits. Now it reads for the counterweight-reel-Earth system:

$$(K_1 + K_2 + U_g)_i = (K_1 + K_2 + U_g)_f$$

where K_1 is the translational kinetic energy of the falling object and K_2 is the rotational kinetic energy of the reel.

$$0 + 0 + m_1 g y_{1i} = \frac{1}{2}m_1 v_{1f}^2 + \frac{1}{2}I_2\omega_{2f}^2 + 0$$

Now note that $\omega = v/r$ as the string unwinds from the reel.

$$mgy_i = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$2mgy_i = mv^2 + I\left(\frac{v^2}{R^2}\right) = v^2\left(m + \frac{I}{R^2}\right)$$

$$v = \sqrt{\frac{2mgy_i}{m + (I/R^2)}} = \sqrt{\frac{2(5.10\text{ kg})(9.80\text{ m/s}^2)(6.00\text{ m})}{5.10\text{ kg} + \dfrac{0.093\,8\text{ kg}\cdot\text{m}^2}{(0.250\text{ m})^2}}}$$

$$= \boxed{9.53\text{ m/s}}$$

P10.56 Each point on the cord moves at a linear speed of $v = \omega r$, where r is the radius of the spool. The energy conservation equation for the counterweight-turntable-Earth system is:

$$(K_1 + K_2 + U_g)_i + W_{\text{other}} = (K_1 + K_2 + U_g)_f$$

Specializing, we have

$$0 + 0 + mgh + 0 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + 0$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{r^2}$$

$$2mgh - mv^2 = I\frac{v^2}{r^2}$$

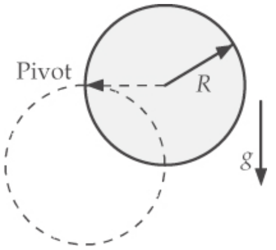
and finally,

$$I = \boxed{mr^2\left(\frac{2gh}{v^2} - 1\right)}$$

P10.57 To identify the change in gravitational energy, think of the height through which the center of mass falls. From the parallel-axis theorem, the moment of inertia of the disk about the pivot point on the circumference is

$$I = I_{\text{CM}} + MD^2 = \frac{1}{2}MR^2 + MR^2$$

$$= \frac{3}{2}MR^2$$



ANS. FIG. P10.57

The pivot point is fixed, so the kinetic energy is entirely rotational around the pivot. The equation for the isolated system (energy) model

$$(K + U)_i = (K + U)_f$$

for the disk-Earth system becomes

$$0 + MgR = \frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega^2 + 0$$

Solving for ω, $\omega = \sqrt{\frac{4g}{3R}}$

(a) At the center of mass, $v = R\omega = \boxed{2\sqrt{\frac{Rg}{3}}}$

(b) At the lowest point on the rim, $v = 2R\omega = \boxed{4\sqrt{\frac{Rg}{3}}}$

(c) For a hoop,

$$I_{\text{CM}} = MR^2 \quad \text{and} \quad I_{\text{min}} = 2MR^2$$

By conservation of energy for the hoop-Earth system, then

$$MgR = \frac{1}{2}\left(2MR^2\right)\omega^2 + 0$$

so $\omega = \sqrt{\frac{g}{R}}$

and the center of mass moves at $v_{\text{CM}} = R\omega = \boxed{\sqrt{gR}}$, slower than the disk.

P10.58 (a) The moment of inertia of the cord on the spool is

$$\frac{1}{2}M\left(R_1^2 + R_2^2\right) = \frac{1}{2}(0.100 \text{ kg})\left[(0.015\,0 \text{ m})^2 + (0.090\,0 \text{ m})^2\right]$$
$$= 4.16 \times 10^{-4} \text{ kg} \cdot \text{m}^2$$

The protruding strand has mass

$$\left(1.00 \times 10^{-2} \text{ kg/m}\right)\left(0.160 \text{ m}\right) = 1.60 \times 10^{-3} \text{ kg}$$

and moment of inertia

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2$$
$$= \left(1.60 \times 10^{-3} \text{ kg}\right)\left[\frac{1}{12}(0.160 \text{ m})^2 + (0.090\,0 \text{ m} + 0.080\,0 \text{ m})^2\right]$$
$$= 4.97 \times 10^{-5} \text{ kg} \cdot \text{m}^2$$

For the whole cord, $I = 4.66 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$. In speeding up, the average power is

$$P = \frac{E}{\Delta t} = \frac{\frac{1}{2}I\omega^2}{\Delta t} = \left[\frac{4.66\times10^{-4}\ \text{kg}\cdot\text{m}^2}{2(0.215\ \text{s})}\right]\left(\frac{2\,500\cdot 2\pi}{60\ \text{s}}\right)^2 = \boxed{74.3\ \text{W}}$$

(b) $P = \tau\omega = (7.65\ \text{N})(0.160\ \text{m} + 0.090\,0\ \text{m})\left(\frac{2\,000\cdot 2\pi}{60\ \text{s}}\right) = \boxed{401\ \text{W}}$

Section 10.9 Rolling Motion of a Rigid Object

P10.59 (a) The kinetic energy of translation is

$$K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}(10.0\ \text{kg})(10.0\ \text{m/s})^2 = \boxed{500\ \text{J}}$$

(b) Call the radius of the cylinder *r*. An observer at the center sees the rough surface and the circumference of the cylinder moving at 10.0 m/s, so the angular speed of the cylinder is

$$\omega = \frac{v_{\text{CM}}}{r} = \frac{10.0\ \text{m/s}}{r}$$

The moment of inertia about an axis through the center of mass is $I_{\text{CM}} = \frac{1}{2}mr^2$, so

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 = \frac{1}{4}(10.0\ \text{kg})(10.0\ \text{m/s})^2$$
$$= \boxed{250\ \text{J}}$$

(c) We can now add up the total energy:

$$K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\ \text{J}}$$

P10.60 Conservation of energy for the sphere rolling without slipping is

$$U_i = K_{\text{translation, f}} + K_{\text{rotation, f}}$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{v}{R}\right)^2 = \frac{7}{10}mv^2$$

which gives $\boxed{v_f = \sqrt{\frac{10}{7}gh}}$

Conservation of energy for the sphere sliding without friction, with $\omega = 0$, is

$$mgh = \frac{1}{2}mv^2$$

which gives $\boxed{v_f = \sqrt{2gh}}$

The time intervals required for the trips follow from $x = 0 + v_{avg}t$:

$$\frac{h}{\sin\theta} = \left(\frac{0+v_f}{2}\right)t \quad \rightarrow \quad t = \frac{2h}{v_f \sin\theta}$$

For rolling we have $t = \left(\frac{2h}{\sin\theta}\right)\sqrt{\frac{10}{7}gh}$

and for sliding, $t = \left(\frac{2h}{\sin\theta}\right)\sqrt{\frac{1}{2}gh}$

$\boxed{\text{The time to roll is longer by a factor of } (0.7/0.5)^{1/2} = 1.18.}$

P10.61 (a) We can consider the weight force acting at the center of mass (gravity) to exert a torque about the point of contact (the axis, in this case) between the disk and the incline. Then, from the particle under a net torque model, we have

$$\tau = I\alpha \quad \text{and} \quad a = R\alpha$$

$$mgR\sin\theta = \left(I_{CM} + mR^2\right)\alpha$$

$$a = \frac{mgR^2\sin\theta}{I_{CM} + mR^2}$$

$$a_{disk} = \frac{mgR^2\sin\theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g\sin\theta}$$

(b) By the same method,

$$a = \frac{mgR^2\sin\theta}{I_{CM} + mR^2}$$

$\boxed{a_{hoop} = \frac{mgR^2\sin\theta}{2mR^2} = \frac{1}{2}g\sin\theta\text{. The acceleration of the hoop is smaller than that of the disk.}}$

(c) Torque about the CM is caused by friction because the lever arm of the weight force is zero:

$$\tau = fR = I\alpha$$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{I\alpha/R}{mg\cos\theta} = \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

P10.62 (a) Both systems of cube-Earth and cylinder-Earth are isolated; therefore, mechanical energy is conserved in both. $\boxed{\text{The cylinder}}$ has extra kinetic energy, in the form of rotational kinetic energy, that is available to be transformed into potential energy, so it travels farther up the incline.

(b) The system of cube-Earth is isolated, so mechanical energy is conserved:

$$K_i = U_f \quad\rightarrow\quad \frac{1}{2}mv^2 = mgd\sin\theta \quad\rightarrow\quad d = \frac{v^2}{2g\sin\theta}$$

Static friction does no work on the cylinder because it acts at the point of contact and not through a distance; therefore, mechanical energy is conserved in the cylinder-Earth system:

$$K_{\text{translation, i}} + K_{\text{rotation, i}} = U_f \rightarrow \frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{1}{2}mr^2\right]\left(\frac{v}{r}\right)^2 = mgd\sin\theta$$

which gives $d = \dfrac{3v^2}{4g\sin\theta}$.

The difference in distance is

$$\frac{3v^2}{4g\sin\theta} - \frac{v^2}{2g\sin\theta} = \boxed{\frac{v^2}{4g\sin\theta}}$$

or, the cylinder travels 50% farther.

(c) $\boxed{\text{The cylinder does not lose mechanical energy because static friction does no work on it. Its rotation means that it has 50\% more kinetic energy than the cube at the start, and so it travels 50\% farther up the incline.}}$

P10.63 (a) $\boxed{\text{The disk reaches the bottom first}}$ because the ratio of its moment of inertia to its mass is smaller than for the hoop; this result is independent of the radius.

(b) Both systems of disk-Earth and hoop-Earth are isolated because static friction does no work because it acts at the point of contact and not through a distance. Mechanical energy is conserved in both systems:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$$

where $\omega = \dfrac{v}{R}$ since no slipping occurs.

Also, $U_i = mgh, U_f = 0$, and $v_i = 0$

Therefore, $\dfrac{1}{2}\left[m + \dfrac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \dfrac{2gh}{\left[1 + \left(I/mR^2\right)\right]}$

For a disk, $I = \dfrac{1}{2}mR^2$, so $v^2 = \dfrac{2gh}{1 + \frac{1}{2}}$ or $\boxed{v_{\text{disk}} = \sqrt{\dfrac{4gh}{3}}}$

For a hoop, $I = mR^2$ so $v^2 = \dfrac{2gh}{2}$ or $\boxed{v_{\text{hoop}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{hoop}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

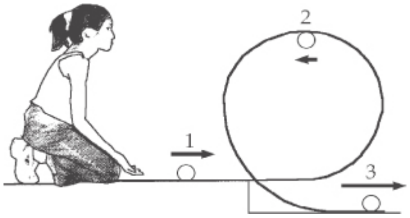
P10.64 (a) Energy conservation for the system of the ball and the Earth between the horizontal section and top of loop:

$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$\begin{aligned} v_2 &= \sqrt{v_1^2 - \frac{6}{5}gy_2} \\ &= \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(0.900\text{ m})} \\ &= \boxed{2.38\text{ m/s}} \end{aligned}$$



ANS. FIG. P10.64

(b) The centripetal acceleration at the top is

$$\frac{v_2^2}{r} = \frac{(2.38\text{ m/s})^2}{0.450\text{ m}} = 12.6\text{ m/s}^2 > g$$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(c) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\begin{aligned} v_3 &= \sqrt{v_1^2 - \frac{6}{5}gy_3} \\ &= \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(-0.200\text{ m})} \\ &= \boxed{4.31\text{ m/s}} \end{aligned}$$

(d) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$\begin{aligned} v_2 &= \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.900\text{ m})} \\ &= \boxed{\sqrt{-1.40\text{ m}^2/\text{s}^2}}\,! \end{aligned}$$

(e) This result is imaginary. In the case where the ball does not roll, the ball starts with less kinetic energy than in part (a) and $\boxed{\text{never makes it to the top of the loop}}$.

P10.65 (a) For the isolated can-Earth system,

$$\Delta K + \Delta U = 0 \rightarrow \left(\frac{1}{2}mv_{\text{CM}}^2 + \frac{1}{2}I\omega^2 - 0\right) + (0 - mgh) = 0$$

which gives

$$I = \frac{2mgh - mv_{\text{CM}}^2}{\omega^2} = \left(2mgh - mv_{\text{CM}}^2\right)\left(\frac{r^2}{v_{\text{CM}}^2}\right) = mr^2\left(\frac{2gh}{v_{\text{CM}}^2} - 1\right)$$

From the particle under constant acceleration model,

$$v_{\text{CM, avg}} = \frac{0 + v_{\text{CM}}}{2} \rightarrow v_{\text{CM}} = 2v_{\text{CM, avg}} = \frac{2d}{\Delta t}$$

Therefore, the moment of inertia is

$$I = mr^2\left(\frac{2gh(\Delta t)^2}{4d^2} - 1\right) = mr^2\left(\frac{2g(d\sin\theta)(\Delta t)^2}{4d^2} - 1\right)$$

$$= mr^2\left(\frac{g(\sin\theta)(\Delta t)^2}{2d} - 1\right)$$

Substitute numerical values:

$$I = (0.215\text{ kg})(0.031\,9\text{ m})^2 \times\left(\frac{(9.80\text{ m/s}^2)(\sin 25.0°)(1.50\text{ s})^2}{2(3.00\text{ m})} - 1\right)$$

$$= \boxed{1.21 \times 10^{-4}\text{ kg}\cdot\text{m}^2}$$

(b) The $\boxed{\text{height of the can}}$ is unnecessary data.

(c) $\boxed{\text{The mass is not uniformly distributed; the density of the metal can is larger than that of the soup.}}$

Additional Problems

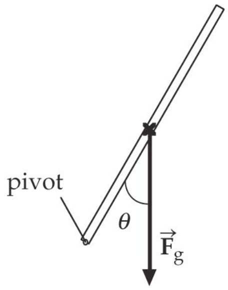
P10.66 When the rod is at angle θ from the vertical, the vertical weight force *mg* is at the same angle from the vertical so that its torque about the pivot is $mg\frac{\ell}{2}\sin\theta$. From the particle under a net torque model,

$$\sum\tau = I\alpha$$

$$mg\frac{\ell}{2}\sin\theta = \frac{1}{3}m\ell^2\alpha$$

$$\alpha = \frac{3}{2}\frac{g}{\ell}\sin\theta \rightarrow a_t = \left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r$$

$$\text{For}\left(\frac{3}{2}\frac{g}{\ell}\sin\theta\right)r > g\sin\theta \rightarrow r > \frac{2}{3}\ell$$



ANS. FIG. P10.66

$\therefore$ About $\boxed{\frac{1}{3}\text{ the length of the chimney}}$ will have a tangential acceleration greater than $g\sin\theta$.

P10.67 (a) The spool starts from rest, with zero rotational kinetic energy, and accelerates to 8.00 rad/s. The work done to accomplish this is given by the work-kinetic energy theorem:

$$W = \Delta K = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \frac{1}{2}I\left(\omega_f^2 - \omega_i^2\right), \quad \text{where} \quad I = \frac{1}{2}mR^2$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(1.00\text{ kg})(0.500\text{ m})^2\left[(8.00\text{ rad/s})^2 - 0\right] = \boxed{4.00\text{ J}}$$

(b) The time interval can be found from

$$\omega_f = \omega_i + \alpha t, \quad \text{where} \quad \alpha = \frac{a}{r} = \frac{2.50\text{ m/s}^2}{0.500\text{ m}} = 5.00\text{ rad/s}^2$$

Therefore,

$$t = \frac{\omega_f - \omega_i}{\alpha} = \frac{8.00\text{ rad/s} - 0}{5.00\text{ rad/s}^2} = \boxed{1.60\text{ s}}$$

(c) The spool turns through angular displacement

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$= 0 + 0 + \frac{1}{2}(5.00\text{ rad/s}^2)(1.60\text{ s})^2 = 6.40\text{ rad}$$

The length pulled from the spool is

$$s = r\theta = (0.500\text{ m})(6.40\text{ rad}) = 3.20\text{ m}$$

When the spool reaches an angular velocity of 8.00 rad/s, 1.60 s will have elapsed and 3.20 m of cord will have been removed from the spool. Remaining on the spool will be $\boxed{0.800\text{ m}}$.

P10.68 (a) We consider the elevator-sheave-counterweight-Earth system, including n passengers, as an isolated system and apply the conservation of mechanical energy. We take the initial configuration, at the moment the drive mechanism switches off, as representing zero gravitational potential energy of the system.

Therefore, the initial mechanical energy of the system [elevator (e), counterweight (c), sheave (s)] is

$$E_i = K_i + U_i = \frac{1}{2}m_e v^2 + \frac{1}{2}m_c v^2 + \frac{1}{2}I_s\omega^2 + 0$$

$$= \frac{1}{2}m_e v^2 + \frac{1}{2}m_c v^2 + \frac{1}{2}\left[\frac{1}{2}m_s r^2\right]\left(\frac{v}{r}\right)^2$$

$$= \frac{1}{2}\left[m_e + m_c + \frac{1}{2}m_s\right]v^2$$

The final mechanical energy of the system is entirely gravitational because the system is momentarily at rest:

$$E_f = K_f + U_f = 0 + m_e g d - m_c g d$$

where we have recognized that the elevator car goes up by the same distance d that the counterweight goes down. Setting the initial and final energies of the system equal to each other, we have

$$\frac{1}{2}\left[m_e + m_c + \frac{1}{2}m_s\right]v^2 = \left(m_e - m_c\right)gd$$

$$\frac{1}{2}\left\{\left[800\text{ kg} + n\left(80.0\text{ kg}\right)\right] + 950\text{ kg} + 140\text{ kg}\right\}\left(3.00\text{ m/s}\right)^2$$
$$= \left[800\text{ kg} + n\left(80.0\text{ kg}\right) - 950\text{ kg}\right](9.80\text{ m/s}^2)d$$

$$\boxed{d = \left(1890 + 80n\right)\left(\frac{0.459\text{m}}{80n - 150}\right)}$$

(b) For $n = 2$: $d = \left(1890 + 80.0\times 2\right)\dfrac{0.459\text{m}}{\left(80.0\times 2 - 150\right)} = \boxed{94.1\text{ m}}$

(c) For $n = 12$: $d = \left(1890 + 80.0\times 12\right)\dfrac{0.459\text{ m}}{\left(80.0\times 12 - 150\right)} = \boxed{1.62\text{ m}}$

(d) For $n = 0$: $d = \left(1890 + 80.0\times 0\right)\dfrac{0.459\text{ m}}{\left(80.0\times 0 - 150\right)} = \boxed{-5.79\text{ m}}$

(e) $\boxed{\text{The raising car will coast to a stop only for } n \geq 2.}$

(f) $\boxed{\text{For } n = 0 \text{ or } n = 1\text{, the mass of the elevator is less than the counterweight, so the car would accelerate upward if released.}}$

(g) For $n \to \infty$, $d \to 80n\left(0.459\text{ m}\right)/\left(80n\right) = \boxed{0.459\text{ m}}$

P10.69 (a) We find the angular speed by integrating the angular acceleration, which is given as $\alpha = -10.0 - 5.00t = \frac{d\omega}{dt}$, where α is in rad/s^2 and *t* is in seconds:

$$\Delta\omega = \int_{65.0}^{\omega} d\omega = \int_0^t [-10.0 - 5.00t]dt$$

$$\omega - 65.0 = -10.0t - 2.50t^2 \rightarrow \omega = 65.0 - 10.0t - 2.50t^2$$

where ω is in rad/s and *t* is in seconds.

For *t* = 3.00 s: $\omega = 65.0 - 10.0(3.00) - 2.50(3.00)^2 = \boxed{12.5\,\text{rad/s.}}$

(b) $\omega = \frac{d\theta}{dt} = 65.0\text{ rad/s} - (10.0\text{ rad/s}^2)t - (2.50\text{ rad/s}^3)t^2$

Suppressing units,

$$\Delta\theta = \int_0^t \omega\, dt = \int_0^t [65.0 - 10.0t - 2.50t^2]dt$$

$$\Delta\theta = 65.0t - 5.00t^2 - (2.50/3)t^3$$

$$\Delta\theta = 65.0t - 5.00t^2 - 0.833t^3$$

At *t* = 3.00 s,

$$\Delta\theta = (65.0\ \text{rad/s})(3.00\text{ s}) - (5.00\ \text{rad/s}^2)(9.00\text{ s}^2) - (0.833\ \text{rad/s}^3)(27.0\text{ s}^3)$$

$$\Delta\theta = \boxed{128\text{ rad}}$$

P10.70 (a) We find the angular speed by integrating the angular acceleration, which is given as $\alpha(t) = A + Bt = \frac{d\omega}{dt}$, where the shaft is turning at angular speed ω at time *t* = 0.

$$\Delta\omega = \int_{\omega(0)}^{\omega(t)} d\omega = \int_0^t [A + Bt]dt$$

$$\omega(t) - \omega(0) = At + \frac{1}{2}Bt^2,\text{ and }\omega(0) = \omega \rightarrow \boxed{\omega(t) = \omega + At + \frac{1}{2}Bt^2}$$

(b) $\frac{d\theta}{dt} = \omega + At + \frac{1}{2}Bt^2$

$$\Delta\theta = \int_0^t \omega(t)\,dt = \int_0^t \left[\omega + At + \frac{1}{2}Bt^2\right]dt$$

$$\Delta\theta = \boxed{\omega t + \frac{1}{2}At^2 + \frac{1}{6}Bt^3}$$

***P10.71** The resistive force on each ball is $R = D\rho A v^2$. Here $v = r\omega$, where *r* is the radius of each ball's path. The resistive torque on each ball is $\tau = rR$, so the total resistive torque on the three-ball system is $\tau_{\text{total}} = 3rR$.

The power required to maintain a constant rotation rate is $P = \tau_{\text{total}}\omega = 3rR\omega$. This required power may be written as

$$P = \tau_{\text{total}}\omega = 3r\left[D\rho A(r\omega)^2\right]\omega = \left(3r^3 DA\omega^3\right)\rho$$

with

$$\omega = \frac{2\pi \text{ rad}}{1 \text{ rev}}\left(\frac{10^3 \text{ rev}}{1 \text{ min}}\right)\left(\frac{1 \text{ min}}{60.0 \text{ s}}\right) = \frac{1\,000\pi}{30.0} \text{ rad/s}$$

Then

$$P = 3(0.100 \text{ m})^3(0.600)\left(4.00\times10^{-4} \text{ m}^2\right)\left(\frac{1\,000\pi}{30.0 \text{ s}}\right)^3 \rho$$

or $P = \left(0.827 \text{ m}^5/\text{s}^3\right)\rho$, where ρ is the density of the resisting medium.

(a) In air, $\rho = 1.20 \text{ kg/m}^3$, and

$$P = \left(0.827 \text{ m}^5/\text{s}^3\right)\left(1.20 \text{ kg/m}^3\right) = 0.992 \text{ N}\cdot\text{m/s} = \boxed{0.992 \text{ W}}$$

(b) In water, $\rho = 1\,000 \text{ kg/m}^3$ and $P = \boxed{827 \text{ W}}$.

***P10.72** Consider the total weight of each hand to act at the center of gravity (midpoint) of that hand. Then the total torque (taking CCW as positive) of these hands about the center of the clock is given by

$$\tau = -m_h g\left(\frac{L_h}{2}\right)\sin\theta_h - m_m g\left(\frac{L_m}{2}\right)\sin\theta_m$$

$$= -\frac{g}{2}\left(m_h L_h \sin\theta_h + m_m L_m \sin\theta_m\right)$$

If we take *t* = 0 at 12 o'clock, then the angular positions of the hands at time *t* are $\theta_h = \omega_h t$, where $\omega_h = \frac{\pi}{6}$ rad/h and $\theta_m = \omega_m t$, where $\omega_m = 2\pi$ rad/h. Therefore,

$$\tau = \left(-4.90\ \text{m/s}^2\right) \times \left[(60.0\ \text{kg})(2.70\ \text{m})\sin\left(\frac{\pi t}{6}\right) + (100\ \text{kg})(4.50\ \text{m})\sin 2\pi t\right]$$

or $$\tau = (-794\ \text{N}\cdot\text{m})\left[\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t\right],$$ where t is in hours.

(a) (i) At 3:00, $t = 3.00$ h, so

$$\tau = (-794\ \text{N}\cdot\text{m})\left[\sin\left(\frac{\pi}{2}\right) + 2.78\sin 6\pi\right] = \boxed{-794\ \text{N}\cdot\text{m}}$$

(ii) At 5:15, $t = 5\ \text{h} + \frac{15}{60}\ \text{h} = 5.25\ \text{h},$ and substitution gives:

$$\tau = \boxed{-2\,510\ \text{N}\cdot\text{m}}$$

(iii) At 6:00, $\tau = \boxed{0\ \text{N}\cdot\text{m}}$

(iv) At 8:20, $\tau = \boxed{-1\,160\ \text{N}\cdot\text{m}}$

(v) At 9:45, $\tau = \boxed{2\,940\ \text{N}\cdot\text{m}}$

(b) The total torque is zero at those times when

$$\sin\left(\frac{\pi t}{6}\right) + 2.78\sin 2\pi t = 0$$

We proceed numerically, to find 0, 0.515 295 5, ..., corresponding to the times

12:00:00 12:30:55 12:58:19 1:32:31 1:57:01

2:33:25 2:56:29 3:33:22 3:56:55 4:32:24

4:58:14 5:30:52 6:00:00 6:29:08 7:01:46

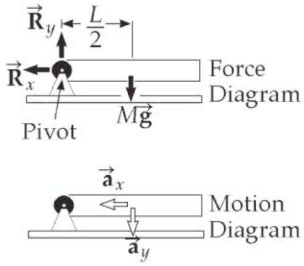
7:27:36 8:03:05 8:26:38 9:03:31 9:26:35

10:02:59 10:27:29 11:01:41 11:29:05

P10.73 (a) Since only conservative forces are acting on the bar, we have conservation of energy of the bar-Earth system:

$$K_i + U_i = K_f + U_f$$

For evaluation of the gravitational energy of the system, a rigid body can be modeled as a particle at its center of mass. Take the zero configuration for potential energy for the bar-Earth system with the bar horizontal.



ANS. FIG. P10.73

Under these conditions, $U_f = 0$ and $U_i = MgL/2$.

Using the conservation of energy equation above,

$$0 + \frac{1}{2}MgL = \frac{1}{2}I\omega_f^2 \quad \text{and} \quad \omega_f = \sqrt{MgL/I}$$

For a bar rotating about an axis through one end, $I = ML^2/3$.

Therefore,

$$\omega_f = \sqrt{\frac{MgL}{\frac{1}{3}ML^2}} = \boxed{\sqrt{\frac{3g}{L}}}$$

Note that we have chosen clockwise rotation as positive.

(b) $\sum \tau = I\alpha$: $\quad Mg\left(\frac{L}{2}\right) = \left(\frac{1}{3}ML^2\right)\alpha$ and $\alpha = \boxed{\frac{3g}{2L}}$

(c) $a_x = -a_c = -r\omega_f^2 = -\left(\frac{L}{2}\right)\left(\frac{3g}{L}\right) = -\frac{3g}{2}$

Since this is **centripetal** acceleration, it is directed along the **negative** horizontal.

$$a_y = -a_t = -r\alpha = \frac{L}{2}\alpha = -\frac{3g}{4}$$

$$\boxed{\vec{a} = -\frac{3}{2}g\hat{\mathbf{i}} - \frac{3}{4}g\hat{\mathbf{j}}}$$

(d) The pivot exerts a force $\vec{\mathbf{F}}$ on the rod. Using Newton's second law, we find

$$F_x = Ma_x = -\frac{3}{2}Mg$$

$$F_y - Mg = Ma_y = -\frac{3}{4}Mg \to F_y = Mg - \frac{3}{4}Mg = \frac{1}{4}Mg$$

$$\boxed{\vec{\mathbf{F}} = M\vec{\mathbf{a}} = -\frac{3}{2}Mg\hat{\mathbf{i}} + \frac{1}{4}Mg\hat{\mathbf{j}}}$$

P10.74 We assume that air resistance has a negligible effect on a drop so that mechanical energy is conserved in the drop-Earth system. The first drop leaving the wheel has a velocity v_1 directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1$$

so $$v_1 = \sqrt{2gh_1} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(0.540\text{ m}\right)} = 3.25\text{ m/s}$$

Similarly, the second drop has a velocity given by

$$v_2 = \sqrt{2gh_2} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(0.510\text{ m}\right)} = 3.16\text{ m/s}$$

From $\omega = \frac{v}{r}$, we find

$$\omega_1 = \frac{v_1}{r} = \frac{3.25\text{ m/s}}{0.381\text{ m}} = 8.53\text{ rad/s}$$

and $$\omega_2 = \frac{v_2}{r} = \frac{3.16\text{ m/s}}{0.381\text{ m}} = 8.29\text{ rad/s}$$

or $$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\Delta\theta} = \frac{\left(8.29\text{ rad/s}\right)^2 - \left(8.53\text{ rad/s}\right)^2}{4\pi} = \boxed{-0.322\text{ rad/s}^2}$$

P10.75 We assume that air resistance has a negligible effect on a drop so that mechanical energy is conserved in the drop-Earth system. At the instant it comes off the wheel, the first drop has a velocity v_1 directed upward. The magnitude of this velocity is found from

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_1^2 + 0 = 0 + mgh_1 \text{ or } v_1 = \sqrt{2gh_1}$$

The angular velocity of the wheel at the instant the first drop leaves is

$$\omega_1 = \frac{v_1}{R} = \sqrt{\frac{2gh_1}{R^2}}$$

Similarly for the second drop: $v_2 = \sqrt{2gh_2}$ and $\omega_2 = \frac{v_2}{R} = \sqrt{\frac{2gh_2}{R^2}}$

The angular acceleration of the wheel is then

$$a = \frac{\omega_2^2 - \omega_1^2}{2\Delta\theta} = \frac{2gh_2/R^2 - 2gh_1/R^2}{2(2\pi)} = \boxed{\frac{g(h_2 - h_1)}{2\pi R^2}}$$

P10.76 (a) Modeling the Earth as a sphere, its rotational kinetic energy is

$$\begin{aligned} K &= \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\omega^2\right) \\ &= \frac{1}{2}\left[\frac{2}{5}\left(5.98\times10^{24}\ \text{kg}\right)\left(6.37\times10^{6}\ \text{m}\right)^2\right]\left(\frac{2\pi}{86\,400\ \text{s}}\right)^2 \\ &= \boxed{2.57\times10^{29}\ \text{J}} \end{aligned}$$

(b) The change in rotational kinetic energy is found by differentiating the equation for rotational kinetic energy with respect to time:

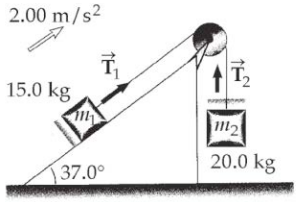
$$\begin{aligned} \frac{dK}{dt} &= \frac{d}{dt}\left[\frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{2\pi}{T}\right)^2\right] \\ &= \frac{1}{5}MR^2(2\pi)^2\left(-2T^{-3}\right)\frac{dT}{dt} \\ &= \frac{1}{5}MR^2\left(\frac{2\pi}{T}\right)^2\left(\frac{-2}{T}\right)\frac{dT}{dt} = K\left(\frac{-2}{T}\right)\frac{dT}{dt} \end{aligned}$$

Substituting,

$$\begin{aligned} \frac{dK}{dt} &= \left(2.57\times10^{29}\ \text{J}\right)\left(\frac{-2}{86\,400\ \text{s}}\right)\left(\frac{10\times10^{-6}\ \text{s}}{3.16\times10^{7}\ \text{s}}\right)\left(86\,400\ \text{s/day}\right) \\ &= \boxed{-1.63\times10^{17}\ \text{J/day}} \end{aligned}$$

P10.77 (a) We apply the $\boxed{\text{particle under a net force}}$ model to each block.

(b) We apply the $\boxed{\text{rigid object under a net torque}}$ model to the pulley.



ANS. FIG. P10.77

(c) We use $\sum F = ma$ for each block to find each string tension. The forces acting on the 15-kg block are its weight, the normal support from the incline, and T_1. Taking the positive x axis as directed up the incline,

$$\sum F_x = ma_x \quad \text{yields:} \qquad -(m_1 g)_x + T_1 = m_1(+a)$$

Solving and substituting known values, we have

$$\begin{aligned} T_1 &= m_1(+a) + (m_1 g)_x \\ &= (15.0\ \text{kg})(2.00\ \text{m/s}^2) + (15.0\ \text{kg})(9.80\ \text{m/s}^2)\sin 37.0° \\ &= \boxed{118\ \text{N}} \end{aligned}$$

(d) Similarly, for the counterweight, we have

$$\sum F_y = ma_y \quad \text{or} \quad T_2 - m_2 g = m_2(-a)$$

$$\begin{aligned} T_2 &= m_2 g + m_2(-a) \\ &= (20.0\ \text{kg})(9.80\ \text{m/s}^2) + (20.0\ \text{kg})(-2.00\ \text{m/s}^2) \\ &= \boxed{156\ \text{N}} \end{aligned}$$

(e) Now for the pulley,

$$\sum \tau = r(T_2 - T_1) = I\alpha = I\,a/r$$

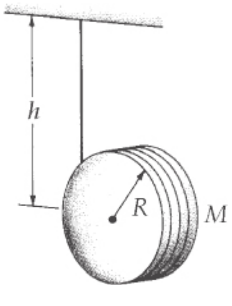
so $$I = \frac{r^2}{a}(T_2 - T_1)$$

where we have chosen to call clockwise positive.

(f) Computing from above, the pulley's rotational inertia is

$$I = \frac{r^2}{a}(T_2 - T_1) = \frac{(156\ \text{N} - 118\ \text{N})(0.250\ \text{m})^2}{2.00\ \text{m/s}^2} = \boxed{1.17\ \text{kg}\cdot\text{m}^2}$$

P10.78 Choosing positive linear quantities to be downwards and positive angular quantities to be clockwise, $\sum F_y = ma_y$ yields



ANS. FIG. P10.78

$$\sum F = Mg - TM = a \quad \text{or} \quad a = \frac{Mg - T}{M}$$

$\sum \tau = I\alpha$ then becomes

$$\sum \tau = TR = I\alpha = \frac{1}{2}MR^2\left(\frac{a}{R}\right) \quad \text{so} \quad a = \frac{2T}{M}$$

(a) Setting these two expressions equal,

$$\frac{Mg - T}{M} = \frac{2T}{M} \quad \text{and} \quad T = \boxed{Mg/3}$$

(b) Substituting back,

$$a = \frac{2T}{M} = \frac{2Mg}{3M} \quad \text{or} \quad a = \boxed{\frac{2}{3}g}$$

(c) Since $v_i = 0$ and $a = \frac{2}{3}g$, $v_f^2 = v_i^2 + 2ah$ gives us $v_f^2 = 0 + 2\left(\frac{2}{3}g\right)h$,

or $v_f = \boxed{\sqrt{4gh/3}}$

(d) Now we verify this answer. Requiring conservation of mechanical energy for the disk-Earth system, we have

$$U_i + K_{\text{rot},\,i} + K_{\text{trans},\,i} = U_f + K_{\text{rot},\,f} + K_{\text{trans},\,f}$$

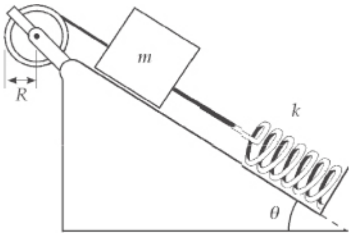
$$mgh + 0 + 0 = 0 + \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega^2 + \frac{1}{2}Mv^2$$

When there is no slipping, $\omega = \frac{v}{R}$ and $v = \sqrt{\frac{4gh}{3}}$.

$\boxed{\text{The answer is the same.}}$

P10.79 The block and end of the spring are pulled a distance d up the incline and then released. The angular speed of the reel and the speed of the block are related by $v = \omega R$. The block-reel-Earth system is isolated, so



ANS. FIG. P10.79

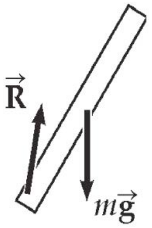
$$\Delta K + \Delta U = 0 \rightarrow K_f - K_i + U_f - U_i = 0$$

$$\left(\frac{1}{2}mv^2 - 0\right) + \left(\frac{1}{2}I\omega^2 - 0\right) + \left(0 - mgd\sin\theta\right) + \left(0 - \frac{1}{2}kd^2\right) = 0$$

$$\frac{1}{2}\omega^2\left(I + mR^2\right) = mgd\sin\theta + \frac{1}{2}kd^2$$

$$\boxed{\omega = \sqrt{\frac{2mgd\sin\theta + kd^2}{I + mR^2}}}$$

P10.80 The center of gravity of the uniform board is at its middle. For the board just starting to move,



ANS. FIG. P10.80

$$\sum\tau = I\alpha:$$

$$mg\left(\frac{\ell}{2}\right)\cos\theta = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3}{2}\left(\frac{g}{\ell}\right)\cos\theta$$

The tangential acceleration of the end is $a_t = \ell\alpha = \frac{3}{2}g\cos\theta$

and its vertical component is $a_y = a_t\cos\theta = \frac{3}{2}g\cos^2\theta$.

If this is greater than g, the board will pull ahead of the falling ball:

(a) $\frac{3}{2}g\cos^2\theta \geq g$ gives $\cos^2\theta \geq \frac{2}{3}$ so $\cos\theta \geq \sqrt{\frac{2}{3}}$ and $\boxed{\theta \leq 35.3°}$

(b) When $\theta = 35.3°$, the cup will land underneath the release point of the ball if $r_c = \ell\cos\theta$.

When $\ell = 1.00$ m and $\theta = 35.3°$,

$$r_c = 1.00\text{ m}\sqrt{\frac{2}{3}} = 0.816\text{ m}$$

so the cup should be

$$\ell - r_c = 1.00\text{ m} - 0.816\text{ m} = \boxed{0.184\text{ m from the moving end}}$$

P10.81 For the isolated sphere-Earth system, energy is conserved, so

$$\Delta U + \Delta K_{\text{rot}} + \Delta K_{\text{trans}} = 0$$

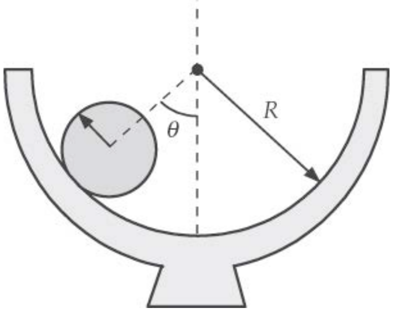
$$mg(R - r)(\cos\theta - 1) + \left[\frac{1}{2}mv^2 - 0\right] + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = 0$$

Substituting $v = r\omega$, we obtain

$$mg(R-r)(\cos\theta-1)+\left[\frac{1}{2}m(r\omega)^2-0\right]+\frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2=0$$

$$mg(R-r)(\cos\theta-1)+\left[\frac{1}{2}+\frac{1}{5}\right]mr^2\omega^2=0$$

$$\boxed{\omega=\sqrt{\left(\frac{10}{7}\right)\frac{(R-r)(1-\cos\theta)g}{r^2}}}$$



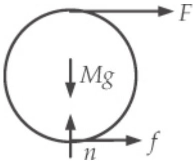
ANS. FIG. P10.81

P10.82 (a) From the particle under a net force model in the *x* direction, we have

$$\sum F_x = F + f = Ma_{CM}$$

From the particle under a net torque model,

$$\sum \tau = FR - fR = I\alpha$$



ANS. FIG. P10.82

Combining the two equations, and noting that $I = \frac{1}{2}MR^2$, gives

$$FR-(Ma_{CM}-F)R=\frac{Ia_{CM}}{R} \qquad \boxed{a_{CM}=\frac{4F}{3M}}$$

(b) Assuming friction is to the right, then

$$f + F = Ma_{CM} = M\left(\frac{4F}{3M}\right)$$

$$\rightarrow f = M\left(\frac{4F}{3M}\right)-F=\boxed{\frac{1}{3}F}$$

The facts that (1) we assumed that friction is to the right in Figure P10.82 and (2) our value for *f* comes out positive indicate that the friction force must indeed be to the right.

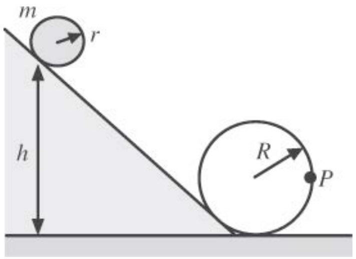
(c) From the kinematic equations,

$$v_f^2 = v_i^2 + 2a(x_f - x_i)$$
$$= 0 + 2ad$$

or

$$v_f = \sqrt{2ad} = \boxed{\sqrt{\frac{8Fd}{3M}}}$$

P10.83 (a) $\Delta K_{\text{rot}} + \Delta K_{\text{trans}} + \Delta U = 0$



ANS. FIG. P10.83

Note that initially the center of mass of the sphere is slightly higher than the distance *h* above the bottom of the loop; and as the mass reaches the top of the loop, this distance above the reference level is 2*R* – *r*, but we are told that *r* << *R*, so we ignore *r* when considering heights for the gravitational potential energy of the sphere-Earth system. The conservation of energy requirement gives

$$mgh = mg(2R) + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

For the sphere $I = \frac{2}{5}mr^2$ and $v = r\omega$, so that the expression becomes

$$gh = 2gR + \frac{7}{10}v^2 \qquad \textbf{[1]}$$

Note that $h = h_{\text{min}}$ when the speed of the sphere at the top of the loop satisfies the condition

$$\sum F = mg = \frac{mv^2}{R} \quad \text{or} \quad v^2 = gR$$

Substituting this into equation [1] gives

$$h_{\text{min}} = 2R + 0.700R \text{ or } \boxed{h_{\text{min}} = 2.70R}$$

(b) When the sphere is initially at $h = 3R$ and finally at point *P*, the conservation of energy equation gives

$$mg3R = mgR + \frac{1}{2}mv^2 + \frac{1}{5}mv^2\text{, or } v^2 = \frac{20}{7}Rg$$

Turning clockwise as it rolls without slipping past point *P*, the sphere is slowing down with counterclockwise angular acceleration caused by the torque of an upward force *f* of static friction. We have

$$\sum F_y = ma_y \quad \rightarrow \quad f - mg = -m\alpha r$$

and $\sum \tau = I\alpha \rightarrow fr = \left(\frac{2}{5}\right)mr^2\alpha.$

Eliminating *f* by substitution yields

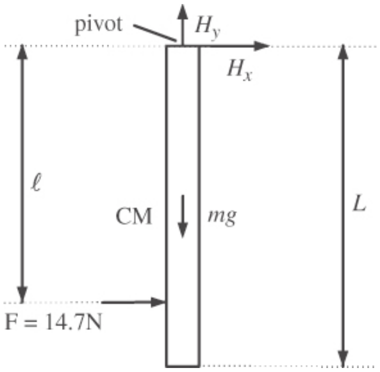
$$\alpha = \frac{5g}{7r} \text{ so that } \sum F_y = \boxed{-\frac{5}{7}mg}$$

P10.84 The length of the rod is *L*, and the horizontal force is applied the vertical distance *L* from the hinge. Consider the free-body diagram shown. The sum of torques about the chosen pivot is

$$\sum \tau = I\alpha \Rightarrow F = \left(\frac{1}{3}mL^2\right)\left(\frac{a_{CM}}{L/2}\right) = \left(\frac{2}{3}mL\right)a_{CM} \qquad [1]$$

(a) $\ell = L = 1.24$ m: In this case, equation [1] becomes

$$a_{CM} = \frac{3F}{2m} = \frac{3(14.7 \text{ N})}{2(0.630 \text{ kg})} = \boxed{35.0 \text{ m/s}^2}$$



ANS. FIG. P10.84

(b) We apply the particle under a net force model in the horizontal direction (see ANS. FIG. P10.84 for the labelling of forces):

$$\sum F_x = ma_{CM} \Rightarrow F + H_x = ma_{CM}$$

or $H_x = ma_{CM} - F$

Thus,

$$H_x = (0.630 \text{ kg})(35.0 \text{ m/s}^2) - 14.7 \text{ N} = +7.35 \text{ N}$$

or $\vec{\mathbf{H}}_x = \boxed{7.35\hat{\mathbf{i}}\text{ N}}$

(c) With $\ell = \frac{1}{2}L = 0.620\text{ m}$, equation [1] yields

$$a_{CM} = \frac{3F}{4m} = \frac{3(14.7\text{ N})}{4(0.630\text{ kg})} = \boxed{17.5\text{ m/s}^2}$$

(d) Again, $\sum F_x = ma_{CM} \Rightarrow H_x = ma_{CM} - F$, so

$$H_x = (0.630\text{ kg})(17.5\text{ m/s}^2) - 14.7\text{ N} = -3.68\text{ N}$$

or $\mathbf{H}_x = \boxed{-3.68\hat{\mathbf{i}}\text{ N}}$

(e) If $H_x = 0$, then $\sum F_x = ma_{CM} \Rightarrow F = ma_{CM}$, or $a_{CM} = \frac{F}{m}$.

Thus, equation [1] becomes

$$F\ell = \left(\frac{2}{3}mL\right)\left(\frac{F}{m}\right)$$

so $\ell = \frac{2}{3}L = \frac{2}{3}(1.24\text{ m}) = \boxed{0.827\text{ m (from the top)}}$

P10.85 Note that when the CM of the falling rod is very near the surface, the velocity of the end of the rod in contact with the surface is a combination of the downward motion of the CM and the upward motion of the rotating end: $v_{end} = v_{CM} - \omega r$. Because the velocity of this end relative to the surface is zero,

$$v_{end} = v_{CM} - \omega(h/2) = 0 \rightarrow v_{CM} = \omega(h/2)t$$

(a) There are no horizontal forces acting on the rod, so the center of mass (CM) will not move horizontally. Rather, the center of mass drops straight downward (distance $h/2$) with the rod rotating about the center of mass as it falls.

From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}Mv_{CM}^2 + \frac{1}{2}\left(\frac{1}{12}Mh^2\right)\left(\frac{v_{CM}}{h/2}\right)^2 = Mg\left(\frac{h}{2}\right)$$

which reduces to

$$\mathrm{v}_{\mathrm{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

(b) In this case, the motion is a pure rotation about a fixed pivot point (the lower end of the rod) with the center of mass moving in a circular path of radius $h/2$. From conservation of energy:

$$K_f + U_{gf} = K_i + U_{gi}$$

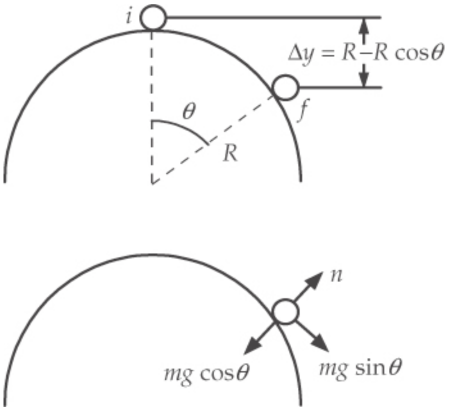
$$\frac{1}{2}I\omega^2 + 0 = 0 + Mg\left(\frac{h}{2}\right) \text{ or}$$

$$\frac{1}{2}\left(\frac{1}{3}Mh^2\right)\left(\frac{\mathrm{v}_{\mathrm{CM}}}{h/2}\right)^2 = Mg\left(\frac{h}{2}\right)$$

which reduces to

$$\mathrm{v}_{\mathrm{CM}} = \boxed{\sqrt{\frac{3gh}{4}}}$$

P10.86 The grape-Earth system is isolated, so mechanical energy in that system is conserved. Between top of the clown's head and the point where the grape leaves the surface:



ANS. FIG. P10.86

$$K_i + U_i = K_f + U_f$$

$$0 + mg\Delta y = \frac{1}{2}m\mathrm{v}_f^2 + \frac{1}{2}I\omega_f^2 + 0$$

$$mgR(1-\cos\theta) = \frac{1}{2}m\mathrm{v}_f^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\left(\frac{\mathrm{v}_f}{R}\right)^2$$

which gives

$$g(1-\cos\theta) = \frac{7}{10}\left(\frac{\mathrm{v}_f^2}{R}\right) \qquad [1]$$

Consider the radial forces acting on the grape:

$$mg\cos\theta - n = \frac{m\mathrm{v}_f^2}{R}$$

At the point where the grape leaves the surface, $n \to 0$. Thus,

$$mg\cos\theta = \frac{mv_f^2}{R} \quad \text{or} \quad \frac{v_f^2}{R} = g\cos\theta$$

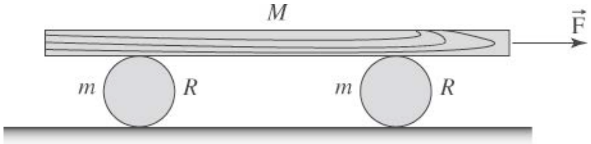
Substituting this into equation [1] gives

$$g - g\cos\theta = \frac{7}{10}g\cos\theta$$

or $$\theta = \cos^{-1}\left(\frac{10}{17}\right) = \boxed{54.0°}$$

Challenge Problems

P10.87 Refer to the force diagrams for the plank and rollers in ANS. FIG. P10.87(b) below. Call f_t the frictional force exerted by each roller backward on the plank. Name as f_b the rolling resistance exerted backward by the ground on each roller.



ANS. FIG. P10.87(a)

For the plank,

$$\sum F_x = ma_x: \quad 6.00\text{ N} - 2f_t = (6.00\text{ kg})a_p \qquad \textbf{[1]}$$

If we think of the motion of a roller as a small rotation about its point of contact with the surface, we see that the center of each roller moves forward only half as far as the plank.

Each roller has acceleration $\frac{a_p}{2}$ and angular acceleration

$$\frac{a_p/2}{5.00\text{ cm}} = \frac{a_p}{0.100\text{ m}}$$

Then for each,

$$\sum F_x = ma_x: \quad +f_t - f_b = (2.00\text{ kg})\frac{a_p}{2} \qquad \textbf{[2]}$$

$$\sum \tau = I\alpha:$$

$$f_t(5.00\text{ cm}) + f_b(5.00\text{ cm}) = \frac{1}{2}(2.00\text{ kg})(5.00\text{ cm})^2 \frac{a_p}{10.0\text{ cm}}$$

So $$f_t + f_b = \left(\frac{1}{2}\text{ kg}\right)a_p \qquad \textbf{[3]}$$

Add equations [2] and [3] to eliminate f_b: $2f_t = (1.50\text{ kg})a_p$

(a) Substituting the value for $2f_t$ into equation [1] gives

$$6.00\text{ N} - (1.50\text{ kg})a_p = (6.00\text{ kg})a_p$$

$$\rightarrow a_p = \frac{6.00\text{ N}}{7.50\text{ kg}} = \boxed{0.800\text{ m/s}^2}$$

(b) For each roller, $a = \frac{a_p}{2} = \boxed{0.400\text{ m/s}^2}$

(c) Substituting back,

$$2f_t = (1.50\text{ kg})(0.800\text{ m/s}^2)$$

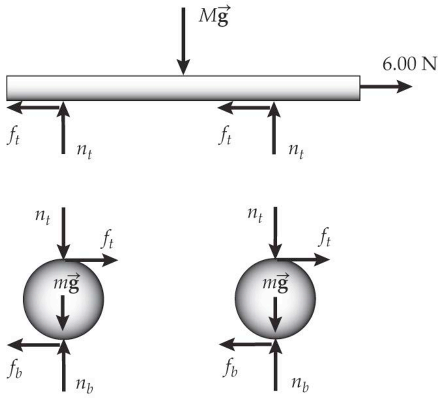
$$f_t = \boxed{0.600\text{ N}}$$

then, from equation [3],

$$0.600\text{ N} + f_b = \left(\frac{1}{2}\text{ kg}\right)(0.800\text{ m/s}^2)$$

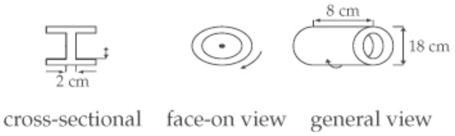
$$f_b = -0.200\text{ N}$$

The negative sign means that the horizontal force of ground on each roller is $\boxed{\text{0.200 N forward}}$ rather than backward as we assumed.



ANS. FIG. P10.87(b)

P10.88 For large energy storage at a particular rotation rate, we want a large moment of inertia. To combine this requirement with small mass, we place the mass as far away from the axis as possible.



ANS. FIG. P10.88

We choose to make the flywheel as a hollow cylinder 18.0 cm in diameter and 8.00 cm long. To support this rim, we place a disk across its center. We assume that a disk 2.00 cm thick will be sturdy enough to support the hollow cylinder securely.

The one remaining adjustable parameter is the thickness of the wall of the hollow cylinder. From Table 10.2, the moment of inertia can be written as

$$\begin{aligned} I_{\text{disk}} + I_{\text{hollow cylinder}} &= \frac{1}{2}M_{\text{disk}}R_{\text{disk}}^2 + \frac{1}{2}M_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\ &= \frac{1}{2}\rho V_{\text{disk}}R_{\text{outer}}^2 + \frac{1}{2}\rho V_{\text{wall}}\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \end{aligned}$$

$$\begin{aligned} &= \frac{\rho}{2}\pi R_{\text{outer}}^2(2.00\text{ cm})R_{\text{outer}}^2 + \frac{\rho}{2}\left[\pi R_{\text{outer}}^2 - \pi R_{\text{inner}}^2\right] \\ &\qquad \times(6.00\text{ cm})\left(R_{\text{outer}}^2 + R_{\text{inner}}^2\right) \\ &= \frac{\rho\pi}{2}\Big[(9.00\text{ cm})^4(2.00\text{ cm}) \\ &\qquad +(6.00\text{ cm})\left[(9.00\text{ cm})^2 - R_{\text{inner}}^2\right]\left[(9.00\text{ cm})^2 + R_{\text{inner}}^2\right]\Big] \\ &= \rho\pi\left[6\,561\text{ cm}^5 + (3.00\text{ cm})\left((9.00\text{ cm})^4 - R_{\text{inner}}^4\right)\right] \\ &= \rho\pi\left[26\,244\text{ cm}^5 - (3.00\text{ cm})R_{\text{inner}}^4\right] \end{aligned}$$

For the required energy storage,

$$\frac{1}{2}I\omega_1^2 = \frac{1}{2}I\omega_2^2 + W_{\text{out}}$$

$$\begin{aligned} &\frac{1}{2}I\left[(800\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]^2 \\ &\qquad -\frac{1}{2}I\left[(600\text{ rev/min})\left(\frac{2\pi\text{ rad}}{60\text{ s}}\right)\right]^2 \\ &= 60.0\text{ J} \end{aligned}$$

$$
\begin{aligned}
\mathrm{I} &= \frac{60.0\ \mathrm{J}}{1\,535/\mathrm{s}^2} \\
&= \left(7.85\times10^{3}\ \mathrm{kg/m^3}\right)\pi\left[26\,244\ \mathrm{cm}^5-(3.00\ \mathrm{cm})\mathrm{R}_{\mathrm{inner}}^{4}\right]
\end{aligned}
$$

$$1.58\times10^{-5}\ \mathrm{m}^5\left(\frac{100\ \mathrm{cm}}{1\ \mathrm{m}}\right)^5 = 26\,244\ \mathrm{cm}^5-(3.00\ \mathrm{cm})\mathrm{R}_{\mathrm{inner}}^{4}$$

$$\mathrm{R}_{\mathrm{inner}} = \left(\frac{26\,244\ \mathrm{cm}^4-15\,827\ \mathrm{cm}^4}{3.00}\right)^{1/4} = 7.68\ \mathrm{cm}$$

The inner radius of the flywheel is 7.68 cm. The mass of the flywheel is then 7.27 kg, found as follows:

$$
\begin{aligned}
\mathrm{M}_{\mathrm{disk}}+\mathrm{M}_{\mathrm{wall}} &= \rho\pi\mathrm{R}_{\mathrm{outer}}^{2}(2.00\ \mathrm{cm}) \\
&\quad+\rho\left[\pi\mathrm{R}_{\mathrm{outer}}^{2}-\pi\mathrm{R}_{\mathrm{inner}}^{2}\right](6.00\ \mathrm{cm}) \\
&= \left(7.86\times10^{3}\ \mathrm{kg/m^3}\right)\pi \\
&\qquad\left[(0.090\ \mathrm{m})^2(0.020\ \mathrm{m})\right. \\
&\qquad\left.+\left[(0.090\ \mathrm{m})^2-(0.076\,8\ \mathrm{m})^2\right](0.060\ \mathrm{m})\right] \\
&= 7.27\ \mathrm{kg}
\end{aligned}
$$

If we made the thickness of the disk somewhat less than 2.00 cm and the inner radius of the cylindrical wall less than 7.68 cm to compensate, the mass could be a bit less than 7.27 kg.

> The flywheel can be shaped like a cup or open barrel, 9.00 cm in outer radius and 7.68 cm in inner radius, with its wall 6 cm high, and with its bottom forming a disk 2.00 cm thick and 9.00 cm in radius. It is mounted to the crankshaft at the center of this disk and turns about its axis of symmetry. Its mass is 7.27 kg. If the disk were made somewhat thinner and the barrel wall thicker, the mass could be smaller.

P10.89 (a) At $t = 0$, $\omega = 3.50\ \mathrm{rad/s} = \omega_0 e^0$. Thus, $\boxed{\omega_0 = 3.50\ \mathrm{rad/s}}$.

At $t = 9.30$ s, $\omega = 2.00\ \mathrm{rad/s} = \omega_0 e^{-\sigma(9.30\mathrm{s})}$.

We now calculate σ: To solve $\omega = \omega_0 e^{-\sigma t}$ for σ, we recall that the natural logarithm function is the inverse of the exponential function.

$$\omega/\omega_0 = e^{-\sigma t} \quad \text{becomes} \quad \ln(\omega/\omega_0) = -\sigma t \quad \text{or} \quad \ln(\omega_0/\omega) = +\sigma t$$

so $\sigma = \left(\frac{1}{t}\right)\ln(\omega_0/\omega) = \left(\frac{1}{9.30\text{ s}}\right)\ln\left(\frac{3.50}{2.00}\right) = \frac{0.560}{9.30\text{ s}} = \boxed{6.02\times10^{-2}\text{s}^{-1}}$

(b) At all times,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left[\omega_0 e^{-\sigma t}\right] = -\sigma\omega_0 e^{-\sigma t}$$

At $t = 3.00$ s,

$$\alpha = -(0.060\ 2\text{ s}^{-1})(3.50\text{ rad/s})e^{-0.181} = \boxed{-0.176\text{ rad/s}^2}$$

(c) From the given equation, we have $d\theta = \omega_0^{e^{-\sigma t}}dt$

and

$$\theta = \int_{0\text{ s}}^{2.50\text{ s}} \omega_0 e^{-\sigma t}dt = \frac{\omega_0}{-\sigma}e^{-\sigma t}\Big|_{0\text{ s}}^{2.50\text{ s}} = \frac{\omega_0}{-\sigma}\left(e^{-2.50\sigma} - 1\right)$$

Substituting and solving,

$$\theta = -58.2(0.860 - 1)\text{ rad} = 8.12\text{ rad}$$

or $\theta = (8.12\text{ rad})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{1.29\text{ rev}}$

(d) The motion continues to a finite limit, as ω approaches zero and t goes to infinity. From part (c), the total angular displacement is

$$\theta = \int_0^\infty \omega_0 e^{-\sigma t}dt = \frac{\omega_0}{-\sigma}e^{-\sigma t}\Big|_0^\infty = \frac{\omega_0}{-\sigma}(0 - 1) = \frac{\omega_0}{\sigma}$$

Substituting,

$$\theta = 58.2\text{ rad} \quad \text{or} \quad \theta = \left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)(58.2\text{ rad}) = \boxed{9.26\text{ rev}}$$

P10.90 (a) If we number the loops of the spiral track with an index n, with the innermost loop having $n = 0$, the radii of subsequent loops as we move outward on the disc is given by $r = r_i + hn$. Along a given radial line, each new loop is reached by rotating the disc through 2π rad. Therefore, the ratio $\theta/2\pi$ is the number of revolutions of the disc to get to a certain loop. This is also the number of that loop, so $n = \theta/2\pi$. Therefore, $r = r_i + h\,\theta/2\pi$.

(b) Starting from $\omega = v/r$, we substitute the definition of angular speed on the left and the result for r from part (a) on the right:

$$\omega = \frac{v}{r} \rightarrow \frac{d\theta}{dt} = \frac{v}{r_i + (h\theta/2\pi)}$$

(c) Rearrange terms in preparation for integrating both sides:

$$\left(r_i + \frac{h}{2\pi}\theta\right)d\theta = vdt$$

and integrate from $\theta = 0$ to $\theta = \theta$ and from $t = 0$ to $t = t$:

$$r_i\theta + \frac{h}{4\pi}\theta^2 = vt$$

We rearrange this equation to form a standard quadratic equation in θ:

$$\frac{h}{4\pi}\theta^2 + r_i\theta - vt = 0$$

The solution to this equation is

$$\theta = \frac{-r_i \pm \sqrt{r_i^2 + \frac{h}{\pi}vt}}{\frac{h}{2\pi}} = \boxed{\frac{2\pi r_i}{h}\left(\sqrt{1 + \frac{vh}{\pi r_i^2}t} - 1\right)}$$

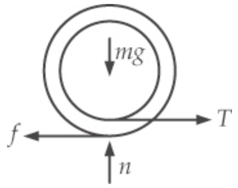
where we have chosen the positive root in order to make the angle θ positive.

(d) We differentiate the result in (c) twice with respect to time to find the angular acceleration, resulting in

$$\boxed{\alpha = -\frac{hv^2}{2\pi r_i^3\left(1 + \frac{vh}{\pi r_i^2}t\right)^{3/2}}}$$

Where we have used $\frac{d}{dx}\sqrt{u} = \frac{1}{2\sqrt{u}}\frac{du}{dx}$. Because this expression involves the time *t*, the angular acceleration is not constant.

P10.91 (a) $\sum F_x = ma_x$ reads $-f + T = ma$. If we take torques around the center of mass, we can use $\sum\tau = I\alpha$, which reads $+fR_2 - TR_1 = I\alpha$. For rolling without slipping, $\alpha = \frac{a}{R_2}$. By substitution,



ANS. FIG. P10.91

$$fR_2 - TR_1 = \frac{Ia}{R_2} = \frac{I}{R_2 m}(T - f)$$

$$fR_2^2m - TR_1R_2m = IT - If$$
$$f\left(I + mR_2^2\right) = T\left(I + mR_1R_2\right)$$
$$f = \left(\frac{I + mR_1R_2}{I + mR_2^2}\right)T$$

(b) Since the answer is positive, the friction force is confirmed to be $\boxed{\text{to the left}}$.

P10.92 (a) From the isolated system model for the block-pulley-Earth system,

$$\Delta K + \Delta U + \Delta E_{\text{int}} = 0$$
$$\left(\frac{1}{2}Mv^2 - 0\right) + \left(\frac{1}{2}I\omega^2 - 0\right) + \left(0 - Mgd\sin\theta\right) + fd = 0$$
$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2 - Mgd\sin\theta + \left(\mu Mg\cos\theta\right)d = 0$$
$$v = \sqrt{\frac{4Mgd\left(\sin\theta - \mu\cos\theta\right)}{2M + m}}$$

(b) From the particle under constant acceleration model for the block,

$$v_f^2 = v_i^2 + 2ad$$
$$a = \frac{v_f^2 - v_i^2}{2d} = \frac{v^2}{2d} = \boxed{\frac{2Mg\left(\sin\theta - \mu\cos\theta\right)}{2M + m}}$$

P10.93 The location of the dog is described by $\theta_d = \left(0.750 \text{ rad/s}\right)t$. For the bone,

$$\theta_b = \frac{1}{3}2\pi \text{ rad} + \frac{1}{2}0.015 \text{ rad/s}^2\ t^2$$

(a) We look for a solution to (suppressing units)

$$0.75t = \frac{2\pi}{3} + 0.007\,5t^2$$
$$0 = 0.007\,5t^2 - 0.75t + 2.09 = 0$$
$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4\left(0.007\,5\right)2.09}}{0.015} = 2.88 \text{ s or } 97.1 \text{ s}$$

The first time the dog reaches the bone is $\boxed{2.88 \text{ s.}}$

(b) If the dog passes the bone, he must run around the merry-go-round again. The dog will draw even with the bone when

$$0.75t = \frac{2\pi}{3} + 2\pi + 0.007\,5t^2 \cdot$$

Solving this equation, we find (suppressing units)

$$t = \frac{0.75 \pm \sqrt{0.75^2 - 4(0.007\,5)8.38}}{0.015} = 12.8 \text{ s} \quad \text{or} \quad 87.2 \text{ s}$$

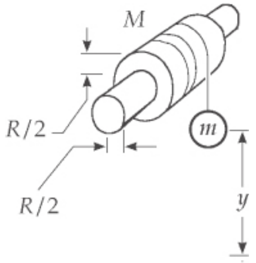
The dog draws even with the bone again at the time of $\boxed{12.8\text{ s.}}$

P10.94 τ_f will oppose the torque due to the hanging object:

$$\sum \tau = I\alpha = TR - \tau_f: \quad \tau_f = TR - I\alpha \qquad \textbf{[1]}$$

Now find T, I, and α in given or known terms and substitute into equation [1].

$$\sum F_y = T - mg = -ma: \;\; T = m(g - a) \qquad \textbf{[2]}$$



ANS. FIG. P10.94

also, $$\Delta y = v_i t + \frac{at^2}{2} \; a = \frac{2y}{t^2} \qquad \textbf{[3]}$$

and $$\alpha = \frac{a}{R} = \frac{2y}{Rt^2} \qquad \textbf{[4]}$$

with $$I = \frac{1}{2}M\left[R^2 + \left(\frac{R}{2}\right)^2\right] = \frac{5}{8}MR^2 \qquad \textbf{[5]}$$

Substituting [2], [3], [4], and [5] into [1], we find

$$\tau_f = m\left(g - \frac{2y}{t^2}\right)R - \frac{5}{8}\frac{MR^2(2y)}{Rt^2} = \boxed{R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P10.2	(a) 0.209 rad/s^2; (b) yes
P10.4	144 rad
P10.6	-2.26×10^2 rad/s^2
P10.8	(a) 3.5 rad; (b) increase by a factor of 4
P10.10	Because the disk's average angular speed does not match the average angular speed expressed as $(\omega_i + \omega_f)/2$ in the model of a rigid object under constant angular acceleration, the angular acceleration of the disk cannot be constant.
P10.12	50.0 rev
P10.14	(a) $\boxed{\omega h^{3/2}\left(\frac{2}{g}\right)^{1/2}}$; (b) 1.16 cm; (c) The deflection is only 0.02% of the original height, so it is negligible in many practical cases; (d) Decrease
P10.16	~10^7 rev/yr
P10.18	(a) 0.605 m/s; (b) 17.3 rad/s; (c) 5.82 m/s; (d) We did not need to know the length of the pedal cranks.
P10.20	(a) 54.3 rev; (b) 12.1 rev/s
P10.22	(a) 5.77 cm; (b) Yes. See P10.20 for full explanation.
P10.24	$\frac{a}{g}\sqrt{1+\pi^2}$
P10.26	(a) $(-2.73\hat{\mathbf{i}} + 1.24\hat{\mathbf{j}})$ m; (b) It is in the second quadrant, at 156°; (c) $(-1.85\hat{\mathbf{i}} - 4.10\hat{\mathbf{j}})$ m/s; (d) It is moving toward the third quadrant, at 246°; (e) $(6.15\hat{\mathbf{i}} - 2.78\hat{\mathbf{j}})$ m/s^2; (f) See ANS. FIG. P10.26; (g) $(24.6\hat{\mathbf{i}} - 11.1\hat{\mathbf{j}})$ N
P10.28	168 N · m
P10.30	(a) 1.03 s; (b) 10.3 rev
P10.32	(a) See ANS. FIG. P10.32; (b) 0.309 m/s^2; (c) $T_1 = 7.67$ N, $T_2 = 9.22$ N
P10.34	(a) For $F = 25.1$ N, $R = 1.00$ m. For $F = 10.0$ N, $R = 25.1$ m; (b) No. Infinitely many pairs of values that satisfy this requirement may exist: for any $F \le 50.0$ N, $R = 25.1$ N · m/F, as long as $R \le 3.00$ m.

P10.36	(a) 1.95 s; (b) If the pulley were massless, the acceleration would be larger by a factor 35/32.5 and the time short by the square root of the factor 32.5/35. That is, the time would be reduced by 3.64%.
P10.38	$10^0\ \text{kg}\cdot\text{m}^2 = 1\ \text{kg}\cdot\text{m}^2$
P10.40	(a) See P10.40(a) for full description; (b) See P10.40(b) for full description
P10.42	$I_{y'} = \int_{\text{all mass}} r^2 dm = \int_0^L x^2 \frac{M}{L} dx = \frac{M}{L}\frac{x^3}{3}\Big\|_0^L = \frac{1}{3}ML^2$
P10.44	(a) 92.0 kg·m^2; (b) 184 J; (c) 6.00 m/s, 4.00 m/s, 8.00 m/s; (d) 184 J; (e) The kinetic energies computed in parts (b) and (d) are the same.
P10.46	$\frac{13}{24}MR^2\omega^2$
P10.48	276 J
P10.50	$v = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2 + \frac{I}{R^2}}}$ and $\omega = \sqrt{\frac{2(m_1 - m_2)gh}{m_1R^2 + m_2R^2 + I}}$
P10.52	The situation is impossible because the range is only 3.86 km, not city-wide.
P10.54	(a) 6.90 J; (b) 8.73 rad/s; (c) 2.44 m/s; (d) The speed it attains in swinging is greater by 1.043 2 times
P10.56	$mr^2\left(\frac{2gh}{v^2} - 1\right)$
P10.58	(a) 74.3 W; (b) 401 W
P10.60	rolling: $v_f = \sqrt{10gh/7}$; sliding: $v_f = \sqrt{2gh}$; The time to roll is longer by a factor of $(0.7/0.5)^{1/2} = 1.18$
P10.62	(a) the cylinder; (b) $v^2/4g\sin\theta$; (c) The cylinder does not lose mechanical energy because static friction does not work on it. Its rotation means that it has 50% more kinetic energy than the cube at the start, and so it travels 50% farther up the incline.
P10.64	(a) 2.38 m/s; (b) The centripetal acceleration at the top is $\frac{v_2^2}{r} = \frac{(2.38\ \text{m/s})^2}{0.450\ \text{m}} = 12.6\ \text{m/s}^2 > g$. Thus, the ball must be in contact with the track, with the track pushing downward on it; (c) 4.31 m/s; (d) $\sqrt{-1.40\,\text{m}^2/\text{s}^2}$; (e) never makes it to the top of the loop

P10.66 $\frac{1}{3}$ the length of the chimney

P10.68 (a) $d = (1890 + 80n)\left(\frac{0.459\,\text{m}}{80n - 150}\right)$; (b) 94.1 m; (c) 1.62 m; (d) –5.79 m; (e) The rising car will coast to a stop only for $n \geq 2$; (f) For $n = 0$ or $n = 1$, the mass of the elevator is less than the counterweight, so the car would accelerate upward if released; (g) 0.459 m

P10.70 $\omega(t) = \omega + At + \frac{1}{2}Bt^2$; (b) $\omega t + \frac{1}{2}At^2 + \frac{1}{6}Bt^3$

P10.72 (a) (i) –794 N·m, (ii) –2 510 N·m, (iii) 0 N· m, (iv) –1 160 N·m, (v) 2 940 N·m; (b) See P10.72(b) for full description.

P10.74 $-0.322\ \text{rad/s}^2$

P10.76 (a) 2.57×10^{29} J; (b) -1.63×10^{17} J/day

P10.78 (a) $Mg/3$; (b) $2g/3$; (c) $\sqrt{4gh/3}$; (d) The answer is the same.

P10.80 (a) $\theta \leq 35.5^\circ$; (b) 0.184 m from the moving end

P10.82 (a) $a_{\text{CM}} = \frac{4F}{3M}$; (b) $\frac{1}{3}F$; (c) $\sqrt{\frac{8Fd}{3M}}$

P10.84 (a) 35.0 m/s^2; (b) $7.35\hat{\mathbf{i}}$ N; (c) 17.5 m/s^2; (d) $-3.68\hat{\mathbf{i}}$ N; (e) 0.827 m (from the top)

P10.86 54.0°

P10.88 See P10.88 for full design and specifications of flywheel.

P10.90 (a) See P10.90(a) for full solution; (b) See P10.90(g) for full solution; (c) $\frac{2\pi r_i}{h}\left(\sqrt{1 + \frac{vh}{\pi r_i^2}t} - 1\right)$; (d) $\alpha = -\frac{hv^2}{2\pi r_i^2\left(1 + \frac{vh}{\pi r_i^2}t\right)^{3/2}}$

P10.92 (a) See P10.92(a) for full explanation; (b) $\frac{2Mg(\sin\theta - \mu\cos\theta)}{2M + m}$

P10.94 $R\left[m\left(g - \frac{2y}{t^2}\right) - \frac{5}{4}\frac{My}{t^2}\right]$

11

Angular Momentum

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ11.1 Answer (b). Her angular momentum stays constant as I is cut in half and ω doubles. Then $\frac{1}{2}I\omega^2$ doubles.

OQ11.2 The angular momentum of the mouse-turntable system is initially zero, with both at rest. The frictionless axle isolates the mouse-turntable system from outside torques, so its angular momentum must stay constant with the value of zero.

(i) Answer (a). The mouse makes some progress north, or counterclockwise.

(ii) Answer (b). The turntable will rotate clockwise. The turntable rotates in the direction opposite to the motion of the mouse, for the angular momentum of the system to remain zero.

(iii) No. Mechanical energy changes as the mouse converts some chemical into mechanical energy, positive for the motions of both the mouse and the turntable.

(iv) No. Linear momentum is not conserved. The turntable has zero momentum while the mouse has a bit of northward momentum. Initially, momentum is zero; later, when the mouse moves north, the fixed axle prevents the turntable from moving south.

(v) Yes. Angular momentum is constant, with the value of zero.

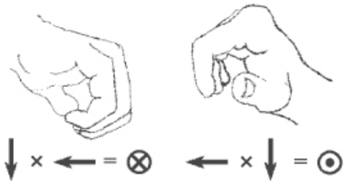
OQ11.3 (i) Answer (a), (ii) Answer (e), (3 m, down) × (2 N, toward you) = 6 N · m, left

OQ11.4 Answer c = e > b = d > a = 0. The unit vectors have magnitude 1, so the magnitude of each cross product is $|1 \cdot 1 \cdot \sin\theta|$ where θ is the angle between the factors. Thus for (a) the magnitude of the cross product is sin 0° = 0. For (b), |sin 135°| = 0.707, (c) sin 90° = 1, (d) sin 45° = 0.707, (e) sin 90° = 1.

OQ11.5 (a) No. (b) No. An axis of rotation must be defined to calculate the torque acting on an object. The moment arm of each force is measured from the axis, so the value of the torque depends on the location of the axis.

OQ11.6 (i) Answer (e). Down–cross–left is away from you: $-\hat{\mathbf{j}} \times \left(-\hat{\mathbf{i}}\right) = -\hat{\mathbf{k}}$, as in the first picture.

(ii) Answer (d). Left–cross–down is toward you: $-\hat{\mathbf{i}} \times \left(-\hat{\mathbf{j}}\right) = \hat{\mathbf{k}}$, as in the second picture.



ANS FIG. OQ11.6

OQ11.7 (i) Answer (a). The angular momentum is constant. The moment of inertia decreases, so the angular speed must increase.

(ii) No. Mechanical energy increases. The ponies must do work to push themselves inward.

(iii) Yes. Momentum stays constant, with the value of zero.

(iv) Yes. Angular momentum is constant with a nonzero value. No outside torque can influence rotation about the vertical axle.

OQ11.8 Answer (d). As long as no net external force, or torque, acts on the system, the linear and angular momentum of the system are constant.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ11.1 The star is isolated from any outside torques, so its angular momentum is conserved as it changes size. As the radius of the star decreases, its moment of inertia decreases, resulting in its angular speed increasing.

CQ11.2 The suitcase might contain a spinning gyroscope. If the gyroscope is spinning about an axis that is oriented horizontally passing through the bellhop, the force he applies to turn the corner results in a torque that could make the suitcase swing away. If the bellhop turns quickly enough, anything at all could be in the suitcase and need not be rotating. Since the suitcase is massive, it will tend to follow an inertial path. This could be perceived as the suitcase swinging away by the bellhop.

CQ11.3 The long pole has a large moment of inertia about an axis along the rope. An unbalanced torque will then produce only a small angular acceleration of the performer-pole system, to extend the time available for getting back in balance. To keep the center of mass above the rope, the performer can shift the pole left or right, instead of having to bend his body around. The pole sags down at the ends to lower the system's center of gravity.

CQ11.4 (a) Frictional torque arises from kinetic friction between the inside of the roll and the child's fingers. As with all friction, the magnitude of the friction depends on the normal force between the surfaces in contact. As the roll unravels, the weight of the roll decreases, leading to a decrease in the frictional force, and, therefore, a decrease in the torque.

(b) As the radius R of the paper roll shrinks, the roll's angular speed $\omega = \frac{v}{R}$ must increase because the speed v is constant.

(c) If we think of the roll as a uniform disk, then its moment of inertia is $I = \frac{1}{2}MR^2$. But the roll's mass is proportional to its base area πR^2; therefore, the moment of inertia is proportional to R^4. The moment of inertia decreases as the roll shrinks. When the roll is given a sudden jerk, its angular acceleration may not be great enough to set the roll moving in step with the paper, so the paper breaks. The roll is most likely to break when its radius is large, when its moment of inertia is large, than when its radius is small, when its moment of inertia is small.

CQ11.5 Work done by a torque results in a change in rotational kinetic energy about an axis. Work done by a force results in a change in translational kinetic energy. Work by either has the same units:

$$W = F\Delta x = [\mathrm{N}][\mathrm{m}] = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

$$W = \tau\Delta\theta = [\mathrm{N}\cdot\mathrm{m}][\mathrm{rad}] = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

CQ11.6 Suppose we look at the motorcycle moving to the right. Its drive wheel is turning clockwise. The wheel speeds up when it leaves the ground. No outside torque about its center of mass acts on the airborne cycle, so its angular momentum is conserved. As the drive wheel's clockwise angular momentum increases, the frame of the cycle acquires counterclockwise angular momentum. The cycle's front end moves up and its back end moves down.

CQ11.7 Its angular momentum about that axis is constant in time. You cannot conclude anything about the magnitude of the angular momentum.

CQ11.8 No. The angular momentum about any axis that does not lie along the instantaneous line of motion of the ball is nonzero.

CQ11.9 The Earth is an isolated system, so its angular momentum is conserved when the distribution of its mass changes. When its mass moves away from the axis of rotation, its moment of inertia increases, its angular speed decreases, so its period increases. Most of the mass of Earth would not move, so the effect would be small: we would not have more hours in a day, but more nanoseconds.

CQ11.10 As the cat falls, angular momentum must be conserved. Thus, if the upper half of the body twists in one direction, something must get an equal angular momentum in the opposite direction. Rotating the lower half of the body in the opposite direction satisfies the law of conservation of angular momentum.

CQ11.11 Energy bar charts are useful representations for keeping track of the various types of energy storage in a system: translational and rotational kinetic energy, various types of potential energy, and internal energy. However, there is only *one type* of angular momentum. Therefore, there is no need for bar charts when analyzing a physical situation in terms of angular momentum.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 11.1 The Vector Product and Torque

P11.1 $$\vec{\mathbf{M}}\times\vec{\mathbf{N}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 2 & -3 & 1 \\ 4 & 5 & -2 \end{vmatrix} = \hat{\mathbf{i}}(6-5) - \hat{\mathbf{j}}(-4-4) + \hat{\mathbf{k}}(10+12) = \boxed{\hat{\mathbf{i}} + 8.00\hat{\mathbf{j}} + 22.0\hat{\mathbf{k}}}$$

P11.2 (a) $$\text{area} = \left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = AB\sin\theta = (42.0\text{ cm})(23.0\text{ cm})\sin(65.0^\circ - 15.0^\circ)$$
$$= \boxed{740\text{ cm}^2}$$

(b) The longer diagonal is equal to the sum of the two vectors.

$$\vec{\mathbf{A}}+\vec{\mathbf{B}} = [(42.0\text{ cm})\cos 15.0^\circ + (23.0\text{ cm})\cos 65.0^\circ]\hat{\mathbf{i}}$$
$$+[(42.0\text{ cm})]\sin 15.0^\circ + (23.0\text{ cm})\sin 65.0^\circ]\hat{\mathbf{j}}$$

$$\vec{\mathbf{A}}+\vec{\mathbf{B}} = (50.3\text{ cm})\hat{\mathbf{i}} + (31.7\text{ cm})\hat{\mathbf{j}}$$
$$\text{length} = \left|\vec{\mathbf{A}}+\vec{\mathbf{B}}\right| = \sqrt{(50.3\text{ cm})^2 + (31.7\text{ cm})^2} = \boxed{59.5\text{ cm}}$$

P11.3 We take the cross product of each term of $\vec{\mathbf{A}}$ with each term of $\vec{\mathbf{B}}$, using the cross-product multiplication table for unit vectors. Then we use the identification of the magnitude of the cross product as $AB\sin\theta$ to find θ. We assume the data are known to three significant digits.

(a) We use the definition of the cross product and note that $\hat{\mathbf{i}}\times\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{j}} = 0$:

$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = (1\hat{\mathbf{i}} + 2\hat{\mathbf{j}})\times(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}})$$
$$\vec{\mathbf{A}}\times\vec{\mathbf{B}} = 2\hat{\mathbf{i}}\times\hat{\mathbf{i}} + 3\hat{\mathbf{i}}\times\hat{\mathbf{j}} - 4\hat{\mathbf{j}}\times\hat{\mathbf{i}} + 6\hat{\mathbf{j}}\times\hat{\mathbf{j}}$$
$$= 0 + 3\hat{\mathbf{k}} - 4(-\hat{\mathbf{k}}) + 0 = \boxed{7.00\hat{\mathbf{k}}}$$

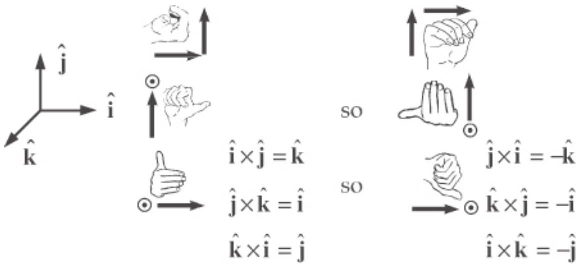
(b) Since $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = AB\sin\theta$, we have

$$\theta = \sin^{-1}\left(\frac{\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|}{AB}\right) = \sin^{-1}\left(\frac{7}{\sqrt{1^2+2^2}\sqrt{2^2+3^2}}\right) = \boxed{60.3^\circ}$$

P11.4 $\left|\hat{\mathbf{i}}\times\hat{\mathbf{i}}\right| = 1\cdot 1\cdot \sin 0° = 0$

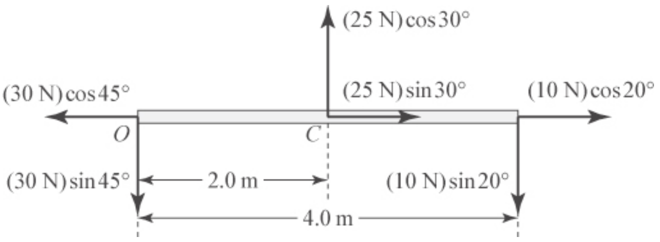
$\hat{\mathbf{j}}\times\hat{\mathbf{j}}$ and $\hat{\mathbf{k}}\times\hat{\mathbf{k}}$ are zero similarly since the vectors being multiplied are parallel.

$\left|\hat{\mathbf{i}}\times\hat{\mathbf{j}}\right| = 1\cdot 1\cdot \sin 90° = 1$



ANS. FIG. P11.4

P11.5 We first resolve all of the forces shown in Figure P11.5 into components parallel to and perpendicular to the beam as shown in ANS. FIG. P11.5.



ANS. FIG. P11.5

(a) The torque about an axis through point *O* is given by

$$\tau_O = +\left[(25\text{ N})\cos 30°\right](2.0\text{ m}) - \left[(10\text{ N})\sin 20°\right](4.0\text{ m}) = \boxed{+30\text{ N}\cdot\text{m}}$$

or $\tau_0 = \boxed{30\text{ N}\cdot\text{m counterclockwise}}$

(b) The torque about an axis through point *C* is given by

$$\tau_C = +\left[(30\text{ N})\sin 45°\right](2.0\text{ m}) - \left[(10\text{ N})\sin 20°\right](2.0\text{ m}) = \boxed{+36\text{ N}\cdot\text{m}}$$

or $\tau_C = \boxed{36\text{ N}\cdot\text{m counterclockwise}}$

P11.6
$$\vec{\mathbf{A}}\cdot\vec{\mathbf{B}} = -3.00(6.00) + 7.00(-10.0) + (-4.00)(9.00) = -124$$

$$AB = \sqrt{(-3.00)^2 + (7.00)^2 + (-4.00)^2}\cdot\sqrt{(6.00)^2 + (-10.0)^2 + (9.00)^2} = 127$$

(a) $\cos^{-1}\left(\dfrac{\vec{\mathbf{A}}\cdot\vec{\mathbf{B}}}{AB}\right) = \cos^{-1}(-0.979) = \boxed{168°}$

(b) $\vec{\mathbf{A}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ -3.00 & 7.00 & -4.00 \\ 6.00 & -10.0 & 9.00 \end{vmatrix} = 23.0\hat{\mathbf{i}} + 3.00\hat{\mathbf{j}} - 12.0\hat{\mathbf{k}}$

$$\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \sqrt{(23.0)^2 + (3.00)^2 + (-12.0)^2} = 26.1$$

$$\sin^{-1}\left(\frac{\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right|}{AB}\right) = \sin^{-1}(0.206) = \boxed{11.9^\circ} \text{ or } 168^\circ$$

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously because sin(180° − θ) = sin θ but cos (180° − θ) = − cos θ; in other words, the vectors can only be at most 180° apart and using the second method cannot distinguish θ from 180° − θ.

P11.7 We are given the condition $\left|\vec{\mathbf{A}}\times\vec{\mathbf{B}}\right| = \vec{\mathbf{A}}\cdot\vec{\mathbf{B}}$.

This says that $AB\sin\theta = AB\cos\theta$

so $\tan\theta = 1$

$\theta = \boxed{45.0^\circ}$ satisfies this condition.

P11.8 (a) The torque acting on the particle about the origin is

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}}\times\vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 6 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}(8-18)$$

$$= \boxed{(-10.0\ \mathrm{N}\cdot\mathrm{m})\hat{\mathbf{k}}}$$

(b) Yes. The point or axis must be on the other side of the line of action of the force, and half as far from this line along which the force acts. Then the lever arm of the force about this new axis will be half as large and the force will produce counterclockwise instead of clockwise torque.

(c) Yes. There are infinitely many such points, along a line that passes through the point described in (b) and parallel the line of action of the force.

(d) Yes, at the intersection of the line described in (c) and the y axis.

(e) $\boxed{\text{No, because there is only one point of intersection of the line described in (d) with the } y \text{ axis.}}$

(f) Let (0, *y*) represent the coordinates of the special axis of rotation located on the *y* axis of Cartesian coordinates. Then the displacement from this point to the particle feeling the force is $\vec{\mathbf{r}}_{\text{new}} = 4\hat{\mathbf{i}} + (6 - y)\hat{\mathbf{j}}$ in meters. The torque of the force about this new axis is

$$\vec{\boldsymbol{\tau}}_{\text{new}} = \vec{\mathbf{r}}_{\text{new}} \times \vec{\mathbf{F}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 4 & 6 - y & 0 \\ 3 & 2 & 0 \end{vmatrix}$$

$$= \hat{\mathbf{i}}(0 - 0) - \hat{\mathbf{j}}(0 - 0) + \hat{\mathbf{k}}(8 - 18 + 3y)$$

$$= (+5\ \text{N} \cdot \text{m})\hat{\mathbf{k}}$$

Then,

$$8 - 18 + 3y = 5 \quad \rightarrow \quad 3y = 15 \quad \rightarrow \quad y = 5$$

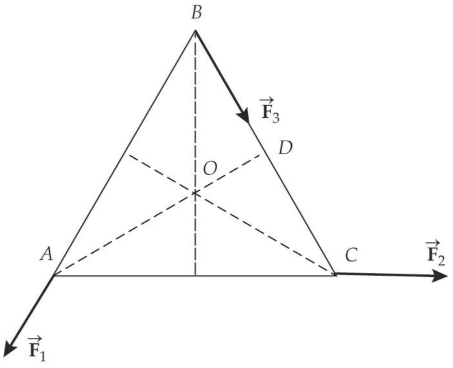
The position vector of the new axis is $\boxed{5.00\hat{\mathbf{j}}\ \text{m}}$.

P11.9 (a) The lever arms of the forces about *O* are all the same, equal to length *OD*, *L*.

If $\vec{\mathbf{F}}_3$ has a magnitude $\boxed{|\vec{\mathbf{F}}_3| = |\vec{\mathbf{F}}_1| + |\vec{\mathbf{F}}_2|}$, the net torque is zero:

$$\sum \tau = F_1 L + F_2 L - F_3 L = F_1 L + F_2 L - (F_1 + F_2)L = 0$$

(b) The torque produced by $\vec{\mathbf{F}}_3$ depends on the perpendicular distance *OD*, therefore translating the point of application of $\vec{\mathbf{F}}_3$ to any other point along *BC* $\boxed{\text{will not change the net torque}}$.



ANS. FIG. P11.9

P11.10 (a) $\boxed{\text{No.}}$

(b) The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero. To check:

$$\begin{aligned}(2\hat{\mathbf{i}}-3\hat{\mathbf{j}}+4\hat{\mathbf{k}})\cdot(4\hat{\mathbf{i}}+3\hat{\mathbf{j}}-\hat{\mathbf{k}}) &= (\hat{\mathbf{i}}\cdot\hat{\mathbf{i}})8+-9(\hat{\mathbf{j}}\cdot\hat{\mathbf{j}})-4(\hat{\mathbf{k}}\cdot\hat{\mathbf{k}})\\ &= 8-9-4=-5\end{aligned}$$

The answer is not zero.

$\boxed{\text{No. The cross product could not work out that way.}}$

Section 11.2 Analysis Model: Nonisolated System (Angular Momentum)

P11.11 Taking the geometric center of the compound object to be the pivot, the angular speed and the moment of inertia are

$$\omega = v/r = (5.00\text{ m/s})/0.500\text{ m} = 10.0\text{ rad/s}$$

and

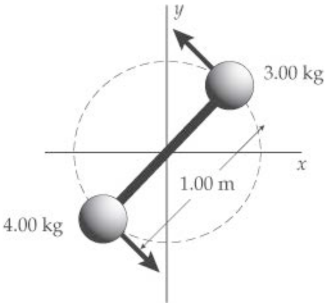
$$\begin{aligned}I = \sum mr^2 &= (4.00\text{ kg})(0.500\text{ m})^2 + (3.00\text{ kg})(0.500\text{ m})^2\\ &= 1.75\text{ kg}\cdot\text{m}^2\end{aligned}$$

By the right-hand rule, we find that the angular velocity is directed out of the plane. So the object's angular momentum, with magnitude

$$L = I\omega = (1.75\text{ kg}\cdot\text{m}^2)(10.0\text{ rad/s})$$

is the vector

$$\boxed{\vec{\mathbf{L}} = (17.5\text{ kg}\cdot\text{m}^2/\text{s})\hat{\mathbf{k}}}$$



ANS. FIG. P11.11

P11.12 We use $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$:

$$\vec{\mathbf{L}} = \left(1.50\hat{\mathbf{i}} + 2.20\hat{\mathbf{j}}\right) \text{ m} \times \left(1.50 \text{ kg}\right)\left(4.20\hat{\mathbf{i}} - 3.60\hat{\mathbf{j}}\right) \text{ m/s}$$

$$\vec{\mathbf{L}} = \left(-8.10\hat{\mathbf{k}} - 13.9\hat{\mathbf{k}}\right) \text{ kg}\cdot\text{m}^2/\text{s} = \boxed{\left(-22.0 \text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.13 We use $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$:

$$\vec{\mathbf{L}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ x & y & 0 \\ mv_x & mv_y & 0 \end{vmatrix} = \hat{\mathbf{i}}(0-0) - \hat{\mathbf{j}}(0-0) + \hat{\mathbf{k}}\left(mxv_y - myv_x\right)$$

$$\vec{\mathbf{L}} = \boxed{m\left(xv_y - yv_x\right)\hat{\mathbf{k}}}$$

P11.14 Whether we think of the Earth's surface as curved or flat, we interpret the problem to mean that the plane's line of flight extended is precisely tangent to the mountain at its peak, and nearly parallel to the wheat field. Let the positive x direction be eastward, positive y be northward, and positive z be vertically upward.

(a) $\vec{\mathbf{r}} = (4.30 \text{ km})\hat{\mathbf{k}} = \left(4.30 \times 10^3 \text{ m}\right)\hat{\mathbf{k}}$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}} = (12\ 000 \text{ kg})\left(-175\hat{\mathbf{i}} \text{ m/s}\right) = -2.10 \times 10^6\hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \left(4.30 \times 10^3\hat{\mathbf{k}} \text{ m}\right) \times \left(-2.10 \times 10^6\hat{\mathbf{i}} \text{ kg}\cdot\text{m/s}\right)$$

$$= \boxed{\left(-9.03 \times 10^9 \text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{j}}}$$

(b) $\boxed{\text{No.}}$ $L = |\vec{\mathbf{r}}||\vec{\mathbf{p}}|\sin\theta = mv(r\sin\theta)$, and $r\sin\theta$ is the altitude of the plane. Therefore, $L = \textit{constant}$ as the plane moves in level flight with constant velocity.

(c) $\boxed{\text{Zero.}}$ The position vector from Pike's Peak to the plane is anti-parallel to the velocity of the plane. That is, it is directed along the same line and opposite in direction. Thus, $L = mvr\sin 180° = 0$.

P11.15 (a) $\boxed{\text{Zero}}$ because $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$ and $\vec{\mathbf{r}} = 0$.

(b) At the highest point of the trajectory,

$$x = \frac{1}{2}R = \frac{v_i^2 \sin 2\theta}{2g} \text{ and}$$

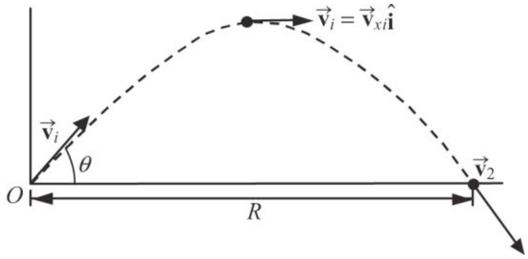
$$y = h_{\text{max}} = \frac{\left(v_i \sin\theta\right)^2}{2g}$$

The angular momentum is then

$$\vec{\mathbf{L}}_1 = \vec{\mathbf{r}}_1 \times m\vec{\mathbf{v}}_1$$

$$= \left[\frac{v_i^2 \sin 2\theta}{2g}\hat{\mathbf{i}} + \frac{(v_i \sin\theta)^2}{2g}\hat{\mathbf{j}}\right] \times m v_{xi}\hat{\mathbf{i}}$$

$$= \boxed{\frac{-m v_i^3 \sin\theta^2 \cos\theta}{2g}\hat{\mathbf{k}}}$$



ANS. FIG. P11.15

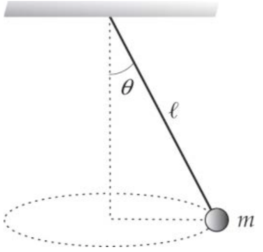
(c) $\vec{\mathbf{L}}_2 = R\hat{\mathbf{i}} \times m\vec{\mathbf{v}}_2$, where $R = \dfrac{v_i^2 \sin 2\theta}{g} = \dfrac{v_i^2(2\sin\theta\cos\theta)}{g}$

$$= mR\hat{\mathbf{i}} \times \left(v_i \cos\theta\,\hat{\mathbf{i}} - v_i \sin\theta\,\hat{\mathbf{j}}\right)$$

$$= -mRv_i \sin\theta\,\hat{\mathbf{k}} = \boxed{\frac{-2m v_i^3 \sin\theta \sin\theta}{g}\hat{\mathbf{k}}}$$

(d) $\boxed{\text{The downward force of gravity exerts a torque in the } -z \text{ direction.}}$

P11.16 We start with the particle under a net force model in the *x* and *y* directions:



ANS. FIG. P11.16

$$\sum F_x = ma_x: \qquad T\sin\theta = \frac{mv^2}{r}$$

$$\sum F_y = ma_y: \qquad T\cos\theta = mg$$

So $\dfrac{\sin\theta}{\cos\theta} = \dfrac{v^2}{rg}$ and $v = \sqrt{rg\dfrac{\sin\theta}{\cos\theta}}$

then $L = rmv\sin 90.0° = rm\sqrt{rg\dfrac{\sin\theta}{\cos\theta}} = \sqrt{m^2 g r^3 \dfrac{\sin\theta}{\cos\theta}}$

and since $r = \ell\sin\theta$,

$$L = \boxed{\sqrt{m^2 g \ell^3 \frac{\sin^4\theta}{\cos\theta}}}$$

P11.17 The angular displacement of the particle around the circle is $\theta = \omega t = \frac{vt}{R}$.

The vector from the center of the circle to the mass is then

$R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$, where *R* is measured from the +*x* axis.

The vector from point *P* to the mass is

$$\vec{\mathbf{r}} = R\hat{\mathbf{i}} + R\cos\theta\,\hat{\mathbf{i}} + R\sin\theta\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{r}} = R\left[\left(1 + \cos\left(\frac{vt}{R}\right)\right)\hat{\mathbf{i}} + \sin\left(\frac{vt}{R}\right)\hat{\mathbf{j}}\right]$$

The velocity is

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = -v\sin\left(\frac{vt}{R}\right)\hat{\mathbf{i}} + v\cos\left(\frac{vt}{R}\right)\hat{\mathbf{j}}$$

So

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$$

$$\vec{\mathbf{L}} = mvR\left[(1+\cos\omega t)\hat{\mathbf{i}} + \sin\omega t\hat{\mathbf{j}}\right]\times\left[-\sin\omega t\hat{\mathbf{i}} + \cos\omega t\hat{\mathbf{j}}\right]$$

$$\vec{\mathbf{L}} = \boxed{mvR\left[\cos\left(\frac{vt}{R}\right) + 1\right]\hat{\mathbf{k}}}$$

P11.18 (a) The net torque on the counterweight-cord-spool system is

$$\tau = \left|\vec{\mathbf{r}} \times \vec{\mathbf{F}}\right| = Rmg\sin\theta$$

$$\tau = 8.00\times10^{-2}\text{ m}(4.00\text{ kg})(9.80\text{ m/s}^2)\sin 90.0° = \boxed{3.14\text{ N}\cdot\text{m}}$$

(b) $L = \sum_i \left|\vec{\mathbf{r}} \times m_i\vec{\mathbf{v}}_i\right| = Rmv + RMv = R(m+M)v$

$$L = (0.080\ 0\text{ m})(4.00\text{ kg} + 2.00\text{ kg})v = \boxed{(0.480\text{ kg}\cdot\text{m})v}$$

(c) $\tau = \frac{dL}{dt} = (0.480\text{ kg}\cdot\text{m})a \quad \rightarrow \quad a = \frac{3.14\text{ N}\cdot\text{m}}{0.480\text{ kg}\cdot\text{m}} = \boxed{6.53\text{ m/s}^2}$

P11.19 Differentiating $\vec{\mathbf{r}} = \left(6.00\hat{\mathbf{i}} + 5.00t\hat{\mathbf{j}}\text{ m}\right)$ with respect to time gives

$$\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} = 5.00\hat{\mathbf{j}}\text{ m/s}$$

so $\vec{\mathbf{p}} = m\vec{\mathbf{v}} = (2.00\text{ kg})\left(5.00\hat{\mathbf{j}}\text{ m/s}\right) = 10.0\hat{\mathbf{j}}\text{ kg}\cdot\text{m/s}$

and $$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 6.00 & 5.00t & 0 \\ 0 & 10.0 & 0 \end{vmatrix} = \boxed{\left(60.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.20 (a) $$\int_0^{\vec{r}} d\vec{\mathbf{r}} = \int_0^t \vec{\mathbf{v}}dt = \ \vec{\mathbf{r}} - 0 = \int_0^t (6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}})dt = \ \vec{\mathbf{r}} = \left(6t^3/3\right)\hat{\mathbf{i}} + \left(2t^2/2\right)\hat{\mathbf{j}}$$

$= \boxed{2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}}$ in meters, where *t* is in seconds.

(b) The particle starts from rest at the origin, starts moving into the first quadrant, and gains speed faster and faster while turning to move more and more nearly parallel to the x axis.

(c) $\vec{\mathbf{a}} = (d\vec{\mathbf{v}}/dt) = (d/dt)(6t^2\ \hat{\mathbf{i}} + 2t\ \hat{\mathbf{j}}) = \boxed{(12t\ \hat{\mathbf{i}} + 2\ \hat{\mathbf{j}})\ \text{m/s}^2}$

(d) $\vec{F} = m\vec{\mathbf{a}} = (5\ \text{kg})(12t\ \hat{\mathbf{i}} + 2\ \hat{\mathbf{j}})\ \text{m/s}^2 = \boxed{(60t\ \hat{\mathbf{i}} + 10\ \hat{\mathbf{j}})\ \text{N}}$

(e) $\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = (2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}) \times (60t\hat{\mathbf{i}} + 10\hat{\mathbf{j}}) = 20t^3\hat{\mathbf{k}} - 60t^3\hat{\mathbf{k}}$

$= \boxed{-40t^3\hat{\mathbf{k}}\ \text{N}\cdot\text{m}}$

(f) $\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}} = (5\ \text{kg})(2t^3\hat{\mathbf{i}} + t^2\hat{\mathbf{j}}) \times (6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}) = 5(4t^4\hat{\mathbf{k}} - 6t^4\hat{\mathbf{k}})$

$= \boxed{-10t^4\hat{\mathbf{k}}\ \text{kg}\cdot\text{m}^2/\text{s}}$

(g) $K = \frac{1}{2}m\vec{\mathbf{v}}\cdot\vec{\mathbf{v}} = \frac{1}{2}(5\ \text{kg})(6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}})\cdot(6t^2\hat{\mathbf{i}} + 2t\hat{\mathbf{j}}) = (2.5)(36t^4 + 4t^2)$

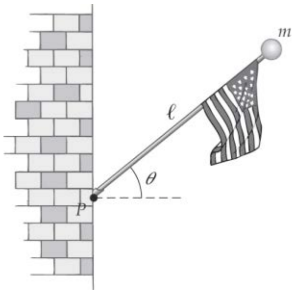
$= \boxed{(90t^4 + 10t^2)\ \text{J}}$

(h) $P = (d/dt)(90t^4 + 10t^2)\ \text{J} = \boxed{(360t^3 + 20t)\ \text{W}}$, all where *t* is in seconds.

P11.21 (a) The vector from *P* to the falling ball is

$$\vec{\mathbf{r}} = \vec{\mathbf{r}}_i + \vec{\mathbf{v}}_i t + \frac{1}{2}\vec{\mathbf{a}}t^2$$

$$\vec{\mathbf{r}} = \left(\ell\cos\theta\ \hat{\mathbf{i}} + \ell\sin\theta\ \hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}$$



ANS. FIG. P11.21

The velocity of the ball is

$$\vec{\mathbf{v}} = \vec{\mathbf{v}}_i + \vec{\mathbf{a}}t = 0 - gt\hat{\mathbf{j}}$$

So $$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$$

$$\vec{\mathbf{L}} = m\left[\left(\ell\cos\theta\,\hat{\mathbf{i}} + \ell\sin\theta\,\hat{\mathbf{j}}\right) + 0 - \left(\frac{1}{2}gt^2\right)\hat{\mathbf{j}}\right]\times\left(-gt\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{L}} = \boxed{-mg\ell\,t\cos\theta\,\hat{\mathbf{k}}}$$

(b) $\boxed{\text{The Earth exerts a gravitational torque on the projectile in the negative } z \text{ direction.}}$

(c) Differentiating with respect to time, we have $\boxed{-mg\ell\cos\theta\,\hat{\mathbf{k}}}$ for the rate of change of angular momentum, which is also the torque due to the gravitational force on the ball.

Section 11.3 Angular Momentum of a Rotating Rigid Object

P11.22 The moment of inertia of the sphere about an axis through its center is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(15.0\text{ kg})(0.500\text{ m})^2 = 1.50\text{ kg}\cdot\text{m}^2$$

Therefore, the magnitude of the angular momentum is

$$L = I\omega = \left(1.50\text{ kg}\cdot\text{m}^2\right)(3.00\text{ rad/s}) = 4.50\text{ kg}\cdot\text{m}^2/\text{s}$$

Since the sphere rotates counterclockwise about the vertical axis, the angular momentum vector is directed upward in the +*z* direction.

Thus,

$$\boxed{\vec{\mathbf{L}} = \left(4.50\text{ kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}}$$

P11.23 The total angular momentum about the center point is given by

$$L = I_h\omega_h + I_m\omega_m$$

For the hour hand: $I_h = \dfrac{m_hL_h^2}{3} = \dfrac{60.0\text{ kg}(2.70\text{ m})^2}{3} = 146\text{ kg}\cdot\text{m}^2$

For the minute hand: $I_m = \dfrac{m_mL_m^2}{3} = \dfrac{100\text{ kg}(4.50\text{ m})^2}{3} = 675\text{ kg}\cdot\text{m}^2$

In addition, $\omega_h = \dfrac{2\pi\text{ rad}}{12\text{ h}}\left(\dfrac{1\text{ h}}{3\,600\text{ s}}\right) = 1.45\times10^{-4}\text{ rad/s}$

while $\omega_m = \dfrac{2\pi\text{ rad}}{1\text{ h}}\left(\dfrac{1\text{ h}}{3600\text{ s}}\right) = 1.75\times10^{-3}\text{ rad/s}$

Thus, $$L = \left(146\ \text{kg}\cdot\text{m}^2\right)\left(1.45\times 10^{-4}\ \text{rad/s}\right) + \left(675\ \text{kg}\cdot\text{m}^2\right)\left(1.75\times 10^{-3}\ \text{rad/s}\right)$$

or $\boxed{L = 1.20\ \text{kg}\cdot\text{m}^2/\text{s}}$. The hands turn clockwise, so their vector angular momentum is $\boxed{\text{perpendicularly into the clock face.}}$

P11.24 We begin with

$$K = \frac{1}{2}I\omega^2$$

And multiply the right-hand side by $\frac{I}{I}$:

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I}$$

Substituting $L = I\omega$ then gives

$$K = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{I^2\omega^2}{I} = \boxed{\frac{L^2}{2I}}$$

P11.25 (a) For an axis of rotation passing through the center of mass, the magnitude of the angular momentum is given by

$$L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}\left(3.00\ \text{kg}\right)\left(0.200\ \text{m}\right)^2\left(6.00\ \text{rad/s}\right)$$
$$= \boxed{0.360\ \text{kg}\cdot\text{m}^2/\text{s}}$$

(b) For a point midway between the center and the rim, we use the parallel-axis theorem to find the moment of inertia about this point. Then,

$$L = I\omega = \left[\frac{1}{2}MR^2 + M\left(\frac{R}{2}\right)^2\right]\omega$$
$$= \frac{3}{4}\left(3.00\ \text{kg}\right)\left(0.200\ \text{m}\right)^2\left(6.00\ \text{rad/s}\right) = \boxed{0.540\ \text{kg}\cdot\text{m}^2/\text{s}}$$

P11.26 (a) Modeling the Earth as a sphere, we first calculate its moment of inertia about its rotation axis.

$$I = \frac{2}{5}MR^2 = \frac{2}{5}\left(5.98\times 10^{24}\ \text{kg}\right)\left(6.37\times 10^{6}\ \text{m}\right)^2$$
$$= 9.71\times 10^{37}\ \text{kg}\cdot\text{m}^2$$

Completing one rotation in one day, Earth's rotational angular speed is

$$\omega = \frac{1\ \text{rev}}{24\ \text{h}} = \frac{2\pi\ \text{rad}}{86\ 400\ \text{s}} = 7.27 \times 10^{5}\ \text{s}^{-1}$$

the rotational angular momentum of the Earth is then

$$\begin{aligned} \mathsf{L} = \mathsf{I}\omega &= \left(9.71 \times 10^{37}\ \text{kg} \cdot \text{m}^2\right)\left(7.27 \times 10^{5}\ \text{s}^{-1}\right) \\ &= \boxed{7.06 \times 10^{33}\ \text{kg} \cdot \text{m}^2/\text{s}} \end{aligned}$$

The Earth turns toward the east, counterclockwise as seen from above north, so the vector angular momentum points north along the Earth's axis, $\boxed{\text{towards the north celestial pole}}$ or nearly toward the star Polaris.

(b) In this case, we model the Earth as a particle, with moment of inertia

$$\begin{aligned} \mathsf{I} = \mathsf{MR}^2 &= \left(5.98 \times 10^{24}\ \text{kg}\right)\left(1.496 \times 10^{11}\ \text{m}\right)^2 \\ &= 1.34 \times 10^{47}\ \text{kg} \cdot \text{m}^2 \end{aligned}$$

Completing one orbit in one year, Earth's orbital angular speed is

$$\omega = \frac{1\ \text{rev}}{365.25\ \text{d}} = \frac{2\pi\ \text{rad}}{(365.25\ \text{d})(86\ 400\ \text{s/d})} = 1.99 \times 10^{-7}\ \text{s}^{-1}$$

the angular momentum of the Earth is then

$$\begin{aligned} \mathsf{L} = \mathsf{I}\omega &= \left(1.34 \times 10^{47}\ \text{kg} \cdot \text{m}^2\right)\left(1.99 \times 10^{-7}\ \text{s}^{-1}\right) \\ &= \boxed{2.66 \times 10^{40}\ \text{kg} \cdot \text{m}^2/\text{s}} \end{aligned}$$

The Earth plods around the Sun, counterclockwise as seen from above north, so the vector angular momentum points north perpendicular to the plane of the ecliptic, $\boxed{\text{toward the north ecliptic pole}}$ or 23.5° away from Polaris, toward the center of the circle that the north celestial pole moves in as the equinoxes precess. The north ecliptic pole is in the constellation Draco.

(c) The periods differ only by a factor of 365 (365 days for orbital motion to 1 day for rotation). Because of the huge distance from the Earth to the Sun, however, the moment of inertia of the Earth around the Sun is six orders of magnitude larger than that of the Earth about its axis.

P11.27 Defining the distance from the pivot to the particle as d, we first find the rotational inertia of the system for each case, from the information $M = 0.100$ kg, $m = 0.400$ kg, and $D = 1.00$ m.

(a) For the meterstick rotated about its center, $I_m = \frac{1}{12}MD^2$.

For the additional particle, $I_w = md^2 = m\left(\frac{1}{2}D^2\right)$.

Together, $I = I_m + I_w = \frac{1}{12}MD^2 + \frac{1}{4}mD^2$, or

$$I = \frac{(0.100\text{ kg})(1.00\text{ m})^2}{12} + \frac{(0.400\text{ kg})(1.00\text{ m})^2}{4} = 0.108\text{ kg}\cdot\text{m}^2$$

And the angular momentum is

$$L = I\omega = \left(0.108\text{ kg}\cdot\text{m}^2\right)\left(4.00\text{ rad/s}\right) = \boxed{0.433\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) For a stick rotated about a point at one end,

$$I_m = \frac{1}{3}mD^2 = \frac{1}{3}\left(0.100\text{ kg}\right)\left(1.00\text{ m}\right)^2 = 0.033\,3\text{ kg}\cdot\text{m}^2$$

For a point mass, $I_w = mD^2 = (0.400\text{ kg})(1.00\text{ m})^2 = 0.400\text{ kg}\cdot\text{m}^2$

so together they have rotational inertia

$$I = I_m + I_w = 0.433\text{ kg}\cdot\text{m}^2$$

and angular momentum

$$L = I\omega = \left(0.433\text{ kg}\cdot\text{m}^2\right)(4.00\text{ rad/s}) = \boxed{1.73\text{ kg}\cdot\text{m}^2/\text{s}}$$

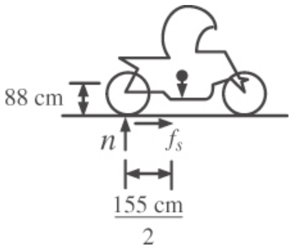
P11.28 We assume that the normal force $n = 0$ on the front wheel. On the bicycle,

$$\sum F_x = ma_x: \quad + f_s = ma_x$$
$$\sum F_y = ma_y: \quad + n - F_g = 0 \rightarrow n = mg$$

We must use the center of mass as the axis in

$$\sum \tau = I\alpha:$$

$$F_g(0) - n(77.5\text{ cm}) + f_s(88\text{ cm}) = 0$$



ANS. FIG. P11.28

We combine the equations by substitution:

$$-mg\left(77.5\text{ cm}\right) + ma_x\left(88\text{ cm}\right) = 0$$

$$a_x = \frac{\left(9.80\text{ m/s}^2\right)77.5\text{ cm}}{88\text{ cm}} = \boxed{8.63\text{ m/s}^2}$$

P11.29 We require $a_c = g = \frac{v^2}{r} = \omega^2 r$:

$$\omega = \sqrt{\frac{g}{r}} = \sqrt{\frac{(9.80\ \text{m/s}^2)}{100\ \text{m}}} = 0.313\ \text{rad/s}$$

$$I = Mr^2 = (5\times10^4\ \text{kg})(100\ \text{m})^2 = 5\times10^8\ \text{kg}\cdot\text{m}^2$$

(a) $L = I\omega = (5\times10^8\ \text{kg}\cdot\text{m}^2)(0.313\ \text{rad/s}) = \boxed{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}$

(b) $\Delta t = \frac{L_f - 0}{\sum\tau} = \frac{1.57\times10^8\ \text{kg}\cdot\text{m}^2/\text{s}}{2(125\ \text{N})(100\ \text{m})} = \boxed{6.26\times10^3\ \text{s}} = 1.74\ \text{h}$

Section 11.4 Analysis Model: Isolated System (Angular Momentum)

P11.30 (a) From conservation of angular momentum for the isolated system of two disks:

$$(I_1 + I_2)\omega_f = I_1\omega_i \quad \text{or} \quad \omega_f = \boxed{\frac{I_1}{I_1 + I_2}\omega_i}$$

This is an example of a totally inelastic collision.

(b) $K_f = \frac{1}{2}(I_1 + I_2)\omega_f^2$ and $K_i = \frac{1}{2}I_1\omega_i^2$

so $$\frac{K_f}{K_i} = \frac{\frac{1}{2}(I_1 + I_2)}{\frac{1}{2}I_1\omega_i^2}\left(\frac{I_1}{I_1 + I_2}\omega_i\right)^2 = \boxed{\frac{I_1}{I_1 + I_2}}$$

P11.31 From conservation of angular momentum,

$$I_i\omega_i = I_f\omega_f:\ (250\ \text{kg}\cdot\text{m}^2)(10.0\ \text{rev/min}) = \left[250\ \text{kg}\cdot\text{m}^2 + (25.0\ \text{kg})(2.00\ \text{m})^2\right]\omega_2$$

$$\omega_2 = \boxed{7.14\ \text{rev/min}}$$

P11.32 (a) Angular momentum is conserved in the puck-rod-putty system because there is no net external torque acting on the system.

$I\omega_{\text{initial}} = I\omega_{\text{final}}$:

$$mR^2\left(\frac{v_i}{R}\right) + m_pR^2(0) = (mR^2 + m_pR^2)\left(\frac{v_f}{R}\right)$$

$$mRv_i = (m + m_p)Rv_f$$

Solving for the final velocity gives

$$v_f = \left(\frac{m}{m + m_p}\right)v_i = \left(\frac{2.40\text{ kg}}{2.40\text{ kg} + 1.30\text{ kg}}\right)(5.00\text{ m/s}) = 3.24\text{ m/s}$$

Then,

$$T = \frac{2\pi R}{v_f} = \frac{2\pi(1.50\text{ m})}{3.24\text{ m/s}} = \boxed{2.91\text{ s}}$$

(b) Yes, because there is no net external torque acting on the puck-rod-putty system.

(c) No, because the pivot pin is always pulling on the rod to change the direction of the momentum.

(d) No. Some mechanical energy is converted into internal energy. The collision is perfectly inelastic.

P11.33 (a) Mechanical energy is not constant; some chemical potential energy in the woman's body is transformed into mechanical energy.

(b) Momentum is not constant. The turntable bearing exerts an external northward force on the axle to prevent the axle from moving southward because of the northward motion of the woman.

(c) Angular momentum is constant because the system is isolated from torque about the axle.

(d) From conservation of angular momentum for the system of the woman and the turntable, we have $L_f = L_i = 0$,

so, $L_f = I_{\text{woman}}\omega_{\text{woman}} + I_{\text{table}}\omega_{\text{table}} = 0$

and $$\omega_{\text{table}} = \left(-\frac{I_{\text{woman}}}{I_{\text{table}}}\right)\omega_{\text{woman}} = \left(-\frac{m_{\text{woman}}r^2}{I_{\text{table}}}\right)\left(\frac{v_{\text{woman}}}{r}\right)$$

$$= -\frac{m_{\text{woman}}rv_{\text{woman}}}{I_{\text{table}}}$$

$$\omega_{\text{table}} = -\frac{60.0\text{ kg}(2.00\text{ m})(1.50\text{ m/s})}{500\text{ kg}\cdot\text{m}^2} = -0.360\text{ rad/s}$$

or $\omega_{\text{table}} = \boxed{0.360\text{ rad/s (counterclockwise)}}$

(e) Chemical energy converted into mechanical energy is equal to

$$\Delta K = K_f - 0 = \frac{1}{2}m_{\text{woman}}v_{\text{woman}}^2 + \frac{1}{2}I\omega_{\text{table}}^2$$

$$\Delta K = \frac{1}{2}(60\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}(500\text{ kg}\cdot\text{m}^2)(0.360\text{ rad/s})^2$$

$$= \boxed{99.9\text{ J}}$$

P11.34 (a) The total angular momentum of the system of the student, the stool, and the weights about the axis of rotation is given by

$$I_{\text{total}} = I_{\text{weights}} + I_{\text{student}} = 2(mr^2) + 3.00\text{ kg}\cdot\text{m}^2$$

Before: $r = 1.00\text{ m}$

Thus, $I_i = 2(3.00\text{ kg})(1.00\text{ m})^2 + 3.00\text{ kg}\cdot\text{m}^2 = 9.00\text{ kg}\cdot\text{m}^2$

After: $r = 0.300\text{ m}$

Thus, $I_f = 2(3.00\text{ kg})(0.300\text{ m})^2 + 3.00\text{ kg}\cdot\text{m}^2 = 3.54\text{ kg}\cdot\text{m}^2$

We now use conservation of angular momentum.

$$I_f\omega_f = I_i\omega_i$$

or $\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.00}{3.54}\right)(0.750\text{ rad/s}) = \boxed{1.91\text{ rad/s}}$

(b) $K_i = \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(9.00\text{ kg}\cdot\text{m}^2)(0.750\text{ rad/s})^2 = \boxed{2.53\text{ J}}$

$K_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}(3.54\text{ kg}\cdot\text{m}^2)(1.91\text{ rad/s})^2 = \boxed{6.44\text{ J}}$

P11.35 (a) We solve by using conservation of angular momentum for the turntable-clay system, which is isolated from outside torques:

$I\omega_{\text{initial}} = I\omega_{\text{final}}$:

$$\frac{1}{2}mR^2\omega_i = \left(\frac{1}{2}mR^2 + m_c r^2\right)\omega_f$$

Solving for the final angular velocity gives

$$\omega_f = \frac{\frac{1}{2}mR^2\omega_i}{\frac{1}{2}mR^2 + m_c r^2} = \frac{\frac{1}{2}(30.0\text{ kg})(1.90\text{ m})^2(4\pi\text{ rad/s})}{\frac{1}{2}(30.0\text{ kg})(1.90\text{ m})^2 + (2.25\text{ kg})(1.80\text{ m})^2}$$

$$= \boxed{11.1\text{ rad/s counterclockwise}}$$

(b) $\boxed{\text{No.}}$ The initial energy is

$$K_i = \frac{1}{2}I\omega_i^2 = \frac{1}{2}\left(\frac{1}{2}mR^2\right)\omega_i^2$$

$$= \frac{1}{2}\left[\frac{1}{2}(30.0\text{ kg})(1.90\text{ m})^2\right](4\pi\text{ rad/s})^2$$

$$= 4\ 276\text{ J}$$

The final mechanical energy is

$$K_f = \frac{1}{2}I\omega_f^2 = \frac{1}{2}\left(\frac{1}{2}mR^2 + m_c r^2\right)\omega_f^2$$

$$= \frac{1}{2}\left[\frac{1}{2}(30.0\text{ kg})(1.90\text{ m})^2 + (2.25\text{ kg})(1.80\text{ m})^2\right] \times (11.1\text{ rad/s})^2$$

$$= 3\ 768\text{ J}$$

$\boxed{\text{Thus 507 J of mechanical energy is transformed into internal energy.}}$ The "angular collision" is completely inelastic.

(c) No. The original horizontal momentum is zero. As soon as the clay has stopped skidding on the turntable, the final momentum is (2.25 kg)(1.80 m)(11.1 rad/s) = 44.9 kg · m/s north. This is the amount of impulse injected by the bearing. The bearing thereafter keeps changing the system momentum to change the direction of the motion of the clay. $\boxed{\text{The turntable bearing promptly imparts an impulse of 44.9 kg} \cdot \text{m/s north into the turntable-clay system, and thereafter keeps changing the system momentum.}}$

P11.36 When they touch, the center of mass is distant from the center of the larger puck by

$$y_{CM} = \frac{0 + (80.0\text{ g})(4.00\text{ cm} + 6.00\text{ cm})}{120\text{ g} + 80.0\text{ g}} = 4.00\text{ cm}$$

(a) $L = r_1 m_1 v_1 + r_2 m_2 v_2 = 0 + (6.00 \times 10^{-2}\text{ m})(80.0 \times 10^{-3}\text{ kg})(1.50\text{ m/s})$

$$= \boxed{7.20 \times 10^{-3}\text{ kg} \cdot \text{m}^2/\text{s}}$$

(b) The moment of inertia about the CM is

$$I = \left(\frac{1}{2}m_1r_1^2 + m_1d_1^2\right) + \left(\frac{1}{2}m_2r_2^2 + m_2d_2^2\right)$$

$$I = \frac{1}{2}(0.120\text{ kg})(6.00\times10^{-2}\text{ m})^2 + (0.120\text{ kg})(4.00\times10^{-2})^2$$

$$+\frac{1}{2}(80.0\times10^{-3}\text{ kg})(4.00\times10^{-2}\text{ m})^2$$

$$+(80.0\times10^{-3}\text{ kg})(6.00\times10^{-2}\text{ m})^2$$

$$I = 7.60\times10^{-4}\text{ kg}\cdot\text{m}^2$$

Angular momentum of the two-puck system is conserved:

$$L = I\omega$$

$$\omega = \frac{L}{I} = \frac{7.20\times10^{-3}\text{ kg}\cdot\text{m}^2/\text{s}}{7.60\times10^{-4}\text{ kg}\cdot\text{m}^2} = \boxed{9.47\text{ rad/s}}$$

P11.37 (a) Taking the origin at the pivot point, note that $\vec{\mathbf{r}}$ is perpendicular to $\vec{\mathbf{v}}$, so $\sin\theta = 1$ and $L_f = L_i = mr\sin\theta = \boxed{mv\ell}$ vertically down.

(b) Taking v_f to be the speed of the bullet and the block together, we first apply conservation of angular momentum: $L_i = L_f$ becomes

$$\ell mv = \ell(m+M)v_f \quad\text{or}\quad v_f = \left(\frac{m}{m+M}\right)v$$

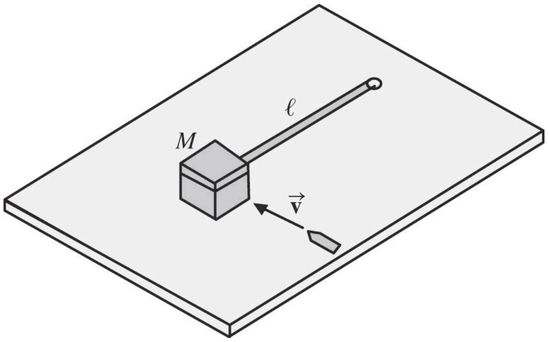
The total kinetic energies before and after the collision are, respectively,

$$K_i = \frac{1}{2}mv^2$$

and $$K_f = \frac{1}{2}(m+M)v_f^2 = \frac{1}{2}(m+M)\left(\frac{m}{m+M}\right)^2v^2 = \frac{1}{2}\left(\frac{m^2}{m+M}\right)v^2$$

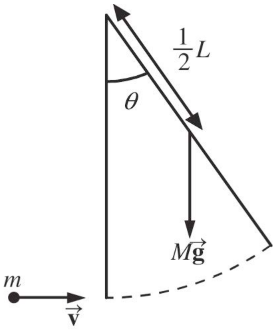
So the fraction of the kinetic energy that is converted into internal energy will be

$$\text{Fraction} = \frac{-\Delta K}{K_i} = \frac{K_i - K_f}{K_i} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}\left(\frac{m^2}{m+M}\right)v^2}{\frac{1}{2}mv^2} = \boxed{\frac{M}{m+M}}$$



ANS. FIG. P11.37

P11.38 (a) Let ω be the angular speed of the signboard when it is vertical.



ANS. FIG. P11.38

$$\frac{1}{2}I\omega^2 = Mgh$$

$$\frac{1}{2}\left(\frac{1}{3}ML^2\right)\omega^2 = Mg\frac{1}{2}L(1-\cos\theta)$$

$$\omega = \sqrt{\frac{3g(1-\cos\theta)}{L}}$$

$$= \sqrt{\frac{3(9.80\text{ m/s}^2)(1-\cos 25.0°)}{0.500\text{ m}}}$$

$$= \boxed{2.35\text{ rad/s}}$$

(b) $I_i\omega_i - mvL = I_f\omega_f$ represents angular momentum conservation for the sign-snowball system. Substituting into the above equation,

$$\left(\frac{1}{3}ML^2 + mL^2\right)\omega_f = \frac{1}{3}ML^2\omega_i - mvL$$

Solving,

$$\omega_f = \frac{\frac{1}{3}ML\omega_i - mv}{\left(\frac{1}{3}M + m\right)L}$$

$$= \frac{\frac{1}{3}(2.40\text{ kg})(0.500\text{ m})(2.347\text{ rad/s}) - (0.400\text{ kg})(1.60\text{ m/s})}{\left[\frac{1}{3}(2.40\text{ kg}) + 0.400\text{ kg}\right](0.500\text{ m})}$$

$$= \boxed{0.498\text{ rad/s}}$$

(c) Let h_{CM} = distance of center of mass from the axis of rotation.

$$h_{CM} = \frac{(2.40\text{ kg})(0.250\text{ m}) + (0.400\text{ kg})(0.500\text{ m})}{2.40\text{ kg} + 0.400\text{ kg}} = 0.285\ 7\text{ m}$$

Applying conservation of mechanical energy,

$$(M+m)gh_{CM}(1-\cos\theta) = \frac{1}{2}\left(\frac{1}{3}ML^2 + mL^2\right)\omega^2$$

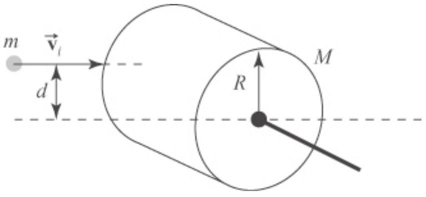
Solving for θ then gives

$$\theta = \cos^{-1}\left[1 - \frac{\left(\frac{1}{3}M + m\right)L^2\omega^2}{2(M+m)gh_{CM}}\right]$$

$$= \cos^{-1}\left\{1 - \frac{\left[\frac{1}{3}(2.40\text{ kg}) + 0.400\text{ kg}\right](0.500\text{ m})^2(0.498\text{ rad/s})^2}{2(2.40\text{ kg} + 0.400\text{ kg})(9.80\text{ m/s}^2)(0.285\ 7\text{ m})}\right\}$$

$$= \boxed{5.58°}$$

P11.39 (a) Consider the system to consist of the wad of clay and the cylinder. No external forces acting on this system have a torque about the center of the cylinder. Thus, angular momentum of the system is conserved about the axis of the cylinder.



ANS. FIG. P11.39

$L_f = L_i$: $I\omega = mv_id$

or $\left[\frac{1}{2}MR^2 + mR^2\right]\omega = mv_id$

Thus, $\omega = \boxed{\dfrac{2mv_id}{(M+2m)R^2}}$

(b) No; some mechanical energy of the system (the kinetic energy of the clay) changes into internal energy.

(c) The linear momentum of the system is not constant. The axle exerts a backward force on the cylinder when the clay strikes.

P11.40 The rotation rate of the station is such that at its rim the centripetal acceleration, a_c, is equal to the acceleration of gravity on the Earth's surface, g. Thus, the normal force from the rim's floor provides centripetal force on any person equal to that person's weight:

$$\sum F_r = ma_r: \quad n = \frac{mv^2}{r} \rightarrow mg = m\omega_i^2 r \rightarrow \omega_i^2 = \frac{g}{r}$$

The space station is isolated, so its angular momentum is conserved. When the people move to the center, the station's moment of inertia decreases, its angular speed increases, and the effective value of gravity increases.

From angular momentum conservation: $I_i\omega_i = I_f\omega_f \rightarrow \dfrac{\omega_f}{\omega_i} = \dfrac{I_i}{I_f}$, where

$$\begin{aligned} I_i &= I_{\text{station}} + I_{\text{people, i}} \\ &= \left[5.00\times10^8\ \text{kg}\cdot\text{m}^2 + 150(65.0\ \text{kg})(100\ \text{m})^2\right] \\ &= 5.98\times10^8\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

$$\begin{aligned} I_f &= I_{\text{station}} + I_{\text{people, f}} \\ &= \left[5.00\times10^8\ \text{kg}\cdot\text{m}^2 + 50(65.0\ \text{kg})(100\ \text{m})^2\right] \\ &= 5.32\times10^8\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

The centripetal acceleration is the effective value of gravity: $a_c \propto g$. Comparing values of acceleration before and during the union meeting, we have

$$\frac{g_f}{g_i} = \frac{a_{c,f}}{a_{c,i}} = \frac{\omega_f^2}{\omega_i^2} = \left(\frac{I_i}{I_f}\right)^2 = \left(\frac{5.98\times10^8}{5.32\times10^8}\right)^2 = 1.26 \rightarrow g_f = 1.26 g_i$$

$$\boxed{\begin{array}{l}\text{When the people move to the center, the angular speed of the} \\ \text{station increases. This increases the effective gravity by 26\%.} \\ \text{Therefore, the ball will not take the same amount of time to drop.}\end{array}}$$

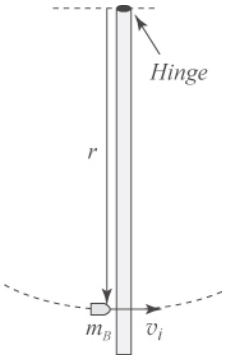
P11.41 (a) $\boxed{\text{Yes}}$, the bullet has angular momentum about an axis through the hinges of the door before the collision.

(b) The bullet strikes the door

$$r = 1.00\text{ m} - 0.100\text{ m} = 0.900\text{ m}$$

from the hinge. Its initial angular momentum is therefore

$$L_i = rp = m_B r v_i$$
$$= (0.005\ 00\text{ kg})(0.900\text{ m}) \times (1.00\times10^3\text{ m/s})$$
$$= \boxed{4.50\text{ kg}\cdot\text{m}^2/\text{s}}$$



ANS. FIG. P11.41

(c) $\boxed{\text{No; in the perfectly inelastic collision kinetic energy is transformed to internal energy.}}$

(d) Apply conservation of angular momentum, $L_i = L_f$:

$$m_B r v_i = I_f \omega_f = (I_{\text{door}} + I_{\text{bullet}})\omega_f$$
$$m_B r v_i = \left(\frac{1}{3}M_{\text{door}}L^2 + m_B r^2\right)\omega_f$$

where $L = 1.00$ m = the width of the door and $r = 0.900$ m [from part (b)]. Solving for the final angular velocity gives,

$$\omega = \frac{m_B r v_i}{\frac{1}{3}M_{\text{door}}L^2 + m_B r^2}$$
$$= \frac{(0.005\ 00\text{ kg})(0.900\text{ m})(1.00\times10^3\text{ m/s})}{\frac{1}{3}(18.0\text{ kg})(1.00\text{ m})^2 + (0.005\ 00\text{ kg})(0.900\text{ m})^2}$$
$$= \boxed{0.749\text{ rad/s}}$$

(e) The kinetic energy of the door-bullet system immediately after impact is

$$KE_f = \frac{1}{2}I_f\omega_f^2$$
$$= \frac{1}{2}\left[\frac{1}{3}(18.0\text{ kg})(1.00\text{ m})^2 + (0.005\ 00\text{ kg})(0.900\text{ m})^2\right] \times (0.749\text{ rad/s})^2$$
$$= \boxed{1.68\text{ J}}$$

The kinetic energy (of the bullet) just before impact was

$$KE_i = \frac{1}{2}m_B v_i^2 = \frac{1}{2}(0.005\ 00\text{ kg})(1.00\times10^3\text{ m/s})^2 = 2.50\times10^3\text{ J}$$

The total energy of the system must be the same before and after the collision, assuming we ignore the energy leaving by mechanical waves (sound) and heat (from the newly-warmer door to the cooler air). The kinetic energies are as follows: $\mathrm{KE_i} = 2.50\times10^3\ \mathrm{J}$ and $\mathrm{KE_f} = 1.68\ \mathrm{J}$.

$$\boxed{\text{Most of the initial kinetic energy is transformed to internal energy in the collision.}}$$

Section 11.5 The Motion of Gyroscopes and Tops

P11.42 Angular momentum of the system of the spacecraft and the gyroscope is conserved. The gyroscope and spacecraft turn in opposite directions.

$$0 = I_1\omega_1 + I_2\omega_2: \qquad -I_1\omega_1 = I_2\frac{\theta}{t}$$

$$\left(-20\ \mathrm{kg\cdot m^2}\right)\left(-100\ \mathrm{rad/s}\right) = \left(5\times10^5\ \mathrm{kg\cdot m^2}\right)\left(\frac{30^\circ}{t}\right)\left(\frac{\pi\ \mathrm{rad}}{180^\circ}\right)$$

$$t = \frac{2.62\times10^5\ \mathrm{s}}{2000} = \boxed{131\ \mathrm{s}}$$

P11.43 We begin by calculating the moment of inertia of the Earth, modeled as a sphere:

$$I = \frac{2}{5}MR^2 = \frac{2}{5}\left(5.98\times10^{24}\ \mathrm{kg}\right)\left(6.37\times10^6\ \mathrm{m}\right)^2$$
$$= 9.71\times10^{37}\ \mathrm{kg\cdot m^2}$$

Earth's rotational angular momentum is then

$$L = I\omega = \left(9.71\times10^{37}\ \mathrm{kg\cdot m^2}\right)\left(\frac{2\pi\ \mathrm{rad}}{86\,400\ \mathrm{s}}\right) = 7.06\times10^{33}\ \mathrm{kg\cdot m^2/s^2}$$

from which we can calculate the torque that is causing the precession:

$$\tau = L\omega_p$$
$$= \left(7.06\times10^{33}\ \mathrm{kg\cdot m^2/s}\right)\left(\frac{2\pi\ \mathrm{rad}}{2.58\times10^4\ \mathrm{yr}}\right)\left(\frac{1\ \mathrm{yr}}{365.25\ \mathrm{d}}\right)\left(\frac{1\ \mathrm{d}}{86\,400\ \mathrm{s}}\right)$$
$$= \boxed{5.45\times10^{22}\ \mathrm{N\cdot m}}$$

Additional Problems

P11.44 (a) Assuming the rope is massless, the tension is the same on both sides of the pulley:

$$\sum \tau = TR - TR = \boxed{0}$$

(b) $\sum \tau = \frac{dL}{dt}$, and since $\sum \tau = 0$, L = constant.

Since the total angular momentum of the system is initially zero, the total angular momentum remains zero, so the $\boxed{\text{monkey and bananas move upward with the same speed}}$ at any instant.

(c) $\boxed{\text{The monkey will not reach the bananas.}}$

ANS. FIG. P11.44

The motions of the monkey and bananas are identical, so the bananas remain out of the monkey's reach—until they get tangled in the pulley. To state the evidence differently, the tension in the rope is the same on both sides. Newton's second law applied to the monkey and bananas give the same acceleration upward.

P11.45 Using conservation of angular momentum, we have

$$L_{\text{aphelion}} = L_{\text{perihelion}} \quad \text{or} \quad \left(mr_a^2\right)\omega_a = \left(mr_p^2\right)\omega_p$$

Thus, $\left(mr_a^2\right)\frac{v_a}{r_a} = \left(mr_p^2\right)\frac{v_p}{r_p}$, giving $r_a v_a = r_p v_p$ or

$$v_a = \frac{r_p}{r_a} v_p = \frac{0.590 \text{ AU}}{35.0 \text{ AU}}(54.0 \text{ km/s}) = \boxed{0.910 \text{ km/s}}$$

P11.46 (a) Momentum is conserved in the isolated system of the two boys:

$$\vec{\mathbf{p}}_i = \vec{\mathbf{p}}_f : m_1 v_1 \,\hat{\mathbf{i}} - m_2 v_2 \,\hat{\mathbf{i}} = \left(m_1 + m_2\right)\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{p}}_i = (45.0 \text{ kg})(8.00 \text{ m/s})\,\hat{\mathbf{i}} - (31.0 \text{ kg})(11.0 \text{ m/s})\,\hat{\mathbf{i}} = (76.0 \text{ kg})\,\vec{\mathbf{v}}_f$$

$$\vec{\mathbf{v}}_f = \boxed{0.250\,\hat{\mathbf{i}} \text{ m/s}}$$

(b) The initial kinetic energy of the system is

$$\begin{aligned} K_i &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\ &= \frac{1}{2}(45.0\text{ kg})(+8.00\text{ m/s})^2 + \frac{1}{2}(31.0\text{ kg})(11.0\text{ m/s})^2 \\ &= 3\ 315.5\text{ J} \end{aligned}$$

and the kinetic energy after the collision is

$$K_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(76.0\text{ kg})(0.250\text{ m/s})^2 = 2.375\text{ J}$$

Thus the fraction remaining is

$$\frac{K_f}{K_i} = \frac{2.375\text{ J}}{3\ 315.5\text{ J}} = \boxed{0.000\ 716} = 0.071\ 6\%$$

(c) The calculation in part (a) still applies: $\vec{\mathbf{v}}_f = \boxed{0.250\ \hat{\mathbf{i}}\text{ m/s}}$

(d) Taking Jacob (m_1 = 45.0 kg) at the origin of a coordinate system, with Ethan (m_2 = 31.0 kg) on the y axis at $y = L$ = 1.20 m, the position of the CM of the boys is

$$y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{m_1(0) + m_2L}{m_1 + m_2} = \frac{m_2L}{m_1 + m_2}$$

$$y_{CM} = \frac{(31.0\text{ kg})(1.20\text{ m})}{45.0\text{ kg} + 31.0\text{ kg}} = 0.489\text{ m}$$

Jacob is y_{CM} = 0.489 m from the CM and Ethan is $(L - y_{CM})$ = $L - m_2L/(m_1 + m_2) = m_1L/(m_1 + m_2) = 0.711\text{ m}$ from the CM. Their angular momentum about the CM is $L = I\omega$:

$$m_1v_1L + m_2v_2(L - y_{CM}) = \left[m_1L^2 + m_2(L - y_{CM})^2\right]\omega$$

$$\rightarrow \omega = \frac{m_1v_1L + m_2v_2(L - y_{CM})}{m_2L^2 + m_2(L - y_{CM})^2}$$

$$\omega = \frac{(45.0\text{ kg})(8.00\text{ m/s})(0.489\text{ m}) + (31.0\text{ kg})(11.0\text{ m/s})(0.711\text{ m})}{(45.0\text{ kg})(0.489\text{ m})^2 + (31.0\text{ kg})(0.711\text{ m})^2}$$

$$\omega = \frac{418\text{ kg}\cdot\text{m}^2/\text{s}}{26.4\text{ kg}\cdot\text{m}^2} = \boxed{15.8\text{ rad/s}}$$

(e) Their kinetic energy after they link arms is

$$K_f = \frac{1}{2}(m_1 + m_2)v_{\text{CM}}^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2}(76.0\text{ kg})(0.250\text{ m/s})^2 + \frac{1}{2}(26.4\text{ kg}\cdot\text{m}^2/\text{s})(15.8\text{ rad/s})^2$$

$$K_f = 3\ 315.5\text{ J}$$

[Note: the result of the calculation of kinetic energy is exactly 3 315.5 J if no round-off is made in the calculation. It can be shown algebraically that the expression for the final kinetic energy is equivalent to the expression for the initial kinetic energy—the student is invited to show this.] Thus the fraction remaining is $K_f/K_i = 3\ 315.5\text{ J}/3\ 315.5\text{ J} = \boxed{1.00} = 100\%$.

(f) In part (b), the boys must necessarily deform as they slam into each other. During this deformation process, mechanical energy is transformed into internal energy. In part (e), there is no deformation involved. The boys simply link hands and some of their translational kinetic energy transforms to rotational kinetic energy, but none is transformed to internal energy.

P11.47 First, we define the following symbols:

I_P = moment of inertia due to mass of people on the equator

I_E = moment of inertia of the Earth alone (without people)

ω = angular velocity of the Earth (due to rotation on its axis)

$T = \frac{2\pi}{\omega}$ = rotational period of the Earth (length of the day)

R = radius of the Earth

The initial angular momentum of the system (before people start running) is

$$L_i = I_P\omega_i + I_E\omega_i = (I_P + I_E)\omega_i$$

When the Earth has angular speed ω, the tangential speed of a point on the equator is $v_t = R\omega$. Thus, when the people run eastward along the equator at speed v relative to the surface of the Earth, their tangential speed is $v_p = v_t + v = R\omega + v$ and their angular speed is

$$\omega_P = \frac{v_p}{R} = \omega + \frac{v}{R}$$

The angular momentum of the system after the people begin to run is

$$L_f = I_P\omega_p + I_E\omega = I_P\left(\omega + \frac{v}{R}\right) + I_E\omega = \left(I_P + I_E\right)\omega + \frac{I_P v}{R}$$

Since no external torques have acted on the system, angular momentum is conserved $\left(L_f = L_i\right)$, giving

$$\left(I_P + I_E\right)\omega + \frac{I_P v}{R} = \left(I_P + I_E\right)\omega_i$$

Thus, the final angular velocity of the Earth is

$$\omega = \omega_i - \frac{I_P v}{\left(I_P + I_E\right)R} = \omega_i\left(1 - x\right), \text{ where } x \equiv \frac{I_P v}{\left(I_P + I_E\right)R\omega_i}$$

The new length of the day is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\omega_i\left(1 - x\right)} = \frac{T_i}{1 - x} \approx T_i\left(1 + x\right)$$

so the increase in the length of the day is

$$\Delta T = T - T_i \approx T_i x = T_i\left[\frac{I_P v}{\left(I_P + I_E\right)R\omega_i}\right]$$

Since $\omega_i = \frac{2\pi}{T_i}$, this may be written as

$$\Delta T \approx \frac{T_i^2 I_P v}{2\pi\left(I_P + I_E\right)R}$$

To obtain a numeric answer, we compute

$$\begin{aligned} I_P = m_p R^2 &= \left[\left(7 \times 10^9\right)\left(55.0\text{ kg}\right)\right]\left(6.37 \times 10^6\text{ m}\right)^2 \\ &= 1.56 \times 10^{25}\text{ kg} \cdot \text{m}^2 \end{aligned}$$

and

$$\begin{aligned} I_E = \frac{2}{5}m_E R^2 &= \frac{2}{5}\left(5.98 \times 10^{24}\text{ kg}\right)\left(6.37 \times 10^6\text{ m}\right)^2 \\ &= 9.71 \times 10^{37}\text{ kg} \cdot \text{m}^2 \end{aligned}$$

Thus,

$$\begin{aligned} \Delta T &\approx \frac{\left(8.64 \times 10^4\text{ s}\right)^2\left(1.56 \times 10^{25}\text{ kg} \cdot \text{m}^2\right)\left(2.5\text{ m/s}\right)}{2\pi\left[\left(1.56 \times 10^{25} + 9.71 \times 10^{37}\right)\text{ kg} \cdot \text{m}^2\right]\left(6.37 \times 10^6\text{ m}\right)} \\ &= \boxed{7.50 \times 10^{-11}\text{ s}} \end{aligned}$$

P11.48 (a) $\left(K+U_g\right)_A = \left(K+U_g\right)_B$

$$0 + mgy_A = \frac{1}{2}mv_B^2 + 0$$

$$v_B = \sqrt{2gy_A} = \sqrt{2\left(9.80\text{ m/s}^2\right)(6.30\text{ m})} = \boxed{11.1\text{ m/s}}$$

(b) $L = mvr = (76.0\text{ kg})\ (11.1\text{ m/s})\ (6.30\text{ m}) = \boxed{5.32\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}$ toward you along the axis of the channel.

(c) The wheels on his skateboard prevent any tangential force from acting on him. Then no torque about the axis of the channel acts on him and his angular momentum is constant. His legs convert chemical into mechanical energy. They do work to increase his kinetic energy. The normal force acts in the upward direction, perpendicular to the direction of motion of the skateboarder.

(d) $L = mvr$: $v = \dfrac{5.32\times10^3\text{ kg}\cdot\text{m}^2/\text{s}}{(76.0\text{ kg})\ (5.85\text{ m})} = \boxed{12.0\text{ m/s}}$

(e) $\left(K+U_g\right)_B + U_{\text{chemical},B} = \left(K+U_g\right)_C$

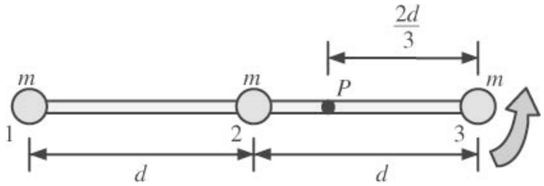
$$\frac{1}{2}(76.0\text{ kg})(11.1\text{ m/s})^2 + 0 + U_{\text{chem}}$$

$$= \frac{1}{2}(76.0\text{ kg})(12.0\text{ m/s})^2 + (76.0\text{ kg})\ (9.80\text{ m/s}^2)\ (0.450\text{ m})$$

$$U_{\text{chem}} = 5.44\text{ kJ} - 4.69\text{ kJ} + 335\text{ J} = \boxed{1.08\text{ kJ}}$$

P11.49 (a) The moment of inertia is given by

$$\begin{aligned} I &= \sum m_i r_i^2 \\ &= m\left(\frac{4d}{3}\right)^2 + m\left(\frac{d}{3}\right)^2 + m\left(\frac{2d}{3}\right)^2 \\ &= \boxed{7m\frac{d^2}{3}} \end{aligned}$$



ANS. FIG. P11.49

(b) Think of the whole weight, $3mg$, acting at the center of gravity.

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} = \left(\frac{d}{3}\right)\left(-\hat{\mathbf{i}}\right) \times 3mg\left(-\hat{\mathbf{j}}\right) = \boxed{\left(mgd\right)\hat{\mathbf{k}}}$$

(c) We find the angular acceleration from

$$\alpha = \frac{\tau}{I} = \frac{3mgd}{7md^2} = \boxed{\frac{3g}{7d}\text{ counterclockwise}}$$

(d) The linear acceleration of particle 3, a distance of $2d/3$ from the pivot, is

$$a = \alpha r_3 = \left(\frac{3g}{7d}\right)\left(\frac{2d}{3}\right) = \boxed{\frac{2g}{7}\text{ upward}}$$

(e) Because the axle is fixed, no external work is performed on the system of the Earth and the three particles, so total mechanical energy is conserved. Rotational kinetic energy will be maximum when the rod has swung to a vertical orientation with the center of gravity directly under the axle. Take gravitational potential energy to be zero when the rod is in its vertical orientation. In the initial horizontal orientation, the center of gravity of the system will be $d/3$ higher:

$$E = \left(K + U\right)_{i\text{ = horizontal}} = \left(K + U\right)_{f\text{ = vertical}}$$

$$0 + \left(3m\right)g\left(\frac{d}{3}\right) = K_f + 0 \rightarrow K_f = \boxed{mgd}$$

(f) In the vertical orientation, the rod has the greatest rotational kinetic energy:

$$K_f = \frac{1}{2}I\omega_f^2$$

$$mgd = \frac{1}{2}\left(7m\frac{d^2}{3}\right)\omega_f^2 \rightarrow \omega_f = \boxed{\sqrt{\frac{6g}{7d}}}$$

(g) The maximum angular momentum of the system is

$$L_f = I\omega_f = \frac{7md^2}{3}\sqrt{\frac{6g}{7d}} = \boxed{\left(\frac{14g}{3}\right)^{1/2} md^{3/2}}$$

(h) The maximum speed of particle 2 is

$$v_f = \omega_f r_2 = \sqrt{\frac{6g}{7d}}\frac{d}{3} = \boxed{\sqrt{\frac{2gd}{21}}}$$

P11.50 (a) The equation simplifies to

$$(1.75\ \text{kg}\cdot\text{m}^2/\text{s} - 0.181\ \text{kg}\cdot\text{m}^2/\text{s})\,\hat{\mathbf{j}} = (0.745\ \text{kg}\cdot\text{m}^2)\vec{\omega}$$

which gives

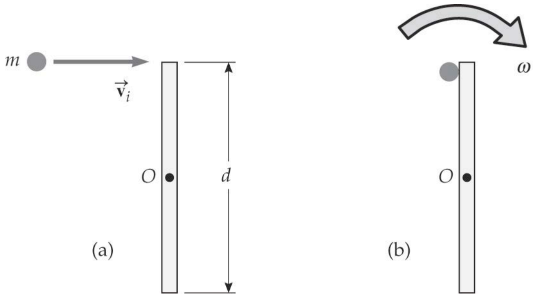
$$\vec{\omega} = \boxed{2.11\hat{\mathbf{j}}\ \text{rad/s}}$$

(b) We take the x axis east, the y axis up, and the z axis south.

The child has moment of inertia 0.730 kg·m^2 about the axis of the stool and is originally turning counterclockwise at 2.40 rad/s. At a point 0.350 m to the east of the axis, he catches a 0.120-kg ball moving toward the south at 4.30 m/s. He continues to hold the ball in his outstretched arm. Find his final angular velocity.

(c) Yes, with the left-hand side representing the final situation and the right-hand side representing the original situation, the equation describes the throwing process.

P11.51 (a) The appropriate model is to treat the projectile and the rod as an $\boxed{\text{isolated system}}$, experiencing no net external torque, or force.



ANS. FIG. P11.51

(b) $L_{\text{total}} = L_{\text{particle}} + L_{\text{rod}}$

$$= \frac{mv_i d}{2} + 0 = \boxed{\frac{mv_i d}{2}}$$

(c) $I_{\text{total}} = I_{\text{particle}} + I_{\text{rod}} = \frac{1}{12}Md^2 + m\left(\frac{d}{2}\right)^2$

$$I_{\text{total}} = \boxed{\frac{d^2(M+3m)}{12}}$$

(d) After the collision, we could express the angular momentum as,

$$L_{\text{total}} = I_{\text{total}}\omega = \boxed{\left(\frac{d^2(M+3m)}{12}\right)\omega}$$

(e) Recognizing that angular momentum is conserved,

$$L_f = L_i$$

$$\left(\frac{d^2(M+3m)}{12}\right)\omega = \frac{mv_i d}{2}$$

$$\omega = \boxed{\frac{6mv_i}{d(M+3m)}}$$

(f) $K = \boxed{\frac{1}{2}mv_i^2}$

(g) $K_{\text{total}} = \frac{1}{2}I_{\text{total}}\omega^2 = \frac{1}{2}\left(\frac{d^2(M+3m)}{12}\right)\left(\frac{6mv_i}{d(M+3m)}\right)^2$

$$K_{\text{total}} = \boxed{\frac{3m^2v_i^2}{2(M+3m)}}$$

(h) The change in mechanical energy is,

$$|\Delta K| = \frac{1}{2}mv_i^2 - \frac{3m^2v_i^2}{2(M+3m)} = \frac{mMv_i^2}{2(M+3m)}$$

Then, the fractional change in the mechanical energy is

$$\frac{\frac{mMv_i^2}{2(M+3m)}}{\frac{1}{2}mv_i^2} = \boxed{\frac{M}{M+3m}}$$

P11.52 (a) The puck's linear momentum is always changing. Its mechanical energy changes as work is done on it. But its angular momentum stays constant because although an external force (the tension of the rope) acts on the puck, no external torques act.

Therefore, L = constant, and at any time,

$$mvr = mv_i r_i$$

giving us

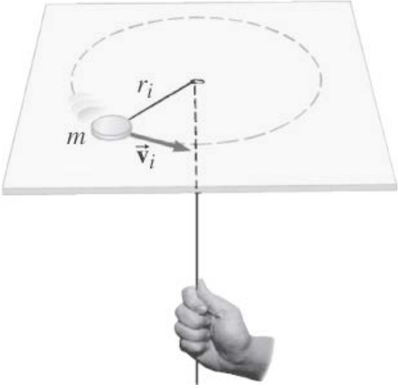
$$v = \frac{v_i r_i}{r} = \frac{(1.50\text{ m/s})(0.300\text{ m})}{0.100\text{ m}} = \boxed{4.50\text{ m/s}}$$

(b) From Newton's second law, the tension is always

$$T = \frac{mv^2}{r} = \frac{(0.050\ 0\text{ kg})(4.50\text{ m/s})^2}{0.100\text{ m}} = \boxed{10.1\text{ N}}$$

(c) The work-kinetic energy theorem identifies the work as

$$W = \Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$
$$= \frac{1}{2}(0.050\ 0\ \text{kg})\left[(4.50\ \text{m/s})^2 - (1.50\ \text{m/s})^2\right]$$
$$= \boxed{0.450\ \text{J}}$$



ANS. FIG. P11.52

P11.53 See ANS. FIG. P11.52 above.

(a) The puck is rotationally isolated because friction is zero and the torque on the puck from the tension in the string is zero:

$$\tau = \left|\vec{\mathbf{r}} \times \vec{\mathbf{F}}\right| = \left|\vec{\mathbf{r}}\right|\left|\vec{\mathbf{F}}\right| \sin 180° = 0$$

therefore, the angular momentum of the puck is conserved as the radius is decreased:

$$L_f = L_i$$
$$mrv = mr_iv_i$$
$$\rightarrow \quad v = \boxed{\frac{r_iv_i}{r}}$$

(b) The net force on the puck is tension:

$$T = \frac{mv^2}{r} = \boxed{\frac{m(r_iv_i)^2}{r^3}}$$

(c) Work is done by the tension force in the *negative-r*, inward direction as the radius decreases [$d\ell = -dr$]:

METHOD 1:

$$W = \int F \cdot d\ell = -\int T dr' = -\int_{r_i}^{r} \frac{m(r_i v_i)^2}{(r')^3} dr' = \left. \frac{m(r_i v_i)^2}{2(r')^2} \right|_{r_i}^{r}$$

$$= \frac{m(r_i v_i)^2}{2}\left(\frac{1}{r^2} - \frac{1}{r_i^2}\right) = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1\right)}$$

METHOD 2:

$$W = \Delta K = \frac{1}{2} m v^2 - \frac{1}{2} m v_i^2 = \boxed{\frac{1}{2} m v_i^2 \left(\frac{r_i^2}{r^2} - 1\right)}$$

P11.54 The description of the problem allows us to assume the asteroid-Earth system is isolated, so angular momentum is conserved ($L_i = L_f$). Let the period of rotation of Earth be *T* before the collision and $T + \Delta T$ after the collision. We have

$$I_E \omega_i = (I_E + I_A)\omega_f$$

$$\frac{2\pi}{T} I_E = \frac{2\pi}{T + \Delta T}(I_E + I_A)$$

$$\frac{T + \Delta T}{T} = \frac{I_E + I_A}{I_E}$$

which gives

$$\frac{\Delta T}{T} = \frac{I_A}{I_E} \quad \rightarrow \quad I_A = I_E \frac{\Delta T}{T}$$

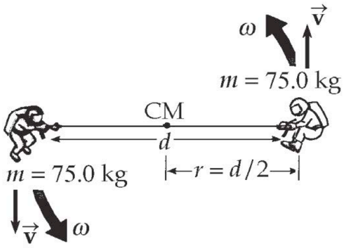
Treating Earth as a solid sphere of mass *M* and radius *R*, its moment of inertia is $\frac{2}{5} MR^2$. The moment of inertia of the asteroid at the equator is mR^2. We have then

$$I_A = I_E \frac{\Delta T}{T} \quad \rightarrow \quad mR^2 = \left(\frac{2}{5} MR^2\right)\left(\frac{\Delta T}{T}\right) \quad \rightarrow \quad m = \frac{2}{5} M\left(\frac{\Delta T}{T}\right)$$

$$m = \frac{2}{5}\left(5.98 \times 10^{24} \text{ kg}\right)\left(\frac{0.500 \text{ s}}{24(3\ 600 \text{ s})}\right) = 1.38 \times 10^{19} \text{ kg}$$

$$\boxed{\text{Life would not go on as normal. An asteroid that would cause a 0.5-s change in the rotation period of the Earth has a mass of } 1.38\times10^{19}\text{ kg and is an order of magnitude larger in diameter than the one that caused the extinction of the dinosaurs.}}$$

P11.55 Both astronauts will speed up equally as angular momentum for the two-astronaut-rope system is conserved in the absence of external torques. We use this principle to find the new angular speed with the shorter tether. Standard equations will tell us the original amount of angular momentum and the original and final amounts of kinetic energy. Then the kinetic energy difference is the work.



ANS. FIG. P11.55

(a) The angular momentum magnitude is $|\vec{L}| = m|\vec{r}\times\vec{v}|$. In this case, $\vec{r}$ and $\vec{v}$ are perpendicular, so the magnitude of L about the center of mass is

$$L = \sum mrv = 2(75.0\text{ kg})(5.00\text{ m})(5.00\text{ m/s})$$
$$= \boxed{3.75\times10^{3}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) The original kinetic energy is

$$K = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = 2\left(\frac{1}{2}\right)(75.0\text{ kg})(5.00\text{ m/s})^2$$
$$= \boxed{1.88\times10^{3}\text{ J}}$$

(c) With a lever arm of zero, the rope tension generates no torque about the center of mass. Thus, the angular momentum for the two-astronaut-rope system is unchanged:

$$L = \boxed{3.75\times10^{3}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(d) Again, $L = 2mrv$, so

$$v = \frac{L}{2mr} = \frac{3.75\times10^{3}\text{ kg}\cdot\text{m}^2/\text{s}}{2(75.0\text{ kg})(2.50\text{ m})} = \boxed{10.0\text{ m/s}}$$

(e) The final kinetic energy is

$$K = 2\left(\frac{1}{2}mv^2\right) = 2\left(\frac{1}{2}\right)(75.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{7.50\times10^{3}\text{ J}}$$

(f) The energy converted by the astronaut is the work he does:

$$W_{nc} = K_f - K_i = 7.50\times 10^3\ \text{J} - 1.88\times 10^3\ \text{J}$$

$$= \boxed{5.62\times 10^3\ \text{J}}$$

P11.56 Please refer to ANS. FIG. P11.55 and the discussion in P11.55 above.

(a) $L_i = 2\left[Mv\left(\dfrac{d}{2}\right)\right] = \boxed{Mvd}$

(b) $K = 2\left(\dfrac{1}{2}Mv^2\right) = \boxed{Mv^2}$

(c) $L_f = L_i = \boxed{Mvd}$

(d) $v_f = \dfrac{L_f}{2Mr_f} = \dfrac{Mvd}{2M\left(\dfrac{d}{4}\right)} = \boxed{2v}$

(e) $K_f = 2\left(\dfrac{1}{2}Mv_f^2\right) = M(2v)^2 = \boxed{4Mv^2}$

(f) If the work performed by the astronaut is made possible entirely by the conversion of chemical energy to mechanical energy, then the necessary chemical potential energy is:

$$W = K_f - K_i = \boxed{3Mv^2}$$

P11.57 (a) At the moment of release, two stones are moving with speed v_0. The total momentum has magnitude $\boxed{2mv_0}$. It keeps this same horizontal component of momentum as it flies away.

(b) The center of mass speed relative to the hunter is $v_{CM} = p/M = 2mv_0/3m = \boxed{2\,v_0/3}$ before the hunter lets go and, as far as horizontal motion is concerned, afterward.

(c) When the bola is first released, the stones are horizontally in line with two at distance ℓ on one side of the center knot and one at distance ℓ on the other side. The center of mass (CM) is then $x_{CM} = (2m\ell - m\ell)/3m = \ell/3$ from the center knot closer to the two stones: the one stone just being released is at distance $r_1 = 4\ell/3$ from the CM, the other two stones are at distance $r_2 = 2\ell/3$ from the CM.

The two stones, moving at v_0, have a relative speed $v_2 = v_0 - 2v_0/3 = v_0/3$ with respect to the CM, and the one stone has relative speed $v_1 = 2v_0/3 - 0 = 2v_0/3$ with respect to the CM. The one stone has angular speed

$$\omega_1 = \frac{v_1}{r_1} = \frac{2v_0/3}{4\ell/3} = \frac{v_0}{2\ell}$$

The other two stones have angular speed

$$\omega_2 = \frac{v_2}{r_2} = \frac{v_0/3}{2\ell/3} = \frac{v_0}{2\ell}$$

which is necessarily the same as that of stone 1: $\omega_1 = \omega_2 = \omega$. The total angular momentum around the center of mass is

$$\begin{aligned}\sum mvr &= mv_1r_1 + 2mv_2r_2\\ &= m(2v_0/3)(4\ell/3) + 2m(v_0/3)(2\ell/3)\\ &= \boxed{4m\ell v_0/3}\end{aligned}$$

The angular momentum remains constant with this value as the bola flies away.

(d) As computed in part (c), the angular speed ω at the moment of release is $v_0/2\ell$. As it moves through the air, the bola keeps constant angular momentum, but its moment of inertia changes to $3m\ell^2$. Then the new angular speed is given by

$$L = I\omega \quad \rightarrow \quad 4m\ell v_0/3 = 3m\ell^2\omega \quad \rightarrow \quad \omega = \boxed{4v_0/9\ell}$$

(e) At the moment of release,

$$K = \frac{1}{2}m(0)^2 + \frac{1}{2}(2m)v_0^2 = \boxed{mv_0^2}$$

(f) As it flies off in its horizontal motion it has kinetic energy

$$\begin{aligned}K &= \frac{1}{2}(3m)(v_{CM})^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}(3m)\left(\frac{2v_0}{3}\right)^2 + \frac{1}{2}(3m\ell^2)\left(\frac{4v_0}{9\ell}\right)^2\\ &= \boxed{\frac{26}{27}mv_0^2}\end{aligned}$$

(g) No horizontal forces act on the bola from outside after release, so the horizontal momentum stays constant. Its center of mass moves steadily with the horizontal velocity it had at release. No torques about its axis of rotation act on the bola, so its spin angular momentum stays constant. Internal forces cannot affect momentum conservation and angular momentum conservation, but they can affect mechanical energy. The cords pull on the stones as the stones rearrange themselves, so the cords must stretch slightly, so that energy of $mv_0^2/27$ changes from mechanical energy into internal energy as the bola takes its stable configuration. In a real situation, air resistance would have an influence on the motion of the stones.

P11.58 (a) Let M = mass of rod and m = mass of each bead. From $I_i\omega_i = I_f\omega_f$ between the moment of release and the moment the beads slide off, we have

$$\left[\frac{1}{12}M\ell^2 + 2mr_1^2\right]\omega_i = \left[\frac{1}{12}M\ell^2 + 2mr_2^2\right]\omega_f$$

When $M = 0.300$ kg, $\ell = 0.500$ m, $r_1 = 0.100$ m, $r_2 = 0.250$ m, and $\omega_i = 36.0$ rad/s, we find

$$[0.006\ 25 + 0.020\ 0m](36.0\text{ rad/s}) = [0.006\ 25 + 0.125m]\omega_f$$

$$\boxed{\omega_f = \frac{36.0(1+3.20m)}{1+20.0m}\text{ rad/s}}$$

(b) The denominator of this fraction always exceeds the numerator, so

ω_f decreases smoothly from a maximum value of 36.0 rad/s for $m = 0$ toward a minimum value of $(36\times 3.2/20) = 5.76$ rad/s as $m \to \infty$.

P11.59 The moment of inertia of the rest of the Earth is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(5.98\times10^{24}\text{ kg})(6.37\times10^6\text{ m})^2$$
$$= 9.71\times10^{37}\text{ kg}\cdot\text{m}^2$$

For the original ice disks,

$$\begin{aligned} I &= \frac{1}{2}Mr^2 = \frac{1}{2}\left(2.30\times10^{19}\text{ kg}\right)\left(6\times10^{5}\text{ m}\right)^2 \\ &= 4.14\times10^{30}\text{ kg}\cdot\text{m}^2 \end{aligned}$$

For the final thin shell of water,

$$\begin{aligned} I &= \frac{2}{3}Mr^2 = \frac{2}{3}\left(2.30\times10^{19}\text{ kg}\right)\left(6.37\times10^{6}\text{ m}\right)^2 \\ &= 6.22\times10^{32}\text{ kg}\cdot\text{m}^2 \end{aligned}$$

Conservation of angular momentum for the spinning planet is expressed by $I_i\omega_i = I_f\omega_f$:

$$\left(4.14\times10^{30}+9.71\times10^{37}\right)\frac{2\pi}{86\,400\text{ s}} = \left(6.22\times10^{32}+9.71\times10^{37}\right)\frac{2\pi}{\left(86\,400\text{ s}+\delta T\right)}$$

$$\left(1+\frac{\delta T}{86\,400\text{ s}}\right)\left(1+\frac{4.14\times10^{30}}{9.71\times10^{37}}\right) = 1+\frac{6.22\times10^{32}}{9.71\times10^{37}}$$

$$\frac{\delta T}{86\,400\text{ s}} = \frac{6.22\times10^{32}}{9.71\times10^{37}} - \frac{4.14\times10^{30}}{9.71\times10^{37}} \to \delta T = 0.550\text{ s}$$

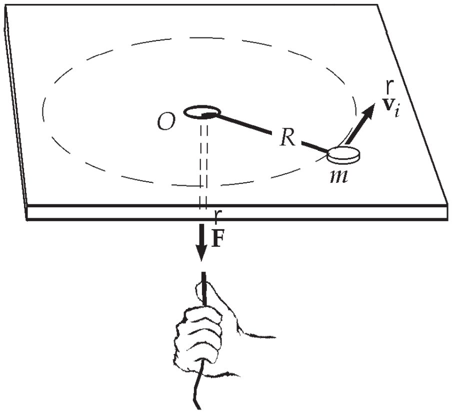
$$\boxed{\text{An increase of } 6.368\times10^{-4}\,\%\text{ or } 0.550\text{ s.}}$$

P11.60 To evaluate the change in kinetic energy of the puck, we first calculate the initial and final moments of inertia of the puck:

$$\begin{aligned} I_i &= mr_i^2 \\ &= (0.120\text{ kg})(0.400\text{ m})^2 \\ &= 1.92\times10^{-2}\text{ kg}\cdot\text{m}^2 \end{aligned}$$

and

$$\begin{aligned} I_f &= mr_f^2 \\ &= (0.120\text{ kg})(0.250\text{ m})^2 \\ &= 7.50\times10^{-3}\text{ kg}\cdot\text{m}^2 \end{aligned}$$



ANS. FIG. P11.60

The initial angular velocity of the puck is given by

$$\omega_i = \frac{v_i}{r_i} = \frac{0.800\text{ m/s}}{0.400\text{ m}} = 2.00\text{ rad/s}$$

Now, use conservation of angular momentum for the system of the puck,

$$\omega_f = \omega_i\left(\frac{I_i}{I_f}\right) = (2.00\ \text{rad/s})\left(\frac{1.92\times10^{-2}\ \text{kg}\cdot\text{m}^2}{7.5\times10^{-3}\ \text{kg}\cdot\text{m}^2}\right) = 5.12\ \text{rad/s}$$

Now,

$$\begin{aligned}\text{work done} = \Delta K &= \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2\\ &= \frac{1}{2}(7.50\times10^{-3}\ \text{kg}\cdot\text{m}^2)(5.12\ \text{rad/s})^2\\ &\quad -\frac{1}{2}(1.92\times10^{-2}\ \text{kg}\cdot\text{m}^2)(2.00\ \text{rad/s})^2\\ &= \boxed{5.99\times10^{-2}\ \text{J}}\end{aligned}$$

Challenge Problems

P11.61 (a) From the particle under a net force model:

$$F = \frac{\Delta p}{\Delta t} \rightarrow f_k = \frac{m(v_f - 0)}{\Delta t} = \frac{mv_f}{\Delta t} \qquad \textbf{[1]}$$

and from the rigid object under a net torque model:

$$\tau = \frac{\Delta L}{\Delta t} \rightarrow -f_k R = \frac{I(\omega_f - \omega_i)}{\Delta t} \qquad \textbf{[2]}$$

Divide [2] by [1]:

$$-R = \frac{I(\omega_f - \omega_i)}{mv_f}$$

Let $v_f = R\omega_f$ for pure rolling:

$$-R = \frac{I(\omega_f - \omega_i)}{m(R\omega_f)}$$

Solve for ω_f:

$$\omega_f = \frac{I\omega_i}{I + mR^2} = \frac{\frac{1}{2}mR^2\omega_i}{\frac{1}{2}mR^2 + mR^2} = \frac{\frac{1}{2}mR^2\omega_i}{\frac{3}{2}mR^2} = \boxed{\frac{1}{3}\omega_i}$$

(b) The fractional change in kinetic energy is

$$\frac{\Delta E}{E} = \frac{\frac{1}{2}I\omega_f^2 + \frac{1}{2}Mv_{CM}^2 - \frac{1}{2}I\omega_i^2}{\frac{1}{2}I\omega_i^2}$$

$$= \frac{\frac{1}{2}\left(\frac{1}{2}MR^2\right)(\omega_i/3)^2 + \frac{1}{2}M(R\omega_i/3)^2 - \frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2}{\frac{1}{2}\left(\frac{1}{2}MR^2\right)\omega_i^2}$$

$$= \boxed{-\frac{2}{3}}$$

(c) $\Delta t = \dfrac{\Delta p}{f} = \dfrac{Mv_f}{\mu Mg} = \dfrac{MR\omega_f}{\mu Mg} = \boxed{\dfrac{R\omega_f}{3\mu g}}$

(d) From the particle under constant acceleration model:

$$\Delta x = v_{avg}\Delta t = \frac{0 + v_f}{2}\Delta t = \frac{1}{2}v_f\Delta t = \frac{1}{2}(R\omega_f)\left[\frac{1}{3}\left(\frac{R\omega_i}{\mu g}\right)\right]$$

$$= \frac{1}{2}\left[R\left(\frac{1}{3}\omega_i\right)\right]\left[\frac{1}{3}\left(\frac{R\omega_i}{\mu g}\right)\right] = \boxed{\frac{R^2\omega_i^2}{18\mu g}}$$

P11.62 (a) After impact, the disk adheres to the stick, so they will rotate about their common center of mass; therefore, we must consider the angular momentum of the system about its CM. First we find the velocity of the CM by writing the equations for momentum conservation:

$$m_d v_{di} + 0 = (m_d + m_s)v_{CM}$$

$$v_{CM} = \frac{m_d}{m_d + m_s}v_{di} = \left(\frac{2.0\text{ kg}}{2.0\text{ kg} + 1.0\text{ kg}}\right)(3.0\text{ m/s}) = 2.0\text{ m/s}$$

The speed of the CM is $\boxed{2.0\text{ m/s}}$.

(b) Locate the center of mass between the disk and the center of the stick at impact:

$$y_{CM} = \frac{m_d r + m_s(0)}{m_d + m_s} = \frac{(2.0\text{ kg})(2.0\text{ m})}{2.0\text{ kg} + 1.0\text{ kg}} = \frac{4}{3}\text{ m}$$

This means at impact the CM is 4/3 meters from the center of the stick; therefore, the disk is 2.0 meters – 4/3 meters = 2/3 meters from the CM at impact. Use the parallel-axis theorem to find the moment of inertia of the system about the CM:

$$I_s = I_{CM} + m_s r_s^2 = 1.33\ \mathrm{kg\cdot m^2} + (1.0\ \mathrm{kg})\left(\frac{4}{3}\ \mathrm{m}\right)^2 = 3.11\ \mathrm{kg\cdot m^2}$$

The moment of inertia of the disk about the CM is

$$I_d = m_d r_d^2 = (2.0\ \mathrm{kg})\left(\frac{2}{3}\ \mathrm{m}\right)^2 = 0.889\ \mathrm{kg\cdot m^2}$$

Angular momentum about the CM is conserved:

$$L = r_d m_d v_d = I_d\omega + I_s\omega = (I_d + I_s)\omega$$

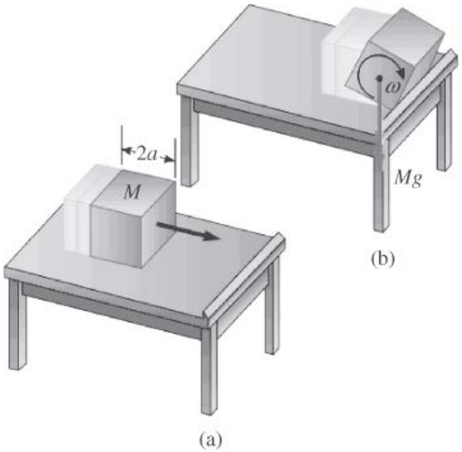
$$\omega = \frac{r_d m_d v_d}{I_d + I_s} = \frac{\left(\frac{2}{3}\ \mathrm{m}\right)(2.0\ \mathrm{kg})(3.0\ \mathrm{m/s})}{0.889\ \mathrm{kg\cdot m^2} + 3.11\ \mathrm{kg\cdot m^2}}$$

$$= \frac{4.0\ \mathrm{kg\cdot m^2/s}}{4.00\ \mathrm{kg\cdot m^2}} = \boxed{1.0\ \mathrm{rad/s}}$$

P11.63 Angular momentum is conserved during the inelastic collision.

$$Mva = I\omega$$

$$\omega = \frac{Mva}{I} = \frac{3v}{8a}$$



ANS. FIG. P11.63

The condition, that the box falls off the table, is that the center of mass must reach its maximum height as the box rotates, $h_{max} = a\sqrt{2}$. Using conservation of energy:

$$\frac{1}{2}I\omega^2 = Mg\left(a\sqrt{2} - a\right)$$

$$\frac{1}{2}\left(\frac{8Ma^2}{3}\right)\left(\frac{3v}{8a}\right)^2 = Mg\left(a\sqrt{2} - a\right)$$

$$v^2 = \frac{16}{3} ga\left(\sqrt{2}-1\right)$$

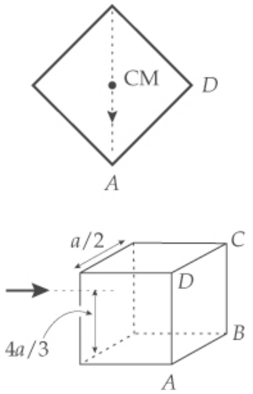
$$v = \boxed{4\left[\frac{ga}{3}\left(\sqrt{2}-1\right)\right]^{1/2}}$$

P11.64 For the cube to tip over, the center of mass (CM) must rise so that it is over the axis of rotation *AB*. To do this, the CM must be raised a distance of $a\left(\sqrt{2}-1\right)$. After the bullet strikes the cube, the system is isolated:

$$K_f + U_f = K_i + U_i$$

$$0 + Mga\left(\sqrt{2}-1\right) = \frac{1}{2} I_{\text{cube}}\omega^2 + 0$$

The moment of inertia of the cube about its CM (from Table 10.2) is



ANS. FIG. P11.64

$$I_{CM} = \frac{1}{12} M\left[(2a)^2 + (2a)^2\right] = \frac{8}{12} Ma^2 = \frac{2}{3} Ma^2$$

The cube rotates about an edge, $\sqrt{2}a$ from the CM. By the parallel-axis theorem,

$$I = I_{CM} + M\left(\sqrt{2}a\right)^2 = \frac{2}{3} Ma^2 + 2Ma^2 = \frac{8}{3} Ma^2$$

From conservation of angular momentum,

$$L_{i\ (\text{bullet})} = L_{i\ (\text{cube})} \rightarrow \frac{4a}{3} mv = \left(\frac{8}{3} Ma^2\right)\omega \quad \rightarrow \quad \omega = \frac{mv}{2Ma}$$

Inserting the expression for ω back into the energy equation, we have

$$Mga\left(\sqrt{2}-1\right) = \frac{1}{2}\left(\frac{8}{3} Ma^2\right)\frac{m^2v^2}{4M^2a^2} \rightarrow v = \boxed{\frac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P11.2	(a) 740 cm^2; (b) 59.5 cm
P11.4	See full solution in P11.4.
P11.6	(a) 168°; (b) 11.9°; (c) the first method
P11.8	(a) $(-10.0\ \text{N}\cdot\text{m})\hat{\mathbf{k}}$; (b) Yes; (c) Yes; (d) Yes; (e) No; (f) $5.00\hat{\mathbf{j}}$ m
P11.10	(a) No; (b) No, the cross product could not work out that way.
P11.12	$\left(-22.0\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}$
P11.14	(a) $\left(-9.03\times10^{9}\text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{j}}$; (b) No; (c) Zero
P11.16	$\sqrt{m^2 g\ell^3 \dfrac{\sin^4\theta}{\cos\theta}}$
P11.18	(a) 3.14 N · m; (b) (0.480 kg · m)v; (c) 6.53 m/s^2
P11.20	(a) $2t^3\hat{\mathbf{i}}+t^2\hat{\mathbf{j}}$; (b) The particle starts from rest at the origin, starts moving into the first quadrant, and gains speed faster while turning to move more nearly parallel to the x axis; (c) $\left(12t\hat{\mathbf{i}}+2\hat{\mathbf{j}}\right)\ \text{m/s}^2$; (d) $\left(60t\hat{\mathbf{i}}+10\hat{\mathbf{j}}\right)$ N; (e) $-40t^3\hat{\mathbf{k}}\ \text{N}\cdot\text{m}$;; (f) $-10t^4\hat{\mathbf{k}}\ \text{kg}\cdot\text{m}^2/\text{s}$; (g) $(90t^4+10t^2)$ J; (h) $(360t^3+20t)$ W
P11.22	$\vec{\mathbf{L}}=\left(4.50\ \text{kg}\cdot\text{m}^2/\text{s}\right)\hat{\mathbf{k}}$
P11.24	$K=\dfrac{1}{2}I\omega^2=\dfrac{1}{2}\dfrac{I^2\omega^2}{I}=\dfrac{L^2}{2I}$
P11.26	(a) $7.06\times10^{33}\text{kg}\cdot\text{m}^2/\text{s}$, toward the north celestial pole; (b) $2.66\times10^{40}\text{kg}\cdot\text{m}^2/\text{s}$, toward the north ecliptic pole; (c) See P11.26(c) for full explanation.
P11.28	8.63 m/s^2
P11.30	(a) $\dfrac{I_1}{I_1+I_2}\omega_i$; (b) $\dfrac{I_1}{I_1+I_2}$
P11.32	(a) 2.91 s; (b) Yes because there is no net external torque acting on the puck-rod-putty system; (c) No because the pivot pin is always pulling on the rod to change the direction of the momentum; (d) No. Some mechanical energy is converted into internal energy. The collision is perfectly inelastic.

P11.34 (a) 1.91 rad/s; (b) 2.53 J, 6.44 J

P11.36 (a) $7.20 \times 10^{-3}\,\mathrm{kg \cdot m^2/s}$; (b) 9.47 rad/s

P11.38 (a) 2.35 rad/s; (b) 0.498 rad/s; (c) 5.58°

P11.40 When the people move to the center, the angular speed of the station increases. This increases the effective gravity by 26%. Therefore, the ball will not take the same amount of time to drop.

P11.42 131 s

P11.44 (a) 0; (b) monkey and bananas move upward with the same speed; (c) The monkey will not reach the bananas.

P11.46 (a) $0.250\hat{\mathbf{i}}$ m/s; (b) 0.000 716; (c) $0.250\hat{\mathbf{i}}$ m/s; (d) 15.8 rad/s; (e) 1.00; (f) See P11.46(f) for full explanation.

P11.48 (a) 11.1 m/s; (b) $5.32 \times 10^{3}\,\mathrm{kg \cdot m^2/s}$; (c) See P11.48(c) for full explanation; (d) 12.0 m/s; (e) 1.08 kJ

P11.50 (a) $2.11\hat{\mathbf{j}}$ rad/s; (b) See P11.50(b) for full problem statement; (c) Yes, with the left-hand side representing the final situation and the right-hand side representing the original situation, the equation describes the throwing process.

P11.52 (a) 4.50 m/s; (b) 10.1 N; (c) 0.450 J

P11.54 An asteroid that would cause a 0.500-s change in the rotation period of the Earth has a mass of 1.38×10^{19} kg and is an order of magnitude larger in diameter than the one that caused the extinction of the dinosaurs.

P11.56 (a) Mvd; (b) Mv^2; (c) Mvd; (d) $2v$; (e) $4Mv^2$; (f) $3Mv^2$

P11.58 (a) $\omega_f = \dfrac{36.0(1+3.20m)}{1+20.0m}$ rad/s; (b) ω_f decreases smoothly from a maximum value of 36.0 rad/s for $m = 0$ toward a minimum value of $(36 \times 3.2/20) = 5.76$ rad/s as $m \to \infty$

P11.60 5.99×10^{-2} J

P11.62 (a) 2.0 m/s; (b) 1.0 rad/s

P11.64 $\dfrac{M}{m}\sqrt{3ga\left(\sqrt{2}-1\right)}$

12

Static Equilibrium and Elasticity

CHAPTER OUTLINE

12.1	Analysis Model: Rigid Object in Equilibrium
12.2	More on the Center of Gravity
12.3	Examples of Rigid Objects in Static Equilibrium
12.4	Elastic Properties of Solids

* An asterisk indicates an item new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ12.1 Answer (b). The skyscraper is about 300 m tall. The gravitational field (acceleration) is weaker at the top by about 900 parts in ten million, by on the order of 10^{-4} times. The top half of the uniform building is lighter than the bottom half by about $(1/2)(10^{-4})$ times. Relative to the center of mass at the geometric center, this effect moves the center of gravity down, by about $(1/2)(10^{-4})(150\text{ m}) \sim 10$ mm.

OQ12.2 Answer (c). Net torque = $(50\text{ N})(2\text{ m}) - (200\text{ N})(5\text{ m}) - (300\text{ N})x = 0$; therefore, $x = 3$ m.

OQ12.3 Answer (a). Our theory of rotational motion does not contradict our previous theory of translational motion. The center of mass of the object moves as if the object were a particle, with all of the forces applied there. This is true whether the object is starting to rotate or not.

OQ12.4 Answer (d). In order for an object to be in equilibrium, it must be in both translational equilibrium and rotational equilibrium. Thus, it must meet two conditions of equilibrium, namely $\vec{\mathbf{F}}_{\text{net}} = 0$ and $\vec{\tau}_{\text{net}} = 0$.

OQ12.5 Answer (b). The lower the center of gravity, the more stable the can. In cases (a) and (c) the center of gravity is above the base by one-half the height of the can. In case (b), the center of gravity is above the base by only a bit more than one-quarter of the height of the can.

OQ12.6 Answer (d). Using the left end of the plank as a pivot and requiring that $\sum \tau = 0$ gives

$$-mg(2.00\text{ m}) + F_2(3.00\text{ m}) = 0$$

or

$$F_2 = \frac{2mg}{3} = \frac{2(20.0\text{ kg})(9.80\text{ m/s}^2)}{3} = 131\text{ N}$$

OQ12.7 Answer: $\tau_D > \tau_C > \tau_E > \tau_B > \tau_A$. The force exerts a counterclockwise torque about pivot D. The line of action of the force passes through C, so the torque about this axis is zero. In order of increasing negative (clockwise) values come the torques about F, E and B essentially together, and A.

OQ12.8 Answer (e). In the problems we study, the forces applied to the object lie in a plane, and the axis we choose is a line perpendicular to this plane, so it appears as a point on the force diagram. It can be chosen anywhere. The algebra of solving for unknown forces is generally easier if we choose the axis where some unknown forces are acting.

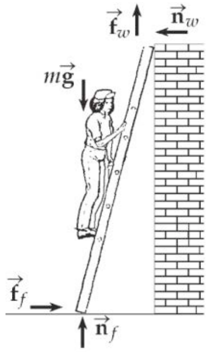
OQ12.9 (i) Answer (b). The extension is directly proportional to the original dimension, according to $F/A = Y\Delta L/L_i$.

(ii) Answer (e). Doubling the diameter quadruples the area to make the extension four times smaller.

OQ12.10 Answer (b). Visualize the ax as like a balanced playground seesaw with one large-mass person on one side, close to the fulcrum, and a small-mass person far from the fulcrum on the other side. Different masses are on the two sides of the center of mass. The mean position of mass is not the median position.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ12.1 The free-body diagram demonstrates that it is necessary to have friction on the ground to counterbalance the normal force of the wall and to keep the base of the ladder from sliding. If there is friction on the floor *and* on the wall, it is not possible to determine whether the ladder will slip, from the equilibrium conditions alone.



ANS. FIG. CQ12.1

CQ12.2 A V-shaped boomerang, a barstool, an empty coffee cup, a satellite dish, and a curving plastic slide at the edge of a swimming pool each have a center of mass that is not within the bulk of the object.

CQ12.3 (a) Consider pushing up with one hand on one side of a steering wheel and pulling down equally hard with the other hand on the other side. A pair of equal-magnitude oppositely-directed forces applied at different points is called a couple.

(b) An object in free fall has a nonzero net force acting on it, but a net torque of zero about its center of mass.

CQ12.4 When one is away from a wall and leans over, one's back moves backward so the body's center of gravity stays over the feet. When standing against a wall and leaning over, the wall prevents the backside from moving backward, so the center of gravity shifts forward. Once your CG is no longer over your feet, gravity contributes to a nonzero net torque on your body and you begin to rotate.

CQ12.5 If an object is suspended from some point and allowed to freely rotate, the object's weight will cause a torque about that point unless the line of action of its weight passes through the point of support. Suspend the plywood from the nail, and hang the plumb bob from the nail. Trace on the plywood along the string of the plumb bob. The plywood's center of gravity is somewhere along that line. Now suspend the plywood with the nail through a different point on the plywood, not along the first line you drew. Again hang the plumb bob from the nail and trace along the string. The center of gravity is located halfway through the thickness of the plywood under the intersection of the two lines you drew.

CQ12.6 She can be correct. Consider the case of a bridge supported at both ends: the sum of the forces on the ends equals the total weight of the bridge. If the dog stands on a relatively thick scale, the dog's legs on the ground might support more of its weight than its legs on the scale. She can check for and if necessary correct for this error by having the dog stand like a bridge with two legs on the scale and two on a book of equal thickness—a physics textbook is a good choice.

CQ12.7 Yes, it can. Consider an object on a spring oscillating back and forth. In the center of the motion both the sum of the torques and the sum of the forces acting on the object are (separately) zero. Again, a meteoroid flying freely through interstellar space feels essentially no forces and keeps moving with constant velocity.

CQ12.8 Shear deformation. Its deformations are parallel to its surface.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

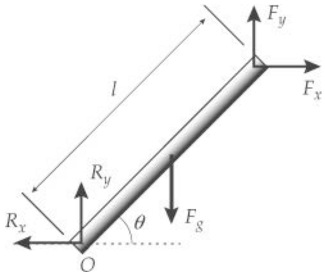
Section 12.1 Analysis Model: Rigid Object in Equilibrium

P12.1 Use distances, angles, and forces as shown in ANS. FIG. P12.1. The conditions of equilibrium are:

$$\sum F_y = 0 \Rightarrow \boxed{F_y + R_y - F_g = 0}$$

$$\sum F_x = 0 \Rightarrow \boxed{F_x - R_x = 0}$$

$$\sum \tau = 0 \Rightarrow \boxed{F_y \ell \cos\theta - F_g\left(\frac{\ell}{2}\right)\cos\theta - F_x \ell \sin\theta = 0}$$



ANS. FIG. P12.1

P12.2 Take torques about P, as shown in ANS. FIG. P12.2.

$$\sum \tau_p = -n_O\left[\frac{\ell}{2} + d\right] + m_1 g\left[\frac{\ell}{2} + d\right] + m_b g d - m_2 g x = 0$$

We want to find x for which $n_O = 0$:

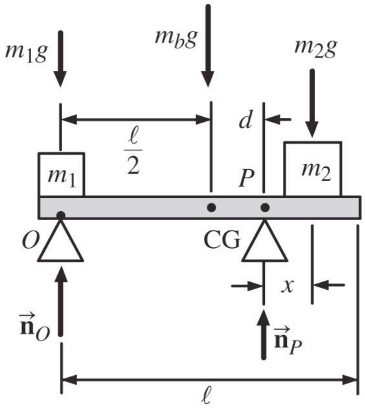
$$x = \frac{(m_1 g + m_b g)d + m_1 g\frac{\ell}{2}}{m_2 g} = \frac{(m_1 + m_b)d + m_1\frac{\ell}{2}}{m_2}$$

For the values given:

$$x = \frac{(m_1 + m_b)d + m_1\dfrac{\ell}{2}}{m_2}$$

$$x = \frac{(5.00\text{ kg} + 3.00\text{ kg})(0.300\text{ m}) + (5.00\text{ kg})\dfrac{1.00\text{ m}}{2}}{15.0\text{ m}}$$

$$x = 0.327\text{ m}$$



ANS. FIG. P12.2

$\boxed{\text{The situation is impossible because } x \text{ is larger than the remaining portion of the beam, which is 0.200 m long.}}$

Section 12.2 More on the Center of Gravity

P12.3 The coordinates of the center of gravity of piece 1 are

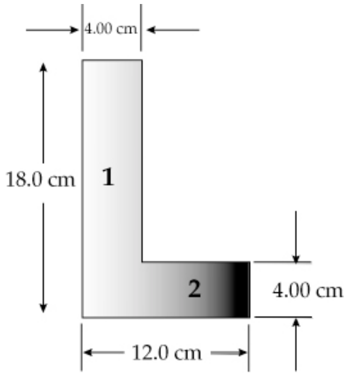
$$x_1 = 2.00\text{ cm and } y_1 = 9.00\text{ cm}$$

The coordinates for piece 2 are

$$x_2 = 8.00\text{ cm and } y_2 = 2.00\text{ cm}$$

The area of each piece is

$$A_1 = 72.0\text{ cm}^2 \text{ and } A_2 = 32.0\text{ cm}^2$$



ANS. FIG. P12.3

And the mass of each piece is proportional to the area. Thus,

$$x_{CG} = \frac{\sum m_i x_i}{\sum m_i} = \frac{\left(72.0\text{ cm}^2\right)\left(2.00\text{ cm}\right)+\left(32.0\text{ cm}^2\right)\left(8.00\text{ cm}\right)}{72.0\text{ cm}^2+32.0\text{ cm}^2}$$

$$= \boxed{3.85\text{ cm}}$$

and

$$y_{CG} = \frac{\sum m_i y_i}{\sum m_i} = \frac{\left(72.0\text{ cm}^2\right)\left(9.00\text{ cm}\right)+\left(32.0\text{ cm}^2\right)\left(2.00\text{ cm}\right)}{104\text{ cm}^2}$$

$$= \boxed{6.85\text{ cm}}$$

P12.4 The definition of the center of gravity as the average position of mass in the set of objects will result in equations about x and y coordinates that we can rearrange and solve to find where the last mass must be.

From $\vec{\mathbf{r}}_{CG} = \frac{\sum m_i \vec{\mathbf{r}}_i}{\sum m_i}$, $\vec{\mathbf{r}}_{CG}\left(\sum m_i\right) = \sum m_i \vec{\mathbf{r}}_i$

We require the center of mass to be at the origin; this simplifies the equation, leaving

$$\sum m_i x_i = 0 \quad \text{and} \quad \sum m_i y_i = 0$$

To find the x coordinate, we substitute the known values:

$$\left(5.00\text{ kg}\right)\left(0\text{ m}\right)+\left(3.00\text{ kg}\right)\left(0\text{ m}\right) + \left(4.00\text{ kg}\right)\left(3.00\text{ m}\right)+\left(8.00\text{ kg}\right)x = 0$$

Solving for x gives $x = -1.50\text{ m}$.

Likewise, to find the y coordinate, we solve:

$$\left(5.00\text{ kg}\right)\left(0\text{ m}\right)+\left(3.00\text{ kg}\right)\left(4.00\text{ m}\right) + \left(4.00\text{ kg}\right)\left(0\text{ m}\right)+\left(8.00\text{ kg}\right)y = 0$$

to find $y = -1.50\text{ m}$

Therefore, a fourth mass of 8.00 kg should be located at

$$\vec{\mathbf{r}}_4 = \boxed{(-1.50\hat{\mathbf{i}} - 1.50\hat{\mathbf{j}})\text{ m}}$$

P12.5 Let σ represent the mass-per-face area. (It would be equal to the material's density multiplied by the constant thickness of the wood.) A vertical strip at position *x*, with width *dx* and height $\frac{(x-3.00)^2}{9}$, has mass

$$dm = \frac{\sigma(x-3.00)^2\,dx}{9}$$

The total mass is

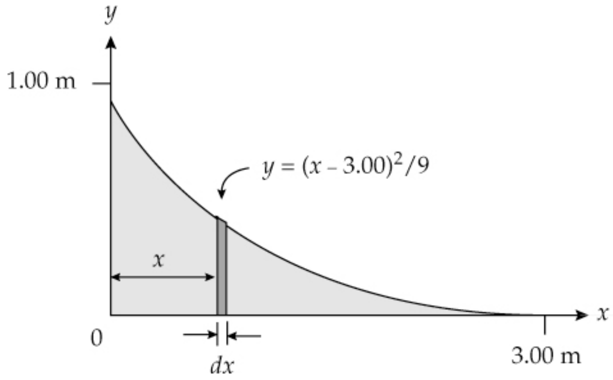
$$M = \int dm = \int_{x=0}^{3.00} \frac{\sigma(x-3)^2\,dx}{9} = \left(\frac{\sigma}{9}\right)\int_0^{3.00}\left(x^2-6x+9\right)dx$$

$$= \left(\frac{\sigma}{9}\right)\left[\frac{x^3}{3}-\frac{6x^2}{2}+9x\right]_0^{3.00} = \sigma$$

The *x* coordinate of the center of gravity is

$$x_{CG} = \frac{\int x\,dm}{M} = \frac{1}{9\sigma}\int_0^{3.00}\sigma x(x-3)^2\,dx = \frac{\sigma}{9\sigma}\int_0^{3.00}\left(x^3-6x^2+9x\right)dx$$

$$= \frac{1}{9}\left[\frac{x^4}{4}-\frac{6x^3}{3}+\frac{9x^2}{2}\right]_0^{3.00} = \frac{6.75\text{ m}}{9.00} = \boxed{0.750\text{ m}}$$



ANS. FIG. P12.5

P12.6 We can visualize this as a whole pizza with mass m_1 and center of gravity located at x_1, plus a hole that has negative mass, $-m_2$, with center of gravity at x_2:

$$x_{CG} = \frac{m_1x_1 - m_2x_2}{m_1 - m_2}$$

Call σ the mass of each unit of pizza area.

$$x_{CG}=\frac{\sigma\pi R^2 0-\sigma\pi\left(\frac{R}{2}\right)^2\left(\frac{-R}{2}\right)}{\sigma\pi R^2-\sigma\pi\left(\frac{R}{2}\right)^2}$$

$$x_{CG}=\frac{R/8}{3/4}=\boxed{\frac{R}{6}}$$

P12.7 In a uniform gravitational field, the center of mass and center of gravity of an object coincide. Thus, the center of gravity of the triangle is located at $x = 6.67$ m, $y = 2.33$ m (see Example 9.12 on the center of mass of a triangle in Chapter 9).

The coordinates of the center of gravity of the three-object system are then:

$$x_{CG}=\frac{\sum m_i x_i}{\sum m_i}$$

$$=\frac{(6.00\text{ kg})(5.50\text{ m})+(3.00\text{ kg})(6.67\text{ m})+(5.00\text{ kg})(-3.50\text{ m})}{(6.00+3.00+5.00)\text{ kg}}$$

$$=\frac{35.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}}=\boxed{2.54\text{ m}}\text{ and}$$

$$y_{CG}=\frac{\sum m_i y_i}{\sum m_i}$$

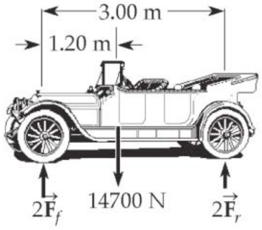
$$=\frac{(6.00\text{ kg})(7.00\text{ m})+(3.00\text{ kg})(2.33\text{ m})+(5.00\text{ kg})(+3.50\text{ m})}{14.0\text{ kg}}$$

$$=\frac{66.5\text{ kg}\cdot\text{m}}{14.0\text{ kg}}=\boxed{4.75\text{ m}}$$

Section 12.3 Examples of Rigid Objects in Static Equilibrium

P12.8 The car's weight is

$$F_g = mg = (1\ 500\text{ kg})(9.80\text{ m/s}^2)$$
$$= 1\ 4700\text{ N}$$



ANS. FIG. P12.8

Call $\vec{F}$ the force of the ground on each of the front wheels and $\vec{R}$ the normal force on each of the rear wheels. If we take torques around the front axle, with counterclockwise in the picture chosen as positive, the equations are as follows:

$$\sum F_x = 0: \quad 0 = 0$$
$$\sum F_y = 0: \quad 2R - 14\ 700\text{N} + 2F = 0$$
$$\sum \tau = 0: \quad +2R(3.00\text{m}) - (14\ 700\text{ N})(1.20\text{m}) + 2F(0) = 0$$

The torque equation gives:

$$R = \frac{17\ 640\text{ N}\cdot\text{m}}{6.00\text{ m}} = 2\ 940\text{ N} = \boxed{2.94\text{ kN}}$$

Then, from the second force equation,

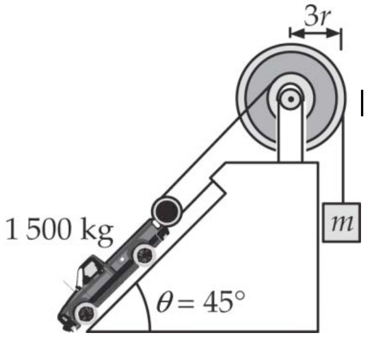
$$2(2.94\text{ kN}) - 14.7\text{ kN} + 2F = 0 \qquad \text{and} \qquad F = \boxed{4.41\text{ kN}}$$

P12.9 The second condition for equilibrium at the pulley is

$$\sum \tau = 0 = mg(3r) - Tr$$

and from equilibrium at the truck, we obtain

$$2T - Mg\sin 45.0° = 0$$
$$T = \frac{Mg\sin 45.0°}{2}$$
$$= \frac{(1\ 500\text{ kg})g\sin 45.0°}{2}$$
$$= 530g\text{ N}$$



ANS. FIG. P12.9

solving for the mass of the counterweight from [1] and substituting gives

$$m = \frac{T}{3g} = \frac{530g}{3g} = \boxed{177\text{ kg}}$$

P12.10 (a) For rotational equilibrium of the lowest rod about its point of support, $\sum \tau = 0$.

$$+(12.0\text{ g})g(3.00\text{ cm}) - m_1 g(4.00\text{ cm}) = 0$$

which gives

$$\boxed{m_1 = 9.00\text{ g}}$$

(b) For the middle rod,

$$+m_2 g(2.00\text{ cm}) - (12.0\text{ g} + 9.0\text{ g})g(5.00\text{ cm}) = 0$$

which gives

$$\boxed{m_2 = 52.5 \text{ g}}$$

(c) For the top rod,

$$(52.5 \text{ g} + 12.0 \text{ g} + 9.0 \text{ g})g(4.00 \text{ cm}) - m_3 g(6.00 \text{ cm}) = 0$$

which gives

$$\boxed{m_3 = 49.0 \text{ g}}$$

P12.11 Since the beam is in equilibrium, we choose the center as our pivot point and require that

$$\sum \tau_{\text{center}} = -F_{\text{Sam}}(2.80 \text{ m}) + F_{\text{Joe}}(1.80 \text{ m}) = 0$$

or

$$F_{\text{Joe}} = 1.56 F_{\text{Sam}} \qquad \textbf{[1]}$$

Also,

$$\sum F_y = 0 \quad \Rightarrow \quad F_{\text{Sam}} + F_{\text{Joe}} = 450 \text{ N} \qquad \textbf{[2]}$$

Substitute equation [1] into [2] to get the following:

$$F_{\text{Sam}} + 1.56 F_{\text{Sam}} = 450 \text{ N} \quad \text{or} \quad F_{\text{Sam}} = \frac{450 \text{ N}}{2.56} = 176 \text{ N}$$

Then, equation [1] yields $F_{\text{Joe}} = 1.56(176 \text{ N}) = 274 \text{ N}$

$$\boxed{\begin{array}{l}\text{Sam exerts an upward force of 176 N.}\\ \text{Joe exerts an upward force of 274 N.}\end{array}}$$

P12.12 (a) To find *U*, measure distances and forces from point *A*. Then, balancing torques,

$$(0.750 \text{ m})U = (29.4 \text{ N})(2.25) \quad \boxed{U = 88.2 \text{ N}}$$

(b) To find *D*, measure distances and forces from point *B*. Then, balancing torques,

$$(0.750 \text{ m})D = (1.50 \text{ m})(29.4 \text{ N}) \quad \boxed{D = 58.8 \text{ N}}$$

Also, notice that $U = D + F_g$, so $\sum F_y = 0$.

P12.13 (a) The wall is frictionless, but it does exert a horizontal normal force, n_w.

$$\sum F_x = f - n_w = 0$$

$$\sum F_y = n_g - 800 \text{ N} - 500 \text{ N} = 0$$

Taking torques about an axis at the foot of the ladder,

$$(800\text{ N})(4.00\text{ m})\sin 30.0° + (500\text{ N})(7.50\text{ m})\sin 30.0° \\ -n_w(15.0\text{ cm})\cos 30.0° = 0$$

Solving the torque equation,

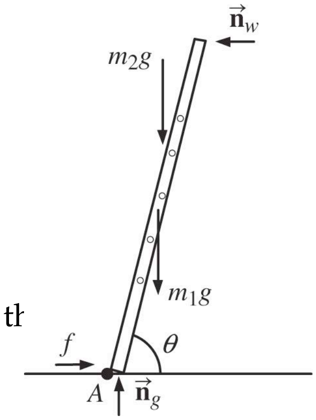
$$n_w = \frac{\left[(4.00\text{ m})(800\text{ N}) + (7.50\text{ m})(500\text{ N})\right]\tan 30.0°}{15.0\text{ m}} = 268\text{ N}$$

Next substitute this value into the F_x equation to find

$f = n_w = \boxed{268\text{ N}}$ in the positive x direction.

Solving the equation $\sum F_y = 0$,

$n_g = \boxed{1\,300\text{ N}}$ in the positive y direction



ANS. FIG. P12.13(b)

(b) Refer to ANS. FIG. P12.13(b) on the right. In th equation $\sum \tau_A = 0$ gives:

$$(9.00\text{ m})(800\text{ N})\sin 30.0° \\ + (7.50\text{ m})(500\text{ N})\sin 30.0° \\ - (15.0\text{ m})(n_w)\sin 60.0° = 0$$

or $n_w = 421\text{ N}$

Since $f = n_w = 421\text{ N}$ and $f = f_{\text{max}} = \mu n_g$, we find

$$\mu = \frac{f_{\text{max}}}{n_g} = \frac{421\text{ N}}{1\,300\text{ N}} = \boxed{0.324}$$

P12.14 (a) The wall is frictionless, but it does exert a horizontal normal force, n_w.

$$\sum F_x = f - n_w = 0 \qquad \textbf{[1]}$$

$$\sum F_y = n_g - m_1 g - m_2 g = 0 \qquad \textbf{[2]}$$

$$\sum \tau_A = -m_1 g\left(\frac{L}{2}\right)\cos\theta - m_2 g x\cos\theta + n_w L\sin\theta = 0$$

From the torque equation,

$$n_w = \left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$$

Then, from equation [1]: $f = n_w = \boxed{\left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta}$

and from equation [2]: $n_g = \boxed{\left(m_1 + m_2\right)g}$

(b) Refer to ANS. FIG. P12.13(b) above. If the ladder is on the verge of slipping when $x = d$, then

$$\mu = \frac{f|_{x=d}}{n_g} = \boxed{\frac{\left(m_1/2 + m_2 d/L\right)\cot\theta}{m_1 + m_2}}$$

P12.15 (a) Vertical forces on one-half of the chain are

$$T_e \sin 42.0° = 20.0\text{ N}$$

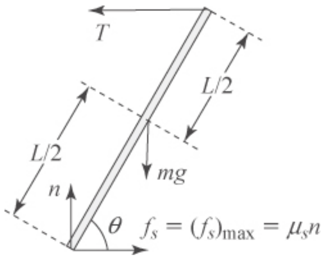
$$\boxed{T_e = 29.9\text{ N}}$$

(b) Horizontal forces on one-half of the chain are

$$T_e \cos 42.0° = T_m$$

$$\boxed{T_m = 22.2\text{ N}}$$

P12.16 (a) See the force diagram shown in ANS. FIG. P12.16.



ANS. FIG. P12.16

(b) Select a pivot point where an unknown force acts so that the force has no torque about that point. Picking the lower end of the beam eliminates torque from the normal force, *n*, and the friction force, *f*.

$$\sum \tau_{\text{lower end}} = 0:$$

$$0 + 0 - mg\left(\frac{L}{2}\cos\theta\right) + T\left(L\sin\theta\right) = 0$$

or

$$T = \frac{mg}{2}\left(\frac{\cos\theta}{\sin\theta}\right) = \boxed{\frac{mg}{2}\cot\theta}$$

(c) From the first condition for equilibrium,

$$\sum F_x = 0 \quad \Rightarrow \quad -T + \mu_s n = 0 \text{ or } T = \mu_s n \qquad \textbf{[1]}$$

$$\sum F_y = 0 \quad \Rightarrow \quad n - mg = 0 \text{ or } n = mg \qquad \textbf{[2]}$$

Substitute equation [2] into [1] to obtain $\boxed{T = \mu_s mg}$.

(d) Equate the results of parts (b) and (c) to obtain $\boxed{\mu_s = \frac{1}{2}\cot\theta}$.

This result is valid only at the critical angle θ where the beam is on the verge of slipping (i.e., where $f_s = (f_s)_{\max}$ is valid).

(e) $\boxed{\text{The ladder slips.}}$ When the base of the ladder is moved to the left, the angle θ decreases. According to the result in part (b), the tension T increases. This requires a larger friction force to balance T, but the static friction force is already at its maximum value in ANS. FIG. P12.16.

P12.17 (a) In Figure P12.17, let the "Single point of contact" be point P, the force the nail exerts on the hammer claws be R, the mass of the hammer (1.00 kg) be M, and the normal force exerted on the hammer at point P be n, while the horizontal static friction exerted by the surface on the hammer at P be f.

Taking moments about P,

$$(R\sin 30.0°)0 + (R\cos 30.0°)(5.00\text{ cm}) + Mg(0) - (150\text{ N})(30.0\text{ cm}) = 0$$

$$R = 1\,039.2\text{ N} = 1.04\text{ kN}$$

The force exerted by the hammer on the nail is equal in magnitude and opposite in direction:

$$\boxed{1.04\text{ kN at }60°\text{ upward and to the right}}$$

(b) From the first condition for equilibrium,

$$\sum F_x = f - R\sin 30.0° + 150\text{ N} = 0 \rightarrow f = 370\text{ N}$$

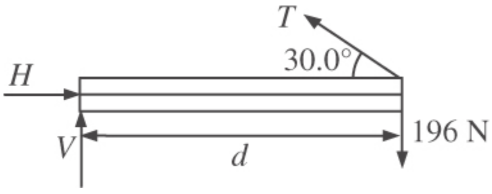
$$\sum F_y = n - Mg - R\cos 30.0° = 0$$

$$\rightarrow n = (1.00\text{ kg})(9.80\text{ m/s}^2) + (1\,040\text{ N})\cos 30.0° = 910\text{ N}$$

$$\boxed{\vec{\mathbf{F}}_{\text{surface}} = \left(370\hat{\mathbf{i}} + 910\hat{\mathbf{j}}\right)\text{ N}}$$

P12.18 (a) See the force diagram in ANS. FIG. P12.18.

(b) The mass M of the beam is 20.0 kg. We consider the torques acting on the beam, about an axis perpendicular



ANS. FIG. P12.18

to the page and through the left end of the horizontal beam.

$$\sum \tau = +(\mathrm{T}\sin 30.0°)\mathrm{d} - \mathrm{Mgd} = 0$$

$$\mathrm{T} = \frac{\mathrm{Mg}}{\sin 30.0°} = \frac{196\text{ N}}{\sin 30.0°} = \boxed{392\text{ N}}$$

(c) From $\sum \mathrm{F}_x = 0,\ \mathrm{H} - \mathrm{T}\cos 30.0° = 0,$

or $\mathrm{H} = (392\text{ N})\cos 30.0° = \boxed{339\text{ N to the right}}$

(d) From $\sum \mathrm{F}_y = 0,\ \mathrm{V} + \mathrm{T}\sin 30.0° - 196\text{ N} = 0,$

or $\mathrm{V} = 196\text{ N} - (392\text{ N})\sin 30.0° = \boxed{0}$

(e) From the same free-body diagram with the axis chosen at the right-hand end, we write

$$\sum \tau = \mathrm{H}(0) - \mathrm{Vd} + \mathrm{T}(0) + 196\mathrm{N}(0) = 0, \quad \text{so} \quad \boxed{\mathrm{V} = 0}$$

(f) From $\sum \mathrm{F}_y = 0,\ \mathrm{V} + \mathrm{T}\sin 30.0° - 196\text{ N} = 0,$

or $\mathrm{T} = 0 + 196\text{ N}/\sin 30.0° = \boxed{392\text{ N}}$

(g) From $\sum \mathrm{F}_x = 0,\ \mathrm{H} - \mathrm{T}\cos 30.0° = 0,$

or $\mathrm{H} = (392\text{ N})\cos 30.0° = \boxed{339\text{ N to the right}}$

(h) $\boxed{\text{The two solutions agree precisely. They are equally accurate.}}$

P12.19 The bridge has mass $M = 2\,000$ kg and the knight and horse have mass $m = 1\,000$ kg. Relative to the hinge end of the bridge, the cable is attached horizontally out a distance $\mathrm{x} = (5.00\text{ m})\cos 20.0° = 4.70\text{ m}$ and vertically up a distance $\mathrm{y} = (5.00\text{ m})\sin 20.0° = 1.71\text{ m}$. The cable then makes the following angle with the vertical wall:

$$\theta = \tan^{-1}\left[\frac{(4.70)\text{ m}}{12.0 - 1.71\text{ m}}\right] = 24.5°$$

Call the force components at the hinge H_x (to the right) and H_y (upward).

(a) Take torques about the hinge end of the bridge:

$$\begin{aligned}&\mathrm{H}_x(0) + \mathrm{H}_y(0) - \mathrm{Mg}(4.00\text{ m})\cos 20.0°\\ &\quad - (\mathrm{T}\sin 24.5°)(1.71\text{ m}) + (\mathrm{T}\cos 24.5°)(4.70\text{ m})\\ &\quad - \mathrm{mg}(7.00\text{ m})\cos 20.0° = 0\end{aligned}$$

which yields $\mathrm{T} = \boxed{27.7\text{ kN}}$

(b) $\sum F_x = 0 \Rightarrow H_x - T\sin 24.5° = 0,$

or $H_x = (27.7\text{ kN})\sin 24.5° = \boxed{11.5\text{ kN (right)}}$

(c) $\sum F_y = 0 \Rightarrow H_y - Mg + T\cos 24.5° - mg = 0$

Thus,

$$H_y = (M+m)g - (27.7\text{ kN})\cos 24.5° = -4.19\text{ kN}$$

$$= \boxed{4.19\text{ kN down}}$$

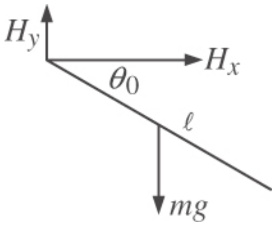
P12.20 (a) $\boxed{\text{No time interval. The horse's feet lose contact with the drawbridge as soon as it begins to move.}}$

From the result of (b) below, the tangential acceleration of the point where the horse stands is

$$a_t = \alpha r = (1.73\text{ rad/s}^2)(7.00\text{ m}) = 12.1\text{ m/s}^2$$

which has a vertical component $a_t\cos 20.0° = 11.4\text{ m/s}^2$, greater than the acceleration of gravity.

(b) Assuming that the bridge does fall from under the horse, its angular acceleration will be caused by torque from the weight of the bridge—if the bridge does not fall out from under the horse, there will be additional torque from the weight of the knight and horse, and the acceleration will be greater.



ANS. FIG. P12.20(b)

$$\sum\tau = I\alpha$$

$$Mg\left(\frac{\ell}{2}\right)\cos\theta_0 = \frac{1}{3}M\ell^2\alpha \rightarrow \alpha = \frac{3g\cos 20.0°}{2(8.00\text{ m})} = \boxed{1.73\text{ rad/s}}$$

As cited in part (a), this results in the bridge falling out from under the horse, so our assumption was justified.

(c) Because there is no friction at the hinge, the bridge-Earth system is isolated, so mechanical energy is conserved. When the bridge strikes the wall:

$$K_i + U_i = K_f + U_f$$

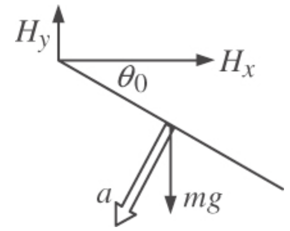
$$Mgh = \frac{1}{2}I\omega^2 \rightarrow Mg\left(\frac{\ell}{2}\right)(1+\sin 20.0°) = \frac{1}{2}\left(\frac{1}{3}M\ell^2\right)\omega^2$$

which gives

$$\omega = \sqrt{\frac{3g\left(1+\sin 20.0^\circ\right)}{8.00\text{ m}}} = \boxed{2.22\text{ rad/s}}$$

(d) The tangential acceleration of the center of mass of the bridge is

$$a_t = \frac{\ell}{2}\alpha = \frac{1}{2}(8.0\text{ m})\left(1.73\text{ rad/s}^2\right)$$
$$= 6.92\text{ m/s}^2$$



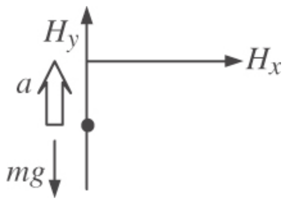
ANS. FIG. P12.20(d)

which is directed 20.0° below the horizontal. By Newton's second law:

$$\sum F_x = Ma_x$$
$$H_x = (2\ 000\text{ kg})\left(6.92\text{ m/s}^2\right)\sin 20.0^\circ$$
$$= 4.72\text{ kN}$$
$$\sum F_y = Ma_y$$
$$H_y - Mg = Ma_y$$
$$H_y = (2\ 000\text{ kg})\left(9.80\text{ m/s}^2\right) + (2\ 000\text{ kg})\left(-6.92\text{ m/s}^2\right)\cos 20.0^\circ$$
$$= \boxed{6.62\text{ kN}}$$

$$\boxed{\text{The force at the hinge is } \left(4.72\hat{\mathbf{i}} + 6.62\hat{\mathbf{j}}\right)\text{ kN.}}$$

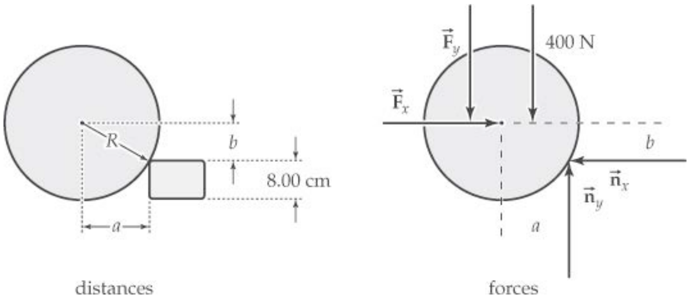
(e) When the bridge strikes the wall, $H_x = 0$ and the hinge supplies a vertical centripetal force:



ANS. FIG. P12.20(e)

$$\sum F_y = Ma_y$$
$$H_y - Mg = Ma_y = M\omega^2\frac{\ell}{2}$$
$$H_y = Mg + M\omega^2\frac{\ell}{2} = M\left(g + \omega^2\frac{\ell}{2}\right)$$
$$H_y = (2\ 000\text{ kg})\left(9.80\text{ m/s}^2 + (2.22\text{ rad/s})^2\frac{8.00\text{ m}}{2}\right)$$
$$H_y = \boxed{59.1\text{ kJ}}$$

P12.21 Call the required force *F*, with components $F_x = F\cos 15.0°$ and $F_y = -F\sin 15.0°$, transmitted to the center of the wheel by the handles.



ANS. FIG. P12.21

Just as the wheel leaves the ground, the ground exerts no force on it.

$$\sum F_x = 0: \quad F\cos 15.0° - n_x = 0 \qquad \textbf{[1]}$$

$$\sum F_y = 0: \quad -F\sin 15.0° - 400\text{ N} + n_y = 0 \qquad \textbf{[2]}$$

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - 8.00\text{ cm} = (20.0 - 8.00)\text{ cm} = 12.0\text{ cm}$$

$$a = \sqrt{R^2 - b^2} = \sqrt{(20.0\text{ cm})^2 - (8.00\text{ cm})^2} = 16.0\text{ cm}$$

(a) $\sum \tau = 0: \quad -F_x b + F_y a + (400\text{ N})a = 0,$ or

$$F\left[-(12.0\text{ cm})\cos 15.0° + (16.0\text{ cm})\sin 15.0°\right] + (400\text{ N})(16.0\text{ cm}) = 0$$

so $$F = \frac{6\,400\text{ N}\cdot\text{cm}}{7.45\text{ cm}} = \boxed{859\text{ N}}$$

(b) Then, using equations [1] and [2],

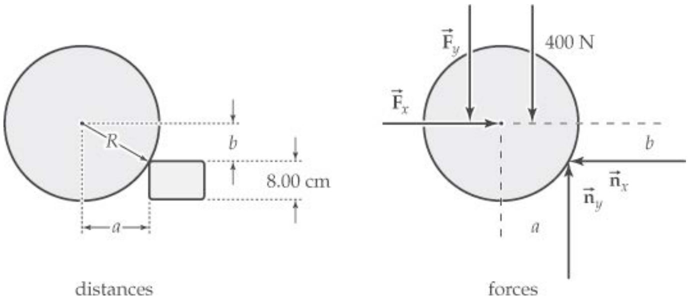
$$n_x = (859\text{ N})\cos 15.0° = 830\text{ N} \text{ and}$$

$$n_y = 400\text{ N} + (859\text{ N})\sin 15.0° = 622\text{ N}$$

$$n = \sqrt{n_x^2 + n_y^2} = \boxed{1.04\text{ kN}}$$

$$\theta = \tan^{-1}\left(\frac{n_y}{n_x}\right) = \tan^{-1}(0.749) = \boxed{36.9° \text{ to the left and upward}}$$

P12.22 Call the required force *F*, with components $F_x = F\cos\theta$ and $F_y = -F\sin\theta$, transmitted to the center of the wheel by the handles.



distances forces

ANS. FIG. P12.22

Just as the wheel leaves the ground, the ground exerts no force on it.

$$\sum F_x = 0: \quad F\cos\theta - n_x = 0 \qquad \textbf{[1]}$$

$$\sum F_y = 0: \quad -F\sin\theta - mg + n_y = 0 \qquad \textbf{[2]}$$

Take torques about its contact point with the brick. The needed distances are seen to be:

$$b = R - h$$

$$a = \sqrt{R^2 - (R-h)^2} = \sqrt{2Rh - h^2}$$

(a) $\sum \tau = 0: \quad -F_x b + F_y a + mga = 0$, or

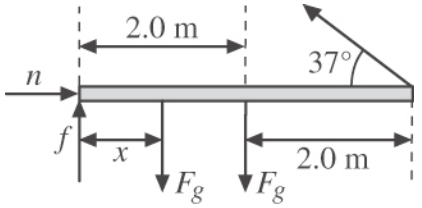
$$F\left[-b\cos\theta + a\sin\theta\right] + mga = 0$$

$$\to F = \frac{mga}{b\cos\theta - a\sin\theta} = \boxed{\frac{mg\sqrt{2Rh-h^2}}{(R-h)\cos\theta - \sqrt{2Rh-h^2}\sin\theta}}$$

(b) Then, using equations [1] and [2],

$$n_x = F\cos\theta = \boxed{\frac{mg\sqrt{2Rh-h^2}\cos\theta}{(R-h)\cos\theta - \sqrt{2Rh-h^2}\sin\theta}}$$

and $$n_y = F\sin\theta + mg = \boxed{mg\left[1 + \frac{\sqrt{2Rh-h^2}\cos\theta}{(R-h)\cos\theta - \sqrt{2Rh-h^2}\sin\theta}\right]}$$



ANS. FIG. P12.23

P12.23 When $x = x_{\text{min}}$, the rod is on the verge of slipping, so

$$f = \left(f_s\right)_{\text{max}} = \mu_s n = 0.50n$$

From $\sum F_x = 0$, $n - T\cos 37° = 0$

or $n = 0.799T$

Thus, $f = 0.50\left(0.799T\right) = 0.399T$

From $\sum F_y = 0$, $f + T\sin 37° - 2F_g = 0$,

or $0.399T + 0.602T - 2F_g = 0$, giving $T = 2.00F_g$

Using $\sum \tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives

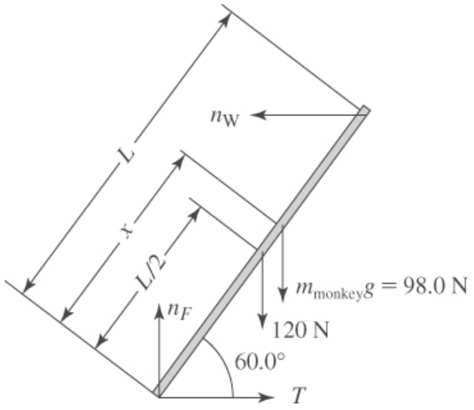
$$-F_g \cdot x_{\text{min}} - F_g\left(2.0\text{ m}\right) + \left[\left(2F_g\right)\sin 37°\right]\left(4.0\text{ m}\right) = 0$$

which reduces to $x_{\text{min}} = \boxed{2.81\text{ m}}$

P12.24 (a) The force diagram is shown in ANS. FIG. P12.24.

(b) From $\sum F_y = 0 \Rightarrow n_F - 120\text{ N} - m_{\text{monkey}}g = 0$

$$n_F = 120\text{ N} + \left(10.0\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = \boxed{218\text{ N}}$$



ANS. FIG. P12.24

(c) When $x = 2L/3$, we consider the bottom end of the ladder as our pivot and obtain

$$\sum \tau\Big|_{\substack{\text{bottom}\\\text{end}}} = 0:$$

$$-(120\text{ N})\left(\frac{\cancel{L}}{2}\cos 60.0°\right) - (98.0\text{ N})\left(\frac{2\cancel{L}}{3}\cos 60.0°\right) + n_W\left(\cancel{L}\sin 60.0°\right) = 0$$

or $$n_W = \frac{\left[60.0\text{ N} + (196/3)\text{ N}\right]\cos 60.0°}{\sin 60.0°} = 72.4\text{ N}$$

Then, $\sum F_x = 0 \Rightarrow T - n_W = 0$ or $T = n_W = \boxed{72.4\text{ N}}$

(d) When the rope is ready to break, $T = n_W = 80.0\text{ N}$. Then $\sum \tau\Big|_{\substack{\text{bottom}\\\text{end}}} = 0$ yields

$$-(120\text{ N})\left(\frac{L}{2}\cos 60.0°\right) - (98.0\text{ N})x\cos 60.0° + (80.0\text{ N})(L\sin 60.0°) = 0$$

or $$x = \frac{\left[(80.0\text{ N})\sin 60.0° - (60.0\text{ N})\cos 60.0°\right]L}{(98.0\text{ N})\cos 60.0°}$$

$$= 0.802L = 0.802(3.00\text{ m}) = \boxed{2.41\text{ m}}$$

(e) If the horizontal surface were rough and the rope removed, a horizontal static friction force directed toward the wall would act on the bottom end of the ladder. Otherwise, the analysis would be much as what is done above. The maximum distance the monkey could climb would correspond to the condition that the friction force have its maximum value, $\mu_s n_F$, so you would need to know the coefficient of static friction between the ladder and the floor to solve part (d).

P12.25 Consider the torques about an axis perpendicular to the page and through the left end of the plank. $\sum \tau = 0$ gives

$$-(700\text{ N})(0.500\text{ m}) - (294\text{ N})(1.00\text{ m}) + (T_1\sin 40.0°)(2.00\text{ m}) = 0$$

or $$T_1 = \boxed{501\text{ N}}$$

Then, $\sum F_x = 0$ gives

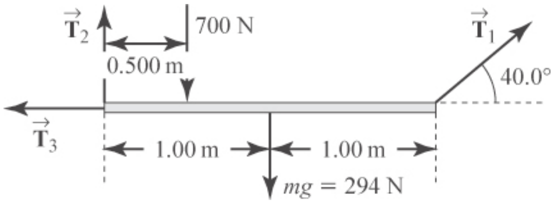
$$-T_3 + T_1 \cos 40.0° = 0$$

or $$T_3 = (501 \text{ N})\cos 40.0° = \boxed{384 \text{ N}}$$

From $\sum F_y = 0$,

$$T_2 - 994 \text{ N} + T_1 \sin 40.0° = 0,$$

or $$T_2 = 994 \text{ N} - (501 \text{ N})\sin 40.0° = \boxed{672 \text{ N}}$$



ANS. FIG. P12.25

Section 12.4 Elastic Properties of Solids

P12.26 Count the wires. If they are wrapped together so that all support nearly equal stress, the number should be

$$\frac{20.0 \text{ kN}}{0.200 \text{ kN}} = 100$$

Since cross-sectional area is proportional to diameter squared, the diameter of the cable will be

$$(1 \text{ mm})\sqrt{100}\ \boxed{\sim 1 \text{ cm}}$$

P12.27 We use $B = -\dfrac{\Delta P}{\Delta V / V_i} = -\dfrac{\Delta P V_i}{\Delta V}$.

(a) $$\Delta V = -\frac{\Delta P V_i}{B} = -\frac{(1.13 \times 10^8 \text{ N/m}^2)(1 \text{ m}^3)}{0.21 \times 10^{10} \text{ N/m}^2} = \boxed{-0.053\,8 \text{ m}^3}$$

(b) The quantity of water with mass 1.03×10^3 kg occupies volume at the bottom: $1 \text{ m}^3 - 0.053\,8 \text{ m}^3 = 0.946 \text{ m}^3$.

So its density is $\dfrac{1.03 \times 10^3 \text{ kg}}{0.946 \text{ m}^3} = \boxed{1.09 \times 10^3 \text{ kg/m}^3}$

(c) $\boxed{\text{With only a 5\% volume change in this extreme case, liquid water is indeed nearly incompressible.}}$

P12.28 (a) We find the maximum force from the equation for stress:

$$\text{stress} = \frac{F}{A} = \frac{F}{\pi r^2}$$

$$F = (\text{stress})\pi\left(\frac{d}{2}\right)^2$$

$$F = \left(1.50\times10^{8}\ \text{N/m}^2\right)\pi\left(\frac{2.50\times10^{-2}\ \text{m}}{2}\right)^2$$

$$F = \boxed{73.6\ \text{kN}}$$

(b) From the definition of Young's modulus,

$$\text{stress} = Y(\text{strain}) = \frac{Y\Delta L}{L_i}$$

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{\left(1.50\times10^{8}\ \text{N/m}^2\right)(0.250\ \text{m})}{1.50\times10^{10}\ \text{N/m}^2} = \boxed{2.50\ \text{mm}}$$

P12.29 From the defining equation for the shear modulus, we find Δx as

$$\Delta x = \frac{hf}{SA} = \frac{\left(5.00\times10^{-3}\ \text{m}\right)(20.0\ \text{N})}{\left(3.0\times10^{6}\ \text{N/m}^2\right)\left(14.0\times10^{-4}\ \text{m}^2\right)} = 2.38\times10^{-5}\,\text{m}$$

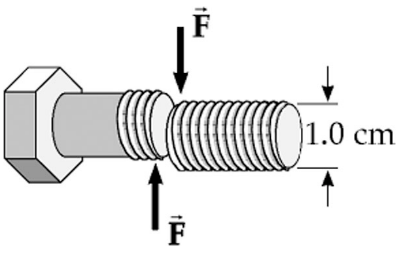
or $\Delta x = \boxed{2.38\times10^{-2}\ \text{mm}}$

P12.30 The definition of Young's modulus, $Y = \dfrac{\text{stress}}{\text{strain}}$, means that *Y* is the slope of the graph:

$$Y = \frac{300\times10^{6}\ \text{N/m}^2}{0.003} = \boxed{1.0\times10^{11}\ \text{N/m}^2}$$

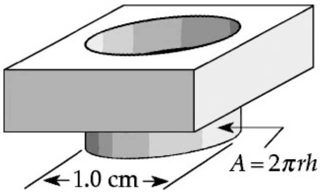
P12.31 (a) From ANS. FIG. P12.31(a),

$$\begin{aligned} F &= \sigma A \\ &= \left(4.00\times10^{8}\ \text{N/m}\right) \\ &\quad\times\left[\pi\left(0.500\times10^{-2}\ \text{m}\right)^2\right] \\ &= \boxed{3.14\times10^{4}\ \text{N}} \end{aligned}$$



ANS. FIG. P12.31(a)

(b) Now the area of the molecular layers sliding over each other is the curved lateral surface area of the cylinder being punched out, a cylinder of radius 0.500 cm and height 0.500 cm. So,



ANS. FIG. P12.31(b)

$$\begin{aligned} F &= \sigma A \\ &= \sigma(h)(2\pi r) \\ &= \left(4.00\times10^{8}\text{ N/m}\right)(2\pi)\left(0.500\times10^{-2}\text{ m}\right) \\ &\qquad\times\left(0.500\times10^{-2}\text{ m}\right) \\ &= \boxed{6.28\times10^{4}\text{ N}} \end{aligned}$$

P12.32 Let *V* represent the original volume. Then, 0.090 0*V* is the change in volume that would occur if the block cracked open. Imagine squeezing the ice, with unstressed volume 1.09*V*, back down to its previous volume, so $\Delta V = -0.090\,0V$. According to the definition of the bulk modulus as given in the chapter text, we have

$$\begin{aligned} \Delta P &= -\frac{B(\Delta V)}{V_i} \\ &= -\frac{\left(2.00\times10^{9}\text{ N/m}^2\right)(-0.090\,0V)}{1.09V} \\ &= \boxed{1.65\times10^{8}\text{ N/m}^2} \end{aligned}$$

P12.33 Young's Modulus is given by $Y = \dfrac{F/A}{\Delta L/L_i}$.

The load force is

$$F = (200\text{ kg})(9.80\text{ m/s}^2) = 1\,960\text{ N}.$$

so $$\Delta L = \frac{FL_i}{AY} = \frac{(1\,960\text{ N})(4.00\text{ m})(1\,000\text{ mm/m})}{\left(0.200\times10^{-4}\text{ m}^2\right)\left(8.00\times10^{10}\text{ N/m}^2\right)} = \boxed{4.90\text{ mm}}$$

P12.34 Part of the load force extends the cable and part compresses the column by the same distance $\Delta\ell$:

$$F = \frac{Y_A A_A \Delta\ell}{\ell_A} + \frac{Y_s A_s \Delta\ell}{\ell_s}$$

from which we obtain

$$\Delta\ell = \frac{F}{Y_A A_A / \ell_A + Y_s A_s / \ell_s}$$

$$= \frac{8\,500\text{ N}}{\left(7\times10^{10}\right)\pi\left(0.162\,4^2 - 0.161\,4^2\right)/4(3.25) + 20\times10^{10}\pi\left(0.012\,7\right)^2/4(5.75)}$$

$$= \boxed{8.60\times10^{-4}\text{ m}}$$

P12.35 Let the 3.00-kg mass be mass #1, with the 5.00-kg mass, mass # 2. Applying Newton's second law to each mass gives

$$m_1 a = T - m_1 g \qquad [1]$$

and $$m_2 a = m_2 g - T \qquad [2]$$

where T is the tension in the wire.

Solving equation [1] for the acceleration gives

$$a = \frac{T}{m_1} - g$$

and substituting this into equation [2] yields

$$\frac{m_2}{m_1}T - m_2 g = m_2 g - T$$

Solving for the tension T gives

$$T = \frac{2m_1 m_2 g}{m_2 + m_1} = \frac{2\left(3.00\text{ kg}\right)\left(5.00\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{8.00\text{ kg}} = 36.8\text{ N}$$

From the definition of Young's modulus, $Y = \dfrac{FL_i}{A(\Delta L)}$, the elongation of the wire is:

$$\Delta L = \frac{TL_i}{YA} = \frac{(36.8\text{ N})(2.00\text{ m})}{\left(2.00\times10^{11}\text{ N/m}^2\right)\pi\left(2.00\times10^{-3}\text{ m}\right)^2}$$

$$= \boxed{0.029\,2\text{ mm}}$$

P12.36 A particle under a net force model:

$$\left|\bar{F}\right| = \frac{m\left|v_f - v_i\right|}{\Delta t}$$

Hence,

$$\left|\bar{F}\right| = \frac{30.0\ \text{kg}\left|-10.0\ \text{m/s} - 20.0\ \text{m/s}\right|}{0.110\ \text{s}} = 8.18\times10^{3}\ \text{N}$$

By Newton's third law, this is also the magnitude of the average force exerted on the spike by the hammer during the blow. Thus, the stress in the spike is

$$\text{Stress} = \frac{F}{A} = \frac{8.18\times10^{3}\ \text{N}}{\pi\left(0.023\,0\ \text{m}\right)^{2}/4} = 1.97\times10^{7}\ \text{N/m}^{2}$$

and the strain is

$$\text{strain} = \frac{\text{stress}}{Y} = \frac{1.97\times10^{7}\ \text{N/m}^{2}}{20.0\times10^{10}\ \text{N/m}^{2}} = \boxed{9.85\times10^{-5}}$$

Additional Problems

P12.37 Let n_A and n_B be the normal forces at the points of support. Then, from the translational equilibrium equation in the y direction, we have

$$\sum F_y = 0: \quad n_A + n_B - \left(8.00\times10^{4}\ \text{kg}\right)g - \left(3.00\times10^{4}\ \text{kg}\right)g = 0$$

Choosing the axis at point A, we find, from the condition for rotational equilibrium:

$$\begin{aligned}&\sum \tau = 0:\\ &\quad -\left(3.00\times10^{4}\ \text{kg}\right)(15.0\ \text{m})g - \left(8.00\times10^{4}\ \text{kg}\right)(25.0\ \text{m})g\\ &\qquad + n_B(50.0\ \text{m}) = 0\end{aligned}$$

We can solve the torque equation directly to find

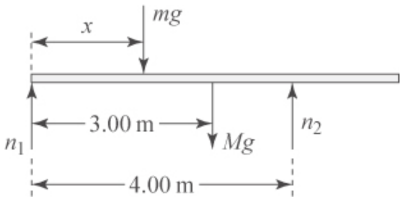
$$\begin{aligned}n_B &= \frac{\left[\left(3.00\times10^{4}\ \text{kg}\right)\left(15.0\ \text{m}\right) + \left(8.00\times10^{4}\ \text{kg}\right)\left(25.0\ \text{m}\right)\right](9.80\ \text{m/s}^2)}{50.0\ \text{m}}\\ &= 4.80\times10^{5}\ \text{N}\end{aligned}$$

Then the force equation gives

$$\begin{aligned}n_A &= (8.00\times10^{4}\ \text{kg} + 3.00\times10^{4}\ \text{kg})(9.80\ \text{m/s}^2) - 4.80\times10^{5}\ \text{N}\\ &= 5.98\times10^{5}\ \text{N}\end{aligned}$$

P12.38 (a) $\boxed{\text{Rigid object in static equilibrium.}}$

(b) ANS. FIG. P12.38 shows the free-body diagram.



ANS. FIG. P12.38

$Mg = (90.0\text{ kg})g = 882$ N, and $mg = (55.0\text{ kg})g = 539$ N.

(c) Note that about the right pivot, only n_1 exerts a clockwise torque, all other forces exert counterclockwise torques except for n_2 which exerts zero torque. $\boxed{\text{The woman is at } x = 0 \text{ when } n_1 \text{ is greatest.}}$ With this location of the woman, the counterclockwise torque about the center of the beam is a maximum. Thus, n_1 must be exerting its maximum clockwise torque about the center to hold the beam in rotational equilibrium.

(d) $\boxed{n_1 = 0}$ As the woman walks to the right along the beam, she will eventually reach a point where the beam will start to rotate clockwise about the rightmost pivot. At this point, the beam is starting to lift up off of the leftmost pivot and the normal force exerted by that pivot will have diminished to zero.

(e) When the beam is about to tip, $n_1 = 0$, and

$$\sum F_y = 0 \text{ gives } 0 + n_2 - Mg - mg = 0, \text{ or}$$

$$n_2 = Mg + mg = 882\text{ N} + 539\text{ N} = \boxed{1.42 \times 10^3\text{ N}}$$

(f) Requiring that the net torque be zero about the right pivot when the beam is about to tip ($n_1 = 0$) gives

$$\sum \tau = n_2(0) + (4.00\text{ m} - x)mg + (4.00\text{ m} - 3.00\text{ m})Mg = 0$$

or $(mg)x = (1.00\text{ m})Mg + (4.00\text{ m})mg$, and

$$x = (1.00\text{ m})\frac{M}{m} + 4.00\text{ m}$$

Thus, $x = (1.00\text{ m})\dfrac{(90.0\text{ kg})}{(55.0\text{ kg})} + 4.00\text{ m} = \boxed{5.64\text{ m}}$

(g) When $n_1 = 0$ and $n_2 = 1.42 \times 10^3$ N, for torque about the left pivot:

$$\sum \tau = 0 - (539 \text{ N})x - (882 \text{ N})(3.00 \text{ m}) + (1.42 \times 10^3 \text{ N})(4.00 \text{ m}) = 0$$

or $$x = \frac{-3.03 \times 10^3 \text{ N} \cdot \text{m}}{-539 \text{ N}} = \boxed{5.62 \text{ m}}$$

which, within limits of rounding errors, is

$\boxed{\text{the same as the answer to part (f)}}$.

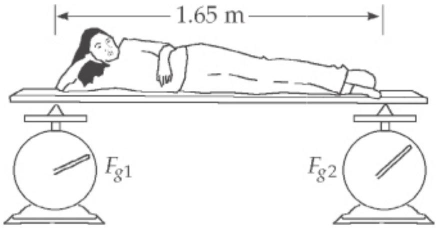
P12.39 $\sum F_y = 0$: $+380 \text{ N} - F_g + 320 \text{ N} = 0$

$F_g = 700 \text{ N}$

Take torques about her feet:

$$\sum \tau = 0: \quad -380 \text{ N}(1.65 \text{ m}) + (700 \text{ N})x + (320 \text{ N})0 = 0$$

$x = \boxed{0.896 \text{ m}}$



ANS. FIG. P12.39

P12.40 When the concrete has cured and the pre-stressing tension has been released, the rod presses in on the concrete and with equal force, T_2, the concrete produces tension in the rod.

(a) In the concrete:

$$\text{stress} = 8.00 \times 10^6 \text{ N/m}^2 = Y \cdot (\text{strain}) = Y\left(\frac{\Delta L}{L_i}\right)$$

Thus,

$$\Delta L = \frac{(\text{stress})L_i}{Y} = \frac{(8.00 \times 10^6 \text{ N/m}^2)(1.50 \text{ m})}{30.0 \times 10^9 \text{ N/m}^2}$$

or $$\Delta L = 4.00 \times 10^{-4} \text{ m} = \boxed{0.400 \text{ mm}}$$

(b) In the concrete:

$$\text{stress} = \frac{T_2}{A_c} = 8.00 \times 10^6 \text{ N/m}^2$$

so

$$T_2 = \left(8.00\times10^{6}\ \text{N/m}^2\right)\left(50.0\times10^{-4}\ \text{m}^2\right) = \boxed{40.0\ \text{kN}}$$

(c) For the rod:

$$\frac{T_2}{A_R} = \left(\frac{\Delta L}{L_i}\right)Y_{\text{steel}} \text{ so } \Delta L = \frac{T_2 L_i}{A_R Y_{\text{steel}}}$$

$$\Delta L = \frac{\left(4.00\times10^{4}\ \text{N}\right)(1.50\ \text{m})}{\left(1.50\times10^{-4}\ \text{m}^2\right)\left(20.0\times10^{10}\ \text{N/m}^2\right)}$$

$$= 2.00\times10^{-3}\ \text{m} = \boxed{2.00\ \text{mm}}$$

(d) The rod in the finished concrete is 2.00 mm longer than its unstretched length. To remove stress from the concrete, one must stretch the rod 0.400 mm farther, by a total of $\boxed{2.40\ \text{mm}}$.

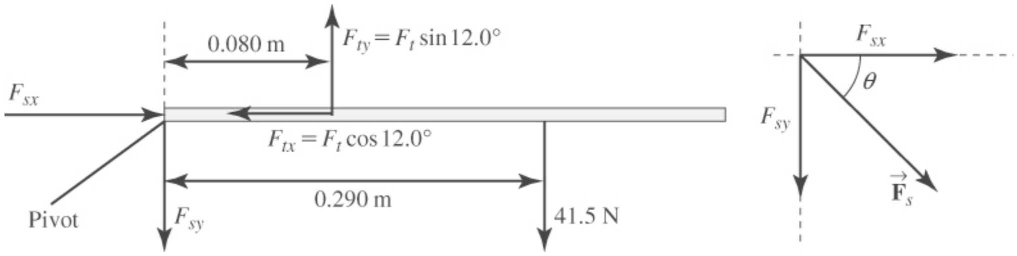
(e) For the stretched rod around which the concrete is poured:

$$\frac{T_1}{A_R} = \left(\frac{\Delta L_{\text{total}}}{L_i}\right)Y_{\text{steel}} \quad \text{or} \quad T_1 = \left(\frac{\Delta L_{\text{total}}}{L_i}\right)A_R Y_{\text{steel}}$$

$$T_1 = \left(\frac{2.40\times10^{-3}\ \text{m}}{1.50\ \text{m}}\right)\left(1.50\times10^{-4}\ \text{m}^2\right)\left(20.0\times10^{10}\ \text{N/m}^2\right)$$

$$= \boxed{48.0\ \text{kN}}$$

P12.41 We reproduce the forces in ANS. FIG. P12.41.



ANS. FIG. P12.41

Requiring that $\sum\tau = 0$, using the shoulder joint at point O as a pivot, gives

$$\sum\tau = \left(F_t \sin 12.0°\right)(0.080\ 0\ \text{m}) - (41.5\ \text{N})(0.290\ \text{m}) = 0$$

or $F_t = \boxed{724\ \text{N}}$

Then

$$\sum F_y = 0 \quad \Rightarrow \quad -F_{sy} + (724\ \text{N})\sin 12.0° - 41.5\ \text{N} = 0$$

yielding $F_{sy} = 109\text{ N}$

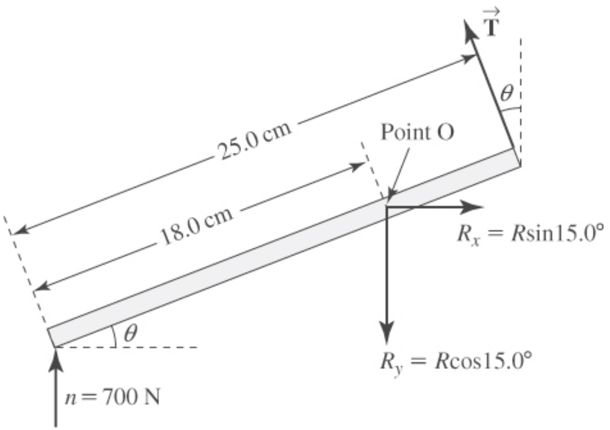
$\sum F_x = 0$ then gives

$$F_{sx} - (724\text{ N})\cos 12.0° = 0, \text{ or } F_{sx} = 708\text{ N}$$

Therefore,

$$F_s = \sqrt{F_{sx}^2 + F_{sy}^2} = \sqrt{(708\text{ N})^2 + (109\text{ N})^2} = \boxed{716\text{ N}}$$

P12.42 In the free-body diagram of the foot given at the right, note that the force $\vec{\mathbf{R}}$ (exerted on the foot by the tibia) has been replaced by its horizontal and vertical components. Employing both conditions of equilibrium (using point O as the pivot point) gives the following three equations:



ANS. FIG. P12.42

$$\sum F_x = 0 \Rightarrow R\sin 15.0° - T\sin\theta = 0$$

or $$R = \frac{T\sin\theta}{\sin 15.0°} \qquad \textbf{[1]}$$

$$\sum F_y = 0 \Rightarrow 700\text{ N} - R\cos 15.0° + T\cos\theta = 0 \qquad \textbf{[2]}$$

$$\sum \tau_O = 0 \Rightarrow -(700\text{ N})[(18.0\text{ cm})\cos\theta] + T(25.0\text{ cm} - 18.0\text{ cm}) = 0$$

or $$T = (1\,800\text{ N})\cos\theta \qquad \textbf{[3]}$$

Substituting equation [3] into equation [1] gives

$$R = \left(\frac{1\,800\text{ N}}{\sin 15.0°}\right)\sin\theta\cos\theta \qquad \textbf{[4]}$$

Substituting equations [3] and [4] into equation [2] yields

$$\left(\frac{1\,800\text{ N}}{\tan 15.0°}\right)\sin\theta\cos\theta - (1\,800\text{ N})\cos^2\theta = 700\text{ N}$$

which reduces to: $\sin\theta\cos\theta = (\tan 15.0°)\cos^2\theta + 0.104\,2$

Squaring this result and using the identity $\sin^2\theta = 1-\cos^2\theta$ gives

$$\left[\tan^2(15.0°)+1\right]\cos^4\theta$$
$$+\left[(2\tan 15.0°)(0.104\,2)-1\right]\cos^2\theta+(0.104\,2)^2=0$$

In this last result, let $u=\cos^2\theta$ and evaluate the constants to obtain the quadratic equation:

$$(1.071\,8)u^2-(0.944\,2)u+0.010\,9=0$$

The quadratic formula yields the solutions $u = 0.869\,3$ and $u = 0.0117$. Thus,

$$\theta=\cos^{-1}\left(\sqrt{0.869\,3}\right)=21.2° \quad \text{or} \quad \theta=\cos^{-1}\left(\sqrt{0.011\,7}\right)=83.8°$$

We ignore the second solution since it is physically impossible for the human foot to stand with the sole inclined at 83.8° to the floor. We are the left with $\theta=\boxed{21.2°}$.

Equation [3] then yields

$$T=(1\,800\text{ N})\cos 21.2°=\boxed{1.68\text{ kN}}$$

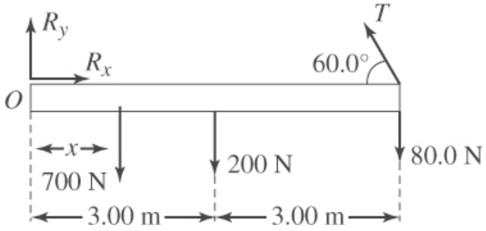
and equation [1] gives

$$R=\frac{(1.68\times10^3\text{ N})\sin 21.2°}{\sin 15.0°}=\boxed{2.34\text{ kN}}$$

P12.43 (a) ANS. FIG. P12.43 shows the force diagram.

(b) If $x = 1.00$ m, then

$$\sum\tau_O=(-700\text{ N})(1.00\text{ m})-(200\text{ N})(3.00\text{ m})$$
$$-(80.0\text{ N})(6.00\text{ m})$$
$$+(T\sin 60.0°)(6.00\text{ m})=0$$



ANS. FIG. P12.43

Solving for the tension gives: $T=\boxed{343\text{ N}}$.

From $\sum F_x=0$, $R_x=T\cos 60.0°=\boxed{171\text{ N}}$.

From $\sum F_y = 0,\ R_y = 980\text{ N} - T\sin 60.0° = \boxed{683\text{ N}}$.

(c) If $T = 900$ N:

$$\sum \tau_O = (-700\text{ N})x - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m})$$
$$+\left[(900\text{ N})\sin 60.0°\right](6.00\text{ m}) = 0$$

Solving for x gives $x = \boxed{5.14\text{ m}}$.

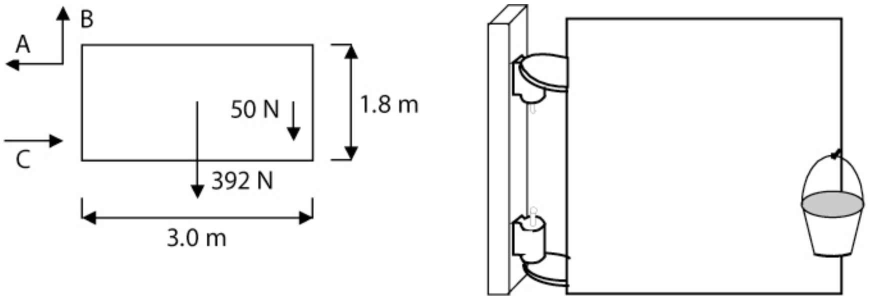
P12.44 (a) See ANS. FIG. P12.44 for the force diagram. See the solution in the textbook. The weight of the uniform gate is 392 N. It is 3.00 m wide. The hinges are separated vertically by 1.80 m. The bucket of grain weighs 50.0 N. One of the hinges, which we suppose is the upper one, supports the whole weight of the gate. Find the components of the forces that both hinges exert on the gate.

(b) From the torque equation,

$$C = \frac{738\text{ N}\cdot\text{m}}{1.8\text{ m}} = 410\text{ N}$$

Then $A = 410$ N. Also $B = 442$ N.

$$\boxed{\begin{array}{l}\text{The upper hinge exerts 410 N to the left and 442 N up.}\\ \text{The lower hinge exerts 410 N to the right.}\end{array}}$$



ANS. FIG. P12.44

P12.45 We know that the direction of the force from the cable at the right end is along the cable, at an angle of θ above the horizontal. On the other end, we do not know magnitude or direction for the hinge force $\vec{\mathbf{R}}$ so we show it as two unknown components.

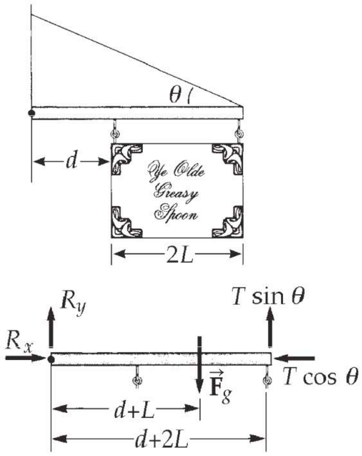
The first condition for equilibrium gives two equations:

$$\sum F_x = 0: \quad +R_s - T\cos\theta = 0$$
$$\sum F_y = 0: \quad +R_y - F_g + T\sin\theta = 0$$

Taking torques about the left end, we find the second condition is

$$\sum \tau = 0:$$

$$R_y(0) + R_s(0) - F_g(d+L) + (0)(T\cos\theta) + (d+2L)(T\sin\theta) = 0$$



ANS. FIG. P12.45

(a) The torque equation gives $T = \boxed{\dfrac{F_g(L+d)}{\sin\theta(2L+d)}}$

(b) Now from the force equations,

$$R_x = \boxed{\frac{F_g(L+d)\cot\theta}{2L+d}} \text{ and } R_y = \boxed{\frac{F_g L}{2L+d}}$$

P12.46 ANS. FIG. P12.46 shows the force diagram.

$\sum \tau_{\text{point O}} = 0$ gives

$$(T\cos 25.0°)\left(\frac{3\ell}{4}\sin 65.0°\right) + (T\sin 25.0°)\left(\frac{3\ell}{4}\cos 65.0°\right)$$

$$= (2\,000\text{ N})(\ell\cos 65.0°) + (1\,200\text{ N})\left(\frac{\ell}{2}\cos 65.0°\right)$$

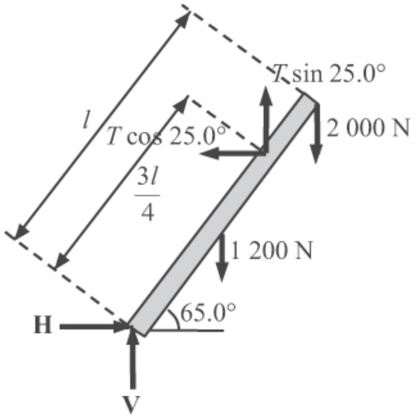
From which, $T = 1\,465\text{ N} = \boxed{1.46\text{ kN}}$

From $\sum F_x = 0$,

$$H = T\cos 25.0° = 1\,328\text{ N}(\text{toward right}) = \boxed{1.33\text{ kN}}$$

From $\sum F_y = 0$,

$$V = 3\,200\text{ N} - T\sin 25.0° = 2\,581\text{ N}(\text{upward}) = \boxed{2.58\text{ kN}}$$



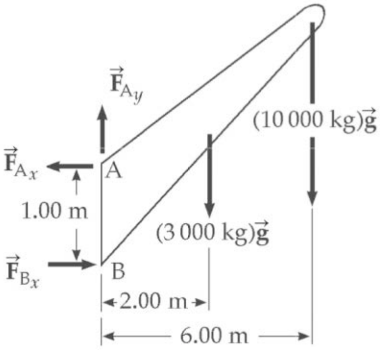
ANS. FIG. P12.46

P12.47 We interpret the problem to mean that the support at point B is frictionless. Then the support exerts a force in the x direction and

$$F_{By} = 0$$
$$\sum F_x = F_{Bx} - F_{Ax} = 0$$
$$F_{Ay} - (3\,000 + 10\,000)g = 0$$

and

$$\sum \tau = -(3\,000g)(2.00) - (10\,000g)(6.00) + F_{Bx}(1.00) = 0$$



ANS. FIG. P12.47

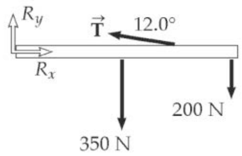
These equations combine to give the magnitudes of the components:

$$F_{Ax} = F_{Bx} = 6.47 \times 10^5\text{ N}$$
$$F_{Ay} = 1.27 \times 10^5\text{ N}$$

$$\boxed{\begin{array}{ll}\text{The forces are:} & \vec{F}_A = (-6.47 \times 10^5\,\hat{i} + 1.27 \times 10^5\,\hat{j})\text{ N} \\ \text{and} & \vec{F}_B = 6.47 \times 10^5\,\hat{i}\text{ N}\end{array}}$$

P12.48 (a) Choosing torques about the hip joint, $\sum \tau = 0$ gives

$$-\frac{L}{2}(350\text{ N}) + (T\sin 12.0°)\left(\frac{2L}{3}\right) - (200\text{ N})L = 0$$



ANS. FIG. P12.48

From which, $T = \boxed{2.71\text{ kN}}$.

(b) Let R_x = compression force along spine, and from $\sum F_x = 0$:

$$R_x = T_x = T\cos 12.0° = \boxed{2.65\text{ kN}}$$

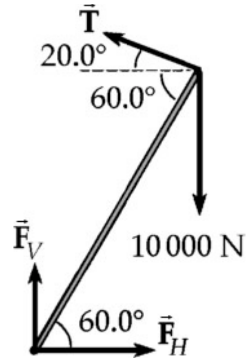
(c) You should lift "with your knees" rather than "with your back." In this situation, with a load weighing only 200 N, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible.

(d) In this situation, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible

P12.49 From ANS. FIG. P12.49, the angle $\vec{\mathbf{T}}$ makes with the rod is $\theta = 60.0° + 20.0° = 80.0°$ and the perpendicular component of $\vec{\mathbf{T}}$ is $T\sin 80.0°$.

Summing torques around the base of the rod, and applying Newton's second law in the horizontal and vertical directions, we have

$$\sum \tau = 0: \quad -(4.00\text{ m})(10\,000\text{ N})\cos 60° + T(4.00\text{ m})\sin 80.0° = 0$$



ANS. FIG. P12.49

(a) Solving the above equation for T gives

$$T = \frac{(10\,000\text{ N})\cos(60.0°)}{\sin(80.0°)} = \boxed{5.08\text{ kN}}$$

(b) In the horizontal direction,

$$\sum F_x = 0: \quad F_H - T\cos(20.0°) = 0$$

so $F_H = T\cos(20.0°) = 4.77\text{ kN}$

(c) From $\sum F_y = 0: \quad F_V + T\sin(20.0°) - 10\,000\text{ N} = 0,$

we find

$$F_V = (10\,000\text{ N}) - T\sin(20.0°) = \boxed{8.26\text{ kN}}$$

P12.50 The cabinet has height $\ell = 1.00$ m, width $w = 0.600$ m, and weight $Mg = 400$ N. The force $F = 300$ N is applied by the worker in the first case at height $h_1 = 0.100$ m and in the second at height $h_2 = 0.650$ m.

Consider the magnitudes of the torques about the lower right front edge of the cabinet from the weight Mg and from the applied force F for the two values of h.

The torque from the weight is the same in each case:

Cases 1 and 2

$$\begin{aligned}\tau_G &= Mg\frac{w}{2}\\ &= (400\text{ N})(0.300\text{ m}) = 120\text{ N}\cdot\text{m}\end{aligned}$$

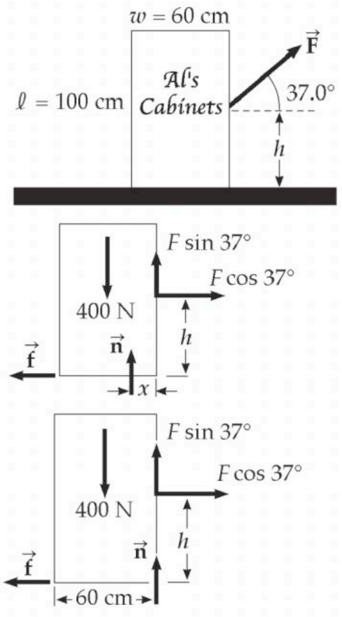
The torque from force F is different in each case:

Case 1

$$\begin{aligned}\tau_F &= (F\cos 37.0°)h_1\\ &= (300\text{ N}\cos 37.0°)(0.100\text{ m})\\ \tau_F &= 24.0\text{ N}\cdot\text{m}\end{aligned}$$

Case 2

$$\begin{aligned}\tau_F &= (F\cos 37.0°)h_2\\ &= (300\text{ N}\cos 37.0°)(0.650\text{ m})\\ \tau_F &= 156\text{ N}\cdot\text{m}\end{aligned}$$

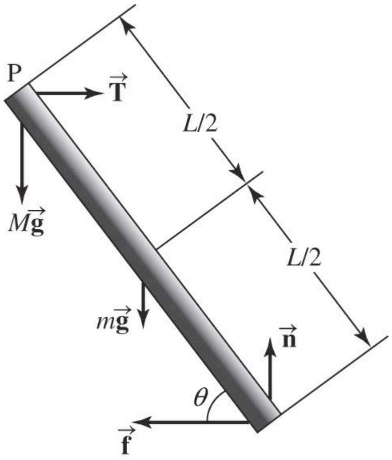


ANS. FIG. P12.50

We see in Case 1 that the counterclockwise torque from the weight is greater than the clockwise torque from the applied force. If the cabinet is to slide without acceleration, the net torque must be zero; this is possible because the normal force from the floor can provide additional clockwise torque. We see in Case 2, however, that the counterclockwise torque from the weight is smaller than the clockwise torque from the applied force, but no other force is available to provide addition counterclockwise torque, so the net torque cannot be zero.

$$\boxed{\text{The situation is impossible because the new technique would tip the cabinet over.}}$$

P12.51 (a) We use $\sum F_x = \sum F_y = \sum \tau = 0$ and choose the axis at the point of contact with the floor to simplify the torque analysis. Since the rope is described as very rough, we will assume that it will never slip on the end of the beam. First, let us determine what friction force at the floor is necessary to put the system in equilibrium; then we can check whether that friction force can be obtained.



ANS. FIG. P12.51

$$\sum F_x = 0: \quad T - f = 0$$

$$\sum F_y = 0: \quad n - Mg - mg = 0$$

$$\sum \tau = 0: \quad Mg(\cos\theta)L + mg(\cos\theta)\frac{L}{2} - T(\sin\theta)L = 0$$

Solving the torque equation, we find $T = \left(M + \frac{1}{2}m\right)g\cot\theta.$

Then the horizontal-force equation implies by substitution that this same expression is equal to *f*. In order for the beam not to slip, we need $f \le \mu_s n.$ Substituting for *n* and *f* from the above equations, we obtain the requirement

$$\mu_s \ge \left[\frac{M + m/2}{M + m}\right]\cot\theta$$

The factor in brackets is always < 1, so if $\mu \ge \cot\theta$ then *M* can be increased without limit. In this case, there is no maximum mass! Otherwise, if $\mu_s < \cot\theta,$ the equality will apply on the verge of slipping, and solving for *M* yields

$$M = \boxed{\frac{m}{2}\left[\frac{2\mu_s\sin\theta - \cos\theta}{\cos\theta - \mu_s\sin\theta}\right]}$$

(b) At the floor, we see that the normal force is in the *y* direction and the friction force is in the −*x* direction. The reaction force exerted by the floor then has magnitude

$$R = \sqrt{n^2 + (\mu_s n)^2} = \boxed{g(M + m)\sqrt{1 + \mu_s^2}}$$

(c) At point *P*, the force of the beam on the rope is in magnitude

$$F = \sqrt{T^2 + (Mg)^2} = \boxed{g\sqrt{M^2 + \mu_s^2(M + m)^2}}$$

P12.52 First, we resolve all forces into components parallel to and perpendicular to the tibia, as shown. Note that $\theta = 40.0°$ and

$$w_y = (30.0\text{ N})\sin 40.0° = 19.3\text{ N}$$
$$F_y = (12.5\text{ N})\sin 40.0° = 8.03\text{ N}$$

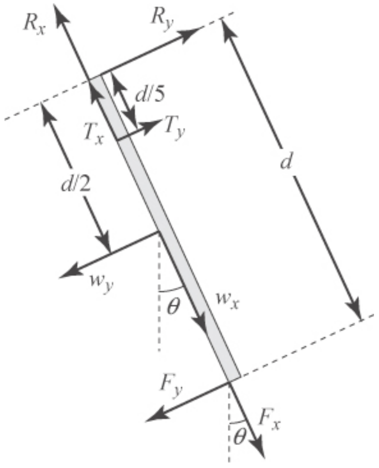
and

$$T_y = T\sin 25.0°$$

Using $\sum\tau = 0$ for an axis perpendicular to the page and through the upper end of the tibia gives

$$(T\sin 25.0°)\frac{d}{5} - (19.3\text{ N})\frac{d}{2} - (8.03\text{ N})d = 0$$

or $T = \boxed{209\text{ N}}$

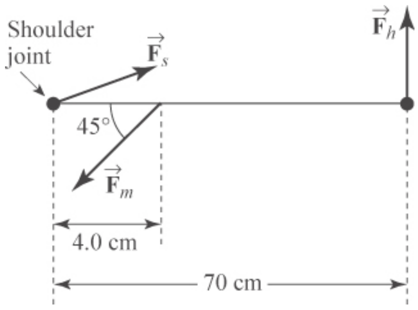


ANS. FIG. P12.52

P12.53 (a) From the symmetry of the situation, we may conclude that the magnitude of the upward force on each hand is half the weight of the athlete: $F_h = 750\text{ N}/2 = 375\text{ N}$. Considering the shoulder joint as the pivot, the second condition of equilibrium gives

$$\sum\tau = 0 \Rightarrow F_h(70.0\text{ cm}) - (F_m\sin 45°)(4.00\text{ cm}) = 0$$

or $$F_m = \frac{(375\text{ N})(70.0\text{ cm})}{(4.00\text{ cm})\sin 45°} = \boxed{9.28\text{ kN}}$$



ANS. FIG. P12.53

(b) The moment arm of the force is no longer 70.0 cm from the shoulder joint but only 49.5 cm:

$$\sum\tau = 0 \Rightarrow F_h(70.0\text{ cm})\sin 45° - (F_m\sin 45°)(4.00\text{ cm}) = 0$$
$$F_m = \frac{(375\text{ N})(70.0\text{ cm})}{(4.00\text{ cm})} = \boxed{6.56\text{ kN}}$$

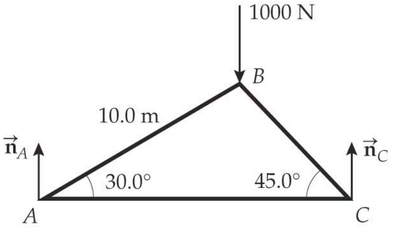
therefore reducing F_m to 6.56 kN.

P12.54 (a) The height of pin *B* is

$$(10.0\text{ m})\sin 30.0° = 5.00\text{ m}$$

The length of bar *BC* is then

$$\overline{\text{BC}} = \frac{5.00\text{ m}}{\sin 45.0°} = 7.07\text{ m}$$



ANS. FIG. P12.54(a)

Consider the entire truss:

$$\sum F_y = n_A - 1\,000\text{ N} + n_C = 0$$

$$\sum \tau_A = -(1\,000\text{ N})10.0\cos 30.0° + n_C\left[10.0\cos 30.0° + 7.07\cos 45.0°\right] = 0$$

Which gives $\boxed{n_C = 634\text{ N}}$.

Then, $\boxed{n_A = 1\,000\text{ N} - n_C = 366\text{ N}}$

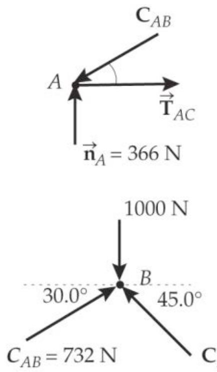
(b) Joint *A*: $\sum F_y = 0$: $-C_{AB}\sin 30.0° + 366\text{ N} = 0$

so $C_{AB} = \boxed{732\text{ N}}$

$\sum F_x = 0$:

$$-C_{AB}\cos 30.0° + T_{AC} = 0$$

$$T_{AC} = (732\text{ N})\cos 30.0° = \boxed{634\text{ N}}$$



ANS. FIG. P12.54(b)

Joint *B*:

$$\sum F_x = 0: \quad (732\text{ N})\cos 30.0° - C_{BC}\cos 45.0° = 0$$

$$C_{BC} = \frac{(732\text{ N})\cos 30.0°}{\cos 45.0°} = \boxed{897\text{ N}}$$

P12.55 Considering the torques about the point at the bottom of the bracket yields:

$$W(0.0500\text{ m}) - F_{\text{hor}}(0.0600\text{ m}) = 0 \text{ so } F_{\text{hor}} = 0.833W$$

(a) With $W = 80.0$ N, $F_{\text{hor}} = 0.833(80\text{ N}) = \boxed{66.7\text{ N}}$.

(b) Differentiate with respect to time: $dF_{\text{hor}}/dt = 0.833\, dW/dt$.

Given that $dW/dt = 0.150$ N/s:

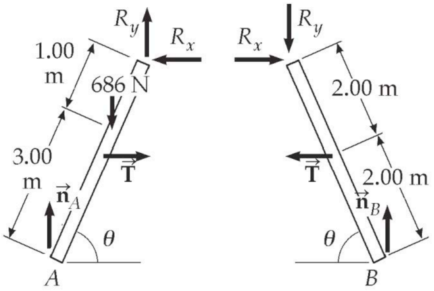
$\boxed{\text{The force exerted by the screw is increasing at the rate } dF_{\text{hor}}/dt = 0.833(0.150\text{ N/s}) = 0.125\text{ N/s.}}$

P12.56 Refer to the solution to P12.57 for a general discussion of the solution.

From the geometry of the ladder, observe that

$$\cos\theta = \frac{1}{4} \rightarrow \theta = 75.5°$$

In the following, we use the variables $m = 70.0$ kg, length $\mathrm{AC} = \mathrm{BC} = \ell = 4.00$ m, and $d = 3.00$ m.



ANS. FIG. P12.56

Consider the net torque about point *A* (on the bottom left side of the ladder) from external forces on the whole ladder. The torques about *A* come from the weight of the painter and the normal force n_B.

$$\sum \tau_A = -\mathrm{mgd}\cos 75.5° + \mathrm{n_B}\frac{\ell}{2} = 0$$

$$\rightarrow \mathrm{n_B} = \frac{2}{\ell}\mathrm{mgd}\cos 75.5° = \frac{2}{\ell}\mathrm{mgd}\left(\frac{1}{4}\right) \rightarrow \mathrm{n_B} = \frac{\mathrm{mgd}}{2\ell}$$

Consider the net torque about point *B* (on the bottom right side of the ladder) from external forces on the whole ladder. The torques about *B* come from the weight of the painter and the normal force n_A.

$$\sum \tau_B = -\mathrm{n_A}\frac{\ell}{2} + \mathrm{mg}\left(\frac{\ell}{2} - \mathrm{d}\cos 75.5°\right) = 0$$

$$\rightarrow \mathrm{n_A}\frac{\ell}{2} = \mathrm{mg}\left(\frac{\ell}{2} - \mathrm{d}\cos 75.5°\right)$$

$$\mathrm{n_A} = \frac{2}{\ell}\mathrm{mg}\left(\frac{\ell}{2} - \mathrm{d}\cos 75.5°\right) = \mathrm{mg}\left(1 - \frac{\mathrm{d}}{2\ell}\right)$$

Consider the torque from external forces about point *C* at the top of the <u>right half</u> of the ladder:

$$\sum \tau_C = -\mathrm{T}\frac{\ell}{2}\sin 75.5° + \mathrm{n_B}\frac{\ell}{4} = 0$$

$$\rightarrow \mathrm{T} = \mathrm{n_B}\frac{1}{2\sin 75.5°} = \frac{\mathrm{mgd}}{2\ell}\,\frac{1}{2\sin 75.5°}$$

$$\rightarrow \mathrm{T} = \frac{\mathrm{mgd}}{4\ell\sin 75.5°}$$

Note that the tension *T* on the right half of the ladder must pull to the left, otherwise it could not contribute a clockwise torque about *C* to balance the counterclockwise torque from n_B.

Now we find the components of the reaction force that the left half of the ladder exerts on the right half. Consider the forces acting on the right half of the ladder:

$$\sum F_x = R_x - T = 0 \rightarrow R_x = T\text{, to the right}$$

$$\sum F_y = R_y + n_B = 0 \rightarrow R_y = -n_B \rightarrow R_y = n_B\text{, downward}$$

Collecting our results, we find

(a) $$T = \frac{mgd}{4\ell \sin 75.5°} = \frac{(70.0\ \text{kg})(9.80\ \text{m/s}^2)(3.00\ \text{m})}{4(4.00\ \text{m})\sin 75.5°} \rightarrow \boxed{T = 133\ \text{N}}$$

(b) $$n_A = mg\left(1 - \frac{d}{2\ell}\right)$$

$$n_A = (70.0\ \text{kg})(9.80\ \text{m/s}^2)\left(1 - \frac{2(3.00\ \text{m})(1/4)}{4.00\ \text{m}}\right) \rightarrow \boxed{n_A = 429\ \text{N}}$$

and

$$n_B = \frac{mgd}{2\ell} = \frac{(70.0\ \text{kg})(9.80\ \text{m/s}^2)(3.00\ \text{m})}{2(4.00\ \text{m})} \rightarrow \boxed{n_B = 257\ \text{N}}$$

(c) The force exerted by the left half of the ladder on the right half is to the right and downward:

$$R_x = T \rightarrow \boxed{R_x = 133\ \text{N, to the right}}$$

and $$R_y = -n_b \rightarrow R_y = -257\ \text{N} \rightarrow \boxed{R_y = 257\ \text{N, downward}}$$

P12.57 From the geometry of Figure P12.56 and ANS. FIG. P12.56, we observe that

$$\cos\theta = \frac{\ell/4}{\ell} = \frac{1}{4}$$

and

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \left(\frac{1}{4}\right)^2} = \sqrt{1 - \frac{1}{16}} = \sqrt{\frac{15}{16}}$$

$$\sin\theta = \frac{\sqrt{15}}{4}$$

(a) Below in part (b) we show that normal force $n_B = mgd/2\ell$. We use this result here to find the tension T in the horizontal bar.

Consider the torque about point C at the top of the right half of the ladder:

$$\sum \tau_C = -T\frac{\ell}{2}\sin\theta + n_B\frac{\ell}{4} = 0$$

$$T = n_B\left(\frac{\ell}{4}\right)\frac{2}{\ell\sin\theta} = \frac{mgd}{2\ell}\left(\frac{\ell}{4}\right)\frac{2}{\ell\sin\theta} = \frac{mgd}{4\ell\sin\theta} = \frac{mgd}{4\ell\left(\sqrt{15}/4\right)}$$

$$\boxed{T = \frac{mgd}{\ell\sqrt{15}}}$$

Note that the tension T on the right half of the ladder must pull to the left, otherwise it could not contribute a clockwise torque about C to balance the counterclockwise torque from n_B.

(b) We now proceed to find the normal forces n_A and n_B.

First, consider the net torque from <u>all forces acting on the ladder</u> about point B at the bottom right side of the whole ladder. Note that tension T on the left half of the ladder and tension T on the right half of the ladder have opposite torques because they have the same moment arms about point B, so their torques cancel (they are forces internal to the system, so they cannot contribute to net torque). In like manner, torques from R_x and R_y on both halves of the ladder cancel in pairs (again, they are internal forces). The only contributing torques come from the weight of the painter and the normal force n_A (these are forces external to the ladder).

$$\sum \tau_B = -n_A\frac{\ell}{2} + mg\left(\frac{\ell}{2} - d\cos\theta\right) = 0$$

$$n_A\frac{\ell}{2} = mg\left(\frac{\ell}{2} - \frac{d}{4}\right)$$

$$n_A\frac{\ell}{2} = mg\left(\frac{2\ell - d}{4}\right)$$

$$n_A = \frac{2}{\ell}mg\left(\frac{2\ell - d}{4}\right)$$

$$\boxed{n_A = \frac{mg(2\ell - d)}{2\ell}}$$

Now, consider the net torque from <u>all forces acting on the ladder</u> about point A on the bottom left side of the whole ladder. Similarly to the case of the torques about point B, the only contributing torques about A come from the weight of the painter and the normal force n_B (again, these are external forces).

$$\sum \tau_A = -mgd\cos\theta + n_B\frac{\ell}{2} = 0$$

$$\to n_B = \frac{2}{\ell}mgd\cos\theta = \frac{2}{\ell}mgd\left(\frac{1}{4}\right) \to \boxed{n_B = \frac{mgd}{2\ell}}$$

(c) Now we find the components of the reaction force that the left half of the ladder exerts on the right half. Consider the forces acting on the right half of the ladder:

$$\sum F_x = R_x - T = 0 \to R_x = T$$

$$\boxed{R_x = \frac{mgd}{\sqrt{15}}, \text{ to the right}}$$

$$\sum F_y = R_y + n_B = 0 \to R_y = -n_B = -\frac{mgd}{2\ell}$$

$$\boxed{R_y = \frac{mgd}{2\ell}, \text{ downward}}$$

P12.58 (a) $F = m\left(\frac{\Delta v}{\Delta t}\right) = (1.00\text{ kg})\frac{(10.0-1.00)\text{ m/s}}{0.002\text{ s}} = \boxed{4\,500\text{ N}}$

(b) $\text{stress} = \frac{F}{A} = \frac{4\,500\text{ N}}{(0.010\text{ m})(0.100\text{ m})} = \boxed{4.50\times10^6\text{ N/m}^2}$

(c) $\boxed{\text{Yes.}}$ This is more than sufficient to break the board.

P12.59 (a) Take both balls together. Their weight is $2mg = 3.33$ N and their CG is at their contact point.

$$\sum F_x = 0: +P_3 - P_1 = 0 \to P_3 = P_1$$

$$\sum F_y = 0: +P_2 - 2mg = 0 \to P_2 = 2mg = \boxed{3.33\text{ N}}$$

For torque about the contact point (CP) between the balls:

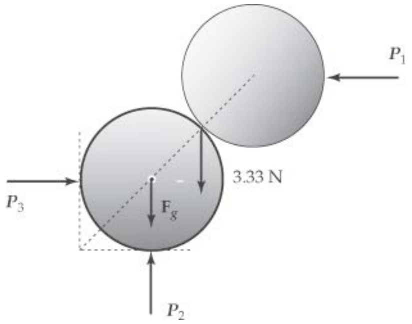
$$\begin{aligned}\sum \tau_{CP} = 0: \quad & P_1(R\cos 45.0°) - P_2(R\cos 45.0°) + P_3(R\cos 45.0°) \\ & - mg(R\cos 45.0°) + mg(R\cos 45.0°) = 0 \\ & \to P_1 - P_2 + P_3 = 0 \to P_1 + P_3 = P_2\end{aligned}$$

Substituting $P_3 = P_1$, we find

$$2P_1 = P_2 = 2mg \to P_1 = mg$$

Therefore,

$$P_1 = P_3 = \boxed{1.67\text{ N}}$$



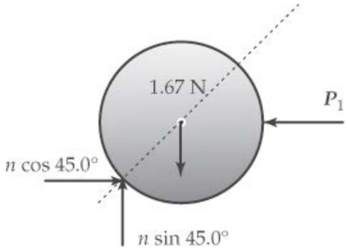
ANS. FIG. P12.59(a)

(b) Take the upper ball. The lines of action of its weight, of P_1, and of the normal force *n* exerted by the lower ball all go through its center, so for rotational equilibrium there can be no frictional force.

$$\sum F_x = 0:\ n\cos 45.0° - P_1 = 0$$

$$n = \frac{1.67\text{ N}}{\cos 45.0°} = \boxed{2.36\text{ N}}$$

$$\sum F_y = 0:\ n\sin 45.0° - 1.67\text{ N} = 0\ \text{ gives the same result.}$$



ANS. FIG. P12.59(b)

P12.60 We will let *F* represent some stretching force and use algebra to combine the Hooke's-law account of the stretching with the Young's-modulus account. Then integration will reveal the work done as the wire extends.

(a) According to Hooke's law, $|\vec{\mathbf{F}}| = k\Delta L$

Young's modulus is defined as $Y = \dfrac{F/A}{\Delta L/L}$

By substitution,

$$Y = k\frac{L}{A} \quad \text{or} \quad k = \boxed{\frac{YA}{L}}$$

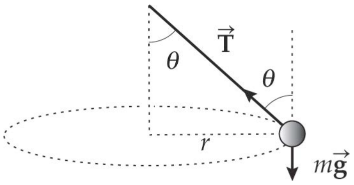
(b) The spring exerts force $-kx$. The outside agent stretching it exerts force $+kx$. We can determine the work done by integrating the force kx over the distance we stretch the wire.

$$W = -\int_0^{\Delta L} F\,dx = -\int_0^{\Delta L}(-kx)dx = \frac{YA}{L}\int_0^{\Delta L} x\,dx = \left[\frac{YA}{L}\left(\frac{1}{2}x^2\right)\right]_{x=0}^{x=\Delta L}$$

Therefore,

$$W = \boxed{\frac{1}{2}YA(\Delta L)^2/L}$$

P12.61 Let θ represent the angle of the wire with the vertical. The radius of the circle of motion is $r = L\sin\theta$, where $L = 0.850$ m.



ANS. FIG. P12.61

For the mass:

$$\sum F_r = ma_r = m\frac{v^2}{r} = mr\omega^2$$

$$T\sin\theta = m[L\sin\theta]\omega^2$$

Further, $\frac{T}{A} = Y\cdot\frac{\Delta L}{L}$ or $T = AY\cdot\frac{\Delta L}{L}$

Thus, $AY\cdot(\Delta L/L) = mL\omega^2$, giving

$$\omega = \sqrt{\frac{AY\cdot(\Delta L/L)}{mL}} = \sqrt{\frac{\pi(3.90\times10^{-4}\text{ m})^2(7.00\times10^{10}\text{ N/m}^2)(1.00\times10^{-3})}{(1.20\text{ kg})(0.850\text{ m})}}$$

or $\omega = \boxed{5.73\text{ rad/s}}$

P12.62 (a), (b) Use the first diagram and sum the torques about the lower front corner of the cabinet.

$$\sum\tau = 0 \Rightarrow -F(1.00\text{ m}) + (400\text{ N})(0.300\text{ m}) = 0$$

yielding $F = \frac{(400\text{ N})(0.300\text{ m})}{1.00\text{ m}} = \boxed{120\text{ N}}$

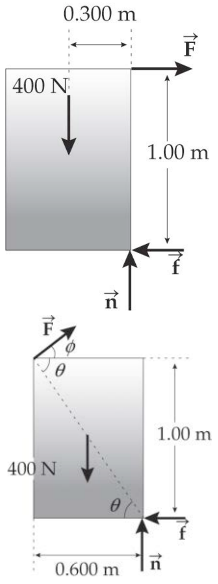
$$\sum F_x = 0 \Rightarrow -f + 120\text{ N} = 0, \quad \text{or} \quad f = 120\text{ N}$$

$$\sum F_y = 0 \Rightarrow -400\text{ N} + n = 0$$

so $n = 400\text{ N}$

Thus,

$$\mu_s = \frac{f}{n} = \frac{120\text{ N}}{400\text{ N}} = \boxed{0.300}$$



ANS. FIG. P12.62

(c) Apply F' at the upper rear corner and directed so $\theta + \phi = 90.0°$ to obtain the largest possible lever arm.

$$\theta = \tan^{-1}\left(\frac{1.00\text{ m}}{0.600\text{ m}}\right) = 59.0°$$

Thus, $\phi = 90.0° - 59.0° = 31.0°$

Sum the torques about the lower front corner of the cabinet:

$$-F'\sqrt{(1.00\text{ m})^2 + (0.600\text{ m})^2} + (400\text{ N})(0.300\text{ m}) = 0$$

so $F' = \dfrac{120\text{ N}\cdot\text{m}}{1.17\text{ m}} = 103\text{ N}$

Therefore, the minimum force required to tip the cabinet is

$$\boxed{\text{103 N applied at 31.0° above the horizontal at the upper left corner}}$$

***P12.63** (a) Consider the torques about an axis perpendicular to the page through the left end of the rod, as shown in ANS. FIG. P12.63.

$$\sum \tau = 0:$$

$$T(6.00\text{ m})\cos 30.0° - (100\text{ N})(3.00\text{ m}) - (500\text{ N})(4.00\text{ m}) = 0$$

then,

$$T = \frac{(100\text{ N})(3.00\text{ m}) + (500\text{ N})(4.00\text{ m})}{(6.00\text{ m})\cos 30.0°}$$

$$= \boxed{443\text{ N}}$$

(b) From the first condition for equilibrium,

$$\sum F_x = 0:$$

$$R_x = T\sin 30.0° = (443\text{ N})\sin 30.0°$$

$$= \boxed{221\text{ N toward the right}}$$

Similarly,

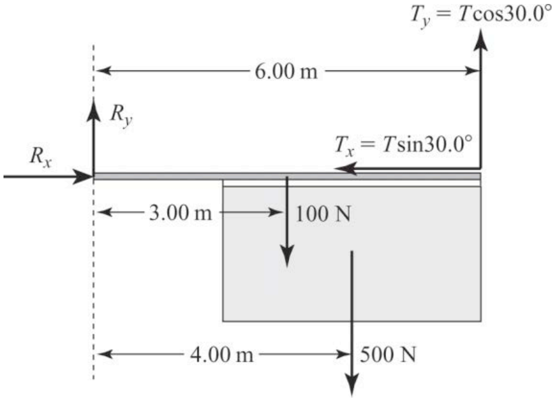
$$\sum F_y = 0:$$

$$R_y + T\cos 30.0° - 100\text{ N} - 500\text{ N} = 0$$

which gives

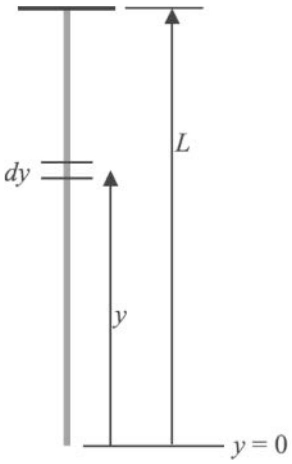
$$R_y = 600\text{ N} - T\cos 30.0° = 600\text{ N} - (443\text{ N})\cos 30.0°$$

$$= \boxed{217\text{ N upward}}$$



ANS. FIG. P12.63

P12.64 Let the original length (when the cable is laid horizontally on a frictionless surface) of an infinitesimal piece of the cable be *dy*. Let the extension of this piece be *dL* when the cable is hung vertically. Then, for the entire cable,



ANS. FIG. P12.64

$$\Delta L = \int dL = \int \frac{F}{AY}dy$$

where *F* is the weight of the cable below a point at position *y*. Evaluating *F*, with μ as the mass per unit length,

$$\Delta L = \int \frac{(\mu y)g}{AY}dy = \frac{\mu g}{AY}\int_0^L y\,dy$$

$$= \frac{\mu g}{AY}\left(\frac{L^2}{2}\right) = \frac{1}{2}\left(\frac{\mu g L^2}{AY}\right)$$

$$\Delta L = \frac{1}{2}\left[\frac{(2.40\text{ kg/m})(9.80\text{ m/s}^2)(500\text{ m})^2}{(2.00\times 10^{11}\text{ N/m}^2)(3.00\times 10^{-4}\text{ m}^2)}\right]$$

$$= 0.049\,0\text{ m} = \boxed{4.90\text{ cm}}$$

Challenge Problems

P12.65 With ℓ as large as possible, n_1 and n_2 will both be large. The equality sign in $f_2 \le \mu_s n_2$ will be true, but the less-than sign will apply in $f_1 < \mu_s n_1$. Take torques about the lower end of the pole.

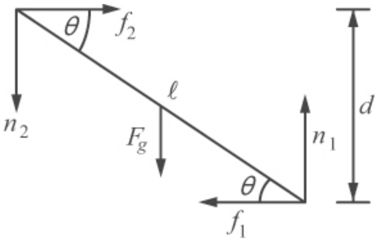
$$n_2 \ell \cos\theta + F_g\left(\frac{1}{2}\ell\right)\cos\theta - f_2 \ell \sin\theta = 0$$

Setting $f_2 = 0.576 n_2$, the torque equation becomes

$$n_2\left(1 - 0.576\tan\theta\right) + \frac{1}{2}F_g = 0$$

Since $n_2 > 0$, it is necessary that

$$1 - 0.576\tan\theta < 0$$
$$\therefore \tan\theta > \frac{1}{0.576} = 1.736$$
$$\therefore \theta > 60.1°$$
$$\therefore \ell = \frac{d}{\sin\theta} < \frac{7.80\text{ ft}}{\sin 60.1°} = \boxed{9.00\text{ ft}}$$



ANS. FIG. P12.65

P12.66 Consider forces and torques on the beam.

$$\sum F_x = 0: \quad R\cos\theta - T\cos 53° = 0$$
$$\sum F_y = 0: \quad R\sin\theta + T\sin 53° - 800\text{ N} = 0$$
$$\sum \tau = 0: \quad (T\sin 53°)(8.00\text{ m}) - (600\text{ N})d - (200\text{ N})(4.00\text{ m}) = 0$$

(a) Suppressing units, we find

$$T = \frac{600d + 800}{8\sin 53°} = \boxed{93.9d + 125,\text{ in N}}$$

(b) From substituting back,

$$R\cos\theta = [93.9d + 125]\cos 53.0°$$
$$R\sin\theta = 800\text{ N} - [93.9d + 125]\sin 53.0°$$

Dividing,

$$\tan\theta = \frac{R\sin\theta}{R\cos\theta} = -\tan 53.0° + \frac{800\text{ N}}{(93.9d+125)\cos 53.0°}$$

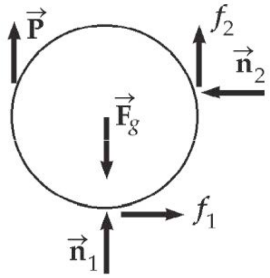
$$\boxed{\tan\theta = \left(\frac{32}{3d+4} - 1\right)\tan 53.0°}$$

(c) To find *R* we can work out $R^2\cos^2\theta + R^2\sin^2\theta = R^2$. From the expressions above for $R\cos\theta$ and $R\sin\theta$,

$$R^2 = T^2\cos^2 53° + T^2\sin^2 53° - 1\,600T\sin 53° + (800\text{ N})^2$$
$$R^2 = T^2 - 1\,600T\sin 53° + 640\,000$$
$$R^2 = (93.9d + 125)^2 - 1\,278(93.9d + 125) + 640\,000$$
$$\boxed{R = \left(8.82\times10^3 d^2 - 9.65\times10^4 d + 4.96\times10^5\right)^{1/2}}$$

(d) $\boxed{\text{As } d \text{ increases, } T \text{ grows larger, } \theta \text{ decreases, and } R \text{ decreases until about } d = 5.4\text{ m, then it increases. Notes as } d \text{ increases, the } d^2 \text{ term predominates.}}$

P12.67 Imagine gradually increasing the force *P*. This will make the force of static friction at the bottom increase, so that the normal force at the wall increases and the friction force at the wall *can* increase. As *P* reaches its maximum value, the cylinder will turn clockwise microscopically to stress the welds at both contact points and make both forces of friction increase to their maximum values.



ANS. FIG. P12.67

When it is on the verge of slipping, the cylinder is in equilibrium.

$$\sum F_x = 0: \quad \rightarrow f_1 = n_2 = \mu_s n_1 \quad \text{and} \quad f_2 = \mu_s n_2$$
$$\sum F_y = 0: \quad \rightarrow P + n_1 + f_2 = F_g$$
$$\sum \tau = 0: \quad \rightarrow -PR + f_1R + f_2R = 0 \rightarrow P = f_1 + f_2$$

As *P* grows, so do f_1 and f_2. Therefore, since $\mu_s = \frac{1}{2}$,

$$f_1 = \frac{n_1}{2} \text{ and } f_2 = \frac{n_2}{2} = \frac{n_1}{4}$$

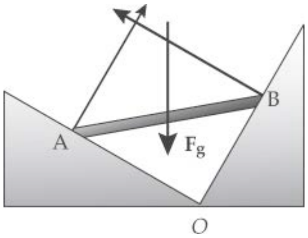
then $$P + n_1 + \frac{n_1}{4} = F_g$$

and $$P = \frac{n_1}{2} + \frac{n_1}{4} = \frac{3}{4}n_1$$

So $P + \frac{5}{4}n_1 = F_g$ becomes $P + \frac{5}{4}\left(\frac{4}{3}P\right) = F_g$ or $\frac{8}{3}P = F_g$.

Therefore, $P = \boxed{\frac{3}{8}F_g}$.

P12.68 (a) Just three forces act on the rod: forces perpendicular to the sides of the trough at A and B, and its weight. The lines of action of the normal forces at A and B will intersect at a point above the rod so that those forces will have no torque about this point. The rod's weight will cause a torque about the point of intersection as in ANS. FIG. P12.68(a), and the rod will not be in equilibrium unless the center of the rod lies vertically below the intersection point, as in ANS. FIG. P12.68(b). All three forces must be concurrent. Then the line of action of the weight is a diagonal of the rectangle formed by the two normal forces, and the rod's center of gravity is vertically above the bottom of the trough.



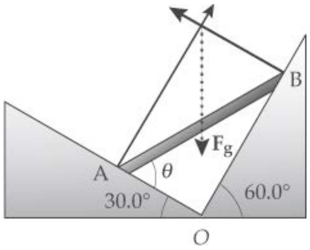
ANS. FIG. P12.68(a)

(b) In ANS. FIG. P12.68(b), $\overline{AO}\cos 30.0° = \overline{BO}\cos 60.0°$ and

$$L^2 = \overline{AO}^2 + \overline{BO}^2 = \overline{AO}^2 + \overline{AO}^2\left(\frac{\cos^2 30.0°}{\cos^2 60.0°}\right)$$

$$\overline{AO} = \frac{L}{\sqrt{1+\left(\frac{\cos 30°}{\cos 60°}\right)^2}} = \frac{L}{2}$$

So $\cos\theta = \dfrac{\overline{\mathrm{AO}}}{L} = \dfrac{1}{2}$ and $\theta = \boxed{60.0°}$



ANS. FIG. P12.68(b)

(c) $\boxed{\text{Unstable.}}$ If the rod is displaced slightly, it will slip until it lies along the left edge of the trough where its center of gravity will be lower.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P12.2 The situation is impossible because x is larger than the remaining portion of the beam, which is 0.200 m long.

P12.4 $x = -1.50$ m; $y = -1.50$ m

P12.6 $\frac{R}{6}$

P12.10 2.94 kN; 4.41 kN

P12.8 (a) $m_1 = 9.00$ g; (b) $m_2 = 52.5$ g; (c) $m_3 = 49.0$ g

P12.12 (a) $U = 88.2$ N; (b) $D = 58.8$ N

P12.14 (a) $\left[\frac{1}{2}m_1 g + \left(\frac{x}{L}\right)m_2 g\right]\cot\theta$, $(m_1 + m_2)g$; (b) $\frac{\left(m_1/2 + m_2 d/L\right)\cot\theta}{m_1 + m_2}$

P12.16 (a) See ANS. FIG. P12.16; (b) $\frac{mg}{2}\cot\theta$; (c) $T = \mu_s mg$; (d) $\mu_s = \frac{1}{2}\cot\theta$; (e) The ladder slips

P12.18 (a) See ANS. FIG. P12.18; (b) 392 N; (c) 339 N to the right; (d) 0; (e) $V = 0$; (f) 392 N; (g) 339 N to the right; (h) The two solutions agree precisely. They are equally accurate.

P12.20 (a) No time interval. The horse's feet lose contact with the drawbridge as soon as it begins to move; (b) 1.73 rad/s; (c) 2.22 rad/s; (d) 6.62 kN. The force at the hinge is $\left(4.72\hat{\mathbf{i}} + 6.62\hat{\mathbf{j}}\right)$kN; (e) 59.1 kJ

P12.22 (a) $\frac{mg\sqrt{2Rh - h^2}}{(R-h)\cos\theta - \sqrt{2Rh - h^2}\sin\theta}$;

(b) $\frac{mg\sqrt{2Rh - h^2}\cos\theta}{(R-h)\cos\theta - \sqrt{2Rh - h^2}\sin\theta}$ and $mg\left[1 + \frac{\sqrt{2Rh - h^2}\cos\theta}{(R-h)\cos\theta - \sqrt{2Rh - h^2}\sin\theta}\right]$

P12.24 (a) See ANS. FIG. P12.24; (b) 218 N; (c) 72.4 N; (d) 2.41 m; (e) See P12.24(e) for full explanation.

P12.26 ~ 1 cm

P12.28 (a) 73.6 kN; (b) 2.50 mm

P12.30 1.0×10^{11} N/m^2

P12.32 1.65×10^{8} N/m^2

P12.34 8.60×10^{-4} m

P12.36	9.85×10^{-5}
P12.38	(a) Rigid object in static equilibrium; (b) See ANS. FIG. P12.38; (c) The woman is at $x = 0$ when n_1 is greatest; (d) $n_1 = 0$; (e) 1.42×10^3 N; (f) 5.64 m; (g) same as answer (f)
P12.40	(a) 0.400 mm; (b) 40.0 kN; (c) 2.00 mm; (d) 2.40 mm; (e) 48.0 kN
P12.42	$\theta = 21.2°$; $T = 1.68$ kN; $R = 2.34$ kN
P12.44	(a) See ANS. FIG. P12.44 for the force diagram and see P12.44(a) for a sample problem statement. (b) The upper hinge exerts 410 N to the left and 442 N up. The lower hinge exerts 410 N to the right.
P12.46	$T = 1.46$ kN; $H = 1.33$ kN; $V = 2.58$ kN
P12.48	(a) 2.71 kN; (b) 2.65 kN; (c) You should lift “with your knees” rather than “with your back”; (d) In this situation, you can make the compressional force in your spine about ten times smaller by bending your knees and lifting with your back as straight as possible.
P12.50	The situation is impossible because the new technique would tip the cabinet over.
P12.52	209 N
P12.54	(a) $n_{\text{C}} = 634$ N, $n_{\text{A}} = 1\,000\text{ N} - n_{\text{C}} = 366$ N; (b) $C_{AB} = 732$ N, $T_{AC} = 634$ N, and $C_{BC} = 897$ N
P12.56	(a) $T = 133$ N; (b) $n_{\text{A}} = 429$ N, $n_{\text{B}} = 257$ N; (c) $R_x = 133$ N, to the right, $R_y = 257$ N, downward
P12.58	(a) 4 500 N; (b) $4.50 \times 10^6\text{ N/m}^2$; (c) yes
P12.60	(a) $\dfrac{YA}{L}$; (b) $YA\,\dfrac{(\Delta L)^2}{2L}$
P12.62	(a and b) 120 N, 0.300; (c) 103 N applied at 31.0° above the horizontal at the upper left corner.
P12.64	4.90 cm
P12.66	(a) $93.9d + 125$, in N; (b) See P12.66(b) for full derivation; (c) See P12.66(c) for full derivation; (d) As d increases, T grows larger, θ decreases, and R decreases until about $d = 5.4$ m, then it increases. Note as d increases, the d^2 term predominates.
P12.68	(a) See P12.68(a) for the full explanation; (b) 60.0°; (c) unstable

13

Universal Gravitation

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ13.1 Answer (c). Ten terms are needed in the potential energy:

$$U = U_{12} + U_{13} + U_{14} + U_{15} + U_{23} + U_{24} + U_{25} + U_{34} + U_{35} + U_{45}$$

OQ13.2 The ranking is a > b = c. The gravitational potential energy of the Earth-Sun system is negative and twice as large in magnitude as the kinetic energy of the Earth relative to the Sun. Then the total energy is negative and equal in absolute value to the kinetic energy.

OQ13.3 Answer (d). The satellite experiences a gravitational force, always directed toward the center of its orbit, and supplying the centripetal force required to hold it in its orbit. This force gives the satellite a centripetal acceleration, even if it is moving with constant angular speed. At each point on the circular orbit, the gravitational force is directed along a radius line of the path, and is perpendicular to the motion of the satellite, so this force does no work on the satellite.

OQ13.4 Answer (d). Having twice the mass would make the surface gravitational field two times larger. But the inverse square law says that having twice the radius would make the surface acceleration due to gravitation four times smaller. Altogether, g at the surface of B becomes $(2\ \text{m/s}^2)(2)/4 = 1\ \text{m/s}^2$.

OQ13.5 Answer (b). Switching off gravity would let the atmosphere evaporate away, but switching off the atmosphere has no effect on the planet's gravitational field.

OQ13.6 Answer (b). The mass of a spherical body of radius R and density ρ is $M = \rho V = \rho(4\pi R^3/3)$. The escape velocity from the surface of this body may then be written in either of the following equivalent forms:

$$v_{esc} = \sqrt{\frac{2GM}{R}} \quad \text{and}\ v_{esc} = \sqrt{\frac{2G}{R}\left(\frac{4\pi\rho R^3}{3}\right)} = \sqrt{\frac{8\pi\rho G R^2}{3}}$$

We see that the escape velocity depends on the three properties (mass, density, and radius) of the planet. Also, the weight of an object on the surface of the planet is $F_g = mg = GMm/R^2$, giving

$$g = GM/R^2 = \frac{G}{R^2}\left[\rho\left(\frac{4\pi R^3}{3}\right)\right] = \frac{4}{3}\pi\rho G R$$

The free-fall acceleration at the planet's surface then depends on the same properties as does the escape velocity. Changing the value of g would necessarily change the escape velocity. Of the listed quantities, the only one that does not affect the escape velocity is the mass of the object.

OQ13.7 (i) Answer (e). According to the inverse square law, $1/4^2 = 16$ times smaller.

(ii) Answer (c). $mv^2/r = GMm/r^2$ predicts that v is proportional to $(1/r)^{1/2}$, so it becomes $(1/4)^{1/2} = 1/2$ as large.

(iii) Answer (a). According to Kepler's third law, $(4^3)^{1/2} = 8$ times larger; also, the circumference is 4 times larger and the speed 1/2 as large: $4/(1/2) = 8$.

OQ13.8 Answer (b). The Earth is farthest from the sun around July 4 every year, when it is summer in the northern hemisphere and winter in the southern hemisphere. As described by Kepler's second law, this is when the planet is moving slowest in its orbit. Thus it takes more time for the planet to plod around the 180° span containing the minimum-speed point.

OQ13.9 The ranking is b > a > c = d > e. The force is proportional to the product of the masses and inversely proportional to the square of the separation distance, so we compute m_1m_2/r^2 for each case: (a) $2\cdot 3/1^2 = 6$ (b) 18 (c) $18/4 = 4.5$ (d) 4.5 (e) $16/4 = 4$.

OQ13.10 Answer (c). The International Space Station orbits just above the atmosphere, only a few hundred kilometers above the ground. This distance is small compared to the radius of the Earth, so the gravitational force on the astronaut is only slightly less than on the ground. We might *think* the gravitational force is zero or nearly zero, because the orbiting astronauts *appear* to be weightless. They and the space station are in free fall, so the normal force of the space station's wall/floor/ceiling on the astronauts is zero; they float freely around the cabin.

OQ13.11 Answer (e). We assume that the elliptical orbit is so elongated that the Sun, at one focus, is almost at one end of the major axis. If the period, T, is expressed in years and the semimajor axis, a, in astronomical units (AU), Kepler's third law states that $T^2 = a^3$. Thus, for Halley's comet, with a period of $T = 76$ y, the semimajor axis of its orbit is

$$a = \sqrt[3]{(76)^2} = 18 \text{ AU}$$

The length of the major axis, and the approximate maximum distance from the Sun, is $2a = 36$ AU.

ANSWERS TO CONCEPTUAL QUESTIONS

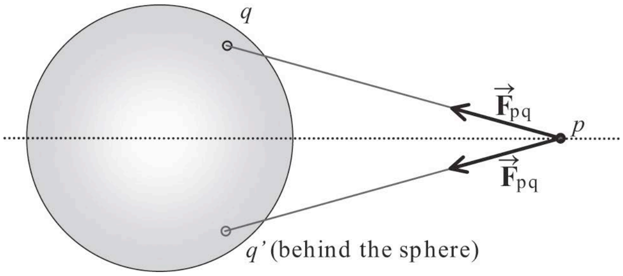
CQ13.1 (a) The gravitational force is conservative. (b) Yes. An encounter with a stationary mass cannot permanently speed up a spacecraft. But Jupiter is moving. A spacecraft flying across its orbit just behind the planet will gain kinetic energy because of the change in potential energy of the spacecraft-planet system. This is a collision because the spacecraft and planet exert forces on each other while they are isolated from outside forces. It is an elastic collision because only conservative forces are involved. (c) The planet loses kinetic energy as the spacecraft gains it.

CQ13.2 Cavendish determined G. Then from $g = \frac{GM}{R^2}$, one may determine the mass of the Earth. The term "weighed" is better expressed as "massed."

CQ13.3 For a satellite in orbit, one focus of an elliptical orbit, or the center of a circular orbit, must be located at the center of the Earth. If the satellite is over the northern hemisphere for half of its orbit, it must be over the southern hemisphere for the other half. We could share with Easter Island a satellite that would look straight down on Arizona each morning and vertically down on Easter Island each evening.

CQ13.4 (a) Every point q on the sphere that does not lie along the axis connecting the center of the sphere and the particle will have companion point q' for which the components of the gravitational force perpendicular to the axis will cancel. Point q' can be found by rotating the sphere through 180° about the axis.

(b) The forces will not necessarily cancel if the mass is not uniformly distributed, unless the center of mass of the nonuniform sphere still lies along the axis.



ANS. FIG. CQ13.4

CQ13.5 The angular momentum of a planet going around a sun is conserved. (a) The speed of the planet is maximum at closest approach. (b) The speed is a minimum at farthest distance. These two points, perihelion and aphelion respectively, are 180° apart, at opposite ends of the major axis of the orbit.

CQ13.6 Set the universal description of the gravitational force, $F_g = \frac{GM_X m}{R_X^2}$,

equal to the local description, $F_g = ma_{\text{gravitational}}$, where M_x and R_x are the mass and radius of planet X, respectively, and m is the mass of a "test particle." Divide both sides by m.

CQ13.7 (a) In one sense, 'no'. If the object is at the very center of the Earth there is no other mass located there for comparison and the formula does not apply in the same way it was being applied while the object was some distance from the center. In another sense, 'yes'. One would have to compare, though, the distance between the object with mass m to the other individual masses that make up the Earth.

(b) The gravitational force of the Earth on an object at its center must be zero, not infinite as one interpretation of Equation 11.1 would suggest. All the bits of matter that make up the Earth pull in different outward directions on the object, causing the net force on it to be zero.

CQ13.8 The escape speed from the Earth is 11.2 km/s and that from the

Moon is 2.3 km/s, smaller by a factor of 5. The energy required—and fuel—would be proportional to v^2, or 25 times more fuel is required to leave the Earth versus leaving the Moon.

CQ13.9 Air resistance causes a decrease in the energy of the satellite-Earth system. This reduces the radius of the orbit, bringing the satellite closer to the surface of the Earth. A satellite in a smaller orbit, however, must travel faster. Thus, the effect of air resistance is to speed up the satellite!

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 13.1 Newton's Law of Universal Gravitation

P13.1 This is a direct application of the equation expressing Newton's law of gravitation:

$$F = \frac{GMm}{r^2} = \left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\frac{(1.50\ \mathrm{kg})\left(15.0\times10^{-3}\ \mathrm{kg}\right)}{\left(4.50\times10^{-2}\ \mathrm{m}\right)^2}$$

$$= \boxed{7.41\times10^{-10}\ \mathrm{N}}$$

P13.2 For two 70-kg persons, modeled as spheres,

$$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)(70\ \mathrm{kg})(70\ \mathrm{kg})}{(2\ \mathrm{m})^2}$$

$$\boxed{\sim10^{-7}\ \mathrm{N}}$$

P13.3 (a) At the midpoint between the two objects, the forces exerted by the 200-kg and 500-kg objects are oppositely directed,

and from $F_g = \dfrac{Gm_1m_2}{r^2}$

we have $\sum F = \dfrac{G(50.0\ \mathrm{kg})(500\ \mathrm{kg} - 200\ \mathrm{kg})}{(2.00\ \mathrm{m})^2} = \boxed{2.50\times10^{-7}\ \mathrm{N}}$

toward the 500-kg object.

(b) At a point between the two objects at a distance d from the 500-kg object, the net force on the 50.0-kg object will be zero when

$$\frac{G(50.0\ \mathrm{kg})(200\ \mathrm{kg})}{(4.00\ \mathrm{m} - d)^2} = \frac{G(50.0\ \mathrm{kg})(500\ \mathrm{kg})}{d^2}$$

To solve, cross-multiply to clear of fractions and take the square root of both sides. The result is

$$d = \boxed{2.45\text{ m from the 500-kg object toward the smaller object}}.$$

P13.4 (a) The Sun-Earth distance is 1.496×10^{11} m and the Earth-Moon distance is 3.84×10^{8} m, so the distance from the Sun to the Moon during a solar eclipse is

$$1.496 \times 10^{11}\text{ m} - 3.84 \times 10^{8}\text{ m} = 1.492 \times 10^{11}\text{ m}$$

The mass of the Sun, Earth, and Moon are

$$M_S = 1.99 \times 10^{30}\text{ kg}$$

$$M_E = 5.98 \times 10^{24}\text{ kg}$$

and $M_M = 7.36 \times 10^{22}\text{ kg}$

We have

$$\begin{aligned} F_{SM} &= \frac{Gm_1m_2}{r^2} \\ &= \frac{\left(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99 \times 10^{30}\text{ kg}\right)\left(7.36 \times 10^{22}\text{ kg}\right)}{\left(1.492 \times 10^{11}\text{ m}\right)^2} \\ &= \boxed{4.39 \times 10^{20}\text{ N}} \end{aligned}$$

(b) $$\begin{aligned} F_{EM} &= \frac{\left(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98 \times 10^{24}\text{ kg}\right)\left(7.36 \times 10^{22}\text{ kg}\right)}{\left(3.84 \times 10^{8}\text{ m}\right)^2} \\ &= \boxed{1.99 \times 10^{20}\text{ N}} \end{aligned}$$

(c) $$\begin{aligned} F_{SE} &= \frac{\left(6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99 \times 10^{30}\text{ kg}\right)\left(5.98 \times 10^{24}\text{ kg}\right)}{\left(1.496 \times 10^{11}\text{ m}\right)^2} \\ &= \boxed{3.55 \times 10^{22}\text{ N}} \end{aligned}$$

(d) The force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon. In a sense, the Moon orbits the Sun more than it orbits the Earth. The Moon's path is everywhere concave toward the Sun. Only by subtracting out the solar orbital motion of the Earth-Moon system do we see the Moon orbiting the center of mass of this system.

P13.5 With one metric ton = 1 000 kg,

$$F = m_1 g = \frac{Gm_1 m_2}{r^2}$$

$$g = \frac{Gm_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(4.00\times10^{7}\ \text{kg}\right)}{\left(100\ \text{m}\right)^2}$$

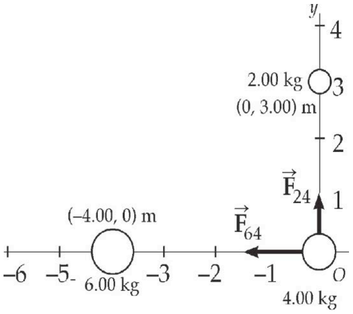
$$= \boxed{2.67\times10^{-7}\ \text{m/s}^2}$$

P13.6 The force exerted on the 4.00-kg mass by the 2.00-kg mass is directed upward and given by

$$\vec{\mathbf{F}}_{12} = G\frac{m_2 m_1}{r_{12}^2}\hat{\mathbf{j}}$$

$$= \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(4.00\ \text{kg}\right)\left(2.00\ \text{kg}\right)}{\left(3.00\ \text{m}\right)^2}\hat{\mathbf{j}}$$

$$= 5.93\times10^{-11}\hat{\mathbf{j}}\ \text{N}$$



ANS. FIG. P13.6

The force exerted on the 4.00-kg mass by the 6.00-kg mass is directed to the left:

$$\vec{\mathbf{F}}_{32} = G\frac{m_2 m_3}{r_{32}^2}\left(-\hat{\mathbf{i}}\right)$$

$$= \left(-6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(4.00\ \text{kg}\right)\left(6.00\ \text{kg}\right)}{\left(4.00\ \text{m}\right)^2}\hat{\mathbf{i}}$$

$$= -10.0\times10^{-11}\hat{\mathbf{i}}\ \text{N}$$

Therefore, the resultant force on the 4.00-kg mass is

$$\vec{\mathbf{F}}_4 = \vec{\mathbf{F}}_{24} + \vec{\mathbf{F}}_{64} = \boxed{\left(-10.0\hat{\mathbf{i}} + 5.93\hat{\mathbf{j}}\right)\times10^{-11}\ \text{N}}$$

***P13.7** The magnitude of the gravitational force is given by

$$F = \frac{Gm_1 m_2}{r^2} = \frac{\left(6.672\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.00\ \text{kg}\right)\left(2.00\ \text{kg}\right)}{\left(0.300\ \text{m}\right)^2}$$

$$= \boxed{2.97\times10^{-9}\ \text{N}}$$

P13.8 Assume the masses of the sphere are the same. Using $F_g = \dfrac{Gm_1 m_2}{r^2}$, we would find that the mass of a sphere is 1.22×10^5 kg! If the spheres have at most a radius of 0.500 m, the density of spheres would be at least $2.34 \times 10^5\ \text{kg/m}^3$, which is ten times the density of the most dense element, osmium.

$\boxed{\text{The situation is impossible because no known element could compose the spheres.}}$

P13.9 We are given $m_1 + m_2 = 5.00$ kg, which means that $m_2 = 5.00\ \text{kg} - m_1$. Newton's law of universal gravitation then becomes

$$F = G\frac{m_1 m_2}{r^2}$$

$$\Rightarrow 1.00 \times 10^{-8}\ \text{N}$$

$$= \left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{m_1\left(5.00\ \text{kg} - m_1\right)}{\left(0.200\ \text{m}\right)^2}$$

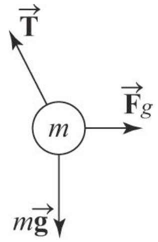
$$\left(5.00\ \text{kg}\right)m_1 - m_1^2 = \frac{\left(1.00 \times 10^{-8}\ \text{N}\right)\left(0.040\,0\ \text{m}^2\right)}{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 6.00\ \text{kg}^2$$

Thus, $m_1^2 - \left(5.00\ \text{kg}\right)m_1 + 6.00\ \text{kg} = 0$

or $\left(m_1 - 3.00\ \text{kg}\right)\left(m_1 - 2.00\ \text{kg}\right) = 0$

giving $\boxed{m_1 = 3.00\ \text{kg, so } m_2 = 2.00\ \text{kg}}$. The answer $m_1 = 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

P13.10 Let θ represent the angle each cable makes with the vertical, L the cable length, x the distance each ball is displaced by the gravitational force, and $d = 1$ m the original distance between them. Then $r = d - 2x$ is the separation of the balls. We have



ANS. FIG. P13.10

$$\sum F_y = 0: \quad T\cos\theta - mg = 0$$

$$\sum F_x = 0: \quad T\sin\theta - \frac{Gmm}{r^2} = 0$$

Then $\tan\theta = \dfrac{Gmm}{r^2 mg}$

$$\frac{x}{\sqrt{L^2 - x^2}} = \frac{Gm}{g\left(d - 2x\right)^2} \quad \rightarrow \quad x\left(d - 2x\right)^2 = \frac{Gm}{g}\sqrt{L^2 - x^2}$$

The factor $\frac{Gm}{g}$ is numerically small. We expect that x is very small compared to both L and d, so we can treat the term $(d - 2x)$ as d, and $(L^2 - x^2)$ as L^2. We then have

$$x(1\ \text{m})^2 = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(100\ \text{kg})}{(9.80\ \text{m/s}^2)}(45.00\ \text{m})$$

$$\boxed{x = 3.06\times10^{-8}\ \text{m}}$$

Section 13.2 Free-Fall Acceleration and the Gravitational Force

P13.11 The distance of the meteor from the center of Earth is $R + 3R = 4R$. Calculate the acceleration of gravity at this distance.

$$g = \frac{GM}{r^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{[4(6.37\times10^6\ \text{m})]^2}$$

$$= \boxed{0.614\ \text{m/s}^2,\ \text{toward Earth}}$$

P13.12 The gravitational field at the surface of the Earth or Moon is given by $g = \frac{GM}{R^2}$.

The expression for density is $\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3}$,

so $M = \frac{4}{3}\pi\rho R^3$

and $g = \frac{G\left(\frac{4}{3}\pi\rho R^3\right)}{R^2} = \frac{4}{3}G\pi\rho R$

Noting that this equation applies to both the Moon and the Earth, and dividing the two equations,

$$\frac{g_M}{g_E} = \frac{\frac{4}{3}G\pi\rho_M R_M}{\frac{4}{3}G\pi\rho_E R_E} = \frac{\rho_M R_M}{\rho_E R_E}$$

Substituting for the fractions,

$$\frac{1}{6}=\frac{\rho_M}{\rho_E}\left(\frac{1}{4}\right)\quad\text{and}\quad\frac{\rho_M}{\rho_E}=\frac{4}{6}=\boxed{\frac{2}{3}}$$

P13.13 (a) For the gravitational force on an object in the neighborhood of Miranda, we have

$$m_{\text{obj}}g=\frac{Gm_{\text{obj}}m_{\text{Miranda}}}{r_{\text{Miranda}}^2}$$

$$g=\frac{Gm_{\text{Miranda}}}{r_{\text{Miranda}}^2}=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.68\times10^{19}\ \text{kg}\right)}{\left(242\times10^3\ \text{m}\right)^2}$$

$$=\boxed{0.076\,1\ \text{m/s}^2}$$

(b) We ignore the difference (of about 4%) in *g* between the lip and the base of the cliff. For the vertical motion of the athlete, we have

$$y_f=y_i+v_{yi}+\frac{1}{2}a_yt^2$$

$$-5\ 000\ \text{m}=0+0+\frac{1}{2}\left(-0.076\ 1\ \text{m/s}^2\right)t^2$$

$$t=\left(\frac{2\left(5\ 000\ \text{m}\right)\text{s}^2}{0.076\ 1\ \text{m}}\right)^{1/2}=\boxed{363\ \text{s}}$$

(c) $x_f=x_i+v_{xi}t+\frac{1}{2}a_xt^2=0+\left(8.50\ \text{m/s}\right)\left(363\ \text{s}\right)+0=\boxed{3.08\times10^3\ \text{m}}$

We ignore the curvature of the surface (of about 0.7°) over the athlete's trajectory.

(d) $v_{xf}=v_{xi}=8.50\ \text{m/s}$

$$v_{yf}=v_{yi}+a_yt=0-\left(0.076\,1\ \text{m/s}^2\right)\left(363\ \text{s}\right)=-27.6\ \text{m/s}$$

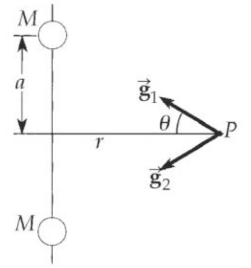
Thus $\vec{\mathbf{v}}_f=\left(8.50\hat{\mathbf{i}}-27.6\hat{\mathbf{j}}\right)\ \text{m/s}=\sqrt{8.50^2+27.6^2}\ \text{m/s}$ at

$\tan^{-1}\left(\frac{27.6\ \text{m/s}}{8.50\ \text{m/s}}\right)=72.9°$ below the *x* axis.

$$\boxed{\vec{\mathbf{v}}_f=28.9\ \text{m/s at }72.9°\text{ below the horizontal}}$$

Section 13.3 Analysis Model: Particle in a Field (Gravitational)

P13.14 (a) $$g_1 = g_2 = \frac{MG}{r^2 + a^2}$$



ANS. FIG. P13.14

$$g_{1y} = -g_{2y}$$

$$g_y = g_{1y} + g_{2y} = 0$$

$$g_{1x} = g_{2x} = g_2 \cos\theta$$

$$\cos\theta = \frac{r}{\left(a^2 + r^2\right)^{1/2}}$$

$$\vec{\mathbf{g}} = 2g_{2x}\left(-\hat{\mathbf{i}}\right)$$

or $$\vec{\mathbf{g}} = \boxed{\frac{2MGr}{\left(r^2 + a^2\right)^{3/2}} \text{ toward the center of mass}}$$

(b) $\boxed{\text{At } r = 0\text{, the fields of the two objects are equal in magnitude and opposite in direction, to add to zero.}}$

(c) $\boxed{\text{As } r \to 0,\ 2MGr(r^2 + a^2)^{-3/2} \text{ approaches } 2MG(0)/a^3 = 0.}$

(d) $\boxed{\text{When } r \text{ is much greater than } a\text{, the angles the field vectors make with the } x \text{ axis become smaller. At very great distances, the field vectors are almost parallel to the axis; therefore, they begin to look like the field vector from a single object of mass } 2M.}$

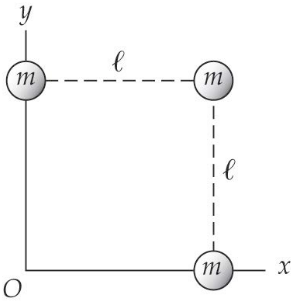
(e) $\boxed{\text{As } r \text{ becomes much larger than } a\text{, the expression approaches } 2MGr(r^2 + 0^2)^{-3/2} = 2MGr/r^3 = 2MG/r^2 \text{ as required.}}$

P13.15 The vector gravitational field at point *O* is given by

$$\vec{\mathbf{g}} = \frac{Gm}{l^2}\hat{\mathbf{i}} + \frac{Gm}{l^2}\hat{\mathbf{j}} + \frac{Gm}{2l^2}\left(\cos 45.0°\hat{\mathbf{i}} + \sin 45.0\hat{\mathbf{j}}\right)$$

so $$\vec{\mathbf{g}} = \frac{Gm}{l^2}\left(1 + \frac{1}{2\sqrt{2}}\right)\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) \text{ or}$$

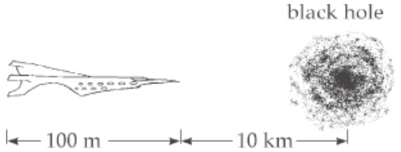
$$\boxed{\vec{\mathbf{g}} = \frac{Gm}{l^2}\left(\sqrt{2} + \frac{1}{2}\right)\text{toward the opposite corner.}}$$



ANS. FIG. P13.15

P13.16 (a) $$F = \frac{GMm}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[100\left(1.99\times10^{30}\ \text{kg}\right)\left(10^3\ \text{kg}\right)\right]}{\left(1.00\times10^4\ \text{m}+50.0\ \text{m}\right)^2}$$

$$= \boxed{1.31\times10^{17}\ \text{N}}$$



ANS. FIG. P13.16

(b) $$\Delta F = \frac{GMm}{r_{\text{front}}^2} - \frac{GMm}{r_{\text{back}}^2}$$

$$\Delta g = \frac{\Delta F}{m} = \frac{GM\left(r_{\text{back}}^2 - r_{\text{front}}^2\right)}{r_{\text{front}}^2 r_{\text{back}}^2}$$

$$\Delta g = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left[100\left(1.99\times10^{30}\ \text{kg}\right)\right]\left[\left(1.01\times10^4\ \text{m}\right)^2-\left(1.00\times10^4\ \text{m}\right)^2\right]}{\left(1.00\times10^4\ \text{m}\right)^2\left(1.01\times10^4\ \text{m}\right)^2}$$

$$\Delta g = \boxed{2.62\times10^{12}\ \text{N/kg}}$$

Section 13.4 Kepler's Laws and the Motion of Planets

P13.17 The gravitational force on mass located at distance *r* from the center of the Earth is $F_g = mg = GM_E m/r^2$. Thus, the acceleration of gravity at this location is $g = GM_E/r^2$. If $g = 9.00\ \text{m/s}^2$ at the location of the satellite, the radius of its orbit must be

$$r = \sqrt{\frac{GM_E}{g}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{9.00\ \text{m/s}^2}}$$

$$= 6.66\times10^6\ \text{m}$$

From Kepler's third law for Earth satellites, $T^2 = 4\pi^2 r^3 GM_E S$, the period is found to be

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}} = 2\pi\sqrt{\frac{(6.66\times 10^6\ \text{m})^3}{(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24}\ \text{kg})}}$$

$$= 5.41\times 10^3\ \text{s}$$

or

$$T = (5.41\times 10^3\ \text{s})\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{1.50\ \text{h} = 90.0\ \text{min}}$$

P13.18 The gravitational force exerted by Jupiter on Io causes the centripetal acceleration of Io. A force diagram of the satellite would show one downward arrow.

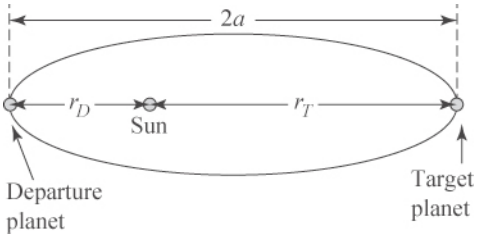
$$\sum F_{\text{on Io}} = M_{\text{Io}}a: \quad \frac{GM_J M_{\text{Io}}}{r^2} = \frac{M_{\text{Io}}v^2}{r} = \frac{M_{\text{Io}}}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r M_{\text{Io}}}{T^2}$$

Thus the mass of Io divides out and we have Kepler's third law with $m << M$,

$$M_J = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(4.22\times 10^8\text{m})^3}{(6.67\times 10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2)(1.77\ \text{d})^2}\left(\frac{1\ \text{d}}{86\,400\ \text{s}}\right)^2$$

and $M_J = \boxed{1.90\times 10^{27}\ \text{kg}}$ (approximately 316 Earth masses)

P13.19 (a) The desired path is an elliptical trajectory with the Sun at one of the foci, the departure planet at the perihelion, and the target planet at the aphelion. The perihelion distance r_D is the radius of the departure planet's orbit, while the aphelion distance r_T is the radius of the target planet's orbit. The semimajor axis of the desired trajectory is then $a = (r_D + r_T)/2$.



ANS. FIG. P13.19

If Earth is the departure planet, $r_D = 1.496\times 10^{11}\ \text{m} = 1.00\ \text{AU}$

With Mars as the target planet,

$$r_T = 2.28\times 10^{11}\text{ m}\left(\frac{1\text{ AU}}{1.496\times 10^{11}\text{ m}}\right) = 1.52\text{ AU}$$

Thus, the semimajor axis of the minimum energy trajectory is

$$a = \frac{r_D + r_T}{2} = \frac{1.00\text{ AU} + 1.52\text{ AU}}{2} = 1.26\text{ AU}$$

Kepler's third law, $T^2 = a^3$, then gives the time for a full trip around this path as

$$T = \sqrt{a^3} = \sqrt{(1.26\text{ AU})^3} = 1.41\text{ yr}$$

so the time for a one-way trip from Earth to Mars is

$$\Delta t = \frac{1}{2}T = \frac{1.41\text{ yr}}{2} = \boxed{0.71\text{ yr}}$$

(b) This trip cannot be taken at just any time. The departure must be timed so that the spacecraft arrives at the aphelion when the target planet is located there.

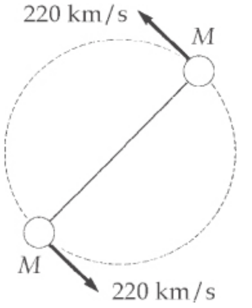
P13.20 (a) The particle does possess angular momentum, because it is not headed straight for the origin.

(b) Its angular momentum is constant. There are no identified outside influences acting on the object.

(c) Since speed is constant, the distance traveled between t_A and t_B is equal to the distance traveled between t_C and t_D. The area of a triangle is equal to one-half its (base) width across one side times its (height) dimension perpendicular to that side.

So $\frac{1}{2}bv_0(t_B - t_A) = \frac{1}{2}bv_0(t_D - t_C)$

states that the particle's radius vector sweeps out equal areas in equal times.



ANS. FIG. P13.21

P13.21 Applying Newton's second law, $\sum F = ma$ yields $F_g = ma_c$ for each star:

$$\frac{GMM}{(2r)^2} = \frac{Mv^2}{r} \quad \text{or} \quad M = \frac{4v^2 r}{G}$$

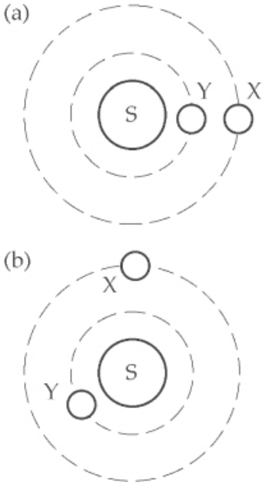
We can write *r* in terms of the period, *T*, by considering the time and distance of one complete cycle. The distance traveled in one orbit is the circumference of the stars' common orbit, so $2\pi r = vT$. Therefore,

$$M = \frac{4v^2 r}{G} = \frac{4v^2}{G}\left(\frac{vT}{2\pi}\right)$$

so,

$$M = \frac{2v^3 T}{\pi G} = \frac{2(220\times 10^3\ \text{m/s})^3(14.4\ \text{d})(86\,400\ \text{s/d})}{\pi(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}$$

$$= \boxed{1.26\times 10^{32}\ \text{kg} = 63.3\ \text{solar masses}}$$



ANS. FIG. P13.22

P13.22 To find the angular displacement of planet Y, we apply Newton's second law:

$$\sum F = ma: \quad \frac{Gm_{\text{planet}}M_{\text{star}}}{r^2} = \frac{m_{\text{planet}}v^2}{r}$$

Then, using $v = r\omega$,

$$\frac{GM_{\text{star}}}{r} = v^2 = r^2\omega^2$$

$$GM_{\text{star}} = r^3\omega^2 = r_x^3\omega_x^2 = r_y^3\omega_y^2$$

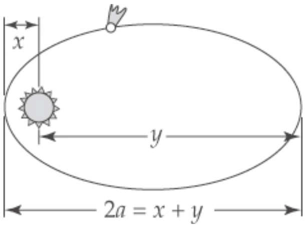
solving for the angular velocity of planet Y gives

$$\omega_y = \omega_x\left(\frac{r_x}{r_y}\right)^{3/2} = \left(\frac{90.0°}{5.00\,\text{yr}}\right)3^{3/2} = \frac{468°}{5.00\,\text{yr}}$$

So, given that there are 360° in one revolution we convert 468° to find that planet *Y* has turned through $\boxed{1.30\ \text{revolutions}}$.

P13.23 By Kepler's third law, $T^2 = ka^3$ (a = semimajor axis). For any object orbiting the Sun, with T in years and a in AU, k = 1.00. Therefore, for Comet Halley, and suppressing units,

$$(75.6)^2 = (1.00)\left(\frac{0.570 + y}{2}\right)^3$$



ANS. FIG. P13.23

The farthest distance the comet gets from the Sun is

$$y = 2(75.6)^{2/3} - 0.570 = \boxed{35.2 \text{ AU}}$$

(out around the orbit of Pluto).

***P13.24** By conservation of angular momentum for the satellite, $r_p v_p = r_a v_a$, or

$$\frac{v_p}{v_a} = \frac{r_a}{r_p} = \frac{2\,289 \text{ km} + 6.37 \times 10^3 \text{ km}}{459 \text{ km} + 6.37 \times 10^3 \text{ km}} = \frac{8\,659 \text{ km}}{6\,829 \text{ km}} = \boxed{1.27}$$

We do not need to know the period.

P13.25 For an object in orbit about Earth, Kepler's third law gives the relation between the orbital period T and the average radius of the orbit ("semimajor axis") as

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right) r^3$$

We assume that the two given distances in the problem statements are the perigee and apogee, respectively.

Thus, if the average radius is

$$r = \frac{r_{\text{min}} + r_{\text{max}}}{2} = \frac{6\,670 \text{ km} + 385\,000 \text{ km}}{2}$$
$$= 1.96 \times 10^5 \text{ km} = 1.96 \times 10^8 \text{ m}$$

The period (time for a round trip from Earth to the Moon) would be

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}}$$
$$= 2\pi\sqrt{\frac{(1.96 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}}$$
$$= 8.63 \times 10^5 \text{ s}$$

The time for a one-way trip from Earth to the Moon is then

$$\Delta t = \frac{1}{2}T = \left(\frac{8.63\times10^{5}\text{ s}}{2}\right)\left(\frac{1\text{ day}}{8.64\times10^{4}\text{ s}}\right) = \boxed{4.99\text{ d}}$$

P13.26 The gravitational force on a small parcel of material at the star's equator supplies the necessary centripetal acceleration:

$$\frac{GM_sm}{R_s^2} = \frac{mv^2}{R_s} = mR_s\omega^2$$

so

$$\omega = \sqrt{\frac{GM_s}{R_s^3}} = \sqrt{\frac{\left(6.67\times10^{-11}\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[2\left(1.99\times10^{30}\text{ kg}\right)\right]}{\left(10.0\times10^{3}\text{ m}\right)^3}}$$

$$\omega = \boxed{1.63\times10^{4}\text{ rad/s}}$$

P13.27 We find the satellite's altitude from

$$\frac{GM_J}{\left(R_J + d\right)^2} = \frac{4\pi^2\left(R_J + d\right)}{T^2}$$

where *d* is the altitude of the satellite above Jupiter's cloud tops. Then,

$$GM_JT^2 = 4\pi^2\left(R_J + d\right)^3$$

$$\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.90\times10^{27}\text{ kg}\right)\left(9.84\times3\,600\right)^2 = 4\pi^2\left(6.99\times10^{7} + d\right)^3$$

which gives

$$d = \boxed{8.92\times10^{7}\text{ m}} = \boxed{89\ 200\text{ km}}\text{ above the planet}$$

P13.28 (a) In $T^2 = 4\pi^2a^3/GM_{\text{central}}$ we take $a = 3.84 \times 10^8$ m.

$$M_{\text{central}} = \frac{4\pi^2a^3}{GT^2}$$

$$= \frac{4\pi^2(3.84\times10^{8}\text{ m})^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(27.3\times86\ 400\text{ s})^2}$$

$$= \boxed{6.02\times10^{24}\text{ kg}}$$

This is a little larger than 5.98×10^{24} kg.

(b) $\boxed{\text{The Earth wobbles a bit as the Moon orbits it, so both objects move nearly in circles about their center of mass, staying on opposite sides of it. The radius of the Moon's orbit is therefore a bit less than the Earth-Moon distance.}}$

P13.29 The speed of a planet in a circular orbit is given by

$$\sum F = ma\text{:}\quad \frac{GM_{\text{sun}}m}{r^2} = \frac{mv^2}{r} \quad \rightarrow \quad v = \sqrt{\frac{GM_{\text{sun}}}{r}}$$

(a) For Mercury, the speed is

$$v_M = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{5.79\times10^{10}\ \text{m}}}$$
$$= 4.79\times10^4\ \text{m/s}$$

and for Pluto,

$$v_p = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{5.91\times10^{12}\ \text{m}}}$$
$$= 4.74\times10^3\ \text{m/s}$$

With greater speed, $\boxed{\text{Mercury will eventually move farther from the Sun than Pluto.}}$

(b) With original distances r_P and r_M perpendicular to their lines of motion, they will be equally far from the Sun at time t, where

$$\sqrt{r_P^2 + v_P^2t^2} = \sqrt{r_M^2 + v_M^2t^2}$$
$$r_P^2 - r_M^2 = \left(v_M^2 - v_P^2\right)t^2$$
$$t = \sqrt{\frac{\left(5.91\times10^{12}\ \text{m}\right)^2 - \left(5.79\times10^{10}\ \text{m}\right)^2}{\left(4.79\times10^4\ \text{m/s}\right)^2 - \left(4.74\times10^3\ \text{m/s}\right)^2}}$$
$$= \sqrt{\frac{3.49\times10^{25}\ \text{m}^2}{2.27\times10^9\ \text{m}^2/\text{s}^2}} = 1.24\times10^8\ \text{s} = \boxed{3.93\ \text{yr}}$$

Section 13.5 Gravitational Potential Energy

P13.30 (a) We compute the gravitational potential energy of the satellite-Earth system from

$$\begin{aligned} U &= -\frac{GM_E m}{r} \\ &= -\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)\left(100\ \mathrm{kg}\right)}{(6.37+2.00)\times10^{6}\ \mathrm{m}} \\ &= \boxed{-4.77\times10^{9}\ \mathrm{J}} \end{aligned}$$

(b), (c) The satellite and Earth exert forces of equal magnitude on each other, directed $\boxed{\text{downward on the satellite and upward on Earth.}}$

The magnitude of this force is

$$\begin{aligned} F &= \frac{GM_E m}{r^2} \\ &= \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)\left(100\ \mathrm{kg}\right)}{\left(8.37\times10^{6}\ \mathrm{m}\right)^2} \\ &= \boxed{569\ \mathrm{N}} \end{aligned}$$

P13.31 The work done by the Moon's gravitational field is equal to the negative of the change of potential energy of the meteor-Moon system:

$$W_{\text{int}} = -\Delta U = -\left(\frac{-Gm_1m_2}{r} - 0\right)$$

$$W_{\text{int}} = \frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(7.36\times10^{22}\ \mathrm{kg})(1.00\times10^{3}\ \mathrm{kg})}{1.74\times10^{6}\ \mathrm{m}}$$

$$= \boxed{2.82\times10^{9}\ \mathrm{J}}$$

***P13.32** The enery required is equal to the change in gravitational potential energy of the object-Earth system:

$$U = -G\frac{Mm}{r} \text{ and } g = \frac{GM_E}{R_E^2} \text{ so that}$$

$$\Delta U = -GMm\left(\frac{1}{3R_E} - \frac{1}{R_E}\right) = \frac{2}{3}mgR_E$$

$$\Delta U = \frac{2}{3}(1\ 000\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)\left(6.37\times10^{6}\ \mathrm{m}\right) = \boxed{4.17\times10^{10}\ \mathrm{J}}$$

P13.33 (a) The definition of density gives

$$\rho = \frac{M_S}{\frac{4}{3}\pi r_E^2} = \frac{3\left(1.99\times 10^{30}\ \text{kg}\right)}{4\pi\left(6.37\times 10^{6}\ \text{m}\right)^3} = \boxed{1.84\times 10^{9}\ \text{kg/m}^3}$$

(b) For an object of mass m on its surface, $mg = GM_S m/R_E^2$. Thus,

$$g = \frac{GM_S}{r_E^2} = \frac{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times 10^{30}\ \text{kg}\right)}{\left(6.37\times 10^{6}\ \text{m}\right)^2}$$

$$= \boxed{3.27\times 10^{6}\ \text{m/s}^2}$$

(c) Relative to $U_g = 0$ at infinity, the potential energy of the object-star system at the surface of the white dwarf is

$$U_g = -\frac{GM_S m}{r_E}$$

$$= -\frac{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times 10^{30}\ \text{kg}\right)\left(1.00\ \text{kg}\right)}{6.37\times 10^{6}\ \text{m}}$$

$$= \boxed{-2.08\times 10^{13}\ \text{J}}$$

P13.34 (a) Energy conservation of the object-Earth system from release to radius r:

$$\left(K + U_g\right)_{\text{altitude } h} = \left(K + U_g\right)_{\text{radius } r}$$

$$0 - \frac{GM_E m}{R_E + h} = \frac{1}{2}mv^2 - \frac{GM_E m}{r}$$

$$v = \left[2GM_E\left(\frac{1}{r} - \frac{1}{R_E + h}\right)\right]^{1/2} = -\frac{dr}{dt}$$

(b) $\int_i^f dt = \int_i^f -\frac{dr}{v} = \int_f^i \frac{dr}{v}$. The time of fall is, suppressing units,

$$\Delta t = \int_{R_E}^{R_E+h}\left[2GM_E\left(\frac{1}{r} - \frac{1}{R_E + h}\right)\right]^{-1/2} dr$$

$$\Delta t = \left(2\times 6.67\times 10^{-11}\times 5.98\times 10^{24}\right)^{-1/2}$$

$$\times \int_{6.37\times 10^{6}\ \text{m}}^{6.87\times 10^{6}\ \text{m}}\left[\left(\frac{1}{r} - \frac{1}{6.87\times 10^{6}\ \text{m}}\right)\right]^{-1/2} dr$$

We can enter this expression directly into a mathematical calculation program.

Alternatively, to save typing we can change variables to $u = \dfrac{r}{10^6}$.

Then

$$\Delta t = \left(7.977\times10^{14}\right)^{-1/2}\int_{6.37}^{6.87}\left(\frac{1}{10^6 u}-\frac{1}{6.87\times10^6}\right)^{-1/2}10^6\,du$$

$$= 3.541\times10^{-8}\frac{10^6}{\left(10^6\right)^{-1/2}}\int_{6.37}^{6.87}\left(\frac{1}{u}-\frac{1}{6.87}\right)^{-1/2}du$$

A mathematics program returns the value 9.596 for this integral, giving for the time of fall

$$\Delta t = 3.541\times10^{-8}\times10^{9}\times9.596 = 339.8 = \boxed{340\ \text{s}}$$

P13.35 (a) Since the particles are located at the corners of an equilateral triangle, the distances between all particle pairs is equal to 0.300 m. The gravitational potential energy of the system is then

$$U_{\text{Tot}} = U_{12}+U_{13}+U_{23} = 3U_{12} = 3\left(-\frac{Gm_1m_2}{r_{12}}\right)$$

$$U_{\text{Tot}} = -\frac{3\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.00\times10^{-3}\ \text{kg}\right)^2}{0.300\ \text{m}}$$

$$= \boxed{-1.67\times10^{-14}\ \text{J}}$$

(b) Each particle feels a net force of attraction toward the midpoint between the other two. Each moves toward the center of the triangle with the same acceleration. They collide simultaneously at the center of the triangle.

Section 13.6 Energy Considerations in Planetary and Satellite Motion

P13.36 We use the isolated system model for energy:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + GM_Em\left(\frac{1}{r_f} - \frac{1}{r_i}\right) = \frac{1}{2}mv_f^2$$

which becomes

$$\frac{1}{2}v_i^2 + GM_E\left(0 - \frac{1}{R_E}\right) = \frac{1}{2}v_f^2$$

or $$v_f^2 = v_1^2 - \frac{2GM_E}{R_E}$$

and $$v_f = \left(v_1^2 - \frac{2GM_E}{R_E}\right)^{1/2}$$

$$v_f = \left\{\left(2.00\times10^4\ \text{m/s}\right)^2 - \left[\frac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{6.37\times10^6\ \text{m}}\right]\right\}^{1/2}$$

$$= \boxed{1.66\times10^4\ \text{m/s}}$$

***P13.37** To determine the energy transformed to internal energy, we begin by calculating the change in kinetic energy of the satellite. To find the initial kinetic energy, we use

$$\frac{v_i^2}{R_E + h} = \frac{GM_E}{\left(R_E + h\right)^2}$$

which gives

$$\begin{aligned}K_i &= \frac{1}{2}mv_i^2 = \frac{1}{2}\left(\frac{GM_Em}{R_E + h}\right)\\ &= \frac{1}{2}\left[\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(500\ \text{kg}\right)}{6.37\times10^6\ \text{m} + 0.500\times10^6\ \text{m}}\right]\\ &= 1.45\times10^{10}\ \text{J}\end{aligned}$$

Also, $$K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}\left(500\ \text{kg}\right)\left(2.00\times10^3\ \text{m/s}\right)^2 = 1.00\times10^9\ \text{J}.$$

The change in gravitational potential energy of the satellite-Earth system is

$$\begin{aligned}\Delta U &= \frac{GM_Em}{R_i} - \frac{GM_Em}{R_f} = GM_Em\left(\frac{1}{R_i} - \frac{1}{R_f}\right)\\ &= \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(500\ \text{kg}\right)\\ &\qquad\times\left(-1.14\times10^{-8}\ \text{m}^{-1}\right)\\ &= -2.27\times10^{9}\ \text{J}\end{aligned}$$

The energy transformed into internal energy due to friction is then

$$\begin{aligned}\Delta E_{\text{int}} &= K_i - K_f - \Delta U = (14.5 - 1.00 + 2.27)\times10^{9}\ \text{J}\\ &= \boxed{1.58\times10^{10}\ \text{J}}\end{aligned}$$

P13.38 To obtain the orbital velocity, we use

$$\sum F = \frac{mMG}{R^2} = \frac{mv^2}{R}$$

or $$v = \sqrt{\frac{MG}{R}}$$

We can obtain the escape velocity from

$$\frac{1}{2}mv_{\text{esc}}^2 = \frac{mMG}{R}$$

or $$v_{\text{esc}} = \sqrt{\frac{2MG}{R}} = \boxed{\sqrt{2}v}$$

P13.39 (a) The total energy of the satellite-Earth system at a given orbital altitude is given by

$$E_{\text{tot}} = -\frac{GMm}{2r}$$

The energy needed to increase the satellite's orbit is then, suppressing units,

$$\begin{aligned}\Delta E &= \frac{GMm}{2}\left(\frac{1}{r_i} - \frac{1}{r_f}\right)\\ &= \frac{\left(6.67\times10^{-11}\right)\left(5.98\times10^{24}\right)}{2}\frac{10^3\ \text{kg}}{10^3\ \text{m}}\left(\frac{1}{6\,370+100} - \frac{1}{6\,370+200}\right)\end{aligned}$$

$$\Delta E = 4.69\times10^{8}\ \text{J} = \boxed{469\ \text{MJ}}$$

(b) $\boxed{\text{Both in the original orbit and in the final orbit, the total energy is negative, with an absolute value equal to the positive kinetic energy. The potential energy is negative and twice as large as the total energy. As the satellite is lifted from the lower to the higher orbit, the gravitational energy increases, the kinetic energy decreases, and the total energy increases. The value of each becomes closer to zero. Numerically, the gravitational energy increases by 938 MJ, the kinetic energy decreases by 469 MJ, and the total energy increases by 469 MJ.}}$

P13.40 (a) The major axis of the orbit is $2a = 50.5$ AU so $a = 25.25$ AU.

Further, in the textbook's diagram of an ellipse, $a + c = 50$ AU, so $c = 24.75$ AU. Then

$$e = \frac{c}{a} = \frac{24.75}{25.25} = \boxed{0.980}$$

(b) In $T^2 = K_s a^3$ for objects in solar orbit, the Earth gives us

$$(1\text{ yr})^2 = K_s(1\text{ AU})^3 \quad K_s = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}$$

Then

$$T^2 = \frac{(1\text{ yr})^2}{(1\text{ AU})^3}(25.25\text{ AU})^3 \quad \rightarrow T = \boxed{127\text{ yr}}$$

(c)
$$U = -\frac{GMm}{r}$$
$$= -\frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\text{ kg})(1.20\times10^{10}\text{ kg})}{50(1.496\times10^{11}\text{ m})}$$
$$= \boxed{-2.13\times10^{17}\text{ J}}$$

***P13.41** For her jump on Earth,

$$\frac{1}{2}mv_i^2 = mgy_f \qquad \textbf{[1]}$$

which gives

$$v_i = \sqrt{2gy_f} = \sqrt{2(9.80\text{ m/s})(0.500\text{ m})} = 3.13\text{ m/s}$$

We assume that she has the same takeoff speed on the asteroid. Here

$$\frac{1}{2}mv_i^2 - \frac{GM_Am}{R_A} = 0 + 0 \qquad \textbf{[2]}$$

The equality of densities between planet and asteroid,

$$\rho = \frac{M_E}{\frac{4}{3}\pi R_E^3} = \frac{M_A}{\frac{4}{3}\pi R_A^3}$$

implies

$$M_A = \left(\frac{R_A}{R_E}\right)^3 M_E \qquad \textbf{[3]}$$

Note also at Earth's surface

$$g = \frac{GM_E}{R_E^2} \qquad \textbf{[4]}$$

Combining the equations [2], [1], [3], and [4] by substitution gives

$$\frac{1}{2}v_i^2 = \frac{GM_A}{R_A}$$

$$\frac{GM_E}{R_E^2}y_f = \frac{GM_ER_A^2}{R_E^3}$$

$$R_A^2 = y_fR_E = (0.500\text{ m})(6.37\times10^6\text{ m})$$

$$R_A = \boxed{1.78\times10^3\text{ m}}$$

***P13.42** For a satellite in an orbit of radius *r* around the Earth, the total energy of the satellite-Earth system is $E = -\frac{GM_E}{2r}$. Thus, in changing from a circular orbit of radius $r = 2R_E$ to one of radius $r = 3R_E$, the required work is

$$W = \Delta E = -\frac{GM_Em}{2r_f} + \frac{GM_Em}{2r_i} = GM_Em\left[\frac{1}{4R_E} - \frac{1}{6R_E}\right] = \boxed{\frac{GM_Em}{12R_E}}$$

***P13.43** (a) The work must provide the increase in gravitational energy:

$$W = \Delta U_g = U_{gf} - U_{gi}$$

$$= -\frac{GM_E M_p}{r_f} + \frac{GM_E M_p}{r_i}$$

$$= -\frac{GM_E M_p}{R_E + y} + \frac{GM_E M_p}{R_E}$$

$$= GM_E M_p\left(\frac{1}{R_E} - \frac{1}{R_E + y}\right)$$

$$= \left(\frac{6.67\times10^{-11}\ \mathrm{N\cdot m^2}}{\mathrm{kg^2}}\right)(5.98\times10^{24}\ \mathrm{kg})(100\ \mathrm{kg})$$

$$\times\left(\frac{1}{6.37\times10^{6}\ \mathrm{m}} - \frac{1}{7.37\times10^{6}\ \mathrm{m}}\right)$$

$$W = \boxed{850\ \mathrm{MJ}}$$

(b) In a circular orbit, gravity supplies the centripetal force:

$$\frac{GM_E M_p}{(R_E + y)^2} = \frac{M_p v^2}{R_E + y}$$

Then,

$$\frac{1}{2}M_p v^2 = \frac{1}{2}\frac{GM_E M_p}{(R_E + y)}$$

So, additional work = kinetic energy required is

$$\Delta W = \frac{1}{2}\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\ \mathrm{kg})(100\ \mathrm{kg})}{7.37\times10^{6}\ \mathrm{m}}$$

$$= \boxed{2.71\times10^{9}\ \mathrm{J}}$$

P13.44 (a) The escape velocity from the solar system, starting at Earth's orbit, is given by

$$v_{\text{solar escape}} = \sqrt{\frac{2M_{\text{Sun}}G}{R_{\text{Sun}}}}$$

$$= \sqrt{\frac{2(1.99\times10^{30}\ \mathrm{kg})(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})}{1.50\times10^{9}\ \mathrm{m}}}$$

$$= \boxed{42.1\ \mathrm{km/s}}$$

(b) Let *x* represent the variable distance from the Sun. Then,

$$v = \sqrt{\frac{2M_{\text{Sun}}G}{x}} \quad \rightarrow \quad x = \frac{v^2}{2M_{\text{Sun}}G}$$

If $v = \dfrac{125\,000 \text{ km}}{3\,600 \text{ s}} = 34.7 \text{ m/s}$, then

$$x = \frac{v^2}{2M_{\text{Sun}}G} = \frac{(34.7 \text{ m/s})^2}{2(1.99\times 10^{30} \text{ kg})(6.67\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)}$$
$$= \boxed{2.20\times 10^{11} \text{ m}}$$

Note that at or beyond the orbit of Mars, 125 000 km/h is sufficient for escape.

P13.45 $F_c = F_G$ gives $\dfrac{mv^2}{r} = \dfrac{GmM_E}{r^2}$

which reduces to $v = \sqrt{\dfrac{GM_E}{r}}$

and period $= \dfrac{2\pi r}{v} = 2\pi r\sqrt{\dfrac{r}{GM_E}}$.

(a) $r = R_E + 200 \text{ km} = 6\,370 \text{ km} + 200 \text{ km} = 6\,570 \text{ km}$

Thus,

$$\text{period} = 2\pi(6.57\times 10^6 \text{ m}) \times \sqrt{\frac{6.57\times 10^6 \text{ m}}{(6.67\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24} \text{ kg})}}$$

$$T = 5.30\times 10^3 \text{ s} = 88.3 \text{ min} = \boxed{1.47 \text{ h}}$$

(b) $$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{(6.67\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times 10^{24} \text{ kg})}{6.57\times 10^6 \text{ m}}} = \boxed{7.79 \text{ km/s}}$$

(c) $K_f + U_f = K_i + U_i +$ energy input gives

$$\text{input} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + \left(\frac{-GM_Em}{r_f}\right) - \left(\frac{-GM_Em}{r_i}\right) \qquad [1]$$

$$r_i = R_E = 6.37 \times 10^6 \text{ m}$$

$$v_i = \frac{2\pi R_E}{86\,400 \text{ s}} = 4.63 \times 10^2 \text{ m/s}$$

Substituting the appropriate values into [1] yields:

$$\text{minimum energy input } = \boxed{6.43 \times 10^9 \text{ J}}$$

P13.46 The gravitational force supplies the needed centripetal acceleration.

Thus, $\dfrac{GM_E m}{(R_E + h)^2} = \dfrac{mv^2}{R_E + h}$ or $v^2 = \dfrac{GM_E}{R_E + h}$

(a) $$T = \frac{2\pi r}{v} = \frac{2\pi(R_E + h)}{\sqrt{\frac{GM_E}{(R_E + h)}}} = \boxed{2\pi\sqrt{\frac{(R_E + h)^3}{GM_E}}}$$

(b) $$v = \boxed{\sqrt{\frac{GM_E}{R_E + h}}}$$

(c) Minimum energy input is

$$\Delta E_{\min} = (K_f + U_{gf}) - (K_i - U_{gi})$$

This choice has the object starting with energy

$$K_i = \frac{1}{2}mv_i^2$$

with $v_i = \dfrac{2\pi R_E}{1.00 \text{ day}} = \dfrac{2\pi R_E}{86\,400 \text{ s}}$ and $U_{gi} = -\dfrac{GM_E m}{R_E}$.

Thus,

$$\Delta E_{\min} = \frac{1}{2}m\left(\frac{GM_E}{R_E + h}\right) - \frac{GM_E m}{R_E + h} - \frac{1}{2}m\left[\frac{4\pi^2 R_E^2}{(86\,400 \text{ s})^2}\right] + \frac{GM_E m}{R_E}$$

or $$\Delta E_{\min} = \boxed{GM_E m\left[\frac{R_E + 2h}{2R_E(R_E + h)}\right] - \frac{2\pi^2 R_E^2 m}{(86\,400 \text{ s})^2}}.$$

P13.47 (a) $\boxed{\text{Gravitational screening does not exist. The presence of the satellite has no effect on the force the planet exerts on the rocket.}}$

(b) The rocket has a gravitational potential energy with respect to Ganymede

$$U_1 = -\frac{Gm_1m_2}{r} = -\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2}\right)m_2\left(1.495\times10^{23}\ \mathrm{kg}\right)}{\left(2.64\times10^{6}\ \mathrm{m}\right)\mathrm{kg}^2}$$

$$U_1 = \left(-3.78\times10^{6}\ \mathrm{m^2/s^2}\right)m_2$$

The rocket's gravitational potential energy with respect to Jupiter at the distance of Ganymede is

$$U_2 = -\frac{Gm_1m_2}{r} = -\frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2}\right)m_2\left(1.90\times10^{27}\ \mathrm{kg}\right)}{\left(1.071\times10^{9}\ \mathrm{m}\right)\mathrm{kg}^2}$$

$$U_2 = \left(-1.18\times10^{8}\ \mathrm{m^2/s^2}\right)m_2$$

To escape from both requires

$$\frac{1}{2}m_2v_{\mathrm{esc}}^2 = +\left[\left(3.78\times10^{6} + 1.18\times10^{8}\right)\ \mathrm{m^2/s^2}\right]m_2$$

$$v_{\mathrm{esc}} = \sqrt{2\times\left(1.22\times10^{8}\ \mathrm{m^2/s^2}\right)} = \boxed{15.6\ \mathrm{km/s}}$$

P13.48 (a) For the satellite $\sum F = ma$; $\dfrac{GmM_E}{r^2} = \dfrac{mv_i^2}{r}$ gives

$$\boxed{v_i = \left(\frac{GM_E}{r}\right)^{1/2}}$$

(b) Conservation of momentum in the forward direction for the exploding satellite gives:

$$\left(\sum mv\right)_i = \left(\sum mv\right)_f$$

$$5mv_i = 4mv + m0$$

$$v = \frac{5}{4}v_i = \boxed{\frac{5}{4}\left(\frac{GM_E}{r}\right)^{1/2}}$$

(c) With velocity perpendicular to radius, the orbiting fragment is at perigee. Its apogee distance and speed are related to r and v by $4mrv = 4mr_fv_f$ and

$$\frac{1}{2}4mv^2 - \frac{GM_E4m}{r} = \frac{1}{2}4mv_f^2 - \frac{GM_E4m}{r_f}$$

Substituting $v_f = \frac{vr}{r_f}$ we have

$$\frac{1}{2}v^2 - \frac{GM_E}{r} = \frac{1}{2}\frac{v^2 r^2}{r_f^2} - \frac{GM_E}{r_f}$$

Further, substituting $v^2 = \frac{25}{16}\frac{GM_E}{r}$ gives

$$\frac{25}{32}\frac{GM_E}{r} - \frac{GM_E}{r} = \frac{25}{32}\frac{GM_E r}{r_f^2} - \frac{GM_E}{r_f}$$

$$\frac{-7}{32r} = \frac{25r}{32r_f^2} - \frac{1}{r_f}$$

Clearing fractions we have $-7r_f^2 = 25r^2 - 32rr_f$, or

$$7\left(\frac{r_f}{r}\right)^2 - 32\left(\frac{r_f}{r}\right) + 25 = 0$$

giving $\frac{r_f}{r} = \frac{+32 \pm \sqrt{32^2 - 4(7)(25)}}{14} = \frac{50}{14}$ or $\frac{14}{14}$.

The latter root describes the starting point.

The outer end of the orbit has $\frac{r_f}{r} = \frac{25}{7}$: $\boxed{r_f = \frac{25r}{7}}$

***P13.49** The height attained is not small compared to the radius of the Earth, so *U* = *mgy* does not apply; $U = -\frac{GM_1 M_2}{r}$ does. From launch to apogee at height *h*, conservation of energy gives

$$K_i + U_i + \Delta E_{\text{mech}} = K_f + U_f$$

$$\frac{1}{2}M_p v_i^2 - \frac{GM_E M_p}{R_E} + 0 = 0 - \frac{GM_E M_p}{R_E + h}$$

The mass of the projectile cancels out, giving

$$\frac{1}{2}v_i^2 - \frac{GM_E}{R_E} = \frac{GM_E}{R_E + h}$$

$$R_E + h = \frac{GM_E}{\frac{1}{2}v_i^2 - \frac{GM_E}{R_E}}$$

$$h = \frac{GM_E}{\frac{1}{2}v_i^2 - \frac{GM_E}{R_E}} - R_E$$

$$= \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\frac{1}{2}\left(10.0\times10^{3}\ \mathrm{m/s}\right)^2 - \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(6.37\times10^{6}\ \mathrm{m}\right)}} - 6.37\times10^{6}\ \mathrm{m}$$

$$= \boxed{2.52\times10^{7}\ \mathrm{m}}$$

Additional Problems

P13.50 (a) When the rocket engine shuts off at an altitude of 250 km, we may consider the rocket to be beyond Earth's atmosphere. Then, its mechanical energy will remain constant from that instant until it comes to rest momentarily at the maximum altitude. That is, $KE_f + PE_f = KE_i + PE_i$, or

$$0 - \frac{GM_E \cancel{m}}{r_{\max}} = \frac{1}{2}\cancel{m}v_i^2 - \frac{GM_E \cancel{m}}{r_i} \qquad \text{or} \qquad \frac{1}{r_{\max}} = -\frac{v_i^2}{2GM_E} + \frac{1}{r_i}$$

With $r_i = R_E + 250\ \mathrm{km} = 6.37\times10^6\ \mathrm{m} + 250\times10^3\ \mathrm{m} = 6.62\times10^6\ \mathrm{m}$ and $v_i = 6.00\ \mathrm{km/s} = 6.00\times10^3\ \mathrm{m/s}$, this gives

$$\frac{1}{r_{\max}} = -\frac{\left(6.00\times10^{3}\ \mathrm{m/s}\right)^2}{2\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)} + \frac{1}{6.62\times10^{6}\ \mathrm{m}}$$
$$= 1.06\times10^{-7}\ \mathrm{m^{-1}}$$

or $r_{\max} = 9.44\times10^6$ m. The maximum distance from Earth's surface is then

$$h_{max} = r_{max} - R_E = 9.44\times10^6\ \mathrm{m} - 6.37\times10^6\ \mathrm{m} = \boxed{3.07\times10^6\mathrm{m}}$$

(b) If the rocket were fired from a launch site on the equator, it would have a significant eastward component of velocity because of the Earth's rotation about its axis. Hence, compared to being fired from the South Pole, the rocket's initial speed would be greater, and $\boxed{\text{the rocket would travel farther from Earth}}$.

P13.51 For a 6.00-km diameter cylinder, $r = 3\,000$ m and to simulate $1g = 9.80\ \text{m/s}^2$,

$$g = \frac{v^2}{r} = \omega^2 r$$

$$\omega = \sqrt{\frac{g}{r}} = \boxed{0.057\,2\ \text{rad/s}}$$

The required rotation rate of the cylinder is $\boxed{\dfrac{1\ \text{rev}}{110\ \text{s}}}$.

(For a description of proposed cities in space, see Gerard K. O'Neill in *Physics Today,* Sept. 1974. and the Wikipedia article on "Rotating Wheel Space Station at http://en.wikipedia.org/wiki/Rotating_wheel_space_station)

***P13.52** To approximate the height of the sulfur, set $\dfrac{mv^2}{2} = mg_{\text{Io}}h$, with $h = 70\,000$ m and $g_{\text{Io}} = \dfrac{GM}{r^2} = 1.79\ \text{m/s}^2$. This gives

$$v = \sqrt{2g_{\text{Io}}h} = \sqrt{2\left(1.79\ \text{m/s}^2\right)\left(70\,000\ \text{m}\right)}$$
$$\approx 500\ \text{m/s}\ \left(\text{over } 1\,000\ \text{mi/h}\right)$$

We can obtain a more precise answer from conservation of energy:

$$\frac{1}{2}mv^2 - \frac{GMm}{r_1} = -\frac{GMm}{r_2}$$

$$\frac{1}{2}v^2 = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(8.90\times10^{22}\ \text{kg}\right) \times\left(\frac{1}{1.82\times10^6\ \text{m}} - \frac{1}{1.89\times10^6\ \text{m}}\right)$$

$$v = \boxed{492\ \text{m/s}}$$

***P13.53** (a) The radius of the satellite's orbit is

$$r = R_E + h = 6.37\times10^6\ \text{m} + 2.80\times10^6\ \text{m} = 9.17\times10^6\ \text{m}$$

Then, modifying Kepler's third law for orbital motion about the Earth rather than the Sun, we have

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 = \frac{4\pi^2\left(9.17\times10^6\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}$$
$$= 7.63\times10^7\ \text{s}^2$$

or $$T = (8.74\times10^3\ \text{s})\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) = \boxed{2.43\ \text{h}}$$

(b) The constant tangential speed of the satellite is

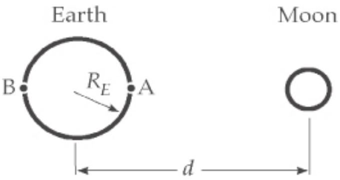
$$v = \frac{2\pi r}{T} = \frac{2\pi(9.17\times10^6\ \text{m})}{8.74\times10^3\ \text{s}} = 6.60\times10^3\ \text{m/s} = \boxed{6.60\ \text{km/s}}$$

(c) The satellite's only acceleration is centripetal acceleration, so

$$a = a_c = \frac{v^2}{r} = \frac{(6.60\times10^3\ \text{m/s})^2}{9.17\times10^6\ \text{m}} = \boxed{4.74\ \text{m/s}^2\ \text{toward the Earth}}$$

P13.54 If one uses the result $v = \sqrt{\frac{GM}{r}}$ and the relation $v = (2\pi r/T)$, one finds the radius of the orbit to be smaller than the radius of the Earth, so the spacecraft would need to be in orbit underground.

P13.55 The acceleration of an object at the center of the Earth due to the gravitational force of the Moon is given by $a = G\frac{M_M}{d^2}$.



ANS. FIG. P13.55

At the point A nearest the Moon,

$$a_+ = G\frac{M_M}{(d-R_E)^2}$$

At the point B farthest from the Moon,

$$a_- = G\frac{M_M}{(d+R_E)^2}$$

From the above, we have

$$\frac{\Delta g_M}{g} = \frac{(a_+ - a_-)}{g} = \frac{GM_M}{g}\left[\frac{1}{(d-R_E)^2} - \frac{1}{(d+R_E)^2}\right]$$

Evaluating this expression, we find across the planet

$$\frac{\Delta g_M}{g} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(7.36\times10^{22}\ \text{kg})}{9.80\ \text{m/s}^2}$$
$$\times\left[\frac{1}{(3.84\times10^8\ \text{m}-6.37\times10^6\ \text{m})^2} - \frac{1}{(3.84\times10^8\ \text{m}+6.37\times10^6\ \text{m})^2}\right]$$
$$= \boxed{2.25\times10^{-7}}$$

P13.56 (a) The only force acting on the astronaut is the normal force exerted on him by the "floor" of the cabin. The normal force supplies the centripetal force:

$$F_c = \frac{mv^2}{r} \qquad \text{and} \qquad n = \frac{mg}{2}$$

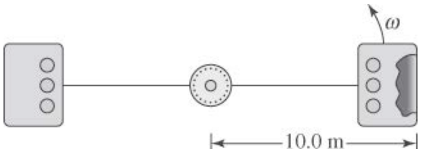
This gives

$$\frac{mv^2}{r} = \frac{mg}{2} \rightarrow v = \sqrt{\frac{gr}{2}}$$

$$v = \sqrt{\frac{(9.80\,\mathrm{m/s^2})(10.0\,\mathrm{m})}{2}} \quad \rightarrow v = 7.00\ \mathrm{m/s}$$

Since $v = r\omega$, we have

$$\omega = \frac{v}{r} = \frac{7.00\ \mathrm{m/s}}{10.0\ \mathrm{m}} = \boxed{0.700\ \mathrm{rad/s}}$$



ANS. FIG. P13.56

(b) Because his feet stay in place on the floor, his head will be moving at the same tangential speed as his feet. However, his feet and his head are travelling in circles of different radii.

(c) If he stands up without holding on to anything with his hands, the only force on his body is radial. Because the wall of the cabin near the traveler's head moves in a smaller circle, it moves at a slower tangential speed than that of the traveler's head so his head moves toward the wall—if he is not careful, there could be a collision. This is an example of the Coriolis force investigated in Section 6.3. Holding onto to a rigid support with his hands will provide a tangential force to the traveler to slow the upper part of his body down.

P13.57 (a) Ignoring air resistance, the energy conservation for the object-Earth system from firing to apex is given by,

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f$$

$$\frac{1}{2}mv_i^2-\frac{GmM_E}{R_E}=0-\frac{GmM_E}{R_E+h}$$

where $\frac{1}{2}mv_{esc}^2=\frac{GmM_E}{R_E}$. Then

$$\frac{1}{2}v_i^2-\frac{1}{2}v_{esc}^2=-\frac{1}{2}v_{esc}^2\frac{R_E}{R_E+h}$$

$$v_{esc}^2-v_i^2=\frac{v_{esc}^2R_E}{R_E+h}$$

$$\frac{1}{v_{esc}^2-v_i^2}=\frac{R_E+h}{v_{esc}^2R_E}$$

$$h=\frac{v_{esc}^2R_E}{v_{esc}^2-v_i^2}-R_E=\frac{v_{esc}^2R_E-v_{esc}^2R_E+v_i^2R_E}{v_{esc}^2-v_i^2}$$

$$h=\frac{R_Ev_i^2}{v_{esc}^2-v_i^2}$$

$$h=\frac{\left(6.37\times10^6\text{ m}\right)\left(8.76\text{ km/s}\right)^2}{\left(11.2\text{ km/s}\right)^2-\left(8.76\text{ km/s}\right)^2}=\boxed{1.00\times10^7\text{ m}}$$

(b) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation:

$$v_i^2=v_{esc}^2\left(1-\frac{R_E}{R_E+h}\right)$$

$$=v_{esc}^2\left(\frac{h}{R_E+h}\right)$$

$$=\left(11.2\times10^3\text{ m/s}\right)^2\left(\frac{2.51\times10^7\text{ m}}{6.37\times10^6\text{ m}+2.51\times10^7\text{ m}}\right)$$

$$=1.00\times10^8\text{ m}^2/\text{s}^2$$

$$v_i=\boxed{1.00\times10^4\text{ m/s}}$$

P13.58 (a) Ignoring air resistance, the energy conservation for the object-Earth system from firing to apex is given by,

$$(K+U_g)_i = (K+U_g)_f$$

$$\frac{1}{2}mv_i^2 - \frac{GmM_E}{R_E} = 0 - \frac{GmM_E}{R_E+h}$$

where $\frac{1}{2}mv_{esc}^2 = \frac{GmM_E}{R_E}$. Then

$$\frac{1}{2}v_i^2 - \frac{1}{2}v_{esc}^2 = -\frac{1}{2}v_{esc}^2\frac{R_E}{R_E+h}$$

$$v_{esc}^2 - v_i^2 = \frac{v_{esc}^2 R_E}{R_E+h}$$

$$\frac{1}{v_{esc}^2 - v_i^2} = \frac{R_E+h}{v_{esc}^2 R_E}$$

$$h = \frac{v_{esc}^2 R_E}{v_{esc}^2 - v_i^2} - R_E = \frac{v_{esc}^2 R_E - v_{esc}^2 R_E + v_i^2 R_E}{v_{esc}^2 - v_i^2}$$

$$\boxed{h = \frac{R_E v_i^2}{v_{esc}^2 - v_i^2}}$$

(b) The fall of the meteorite is the time-reversal of the upward flight of the projectile, so it is described by the same energy equation. From (a) above, replacing v_i with v_f, we have

$$v_f^2 = v_{esc}^2 - v_{esc}^2\frac{R_E}{R_E+h}$$

$$v_f^2 = v_{esc}^2\left(1 - \frac{R_E}{R_E+h}\right)$$

$$\boxed{v_f = v_{esc}\sqrt{\frac{h}{R_E+h}}}$$

(c) With $v_i << v_{esc}$, $h \approx \frac{R_E v_i^2}{v_{esc}^2} = \frac{R_E v_i^2 R_E}{2GM_E}$. But $g = \frac{GM_E}{R_E^2}$, so $h = \frac{v_i^2}{2g}$, in agreement with $0^2 = v_i^2 + 2(-g)(h-0)$.

P13.59 (a) Let *R* represent the radius of the asteroid. Then its volume is $\frac{4}{3}\pi R^3$ and its mass is $\rho\frac{4}{3}\pi R^3$. For your orbital motion, $\sum F = ma$ gives

$$\frac{Gm_1m_2}{R^2} = \frac{m_2v^2}{R} \quad \rightarrow \quad \frac{G\rho 4\pi R^3}{3R^2} = \frac{v^2}{R}$$

solving for *R*,

$$\begin{aligned} R &= \left(\frac{3v^2}{G\rho 4\pi}\right)^{1/2} \\ &= \left[\frac{3(8.50 \text{ m/s})^2}{(6.67\times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(1\,100 \text{ kg/m}^3)4\pi}\right]^{1/2} \\ &= \boxed{1.53\times 10^4 \text{ m}} \end{aligned}$$

(b) $$\rho\frac{4}{3}\pi R^3 = (1\,100 \text{ kg/m}^3)\frac{4}{3}\pi(1.53\times 10^4 \text{ m})^3 = \boxed{1.66\times 10^{16} \text{ kg}}$$

(c) $$v = \frac{2\pi R}{T} \quad T = \frac{2\pi R}{v} = \frac{2\pi(1.53\times 10^4 \text{ m})}{8.5 \text{ m/s}} = \boxed{1.13\times 10^4 \text{ s}} = 3.15 \text{ h}$$

(d) For an illustrative model, we take your mass as 90.0 kg and assume the asteroid is originally at rest. Angular momentum is conserved for the asteroid-you system:

$$\begin{aligned} \sum L_i &= \sum L_f \\ 0 &= m_2vR - I\omega \\ 0 &= m_2vR - \frac{2}{5}m_1R^2\frac{2\pi}{T_{\text{asteroid}}} \\ m_2v &= \frac{4\pi}{5}\frac{m_1R}{T_{\text{asteroid}}} \\ T_{\text{asteroid}} &= \frac{4\pi m_1R}{5m_2v} = \frac{4\pi(1.66\times 10^{16} \text{ kg})(1.53\times 10^4 \text{ m})}{5(90.0 \text{ kg})(8.50 \text{ m/s})} \\ &= 8.37\times 10^{17} \text{ s} = 26.5 \text{ billion years} \end{aligned}$$

$$\boxed{\text{Thus your running does not produce significant rotation of the asteroid if it is originally stationary and does not significantly affect any rotation it does have.}}$$

This problem is realistic. Many asteroids, such as Ida and Eros, are roughly 30 km in diameter. They are typically irregular in shape and not spherical. Satellites such as Phobos (of Mars), Adrastea (of Jupiter), Calypso (of Saturn), and Ophelia (of Uranus) would allow a visitor the same experience of easy orbital motion. So would many Kuiper Belt objects.

P13.60 (a) $\boxed{\text{The two appropriate isolated system models are conservation of momentum and conservation of energy applied to the system consisting of the two spheres.}}$

(b) Applying conservation of momentum to the system, we find

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$0 + 0 = M\,\vec{\mathbf{v}}_{1f} + 2M\,\vec{\mathbf{v}}_{2f}$$

$$\vec{\mathbf{v}}_{1f} = \boxed{-2\vec{\mathbf{v}}_{2f}}$$

(c) Applying conservation of energy to the system, we find

$$K_i + U_i + \Delta E = K_f + U_f$$

$$0 - \frac{Gm_1m_2}{r_i} + 0 = \frac{1}{2}m_1v_{1f}^2 + \frac{1}{2}m_2v_{2f}^2 - \frac{Gm_1m_2}{r_f}$$

$$-\frac{GM(2M)}{12R} = \frac{1}{2}Mv_{1f}^2 + \frac{1}{2}(2M)v_{2f}^2 - \frac{GM(2M)}{4R}$$

$$\frac{1}{2}Mv_{1f}^2 = \frac{GM}{2R} - \frac{GM}{6R} - v_{2f}^2$$

$$v_{1f} = \boxed{\sqrt{\frac{2GM}{3R} - 2v_{2f}^2}}$$

(d) Combining the results for parts (b) and (c),

$$2v_{2f} = \sqrt{\frac{2GM}{3R} - 2v_{2f}^2}$$

$$6v_{2f}^2 = \frac{2GM}{3R}$$

$$v_2 = \boxed{\frac{1}{3}\sqrt{G\frac{M}{R}}} \qquad v_1 = \boxed{\frac{2}{3}\sqrt{G\frac{M}{R}}}$$

P13.61 (a) At infinite separation $U = 0$ and at rest $K = 0$. Since the system is isolated, the energy and momentum of the two-planet system is conserved. We have

$$0 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{d} \qquad \textbf{[1]}$$

and

$$0 = m_1 v_1 - m_2 v_2 \qquad \textbf{[2]}$$

because the initial momentum of the system is zero.

Combine equations [1] and [2]:

$$\boxed{v_1 = m_2\sqrt{\frac{2G}{d(m_1 + m_2)}}} \quad \text{and} \quad \boxed{v_2 = m_1\sqrt{\frac{2G}{d(m_1 + m_2)}}}$$

The relative velocity is then

$$v_r = v_1 - (-v_2) = \boxed{\sqrt{\frac{2G(m_1 + m_2)}{d}}}$$

(b) The instant before the collision, the distance between the planets is $d = r_1 + r_2$. Substitute given numerical values into the equation found for v_1 and v_2 in part (a) to find

$$v_1 = 1.03 \times 10^4 \text{ m/s} \quad \text{and} \quad v_2 = 2.58 \times 10^3 \text{ m/s}$$

Therefore,

$$K_1 = \frac{1}{2}m_1 v_1^2 = \boxed{1.07 \times 10^{32}\text{ J}} \text{ and } K_2 = \frac{1}{2}m_2 v_2^2 = \boxed{2.67 \times 10^{31}\text{ J}}$$

P13.62 (a) The free-fall acceleration produced by the Earth is

$$g = \frac{GM_E}{r^2} = GM_E r^{-2} \text{ (directed downward)}$$

Its rate of change is

$$\frac{dg}{dr} = GM_E(-2)r^{-3} = -2GM_E r^{-3}$$

The minus sign indicates that *g* decreases with increasing height. At the Earth's surface,

$$\boxed{\frac{dg}{dr} = -\frac{2GM_E}{R_E^3}}$$

(b) For small differences,

$$\frac{|\Delta g|}{\Delta r} = \frac{|\Delta g|}{h} = \frac{2GM_E}{R_E^3}$$

Thus,

$$\boxed{\left|\Delta g\right| = \frac{2GM_E h}{R_E^3}}$$

(c) $$\left|\Delta g\right| = \frac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(6.00\ \text{m}\right)}{\left(6.37\times10^{6}\ \text{m}\right)^3}$$

$$= \boxed{1.85\times10^{-5}\ \text{m/s}^2}$$

P13.63 (a) Each bit of mass *dm* in the ring is at the same distance from the object at *A*. The separate contributions $-\frac{Gmdm}{r}$ to the system energy add up to $-\frac{GmM_{\text{ring}}}{r}$. When the object is at *A*, this is

$$-\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1\,000\ \text{kg})(2.36\times10^{20}\ \text{kg})}{\sqrt{\left(1.00\times10^{8}\ \text{m}\right)^2+\left(2.00\times10^{8}\ \text{m}\right)^2}} = \boxed{-7.04\ \text{N}}$$

(b) When the object is at the center of the ring, the potential energy of the system is

$$-\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1\,000\ \text{kg}\right)\left(2.36\times10^{20}\ \text{kg}\right)}{1.00\times10^{8}\ \text{m}}$$

$$= \boxed{-1.57\times10^{5}\ \text{J}}$$

(c) Total energy of the object-ring system is conserved:

$$\left(K+U_g\right)_A = \left(K+U_g\right)_B$$

$$0-7.04\times10^{4}\ \text{J} = \frac{1}{2}\left(1\,000\ \text{kg}\right)v_B^2 - 1.57\times10^{5}\ \text{J}$$

$$v_B = \left(\frac{2\times8.70\times10^{4}\ \text{J}}{1\,000\ \text{kg}}\right)^{1/2} = \boxed{13.2\ \text{m/s}}$$

***P13.64** The original orbit radius is

$$r = a = 6.37\times10^{6}\ \text{m} + 500\times10^{3}\ \text{m} = 6.87\times10^{6}\ \text{m}$$

The original energy is

$$E_i = -\frac{GMm}{2a}$$

$$= -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(10^{4}\ \text{kg}\right)}{2\left(6.87\times10^{6}\ \text{m}\right)}$$

$$= -2.90\times10^{11}\ \text{J}$$

We assume that the perigee distance in the new orbit is 6.87×10^6 m. Then the major axis is $2a = 6.87 \times 10^6 \text{ m} + 2.00 \times 10^7 \text{ m} = 2.69 \times 10^7 \text{ m}$ and the final energy is

$$\begin{aligned} E_f &= -\frac{GMm}{2a} \\ &= -\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(10^4 \text{ kg})}{2.69 \times 10^7 \text{ m}} \\ &= -1.48 \times 10^{11} \text{ J} \end{aligned}$$

The energy input required from the engine is

$$E_f - E_i = -1.48 \times 10^{11} \text{ J} - (-2.90 \times 10^{11} \text{ J}) = \boxed{1.42 \times 10^{11} \text{ J}}$$

P13.65 From the walk, $2\pi r = 25\,000$ m. Thus, the radius of the planet is

$$r = \frac{25\,000 \text{ m}}{2\pi} = 3.98 \times 10^3 \text{ m}$$

From the drop:

$$\Delta y = \frac{1}{2} g t^2 = \frac{1}{2} g (29.2 \text{ s})^2 = 1.40 \text{ m}$$

so,

$$g = \frac{2(1.40 \text{ m})}{(29.2 \text{ s})^2} = 3.28 \times 10^{-3} \text{ m/s}^2 = \frac{MG}{r^2}$$

which gives

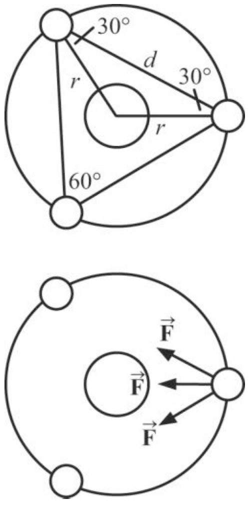
$$M = \boxed{7.79 \times 10^{14} \text{ kg}}$$

P13.66 The distance between the orbiting stars is $d = 2r\cos 30° = \sqrt{3}r$ since $\cos 30° = \frac{\sqrt{3}}{2}$. The net inward force on one orbiting star is

$$\frac{Gmm}{d^2}\cos 30° + \frac{GMm}{r^2} + \frac{Gmm}{d^2}\cos 30° = \frac{mv^2}{r}$$

$$\frac{Gm2\cos 30°}{3r^2} + \frac{GM}{r^2} = \frac{4\pi^2 r^2}{rT^2}$$

$$G\left(\frac{m}{\sqrt{3}} + M\right) = \frac{4\pi^2 r^3}{T^2}$$



ANS. FIG. P13.66

solving for the period gives

$$T^2 = \frac{4\pi^2 r^3}{G\left(M + m/\sqrt{3}\right)}$$

$$T = 2\pi\left(\frac{r^3}{G\left(M + m/\sqrt{3}\right)}\right)^{1/2}$$

P13.67 (a) We find the period from

$$T = \frac{2\pi r}{v} = \frac{2\pi\left(30\,000 \times 9.46 \times 10^{15}\ \text{m}\right)}{2.50 \times 10^5\ \text{m/s}} = 7 \times 10^{15}\ \text{s}$$

$$= \boxed{2 \times 10^8\ \text{yr}}$$

(b) We estimate the mass of the Milky Way from

$$M = \frac{4\pi^2 a^3}{GT^2} = \frac{4\pi^2\left(30\,000 \times 9.46 \times 10^{15}\ \text{m}\right)^3}{\left(6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.13 \times 10^{15}\ \text{s}\right)^2},$$

$$= 2.66 \times 10^{41}\ \text{kg}$$

or $\boxed{\text{about } 10^{41}\ \text{kg}}$

Note that this is the mass of the galaxy contained within the Sun's orbit of the galactic center. Recent studies show that the true mass of the galaxy, including an extended halo of dark matter, is at least an order of magnitude larger than our estimate.

(c) A solar mass is about 1×10^{30} kg: $10^{41}/10^{30} = 10^{11}$

The number of stars is $\boxed{\text{on the order of } 10^{11}}$.

P13.68 Energy conservation for the two-sphere system from release to contact:

$$-\frac{Gmm}{R} = -\frac{Gmm}{2r} + \frac{1}{2}mv^2 + \frac{1}{2}mv^2$$

$$Gm\left(\frac{1}{2r} - \frac{1}{R}\right) = v^2 \quad \rightarrow \quad v = \left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2}$$

(a) The injected momentum is the final momentum of each sphere,

$$mv = m^{2/2}\left(Gm\left[\frac{1}{2r} - \frac{1}{R}\right]\right)^{1/2} = \boxed{\left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}}$$

(b) If they now collide elastically each sphere reverses its velocity to receive impulse

$$mv - (-mv) = 2mv = \boxed{2\left[Gm^3\left(\frac{1}{2r} - \frac{1}{R}\right)\right]^{1/2}}$$

P13.69 (a) The net torque exerted on the Earth is zero. Therefore, the angular momentum of the Earth is conserved. We use this to find the speed at aphelion:

$$mr_a v_a = mr_p v_p$$

and

$$v_a = v_p\left(\frac{r_p}{r_a}\right) = (3.027 \times 10^4 \text{ m/s})\left(\frac{1.471}{1.521}\right) = \boxed{2.93 \times 10^4 \text{ m/s}}$$

(b) $K_p = \frac{1}{2}mv_p^2 = \frac{1}{2}(5.98 \times 10^{24} \text{ kg})(3.027 \times 10^4 \text{ m/s})^2 = \boxed{2.74 \times 10^{33} \text{ J}}$

$$U_p = -\frac{GmM}{r_p}$$

$$= -\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(1.99 \times 10^{30} \text{ kg})}{1.471 \times 10^{11} \text{ m}}$$

$$= \boxed{-5.40 \times 10^{33} \text{ J}}$$

(c) Using the same form as in part (b),

$$K_a = \boxed{2.57 \times 10^{33} \text{ J}} \text{ and } U_a = \boxed{-5.22 \times 10^{33} \text{ J}}$$

(d) Compare to find that

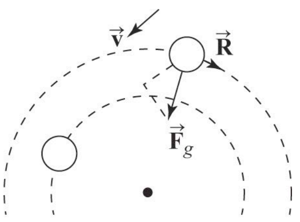
$$K_p + U_p = \boxed{-2.66 \times 10^{33} \text{ J}} \text{ and } K_a + U_a = \boxed{-2.65 \times 10^{33} \text{ J}}.$$

$\boxed{\text{They agree,}}$ with a small rounding error.

P13.70 For both circular orbits,

$$\sum F = ma: \frac{GM_E m}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_E}{r}}$$



ANS. FIG. P13.70

(a) The original speed is

$$v_i = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{6.37\times10^{6}\ \text{m}+2.00\times10^{5}\ \text{m}}}$$

$$= \boxed{7.79\times10^{3}\ \text{m/s}}$$

(b) The final speed is

$$v_i = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{6.47\times10^{6}\ \text{m}}}$$

$$= \boxed{7.85\times10^{3}\ \text{m/s}}$$

The energy of the satellite-Earth system is

$$K+U_g = \frac{1}{2}mv^2-\frac{GM_Em}{r} = \frac{1}{2}m\frac{GM_E}{r}-\frac{GM_E}{r} = -\frac{GM_Em}{2r}$$

(c) Originally,

$$E_i = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{2\left(6.57\times10^{6}\ \text{m}\right)}$$

$$= \boxed{-3.04\times10^{9}\ \text{J}}$$

(d) Finally,

$$E_f = -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{2\left(6.47\times10^{6}\ \text{m}\right)}$$

$$= \boxed{-3.08\times10^{9}\ \text{J}}$$

(e) Thus the object speeds up as it spirals down to the planet. The loss of gravitational energy is so large that the total energy decreases by

$$E_i-E_f = -3.04\times10^{9}\ \text{J}-\left(-3.08\times10^{9}\ \text{J}\right) = \boxed{4.69\times10^{7}\ \text{J}}$$

(f) The only forces on the object are the backward force of air resistance *R*, comparatively very small in magnitude, and the force of gravity. Because the spiral path of the satellite is not perpendicular to the gravitational force,

$$\boxed{\text{one component of the gravitational force pulls forward on the satellite}}$$

to do positive work and make its speed increase.

P13.71 The centripetal acceleration of the blob comes from gravitational acceleration:

$$\frac{v^2}{r} = \frac{M_c G}{r^2} = \frac{4\pi^2 r^2}{T^2 r}$$

$$GM_c T^2 = 4\pi^2 r^3$$

Solving for the radius gives

$$r = \left[\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(20)(1.99\times10^{30}\ \text{kg})(5.00\times10^{-3}\ \text{s})^2}{4\pi^2}\right]^{1/3}$$

$$r_{\text{orbit}} = \boxed{119\ \text{km}}$$

P13.72 From Kepler's third law, minimum period means minimum orbit size. The "treetop satellite" in Problem 38 has minimum period. The radius of the satellite's circular orbit is essentially equal to the radius *R* of the planet.

$$\sum F = ma\text{:}\quad \frac{GMm}{R^2} = \frac{mv^2}{R} = \frac{m}{R}\left(\frac{2\pi R}{T}\right)^2$$

$$G\rho V = \frac{R^2(4\pi^2 R^2)}{RT^2}$$

$$G\rho\left(\frac{4}{3}\pi R^3\right) = \frac{4\pi^2 R^3}{T^2}$$

The radius divides out: $T^2 G\rho = 3\pi \quad \rightarrow \quad \boxed{T = \sqrt{\frac{3\pi}{G\rho}}}$

P13.73 Let *m* represent the mass of the meteoroid and v_i its speed when far away. No torque acts on the meteoroid, so its angular momentum is conserved as it moves between the distant point and the point where it grazes the Earth, moving perpendicular to the radius:

ANS. FIG. P13.73

$$L_i = L_f\text{:}\quad m\vec{\mathbf{r}}_i \times \vec{\mathbf{v}}_i = m\vec{\mathbf{r}}_f \times \vec{\mathbf{v}}_f$$

$$m(3R_E v_i) = mR_E v_f$$

$$v_f = 3v_i$$

Now, the energy of the meteoroid-Earth system is also conserved:

$$\left(K+U_g\right)_i=\left(K+U_g\right)_f:\ \frac{1}{2}mv_i^2+0=\frac{1}{2}mv_f^2-\frac{GM_Em}{R_E}$$

$$\frac{1}{2}v_i^2=\frac{1}{2}\left(9v_i^2\right)-\frac{GM_E}{R_E}$$

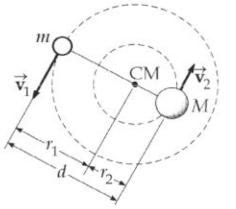
$$\frac{GM_E}{R_E}=4v_i^2: \qquad \boxed{v_i=\sqrt{\frac{GM_E}{4R_E}}}$$

P13.74 If we choose the coordinate of the center of mass at the origin, then

$$0=\frac{\left(Mr_2-mr_1\right)}{M+m} \qquad \text{and} \qquad Mr_2=mr_1$$

(Note: this is equivalent to saying that the net torque must be zero and the two experience no angular acceleration.) For each mass $F = ma$ so

$$mr_1\omega_1^2=\frac{MGm}{d^2} \qquad \text{and} \qquad Mr_2\omega_2^2=\frac{MGm}{d^2}$$



ANS. FIG. P13.74

Combining these two equations and using $d = r_1 + r_2$ gives

$$\left(r_1+r_2\right)\omega^2=\frac{\left(M+m\right)G}{d^2} \text{ with}$$

$$\omega_1=\omega_2=\omega$$

and

$$T=\frac{2\pi}{\omega}$$

we find

$$\boxed{T^2=\frac{4\pi^2d^3}{G\left(M+m\right)}}$$

P13.75 The gravitational forces the particles exert on each other are in the x direction. They do not affect the velocity of the center of mass. Energy is conserved for the pair of particles in a reference frame coasting along with their center of mass, and momentum conservation means that the identical particles move toward each other with equal speeds in this frame:

$$U_{gi} + K_i + K_i = U_{gf} + K_f + K_f$$

$$-\frac{Gm_1m_2}{r_i} + 0 = -\frac{Gm_1m_2}{r_f} + \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$$

$$-\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1\ 000\ \text{kg})^2}{20.0\ \text{m}}$$

$$= -\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1\ 000\ \text{kg})^2}{2.00\ \text{m}} + 2\left(\frac{1}{2}\right)(1\ 000\ \text{kg})v^2$$

$$\left(\frac{3.00\times10^{-5}\ \text{J}}{1\ 000\ \text{kg}}\right)^{1/2} = v = 1.73\times10^{-4}\ \text{m/s}$$

$\boxed{\text{Then their vector velocities are } (800+1.73\times10^{-4})\,\hat{\mathbf{i}}\ \text{m/s and } (800-1.73\times10^{-4})\hat{\mathbf{i}}\ \text{m/s for the trailing particle and the leading particle, respectively.}}$

P13.76 (a) The gravitational force exerted on m by the Earth (mass M_E) accelerates m according to $g_2 = \frac{GM_E}{r^2}$. The equal-magnitude force exerted on the Earth by m produces acceleration of the Earth given by $g_1 = \frac{Gm}{r^2}$. The acceleration of relative approach is then

$$g_2 + g_1 = \frac{Gm}{r^2} + \frac{GM_E}{r^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg} + m\right)}{\left(1.20\times10^7\ \text{m}\right)^2}$$

$$= \boxed{\left(2.77\ \text{m/s}^2\right)\left(1 + \frac{m}{5.98\times10^{24}\ \text{kg}}\right)}$$

(b) and (c) Here m = 5 kg and m = 2000 kg are both negligible compared to the mass of the Earth, so the acceleration of relative approach is just $\boxed{2.77\ \text{m/s}^2}$.

(d) Substituting $m = 2.00 \times 10^{24}$ kg into the expression for $(g_1 + g_2)$ above gives

$$g_1 + g_2 = \boxed{3.70\ \mathrm{m/s^2}}$$

(e) Any object with mass small compared to the Earth starts to fall with acceleration $2.77\ \mathrm{m/s^2}$. As m increases to become comparable to the mass of the Earth, the acceleration increases, and can become arbitrarily large. It approaches a direct proportionality to m.

P13.77 For the Earth, $\sum F = ma$: $\dfrac{GM_s m}{r^2} = \dfrac{mv^2}{r} = \dfrac{m}{r}\left(\dfrac{2\pi r}{T}\right)^2$

Then $GM_s T^2 = 4\pi^2 r^3$

Also, the angular momentum $L = mvr = m\dfrac{2\pi r}{T}r$ is a constant for the Earth. We eliminate $r = \sqrt{\dfrac{LT}{2\pi m}}$ between the equations:

$$GM_s T^2 = 4\pi^2\left(\frac{LT}{2\pi m}\right)^{3/2} \quad \text{gives} \quad GM_s T^{1/2} = 4\pi^2\left(\frac{L}{2\pi m}\right)^{3/2}$$

Now the rates of change with time *t* are described by

$$GM_s\left(\frac{1}{2}T^{-1/2}\frac{dT}{dt}\right) + G\left(1\frac{dM_s}{dt}T^{1/2}\right) = 0$$

or

$$\frac{dT}{dt} = -\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right) \approx \frac{\Delta T}{\Delta t}$$

which gives

$$\Delta T \approx -\Delta t\frac{dM_s}{dt}\left(2\frac{T}{M_s}\right)$$

$$= -(5\,000\ \mathrm{yr})\left(\frac{3.16\times10^7\ \mathrm{s}}{1\ \mathrm{yr}}\right)\left(-3.64\times10^9\ \mathrm{kg/s}\right)$$

$$\times\left(2\frac{1\ \mathrm{yr}}{1.99\times10^{30}\ \mathrm{kg}}\right)$$

$$\Delta T = \boxed{5.78\times10^{-10}\ \mathrm{s}}$$

Challenge Problems

P13.78 Let m represent the mass of the spacecraft, r_E the radius of the Earth's orbit, and x the distance from Earth to the spacecraft.

The Sun exerts on the spacecraft a radial inward force of

$$F_s = \frac{GM_s m}{(r_E - x)^2}$$

while the Earth exerts on it a radial outward force of

$$F_E = \frac{GM_E m}{x^2}$$

The net force on the spacecraft must produce the correct centripetal acceleration for it to have an orbital period of 1.000 year.

Thus, $F_S - F_E = \dfrac{GM_S m}{(r_E - x)^2} - \dfrac{GM_E m}{x^2} = \dfrac{mv^2}{(r_E - x)} = \dfrac{m}{(r_E - x)}\left[\dfrac{2\pi(r_E - x)}{T}\right]^2$

which reduces to

$$\frac{GM_S}{(r_E - x)^2} - \frac{GM_E}{x^2} = \frac{4\pi^2(r_E - x)}{T^2} \qquad \textbf{[1]}$$

Cleared of fractions, this equation would contain powers of x ranging from the fifth to the zeroth. We do not solve it algebraically. We may test the assertion that x is 1.48×10^9 m by substituting it into the equation, along with the following data: $M_S = 1.99 \times 10^{30}$ kg, $M_E = 5.974 \times 10^{24}$ kg, $r_E = 1.496 \times 10^{11}$ m, and $T = 1.000 \text{ yr} = 3.156 \times 10^7$ s.

With $x = 1.48 \times 10^9$ m, the result is

$$6.053 \times 10^{-3} \text{ m/s}^2 - 1.82 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

or

$$5.870\,9 \times 10^{-3} \text{ m/s}^2 \approx 5.870\,8 \times 10^{-3} \text{ m/s}^2$$

$$\boxed{\text{To three-digit precision, the solution is } 1.48 \times 10^9 \text{ m.}}$$

As an equation of fifth degree, equation [1] has five roots. The Sun-Earth system has five Lagrange points, all revolving around the Sun synchronously with the Earth. The SOHO and ACE satellites are at one. Another is beyond the far side of the Sun. Another is beyond the night side of the Earth. Two more are on the Earth's orbit, ahead of the planet and behind it by 60°. The twin satellites of NASA's STEREO mission, giving three-dimensional views of the Sun from orbital positions ahead of and trailing Earth, passed through these Lagrange points in 2009. The Greek and Trojan asteroids are at the co-orbital Lagrange points of the Jupiter-Sun system.

P13.79 (a) From the data about perigee, the energy of the satellite-Earth system is

$$E = \frac{1}{2}mv_p^2 - \frac{GM_Em}{r_p} = \frac{1}{2}(1.60\text{ kg})(8.23\times10^3\text{ m/s})^2 - \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(1.60\text{ kg})}{7.02\times10^6\text{ m}}$$

or $E = \boxed{-3.67\times10^7\text{ J}}$

(b) $$\begin{aligned} L &= mvr\sin\theta = mv_pr_p\sin 90.0° \\ &= (1.60\text{ kg})(8.23\times10^3\text{ m/s})(7.02\times10^6\text{ m}) \\ &= \boxed{9.24\times10^{10}\text{ kg}\cdot\text{m}^2/\text{s}} \end{aligned}$$

(c) Since both the energy of the satellite-Earth system and the angular momentum of the Earth are conserved,

at apogee we must have

$$\frac{1}{2}mv_a^2 - \frac{GMm}{r_a} = E$$

and $mv_ar_a\sin 90.0° = L$

Thus,

$$\frac{1}{2}(1.60\text{ kg})v_a^2 - \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\text{ kg})(1.60\text{ kg})}{r_a} = -3.67\times10^7\text{ J}$$

and $(1.60\text{ kg})v_ar_a = 9.24\times10^{10}\text{ kg}\cdot\text{m}^2/\text{s}$

Solving simultaneously, and suppressing units,

$$\frac{1}{2}(1.60)v_a^2 - \frac{(6.67\times10^{-11})(5.98\times10^{24})(1.60)(1.60)v_a}{9.24\times10^{10}} = -3.67\times10^7$$

which reduces to

$$0.800v_a^2 - 11\,046v_a + 3.672\,3\times10^7 = 0$$

so $$v_a = \frac{11\,046 \pm \sqrt{(11\,046)^2 - 4(0.800)(3.672\,3\times10^7)}}{2(0.800)}$$

This gives $v_a = 8\ 230$ m/s or $\boxed{5\ 580 \text{ m/s}}$. The smaller answer refers to the velocity at the apogee while the larger refers to perigee.

Thus,

$$r_a = \frac{L}{mv_a} = \frac{9.24\times 10^{10}\ \text{kg}\cdot\text{m}^2/\text{s}}{(1.60\ \text{kg})(5.58\times 10^3\ \text{m/s})} = \boxed{1.04\times 10^7\ \text{m}}$$

(d) The major axis is $2a = r_p + r_a$, so the semimajor axis is

$$a = \frac{1}{2}\left(7.02\times 10^6\ \text{m} + 1.04\times 10^7\ \text{m}\right) = \boxed{8.69\times 10^6\ \text{m}}$$

(e) $$T = \sqrt{\frac{4\pi^2 a^3}{GM_E}} = \sqrt{\frac{4\pi^2\left(8.69\times 10^6\ \text{m}\right)^3}{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times 10^{24}\ \text{kg}\right)}}$$

$$T = 8\ 060\ \text{s} = \boxed{134\ \text{min}}$$

***P13.80** (a) Energy of the spacecraft-Mars system is conserved as the spacecraft moves between a very distant point and the point of closest approach:

$$0 + 0 = \frac{1}{2}mv_r^2 - \frac{GM_{\text{Mars}}m}{r}$$

$$v_r = \sqrt{\frac{2GM_{\text{Mars}}}{r}}$$

After the engine burn, for a circular orbit we have

$$\sum F = ma: \qquad \frac{GM_{\text{Mars}}m}{r^2} = \frac{mv_0^2}{r}$$

$$v_0 = \sqrt{\frac{GM_{\text{Mars}}}{r}}$$

The percentage reduction from the original speed is

$$\frac{v_r - v_0}{v_r} = \frac{\sqrt{2}v_0 - v_0}{\sqrt{2}v_0} = \frac{\sqrt{2}-1}{\sqrt{2}}\times 100\% = \boxed{29.3\%}$$

(b) The answer to part (a) applies with $\boxed{\text{no changes}}$, as the solution to part (a) shows.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P13.2 $\sim 10^{-7}$ N

P13.4 (a) 4.39×10^{20} N; (b) 1.99×10^{20} N; (c) 3.55×10^{22} N; (d) The force exerted by the Sun on the Moon is much stronger than the force of the Earth on the Moon.

P13.6 $\left(-10.0\hat{\mathbf{i}} + 5.93\hat{\mathbf{j}}\right) \times 10^{-11}$ N

P13.8 The situation is impossible because no known element could compose the spheres.

P13.10 3.06×10^{-8} m

P13.12 $\frac{2}{3}$

P13.14 (a) $\frac{2MGr}{\left(r^2 + a^2\right)^{3/2}}$ toward the center of mass; (b) At $r = 0$, the fields of the two objects are equal in magnitude and opposite in direction, to add to zero; (c) As $r \to 0, 2MGr\left(r^2 + a^2\right)^{-3/2}$ approaches $2MG(0)/a^3 = 0$; (d) When r is much greater than a, the angles the field vectors make with the x axis become smaller. At very great distances, the field vectors are almost parallel to the axis; therefore they begin to look like the field vector from a single object of mass $2M$; (e) As r becomes much larger than a, the expression approaches $2MGr\left(r^2 + 0^2\right)^{-3/2} = 2MGr/r^3 = 2MG/r^2$ as required.

P13.16 (a) 1.31×10^{17} N; (b) 2.62×10^{12} N/kg

P13.18 1.90×10^{27} kg

P13.20 (a) The particle does posses angular momentum because it is not headed straight for the origin. (b) Its angular momentum is constant. There are no identified outside influences acting on the object. (c) See P13.20(c) for full explanation.

P13.22 1.30 revolutions

P13.24 1.27

P13.26 1.63×10^{4} rad/s

P13.28 (a) 6.02×10^{24} kg; (b) The Earth wobbles a bit as the Moon orbits it, so both objects move nearly in circles about their center of mass, staying on opposite sides of it.

P13.30 (a) -4.77×10^9 J; (b) 569 N

P13.32 4.17×10^{10} J

P13.34 (a) See P13.34 for full description; (b) 340 s

P13.36 1.66×10^4 m/s

P13.38 $\sqrt{2}v$

P13.40 (a) 0.980; (b) 127 yr; (c) -2.13×10^{17} J

P13.42 $\dfrac{GM_Em}{12R_E}$

P13.44 (a) 42.1 km/s; (b) 2.20×10^{11} m

P13.46 (a) $2\pi\sqrt{\dfrac{(R_E + h)^3}{GM_E}}$; (b) $\sqrt{\dfrac{GM_E}{R_E + h}}$; (c) $GM_Em\left[\dfrac{R_E + 2h}{2R_E(R_E + h)}\right] - \dfrac{2\pi^2R_E^2m}{(86\ 400\text{ s})^2}$

P13.48 (a) $v_i = \left(\dfrac{GM_E}{r}\right)^{1/2}$; (b) $\dfrac{5}{4}\left(\dfrac{GM_E}{r}\right)^{1/2}$; (c) $r_f = \dfrac{25r}{7}$

P13.50 (a) 3.07×10^6 m; (b) the rocket would travel farther from Earth

P13.52 492 m/s

P13.54 If one uses the result $v = \sqrt{\dfrac{GM}{r}}$ and the relation $v = (2\pi\tau/T)$, one finds the radius of the orbit to be smaller than the radius of the Earth, so the spacecraft would need to be in orbit underground.

P13.56 (a) 0.700 rad/s; (b) Because his feet stay in place on the floor, his head will be moving at the same tangential speed as his feet. However, his feet and his head are travelling in circles of different radii; (c) If he's not careful, there could be a collision between his head and the wall (see P13.56 for full explanation)

P13.58 (a) $h = \dfrac{R_Ev_i^2}{v_{esc}^2 - v_i^2}$; (b) $v_f = v_{esc}\sqrt{\dfrac{h}{R_E + h}}$; (c) With $v_1 << v_{esc}, h \approx \dfrac{R_Ev_i^2}{v_{esc}^2} = \dfrac{R_Ev_i^2R_E}{2GM_E}$. But $g = \dfrac{GM_E}{R_E^2}$, so $h = \dfrac{v_i^2}{2g}$ in agreement with $0^2 = v_i^2 + 2(-g)(h - 0)$.

P13.60 (a) The two appropriate isolated system models are conservation of momentum and conservation of energy applied to the system consisting of the two spheres; (b) $-2\vec{\mathbf{v}}_{2f}$; (c) $\sqrt{\frac{2GM}{3R}-2v_{2f}^2}$;

(d) $v_2 = \frac{1}{3}\sqrt{G\frac{M}{R}}$, $v_1 = \frac{2}{3}\sqrt{G\frac{M}{R}}$

P13.62 (a) $\frac{dg}{dr} = -\frac{2GM_E}{R_E^3}$; (b) $|\Delta g| = \frac{2GM_E h}{R_E^3}$; (c) 1.85×10^{-5} m/s^2

P13.64 1.42×10^{11} J

P13.66 See P13.66 for the full answer.

P13.68 (a) $mv = \left[GM^3\left(\frac{1}{2r}-\frac{1}{R}\right)\right]^{1/2}$; (b) $2\left[GM^3\left(\frac{1}{2r}-\frac{1}{R}\right)\right]^{1/2}$

P13.70 (a) 7.79×10^3 m/s; (b) 7.85×10^3 m/s; (c) -3.04×10^9 J; (d) -3.08×10^9 J; (e) 4.69×10^7 J; (f) one component of the gravitational force pulls forward on the satellite

P13.72 See P13.72 for full description.

P13.74 See P13.74 for full description.

P13.76 (a) $\left(2.77\text{ m/s}^2\right)\left(1+\frac{m}{5.98\times 10^{24}\text{ kg}}\right)$; (b and c) 2.77 m/s^2; (d) 3.70 m/s^2;

(e) Any object with mass small compared to the Earth starts to fall with acceleration 2.77 m/s^2. As m increases to become comparable to the mass of the Earth, the acceleration increases and can become arbitrarily large. It approaches a direct proportionality to m.

P13.78 See P13.78 for full description.

P13.80 (a) 29.3%; (b) no changes

14

Fluid Mechanics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ14.1 Answer (c). Both must be built the same. A dam must be constructed to withstand the pressure at the bottom of the dam. The pressure at the bottom of a dam due to water is $P = \rho g h,$ where h is the height of the water. If both reservoirs are equally high (meaning the water is equally deep), the pressure is the same regardless of width.

OQ14.2 Answer (b), (e). The buoyant force on an object is equal to the weight of the volume of water displaced by that object.

OQ14.3 Answer (d), (e). The buoyant force on the block is equal to the WEIGHT of the volume of water it displaces.

OQ14.4 Answer (b). The apple does not change volume appreciably in a dunking bucket, and the water also keeps constant density. Then the buoyant force is constant at all depths.

OQ14.5 Answer (c). The water keeps nearly constant density as it increases in pressure with depth. The beach ball is compressed to smaller volume as you take it deeper, so the buoyant force decreases. Note that the situation this question considers is different from that of OQ14.2. In OQ14.2, the beach ball is fully inflated at a pressure higher than 1 atm, and the tension from the plastic balances the excess pressure. So even when the ball is 1 m under water, the water pressure increases, so the plastic tension decreases, but the inside pressure remains practically constant, hence no volume change.

OQ14.6 Answer (a), (c). Both spheres have the same volume, so the buoyant force is the same on each. The lead sphere weighs more, so its string tension must be greater.

OQ14.7 Answer (c). The absolute pressure at depth h below the surface of a fluid having density ρ is $P = P_0 + \rho g h,$ where P_0 is the pressure at the upper surface of that fluid. The fluid in each of the three vessels has density $\rho = \rho_{\text{water}}$, the top of each vessel is open to the atmosphere so that $P_0 = P_{\text{atm}}$ in each case, and the bottom is at the same depth h below the upper surface for the three vessels. Thus, the pressure P at the bottom of each vessel is the same.

OQ14.8 Answer (b). Ice on the continent of Antarctica is above sea level. At the north pole, the melting of the ice floating in the ocean will not raise the ocean level (see OQ14.15).

OQ14.9 Answer (c). The normal force from the bottom plus the buoyant force from the water together balance the weight of the boat.

OQ14.10 (i) Answer (b). (ii) Answer (c). When the steel is underwater, the water exerts on the steel a buoyant force that was not present when the steel was on top surrounded by air. Thus, slightly less wood will be below the water line on the wooden block. It will float higher. In both orientations the compound floating object displaces its own weight of water, so it displaces equal volumes of water. The water level in the tub will be unchanged when the object is turned over.

OQ14.11 Answer (b). The excess pressure is transmitted undiminished throughout the container. It will compress air inside the wood. The water driven into the pores of the wood raises the block's average density and makes if float lower in the water. Add some thumbtacks to reach neutral buoyancy and you can make the wood sink or rise at will by subtly squeezing a large clear–plastic soft–drink bottle. René Descartes invented this toy or trick, called a Cartesian diver.

OQ14.12 Answer (b). The level of the pond falls. This is because the anchor displaces more water while in the boat. A floating object displaces a volume of water whose weight is equal to the weight of the object. A submerged object displaces a volume of water equal to the volume of the object. Because the density of the anchor is greater than that of water, a volume of water that weighs the same as the anchor will be greater than the volume of the anchor.

OQ14.13 Answer: (b) = (d) = (e) > (a) > (c). Objects (a) and (c) float, and (e) barely floats (we ignore the thin-walled bottle). On them the buoyant forces are equal to the gravitational forces exerted on them, so the ranking is (e) greater than (a) and (e) greater than (c). Objects (b) and (d) sink, and have volumes equal to (e), so they feel equal-size buoyant forces: (e) = (b) = (d).

OQ14.14 Answer (d). You want the water drop-Earth system to have four times the gravitational potential energy, relative to where the water drop leaves the nozzle, as a water drop turns around at the top of the fountain. Therefore, you want it to start out with four times the kinetic energy, which means with twice the speed at the nozzle. Given the constant volume flow rate Av, you want the area to be two times smaller. If the nozzle has a circular opening, you need to decrease its radius only by the square root of two.

OQ14.15 Answer (c). The water level stays the same. The solid ice displaced its own mass of liquid water. The meltwater does the same.

OQ14.16 Answer (e). Since the pipe is horizontal, each part of it is at the same vertical level or has the same y coordinate. Thus, from Bernoulli's equation $P+\frac{1}{2}\rho v^2+\rho g y=\text{constant}$, we see that the sum of the pressure and the kinetic energy per unit volume ($P+\frac{1}{2}\rho v^2$) must also be constant throughout the pipe.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ14.1 The horizontal force exerted by the outside fluid, on an area element of the object's side wall, has equal magnitude and opposite direction to the horizontal force the fluid exerts on another element diametrically opposite the first.

CQ14.2 The weight depends upon the total volume of water in the glass. The pressure at the bottom depends only on the depth. With a cylindrical glass, the water pushes only horizontally on the side walls and does not contribute to an extra downward force above that felt by the base. On the other hand, if the glass is wide at the top with a conical shape, the water pushes outward and downward on each bit of side wall. The downward components add up to an extra downward force, more than that exerted on the small base area.

CQ14.3 The air in your lungs, the blood in your arteries and veins, and the protoplasm in each cell exert nearly the same pressure, so that the wall of your chest can be in equilibrium.

CQ14.4 Yes. The propulsive force of the fish on the water causes the scale reading to fluctuate. Its average value will still be equal to the total weight of bucket, water, and fish. In other words, the center of mass of the fish-water-bucket system is moving around when the fish swims. Therefore, the net force acting on the system cannot be a constant. Apart from the weights (which are constants), the vertical force from the scale is the only external force on the system: it changes as the center of mass moves (accelerates). So the scale reading changes.

CQ14.5 (a) The greater air pressure inside the spacecraft causes air to be expelled through the hole.

(b) Clap your shoe or wallet over the hole, or a seat cushion, or your hand. Anything that can sustain a force on the order of 100 N is strong enough to cover the hole and greatly slow down the escape of the cabin air. You need not worry about the air rushing out instantly, or about your body being "sucked" through the hole, or about your blood boiling or your body exploding. If the cabin pressure drops a lot, your ears will pop and the saliva in your mouth may boil—at body temperature—but you will still have a couple of minutes to plug the hole and put on your emergency oxygen mask. Passengers who have been drinking carbonated beverages may find that the carbon dioxide suddenly comes out of solution in their stomachs, distending their vests, making them belch, and all but frothing from their ears; so you might warn them of this effect.

CQ14.6 The rapidly moving air above the ball exerts less pressure than the atmospheric pressure below the ball. This can give substantial lift to balance the weight of the ball.

CQ14.7 Imagine there have been large water demands and the water vessel at the top is half full. The depth of water from the upper water surface to the ground is still large. Therefore, the pressure at the base of the water is only slightly reduced from that due to a full tank, resulting in adequate water pressure at residents' faucets. If the water tank were a tall cylinder, a half-full tank would be only half as deep and the pressure at residents' faucets would be only half as great. Also, the water level in a tall cylinder would drop faster, because its cross-sectional area is smaller, so it would have to be replaced more often.

CQ14.8 Like the ball, the balloon will remain in front of you. It will not bob up to the ceiling. Air pressure will be no higher at the floor of the sealed car than at the ceiling. The balloon will experience no buoyant force. You might equally well switch off gravity. In the freely falling elevator, everything is effectively "weightless," so the air does not exert a buoyant force on anything.

CQ14.9 (a) Yes. (b) Yes. (c) The buoyant force is a conservative force. It does positive work on an object moving upward in a fluid and an equal amount of negative work on the object moving down between the same two elevations. [Note that mechanical energy, $K + U$, is not conserved here because of viscous drag from the water.] Potential energy is not associated with the object on which the buoyant force acts, but with the system of objects interacting by the buoyant force. This system is the immersed object and the fluid.

CQ14.10 The metal is more dense than water. If the metal is sufficiently thin, it can float like a ship, with the lip of the dish above the water line. Most of the volume below the water line is filled with air. The mass of the dish divided by the volume of the part below the water line is just equal to the density of water. Placing a bar of soap into this space to replace the air raises the average density of the compound object and the density can become greater than that of water. The dish sinks with its cargo.

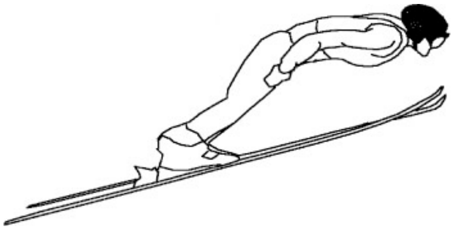
CQ14.11 Use a balance to determine its mass. Then partially fill a graduated cylinder with water. Immerse the rock in the water and determine the volume of water displaced. Divide the mass by the volume and you have the density. It may be more precise to hang the rock from a string, measure the force required to support it under water, and subtract to find the buoyant force. The buoyant force can be thought of as the weight of so many grams of water, which is that number of cubic centimeters of water, which is the volume of the submerged rock. This volume with the actual rock mass tells you its density.

CQ14.12 The diet drink fluid has no dissolved sugar, so its density is less than that of the regular drink. Try it.

CQ14.13 At lower elevation the water pressure is greater because pressure increases with increasing depth below the water surface in the reservoir (or water tower). The penthouse apartment is not so far below the water surface. The pressure behind a closed faucet is weaker there and the flow weaker from an open faucet. Your fire department likely has a record of the precise elevation of every fire hydrant.

CQ14.14 The boat floats higher in the ocean than in the inland lake. According to Archimedes's principle, the magnitude of buoyant force on the ship is equal to the weight of the water displaced by the ship. Because the density of salty ocean water is greater than fresh lake water, less ocean water needs to be displaced to enable the ship to float.

CQ14.15 The ski jumper gives her body the shape of an airfoil. She deflects the air stream downward as it rushes past and the airstream deflects her upward by Newton's third law. The air exerts on her a lift force, giving her a higher and longer trajectory.



ANS FIG. CQ14.15

CQ14.16 When taking off into the wind, the increased airspeed over the wings gives a larger lifting force, enabling the pilot to take off in a shorter length of runway.

CQ14.17 A breeze from any direction speeds up to go over the mound and the air pressure drops. Air then flows through the burrow from the lower entrance to the upper entrance.

CQ14.18 (a) Since the velocity of the air in the right-hand section of the pipe is lower than that in the middle, the pressure is higher.

(b) The equation that predicts the same pressure in the far right- and left-hand sections of the tube assumes laminar flow without viscosity. The equation also assumes the fluid is incompressible, but air is not. Also, the left-hand tube is open to the atmosphere while the right-hand tube is not. Internal friction will cause some loss of mechanical energy, and turbulence will also progressively reduce the pressure. If the pressure at the left were not lower than at the right, the flow would stop.

CQ14.19 The stored corn in the silo acts as a fluid: the greater the depth, the greater the pressure on the sides of the silo. The metal bands are placed closer, or doubled, at lower portions to provide more force to balance the force from the greater pressure.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 14.1 Pressure

P14.1 We shall assume that each chair leg supports one-fourth of the total weight so the normal force each leg exerts on the floor is $n = mg/4$. The pressure of each leg on the floor is then

$$P_{\text{leg}} = \frac{n}{A_{\text{leg}}} = \frac{mg/4}{\pi r^2} = \frac{(95.0\text{ kg})(9.80\text{ m/s}^2)}{4\pi(0.500\times 10^{-2}\text{ m})^2} = \boxed{2.96\times 10^6\text{ Pa}}$$

P14.2 (a) If the particles in the nucleus are closely packed with negligible space between them, the average nuclear density should be approximately that of a proton or neutron. That is

$$\rho_{\text{nucleus}} \approx \frac{m_{\text{proton}}}{V_{\text{proton}}} = \frac{m_{\text{proton}}}{4\pi r^3/3} \sim \frac{3(1.67\times 10^{-27}\text{ kg})}{4\pi(1\times 10^{-15}\text{ m})^3}$$

$$\boxed{\sim 4\times 10^{17}\text{ kg/m}^3}$$

(b) The density of an atom is about 10^{14} times greater than the density of iron and other common solids and liquids. This shows that an atom is mostly empty space. Liquids and solids, as well as gases, are mostly empty space.

P14.3 (a) $P = \dfrac{F}{A} = \dfrac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{\pi(0.500\times 10^{-2}\text{ m})^2} = \boxed{6.24\times 10^6\text{ N/m}^2}$

(b) The pressure from the heel might damage the vinyl floor covering.

P14.4 The Earth's surface area is $4\pi R^2$. The force pushing inward over this area amounts to

$$F = P_0 A = P_0\left(4\pi R^2\right)$$

This force is the weight of the air:

$$F_g = mg = P_0\left(4\pi R^2\right)$$

so, assuming g is everywhere the same, the mass of the air is

$$m = \frac{P_0\left(4\pi R^2\right)}{g}$$

$$= \frac{\left(1.013\times 10^5\ \text{N/m}^2\right)\left[4\pi\left(6.37\times 10^6\ \text{m}\right)^2\right]}{9.80\ \text{m/s}^2}$$

$$= \boxed{5.27\times 10^{18}\ \text{kg}}$$

P14.5 The definition of density, $\rho = m/V$, is often most directly useful in the form $m = \rho V$.

$$V = \ell wh \quad \text{so} \quad m = \rho V = \rho \ell wh$$

Thus $m = (19.3\times 10^3\ \text{kg/m}^3)(4.50\ \text{cm})(11.0\ \text{cm})(26.0\ \text{cm})$

$$= (19.3\times 10^3\ \text{kg/m}^3)(1\ 290\ \text{cm}^3)(1\ \text{m}^3/10^6\ \text{cm}^3) = \boxed{24.8\ \text{kg}}$$

Section 15.2 Variation of Pressure with Depth

P14.6 (a) Suppose the "vacuum cleaner" functions as a high–vacuum pump. The air below the brick will exert on it a lifting force

$$F = PA = \left(1.013\times 10^5\ \text{Pa}\right)\left[\pi\left(1.43\times 10^{-2}\ \text{m}\right)^2\right] = \boxed{65.1\ \text{N}}$$

(b) The octopus can pull the bottom away from the top shell with a force that could be no larger than

$$F = PA = \left(P_0 + \rho gh\right)A$$

$$= \left[1.013\times 10^5\ \text{Pa} + \left(1\ 030\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(32.3\ \text{m}\right)\right] \times\left[\pi\left(1.43\times 10^{-2}\ \text{m}\right)^2\right]$$

$$F = \boxed{275\ \text{N}}$$

P14.7 Assuming the spring obeys Hooke's law, the increase in force on the piston required to compress the spring an additional amount Δx is

$$\Delta F = F - F_0 = \left(P - P_0\right)A = k\left(\Delta x\right)$$

The gauge pressure at depth h beneath the surface of a fluid is

$$P - P_0 = \rho gh$$

so we have

$$\rho g h A = k(\Delta x)$$

or the required depth is

$$h = k(\Delta x)/\rho g A$$

If $k = 1\,250$ N/m, $A = \pi d^2/4$, $d = 1.20 \times 10^{-2}$ m, and the fluid is water ($\rho = 1.00 \times 10^3$ kg/m^3), the depth required to compress the spring an additional $\Delta x = 0.750 \times 10^{-2}$ m is

$$h = \boxed{8.46 \text{ m}}$$

P14.8 Since the pressure is the same on both sides, $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$, and

in this case, $\dfrac{15\,000 \text{ N}}{200 \text{ cm}^2} = \dfrac{F_2}{3.00 \text{ cm}^2}$ or $F_2 = \boxed{225 \text{ N}}$

P14.9 $F_g = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = 784 \text{ N}$

When the cup barely supports the student, the normal force of the ceiling is zero and the cup is in equilibrium.

$$F_g = F = PA = (1.013 \times 10^5 \text{ Pa})A$$

$$A = \frac{F_g}{P} = \frac{784 \text{ N}}{1.013 \times 10^5 \text{ Pa}} = \boxed{7.74 \times 10^{-3} \text{ m}^2}$$

P14.10 The pressure on the bottom due to the water is $P_b = \rho g z = 1.96 \times 10^4$ Pa.

(a) The force exerted by the water on the bottom is then

$$F_b = P_b A = (1.96 \times 10^4 \text{ Pa})(30.0 \text{ m})(10.0 \text{ m})$$
$$= \boxed{5.88 \times 10^6 \text{ N down}}$$

Pressure varies with depth. On a strip of height dz and length L, the force is $dF = PdA = PLdz = \rho gzLdz$, which gives the integral

$$F = \int_0^h \rho g z L dz = \frac{1}{2}\rho g L h^2 = \left(\frac{1}{2}\rho g h\right) L h = P_{\text{average}} A$$

(b) On each end,

$$F = P_{\text{average}} A = (9.80 \times 10^3 \text{ Pa})(20.0 \text{ m}^2) = \boxed{196 \text{ kN outward}}$$

(c) On the side,

$$F = P_{\text{average}} A = (9.80 \times 10^3 \text{ Pa})(60.0 \text{ m}^2) = \boxed{588 \text{ kN outward}}$$

P14.11 (a) At a depth of 27.5 m, the absolute pressure is

$$\begin{aligned} P = P_0 + \rho g h &= 101.3\times10^3 \text{ Pa} \\ &\quad + \left(1.00\times10^3 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(27.5 \text{ m}) \\ &= \boxed{3.71\times10^5 \text{ Pa}} \end{aligned}$$

(b) The inward force the water will exert on the window is

$$\begin{aligned} F = PA = P\left(\pi r^2\right) &= \left(3.71\times10^5 \text{ Pa}\right)\pi\left(\frac{35.0\times10^{-2} \text{ m}}{2}\right)^2 \\ &= \boxed{3.57\times10^4 \text{ N}} \end{aligned}$$

P14.12 We imagine Superman can produce a perfect vacuum in the straw. Take point 1, at position $y_1 = 0$, to be at the water's surface and point 2, at position y_2 = length of straw, to be at the upper end of the straw. What is the greatest length of straw that will allow Superman to drink? Solve for y_2:

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

$$1.013 \times 10^5 \text{ Pa} + 0 = 0 + (10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)y_2$$

or $y_2 = 10.3$ m.

$$\boxed{\begin{array}{l}\text{The situation is impossible because the longest straw Superman can} \\ \text{use and still get a drink is less than 12.0 m.}\end{array}}$$

***P14.13** The excess water pressure (over air pressure) halfway down is

$$\begin{aligned} P_{\text{gauge}} = \rho g h &= \left(1\,000 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(1.20 \text{ m}) \\ &= 1.18\times10^4 \text{ Pa} \end{aligned}$$

The force on the wall due to the water is

$$\begin{aligned} F = P_{\text{gauge}}A &= \left(1.18\times10^4 \text{ Pa}\right)(2.40 \text{ m})(9.60 \text{ m}) \\ &= \boxed{2.71\times10^5 \text{ N}} \end{aligned}$$

horizontally toward the back of the hole.

***P14.14** We first find the absolute pressure at the interface between oil and water:

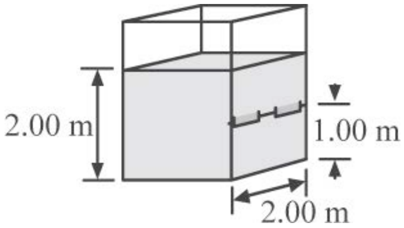
$$\begin{aligned} P_1 &= P_0 + \rho_{\text{oil}} g h_{\text{oil}} \\ &= 1.013\times10^5 \text{ Pa} + \left(700 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)(0.300 \text{ m}) \\ &= 1.03\times10^5 \text{ Pa} \end{aligned}$$

This is the pressure at the top of the water. To find the absolute pressure at the bottom, we use

$$\begin{aligned} P_2 &= P_1 + \rho_{\text{water}} g h_{\text{water}} \\ &= 1.03\times 10^5\ \text{Pa} + \left(10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(0.200\ \text{m}) \\ &= \boxed{1.05\times 10^5\ \text{Pa}} \end{aligned}$$

P14.15 The air outside and water inside both exert atmospheric pressure, so only the excess water pressure *ρgh* counts for the net force. Take a strip of hatch between depth *h* and *h* + *dh*. It feels force

$$dF = PdA = \rho g h(2.00\ \text{m})dh$$



ANS. FIG. P14.15

(a) The total force is

$$F = \int dF = \int_{h=1.00\ \text{m}}^{2.00\ \text{m}} \rho g h(2.00\ \text{m})dh$$

$$F = \rho g(2.00\ \text{m})\left.\frac{h^2}{2}\right|_{1.00\ \text{m}}^{2.00\ \text{m}} = \left(1\ 000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\frac{(2.00\ \text{m})}{2} \times\left[(2.00\ \text{m})^2 - (1.00\ \text{m})^2\right]$$

$$F = \boxed{29.4\ \text{kN}\ (\text{to the right})}$$

(b) The lever arm of *dF* is the distance $(h - 1.00\ \text{m})$ from hinge to strip:

$$\tau = \int d\tau = \int_{h=1.00\ \text{m}}^{2.00\ \text{m}} \rho g h(2.00\ \text{m})(h-1.00\ \text{m})dh$$

$$\tau = \rho g(2.00\ \text{m})\left[\frac{h^3}{3} - (1.00\ \text{m})\frac{h^2}{2}\right]_{1.00\ \text{m}}^{2.00\ \text{m}}$$

$$\tau = \left(1\ 000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m})\left(\frac{7.00\ \text{m}^3}{3} - \frac{3.00\ \text{m}^3}{2}\right)$$

$$\tau = \boxed{16.3\ \text{kN}\cdot\text{m counterclockwise}}$$

P14.16 The air outside and water inside both exert atmospheric pressure, so only the excess water pressure *ρgh* counts for the net force.

(a) At a distance *y* from the top of the water, take a strip of hatch between depth *y* and *y* + *dy*. It feels force

$$dF = PdA = Pwdy = (\rho g y w)dy$$

The total force is

$$F = \int_{d-h}^{d} \rho g w y\,dy = \frac{1}{2}\rho g w y^2\Big|_{d-h}^{d} = \frac{1}{2}\rho g w\left[d^2 - (d-h)^2\right]$$

$$\boxed{F = \frac{1}{2}\rho g w h(2d-h)}$$

(b) The lever arm of dF is the distance $[y - (d - h)]$ from hinge to strip:

$$\tau = \int_{d-h}^{d} \rho g w y\left[y-(d-h)\right]dy = \rho g w \int_{d-h}^{d}\left[y^2 - y(d-h)\right]dy$$

$$= \rho g w\left[\frac{y^3}{3} - \frac{(d-h)y^2}{2}\right]_{d-h}^{d}$$

$$= \frac{\rho g w}{6}\left[2d^3 - 2(d-h)^3 - 3(d-h)d^2 + 3(d-h)^3\right]$$

$$= \frac{\rho g w}{6}\left[2d^3 - 3(d-h)d^2 + (d-h)^3\right]$$

$$= \frac{\rho g w}{6}\left[2d^3 - 3d^3 + 3d^2h + d^3 - 3d^2h + 3dh^2 - h^3\right]$$

$$\tau = \frac{\rho g w}{6}\left[+3dh^2 - h^3\right]$$

$$\boxed{\tau = \frac{1}{2}\rho g w\left(dh^2 - \frac{1}{3}h^3\right)}$$

***P14.17** The fluid in the hydraulic jack is originally exerting the same pressure as the air outside. This pressure P_0 results in zero net force on either piston. For the equilibrium of piston 2 we require

$$500\text{ lb} = (P - P_0)A = (P - P_0)\pi\left(\frac{1.50\text{ in.}}{2}\right)^2$$

Let F_1 represent the force the lever bar exerts on piston 1. Then similarly

$$F_1 = (P - P_0)\pi\left(\frac{0.250\text{ in.}}{2}\right)^2$$

We ignore the weights of the pistons, sliding friction, and the slight difference in fluid pressure P due to the height difference between points 1 and 2. By division,

$$\frac{F_1}{500\text{ lb}} = \left(\frac{0.250\text{ in.}}{1.50\text{ in.}}\right)^2 \quad \rightarrow \quad F_1 = \frac{500\text{ lb}}{36.0}$$

We say the hydraulic lift has an ideal mechanical advantage of 36. Next for the lever bar we ignore weight and friction, assume equilibrium, and take torques about the fixed hinge.

$$\sum \tau = 0 \text{ gives } F_1(2.00\text{ in.}) - F(12.0\text{ in.}) = 0\text{, or } F = \frac{F_1}{6}.$$

The lever has an ideal mechanical advantage of 6. By substitution,

$$F = \frac{500\text{ lb}}{36\cdot 6} = \boxed{2.31\text{ lb}}$$

P14.18 The bell is uniformly compressed, so we can model it with any shape. We choose a sphere of diameter 3.00 m.

The pressure on the ball is given by $P = P_{\text{atm}} + \rho_w gh,$ so the change in pressure on the ball from when it is on the surface of the ocean to when it is at the bottom of the ocean is $\Delta P = \rho_w gh.$

In addition,

$$\Delta V = \frac{-V\Delta P}{B} = \frac{-\rho_w ghV}{B} = -\frac{4\pi\rho_w ghr^3}{3B}$$

where B is the bulk modulus. Substituting,

$$\Delta V = -\frac{4\pi\left(1\ 030\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)\left(1\ 000\text{ m}\right)\left(1.50\text{ m}\right)^3}{3\left(14.0\times 10^{10}\text{ Pa}\right)}$$

$$\Delta V = -1.02\times 10^{-3}\text{ m}^3$$

From $V = \frac{4}{3}\pi r^3 \to dV = 4\pi r^2 dr,$ we use $r = 1.50$ m, set $dV = \Delta V$, and solve for dr:

$$dr = -3.60\times 10^{-5}\text{ m}$$

Therefore, the diameter decreases by $\boxed{0.072\ 1\text{ mm.}}$

Section 14.3 Pressure Measurements

P14.19 A drop of 20.0 mm of mercury is a pressure change of

$$\Delta P = \rho g\Delta h = \left(13.6\times 10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)\left(-20.0\times 10^{-3}\text{ m}\right)$$
$$= -2.66\times 10^3\text{ Pa}$$

$$P = P_0 + \Delta P_0 = \left(1.013 - 0.026\ 6\right)\times 10^5\text{ Pa} = \boxed{0.986\times 10^5\text{ Pa}}$$

P14.20 (a) $P = P_0 + \rho gh$ and the gauge pressure is

$$P - P_0 = \rho gh = (1\ 000\ \text{kg})(9.8\ \text{m/s}^2)(0.160\ \text{m})$$
$$= \boxed{1.57\ \text{kPa}} = (1.57\times10^3\ \text{Pa})\left(\frac{1\ \text{atm}}{1.013\times10^5\ \text{Pa}}\right)$$
$$= \boxed{0.015\ 5\ \text{atm}}$$

It would lift a mercury column to height

$$h = \frac{P - P_0}{\rho g} = \frac{1\ 568\ \text{Pa}}{(13\ 600\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = \boxed{11.8\ \text{mm}}$$

(b) Blockage of the fluid within the spinal column or between the skull and the spinal column would prevent the fluid level from rising.

P14.21 (a) To find the height of the column of wine, we use

$$P_0 = \rho gh$$

then

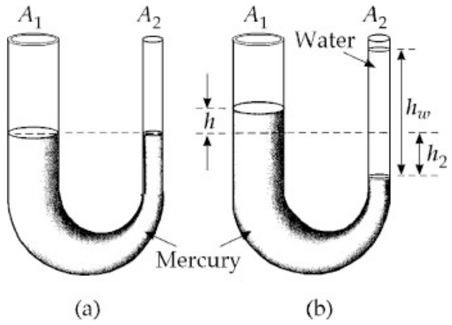
$$h = \frac{P_0}{\rho g}$$
$$= \frac{1.013\times10^5\ \text{Pa}}{(0.984\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)}$$
$$= \boxed{10.5\ \text{m}}$$

ANS. FIG. P14.21

(b) No. The vacuum is not as good because some alcohol and water will evaporate. The equilibrium vapor pressures of alcohol and water are higher than the vapor pressure of mercury.

P14.22 (a) Using the definition of density, we have

$$h_w = \frac{m_{\text{water}}}{A_2\rho_{\text{water}}}$$
$$= \frac{100\ \text{g}}{(5.00\ \text{cm}^2)(1.00\ \text{g/cm}^3)}$$
$$= \boxed{20.0\ \text{cm}}$$



ANS. FIG. P14.22

(b) ANS. FIG. P14.22 (b) represents the situation after the water is added. A volume (A_2h_2) of mercury has been displaced by water in the right tube. The additional volume of mercury now in the left tube is A_1h. Since the total volume of mercury has not changed,

$$A_2h_2 = A_1h \quad \text{or} \quad h_2 = \frac{A_1}{A_2}h \qquad \textbf{[1]}$$

At the level of the mercury–water interface in the right tube, we may write the absolute pressure as:

$$P = P_0 + \rho_{\text{water}}gh_w$$

The pressure at this same level in the left tube is given by

$$P = P_0 + \rho_{\text{Hg}}g(h + h_2) = P_0 + \rho_{\text{water}}gh_w$$

which, using equation [1] above, reduces to

$$\rho_{\text{Hg}}h\left[1 + \frac{A_1}{A_2}\right] = \rho_{\text{water}}h_w$$

or $$h = \frac{\rho_{\text{water}}h_w}{\rho_{\text{Hg}}(1 + A_1/A_2)}.$$

Thus, the level of mercury has risen a distance of

$$h = \frac{(1.00\ \text{g/cm}^3)(20.0\ \text{cm})}{(13.6\ \text{g/cm}^3)(1 + 10.0/5.00)}$$

$\boxed{h = 0.490\ \text{cm}}$ above the original level.

P14.23 (a) We can directly write the bottom pressure as $P = P_0 + \rho gh$, or we can say that the bottom of the tank must support the weight of the water:

$$PA - P_0A = m_{\text{water}}g = \rho Vg = \rho Ahg$$

which gives again

$$P = P_0 + \rho gh$$

The absolute pressure at depth $h = 1.50$ m is

$$P = P_0 + \rho gh = 101.3\ \text{kPa} + (1\ 000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.50\ \text{m})$$

$$= \boxed{116\ \text{kPa}}$$

(b) Now the bottom of the tank must support the weight of the whole contents. Before the people enter, $P = 116$ kPa. Afterwards,

$$\Delta P = \frac{Mg}{A} = \frac{(150\text{ kg})(9.80\text{ m/s}^2)}{\pi(3.00\text{ m})^2} = \boxed{52.0\text{ Pa}}$$

P14.24 (a) We can directly write the bottom pressure as $P = P_0 + \rho gh$, or we can say that the bottom of the tank must support the weight of the water:

$$PA - P_0 A = m_{\text{water}} g = \rho Vg = \rho Ahg$$

which gives again

$$\boxed{P = P_0 + \rho gh}$$

(b) Now, the bottom of the tank must support the weight of the whole contents:

$$P_b A - P_0 A = m_{\text{water}} g + Mg = \rho Vg + Mg = \rho Ahg + Mg$$

and this gives

$$P_b = P_0 + \rho hg + Mg/A$$

Then

$$\Delta P = P_b - P = \boxed{\frac{Mg}{A}}$$

Section 14.4 Buoyant Forces and Archimedes's Principle

P14.25 At equilibrium $\sum F = 0$ or $F_{\text{app}} + mg = B$,

where B is the buoyant force.

The applied force is $F_{\text{app}} = B - mg$,

where $B = V(\rho_{\text{water}})g$

and $m = V\rho_{\text{ball}}$

So, $F_{\text{app}} = Vg(\rho_{\text{water}} - \rho_{\text{ball}}) = \frac{4}{3}\pi r^3 g(\rho_{\text{water}} - \rho_{\text{ball}})$:

$$F_{\text{app}} = \frac{4}{3}\pi(1.90\times10^{-2}\text{ m})^3(9.80\text{ m/s}^2)(10^3\text{ kg/m}^3 - 84.0\text{ kg/m}^3)$$
$$= \boxed{0.258\text{ N down}}$$

P14.26 Refer to Figure P14.26. We observe from the left-hand diagram,

$$\sum F_y = 0 \quad \rightarrow \quad T_1 = F_g = m_{\text{object}}g = \rho_{\text{object}}gV_{\text{object}}$$

and from the right-hand diagram,

$$\sum F_y = 0 \quad \rightarrow \quad T_2 + B = F_g \quad \rightarrow \quad T_2 + B = T_1$$

which gives

$$T_2 - T_1 = B$$

where the buoyant force is

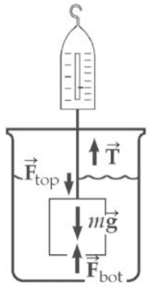
$$B = m_{\text{water}}g = \rho_{\text{w}}V_{\text{object}}g$$

Now the density of the object is

$$\rho_{\text{object}} = \frac{m_{\text{object}}}{V_{\text{object}}} = \frac{T_1/g}{B/(\rho_w g)} = \frac{\rho_w T_1}{B}$$

$$\rho_{\text{object}} = \frac{\rho_w T_1}{T_1 - T_2} = \frac{(1\ 000\ \text{kg/m}^3)(5.00\ \text{N})}{1.50\ \text{N}} = \boxed{3.33 \times 10^3\ \text{kg/m}^3}$$

P14.27 (a) We start with $P = P_0 + \rho gh$.



ANS. FIG. P14.27

Taking $P_0 = 1.013 \times 10^5\ \text{N/m}^2$,

$\rho_{\text{water}} = 1\ 000\ \text{kg/m}^3$, and $h = 5.00$ cm,

we find $P_{\text{top}} = 1.017\ 9 \times 10^5\ \text{N/m}^2$.

For $h = 17.0$ cm, we get

$$P_{\text{bot}} = 1.029\ 7 \times 10^5\ \text{N/m}^2$$

Since the areas of the top and bottom are

$$A = (0.100\ \text{m})^2 = 10^{-2}\ \text{m}^2$$

we find

$$F_{\text{top}} = P_{\text{top}}A = \boxed{1.017\ 9 \times 10^3\ \text{N}}$$

and $F_{\text{bot}} = \boxed{1.029\ 7 \times 10^3\ \text{N}}$.

(b) The tension in the string is the scale reading:

$$T = Mg - B$$

where

$$B = \rho_w Vg = (10^3\ \text{kg/m}^3)(1.20 \times 10^{-3}\ \text{m}^3)(9.80\ \text{m/s}^2) = 11.8\ \text{N}$$

and

$$Mg = (10.0\ \text{kg})(9.80\ \text{m/s}^2) = 98.0\ \text{N}$$

Therefore,

$$T = Mg - B = 98.0\ \text{N} - 11.8\ \text{N} = \boxed{86.2\ \text{N}}$$

(c) $F_{\text{bot}} - F_{\text{top}} = (1.0297 - 1.0179) \times 10^3\ \text{N} = \boxed{11.8\ \text{N}}$

which is equal to B found in part (b).

P14.28 (a) The balloon is nearly in equilibrium:

$$\sum F_y = ma_y \Rightarrow B - \left(F_g\right)_{\text{helium}} - \left(F_g\right)_{\text{payload}} = 0$$

or $$\rho_{\text{air}} gV - \rho_{\text{helium}} gV - m_{\text{payload}} g = 0$$

This reduces to

$$\begin{aligned} m_{\text{payload}} &= (\rho_{\text{air}} - \rho_{\text{helium}})V \\ &= (1.29\ \text{kg/m}^3 - 0.179\ \text{kg/m}^3)(400\ \text{m}^3) \\ m_{\text{payload}} &= \boxed{444\ \text{kg}} \end{aligned}$$

(b) Similarly,

$$\begin{aligned} m_{\text{payload}} &= (\rho_{\text{air}} - \rho_{\text{hydrogen}})V \\ &= (1.29\ \text{kg/m}^3 - 0.089\ 9\ \text{kg/m}^3)(400\ \text{m}^3) \\ m_{\text{payload}} &= \boxed{480\ \text{kg}} \end{aligned}$$

The surrounding air does the lifting, nearly the same for the two balloons.

P14.29 (a) The cube has sides of length L. When floating, the horizontal top surface lies a distance h above the water's surface. The buoyant force supports the weight of the block:

$$B = \rho_{\text{water}} V_{\text{object}} g = \rho_{\text{water}} L^2 (L - h) g = \rho_{\text{wood}} L^3 g$$

Solve for h:

$$\begin{aligned} h &= L - L(\rho_{\text{wood}} / \rho_{\text{water}}) = L(1 - \rho_{\text{wood}} / \rho_{\text{water}}) \\ &= (20.0\ \text{cm})(1 - 0.650) = \boxed{7.00\ \text{cm}} \end{aligned}$$

(b) The buoyant force supports the weight of both blocks:

$$B = F_g + Mg, \text{ where } M = \text{mass of lead}$$

$$\rho_{\text{water}} L^3 g = \rho_{\text{wood}} L^3 g + Mg \quad \rightarrow \quad M = (\rho_{\text{water}} - \rho_{\text{wood}}) L^3$$

$$M = (1.00\ \text{kg/m}^3 - 0.650\ \text{kg/m}^3)(20.0\ \text{m})^3 = \boxed{2.80\ \text{kg}}$$

P14.30 By Archimedes's principle, the weight of the 50 planes is equal to the weight of a horizontal slice of water 11.0 cm thick and circumscribed by the water line:

$$\Delta B = \rho_{\text{water}} g(\Delta V)$$

$$50\left(2.90\times 10^4\ \text{kg}\right)g = \left(1030\ \text{kg/m}^3\right)g\left(0.110\ \text{m}\right)A$$

giving $A = \boxed{1.28\times 10^4\ \text{m}^2}$. The acceleration of gravity does not affect the answer.

P14.31 (a) The buoyant force of glycerin supports the weight of the sphere which is supported by the buoyant force of water.

$$B = \rho_{\text{glycerin}}\left(0.40V\right) = \rho_{\text{water}}\frac{V}{2}$$

$$\boxed{\rho_{\text{glycerin}} = \frac{\rho_{\text{water}}}{2\left(0.40\right)} = \frac{1\,000\ \text{kg/m}^3}{0.80} = 1\,250\ \text{kg/m}^3}$$

(b) The buoyant force from the water supports the weight of the sphere:

$$B = F_g$$

$$B = \rho_{\text{water}}\frac{V}{2} = \rho_{\text{sphere}}V$$

$$\boxed{\rho_{\text{sphere}} = \frac{\rho_{\text{water}}}{2} = 500\ \text{kg/m}^3}$$

P14.32 Constant velocity implies zero acceleration, which means that the submersible is in equilibrium under the gravitational force, the upward buoyant force, and the upward resistance force:

$$\sum F_y = ma_y = 0:$$

$$-\left(1.20\times 10^4\ \text{kg} + m\right)g + \rho_w gV + 1\,100\ \text{N} = 0$$

where m is the mass of the added water and V is the sphere's volume. Substituting,

$$1.20\times 10^4\ \text{kg} + m$$

$$= \left(1.03\times 10^3\ \text{kg/m}^3\right)\left[\frac{4}{3}\pi\left(1.50\ \text{m}\right)^3\right] + \frac{1\,100\ \text{N}}{9.80\ \text{m/s}^2}$$

so $m = \boxed{2.67\times 10^3\text{ kg}}$.

***P14.33** (a) While the system floats, $B = w_{\text{total}} = w_{\text{block}} + w_{\text{steel}}$, or

$$\rho_w \cancel{g} V_{\text{submerged}} = \rho_b \cancel{g} V_b + m_{\text{steel}}\cancel{g}$$

When $m_{\text{steel}} = 0.310\text{ kg}$, $V_{\text{submerged}} = V_b = 5.24\times 10^{-4}\text{ m}^3$ giving

$$\rho_b = \frac{\rho_w V_b - m_{\text{steel}}}{V_b} = \rho_w - \frac{m_{\text{steel}}}{V_b}$$

$$= 1.00\times 10^3\text{ kg/m}^3 - \frac{0.310\text{ kg}}{5.24\times 10^{-4}\text{ m}^3}$$

$$= \boxed{408\text{ kg/m}^3}$$

(b) If the total weight of the block + steel system is reduced, by having $m_{\text{steel}} < 0.310$ kg, a smaller buoyant force is needed to allow the system to float in equilibrium. $\boxed{\text{Thus, the block will displace a smaller volume of water and will be only partially submerged in the water.}}$

(c) The block is fully submerged when $m_{\text{steel}} = 0.310$ kg. The mass of the steel object can increase slightly above this value without causing it and the block to sink to the bottom. As the mass of the steel object is gradually increased above 0.310 kg, the steel object begins to submerge, displacing additional water, and providing a slight increase in the buoyant force. With a density of about eight times that of water, the steel object will be able to displace approximately 0.310 kg/8 = 0.039 kg of additional water before it becomes fully submerged. At this point, the steel object will have a mass of about 0.349 kg and will be unable to displace any additional water. Any further increase in the mass of the object causes it and the block to sink to the bottom. In conclusion,

$$\boxed{\text{the block + steel system will sink if } m_{\text{stee}} \geq 0.350\text{ kg.}}$$

P14.34 (a) $\sum F_y = 0$: $B - T - F_g = 0 \rightarrow B - 15.0\text{ N} - 10.0\text{ N} = 0$

$\boxed{B = 25.0\text{ N}}$

(b) The oil pushes $\boxed{\text{horizontally inward}}$ on each side of the block.

(c) The string tension increases. The water under the block pushes up on the block more strongly than before because the water is under higher pressure due to the weight of the oil above it.

(d) The pressure of the oil's weight on the water is $P = \rho_{\text{oil}}gh$, where h is the height of the oil. This pressure is transmitted to the bottom of the block, so the extra upward force on the block is $F_{\text{oil}} = PA = \rho_{\text{oil}}ghA = \rho_{\text{oil}}g\Delta V$, where $\Delta V = hA$ is the volume of the block below the top surface of the oil.

The force from the oil and the buoyant force of water balance the tension and the weight of the block:

$$\begin{aligned}\sum F_y = 0:\quad & F_{\text{oil}} + B - T - F_g = 0\\ & F_{\text{oil}} + 25.0\text{ N} - 60.0\text{ N} - 15.0\text{ N} = 0\\ & F_{\text{oil}} = 50.0\text{ N}\end{aligned}$$

The ratio of F_{oil} and B are

$$\frac{F_{\text{up}}}{B} = \frac{\rho_{\text{oil}}g\Delta V}{\rho_{\text{water}}g(V/4)} \rightarrow \frac{\Delta V}{V} = \frac{F_{\text{up}}}{4B}\frac{\rho_{\text{water}}}{\rho_{\text{oil}}}$$

$$\frac{\Delta V}{V} = \frac{50.0\text{ N}}{4(25.0\text{ N})}\frac{1\,000\text{ kg/m}^3}{800\text{ kg/m}^3} = 0.625$$

The additional fraction of the block's volume below the top surface of the oil is $\boxed{62.5\%.}$

P14.35 (a) Since the balloon is fully submerged in air, $V_{\text{submerged}} = V_b = 325\text{ m}^3$, and

$$\begin{aligned}B &= \rho_{\text{air}}gV_b = (1.20\text{ kg/m}^3)(9.80\text{ m/s}^2)(325\text{ m}^3)\\ &= \boxed{3.82\times10^3\text{ N}}\end{aligned}$$

(b) $$\begin{aligned}\sum F_y &= B - w_b - w_{\text{He}} = B - m_bg - \rho_{\text{He}}gV_b = B - (m_b + \rho_{\text{He}}V)g\\ &= 3.82\times10^3\text{ N}\\ &\quad -\left[226\text{ kg} + (0.179\text{ kg/m}^3)(325\text{ m}^3)\right](9.80\text{ m/s}^2)\\ &= \boxed{+1.04\times10^3\text{ N}}\end{aligned}$$

Since $\sum F_y = ma_y > 0$, a_y will be positive (upward), and $\boxed{\text{the balloon rises}}$.

(c) If the balloon and load are in equilibrium,

$$\sum F_y = (B - w_b - w_{\text{He}}) - w_{\text{load}} = 0$$

and

$$w_{\text{load}} = \left(B - w_b - w_{\text{He}}\right) = 1.04 \times 10^3\ \text{N}$$

Thus, the mass of the load is

$$m_{\text{load}} = \frac{w_{\text{load}}}{g} = \frac{1.04 \times 10^3\ \text{N}}{9.80\ \text{m/s}^2} = \boxed{106\ \text{kg}}$$

P14.36 Let A represent the horizontal cross-sectional area of the rod, which we presume to be constant. The rod is in equilibrium:

$$\sum F_y = 0: \quad -mg + B = 0 = -\rho_0 V_{\text{whole rod}} g + \rho_{\text{fluid}} V_{\text{immersed}} g$$

$$\rho_0 A L g = \rho A\left(L - h\right) g$$

The density of the liquid is $\rho = \dfrac{\rho_0 L}{L - h}$.

P14.37 We use the result of Problem 14.36. For the rod floating in a liquid of density 0.98 g/cm^3,

$$\rho = \rho_0 \frac{L}{L - h}$$

$$0.98\ \text{g/cm}^3 = \frac{\rho_0 L}{\left(L - 0.2\ \text{cm}\right)}$$

$$\left(0.98\ \text{g/cm}^3\right)L - \left(0.98\ \text{g/cm}^3\right)0.2\ \text{cm} = \rho_0 L$$

For floating in the dense liquid,

$$1.14\ \text{g/cm}^3 = \frac{\rho_0 L}{\left(L - 1.80\ \text{cm}\right)}$$

$$\left(1.14\ \text{g/cm}^3\right)L - \left(1.14\ \text{g/cm}^3\right)\left(1.80\ \text{cm}\right) = \rho_0 L$$

(a) By substitution, and suppressing units,

$$1.14L - 1.14(1.80) = 0.98L - 0.200(0.98)$$

$$0.16L = 1.856$$

$$L = \boxed{11.6\ \text{cm}}$$

(b) Substituting back,

$$\left(0.98\ \text{g/cm}^3\right)\left(11.6\ \text{cm} - 0.200\ \text{cm}\right) = \rho_0\left(11.6\ \text{cm}\right)$$

$$\rho_0 = \boxed{0.963\ \text{g/cm}^3}$$

(c) $\boxed{\text{No; the density } \rho \text{ is not linear in } h.}$

P14.38 (a) We can estimate the total buoyant force of the 600 toy balloons as

$$\begin{aligned} B_{\text{total}} &= 600 \cdot B_{\substack{\text{single}\\ \text{balloon}}} = 600\left(\rho_{\text{air}} g V_{\text{balloon}}\right) \\ &= 600\left[\rho_{\text{air}} g\left(\frac{4\pi}{3} r^3\right)\right] \\ &= 600\left[\left(1.20\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\frac{4\pi}{3}(0.50\ \text{m})^3\right] \\ &= 3.7 \times 10^3\ \text{N} = \boxed{3.7\ \text{kN}} \end{aligned}$$

(b) We estimate the net upward force by applying Newton's second law in the vertical direction:

$$\begin{aligned} \sum F_y &= B_{\text{total}} - m_{\text{total}} g \\ &= 3.7 \times 10^3\ \text{N} - 600(0.30\ \text{kg})\left(9.8\ \text{m/s}^2\right) \\ &= 1.9 \times 10^3\ \text{N} = \boxed{1.9\ \text{kN}} \end{aligned}$$

This net force was sufficient to lift Ashpole, his parachute, and other supplies.

(c) $\boxed{\text{Atmospheric pressure at this high altitude is much lower than at Earth's surface}}$, so the balloons expanded and eventually burst.

P14.39 We assume that the mass of the balloon envelope is included in the 400 kg. We assume that the 400-kg total load is much denser than air and so has negligible volume compared to the helium. At $z = 8\,000$ m, the density of air is

$$\begin{aligned} \rho_{\text{air}} &= \rho_0 e^{-z/8\,000} = (1.20\ \text{kg/m}^3)e^{-1} \\ &= (1.20\ \text{kg/m}^3)(0.368) \\ &= 0.441\ \text{kg/m}^3 \end{aligned}$$

Think of the balloon reaching equilibrium at this height. The weight of its payload is $Mg = (400\ \text{kg})(9.80\ \text{m/s}^2) = 3\,920$ N. The weight of the helium in it is $mg = \rho_{\text{He}} Vg$.

$$\sum F_y = 0 \quad \rightarrow \quad +\rho_{\text{air}} Vg - Mg - \rho_{\text{He}} Vg = 0$$

Solving,

$$(\rho_{\text{air}} - \rho_{\text{He}})V = M$$

and

$$V = \frac{M}{\rho_{\text{air}} - \rho_{\text{He}}} = \frac{400\ \text{kg}}{(0.441 - 0.179)\ \text{kg/m}^3} = \boxed{1.52 \times 10^3\ \text{m}^3}$$

Section 14.5 Fluid Dynamics
Section 14.6 Bernoulli's Equation

P14.40 (a) The cross-sectional area of the hose is

$$A = \pi r^2 = \pi d^2/4 = \pi(2.74\ \text{cm})^2/4$$

and the volume flow rate (volume per unit time) is

$$Av = 25.0\ \text{L}/1.50\ \text{min}$$

Thus,

$$\begin{aligned} v &= \frac{25.0\ \text{L}/1.50\ \text{min}}{A} \\ &= \left(\frac{25.0\ \cancel{\text{L}}}{1.50\ \cancel{\text{min}}}\right)\left[\frac{4}{\pi\cdot(2.74)^2\ \text{cm}^2}\right]\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)\left(\frac{10^3\ \text{cm}^3}{1\ \cancel{\text{L}}}\right) \\ &= (47.1\ \text{cm/s})\left(\frac{1\ \text{m}}{10^2\ \text{cm}}\right) = \boxed{0.471\ \text{m/s}} \end{aligned}$$

(b) $\dfrac{A_2}{A_1} = \left(\dfrac{\pi d_2^2}{4}\right)\left(\dfrac{4}{\pi d_1^2}\right) = \left(\dfrac{d_2}{d_1}\right)^2 = \left(\dfrac{1}{3}\right)^2 = \dfrac{1}{9}$ or $A_2 = \dfrac{A_1}{9}$

Then from the equation of continuity, $A_2 v_2 = A_1 v_1$, we find

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 = 9(0.471\ \text{m/s}) = \boxed{4.24\ \text{m/s}}$$

P14.41 Assuming the top is open to the atmosphere, then

$$P_1 = P_0$$

Note $P_2 = P_0$. The water pushes on the air just as hard as the air pushes on the water.

$$\text{Flow rate} = 2.50\times10^{-3}\ \text{m}^3/\text{min} = 4.17\times10^{-5}\ \text{m}^3/\text{s}$$

(a) $A_1 >> A_2$ so $v_1 << v_2$

Assuming $v_1 = 0$,

$$P_1 + \frac{\rho v_1^2}{2} + \rho g y_1 = P_2 + \frac{\rho v_2^2}{2} + \rho g y_2$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2(9.80\ \text{m/s}^2)(16.0\ \text{m})} = \boxed{17.7\ \text{m/s}}$$

(b) $\text{Flow rate} = A_2 v_2 = \left(\dfrac{\pi d^2}{4}\right)(17.7\ \text{m/s}) = 4.17\times10^{-5}\ \text{m}^3/\text{s}$

$$d = \boxed{1.73\times10^{-3}\ \text{m}} = 1.73\ \text{mm}$$

P14.42 (a) The mass flow rate and the volume flow rate are constant:

$$\rho A_1 v_1 = \rho A_2 v_2 \quad \rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2$$

Substituting,

$$(3.00\ \text{cm})^2 v_1 = (1.50\ \text{cm})^2 v_2 \quad \rightarrow \quad v_2 = 4v_1$$

For ideal flow,

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$1.75\times10^4\ \text{Pa} + 0 + \frac{1}{2}\left(1\,000\ \text{kg/m}^3\right)\left(v_1\right)^2$$
$$= 1.20\times10^4\ \text{Pa} + (1000)(9.8)(0.250)\ \text{Pa}$$
$$+ \frac{1}{2}\left(1\,000\ \text{kg/m}^3\right)(4v_1)^2$$

Solving for v_1 gives

$$v_1 = \sqrt{\frac{3\ 050\ \text{Pa}}{7\,500\ \text{kg/m}^3}} = \boxed{0.638\ \text{m/s}}$$

(b) From part (a), we have

$$v_2 = 4v_1 = \boxed{2.55\ \text{m/s}}$$

(c) The volume flow rate is

$$\pi r_1^2 v_1 = \pi(0.030\ 0\ \text{m})^2(0.638\ \text{m/s}) = \boxed{1.80\times10^{-3}\ \text{m}^3/\text{s}}$$

P14.43 The volume flow rate is

$$\frac{\Delta V}{\Delta t} = \frac{125\ \text{cm}^3}{16.3\ \text{s}} = 7.67\ \text{cm}^3/\text{s} = Av_1$$

where $d = 0.96$ cm and $A = \pi r^2 = 0.724\ \text{cm}^2$. The speed at the top of the falling column is

$$v_1 = \frac{\Delta V/\Delta t}{A} = \frac{7.67\ \text{cm}^3/\text{s}}{0.724\ \text{cm}^3} = 10.6\ \text{cm/s}$$

Take point 2 at 13 cm below:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\begin{aligned} &P_0 + \left(1\,000 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)0.130 \text{ m} \\ &\qquad + \frac{1}{2}\left(1\,000 \text{ kg/m}^3\right)\left(0.106 \text{ m/s}\right)^2 \\ &= P_0 + 0 + \frac{1}{2}\left(1\,000 \text{ kg/m}^3\right)v_2^2 \end{aligned}$$

solving for the velocity gives

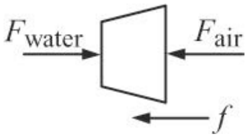
$$v_2 = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(0.130 \text{ m}\right) + \left(0.106 \text{ m/s}\right)^2} = 1.60 \text{ m/s}$$

The volume flow rate is constant:

$$7.67 \text{ cm}^3/\text{s} = \pi\left(\frac{d}{2}\right)^2 160 \text{ cm/s}$$

$$d = \boxed{0.247 \text{ cm}}$$

P14.44 Take point ① at the free surface of the water in the tank and ② inside the nozzle.



ANS. FIG. P14.44

(a) With the cork in place,

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

becomes

$$P_0 + \left(1\,000 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)\left(7.50 \text{ m}\right) + 0 = P_2 + 0 + 0$$

$$P_2 - P_0 = 7.35 \times 10^4 \text{ Pa}$$

For the stopper,

$$\sum F_x = 0$$

$$F_{\text{water}} - F_{\text{air}} - f = 0$$

$$P_2 A - P_0 A = f$$

$$f = \left(7.35 \times 10^4 \text{ Pa}\right)\pi\left(0.011\,0 \text{ m}\right)^2 = \boxed{27.9 \text{ N}}$$

(b) Now Bernoulli's equation gives

$$P_0 + 7.35 \times 10^4 \text{ Pa} + 0 = P_0 + 0 + \frac{1}{2}\left(1\,000 \text{ kg/m}^3\right)v_2^2$$

$$v_2 = 12.1 \text{ m/s}$$

The quantity leaving the nozzle in 2 h is

$$\begin{aligned}\rho V &= \rho A v_2 t\\ &= \left(1\ 000\ \text{kg/m}^3\right)\pi\left(0.011\ 0\ \text{m}\right)^2\left(12.1\ \text{m/s}\right)\left(7\ 200\ \text{s}\right)\\ &= \boxed{3.32\times10^4\ \text{kg}}\end{aligned}$$

(c) Take point 1 in the wide hose and 2 just outside the nozzle. Applying the continuity equation:

$$A_1v_1 = A_2v_2$$

$$\pi\left(\frac{6.60\ \text{cm}}{2}\right)^2 v_1 = \pi\left(\frac{2.20\ \text{cm}}{2}\right)^2\left(12.1\ \text{m/s}\right)$$

$$v_1 = \frac{12.1\ \text{m/s}}{9} = 1.35\ \text{m/s}$$

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$\begin{aligned}P_1 + 0 + \frac{1}{2}\left(1\ 000\ \text{kg/m}^3\right)\left(1.35\ \text{m/s}\right)^2\\ = P_0 + 0 + \frac{1}{2}\left(1\ 000\ \text{kg/m}^3\right)\left(12.1\ \text{m/s}\right)^2\end{aligned}$$

$$P_1 - P_0 = 7.35\times10^4\ \text{Pa} - 9.07\times10^2\ \text{Pa} = \boxed{7.26\times10^4\ \text{Pa}}$$

P14.45 (a) Between sea surface and clogged hole:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$1\ \text{atm} + 0 + \left(1\ 030\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(2.00\ \text{m}\right) = P_2 + 0 + 0$$

$$P_2 = 1\ \text{atm} + 20.2\ \text{kPa}$$

The air on the back of his hand pushes opposite the water, so the net force on his hand is

$$F = PA = \left(20.2\times10^3\ \text{N/m}^2\right)\left(\frac{\pi}{4}\right)\left(1.2\times10^{-2}\ \text{m}\right)^2$$

$$F = \boxed{2.28\ \text{N}}\ \text{toward Holland}$$

(b) Now, Bernoulli's equation gives

$$1\ \text{atm} + 0 + 20.2\ \text{kPa} = 1\ \text{atm} + \frac{1}{2}\left(1\ 030\ \text{kg/m}^3\right)v_2^2 + 0$$

$$v_2 = 6.26\ \text{m/s}$$

The volume rate of flow is

$$A_2 v_2 = \frac{\pi}{4}\left(1.2\times10^{-2}\ \text{m}\right)^2\left(6.26\ \text{m/s}\right) = 7.08\times10^{-4}\ \text{m}^3/\text{s}$$

One acre–foot is $4\ 047\ \text{m}^2 \times 0.304\ 8\ \text{m} = 1\ 234\ \text{m}^3$.

Requiring $\dfrac{1\ 234\ \text{m}^3}{7.08\times10^{-4}\ \text{m}^3/\text{s}} = \boxed{1.74\times10^6\ \text{s}} = 20.2$ days.

P14.46 (a) Power is the rate of energy flow as a function of time:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta mgh}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)gh = Rgh$$

(b) The power delivered by the Grand Coulee dam is

$$P_{\text{EL}} = 0.85\left(8.50\times10^5\ \text{kg/s}\right)\left(9.80\ \text{m/s}^2\right)\left(87.0\ \text{m}\right) = \boxed{616\ \text{MW}}$$

P14.47 (a) The cross-sectinal area is the same everywhere, so the speed is the same everywhere:

$$\left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{river}} = \left(P + \frac{1}{2}\rho v^2 + \rho g y\right)_{\text{rim}}$$

$$P + 0 + \rho g\left(564\ \text{m}\right) = 1\ \text{atm} + 0 + \rho g\left(2\ 096\ \text{m}\right)$$

$$P = 1\ \text{atm} + \left(1\ 000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(1\ 532\ \text{m}\right)$$

$$= \boxed{1\ \text{atm} + 15.0\ \text{MPa}}$$

(b) The volume flow rate is $4\ 500\ \text{m}^3/d = Av = \dfrac{\pi d^2 v}{4}$.

$$v = \left(4\ 500\ \text{m}^3/\text{d}\right)\left(\frac{1\ \text{d}}{86\ 400\ \text{s}}\right)\left(\frac{4}{\pi\left(0.150\ \text{m}\right)^2}\right) = \boxed{2.95\ \text{m/s}}$$

P14.48 (a) The volume flow rate is the same at the two points: $A_1 v_1 = A_2 v_2$:

$$\pi\left(1\ \text{cm}\right)^2 v_1 = \pi\left(0.5\ \text{cm}\right)^2 v_2 \quad \rightarrow \quad v_2 = 4v_1$$

We assume the tubes are at the same elevation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_1 - P_2 = \Delta P = \frac{1}{2}\rho(4v_1)^2 + 0 - \frac{1}{2}\rho v_1^2$$

$$\Delta P = \frac{1}{2}(850\ \text{kg/m}^3)\ 15 v_1^2$$

$$v_1 = \left(0.012\ 5\ \text{m/s}\right)\sqrt{\Delta P}$$

where the pressure is in pascals.

The volume flow rate is

$$\pi(0.01\text{ m})^2(0.0125\text{ m/s})\sqrt{\Delta P}$$
$$= \boxed{(3.93\times10^{-6}\text{ m}^3/\text{s})\sqrt{\Delta P},\quad \text{where } \Delta P \text{ is in pascals}}$$

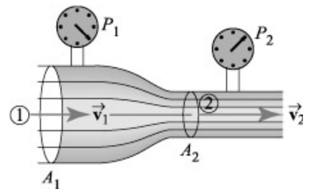
(b) For $\Delta P = 6.00$ kPa,

$$(3.93\times10^{-6}\text{ m}^3/\text{s})\sqrt{6\ 000\text{ Pa}} = \boxed{0.305\text{ L/s}}$$

(c) With pressure difference 2 times larger, the flow rate is larger by the square root of 2:

$$\sqrt{2}(0.305\text{ L/s}) = \boxed{0.431\text{ L/s}}$$

P14.49 (a) Since the tube is horizontal, $y_1 = y_2$ and the gravity terms in Bernoulli's equation cancel, leaving



ANS. FIG. P14.49

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or

$$v_2^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho} = \frac{2(1.20\times10^3\text{ Pa})}{7.00\times10^2\text{ kg/m}^3}$$

and

$$v_2^2 - v_1^2 = 3.43\text{ m}^2/\text{s}^2 \qquad \textbf{[1]}$$

From the continuity equation, $A_1v_1 = A_2v_2$, we find

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{2.40\text{ cm}}{1.20\text{ cm}}\right)^2 v_1$$

or

$$v_2 = 4v_1 \qquad \textbf{[2]}$$

Substituting equation [2] into [1] yields $15v_1^2 = 3.43\text{ m}^2/\text{s}^2$ and

$$v_1 = 0.478\text{ m/s}$$

Then, equation [2] gives

$$v_2 = 4(0.478\text{ m/s}) = \boxed{1.91\text{ m/s}}$$

(b) The volume flow rate is

$$A_1 v_1 = A_2 v_2 = \left(\pi r_2^2\right) v_2 = \pi\left(1.20\times10^{-2}\ \text{m}\right)^2\left(1.91\ \text{m/s}\right)$$
$$= \boxed{8.64\times10^{-4}\ \text{m}^3/\text{s}}$$

P14.50 (a) For upward flight of a water-drop projectile from geyser vent to fountain–top, $v_{yf}^2 = v_{yi}^2 + 2a_y\Delta y$.

Then $0 = v_i^2 + 2\left(-9.80\ \text{m/s}^2\right)\left(+40.0\ \text{m}\right)$ and $v_i = \boxed{28.0\ \text{m/s}}$.

(b) Between geyser vent and fountain-top:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Air is so low in density that very nearly $P_1 = P_2 = 1$ atm. Then,

$$\frac{1}{2}v_1^2 + 0 = 0 + \left(9.80\ \text{m/s}^2\right)\left(40.0\ \text{m}\right)$$

$$v_1 = \boxed{28.0\ \text{m/s}}$$

(c) $\boxed{\text{The answers agree precisely. The models are consistent with each other.}}$

(d) Between the chamber and the fountain-top:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_1 + 0 + \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(-175\ \text{m}\right)$$
$$= P_0 + 0 + \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(+40.0\ \text{m}\right)$$
$$P_1 - P_0 = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(215\ \text{m}\right) = \boxed{2.11\ \text{MPa}}$$

Section 14.7 Other Applications of Fluid Dynamics

P14.51 The assumption of incompressibility is surely unrealistic, but allows an estimate of the speed. From Bernoulli's equation,

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$1.00\ \text{atm} + 0 + 0 = 0.287\ \text{atm} + 0 + \frac{1}{2}\left(1.20\ \text{kg/m}^3\right)v_2^2$$

solving for the velocity gives

$$v_2 = \sqrt{\frac{2(1.00 - 0.287)(1.013 \times 10^5\ \text{N/m}^2)}{1.20\ \text{kg/m}^3}} = \boxed{347\ \text{m/s}}$$

P14.52 (a) Force balance requires that

$$Mg = (P_1 - P_2)A$$

$$\frac{(16\,000\ \text{kg})(9.80\ \text{m/s}^2)}{2(40.0\ \text{m}^2)} = 7.00 \times 10^4\ \text{Pa} - P_2$$

$$\therefore P_2 = 7.0 \times 10^4\ \text{Pa} - 0.196 \times 10^4\ \text{Pa} = \boxed{6.80 \times 10^4\ \text{Pa}}$$

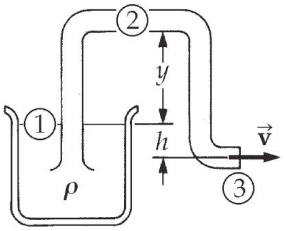
(b) Higher. With the inclusion of another upward force due to deflection of air downward, the pressure difference does not need to be as great to keep the airplane in flight.

P14.53 (a) We use Bernoulli's equation,

$$P_0 + \rho g h + 0 = P_0 + 0 + \frac{1}{2}\rho v_3^2$$

which gives $v_3 = \sqrt{2gh}$.

If $h = 1.00$ m, then $v_3 = \boxed{4.43\ \text{m/s}}$.



ANS. FIG. P14.53

(b) Again, from Bernoulli's equation,

$$P + \rho g y + \frac{1}{2}\rho v_2^2 = P_0 + 0 + \frac{1}{2}\rho v_3^2$$

Since $v_2 = v_3$,

$$P = P_0 - \rho g y$$

Since $P \geq 2.3$ kPa, the greatest possible siphon height is given by

$$y \leq \frac{P_0 - P}{\rho g} = \frac{1.013 \times 10^5\ \text{Pa} - 2.30 \times 10^3\ \text{Pa}}{(10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = \boxed{10.1\ \text{m}}$$

P14.54 Take points 1 and 2 in the air just inside and outside the window pane.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

$$P_0 + 0 = P_2 + \frac{1}{2}(1.20\ \text{kg/m}^3)(11.2\ \text{m/s})^2 \rightarrow P_2 = P_0 - 75.3\ \text{Pa}$$

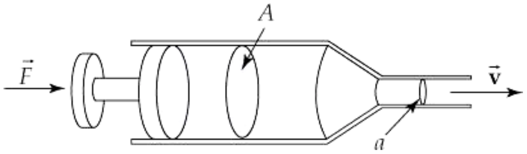
(a) The total force exerted by the air is outward,

$$P_1A - P_2A = P_0A - P_0A + \left(75.3\ \text{N/m}^2\right)(4.00\ \text{m})(1.50\ \text{m})$$
$$= \boxed{452\ \text{N outward}}$$

(b) $$P_1A - P_2A = \frac{1}{2}\rho v_2^2 A = \frac{1}{2}\left(1.20\ \text{kg/m}^3\right)(22.4\ \text{m/s})^2(4.00\ \text{m})(1.50\ \text{m})$$
$$= \boxed{1.81\ \text{kN outward}}$$

P14.55 In the reservoir, the gauge pressure is

$$\Delta P = \frac{2.00\ \text{N}}{2.50\times10^{-5}\ \text{m}^2} = 8.00\times10^4\ \text{Pa}$$



ANS. FIG. P14.55

From the equation of continuity, we have

$$A_1v_1 = A_2v_2$$
$$\left(2.50\times10^{-5}\ \text{m}^2\right)v_1 = \left(1.00\times10^{-8}\ \text{m}^2\right)v_2 \quad \text{so} \quad v_1 = \left(4.00\times10^{-4}\right)v_2$$

Thus, v_1^2 is negligible in comparison to v_2^2. In Bernoulli's equation, $(P_1 - P_2) + \frac{1}{2}\rho v_1^2 + \rho g y_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2$, the term in v_1^2 is essentially zero and the terms in y_1 and y_2 cancel each other. Then,

$$v_2 = \left(\frac{2(P_1 - P_2)}{\rho}\right)^{1/2} = \sqrt{\frac{2(8.00\times10^4\ \text{Pa})}{1\,000\ \text{kg/m}^3}} = \boxed{12.6\ \text{m/s}}$$

Additional Problems

***P14.56** The water exerts a buoyant force on the air, given by

$$B = \rho_{\text{fluid}}gV = \left(1\,000\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(10.0\ \text{L})\left(\frac{1\ \text{m}^3}{10^3\ \text{L}}\right)$$
$$= 98.0\ \text{N up}$$

The weight of the air is

$$F_g = \rho gV = \left(2.40\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(10.0\times10^{-3}\ \text{m}^3\right)$$
$$= 0.235\ \text{N down}$$

To transport the air down at constant speed requires a downward force D in $+98.0\ \text{N} - 0.235\ \text{N} - D = 0$, $D = 97.8$ N, and work

$$W = \vec{\mathbf{D}}\cdot\vec{\mathbf{d}} = (97.8\ \text{N})(10.3\ \text{m})\cos 0° = \boxed{1.01\ \text{kJ}}$$

P14.57 (a) At a depth of 1 000 m,

$$P = P_0 + \rho g h = 1.013\times 10^5\ \text{Pa} + \left(1\,030\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(1\,000\ \text{m}\right)$$

$$P = \boxed{1.02\times 10^7\ \text{Pa}}$$

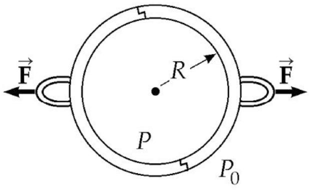
(b) The buoyant force on the submarine at this depth is

$$B = \rho g V = \rho g \frac{4}{3}\pi r^3 = \left(1\,030\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\frac{4}{3}\pi (2.50\ \text{m})^3$$

$$= \boxed{6.61\times 10^5\ \text{N}}$$

P14.58 The pressure on the surface of the two hemispheres is constant at all points, and the force on each element of surface area is directed along the radius of the hemispheres. The applied force along the axis must balance the force on the "effective" area, which is the projection of the actual surface onto a plane perpendicular to the *x* axis, $A = \pi R^2$. Therefore,

$$F = \boxed{\left(P_0 - P\right)\pi R^2}$$



ANS. FIG. P14.58

P14.59 (a) The weight of the ball must be equal to the buoyant force of the water:

$$1.26\ \text{kg}\, g = \rho_{\text{water}} \frac{4}{3}\pi r_{\text{outer}}^3 g$$

$$r_{\text{outer}} = \left(\frac{3\times 1.26\ \text{kg}}{4\pi\ 1\,000\ \text{kg/m}^3}\right)^{1/3} = \boxed{6.70\ \text{cm}}$$

(b) The mass of the ball is determined by the density of aluminum:

$$m = \rho_{\text{Al}} V = \rho_{\text{Al}}\left(\frac{4}{3}\pi r_0^3 - \frac{4}{3}\pi r_i^3\right)$$

$$1.26\ \text{kg} = 2700\ \text{kg/m}^3\left(\frac{4}{3}\pi\right)\left(\left(0.067\ \text{m}\right)^3 - r_i^3\right)$$

$$1.11\times 10^{-4}\ \text{m}^3 = 3.01\times 10^{-4}\ \text{m}^3 - r_i^3$$

$$r_i = \left(1.89\times 10^{-4}\ \text{m}^3\right)^{1/3} = \boxed{5.74\ \text{cm}}$$

P14.60 (a) $\boxed{\text{A particle in equilibrium model}}$

(b) When the balloon comes into equilibrium, we must have

$$\boxed{\sum F_y = B - F_b - F_{\text{He}} - F_s = 0}$$

where B is the buoyant force, F_b the weight of the balloon, F_{He} the weight of the helium, and F_s the weight of the segment of string above the ground.

(c) Write expressions for each of the terms in the force equation:

$$B = \rho_{\text{air}} V g = \rho_{\text{air}} \frac{4}{3}\pi r^3 g$$

$$F_b = m_b g$$

$$F_{\text{He}} = \rho_{\text{He}} V g = \rho_{\text{He}} \frac{4}{3}\pi r^3 g$$

and $F_s = m_s g;$ where $m_s = m\dfrac{h}{\ell}$

Therefore, we have

$$\rho_{\text{air}} V \cancel{g} - m_b \cancel{g} - \rho_{\text{He}} V \cancel{g} - m_s \cancel{g} = 0$$

or $m_s = \left(\rho_{\text{air}} - \rho_{\text{He}}\right)V - m_b \rightarrow \boxed{m_s = \left(\rho_{\text{air}} - \rho_{\text{He}}\right)\frac{4}{3}\pi r^2 - m_b}$

(d) $m_s = \left[(1.20 - 0.179)\ \text{kg/m}^3\right]\left[\dfrac{4}{3}\pi(0.400\ \text{m})^3\right] - 0.250\ \text{kg}$

$= \boxed{0.023\ 7\ \text{kg}}$

(e) $m_s = m\dfrac{h}{\ell} \rightarrow h = \ell\dfrac{m_s}{m} = (2.00\ \text{m})\dfrac{0.023\ 7\ \text{kg}}{0.050\ 0\ \text{kg}} = \boxed{0.948\ \text{m}}$

P14.61 Consider the diagram in ANS. FIG. P14.61 and apply Bernoulli's equation to points A and B, taking $y = 0$ at the level of point B, and recognizing that v_A is approximately zero. This gives:

$$P_A + \frac{1}{2}\rho_w (0)^2 + \rho_w g\left(h - L\sin\theta\right) = P_B + \frac{1}{2}\rho_w v_B^2 + \rho_w g(0)$$

Now, recognize that $P_A = P_B = P_{\text{atmosphere}}$ since both points are open to the atmosphere (neglecting variation of atmospheric pressure with altitude). Thus, we obtain

$$v_B = \sqrt{2g\left(h - L\sin\theta\right)}$$

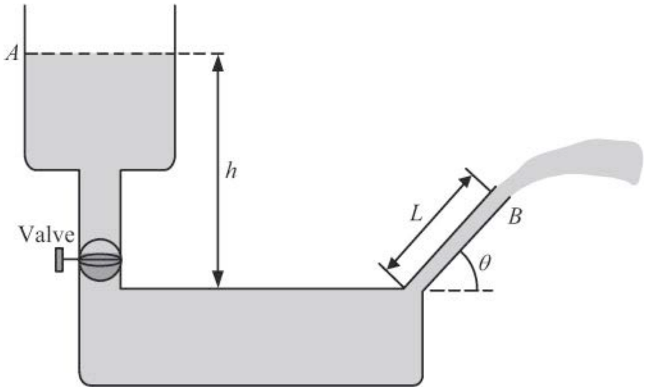
Now the problem reduces to one of projectile motion with $v_{yi} = v_B \sin\theta$. Then using, $v_{yf}^2 = v_{yi}^2 + 2a(\Delta y)$, where $y = y_{\max}, v_{yf} = 0$, and $a = -g$, we find

$$\Delta y = \frac{0 - v_{yi}{}^2}{2a} = \frac{-v_B{}^2 \sin^2\theta}{2(-g)} = \frac{\left[2g(h - L\sin\theta)\right]\sin^2\theta}{2g}$$

$$\Delta y = (h - L\sin\theta)\sin^2\theta$$

$$\Delta y = \left[10.0\text{ m} - (2.00\text{ m})\sin 30.0°\right]\sin^2 30.0°$$

$$y_{\max} = \boxed{2.25\text{ m (above the level where the water emerges)}}$$



ANS. FIG. P14.61

P14.62 The "balanced" condition is one in which the apparent weight of the body equals the apparent weight of the weights. This condition can be written as

$$F_g - B = F_g' - B'$$

where B and B' are the buoyant forces on the body and weights, respectively. The buoyant force experienced by an object of volume V in air equals

$$\text{Buoyant force} = (\text{Volume of object})\rho_{\text{air}}g$$

so we have $B = V\rho_{\text{air}}g$ and

$$B' = \left(\frac{F_g'}{\rho g}\right)\rho_{\text{air}}g$$

Therefore, $F_g = F_g' + \left(V - \dfrac{F_g'}{\rho g}\right)\rho_{\text{air}}g$.

P14.63 Assume $v_{\text{inside}} \approx 0$. From Bernoulli's equation,

$$P + 0 + 0 = 1\text{ atm} + \frac{1}{2}\left(1\ 000\text{ kg/m}^3\right)\left(30.0\text{ m/s}\right)^2 + \left(1\ 000\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)\left(0.500\text{ m}\right)$$

$$P_{\text{gauge}} = P - 1\text{ atm} = 4.50\times10^5\text{ Pa} + 4.90\times10^3\text{ Pa} = \boxed{455\text{ kPa}}$$

P14.64 Let the ball be released at point 1, enter the liquid at point 2, attain maximum depth at point 3, and pop through the surface on the way up at point 4.

(a) Energy conservation for the fall through the air:

$$K_i + U_i = K_f + U_f$$

$$0 + mgy_1 = \frac{1}{2}mv_2^2$$

$$v_2 = \sqrt{2gy_1} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(3.30\text{ m}\right)} = \boxed{8.04\text{ m/s}}$$

(b) $\boxed{\text{The gravitational force and the buoyant force.}}$

The gravitational force is

$$mg = (2.10\text{ kg})(9.80\text{ N/kg}) = 20.6\text{ N down}$$

and the buoyant force is

$$m_{\text{fluid}}g = \rho_{\text{fluid}}V_{\text{object}}g = \rho_{\text{fluid}}(4/3)\pi r^3 g = \left(1\ 230\text{ kg/m}^3\right)(4\pi/3)(0.090\ 0\text{ m})^3\left(9.80\text{ m/s}^2\right) = 36.8\text{ N up}$$

(c) The buoyant force is greater than the gravitational force.

$\boxed{\text{The net upward force on the ball brings its downward motion to a stop.}}$

We choose to use the work-kinetic energy theorem.

$$\frac{1}{2}mv_2^2 + F_{net}\cdot\Delta y = \frac{1}{2}mv_3^2$$

$$\frac{1}{2}(2.10\text{ kg})(8.04\text{ m/s})^2 + (36.8\text{ N} - 20.6\text{ N})(-\Delta y) = 0$$

$$\Delta y = 67.9\text{ J}/16.2\text{ N} = \boxed{4.18\text{ m}}$$

(d) The same net force acts on the ball over the same distance as it moves down and as it moves up, to produce the same speed change. Thus $v_4 = \boxed{8.04 \text{ m/s}}$.

(e) $\boxed{\text{The time intervals are equal}}$, because the ball moves with the same range of speeds over equal distance intervals.

(f) $\boxed{\begin{array}{l}\text{With friction present, } \Delta t_{\text{down}} \text{ is less than } \Delta t_{\text{up}}\text{. The magnitude} \\ \text{of the ball's acceleration on the way down is greater than its} \\ \text{acceleration on the way up. The two motions cover equal} \\ \text{distances and both have zero speed at one end point, so the} \\ \text{downward trip with larger-magnitude acceleration must take} \\ \text{less time.}\end{array}}$

P14.65 At equilibrium, $\sum F_y = 0$: $B - F_{\text{spring}} - F_{g,\text{ He}} - F_{g,\text{ balloon}} = 0$

giving $F_{\text{spring}} = kL = B - \left(m_{\text{He}} + m_{\text{balloon}}\right)g$

But B = weight of displaced air = $\rho_{\text{air}}Vg$

and $m_{\text{He}} = \rho_{\text{He}}V$

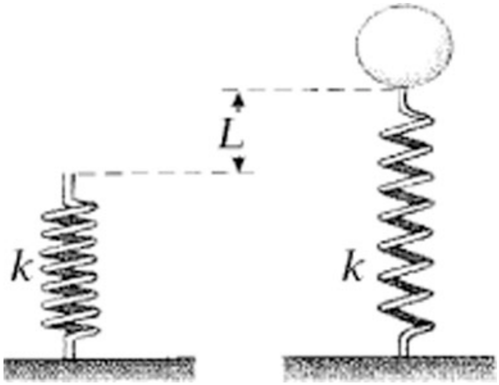
Therefore, we have $kL = \rho_{\text{air}}Vg - \rho_{\text{He}}Vg - m_{\text{balloon}}g$

or
$$L = \frac{\left(\rho_{\text{air}} - \rho_{\text{He}}\right)V - m_{\text{balloon}}}{k}g$$

From the data given,

$$L = \frac{\left[(1.20 - 0.179)\text{ kg/m}^3\right]\left(5.00\text{ m}^3\right) - 2.00\times 10^{-3}\text{ kg}}{90.0\text{ N/m}}\left(9.80\text{ m/s}^2\right)$$

Thus, this gives $\boxed{L = 0.556\text{ m.}}$



ANS. FIG. P14.65

***P14.66** Consider spherical balloons of radius 12.5 cm containing helium at STP and immersed in air at 0°C and 1 atm. If the rubber envelope has mass 5.00 g, the upward force on each is

$$F_{up} = B - F_{g,\text{He}} - F_{g,\text{env}} = \rho_{\text{air}} Vg - \rho_{\text{He}} Vg - m_{\text{env}} g$$

$$F_{up} = (\rho_{\text{air}} - \rho_{\text{He}})\left(\frac{4}{3}\pi r^3\right) g - m_{\text{env}} g$$

$$F_{up} = \left[(1.29 - 0.179)\ \text{kg/m}^3\right]\left[\frac{4}{3}\pi (0.125\ \text{m})^3\right](9.80\ \text{m/s}^2) - (5.00 \times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2) = 0.040\,1\ \text{N}$$

If your weight (including harness, strings, and submarine sandwich) is

$$70.0\ \text{kg}(9.80\ \text{m/s}^2) = 686\ \text{N}$$

you need this many balloons:

$$\frac{686\ \text{N}}{0.040\,1\ \text{N}} = 17\,000 \boxed{\sim 10^4}$$

P14.67 The buoyant force B supports the weights of the raft and the boy. Using M = mass of boy, V = volume of raft, ρ_{st} = density of Styrofoam, and ρ_{w} = density of water, and the volume of the raft is

$$V = (1.00\ \text{m})(1.00\ \text{m})(0.050\ \text{m}) = 0.050\ \text{m}^3$$

From Newton's second law,

$$\sum F_y = 0: \quad B - Mg - \rho_{\text{st}} gV = 0 \quad \rightarrow \quad \rho_{\text{w}} gV - Mg - \rho_{\text{st}} gV = 0$$

Solving for ρ_{st} we get

$$\rho_{\text{st}} = \rho_{\text{w}} - \frac{M}{V} = 1\,000\ \text{kg/m}^3 - \left(\frac{42.0\ \text{kg}}{0.050\,0\ \text{m}^3}\right) = \boxed{160\ \text{kg/m}^3}$$

***P14.68** (a) The blood flowing through the artery is similar to water flowing through a pipe. We substitute numerical values into the equation for the Reynolds number:

$$\text{Re} = \frac{(1.06 \times 10^3\ \text{kg/m}^3)(6.70 \times 10^{-2}\ \text{m/s})(3.00 \times 10^{-2}\ \text{m})}{3.00 \times 10^{-3}\ \text{Pa} \cdot \text{s}}$$
$$= 710$$

Because this result is less than 2 300, the flow is laminar.

(b) Denote the situation in part (a) using subscripts 1. In the expression for the Reynolds number for the capillary, which we denote as situation 2, incorporate the continuity equation for fluids as the blood flows into the smaller blood vessel:

$$\text{Re}_2 = \frac{\rho v_2 d_2}{\mu} = \frac{\rho v_1\left(\frac{A_1}{A_2}\right)(2r_2)}{\mu} = \frac{\rho v_1\left(\frac{\pi r_1^2}{\pi r_2^2}\right)(2r_2)}{\mu} = \frac{2\rho v_1 r_1^2}{\mu r_2}$$

Solve the resulting equation for the radius of the capillary:

$$r_2 = \frac{2\rho v_1 r_1^2}{\mu(\text{Re}_2)}$$

Substitute numerical values, including a Reynolds number representing turbulent flow:

$$r_2 = \frac{2\left(1.06\times 10^3\ \text{kg/m}^3\right)\left(6.70\times 10^{-2}\ \text{m/s}\right)\left(1.50\times 10^{-2}\ \text{m}\right)^2}{\left(3.00\times 10^{-3}\ \text{Pa}\cdot\text{s}\right)(4\ 000)}$$

$$= \boxed{2.66\times 10^{-3}\ \text{m}}$$

(c) The situation in the human body is not represented by a large artery feeding into a single capillary as in part (b). The artery branches into smaller vessels and eventually into approximately 10 billion capillaries. Even though the radius of each capillary is very small, the overall area through which the blood flows in all the capillaries is larger than the area of the artery in part (a). Consequently, in the expression for the Reynolds number, both the speed of the blood and the diameter is very small for each capillary, representing a very low value for the Reynolds number and, consequently, laminar flow.

P14.69 (a) $P = \rho g h$ gives $1.013\times 10^5\ \text{Pa} = \left(1.29\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)h.$

$h = \boxed{8.01\ \text{km}}$

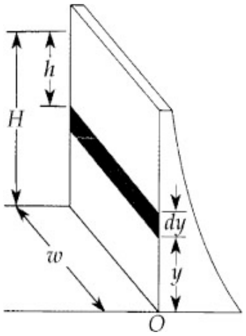
(b) For Mt. Everest, 29 300 ft = 8.88 km, $\boxed{\text{Yes}}$.

P14.70 (a) The torque is

$$\tau = \int d\tau = \int r\,dF$$

From ANS. FIG. P14.70,

$$\tau = \int_0^H y\left[\rho g\left(H - y\right)w\,dy\right] = \boxed{\frac{1}{6}\rho g w H^3}$$



ANS. FIG. P14.70

(b) The total force is given as $\frac{1}{2}\rho g w H^2$.

If this were applied at a height y_{eff} such that the torque remains unchanged, we have

$$\frac{1}{6}\rho g w H^3 = y_{eff}\left[\frac{1}{2}\rho g w H^2\right] \quad \text{and} \quad y_{eff} = \boxed{\frac{1}{3}H}$$

P14.71 Looking first at the top scale and the iron block, we have

$$T + B = F_{g,\text{ iron}}$$

where T is the tension in the spring scale, B is the buoyant force, and $F_{g,\text{ iron}}$ is the weight of the iron block. Now if m_{iron} is the mass of the iron block, we have

$$m_{\text{iron}} = \rho_{\text{iron}} V$$

so $$V = \frac{m_{\text{iron}}}{\rho_{\text{iron}}} = V_{\text{displaced oil}}$$

Then, $$B = \rho_{\text{oil}} V_{\text{iron}} g$$

Therefore,

$$T = F_{g,\text{ iron}} - \rho_{\text{oil}} V_{\text{iron}} g = m_{\text{iron}} g - \rho_{\text{oil}} \frac{m_{\text{iron}}}{\rho_{\text{iron}}}$$

or

$$\begin{aligned} T &= \left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{iron}}}\right) m_{\text{iron}} g \\ &= \left(1 - \frac{916\text{ kg/m}^3}{7\,860\text{ kg/m}^3}\right)(2.00\text{ kg})\left(9.80\text{ m/s}^2\right) \\ &= \boxed{17.3\text{ N}} \end{aligned}$$

Next, we look at the bottom scale which reads *n* (i.e., exerts an upward normal force *n* on the system). Consider the external vertical forces acting on the beaker-oil-iron combination.

$$\sum F_y = 0 \text{ gives}$$

$$T + n - F_{\text{g,beaker}} - F_{\text{g,oil}} - F_{\text{g,iron}} = 0 \rightarrow$$

$$n = \left(m_{\text{beaker+}} m_{\text{oil+}} m_{\text{iron}}\right)g - T = (5.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) - 17.3\ \text{N}$$

Thus, $\boxed{n = 31.7\ \text{N}}$ is the lower scale reading.

P14.72 Looking at the top scale and the iron block:

$$T + B = F_{g,\ \text{Fe}}, \qquad \text{where} \qquad B = \rho_{\text{o}} V_{\text{Fe}} g = \rho_{\text{o}}\left(\frac{m_{\text{Fe}}}{\rho_{\text{Fe}}}\right)g$$

is the buoyant force exerted on the iron block by the oil.

Thus, $T = F_{g,\ \text{Fe}} - B = m_{\text{Fe}}g - \rho_{\text{o}}\left(\dfrac{m_{\text{Fe}}}{\rho_{\text{Fe}}}\right)g$,

or $\quad T = \boxed{\left(1 - \dfrac{\rho_{\text{o}}}{\rho_{\text{Fe}}}\right)m_{\text{Fe}}g}$ is the reading on the top scale.

Now, consider the bottom scale, which exerts an upward force of *n* on the beaker-oil-iron combination.

$$\sum F_y = 0:$$

$$T + n - F_{g,\text{beaker}} - F_{g,\text{oil}} - F_{g,\text{Fe}} = 0$$
$$n = (m_b + m_o + m_{\text{Fe}})g - T$$
$$n = (m_b + m_o + m_{\text{Fe}})g - \left(1 - \frac{\rho_o}{\rho_{\text{Fe}}}\right)m_{\text{Fe}}g$$

or the reading of the bottom scale is

$$n = \boxed{\left[m_b + m_{\text{o}} + \left(\frac{\rho_{\text{o}}}{\rho_{\text{Fe}}}\right)m_{\text{Fe}}\right]g}$$

P14.73 Let *f* represent the fraction of the volume *V* occupied by zinc in the new coin. We have $m = \rho V$ for both coins:

$$3.083\ \text{g} = (8.920\ \text{g/cm}^3)V$$

and $\quad 2.517\ \text{g} = \left(7.133\ \text{g/cm}^3\right)(fV) + \left(8.920\ \text{g/cm}^3\right)(1-f)V$

By substitution,

$$2.517\text{ g} = (7.133\text{ g/cm}^3)fV + 3.083\text{ g} - (8.920\text{ g/cm}^3)fV$$

$$fV = \frac{3.083\text{ g} - 2.517\text{ g}}{8.920\text{ g/cm}^3 - 7.133\text{ g/cm}^3}$$

and again substituting to eliminate the volume,

$$f = \frac{0.566\text{ g}}{1.787\text{ g/cm}^3}\left(\frac{8.920\text{ g/cm}^3}{3.083\text{ g}}\right) = 0.916\,4 = \boxed{91.64\%}$$

***P14.74** Let ℓ represent the length below water at equilibrium and *M* the tube's mass:

$$\sum F_y = 0 \qquad \text{gives} \qquad -Mg + \rho\pi r^2\ell g = 0$$

Now with any excursion *x* from equilibrium:

$$-Mg + \rho\pi r^2(\ell - x)g = Ma$$

Subtracting the equilibrium equation gives:

$$-\rho\pi r^2 gx = Ma$$

$$a = -\left(\frac{\rho\pi r^2 g}{M}\right)x = -\omega^2 x$$

The opposite direction and direct proportionality of *a* to *x* imply SHM with angular frequency

$$\omega = \sqrt{\frac{\rho\pi r^2 g}{M}}$$

$$T = \frac{2\pi}{\omega} = \boxed{\frac{2}{r}\sqrt{\frac{\pi M}{\rho g}}}$$

P14.75 Pascal's principle, $\frac{F_1}{A_1} = \frac{F_2}{A_2}$, or $\frac{F_{\text{pedal}}}{A_{\substack{\text{Master}\\ \text{cylinder}}}} = \frac{F_{\text{brake}}}{A_{\substack{\text{brake}\\ \text{cylinder}}}}$, gives

$$F_{\text{brake}} = \left(\frac{A_{\text{brake cylinder}}}{A_{\text{master cylinder}}}\right)F_{\text{pedal}} = \left(\frac{6.4\text{ cm}^2}{1.8\text{ cm}^2}\right)(44\text{ N}) = 156\text{ N}$$

This is the normal force exerted on the brake shoe. The frictional force is

$$f = \mu_k n = 0.50(156\text{ N}) = 78\text{ N}$$

and the torque is $\tau = f \cdot r_{\text{drum}} = (78\text{ N})(0.34\text{ m}) = \boxed{27\text{ N}\cdot\text{m}}$.

***P14.76** (a) Since the upward buoyant force is balanced by the weight of the sphere,

$$m_1 g = \rho V g = \rho\left(\frac{4}{3}\pi R^3\right)g$$

In this problem, $\rho = 0.789\,45\ \text{g/cm}^3$ at 20.0°C, and $R = 1.00$ cm, so we find

$$m_1 = \rho\left(\frac{4}{3}\pi R^3\right) = \left(0.789\,45\ \text{g/cm}^3\right)\left[\frac{4}{3}\pi(1.00\ \text{cm})^3\right]$$
$$= \boxed{3.307\ \text{g}}$$

(b) Following the same procedure as in part (a), with $\rho' = 0.780\,97\ \text{g/cm}^3$ at 30.0°C, we find

$$m_2 = \rho'\left(\frac{4}{3}\pi R^3\right) = \left(0.780\,97\ \text{g/cm}^3\right)\left[\frac{4}{3}\pi(1.00\ \text{cm})^3\right]$$
$$= \boxed{3.271\ \text{g}}$$

(c) When the first sphere is resting on the bottom of the tube, $n + B = F_{g1} = m_1 g$, where n is the normal force.

Since $B = \rho' V g$,

$$n = m_1 g - \rho' V g$$
$$= \left[3.307\ \text{g} - \left(0.780\,97\ \text{g/cm}^3\right)\frac{4}{3}\pi(1.00\ \text{cm})^3\right]\left(980\ \text{cm/s}^2\right)$$
$$n = 34.8\ \text{g}\cdot\text{cm/s}^2 = \boxed{3.48\times 10^{-4}\ \text{N}}$$

P14.77 The disk (mass $M = 10.0$ kg, radius $R = 0.250$ m) has moment of inertia $I = \frac{1}{2}MR^2$. The disk slows from $\omega_i = 300$ rev/min to $\omega_f = 0$ in time interval $\Delta t = 60.0$ s. Its angular acceleration is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{-\omega_i}{\Delta t}$$

Frictional torque from the brake pad slows the wheel. Friction has moment arm $d = 0.220$ m. The relation between friction and angular acceleration is

$$\sum\tau = I\alpha:\quad -fd = I\alpha \;\rightarrow\; f = -\frac{I}{d}\alpha = -\frac{\frac{1}{2}MR^2}{d}\left(\frac{-\omega_i}{\Delta t}\right)$$

$$\rightarrow f = \frac{MR^2\omega_i}{2d\Delta t}$$

The normal force and coefficient of friction $(\mu_k = 0.500)$ between the brake pad and the disk determine the amount of friction. We can write an expression for the normal force:

$$f = \mu_k n \quad \rightarrow \quad n = \frac{f}{\mu_k} = \frac{MR^2\omega_i}{2\mu_k d\Delta t}$$

The pressure of the brake fluid acting on a piston of area A (diameter $D = 5.00$ cm, radius $r = D/2 = 0.0250$ m) produces the normal force that the brake pad exerts on the disk. The pressure in the brake fluid is

$$P = \frac{n}{\pi r^2} = \frac{MR^2\omega_i}{(2\mu_k d\Delta t)\pi r^2}$$

$$P = \frac{(10.0\ \text{kg})(0.250\ \text{m})^2\left[\left(\frac{300\ \text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60.0\ \text{s}}\right)\right]}{2(0.500)(0.220\ \text{m})(60.0\ \text{s})\pi(0.0250\ \text{m})^2}$$
$$= \boxed{758\ \text{Pa}}$$

P14.78 (a) Since the pistol is fired horizontally, the emerging water stream has initial velocity components of $(v_{0x} = v_{\text{nozzle}}, v_{0y} = 0)$. Then, $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, with $a_y = -g$, gives the time of flight as

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-1.50\ \text{m})}{-9.80\ \text{m/s}^2}} = \boxed{0.553\ \text{s}}$$

(b) With $a_x = 0$, and $v_{0x} = v_{\text{nozzle}}$, the horizontal range of the emergent stream is $\Delta x = v_{\text{nozzle}}t$, where t is the time of flight from above. Thus, the speed of the water emerging from the nozzle is

$$v_{\text{nozzle}} = \frac{\Delta x}{t} = \frac{8.00\ \text{m}}{0.553\ \text{s}} = \boxed{14.5\ \text{m/s}}$$

(c) From the equation of continuity, $A_1v_1 = A_2v_2$, the speed of the water in the larger cylinder is $v_1 = (A_2/A_1)v_2 = (A_2/A_1)v_{\text{nozzle}}$, or

$$v_1 = \left(\frac{\pi r_2^2}{\pi r_1^2}\right)v_{\text{nozzle}} = \left(\frac{r_2}{r_1}\right)^2 v_{\text{nozzle}} = \left(\frac{1.00\ \text{mm}}{10.0\ \text{mm}}\right)^2(14.5\ \text{m/s})$$
$$= \boxed{0.145\ \text{m/s}}$$

(d) The pressure at the nozzle is atmospheric pressure, or $\boxed{P_2 = 1.013 \times 10^5\ \text{Pa}}$.

(e) With the two cylinders horizontal, $y_1 = y_2$ and gravity terms from Bernoulli's equation can be neglected, leaving

$$P_1 + \frac{1}{2}\rho_w v_1^2 = P_2 + \frac{1}{2}\rho_w v_2^2$$

so the needed pressure in the larger cylinder is

$$\begin{aligned} P_1 &= P_2 + \frac{\rho_w}{2}\left(v_2^2 - v_1^2\right) \\ &= 1.013\times 10^5\ \text{Pa} \\ &\qquad + \frac{1.00\times 10^3\ \text{kg/m}^3}{2}\left[(14.5\ \text{m/s})^2 - (0.145\ \text{m/s})^2\right] \end{aligned}$$

or

$$P_1 = \boxed{2.06\times 10^5\ \text{Pa}}$$

(f) To create an overpressure of $\Delta P = 2.06\times 10^5\ \text{Pa} = 1.05\times 10^5\ \text{Pa}$ in the larger cylinder, the force that must be exerted on the piston is

$$\begin{aligned} F_1 &= (\Delta P)A_1 = (\Delta P)\left(\pi r_1^2\right) \\ &= \left(1.05\times 10^5\ \text{Pa}\right)\pi\left(1.00\times 10^{-2}\ \text{m}\right)^2 \\ &= \boxed{33.0\ \text{N}} \end{aligned}$$

P14.79 Energy for the fluid-Earth system is conserved.

$$\begin{aligned} &(K+U)_i = (K+U)_f \\ &0 + \frac{mgL}{2} + 0 = \frac{1}{2}mv^2 + 0 \\ &v = \sqrt{gL} = \sqrt{(2.00\ \text{m})\left(9.80\ \text{m/s}^2\right)} = \boxed{4.43\ \text{m/s}} \end{aligned}$$

P14.80 (a) The flow rate, Av, as given may be expressed as follows:

$$\frac{25.0\ \text{liters}}{30.0\ \text{s}} = 0.833\ \text{liters/s} = 833\ \text{cm}^3/\text{s}$$

The area of the faucet tap is $\pi\ \text{cm}^2$, so we can find the velocity as

$$v = \frac{\text{flow rate}}{A} = \frac{833\ \text{cm}^3/\text{s}}{\pi\ \text{cm}^2} = 265\ \text{cm/s} = \boxed{2.65\ \text{m/s}}$$

(b) We choose point 1 to be in the entrance pipe and point 2 to be at the faucet tap. $A_1v_1 = A_2v_2$ gives $v_1 = 0.295\ \text{m/s}$. Bernoulli's equation is:

$$P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) + \rho g\left(y_2 - y_1\right)$$

and gives

$$P_1 - P_2 = \frac{1}{2}\left(10^3\ \text{kg/m}^3\right)\left[\left(2.65\ \text{m/s}\right)^2 - \left(0.295\ \text{m/s}\right)^2\right]$$
$$+\left(10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}\right)\left(2.00\ \text{m}\right)$$

or $\quad P_{\text{gauge}} = P_1 - P_2 = \boxed{2.31\times 10^4\ \text{Pa}}$.

P14.81 (a) Consider the pressure at points A and B in ANS. FIG. P14.81(b).

Using the left tube: $P_A = P_{\text{atm}} + \rho_{\text{w}} g\left(L - h\right)$

Using the right tube: $P_B = P_{\text{atm}} + \rho_{\text{o}} gL$

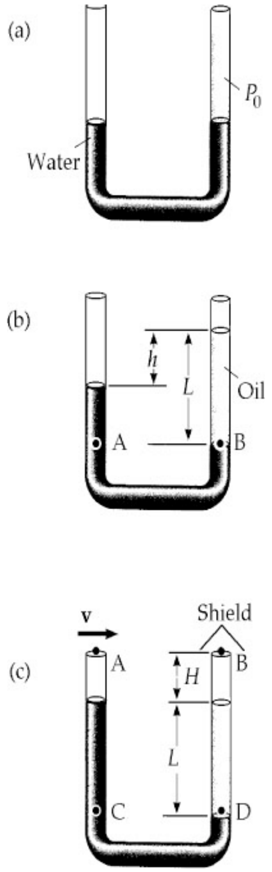
But Pascal's principle says that $P_A = P_B$.

Therefore, $P_{\text{atm}} + \rho_{\text{w}} g\left(L - h\right) = P_{\text{atm}} + \rho_{\text{o}} gL$

or $\quad \rho_{\text{w}} h = \left(\rho_{\text{w}} - \rho_{\text{o}}\right)L,$ giving

$$h = \left(\frac{\rho_{\text{w}} - \rho_{\text{o}}}{\rho_{\text{w}}}\right)L$$
$$= \left(\frac{1\,000\ \text{kg/m}^3 - 750\ \text{kg/m}^3}{1\,000\ \text{kg/m}^3}\right)\left(5.00\ \text{cm}\right)$$
$$= \boxed{1.25\ \text{cm}}$$

(b) Consider part (c) of the diagram showing the situation when the air flow over the left tube equalizes the fluid levels in the two tubes. First, apply Bernoulli's equation to points A and B ($y_A = y_B$, $v_A = v$, and $v_B = 0$).



ANS. FIG. P14.81

This gives:

$$P_A + \frac{1}{2}\rho_a v^2 + \rho_a g y_A$$
$$= P_B + \frac{1}{2}\rho_a\left(0\right)^2 + \rho_a g y_B$$

and since $y_A = y_B$, this reduces to

$$P_B - P_A = \frac{1}{2}\rho_a v^2 \qquad \textbf{[1]}$$

Now consider points C and D, both at the level of the oil-water interface in the right tube. Using the variation of pressure with depth in static fluids, we have

$$P_C = P_A + \rho_\mathrm{a} gH + \rho_\mathrm{w} gL$$

and $P_D = P_B + \rho_\mathrm{a} gH + \rho_\mathrm{o} gL$

But Pascal's principle says that $P_C = P_D$. Equating these two gives:

$$P_B + \rho_\mathrm{a} gH + \rho_\mathrm{o} gL = P_A + \rho_\mathrm{a} gH + \rho_\mathrm{w} gL$$

or $P_B - P_A = \left(\rho_\mathrm{w} - \rho_\mathrm{o}\right) gL$ **[2]**

Substitute equation [1] for $P_B - P_A$ into [2] to obtain

$$\frac{1}{2}\rho_\mathrm{a} v^2 = \left(\rho_\mathrm{w} - \rho_\mathrm{o}\right) gL$$

or

$$v = \sqrt{\frac{2gL\left(\rho_\mathrm{w} - \rho_\mathrm{o}\right)}{\rho_\mathrm{a}}}$$

$$= \sqrt{2\left(9.80\ \mathrm{m/s^2}\right)\left(0.050\ 0\ \mathrm{m}\right)\left(\frac{1\ 000\ \mathrm{kg/m^3} - 750\ \mathrm{kg/m^3}}{1.20\ \mathrm{kg/m^3}}\right)}$$

$$v = \boxed{14.3\ \mathrm{m/s}}$$

P14.82 Take point ① at the free water surface in the tank and point ② at the bottom end of the tube:

$$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

$$P_0 + \rho g d + 0 = P_0 + 0 + \frac{1}{2}\rho v_2^2$$

$$v_2 = \sqrt{2gd}$$

The volume flow rate is $\frac{V}{t} = \frac{Ah}{t} = v_2 A'$. Then $t = \frac{Ah}{v_2 A'} = \frac{Ah}{A'\sqrt{2gd}}$.

P14.83 (a) For diverging streamlines that pass just above and just below the hydrofoil, we have

$$P_t + \rho g y_t + \frac{1}{2}\rho v_t^2 = P_b + \rho g y_b + \frac{1}{2}\rho v_b^2$$

Ignoring the buoyant force means taking $y_t \approx y_b$:

$$P_t + \frac{1}{2}\rho\left(nv_b\right)^2 = P_b + \frac{1}{2}\rho v_b^2$$

$$P_b - P_t = \frac{1}{2}\rho v_b^2\left(n^2 - 1\right)$$

The lift force is $\left(P_b - P_t\right)A = \dfrac{1}{2}\rho v_b^2\left(n^2 - 1\right)A.$

(b) For liftoff,

$$\frac{1}{2}\rho v_b^2\left(n^2 - 1\right)A = Mg$$

$$v_b = \left(\frac{2Mg}{\rho\left(n^2 - 1\right)A}\right)^{1/2}$$

The speed of the boat relative to the shore must be nearly equal to this speed of the water below the hydrofoil relative to the boat.

***P14.84** First, consider the path from the viewpoint of projectile motion to find the speed at which the water emerges from the tank. From $\Delta y = v_{yi}t + \dfrac{1}{2}a_y t^2$ with $v_{yi} = 0$, $\Delta y = -1.00$ m, and $a_y = -g$, we find the time of flight as

$$t = \sqrt{\frac{2\left(\Delta y\right)}{a_y}} = \sqrt{\frac{2.00\text{ m}}{g}} = 0.452\text{ s}$$

From the horizontal motion, the speed of the water coming out of the hole is

$$v_2 = v_{xi} = \frac{\Delta x}{t} = \frac{0.600\text{ m}}{0.452\text{ s}} = 1.33\text{ m/s}$$

We now use Bernoulli's equation, with point 1 at the top of the tank and point 2 at the level of the hole. With $P_1 = P_1 = P_{\text{atm}}$ and $v_1 \approx 0$, this gives

$$\rho g y_1 = \rho g y_2 + \frac{1}{2}\rho v_2^2$$

or

$$h = y_1 - y_2 = \frac{v_2^2}{2g} = \frac{\left(1.33\text{ m/s}\right)^2}{2g} = 9.00\times 10^{-2}\text{ m} = \boxed{9.00\text{ cm}}$$

Challenge Problems

P14.85 Let s stand for the edge of the cube, h for the depth of immersion, ρ_{ice} for the density of the ice, ρ_{w} for the density of water, and ρ_{al} for the density of the alcohol.

(a) According to Archimedes's principle, at equilibrium we have

$$B = F_{\text{g}}$$

$$\rho_{\text{w}} g h s^2 = \rho_{\text{ice}} g s^3 \Rightarrow h = s\frac{\rho_{\text{ice}}}{\rho_{\text{w}}}$$

With $\rho_{\text{ice}} = 0.917\times 10^3\ \text{kg/m}^3$,

$$\rho_{\text{w}} = 1.00\times 10^3\ \text{kg/m}^3$$

and $s = 20.0$ mm,

we obtain $h = 20.0(0.917) = 18.34\ \text{mm} \approx \boxed{18.3\ \text{mm}}$

(b) We assume that the top of the cube is still above the alcohol surface. Letting h_{al} stand for the thickness of the alcohol layer, we have

$$\rho_{\text{al}} g s^2 h_{\text{al}} + \rho_{\text{w}} g s^2 h_{\text{w}} = \rho_{\text{ice}} g s^3 \quad \text{so} \quad h_{\text{w}} = \left(\frac{\rho_{\text{ice}}}{\rho_{\text{w}}}\right)s - \left(\frac{\rho_{\text{al}}}{\rho_{\text{w}}}\right)h_{\text{al}}.$$

With $\rho_{\text{al}} = 0.806\times 10^3\ \text{kg/m}^3$

and $h_{\text{al}} = 5.00$ mm,

we obtain $h_{\text{w}} = 18.34 - 0.806(5.00) = 14.31\ \text{mm} \approx \boxed{14.3\ \text{mm}}$.

To check our assumption above, the bottom of the cube is below the top surface of the alcohol 14.4 mm + 5.00 mm = 19.3 mm, so the top of the cube is above the surface of the alcohol 20.0 mm – 19.3 mm = 0.7 mm. The assumption was valid.

(c) Here , $h'_{\text{w}} = s - h'_{\text{al}}$, so Archimedes's principle gives

$$\rho_{\text{al}} g s^2 h'_{\text{al}} + \rho_{\text{w}} g s^2 \left(s - h'_{\text{al}}\right) = \rho_{\text{ice}} g s^3 \Rightarrow \rho_{\text{al}} h'_{\text{al}} + \rho_{\text{w}}\left(s - h'_{\text{al}}\right) = \rho_{\text{ice}} s$$

$$h'_{\text{al}} = s\frac{\left(\rho_{\text{w}} - \rho_{\text{ice}}\right)}{\left(\rho_{\text{w}} - \rho_{\text{al}}\right)} = (20.0\ \text{mm})\frac{(1.000 - 0.917)}{(1.000 - 0.806)}$$

$$= 8.557 \approx \boxed{8.56\ \text{mm}}$$

P14.86 Assume the top of the barge without the pile of iron has height H_0 above the surface of the water. When a mass of iron M_{Fe} is added to the barge, the barge sinks a distance ΔH until the buoyant force from the water equals the additional weight of the iron. The barge is a square with sides of length L, so the volume of displaced water is $L^2\Delta H$, and the buoyant force supporting the extra weight is

$$B = \left(\rho_{\mathrm{w}} L^2 \Delta H\right) g = M_{\mathrm{Fe}} g$$

where ρ_{w} is the density of water.

The scrap iron pile has the shape of a cone, and the volume of a cone of base radius R and central height h is $V_{\mathrm{cone}} = \pi R^2 h/3$; therefore, the mass of the iron is $M_{\mathrm{Fe}} = \rho_{\mathrm{Fe}} \pi R^2 h / 3$, where ρ_{Fe} is the density of iron. We find the distance the barge sinks with a pile of iron:

$$B = \left(\rho_{\mathrm{w}} L^2 \Delta H\right) g = M_{\mathrm{Fe}} g$$

$$\left(\rho_{\mathrm{w}} L^2 \Delta H\right) g = \left(\rho_{\mathrm{Fe}} \pi R^2 h / 3\right) g \to \Delta H = \left(\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{w}}}\right)\left(\frac{\pi}{3}\right)\left(\frac{R^2}{L^2}\right) h$$

If the iron is piled to a height h, the barge will sink by the distance ΔH, so the distance from the water level to the top of the iron pile is $D_{\mathrm{top}} = H_0 - \Delta H + h$.

For the situation of the problem, side $L = 2r$, and the initial conical pile of scrap iron has radius $R = r$ and height is $h = r$. The distance the barge sinks is

$$\Delta H = \left(\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{w}}}\right)\left(\frac{\pi}{3}\right)\left(\frac{R^2}{L^2}\right) h$$

$$\Delta H = \left(\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{w}}}\right)\left(\frac{\pi}{3}\right)\left(\frac{r^2}{(2r)^2}\right) r = \left(\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{w}}}\right)\left(\frac{\pi}{3}\right)\left(\frac{r^2}{4r^2}\right) r = \left(\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{w}}}\right)\left(\frac{\pi}{12}\right) r$$

and the height of the top of the pile above the water is

$$D_{\mathrm{top}} = H_0 - \Delta H + h = H_0 - \left(\frac{\rho_{\mathrm{Fe}}}{\rho_{\mathrm{w}}}\right)\left(\frac{\pi}{12}\right) r + r$$

For $\rho_{\mathrm{w}} = 1.00 \times 10^3\ \mathrm{kg/m^3}$ and $\rho_{\mathrm{Fe}} = 7.86 \times 10^3\ \mathrm{kg/m^3}$, this expression becomes

$$D_{\mathrm{top}} = H_0 - \left(\frac{7.86 \times 10^3}{1.00 \times 10^3}\right)\left(\frac{\pi}{12}\right) r + r = H_0 - 2.06r + r$$

$$D_{\mathrm{top}} = H_0 - 1.06r$$

This distance is too large to allow the barge to go under the bridge:

$$D_{\text{top}} = H_0 - 1.06r \geq D_{\text{bridge}}$$

When the pile is reduced to a height h', but still with the same base radius $R = r$, the distance the barge sinks is

$$\Delta H = \left(\frac{\rho_{\text{Fe}}}{\rho_{\text{w}}}\right)\left(\frac{\pi}{12}\right)h' = 2.06h'$$

The height of the top of the pile above the water is now

$$D'_{\text{top}} = H_0 - \Delta H + h' = H_0 - 2.06h' + h' = H_0 - 1.06h'$$

but this means the top of the pile is now higher! To check this, recall that the height of the pile is reduced, so $h' < r$:

$$D'_{\text{top}} > D_{\text{top}}$$

$$H_0 - 1.06h' > H_0 - 1.06r \quad \rightarrow \quad -1.06h' > -1.06r \quad \rightarrow \quad h' < r$$

which is true.

$$\boxed{\text{The situation is impossible because lowering the height of the iron pile on the barge while keeping the base radius the same results in the top of the pile rising higher above the water level.}}$$

P14.87 The incremental version of $P - P_0 = \rho g y$ is $dP = -\rho g dy$.

We assume that the density of air is proportional to pressure,

or $\dfrac{P}{\rho} = \dfrac{P_0}{\rho_0}$. Combining these two equations we have

$$dP = -P\frac{\rho_0}{P_0}g\,dy$$

Integrating both sides,

$$\int_{P_0}^{P}\frac{dP}{P} = -g\frac{\rho_0}{P_0}\int_0^y dy$$

gives $$\ln\left(\frac{P}{P_0}\right) = -\frac{\rho_0 g y}{P_0}$$

Defining $\alpha = \dfrac{\rho_0 g}{P_0}$ then gives $P = P_0 e^{-\alpha y}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P14.2	(a) ~4×10^{17} kg/m^3; (b) See P14.2 for the full description.
P14.4	5.27×10^{18} kg
P14.6	(a) 65.1 N; (b) 275 N
P14.8	225 N
P14.10	(a) 5.88×10^6 N down; (b) 196 kN outward; (c) 588 kN outward
P14.12	The situation is impossible because the longest straw Superman can use and still get a drink is less than 12.0 m.
P14.14	1.05×10^5 Pa
P14.16	(a) $F = \frac{1}{2}\rho g w h\left(2d - h\right)$; (b) $\tau = \frac{1}{2}\rho g h\left(dh^2 - \frac{1}{3}h^3\right)$
P14.18	0.072 1 mm
P14.20	(a) 14.7 kPa, 0.015 5 atm, 11.8 m; (b) Blockage of the fluid within the spinal column or between the skull and the spinal column would prevent the fluid level from rising.
P14.22	(a) 20.0 cm; (b) 0.490 cm
P14.24	(a) $P = P_0 + \rho g h$; (b) Mg/A
P14.26	3.33×10^3 kg/m^3
P14.28	(a) 444 kg; (b) 480 kg
P14.30	1.28×10^4 m^2
P14.32	2.67×10^3 kg
P14.34	(a) $B = 25.0$ N; (b) horizontally inward; (c) The string tension increases. The water under the block pushes up on the block more strongly than before because the water is under higher pressure due to the weight of the oil above it; (d) 62.5%
P14.36	See P14.36 for the full derivation.
P14.38	(a) 3.7 kN; (b) 1.9 kN; (c) Atmospheric pressure at this high altitude is much lower than at the Earth's surface
P14.40	(a) 0.471 m/s; (b) 4.24 m/s
P14.42	(a) 0.638 m/s; (b) 2.55 m/s; (c) 1.80×10^{-3} m^3/s
P14.44	(a) 27.9 N; (b) 3.32×10^4 kg; (c) 7.26×10^4 Pa

P14.46 (a) $P = \dfrac{\Delta E}{\Delta t} = \dfrac{\Delta mgh}{\Delta t} = \left(\dfrac{\Delta m}{\Delta t}\right)gh = Rgh$; (b) 616 MW

P14.48 (a) $\left(3.93\times10^{-6}\ \text{m}^3/\text{s}\right)\sqrt{\Delta P}$ where ΔP is in pascal; (b) 0.305 L/s; (c) 0.431 L/s

P14.50 (a) 28.0 m/s; (b) 28.0 m/s; (c) The answers agree precisely. The models are consistent with each other. (d) 2.11 MPa

P14.52 (a) 6.80×10^4 Pa; (b) Higher. With the inclusion of another upward force due to deflection of air downward, the pressure difference does not need to be as great to keep the airplane in flight.

P14.54 (a) 452 N outward; (b) 1.81 kN outward

P14.56 1.01 kJ

P14.58 $\left(P_0 - P\right)\pi R^2$

P14.60 (a) A particle in equilibrium model; (b) $\sum F_y = B - F_b - F_{\text{He}} - F_s = 0$; (c) $m_s = \left(\rho_{\text{air}} - \rho_{\text{He}}\right)\dfrac{4}{3}\pi r^2 - m_b$; (d) 0.023 7 kg; (e) 0.948 m

P14.62 See P14.62 for full description.

P14.64 (a) 8.04 m/s; (b) The gravitational force and the buoyant force; (c) The net upward force on the ball brings it downward motion to a stop, 4.18 m; (d) 8.04 m/s; (e) The time intervals are equal; (f) See P14.64(f) for a full conceptual argument.

P14.66 $\sim 10^4$

P14.68 (a) See P14.68(a) for full description; (b) 2.66×10^{-3} m; (c) The situation in the human body is not represented by a large artery feeding into a single capillary as in part (b). See P14.68(c) for full explanation.

P14.70 (a) $\dfrac{1}{6}\rho g w H^3$; (b) $\dfrac{1}{3}H$

P14.72 $T = \left(1 - \dfrac{\rho_{\text{o}}}{\rho_{\text{Fe}}}\right)m_{\text{Fe}}g,\ n = \left[m_b + m_{\text{o}} + \left(\dfrac{\rho_{\text{o}}}{\rho_{\text{Fe}}}\right)\right]g$

P14.74 $\dfrac{2}{r}\sqrt{\dfrac{\pi M}{\rho g}}$

P14.76 (a) 3.307 g; (b) 3.271 g; (c) 3.48×10^{-4} N

P14.78 (a) 0.553 s; (b) 14.5 m/s; (c) 0.145 m/s; (d) $P_2 = 1.013 \times 10^5$ Pa; (e) 2.06×10^5 Pa; (f) 33.0 N

P14.80 (a) 2.65 m/s; (b) 2.31×10^4 Pa

P14.82 See P14.82 for the full answer.

P14.84 9.00 cm

P14.86 The situation is impossible because lowering the height of the iron pile on the barge while keeping the base radius the same results in the top of the pile rising higher above the water level.

15

Oscillatory Motion

CHAPTER OUTLINE

15.1 Motion of an Object Attached to a Spring
15.2 Analysis Model: Particle in Simple Harmonic Motion
15.3 Energy of the Simple Harmonic Oscillator
15.4 Comparing Simple Harmonic Motion with Uniform Circular Motion
15.5 The Pendulum
15.6 Damped Oscillations
15.7 Forced Oscillations

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ15.1 Answer (d). The period of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, and its frequency is $f = 1/T = (1/2\pi)\sqrt{g/\ell}$. Thus, if the length is doubled so $\ell' = 2\ell$, the new frequency is

$$f' = \frac{1}{2\pi}\sqrt{\frac{g}{\ell'}} = \frac{1}{2\pi}\sqrt{\frac{g}{2\ell}} = \frac{1}{\sqrt{2}}\left(\frac{1}{2\pi}\sqrt{\frac{g}{\ell}}\right) = \frac{f}{\sqrt{2}}$$

OQ15.2 Answer (c). The equilibrium position is 15 cm below the starting point. The motion is symmetric about the equilibrium position, so the two turning points are 30 cm apart.

OQ15.3 Answer (a). In this spring-mass system, the total energy equals the elastic potential energy at the moment the mass is temporarily at rest at $x = A = 6$ cm (i.e., at the extreme ends of the simple harmonic motion). Thus, $E = kA^2/2$ and we see that as long as the spring constant k and the amplitude A remain unchanged, the total energy is unchanged.

OQ15.4 Answer (c). The total energy of the object-spring system is $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$. When the kinetic energy is twice the potential energy, $\frac{1}{2}mv^2 = 2\left(\frac{1}{2}kx^2\right) = kx^2$, and the total energy is

$$\frac{1}{2}kA^2 = kx^2 + \frac{1}{2}kx^2 \quad \rightarrow \quad \frac{1}{2}kA^2 = \frac{3}{2}kx^2 \quad \rightarrow \quad x = \frac{A}{\sqrt{3}}$$

OQ15.5 Answer (d). When the object is at its maximum displacement, the magnitude of the force exerted on it by the spring is $F_s = k|x_{\max}| = (8.0\ \text{N/m})(0.10\ \text{m}) = 0.80\ \text{N}.$ This force will give the mass an acceleration of $a = F_s/m = 0.80\ \text{N}/0.40\ \text{kg} = 2.0\ \text{m/s}^2$.

OQ15.6 Answer (a). The car will continue to compress the spring until all of the car's original kinetic energy has been converted into elastic potential energy within the spring, i.e., until $\frac{1}{2}kx^2 = \frac{1}{2}mv_i^2$, or

$$x = v_i\sqrt{\frac{m}{k}} = (2.0\ \text{m/s})\sqrt{\frac{3.0\times10^5\ \text{kg}}{2.0\times10^6\ \text{N/m}}} = 0.77\ \text{m}$$

OQ15.7 Answer (c). When an object undergoes simple harmonic motion, the position as a function of time may be written as $x = A\cos\omega t = A\cos(2\pi ft)$. Comparing this to the given relation, we see that the frequency of vibration is $f = 3$ Hz, and the period is $T = 1/f = 1/3$.

***OQ15.8** Answer (b). The frequency of vibration is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Thus, increasing the mass by a factor of 9 will decrease the frequency to 1/3 of its original value.

OQ15.9 Answer (a). Higher frequency. When it supports your weight, the center of the diving board flexes down less than the end does when it supports your weight—this is similar to a spring that stretches a smaller distance for the same force: its spring constant is greater because the displacement is smaller. Therefore, the stiffness constant describing the center of the board is greater than the stiffness constant describing the end. And then $f = \left(\frac{1}{2\pi}\right)\sqrt{\frac{k}{m}}$ is greater for you bouncing on the center of the board.

OQ15.10 (i) Answer (c). At 40 cm we have the midpoint between the turning points, so it is the equilibrium position and the point of maximum speed, and therefore, maximum momentum.

(ii) Answer (c). The position of maximum speed is also the position of maximum kinetic energy.

(iii) Answer (e). The total energy of the system is conserved, so it is the same at every position.

OQ15.11 The ranking is (c) > (e) > (a) = (b) > (d). The amplitude does not affect the period in simple harmonic motion; neither do constant forces that offset the equilibrium position. Thus (a) and (b) have equal periods. The period is proportional to the square root of mass divided by spring constant. So (c), with larger mass, has a larger period than (a). And (d) with greater stiffness has smaller period. In situation (e) the motion is not quite simple harmonic, but has slightly smaller angular frequency and so a slightly longer period.

OQ15.12 (a) Yes. In simple harmonic motion, one-half of the time, the velocity is in the same direction as the displacement away from equilibrium.

(b) Yes. Velocity and acceleration are in the same direction half the time.

(c) No. The spring force and, therefore, the acceleration are always opposite to the position vector, and never in the same direction.

OQ15.13 Answer (d). We assume that the coils of the spring do not hit one another. When the spring with two blocks is set into oscillation in space, the coil in the center of the spring does not move. We can imagine clamping the center coil in place without affecting the motion. We can effectively duplicate the motion of each individual block in space by hanging a single block on a half-spring here on Earth. The half-spring with its center coil clamped—or its other half cut off—has twice the spring constant as the original uncut spring because an applied force of the same size would produce only one-half the extension distance. Thus the oscillation frequency in space is $\left(\frac{1}{2\pi}\right)\left(\frac{2k}{m}\right)^{1/2} = \sqrt{2}f$. The absence of a force required to support the vibrating system in orbital free fall has no effect on the frequency of its vibration.

OQ15.14 Answer (d) is the only false statement. At the equilibrium position, $x = 0$, the elastic potential energy of the system $\left(PE_s = \frac{1}{2}kx^2\right)$ is a minimum and the kinetic energy is a maximum.

OQ15.15 (i) Answer (e). We have $T_i = 2\pi\sqrt{\frac{L_i}{g}}$ and

$$T_f = 2\pi\sqrt{\frac{L_f}{g}} = 2\pi\sqrt{\frac{4L_i}{g}} = 2T_i.$$ The period becomes larger by a factor of 2, to become 5 s.

(ii) Answer (c). Changing the mass has no effect on the period of a simple pendulum.

OQ15.16 (i) Answer (b). The upward acceleration has the same effect as an increased gravitational acceleration.

(ii) Answer (a). The downward acceleration has the same effect as a decreased gravitational acceleration.

(iii) Answer (c). The absence of acceleration means that the effective gravitational field is the same as that for a stationary elevator.

OQ15.17 (i) Answer (c). At 120 cm we have the midpoint between the turning points, so it is the equilibrium position and the point of maximum speed.

(ii) Answer (a). In simple harmonic motion the acceleration is maximum when the displacement from equilibrium is maximum.

(iii) Answer (a), by the same logic as in part (ii).

ANSWERS TO CONCEPTUAL QUESTIONS

CQ15.1 An imperceptibly slight breeze blowing over the edge of a leaf can produce fluttering in the same way that a breeze can cause a flag to flap. As a leaf twists in the wind, the fibers in its stem provide a restoring torque. If the frequency of the breeze matches the natural frequency of vibration of one particular leaf as a torsional pendulum, that leaf can be driven into a large-amplitude resonance vibration. Note that it is not the *size* of the driving force that sets the leaf into resonance, but the *frequency* of the driving force. If the frequency changes, another leaf will be set into resonant oscillation.

CQ15.2 (a) No. Since the acceleration is not constant in simple harmonic motion, none of the equations in Table 2.2 are valid.

(b)

Equation	*Information given by equation*
$x(t) = A\cos(\omega t + \phi)$	position as a function of time

$v(t) = -\omega A \sin(\omega t + \phi)$	velocity as a function of time
$v(x) = \pm\omega\left(A^2 - x^2\right)^{1/2}$	velocity as a function of position
$a(t) = -\omega^2 A \cos(\omega t + \phi)$	acceleration as a function of time
$a(t) = -\omega^2 x(t)$	acceleration as a function of position

(c) The angular frequency ω appears in every equation.

CQ15.3 (a) The general equation of position is $x(t) = A\cos(\omega t + \phi)$. If $x = -A\cos(\omega t)$, then $\phi = \pi$, or equally well, $\phi = -\pi$.

(b) At $t = 0$, the particle is at its turning point on the negative side of equilibrium, at $x = -A$.

CQ15.4 We assume the diameter of the bob is not very small compared to the length of the cord supporting it. As the water leaks out, the center of mass of the bob moves down, increasing the effective length of the pendulum and slightly lowering its frequency. As the last drops of water dribble out, the center of mass of the bob moves back up to the center of the sphere, and the pendulum frequency quickly increases to its original value.

CQ15.5 (a) No force is exerted on the particle. The particle moves with constant velocity.

(b) The particle feels a constant force toward the left. It moves with constant acceleration toward the left. If its initial push is toward the right, it will slow down, turn around, and speed up in motion toward the left. If its initial push is toward the left, it will just speed up.

(c) A constant force toward the right acts on the particle to produce constant acceleration toward the right.

(d) The particle moves in simple harmonic motion about the lowest point of the potential energy curve.

CQ15.6 Most everyday vibrations are damped, they eventually die down as their energy is transferred to their surroundings. However, as you will learn later, atoms in the molecules have vibration modes that do not damp out.

CQ15.7 The mechanical energy of a damped oscillator changes back and forth between kinetic and potential while it gradually and permanently decreases and transforms to internal energy.

CQ15.8 Yes. An oscillator with damping can vibrate at resonance with amplitude that remains constant in time. Without damping, the amplitude would increase without limit at resonance.

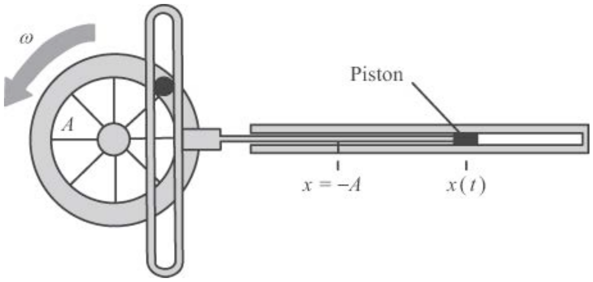
CQ15.9 No. If the resistive force is large compared to the restoring force of the spring (in particular, if $b^2 > 4mk$), the system will be overdamped and will not oscillate.

CQ15.10 The period of a pendulum depends on the acceleration of gravity: $T = 2\pi\sqrt{\frac{L}{g}}$. If the acceleration of gravity is different at the top of the mountain, the period is different and the pendulum does not keep perfect time. Two things can effect the acceleration of gravity, the top of the mountain is farther from the center of the Earth, and the nearby large mass of the mountain under the pendulum.

CQ15.11 Neither are examples of simple harmonic motion, although they are both periodic motion. In neither case is the acceleration proportional to the displacement from an equilibrium position. Neither motion is so smooth as SHM. The ball's acceleration is very large when it is in contact with the floor, and the student's when the dismissal bell rings.

CQ15.12 The motion will be periodic—that is, it will repeat, though it is not harmonic at large angles. The period is nearly constant as the angular amplitude increases through small values; then the period becomes noticeably larger as θ increases farther.

CQ15.13 The angle of the crank pin is $\theta = \omega t$. Its x coordinate is $x = A\cos\theta = A\cos\omega t$, where A is the distance from the center of the wheel to the crank pin. This is of the form $x = A\cos(\theta t + \phi)$, so the yoke and piston move with simple harmonic motion.



ANS FIG. CQ15.13

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 15.1 Motion of an Object Attached to a Spring

P15.1 (a) Taking to the right as positive, the spring force acting on the block at the instant of release is

$$\begin{aligned} F_s &= -kx_i = -(130\ \text{N/m})(+0.13\ \text{m}) \\ &= -17\ \text{N} \quad \text{or} \quad \boxed{17\ \text{N to the left}} \end{aligned}$$

(b) At this instant, the acceleration is

$$a = \frac{F_s}{m} = \frac{-17\ \text{N}}{0.60\ \text{kg}} = -28\ \text{m/s}^2$$

or $\boxed{a = 28\ \text{m/s}^2\ \text{to the left}}$

P15.2 When the object comes to equilibrium (at distance y_0 *below* the unstretched position of the end of the spring), $\sum F_y = -k(-y_0) - mg = 0$ and the force constant is

$$k = \frac{mg}{y_0} = \frac{(4.25\ \text{kg})(9.80\ \text{m/s}^2)}{2.62 \times 10^{-2}\ \text{m}} = 1.59 \times 10^3\ \text{N} = \boxed{1.59\ \text{kN/m}}$$

Section 12.2 Analysis Model: Particle in Simple Harmonic Motion

P15.3 The spring constant is found from

$$k = \frac{F_s}{x} = \frac{mg}{x} = \frac{(0.010\ \text{kg})(9.80\ \text{m/s}^2)}{3.9 \times 10^{-2}\ \text{m}} = 2.5\ \text{N/m}$$

When the object attached to the spring has mass m = 25 g, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\ \text{kg}}{2.5\ \text{N/m}}} = \boxed{0.63\ \text{s}}$$

P15.4 (a) The equation for the piston's position is given as

$$x = (5.00\ \text{cm})\cos\left(2t + \frac{\pi}{6}\right)$$

At $t = 0$,

$$x = (5.00 \text{ cm}) \cos\left(\frac{\pi}{6}\right) = \boxed{4.33 \text{ cm}}$$

(b) Differentiating the equation for position with respect to time gives us the piston's velocity:

$$v = \frac{dx}{dt} = -(10.0 \text{ cm/s}) \sin\left(2t + \frac{\pi}{6}\right)$$

At $t = 0$, $v = \boxed{-5.00 \text{ cm/s}}$

(c) Differentiating again gives its acceleration:

$$a = \frac{dv}{dt} = -(20.0 \text{ cm/s}^2) \cos\left(2t + \frac{\pi}{6}\right)$$

At $t = 0$, $a = \boxed{-17.3 \text{ cm/s}^2}$

(d) The period of motion is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2} = \boxed{3.14 \text{ s}}$$

(e) We read the amplitude directly from the equation for x:

$$A = \boxed{5.00 \text{ cm}}$$

P15.5 $x = (4.00 \text{ m})\cos(3.00\pi t + \pi)$; compare this with $x = A\cos(\omega t + \phi)$ to find

(a) $\omega = 2\pi f = 3.00\pi$ or $\boxed{f = 1.50 \text{ Hz}}$

(b) $T = \dfrac{1}{f} = \boxed{0.667 \text{ s}}$

(c) $A = \boxed{4.00 \text{ m}}$

(d) $\phi = \boxed{\pi \text{ rad}}$

(e) $x(t = 0.250 \text{ s}) = (4.00 \text{ m}) \cos(1.75\pi) = \boxed{2.83 \text{ m}}$

P15.6 From the information given, we write the equation for position as $x = A\cos\omega t$, with the amplitude given as $A = 0.050\,0$ m. Differentiating gives us the piston's velocity,

$$v = -A\omega \sin\omega t$$

and differentiating again gives its acceleration

$$a = -A\omega^2 \cos \omega t$$

Then, if $f = 3600$ rev/min = 60 Hz, then $\omega = 2\pi f = 120\pi \text{ s}^{-1}$

(a) $v_{\max} = \omega A = (120\pi)(0.050\ 0) \text{ m/s} = \boxed{18.8 \text{ m/s}}$

(b) $a_{\max} = \omega^2 A = (120\pi)^2 (0.050\ 0) \text{ m/s}^2 = \boxed{7.11 \text{ km/s}^2}$

15.7 The 0.500 s must elapse between one turning point and the other. Thus the period is 1.00 s.

$$\omega = \frac{2\pi}{T} = 6.28 \text{ s}^{-1}$$

and $v_{\max} = \omega A = (6.28 \text{ s}^{-1})(0.100 \text{ m}) = \boxed{0.628 \text{ m/s}}$.

P15.8 (a) From the information given,

$$T = \frac{12.0 \text{ s}}{5} = \boxed{2.40 \text{ s}}$$

(b) $f = \frac{1}{T} = \frac{1}{2.40} = \boxed{0.417 \text{ Hz}}$

(c) $\omega = 2\pi f = 2\pi(0.417) = \boxed{2.62 \text{ rad/s}}$

P15.9 An object hanging from a vertical spring moves with simple harmonic motion just like an object moving without friction attached to a horizontal spring. We are given the period, which is related to the frequency of motion by $T = 1/f$. Then, since $\omega = 2\pi f = \sqrt{\frac{k}{m}}$,

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$

Solving for k,

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (7.00 \text{ kg})}{(2.60 \text{ s})^2} = \boxed{40.9 \text{ N/m}}$$

***P15.10** For a simple harmonic oscillator, the maximum speed occurs at the equilibrium position and is given by Equation 15.17:

$$v_{\max} = A\sqrt{\frac{k}{m}}$$

Thus,

$$m = \frac{kA^2}{v_{\max}^2} = \frac{(16.0\ \text{N/m})(0.200\ \text{m})^2}{(0.400\ \text{m/s})^2} = 4.00\ \text{kg}$$

and

$$F_g = mg = (4.00\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{39.2\ \text{N}}$$

***P15.11** The mass of the cube is

$$m = \rho V = (2.70\times 10^3\ \text{kg/m}^3)(0.015\ \text{m})^3 = 9.11\times 10^{-3}\ \text{kg}$$

The spring constant of the strip of steel is

$$k = \frac{F}{x} = \frac{1.43\ \text{N}}{0.027\,5\ \text{m}} = 52.0\ \text{N/m}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{52.0\ \text{N/m}}{9.11\times 10^{-3}\ \text{kg}}} = \boxed{12.0\ \text{Hz}}$$

P15.12 (a) The spring constant of this spring is

$$k = \frac{F}{x} = \frac{mg}{x} = \frac{(0.450\ \text{kg})(9.80\ \text{m/s}^2)}{0.350\ \text{m}} = 12.6\ \text{N/m}$$

We take the x axis pointing downward, so $\phi = 0$.

$$x = A\cos\omega t = (18.0\ \text{cm})\cos\left[\sqrt{\frac{12.6\ \text{N/m}}{0.450\ \text{kg}}}(84.4\ \text{s})\right]$$

$$= (18.0\ \text{cm})\cos(446.6\ \text{rad}) = \boxed{15.8\ \text{cm}}$$

(b) Now $446.6\ \text{rad} = 71\times 2\pi + 0.497\ \text{rad}$. In each cycle the object moves 4(18) = 72 cm, so it has moved
$71(72\ \text{cm}) + (18 - 15.8)\ \text{cm} = \boxed{51.1\ \text{m}}$

(c) By the same steps, $k = \dfrac{(0.440\ \text{kg})(9.80\ \text{m/s}^2)}{0.355\ \text{m}} = 12.1\ \text{N/m}$.

$$x = A\cos\sqrt{\frac{k}{m}}t = (18.0\ \text{cm})\cos\left[\sqrt{\frac{12.1\ \text{N/m}}{0.440\ \text{kg}}}(84.4\ \text{s})\right]$$

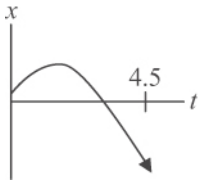
$$= (18.0\ \text{cm})\cos(443.4\ \text{rad}) = \boxed{-15.9\ \text{cm}}$$

(d) $443.4\ \text{rad} = 70.569(2\pi)$

Distance moved $= 70.569(0.72\ \text{m}) = \boxed{50.8\ \text{m}}$

(e) $\boxed{\text{The patterns of oscillation diverge from each other, starting out in phase but becoming completely out of phase. To calculate the future we would need } \textit{exact} \text{ knowledge of the present, an impossibility.}}$

P15.13 (a) For constant acceleration position is given as a function of time by

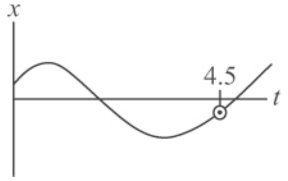


ANS. FIG. P15.13(a, b)

$$\begin{aligned}x &= x_i + v_{xi}t + \frac{1}{2}a_x t^2\\ &= 0.270\text{ m} + (0.140\text{ m/s})(4.50\text{ s})\\ &\qquad + \frac{1}{2}(-0.320\text{ m/s}^2)(4.50\text{ s})^2\\ &= \boxed{-2.34\text{ m}}\end{aligned}$$

(b) $v_x = v_{xi} + a_x t = 0.140\text{ m/s} - (0.320\text{ m/s}^2)(4.50\text{ s}) = \boxed{-1.30\text{ m/s}}$

(c) For simple harmonic motion we have instead



ANS. FIG. P15.13(c, d)

$$x = A\cos(\omega t + \phi)$$

and $v = -A\omega\sin(\omega t + \phi)$

where $a = -\omega^2 x$, so that $-0.320\text{ m/s}^2 = -\omega^2(0.270\text{ m})$, $\omega = 1.09\text{ rad/s}$.

At $t = 0$, $0.270\text{ m} = A\cos\phi$ and $0.140\text{ m/s} = -A(1.09\text{ s}^{-1})\sin\phi$.

Dividing gives $\dfrac{0.140\text{ m/s}}{0.270\text{ m}} = -(1.09\text{ s}^{-1})\tan\phi$, $\tan\phi = -0.476$, $\phi = -25.5°$. Still at $t = 0$, $0.270\text{ m} = A\cos(-25.5°)$, $A = 0.299\text{ m}$.

Now at $t = 4.50$ s,

$$\begin{aligned}x &= (0.299\text{ m})\cos[(1.09\text{ rad/s})(4.50\text{ s}) - 25.5°]\\ &= (0.299\text{ m})\cos(4.90\text{ rad} - 25.5°)\\ &= (0.299\text{ m})\cos 255°\\ &= \boxed{-0.076\ 3\text{ m}}\end{aligned}$$

(d) $v = -(0.299\text{ m})(1.09\text{ s}^{-1})\sin 255° = \boxed{+0.315\text{ m/s}}$

P15.14 (a) Since the collision is perfectly elastic, the ball will rebound to the height of 4.00 m and then repeat the motion over and over again. Thus, the $\boxed{\text{motion is periodic}}$.

(b) To determine the period, we use $x = \frac{1}{2}gt^2$. The time for the ball to hit the ground is

$$t = \sqrt{\frac{2x}{g}} = \sqrt{\frac{2(4.00\text{ m})}{9.80\text{ m/s}^2}} = 0.904\text{ s}$$

This equals one-half the period, so $T = 2(0.904\text{ s}) = \boxed{1.81\text{ s}}$.

(c) $\boxed{\begin{array}{l}\text{The motion is not simple harmonic. The net force action on the}\\ \text{ball is a constant given by } F = -mg \text{ (except when it is in contact}\\ \text{with the ground), which is not in the form of Hooke's law.}\end{array}}$

P15.15 The period of the oscillation is
$T = 1/f = 1/1.50\text{ Hz} = 1/(3/2\text{ s}^{-1}) = 2/3\text{ s}.$

(a) At $t = 0$, $x = 0$ and v is positive (to the right). Therefore, this situation corresponds to $x = A\sin\omega t$ and $v = v_i\cos\omega t$. Since $f = 1.50$ Hz, $\omega = 2\pi f = 3.00\pi$, and $A = 2.00$ cm:

$$\boxed{x = 2.00\cos(3.00\pi t - 90°) = 2.00\sin 3.00\pi t}$$

where x is in centimenters and t is in seconds.

(b) $v_{\text{max}} = v_i = A\omega = 2.00(3.00\pi) = 6.00\pi\text{ cm/s} = \boxed{18.8\text{ cm/s}}$

(c) The particle has this speed at $t = 0$ and next after half a period:

$$t = \frac{T}{2} = \boxed{\frac{1}{3}\text{ s}}$$

(d) $a_{\text{max}} = A\omega^2 = 2.00(3.00\pi)^2 = 18.0\pi^2\text{ cm/s}^2 = \boxed{178\text{ cm/s}^2}$

(e) This positive value of maximum acceleration first occurs when the particle is reversing its direction on the negative x axis, three-quarters of a period after $t = 0$: at $t = \frac{3}{4}T = \boxed{0.500\text{ s}}$.

(f) Since $T = \frac{2}{3}$ s and $A = 2.00$ cm, the particle will travel 8.00 cm in one cycle. Hence, in $1.00\text{ s} = \frac{3}{2}T = 1\tfrac{1}{2}$ cycles, the particle will travel $8.00\text{ cm} + 4.00\text{ cm} = \boxed{12.0\text{ cm}}$.

***P15.16** The proposed solution,

$$x(t) = x_i \cos \omega t + \left(\frac{v_i}{\omega}\right) \sin \omega t$$

implies velocity

$$v = \frac{dx}{dt} = -x_i\omega \sin \omega t + v_i \cos \omega t$$

and acceleration

$$\begin{aligned} a &= \frac{dv}{dt} = -x_i\omega^2 \cos \omega t - v_i\omega \sin \omega t \\ &= -\omega^2\left(x_i \cos \omega t + \left(\frac{v_i}{\omega}\right) \sin \omega t\right) = -\omega^2 x \end{aligned}$$

(a) The acceleration being a negative constant times position means we do have SHM, and its angular frequency is ω. At $t = 0$ the equations reduce to $x = x_i$ and $v = v_i$, so they satisfy all the requirements.

(b) $v^2 - ax = v^2 - (-\omega^2 x)x = v^2 + \omega^2 x^2$

$$\begin{aligned} &= (-x_i\omega \sin \omega t + v_i \cos \omega t)^2 + \omega^2\left(x_i \cos \omega t + \left(\frac{v_i}{\omega}\right) \sin \omega t\right)^2 \\ &= x_i^2\omega^2 \sin^2 \omega t - 2x_i v_i\omega \sin \omega t \cos \omega t + v_i^2 \cos^2 \omega t \\ &\qquad + x_i^2\omega^2 \cos^2 \omega t + 2x_i v_i\omega \cos \omega t \sin \omega t \\ &\qquad + v_i^2 \sin^2 \omega t \\ &= x_i^2\omega^2 + v_i^2 \end{aligned}$$

So the expression $v^2 - ax$ is constant in time because all the parameters in the final equivalent expression $x_i^2\omega^2 + v_i^2$ are constant. Because $v^2 - ax$ must have the same value at all times, it must be equal to the value at $t = 0$, so $v^2 - ax = v_i^2 - a_i x_i$. If we evaluate $v^2 - ax$ at a turning point where $v = 0$ and $x = A$, it is $v^2 - ax = v^2 + \omega^2 x^2 = 0^2 + \omega^2(A^2) = \omega^2 A^2$. Thus it is proved.

***P15.17** (a) The distance traveled in one cycle is four times the amplitude of motion, or $\boxed{20.0\text{ cm}}$.

(b) $v_{\max} = \omega A = 2\pi fA = 2\pi(3.00\text{ Hz})(5.00\text{ cm}) = \boxed{94.2\text{ cm/s}}$

This occurs as the particle passes through equilibrium.

(c) $a_{\max} = \omega^2 A = \left(2\pi f\right)^2 A = \left[2\pi(3.00\text{ Hz})\right]^2(0.05\text{ m}) = \boxed{17.8\text{ m/s}^2}$

This occurs at maximum excursion from equilibrium.

P15.18 $m = 1.00$ kg, $k = 25.0$ N/m, and $A = 3.00$ cm. At $t = 0$, $x = -3.00$ cm.

(a) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{25.0\text{ N/m}}{1.00\text{ kg}}} = 5.00\text{ rad/s}$

so that, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{5.00} = \boxed{1.26\text{ s}}$

(b) $v_{\max} = A\omega = \left(3.00\times10^{-2}\text{ m}\right)\left(5.00\text{ rad/s}\right) = \boxed{0.150\text{ m/s}}$

$a_{\max} = A\omega^2 = \left(3.00\times10^{-2}\text{ m}\right)\left(5.00\text{ rad/s}\right)^2 = \boxed{0.750\text{ m/s}^2}$

(c) Because $x = -3.00$ cm and $v = 0$ at $t = 0$, the required solution is $x = -A\cos\omega t$, or

$\boxed{x = 3.00\cos\left(5.00t + \pi\right)}$

Then, $v = \dfrac{dx}{dt} = \boxed{-15.0\sin\left(5.00t + \pi\right)}$

and $a = \dfrac{dv}{dt} = \boxed{-75.0\cos\left(5.00t + \pi\right)}$

where x is in cm, v is in cm/s, and a is in cm/s^2.

P15.19 $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{8.00\text{ N/m}}{0.500\text{ kg}}} = 4.00\text{ s}^{-1}$. Assuming the position of the object is at the origin at $t = 0$, position is given by $x = 10.0\sin\left(4.00t\right)$, where x is in cm. From this, we find that $v = 40.0\cos\left(4.00t\right)$, where v is in cm/s, and $a = -160\sin\left(4.00t\right)$, where a is in cm/s^2.

(a) $v_{\max} = \omega A = \left(4.00\text{ rad/s}\right)\left(10.0\text{ cm}\right) = \boxed{40.0\text{ cm/s}}$

(b) $a_{\max} == \omega^2 A = \left(4.00\text{ rad/s}\right)^2\left(10.0\text{ cm}\right) = \boxed{160\text{ cm/s}^2}$

From our assumed expression for x, we solve for the time t:

$$t = \left(\frac{1}{4.00\text{ Hz}}\right)\sin^{-1}\left(\frac{x}{10.0\text{ cm}}\right)$$

When $x = 6.00$ cm, $t = \left(\frac{1}{4.00\text{ Hz}}\right)\sin^{-1}\left(\frac{6.00\text{ cm}}{10.0\text{ cm}}\right) = 0.161$ s.

We find then that at that time:

(c) $v = (40.0\text{ cm/s})\cos\left[(4.00\text{ Hz})(0.161\text{ s})\right] = \boxed{32.0\text{ cm/s}}$ and

(d) $a = -(160\text{ cm/s}^2)\sin\left[(4.00\text{ Hz})(0.161\text{ s})\right] = \boxed{-96.0\text{ cm/s}^2}$

(e) Using $t = \left(\frac{1}{4.00\text{ Hz}}\right)\sin^{-1}\left(\frac{x}{10.0\text{ cm}}\right)$ we find that when $x = 0$, $t = 0$, and when $x = 8.00$ cm, $t = 0.232$ s. Therefore, $\Delta t = \boxed{0.232\text{ s}}$.

P15.20 (a) $\boxed{\text{Yes.}}$

(b) We assume that the mass of the spring is negligible and that we are on Earth. Let m represent the mass of the object. Its hanging at rest is described by

$$\sum F = 0 \rightarrow kx - mg = 0 \rightarrow k = \frac{mg}{x}, \text{ where } x = 18.3\text{ cm}$$

To find the period, we must find the angular frequency $T = \frac{2\pi}{\omega}$.

We do not know the mass, but we do not need it because

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{mg}{x}\frac{1}{m}} = \sqrt{\frac{g}{x}}$$

From our value for x, we find

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{x}{g}} = 2\pi\sqrt{\frac{0.183\text{ m}}{9.80\text{ m/s}^2}} = 0.859\text{ s}$$

$\boxed{\text{We see that finding the period does not depend on knowing the mass: } T = 0.859\text{ s.}}$

Section 15.3 Energy of the Simple Harmonic Oscillator

P15.21 Choose the car with its shock-absorbing bumper as the system; by conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2:$$

$$v = x\sqrt{\frac{k}{m}} = \left(3.16\times10^{-2}\text{ m}\right)\sqrt{\frac{5.00\times10^6\text{ N/m}}{10^3\text{ kg}}} = \boxed{2.23\text{ m/s}}$$

P15.22 We are given $m = 200$ g, $T = 0.250$ s, $E = 2.00$ J, and

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.250} = 25.1\text{ rad/s}$$

(a) $k = m\omega^2 = \left(0.200\text{ kg}\right)\left(25.1\text{ rad/s}\right)^2 = \boxed{126\text{ N/m}}$

(b) $E = \dfrac{kA^2}{2} \Rightarrow A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2\left(2.00\text{ J}\right)}{126\text{ N/m}}} = \boxed{0.178\text{ m}}$

***P15.23** (a) Energy is conserved for the block-spring system between the maximum-displacement and the half-maximum points:

$$(K+U)_i = (K+U)_f$$

$$0 + \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}(6.50\text{ N/m})(0.100\text{ m})^2 = \frac{1}{2}m(0.300\text{ m/s})^2 + \frac{1}{2}(6.50\text{ N/m})\left(5.00\times10^{-2}\text{ m}\right)^2$$

$$3.25\times10^{-2}\text{ J} = \frac{1}{2}m(0.300\text{ m/s})^2 + 8.12\times10^{-3}\text{ J}$$

giving $m = \dfrac{2\left(2.44\times10^{-2}\text{ J}\right)}{9.0\times10^{-2}\text{ m}^2/\text{s}^2} = \boxed{0.542\text{ kg}}$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{6.50\text{ N/m}}{0.542\text{ kg}}} = 3.46\text{ rad/s}$

Then, $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi\text{ rad}}{3.46\text{ rad/s}} = \boxed{1.81\text{ s}}$

(c) $a_{\max} = A\omega^2 = (0.100\text{ m})(3.46\text{ rad/s})^2 = \boxed{1.20\text{ m/s}^2}$

***P15.24** (a) The mechanical energy of the system is equal to the potential energy stored in the spring at maximum amplitude:

$$E = \frac{kA^2}{2} = \frac{(250 \text{ N/m})(3.50 \times 10^{-2} \text{ m})^2}{2} = \boxed{0.153 \text{ J}}$$

(b) $v_{\max} = A\omega$, where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}} = 22.4 \text{ s}^{-1}$,

giving $v_{\max} = \boxed{0.784 \text{ m/s}}$

(c) $a_{\max} = A\omega^2 = (3.50 \times 10^{-2} \text{ m})(22.4 \text{ s}^{-1})^2 = \boxed{17.5 \text{ m/s}^2}$

***P15.25** Model the oscillator as a block-spring system. From energy considerations,

$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

with $v_{\max} = \omega A$ and $v = \frac{\omega A}{2}$, so

$$\left(\frac{\omega A}{2}\right)^2 + \omega^2 x^2 = \omega^2 A^2$$

From this we find

$$x^2 = \frac{3}{4} A^2$$

and since $A = 3.00$ cm,

$$x = \pm\frac{\sqrt{3}}{2} A = \boxed{\pm 2.60 \text{ cm}}$$

***P15.26** (a) $E = \frac{1}{2}kA^2$, so if $A' = 2A$, $E' = \frac{1}{2}k(A')^2 = \frac{1}{2}k(2A)^2 = 4E$

Therefore $\boxed{E \text{ increases by factor of 4.}}$

(b) $v_{\max} = \sqrt{\frac{k}{m}}A$, so if A is doubled, $\boxed{v_{\max} \text{ is doubled.}}$

(c) $a_{\max} = \frac{k}{m}A$, so if A is doubled, $\boxed{a_{\max} \text{ also doubles.}}$

(d) $T = 2\pi\sqrt{\frac{m}{k}}$ is independent of A, so $\boxed{\text{the period is unchanged.}}$

P15.27 (a) $E = \frac{1}{2}kA^2 = \frac{1}{2}(35.0\ \text{N/m})(4.00\times10^{-2}\ \text{m})^2 = \boxed{28.0\ \text{mJ}}$

(b) $|v| = \omega\sqrt{A^2 - x^2} = \sqrt{\frac{k}{m}}\sqrt{A^2 - x^2}$

$$|v| = \sqrt{\frac{35.0\ \text{N/m}}{50.0\times10^{-3}\ \text{kg}}}\sqrt{(4.00\times10^{-2}\ \text{m})^2 - (1.00\times10^{-2}\ \text{m})^2}$$
$$= \boxed{1.02\ \text{m/s}}$$

(c) $\frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$

$$= \frac{1}{2}(35.0\ \text{N/m})\left[(4.00\times10^{-2}\ \text{m})^2 - (3.00\times10^{-2}\ \text{m})^2\right]$$
$$= \boxed{12.2\ \text{mJ}}$$

(d) $\frac{1}{2}kx^2 = E - \frac{1}{2}mv^2 = 28.0\ \text{mJ} - 12.2\ \text{mJ} = \boxed{15.8\ \text{mJ}}$

P15.28 (a) $k = \frac{|F|}{|x|} = \frac{20.0\ \text{N}}{0.200\ \text{m}} = \boxed{100\ \text{N/m}}$

(b) $\omega = \sqrt{\frac{k}{m}} = \sqrt{50.0}\ \text{rad/s}$ so $f = \frac{\omega}{2\pi} = \boxed{1.13\ \text{Hz}}$

(c) $v_{\text{max}} = \omega A = \sqrt{50.0}(0.200) = \boxed{1.41\ \text{m/s}}$

(d) Maximum speed occurs when the object passes through its equilibrium position, at $\boxed{x = 0}$.

(e) $a_{\text{max}} = \omega^2 A = 50.0(0.200) = \boxed{10.0\ \text{m/s}^2}$

(f) Maximum acceleration occurs where the object reverses direction, which is where its distance from equilibrium is a maximum, at $x = \pm A = \boxed{\pm 0.200\ \text{m}}$.

(g) $E = \frac{1}{2}kA^2 = \frac{1}{2}(100\ \text{N/m})(0.200\ \text{m})^2 = \boxed{2.00\ \text{J}}$

(h) $|v| = \omega\sqrt{A^2 - x^2} = (\sqrt{50.0}\ \text{rad/s})\sqrt{\frac{8}{9}(0.200)^2}\ \text{m} = \boxed{1.33\ \text{m/s}}$

(i) $|a| = \omega^2 x = (50.0\ \text{rad}^2/\text{s}^2)\left(\frac{0.200\ \text{m}}{3}\right) = \boxed{3.33\ \text{m/s}^2}$

P15.29 (a) Energy is conserved by an isolated simple harmonic oscillator:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \to \frac{1}{2}mv^2 = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$\to \frac{1}{2}mv^2 = \frac{1}{2}k\left(A^2 - x^2\right)$$

When $x = A/3$,

$$\frac{1}{2}mv^2 = \frac{1}{2}k\left(A^2 - x^2\right) = \frac{1}{2}k\left[A^2 - \left(\frac{A}{3}\right)^2\right] = \frac{1}{2}kA^2\left[1 - \frac{1}{9}\right]$$

$$\frac{1}{2}mv^2 = \frac{1}{2}kA^2\frac{8}{9} = \boxed{\frac{8}{9}E}$$

(b) When $x = A/3$,

$$\frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{A}{3}\right)^2 = \frac{1}{9}\left(\frac{1}{2}kA^2\right) = \boxed{\frac{1}{9}E}$$

(c) $\frac{1}{2}kA^2 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kx^2\right) + \frac{1}{2}kx^2$

$$\frac{1}{2}kA^2 = \frac{3}{4}kx^2 \to x = \boxed{\pm\sqrt{\frac{2}{3}}A}$$

(d) $\boxed{\text{No.}}$ The maximum potential energy of the system is equal to the total energy of the system: kinetic plus potential energy. Because the total energy must remain constant, the kinetic energy can never be greater than the maximum potential energy.

P15.30 (a) $\boxed{\text{Particle under constant acceleration.}}$

(b) $y_{fi} = y + v_{yi}t + \frac{1}{2}a_y t^2$:

$$-11.0\text{ m} = 0 + 0 + \frac{1}{2}\left(-9.80\text{ m/s}^2\right)t^2$$

$$t = \sqrt{\frac{22.0\text{ m}}{9.80\text{ m/s}^2}} = \boxed{1.50\text{ s}}$$

(c) The system of the bungee jumper, the spring (cord), and the Earth is $\boxed{\text{isolated}}$.

(d) The system is isolated, so energy is conserved within the system. Take the initial point where she steps off the bridge and the final point at the bottom of her motion.

$$\left(K+U_g+U_s\right)_i=\left(K+U_g+U_s\right)_f$$

$$0+mgy+0=0+0+\frac{1}{2}kx^2$$

$$(65.0\text{ kg})\left(9.80\text{ m/s}^2\right)(36.0\text{ m})=\frac{1}{2}k(25.0\text{ m})^2$$

which gives $k=\boxed{73.4\text{ N/m}}$

(e) The spring extension at equilibrium is

$$x=\frac{F}{k}=\frac{mg}{k}=\frac{(65.0\text{ kg})\left(9.80\text{ m/s}^2\right)}{73.4\text{ N/m}}=8.68\text{ m}$$

so this point is $11.0+8.68\text{ m}=\boxed{19.7\text{ m below the bridge}}$ and the amplitude of her oscillation is 36.0 m – 19.7 m = 16.3 m.

(f) $\omega=\sqrt{\dfrac{k}{m}}=\sqrt{\dfrac{73.4\text{ N/m}}{65.0\text{ kg}}}=\boxed{1.06\text{ rad/s}}$

(g) Set $x = 0$ at the equilibrium position of the bungee jumper on the spring. Relative to the equilibrium position, the lowest part of the drop corresponds to $x = +16.3$ m—we have taken down as positive—and the point in the drop where the spring begins to stretch is at $x = –8.68$ m. Take the phase as zero at maximum downward extension ($x = +16.3$ m). We find that the phase, ωt, was 25 m higher where $x = –8.68$ (above the equilibrium point):

$x=A\cos\omega t$: at time $t = 0$, $x=(16.3\text{ m})\cos 0=16.3\text{ m}$, and when $x=-8.68=16.3\cos(\omega t)$, $\omega t=\pm 122°=\pm 2.13$ rad. Which sign do we pick for ωt? From $v=\dfrac{dx}{dt}=-\omega A\sin\omega t$, at $x = –8.68$ m, v is downward, which means by our choice of positive direction, v is positive. Pick $\omega t=-2.13$ rad:

$v=-\omega A\sin(-2.13\text{ rad})=+\omega A(0.848)$, which is positive.

Therefore, $\omega t=1.06t=-2.13\text{ rad}\rightarrow t=\dfrac{-2.13\text{ rad}}{1.06}=-2.01\text{ s}$,

meaning $t = –2.01$ s when the spring begins to stretch and $t = 0$ when the jumper reaches the bottom of the jump: then $\boxed{+2.01\text{ s}}$ is the time over which the spring stretches.

(h) total time $=1.50\text{ s}+2.01\text{ s}=\boxed{3.50\text{ s}}$

P15.31 (a) $F = k|x| = (83.8\ \text{N/m})(5.46\times10^{-2}\ \text{m}) = \boxed{4.58\ \text{N}}$

(b) $E = U_s = \frac{1}{2}kx^2 = \frac{1}{2}(83.8\ \text{N/m})(5.46\times10^{-2}\ \text{m})^2 = \boxed{0.125\ \text{J}}$

(c) While the block was held stationary at $x = 5.46$ cm, $\sum F_x = -F_s + F = 0,$ or the spring force was equal in magnitude and oppositely directed to the applied force. When the applied force is suddenly removed, there is a net force $F_s = 4.58$ N directed toward the equilibrium position acting on the block. This gives the block an acceleration having magnitude

$$|a| = \frac{F_s}{m} = \frac{4.58\ \text{N}}{0.250\ \text{kg}} = \boxed{18.3\ \text{m/s}^2}$$

(d) At the equilibrium position, $PE_s = 0$, so the block has kinetic energy $K = E = 0.125$ J and speed

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.125\ \text{J})}{0.250\ \text{kg}}} = \boxed{1.00\ \text{m/s}}$$

(e) $\boxed{\text{Smaller.}}$ Friction would transform some kinetic energy into internal energy.

(f) $\boxed{\text{The coefficient of kinetic friction between the block and surface.}}$

(g) The block will come to a stop after sliding through distance $d = x = 0.054\ 6$ m.

$$\Delta E_{\text{mech}} = \Delta K + \Delta U = -f_k d$$

$$0 + \left(0 - \frac{1}{2}kx^2\right) = -f_k d = -\mu_k mgd \rightarrow \mu_k = \frac{kx^2}{2mgd} = \frac{kx^2}{2mgx} = \frac{kx}{2mg}$$

$$\rightarrow \mu_k = \frac{(83.8\ \text{N/m})(0.054\ 6\ \text{m})}{2(0.250\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{0.934}$$

P15.32 (a) At the equilibrium position, the total energy of the system is in the form of kinetic energy and $mv_{\text{max}}^2/2 = E,$ so the maximum speed is

$$v_{\text{max}} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(5.83\ \text{J})}{0.326\ \text{kg}}} = \boxed{5.98\ \text{m/s}}$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant of the spring is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2 (0.326 \text{ kg})}{(0.250 \text{ s})^2} = \boxed{206 \text{ N/m}}$$

(c) At the turning points, $x = \pm A$, the total energy of the system is in the form of elastic potential energy, or $E = KA^2/2$, giving the amplitude as

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(5.83 \text{ J})}{206 \text{ N/m}}} = \boxed{0.238 \text{ m}}$$

Section 15.4 Comparing Simple Harmonic Motion with Uniform Circular Motion

P15.33 (a) $\boxed{\text{The motion is simple harmonic because the tire is rotating with constant angular velocity and you see the projection of the motion of the bump in a plane perpendicular to the tire.}}$

(b) Since the car is moving with speed $v = 3.00$ m/s, and its radius is 0.300 m, we have

$$\omega = \frac{3.00 \text{ m/s}}{0.300 \text{ m}} = 10.0 \text{ rad/s}$$

Therefore, the period of the motion is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(10.0 \text{ rad/s})} = \boxed{0.628 \text{ s}}$$

Section 15.5 The Pendulum

P15.34 The period in Tokyo is $T_T = 2\pi\sqrt{\frac{L_T}{g_T}}$, and the period in Cambridge is

$$T_C = 2\pi\sqrt{\frac{L_C}{g_C}}.$$

We know that $T_T = T_C = 2.00\text{ s}$, which means that $\frac{L_T}{g_T} = \frac{L_C}{g_C}$,

or $\frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.994\ 2}{0.992\ 7} = \boxed{1.001\ 5}$

P15.35 The period of a pendulum is the time for one complete oscillation and is given by $T = 2\pi\sqrt{\ell/g}$, where ℓ is the length of the pendulum.

(a) $T = \left(\frac{3.00\text{ min}}{120\text{ oscillations}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{1.50\text{ s}}$

(b) The length of the pendulum is

$$\ell = g\left(\frac{T^2}{4\pi^2}\right) = \left(9.80\text{ m/s}^2\right)\left(\frac{(1.50\text{ s})^2}{4\pi^2}\right) = \boxed{0.559\text{ m}}$$

P15.36 Referring to ANS. FIG. P15.36, we have

$$F = -mg\sin\theta \quad\text{and}\quad \tan\theta = \frac{x}{R}$$

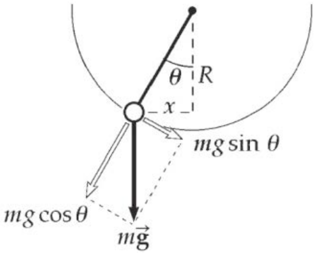
For small displacements,

$$\tan\theta \approx \sin\theta \quad\text{and}\quad F = -\frac{mg}{R}x = -kx$$

Since the restoring force is proportional to the displacement from equilibrium, the motion is simple harmonic motion.

Comparing to $F = -m\omega^2 x$ shows

$$\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{R}}}$$



ANS. FIG. P15.36

P15.37 *f* = 0.450 Hz, *d* = 0.350 m, and *m* = 2.20 kg. Now,

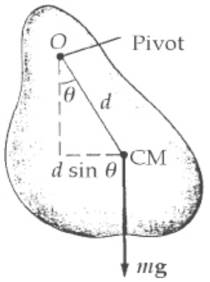
$$T = \frac{1}{f}$$

$$T = 2\pi\sqrt{\frac{I}{mgd}} \rightarrow T^2 = \frac{4\pi^2 I}{mgd}$$

Solving for the moment of inertia, we obtain

$$I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} = \frac{(2.20\text{ kg})\left(9.80\text{ m/s}^2\right)(0.350\text{ m})}{4\pi^2\left(0.450\text{ s}^{-1}\right)^2}$$

$$= \boxed{0.944\text{ kg}\cdot\text{m}^2}$$



ANS. FIG. P15.37

P15.38 Please see ANS. FIG. P15.37. For a physical pendulum,

$$T = \frac{1}{f}$$

$$T = 2\pi\sqrt{\frac{I}{mgd}} \rightarrow T^2 = \frac{4\pi^2 I}{mgd}$$

$$\rightarrow I = T^2\frac{mgd}{4\pi^2} = \left(\frac{1}{f}\right)^2\frac{mgd}{4\pi^2} \rightarrow \boxed{I = \frac{mgd}{4\pi^2 f^2}}$$

***P15.39** We solve $\omega = \dfrac{2\pi}{T}$ for $T = \dfrac{2\pi}{\omega} = \dfrac{2\pi}{4.43} = \boxed{1.42\text{ s}}$

We then solve $\omega = \sqrt{\dfrac{g}{L}}$ for $L = \dfrac{g}{\omega^2} = \dfrac{9.80\text{ m/s}^2}{(4.43\text{ rad/s})^2} = \boxed{0.499}$

P15.40 (a) The parallel-axis theorem gives $I = I_{\text{CM}} + md^2$,

so $T = 2\pi\sqrt{\dfrac{I}{mgd}} = \boxed{2\pi\sqrt{\dfrac{I_{\text{CM}} + md^2}{mgd}}}$

(b) When *d* is very large $T \rightarrow 2\pi\sqrt{\dfrac{d}{g}}$ gets large.

When *d* is very small $T \rightarrow 2\pi\sqrt{\dfrac{I_{\text{CM}}}{mgd}}$ gets large.

So there must be a minimum, found by

$$\frac{dT}{dd} = 0 = \frac{d}{dd}2\pi\left(I_{\text{CM}} + md^2\right)^{1/2}\left(mgd\right)^{-1/2}$$

$$= 2\pi\left(I_{\text{CM}} + md^2\right)^{1/2}\left(-\frac{1}{2}\right)\left(mgd\right)^{-3/2}mg$$

$$+ 2\pi\left(mgd\right)^{-1/2}\left(\frac{1}{2}\right)\left(I_{\text{CM}} + md^2\right)^{-1/2}2md$$

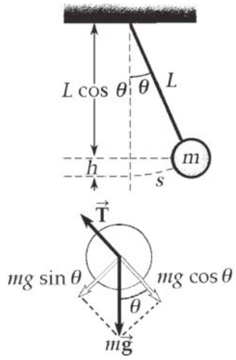
$$= \frac{-\pi\left(I_{\text{CM}} + md^2\right)mg}{\left(I_{\text{CM}} + md^2\right)^{1/2}\left(mgd\right)^{3/2}} + \frac{2\pi\,md\,mgd}{\left(I_{\text{CM}} + md^2\right)^{1/2}\left(mgd\right)^{3/2}} = 0$$

This requires

$$-I_{\text{CM}} - md^2 + 2md^2 = 0$$

or $\boxed{I_{\text{CM}} = md^2}$

P15.41 Using the simple harmonic motion model:



ANS. FIG. P15.41

$$A = r\theta = (1.00\text{ m})\left[(15.0^\circ)\frac{\pi}{180^\circ}\right] = 0.262\text{ m}$$

$$\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.80\text{ m/s}^2}{1.00\text{ m}}} = 3.13\text{ rad/s}$$

(a) $v_{\text{max}} = A\omega = (0.262\text{ m})(3.13\text{ s}^{-1})$

$$= \boxed{0.820\text{ m/s}}$$

(b) For simple harmonic motion, the maximum acceleration

$$a_{\text{max}} = A\omega^2 = (0.262\text{ m})(3.13\text{ s}^{-1})^2 = 2.57\text{ m/s}^2$$

which is equal to the maximum tangential acceleration, occurs at the extreme ends of the swing:

$$a_{\text{t}} = r\alpha \rightarrow \alpha = \frac{a_{\text{t}}}{r} = \frac{2.57\text{ m/s}^2}{1.00\text{ m}} = \boxed{2.57\text{ rad/s}^2}$$

(c) The maximum restoring force causes the maximum acceleration:

$$F = ma_{\text{max}} = 0.25\text{ kg}\,(2.57\text{ m/s}^2) = \boxed{0.641\text{ N}}$$

(d) (a) Applying energy conservation to the isolated pendulum-Earth system:

$$K_i + U_i = K_f + U_f \rightarrow mgh = \frac{1}{2}mv^2$$

and $h = L(1-\cos\theta)$,

then

$$v_{\text{max}} = \sqrt{2gh} = \sqrt{2gL(1-\cos\theta)}$$
$$= \sqrt{2(9.80\text{ m/s}^2)(1.00\text{ m})(1-\cos 15.0^\circ)}$$
$$= \boxed{0.817\text{ m/s}}$$

P15.42 (a) The parallel-axis theorem gives:

$$I = I_{\text{CM}} + Md^2 = \frac{1}{12}ML^2 + Md^2$$

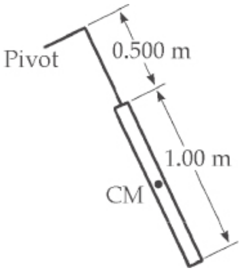
$$= \frac{1}{12}M(1.00\text{ m})^2 + M(1.00\text{ m})^2$$

$$= M\left(\frac{13}{12}\text{ m}^2\right)$$

$$T = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{M(13/12\text{ m}^2)}{Mg(1.00\text{ m})}}$$

$$= 2\pi\sqrt{\frac{13/12\text{ m}}{9.80\text{ m/s}^2}}$$

$$= \boxed{2.09\text{ s}}$$



ANS. FIG. P15.42

(b) For the simple pendulum,

$$T = 2\pi\sqrt{\frac{1.00\text{ m}}{9.80\text{ m/s}^2}} = 2.01\text{ s}$$

$$\text{difference} = \frac{2.09\text{ s} - 2.01\text{ s}}{2.01\text{ s}} = \boxed{4.08\%}$$

P15.43 (a) The string tension must support the weight of the bob, accelerate it upward, and also provide the restoring force, just as if the elevator were at rest in a gravity field of (9.80 + 5.00) m/s^2. Thus the period is

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{5.00\text{ m}}{14.8\text{ m/s}^2}}$$

$$T = \boxed{3.65\text{ s}}$$

(b) $$T = 2\pi\sqrt{\frac{5.00\text{ m}}{(9.80\text{ m/s}^2 - 5.00\text{ m/s}^2)}} = \boxed{6.41\text{ s}}$$

(c) $$g_{\text{eff}} = \sqrt{(9.80\text{ m/s}^2)^2 + (5.00\text{ m/s}^2)^2} = 11.0\text{ m/s}^2$$

$$T = 2\pi\sqrt{\frac{5.00\text{ m}}{11.0\text{ m/s}^2}} = \boxed{4.24\text{ s}}$$

P15.44 (a) From $T = \dfrac{\text{total measured time}}{50}$,

the measured periods are:

Length, L (m)	1.000	0.750	0.500
Period, T (s)	2.00	1.73	1.42

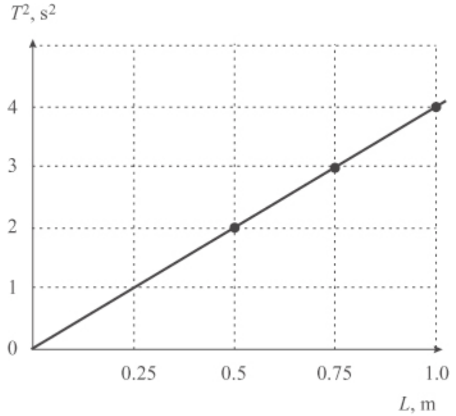
(b) $T = 2\pi\sqrt{\dfrac{L}{g}}$ so $g = \dfrac{4\pi^2 L}{T^2}$

The calculated values for g are:

Period, T (s)	2.00	1.73	1.42
g (m/s^2)	9.87	9.89	9.79

Thus, $g_{\text{avg}} = 9.85\ \text{m/s}^2$

$\boxed{\text{This agrees with the accepted value of } g = 9.80\ \text{m/s}^2 \text{ within } 0.5\%.}$



ANS. FIG. P15.44

(c) From $T^2 = \left(\dfrac{4\pi^2}{g}\right)L$, the slope of T^2 versus L graph is

$$\frac{4\pi^2}{g} = 3.97\ \text{s}^2/\text{m}$$

Thus, $g = \dfrac{4\pi^2}{\text{slope}} = \boxed{9.94\ \text{m/s}^2}$. This is within 1.5% of the accepted value for *g*.

P15.45 The period of oscillation for the watch balance wheel is $T = 0.250$ s. Modeling the 20.0-g mass as a particle, we find the moment of inertia from $I = mr^2$.

(a) $I = mr^2 = \left(2.00\times10^{-2}\ \text{kg}\right)\left(5.00\times10^{-3}\ \text{m}\right)^2 = \boxed{5.00\times10^{-7}\ \text{kg}\cdot\text{m}^2}$

(b) To find the torsion constant, we use Equation 15.29 for the motion of a torsional pendulum,

$$I\frac{d^2\theta}{dt^2} = -\kappa\theta$$

where

$$\sqrt{\frac{\kappa}{I}} = \omega = \frac{2\pi}{T}$$

Solving for the torsion constant gives

$$\kappa = I\omega^2 = \left(5.00\times10^{-7}\ \text{kg}\cdot\text{m}^2\right)\left(\frac{2\pi}{0.250\ \text{s}}\right)^2 = \boxed{3.16\times10^{-4}\ \frac{\text{N}\cdot\text{m}}{\text{rad}}}$$

Section 15.6 Damped Oscillations

P15.46 We are given $\theta_i = 15.0°$, and $\theta(t = 1\ 000\ \text{s}) = 5.50°$. We then use Equation 15.32 for damped oscillations:

$$x = Ae^{-bt/2m}$$

Substituting,

$$\frac{x_{1\,000}}{x_i} = \frac{Ae^{-bt/2m}}{A} = \frac{5.50}{15.0} = e^{-b(1\,000)/2m}$$

$$\ln\left(\frac{5.50}{15.0}\right) = -1.00 = \frac{-b(1\ 000)}{2m}$$

which gives

$$\frac{b}{2m} = \boxed{1.00\times10^{-3}\ \text{s}^{-1}}$$

P15.47 If the oscillation was undamped, its frequency would be

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.05\times 10^4\ \text{N/m}}{10.6\ \text{kg}}} = 44.0\ \text{s}^{-1}$$

(a) With damping, the frequency becomes

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} = \sqrt{\left(44\ \frac{1}{\text{s}}\right)^2 - \left(\frac{3\ \text{kg}}{2\times 10.6\ \text{kg}}\right)}$$

$$= \sqrt{1\,933.96 - 0.02} = 44.0\ \text{s}^{-1}$$

$$f = \frac{\omega}{2\pi} = \frac{44.0\ \text{s}^{-1}}{2\pi} = \boxed{7.00\ \text{Hz}}$$

(b) In $x = A_0 e^{-bt/2m}\cos(\omega t + \phi)$ over one cycle, a time $T = \frac{2\pi}{\omega}$, the amplitude changes from A_0 to $A_0 e^{-b2\pi/2m\omega}$ for a fractional decrease of

$$\frac{A_0 - A_0 e^{-\pi b/m\omega}}{A_0} = 1 - e^{-\pi 3/(10.6\cdot 44.0)} = 1 - e^{-0.020\,2} = 1 - 0.979\,98$$

$$= 0.020\ 0 = \boxed{2.00\%}$$

(c) The energy is proportional to the square of the amplitude, so its fractional rate of decrease is twice as fast:

$$E = \frac{1}{2}kA^2 = \frac{1}{2}kA_0^2 e^{-2bt/2m} = E_0 e^{-bt/m}$$

We specify

$$(0.050\ 0)E_0 = E_0 e^{-3t/10.6}$$

$$0.050\ 0 = e^{-3t/10.6}$$

$$e^{+3t/10.6} = 20.0$$

$$\frac{3t}{10.6} = \ln 20.0 = 3.00$$

$$t = \boxed{10.6\ \text{s}}$$

P15.48 The total energy is $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$.

Taking the time derivative gives $\frac{dE}{dt} = mv\frac{d^2x}{dt^2} + kxv$.

Then, substituting from Equation 15.31, $\frac{md^2x}{dt^2} = -kx - bv,$ gives

$$\frac{dE}{dt} = v\left(-kx - bv\right) + kvx$$

Thus, $$\boxed{\frac{dE}{dt} = -bv^2 < 0}$$

We have proved that the mechanical energy of a damped oscillator is always decreasing.

P15.49 To show that $x = Ae^{-bt/2m}\cos\left(\omega t + \phi\right)$

is a solution of $-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$ **[1]**

where $\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$ and $b^2 < 4mk$ so that ω is real, **[2]**

we take $x = Ae^{-bt/2m}\cos\left(\omega t + \phi\right)$ and compute **[3]**

$$\frac{dx}{dt} = Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos\left(\omega t + \phi\right) - Ae^{-bt/2m}\omega\sin\left(\omega t + \phi\right) \quad \textbf{[4]}$$

$$\frac{d^2x}{dt^2} = -\frac{b}{2m}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos\left(\omega t + \phi\right) - Ae^{-bt/2m}\omega\sin\left(\omega t + \phi\right)\right]$$

$$-\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\omega\sin\left(\omega t + \phi\right) + Ae^{-bt/2m}\omega^2\cos\left(\omega t + \phi\right)\right] \quad \textbf{[5]}$$

We substitute [3] and [4] into the left side of [1], and [5] into the right side of [1]:

$$-kAe^{-bt/2m}\cos\left(\omega t + \phi\right) + \frac{b^2}{2m}Ae^{-bt/2m}\cos\left(\omega t + \phi\right)$$
$$+ b\omega Ae^{-bt/2m}\sin\left(\omega t + \phi\right)$$
$$= -\frac{b}{2}\left[Ae^{-bt/2m}\left(-\frac{b}{2m}\right)\cos\left(\omega t + \phi\right) - Ae^{-bt/2m}\omega\sin\left(\omega t + \phi\right)\right]$$
$$+ \frac{b}{2}Ae^{-bt/2m}\omega\sin\left(\omega t + \phi\right) - m\omega^2 Ae^{-bt/2m}\cos\left(\omega t + \phi\right)$$

We then compare the coefficients of the $Ae^{-bt/2m}\cos\left(\omega t + \phi\right)$ and the $Ae^{-bt/2m}\sin\left(\omega t + \phi\right)$ terms.

The cosine term is

$$-k+\frac{b^2}{2m}=-\frac{b}{2}\left(-\frac{b}{2m}\right)-m\omega^2=\frac{b^2}{4m}-m\left(\frac{k}{m}-\frac{b^2}{4m^2}\right)=-k+\frac{b^2}{2m}$$

and the sine term is

$$b\omega=+\frac{b}{2}(\omega)+\frac{b}{2}(\omega)=b\omega$$

Since the coefficients are equal, $x=Ae^{-bt/2m}\cos(\omega t+\phi)$ is a solution of the equation.

Section 15.7 Forced Oscillations

P15.50 (a) For resonance, her frequency must match:

$$f_0=\frac{\omega_0}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=\frac{1}{2\pi}\sqrt{\frac{7.00\times10^2\ \text{N/m}}{12.5\ \text{kg}}}=\boxed{1.19\ \text{Hz}}$$

(b) From $x=A\cos\omega t$, $v=\dfrac{dx}{dt}=-A\omega\sin\omega t$, and $a=\dfrac{dv}{dt}=-A\omega^2\cos\omega t$, the maximum acceleration is $A\omega^2$. When this becomes equal to the acceleration due to gravity, the normal force exerted on her by the mattress will drop to zero at one point in the cycle:

$$A\omega^2=g\quad\text{or}\quad A=\frac{g}{\omega^2}=\frac{g}{k/m}=\frac{gm}{k}$$

$$A=\frac{(9.80\ \text{m/s}^2)(12.5\ \text{kg})}{7.00\times10^2\ \text{N/m}}=\boxed{17.5\ \text{cm}}$$

P15.51 The pendulum is resonating with the beeper. The beeper must vibrate at the frequency of a simple pendulum of frequency 1.50 Hz:

$$\omega=2\pi f=\sqrt{\frac{g}{L}}\rightarrow L=\frac{g}{(2\pi f)^2}=\frac{9.80\ \text{m/s}^2}{[2\pi(1.50\ \text{Hz})]^2}$$

$$=0.110\ \text{m}=\boxed{11.0\ \text{cm}}$$

P15.52 From the equation for the amplitude of a driven oscillator with no damping,

$$A = \frac{F_0/m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2}} = \frac{F_0/m}{\omega^2 - \omega_0^2}$$

which gives

$$F_0 = mA\left(\omega^2 - \omega_0^2\right)$$

The driving frequency is

$$\omega^2 = (2\pi f)^2 = \left[2\pi\left(10.0\ \text{s}^{-1}\right)\right]^2 = 3.95\times 10^3\ \text{s}^{-2}$$

and the natural frequency of the oscillator is

$$\omega_0^2 = \frac{k}{m} = \frac{200\ \text{N/m}}{40.0\ \text{N}/9.80\ \text{m/s}^2} = 49.0\ \text{s}^{-2}$$

Substituting gives us a driving force of

$$F_0 = \left(\frac{40.0\ \text{N}}{9.80\ \text{m/s}^2}\right)\left(2.00\times 10^{-2}\ \text{m}\right)\left(3.95\times 10^3\ \text{s}^{-2} - 49.0\ \text{s}^{-2}\right)$$

$$= \boxed{318\ \text{N}}$$

P15.53 We are given $F = 3.00 \sin(2\pi t)$, $k = 20.0$ N/m, and $m = 2.00$ kg.

(a) $$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{20.0\ \text{N/m}}{2.00\ \text{kg}}} = \boxed{3.16\ \text{s}^{-1}}$$

(b) From $F = 3.00 \sin(2\pi t)$, the angular frequency of the force is

$$\omega = 2\pi = \boxed{6.28\ \text{s}^{-1}}$$

(c) From equation 15.36, the amplitude A of a driven oscillator, with $b = 0$, gives

$$A = \frac{F_0/m}{\omega^2 - \omega_0^2} = \frac{(3.00\ \text{N/m})/(2.00\ \text{kg})}{\left(6.28\ \text{s}^{-1}\right)^2 - \left(3.16\ \text{s}^{-1}\right)^2} = 0.050\ 9\ \text{m} = \boxed{5.09\ \text{cm}}$$

P15.54 We start with Equation 15.34, $F_0 \sin \omega t - kx = m\dfrac{d^2x}{dt^2}$ **[1]**

Equation 15.35 gives the solution to this equation as

$$x = A\cos(\omega t + \phi)$$ **[2]**

differentiating,

$$\frac{dx}{dt} = -A\omega \sin(\omega t + \phi) \qquad \textbf{[3]}$$

and differentiating again,

$$\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \phi) \qquad \textbf{[4]}$$

Substituting [2] and [4] into [1]:

$$F_0 \sin \omega t - kA \cos(\omega t + \phi) = m(-A\omega^2) \cos(\omega t + \phi)$$

Solving for the amplitude:

$$(kA - mA\omega^2) \cos(\omega t + \phi) = F_0 \sin \omega t = -F_0 \cos(\omega t + 90°)$$

These will be equal, provided only that ϕ must be +90° and

$$kA - mA\omega^2 = -F_0$$

Thus, $A = \dfrac{F_0/m}{\omega^2 - \omega_0^2}$, where $\omega_0 = \sqrt{\dfrac{k}{m}}$.

P15.55 We use the equation for the amplitude of forced oscillations,

$$A = \frac{F_{\text{ext}}/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + (b\omega/m)^2}}$$

With $b = 0$,

$$A = \frac{F_{\text{ext}}/m}{\sqrt{(\omega^2 - \omega_0^2)^2}} = \frac{F_{\text{ext}}/m}{\pm(\omega^2 - \omega_0^2)} = \pm\frac{F_{\text{ext}}/m}{\omega^2 - \omega_0^2}$$

Thus,

$$\omega^2 = \omega_0^2 \pm \frac{F_{\text{ext}}/m}{A} = \frac{k}{m} \pm \frac{F_{\text{ext}}}{mA}$$

$$= \frac{6.30 \text{ N/m}}{0.150 \text{ kg}} \pm \frac{1.70 \text{ N}}{(0.150 \text{ kg})(0.440 \text{ m})}$$

This yields $\omega = 8.23$ rad/s or $\omega = 4.03$ rad/s. Then,

$$f = \frac{\omega}{2\pi} \text{ gives either } f = \boxed{1.31 \text{ Hz}} \quad \text{or} \quad f = \boxed{0.641 \text{ Hz}}$$

Additional Problems

P15.56 Deuterium is the isotope of the element hydrogen with atoms having nuclei consisting of one proton and one neutron. For brevity we refer to the molecule formed by two deuterium atoms as *D* and to the diatomic molecule of hydrogen-1 as *H*, with $M_D = 2M_H$.

$$\frac{\omega_D}{\omega_H} = \frac{\sqrt{k/M_D}}{\sqrt{k/M_H}} = \sqrt{\frac{M_H}{M_D}} = \sqrt{\frac{1}{2}}$$

$$f_D = \frac{f_H}{\sqrt{2}} = \frac{1.30\times 10^{14}\ \text{Hz}}{\sqrt{2}} = \boxed{0.919\times 10^{14}\ \text{Hz}}$$

P15.57 From $a = -\omega^2 x$, the maximum acceleration is given by $a_{\text{max}} = \omega^2 A$. Then $108\ \text{cm/s}^2 = \omega^2\ (12.0\ \text{cm})$, giving $\omega = 3.00$ rad/s.

(a) $T = 1/f = 2\pi/\omega = 2\pi/\left(3.00\ \text{s}^{-1}\right) = \boxed{2.09\ \text{s}}$

(b) $f = \omega/2\pi = \left(3.00\ \text{s}^{-1}\right)/2\pi = \boxed{0.477\ \text{Hz}}$

(c) $v_{\text{max}} = \omega A = \left(3\ \text{s}^{-1}\right)\left(12.0\ \text{cm}\right) = \boxed{36.0\ \text{cm/s}}$

(d) $E = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}m\left(0.360\ \text{m/s}\right)^2$

$$= \boxed{0.064\ 8m,\ \text{where } E \text{ is in joules and } m \text{ is in kg}}$$

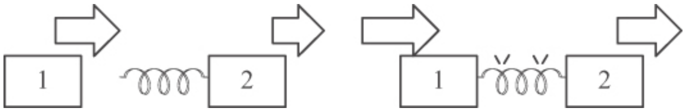
(e) From $\omega^2 = \dfrac{k}{m}$, we obtain

$$k = \omega^2 m = \left(3.00\ \text{s}^{-1}\right)^2 m$$

$$= \boxed{9.00m,\ \text{where } k \text{ is in newtons/meter and } m \text{ is in kg}}$$

(f) $\boxed{\text{Period, frequency, and maximum speed are all independent of mass in this situation. The energy and the force constant are directly proportional to mass.}}$

P15.58 (a) Consider the first process of spring compression. It continues as long as glider 1 is moving faster than glider 2. The spring instantaneously has maximum compression when both gliders are moving with the same speed v_a.



ANS. FIG. P15.58(a)

Momentum conservation then gives

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

$$(0.240\text{ kg})(0.740\text{ m/s}) + (0.360\text{ kg})(0.12\text{ m/s}) = (0.240\text{ kg})v_a + (0.360\text{ kg})v_a$$

$$\frac{0.220\ 8\text{ kg}\cdot\text{m/s}}{0.600\text{ kg}} = v_a$$

$$v_a = \boxed{0.368\text{ m/s}}$$

(b) From energy conservation, we have

$$(K_1 + K_2 + U_s)_i = (K_1 + K_2 + U_s)_f$$

$$\frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 + 0 = \frac{1}{2}(m_1 + m_2)v_a^2 + \frac{1}{2}kx^2$$

$$\frac{1}{2}(0.240\text{ kg})(0.740\text{ m/s})^2 + \frac{1}{2}(0.360\text{ kg})(0.120\text{ m/s})^2$$

$$= \frac{1}{2}(0.600\text{ kg})(0.368\text{ m/s})^2 + \frac{1}{2}(45.0\text{ N/m})x^2$$

$$0.068\ 3\text{ J} = 0.040\ 6\text{ J} + \frac{1}{2}(45.0\text{ N/m})x^2$$

$$x = \left(\frac{2(0.027\ 7\text{ J})}{45.0\text{ N/m}}\right)^{1/2} = 0.035\ 1\text{ m} = \boxed{3.51\text{ cm}}$$

(c) $\frac{1}{2}m_{\text{tot}}v_{\text{CM}}^2 = \frac{1}{2}(0.600\text{ kg})(0.368\text{ m/s})^2 = 0.040\ 6\text{ J} = \boxed{40.6\text{ mJ}}$

(d) $\frac{1}{2}kx^2 = \frac{1}{2}(45.0\text{ N/m})(0.0351\text{ m})^2 = 0.027\ 7\text{ J} = \boxed{27.7\text{ mJ}}$

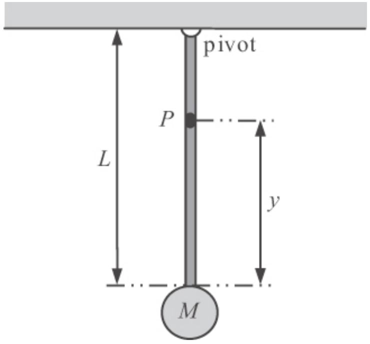
P15.59 Let *F* represent the tension in the rod.

(a) At the pivot,

$$F = Mg + Mg = \boxed{2Mg}$$

(b) A fraction of the rod's weight $Mg\left(\frac{y}{L}\right)$ as well as the weight of the ball pulls down on point *P*. Thus, the tension in the rod at point *P* is

$$F = Mg\left(\frac{y}{L}\right) + Mg = \boxed{Mg\left(1 + \frac{y}{L}\right)}$$



ANS. FIG. P15.59

(c) Relative to the pivot, $I = I_{\text{rod}} + I_{\text{ball}} = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2$.

For the physical pendulum, $T = 2\pi\sqrt{\frac{I}{mgd}}$, where $m = 2M$ and d is the distance from the pivot to the center of mass of the rod and ball combination. Therefore,

$$d = \frac{M(L/2) + ML}{M + M} = \frac{3L}{4}$$

and $T = 2\pi\sqrt{\frac{(4/3)ML^2}{(2M)g(3L/4)}} = \boxed{\frac{4\pi}{3}\sqrt{\frac{2L}{g}}}$.

(d) For $L = 2.00$ m, $T = \frac{4\pi}{3}\sqrt{\frac{2(2.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{2.68\text{ s}}$.

P15.60 (a) From $a = -\omega^2 x$, the maximum acceleration is given by $a_{\max} = \omega^2 A$. As A increases, the maximum acceleration increases. When it becomes greater than the free-fall acceleration, the rock will no longer stay in contact with the vibrating ground, but lag behind as the ground moves down with greater acceleration. We have then

$$A = \frac{g}{\omega^2} = \frac{g}{(2\pi f)^2} = \frac{9.80\text{ m/s}^2}{\left[2\pi(2.40\text{ s}^{-1})\right]^2} = \boxed{4.31\text{ cm}}$$

(b) $\boxed{\text{When the rock is on the point of lifting off, the surrounding water is also barely in free fall. No pressure gradient exists in the water, so no buoyant force acts on the rock. The effect of the surrounding water disappears at that instant.}}$

***P15.61** For the resonance vibration with the occupants in the car, we have for the spring constant of the suspension:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$\begin{aligned} k &= 4\pi^2 f^2 m = 4\pi^2(1.80\text{ s}^{-1})^2\left[1\,130\text{ kg} + 4(72.4\text{ kg})\right] \\ &= 1.82\times 10^5\text{ kg/s}^2 \end{aligned}$$

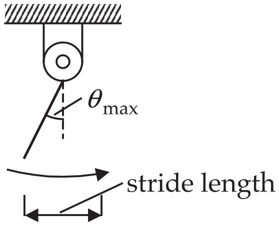
Now as the occupants exit,

$$x = \frac{F}{k} = \frac{4(72.4\text{ kg})(9.8\text{ m/s}^2)}{1.82\times10^5\text{ kg/s}^2} = \boxed{1.56\times10^{-2}\text{ m}}$$

***P15.62** (a) The period of the swinging rod is

$$T = 2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{(1/3)m\ell^2}{mg\ell/2}}$$

$$= 2\pi\sqrt{\frac{2\ell}{3g}}$$



ANS. FIG. P15.62

The time for one half a cycle is $\frac{T}{2} = \pi\sqrt{\frac{2\ell}{3g}}$.

The distance traveled in this time is the stride length $2\ell\sin\theta_{\text{max}}$, so the speed is

$$\frac{d}{t} = \frac{2\ell\sin\theta_{\text{max}}}{\pi\sqrt{2\ell/3g}} = \frac{\sqrt{2\ell 3g}\,\sin\theta_{\text{max}}}{\pi} = \frac{\sqrt{6g\ell}\,\sin\theta_{\text{max}}}{\pi}$$

(b) We use the more precise expression

$$\frac{\sqrt{6g\ell\cos(\theta_{\text{max}}/2)}\,\sin\theta_{\text{max}}}{\pi}$$

$$= \frac{\sqrt{6(9.80\text{ m/s}^2)(0.850\text{ m})\cos 14.0^\circ}\,\sin 28.0^\circ}{\pi}$$

$$= \boxed{1.04\text{ m/s}}$$

(c) With

$$v_{\text{old}} = \frac{\sqrt{6g\ell_{\text{old}}\cos(\theta_{\text{max}}/2)}\,\sin\theta_{\text{max}}}{\pi}$$

$$v_{\text{new}} = \frac{\sqrt{6g\ell_{\text{new}}\cos(\theta_{\text{max}}/2)}\,\sin\theta_{\text{max}}}{\pi}$$

dividing gives

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\ell_{\text{new}}}}{\sqrt{\ell_{\text{old}}}} = 2$$

$$\frac{\ell_{\text{new}}}{0.850\text{ m}} = 2^2$$

$$\ell_{\text{new}} = \boxed{3.40\text{ m}}$$

***P15.63** From $T = 2\pi\sqrt{\frac{L}{g}}$, the length of a pendulum with period T is $L = \frac{gT^2}{4\pi^2}$.

(a) On Earth, with T = 1.0 s,

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.8\text{ m/s}^2)(1.0\text{ s})^2}{4\pi^2} = 0.25\text{ m} = \boxed{25\text{ cm}}$$

(b) If T = 1.0 s on Mars, then

$$L = \frac{g_{\text{Mars}}T^2}{4\pi^2} = \frac{(3.7\text{ m/s}^2)(1.0\text{ s})^2}{4\pi^2} = 0.094\text{ m} = \boxed{9.4\text{ cm}}$$

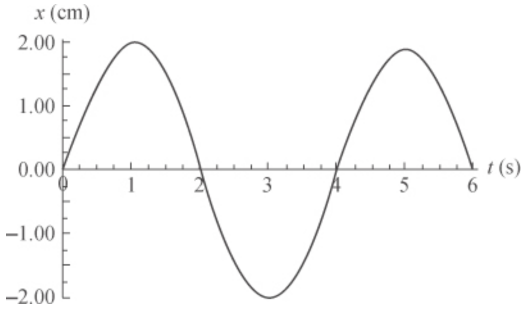
(c) and (d) The period of an object on a spring is $T = 2\pi\sqrt{\frac{m}{k}}$, which is independent of the local free-fall acceleration. Thus, the same mass will work on Earth and on Mars. This mass is

$$m = \frac{kT^2}{4\pi^2} = \frac{(10\text{ N/m})(1.0\text{ s})^2}{4\pi^2} = \boxed{0.25\text{ kg}}$$

P15.64 (a) The amplitude is the magnitude of the maximum displacement from equilibrium (at x = 0). Thus, $\boxed{A = 2.00\text{ cm}}$.

(b) The period is the time for one full cycle of the motion. Therefore, $\boxed{T = 4.00\text{ s}}$.

(c) The angular frequency is $\omega = \frac{2\pi}{T} = \frac{2\pi}{4.00\text{ s}} = \boxed{\frac{\pi}{2}\text{ rad/s}}$.



ANS. FIG. P15.64

(d) The maximum speed is

$$v_{\max} = \omega A = \left(\frac{\pi}{2}\text{ rad/s}\right)(2.00\text{ cm}) = \boxed{\pi\text{ cm/s}}$$

(e) The maximum acceleration is

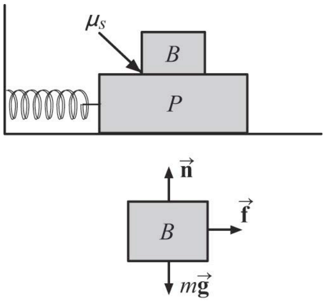
$$a_{\max} = \omega^2 A = \left(\frac{\pi}{2}\ \text{rad/s}\right)^2 (2.00\ \text{cm}) = \boxed{4.93\ \text{cm/s}^2}$$

(f) The general equation for position as a function of time for an object undergoing simple harmonic motion with $x = 0$ when $t = 0$ and x increasing positively is $x = A\sin\omega t$. For this oscillator, this becomes

$$\boxed{x = 2.00\sin\left(\frac{\pi}{2}t\right),\ \text{where } x \text{ is in centimeters and } t \text{ in seconds.}}$$

P15.65 The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating Block B at this rate. Thus, if Block B is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg$$
$$= m\left(4\pi^2 Af^2\right)$$



ANS. FIG. P15.65

which gives a maximum amplitude of oscillation of

$$A = \frac{\mu_s g}{4\pi^2 f^2} = \frac{(0.600)(980\ \text{cm/s}^2)}{4\pi^2(1.50\ \text{s}^{-1})^2} = \boxed{6.62\ \text{cm}}$$

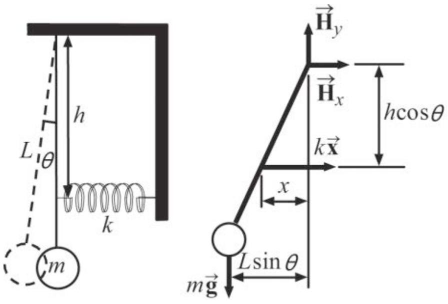
P15.66 Refer to ANS. FIG. P15.65. The maximum acceleration of the oscillating system is $a_{\max} = A\omega^2 = 4\pi^2 Af^2$. The friction force exerted between the two blocks must be capable of accelerating Block B at this rate. Thus, if Block B is about to slip,

$$f = f_{\max} = \mu_s n = \mu_s mg = m\left(4\pi^2 Af^2\right)$$

which gives a maximum amplitude of oscillation of

$$A = \boxed{\frac{\mu_s g}{4\pi^2 f^2}}$$

P15.67 We draw a free-body diagram of the pendulum in ANS. FIG. P15.67. The force $\vec{\mathbf{H}}$ exerted by the hinge causes no torque about the axis of rotation.



ANS. FIG. P15.67

$$\tau = I\alpha \quad \text{and} \quad \frac{d^2\theta}{dt^2} = -\alpha$$

$$\tau = MgL\sin\theta + kxh\cos\theta$$

$$= -I\frac{d^2\theta}{dt^2}$$

For small-amplitude vibrations, use the approximations: $\sin\theta \approx \theta$, $\cos\theta \approx 1$, and $x \approx s = h\theta$.

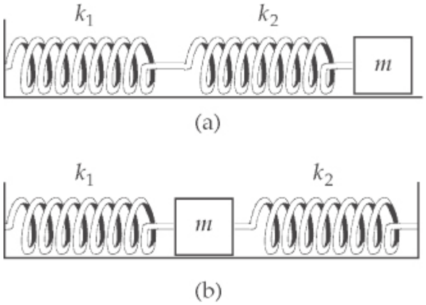
Therefore,

$$\frac{d^2\theta}{dt^2} = -\left(\frac{MgL + kh^2}{I}\right)\theta = -\omega^2\theta$$

$$\omega = \sqrt{\frac{MgL + kh^2}{ML^2}} = 2\pi f$$

$$f = \boxed{\frac{1}{2\pi L}\sqrt{gL + \frac{kh^2}{M}}}$$

P15.68 (a) When the mass is displaced a distance x from equilibrium, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2.



ANS. FIG. P15.68

By Newton's third law, we expect

$$k_1x_1 = k_2x_2$$

When this is combined with the requirement that

$$x = x_1 + x_2$$

we find $x_1 = \left[\frac{k_2}{k_1 + k_2}\right]x$.

The force on either spring is given by $F_1 = \left[\frac{k_1k_2}{k_1 + k_2}\right]x = ma$

where a is the acceleration of the mass m.

This is in the form $F = k_{\text{eff}}x = ma$

and $T = 2\pi\sqrt{\dfrac{m}{k_{\text{eff}}}} = \boxed{2\pi\sqrt{\dfrac{m(k_1+k_2)}{k_1k_2}}}$.

(b) In this case each spring is distorted by the distance x which the mass is displaced. Therefore, the restoring force is

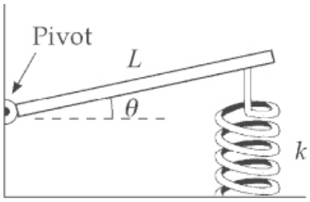
$$F = -(k_1 + k_2)x \quad \text{and} \quad k_{eff} = k_1 + k_2$$

so that $T = \boxed{2\pi\sqrt{\dfrac{m}{k_1+k_2}}}$.

P15.69 At equilibrium, we have

$$\sum\tau = 0 - mg\left(\frac{L}{2}\right) + kx_0L$$

where x_0 is the equilibrium compression.



ANS. FIG. P15.69

After displacement by a small angle (we assume cos $\theta \approx 1$),

$$\sum\tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k(x_0 - L\theta)L = -k\theta L^2$$

But,

$$\sum\tau = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

so $$\frac{d^2\theta}{dt^2} = -\frac{3k}{m}\theta$$

Comparing this result to the general form for simple harmonic motion in which the angular acceleration is opposite in direction and proportional to the displacement,

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

we find that

$$\omega^2 = \frac{3k}{m} \rightarrow \omega = \sqrt{\frac{3k}{m}} = \sqrt{\frac{3(100\text{ N/m})}{5.00\text{ kg}}} = \boxed{7.75\text{ s}^{-1}}$$

P15.70 Please refer to ANS. FIG. P15.69. At equilibrium, we have

$$\sum \tau = 0 - mg\left(\frac{L}{2}\right) + kx_0 L$$

where x_0 is the equilibrium compression.

After displacement by a small angle (we assume $\cos\theta \approx 1$),

$$\sum \tau = -mg\left(\frac{L}{2}\right) + kxL = -mg\left(\frac{L}{2}\right) + k\left(x_0 - L\theta\right)L = -k\theta L^2$$

But,

$$\sum \tau = I\alpha = \frac{1}{3}mL^2\frac{d^2\theta}{dt^2}$$

Comparing this result to the general form for simple harmonic motion in which the angular acceleration is opposite in direction and proportional to the displacement,

$$\frac{d^2\theta}{dt^2} = -\omega^2\theta$$

we find that

$$\omega^2 = \frac{3k}{m} \to \boxed{\omega = \sqrt{\frac{3k}{m}}}$$

P15.71 As it passes through equilibrium, the 4.00-kg object has speed

$$v_{\max} = \omega A = \sqrt{\frac{k}{m}}A = \sqrt{\frac{100\ \text{N/m}}{4.00\ \text{kg}}}\left(2.00\ \text{m}\right) = 10.0\ \text{m/s}$$

In the completely inelastic collision, momentum of the two-object system is conserved. So the new 10.0-kg object starts its oscillation with a new maximum speed given by

$$\left(4.00\ \text{kg}\right)\left(10.0\ \text{m/s}\right) + \left(6.00\ \text{kg}\right)0 = \left(10.0\ \text{kg}\right)v_{\max}$$
$$v_{\max} = 4.00\ \text{m/s}$$

(a) The system consisting of the two objects, the spring, and the Earth, is isolated, so mechanical energy is conserved. The new amplitude is given by

$$\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$$

$$\left(10.0\ \text{kg}\right)\left(4.00\ \text{m/s}\right)^2 = \left(100\ \text{N/m}\right)A^2$$

$$A = \boxed{1.26\ \text{m}}$$

(b) The old period was $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{4.00\text{ kg}}{100\text{ N/m}}} = 1.26\text{ s}$.

The new period is $T = 2\pi\sqrt{\frac{10}{100}\text{ s}^2} = 1.99\text{ s}$.

The period of the system has changed by a factor of

$$\frac{f_{\text{new}}}{f_{\text{old}}} = \frac{1.99\text{ s}}{1.26\text{ s}} = \boxed{1.58}$$

(c) The old energy was $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}(4.00\text{ kg})(10.0\text{ m/s})^2 = 200\text{ J}$.

The new mechanical energy is $\frac{1}{2}(10.0\text{ kg})(4.00\text{ m/s})^2 = 80.0\text{ J}$.

The energy has $\boxed{\text{decreased by 120 J}}$.

(d) $\boxed{\text{Mechanical energy is transformed into internal energy in the perfectly inelastic collision.}}$

P15.72 (a) $\sum\vec{\mathbf{F}} = -2T\sin\theta\,\hat{\mathbf{j}}$

where $\theta = \tan^{-1}\left(\frac{y}{L}\right)$.

ANS. FIG. P15.72

Therefore, for a small displacement,

$$\sin\theta \approx \tan\theta = \frac{y}{L} \qquad \text{and} \qquad \boxed{\sum\vec{\mathbf{F}} = \frac{-2Ty}{L}\hat{\mathbf{j}}}$$

(b) The total force exerted on the ball is opposite in direction and proportional to its displacement from equilibrium, so the ball moves with simple harmonic motion. For a spring system,

$$\sum\vec{\mathbf{F}} = -k\vec{\mathbf{x}} \quad \text{becomes here} \quad \sum\vec{\mathbf{F}} = -\frac{2T}{L}\vec{\mathbf{y}}.$$

Therefore, the effective spring constant is $\frac{2T}{L}$ and

$$\boxed{\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2T}{mL}}}$$

P15.73 One can write the following equations of motion:

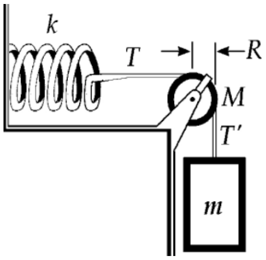
$$T - kx = 0 \qquad \text{(describes the spring)}$$

$$mg - T' = ma = m\frac{d^2x}{dt^2}$$

(for the hanging object)

$$R(T' - T) = I\frac{d^2\theta}{dt^2} = \frac{I}{R}\frac{d^2x}{dt^2}$$

(for the pulley)



ANS. FIG. P15.73

with $I = \frac{1}{2}MR^2$.

Combining these equations gives the equation of motion:

$$\left(m + \frac{1}{2}M\right)\frac{d^2x}{dt^2} + kx = mg$$

The solution is $x(t) = A\sin\omega t + \frac{mg}{k}$ (where $\frac{mg}{k}$ arises because of the extension of the spring due to the weight of the hanging object), with angular frequency

$$\omega = \sqrt{\frac{k}{m + \frac{1}{2}M}} = \sqrt{\frac{2k}{2m + M}}$$

(a) For $k = 100$ N/m and $m = 0.200$ kg,

$$\boxed{\omega = \sqrt{\frac{200}{0.400 + M}}, \text{ where } \omega \text{ is in s}^{-1} \text{and } M \text{ is in kilograms.}}$$

(b) The highest possible value occurs when $M = 0$: $\omega = \boxed{22.4\text{ s}^{-1}}$.

(c) The angular frequency is independent of the radius of the pulley: $\omega = \boxed{22.4\text{ s}^{-1}}$

P15.74 Suppose a 100-kg biker compresses the suspension 2.00 cm.

Then,

$$k = \frac{F}{x} = \frac{980\text{ N}}{2.00 \times 10^{-2}\text{ m}} = 4.90 \times 10^4\text{ N/m}$$

If total mass of motorcycle and biker is 500 kg, the frequency of free vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.90\times10^4\ \text{N/m}}{500\ \text{kg}}} = 1.58\ \text{Hz}$$

$$\boxed{\text{If he encounters washboard bumps at the same frequency as the free vibration, resonance will make the motorcycle bounce a lot. It may bounce so much as to interfere with the rider's control of the machine.}}$$

Assuming a speed of 20.0 m/s, we find these ridges are separated by

$$\frac{20.0\ \text{m/s}}{1.58\ \text{s}^{-1}} = 12.7\ \text{m}\ \boxed{\sim 10^1\ \text{m}}$$

In addition to this vibration mode of bouncing up and down as one unit, the motorcycle can also vibrate at higher frequencies by rocking back and forth between front and rear wheels, by having just the front wheel bounce inside its fork, or by doing other things. Other spacing of bumps will excite all of these other resonances.

P15.75 (a) $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2.23\ \text{m}}{9.80\ \text{m/s}^2}} = \boxed{3.00\ \text{s}}$

(b) $E = \frac{1}{2}mv^2 = \frac{1}{2}(6.74\ \text{kg})(2.06\ \text{m/s})^2 = \boxed{14.3\ \text{J}}$

(c) For a system of an isolated pendulum-Earth, mechanical energy is conserved. Relate the pendulum bob at the lowest point to the highest point:

$$\Delta K + \Delta U_g = 0$$

$$\left(0 - \frac{1}{2}mv^2\right) + (mgh - 0) = 0$$

$$mgh = \frac{1}{2}mv^2$$

$$h = \frac{v^2}{2g} = \frac{(2.06\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 0.217\ \text{m}$$

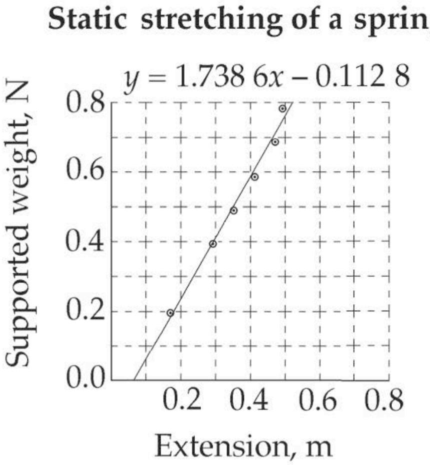
and

$$h = L - L\cos\theta = L(1-\cos\theta)$$

$$\cos\theta = 1 - \frac{h}{L} = 1 - \frac{0.217\ \text{m}}{2.23\ \text{m}}$$

$$\boxed{\theta = 25.5°}$$

P15.76 (a) The graph of Mg versus x is shown in ANS. FIG. P15.76(a).



ANS. FIG. P15.76(a)

(b) Assuming a Hooke's Law type spring, $F = Mg = kx$, and empirically

$$Mg = 1.74x - 0.113$$

so $k = \boxed{1.74\ \text{N/m} \pm 6\%}$

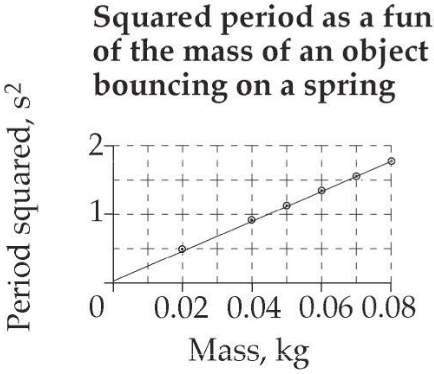
(c)

M, kg	x, m	Mg, N
0.020 0	0.17	0.196
0.040 0	0.293	0.392
0.050 0	0.353	0.49
0.060 0	0.413	0.588
0.070 0	0.471	0.686
0.080 0	0.493	0.784

(d)

Time, s	T, s	M, kg	T^2, s^2
7.03	0.703	0.020 0	0.494
9.62	0.962	0.040 0	0.925
10.67	1.067	0.050 0	1.138
11.67	1.167	0.060 0	1.362
12.52	1.252	0.070 0	1.568
13.41	1.341	0.080 0	1.798

(e) The graph of T^2 versus M is shown in ANS. FIG. P15.76(e).



ANS. FIG. P15.76(e)

(f) We may write the equation as theoretically

$$T^2 = \frac{4\pi^2}{k}M + \frac{4\pi^2}{3k}m_s$$

and empirically

$$T^2 = 21.7\ M + 0.058\ 9$$

so

$$k = \frac{4\pi^2}{21.7} = \boxed{1.82\ \text{N/m} \pm 3\%}$$

(g) The k values $1.74\ \text{N/m} \pm 6\%$

and $1.82\ \text{N/m} \pm 3\%$ differ by 4% so $\boxed{\text{they agree}}$.

(h) Utilizing the axis-crossing point,

$$m_s = 3\left(\frac{0.0589}{21.7}\right)\text{ kg} = \boxed{8\text{ grams} \pm 12\%}$$

$\boxed{\text{in agreement}}$ with 7.4 grams.

P15.77 The free-body diagram in ANS. FIG. P15.77 shows the forces acting on the balloon when it is displaced distance $s = L\theta$ along the circular arc it follows. The net force tangential to this path is

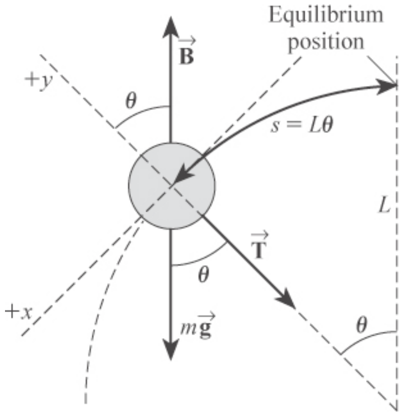
$$F_{\text{net}} = \sum F_x = -B\sin\theta + mg\sin\theta = -(B - mg)\sin\theta$$

For small angles, $\sin\theta \approx \theta = s/L$

Also, $mg = (\rho_{\text{He}}V)g$

and the buoyant force is $B = (\rho_{\text{air}}V)g$. Thus, the net restoring force acting on the balloon is

$$F_{\text{net}} \approx -\left[\frac{(\rho_{\text{air}} - \rho_{\text{He}})Vg}{L}\right]s$$



ANS. FIG. P15.77

Observe that this is in the form of Hooke's law, $F = -ks$, with

$$k = (\rho_{\text{air}} - \rho_{\text{He}})Vg/L$$

Thus, the motion will be simple harmonic and the period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\rho_{\text{He}}V}{(\rho_{\text{air}} - \rho_{\text{He}})Vg/L}}$$

$$= 2\pi\sqrt{\left(\frac{\rho_{\text{He}}}{\rho_{\text{air}} - \rho_{\text{He}}}\right)\frac{L}{g}}$$

This yields

$$T = 2\pi\sqrt{\left(\frac{0.179\ \text{kg/m}^3}{1.20\ \text{kg/m}^3 - 0.179\ \text{kg/m}^3}\right)\frac{(3.00\ \text{m})}{(9.80\ \text{m/s}^2)}} = \boxed{1.46\ \text{s}}$$

P15.78 (a) We require $Ae^{-bt/2m} = \frac{A}{2} \quad \to \quad e^{+bt/2m} = 2,$

or $\frac{bt}{2m} = \ln 2$

or $\frac{0.100\ \text{kg/s}}{2(0.375\ \text{kg})}t = 0.693$

which gives $t = \boxed{5.20\ \text{s}}$.

The spring constant is irrelevant.

(b) We can evaluate the energy at successive turning points, where

$\cos(\omega t + \phi) = \pm 1$ and the energy is $\frac{1}{2}kx^2 = \frac{1}{2}kA^2e^{-bt/m}$.

We require $\frac{1}{2}kA^2e^{-bt/m} = \frac{1}{2}\left(\frac{1}{2}kA^2\right)$

or $e^{+bt/m} = 2$

which gives

$$t = \frac{m(\ln 2)}{b} = \frac{(0.375\ \text{kg})(0.693)}{0.100\ \text{kg/s}} = \boxed{2.60\ \text{s}}$$

(c) From $E = \frac{1}{2}kA^2$, the fractional rate of change of energy over time is

$$\frac{dE/dt}{E} = \frac{(d/dt)\left(\frac{1}{2}kA^2\right)}{\frac{1}{2}kA^2} = \frac{\frac{1}{2}k(2A)(dA/dt)}{\frac{1}{2}kA^2} = 2\frac{dA/dt}{A}$$

which gives $\boxed{\frac{dA/dt}{A} = \frac{1}{2}\frac{dE/dt}{E}}$.

which is twice as fast as the fractional rate of change in amplitude.

P15.79 (a) $x = A\cos(\omega t + \phi) \rightarrow v = -\omega A \sin(\omega t + \phi)$

We have at, $t = 0$, $v = -\omega A \sin\phi = -v_{\max}$.

This requires $\phi = 90°$, so $x = A\cos(\omega t + 90°)$

$\rightarrow x = A\cos\left(\omega t + \dfrac{\pi}{2}\right)$.

Numerically we have $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{50.0\ \text{N/m}}{0.500\ \text{kg}}} = 10.0\ \text{s}^{-1}$

and $v_{\max} = \omega A \rightarrow 20.0\ \text{m/s} = (10.0\ \text{s}^{-1})A \rightarrow A = 2.00\ \text{m}$.

So $\boxed{x = 2\cos\left(10t + \dfrac{\pi}{2}\right)}$, where x is in meters and t in seconds.

(b) Using $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$, we require $\frac{1}{2}kx^2 = 3\left(\frac{1}{2}mv^2\right)$

which implies $\frac{1}{3}\left(\frac{1}{2}kx^2\right) + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 \rightarrow \frac{4}{3}x^2 = A^2$

which gives $x = \pm\sqrt{\dfrac{3}{4}}A = \pm 0.866(2.00\ \text{m}) = \boxed{\pm 1.73\ \text{m}}$

(c) The particle's position is given by $x = 2\cos\left(10t + \dfrac{\pi}{2}\right)$.

The particle is at $x = 0$ when

$$10t + \frac{\pi}{2} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots \rightarrow 10t = 0, \pi, 2\pi, 4\pi\ldots$$

At $t = 0$, the particle is at the origin, but moving to the left. The next time the particle is at the origin is when $10t = \pi$ when it is moving to the right.

The particle is first at $x = 1.00$ m when $10t + \dfrac{\pi}{2} = \dfrac{3\pi}{2} + \dfrac{\pi}{3} = \dfrac{11\pi}{6}$.

So then, $10t = \dfrac{4\pi}{3}$.

The minimum time required for the particle to move from $x = 0$ to $x = 1.00$ m is

$$10\Delta t = \frac{4\pi}{3} - \pi = \frac{\pi}{3} \rightarrow \Delta t = \frac{\pi}{30} = \boxed{0.105\ \text{s} = 105\ \text{ms}}$$

(d) $\omega = \sqrt{\dfrac{g}{L}} \rightarrow L = \dfrac{g}{\omega^2} = \dfrac{9.80\text{ m/s}^2}{\left(10\text{ s}^{-1}\right)^2} = \boxed{0.098\ 0\text{ m}}$

P15.80 (a) The block moves with the board in what we take as the positive x direction, stretching the spring until the spring force $-kx$ is equal in magnitude to the maximum force of static friction:

$$kx = \mu_s n = \mu_s mg$$

This occurs at $x = \dfrac{\mu_s mg}{k}$.

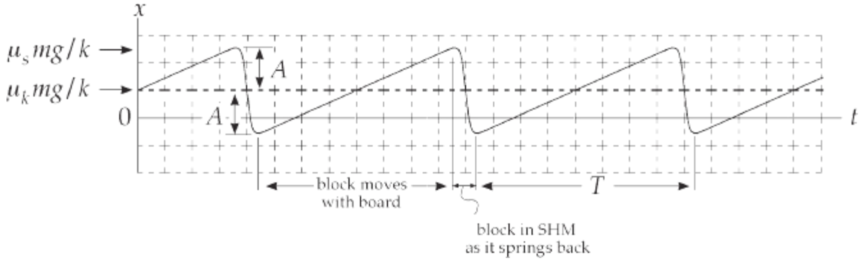
(b) Since v is small, the block is nearly at the rest at this break point. It starts almost immediately to move back to the left, the forces on it being $-kx$ and $+\mu_k mg$. While it is sliding the net force exerted on it can be written as

$$\begin{aligned} F_{\text{net}} &= -kx + \mu_k mg = -kx + \frac{k\mu_k mg}{k} = -k\left(x - \frac{\mu_k mg}{k}\right) \\ &= -kx_{rel} \end{aligned}$$

where x_{rel} is the excursion of the block away from the point $\dfrac{\mu_k mg}{k}$.

Conclusion: the block goes into simple harmonic motion centered about the equilibrium position where the spring is stretched by $\dfrac{\mu_k mg}{k}$.

(c) The graph of the motion looks as shown in ANS. FIG. P15.80(c):



ANS. FIG. P15.80(c)

(d) The amplitude of its motion is its original displacement, $A = \dfrac{\mu_s mg}{k} - \dfrac{\mu_k mg}{k}$, because the block has been pulled out to $x = \dfrac{\mu_s mg}{k}$, then it goes into simple harmonic motion centered about $x = \dfrac{\mu_k mg}{k}$.

It first comes to rest at spring extension $\frac{\mu_k mg}{k} - A = \frac{(2\mu_k - \mu_s)mg}{k}$.

Almost immediately at this point it latches onto the slowly-moving board to move with the board. The board exerts a force of static friction on the block, and the cycle continues.

(e) The time during each cycle when the block is moving with the board is $\frac{2A}{v} = \frac{2(\mu_s - \mu_k)mg}{kv}$. The time for which the block is springing back is one half a cycle of simple harmonic motion, $\frac{1}{2}\left(2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{m}{k}}\right)$ (because the block slides from $+A$ to $-A$ during its SHM). We ignore the times at the end points of the motion when the speed of the block changes from v to 0 and from 0 to v. Since v is small compared to $\frac{2A}{\pi\sqrt{m/k}}$, these times are negligible. Then the period is

$$T = \frac{2(\mu_s - \mu_k)mg}{kv} + \pi\sqrt{\frac{m}{k}}$$

P15.81 (a) Let ℓ represent the length below water at equilibrium and M the tube's mass:

$$\sum F_y = 0 \Rightarrow -Mg + \rho\pi r^2 \ell g = 0$$

Now with any excursion x from equilibrium

$$-Mg + \rho\pi r^2(\ell - x)g = Ma$$

Subtracting the equilibrium equation gives

$$-\rho\pi r^2 gx = Ma \to a = -\left(\frac{\rho\pi r^2 g}{M}\right)x$$

$$\boxed{\text{The opposite direction and direct proportionality of } a \text{ to } x \text{ imply SHM.}}$$

(b) For SHM, $F = -kx = ma \to a = -(k/m)x = -\omega^2 x$: the coefficient of x is the square of the angular frequency:

$$\omega = \sqrt{\frac{\rho\pi r^2 g}{M}} \to T = \frac{2\pi}{\omega} = \boxed{\frac{2}{r}\sqrt{\frac{\pi M}{\rho g}}}$$

P15.82 From the oscillator information, find the natural frequency of the oscillator:

$$\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{10.0 \text{ N/m}}{0.001 \text{ kg}}} = 100 \text{ s}^{-1}$$

From the measurement information, find the value of $b/2m$:

$$\frac{x_{\max}(23.1 \text{ ms})}{x_{\max}(0)} = 0.250 = \frac{Ae^{-(b/2m)(0.023\ 1 \text{ s})}}{A(e^0)} = e^{-(b/2m)(0.023\ 1 \text{ s})}$$

Solving,

$$\frac{b}{2m} = -\frac{\ln(0.250)}{0.0231 \text{ s}} = 60.0 \text{ s}^{-1}$$

$$\boxed{\text{If the damping constant is doubled, } b/2m = 120 \text{ s}^{-1}\text{. In this case, however, } b/2m > \omega_0 \text{ and the system is overdamped. Your design objective is not met because the system does not oscillate.}}$$

P15.83 The effective spring constant of a ball is

$$k = \frac{|F|}{|x|} = \frac{1.60 \times 10^3 \text{ N}}{0.200 \times 10^{-3} \text{ m}} = 80.0 \text{ MN/m}$$

The half-cycle is from the equilibrium position of the model spring to maximum compression and back to equilibrium again. The time is one-half the period:

$$\frac{1}{2}T = \frac{1}{2}(2\pi)\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{0.067\ 4 \text{ kg}}{80.0 \times 10^6 \text{ N/m}}} = \boxed{9.12 \times 10^{-5} \text{ s}}$$

Challenge Problems

P15.84 (a) $\Delta K + \Delta U = 0$

Thus, $K_{\text{top}} + U_{\text{top}} = K_{\text{bot}} + U_{\text{bot}}$

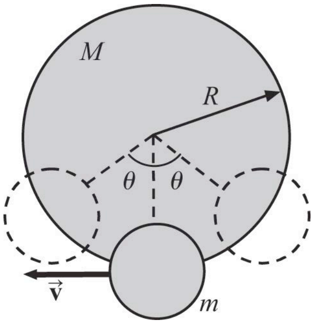
where $K_{\text{top}} = U_{\text{bot}} = 0.$

Therefore, $mgh = \frac{1}{2}I\omega^2$, but

$$h = R - R\cos\theta = R(1-\cos\theta)$$

$$\omega = \frac{v}{R}$$

and $I = \frac{MR^2}{2} + \frac{mr^2}{2} + mR^2$



ANS. FIG. P15.84

Substituting, we find

$$mgR(1-\cos\theta) = \frac{1}{2}\left(\frac{MR^2}{2} + \frac{mr^2}{2} + mR^2\right)\frac{v^2}{R^2}$$

$$mgR(1-\cos\theta) = \left[\frac{M}{4} + \frac{mr^2}{4R^2} + \frac{m}{2}\right]v^2$$

and $v^2 = 4gR\left(\frac{1-\cos\theta}{M/m + r^2/R^2 + 2}\right)$, so

$$\boxed{v = 2\left[\frac{Rg(1-\cos\theta)}{M/m + r^2/R^2 + 2}\right]^{1/2}}$$

(b) $T = 2\pi\sqrt{\frac{I}{m_T g d_{\text{CM}}}}$

Substituting $m_T = m + M$ and solving for d_{CM} gives

$$d_{\text{CM}} = \frac{mR + M(0)}{m+M}$$

The period is then

$$T = 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + \frac{1}{2}mr^2 + mR^2}{mgR}} = 2\pi\sqrt{\frac{\frac{1}{2}(MR^2 + 2mR^2 + mr^2)}{mgR}}$$

$$= \boxed{2\pi\left[\frac{(M+2m)R^2 + mr^2}{2mgR}\right]^{1/2}}$$

P15.85 (a) Total energy $= \frac{1}{2}kA^2 = \frac{1}{2}(100\ \text{N/m})(0.200\ \text{m})^2 = 2.00\ \text{J}$.

At equilibrium, the total energy is:

$$\frac{1}{2}(m_1 + m_2)v^2 = \frac{1}{2}(16.0\ \text{kg})v^2 = (8.00\ \text{kg})v^2$$

Therefore,

$$(8.00\ \text{kg})v^2 = 2.00\ \text{J}\text{, and } v = \boxed{0.500\ \text{m/s}}$$

This is the speed of m_1 and m_2 at the equilibrium point. Beyond this point, the mass m_2 moves with the constant speed of 0.500 m/s while mass m_1 starts to slow down due to the restoring force of the spring.

(b) The energy of the m_1-spring system at equilibrium is:

$$\frac{1}{2}m_1v^2 = \frac{1}{2}(9.00\ \text{kg})(0.500\ \text{m/s})^2 = 1.125\ \text{J}$$

This is also equal to $\frac{1}{2}k(A')^2$, where A' is the amplitude of the m_1-spring system.

Therefore,

$$\frac{1}{2}(100)(A')^2 = 1.125 \text{ or } A' = 0.150\ \text{m}$$

The period of the m_1-spring system is

$$T = 2\pi\sqrt{\frac{m_1}{k}} = 2\pi\sqrt{\frac{9.00\ \text{kg}}{100\ \text{N/m}}} = 1.885\ \text{s}$$

and it takes $\frac{1}{4}T = 0.471\ \text{s}$ after it passes the equilibrium point for the spring to become fully stretched the first time. The distance separating m_1 and m_2 at this time is

$$D = v\left(\frac{T}{4}\right) - A' = (0.500\ \text{m/s})(0.471\ \text{s}) - 0.150\ \text{m}$$

$$= 0.085\ 6\ \text{m} = \boxed{8.56\ \text{cm}}$$

P15.86 The time interval for your competitor's package to arrive is half of the orbital period found from Kepler's third law, Equation 13.11:

$$\Delta t = \frac{1}{2}T = \frac{1}{2}\sqrt{\frac{4\pi^2}{GM_E}(R_E)^3} = \pi\sqrt{\frac{R_E^3}{GM_E}}$$

Now, consider your proposal. The force on the package at an arbitrary position r is

$$F_g = -G\frac{M_{\text{closer than } r}m}{r^2} = -G\frac{m}{r^2}\frac{\left(\frac{4}{3}\pi r^3\right)}{\left(\frac{4}{3}\pi R_E^3\right)}M_E = -G\frac{M_E m}{R_E^3}r$$

This force is of the form of Hooke's law! The "spring constant" for this motion is

$$k = G\frac{M_E m}{R_E{}^3}$$

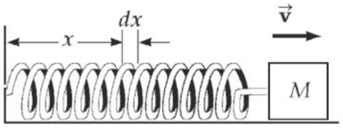
Because the force on the package is a Hooke's-law force, the package will oscillate between opposite points on the Earth in simple harmonic motion. To deliver the package to the other side of the Earth, someone must grab the package before it begins its return journey. The time interval for the package to travel to the other side of the Earth is half of a period of oscillation:

$$\Delta t = \frac{1}{2}T = \frac{1}{2}\left[2\pi\sqrt{\frac{m}{k}}\right] = \frac{1}{2}\left[2\pi\sqrt{m\left(G\frac{M_E m}{R_E^3}\right)^{-1}}\right] = \pi\sqrt{\frac{R_E^3}{GM_E}}$$

$$\boxed{\text{This is exactly the same time interval as for your competitor, so you have no advantage! In fact, you have the disadvantage of the initial capital outlay to bore through the entire Earth!}}$$

P15.87 (a) For each segment of the spring:

$$dK = \frac{1}{2}(dm)v_x^2$$



ANS. FIG. P15.87

Also,

$$v_x = \frac{x}{\ell}v \quad \text{and} \quad dm = \frac{m}{\ell}dx$$

Therefore, the total kinetic energy of the block-spring system is

$$K = \frac{1}{2}Mv^2 + \frac{1}{2}\int_0^\ell\left(\frac{x^2v^2}{\ell^2}\right)\frac{m}{\ell}dx = \boxed{\frac{1}{2}\left(M + \frac{m}{3}\right)v^2}$$

(b) $\omega = \sqrt{\dfrac{k}{m_{\text{eff}}}}$ and $\dfrac{1}{2}m_{\text{eff}}v^2 = \dfrac{1}{2}\left(M + \dfrac{m}{3}\right)v^2$

Therefore,

$$T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{M + m/3}{k}}}$$

P15.88 (a) Note that as the spring passes through the vertical position, the object is moving in a circular arc of radius $L - y_f$, where the y coordinate of the object at this point must be negative ($y_f < 0$). When the object is at y_f, the spring is stretched $x = y_f - L$. At position y_f, the spring is stretched and exerting an upward tension force of magnitude greater than the object's weight. This is necessary so the object experiences a net force toward the pivot to supply the needed centripetal acceleration in this position. This is summarized by Newton's second law applied to the object at this point, stating (remember, y_f is negative)

$$\sum F_y = ma \to -ky_f - mg = \frac{mv^2}{L - y_f} \qquad \textbf{[1]}$$

The system is isolated, so conservation of energy requires that

$$E = KE_i + PE_{g,i} + PE_{s,i} = KE_f + PE_{g,f} + PE_{s,f}$$

or

$$E = 0 + mgL + 0 = \frac{1}{2}mv^2 + mgy_f + \frac{1}{2}ky_f^2$$

reducing to

$$2mg\left(L - y_f\right) = mv^2 + ky_f^2 \qquad \textbf{[2]}$$

From equation [1], observe that $mv^2 = -(L - y_f)(ky_f + mg)$. Substituting this into equation [2] gives

$$2mg(L - y_f) = -(L - y_f)(ky_f + mg) + ky_f$$

After expanding and regrouping terms, this becomes

$$(2k)y_f^2 + (3mg - kL)y_f + (-3mgL) = 0$$

which is a quadratic equation $ay_f^2 + by_f + c = 0$, with

$$a = 2k = 2\left(1250\text{ N/m}\right) = 2.50 \times 10^3\text{ N/m}$$

$$b = 3mg - kL = 3(5.00\ \text{kg})(9.80\ \text{m/s}^2) - (1\,250\ \text{N/m})(1.50\ \text{m})$$
$$= -1.73\times 10^3\ \text{N}$$

and

$$c = -3mgL = -3(5.00\ \text{kg})(9.80\ \text{m/s}^2)(1.50\ \text{m}) = -221\ \text{N}\cdot\text{m}$$

Applying the quadratic formula, keeping only the negative solution [see the discussion in part (a)], and suppressing units, gives

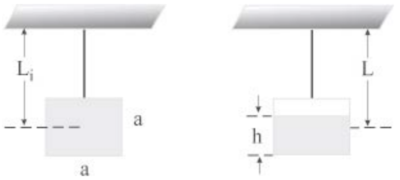
$$y_f = \frac{-b-\sqrt{b^2-4ac}}{2a}$$
$$= \frac{-(-1.73\times 10^3)-\sqrt{(-1.73\times 10^3)^2-4(2.50\times 10^3)(-221)}}{2(2.50\times 10^3)}$$

or $\boxed{y_f = -0.110\ \text{m}}$

(b) Because the length of this pendulum varies and is longer throughout its motion than a simple pendulum of length *L*, $\boxed{\text{its period will be longer}}$ than that of a simple pendulum.

P15.89 (a) The period of the pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$



ANS. FIG. P15.89

and changes as

$$\frac{dT}{dt} = \frac{\pi}{\sqrt{g}}\frac{1}{\sqrt{L}}\frac{dL}{dt} \qquad \textbf{[1]}$$

We need to find *L* (*t*) and $\frac{dL}{dt}$. From the diagram in ANS. FIG. P15.89(a),

$$L = L_i + \frac{a}{2} - \frac{h}{2} \qquad \text{and} \qquad \frac{dL}{dt} = -\left(\frac{1}{2}\right)\frac{dh}{dt}$$

But $\frac{dM}{dt} = \rho\frac{dV}{dt} = -\rho A\frac{dh}{dt}$. Therefore,

$$\frac{dh}{dt} = -\frac{1}{\rho A}\frac{dM}{dt} \qquad \rightarrow \qquad \frac{dL}{dt} = \left(\frac{1}{2\rho A}\right)\frac{dM}{dt} \qquad \textbf{[2]}$$

Also,

$$\int_{L_i}^{L} dL = \left(\frac{1}{2\rho A}\right)\left(\frac{dM}{dt}\right)t = L - L_i \qquad \textbf{[3]}$$

Substituting equations [2] and [3] into [1] gives:

$$\frac{dT}{dt} = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\frac{1}{\sqrt{L_i + \left(t/2\rho a^2\right)\left(dM/dt\right)}}$$

Integrating, we get

$$T = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\int_0^t \frac{dt}{\sqrt{L_i + \left(t/2\rho a^2\right)\left(dM/dt\right)}}$$

$$T = \frac{\pi}{\sqrt{g}}\left(\frac{1}{2\rho a^2}\right)\left(\frac{dM}{dt}\right)\frac{2\sqrt{L_i + \left(t/2\rho a^2\right)\left(dM/dt\right)}}{\left(1/2\rho a^2\right)\left(dM/dt\right)}$$

$$T = \boxed{\frac{2\pi}{\sqrt{g}}\sqrt{L_i + \frac{1}{2\rho a^2}\left(\frac{dM}{dt}\right)t}}$$

(b) When the liquid is gone, the CM of the bob is suddenly again at the center of the cube. We had ignored the mass of the cube up until now since it was small compared to the mass of the liquid. Thus, once the liquid is gone, $L = L_i$.

$$T = \boxed{2\pi\sqrt{\frac{L_i}{g}}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P15.2 1.59 k N/m

P15.4 (a) 4.33 cm; (b) –5.00 cm/s; (c) -17.3 cm/s^2; (d) 3.14 s; (e) 5.00 cm

P15.6 (a) 18.8 m/s; (b) 7.11 km/s^2

P15.8 (a) 2.40 s; (b) 0.417 Hz; (c) 2.62 rad/s

P15.10 39.2 N

P15.12 (a) 15.8 cm; (b) 51.1 m; (c) –15.9 cm; (d) 50.8 m; (e) The patterns of oscillation diverge from each other, starting out in phase but becoming completely out of phase. To calculate the future, we would need *exact* knowledge of the present; an impossibility.

P15.14 (a) motion is periodic; (b) 1.81 s; (c) The motion is not simple harmonic. The net force acting on the ball is a constant given by $F = -mg$ (except when it is in contact with the ground), which is not in the form of Hooke's law.

P15.16 (a) See P15.16(a) for complete solution; (b) See P15.16(b) for complete solution

P15.18 (a) 1.26 s; (b) 0.150 m/s, 0.750 m/s^2; (c) $x = 3.00\cos\left(5.00t + \pi\right)$, $-15.0\sin\left(5.00t + \pi\right)$, and $-75.0\cos\left(5.00t + \pi\right)$

P15.20 (a) yes; (b) We see that finding the period does not depend on knowing the mass: $T = 0.859$ s.

P15.22 (a) 126 N/m; (b) 0.178 m

P15.24 (a) 0.153 J; (b) 0.784 m/s; (c) 17.5 m/s^2

P15.26 (a) E increases by a factor of 4; (b) $v_{\max}$ is doubled; (c) $a_{\max}$ also doubles; (d) the period is unchanged.

P15.28 (a) 100 N/m; (b) 1.13 Hz; (c) 1.41 m/s; (d) $x = 0$; (e) 10.0 m/s^2; (f) ±0.200 m; (g) 2.00 J; (h) 1.33 m/s; (i) 3.33 m/s^2

P15.30 (a) Particle under constant acceleration; (b) 1.50 s; (c) isolated; (d) 73.4 N/m; (e) 19.7 m below the bridge; (f) 1.06 rad/s; (g) +2.01 s; (h) 3.50 s

P15.32 (a) 5.98 m/s; (b) 206 N/m; (c) 0.238 m

P15.34 1.001 5

P15.36 $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{g}{R}}$

P15.38 $I = \dfrac{mgd}{4\pi^2 f^2}$

P15.40 (a) $2\pi\sqrt{\dfrac{\left(I_{\text{CM}} + md^2\right)}{mgd}}$; (b) $I_{\text{CM}} = md^2$

P15.42 (a) 2.09 s; (b) 4.08%

P15.44 For Length, L (m): 1.000, 0.750, 0.500 and Period, T (s): 2.00, 1.73, 1.42; (b) For Period T(s): 2.00, 1.73, 1.42 and g (m/s^2): 9.87, 9.89, 9.79. This agrees with the accepted value of $g = 9.80$ m/s^2 within 0.5%; (c) 9.94 m/s^2

P15.46 $1.00 \times 10^{-3}\ \text{s}^{-1}$

P15.48 $\dfrac{dE}{dt} = -bv^2 < 0$

P15.50 (a) 1.19 Hz; (b) 17.5 cm

P15.52 318 N

P15.54 See P15.54 for complete solution.

P15.56 0.919×10^{14} Hz

P15.58 (a) 0.368 m/s; (b) 3.51 cm; (c) 40.6 mJ; (d) 27.7 mJ

P15.60 (a) 4.31 cm; (b) When the rock is on the point of lifting off, the surrounding water is also barely in free fall. No pressure gradient exists in the water, so no buoyant force acts on the rock. The effect of the surrounding water disappears at that instant.

P15.62 (a) See P15.62(a) for complete solution; (b) 1.04 m/s; (c) 3.40 m

P15.64 (a) $A = 2.00$ cm; (b) $T = 4.00$ s; (c) $\dfrac{\pi}{2}$ rad/s; (d) π cm/s; (e) 4.93 cm/s^2; (f) $x = 2.00\sin\left(\dfrac{\pi}{2}t\right)$, where x is in centimeters and t is in seconds

P15.66 $\dfrac{\mu_s g}{4\pi^2 f^2}$

P15.68 (a) $2\pi\sqrt{\dfrac{m\left(k_1 + k_2\right)}{k_1 k_2}}$; (b) $2\pi\sqrt{\dfrac{m}{\left(k_1 + k_2\right)}}$

P15.70 $\omega = \sqrt{\dfrac{3k}{m}}$

P15.72 (a) $\sum \vec{\mathbf{F}} = \frac{-2Ty}{L}\hat{\mathbf{j}}$; (b) $\omega = \sqrt{\frac{2T}{mL}}$

P15.74 If he encounters washboard bumps at the same frequency as the free vibration, resonance will make the motorcycle bounce a lot. It may bounce so much as to interfere with the rider's control of the machine; $\sim 10^1$ m.

P15.76 (a) See ANS. FIG. P15.76(a); (b) 1.74 N/m ± 6%; (c) See table in P15.76(c); (d) See table in P15.76(d); (e) See ANS. FIG. P15.64(e); (f) 1.82 N/m ± 3%; (g) they agree; (h) 8 grams ± 12% in agreement

P15.78 (a) 5.20 s; (b) 2.60 s; (c) $\frac{dA/dt}{A} = \frac{1}{2}\frac{dE/dt}{E}$

P15.80 See P15.80 for complete solution.

P15.82 If the damping constant is doubled, $b/2m = 120\ \text{s}^{-1}$. In this case, however, $b/2m > \omega_0$ and the system is overdamped. Your design objective is not met because the system does not oscillate.

P15.84 (a) $v = 2\left[\frac{Rg(1-\cos\theta)}{M/m + r^2/R^2 + 2}\right]^{1/2}$; (b) $2\pi\left[\frac{(M+2m)R^2 + mr^2}{2mgR}\right]^{1/2}$

P15.86 This is exactly the same time interval as for your competitor, so you have no advantage! In fact, you have the disadvantage of the initial capital outlay to bore through the entire Earth!

P15.88 (a) $y_f = -0.110\ \text{m}$; (b) its period will be longer

16

Wave Motion

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ16.1 (i) Answer (a). As the wave passes from the massive string to the less massive string, the wave speed will increase according to $v = \sqrt{\dfrac{T}{\mu}}.$

(ii) Answer (c). The frequency will remain unchanged. The rate at which crests come up to the boundary is the same rate at which they leave the boundary.

(iii) Answer (a). Since $v = f\lambda,$ the wavelength must increase.

OQ16.2 (i) Answer (a). Higher tension makes wave speed higher.

(ii) Answer (b). Greater linear density makes the wave move more slowly.

OQ16.3 (i) The ranking is (c) = (d) > (e) > (b) > (a). Look at the coefficients of the sine and cosine functions: (a) 4, (b) 6, (c) 8, (d) 8, (e) 7.

(ii) The ranking is (c) > (a) = (b) > (d) > (e). Look at the coefficients of x. Each is the wave number, $2\pi/\lambda$, so the smallest k goes with the largest wavelength.

(iii) The ranking is (e) > (d) > (a) = (b) = (c). Look at the coefficients of t. The absolute value of each is the angular frequency $\omega = 2\pi f$.

(iv) The ranking is (a) = (b) = (c) > (d) > (e). Period is the reciprocal of frequency, so the ranking is the reverse of that in part (iii).

(v) The ranking is (c) > (a) = (b) = (d) > (e). From $v = f\lambda = \omega / k,$ we compute the absolute value of the ratio of the coefficient of t to the coefficient of x in each case: (a) 5, (b) 5, (c) 7.5, (d) 5, (e) 4.

OQ16.4 Answer (b). From $v = \sqrt{\frac{T}{\mu}},$ we must increase the tension by a factor of 4 to make v double.

OQ16.5 Answer (b). Wave speed is inversely proportional to the square root of linear density.

OQ16.6 Answer (b). Not all waves are sinusoidal. A sinusoidal wave is a wave of a single frequency. In general, a wave can be a superposition of many sinusoidal waves.

OQ16.7 (a) through (d): Yes to all. The maximum element speed and the wave speed are related by $v_{y,\max} = \omega A = 2\pi f A = 2\pi v A / \lambda.$ Thus the amplitude or the wavelength of the wave can be adjusted to make either $v_{y,\max}$ or v larger.

OQ16.8 Answer (c). The power carried by a wave is proportional to its frequency, wave speed, and the square of its amplitude. If the frequency does not change, the amplitude is increased by a factor of $\sqrt{2}.$ The wave speed does not change.

OQ16.9 Answer (c). The distance between two successive peaks is the wavelength: $\lambda = 2$ m, and the frequency is 4 Hz. The frequency, wavelength, and speed of a wave are related by the equation $f\lambda = v.$

ANSWERS TO CONCEPTUAL QUESTIONS

CQ16.1 Longitudinal waves depend on the compressibility of the fluid for their propagation. Transverse waves require a restoring force in response to shear strain. Fluids do not have the underlying structure to supply such a force. A fluid cannot support static shear. A viscous fluid can temporarily be put under shear, but the higher its viscosity the more quickly it converts kinetic energy into internal energy. A local vibration imposed on it is strongly damped, and not a source of wave propagation.

CQ16.2 The type of wave you generate depends upon the direction of the disturbance (vibration) you generate and the direction of its travel (propagation).

(a) To use a spring (or slinky) to create a longitudinal wave, pull a few coils back and release.

(b) For a transverse wave, jostle the end coil side to side.

CQ16.3 It depends on from what the wave reflects. If reflecting from a less dense string, the reflected part of the wave will be right side up. A wave inverts when it reflects off a medium in which the wave speed is smaller.

CQ16.4 The speed of a wave on a "massless" string would be infinite!

CQ16.5 Since the frequency is 3 cycles per second, the period is 1/3 second = 333 ms.

CQ16.6 (a) and (b) Each element of the rope must support the weight of the rope below it. The tension increases with height. (It increases linearly, if the rope does not stretch.) Then the wave speed $v = \sqrt{\dfrac{T}{\mu}}$ increases with height.

CQ16.7 As the pulse moves down the string, the elements of the string itself move side to side. Since the medium—here, the string—moves perpendicular to the direction of wave propagation, the wave is transverse by definition.

CQ16.8 No. The vertical speed of an element will be the same on any string because it depends only on frequency and amplitude:

$$v_{y,\max} = \omega A = 2\pi f A$$

The elements of strings with different wave speeds will have the same maximum vertical speed.

CQ16.9 (a) Let $\Delta t = t_s - t_p$ represent the difference in arrival times of the two waves at a station at distance $d = v_s t_s = v_p t_p$ from the focus. Then $d = \Delta t\left(\dfrac{1}{v_s} - \dfrac{1}{v_p}\right)^{-1}$.

(b) Knowing the distance from the first station places the focus on a sphere around it. A measurement from a second station limits it to another sphere, which intersects with the first in a circle. Data from a third non-collinear station will generally limit the possibilities to a point.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 16.1 Propagation of a Disturbance

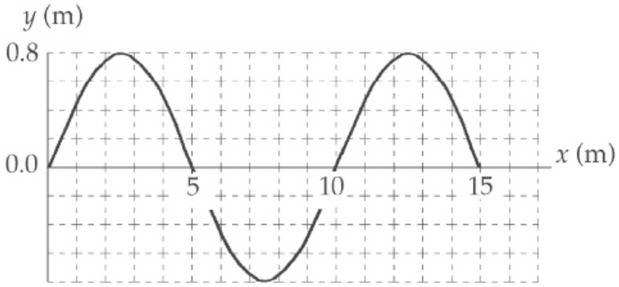
P16.1 The distance the waves have traveled is $d = (7.80 \text{ km/s})t = (4.50 \text{ km/s})(t + 17.3 \text{ s})$, where t is the travel time for the faster wave.

Then, $$(7.80 - 4.50)(\text{km/s})t = (4.50 \text{ km/s})(17.3 \text{ s})$$

or $$t = \frac{(4.50 \text{ km/s})(17.3 \text{ s})}{(7.80 - 4.50) \text{ km/s}} = 23.6 \text{ s}$$

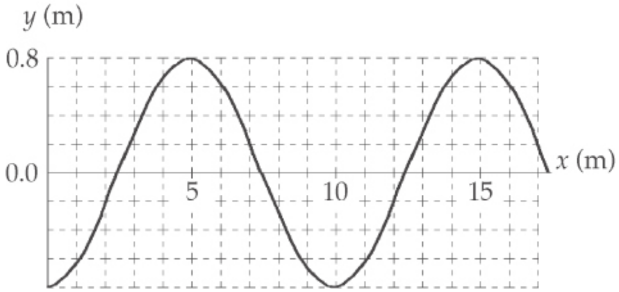
and the distance is $d = (7.80 \text{ km/s})(23.6 \text{ s}) = \boxed{184 \text{ km}}$

P16.2 (a) ANS. FIG. P16.2(a) shows the sketch of $y(x,t)$ at $t = 0$.



ANS. FIG. P16.2(a)

(b) ANS. FIG. P16.2(b) shows the sketch of $y(x,t)$ at $t = 2.00$ s.



ANS. FIG. P16.2(b)

(c) $\boxed{\text{The graph in ANS. FIG. P16.2(b) has the same amplitude and wavelength as the graph in ANS. FIG. P16.2(a). It differs just by being shifted toward larger } x \text{ by 2.40 m.}}$

(d) $\boxed{\text{The wave has travelled } d = vt = 2.40 \text{ m to the right.}}$

P16.3 We obtain a function of the same shape by writing

$$y(x,t)=\frac{6}{\left[(x-x_0)^2+3\right]}$$

where the center of the pulse is at $x_0 = 4.50t$. Thus, we have

$$\boxed{y=\frac{6}{\left[(x-4.50t)^2+3\right]}}$$

Note that for y to stay constant as t increases, x must increase by $4.50t$, as it should to describe the wave moving at 4.50 m/s.

P16.4 (a) The $\boxed{\text{longitudinal P wave}}$ travels a shorter distance and is moving faster, so it will arrive at point *B* first.

(b) The P wave that travels through the Earth must travel

a distance of $2R\sin 30.0° = 2\left(6.37\times10^6\text{ m}\right)\sin 30.0° = 6.37\times10^6\text{ m}$

at a speed of 7 800 m/s.

Therefore, it takes $\Delta t_P = \dfrac{6.37\times10^6\text{ m}}{7\,800\text{ m/s}} \simeq 817\text{ s}.$

The Rayleigh wave that travels along the Earth's surface must travel a distance of

$$s = R\theta = R\left(\frac{\pi}{3}\text{ rad}\right) = 6.67\times10^6\text{ m}$$

at a speed of 4 500 m/s.

Therefore, it takes $\Delta t_S = \dfrac{6.67\times10^6\text{ m}}{4\,500\text{ m/s}} \simeq 1\,482\text{ s}.$

The time difference is $\Delta T = \Delta t_S - \Delta t_P = \boxed{666\text{ s}} = 11.1\text{ min}.$

Section 16.2 Analysis Model: Traveling Wave

P16.5 Compare the specific equation to the general form:

$$y = (0.020\,0\text{ m})\sin(2.11x - 3.62t) = y = A\sin(kx - \omega t + \phi)$$

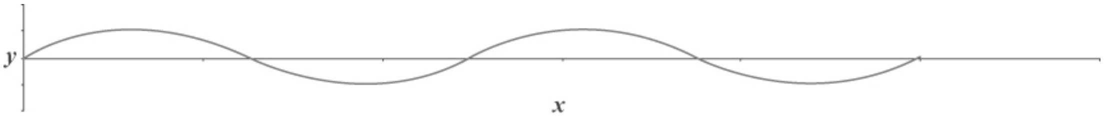
(a) $A = \boxed{2.00\text{ cm}}$

(b) $k = 2.11 \text{ rad/m} \rightarrow \lambda = \dfrac{2\pi}{k} = \boxed{2.98 \text{ m}}$

(c) $\omega = 3.62 \text{ rad/s} \rightarrow f = \dfrac{\omega}{2\pi} = \boxed{0.576 \text{ Hz}}$

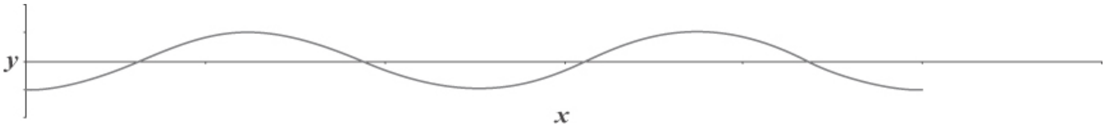
(d) $v = f\lambda = \dfrac{\omega}{2\pi}\dfrac{2\pi}{k} = \dfrac{3.62}{2.11} = \boxed{1.72 \text{ m/s}}$

P16.6 (a) ANS. FIG. P16.6(a) shows the snapshot of a wave on a string.



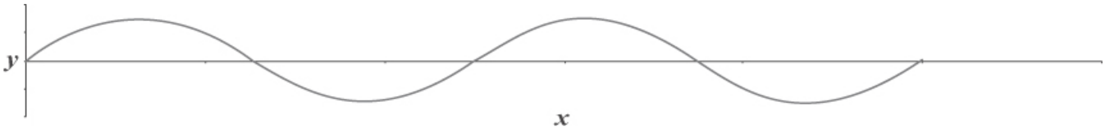
ANS. FIG. P16.6(a)

(b) ANS. FIG. P16.6(b) shows the wave from part (a) one-quarter period later



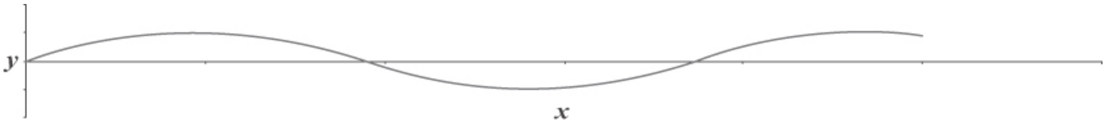
ANS. FIG. P16.6(b)

(c) ANS. FIG. P16.6(c) shows a wave with an amplitude 1.5 times larger than the wave in part (a).



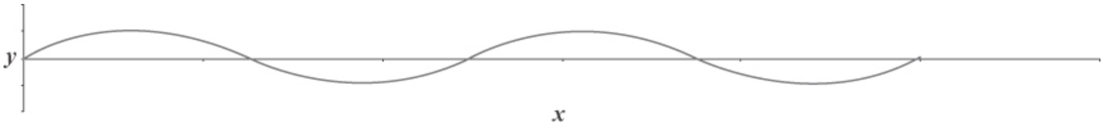
ANS. FIG. P16.6(c)

(d) ANS. FIG. P16.6(d) shows a wave with wavelength 1.5 times larger than the wave in part (a).



ANS. FIG. P16.6 (d)

(e) ANS. FIG. P16.6(e) shows a wave with frequency 1.5 times larger than the wave in part (a): The wave appears the same as in ANS. FIG. P16.6(a) because this is a snapshot of a given moment.



ANS. FIG. P16.6(e)

P16.7 The frequency of the wave is

$$f = \frac{40.0\text{ vibrations}}{30.0\text{ s}} = \frac{4}{3}\text{ Hz}$$

as the wave travels 425 cm in 10.0 s, its speed is

$$v = \frac{425\text{ cm}}{10.0\text{ s}} = 42.5\text{ cm/s}$$

and its wavelength is therefore

$$\lambda = \frac{v}{f} = \frac{42.5\text{ cm/s}}{1.33\text{ Hz}} = 31.9\text{ cm} = \boxed{0.319\text{ m}}$$

P16.8 Using data from the observations, we have $\lambda = 1.20$ m and

$$f = \frac{8.00\text{ crests}}{12.0\text{ s}} = \frac{8.00\text{ cycles}}{12.0\text{ s}} = \frac{8.00}{12.0}\text{ Hz}$$

Therefore, $v = \lambda f = (1.20\text{ m})\left(\frac{8.00}{12.0}\text{ Hz}\right) = \boxed{0.800\text{ m/s}}$.

P16.9 (a) We note that $\sin\theta = -\sin(-\theta) = \sin(-\theta+\pi)$, so the given wave function can be written as

$$y(x,t) = (0.350)\sin(-10\pi t + 3\pi x + \pi - \pi/4)$$

Comparing, $10\pi t - 3\pi x + \pi/4 = kx - \omega t + \phi$. For constant phase, x must increase as t increases, so the wave travels in the positive x direction. Comparing the specific form to the general form, we find that

$$v = \frac{\omega}{k} = \frac{10\pi}{3\pi} = 3.33\text{ m/s}.$$

Therefore, the velocity is $\boxed{\left(3.33\hat{\mathbf{i}}\right)\text{ m/s}}$.

(b) Substituting $t = 0$ and $x = 0.100$ m, we have

$$y(0.100\ 0) = (0.350\text{ m})\sin\left(-0.300\pi + \frac{\pi}{4}\right) = -0.054\ 8\text{ m}$$
$$= \boxed{-5.48\text{ cm}}$$

(c) $k = \dfrac{2\pi}{\lambda} = 3\pi$: $\lambda = \boxed{0.667\text{ m}}$ $\quad \omega = 2\pi f = 10\pi$: $f = \boxed{5.00\text{ Hz}}$

(d) $v_y = \dfrac{\partial y}{\partial t} = (0.350)(10\pi)\cos\left(10\pi t - 3\pi x + \dfrac{\pi}{4}\right)$

$$v_{y,\text{ max}} = (10\pi)(0.350) = \boxed{11.0\text{ m/s}}$$

P16.10 The speed of waves along this wire is

$$v = f\lambda = (4.00\text{ Hz})(60.0\text{ cm}) = 240\ \text{cm/s} = \boxed{2.40\ \text{m/s}}$$

P16.11 (a) $\omega = 2\pi f = 2\pi\left(5.00\text{ s}^{-1}\right) = \boxed{31.4\ \text{rad/s}}$

(b) $\lambda = \dfrac{v}{f} = \dfrac{20.0\text{ m/s}}{5.00\text{ s}^{-1}} = 4.00\text{ m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{4.00\text{ m}} = \boxed{1.57\text{ rad/m}}$$

(c) In $y = A\sin(kx - \omega t + \phi)$ we take $A = 12.0$ cm. At $x = 0$ and $t = 0$ we have $y = (12.0\text{ cm})\sin\phi$. To make this fit $y = 0$, we take $\phi = 0$. Then

$$\boxed{y = 0.120\sin(1.57x - 31.4t)\text{, where } x \text{ and } y \text{ are in meters and } t \text{ is in seconds}}$$

(d) The transverse velocity is $\dfrac{\partial y}{\partial t} = -A\omega\cos(kx - \omega t)$.

Its maximum magnitude is

$$A\omega = (12.0\text{ cm})(31.4\ \text{rad/s}) = \boxed{3.77\ \text{m/s}}$$

(e) $a_y = \dfrac{\partial v_y}{\partial t} = \dfrac{\partial}{\partial t}\left[-A\omega\cos(kx - \omega t)\right] = -A\omega^2\sin(kx - \omega t)$

The maximum value is $A\omega^2 = (0.120\text{ m})\left(31.4\text{ s}^{-1}\right)^2 = \boxed{118\text{ m/s}^2}$

P16.12 At time t, the motion at point A, where $x = 0$, is

$$y_A = (1.50\text{ cm})\cos(-50.3t)$$

At point B, the motion is

$$y_B = (15.0\text{ cm})\cos(15.7x_B - 50.3t) = (15.0\text{ cm})\cos\left(-50.3t \pm \frac{\pi}{3}\right)$$

which implies

$$15.7x_B = (15.7\text{ m}^{-1})x_B = \pm\frac{\pi}{3}$$

or $$x_B = -0.066\,7\text{ m} = \boxed{\pm 6.67\text{ cm}}$$

P16.13 (a) $f = \dfrac{v}{\lambda} = \dfrac{(1.00\text{ m/s})}{2.00\text{ m}} = \boxed{0.500\text{ Hz}}$

(b) $\omega = 2\pi f = 2\pi(0.500/\text{s}) = \pi/\text{s} = \boxed{3.14\text{ rad/s}}$

(c) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{2.00\text{ m}} = \pi/\text{m} = \boxed{3.14\text{ rad/m}}$

(d) $y = A\sin(kx - \omega t + \phi)$ becomes

$$y = \boxed{0.100\sin(\pi x - \pi t)}$$

(e) For $x = 0$ the wave function requires

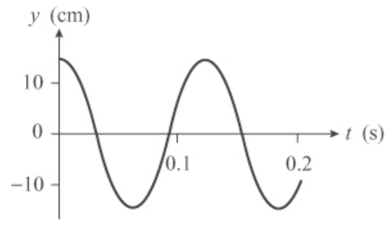
$$\boxed{y = 0.100\sin(\pi t)}$$

(f) $\boxed{y = 0.100\sin(4.71 - \pi t)}$

(g) $v_y = \dfrac{\partial y}{\partial t} = 0.100\text{ m}(-3.14/\text{s})\cos(3.14x/\text{m} - 3.14t/\text{s})$

The cosine varies between +1 and −1, so maximum $v_y = \boxed{0.314\text{ m/s}}$.

P16.14 (a) ANS. FIG. P16.14 shows the y vs. t plot of the given wave.



ANS. FIG. P16.14

(b) The time from one peak to the next one is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{50.3\ \text{s}^{-1}} = \boxed{0.125\ \text{s}}$$

(c) $\boxed{\text{This agrees with the period found in the example in the text.}}$

P16.15 The wave function is given as

$$y = (0.120\ \text{m})\sin\left(\frac{\pi}{8}x + 4\pi t\right)$$

(a) We differentiate the wave function with respect to time to obtain the velocity:

$$v = \frac{\partial y}{\partial t}:\ \ v = (0.120)(4\pi)\cos\left(\frac{\pi}{8}x + 4\pi t\right)$$

$$v(0.200\ \text{s}, 1.60\ \text{m}) = \boxed{-1.51\ \text{m/s}}$$

(b) Differentiating the velocity function gives the acceleration:

$$a = \frac{\partial v}{\partial t}:\ \ a = (-0.120\ \text{m})(4\pi)^2\sin\left(\frac{\pi}{8}x + 4\pi t\right)$$

$$a(0.200\ \text{s}, 1.60\ \text{m}) = \boxed{0}$$

(c) $k = \frac{\pi}{8} = \frac{2\pi}{\lambda}:\ \ \lambda = \boxed{16.0\ \text{m}}$

(d) $\omega = 4\pi = \frac{2\pi}{T}:\ \ T = \boxed{0.500\ \text{s}}$

(e) $v = \frac{\lambda}{T} = \frac{16.0\ \text{m}}{0.500\ \text{s}} = \boxed{32.0\ \text{m/s}}$

P16.16 (a) At $x = 2.00$ m, $y = \boxed{0.100\sin(1.00 - 20.0t)}$. Because this disturbance varies sinusoidally in time, it describes simple harmonic motion.

(b) At $x = 2.00$ m, compare $y = 0.100\sin(1.00 - 20.0t)$ to $A\cos(\omega t + \phi)$:

$$\begin{aligned} y &= 0.100\sin(1.00 - 20.0t) = -0.100\sin(20.0t - 1.00) \\ &= 0.100\cos(20.0t - 1.00 + \pi) \\ &= 0.100\cos(20.0t + 2.14) \end{aligned}$$

so $\omega = 20.0\ \text{rad/s}$ and $f = \frac{\omega}{2\pi} = \boxed{3.18\ \text{Hz}}$

P16.17 The wave function is: $y = 0.25 \sin(0.30x - 40t)$ m

Compare this with the general expression $y = A \sin(kx - \omega t)$:

(a) $A = \boxed{0.250\text{ m}}$

(b) $\omega = \boxed{40.0\text{ rad/s}}$

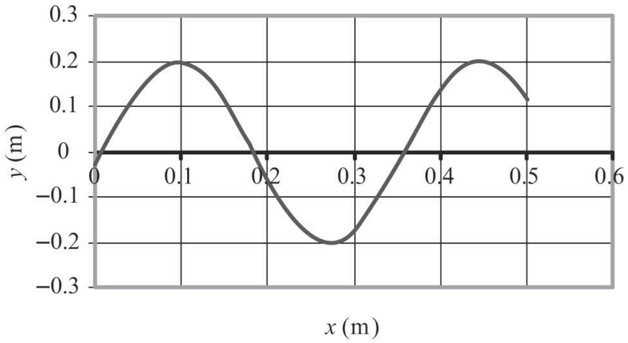
(c) $k = \boxed{0.300\text{ rad/m}}$

(d) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{0.300\text{ rad/m}} = \boxed{20.9\text{ m}}$

(e) $v = f\lambda = \left(\dfrac{\omega}{2\pi}\right)\lambda = \left(\dfrac{40.0\text{ rad/s}}{2\pi}\right)(20.9\text{ m}) = \boxed{133\text{ m/s}}$

(f) The wave moves to the right, $\boxed{\text{in the} + x\text{ direction}}$.

P16.18 (a) ANS. FIG. P16.18(a) shows a sketch of the wave at $t = 0$.



ANS FIG. P16.18(a)

(b) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.350\text{ m}} = \boxed{18.0\text{ rad/m}}$

(c) $T = \dfrac{1}{f} = \dfrac{1}{12.0/\text{s}} = \boxed{0.083\ 3\text{ s}}$

(d) $\omega = 2\pi f = 2\pi 12.0/\text{s} = \boxed{75.4\text{ rad/s}}$

(e) $|v| = f\lambda = (12.0/\text{s})(0.350\text{ m}) = \boxed{4.20\text{ m/s}}$

(f) $y = A\sin(kx + \omega t + \phi)$ specializes to

$$\boxed{y = (0.200\text{ m})\sin(18.0x/\text{m} + 75.4t/\text{s} + \phi)}$$

(g) At $x = 0$, $t = 0$ we require

$$-3.00\times 10^{-2}\ \text{m} = (0.200\ \text{m})\sin(+\phi)$$

$$\phi = -8.63° = -0.151\ \text{rad}$$

so

$\boxed{y(x,\ t) = 0.200\sin(18.0x + 75.4t - 0.151),\ \text{where } x \text{ and } y \text{ are in meters and } t \text{ is in seconds.}}$

P16.19 Using the traveling wave model, we can put constants with the right values into $y = A\sin(kx + \omega t + \phi)$ to have the mathematical representation of the wave. We have the same (positive) signs for both kx and ωt so that a point of constant phase will be at a decreasing value of x as t increases—that is, so that the wave will move to the left.

The amplitude is $A = y_{\max} = 8.00\ \text{cm} = 0.080\ 0\ \text{m}$

The wave number is $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{0.800\ \text{m}} = 2.50\pi\ \text{m}^{-1}$

The angular frequency is $\omega = 2\pi f = 2\pi(3.00\ \text{s}^{-1}) = 6.00\pi\ \text{rad/s}$

(a) In $y = A\sin(kx + \omega t + \phi)$, choosing $\phi = 0$ will make it true that $y(0, 0) = 0$. Then the wave function becomes upon substitution of the constant values for this wave

$$\boxed{y = (0.080\ 0)\sin(2.50\pi x + 6.00\pi t)}$$

(b) In general, $y = (0.080\ 0)\sin(2.50\pi x + 6.00\pi t + \phi)$

If $y(x, 0) = 0$ at $x = 0.100$ m, we require

$$0 = (0.080\ 0)\sin(2.50\pi + \phi)$$

so we must have the phase constant be $\phi = -0.250\pi$ rad.

Therefore, the wave function for all values of x and t is

$\boxed{y = 0.080\ 0\sin(2.50\pi x + 6.00\pi t - 0.250\pi),\ \text{where } x \text{ and } y \text{ are in meters and } t \text{ is in seconds.}}$

P16.20 (a) Let us write the wave function as $y(x,t) = A\sin(kx + \omega t + \phi)$.

We have $y_i = y(0, 0) = A\sin\phi = 0.020\ 0\ \text{m}$

and $v_i = v(0, 0) = \left.\dfrac{\partial y}{\partial t}\right|_{0,\,0} = A\omega\cos\phi = -2.00\ \text{m/s}.$

Also, $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.025\ 0\ \text{s}} = 80.0\pi\ \text{s}^{-1}$.

Use the identity $\sin^2\phi + \cos^2\phi = 1$ and the expressions for y_i and v_i:

$$\frac{(A\sin\phi)^2}{A^2} + \frac{(A\omega\cos\phi)^2}{A^2\omega^2} = 1$$

$$(A\sin\phi)^2 + \frac{(A\omega\cos\phi)^2}{\omega^2} = A^2$$

$$A^2 = y_i^2 + \left(\frac{v_i}{\omega}\right)^2 = (0.020\ 0\ \text{m})^2 + \left(\frac{-2.00\ \text{m/s}}{80.0\pi\ \text{s}^{-1}}\right)^2$$

$$A = \boxed{0.021\ 5\ \text{m}}$$

(b) $\frac{\omega y_i}{v_i} = \frac{\omega(A\sin\phi)}{\omega A\cos\phi} = \tan\phi \rightarrow \tan\phi = \frac{80.0\pi(0.020\ 0)}{-2.00} = -2.51$

Your calculator's answer $\phi = \tan^{-1}(-2.51) = -1.19$ rad is an angle in the fourth quadrant with a negative sine and positive cosine, just the reverse of what is required. Recall on the unit circle, an angle with a negative tangent can be in either the second or fourth quadrant. The sine is positive and the cosine is negative in the second quadrant. The angle in the second quadrant is

$$\phi = \pi - 1.19\ \text{rad} = \boxed{1.95\ \text{rad}}$$

(c) $v_{y,\ \text{max}} = A\omega = (0.021\ 5\ \text{m})(80.0\pi/\text{s}) = \boxed{5.41\ \text{m/s}}$

(d) $\lambda = v_x T = (30.0\ \text{m/s})(0.025\ 0\ \text{s}) = 0.750\ \text{m}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.750\ \text{m}} = 8.38\ \text{m}^{-1}, \qquad \omega = 80.0\pi\ \text{s}^{-1}$$

$$\boxed{y(x,\ t) = (0.021\ 5)\sin(8.38x + 80.0\pi t + 1.95)}$$

Section 16.3 The Speed of Transverse Waves on Strings

P16.21 If the tension in the wire is T, the tensile stress is

$$\text{stress} = \frac{T}{A} \qquad \text{so} \qquad T = A(\text{stress})$$

The speed of transverse waves in the wire is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{A(\text{Stress})}{m/L}} = \sqrt{\frac{\text{Stress}}{m/AL}} = \sqrt{\frac{\text{Stress}}{m/\text{Volume}}} = \sqrt{\frac{\text{Stress}}{\rho}}$$

where ρ is the density. The maximum velocity occurs when the stress is a maximum:

$$v_{\max} = \sqrt{\frac{2.70 \times 10^{8}\ \text{Pa}}{7860\ \text{kg/m}^{3}}} = \boxed{185\ \text{m/s}}$$

P16.22 The speed is given by

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1\,350\ \text{kg}\cdot\text{m/s}^{2}}{5.00 \times 10^{-3}\ \text{kg/m}}} = \boxed{520\ \text{m/s}}$$

P16.23 The two wave speeds can be written as

$$v_1 = \sqrt{T_1/\mu} \qquad \text{and} \qquad v_2 = \sqrt{T_2/\mu}$$

Since μ is constant, $\mu = \frac{T_2}{v_2^2} = \frac{T_1}{v_1^2}$, and

$$T_2 = \left(\frac{v_2}{v_1}\right)^2 T_1 = \left(\frac{30.0\ \text{m/s}}{20.0\ \text{m/s}}\right)^2 (6.00\ \text{N}) = \boxed{13.5\ \text{N}}$$

P16.24 (a) For the first equation,

$$f = \frac{1}{T} \quad\rightarrow\quad T = \frac{1}{f} \quad\rightarrow\quad [T] = \frac{1}{[f]} = \frac{1}{\text{T}^{-1}} = \text{T}$$

$\boxed{\text{units are seconds}}$

$$v = \sqrt{\frac{T}{\mu}} \quad\rightarrow\quad T = \mu v^2 \rightarrow \quad [T] = \left[\mu v^2\right] = \frac{\text{M}}{\text{L}}\left(\frac{\text{L}}{\text{T}}\right)^2 = \frac{\text{ML}}{\text{T}^2}$$

$\boxed{\text{units are newtons}}$

(b) $\boxed{\text{The first } T \text{ is period of time; the second is force of tension.}}$

P16.25 The down and back distance is 4.00 m + 4.00 m = 8.00 m.

The speed is then $v = \frac{d_{\text{total}}}{t} = \frac{4(8.00\text{ m})}{0.800\text{ s}} = 40.0\ \text{m/s} = \sqrt{\frac{T}{\mu}}$.

Now, $\mu = \frac{0.200\text{ kg}}{4.00\text{ m}} = 5.00 \times 10^{-2}\ \text{kg/m}$.

So $T = \mu v^2 = \left(5.00 \times 10^{-2}\ \text{kg/m}\right)\left(40.0\text{ m/s}\right)^2 = \boxed{80.0\text{ N}}$.

P16.26 (a) To write the equation, we determine the angular frequency and wave number:

$$\omega = 2\pi f = 2\pi\left(500\text{ Hz}\right) = 3\ 140\ \text{rad/s}$$

$$k = \frac{\omega}{v} = \frac{3\ 140}{196} = 16.0\ \text{m}^{-1}$$

$\boxed{y = \left(2.00 \times 10^{-4}\right)\sin\left(16.0x - 3\ 140t\right)}$, where y and x are in meters and t is in seconds.

(b) $v = 196\ \text{m/s} = \sqrt{\frac{T}{4.10 \times 10^{-3}\ \text{kg/m}}} \rightarrow T = \boxed{158\text{ N}}$

P16.27 The total time interval is the sum of the two time intervals.

In each wire

$$\Delta t = \frac{L}{v} = L\sqrt{\frac{\mu}{T}}$$

Let A represent the cross-sectional area of one wire. The mass of one wire can be written both as $m = \rho V = \rho AL$ and also as $m = \mu L$.

Then we have $\mu = \rho A = \frac{\pi\rho d^2}{4}$.

Thus, $\Delta t = L\left(\frac{\pi\rho d^2}{4T}\right)^{1/2}$

For copper,

$$\Delta t = \left(20.0\text{ m}\right)\left[\frac{(\pi)\left(8\ 920\text{ kg/m}^3\right)\left(1.00 \times 10^{-3}\text{ m}\right)^2}{(4)(150\text{ N})}\right]^{1/2} = 0.137\text{ s}$$

For steel,

$$\Delta t = (30.0\text{ m})\left[\frac{(\pi)(7\ 860\text{ kg/m}^3)(1.00\times10^{-3}\text{ m})^2}{(4)(150\text{ N})}\right]^{1/2} = 0.192\text{ s}$$

The total time interval is $0.137 + 0.192 = \boxed{0.329\text{ s}}$

P16.28 The tension in the string is $T = mg$, where g is the acceleration of gravity on the Moon, about one-sixth that of Earth. From the data given, what is the acceleration of gravity on the Moon?

The wave speed is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg}{m/L}} = \sqrt{\frac{MgL}{m}} = \frac{L}{t} \rightarrow \frac{MgL}{m} = \frac{L^2}{t^2} \rightarrow g = \frac{mL}{Mt^2}$$

$$g = \frac{mL}{Mt^2} = \frac{(4.00\times10^{-3}\text{ kg})(1.60\text{ m})}{(3.00\text{ kg})(26.1\times10^{-3}\text{ s})^2} = 3.13\text{ m/s}^2$$

$\boxed{\text{The calculated gravitational acceleration of the Moon is almost twice that of the accepted value.}}$

P16.29 (a) The tension in the string is

$$F = mg = (3.00\text{ kg})(9.80\text{ m/s}^2) = 29.4\text{ N}$$

Then, from $v = \sqrt{\frac{F}{\mu}}$, the mass per unit length is

$$\mu = \frac{F}{v^2} = \frac{29.4\text{ N}}{(24.0\text{ m/s})^2} = \boxed{0.0510\text{ kg/m}}$$

(b) When $m = 2.00$ kg, the tension is

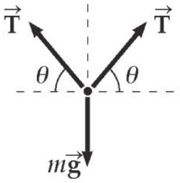
$$F = mg = (2.00\text{ kg})(9.80\text{ m/s}^2) = 19.6\text{ N}$$

and the speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{19.6\text{ N}}{0.0510\text{ kg/m}}} = \boxed{19.6\text{ m/s}}$$

P16.30 From the free-body diagram $mg = 2T\sin\theta$

$$T = \frac{mg}{2\sin\theta}$$



ANS. FIG. P16.30

The angle θ is found from

$$\cos\theta = \frac{3L/8}{L/2} = \frac{3}{4}$$

$$\therefore \theta = 41.4°$$

(a) $$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{2\mu\sin\theta}}$$

$$= \sqrt{\frac{mg}{2\mu\sin 41.4°}} = \left(\sqrt{\frac{9.80\ \text{m/s}^2}{2\left(8.00\times10^{-3}\ \text{kg/m}\right)\sin 41.4°}}\right)\sqrt{m}$$

or $\boxed{v = (30.4\ )\sqrt{m}\text{, where } v \text{ is in meters per second and } m \text{ is in kilograms.}}$

(b) $v = 60.0 = 30.4\sqrt{m}$ and $\boxed{m = 3.89\ \text{kg}}$

P16.31 We use $v = \sqrt{\frac{T}{\mu}}$ to solve for the tension:

$$T = \mu v^2 = \rho A v^2 = \rho\pi r^2 v^2$$

$$T = \left(8920\ \text{kg/m}^3\right)(\pi)\left(7.50\times10^{-4}\ \text{m}\right)^2\left(200\ \text{m/s}\right)^2$$

$$T = \boxed{631\ \text{N}}$$

Section 16.5 Rate of Energy Transfer by Sinusoidal Waves on Strings

P16.32 (a) As for a string wave, the rate of energy transfer is proportional to the square of the amplitude and to the speed. The rate of energy transfer stays constant because each wavefront carries constant energy and the frequency stays constant. As the speed drops the amplitude must increase.

(b) We write $P = FvA^2$, where F is some constant. With no absorption of energy,

$$Fv_{\text{granite}}A^2_{\text{granite}} = Fv_{\text{mudfill}}A^2_{\text{mudfill}}$$

$$\frac{A_{\text{mudfill}}}{A_{\text{granite}}} = \sqrt{\frac{v_{\text{granite}}}{v_{\text{mudfill}}}} = \sqrt{\frac{v_{\text{granite}}}{v_{\text{granite}}/25.0}} = \sqrt{\frac{25.0v_{\text{granite}}}{v_{\text{granite}}}} = 5.00$$

$\boxed{\text{The amplitude increases by 5.00 times.}}$

P16.33 We are given T = constant; we use the equation for the speed of a wave on a string, $v = \sqrt{\frac{T}{\mu}}$, and the power supplied to a vibrating string, $P = \frac{1}{2}\mu\omega^2A^2v$.

(a) If L is doubled, μ is still the same, so v remains constant: therefore P is constant: $\boxed{1}$.

(b) If A is doubled and ω is halved, $P \propto \omega^2A^2$ remains constant: $\boxed{1}$.

(c) If λ and A are doubled, the product $\omega^2A^2 \propto \frac{A^2}{\lambda^2}$ remains constant, so $\boxed{1}$.

(d) If L and λ are halved, μ is still the same, and $\omega^2 \propto \frac{1}{\lambda^2}$ is quadrupled, so P $\boxed{\text{is increased by a factor of 4}}$.

P16.34 We will use the expression for power carried by a wave on a string. The wave speed is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100\text{ N}}{4.00\times10^{-2}\text{ kg/m}}} = 50.0\text{ m/s}$

From $P = \frac{1}{2}\mu\omega^2A^2v$, we have

$$\omega^2 = \frac{2P}{\mu A^2v} = \frac{2(300\text{ N}\cdot\text{m/s})}{(4.00\times10^{-2}\text{ kg/m})(5.00\times10^{-2}\text{ m})^2(50.0\text{ m/s})}$$

Computing,

$$\omega = 346\text{ rad/s} \quad\text{and}\quad f = \frac{\omega}{2\pi} = \boxed{55.1\text{ Hz}}$$

P16.35 Comparing the given wave function, $y = (0.15)\sin(0.80x - 50t)$, with the general wave function, $y = A\sin(kx - \omega t)$, we have $k = 0.80$ rad/m, $\omega = 50$ rad/s, and $A = 0.15$ m.

(a) $v = f\lambda = \frac{\omega}{2\pi}\frac{2\pi}{k} = \frac{\omega}{k} = \frac{50.0}{0.800}\ \text{m/s} = \boxed{62.5\ \text{m/s}}$

(b) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.800}\ \text{m} = \boxed{7.85\ \text{m}}$

(c) $f = \frac{50.0}{2\pi} = \boxed{7.96\ \text{Hz}}$

(d) $P = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}(12.0\times10^{-3})(50.0)^2(0.150)^2(62.5)\ \text{W} = \boxed{21.1\ \text{W}}$

P16.36 The frequency and angular frequency of the wave are

$$f = \frac{v}{\lambda} = \frac{30.0\ \text{m/s}}{0.500\ \text{s}} = 60.0\ \text{Hz} \quad \text{and} \quad \omega = 2\pi f = 120\pi\ \text{rad/s}$$

The power that is required is then

$$\begin{aligned}P &= \frac{1}{2}\mu\omega^2A^2v\\ &= \frac{1}{2}\left(\frac{0.180\ \text{kg}}{3.60\ \text{m}}\right)(120\pi\ \text{rad/s})^2(0.100\ \text{m})^2(30.0\ \text{m/s})\\ &= \boxed{1.07\ \text{kW}}\end{aligned}$$

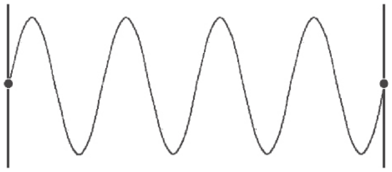
P16.37 We are given $\mu = 30.0\ \text{g/m} = 30.0\times10^{-3}\ \text{kg/m}$, with

$$\begin{aligned}&\lambda = 1.50\ \text{m}\\ &f = 50.0\ \text{Hz:}\quad \omega = 2\pi f = 314\ \text{s}^{-1}\\ &2A = 0.150\ \text{m:}\quad A = 7.50\times10^{-2}\ \text{m}\end{aligned}$$

(a) From $y = A\sin\left(\frac{2\pi}{\lambda}x - \omega t\right)$, $\boxed{y = (0.075)\sin(4.19x - 314t)}$

(b) $P = \frac{1}{2}\mu\omega^2A^2v = \frac{1}{2}(30.0\times10^{-3})(314)^2(7.50\times10^{-2})^2\left(\frac{314}{4.19}\right)\ \text{W}$

$\boxed{P = 625\ \text{W}}$



ANS. FIG. P16.37

P16.38 Originally,

$$P_0 = \frac{1}{2}\mu\omega^2 A^2 v$$

$$P_0 = \frac{1}{2}\mu\omega^2 A^2 \sqrt{\frac{T}{\mu}}$$

$$P_0 = \frac{1}{2}\omega^2 A^2 \sqrt{T\mu}$$

The doubled string will have doubled mass per length. Presuming that we hold tension constant, it can carry power larger by $\sqrt{2}$ times:

$$P = \frac{1}{2}\omega^2 A^2 \sqrt{T(2\mu)} = \sqrt{2}\left(\frac{1}{2}\omega^2 A^2 \sqrt{T\mu}\right) = \boxed{\sqrt{2}P_0}$$

P16.39 Comparing

$$y = 0.350\sin\left(10\pi t - 3\pi x + \frac{\pi}{4}\right)$$

with

$$y = A\sin(kx - \omega t + \phi) = A\sin(\omega t - kx - \phi + \pi)$$

we have

$$k = 3\pi\ \text{m}^{-1},\ \omega = 10\pi\ \text{s}^{-1},\ \text{and } A = 0.350\ \text{m}$$

Then,

$$v = f\lambda = (2\pi f)\left(\frac{\lambda}{2\pi}\right) = \frac{\omega}{k} = \frac{10\pi\ \text{s}^{-1}}{3\pi\ \text{m}^{-1}} = 3.33\ \text{m/s}$$

(a) The rate of energy transport is

$$\begin{aligned} P &= \frac{1}{2}\mu\omega^2 A^2 v \\ &= \frac{1}{2}(75\times 10^{-3}\ \text{kg/m})(10\pi\ \text{s}^{-1})^2(0.350\ \text{m})^2(3.33\ \text{m/s}) \\ &= \boxed{15.1\ \text{W}} \end{aligned}$$

(b) Recall that $vT = \lambda$. The energy per cycle is

$$\begin{aligned} E_\lambda &= PT = \frac{1}{2}\mu\omega^2 A^2 \lambda \\ &= \frac{1}{2}(75.0\times 10^{-3}\ \text{kg/m})(10\pi\ \text{s}^{-1})^2(0.350\ \text{m})^2\left(\frac{2\pi}{3\pi\ \text{m}^{-1}}\right) \\ &= \boxed{3.02\ \text{J}} \end{aligned}$$

P16.40 Suppose that no energy is absorbed or carried down into the water. Then a fixed amount of power is spread thinner farther away from the source. It is spread over the circumference $2\pi r$ of an expanding circle. The power-per-width across the wave front

$$\frac{P}{2\pi r}$$

is proportional to amplitude squared, so amplitude is proportional to

$$\sqrt{\frac{P}{2\pi r}}$$

Section 16.6 The Linear Wave Equation

P16.41 The important thing to remember with partial derivatives is that **you treat all variables as constants, except the single variable of interest**. Keeping this in mind, we must apply two standard rules of differentiation to the function $y = \ln[b(x - vt)]$:

$$\frac{\partial}{\partial x}\left[\ln f(x)\right] = \frac{1}{f(x)}\frac{\partial\left[f(x)\right]}{\partial x} \qquad \textbf{[1]}$$

$$\frac{\partial}{\partial x}\left[\frac{1}{f(x)}\right] = \frac{\partial}{\partial x}\left[f(x)\right]^{-1} = (-1)\left[f(x)\right]^{-2}\frac{\partial\left[f(x)\right]}{\partial x}$$

$$= -\frac{1}{\left[f(x)\right]^2}\frac{\partial\left[f(x)\right]}{\partial x} \qquad \textbf{[2]}$$

Applying [1],

$$\frac{\partial y}{\partial x} = \left(\frac{1}{b(x-vt)}\right)\frac{\partial(bx - bvt)}{\partial x} = \left(\frac{1}{b(x-vt)}\right)(b) = \frac{1}{x-vt}$$

Applying [2],

$$\frac{\partial^2 y}{\partial x^2} = -\frac{1}{(x-vt)^2}$$

In a similar way,

$$\frac{\partial y}{\partial t} = \frac{-v}{x-vt} \quad \text{and} \quad \frac{\partial^2 y}{\partial t^2} = \frac{v^2}{(x-vt)^2}$$

From the second-order partial derivatives, we see that it is true that

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

so the proposed function **is** one solution to the wave equation.

P16.42 (a) $A = (7.00 + 3.00)(4.00)$ yields $\boxed{A = 40.0}$

(b) $\boxed{A = 7.00,\ B = 0,\ \text{and } C = 3.00}$

(c) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-dimensional space. All of their components must be equal, so all coefficients of the unit vectors must be equal.

(d) $\boxed{A = 0}$ $\boxed{B = 7.00}$ in meters, $\boxed{C = 3.00}$ in m^{-1}, $\boxed{D = 4.00}$ in s^{-1}, $\boxed{E = 2.00}$ in rad.

(e) Identify corresponding parts. In order for two functions to be identically equal, corresponding parts must be identical. The argument of the sine function must have no units, or be equivalent to units of radians.

P16.43 The linear wave equation is $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$.

If $y = e^{b(x-vt)}$

Then $\dfrac{\partial y}{\partial t} = -bve^{b(x-vt)}$ and $\dfrac{\partial y}{\partial x} = be^{b(x-vt)}$

$\dfrac{\partial^2 y}{\partial t^2} = b^2v^2e^{b(x-vt)}$ and $\dfrac{\partial^2 y}{\partial x^2} = b^2e^{b(x-vt)}$

Therefore, $\dfrac{\partial^2 y}{\partial t^2} = v^2\dfrac{\partial^2 y}{\partial x^2}$, demonstrating that $e^{b(x-vt)}$ is a solution.

P16.44 (a) From $y = x^2 + v^2t^2$,

evaluate $\dfrac{\partial y}{\partial x} = 2x$ and $\dfrac{\partial^2 y}{\partial x^2} = 2$

Also, $\dfrac{\partial y}{\partial t} = v^2 2t$ and $\dfrac{\partial^2 y}{\partial t^2} = 2v^2$

Does $\frac{\partial^2 y}{\partial t^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$?

By substitution, we must test $2 = \frac{1}{v^2}\left(2v^2\right)$ and this is true, so the wave function does satisfy the wave equation.

(b) Note

$$\begin{aligned}\frac{1}{2}(x+vt)^2 + \frac{1}{2}(x-vt)^2 &= \frac{1}{2}x^2 + xvt + \frac{1}{2}v^2t^2 + \frac{1}{2}x^2 - xvt + \frac{1}{2}v^2t^2\\ &= x^2 + v^2t^2\end{aligned}$$

as required. So

$$\boxed{f(x+vt) = \frac{1}{2}(x+vt)^2} \quad \text{and} \quad \boxed{g(x-vt) = \frac{1}{2}(x-vt)^2}$$

(c) $y = \sin x \cos vt$ makes

$$\frac{\partial y}{\partial x} = \cos x \cos vt \qquad \frac{\partial^2 y}{\partial x^2} = -\sin x \cos vt$$

$$\frac{\partial y}{\partial t} = -v \sin x \sin vt \qquad \frac{\partial^2 y}{\partial t^2} = -v^2 \sin x \cos vt$$

Then $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ becomes $-\sin x \cos vt = \frac{-1}{v^2}v^2 \sin x \cos vt$ which is true, as required.

Note $\quad \sin(x+vt) = \sin x \cos vt + \cos x \sin vt$

$\quad \sin(x-vt) = \sin x \cos vt - \cos x \sin vt$

So $\quad \sin x \cos vt = f(x+vt) + g(x-vt)$ with

$$\boxed{f(x+vt) = \frac{1}{2}\sin(x+vt)} \quad \text{and} \quad \boxed{g(x-vt) = \frac{1}{2}\sin(x-vt)}$$

Additional Problems

P16.45 The equation $v = \lambda f$ is a special case of

speed = (cycle length)(repetition rate)

Thus,

$$v = \left(19.0\times10^{-3}\ \text{m/frame}\right)\left(24.0\ \text{frames/s}\right) = \boxed{0.456\ \text{m/s}}$$

P16.46 Assume a typical distance between adjacent people ~ 1 m.

Then the wave speed is $v = \dfrac{\Delta x}{\Delta t} \sim \dfrac{1\text{ m}}{0.1\text{ s}} \sim 10\text{ m/s}$.

Model the stadium as a circle with a radius of order 100 m. Then, the time for one circuit around the stadium is

$$T = \frac{2\pi r}{v} \sim \frac{2\pi\left(10^2\right)}{10\text{ m/s}} = 63\text{ s}\ \boxed{\sim 1\text{ min}}$$

P16.47 The speed of the wave on the rope is $v = \sqrt{\dfrac{T}{\mu}}$ and in this case $T = mg$;

therefore, $m = \dfrac{\mu v^2}{g}$.

Now $v = f\lambda$ implies $v = \dfrac{\omega}{k}$ so that

$$m = \frac{\mu}{g}\left(\frac{\omega}{k}\right)^2 = \frac{0.250\text{ kg/m}}{9.80\text{ m/s}^2}\left[\frac{18\pi\text{ s}^{-1}}{0.750\pi\text{ m}^{-1}}\right]^2 = \boxed{14.7\text{ kg}}$$

***P16.48** $v = \dfrac{2d}{t}$ gives

$$d = \frac{vt}{2} = \frac{1}{2}\left(6.50\times 10^3\text{ m/s}\right)(1.85\text{ s}) = \boxed{6.01\text{ km}}$$

P16.49 The block-cord-Earth system is isolated, so energy is conserved as the block moves down distance x:

$$\Delta K + \Delta U = 0 \rightarrow$$

$$\left(K + U_g + U_s\right)_{\text{top}} = \left(K + U_g + U_s\right)_{\text{bottom}}$$

$$0 + Mgx + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$x = \frac{2Mg}{k}$$

(a) $T = kx = 2Mg = 2(2.00\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{39.2\text{ N}}$

(b) $L = L_0 + x = L_0 + \dfrac{2Mg}{k}$

$$L = 0.500\text{ m} + \frac{39.2\text{ N}}{100\text{ N/m}} = \boxed{0.892\text{ m}}$$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}}$

$$v = \sqrt{\frac{39.2\text{ N} \times 0.892\text{ m}}{5.0 \times 10^{-3}\text{ kg}}}$$

$$v = \boxed{83.6\text{ m/s}}$$

P16.50 The block-cord-Earth system is isolated, so energy is conserved as the block moves down distance *x*:

$$\Delta K + \Delta U = 0 \rightarrow$$

$$\left(K + U_g + U_s\right)_{\text{top}} = \left(K + U_g + U_s\right)_{\text{bottom}}$$

$$0 + Mgx + 0 + 0 = 0 + 0 + \frac{1}{2}kx^2$$

$$Mgx = \frac{1}{2}kx^2$$

(a) $T = kx = \boxed{2Mg}$

(b) $L = L_0 + x = \boxed{L_0 + \dfrac{2Mg}{k}}$

(c) $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{TL}{m}} = \boxed{\sqrt{\dfrac{2Mg}{m}\left(L_0 + \dfrac{2Mg}{k}\right)}}$

P16.51 (a) The wave function becomes

$$0.175\text{ m} = (0.350\text{ m})\sin\left[(99.6\text{ rad/s})t\right]$$

or $\sin\left[(99.6\text{ rad/s})t\right] = 0.500$

The smallest two angles for which the sine function is 0.500 are 30.0° and 150°, i.e., 0.523 6 rad and 2.618 rad.

$$(99.6\text{ rad/s})t_1 = 0.523\,6\text{ rad, thus } t_1 = 5.26\text{ ms}$$

$$(99.6\text{ rad/s})t_2 = 2.618\text{ rad, thus } t_2 = 26.3\text{ ms}$$

$$\Delta t \equiv t_2 - t_1 = 26.3\text{ ms} - 5.26\text{ ms} = \boxed{21.0\text{ ms}}$$

(b) Distance traveled by the wave

$$= \left(\frac{\omega}{k}\right)\Delta t = \left(\frac{99.6\text{ rad/s}}{1.25\text{ rad/m}}\right)\left(21.0 \times 10^{-3}\text{ s}\right) = \boxed{1.68\text{ m}}$$

P16.52 (a) From $y = (0.150\text{ m})\sin(0.800x - 50.0t) = A\sin(kx - \omega t)$

we compute

$$\partial y/\partial t = (0.150\text{ m})(-50.0\text{ s}^{-1})\cos(0.800x - 50.0t)$$

and $a = \partial^2 y/\partial t^2 = -(0.150\text{ m})(-50.0\text{ s}^{-1})^2\sin(0.800x - 50.0t)$

Then $a_{\max} = (0.150\text{ m})(50.0\text{ s}^{-1})^2 = \boxed{375\text{ m/s}^2}$

(b) For the 1.00-cm segment with maximum force acting on it,

$$\sum F = ma = \left(\frac{12.0\times10^{-3}\text{ kg}}{100\text{ cm}}\right)(1.00\text{ cm})(375\text{ m/s}^2) = \boxed{0.045\ 0\text{ N}}$$

(c) To find the tension in the string, we first compute the wave speed

$$v = \lambda f = \frac{\omega}{k} = \frac{50.0\text{ s}^{-1}}{0.800\text{ m}^{-1}} = 62.5\text{ m/s}$$

then,

$$v = \sqrt{\frac{T}{\mu}}\text{ gives } T = \mu v^2 = \left(\frac{12.0\times10^{-3}\text{ kg}}{1.00\text{ m}}\right)(62.5\text{ m/s})^2 = \boxed{46.9\text{ N}}$$

$\boxed{\text{The maximum transverse force is very small compared to the tension, more than a thousand times smaller.}}$

P16.53 Assuming the incline to be frictionless and taking the positive x direction to be up the incline:

$$\sum F_x = T - Mg\sin\theta = 0$$

or the tension in the string is $T = Mg\sin\theta$.

The speed of transverse waves in the string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg\sin\theta}{m/L}} = \sqrt{\frac{MgL\sin\theta}{m}}$$

The time interval for a pulse to travel the string's length is

$$\Delta t = \frac{L}{v} = L\sqrt{\frac{m}{MgL\sin\theta}} = \boxed{\sqrt{\frac{mL}{Mg\sin\theta}}}$$

P16.54 (a) $\boxed{\text{The energy a wave crest carries is constant in the absence of absorption. Then the rate at which energy passes a stationary point, which is the power of the wave, is constant.}}$

(b) $\boxed{\text{The power is proportional to the square of the amplitude and to the wave speed. The speed decreases as the wave moves into shallower water near shore, so the amplitude must increase.}}$

(c) For the wave described, with a single direction of energy transport, the power is the same at the deep-water location ① and at the place ② with depth 9.00 m. Because power is proportional to the square of the amplitude and the wave speed, to express the constant power we write,

$$A_1^2 v_1 = A_2^2 v_2 = A_2^2\sqrt{gd_2}$$

$$(1.80\text{ m})^2(200\text{ m/s}) = A_2^2\sqrt{(9.80\text{ m/s}^2)(9.00\text{ m})}$$

$$= A_2^2(9.39\text{ m/s})$$

$$A_2 = 1.80\text{ m}\left(\frac{200\text{ m/s}}{9.39\text{ m/s}}\right)^{1/2}$$

$$= \boxed{8.31\text{ m}}$$

(d) $\boxed{\text{As the water depth goes to zero, our model would predict zero speed and infinite amplitude. In fact the amplitude must be finite as the wave comes ashore. As the speed decreases the wavelength also decreases. When it becomes comparable to the water depth, or smaller, our formula } \sqrt{gd} \text{ for wave speed no longer applies.}}$

P16.55 Let M = mass of block, m = mass of string. For the block, $\sum F = ma$ implies $T = \frac{mv_b^2}{r} = m\omega^2 r$. The speed of a wave on the string is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{M\omega^2 r}{m/r}} = r\omega\sqrt{\frac{M}{m}}$$

the travel time of the wave on the string is given by

$$\Delta t = \frac{r}{v} = \frac{1}{\omega}\sqrt{\frac{m}{M}}$$

and the angle through which the block rotates is

$$\Delta\theta = \omega\,\Delta t = \sqrt{\frac{m}{M}} = \sqrt{\frac{0.003\ 2\text{ kg}}{0.450\text{ kg}}} = \boxed{0.084\ 3\text{ rad}}$$

P16.56 The transverse wave velocity in the string is $v_{\text{trans}} = \sqrt{\dfrac{T}{\mu}}$,

where *T* is the tension in the cord, and *μ* is the mass per unit length of the cord. The tension *T* is generated by the centripetal force holding the mass and cord in uniform circular motion at the angular velocity *ω*; thus:

$$T = F_c = M\frac{v^2}{r} = M\omega^2 r$$

where we note that *M* is the mass of the block.

The mass density of the cord is $\mu = \dfrac{m}{r}$; thus, the transverse wave velocity is

$$v_{\text{trans}} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\left(M\omega^2 r\right)}{\left(\dfrac{m}{r}\right)}} = \sqrt{\frac{\left(M\omega^2 r^2\right)}{(m)}} = \omega r\sqrt{\frac{M}{m}}$$

Now the transverse wave travels a distance *r* (the length of the cord) at a uniform velocity v_{trans}; thus, distance = $r = v_{\text{trans}}t$, and therefore,

$$t = \frac{r}{v_{\text{trans}}} = \frac{r}{\left(\omega r\sqrt{\dfrac{M}{m}}\right)} = \boxed{\frac{1}{\omega}\sqrt{\frac{m}{M}}}$$

which we may solve numerically:

$$t = \frac{1}{\omega}\sqrt{\frac{m}{M}} = \frac{1}{(10.0\ \text{rad/s})}\sqrt{\frac{0.003\ 20\ \text{kg}}{0.450\ \text{kg}}} = \boxed{8.43\times 10^{-3}\ \text{s}}$$

[See Note to P16.57.]

P16.57 The transverse wave velocity in the string is $v_{\text{trans}} = \sqrt{\dfrac{T}{\mu}}$,

where *T* is the tension in the cord, and *μ* is the mass per unit length of the cord. The tension *T* is generated by the centripetal force holding the mass and cord in uniform circular motion at the angular velocity, *ω*; thus

$$T = F_c = M\frac{v^2}{r} = M\omega^2 r$$

where we note that *M* is the mass of the block, and the mass density of the cord is $\mu = \frac{m}{r}$. Thus transverse wave velocity is

$$v_{\text{trans}} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\left(M\omega^2 r\right)}{\left(\frac{m}{r}\right)}} = \sqrt{\frac{\left(M\omega^2 r^2\right)}{(m)}} = \omega r\sqrt{\frac{M}{m}}$$

Now the transverse wave travels a distance r (the length of the cord) at a uniform velocity v_{trans}; thus, distance $= r = v_{\text{trans}}t$, and therefore,

$$t = \frac{r}{v_{\text{trans}}} = \frac{r}{\left(\omega r\sqrt{\frac{M}{m}}\right)} = \boxed{\frac{1}{\omega}\sqrt{\frac{m}{M}}}$$

[Note: To solve this problem without integration of the mass density μ over the length of the cord to include the cord's own mass as a contribution to its own tension, and thus to a ***nonuniform tension*** along the length of the cord (and thus also to a ***nonuniform wave velocity*** along the cord), we must assume that the mass of the cord m is very small compared to the mass of the block M. In such a case, the mass of the cord does not contribute to the centripetal force, or as a result, to the tension on the cord itself. The only role the cord's mass will then play is in generating the linear density in the transverse wave velocity equation. To be forced to include mass of the cord in the centripetal force calculation is a ***significantly*** more difficult problem and is not attempted here.]

P16.58 (a) In $P = \frac{1}{2}\mu\omega^2 A^2 v$ where v is the wave speed, the quantity ωA is the maximum particle speed $v_{y,\max}$. We have $\mu = 0.500 \times 10^{-3}$ kg/m and

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20.0\text{ N}}{0.500\times 10^{-3}\text{ kg/m}}} = 200\text{ m/s}$$

Then

$$P = \frac{1}{2}\left(0.500\times 10^{-3}\text{ kg/m}\right)v_{y,\max}^2\left(200\text{ m/s}\right)$$

$$\boxed{P = 0.050\,0\, v_{y,\max}^2\text{, where } P \text{ is in watts and } v_{y,\max} \text{ is in meters per second}}$$

(b) $\boxed{\text{The power is proportional to the square of the maximum particle speed.}}$

(c) In time $t = (3.00\text{ m})/v = (3.00\text{ m})/(200\text{ m/s}) = 1.50 \times 10^{-2}$ s, all the energy in a 3.00-m length of string goes past a point. Therefore, the amount of this energy is

$$E = Pt = \left(0.050\ 0\text{ kg/s}\right)v_{y,\max}^2\ (0.015\text{ s}) = \left(7.50\times 10^{-4}\text{ kg}\right)v_{y,\max}^2$$

The mass of this section is

$$m_{3.00-m} = \left(0.500\times 10^{-3}\text{ kg/m}\right)\left(3.00\text{ m}\right) = 1.50\times 10^{-3}\text{ kg}$$

so $\dfrac{1}{2}m_{3.00-m} = 7.50\times 10^{-4}$ kg

$\boxed{E = (7.5\times 10^{-4})\ v_{y,\max}^2\text{, where } E \text{ is in joules and } v_{y,\max} \text{ is in meters per second.}}$

(d) $\boxed{\dfrac{1}{2}mv_{y,\max}^2}$

(e) $E = Pt = (0.050\ 0\text{ kg/s})\ v_{y,\max}^2\ (6.00\text{ s})$

$\rightarrow \boxed{E\ =\ 0.300\ v_{y,\max}^2 \text{ where } E \text{ is in joules and } v_{y,\max} \text{ is in meters per second.}}$

P16.59 (a) $\mu = \dfrac{dm}{dL} = \rho A\dfrac{dx}{dx} = \rho A$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho A}} = \sqrt{\frac{T}{\left[\rho\left(ax+b\right)\right]}} = \sqrt{\frac{T}{\left[\rho\left(10^{-3}x + 10^{-2}\right)\text{cm}^2\right]}}$$

With all SI units,

$$\boxed{v = \sqrt{\frac{T}{\left[\rho\left(1.00\times 10^{-5}x + 1.00\times 10^{-6}\right)\right]}} \text{ where } x \text{ is in meter, } T \text{ is in newtons, and } v \text{ is in meters per second.}}$$

(b) $$v(0) = \sqrt{\frac{24.0}{\left[\left(2700\right)\left(0+10^{-6}\right)\right]}} = \boxed{94.3\ \text{m/s}}$$

$$v(10.0\ m) = \sqrt{\frac{24.0}{\left[\left(2700\right)\left(10^{-4}+10^{-6}\right)\right]}} = \boxed{9.38\ \text{m/s}}$$

P16.60 Imagine a short transverse pulse traveling from the bottom to the top of the rope. When the pulse is at position *x* above the lower end of the rope, the wave speed of the pulse is given by $v = \sqrt{\frac{T}{\mu}}$, where $T = \mu x g$ is the tension required to support the weight of the rope below position *x*.

Therefore, $v = \sqrt{gx}$.

But $v = \frac{dx}{dt}$, so that $dt = \frac{dx}{\sqrt{gx}}$

and $t = \int_0^L \frac{dx}{\sqrt{gx}} = \frac{1}{\sqrt{g}} \left.\frac{\sqrt{x}}{\frac{1}{2}}\right|_0^L \approx \boxed{2\sqrt{\frac{L}{g}}}$

P16.61 (a) $P(x) = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\mu\omega^2 A_0^2 e^{-2bx}\left(\frac{\omega}{k}\right) = \boxed{\frac{\mu\omega^3}{2k} A_0^2 e^{-2bx}}$

(b) $P(0) = \boxed{\frac{\mu\omega^3}{2k} A_0^2}$

(c) $\frac{P(x)}{P(0)} = \boxed{e^{-2bx}}$

P16.62 $v = \left(\frac{4\ 450 \times 10^3\ \text{m}}{5.88\ \text{h}}\right)\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) = 210\ \text{m/s}$

$$d_{\text{avg}} = \frac{v^2}{g} = \frac{(210\ \text{m/s})^2}{9.80\ \text{m/s}^2} = 4\ 500\ \text{m}$$

$\boxed{\text{The given speed corresponds to an ocean depth that is greater than the average ocean depth, about 4 280 m.}}$

P16.63 Young's modulus for the wire may be written as $Y = \frac{T/A}{\Delta L/L}$, where *T* is the tension maintained in the wire and ΔL is the elongation produced by this tension. Also, the mass density of the wire may be expressed as

$$\rho = \frac{\mu}{A}$$

The speed of transverse waves in the wire is then

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T/A}{\mu/A}} = \sqrt{\frac{Y(\Delta L/L)}{\rho}}$$

and the strain in the wire is $\frac{\Delta L}{L} = \frac{\rho v^2}{Y}$.

If the wire is made of aluminum and $v = 100$ m/s, the strain is

$$\frac{\Delta L}{L} = \frac{\left(2.70\times 10^3\ \text{kg/m}^3\right)\left(100\ \text{m/s}\right)^2}{7.00\times 10^{10}\ \text{N/m}^2} = \boxed{3.86\times 10^{-4}}$$

Challenge Problems

P16.64 Refer to Problem 60. At distance x from the bottom, the tension is $T = \left(\frac{mxg}{L}\right) + Mg$, so the wave speed is:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} = \sqrt{xg + \left(\frac{MgL}{m}\right)} = \frac{dx}{dt} \rightarrow dt = \frac{dx}{\sqrt{xg + \left(\frac{MgL}{m}\right)}}$$

(a) Then

$$t = \int_0^t dt = \int_0^L \left[xg + \left(\frac{MgL}{m}\right)\right]^{-1/2} dx$$

gives $$t = \frac{1}{g}\left.\frac{\left[xg + \left(MgL/m\right)\right]^{1/2}}{\frac{1}{2}}\right|_{x=0}^{x=L}$$

$$t = \frac{2}{g}\left[\left(Lg + \frac{MgL}{m}\right)^{1/2} - \left(\frac{MgL}{m}\right)^{1/2}\right]$$

$$\boxed{t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M+m} - \sqrt{M}\right)}$$

(b) When $M = 0$,

$$t = 2\sqrt{\frac{L}{g}}\left(\frac{\sqrt{m} - 0}{\sqrt{m}}\right) = \boxed{2\sqrt{\frac{L}{g}}}$$

(c) As $m \to 0$ we expand

$$\sqrt{M+m} = \sqrt{M}\left(1+\frac{m}{M}\right)^{1/2} = \sqrt{M}\left(1+\frac{1}{2}\frac{m}{M}-\frac{1}{8}\frac{m^2}{M^2}+\cdots\right)$$

to obtain $$t = 2\sqrt{\frac{L}{mg}}\left(\sqrt{M}+\frac{1}{2}\left(\frac{m}{\sqrt{M}}\right)-\frac{1}{8}\left(m^2/M^{3/2}\right)+\cdots-\sqrt{M}\right)$$

$$t \approx 2\sqrt{\frac{L}{g}}\left(\frac{1}{2}\sqrt{\frac{m}{M}}\right) = \boxed{\sqrt{\frac{mL}{Mg}}}$$

where we neglect terms $\frac{1}{8}\left(\frac{m^2}{M^{3/2}}\right)$ and higher because terms with m^2 and higher powers are very small.

P16.65 (a) Refer to Problem 60. From the definition of velocity, find the relationship between the position x of the pulse and the time interval Δt required to reach that position from the bottom of the rope:

$$v = \frac{dx}{dt} \quad\rightarrow\quad dt = \frac{dx}{v} = \frac{dx}{\sqrt{gx}} \quad\rightarrow\quad \Delta t = \int\frac{dx}{\sqrt{gx}} \quad\rightarrow\quad \Delta t = 2\sqrt{\frac{x}{g}}$$

Evaluate this time interval for $x = \frac{L}{2}$:

$$\Delta t = 2\sqrt{\frac{L/2}{g}} = 2\sqrt{\frac{L}{2g}} = \frac{1}{\sqrt{2}}\left(2\sqrt{\frac{L}{g}}\right) = \boxed{0.707\left(2\sqrt{\frac{L}{g}}\right)}$$

(b) Solve the expression from part (a) for x and substitute the given time interval:

$$x = \frac{g(\Delta t)^2}{4} = \frac{g\left(\sqrt{L/2}\right)^2}{4} = \frac{g}{4}\frac{L}{g} = \boxed{\frac{L}{4}}$$

P16.66 (a) $\mu(x)$ is a linear function, so it is of the form $\mu(x) = mx + b.$

To have $\mu(0) = \mu_0$ we require $b = \mu_0.$ Then $\mu(L) = \mu_L = mL + \mu_0$

so $m = \frac{\mu_L - \mu_0}{L}.$

Then $\boxed{\mu(x) = \frac{(\mu_L - \mu_0)x}{L} + \mu_0}.$

(b) Imagine the crest of a short transverse pulse traveling from one end of the string to the other. Consider the pulse to be at position

x. From $v = \dfrac{dx}{dt}$, the time interval required to move from x to $x + dx$ is $\dfrac{dx}{v}$. The time interval required to move from 0 to L is

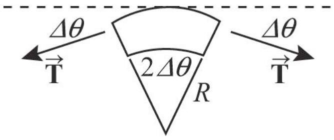
$$\Delta t = \int_0^L \frac{dx}{v} = \int_0^L \frac{dx}{\sqrt{T/\mu}} = \frac{1}{\sqrt{T}} \int_0^L \sqrt{\mu(x)}\,dx$$

$$\Delta t = \frac{1}{\sqrt{T}} \int_0^L \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0 \right)^{1/2} \left(\frac{\mu_L - \mu_0}{L} \right) dx \left(\frac{L}{\mu_L - \mu_0} \right)$$

$$\Delta t = \frac{1}{\sqrt{T}} \left(\frac{L}{\mu_L - \mu_0} \right) \left(\frac{(\mu_L - \mu_0)x}{L} + \mu_0 \right)^{3/2} \frac{1}{\left(\frac{3}{2}\right)} \Bigg|_0^L$$

$$\boxed{\Delta t = \frac{2L}{3\sqrt{T}(\mu_L - \mu_0)} \left(\mu_L^{3/2} - \mu_0^{3/2} \right)}$$

P16.67 (a) Consider a short section of chain at the top of the loop. A free-body diagram is shown. Its length is $s = R(2\Delta\theta)$ and its mass is $\mu R 2\Delta\theta$. In the frame of reference of the center of the loop, Newton's second law is



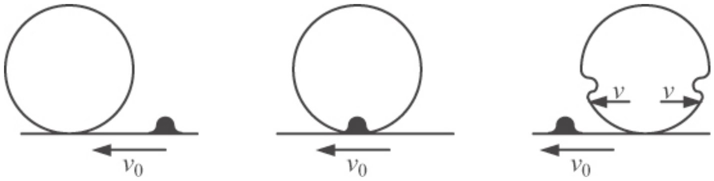
ANS. FIG. P16.67(a)

$$\sum F_y = ma_y: \quad 2T\sin\Delta\theta \text{ down} = \frac{mv_0^2}{R} \text{ down} = \frac{\mu R 2\Delta\theta v_0^2}{R}$$

For a very short section, $\sin\Delta\theta = \Delta\theta$ and $\boxed{T = \mu v_0^2}$

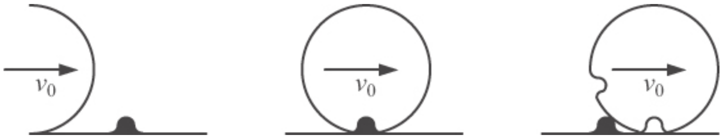
(b) The wave speed is $v = \sqrt{\dfrac{T}{\mu}} = \boxed{v_0}$

(c) In the frame of reference of the center of the loop, each pulse moves with equal speed clockwise and counterclockwise (ANS. FIG. P16.67(c1)).



ANS. FIG. P16.67(c1)

In the frame of reference of the ground, once pulse moves backward, clockwise, at speed $v_0 + v = 2v_0$ and the other forward, counterclockwise, at $v_0 - v = 0$ (ANS. FIG. P16.67(c2))



ANS. FIG. P16.67(c2)

$\boxed{\text{While the loop makes one revolution, the one pulse traveling clockwise makes two revolutions and the other pulse traveling counterclockwise does not move around the loop. The counterclockwise pulse it is generated at the 6 o'clock position, and it will stay at the 6 o'clock position.}}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P16.2 (a) See ANS. FIG. P16.2(a); (b) See ANS. FIG. P16.2(b); (c) The graph in ANS. FIG. P16.2(b) has the same amplitude and wavelength as the graph in ANS. FIG. P16.2(a). It differs just by being shifted toward larger x by 2.40 m; (d) The wave has traveled $d = vt = 2.40$ m to the right.

P16.4 (a) longitudinal P wave; (b) 666 s

P16.6 (a) See ANS. FIG. P16.6(a); (b) See ANS. FIG. P16.6(b); (c) See ANS. FIG. P16.6(c); (d) See ANS. FIG. P16.6(d); (e) See ANS. FIG. P16.6(e)

P16.8 0.800 m/s

P16.10 2.40 m/s

P16.12 ±6.67 cm

P16.14 (a) See ANS FIG P16.14; (b) 0.125 s; (c) This agrees with the period found in the example in the text.

P16.16 (a) 0.100 sin (1.002–20.0t); (b) 3.18 Hz

P16.18 (a) See ANS FIG P13.12(a); (b) 18.0 rad/m; (c) 0.083 3 s; (d) 75.4 rad/s; (e) 4.20 m/s; (f) $y = (0.200\ \text{m})\sin\left(18.0x/\text{m} + 75.4t/\text{s} + \phi\right)$; (g) $y(x, t) = 0.200 \sin(18.0x + 75.4t - 0.151)$, where x and y are in meters and t is in seconds.

P16.20 (a) 0.021 5 m; (b) 1.95 rad; (c) 5.41 m/s;
(d) $y(x,t) = (0.021\ 5)\sin(8.38x + 80.0\pi t + 1.95)$

P16.22 520 m/s

P16.24 (a) units are seconds and newtons; (b) The first T is period of time; the second is force of tension.

P16.26 (a) $y = (2.00 \times 10^{-4})\sin(16.0x - 3\ 140t)$, where y and x are in meters and t is in seconds; (b) 158 N

P16.28 The calculated gravitational acceleration of the Moon is almost twice that of the accepted value.

P16.30 (a) $v = (30.4)\sqrt{m}$ where v is in meters per second and m is in kilograms; (b) $m = 3.89$ kg

P16.32 (a) As for a string wave, the rate of energy transfer is proportional to the square of the amplitude to the speed. The rate of energy transfer stays constant because each wavefront carries constant energy, and the frequency stays constant. As the speed drops, the amplitude must increase; (b) The amplitude increases by 5.00 times

P16.34 55.1 Hz

P16.36 1.07 kW

P16.38 $\sqrt{2}P_0$

P16.40 See P16.40 for the full explanation.

P16.42 (a) $A = 40.0$; (b) $A = 7.00$, $B = 0$, and $C = 3.00$; (c) In order for two vectors to be equal, they must have the same magnitude and the same direction in three-directional space. All of their components must be equal, so all coefficients of the unit vectors must be equal; (d) $A = 0$, $B = 7.00$, $C = 3.00$, $D = 4.00$, $E = 2.00$; (e) Identify corresponding parts. In order for two functions to be identically equal, corresponding parts must be identical. The argument of the sine function must have no units or be equal to units of radians.

P16.44 (a) See P16.44(a) for full explanation; (b) $f(x+vt) = \frac{1}{2}(x+vt)^2$ and $g(x-vt) = \frac{1}{2}(x-vt)^2$; (c) $f(x+vt) = \frac{1}{2}\sin(x+vt)$ and $g(x-vt) = \frac{1}{2}\sin(x-vt)$

P16.46 ~1 min

P16.48 6.01 km

P16.50 (a) $2\,Mg$; (b) $L_0 + \frac{2\,Mg}{k}$; (c) $\sqrt{\frac{2\,Mg}{k}\left(L_0 + \frac{2\,Mg}{k}\right)}$

P16.52 (a) 375 m/s^2; (b) 0.045 0 N; (c) 46.9 N. The maximum transverse force is very small compared to the tension, more than a thousand times smaller.

P16.54 (a) The energy a wave crest carries is constant in the absence of absorption. Then the rate at which energy passes a stationary point, which is the power of the wave, is constant; (b) The power is proportional to the square of the amplitude and to the wave speed. The speed decreases as the wave moves into shallower water near shore, so the amplitude must increase; (c) 8.31 m; (d) As the water depth goes to zero, our model would predict zero speed and infinite amplitude. In fact, the amplitude must be finite as the wave comes ashore. As the speed decreases, the wavelength also decreases. When it becomes comparable to the water depth, or smaller, our formula $\sqrt{gd}$ for wave speed no longer applies.

P16.56 8.43×10^{-3} s

P16.58 (a) $P = 0.050\ 0\ v_{y,\max}^2$ where *P* is in watts and $v_{y,\max}$ is in meters per second; (b) The power is proportional to the square of the maximum particle speed; (c) $E = \left(7.50\times10^{-4}\right)v_{y,\max}^2$ where *E* is in joules and $v_{y,\max}$ is in meters per second; (d) $\frac{1}{2}mv_{y,\max}^2$; (e) $E = 0.300v_{y,\max}^2$ where *E* is in joules and $v_{y,\max}$ is in meters per second

P16.60 $2\sqrt{\frac{L}{g}}$

P16.62 The given speed corresponds to an ocean depth that is greater than the average ocean depth, about 4 280 m.

P16.64 (a) $t = 2\sqrt{\frac{L}{g}}\left(\sqrt{M+m}-\sqrt{M}\right)$; (b) $2\sqrt{\frac{L}{g}}$; (c) $\sqrt{\frac{mL}{Mg}}$

P16.66 (a) $\mu(x) = \frac{\left(\mu_L-\mu_0\right)x}{L}+\mu_0$; (b) $\Delta t = \frac{2L}{3\sqrt{T}\left(\mu_L-\mu_0\right)}\left(\mu_L^{3/2}-\mu_0^{3/2}\right)$

17

Sound Waves

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ17.1 Answer (b). The typically higher density would by itself make the speed of sound lower in a solid compared to a gas.

OQ17.2 Answer (e). The speed of sound in air, at atmospheric pressure, is determined by the temperature of the air and does not depend on the frequency of the sound. Sound from siren A will have a wavelength that is half the wavelength of the sound from B, but the speed of the sound (the product of frequency times wavelength) will be the same for the two sirens.

OQ17.3 Answer (c). The ambulance driver, sitting at a fixed distance from the siren, hears the actual frequency emitted by the siren. However, the distance between you and the siren is decreasing, so you will detect a frequency higher than the actual 500 Hz.

OQ17.4 Answer (d). When a sound wave travels from air into water, several properties will change. The wave speed will increase as the wave crosses the boundary into the water causing the spacing between crests (the wavelength) to increase, because crests move away from the boundary faster than they move up to the boundary. The sound intensity in the water will be less than it was in air because some sound is reflected by the water surface. However, the frequency (number of crests passing each second) will be unchanged, since a crest moves away from the boundary every time a crest arrives at the boundary.

OQ17.5 Answer (d). The drop in intensity is what we should expect according to the inverse-square law:

$$\frac{I_1}{I_2}=\frac{r_1^2}{r_2^2}: \quad \frac{2\ \mu\text{W}/\text{m}^2}{0.2\ \mu\text{W}/\text{m}^2}=10=\frac{(950\text{ m})^2}{(300\text{ m})^2}$$

OQ17.6 Answer (d). We have f_s = 1 000 Hz, v = 343 m/s, v_o = –30 m/s, v_s = 50 m/s. We find

$$f'=\frac{f(v+v_o)}{(v-v_s)}=\frac{(1\ 000\text{ Hz})[(343\text{ m/s})+(-30\text{ m/s})]}{343\text{ m/s}-50\text{ m/s}}$$
$$=1\ 068\text{ Hz}$$

OQ17.7 Answer (b). A sound wave is a longitudinal vibration that is propagated through a material medium.

OQ17.8 (i) Answer (b). The frequency increases by a factor of 2 because the wave speed, which is dependent only on the medium through which the wave travels, remains constant.

(ii) Answer (c).

OQ17.9 Answer (a) We suppose that a point source has no structure, and radiates sound equally in all directions (isotropically). The sound wavefronts are expanding spheres, so the area over which the sound energy spreads increases according to $A=4\pi r^2$. Thus, if the distance is tripled, the area increases by a factor of nine, and the new intensity will be one-ninth of the old intensity. This answer according to the inverse-square law applies if the medium is uniform and unbounded. For contrast, suppose that the sound is confined to move in a horizontal layer. (Thermal stratification in an ocean can have this effect on sonar "pings.") Then the area over which the sound energy is dispersed will only increase according to the circumference of an expanding circle: $A=2\pi rh$, and so three times the distance will result in one-third the intensity. In the case of an entirely enclosed speaking tube (such as a ship's telephone), the area perpendicular to the energy flow stays the same, and increasing the distance will not change the intensity appreciably.

OQ17.10 (i) Answer (c). Both observer and source have equal speeds in opposite directions relative to the medium, so in $f'=(v+v_o)/(v-v_s)$ we would have something like (343 – 25)f/(343 – 25) = f.

(ii) Answer (a). The speed of the medium adds to the speed of sound as far as the observer is concerned, to cause an increase in $\lambda = v/f$. The wind "stretches" the wavelength out.

(iii) Answer (a).

OQ17.11 In order of decreasing size we have (b) > (d) > (a) > (c) > (e). In $f' = f[(v+v_o)]/[(v-v_s)]$ we can consider the size of the fraction $(v+v_o)/(v-v_s)$ in each case, where the positive direction for the observer is toward the source, the positive direction for the source is toward the observer: (a) 343 / 343 = 1, (b) 343 / (343 – 25) = 1.08, (c) 343 / (343 + 25) = 0.932, (d) (343 + 25) / 343 = 1.07, (e) (343 – 25) / 343 = 0.927.

OQ17.12 Answer (c). The intensity is about 10^{-13} W / m^2.

OQ17.13 Answer (c). Doubling the power output of the source will double the intensity of the sound at the observer's location. The original decibel level of the sound is $\beta = 10 \cdot \log(I/I_0)$. After doubling the power output and intensity, the new decibel level will be

$$\begin{aligned}\beta' &= 10 \cdot \log(2I/I_0) = 10 \cdot \log\left[2(I/I_0)\right] = 10 \cdot \left[\log(2) + \log(I/I_0)\right] \\ &= 10 \cdot \log(2) + \beta\end{aligned}$$

so the increase in decibel level is $\beta' - \beta = 10 \cdot \log(2) = 3.0\text{ dB}$, making (c) the correct answer.

OQ17.14 Answer (c). The threshold of human hearing is defined as 0 dB; the average person cannot hear sound with a lower intensity level. Normal conversation has an intensity level of about 60 dB.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ17.1 For the sound from a source not to shift in frequency, the radial velocity of the source relative to the observer must be zero; that is, the source must not be moving toward or away from the observer. The source can be moving in a plane perpendicular to the line between it and the observer. Other possibilities: The source and observer might both have zero velocity. They might have equal velocities relative to the medium. The source might be moving around the observer on a sphere of constant radius. Even if the source speeds up on the sphere, slows down, or stops, the frequency heard will be equal to the frequency emitted by the source.

CQ17.2 The speed of sound in air is proportional to the square-root of the absolute temperature, $\sqrt{T}$. The speed of sound is greater in warmer air, so the pulse from the camera would return sooner than it would on a cooler day from an object at the same distance. The camera would interpret an object as being closer than it actually is on a hot day.

CQ17.3 The speed of sound to two significant figures is 340 m/s. Let's assume that you can measure time to $\frac{1}{10}$ second by using a stopwatch. To get a speed to two significant figures, you need to measure a time of at least 1.0 seconds. Since $d = vt$, the minimum distance is 340 meters.

CQ17.4 When listening, you are approximately the same distance from all of the members of the group. If different frequencies traveled at different speeds, then you might hear the higher pitched frequencies before you heard the lower ones produced at the same time.

CQ17.5 The speed of light is so high that the arrival of the flash is practically simultaneous with the lightning discharge. Thus, the delay between the flash and the arrival of the sound of thunder is the time sound takes to travel the distance separating the lightning from you. By counting the seconds between the flash and thunder and knowing the approximate speed of sound in air, you have a rough measure of the distance to the lightning bolt.

CQ17.6 Both. There are actually two Doppler shifts. The first shift arises from the source (you) moving toward the observer (the cliff). The second arises from the observer (you) moving toward the source (the cliff). If, instead of a cliff, there is a spacecraft moving toward you, then there are shifts due to moving source (you) and moving observer (the spacecraft) before reflection, and moving source (the spacecraft) and moving observer (you) after reflection.

CQ17.7 A beam of radio waves of known frequency is sent toward a speeding car, which reflects the beam back to a detector in the police car. The amount the returning frequency has been shifted depends on the velocity of the oncoming car.

CQ17.8 Our brave Siberian saw the first wave he encountered, light traveling at 3.00×10^8 m/s. At the same moment, infrared as well as visible light began warming his skin, but some time was required to raise the temperature of the outer skin layers before he noticed it. The meteor produced compressional waves in the air and in the ground. The wave in the ground, which can be called either sound or a seismic wave, traveled much faster than the wave in air, since the ground is much stiffer against compression. Our witness received it next and noticed it as a little earthquake. He was no doubt unable to distinguish the P and S waves from each other. The first air-compression wave he received was a shock wave with an amplitude on the order of meters. It transported him off his doorstep. Then he could hear some additional direct sound, reflected sound, and perhaps the sound of the falling trees.

CQ17.9 If an object is a half meter from the sonic ranger, then the sensor would have to measure how long it would take for a sound pulse to travel one meter. Because sound of any frequency moves at about 343 m/s, the sonic ranger would have to be able to measure a time difference of under 0.003 seconds. This small time measurement is possible with modern electronics, but it would be more expensive to outfit sonic rangers with the more sensitive equipment than it is to print "do not use to measure distances less than $\frac{1}{2}$ meter" in the users' manual.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 17.1 Pressure Variations in Sound Waves

P17.1 (a) $A = \boxed{2.00\ \mu\text{m}}$

(b) $\lambda = \dfrac{2\pi}{15.7} = 0.400\ \text{m} = \boxed{40.0\ \text{cm}}$

(c) $v = \dfrac{\omega}{k} = \dfrac{858}{15.7} = \boxed{54.6\ \text{m/s}}$

(d) $s = 2.00\cos\left[(15.7)(0.050\ 0) - (858)(3.00\times10^{-3})\right] = \boxed{-0.433\ \mu\text{m}}$

(e) $v_{\text{max}} = A\omega = (2.00\ \mu\text{m})(858\ \text{s}^{-1}) = \boxed{1.72\ \text{mm/s}}$

P17.2 (a) $\Delta P = (1.27\ \text{Pa})\sin\left(\dfrac{\pi x}{\text{m}} - \dfrac{340\pi t}{\text{s}}\right)$ (SI units)

The pressure amplitude is: $\Delta P_{\text{max}} = \boxed{1.27\ \text{Pa}}$

(b) $\omega = 2\pi f = 340\pi/\text{s}$, so $f = \boxed{170\ \text{Hz}}$

(c) $k = \dfrac{2\pi}{\lambda} = \pi/\text{m}$, giving $\lambda = \boxed{2.00\ \text{m}}$

(d) $v = \lambda f = (2.00\ \text{m})(170\ \text{Hz}) = \boxed{340\ \text{m/s}}$

P17.3 We write the pressure variation as $\Delta P = \Delta P_{\text{max}} \sin(kx - \omega t)$. Note that

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{(0.100\text{ m})} = 62.8\text{ m}^{-1}$$

and $$\omega = \frac{2\pi v}{\lambda} = \frac{2\pi(343\text{ m/s})}{(0.100\text{ m})} = 2.16 \times 10^4\text{ s}^{-1}.$$

Therefore,

$$\boxed{\Delta P = 0.200\sin\left[62.8x - 2.16 \times 10^4 t\right]}$$

where ΔP is in Pa, *x* is in meters, and *t* is in seconds.

Section 17.2 Speed of Sound Waves

P17.4 We use $\Delta P_{\text{max}} = \rho v \omega s_{\text{max}} = \rho v\left(\frac{2\pi v}{\lambda}\right)s_{\text{max}}$:

$$\lambda_{\text{min}} = \frac{2\pi\rho v^2 s_{\text{max}}}{\Delta P_{\text{max}}} = \frac{2\pi(1.20\text{ kg/m}^3)(343\text{ m/s})^2(5.50 \times 10^{-6}\text{ m})}{0.840\text{ Pa}} = \boxed{5.81\text{ m}}$$

***P17.5** $$\Delta P_{\text{max}} = \rho\omega v s_{\text{max}} = (1.20\text{ kg/m}^3)\left[2\pi(2\,000\text{ s}^{-1})\right](343\text{ m/s})(2.00 \times 10^{-8}\text{ m})$$

$$\Delta P_{\text{max}} = \boxed{0.103\text{ Pa}}$$

P17.6 The speed of longitudinal waves in a fluid is $v = \sqrt{B/\rho}$. Considering the Earth's crust to consist of a very viscous fluid, our estimate of the average bulk modulus of the material in Earth's crust is

$$B = \rho v^2 = (2\,500\text{ kg/m}^3)(7 \times 10^3\text{ m/s})^2 = \boxed{1 \times 10^{11}\text{ Pa}}$$

P17.7 The sound pulse must travel 150 m before reflection and 150 m after reflection. We have *d* = *vt*:

$$t = \frac{d}{v} = \frac{300\text{ m}}{1\,533\text{ m/s}} = \boxed{0.196\text{ s}}$$

P17.8 (a) $$\boxed{\begin{array}{l}\text{The speed gradually changes from}\\ \qquad v=(331\text{ m/s})\left(1+\dfrac{27.0^\circ\text{C}}{273^\circ\text{C}}\right)^{1/2}=347\text{ m/s}\\ \text{to}\qquad v=(331\text{ m/s})\left(1+\dfrac{0^\circ\text{C}}{273^\circ\text{C}}\right)^{1/2}=331\text{ m/s}\\ \text{a 4.6\% decrease. The cooler air at the same pressure is}\\ \text{more dense.}\end{array}}$$

(b) $\boxed{\text{The frequency is unchanged because every wave crest in the hot air becomes one crest without delay in the cold air.}}$

(c) $$\boxed{\begin{array}{l}\text{The wavelength decreases by 4.6\%, from}\\ \qquad v/f=(347\text{ m/s})/(4000/\text{s})=86.7\text{ mm}\\ \text{to}\qquad v/f=(331\text{ m/s})/(4000/\text{s})=82.8\text{ mm}\\ \text{The crests are more crowded together when they move}\\ \text{more slowly.}\end{array}}$$

P17.9 (a) If $f=2.40$ MHz, $$\lambda=\frac{v}{f}=\frac{1\,500\text{ m/s}}{2.40\times10^6\text{ s}^{-1}}=\boxed{0.625\text{ mm}}$$

(b) If $f=1.00$ MHz, $$\lambda=\frac{v}{f}=\frac{1\,500\text{ m/s}}{10^6\text{ s}^{-1}}=\boxed{1.50\text{ mm}}$$

If $f=20.0$ MHz, $$\lambda=\frac{1\,500\text{ m/s}}{2\times10^7\text{ s}^{-1}}=\boxed{75.0\ \mu\text{m}}$$

P17.10 $\Delta P_{\text{max}}=\rho v\omega s_{\text{max}}$

$$s_{\text{max}}=\frac{\Delta P_{\text{max}}}{\rho v\omega}=\frac{4.00\times10^{-3}\text{ N/m}^2}{(1.20\text{ kg/m}^3)(343\text{ m/s})(2\pi)(10.0\times10^3\text{ s}^{-1})}$$
$$=\boxed{1.55\times10^{-10}\text{ m}}$$

P17.11 (a) Since $v_{\text{light}}>>v_{\text{sound}}$, and assuming that the speed of sound is constant through the air between the lightning strike and the observer, we have

$$d\approx(343\text{ m/s})(16.2\text{ s})=\boxed{5.56\text{ km}}$$

(b) $\boxed{\text{No, we do not need to know the value of the speed of light. The speed of light is much greater than the speed of sound, so the time interval required for the light to reach you is negligible compared to the time interval for the sound.}}$

P17.12 It is easiest to solve part (b) first:

(b) The distance the sound travels to the plane is

$$d_s = \sqrt{h^2 + \left(\frac{h}{2}\right)^2} = \frac{h\sqrt{5}}{2}$$

The sound travels this distance in 2.00 s, so

$$d_s = \frac{h\sqrt{5}}{2} = (343\ \text{m/s})(2.00\ \text{s}) = 686\ \text{m}$$

giving the altitude of the plane as $h = \dfrac{2(686\ \text{m})}{\sqrt{5}} = \boxed{614\ \text{m}}$

(a) The distance the plane has traveled in 2.00 s is

$$v(2.00\ \text{s}) = \frac{h}{2} = 307\ \text{m}$$

Thus, the speed of the plane is:

$$v = \frac{307\ \text{m}}{2.00\ \text{s}} = \boxed{153\ \text{m/s}}$$

P17.13 Sound takes this time to reach the man: $\Delta t_\text{s} = \dfrac{d-h}{v}$. The minimum time interval between when a warning is shouted and when the man responds to the warning is $\Delta t_\text{min} = \Delta t_s + \Delta t$.

Since the whole time interval to fall is given by

$$\Delta y = (d-h) = \frac{1}{2} g \Delta t_f^2 \rightarrow \Delta t_f = \sqrt{\frac{2(d-h)}{g}}$$

The warning needs to come at least

$$\Delta T = \Delta t_f - \Delta t - \Delta t_s = \sqrt{\frac{2(d-h)}{g}} - \Delta t - \frac{d-h}{v}$$

into the fall, when the pot is at the position

$$y_f = y_i + v_{yi}\Delta T - \frac{1}{2} g \Delta T^2$$

$$y_f = 20.0\ \text{m} - \frac{1}{2}(9.80\ \text{m/s}^2) \times \left(\sqrt{\frac{2(20.0\ \text{m} - 1.75\ \text{m})}{g}} - 0.300\ \text{s} - \frac{20.0\ \text{m} - 1.75\ \text{m}}{343\ \text{m/s}}\right)^2$$

$y_f = \boxed{7.82\ \text{m}}$ above the ground.

P17.14 Sound takes this time to reach the man: $\Delta t_s = \dfrac{d-h}{v}$. The minimum time interval between when a warning is shouted and when the man responds to the warning is $\Delta t_{\text{min}} = \Delta t_{\text{s}} + \Delta t$.

Since the whole time interval to fall is given by

$$\Delta y = (d-h) = \frac{1}{2} g\Delta t_f^2 \rightarrow \Delta t_f = \sqrt{\frac{2(d-h)}{g}}$$

The warning needs to come at least

$$\Delta T = \Delta t_f - \Delta t - \Delta t_s = \sqrt{\frac{2(d-h)}{g}} - \Delta t - \frac{d-h}{v}$$

into the fall, when the pot is at the position

$$y_f = y_i + v_{yi}\Delta T - \frac{1}{2} g\Delta T^2$$

$$y_f = \boxed{d - \frac{1}{2} g\left(\sqrt{\frac{2(d-h)}{g}} - \Delta t - \frac{d-h}{v}\right)^2} \text{ above the ground.}$$

P17.15 (a) At 9 000 m, $\Delta T = (9\ 000 \text{ m})\left(\dfrac{-1.00°\text{C}}{150 \text{ m}}\right) = -60.0°\text{C}$, so $T = -30.0°\text{C}$.

Using the chain rule,

$$\frac{dv}{dt} = \frac{dv}{dT_{\text{C}}}\frac{dT_{\text{C}}}{dx}\frac{dx}{dt} = v\frac{dv}{dT_{\text{C}}}\frac{dT_{\text{C}}}{dx} = v(0.607)\left(\frac{1}{150}\right) = \frac{v}{247}$$

so $dt = (247 \text{ s})\dfrac{dv}{v}$. Integrating,

$$\int_0^t dt = (247 \text{ s})\int_{v_i}^{v_f} \frac{dv}{v}$$

$$t = (247 \text{ s})\ln\left(\frac{v_f}{v_i}\right) = (247 \text{ s})\ln\left[\frac{331.5 + 0.607(30.0)}{331.5 + 0.607(-30.0)}\right]$$

which gives $t = \boxed{27.2 \text{ s}}$ for sound to reach the ground.

(b) $t = \dfrac{h}{v} = \dfrac{9\,000 \text{ m}}{331.5 \text{ m/s} + 0.607(30.0°\text{C})} = \boxed{25.7 \text{ s}}$

$\boxed{\text{The time interval in (a) is longer.}}$

P17.16 Since $\cos^2\theta + \sin^2\theta = 1$, $\sin\theta = \pm\sqrt{1-\cos^2\theta}$ (each sign applying half the time),

$$\Delta P = \Delta P_{\text{max}} \sin(kx - \omega t) = \pm\rho v\omega s_{\text{max}}\sqrt{1-\cos^2(kx-\omega t)}$$

Therefore,

$$\Delta P = \pm\,\rho v\omega\,\sqrt{s_{\text{max}}^2 - s_{\text{max}}^2\cos^2(kx-\omega t)} = \pm\,\rho v\omega\,\sqrt{s_{\text{max}}^2 - s^2}$$

P17.17 (a) The two pulses travel the same distance, and so the one that travels at the highest velocity will arrive first. Because the speed of sound in air is 343 m / s and the speed of sound in the iron rod is 5 950 m / s,

$\boxed{\text{the pulse travelling through the iron rail will arrive first}}$.

(b) For each of the pulses $t = \dfrac{L}{v}$.

Therefore,

$$t_{\text{rod}} = \frac{L}{v_{\text{rod}}} = \frac{8.50\text{ m}}{5\,950\text{ m/s}} = 1.43\text{ milliseconds}$$

and $$t_{\text{air}} = \frac{L}{v_{\text{air}}} = \frac{8.50\text{ m}}{343\text{ m/s}} = 24.78\text{ milliseconds}$$

The difference between their two arrival times is

$$\Delta t = t_{\text{air}} - t_{\text{rod}} = 24.78\text{ ms} - 1.43\text{ ms} = \boxed{23.4\text{ ms}}$$

P17.18 Let d_1 represent the cowboy's distance from the nearer canyon wall and d_2 his distance from the farther cliff. The sound for the first echo travels distance $2d_1$. For the second, $2d_2$. For the third, $2d_1 + 2d_2$. For the fourth echo, $2d_1 + 2d_2 + 2d_1$. The time interval between the shot and the first echo is $\Delta t_1 = 2d_1 / v$, between the shot and the second echo is $\Delta t_2 = 2d_2 / v$, and so on.

Then

$$\Delta t_2 - \Delta t_1 = \frac{2d_2 - 2d_1}{343\text{ m/s}} = 1.92\text{ s} \text{ and}$$

$$\Delta t_3 - \Delta t_2 = 1.47\text{ s} \quad\rightarrow\quad \frac{(2d_1 + 2d_2) - 2d_2}{343\text{ m/s}} = \frac{2d_1}{343\text{ m/s}} = 1.47\text{ s}$$

Thus, $d_1 = \frac{1}{2}(343\text{ m/s})(1.47\text{ s}) = 252\text{ m}$, and $\Delta t_1 = 1.47\text{ s}$

From above,

$$\Delta t_2 - \Delta t_1 = 1.92\text{ s} \quad \rightarrow \quad \frac{2d_2}{343\text{ m/s}} = 1.92\text{ s} + 1.47\text{ s}$$

which gives $d_2 = 581\text{ m}$

(a) So, $d_1 + d_2 = \boxed{833\text{ m}}$.

(b) $$\frac{2d_1 + 2d_2 + 2d_1 - (2d_1 + 2d_2)}{343\text{ m/s}} = \frac{2d_2}{343\text{ m/s}} = \boxed{1.47\text{ s}}$$

Section 17.3 Intensity of Periodic Sound Waves

P17.19 We use Equation 17.14:

$$\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right) = (10\text{ dB})\log\left(\frac{4.00\times 10^{-6}\text{ W/m}^2}{1.00\times 10^{-12}\text{ W/m}^2}\right)$$
$$= \boxed{66.0\text{ dB}}$$

P17.20 The sound power incident on the eardrum is $P = IA$, where I is the intensity of the sound and $A = 5.00 \times 10^{-5}\text{ m}^2$ is the area of the eardrum.

(a) At the threshold of pain, $I = 1.00\text{ W/m}^2$.

Thus, $P = IA = (5.00\times 10^{-5}\text{ m}^2)(1.00\text{ W/m}^2) = \boxed{5.00\times 10^{-5}\text{ W}}$

(b) Energy transfer can be obtained from power by $P = \frac{E}{\Delta t} \rightarrow E = P\Delta t$. Thus,

$$E = P\Delta t = (5.00\times 10^{-5}\text{ J/s})(60.0\text{ s}) = \boxed{3.00\times 10^{-3}\text{ J}}$$

P17.21 We use $I = \frac{1}{2}\rho\omega^2 s_{\text{max}}^2 v$.

(a) At $f = 2\,500$ Hz, the frequency is increased by a factor of 2.50, so the intensity (at constant s_{max}) increases by $(2.50)^2 = 6.25$.

Therefore, $6.25(0.600) = \boxed{3.75\text{ W/m}^2}$

(b) The changes cancel each other: frequency $f \rightarrow f' = f/2$, and displacement amplitude $s_{\text{max}} \rightarrow s'_{\text{max}} = 2s_{\text{max}}$

originial intensity: $I = \frac{1}{2}\rho\omega^2 s_{\text{max}}^2 v = 0.600\ \text{W/m}^2$

new intensity: $I' = \frac{1}{2}\rho\omega'^2 s'^2_{\text{max}} v = \frac{1}{2}\rho\left(\frac{\omega}{2}\right)^2\left(2s_{\text{max}}\right)^2 v = \frac{1}{2}\rho\omega^2 s_{\text{max}}^2 v$

$$= \boxed{600\ \text{W/m}^2}$$

P17.22 The original intensity is $I_1 = \frac{1}{2}\rho\omega^2 s_{\text{max}}^2 v = 2\pi^2\rho v f^2 s_{\text{max}}^2$

(a) If the frequency is increased to f' while a constant displacement amplitude is maintained, the new intensity is

$$I_2 = 2\pi^2\rho v\left(f'\right)^2 s_{\text{max}}^2 \quad \text{so} \quad \frac{I_2}{I_1} = \frac{2\pi^2\rho v\left(f'\right)s_{\text{max}}^2}{2\pi^2\rho v f^2 s_{\text{max}}^2} = \left(\frac{f'}{f}\right)^2$$

or $\boxed{I_2 = \left(\frac{f'}{f}\right)^2 I_1}$

(b) If the frequency is reduced to $f' = \frac{f}{2}$ while the displacement amplitude is doubled, the new intensity is

$$I_2 = 2\pi^2\rho v\left(\frac{f}{2}\right)^2\left(2s_{\text{max}}\right)^2 = 2\pi^2\rho v f^2 s_{\text{max}}^2 = I_1$$

or the $\boxed{\text{intensity is unchanged}}$.

P17.23 In terms of their intensities, the difference in the decibel level of two sounds is

$$\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\frac{I_2}{I_0}\right) - (10\ \text{dB})\log\left(\frac{I_1}{I_0}\right)$$

$$= (10\ \text{dB})\log\left(\frac{I_2}{\cancel{I_0}}\cdot\frac{\cancel{I_0}}{I_1}\right) = (10\ \text{dB})\log\left(\frac{I_2}{I_1}\right)$$

Thus, $\frac{I_2}{I_1} = 10^{(\beta_2-\beta_1)/10}$ or $I_2 = I_1 \times 10^{(\beta_2-\beta_1)/10}$

If $\beta_2 - \beta_1 = 30\ \text{dB}$ and $I_1 = 3.0\times10^{-11}\ \text{W/m}^2$, then

$$I_2 = \left(3.0\times10^{-11}\ \text{W/m}^2\right)\times10^3 = \boxed{3.0\times10^{-8}\ \text{W/m}^2}$$

P17.24 The intensity is given by $I = \frac{P_{\text{avg}}}{4\pi r^2}$.

The power is not given, but the intensity at a known distance is $I = \frac{P_{\text{avg}}}{4\pi r^2}$, which gives

$$P_{\text{avg}} = I(r)4\pi r^2 = 4\pi\left(0.25\ \text{W/m}^2\right)\left(16\ \text{m}\right)^2 = 804.2\ \text{W}$$

which can then be substituted back into the same equation:

$$I = \frac{P_{\text{avg}}}{4\pi r^2} = \frac{804.2\ \text{W}}{4\pi\left(28\ \text{m}\right)^2} = \boxed{0.082\ \text{W/m}^2}$$

P17.25 (a) From the sound level equation,

$$120\ \text{dB} = \left(10\ \text{dB}\right)\log\left[\frac{I}{10^{-12}\ \text{W/m}^2}\right]$$

$$I = 1.00\ \text{W/m}^2 = \frac{P}{4\pi r^2}$$

$$r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{6.00\ \text{W}}{4\pi\left(1.00\ \text{W/m}^2\right)}} = \boxed{0.691\ \text{m}}$$

We have assumed the speaker is an isotropic point source.

(b) Again from the sound level equation,

$$0\ \text{dB} = \left(10\ \text{dB}\right)\log\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$$

$$I = 1.00\times 10^{-12}\ \text{W/m}^2$$

$$r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{6.00\ \text{W}}{4\pi\left(1.00\times 10^{-12}\ \text{W/m}^2\right)}} = \boxed{691\,\text{km}}$$

We have assumed a uniform medium that absorbs no energy.

P17.26 The decibel level due to the first siren is

$$\beta_1 = \left(10\ \text{dB}\right)\log\left(\frac{100.0\ \text{W/m}^2}{1.0\times 10^{-12}\ \text{W/m}^2}\right) = 140\ \text{dB}$$

Thus, the decibel level of the sound from the ambulance is

$$\beta_2 = \beta_1 + 10\ \text{dB} = 140\ \text{dB} + 10\ \text{dB} = \boxed{150\ \text{dB}}$$

***P17.27** (a) The intensity of sound at 10 km from the horn (where $\beta = 50$ dB) is

$$I = I_0 10^{\beta/10} = \left(1.0\times10^{-12}\ \text{W/m}^2\right)10^{5.0} = 1.0\times10^{-7}\ \text{W/m}^2$$

Thus, from $I = \dfrac{P}{4\pi r^2}$, the power emitted by the source is

$$P = 4\pi r^2 I = 4\pi\left(10.0\times10^3\ \text{m}\right)^2\left(1.0\times10^{-7}\ \text{W/m}^2\right) = 126\ \text{W}$$

(b) At $r = 50$ m, the intensity of the sound will be

$$I = \frac{P}{4\pi r^2} = \frac{1.3\times10^2\ \text{W}}{4\pi(50\ \text{m})^2} = 4.0\times10^{-3}\ \text{W/m}^2$$

and the sound level is

$$\beta = (10\ \text{dB})\log\left(\frac{I}{I_0}\right) = (10\ \text{dB})\log\left(\frac{4.0\times10^{-3}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right)$$

$$= \boxed{96\ \text{dB}}$$

P17.28 (a) The sound intensity inside the church is given by

$$\beta = (10\ \text{dB})\log\left(\frac{I}{I_0}\right)$$

$$101\ \text{dB} = (10\ \text{dB})\log\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$$

$$I = 10^{10.1}\left(10^{-12}\ \text{W/m}^2\right) = 10^{-1.90}\ \text{W/m}^2 = 0.012\ 6\ \text{W/m}^2$$

We suppose that sound comes perpendicularly out through the windows and doors. Then, the radiated power is

$$P = IA = \left(0.012\,6\ \text{W/m}^2\right)\left(22.0\ \text{m}^2\right) = 0.277\ \text{W}$$

Are you surprised by how small this is? The energy radiated in 20.0 minutes is

$$E = Pt = (0.277\ \text{J/s})(20.0\ \text{min})\left(\frac{60.0\ \text{s}}{1.00\ \text{min}}\right) = \boxed{332\ \text{J}}$$

(b) If the ground reflects all sound energy headed downward, the sound power, $P = 0.277$ W, covers the area of a hemisphere. One kilometer away, this area is

$$A = 2\pi r^2 = 2\pi(1\,000\ \text{m})^2 = 2\pi\times10^6\ \text{m}^2$$

The intensity at this distance is

$$I = \frac{P}{A} = \frac{0.277\ \text{W}}{2\pi \times 10^6\ \text{m}^2} = 4.41 \times 10^{-8}\ \text{W/m}^2$$

and the sound intensity level is

$$\beta = (10\ \text{dB})\log\left(\frac{4.41 \times 10^{-8}\ \text{W/m}^2}{1.00 \times 10^{-12}\ \text{W/m}^2}\right) = \boxed{46.4\ \text{dB}}$$

P17.29 (a) For the initial low note the wavelength is

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{146.8\ /\ \text{s}} = \boxed{2.34\ \text{m}}$$

(b) For the final high note $\lambda = \frac{343\ \text{m/s}}{880\ /\ \text{s}} = \boxed{0.390\ \text{m}}$

We observe that the ratio of the frequencies of these two notes is $\frac{880\ \text{Hz}}{146.8\ \text{Hz}} = 5.99,$ nearly equal to a small integer. This fact is associated with the consonance of the notes D and A.

(c, d) The intensity level for both notes is the same 75.0 dB:

$$\beta = (10\ \text{dB})\log\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right) = 75\ \text{dB}$$

gives $I = 3.16 \times 10^{-5}\ \text{W/m}^2$

Therefore, the pressure amplitude for both low and high notes is the same, and $I = \frac{\Delta P_{\text{max}}^2}{2\rho v}$ gives

$$\begin{aligned}\Delta P_{\text{max}} &= \sqrt{2\rho v I}\\ &= \sqrt{2(1.20\ \text{kg/m}^3)(343\ \text{m/s})(3.16 \times 10^{-5}\ \text{W/m}^2)}\\ &= \boxed{0.161\ \text{Pa}}\end{aligned}$$

(e) $I = \frac{1}{2}\rho v(\omega s_{\text{max}})^2 = \frac{1}{2}\rho v 4\pi^2 f^2 s_{\text{max}}^2 \rightarrow s_{\text{max}} = \sqrt{\frac{I}{2\pi^2 \rho v f^2}} = \frac{1}{f}\sqrt{\frac{I}{2\pi^2 \rho v}}$

We see that for the same intensity level, the displacement amplitude is inversely proportional to the frequency.

For the low note,

$$s_{\max} = \frac{1}{146.8/\text{s}}\sqrt{\frac{3.16\times10^{-5}\ \text{W/m}^2}{2\pi^2\,(1.20\ \text{kg/m}^3)(343\ \text{m/s})}}$$

$$= \frac{6.24\times10^{-5}\ \text{m/s}}{146.8\ \text{s}^{-1}} = \boxed{4.25\times10^{-7}\ \text{m}}$$

(f) For the high note,

$$s_{\max} = \frac{6.24\times10^{-5}\ \text{m/s}}{880\ \text{s}^{-1}} = \boxed{7.09\times10^{-8}\ \text{m}}$$

P17.30 We begin with $\beta_2 = (10\ \text{dB})\log\left(\frac{I_2}{I_0}\right)$ and $\beta_1 = (10\ \text{dB})\log\left(\frac{I_1}{I_0}\right)$, so

$$\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\frac{I_2}{I_1}\right)$$

Also, $I_2 = \frac{P}{4\pi r_2^2}$ and $I_1 = \frac{P}{4\pi r_1^2}$, giving $\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2$

Then, $\beta_2 - \beta_1 = (10\ \text{dB})\log\left(\frac{r_1}{r_2}\right)^2 = \boxed{20\log\left(\frac{r_1}{r_2}\right)}$

P17.31 From $\beta = 10\ \log\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$, we have

$$I = [10^{\beta/10}](10^{-12}\ \text{W/m}^2)$$

(a) For your baby,

$$I_b = (10^{75.0/10})(10^{-12}\ \text{W/m}^2) = 3.16\times10^{-5}\ \text{W/m}^2$$

For the music,

$$I_m = (10^{80.0/10})(10^{-12}\ \text{W/m}^2) = 10.0\times10^{-5}\ \text{W/m}^2$$

The combined intensity is

$$\begin{aligned} I_{\text{total}} &= I_m + I_b \\ &= 10.0\times10^{-5}\ \text{W/m}^2 + 3.16\times10^{-5}\ \text{W/m}^2 \\ &= \boxed{13.2\times10^{-5}\ \text{W/m}^2} \end{aligned}$$

(b) The combined sound level is then

$$\beta_{\text{total}} = 10\log\left(\frac{I_{\text{total}}}{10^{-12}\ \text{W/m}^2}\right) = 10\log\left(\frac{1.32\times10^{-4}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right)$$

$$= \boxed{81.2\ \text{dB}}$$

P17.32 The speakers broadcast equally in all directions, so the intensity of sound is inversely proportional to the square of the distance from its source.

(a) $r_{AC} = \sqrt{3.00^2 + 4.00^2}\ \text{m} = 5.00\ \text{m}$

$$I = \frac{P}{4\pi r^2} = \frac{1.00\times10^{-3}\ \text{W}}{4\pi(5.00\ \text{m})^2} = 3.18\times10^{-6}\ \text{W/m}^2$$

$$\beta = (10\ \text{dB})\log\left(\frac{3.18\times10^{-6}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right)$$

$$\beta = (10\ \text{dB})\ 6.50 = \boxed{65.0\,\text{dB}}$$

(b) $r_{BC} = 4.47\ \text{m}$

$$I = \frac{1.50\times10^{-3}\ \text{W}}{4\pi(4.47\ \text{m})^2} = 5.97\times10^{-6}\ \text{W/m}^2$$

$$\beta = (10\ \text{dB})\log\left(\frac{5.97\times10^{-6}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right)$$

$$\beta = \boxed{67.8\ \text{dB}}$$

(c) $I = 3.18\ \mu\text{W/m}^2 + 5.97\ \mu\text{W/m}^2$

$$\beta = (10\ \text{dB})\log\left(\frac{9.15\times10^{-6}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right) = \boxed{69.6\ \text{dB}}$$

P17.33 The sound intensity at distance d_1 is, suppressing units,

$$I_1 = \frac{\Delta P_{\max}^2}{2\rho v} = \frac{(10.0)^2}{2(1.20)(343)} = 0.121\ \text{W/m}^2$$

If air does not absorb sound energy, the intensity of sound is inversely proportional to the square of the distance from its source. The intensity at distance d_2 is

$$I_2 = \left(\frac{d_1}{d_2}\right)^2 I_1 = \left(\frac{500\ \text{m}}{4\ 000\ \text{m}}\right)^2 I_1 = \frac{1}{64}(0.121\ \text{W/m}^2)$$

$$= 1.89\times10^{-3}\ \text{W/m}^2$$

which has an intensity level of

$$\beta_2 = (10\ \text{dB})\log\left(\frac{I_2}{I_0}\right) = (10\ \text{dB})\log\left(\frac{1.89\times 10^{-3}\ \text{W/m}^2}{10^{-12}\ \text{W/m}^2}\right)$$

$$= 92.77\ \text{dB}$$

Allowing for absorption of the wave over the distance traveled,

$$\beta_2' = \beta_2 - (7.00\ \text{dB/km})(3.50\ \text{km}) = \boxed{68.3\ \text{dB}}$$

This is equivalent to the sound intensity level of heavy traffic.

P17.34 (a) The energy transferred by sound from the explosion is

$$T_{MW} = P\Delta t = 4\pi r^2 I\Delta t$$

$$= 4\pi(100\ \text{m})^2(7.00\times 10^{-2}\ \text{W/m}^2)(0.200\ \text{s})$$

$$= \boxed{1.76\ \text{kJ}}$$

(b) $\beta = (10\ \text{dB})\log\left(\dfrac{7.00\times 10^{-2}}{1.00\times 10^{-12}}\right) = \boxed{108\ \text{dB}}$

P17.35 From the definition of sound level,

$$\beta = 10\log\left(\frac{I}{10^{-12}\ \text{W/m}^2}\right)$$

we can compute the intensities corresponding to each of the levels mentioned as $I = [10^{\beta/10}]10^{-12}\ \text{W/m}^2$.

They are $I_{120} = 1\ \text{W/m}^2$

$I_{100} = 10^{-2}\ \text{W/m}^2$

and $I_{10} = 10^{-11}\ \text{W/m}^2$

(a) The power passing through any sphere around the source is $Power = 4\pi r^2 I$. If we ignore absorption of sound by the medium, conservation of energy for the sound wave as a system requires that $r_{120}^2 I_{120} = r_{100}^2 I_{100} = r_{10}^2 I_{10}$. Then

$$r_{100} = r_{120}\sqrt{\frac{I_{120}}{I_{100}}} = (3.00\ \text{m})\sqrt{\frac{1\ \text{W/m}^2}{10^{-2}\ \text{W/m}^2}} = \boxed{30.0\ \text{m}}$$

(b) $r_{10} = r_{120}\sqrt{\dfrac{I_{120}}{I_{10}}} = (3.00\ \text{m})\sqrt{\dfrac{1\ \text{W/m}^2}{10^{-11}\ \text{W/m}^2}} = \boxed{9.49\times 10^5\ \text{m}}$

P17.36 $\boxed{\text{We assume that both lawn mowers are equally loud and approximately the same distance away. We found in Example 17.3 that a sound of twice the intensity results in an increase in sound level of 3 dB. We also see from the What If? section of that example that a doubling of loudness requires a 10-dB increase in sound level. Therefore, the sound of two lawn mowers will not be twice the loudness, but only a little louder than one!}}$

Section 17.4 The Doppler Effect

P17.37 The source and detector of waves are both moving with respect to the medium in which the waves are travelling.

(a) The general form of the Doppler equation is:

$$f' = \left(\frac{v + v_o}{v - v_s}\right) f$$

where the positive signs for v_o and v_s are for source or observer approaching each other. When the ambulance is approaching the car from behind:

$$f' = \left(\frac{v + v_O}{v - v_S}\right) f = \left(\frac{343 + (-25)}{343 - (+42)}\right) 450\text{ Hz} = (1.056)450\text{ Hz}$$

$$= \boxed{475\text{ Hz}}$$

(b) When the ambulance is moving away in front of the moving car:

$$f' = \left(\frac{v + v_O}{v - v_S}\right) f = \left(\frac{343 + (+25)}{343 - (-42)}\right) 450\text{ Hz} = (0.956)450\text{ Hz}$$

$$= \boxed{430\text{ Hz}}$$

P17.38 The *half angle* of the shock wave cone is given by $\sin\theta = \dfrac{v_{\text{light}}}{v_S}$.

$$v_S = \frac{v_{\text{light}}}{\sin\theta} = \frac{2.25 \times 10^8\text{ m/s}}{\sin(53.0°)} = \boxed{2.82 \times 10^8\text{ m/s}}$$

P17.39 (a) The Doppler-shifted frequency is found from

$$f' = \frac{f(v+v_o)}{(v-v_s)}$$

$$= (2\,500\text{ Hz})\frac{(343+25.0)}{(343-40.0)} = \boxed{3.04\text{ kHz}}$$

(b) After the police car passes,

$$f' = (2\,500\text{ Hz})\left(\frac{343+(-25.0)}{343-(-40.0)}\right) = \boxed{2.08\text{ kHz}}$$

(c) While the police car overtakes the driver,

$$f' = (2\,500\text{ Hz})\left(\frac{343+(-25.0)}{343-40.0}\right) = \boxed{2.62\text{ kHz}}$$

After the police car passes,

$$f' = (2\,500\text{ Hz})\left(\frac{343+25.0}{343-(-40.0)}\right) = \boxed{2.40\text{ kHz}}$$

P17.40 (a) Equation 17.19, $f' = f\left(\frac{v+v_o}{v-v_s}\right)$, applies to an observer on $\boxed{\text{B}}$ because B is receiving sound from source A.

(b) The sign of v_s should be $\boxed{\text{positive}}$ because the source is moving toward the observer, resulting in an increase in frequency.

(c) The sign of v_o should be $\boxed{\text{negative}}$ because the observer is moving away from the source, resulting in a decrease in frequency.

(d) The speed of sound should be that of the medium of seawater, $\boxed{1\,533\text{ m/s}}$.

(e) $$f_o = f_s\left(\frac{v+v_o}{v-v_s}\right) = (5.27\times 10^3\text{ Hz})\left[\frac{(1\,533\text{ m/s})+(-3.00\text{ m/s})}{(1\,533\text{ m/s})-(+11.0\text{ m/s})}\right]$$

$$= \boxed{5.30\times 10^3\text{ Hz}}$$

P17.41 (a) The maximum speed of the speaker is described by

$$\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$$

$$v_{\text{max}} = \sqrt{\frac{k}{m}}A = \sqrt{\frac{20.0\ \text{N/m}}{5.00\ \text{kg}}}(0.500\ \text{m}) = 1.00\ \text{m/s}$$

The frequencies heard by the stationary observer range from

$$f'_{\text{max}} = f\left(\frac{v}{v - v_{\text{max}}}\right) = 440\ \text{Hz}\left(\frac{343}{343 - 1.00}\right) = \boxed{441\ \text{Hz}}$$

to

(b) $$f'_{\text{min}} = f\left(\frac{v}{v + v_{\text{max}}}\right) = 440\ \text{Hz}\left(\frac{343}{343 + 1.00}\right) = \boxed{439\ \text{Hz}}$$

(c) $$\beta = (10\ \text{dB})\log\left(\frac{I}{I_0}\right) = (10\ \text{dB})\log\left(\frac{P/4\pi r^2}{I_0}\right)$$

The maximum intensity level $\beta_{\text{max}} = 60.0$ dB occurs at $r = r_{\text{min}} = 1.00$ m. The minimum intensity level occurs when the speaker is farthest from the listener, i.e., when $r = r_{\text{max}} = r_{\text{min}} + 2A = 2.00$ m.

Thus, $\beta_{\text{max}} - \beta_{\text{min}} = (10\ \text{dB})\log\left(\frac{P}{4\pi I_0 r_{\text{min}}^2}\right) - (10\ \text{dB})\log\left(\frac{P}{4\pi I_0 r_{\text{max}}^2}\right)$

or

$$\beta_{\text{max}} - \beta_{\text{min}} = (10\ \text{dB})\log\left(\frac{P}{4\pi I_0 r_{\text{min}}^2}\frac{4\pi I_0 r_{\text{max}}^2}{P}\right)$$

$$= (10\ \text{dB})\log\left(\frac{r_{\text{max}}^2}{r_{\text{min}}^2}\right) = (20\ \text{dB})\log\left(\frac{r_{\text{max}}}{r_{\text{min}}}\right)$$

This gives:

$$60.0\ \text{dB} - \beta_{\text{min}} = (20\ \text{dB})\log(2.00) = 6.02\ \text{dB}$$

or $\beta_{\text{min}} = \boxed{54.0\ \text{dB}}$

P17.42 The maximum speed of the speaker is described by

$$\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$$

$$v_{\text{max}} = \sqrt{\frac{k}{m}}A$$

The frequencies heard by the stationary observer range from

(a) $f'_{\text{max}} = \boxed{\dfrac{vf}{v - A\sqrt{\dfrac{k}{m}}}}$ to (b) $f'_{\text{max}} = \boxed{\dfrac{vf}{v + A\sqrt{\dfrac{k}{m}}}}$

where *v* is the speed of sound.

(c) $\beta = (10\text{ dB})\log\left(\dfrac{I}{I_0}\right) = (10\text{ dB})\log\left(\dfrac{P/4\pi r^2}{I_0}\right)$

The maximum intensity level $\beta_{\text{max}} = \beta$ occurs at $r = r_{\text{min}} = d$. The minimum intensity level occurs when the speaker is farthest from the listener, i.e., when $r = r_{\text{max}} = r_{\text{min}} + 2A = d + 2A$.

Thus,

$$\beta_{\text{max}} - \beta_{\text{min}} = (10\text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\text{min}}^2}\right) - (10\text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\text{max}}^2}\right)$$

or

$$\beta_{\text{max}} - \beta_{\text{min}} = (10\text{ dB})\log\left(\frac{P}{4\pi I_0 r_{\text{min}}^2}\frac{4\pi I_0 r_{\text{max}}^2}{P}\right)$$
$$= (10\text{ dB})\log\left(\frac{r_{\text{max}}^2}{r_{\text{min}}^2}\right) = (20\text{ dB})\log\left(\frac{r_{\text{max}}}{r_{\text{min}}}\right)$$

This gives:

$$\beta - \beta_{\text{min}} = (20\text{ dB})\log\left(\frac{d + 2A}{d}\right)$$

or $\beta_{\text{min}} = \boxed{\beta - (20\text{ db})\log\left(1 + \dfrac{2A}{d}\right)}$

P17.43 (a) $\omega = 2\pi f = 2\pi\left(\dfrac{115\text{ min}^{-1}}{60.0\text{ s/min}}\right) = 12.0\text{ rad/s}$

$v_{\text{max}} = \omega A = (12.0\text{ rad/s})(1.80 \times 10^{-3}\text{ m}) = \boxed{0.021\,7\text{ m/s}}$

(b) The heart wall is a moving observer:

$$\Delta f' = f' - f = f\left(\frac{v + v_O}{v} - f\right) - f = f\left(\frac{v_O}{v}\right)$$
$$= (2\,000\,000\text{ Hz})\left(\frac{0.021\,7}{1\,500}\right) = \boxed{28.9\text{ Hz}}$$

(c) Now, the heart wall is a moving source:

$$\Delta f'' = f'\left(\frac{v}{v-v_s}\right) - f = f\left(\frac{v+v_o}{v}\right)\left(\frac{v}{v-v_s}\right) - f$$

$$\Delta f'' = f\left[\left(\frac{v(v+v_o)}{v(v-v_s)}\right) - \left(\frac{v(v-v_s)}{v(v-v_s)}\right)\right] = f\left[\frac{v_o+v_s}{v-v_s}\right]$$

Since the velocities of the source and the observer in these expressions are both referring to the movement of the heart wall, and the velocity of the sound wave is much greater than those velocities, we may approximate:

$$\Delta f'' \cong f\left[\frac{2v_s}{v}\right]$$

$$\Delta f'' \cong (2.00\times 10^6\text{ Hz})\left[\frac{0.043\ 4}{1\ 500}\right] = \boxed{57.9\text{ Hz}}$$

P17.44 The apparent frequency drops because of the Doppler effect. Using a *T* subscript for the situation when the athlete moves *toward* the horn, and *A* for movement away from the horn, we have,

$$\frac{f'_A}{f'_T} = \frac{\left(\dfrac{v+v_{OA}}{v-v_S}\right)f}{\left(\dfrac{v+v_{OT}}{v-v_S}\right)f} = \frac{v+v_{OA}}{v+v_{OT}} = \frac{v+(-v_O)}{v+(+v_O)} = \frac{v-v_O}{v+v_O}$$

where v_0 is the constant speed of the athlete. Setting this ratio equal to 5/6, we have

$$\frac{5}{6} = \frac{v-v_O}{v+v_O} \quad\rightarrow\quad 5v+5v_O = 6v-6v_O \quad\rightarrow\quad 11v_O = v$$

Solving for the speed of the athlete,

$$v_O = \frac{v}{11} = \frac{343\text{ m/s}}{11} = 31.2\text{ m/s}$$

$\boxed{\text{This is much faster than a human athlete can run.}}$

P17.45 Let v_a represent the magnitude of the velocity of the ambulance.

As it approaches you hear the frequency $f' = v\left(\dfrac{v}{v-v_a}\right)f = 560\text{ Hz}.$

The negative sign appears because the source is moving toward the observer. The opposite sign with source velocity magnitude describes the ambulance moving away. As the ambulance recedes, the Doppler-shifted frequency is

$$f'' = \left(\frac{v}{v + v_a}\right) f = 480\text{ Hz}.$$

Solving the second of these equations for f and substituting into the other gives

$$f' = \left(\frac{v}{v - v_a}\right)\left(\frac{v + v_a}{v}\right) f'' \quad \text{or} \quad f'v - f'v_a = vf'' + v_a f''$$

so the speed of the source is

$$v_a = \frac{v\left(f' - f''\right)}{f' + f''} = \frac{(343\text{ m/s})(560\text{ Hz} - 480\text{ Hz})}{560\text{ Hz} + 480\text{ Hz}} = \boxed{26.4\text{ m/s}}$$

P17.46 We first determine how fast the tuning fork is falling to emit sound with apparent frequency 485 Hz. Call the magnitude of its velocity v_{fall}. The tuning fork source is moving away from the listener, so $v_s = -v_{\text{fall}}$.

Therefore, we use the equation $f' = \left(\frac{v}{v + v_{\text{fall}}}\right) f$

Solving for v_{fall} gives $\frac{v + v_{\text{fall}}}{v} = \frac{f}{f'}$ and $v_{\text{fall}} = v\left(\frac{f}{f'} - 1\right)$.

Substituting, we have $v_{\text{fall}} = \left(\frac{512\text{ Hz}}{485\text{ Hz}} - 1\right)(343\text{ m/s}) = 19.1\text{ m/s}$.

The time interval required for the tuning fork to reach this speed, from the particle under constant acceleration model, is given by

$$v_y = 0 + a_y t \quad \text{as} \quad t = v_y/a_y = (19.1\text{ m/s})/(9.80\text{ m/s}^2) = 1.95\text{ s}$$

The distance that the fork has fallen is

$$\Delta y = 0 + \frac{1}{2}a_y t^2 = \frac{1}{2}(9.80\text{ m/s}^2)(1.95\text{ s})^2 = 18.6\text{ m}$$

At this moment, the fork would appear to ring at 485 Hz to a stationary observer just above the fork. However, some additional time is required for the waves to reach the point of release. The fork is moving down, but the sound it radiates still travels away from its instantaneous position at 343 m/s. From the traveling wave model, the time interval it takes to return to the listener is

$$\Delta t = \Delta y/v = 18.6\text{ m}/(343\text{ m/s}) = 0.054\,2\text{ s}$$

Over a total time $t + \Delta t = 1.95\text{ s} + 0.054\,2\text{ s} = 2.00\text{ s}$, the fork falls a total distance

$$d_{\text{total}} = \frac{1}{2} g t_{\text{total fall}}^2 = \boxed{19.7\text{ m}}$$

P17.47 (a) We find the shock angle from

$$\theta = \sin\left(\frac{v}{v_S}\right)^{-1} = \sin\left(\frac{1}{3.00}\right)^{-1} = 19.5°$$

from $\tan\theta = \frac{h}{x}$,

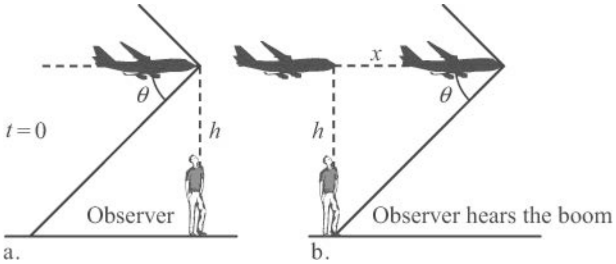
$$x = \frac{h}{\tan\theta} = \frac{20\,000\text{ m}}{\tan 19.5°} = 5.66 \times 10^4\text{ m} = 56.6\text{ km}$$

It takes the plane

$$t = \frac{x}{v_S} = \frac{5.66 \times 10^4\text{ m}}{3.00(335\text{ m/s})} = \boxed{56.3\text{ s}}$$

to travel this distance.

(b) From part (a), $x = \boxed{56.6\text{ km}}$



ANS. FIG. P17.47

Additional Problems

***P17.48** The size of the insect detected by the bat will be comparable to the wavelength of sound emitted by the bat:

$$\lambda = \frac{v}{f} = \frac{340\text{ m/s}}{60.0 \times 10^3\text{ s}^{-1}} = \boxed{5.67\text{ mm}}$$

***17.49** At normal body temperature of $T = 37.0°\text{C}$, the speed of sound in air is

$$v = (331\text{ m/s})\sqrt{1+\frac{T_C}{273}} = (331\text{ m/s})\sqrt{1+\frac{37.0}{273}} = 353\text{ m/s}$$

and the wavelength of sound having a frequency of $f = 20\ 000$ Hz is

$$\lambda = \frac{v}{f} = \frac{353\text{ m/s}}{20\ 000\text{ Hz}} = 1.76\times10^{-2}\text{ m} = \boxed{1.76\text{ cm}}$$

Thus, the diameter of the eardrum is $\boxed{1.76\text{ cm.}}$

P17.50 (a) The wavelength of the note is

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{1\,480\text{ s}^{-1}} = \boxed{0.232\text{ m}}$$

(b) We find the intensity of the 81.0 dB sound from

$$\beta = 81.0\text{ dB} = (10\text{ dB})\log\left[\frac{I}{10^{-12}\text{ W/m}^2}\right]$$

Then,

$$I = \left(10^{-12}\text{ W/m}^2\right)10^{8.10} = 10^{-3.90}\text{ W/m}^2 = 1.26\times10^{-4}\text{ W/m}^2$$
$$= \frac{1}{2}\rho v\omega^2 s_{\max}^2$$

Which gives a displacement amplitude of

$$s_{\max} = \sqrt{\frac{2I}{\rho v\omega^2}} = \sqrt{\frac{2\left(1.26\times10^{-4}\text{ W/m}^2\right)}{\left(1.20\text{ kg/m}^3\right)\left(343\text{ m/s}\right)4\pi^2\left(1\,480\text{ s}^{-1}\right)^2}}$$
$$= \boxed{8.41\times10^{-8}\text{ m}}$$

(c) The wavelength of the F above high C is

$$\lambda' = \frac{v}{f'} = \frac{343\text{ m/s}}{1\,397\text{ s}^{-1}} = 0.246\text{ m}$$

and the change in wavelength is

$$\Delta\lambda = \lambda' - \lambda = 0.246\text{ m} - 0.232\text{ m} = \boxed{13.8\text{ mm}}$$

P17.51 The trucks form a train analogous to a wave train of crests with speed $v = 19.7\text{ m/s}$ and unshifted frequency $f = \dfrac{2}{3.00\text{ min}} = 0.667\text{ min}^{-1}$.

(a) The cyclist as observer measures a lower Doppler-shifted frequency:

$$f' = f\left(\frac{v+v_o}{v}\right) = \left(0.667\text{ min}^{-1}\right)\left(\frac{19.7+(-4.47)}{19.7}\right)$$

$$= \boxed{0.515/\text{min}}$$

(b) $$f'' = f\left(\frac{v+v_o'}{v}\right) = \left(0.667\text{ min}^{-1}\right)\left(\frac{19.7+(-1.56)}{19.7}\right) = \boxed{0.614/\text{min}}$$

The cyclist's speed has decreased very significantly, but there is only a modest increase in the frequency of trucks passing him.

P17.52 We calculate the intensity of the speaker from

$$103\text{ dB} = (10\text{ dB})\log\left(\frac{I}{10^{-12}\text{ W/m}^2}\right)$$

which gives $I = 2.00\times10^{-2}\text{ W/m}^2$

(a) We find the sound power output from

$$I = \frac{P}{4\pi r^2}$$

which gives

$$P = 4\pi r^2 I = 4\pi(1.60\text{ m})^2\left(2.00\times10^{-2}\text{ W/m}^2\right) = \boxed{0.642\text{ W}}$$

(b) The efficiency of the speaker is

$$e = \frac{P_{out}}{P_{in}} = \frac{0.642\text{ W}}{150\text{ W}} = \boxed{0.004} \text{ or } \boxed{0.4\%}$$

P17.53 The flow of traffic at night is 1/20th that of the afternoon, so

$$P_2 = \frac{1}{20.0}P_1$$

The difference in sound level is

$$\beta_1 - \beta_2 = 10\log\left(\frac{P_1}{P_2}\right)$$

solving for the sound level at night gives

$$80.0-\beta_2 = 10\log(20.0) = +13.0$$

$$\beta_2 = \boxed{67.0\text{ dB}}$$

***P17.54** (a) We have $f' = \dfrac{fv}{v-u}$ and $f'' = \dfrac{fv}{v-(-u)}$. We then have

$$f'-f'' = fv\left(\frac{1}{v-u}-\frac{1}{v+u}\right)$$

$$\Delta f = \frac{fv(v+u-v+u)}{v^2-u^2} = \frac{2uvf}{v^2\left(1-\dfrac{u^2}{v^2}\right)} = \boxed{\frac{2\left(\dfrac{u}{v}\right)}{1-\dfrac{u^2}{v^2}}f}$$

(b) $130\text{ km/h} = 36.1\text{ m/s}$

$$\Delta f = \frac{2(36.1\text{ m/s})(400\text{ Hz})}{(340\text{ m/s})\left[1-\dfrac{(36.1\text{ m/s})^2}{(340\text{ m/s})^2}\right]} = \boxed{85.9\text{ Hz}}$$

***P17.55** The sound speed is

$$v = 331\text{ m/s} + (0.600\text{ m/s}\cdot{}^\circ\text{C})(26.0^\circ\text{C}) = 347\text{ m/s}$$

(a) Let t represent the time for the echo to return. Then

$$d = \frac{1}{2}vt = \frac{1}{2}(347\text{ m/s})\left(24.0\times10^{-3}\text{ s}\right) = \boxed{4.16\text{ m}}$$

(b) Let Δt represent the duration of the pulse:

$$\Delta t = \frac{10\lambda}{v} = \frac{10\lambda}{f\lambda} = \frac{10}{f} = \frac{10}{22.0\times10^6\text{ s}^{-1}} = \boxed{0.455\ \mu\text{s}}$$

(c) $$L = 10\lambda = \frac{10v}{f} = \frac{10(347\text{ m/s})}{22.0\times10^6\text{ s}^{-1}} = \boxed{0.158\text{ mm}}$$

P17.56 (a) The sound “pressure” is extra tensile stress for one-half of each cycle. When it becomes (0.500%)(13.0×10^{10} Pa) = 6.50×10^8 Pa, the rod will break. Then, $\Delta P_{\max} = \rho v\omega s_{\max}$ and

$$s_{\max} = \frac{\Delta P_{\max}}{\rho v\omega} = \frac{6.50\times10^8\text{ N/m}^2}{\left(8.92\times10^3\text{ kg/m}^3\right)\left(5\,010\text{ m/s}\right)\left(2\pi 500\text{ s}^{-1}\right)}$$

$$= \boxed{4.63\text{ mm}}$$

(b) From $s = s_{\max}\cos(kx-\omega t)$, differentiating gives

$$v = \frac{\partial s}{\partial t} = -\omega s_{\max}\sin(kx-\omega t)$$

then

$$v_{\max} = \omega s_{\max} = \left(2\pi 500\ \text{s}^{-1}\right)\left(4.63\ \text{mm}\right) = \boxed{14.5\ \text{m/s}}$$

(c) $$I = \frac{1}{2}\rho v\left(\omega s_{\max}\right)^2 = \frac{1}{2}\rho v v_{\max}^2$$

$$= \frac{1}{2}\left(8.92\times 10^3\ \text{kg/m}^3\right)\left(5\,010\ \text{m/s}\right)\left(14.5\ \text{m/s}\right)^2$$

$$= \boxed{4.73\times 10^9\ \text{W/m}^2}$$

P17.57 The gliders stick together and move with final speed given by momentum conservation for the two-glider system:

$$m_1v_1 + m_2v_2 = m_1v_1 + 0 = \left(m_1 + m_2\right)v$$

$$v = \frac{m_1v_1}{m_1 + m_2} = \frac{\left(0.150\ \text{kg}\right)\left(2.30\ \text{m/s}\right)}{0.150\ \text{kg} + 0.200\ \text{kg}} = 0.986\ \text{m/s}$$

The missing mechanical energy is

$$\Delta K = \frac{1}{2}m_1v_1^2 - \frac{1}{2}\left(m_1 + m_2\right)v^2$$

$$= \frac{1}{2}\left(0.150\ \text{kg}\right)\left(2.30\ \text{m/s}\right)^2 - \frac{1}{2}\left(0.350\ \text{kg}\right)\left(0.986\ \text{m/s}\right)^2$$

$$= 0.227\ \text{J}$$

We imagine one-half of 227 mJ going into internal energy and half into sound radiated isotropically in 7.00 ms. Its intensity 0.800 m away is

$$I = \frac{E}{At} = \frac{\frac{1}{2}\left(0.227\ \text{J}\right)}{4\pi\left(0.800\ \text{m}\right)^2\left(7.00\times 10^{-3}\ \text{s}\right)} = 2.01\ \text{W/m}^2$$

Its intensity level is

$$\beta = \left(10\ \text{dB}\right)\log\left(\frac{2.01\ \text{W/m}^2}{1.00\times 10^{-12}\ \text{W/m}^2}\right) = 123\ \text{dB}$$

It is unreasonable, implying a sound level of 123 dB. Nearly all of the decrease in mechanical energy becomes internal energy in the latch.

P17.58 (a) The wave moves outward equally in all directions. (We can tell it is outward because of the negative sign in $1.36r - 2\ 030t$.)

(b) $\boxed{\text{Its amplitude is inversely proportional to its distance from the center. Its intensity is proportional to the square of the amplitude, so the intensity follows the inverse-square law, with no absorption of energy by the medium.}}$

(c) $\boxed{\text{Its speed is constant at } v = f\lambda = \omega/k = (2030/\text{s})/(1.36/\text{m}) = 1.49\text{ km/s. By comparison to the table in the chapter, it can be moving through water at 25°C, and we assume that it is.}}$

(d) $\boxed{\text{Its frequency is constant at } (2030/\text{s})/2\pi = 323\text{ Hz.}}$

(e) $\boxed{\text{Its wavelength is constant at } 2\pi/k = 2\pi/(1.36/\text{m}) = 4.62\text{ m.}}$

(f) Its pressure amplitude is (25.0 Pa$/r$). Its intensity at this distance is

$$I = \frac{\Delta P_{\text{max}}^2}{2\rho v} = \frac{\left[\left(25\text{ N/m}^2\right)/r\right]^2}{2(1000\text{ kg/m}^3)(1490\text{ m/s})} = \frac{209\ \mu\text{W/m}^2}{r^2}$$

so the power of the source and the net power of the wave at all distances is

$$P = I4\pi r^2 = \left(\frac{2.09\times10^{-4}\text{ W/m}^2}{r^2}\right)4\pi r^2 = 2.63\text{ mW}$$

(g) $\boxed{\text{Its intensity follows the inverse-square law; at } r = 1\text{ m, the intensity is } 209\ \mu\text{W/m}^2.}$

P17.59 (a) The speed of a compression wave in a bar is

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{20.0\times10^{10}\ \text{N/m}^2}{7\,860\ \text{kg/m}^3}} = \boxed{5.04\times10^3\ \text{m/s}}$$

(b) The signal to stop passes between layers of atoms as a sound wave, reaching the back end of the bar in time interval

$$\Delta t = \frac{L}{v} = \frac{0.800\text{ m}}{5.04\times10^3\ \text{m/s}} = \boxed{1.59\times10^{-4}\text{ s}}$$

(c) As described by Newton's first law, the rearmost layer of steel has continued to move forward with its original speed v_i for this time, compressing the bar by

$$\Delta L = v_i\Delta t = \left(12.0\ \text{m/s}\right)\left(1.59\times10^{-4}\ \text{s}\right) = 1.90\times10^{-3}\ \text{m}$$
$$= \boxed{1.90\text{ mm}}$$

(d) The strain in the rod is $\frac{\Delta L}{L} = \frac{1.90 \times 10^{-3}\ \text{m}}{0.800\ \text{m}} = \boxed{2.38 \times 10^{-3}}$

(e) The stress in the rod is

$$\sigma = Y\left(\frac{\Delta L}{L}\right) = \left(20.0 \times 10^{10}\ \text{N/m}^2\right)\left(2.38 \times 10^{-3}\right)$$
$$= \boxed{4.76 \times 10^{8}\ \text{N/m}^2}$$

Since $\sigma > 400\ \text{MPa}$, the rod will be permanently distorted.

(f) We go through the same steps as in parts (a) through (e), but use algebraic expressions rather than numbers:

The speed of sound in the rod is $v = \sqrt{\frac{Y}{\rho}}$

The back end of the rod continues to move forward at speed v_i for a time interval of $\Delta t = \frac{L}{v} = L\sqrt{\frac{\rho}{Y}}$, traveling distance $\Delta L = v_i \Delta t$ after the front end hits the wall.

The strain in the rod is $\frac{\Delta L}{L} = \frac{v_i t}{L} = v_i\sqrt{\frac{\rho}{Y}}$

The stress is then $\sigma = Y\left(\frac{\Delta L}{L}\right) = Yv_i\sqrt{\frac{\rho}{Y}} = v_i\sqrt{\rho Y}$

For this to be less than the yield stress, σ_y, it is necessary that the maximum speed be

$$\boxed{\frac{\sigma_y}{\sqrt{\rho Y}}}$$

P17.60 (a) Model your loud, sharp sound impulse as a single narrow peak in a graph of air pressure versus time. It is a noise with no frequency, wavelength, or period. It radiates away from you in all directions and some of it is incident on each one of the solid vertical risers of the bleachers.

The repeated reflections from the steps create a repetition frequency so that the ear / brain combination assigns a pitch to the sound heard by the listener.

Suppose that, at the ambient temperature, sound moves at 343 m/s; and suppose that the horizontal width of each row of seats is 60 cm. Then there is a time delay of

$$\frac{0.60\text{ m}}{343\text{ m/s}} = 0.001\ 7\text{ s}$$

between your sound impulse reaching each riser and the next. Whatever its material, each will reflect much of the sound that reaches it. The reflected wave sounds very different from the sharp pop you made.

(b) If there are twenty rows of seats, you hear from the bleachers a tone with twenty crests, each separated from the next in time by

$$\frac{2(0.60\text{ m})}{343\text{ m/s}} = 0.003\ 5\text{ s}$$

This is the extra time for it to cross the width of one seat twice, once as an incident pulse and once again after its reflection. Thus, you hear a sound of definite pitch, with a period of about 0.0035 s, and frequency,

$$\frac{1}{0.003\,5\text{ s}} = 290\text{ Hz}\ \boxed{\sim\text{a few hundred Hz}}$$

(c) Wavelength

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{290\text{ s}^{-1}} = 1.2\text{ m} \sim \boxed{1\text{ m}}$$

(d) and duration

$$20(0.003\ 5\text{ s}) \sim \boxed{0.1\text{ s}}$$

P17.61 Let f_e = 1 800 Hz represent the emitted frequency; v_e the speed of the skydiver; and f_g = 2 150 Hz the frequency of the wave crests reaching the ground.

(a) The skydiver source is moving toward the stationary ground, so we rearrange the equation $f_g = f_e\left(\frac{v}{v - v_e}\right)$ to give

$$v_e = v\left(1 - \frac{f_e}{f_g}\right) = (343\text{ m/s})\left(1 - \frac{1\ 800\text{ Hz}}{2\ 150\text{ Hz}}\right) = \boxed{55.8\text{ m/s}}$$

(b) The ground now becomes a stationary source, reflecting crests with the 2 150-Hz frequency at which they reach the ground, and sending them to a moving observer, who receives them at the rate

$$f_{e2} = f_g\left(\frac{v+v_e}{v}\right) = (2\ 150\ \text{Hz})\left(\frac{343\ \text{m/s} + 55.8\ \text{m/s}}{343\ \text{m/s}}\right)$$

$$= \boxed{2\ 500\ \text{Hz}}$$

P17.62 (a) The distance is larger by $240/60 = 4$ times. The intensity is 16 times smaller at the larger distance because the sound power is spread over a 42 times larger area.

(b) The amplitude is 4 times smaller at the larger distance because intensity is proportional to the square of amplitude.

(c) The extra distance is $(240-60)/45 = 4$ wavelengths. The phase is the same at both points because they are separated by an integer number of wavelengths.

P17.63 (a) If the velocity of the insect is v_x,

$$40.4\ \text{kHz} = (40.0\ \text{kHz})\frac{(343\ \text{m/s} + 5.00\ \text{m/s})(343\ \text{m/s} - v_x)}{(343\ \text{m/s} - 5.00\ \text{m/s})(343\ \text{m/s} + v_x)}$$

Solving, $v_x = \boxed{3.29\ \text{m/s}}$.

(b) Therefore, $\boxed{\text{the bat is gaining on its prey at 1.71 m/s}}$.

P17.64 When the observer is moving in front of and in the same direction as the source, $f' = f\dfrac{v - v_O}{v - v_S}$, where v_O and v_S are measured relative to the medium in which the sound is propagated. In this case the ocean current is opposite the direction of travel of the ships, and

$$v_O = 45.0\ \text{km/h} - (-10.0\ \text{km/h}) = 55.0\ \text{km/h} = 15.3\ \text{m/s}, \text{ and}$$

$$v_S = 64.0\ \text{km/h} - (-10.0\ \text{km/h}) = 74.0\ \text{km/h} = 20.55\ \text{m/s}$$

Therefore,

$$f' = (1\ 200.0\ \text{Hz})\frac{(1\ 533\ \text{m/s}) - (15.3\ \text{m/s})}{(1\ 533\ \text{m/s}) - (20.55\ \text{m/s})} = \boxed{1\ 204.2\ \text{Hz}}$$

P17.65 (a) If the police car were at rest, the wavelength in air of its siren would be

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{1\,000\ \text{s}^{-1}} = \boxed{0.343\ \text{m}}$$

(b) In front of the police car,

$$\lambda' = \frac{v}{f'} = \frac{v}{f}\left(\frac{v - v_S}{v}\right) = \frac{(343 - 40.0)\ \text{m/s}}{1\,000\ \text{s}^{-1}} = \boxed{0.303\ \text{m}}$$

(c) Behind the police car,

$$\lambda'' = \frac{v}{f''} = \frac{v}{f}\left(\frac{v + v_S}{v}\right) = \frac{(343 + 40.0)\ \text{m/s}}{1\,000\ \text{s}^{-1}} = \boxed{0.383\ \text{m}}$$

(d) The frequency heard by the speeder is

$$f' = f\left(\frac{v - v_O}{v - v_S}\right) = (1\ 000\ \text{Hz})\frac{(343 - 30.0)\ \text{m/s}}{(343 - 40.0)\ \text{m/s}} = \boxed{1.03\ \text{kHz}}$$

P17.66 (a) $\boxed{\text{The sound through the metal arrives first because it moves faster than sound in air.}}$

(b) Each travel time is individually given by $t = L/v$. Then the delay between the pulses' arrivals is $\Delta t = L\left(\frac{1}{v_{\text{air}}} - \frac{1}{v_{\text{cu}}}\right) = L\frac{v_{\text{cu}} - v_{\text{air}}}{v_{\text{air}}v_{\text{cu}}}$ and the length of the bar is

$$L = \frac{v_{\text{air}}v_{\text{cu}}}{v_{\text{cu}} - v_{\text{air}}}\Delta t = \frac{(343\ \text{m/s})(3.56\times 10^3\ \text{m/s})}{(3\,560 - 343)\ \text{m/s}}\Delta t$$

$$\boxed{L = 380\Delta t,\ \text{where } \Delta t \text{ is seconds and the length is in meters.}}$$

(c) $L = (380\ \text{m/s})(0.127\ \text{s}) = \boxed{48.2\ \text{m}}$

(d) $\boxed{\text{The answer becomes } L = \dfrac{\Delta t}{\dfrac{1}{343} - \dfrac{1}{v_r}},\ \text{where } v_r \text{ is the speed of sound in the rod in meters per second, } \Delta t \text{ is in seconds, and } L \text{ is in meters.}}$

(e) $\boxed{\text{As } v_r \text{ goes to infinity, the travel time in the rod becomes negligible. The answer approaches } 343\Delta t\text{, which is just the distance that the sound travels in air during the delay time.}}$

P17.67 (a) The Mach angle in the air is

$$\theta = \sin^{-1}\left(\frac{v_{\text{sound}}}{v_{\text{obj}}}\right) = \sin^{-1}\left(\frac{343}{20.0\times10^3}\right) = \boxed{0.983°}$$

(b) At impact with the ocean,

$$\theta' = \sin^{-1}\left(\frac{1\,533}{20.0\times10^3}\right) = \boxed{4.40°}$$

P17.68 The time interval required for a sound pulse to travel a distance L at a speed v is given by $t = \frac{L}{v} = \frac{L}{\sqrt{Y/\rho}}$. Using this expression, we find the travel time in each rod.

$$t_1 = L_1\sqrt{\frac{\rho_1}{Y_1}} = L_1\sqrt{\frac{2.70\times10^3 \text{ kg/m}^3}{7.00\times10^{10} \text{ N/m}^2}} = L_1(1.96\times10^{-4} \text{ s/m})$$

$$t_2 = (1.50 - L_1)\sqrt{\frac{11.3\times10^3 \text{ kg/m}^3}{1.60\times10^{10} \text{ N/m}^2}}$$
$$= 1.26\times10^{-3} \text{ s} - (8.40\times10^{-4} \text{ s/m})L_1$$

$$t_3 = (1.50 \text{ m})\sqrt{\frac{8.80\times10^3 \text{ kg/m}^3}{11.0\times10^{10} \text{ N/m}^2}} = 4.24\times10^{-4} \text{ s}$$

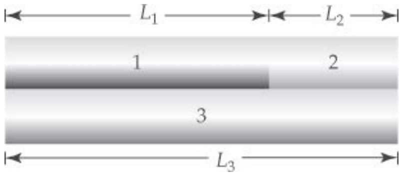
We require $t_1 + t_2 = t_3$, or

$$(1.96\times10^{-4} \text{ s/m})L_1 + (1.26\times10^{-3} \text{ s}) - (8.40\times10^{-4} \text{ s/m})L_1 = 4.24\times10^{-4} \text{ s}$$

This gives

$$L_1 = 1.30 \text{ m and } L_2 = (1.50 \text{ m}) - (1.30 \text{ m}) = 0.201 \text{ m}$$

The ratio of lengths is $\frac{L_1}{L_2} = \boxed{6.45}$.



ANS. FIG. P17.68

P17.69 For the longitudinal wave $v_L = \left(\dfrac{Y}{\rho}\right)^{1/2}$

For the transverse wave $v_T = \left(\dfrac{T}{\mu}\right)^{1/2}$

If we require $\dfrac{v_L}{v_T} = 8.00$, we have $T = \dfrac{\mu Y}{64.0\rho}$ where $\mu = \dfrac{m}{L}$ and

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{m}{\pi r^2 L}$$

This gives

$$T = \frac{\pi r^2 Y}{64.0} = \frac{\pi\left(2.00\times10^{-3}\ \text{m}\right)^2\left(6.80\times10^{10}\ \text{N/m}^2\right)}{64.0}$$

$$= \boxed{1.34\times10^4\ \text{N}}$$

***P17.70** (a) Sound moves upwind with speed (343 – 15) m/s = 328 m/s. Crests pass a stationary upwind point at frequency 900 Hz. Then

$$\lambda = \frac{v}{f} = \frac{328\ \text{m/s}}{900\ \text{s}^{-1}} = \boxed{0.364\ \text{m}}$$

(b) By similar logic, $\lambda = \dfrac{v}{f} = \dfrac{(343+15)\ \text{m/s}}{900\ \text{s}^{-1}} = \boxed{0.398\ \text{m}}$

(c) The source is moving through the air at 15 m/s toward the observer. The observer is stationary relative to the air.

$$f' = f\left(\frac{v+v_o}{v-v_s}\right) = (900\ \text{Hz})\left(\frac{343\ \text{m/s}+0}{343\ \text{m/s}-15.0\ \text{m/s}}\right) = \boxed{941\ \text{Hz}}$$

(d) The source is moving through the air at 15 m/s away from the downwind firefighter. Her speed relative to the air is 30 m/s toward the source.

$$f' = f\left(\frac{v+v_o}{v-v_s}\right) = (900\ \text{Hz})\left(\frac{343\ \text{m/s}+30.0\ \text{m/s}}{343\ \text{m/s}-(-15.0\ \text{m/s})}\right)$$

$$= (900\ \text{Hz})\left(\frac{373\ \text{m/s}}{358\ \text{m/s}}\right) = \boxed{938\ \text{Hz}}$$

Challenge Problems

P17.71 (a) If $v_O = 0$ m/s, then $f' = \dfrac{v}{v - v_S \cos\theta_S} f$.

Also, when the train is 40.0 m from the intersection, and the car is 30.0 m from the intersection,

$$\cos\theta_S = \frac{4}{5}$$

so $$f' = \frac{343 \text{ m/s}}{343 \text{ m/s} - 0.800(25.0 \text{ m/s})}(500 \text{ Hz}),$$

or $$f' = \boxed{531 \text{ Hz}}.$$

(b) Note that as the train approaches, passes, and departs from the intersection, θ_S varies from 0° to 180° and the frequency heard by the observer varies between the limits

$$f'_{\text{max}} = \frac{v}{v - v_S \cos 0°} f = \frac{343 \text{ m/s}}{343 \text{ m/s} - 25.0 \text{ m/s}}(500 \text{ Hz})$$
$$= \boxed{539 \text{ Hz}}$$

to

$$f'_{\text{min}} = \frac{v}{v - v_S \cos 180°} f = \frac{343 \text{ m/s}}{343 \text{ m/s} + 25.0 \text{ m/s}}(500 \text{ Hz})$$
$$= \boxed{466 \text{ Hz}}$$

(c) Now $v_O = +40.0$ m/s, and the train is 40.0 m from the intersection, and the car is 30.0 m from the intersection, so

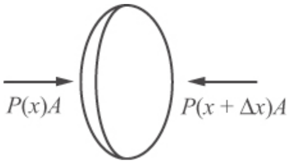
$$\cos\theta_O = \frac{3}{5}$$

$$f' = \frac{343 \text{ m/s} + 0.600(40.0 \text{ m/s})}{343 \text{ m/s} - 0.800(25.0 \text{ m/s})}(500 \text{ Hz}) = \boxed{568 \text{ Hz}}$$

P17.72 (a) ANS. FIG. P17.72 shows a force diagram of an element of gas indicating the forces exerted on the left and right surfaces due to the pressure of the gas on either side of the element.

(b) Let $P(x)$ represent absolute pressure as a function of x. The net force to the right on the chunk of air is $+P(x)A - P(x + \Delta x)A$. Atmospheric pressure subtracts out, leaving

$$\left[-\Delta P(x + \Delta x) + \Delta P(x)\right]A = -\frac{\partial \Delta P}{\partial x}\Delta x A$$



ANS. FIG. P17.72

The mass of the air is $\Delta m = \rho \Delta V = \rho A \Delta x$ and its acceleration is $\frac{\partial^2 s}{\partial t^2}$. So Newton's second law becomes

$$-\frac{\partial \Delta P}{\partial x}\Delta x A = \rho A \Delta x \frac{\partial^2 s}{\partial t^2}$$

(c) From the result above, we have

$$-\frac{\partial \Delta P}{\partial x}\Delta x A = \rho A \Delta x \frac{\partial^2 s}{\partial t^2} \quad \rightarrow \quad -\frac{\partial \Delta P}{\partial x} = \rho \frac{\partial^2 s}{\partial t^2}$$

Substituting $\Delta P = -(B \partial s / \partial x)$ (Eq. 17.3), we have

$$-\frac{\partial}{\partial x}\left(-B\frac{\partial s}{\partial x}\right) = \rho \frac{\partial^2 s}{\partial t^2} \quad \rightarrow \quad \frac{B}{\rho}\frac{\partial^2 s}{\partial x^2} = \frac{\partial^2 s}{\partial t^2}$$

(d) Into this wave equation we substitute a trial solution $s(x,\ t) = s_{\text{max}} \cos(kx - \omega t)$. We find

$$\frac{\partial s}{\partial x} = -k s_{\text{max}} \sin(kx - \omega t)$$
$$\frac{\partial^2 s}{\partial x^2} = -k^2 s_{\text{max}} \cos(kx - \omega t)$$
$$\frac{\partial s}{\partial t} = +\omega s_{\text{max}} \sin(kx - \omega t)$$
$$\frac{\partial^2 s}{\partial t^2} = -\omega^2 s_{\text{max}} \cos(kx - \omega t)$$

$\frac{B}{\rho}\frac{\partial^2 s}{\partial x^2} = \frac{\partial^2 s}{\partial t^2}$ becomes

$$-\frac{B}{\rho} k^2 s_{\text{max}} \cos(kx - \omega t) = -\omega^2 s_{\text{max}} \cos(kx - \omega t)$$

This is true provided that $\frac{B}{\rho}k^2 = \omega^2 \rightarrow \frac{\omega}{k} = \sqrt{\frac{B}{\rho}}$, that is, provided it propagates with speed $v = \sqrt{\frac{B}{\rho}}$.

P17.73 Figure 17.10 shows that each wavefront that passes the observer is spherical. Let T represent the period of the source vibration, and T_{MW} be the energy put into each wavefront during one vibration. Then $(Power)_{\text{avg}} = \frac{T_{\text{MW}}}{T}$. At the moment when the observer is at distance r in front of the source, he is receiving a spherical wavefront of radius $R_w = v\Delta t$, where Δt is the time interval since this energy was radiated. Since the wavefront was radiated, the source has moved forward distance $d_s = v_s \Delta t$, so the total distance the wavefront has traveled is

$$R_w = r + d_s \quad \rightarrow \quad v\Delta t = r + v_s \Delta t$$

therefore,

$$\Delta t = \frac{r}{v - v_S}$$

The surface area of the sphere is $4\pi R_w^2 = 4\pi (v\Delta t)^2 = \frac{4\pi v^2 r^2}{(v - v_S)^2}$. The energy per unit area emitted during one cycle and carried by one spherical wavefront is uniform with the value

$$I = \frac{T_{\text{MW}}}{A} = \frac{(Power)_{\text{avg}} T (v - v_S)^2}{4\pi v^2 r^2}$$

The energy carried by the wavefront passes the observer in the time interval $T' = 1/f'$, where f' is the Doppler-shifted frequency

$$f' = f\left(\frac{v}{v - v_S}\right) = \frac{v}{T(v - v_S)}$$

so the observer receives a wave with intensity

$$I = \left(\frac{T_{\text{MW}}}{A}\right)\frac{1}{T'} = \left(\frac{T_{\text{MW}}}{A}\right) f' = \left(\frac{(Power)_{\text{avg}} T (v - v_S)^2}{4\pi v^2 r^2}\right)\left(\frac{v}{T(v - v_S)}\right)$$

$$I = \boxed{\frac{(Power)_{\text{avg}}}{4\pi r^2}\left(\frac{v - v_S}{v}\right)}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P17.2 (a) 1.27 Pa; (b) 170 Hz; (c) 2.00 m; (d) 340 m/s

P17.4 5.81 m

P17.6 1×10^{11} Pa

P17.8 (a) The speed gradually changes from $v = (331\text{ m/s})(1 + 27°\text{C}/273°\text{C})^{1/2} = 347$ m/s to $(331\text{ m/s})(1 + 0/273°\text{C})^{1/2} = 331$ m/s, a 4.6% decrease. The cooler air at the same pressure is more dense; (b) The frequency is unchanged because every wave crest in the hot air becomes one crest without delay in the cold air; (c) The wavelength decreases by 4.6%, from *v/f* = (347 m/s) (4 000/s) = 86.7 mm to (331 m/s)(4 000/s) = 82.8 mm. The crests are more crowded together when they move more slowly.

P17.10 1.55×10^{-10} m

P17.12 (a) 153 m/s; (b) 614 m

P17.14 $d - \frac{1}{2}g\left(\sqrt{\frac{2(d-h)}{g}} - \Delta t - \frac{d-h}{v}\right)^2$ above the ground

P17.16 See P17.16 for complete solution.

P17.18 (a) 833 m; (b) 1.47 s

P17.20 (a) 5.00×10^{-5} W; (b) 3.00×10^{-3} J

P17.22 (a) $I_2 = \left(\frac{f'}{f}\right)^2 I_1$; (b) intensity is unchanged

P17.24 0.082 W/m^2

P17.26 150 dB

P17.28 (a) 332 J; (b) 46.4 dB

P17.30 $20\log\left(\frac{r_1}{r_2}\right)$

P17.32 (a) 65.0 dB; (b) 67.8 dB; (c) 69.6 dB

P17.34 (a) 1.76 kJ; (b) 108 dB

P17.36 We assume that both lawn mowers are equally loud and approximately the same distance away. We found in Example 17.3 that a sound of twice the intensity results in an increase in sound level of 3 dB. We also see from the What If? section of that example that a doubling of loudness requires a 10-dB increase in sound level. Therefore, the sound of two lawn mowers will not be twice the loudness, but only a little louder than one!

P17.38 2.82×10^8 m/s

P17.40 (a) B; (b) positive; (c) negative; (d) 1 533 m/s; (e) 5.30×10^3 Hz

P17.42 (a) $\dfrac{vf}{v - A\sqrt{\dfrac{k}{m}}}$; (b) $\dfrac{vf}{v + A\sqrt{\dfrac{k}{m}}}$; (c) $\beta - (20\text{ dB})\log\left(1 + \dfrac{2A}{d}\right)$

P17.44 This is much faster than a human athlete can run.

P17.46 19.7 m

P17.48 5.67 mm

P17.50 (a) 0.232 m; (b) 8.41×10^{-8} m; (c) 13.8 mm

P17.52 0.642 W

P17.54 (a) $\dfrac{2\frac{u}{v}}{1 - \frac{u^2}{v^2}} f$; (b) 85.9 Hz

P17.56 (a) 4.63 mm; (b) 14.5 m/s; (c) 4.73×10^9 W/m^2

P17.58 (a) The wave moves outward equally in all directions; (b) Its amplitude is inversely proportional to its distance from the center. Its intensity is proportional to the square of the amplitude, so the intensity follows the inverse-square law, with no absorption of energy by the medium; (c) Its speed is constant $v = f\lambda = \omega/k = (2\,030/\text{s})(1.36/\text{m}) = 1.49\,\text{km/s}$. By comparison to the table, it can be moving through water at 25° C, and we assume it is; (d) Its frequency is constant at $(2\ 030/\text{s})/2\pi = 323$ Hz; (e) Its wavelength is constant at $2\pi/\text{k} = 2\pi/(1.36/\text{m}) = 4.62$ m;

(f) $P = I4\pi r^2 = \left(\dfrac{2.09 \times 10^{-4}\ \text{W/m}^2}{r^2}\right)4\pi r^2 = 2.63$ mW; (g) Its intensity follows the inverse-square law; at $r = 1$ m, the intensity is $209\ \mu\text{W/m}^2$

P17.60 (a) The repeated reflections from the steps create a repetition frequency so that the ear/brain combination assigns a pitch to the sound heard by the listener; (b) ~ a few hundred Hz; (c) ~ 1 m; (d) ~ 0.1 s

P17.62 (a) The distance is larger by 240 / 60 = 4 times. The intensity is 16 times smaller at the larger distance because the sound power is spread over a 4^2 times larger area; (b) The amplitude is 4 times smaller at the larger distance because intensity is proportional to the square of amplitude; (c) The extra distance is (240 – 60) / 45 = 4 wavelengths. The phase is the same at both points because they are separated by an integer number of wavelengths

P17.64 1 204.2 Hz

P17.66 (a) The sound through the metal arrives first because it moves faster than sound in air; (b) $L = 380\Delta t$, where Δt is in seconds and the length is in meters; (c) 48.2 m; (d) The answer becomes $L = \dfrac{\Delta t}{\dfrac{1}{343} - \dfrac{1}{v_r}}$ where v_r is the speed of sound in the rod in meters per second, Δt is in seconds, and *L* is in meters; (e) As v_r goes to infinity, the travel time in the rod becomes negligible. The answer approaches $343\Delta t$ which is just the distance that the sound travels in air during the delay time

P17.68 6.45

P17.70 (a) 0.364 m; (b) 0.398 m; (c) 941 Hz; (d) 938 Hz

P17.72 (a) See ANS. FIG P17.72; (b) See P17.72(b) for full explanation; (c) See P17.72(c) for full explanation; (d) See P17.72(d) for full explanation.

18

Superposition and Standing Waves

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ18.1 The ranking is (d) > (a) = (c) > (b). In the starting situation, the waves interfere constructively. When the sliding section is moved out by 0.1 m, the wave going through it has an extra path length of 0.2 m = $\lambda/4$, to show partial interference. When the slide has come out 0.2 m from the starting configuration, the extra path length is 0.4 m = $\lambda/2$, for destructive interference. Another 0.1 m and we are at $r_2 - r_1 = 3\lambda/4$ for partial interference as before. At last, another equal step of sliding and one wave travels one wavelength farther to interfere constructively.

OQ18.2 The fundamental frequency is described by

$$f_1 = \frac{v}{2L}, \text{ where } v = \left(\frac{T}{\mu}\right)^{1/2}$$

(i) Answer (e). If L is doubled, then the wavelength of the fundamental frequency is doubled, then $f = v/\lambda$ will be reduced by a factor of $\frac{1}{2}$.

(ii) Answer (d). If μ is doubled, then the speed is reduced by a factor of $\frac{1}{\sqrt{2}}$, so $f = v/\lambda$ will be reduced by a factor of $\frac{1}{\sqrt{2}}$.

(iii) Answer (b). If T is doubled, then the speed is increased by a factor of $\sqrt{2}$, so $f = v/\lambda$ will increase by a factor of $\sqrt{2}$.

OQ18.3 Answer (c). The two waves must have slightly different amplitudes at P because of their different distances, so they cannot cancel each other exactly.

OQ18.4 (i) Answer (e). If the end is fixed, there is inversion of the pulse upon reflection. Thus, when they meet, they cancel and the amplitude is zero.

(ii) Answer (c). If the end is free, there is no inversion on reflection. When they meet, the amplitude is $2A = 2(0.1 \text{ m}) = 0.2 \text{ m}$.

OQ18.5 Answer (a). At resonance, a tube closed at one end and open at the other forms a standing wave pattern with a node at the closed end and antinode at the open end. In the fundamental mode (or first harmonic), the length of the tube closed at one end is a quarter wavelength ($L = \lambda_1/4$ or $\lambda_1 = 4L$). Therefore, for the given tube, $\lambda_1 = 4(0.580 \text{ m}) = 2.32 \text{ m}$ and the fundamental frequency is

$$f_1 = \frac{v}{\lambda_1} = \frac{343 \text{ m/s}}{2.32 \text{ m}} = 148 \text{ Hz}$$

OQ18.6 Answer (e). The number of beats per second (the beat frequency) equals the difference in the frequencies of the two tuning forks. Thus, if the beat frequency is 5 Hz and one fork is known to have a frequency of 245 Hz, the frequency of the second fork could be either $f_2 = 245 \text{ Hz} - 5 \text{ Hz} = 240 \text{ Hz}$ or $f_2 = 245 \text{ Hz} + 5 \text{ Hz} = 250 \text{ Hz}$. This means that the best answer for the question is choice (e), since choices (a) and (d) are both possibly correct.

OQ18.7 Answer (d). The tape will reduce the frequency of the fork, leaving the string frequency unchanged. If the bit of tape is small, the fork must have started with a frequency 4 Hz below that of the string, to end up with a frequency 5 Hz below that of the string. The string frequency is 262 + 4 = 266 Hz.

OQ18.8 Answer (c). The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonics will not be excited because they have a node at the point where the string exhibits its maximum displacement.

OQ18.9 Answer (d). The energy has not disappeared, but is still carried by the wave pulses. Each element of the string still has kinetic energy. This is similar to the motion of a simple pedulum. The pendulum does not stop at its equilibrium position during oscillation—likewise the elements of the string do not stop at the equilibrium position of the string when these two waves superimpose.

OQ18.10 Answer (c). On a string fixed at both ends, a standing wave with three nodes is the second harmonic: there is a node on each end and one in the middle, so it has two antinodes because there is an antinode between each pair of nodes. The number of antinodes is the same as the harmonic number. Doubling the frequency gives the fourth harmonic, therefore four antinodes.

OQ18.11 Answers (b) and (e). The strings have different linear densities and are stretched to different tensions, so they carry string waves with different speeds and vibrate with different fundamental frequencies. They are all equally long, so the string waves have equal fundamental wavelengths. They all radiate sound into air, where the sound moves with the same speed for different sound wavelengths.

OQ18.12 Answer (d). The resultant amplitude is greater than either individual amplitude, wherever the two waves are nearly enough in phase that $2A\cos(\phi/2)$ is greater than A. This condition is satisfied whenever the absolute value of the phase difference ϕ between the two waves is less than 120°.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ18.1 The resonant frequency depends on the length of the pipe. Thus, changing the length of the pipe will cause different frequencies to be emphasized in the resulting sound.

CQ18.2 No. The total energy of the pair of waves remains the same. Energy missing from zones of destructive interference appears in zones of constructive interference.

CQ18.3 What is needed is a tuning fork—or other pure-tone generator—of the desired frequency. Strike the tuning fork and pluck the corresponding string on the piano at the same time. If they are precisely in tune, you will hear a single pitch with no amplitude modulation. If the two frequences are a bit off, you will hear beats. As they vibrate, retune the piano string until the beat frequency goes to zero.

CQ18.4 Damping, and nonlinear effects in the vibration, transform the energy of vibration into internal energy.

CQ18.5 (a) The tuning fork hits the paper repetitively to make a sound like a buzzer, and the paper efficiently moves the surrounding air. The tuning fork will vibrate audibly for a shorter time.

(b) Instead of just radiating sound very softly into the surrounding air, the tuning fork makes the chalkboard vibrate. With its large area this stiff sounding board radiates sound into the air with higher power. So it drains away the fork's energy of vibration faster and the fork stops vibrating sooner.

(c) The tuning fork in resonance makes the column of air vibrate, especially at the antinode of displacement at the top of the tube. Its area is larger than that of the fork tines, so it radiates louder sound into the environment. The tuning fork will not vibrate for so long.

(d) The cardboard acts to cut off the path of air flow from the front to the back of a single tine. When a tine moves forward, the high pressure air in front of the tine can simply move to fill in the lower pressure area behind the tine. This "sloshing" of the air back and forth does not contribute to sound radiation and results in low intensity of sound actually leaving the tine. By cutting off this "sloshing" path by bringing the cardboard near, the tine becomes a more efficient radiator. This is the same theory as that involved with placing loudspeakers on baffles. A speaker enclosure for a loudspeaker is equivalent to an infinite baffle because there is no path the high pressure air can find to cancel the lower pressure air on the other side of the speaker.

CQ18.6 The loudness varies because of beats. The propellers are rotating at slightly different frequencies.

CQ18.7 Walking makes the person's hand vibrate a little. If the frequency of this motion is equal to the natural frequency of coffee sloshing from side to side in the cup, then a large-amplitude vibration of the coffee will build up in resonance. To get off resonance and back to the normal case of a small-amplitude disturbance producing a small-amplitude result, the person can walk faster, walk slower, or get a larger or smaller cup. You do not need a cover on your cup.

CQ18.8 Consider the level of fluid in the bottle to be adjusted so that the air column above it resonates at the first harmonic. This is given by $f=\dfrac{v}{4L}$. This equation indicates that as the length L of the column increases (fluid level decreases), the resonant frequency decreases.

CQ18.9 No. Waves with all waveforms interfere. Waves with other wave shapes are also trains of disturbance that add together when waves from different sources move through the same medium at the same time.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 18.1 Analysis Model: Waves in Interference

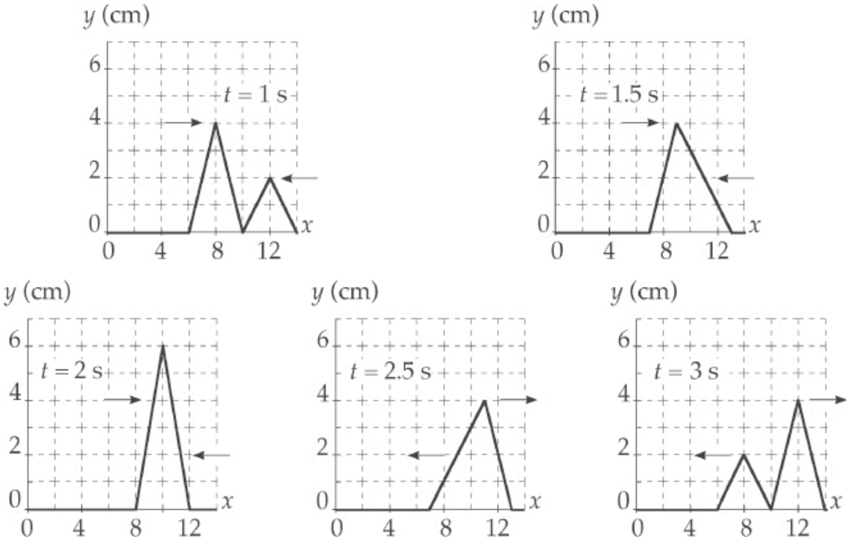
P18.1 Suppose the waves are sinusoidal. The sum is

$$(4.00\text{ cm})\sin(kx-\omega t)+(4.00\text{ cm})\sin(kx-\omega t+90.0°)$$
$$=2(4.00\text{ cm})\sin(kx-\omega t+45.0°)\cos 45.0°$$

So the amplitude of the resultant wave is

$$(8.00\text{ cm})\cos 45.0° = \boxed{5.66\text{ cm}}$$

P18.2 ANS. FIG. P18.2 shows the sketches at each of the times.



ANS. FIG. P18.2

P18.3 The superposition of the waves is given by

$$y=y_1+y_2=3.00\cos(4.00x-1.60t)+4.00\sin(5.00x-2.00t)$$

evaluated at the given *x* values.

(a) At $x = 1.00$, $t = 1.00$, the superposition of the two waves gives

$$y = 3.00\cos\left[4.00(1.00) - 1.60(1.00)\right] + 4.00\sin\left[5.00(1.00) - 2.00(1.00)\right]$$
$$= 3.00\cos(2.40 \text{ rad}) + 4.00\sin(3.00 \text{ rad}) = \boxed{-1.65 \text{ cm}}$$

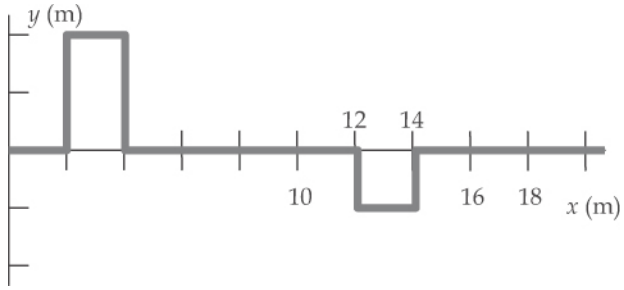
(b) At $x = 1.00$, $t = 0.500$, the superposition of the two waves gives

$$y = 3.00\cos\left[4.00(1.00) - 1.60(0.500)\right] + 4.00\sin\left[5.00(1.00) - 2.00(0.500)\right]$$
$$= 3.00\cos(3.20 \text{ rad}) + 4.00\sin(4.00 \text{ rad}) = \boxed{-6.02 \text{ cm}}$$

(c) At $x = 0.500$, $t = 0$, the superposition of the two waves gives

$$y = 3.00\cos\left[4.00(1.00) - 1.60(0)\right] + 4.00\sin\left[5.00(1.00) - 2.00(0)\right]$$
$$= 3.00\cos(2.00 \text{ rad}) + 4.00\sin(2.50 \text{ rad}) = \boxed{+1.15 \text{ cm}}$$

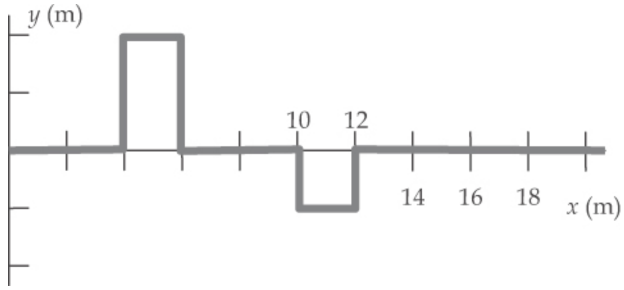
P18.4 (a) The graph at time $t = 0.00$ seconds is shown in ANS. FIG. P18.4(a)



ANS. FIG. P18.4(a)

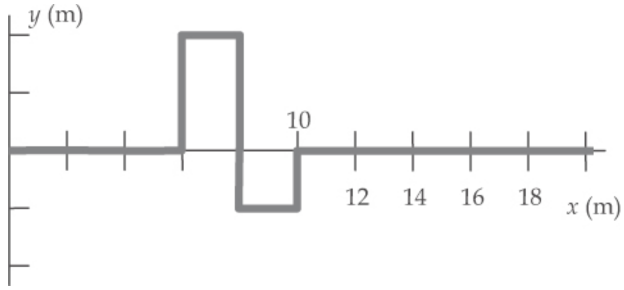
The pulse initially on the left will move to the right at 1.00 m/s, and the one initially at the right will move toward the left at the same rate, as follows:

ANS. FIG. P18.4(b) shows the pulses at time $t = 2.00$ seconds



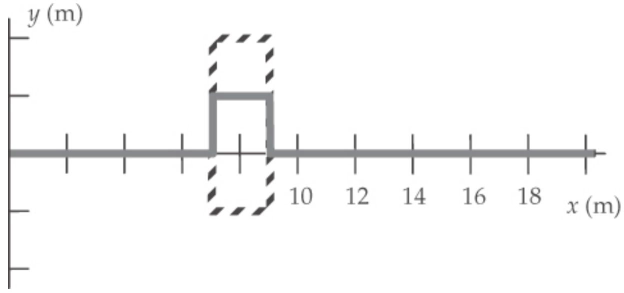
ANS. FIG. P18.4(b)

ANS. FIG. P18.4(c) shows the waves at time t = 4.00 seconds, immediately before they overlap.



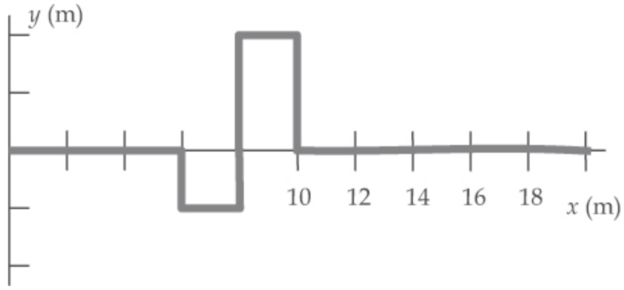
ANS. FIG. P18.4(c)

ANS. FIG. P18.4(d) shows the pulses at time t = 5.00 seconds, while the two pulses are fully overlapped. The two pulses are shown as dashed lines.



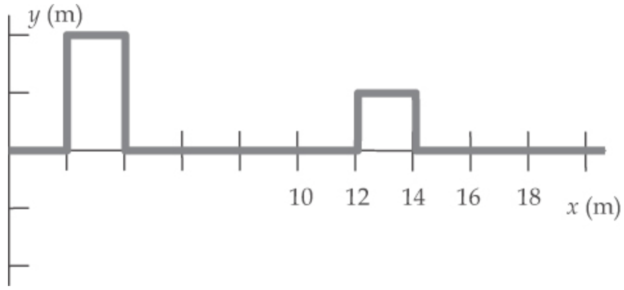
ANS. FIG. P18.4(d)

ANS. FIG. P18.4(e) shows the pulses at time At time t = 6.00 seconds, immediately after they completely pass.



ANS. FIG. P18.4(e)

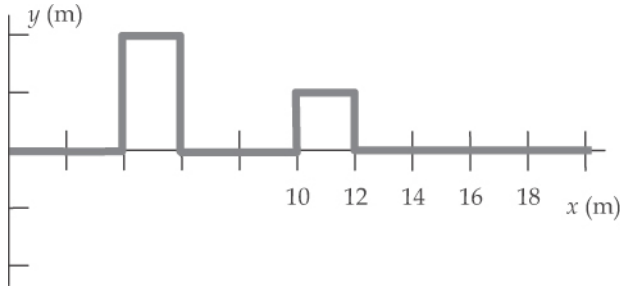
(b) If the pulse to the right is inverted, ANS. FIG. P18.4(f) shows the pulses at time *t* = 0.00 seconds.



ANS. FIG. P18.4(f)

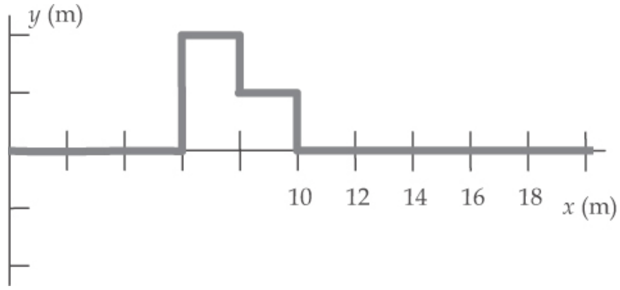
The pulse initially on the left will move to the right at 1.00 m / s, and the one initially at the right will move toward the left at the same rate, as follows:

ANS. FIG. P18.4(g) shows the two pulses at time *t* = 2.00 seconds



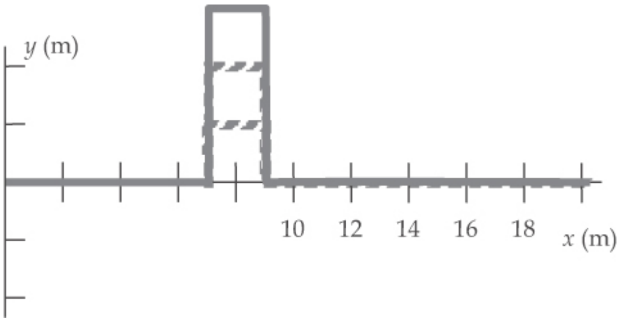
ANS. FIG. P18.4(g)

ANS. FIG. P18.4(h) shows the two pulses at time *t* = 4.00 seconds, immediately before they overlap.



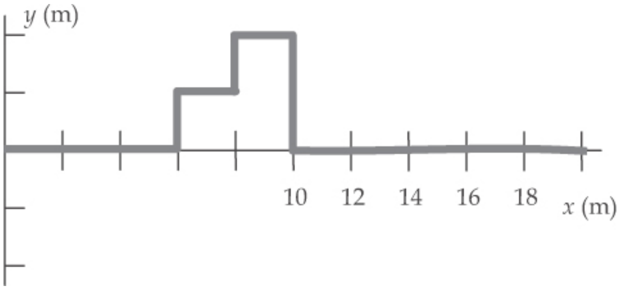
ANS. FIG. P18.4(h)

ANS. FIG. P18.4(i) shows the two pulses at time t = 5.00 seconds, while the two pulses are fully overlapped. The two pulses are shown as dashed lines.



ANS. FIG. P18.4(i)

ANS. FIG. P18.4(j) shows the two pulses at time t = 6.00 seconds, immediately after they completely pass.



ANS. FIG. P18.4(j)

***P18.5** Waves reflecting from the near end travel 28.0 m (14.0 m down and 14.0 m back), while waves reflecting from the far end travel 66.0 m. The path difference for the two waves is:

$$\Delta r = 66.0\text{ m} - 28.0\text{ m} = 38.0\text{ m}$$

Since $\lambda = \dfrac{v}{f}$,

Then

$$\frac{\Delta r}{\lambda} = \frac{(\Delta r)f}{v} = \frac{(38.0\text{ m})(246\text{ Hz})}{343\text{ m/s}} = 27.254$$

or $\Delta r = 27.254\lambda$

The phase difference between the two reflected waves is then

$$\phi = (0.254)(1\text{ cycle}) = (0.254)(2\pi\text{ rad}) = 1.594\text{ rad} = \boxed{91.3°}$$

P18.6 The wavelength of the sound emitted by the speaker is

$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{756\ \text{Hz}} \simeq 0.454\ \text{m}$$

Raising the sliding section by Δh changes the path through that section by $2\Delta h$, because sound must travel up and down through the addition distance.

(a) If constructive interference currently exists, this can be changed to destructive interference by increasing the path distance through the sliding section by $\lambda/2$, which means raising it by $\boxed{\lambda/4 = 0.113\ \text{m}}$.

(b) To move from constructive interference to the next occurrence of constructive interference, one should increase the path distance through the sliding section by λ, which means raising it by $\boxed{\lambda / 2 = 0.227\ \text{m}}$.

P18.7 (a) At constant phase, $\phi = 3x - 4t$ will be constant. Then $x = \dfrac{\phi + 4t}{3}$ will change: the wave moves. As t increases in this equation, x increases, so the first wave moves to the right, in the $\boxed{+x\ \text{direction}}$.

In the same way, in the second case $x = \dfrac{\phi - 4t + 6}{3}$. As t increases, x must decrease, so the second wave moves to the left, in the $\boxed{-x\ \text{direction}}$.

(b) We require that $y_1 + y_2 = 0$.

$$\frac{5}{(3x - 4t)^2 + 2} + \frac{-5}{(3x + 4t - 6)^2 + 2} = 0$$

This can be written as

$$(3x - 4t)^2 = (3x + 4t - 6)^2$$

Solving for the positive root, $8t = 6$, or

$$\boxed{t = 0.750\ \text{s}}$$

(c) The negative root yields

$$(3x - 4t) = -(3x + 4t - 6)$$

The time terms cancel, leaving $\boxed{x = 1.00\ \text{m}}$. At this point, the waves **always** cancel.

P18.8 (a) $\Delta x = \sqrt{9.00\text{ m}^2 + 4.00\text{ m}^2} - 3.00\text{ m} = \sqrt{13\text{ m}^2} - 3.00\text{ m} = 0.606\text{ m}$

The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{300\text{ Hz}} = 1.14\text{ m}$.

Thus, $\dfrac{\Delta x}{\lambda} = \dfrac{0.606}{1.14} = 0.530$ of a waves,

or $\Delta\phi = 2\pi(0.530) = \boxed{3.33\text{ rad}}$.

(b) For destructive interference, we want

$$\frac{\Delta x}{\lambda} = 0.500 \rightarrow \lambda = \frac{\Delta x}{0.500} = 2\Delta x$$

The frequency is $f = \dfrac{v}{\lambda} = \dfrac{v}{2\Delta x} = \dfrac{343\text{ m/s}}{2(0.606\text{ m})} = \boxed{283\text{ Hz}}$.

P18.9 The sum of two waves traveling in the same direction that have the same amplitude A_0, angular frequency ω, and wave number k but are different in phase ϕ have the resultant wave function in the form

$$y = 2A_0 \cos\left(\frac{\phi}{2}\right) \sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

(a) $A = 2A_0 \cos\left(\dfrac{\phi}{2}\right) = 2(5.00\text{ m}) \cos\left[\dfrac{-\pi/4}{2}\right] = \boxed{9.24\text{ m}}$

(b) $f = \dfrac{\omega}{2\pi} = \dfrac{1\,200\pi\text{ rad/s}}{2\pi} = \boxed{600\text{ Hz}}$

P18.10 Consider the geometry of the situation shown on the right. The path difference for the sound waves at the location of the man is

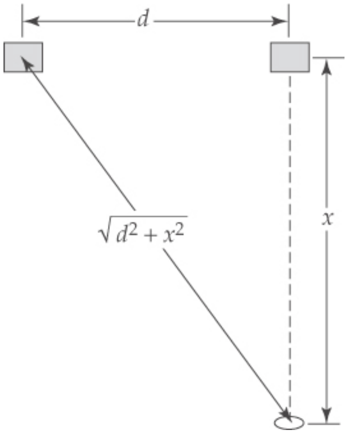
$$\Delta r = \sqrt{d^2 + x^2} - x$$

For a minimum, this path difference must equal a half-integral number of wavelengths:

$$\sqrt{d^2 + x^2} - x = \left(n + \tfrac{1}{2}\right)\lambda \qquad n = 0, 1, 2, \ldots$$

Solve for x:

$$x = \frac{d^2 - \left[\left(n + \frac{1}{2}\right)\lambda\right]^2}{2\left(n + \frac{1}{2}\right)\lambda}$$



ANS. FIG. P18.10

In order for *x* to be positive, we must have

$$\left[\left(n+\frac{1}{2}\right)\lambda\right]^2 < d^2 \quad \rightarrow \quad n < \frac{d}{\lambda} - \frac{1}{2} = \frac{df}{v} - \frac{1}{2}$$

Substitute numerical values:

$$n < \frac{(4.00\ \text{m})(200\ \text{Hz})}{343\ \text{m/s}} - \frac{1}{2} = 1.83$$

The only values of *n* that satisfy this requirement are $n = 0$ and $n = 1$.

Therefore,

$$\boxed{\begin{array}{l}\text{the man walks through only } \textit{two} \text{ minima;}\\ \text{a third minimum is impossible}\end{array}}$$

***P18.11** At any time and place, the phase shift between the waves is found by subtracting the phases of the two waves, $\Delta\phi = \phi_1 - \phi_2$.

$$\Delta\phi = (20.0\ \text{rad/cm})x - (32.0\ \text{rad/s})t$$
$$- [(25.0\ \text{rad/cm})x - (40.0\ \text{rad/s})t]$$

Collecting terms,

$$\Delta\phi = -(5.00\ \text{rad/cm})x + (8.00\ \text{rad/s})t$$

(a) At $x = 5.00$ cm and $t = 2.00$ s, the phase difference is

$$\Delta\phi = (-5.00\ \text{rad/cm})(5.00\ \text{cm}) + (8.00\ \text{rad/s})(2.00\ \text{s})$$

$$\Delta\phi = 9.00\ \text{radians} = 516° = \boxed{156°}$$

(b) The sine functions repeat whenever their arguments change by an integer number of cycles, an integer multiple of 2π radians. Then the phase shift equals $\pm\pi$ whenever $\Delta\phi = \pi + 2n\pi$, for all integer values of *n*. Substituting this into the phase equation, we have

$$\pi + 2n\pi = -(5.00\ \text{rad/cm})x + (8.00\ \text{rad/s})t$$

At $t = 2.00$ s,

$$\pi + 2n\pi = -(5.00\ \text{rad/cm})x + (8.00\ \text{rad/s})(2.00\ \text{s})$$

or $(5.00\ \text{rad/cm})x = (16.0 - \pi - 2n\pi)\ \text{rad}$

The smallest positive value of *x* is found when $n = 2$:

$$x = \frac{(16.0 - 5\pi)\ \text{rad}}{5.00\ \text{rad/cm}} = \boxed{0.058\ 4\ \text{cm}}$$

***P18.12** $2A_0 \cos\left(\frac{\phi}{2}\right) = A_0$ so $\frac{\phi}{2} = \cos^{-1}\left(\frac{1}{2}\right) = 60.0° = \frac{\pi}{3}$

Thus, the phase difference is

$$\phi = 120° = \frac{2\pi}{3}$$

This phase difference results if the time delay is

$$\frac{T}{3} = \frac{1}{3f} = \frac{\lambda}{3v}$$

$$\text{Time delay } = \frac{3.00\text{ m}}{3(2.00\text{ m/s})} = \boxed{0.500\text{ s}}$$

P18.13 (a) First we calculate the wavelength: $\lambda = \frac{v}{f} = \frac{344\ \text{m/s}}{21.5\text{ Hz}} = 16.0\text{ m}$

Then we note that the path difference equals

$$9.00\text{ m} - 1.00\text{ m} = \frac{1}{2}\lambda$$

Point A is one-half wavelength farther from one speaker than from the other. The waves from the two sources interfere destructively, so the receiver records a minimum in sound intensity.

(b) We choose the origin at the midpoint between the speakers. If the receiver is located at point (*x*, *y*), then we must solve:

$$\sqrt{(x+5.00)^2 + y^2} - \sqrt{(x-5.00)^2 + y^2} = \frac{1}{2}\lambda$$

Then,

$$\sqrt{(x+5.00)^2 + y^2} = \sqrt{(x-5.00)^2 + y^2} + \frac{1}{2}\lambda$$

Square both sides and simplify to get

$$20.0x - \frac{\lambda^2}{4} = \lambda\sqrt{(x-5.00)^2 + y^2}$$

Upon squaring again, this reduces to

$$400x^2 - 10.0\lambda^2 x + \frac{\lambda^4}{16.0} = \lambda^2(x-5.00)^2 + \lambda^2 y^2$$

Substituting, $\lambda = 16.0$ m, and reducing,

$$9.00x^2 - 16.0y^2 = 144$$

Note that the equation $9.00x^2 - 16.0y^2 = 144$ represents two hyperbolas: one passes through the x axis at $x = +4.00$ m; the second, which is the mirror image of the first, passes through $x = -4.00$ m to the left of the y axis.

(c) Solve for y in terms of x:

$$9x^2 - 16y^2 = 144$$

Then

$$y = \pm\sqrt{\frac{9}{16}x^2 - 9} = \pm\frac{3}{4}x\sqrt{1 - \frac{16}{x^2}}$$

$$y = \pm\frac{3}{4}x\sqrt{1 - \frac{16}{x^2}}$$

For very large x, the square root term approaches 1:

$$y = \pm\frac{3}{4}x\sqrt{1 - \frac{16}{x^2}} \quad \rightarrow \quad y = \pm\frac{3}{4}x$$

To the right of the origin, for large x the hyperbola approaches the shape of a straight line above and below the x axis.

Yes; the limiting form of the path is two straight lines through the origin with slope ±0.75.

Section 18.2 Standing Waves

P18.14 (a) From the resultant wave $y = 2A\sin\left(kx + \frac{\phi}{2}\right)\cos\left(\omega t - \frac{\phi}{2}\right)$,

the shape of the wave form is determined by the term

$\sin\left(kx + \frac{\phi}{2}\right)$.

The nodes are located at $kx + \frac{\phi}{2} = n\pi$, or where $x = \frac{n\pi}{k} - \frac{\phi}{2k}$.

The separation of adjacent nodes is

$$\Delta x = \left[(n+1)\frac{\pi}{k} - \frac{\phi}{2k}\right] - \left[\frac{n\pi}{k} - \frac{\phi}{2k}\right] = \frac{\pi}{k} = \frac{\lambda}{2}$$

The nodes are still separated by half a wavelength.

(b) $\boxed{\text{Yes. The nodes are located at } kx + \frac{\phi}{2} = n\pi\text{, so that } x = \frac{n\pi}{k} - \frac{\phi}{2k}\text{, which means that each node is shifted } \frac{\phi}{2r} \text{ to the left by the phase difference between the traveling waves in comparison to the case in which } \phi = 0.}$

P18.15 $y = (1.50\text{ m})\sin(0.400x)\cos(200t) = 2A_0 \sin kx \cos \omega t$

Compare corresponding parts:

(a) $k = \frac{2\pi}{\lambda} = 0.400\ \text{rad/m}$

$$\lambda = \frac{2\pi}{0.400\ \text{rad/m}} = \boxed{15.7\ \text{m}}$$

(b) $\omega = 2\pi f$ so $f = \frac{\omega}{2\pi} = \frac{200\ \text{rad/s}}{2\pi\ \text{rad}} = \boxed{31.8\ \text{Hz}}$

(c) The speed of waves in the medium is

$$v = \lambda f = \frac{\lambda}{2\pi} 2\pi k f = \frac{\omega}{k} = \frac{200\ \text{rad/s}}{0.400\ \text{rad/m}} = \boxed{500\ \text{m/s}}$$

P18.16 From $y = 2A_0 \sin kx \cos \omega t,$ we find

$$\frac{\partial y}{\partial x} = 2A_0 k \cos kx \cos \omega t \qquad \frac{\partial y}{\partial t} = -2A_0 \omega \sin kx \sin \omega t$$

$$\frac{\partial^2 y}{\partial x^2} = -2A_0 k^2 \sin kx \cos \omega t$$

$$\frac{\partial^2 y}{\partial t^2} = -2A_0 \omega^2 \sin kx \cos \omega t$$

Substitution into the wave equation gives

$$-2A_0 k^2 \sin kx \cos \omega t = \left(\frac{1}{v^2}\right)\left(-2A_0 \omega^2 \sin kx \cos \omega t\right)$$

This is satisfied, provided that $v = \frac{\omega}{k}$. But this is true, because

$$v = \lambda f = \frac{\lambda}{2\pi} 2\pi f = \frac{\omega}{k}$$

P18.17 $y_1 = 3.00 \sin\left[\pi\left(x + 0.600t\right)\right]$; $y_2 = 3.00 \sin\left[\pi\left(x - 0.600t\right)\right]$

$$y = y_1 + y_2 = \left[3.00 \sin\left(\pi x\right)\cos\left(0.600\pi t\right) + 3.00 \sin\left(\pi x\right)\cos\left(0.600\pi t\right)\right]$$

$$y = \left(6.00 \text{ cm}\right)\sin\left(\pi x\right)\cos\left(0.600\pi t\right)$$

We can take $\left|\cos\left(0.600\pi t\right)\right| = 1$ to get the maximum *y*.

(a) At $x = 0.250$ cm, $\left|y_{\max}\right| = \left|\left(6.00 \text{ cm}\right)\sin\left(0.250\pi\right)\right| = \boxed{4.24 \text{ cm}}$

(b) At $x = 0.500$ cm, $\left|y_{\max}\right| = \left|\left(6.00 \text{ cm}\right)\sin\left(0.500\pi\right)\right| = \boxed{6.00 \text{ cm}}$

(c) At $x = 1.50$ cm, $\left|y_{\max}\right| = \left|\left(6.00 \text{ cm}\right)\sin\left(1.50\pi\right)\right| = \boxed{6.00 \text{ cm}}$

(d) The antinodes occur where

$$\sin\left(\pi x\right) = \pm 1 \quad \rightarrow \quad \pi x = n\frac{\pi}{2}$$

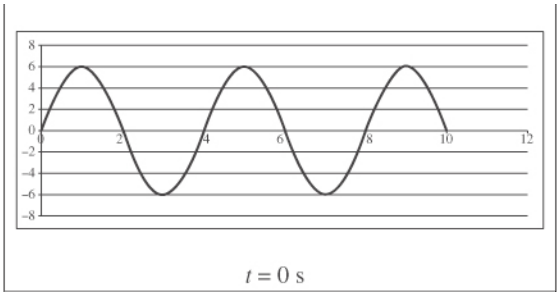
or where $x = \dfrac{n}{2}$, where $n = 1,\ 3,\ 5,\ 7,\ldots$ and *x* is in centimeters.

$$n = 1\text{:}\quad x_1 = \frac{1}{2} = \boxed{0.500 \text{ cm}} \quad \text{as in (b)}$$

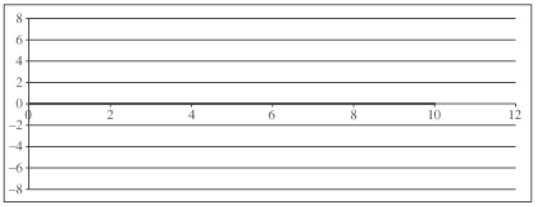
$$n = 3\text{:}\quad x_2 = \frac{3}{2} = \boxed{1.50 \text{ cm}} \quad \text{as in (c)}$$

$$n = 5\text{:}\quad x_3 = \frac{5}{2} = \boxed{2.50 \text{ cm}}$$

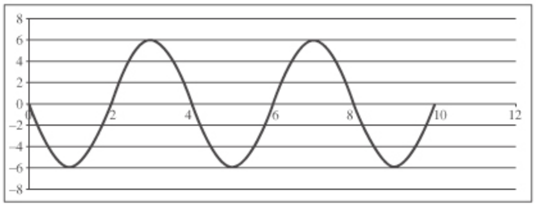
P18.18 (a) ANS. FIG. P18.18 shows the graphs for *t* = 0, *t* = 5 ms, *t* =10 ms, *t* = 15 ms, and *t* = 20 ms. The units of the *x* and *y* axes are meters.



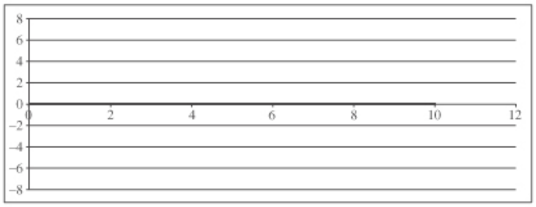
t = 0 s



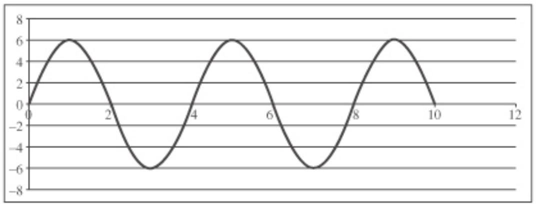
t = 5 ms



t = 10 ms



t = 15 ms



t = 20 ms

ANS. FIG. P18.18

(b) In any one picture, the wavelength is the smallest distance along the x axis that contains a nonrepeating shape. The wavelength is $\lambda = 4$ m.

(c) The frequency is the inverse of the period. The period is the time the wave takes to go from a full amplitude starting shape to the inversion of that shape and then back to the original shape. The period is the time interval between the top and bottom graphs: 20 ms. The frequency is $1/0.020$ s = 50 Hz.

(d) 4 m. By comparison with the wave function $y = (2A \sin kx) \cos \omega t$, we identify $k = \pi/2$, and then compute $\lambda = 2\pi/k$.

(e) $\boxed{\begin{array}{l}\text{50 Hz. By comparison with the wave function}\\ y=(2A\sin kx)\cos\omega t,\\ \text{we identify } \omega=2\pi f=100\pi.\end{array}}$

P18.19 The facing speakers produce a standing wave in the space between them, with the spacing between nodes being

$$d_{\text{NN}}=\frac{\lambda}{2}=\frac{v}{2f}=\frac{343\ \text{m/s}}{2\left(800\ \text{s}^{-1}\right)}=0.214\ \text{m}$$

If the speakers vibrate in phase, the point halfway between them is an antinode of pressure at a distance from either speaker of

$$\frac{1.25\ \text{m}}{2}=0.625\ \text{m}$$

Then there is a node one-quarter of a wavelength away at

$$0.625-\frac{0.214}{2}=\boxed{0.518\ \text{m}}$$

from either speaker, after which, there is a node every half-wavelength:

a node at $0.518\ \text{m}-0.214\ \text{m}=\boxed{0.303\ \text{m}}$

a node at $0.303\ \text{m}-0.214\ \text{m}=\boxed{0.089\ 1\ \text{m}}$

a node at $0.518\ \text{m}+0.214\ \text{m}=\boxed{0.732\ \text{m}}$

a node at $0.732\ \text{m}+0.214\ \text{m}=\boxed{0.947\ \text{m}}$

and a node at $0.947\ \text{m}+0.214\ \text{m}=\boxed{1.16\ \text{m}}$ from either speaker.

Section 18.3 Analysis Model: Waves Under Boundary Conditions

***P18.20** We are given L = 120 cm, f = 120 Hz.

(a) For four segments, $L=2\lambda$ or $\lambda=60.0\ \text{cm}=\boxed{0.600\ \text{m}}$.

(b) $v=\lambda f=72.0\ \text{m/s}$, $f_1=\dfrac{v}{2L}=\dfrac{72.0\ \text{m/s}}{2(1.20\ \text{m})}=\boxed{30.0\ \text{Hz}}$

P18.21 Using L_v for the vibrating portion of the string of total length L,

$$f = \frac{v}{2L_v} = \frac{1}{2L_v}\sqrt{\frac{T}{\mu}} = \frac{1}{2L_v}\sqrt{\frac{MgL}{m}}$$

$$= \frac{1}{2(4.00\text{ m})}\sqrt{\frac{(4.00\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m})}{0.008\ 00\text{ kg}}} = \boxed{19.6\text{ Hz}}$$

P18.22 The frequency of vibration of a string is determined by the wave speed and the wavelength of the standing wave on the string. The length of the string and mode number n determines the size of the allowed wavelengths:

$$\lambda = 2L/n$$

$$f = \frac{v}{\lambda} = \frac{v}{2L/n} = n\frac{v}{2L}$$

As long as the wave speed does not change,

$$f \propto \frac{n}{L}$$

and so we may compare frequencies of vibrations for different modes and lengths of string:

$$\frac{f_2}{f_1} = \frac{n_2L_1}{n_1L_2}$$

When the string is pressed down on the fret, the wave speed on the string remains the same, but the length of the vibrating string is smaller. When the string is plucked, it vibrates at the fundamental frequency ($n = 1$) corresponding to the shorter length of the string. We can compare frequencies and length of vibrating string thus:

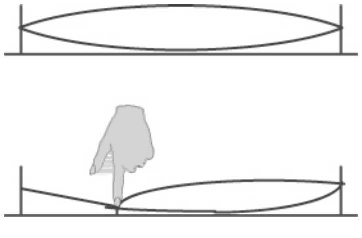
$$\frac{f_2}{f_1} = \frac{n_2L_1}{n_1L_2}$$

For the original length of string, $L_1 = L = 0.640$ m, $n_1 = 1$, and $f_1 = 330$ Hz.

(a) When the string is stopped at the fret, $L_2 = \frac{2}{3}L_1$, and $n_1 = n_2 = 1$.

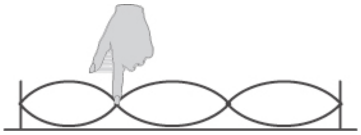
$$\frac{f_2}{f_1} = \frac{n_2L_1}{n_1L_2} = \frac{(1)L_1}{(1)\left(\frac{2}{3}L_1\right)} = \frac{3}{2}$$

$$f_2 = \frac{3}{2}f_1 = \boxed{495\text{ Hz}}$$



ANS. FIG. P18.22(a)

(b) The light touch at a point one-third of the way along the string forces the point of contact to be a node while still allowing the entire string to vibrate. The whole string vibrates in three loops; therefore, the string vibrates in its third resonance possibility ($n = 3$):

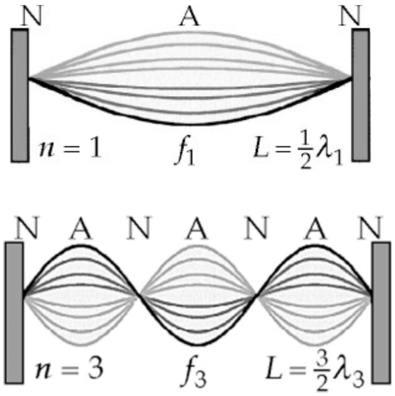


ANS. FIG. P18.22(b)

$$\frac{f_2}{f_1} = \frac{n_2 L_1}{n_1 L_2}$$

$$\frac{f_2}{f_1} = \frac{(3)L_1}{(1)L_1} \rightarrow f_2 = 3f_1 = \boxed{990\ \text{Hz}}$$

P18.23 When the string vibrates in the lowest frequency mode, the length of string forms a standing wave where $L = \lambda/2$, so the fundamental harmonic wavelength is



ANS. FIG. P18.23

$$\lambda = 2L = 2(0.700\ \text{m})$$
$$= 1.40\ \text{m}$$

and the speed is

$$v = \lambda f = (220\ \text{s}^{-1})(1.40\ \text{m})$$
$$= 308\ \text{m/s}$$

(a) From the tension equation

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{m/L}}$$

we get $T = v^2 m/L$,

or $$T = \frac{(308\ \text{m/s})^2 (1.20 \times 10^{-3}\ \text{kg})}{0.700\ \text{m}} = \boxed{163\ \text{N}}$$

(b) For the third harmonic, the tension, linear density, and speed are the same, but the string vibrates in three segments. Thus, the wavelength is one third as long as in the fundamental.

$$\lambda_3 = \lambda_1/3$$

From the equation $v = f\lambda$, we find the frequency is three times as high.

$$f_3 = \frac{v}{\lambda_3} = 3\frac{v}{\lambda_1} = 3f_1 = \boxed{660\ \text{Hz}}$$

P18.24 (a) Because the string is taut and is fixed at both ends, any standing waves will have nodes (which are multiples of $\lambda/2$ apart). The wavelengths of all possible modes on the string are:

$$\lambda_n = \frac{2L}{n}, \quad \text{where } n = 1, 2, 3, \ldots$$

The fundamental ($n = 1$) wavelength must then have a wavelength λ exactly twice the string length, or

$$\lambda_1 = \frac{2L}{1} = 2(2.60\text{ m}) = \boxed{5.20\text{ m}}$$

(b) $\boxed{\text{No. We do not know the speed of waves on the string.}}$ To obtain the frequencies on the string,

$$f_n = n\frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

it is necessary to have either the wave velocity v or the tension T and mass density μ of the string. We do not know these; therefore, it is not possible to find the frequency of this mode on the string.

P18.25 Because the piano string is fixed at both end, it will have nodes at each end, and also a node between the two antinodes. Thus, this standing wave pattern represents one full wavelength.

(a) Thus, this is $\boxed{\text{second harmonic}}$.

(b) And, because $\lambda_n = \frac{2L}{n}$, where $n = 1, 2, 3, \ldots$

The wavelength is $\lambda_2 = \frac{2L}{n} = \frac{2(74.0\text{ cm})}{2} = \boxed{74.0\text{ cm}}$.

(c) Because nodes are at both ends and in the middle, the number of nodes is $\boxed{3}$.

P18.26 The wave speed is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{20.0\text{ N}}{9.00\times10^{-3}\text{ kg/m}}} = 47.1\text{ m/s}$$

For a vibrating string of length L fixed at both ends, there are nodes at both ends. The wavelength of the fundamental is $\lambda = 2d_{NN} = 2L = 0.600$ m, and the frequency is

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L} = \frac{47.1\text{ m/s}}{0.600\text{ m}} = \boxed{78.6\text{ Hz}}$$

After NAN, the next three vibration possibilities read NANAN, NANANAN, and NANANANAN. Each has just one more node and one more antinode than the one before. Respectively, these string waves have wavelengths of one-half, one-third, and one-quarter of 60.0 cm. The harmonic frequencies are

$$f_2 = 2f_1 = \boxed{157\ \text{Hz}}$$

$$f_3 = 3f_1 = \boxed{236\ \text{Hz}}$$

$$f_4 = 4f_1 = \boxed{314\ \text{Hz}}$$

P18.27 (a) Let *n* be the number of nodes in the standing wave resulting from the 25.0-kg mass. Then *n* + 1 is the number of nodes for the standing wave resulting from the 16.0-kg mass. For standing waves, $\lambda = \frac{2L}{n}$ and the frequency is $f = \frac{v}{\lambda}$. The frequency does not change as the masses are changed.

Thus, $f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}}$ and also $f = \frac{n+1}{2L}\sqrt{\frac{T_{n+1}}{\mu}}$.

Equating the expressions for *f*, we have

$$\frac{n+1}{n} = \sqrt{\frac{T_n}{T_{n+1}}} = \sqrt{\frac{(25.0\ \text{kg})g}{(16.0\ \text{kg})g}} = \frac{5}{4}$$

Therefore, 4*n* + 4 = 5*n*, or *n* = 4. Using either expression for *f*, we find

$$f = \frac{4}{2(2.00\ \text{m})}\sqrt{\frac{(25.0\ \text{kg})(9.80\ \text{m/s}^2)}{0.002\,00\ \text{kg/m}}} = \boxed{350\ \text{Hz}}$$

(b) For tension $T_n = mg$, we write

$$f = \frac{n}{2L}\sqrt{\frac{T_n}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}} \quad \rightarrow \quad m = \frac{4L^2 f^2 \mu}{n^2 g}$$

We solve for *m* for *n* = 1:

$$m = \frac{4(2.00\ \text{m})^2(350\ \text{Hz})^2(0.002\,00\ \text{kg/m})}{(1)^2(9.80\ \text{m/s}^2)} = \boxed{400\ \text{kg}}$$

***P18.28** (a) For a standing wave of 6 loops, $6(\lambda/2) = L$, or

$$\lambda = L/3 = (2.00\ \text{m})/3$$

The speed of the waves in the string is then

$$v = \lambda f = \left(\frac{2.00\text{ m}}{3}\right)\left(150\text{ Hz}^{-1}\right) = 1.00\times 10^{2}\text{ m/s}$$

Since the tension in the string is

$$F = mg = (5.00\text{ kg})\left(9.80\text{ m/s}^2\right) = 49.0\text{ N}$$

$v = \sqrt{\dfrac{F}{\mu}}$ gives

$$\mu = \frac{F}{v^2} = \frac{49.0\text{ N}}{\left(1.00\times 10^{2}\text{ m/s}\right)^2} = \boxed{4.90\times 10^{-3}\text{ kg/m}}$$

(b) If *m* = 45.0 kg, then

$$F = mg = (45.0\text{ kg})\left(9.80\text{ m/s}^2\right) = 4.41\times 10^{2}\text{ N}$$

and

$$v = \sqrt{\frac{4.41\times 10^{2}\text{ N}}{4.90\times 10^{-3}\text{ kg/m}}} = 3.00\times 10^{2}\text{ m/s}$$

Thus, the wavelength will be

$$\lambda = \frac{v}{f} = \frac{3.00\times 10^{2}\text{ m/s}}{150\text{ Hz}} = 2.00\text{ m}$$

and the number of loops is

$$n = \frac{L}{\lambda/2} = \frac{2.00\text{ m}}{1.00\text{ m}} = \boxed{2}$$

(c) If *m* = 10.0 kg, the tension is

$$F = mg = (10.0\text{ kg})\left(9.80\text{ m/s}^2\right) = 98.0\text{ N}$$

and

$$v = \sqrt{\frac{98.0\text{ N}}{4.90\times 10^{-3}\text{ kg/m}}} = 1.41\times 10^{2}\text{ m/s}$$

Then,

$$\lambda = \frac{v}{f} = \frac{1.41\times 10^{2}\text{ m/s}}{150\text{ Hz}} = 0.943\text{ m}$$

and $n = \dfrac{L}{\lambda/2} = \dfrac{2.00\text{ m}}{0.471\text{ m}}$ is not an integer,

so $\boxed{\text{no standing wave will form}}$.

P18.29 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an antinode halfway between A and B. Thus,

$$\frac{\lambda}{2}=\overline{AB}=\frac{L}{\cos\theta} \quad \text{or} \quad \lambda=\frac{2L}{\cos\theta}$$

Since the fundamental frequency is *f*, the wave speed in this segment of string is

$$v=\lambda f=\frac{2Lf}{\cos\theta}$$

Because of the pulley, the string has tension *T* = *Mg*.

Also,

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{Mg}{m/\overline{AB}}}=\sqrt{\frac{MgL}{m\cos\theta}}$$

Thus,

$$\frac{2Lf}{\cos\theta}=\sqrt{\frac{MgL}{m\cos\theta}} \quad \text{or} \quad \frac{4L^2f^2}{\cos^2\theta}=\frac{MgL}{m\cos\theta}$$

and the mass of string above the rod is:

$$m=\frac{Mg\cos\theta}{4f^2L}=\frac{(1.00\text{ kg})(9.80\text{ m/s}^2)\cos 35.0°}{4(60.0\text{ Hz})^2(0.300\text{ m})}=\boxed{1.86\text{ g}}$$

P18.30 In the fundamental mode, the string above the rod has only two nodes, at A and B, with an anti-node halfway between A and B. Thus,

$$\frac{\lambda}{2}=\overline{AB}=\frac{L}{\cos\theta} \quad \text{or} \quad \lambda=\frac{2L}{\cos\theta}$$

Since the fundamental frequency is *f*, the wave speed in this segment of string is

$$v=\lambda f=\frac{2Lf}{\cos\theta}$$

Because of the pulley, the string has tension *T* = *Mg*.

Also,

$$v=\sqrt{\frac{T}{\mu}}=\sqrt{\frac{Mg}{m/\overline{AB}}}=\sqrt{\frac{MgL}{m\cos\theta}}$$

Thus,

$$\frac{2Lf}{\cos\theta}=\sqrt{\frac{MgL}{m\cos\theta}} \quad \text{or} \quad \frac{4L^2f^2}{\cos^2\theta}=\frac{MgL}{m\cos\theta}$$

and the mass of string above the rod is:

$$\boxed{m = \frac{Mg\cos\theta}{4f^2L}}$$

P18.31 When the open string vibrates in its fundamental mode it produces concert G. When concert A is played, the shorter length of string vibrates in its fundamental mode also.

(a) $\lambda_G = 2L_G = \frac{v}{f_G}$; $\lambda_A = 2L_A = \frac{v}{f_A}$, and $\frac{L_A}{L_G} = \frac{f_G}{f_A}$

$$L_G - L_A = L_G - \left(\frac{L_A}{L_G}\right)L_G = L_G - \left(\frac{f_G}{f_A}\right)L_G = L_G\left(1 - \frac{f_G}{f_A}\right)$$

$$L_G - L_A = (0.350\text{ m})\left(1 - \frac{392}{440}\right) = 0.038\ 2\text{ m}$$

Thus, $L_A = L_G - 0.038\ 2\text{ m} = 0.350\text{ m} - 0.038\ 2\text{ m} = 0.312\text{ m}$,

or the finger should be placed $\boxed{\text{31.2 cm from the bridge}}$.

(b) If the position of the finger is correct within *dL* = 0.600 cm when the note is played, by how much can the tension be off so that the note is the same? We want to find the maximum allowable percentage change in tension, *dT* / *T*, that will compensate for a small percentage change in position, *dL* / *L*, so that the change in the fundamental frequency, *df*, is zero.

From the expression for the fundamental frequency,

$$f = \frac{v}{2L} = \frac{1}{2L}\sqrt{\frac{T}{\mu}}, \quad \text{we require } df = 0.$$

$$df = \frac{-dL}{2L^2}\sqrt{\frac{T}{\mu}} + \frac{1}{2L}\frac{1}{2}\frac{dT}{\sqrt{T\mu}} = 0 \quad \rightarrow \quad \frac{dL}{2L^2}\sqrt{\frac{T}{\mu}} = \frac{1}{4L}\frac{dT}{\sqrt{T\mu}}$$

$$\rightarrow \quad \frac{dL}{L}\sqrt{\frac{T}{\mu}} = \frac{1}{2}\sqrt{\frac{T}{\mu}}\frac{dT}{T} \quad \rightarrow \quad \frac{dT}{T} = 2\frac{dL}{L}$$

$$= 2\left(\frac{0.600\text{ cm}}{31.2\text{ cm}}\right) \quad \rightarrow \quad \boxed{3.85\%}$$

P18.32 Let $m = \rho V$ represent the mass of the copper cylinder. The original tension in the wire is $T_1 = mg = \rho V g$. The water exerts a buoyant force $\rho_{\text{water}}\left(\frac{V}{2}\right)g$ on the cylinder, to reduce the tension to

$$T_2 = \rho V g - \rho_{\text{water}}\left(\frac{V}{2}\right)g = \left(\rho - \frac{\rho_{\text{water}}}{2}\right)Vg$$

The speed of a wave on the string changes from $\sqrt{\frac{T_1}{\mu}}$ to $\sqrt{\frac{T_2}{\mu}}$. The frequency changes from

$$f_1 = \frac{v_1}{\lambda} = \sqrt{\frac{T_1}{\mu}}\frac{1}{\lambda} \quad \text{to} \quad f_2 = \sqrt{\frac{T_2}{\mu}}\frac{1}{\lambda}$$

where we assume $\lambda = 2L$ is constant.

Then

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{\frac{\rho - \rho_{\text{water}}/2}{\rho}} = \sqrt{\frac{8.92 - 1.00/2}{8.92}}$$

and

$$f_2 = (300\text{ Hz})\sqrt{\frac{8.42}{8.92}} = \boxed{291\text{ Hz}}$$

P18.33 Comparing $y = 0.002\,00 \sin(\pi x)\cos(100\pi t)$ with $y = 2A \sin kx \cos \omega t$,

we find $k = \frac{2\pi}{\lambda} = \pi\text{ m}^{-1} \rightarrow \lambda = 2.00\text{ m}$, and

$\omega = 2\pi f = 100\pi\text{ s}^{-1} \rightarrow f = 50.0\text{ Hz}$

(a) The distance between adjacent nodes is $d_{\text{NN}} = \frac{\lambda}{2} = 1.00\text{ m}$,

and on the string there are $\frac{L}{d_{\text{NN}}} = \frac{3.00\text{ m}}{1.00\text{ m}} = \boxed{3\text{ loops}}$.

(b) For the speed we have $v = \omega/k = 100\pi\text{ s}^{-1}/\pi\text{ m}^{-1} = 100\text{ m/s}$.

In the simplest standing wave vibration, $d_{\text{NN}} = 3.00\text{ m} = \frac{\lambda_b}{2}$,

$\lambda_b = 6.00\text{ m}$, and $f_b = \frac{v_a}{\lambda_b} = \frac{100\text{ m/s}}{6.00\text{ m}} = \boxed{16.7\text{ Hz}}$.

(c) In $v_0 = \sqrt{\dfrac{T_0}{\mu}}$, if the tension increases to $T_c = 9T_0$ and the string does not stretch, the speed increases to

$$v_c = \sqrt{\frac{9T_0}{\mu}} = 3\sqrt{\frac{T_0}{\mu}} = 3v_0 = 3(100\text{ m/s}) = 300\text{ m/s}$$

Then $\lambda_c = \dfrac{v_c}{f_a} = \dfrac{300\text{ m/s}}{50\text{ Hz}} = 6.00\text{ m}$, $\quad d_{\text{NN}} = \dfrac{\lambda_c}{2} = 3.00\text{ m}$,

and $\boxed{\text{one loop}}$ fits onto the string.

Section 18.4 Resonance

P18.34 The wave speed is $v = \sqrt{gd} = \sqrt{(9.80\text{ m/s}^2)(36.1\text{ m})} = 18.8\text{ m/s}$.

The bay has one end open and one closed. Its simplest resonance is with a node of horizontal velocity, which is also an antinode of vertical displacement, at the head of the bay and an antinode of velocity, which is a node of displacement, at the mouth.

Then, $\quad d_{\text{NA}} = 210 \times 10^3\text{ m} = \dfrac{\lambda}{4}$

and $\quad \lambda = 840 \times 10^3\text{ m}$.

Therefore, the period is

$$T = \frac{1}{f} = \frac{\lambda}{v} = \frac{840 \times 10^3\text{ m}}{18.8\text{ m/s}} = 4.47 \times 10^4\text{ s} = \boxed{12\text{ h, }24\text{ min}}$$

$\boxed{\text{The natural frequency of the water sloshing in the bay agrees precisely with that of lunar excitation, so we identify the extra-high tides as amplified by resonance.}}$

P18.35 (a) The wave speed is $v = \dfrac{9.15\text{ m}}{2.50\text{ s}} = \boxed{3.66\text{ m/s}}$.

(b) There are antinodes at both ends of the pond, so the distance between adjacent antinodes is $d_{\text{AA}} = \dfrac{\lambda}{2} = 9.15\text{ m}$ and the wavelength is $\lambda = 18.3\text{ m}$

The frequency is then $f = \frac{v}{\lambda} = \frac{3.66\text{ m/s}}{18.3\text{ m}} = \boxed{0.200\text{ Hz}}$.

We have assumed the wave speed on water is the same for all wavelengths.

P18.36 The distance between adjacent nodes is one-quarter of the circumference.

$$d_{\text{NN}} = d_{\text{AA}} = \frac{\lambda}{2} = \frac{20.0\text{ cm}}{4} = 5.00\text{ cm}$$

so $\lambda = 10.0\text{ cm}$

and $f = \frac{v}{\lambda} = \frac{900\text{ m/s}}{0.100\text{ m}} = 9\,000\text{ Hz} = \boxed{9.00\text{ kHz}}$

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

Section 18.5 Standing Waves in Air Columns

***P18.37** Assuming an air temperature of $T = 37.0°\text{C} = 310\text{ K}$, the speed of sound inside the pipe is

$$v = 331\text{ m/s} + (0.600\text{ m/s}\cdot°\text{C})(37.0°\text{C}) = 353\text{ m/s}$$

In the fundamental resonant mode, the wavelength of sound waves in a pipe closed at one end is $\lambda = 4L$. Thus, for the whooping crane,

$$\lambda = 4(5.00\text{ ft}) = 2.00\times 10^1\text{ ft}$$

and

$$f = \frac{v}{\lambda} = \frac{(353\text{ m/s})}{(2.00\times 10^1\text{ ft})}\left(\frac{3.281\text{ ft}}{1\text{ m}}\right) = \boxed{57.9\text{ Hz}}$$

18.38 At $T = 37.0°\text{C} = 310\text{ K}$, the speed of sound in air is

$$v = (331\text{ m/s})\sqrt{1+\frac{T_C}{273}} = (331\text{ m/s})\sqrt{1+\frac{37.0}{273}} = 353\text{ m/s}$$

Thus, the wavelength of 3 000-Hz sound is

$$\lambda = \frac{v}{f} = \frac{353\text{ m/s}}{3\,000\text{ Hz}} = 0.118\text{ m}$$

For the fundamental resonant mode in a pipe closed at one end, the length required is

$$L = \frac{\lambda}{4} = \frac{0.118\text{ m}}{4} = 0.029\ 4\text{ m} = \boxed{2.94\text{ cm}}$$

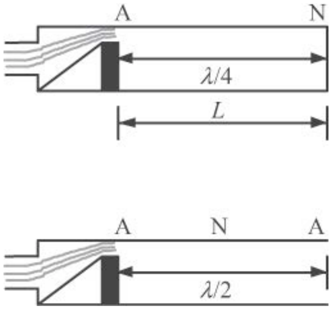
P18.39 (a) For the fundamental mode in a closed pipe, $\lambda = 4L$, as in the diagram.

But $v = f\lambda$, therefore $L = \dfrac{v}{4f}$.

so, $$L = \frac{343\ \text{m/s}}{4\left(240\ \text{s}^{-1}\right)} = \boxed{0.357\ \text{m}}$$

(b) For an open pipe, $\lambda = 2L$, as in the diagram.

So, $$L = \frac{v}{2f} = \frac{343\ \text{m/s}}{2\left(240\ \text{s}^{-1}\right)} = \boxed{0.715\text{ m}}$$



ANS. FIG. P18.39

P18.40 The 32.0-cm length corresponds to $d_{\text{AA}} = 0.320$ m, which gives a wavelength of

$$\lambda = 2d_{AA} = 2(0.320\text{ m}) = 0.640\text{ m}$$

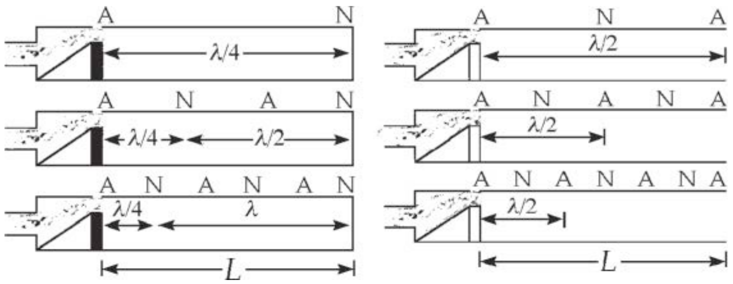
(a) The frequency of the lowest note is

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.640\text{ m}} = \boxed{536\text{ Hz}}$$

(b) For a 4 000 Hz high note,

$$d_{AA} = \frac{\lambda}{2} = \frac{v}{2f} = \frac{343\text{ m/s}}{8\ 000\text{ Hz}} = 0.042\ 9\text{ m} = \boxed{42.9\text{ mm}}$$

P18.41 (a) The wavelength is $\lambda = \dfrac{v}{f} = \dfrac{343\text{ m/s}}{261.6\text{/s}} = 1.31$ m,



ANS. FIG. P18.41

so the length of the open pipe vibrating in its simplest (ANA) mode is

$$d_{\text{A to A}} = \frac{1}{2}\lambda = \boxed{0.656\ \text{m}}$$

(b) A closed pipe has (NA) for its simplest resonance, (NANA) for the second, and (NANANA) for the third, equal to 5/4 wavelengths.

Here, the pipe length is $5d_{\text{N to A}} = \dfrac{5\lambda}{4} = \dfrac{5}{4}(1.31\ \text{m}) = \boxed{1.64\ \text{m}}$

P18.42 At T_C = 0.00°C, the speed of sound is

$$v = 331\ \text{m/s}\sqrt{1+\frac{T_C}{273}} = 331\ \text{m/s}$$

(a) For a pipe closed at one end,

$$f_1 = \frac{v}{4L} = \frac{331\ \text{m/s}}{4(4.88\ \text{m})} = \boxed{17.0\ \text{Hz}}$$

(b) For a pipe open at each end,

$$f_1 = \frac{v}{2L} = \frac{331\ \text{m/s}}{2(4.88\ \text{m})} = \boxed{33.9\ \text{Hz}}$$

(c) At T_C = 20.0°C, the speed of sound is v = 343 m/s.

closed at one end:

$$f_1 = \frac{v}{4L} = \frac{343\ \text{m/s}}{4(4.88\ \text{m})} = \boxed{17.6\ \text{Hz}}$$

open at each end:

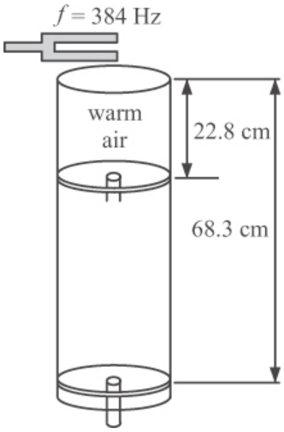
$$f_1 = \frac{v}{2L} = \frac{343\ \text{m/s}}{2(4.88\ \text{m})} = \boxed{35.1\ \text{Hz}}$$

P18.43 For resonance in a narrow tube open at one end,

$$f = n\frac{v}{4L}\,(n = 1,\ 3,\ 5,\ \ldots)$$

(a) The node–node distance is

$$d_{NN} = 68.3\ \text{cm} - 22.8\ \text{cm} = 45.5\ \text{cm}$$



ANS. FIG. P18.43

This distance is equal to half the wavelength, so,

$$\begin{aligned} v &= \lambda f = 2d_{\text{NN}} f \\ &= 2(0.455\text{ m})(384\text{ Hz}) \\ &= \boxed{349\text{ m/s}} \end{aligned}$$

(b) Resonance will be established when the tube length has increased by another half wavelength: 68.3 cm + 45.5 = 113.8 = $\boxed{1.14\text{ m}}$

P18.44 The tube acts as a pipe open at one end and closed at the other. Resonance frequencies are odd harmonics. The length corresponding to the fundamental satisfies

$$f_1 = \frac{v}{4L}:\quad L_1 = \frac{v}{4f_1} = \frac{343\text{ m/s}}{4\left(512\text{ s}^{-1}\right)} = 0.167\text{ m}$$

Since *L* > 20.0 cm, the *next* two modes will be observed,

corresponding to $f_2 = \dfrac{3v}{4L_2}$ and $f_3 = \dfrac{5v}{4L_3}$,

or $L_2 = \dfrac{3v}{4f_2} = 3f_1 = \boxed{0.502\text{ m}}$ and $L_3 = \dfrac{5v}{4f_3} = 5f_1 = \boxed{0.837\text{ m}}$.

P18.45 (a) For the fundamental mode of an open tube,

$$L = \frac{\lambda_1}{2} = \frac{v}{2f_1} = \frac{343\text{ m/s}}{2\left(880\text{ s}^{-1}\right)} = \boxed{0.195\text{ m}}$$

(b) $v = 331\text{ m/s}\sqrt{1 + \dfrac{(-5.00)}{273}} = 328\text{ m/s}$

We ignore the thermal expansion of the metal.

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \frac{328\text{ m/s}}{2(0.195\text{ m})} = \boxed{841\text{ Hz}}$$

The flute is flat by a semitone.

P18.46 For a closed box, the resonant frequencies will have nodes at both sides, so the permitted wavelengths will be $\lambda = \dfrac{2L}{n}$, (*n* = 1, 2, 3, …),

i.e., $L = \dfrac{n\lambda}{2} = \dfrac{nv}{2f}$ and $f = \dfrac{nv}{2L}$.

Therefore, with L = 0.860 m, L' = 2.10 m, and v = 355 m/s, the resonant frequencies are

$$f_n = \boxed{n(206\text{ Hz})} \quad \text{for } L = 0.860 \text{ m for each } n \text{ from 1 to 9}$$

and $$f'_n = \boxed{n(84.5\text{ Hz})} \quad \text{for } L' = 2.10 \text{ m for each } n \text{ from 2 to 23.}$$

***P18.47** $\frac{\lambda}{2} = d_{\text{AA}} = \frac{L}{n}$ or $L = \frac{n\lambda}{2}$ for n = 1, 2, 3, …

Since $\lambda = \frac{v}{f}$, $L = n\left(\frac{v}{2f}\right)$ for n = 1, 2, 3, …

With v = 343 m/s and f = 680 Hz,

$$L = n\left(\frac{343\text{ m/s}}{2(680\text{ Hz})}\right) = n(0.252\text{ m}) \quad \text{for } n = 1, 2, 3, \ldots$$

Possible lengths for resonance are:

$$L = \boxed{0.252\text{ m}, 0.504\text{ m}, 0.757\text{ m}, \ldots, n(0.252)\text{ m}}$$

P18.48 (a) The open ends of the tunnel are antinodes, so d_{AA} = 2 000 m/n, with n = 1, 2, 3,…

Then

$$\lambda = 2d_{AA} = 4\ 000\text{ m}/n$$

and

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{4\ 000\text{ m}/n} = \boxed{0.085\ 8n\text{ Hz, with } n = 1, 2, 3, \ldots}$$

(b) $\boxed{\text{It is a good rule. Any car horn would produce several or many of the closely-spaced resonance frequencies of the air in the tunnel, so it would be greatly amplified. Other drivers might be frightened directly into dangerous behavior, or might blow their horns also.}}$

P18.49 The wavelength of the sound from the tuning fork is $\lambda = \frac{v}{f}$. The cylinder is a pipe open at the top and closed at the water surface; its resonance patterns are AN, ANAN, ANANAN, etc. Resonance occurs each time the height of the air column changes by half a wavelength: $\Delta h = \frac{v}{2f}$. The volume of the pipe between these two water levels is $\pi r^2 \Delta h$, which is also equal to the amount of water that has entered the pipe at rate *R* in a time interval Δt and has filled this volume. Therefore,

$$R\Delta t = \pi r^2 \Delta h = \frac{\pi r^2 v}{2f} \quad \rightarrow \quad \Delta t = \frac{\pi r^2 v}{2Rf}$$

$$\Delta t = \frac{\pi r^2 v}{2Rf} = \frac{\pi (0.050\ 0 \text{ m})^2 (343 \text{ m/s})}{2(1.00 \text{ L/min})(512 \text{ Hz})}\left(\frac{1 \text{ L}}{10^3 \text{ cm}^3}\right)\left(\frac{100 \text{ cm}}{\text{m}}\right)^3$$

$$= (2.63 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{158 \text{ s}}$$

P18.50 The wavelength of the sound from the tuning fork is $\lambda = \frac{v}{f}$. The cylinder is a pipe open at the top and closed at the water surface; its resonance patterns are AN, ANAN, ANANAN, etc. Resonance occurs each time the height of the air column changes by half a wavelength: $\Delta h = \frac{v}{2f}$. The volume of the pipe between these two water levels is $\pi r^2 \Delta h$, which is also equal to the amount of water that has entered the pipe at rate *R* in a time interval Δt and has filled this volume. Therefore,

$$R\Delta t = \pi r^2 \Delta h = \frac{\pi r^2 v}{2f} \rightarrow \Delta t = \boxed{\frac{\pi r^2 v}{2Rf}}$$

P18.51 For both open and closed pipes, resonant frequencies are equally spaced as numbers. The set of resonant frequencies then must be 650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, 50 Hz. These are odd-integer multipliers of the fundamental frequency of $\boxed{50.0 \text{ Hz}}$. Then the pipe length is $d_{\text{NA}} = \frac{\lambda}{4} = \frac{v}{4f} = \frac{343 \text{ m/s}}{4(50.0 \text{ s}^{-1})} = \boxed{1.72 \text{ m}}$.

P18.52 For an air column of length 0.730 m, the column may be open ended or closed at one end. For a column open at both ends:

$$f_n = n\frac{v}{2L} \quad \text{where } n = 1, 2, 3, \ldots$$

$$f_n = n\frac{v}{2L} = n\frac{343 \text{ m/s}}{2(0.730 \text{ m})} = n(235 \text{ Hz}) \quad \text{where } n = 1, 2, 3, \ldots$$

And thus 235 Hz belongs to the harmonic series of an open column (with *n* = 1), but 587 Hz does not match this harmonic series. Similarly, for a column open only at one end:

$$f_n = n\frac{v}{4L}, \quad \text{where } n = 1, 3, 5,\ldots \text{ (only odd harmonics)}$$

$$f_n = n\frac{v}{4L} = n\frac{343\text{ m/s}}{4(0.730\text{ m})} = n(117.5\text{ Hz}), \quad \text{where } n = 1, 3, 5,\ldots$$

and 587 Hz belongs to the harmonic series of a column open at only one end (for *n* = 5), but 235 Hz does not match this harmonic series. $\boxed{\text{Therefore, it is impossible because a single column could not produce both frequencies.}}$

P18.53 (a) The well acts like a pipe open at one end and closed at the other. The normal modes of vibrations of such a pipe are odd harmonics of a fundamental. Call *L* the depth of the well and *v* the speed of sound.

Then for some integer *n,*

$$L = (2n-1)\frac{\lambda_1}{4} = (2n-1)\frac{v}{4f_1} = \frac{(2n-1)(343\text{ m/s})}{4(51.87\text{ s}^{-1})}$$

and for the next resonance

$$L = [2(n+1)-1]\frac{\lambda_2}{4} = (2n+1)\frac{v}{4f_2} = \frac{(2n+1)(343\text{ m/s})}{4(59.85\text{ s}^{-1})}$$

Thus,

$$\frac{(2n-1)(343\text{ m/s})}{4(51.87\text{ s}^{-1})} = \frac{(2n+1)(343\text{ m/s})}{4(59.85\text{ s}^{-1})}$$

and we require an *integer* solution to $\frac{2n+1}{59.85} = \frac{2n-1}{51.87}$.

The equation gives *n* = 7, which gives us

$$L = \frac{[2(7)-1](343\text{ m/s})}{4(51.87\text{ s}^{-1})} = \frac{[2(7)+1](343\text{ m/s})}{4(59.85\text{ s}^{-1})} = \boxed{21.5\text{ m}}$$

(b) The first harmonic (fundamental frequency) of the well is f_1 = *v*/4*L* = (343 m/s)/[4(21.5 m)] = 3.99 Hz; its pattern is AN. The 3rd harmonic pattern is ANAN, the 5th is ANANAN, etc. We can see that a pattern with *n* antinodes is the (2*n* – 1)th harmonic. The frequency 51.87 Hz = 13(3.99 Hz) is the 13th harmonic: 13 = 2(7) – 1, so the standing wave has $\boxed{\text{7 antinodes.}}$

Section 18.6 Standing Waves in Rods and Membranes

P18.54 When the rod is clamped at one-quarter of its length, the vibration pattern reads ANANA and the rod length is $L = 2d_{\text{AA}} = \lambda$.

Therefore, $$L = \frac{v}{f} = \frac{5\,100\text{ m/s}}{4\,400\text{ Hz}} = \boxed{1.16\text{ m}}$$

P18.55 (a) The mode pattern is ANA, corresponding to the fundamental mode. The length of the rod is one wavelength:

$$f = \frac{v}{2L} = \frac{5\,100}{(2)(1.60)} = \boxed{1.59\text{ kHz}}$$

(b) Since it is held in the center, there must be a node in the center as well as antinodes at the ends. The even harmonics have an antinode at the center so only $\boxed{\text{the odd harmonics}}$ are present.

(c) The wavelength is the same as in (a):

$$f = \frac{v'}{2L} = \frac{3\,560}{(2)(1.60)} = \boxed{1.11\text{ kHz}}$$

Section 18.7 Beats: Interference in Time

P18.56 (a) The string could be tuned to either $\boxed{\text{521 Hz or 525 Hz}}$ from this evidence.

(b) Tightening the string raises the wave speed and frequency. If the frequency were originally 521 Hz, the beats would slow down.

Instead, the frequency must have started at 525 Hz to become $\boxed{\text{526 Hz}}$.

(c) From $f = \dfrac{v}{\lambda} = \dfrac{\sqrt{T/\mu}}{2L} = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$,

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}} \quad \text{and} \quad T_2 = \left(\frac{f_2}{f_1}\right)^2 T_1.$$

The fractional change that should be made in the tension is then

$$\text{fractional change} = \frac{T_2 - T_1}{T_1} = \frac{T_2}{T_1} - 1 = T_2 = \left(\frac{f_2}{f_1}\right)^2 - 1$$

$$= \left(\frac{523}{526}\right)^2 - 1 = -0.011\,4 = -1.14\%$$

The tension should be $\boxed{\text{reduced by } 1.14\%}$.

P18.57 Combining the velocity and the tension equations $v = f\lambda$ and $v = \sqrt{T/\mu}$, we find that the frequency is

$$f = \sqrt{\frac{T}{\mu\lambda^2}}$$

Since μ and λ are constant, we can apply that equation to both frequencies, and then divide the two equations to get the proportion

$$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2}}$$

With $f_1 = 110$ Hz, $T_1 = 600$ N, and $T_2 = 540$ N we have

$$f_2 = (110\text{ Hz})\sqrt{\frac{540\text{ N}}{600\text{ N}}} = 104.4\text{ Hz}$$

The beat frequency is

$$f_b = |f_1 - f_2| = 110\text{ Hz} - 104.4\text{ Hz} = \boxed{5.64\text{ beats/s}}$$

P18.58 We use $f' = \left(\frac{v + v_O}{v - v_S}\right)f$. The observer is stationary, so $v_0 = 0$.

For the approaching train $v_s = +8.00$ m/s; the frequency arriving at the observer is

$$f_1' = \left(\frac{v}{v - v_S}\right)f = \left(\frac{343\text{ m/s}}{343\text{ m/s} - 8.00\text{ m/s}}\right)f = \left(\frac{343\text{ m/s}}{335\text{ m/s}}\right)f$$

For the receding train is $v_s = -8.00$ m/s; the frequency arriving at the observer is

$$f_2' = \left(\frac{v}{v - v_S}\right)f = \left(\frac{343\text{ m/s}}{343\text{ m/s} - (-8.00\text{ m/s})}\right)f = \left(\frac{343\text{ m/s}}{351\text{ m/s}}\right)f$$

The beat frequency between the waves emanating from the trains is

$$f_{\text{beat}} = |f_1 - f_2|$$

and, because the receding train produces a lower frequency,

$$f_1' - f_2' = f_{\text{beat}} \rightarrow \left(\frac{343\text{ m/s}}{335\text{ m/s}}\right)f - \left(\frac{343\text{ m/s}}{351\text{ m/s}}\right)f = 4.00\text{ Hz}$$

$$\left[\left(\frac{343\text{ m/s}}{335\text{ m/s}}\right) - \left(\frac{343\text{ m/s}}{351\text{ m/s}}\right)\right]f = 4.00\text{ Hz} \rightarrow f = \boxed{85.7\text{ Hz}}$$

P18.59 The source moves toward the wall:

$$v_s = +v_{\text{student}},\ v_0 = 0,\quad \text{and}\quad f' = f\frac{(v + v_o)}{(v - v_s)} = f\frac{v}{(v - v_{\text{student}})}.$$

The wall acts as stationary source, reflecting the wave of frequency f'. The observe moves toward the source: $v_s = 0$, $v_0 = +v_{\text{student}}$, and

$$f'' = f'\frac{(v + v_o)}{(v - v_s)} = f'\frac{(v + v_s)}{v} = f\frac{v}{(v - v_{\text{student}})}\frac{(v + v_{\text{student}})}{v}$$

$$= f\frac{(v + v_{\text{student}})}{(v - v_{\text{student}})}$$

(a) When the student walks toward the wall f'' is larger than f; the beat frequency is

$$f_b = |f'' - f| = f\frac{(v + v_{\text{student}})}{(v - v_{\text{student}})} - f = f\left[\frac{(v + v_{\text{student}})}{(v - v_{\text{student}})} - 1\right]$$

$$= f\frac{2v_{\text{student}}}{(v - v_{\text{student}})}$$

$$f_b = (256\text{ Hz})\frac{2(1.33\text{ m/s})}{(343\text{ m/s} - 1.33\text{ m/s})} = \boxed{1.99\text{ Hz}}$$

(b) When he is moving away from the wall, the sign of v_{student} changes and f'' is smaller than f:

$$f_b = |f'' - f| = f - f\frac{(v - v_{\text{student}})}{(v + v_{\text{student}})} = f\left[1 - \frac{(v - v_{\text{student}})}{(v + v_{\text{student}})}\right]$$

$$= f\frac{2v_{\text{student}}}{(v + v_{\text{student}})}$$

Solving for v_{student} gives

$$v_{\text{student}} = \frac{f_b v}{2f - f_b} = \frac{(5\text{ Hz})(343\text{ m/s})}{(2)(256\text{ Hz}) - 5\text{ Hz}} = \boxed{3.38\text{ m/s}}$$

Section 18.8 Nonsinusoidal Wave Patterns

***P18.60** We list the frequencies of the harmonics of each note in Hz:

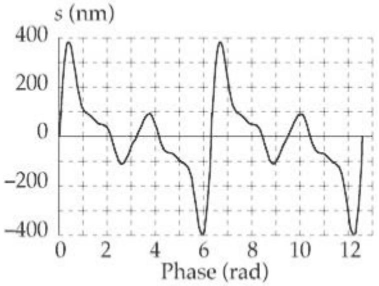
	Harmonic				
Note	1	2	3	4	5
A	440.00	880.00	1 320.0	1 760.0	2 200.0
C#	554.37	1 108.7	1 663.1	2 217.5	2 771.9
E	659.26	1 318.5	1 977.8	2 637.0	3 296.3

The second harmonic of E is close the the third harmonic of A, and the fourth harmonic of C# is close to the fifth harmonic of A.

P18.61 We evaluate

$$\begin{aligned} s &= 100 \sin\theta + 157 \sin 2\theta + 62.9 \sin 3\theta + 105 \sin 4\theta \\ &\quad + 51.9 \sin 5\theta + 29.5 \sin 6\theta + 25.3 \sin 7\theta \end{aligned}$$

where s represents particle displacement in nanometers and θ represents the phase of the wave in radians. As θ advances by 2π, time advances by (1/523) s. The resultant waveform is shown below in ANS. FIG. P18.61.



ANS. FIG. P18.61

Additional Problems

***P18.62** (a) The fundamental wavelength of the pipe open at both ends is $\lambda = 2L = v / f_1$. Since the speed of sound is 331 m/s at 0°C, the length of the pipe is

$$L = \frac{v}{2f_1} = \frac{331\text{ m/s}}{2(300\text{ Hz})} = \boxed{0.552\text{ m}}$$

(b) At $T = 30.0°\text{C} = 303\text{ K}$,

$$v = (331\text{ m/s})\sqrt{\frac{T_K}{273}} = (331\text{ m/s})\sqrt{\frac{303}{273}} = 349\text{ m/s}$$

and

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \frac{349\text{ m/s}}{2(0.552\text{ m})} = \boxed{316\text{ Hz}}$$

***P18.63** The second standing wave mode of the air in the pipe reads ANAN,

with $d_{\text{NA}} = \dfrac{\lambda}{4} = \dfrac{1.75\text{ m}}{3}$,

so $\lambda = 2.33\text{ m}$

and $f = \dfrac{v}{\lambda} = \dfrac{343\text{ m/s}}{2.33\text{ m}} = 147\text{ Hz}.$

For the string, λ and *v* are different but *f* is the same.

$$\frac{\lambda}{2} = d_{\text{NN}} = \frac{0.400\text{ m}}{2}$$

so $\lambda = 0.400\text{ m}.$

$$v = \lambda f = (0.400\text{ m})(147\text{ Hz}) = 58.8\text{ m/s} = \sqrt{\frac{T}{\mu}}$$

$$T = \mu v^2 = (9.00\times 10^{-3}\text{ kg/m})(58.8\text{ m/s})^2 = \boxed{31.1\text{ N}}$$

P18.64 The beat frequency between the waves emanating from the two strings is

$$f_{beat} = |f_1 - f_2|$$

and, because the decrease in tension causes the second frequency to be lower,

$$f_2 = f_1 - f_{beat} = (150\text{ Hz}) - (4\text{ Hz}) = \boxed{146\text{ Hz}}$$

P18.65 At point *D*, the distance of the ship from point *A* is

$$d_1 = \sqrt{d_2^2 + (800\text{ m})^2} = \sqrt{(600\text{ m})^2 + (800\text{ m})^2} = 1\,000\text{ m}$$

Since destructive interference occurs for the first time when the ship reaches *D*, it is necessary that $d_1 - d_2 = \lambda/2$ or

$$\lambda = 2(d_1 - d_2) = 2(1\,000\text{ m} - 600\text{ m}) = \boxed{800\text{ m}}$$

P18.66 According to Equation 18.6, the natural frequencies of vibration of a string fixed at both ends are given by

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} = \frac{n}{2(2.00\text{ m})}\sqrt{\frac{20.0\text{ N}}{\left(\dfrac{0.100\text{ kg}}{2.00\text{ m}}\right)}} = n(5.00\text{ Hz})$$

where *n* = 1, 2, 3, ...

(a) $f_1 = \boxed{5.0\text{ Hz}}, f_2 = \boxed{10.0\text{ Hz}}, f_3 = \boxed{15.0\text{ Hz}}$

(b) This could be any mode that has a node 0.400 m from an end. If *D* = 0.400 m is the distance between adjacent nodes (the distance across a pair of nodes), $d_{\text{NN}} = D = \lambda/2$, and its wavelength is 0.800 m:

$$\frac{\lambda}{2} = D \quad \rightarrow \quad \lambda = 2D = 2(0.400\text{ m})$$

$$\lambda = \frac{2L}{n} \quad \rightarrow \quad n = \frac{2L}{\lambda} = \frac{2L}{2D} = \frac{L}{D} = \frac{2.00\text{ m}}{0.400\text{ m}} = 5$$

This mode corresponds to the 5th harmonic: $f_5 = 5(5.00\text{ Hz}) = 25.0\text{ Hz}$. But *D* could be the distance across two pairs of nodes (from node to node to node), $d_{\text{NNN}} = D = 2(\lambda/2)$, or three pairs, d_{NNN}, or across *N* pairs of nodes:

$$N\frac{\lambda}{2} = D \quad \rightarrow \quad \lambda = 2D/N$$

then,

$$n = \frac{2L}{\lambda} = \frac{2L}{(2D/N)} = N\frac{L}{D} = N\frac{2.00\text{ m}}{0.400\text{ m}} = 5N$$

and so on, corresponding to the 10th, or the 15th harmonic, etc.

$$\boxed{\text{The frequency could be the fifth state at 25.0 Hz or any integer multiple, such as the tenth state at 50.0 Hz, the fifteenth state at 75.0 Hz, and so on.}}$$

P18.67 When the string is plucked, nodes occur on the ends because they are fixed. The plucked guitar string vibrates in its fundamental mode with a wavelength equal to twice the length of the string. For the 2 349-Hz note, the length of the vibrating string is L = 21.4 cm. For the 2 217-Hz note, the length of the vibrating string is $L + x$, where x is the distance to the next fret. We wish to solve for x.

We assume the wave speed is the same on each string. Compare the frequencies to the lengths of vibrating string:

$$f_1 = 2349\ \text{Hz} = \frac{v}{2L}$$

$$f_2 = 2217\ \text{Hz} = \frac{v}{2(L+x)}$$

Taking the ratio,

$$\frac{f_1}{f_2} = \frac{L+x}{L} = 1 + \frac{x}{L}$$

$$x = L\left(\frac{f_1}{f_2} - 1\right) = (21.4\ \text{cm})\left(\frac{2\ 349\ \text{Hz}}{2\ 217\ \text{Hz}} - 1\right) = \boxed{1.27\ \text{cm}}$$

P18.68 (a) The frequency of the normal mode produces a sound wave of the same frequency. For the same frequency, wavelength is proportional to wave speed. On the string, the wave speed is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{(48.0\ \text{N})}{\left(\dfrac{4.80 \times 10^{-3}\ \text{kg}}{2.00\ \text{m}}\right)}} = 141\ \text{m/s}$$

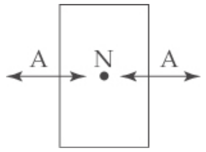
which is smaller than the speed of sound (343 m/s).

The wavelength in air of the sound produced by the string is $\boxed{\text{larger}}$ because the wave speed is larger.

(b) $$\frac{\lambda_{\text{air}}}{\lambda_{\text{string}}} = \frac{v_{\text{air}}/f}{v_{\text{string}}/f} = \frac{v_{\text{air}}}{v_{\text{string}}} = \frac{343\ \text{m/s}}{141\ \text{m/s}} = \boxed{2.43}$$

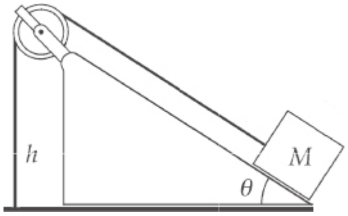
P18.69 $d_{\text{AA}} = \dfrac{\lambda}{2} = 7.05 \times 10^{-3}\ \text{m}$ is the distance between antinodes. Then $\lambda = 14.1 \times 10^{-3}\ \text{m}$,

and $f = \dfrac{v}{\lambda} = \dfrac{3.70 \times 10^3\ \text{m/s}}{14.1 \times 10^{-3}\ \text{m}} = \boxed{2.62 \times 10^5\ \text{Hz}}$,



ANS. FIG. P18.69

The crystal can be tuned to vibrate at 2^{18} Hz, so that binary counters can derive from it a signal at precisely 1 Hz.



ANS. FIG. P18.70

P18.70 (a) $\boxed{\text{The particle under constant acceleration model}}$

(b) $\boxed{\text{Waves under boundary conditions model}}$

(c) For the block:

$$\sum F_x = T - Mg\sin\theta = 0$$

so $T = \boxed{Mg\sin\theta}$.

(d) The length of the section of string parallel to the incline is $\dfrac{h}{\sin\theta}$.

The total length of the string is then

$$L = \frac{h}{\sin\theta} + h = \frac{h}{\sin\theta} + \frac{h\sin\theta}{\sin\theta} = \boxed{h\left(\frac{1+\sin\theta}{\sin\theta}\right)}$$

(e) The mass per unit length of the string is

$$\mu = \frac{m}{L} = \frac{m}{h\left(\dfrac{1+\sin\theta}{\sin\theta}\right)} = \boxed{\frac{m\sin\theta}{h(1+\sin\theta)}}$$

(f) The speed of waves in the string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{Mg\sin\theta}{\left[\dfrac{m\sin\theta}{h(1+\sin\theta)}\right]}} = \boxed{\sqrt{\frac{Mgh}{m}(1+\sin\theta)}}$$

(g) The fundamental mode vibrates at the lowest frequency. In the fundamental mode, the segment of length *h* vibrates as one loop. The distance between adjacent nodes is then $d_{\text{NN}} = \dfrac{\lambda}{2} = h$, so the wavelength is $\lambda = 2h$.

The frequency is $f = \dfrac{v}{\lambda} = \dfrac{1}{2h}\sqrt{\dfrac{Mgh}{m}(1+\sin\theta)} = \boxed{\sqrt{\dfrac{Mg}{4mh}(1+\sin\theta)}}$.

(h) $$f = \sqrt{\frac{Mg}{4mh}(1+\sin\theta)} = \sqrt{\frac{(1.50\text{ kg})(9.80\text{ m/s}^2)}{4(0.750\times10^{-3}\text{ kg})(0.500\text{ m})}(1+\sin 30.0°)}$$

$$= \boxed{121\text{ Hz}}$$

(i) The fundamental mode has a wavelength twice the length of the sloped section of string, $\lambda = 2\frac{h}{\sin\theta}$. Its frequency is

$$f = \frac{v}{\lambda} = \frac{1}{\left(2\frac{h}{\sin\theta}\right)}\sqrt{\frac{Mgh}{m}(1+\sin\theta)} = \sin\theta\sqrt{\frac{Mg}{4mh}(1+\sin\theta)}$$

$$f = \sin 30.0°(121\text{ Hz}) = \boxed{60.6\text{ Hz}}$$

***P18.71** For the wire, $\mu = \frac{0.010\ 0\text{ kg}}{2.00\text{ m}} = 5.00\times10^{-3}\text{ kg/m}$:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{200\text{ kg}\cdot\text{m/s}^2}{5.00\times10^{-3}\text{ kg/m}}} = 200\text{ m/s}$$

If it vibrates in its simplest state, $d_{\text{NN}} = 2.00\text{ m} = \frac{\lambda}{2}$:

$$f = \frac{v}{\lambda} = \frac{200\text{ m/s}}{4.00\text{ m}} = 50.0\text{ Hz}$$

(a) The tuning fork can have frequencies $\boxed{45.0\text{ Hz or }55.0\text{ Hz.}}$

(b) If *f* = 45.0 Hz, and $v = f\lambda = (45.0\text{ s}^{-1})(4.00\text{ m}) = 180\text{ m/s}$, then

$$T = v^2\mu = (180\text{ m/s})^2(5.00\times10^{-3}\text{ kg/m}) = \boxed{162\text{ N}}$$

or if *f* = 55.0 Hz,

$$T = v^2\mu = f^2\lambda^2\mu = (55.0\text{ s}^{-1})^2(4.00\text{ m})^2(5.00\times10^{-3}\text{ kg/m})$$
$$= \boxed{242\text{ N}}$$

***P18.72** We are told that the man's ears are at the same level as the lower speaker. Sound from the upper speaker is delayed by traveling the extra distance $\Delta r = \sqrt{L^2 + d^2} - L$.

He hears a minimum when $\Delta r = (2n-1)\left(\frac{\lambda}{2}\right)$, with *n* = 1, 2, 3, …

Then,

$$\sqrt{L^2 + d^2} - L = \left(n - \frac{1}{2}\right)\left(\frac{v}{f}\right)$$

$$\sqrt{L^2+d^2} = \left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)+L$$

$$L^2+d^2 = \left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2+2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L+L^2$$

$$d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2 = 2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)L \qquad \textbf{[1]}$$

Equation [1] gives the distances from the lower speaker at which the man will hear a minimum. The path difference Δr starts from nearly zero when the man is very far away and increases to *d* when *L* = 0.

(a) The number of minima he hears is the greatest integer value for which $L \geq 0$. This is the same as the greatest integer solution to

$$d \geq \left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)$$

or

$$\boxed{\begin{aligned}\text{number of minima heard } &= n_{\text{max}}\\ &= \text{greatest integer } \leq d\left(\frac{f}{v}\right)+\frac{1}{2}\end{aligned}}$$

(b) From equation [1], the distances at which minima occur are given by

$$\boxed{L_n = \frac{d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2}{2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)}, \text{ where } n = 1,\ 2,\ \ldots,\ n_{\text{max}}}$$

P18.73 (a) The tension on the string defines the wave velocity on the string, and thus also the frequencies, wavelengths, and number of nodes of the standing waves. The tension on the string in Figure 18.11a is:

$T_1 = mg$, where *m* is the mass of the sphere.

The tension on the string in Figure 18.11b, must also include the buoyant force on the sphere:

$$T_2 = mg - B = mg - \rho_{\text{water}}gV_{\text{sphere}} = mg - \rho_{\text{water}}g\left(\frac{4}{3}\pi r^3\right)$$

Notice that the number of antinodes *n* is exactly the number of half wavelengths of standing waves on the string (i.e. there are two antinodes (and one full wavelength) on the string in Figure 18.11a, and there are five antinodes (and two and a half full wavelengths) in Figure 18.11b). From Equations 18.5 and 18.6 we have

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} = \frac{n}{2L}\sqrt{\frac{T}{\mu}},\ n = 1,\ 2,\ 3,\ 4,\ldots$$

The frequency of oscillation is the same in both cases because it is defined by the moving blade to the left. In addition, neither the total length of the string *L* nor the string density *μ* changes between the two cases:

$$f = \frac{n_1}{2L}\sqrt{\frac{T_1}{\mu}} \quad \text{and} \quad f = \frac{n_2}{2L}\sqrt{\frac{T_2}{\mu}}$$

therefore,

$$2Lf\sqrt{\mu} = n_1\sqrt{T_1} = n_2\sqrt{T_2}$$

Or equivalently,

$$T_2 = \left(\frac{n_1}{n_2}\right)^2 T_1 = \left(\frac{n_1}{n_2}\right)^2 mg$$

But we have already obtained the value for tension above in terms of the buoyant force and thus the radius of the sphere.

$$T_2 = \left(\frac{n_1}{n_2}\right)^2 T_1 = \left(\frac{n_1}{n_2}\right)^2 mg = mg - \rho_{\text{water}}g\left(\frac{4}{3}\pi r^3\right)$$

The radius of the sphere *r* may now be solved in terms of the number of antinodes n_2 (and the other parameters, n_1, *m*, *g*, and ρ_{water} which are all constants, or already defined in the problem).

$$\rho_{\text{water}}g\left(\frac{4}{3}\pi r^3\right) = mg\left(1 - \frac{n_1^2}{n_2^2}\right) \quad \rightarrow \quad r^3 = \frac{3m}{4\pi\rho_{\text{water}}}\left(1 - \frac{n_1^2}{n_2^2}\right)$$

solving for *r* gives

$$r = \left[\left(\frac{3m}{4\pi\rho_{\text{water}}}\right)\left(1 - \frac{n_1^2}{n_2^2}\right)\right]^{1/3}$$

$$= \left\{\left[\frac{3(2.00\ \text{kg})}{4\pi(10^3\ \text{kg/m}^3)}\right]\left(1 - \frac{4}{n^2}\right)\right\}^{1/3} = 0.078\ 2\left(1 - \frac{4}{n^2}\right)^{1/3}$$

where *r* is in meters.

(b) Because the factor inside the cube root

$$\left(1-\frac{4}{n^2}\right)^{1/3}$$

will be either zero or negative, which are each meaningless results, for *n* = 1 and 2, the minimum allowed value of *n* for a sphere of nonzero size is $n = \boxed{3}$.

(c) Because the mass of the sphere is held constant, while its radius (and thus also volume and density) is changed, there will reach a point where the density of the sphere reaches the density of water, and thus the sphere will float, so that it will no longer be fully immersed in the water. After this point, the sphere will float on the water, and will not produce further standing waves.

The limiting condition is $\rho_{\text{sphere}} = \rho_{\text{water}} = 1.00\times 10^3\ \text{kg/m}^3$.

But $\rho_{\text{sphere}} = \dfrac{m}{V} = \dfrac{m}{\frac{4}{3}\pi r^3}$ which may be rearranged to solve for *r*.

$$\frac{4}{3}\pi r^3 = \frac{m}{\rho_{\text{sphere}}} = \frac{m}{\rho_{\text{water}}} \quad \rightarrow \quad r = \left(\frac{3m}{4\pi\rho_{\text{water}}}\right)^{1/3}$$

and substituting in numerically:

$$r = \left(\frac{3m}{4\pi\rho_{\text{water}}}\right)^{1/3} = \left(\frac{3(2.00\ \text{kg})}{4\pi\left(1.0\times 10^3\ \text{kg/m}^3\right)}\right)^{1/3}$$
$$= \left(4.766\times 10^{-4}\ \text{m}^3\right)^{1/3} = \boxed{0.078\ 2\ \text{m}}$$

is the limiting (maximum) radius for which the sphere will stay totally immersed.

(d) $\boxed{\text{The sphere floats on the water.}}$

P18.74 (a) The wavelength is twice the length of string from the top end to the yo-yo: $\lambda = 2L$. The length *L* changes in time because the yo-yo is a particle under constant acceleration: $L = L_0 + \frac{1}{2}at^2$, where L_0 is the length of the string at *t* = 0 and *a* is the acceleration of the yo-yo. Therefore,

$$\boxed{\begin{aligned}\frac{d\lambda}{dt} &= \frac{d}{dt}(2L) = \frac{d}{dt}\left[2\left(L_0 + \frac{1}{2}at^2\right)\right] = 2at \\ &= 2\left(0.800\ \text{m/s}^2\right)(1.20\ \text{s}) = 1.92\ \text{m/s}\end{aligned}}$$

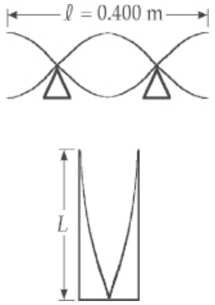
(b) For the second harmonic, the wavelength is equal to the length of the string. Therefore,

$$\frac{d\lambda}{dt} = \frac{d}{dt}L = \frac{d}{dt}\left(L_0 + \frac{1}{2}at^2\right) = at$$
$$= \left(0.800\ \text{m/s}^2\right)(1.20\ \text{s})$$
$$= \boxed{0.960\ \text{m/s, half as much as for the first harmonic}}.$$

(c) $\boxed{\text{Yes. A yo-yo of different mass will hold the string under different tension to make each string wave vibrate with a different frequency, but the geometrical argument given in part (a) still applies to the wavelength.}}$

(d) $\boxed{\text{Yes, for the same reason as in (c): the geometrical argument given in part (b) still applies to the wavelength.}}$

P18.75 $f = 87.0$ Hz. The speed of sound in air is $v_a = 343$ m/s.



ANS. FIG. P18.75

(a) The pattern on the bar (see upper figure at right) is ANANA, corresponding to the second harmonic. The wavelength on the bar is $\lambda_b = L$,

$$v = f\lambda_b = \left(87.0\ \text{s}^{-1}\right)(0.400\ \text{m})$$
$$= \boxed{34.8\ \text{m/s}}$$

(b) With $\lambda_a = 4L$ and $v_a = \lambda_a f$,

$$L = \frac{v_a}{4f} = \frac{343\ \text{m/s}}{4\left(87.0\ \text{s}^{-1}\right)} = \boxed{0.986\ \text{m}}$$

P18.76 (a) $$\mu = \frac{5.50\times10^{-3}\ \text{kg}}{0.860\ \text{m}} = 6.40\times10^{-3}\ \text{kg/m}$$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{1.30\ \text{kg}\cdot\text{m/s}^2}{6.40\times10^{-3}\ \text{kg/m}}} = \boxed{14.3\ \text{m/s}}$$

(b) The distance between a node and its adjacent antinode is one-quarter of a wavelength. In order for there to be a node at the bottom and an antinode at the top, the string can contain only an odd number of node-antinode pairs.

The simplest pattern is (top to bottom) AN = one node-antinode pair:

$$\frac{\lambda_1}{4} = L = \boxed{86.0\text{ cm}}$$

The next simplest pattern is ANAN = AN + NA + AN = three node-antinode pairs:

$$3\frac{\lambda_3}{4} = L \quad \rightarrow \quad \frac{\lambda_3}{4} = \frac{L}{3} = \boxed{28.7\text{ cm}}$$

The next simplest pattern is ANANAN = AN + NA + AN + NA + AN = five node-antinode pairs:

$$5\frac{\lambda_5}{4} = L \quad \rightarrow \quad \frac{\lambda_5}{4} = \frac{L}{5} = \boxed{17.2\text{ cm}}$$

(c) The distance between node and an antinode is $\frac{\lambda}{4}$. The corresponding frequency is

$$f_n = \frac{v}{4\left(\frac{\lambda_n}{4}\right)}: \quad \begin{cases} f_1 = \dfrac{v}{4\left(\frac{\lambda_1}{4}\right)} = \dfrac{14.3\text{ m/s}}{4(0.860\text{ m})} = \boxed{4.14\text{ Hz}} \\ f_3 = \dfrac{v}{4\left(\frac{\lambda_3}{4}\right)} = \dfrac{14.3\text{ m/s}}{4(0.287\text{ m})} = \boxed{12.4\text{ Hz}} \\ f_5 = \dfrac{v}{4\left(\frac{\lambda_5}{4}\right)} = \dfrac{14.3\text{ m/s}}{4(0.172\text{ m})} = \boxed{20.7\text{ Hz}} \end{cases}$$

P18.77 We consider velocities of approach and of recession separately in the Doppler equation, after we observe from our beat equation $f_b = |f_1 - f_2| = |f - f'|$ that the moving train must have an apparent frequency of either $f' = 182$ Hz or $f' = 178$ Hz.

We let v_t represent the magnitude of the train's velocity. If the train is moving away from the station, the apparent frequency is 178 Hz, lower, as described by

$$f' = \frac{v}{v + v_t}$$

and the train is **moving away** at

$$v_t = v\left(\frac{f}{f'} - 1\right) = (343\text{ m/s})\left(\frac{180\text{ Hz}}{178\text{ Hz}} - 1\right) = 3.85\text{ m/s}$$

If the train is pulling into the station, then the apparent frequency is 182 Hz. Again from the Doppler shift,

$$f' = \frac{v}{v - v_s}$$

The train is **approaching** at

$$v_s = v\left(1 - \frac{f}{f'}\right) = (343\text{ m/s})\left(1 - \frac{180\text{ Hz}}{182\text{ Hz}}\right)$$

$$v_s = 3.77\text{ m/s}$$

The moving train has a velocity of either $\boxed{\text{3.85 m/s away from the station or 3.77 m/s toward the station}}$.

***P18.78** (a) Use the Doppler formula:

$$f' = f\frac{(v \pm v_0)}{(v \mp v_s)}$$

with $f_1' =$ frequency of the speaker in front of student and

$f_2' =$ frequency of the speaker behind the student.

$$f_1' = (456\text{ Hz})\frac{(343\text{ m/s} + 1.50\text{ m/s})}{(343\text{ m/s} - 0)} = 458\text{ Hz}$$

$$f_2' = (456\text{ Hz})\frac{(343\text{ m/s} - 1.50\text{ m/s})}{(343\text{ m/s} + 0)} = 454\text{ Hz}$$

Therefore, $f_b = f_1' - f_2' = \boxed{3.99\text{ Hz}}$.

(b) The waves broadcast by both speakers have

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{456\text{ s}^{-1}} = 0.752\text{ m}$$

The standing wave between them has $d_{AA} = \frac{\lambda}{2} = 0.376\text{ m}$.

The student walks from one maximum to the next in time

$\Delta t = \frac{0.376\text{ m}}{1.50\text{ m/s}} = 0.251\text{ s}$, so the frequency at which she hears maxima is

$$f = \frac{1}{T} = \frac{1}{0.251\text{ s}} = \boxed{3.99\text{ Hz}}$$

***P18.79** As in Problem 32, we let $m = \rho V$ represent the mass of the copper cylinder. The original tension in the wire is $T_1 = mg = \rho Vg$. The water exerts a buoyant force $\rho_{\text{water}}(nV)g$ on the copper object, where *n* is the fraction of the object that is submerged, to reduce the tension to

$$T_2 = \rho Vg - \rho_{\text{water}}(nV)g = (\rho - n\rho_{\text{water}})Vg$$

The speed of a wave on the string changes from $\sqrt{\frac{T_1}{\mu}}$ to $\sqrt{\frac{T_2}{\mu}}$. The frequency changes from

$$f_1 = \frac{v_1}{\lambda} = \left(\frac{1}{\lambda}\right)\sqrt{\frac{T_1}{\mu}} \quad \text{to} \quad f_2 = \left(\frac{1}{\lambda}\right)\sqrt{\frac{T_2}{\mu}}$$

where we assume $\lambda = 2L$ is constant.

Then

$$\frac{f_2}{f_1} = \sqrt{\frac{T_2}{T_1}}$$

and

$$f_2 = f_1\sqrt{\frac{\rho - n\rho_{\text{water}}}{\rho}}$$

The frequency decreases as the fraction of the object that is submerged increases, with the lowest frequency occurring when the object is completely submerged, or $n = 1$:

$$f_2 = f_1\sqrt{\frac{\rho - n\rho_{\text{water}}}{\rho}} = (300\ \text{Hz})\sqrt{\frac{8.92 - (1.00)1.00}{8.92}}$$

$$= (300\ \text{Hz})\sqrt{\frac{7.92}{8.92}} = \boxed{283\ \text{Hz}}$$

P18.80 (a) Since the first node is at the weld, the wavelength in the thin wire is 2*L* or 80.0 cm. The frequency and tension are the same in both sections, so

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2(0.400\ \text{m})}\sqrt{\frac{4.60\ \text{N}}{2.00\times 10^{-3}\ \text{kg/m}}} = \boxed{59.9\ \text{Hz}}$$

(b) As the thick wire is twice the diameter, the linear density is 4 times that of the thin wire, or $\mu' = 8.00\ \text{g/m}$.

so

$$L' = \frac{1}{2f}\sqrt{\frac{T}{\mu'}}$$

$$= \left[\frac{1}{(2)(59.9\ \text{s}^{-1})}\right]\sqrt{\frac{4.60\ \text{N}}{8.00\times 10^{-3}\ \text{kg/m}}} = \boxed{20.0\ \text{cm}}$$

or half the length of the thin wire.

P18.81 The wavelength stays constant at $\lambda_1 = 2L$ while the wavespeed rises according to

$$v = (T/\mu)^{1/2} = [(15.0 + 10.0t/3.50)/\mu]^{1/2}$$

so the frequency rises as $f = v/\lambda = [(15.0 + 10.0t/3.50)/\mu]^{1/2}/2L$.

The number of cycles is $N = dt/T = f\,dt$ in each incremental bit of time, or altogether

$$N = \frac{1}{2L\sqrt{\mu}}\int_0^{3.5}\left(15.0 + \frac{10.0}{3.50}t\right)^{1/2}dt$$

$$= \frac{1}{2L\sqrt{3.50\mu}}\int_0^{3.5}(52.5 + 10.0t)^{1/2}dt$$

$$N = \frac{1}{2L\sqrt{3.50\mu}}\,\frac{1}{10.0(3/2)}(52.5 + 10.0t)^{3/2}\Big|_0^{3.5}$$

$$= \frac{1}{30L\sqrt{3.50\mu}}(52.5 + 10.0t)^{3/2}\Big|_0^{3.5}$$

$$N = \frac{1}{30.0(0.480\ \text{m})\sqrt{3.50(1.60\times 10^{-3}\ \text{kg/m})}} \times\left[(52.5 + 35.0)^{3/2} - (52.5)^{3/2}\right]$$

$$N = \boxed{407\ \text{cycles}}$$

P18.82 We use the basic relationship $f = \dfrac{n}{2L}\sqrt{\dfrac{T}{\mu}}$.

(a) Changing the length does not change the tension or the mass per unit length, so the wave speed is the same.

$$\frac{f'}{f} = \frac{L}{L'} = \frac{L}{2L} = \frac{1}{2}$$

The $\boxed{\text{frequency should be halved}}$ to get the same number of antinodes for twice the length.

(b) $\dfrac{n'}{n}=\sqrt{\dfrac{T}{T'}}$ so $\dfrac{T'}{T}=\left(\dfrac{n}{n'}\right)^2=\left[\dfrac{n}{n+1}\right]^2$

The tension must be $T'=\boxed{\left[\dfrac{n}{n+1}\right]^2 T}$.

(c) $\dfrac{f'}{f}=\dfrac{n'L}{nL'}\sqrt{\dfrac{T'}{T}}$ so $\dfrac{T'}{T}=\left(\dfrac{nf'L'}{n'fL}\right)^2=\left[\left(\dfrac{n}{n'}\right)\left(\dfrac{f'}{f}\right)\left(\dfrac{L'}{L}\right)\right]^2$

$\dfrac{T'}{T}=\left(\dfrac{3}{2\cdot 2}\right)^2 \quad\rightarrow\quad \boxed{\dfrac{T'}{T}=\dfrac{9}{16}}$ to get twice as many antinodes.

P18.83 We look for a solution of the form

$$5.00\sin(2.00x-10.0t)+10.0\cos(2.00x-10.0t)$$
$$=A\sin(2.00x-10.0t+\phi)$$
$$=A\sin(2.00x-10.0t)\cos\phi+A\cos(2.00x-10.0t)\sin\phi$$

This will be true if both $5.00=A\cos\phi$ and $10.0=A\sin\phi$, requiring

$$(5.00)^2+(10.0)^2=A^2 \quad\rightarrow\quad A=11.2,\ \text{and}$$

$$\tan\phi=\frac{10.0}{5.00}=2.00 \quad\rightarrow\quad \phi=63.4°$$

(a) $\boxed{\text{From above, we were able to find values for } A \text{ and } \phi\text{; therefore, the resultant wave is sinusoidal.}}$

(b) From above $\boxed{A=11.2}$ and $\boxed{\phi=63.4°}$.

P18.84 The speed of sound at Celsius temperature T_C is

$$v=(331\text{ m/s})\sqrt{1+\frac{T_C}{273°\text{C}}}$$

At 20.0°C, the speed of sound is 343 m/s.

(a) For a pipe open at both ends, the fundamental frequency (n = 1) is

$$f_1=\frac{v}{2L} \quad\rightarrow\quad L=\frac{v}{2f_1}=\frac{343\text{ m/s}}{2(261.6\text{ Hz})}=\boxed{0.656\text{ m}}$$

(b) The speed of sound in the colder room is smaller because the temperature is lower. The fundamental frequency of the pipe played in that room, call it f'_1, is smaller because the frequency of a standing wave is proportional to the wave speed. The beat frequency is

$$f_{\text{beat}} = \left|f'_1 - f'_1\right| = f_1 - f'_1$$

which gives

$$f_1 = f_1 - f_{\text{beat}} = 261.6 - 3.00 = 258.6 \text{ Hz}$$

because $f_1 = 261.6$ Hz is larger than f'_1.

The lengths of the flutes are the same, so compare frequencies and wave speeds:

$$f_1 = \frac{v}{2L} \quad \rightarrow \quad \frac{f'_1}{f_1} = \frac{v'}{v}$$

Solving for the wave speed gives

$$v' = v\frac{f'_1}{f_1} = (343 \text{ m/s})\left(\frac{258.6 \text{ Hz}}{261.6 \text{ Hz}}\right) = 339 \text{ m/s}$$

The wave speed depends on the temperature:

$$v = (331 \text{ m/s})\sqrt{1 + \frac{T_{\text{C}}}{273°}} = 339 \text{ m/s}$$

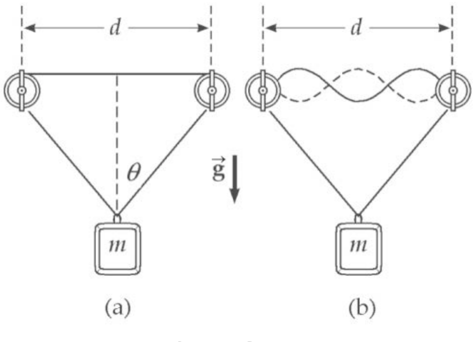
Solving for the temperature gives

$$T_C = 273°\left[\left(\frac{339 \text{ m/s}}{331 \text{ m/s}}\right)^2 - 1\right] = \boxed{13.5°\text{C}}$$

P18.85 (a) Let θ represent the angle each slanted rope makes with the vertical. In the diagram, observe that:

$$\sin\theta = \frac{1.00 \text{ m}}{1.50 \text{ m}} = \frac{2}{3}$$

or $\theta = 41.8°$



(a) (b)

ANS. FIG. P18.85

Considering the mass,

$$\sum F_y = 0: \quad 2T\cos\theta = mg$$

or $$T = \frac{(12.0\text{ kg})(9.80\text{ m/s}^2)}{2\cos 41.8°} = \boxed{78.9\text{ N}}$$

(b) The speed of transverse waves in the string is

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{78.9\text{ N}}{0.001\,00\text{ kg/m}}} = 281\text{ m/s}$$

For the standing wave pattern shown (3 loops), $d = \frac{3}{2}\lambda$,

or $$\lambda = \frac{2(2.00\text{ m})}{3} = 1.33\text{ m}.$$

Thus, the required frequency is $f = \frac{v}{\lambda} = \frac{281\text{ m/s}}{1.33\text{ m}} = \boxed{211\text{ Hz}}$.

P18.86 (a) Let θ (refer to ANS. FIG. P18.85) represent the angle each slanted rope makes with the vertical. In the diagram, observe that:

$$\sin\theta = \frac{d/2}{(L-d)/2} = \frac{d}{L-d}$$

and

$$\cos\theta = \sqrt{1-\sin^2\theta} = \left[1-\left(\frac{d}{L-d}\right)^2\right]^{\frac{1}{2}}$$

$$\cos\theta = \left[\frac{(L^2-2dL+d^2)-d^2}{(L-d)^2}\right]^{\frac{1}{2}}$$

$$\cos\theta = \frac{\sqrt{L^2-2dL}}{L-d}$$

Considering the mass,

$$\sum F_y = 0:\quad 2T\cos\theta = mg \rightarrow T = \frac{mg}{2\cos\theta}$$

or $$T = \boxed{\frac{mg(L-d)}{2\sqrt{L^2-2dL}}}.$$

(b) The speed of transverse waves in the string is $v = \sqrt{\frac{T}{\mu}}$.

For the standing wave pattern shown (3 loops), $d = \frac{3}{2}\lambda$, or $\lambda = \dfrac{2d}{3}$.

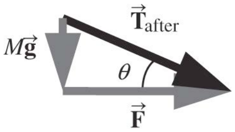
Thus, the required frequency is $f = \dfrac{v}{\lambda} = \boxed{\dfrac{3}{2d}\sqrt{\dfrac{mg(L-d)}{2\mu\sqrt{L^2-2dL}}}}$.

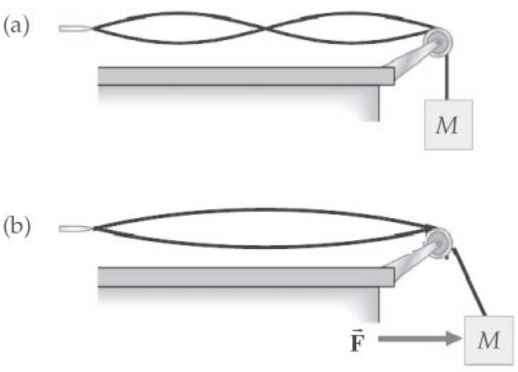
Challenge Problems

P18.87 The idea is that the tension on the string after the force is applied is the vector sum of the wind force $\vec{\mathbf{F}}$ and the weight $M\vec{\mathbf{g}}$ of the mass.

$$\vec{\mathbf{T}}_{\text{after}} = \vec{\mathbf{F}} + M\vec{\mathbf{g}}$$

Notice that this forms a right triangle:





ANS. FIG. P18.87

The relationship between the driving frequency and the string tension is

$$f = \frac{n}{2L}\sqrt{\frac{T}{\mu}}$$

Before and after the application of the wind force, the frequency *f*, string mass density *μ*, and string length *L* are all held constant. Thus, the string tension *T* is a function of only one variable, *n*.

$$f = \frac{n_1}{2L}\sqrt{\frac{T_1}{\mu}}, \quad f = \frac{n_2}{2L}\sqrt{\frac{T_2}{\mu}}; \quad \text{thus} \quad 2Lf\sqrt{\mu} = n_1\sqrt{T_1} = n_2\sqrt{T_2}$$

where $n_1 = 2$ and $n_2 = 1$:

$$T_{\text{after}} = T_2 = T_1\left(\frac{n_1}{n_2}\right)^2 = Mg\left(\frac{n_1}{n_2}\right)^2 = Mg\left(\frac{2}{1}\right)^2 = 4Mg$$

From the geometry of the right triangle,

$$T_2^2 = F^2 + (Mg)^2$$

$$(4Mg)^2 = F^2 + (Mg)^2 \rightarrow F^2 = 16(Mg)^2 - (Mg)^2 = 15(Mg)^2$$

$$F = \boxed{\sqrt{15}Mg}$$

P18.88 Equation 18.13 is

$$\begin{aligned} y(t) &= \sum(A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t) \\ &= \sum(A_n \sin n\omega t + B_n \cos n\omega t) \end{aligned}$$

(a) Multiplying by sin *mωt* gives:

$$y(t)\sin m\omega t = \sum \sin m\omega t(A_n \sin n\omega t + B_n \cos n\omega t)$$

Integrating over one period *T* gives:

$$\int_0^T y(t)\sin m\omega t dt = \sum\int_0^T A_n(\sin n\omega t)(\sin m\omega t)dt$$

$$+\sum\int_0^T B_n(\cos n\omega t)(\sin m\omega t)dt \qquad [1]$$

Inspecting the left-hand side of the equation, we note that *y*(*t*) is a positive constant *A* for half of the period *T*, and an equal but negative constant –*A* for the other half period:

$$\int_0^T y(t)\sin m\omega t dt = \int_0^{T/2} A\sin m\omega t dt + \int_{T/2}^T -A\sin m\omega t dt$$

If we look at the first of the two integrals on the right:

$$\int_0^{T/2} A\sin m\omega t dt = -\frac{A}{m\omega}\cos m\omega t\Big|_0^{T/2}$$

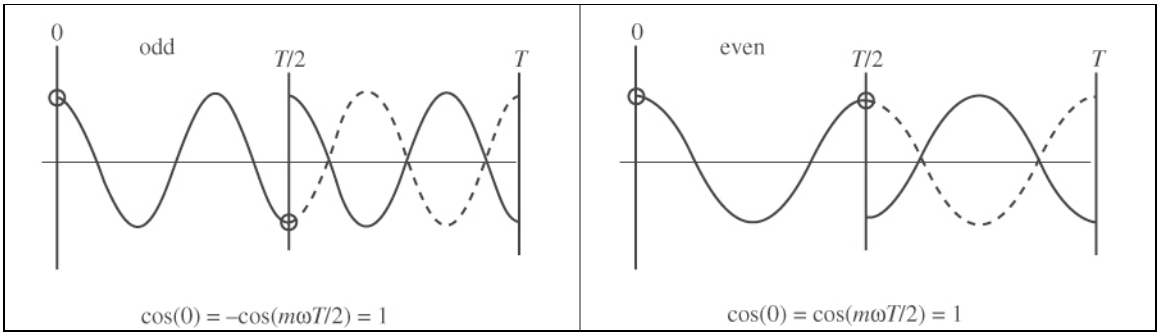
$$= -\frac{A}{m\omega}\left[\cos m\omega\left(\frac{T}{2}\right) - \cos m\omega(0)\right]$$

which gives different answers depending on whether *m* is even or odd:

If *m* is odd: $= -\dfrac{A}{m\omega}[(-1) - (1)] = \boxed{\dfrac{2A}{m\omega}}$

If *m* is even: $= -\dfrac{A}{m\omega}[(1) - (1)] = \boxed{0}$

(because we are integrating over half periods).



(a) **(b)**

ANS. FIG. P18.88

The second of the two integrals on the right gives a similar result:

$$\int_{T/2}^{T} -A\sin m\omega t dt = -\left(-\frac{A}{m\omega}\right)\cos m\omega t\Big|_{T/2}^{T}$$

$$= +\frac{A}{m\omega}\left[\cos m\omega(T) - \cos m\omega\left(\frac{T}{2}\right)\right]$$

$$= \left\{\begin{matrix} \left(\dfrac{2A}{m\omega}\right) & \text{odd} \\ (0) & \text{even} \end{matrix}\right\}$$

Thus,

$$\int_0^T y(t)\sin m\omega t dt = \int_0^{T/2} A\sin m\omega t dt + \int_{T/2}^{T} -A\sin m\omega t dt$$

$$= \left\{\begin{matrix} \dfrac{2A}{m\omega} & m\text{ odd} \\ 0 & m\text{ even} \end{matrix}\right\} + \left\{\begin{matrix} \dfrac{2A}{m\omega} & m\text{ odd} \\ 0 & m\text{ even} \end{matrix}\right\}$$

Putting everything together, we have shown that

$$\int_0^T y(t)\sin m\omega t dt = \begin{cases} \dfrac{4A}{m\omega} & m\text{ odd} \\ 0 & m\text{ even} \end{cases}$$

(b) We can analyze the terms involving B_n on the right hand side of eqn. [1] above:

$$\sum \int_0^T B_n(\cos n\omega t)(\sin m\omega t) dt$$

Using the trigonometric identity

$$\cos\alpha\sin\beta = \frac{1}{2}\sin(\alpha+\beta) - \frac{1}{2}\sin(\alpha-\beta)$$

we have

$$\sum\int_0^T B_n(\cos n\omega t)(\sin m\omega t)\,dt$$

$$= \sum\frac{1}{2}B_n\int_0^T\left[\sin(n\omega t + m\omega t) - \sin(n\omega t - m\omega t)\right]dt$$

$$= \sum\frac{1}{2}B_n\int_0^T\left[\sin(n+m)\omega t - \sin(n-m)\omega t\right]dt$$

The sine function, whether the terms are (*n* + *m*) or (*n* – *m*), it will always integrate to zero over any full multiple of a period:

$$= \sum\frac{1}{2}B_n\left[\sin(n+m)\omega t - \sin(n-m)\omega t\right]\Big|_0^T = \sum\frac{1}{2}B_n(0) = 0$$

Thus, all the terms involving B_n on the right hand side of eqn. [1] are equal to zero:

$$\sum\int_0^T B_n(\cos n\omega t)(\sin m\omega t)\,dt = 0$$

(c) For all the terms on the right hand side of eqn.(1) with A_n:

$$\sum\int_0^T A_n(\sin n\omega t\ )(\sin m\omega t)\,dt$$

Using the trigonometric identity

$$\sin\alpha\sin\beta = \frac{1}{2}\cos(\alpha-\beta) - \frac{1}{2}\cos(\alpha+\beta)$$

we have

$$\sum\int_0^T A_n(\sin n\omega t\ )(\sin m\omega t)\,dt$$

$$= \sum\frac{1}{2}A_n\int_0^T\left[\cos(n\omega t - m\omega t) - \cos(n\omega t + m\omega t)\right]dt$$

$$= \sum\frac{1}{2}A_n\int_0^T\left[\cos(n-m)\omega t - \cos(n+m)\omega t\right]dt$$

which can be integrated and evaluated at 0 and *T*:

$$= \sum\frac{1}{2}A_n\left[\frac{1}{(n-m)\omega}\sin(n-m)\,\omega t - \frac{1}{(n+m)\omega}\sin(n+m)\omega t\right]_0^T$$

The second term, when evaluated at 0 and *T*, always gives zero. The same is true for the first term for all values of *n* except where *n* = *m*. Thus, all the terms on the right hand side of eqn. (1) with A_n are zero except when *m* = *n*.

(d) For *n* = *m*, we will do the integration separately:

$$\sum\int_0^T A_n(\sin n\omega t)(\sin m\omega t)dt+\sum\int_0^T B_n(\cos n\omega t)(\sin m\omega t)dt$$

$$=\sum\int_0^T A_n(\sin n\omega t)(\sin m\omega t)dt+0$$

$$=\frac{1}{2}A_{n=m}\int_0^T[\cos(n-m)\omega t]dt=\frac{1}{2}\left[A_m\int_0^T\cos(0)dt\right]$$

$$=\frac{1}{2}A_m\int_0^T(1)dt=\frac{A_m}{2}[T-0]=\frac{1}{2}A_mT$$

Thus, the entire right side reduces to $\frac{1}{2}A_mT$.

(e) Starting with our original Equation 18.13:

$$y(t)=\sum\left(A_n\sin n\omega t\ +\ B_n\cos n\omega t\right)$$

notice that *y*(*t*) is an odd function of *t*: *y*(*t*) = –*y*(*t*), and the sine function is also odd, but the cosine function is even. From these observations, we can conclude that there are no cosine terms in the Fourier series expansion of *y*(*t*); therefore, all the B_n = 0. Thus,

$$y(t)=\sum A_n\sin n\omega t$$

But we have shown in part (a) above that:

$$\int_0^T y(t)\sin m\omega t dt=\frac{4A}{m\omega}$$

where *m* must be odd, and in part (d) that:

$$\int_0^T y(t)\sin m\omega t dt=\sum\int_0^T\sin m\omega t\left(A_n\sin n\omega t+B_n\cos n\omega t\right)dt$$

$$=\frac{1}{2}A_mT$$

where *n* = *m.*

Thus, for each A_n term: $\frac{1}{2}A_nT=\frac{4A}{n\omega}$. And because $\omega=\frac{2\pi}{T}$,

$$\frac{4A}{n\omega}=\frac{1}{2}A_nT\to A_n=\frac{8A}{n\omega T}=\frac{4A}{n\pi}\left(\frac{2\pi}{\omega T}\right)=\frac{4A}{n\pi}$$

which we substitute in to give:

$$\boxed{y(t) = \sum_{n} \frac{4A}{n\pi} \sin n\omega t}$$

where the summation is only over odd values of *n*.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P18.2 See ANS. FIG. P18.2.

P18.4 (a) See ANS. FIG. P18.4 (a-e); (b) See ANS. FIG. P18.4 (f-j)

P18.6 (a) $\lambda/4 = 0.113$ m; (b) $\lambda/2 = 0.227$ m

P18.8 (a) 3.33 rad; (b) 283 Hz

P18.10 The man walks only through two minima; a third minimum is impossible.

P18.12 0.500 s

P18.14 (a) The separation of adjacent nodes is $\Delta x = \frac{\pi}{k} = \frac{\lambda}{2}$. The nodes are still separated by half a wavelength; (b) Yes. The nodes are located at $kx + \frac{\phi}{2} = n\pi$, so that $x = \frac{n\pi}{k} - \frac{\phi}{2k}$, which means that each node is shifted $\frac{\phi}{2k}$ to the left by the phase difference between the traveling waves in comparison to the case in which $\phi = 0$.

P18.16 See P18.16 for full verification.

P18.18 (a) See ANS. FIG. P18.18; (b) In any one picture, the wavelength is the smallest distance along the x axis that contains a nonrepeating shape. The wavelength is $\lambda = 4$ m; (c) The frequency is the inverse of the period. The period is the time the wave takes to go from a full amplitude starting shape to the inversion of that shape and then back to the original shape. The period is the time interval between the top and bottom graphs: 20 ms. The frequency is 1/0.020 s = 50 Hz; (d) 4 m. By comparison with the wave function $y = (2A\sin kx)\cos\omega t$, we identify $k = \pi/2$, and then compute $\lambda = 2\pi/k$; (e) 50 Hz. By comparison with the wave function $y = (2A\sin kx)\cos\omega t$, we identify $\omega = 2\pi f = 100\pi$.

P18.20 (a) 0.600 m; (b) 30.0 Hz

P18.22 (a) 495 Hz; (b) 990 Hz

P18.24 (a) 5.20 m; (b) No. We do not know the speed of waves on the string.

P18.26 (a) 78.6 Hz; (b) 157 Hz, 236 Hz, 314 Hz

P18.28 (a) 4.90×10^{-3} kg/m; (b) 2; (c) no standing wave will form

P18.30 $m = \frac{Mg\cos\theta}{4f^2L}$

P18.32 291 Hz

P18.34 12 h, 24 min. The natural frequency of the water sloshing in the bay agrees precisely with that of lunar excitation, so we identify the extra-high tides as amplified by resonance.

P18.36 9.00 Hz

P18.38 2.94 cm

P18.40 (a) 536 Hz; (b) 42.9 mm

P18.42 (a) 17.0 Hz; (b) 33.9 Hz; (c) 17.6 Hz, 35.1 Hz

P18.44 0.502 m and 0.837 m

P18.46 *n*(206 Hz) and *n*(84.5 Hz)

P18.48 (a) 0.085 8*n* Hz, with $n = 1, 2, 3 \ldots$; (b) It is a good rule. A car horn would produce several or many of the closely-spaced resonance frequencies of the air in the tunnel, so it would be great amplified.

P18.50 $\dfrac{\pi r^2 v}{2Rf}$

P18.52 It is impossible because a single column could not produce both frequencies.

P18.54 1.16 m

P18.56 (a) 521 Hz or 525 Hz; (b) 526 Hz; (c) reduced by 1.14%

P18.58 85.7 Hz

P18.60 See P18.60 for a table of the frequencies of the harmonics of each note. The second harmonic of E is close to the third harmonic of A, and the fourth harmonic of C# is close to the fifth harmonic of A.

P18.62 (a) 0.522 m; (b) 316 Hz

P18.64 146 Hz

P18.66 (a) 5.0 Hz, 10.0 Hz, 15.0 Hz; (b) The frequency could be the fifth state at 25.0 Hz or any integer multiple, such as the tenth state at 50.0 Hz, the fifteenth state at 75.0 Hz, and so on.

P18.68 (a) larger; (b) 2.43

P18.70 (a) the particle under constant acceleration model; (b) waves under boundary conditions model; (c) $Mg \sin\theta$; (d) $h\left(\dfrac{1+\sin\theta}{\sin\theta}\right)$; (e) $\dfrac{m\sin\theta}{h(1+\sin\theta)}$; (f) $\sqrt{\dfrac{Mgh}{m}(1+\sin\theta)}$; (g) $\sqrt{\dfrac{Mg}{4mh}(1+\sin\theta)}$; (h) 121 Hz; (i) 60.6 Hz

P18.72 (a) greatest integer $\leq d\left(\dfrac{f}{v}\right)+\dfrac{1}{2}$;

(b) $L_n = \dfrac{d^2-\left(n-\frac{1}{2}\right)^2\left(\frac{v}{f}\right)^2}{2\left(n-\frac{1}{2}\right)\left(\frac{v}{f}\right)}$, where $n = 1, 2, \ldots, n_{\max}$

P18.74 (a) $\dfrac{d\lambda}{dt}=\dfrac{d}{dt}(2L)=\dfrac{d}{dt}\left[2\left(L_0+\dfrac{1}{2}at^2\right)\right]=2at=2\left(0.800\text{ m/s}^2\right)(1.20\text{ s})=1.92$ m/s; (b) 0.960 m/s, half as much as for the first harmonic; (c) Yes. A yo-yo of different mass will hold the string under different tension to make each string wave vibrate with a different frequency, but the geometrical argument given in part (a) still applies to the wavelength; (d) Yes, for the same reason as (c); the geometrical argument given in part (b) still applies to the wavelength.

P18.76 (a) 14.3 m/s; (b) 86.0 cm, 28.7 cm, 17.2 cm; (c) 4.14 Hz, 12.4 Hz, 20.7 Hz

P18.78 (a) 3.99 Hz; (b) 3.99 Hz

P18.80 (a) 59.9 Hz; (b) 20.0 cm

P18.82 (a) frequency should be halved; (b) $\left[\dfrac{n}{n+1}\right]^2 T$; (c) $\dfrac{T'}{T}=\dfrac{9}{16}$

P18.84 (a) 0.656 m; (b) 13.5° C

P18.86 (a) $\dfrac{mg(L-d)}{2\sqrt{L^2-2dL}}$; (b) $\dfrac{3}{2d}\sqrt{\dfrac{mg(L-d)}{2\mu\sqrt{L^2-2dL}}}$

P18.88 (a) see P18.88(a) for full explanation; (b) see P18.88(b) for full explanation; (c) See P18.88(c) for full explanation; (d) see P18.88(d) for full explanation; (e) see P18.88(e) for full explanation.

19

Temperature

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ19.1 Answer (b). The markings are now farther apart than intended, so measurements made with the heated steel tape will be too short—but only by a factor of 5×10^{-5} of the measured length.

OQ19.2 Answer (d). Remember that one must use absolute temperatures and pressures in the ideal gas law. Thus, the original temperature is $T_K = T_C + 273.15 = 25 + 273.15 = 298$ K, and with the mass of the gas constant, the ideal gas law gives

$$T_2 = \left(\frac{P_2}{P_1}\right)\left(\frac{V_2}{V_1}\right)T_1 = \left(\frac{1.07\times10^6\ \text{Pa}}{5.00\times10^6\ \text{Pa}}\right)(3.00)(298\ \text{K}) = 191\ \text{K}$$

OQ19.3 Answer (d). From the ideal gas law, with the mass of the gas constant, $P_2V_2/T_2 = P_1V_2/T_1$. Thus,

$$P_2 = \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right)P_1 = \left(\frac{1}{2}\right)(4)P_1 = 2P_1$$

OQ19.4 Answer (a). As the temperature increases, the brass expands. This would effectively increase the distance d from the pivot point to the center of mass of the pendulum, and also increase the moment of inertia of the pendulum. Since the moment of inertia is proportional to d^2, and the period of a physical pendulum is $T = 2\pi\sqrt{\dfrac{I}{mgd}}$, the period would increase, and the clock would run slow.

OQ19.5 Answer (c). $T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(162 - 32) = 72.2°\text{C}$, then,

$$T_K = T_C + 273.15 = 72.2 + 273.15 = 345\ \text{K}$$

OQ19.6 Answer (c). From the ideal gas law, with the mass of the gas constant, $P_2V_2/T_2 = P_1V_2/T_1$. Thus,

$$V_2 = \left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)V_1 = (4)(1)(0.50\ \text{m}^3) = 2.0\ \text{m}^3$$

OQ19.7 Answer (d). If glass were to expand more than the liquid, the liquid level would fall relative to the tube wall as the thermometer is warmed. If the liquid and the tube material were to expand by equal amounts, the thermometer could not be used because the liquid level would not change with temperature.

OQ19.8 The ranking is (a) = (b) = (d) > (e) > (c). We think about nRT/V in each case. Since R is constant, we need only think about nT/V, and units of $\text{mmol}\cdot\text{K}/\text{cm}^3$ are as convenient as any: (a) $2\cdot 3/1 = 6$, (b) 6, (c) 4, (d) 6, (e) 5.

OQ19.9 Answer (d). Cylinder A must be at lower pressure. If the gas is thin, $PV = nRT$ applies to both with the same value of nRT for both. Then A will be at one-third the absolute pressure of B.

OQ19.10 (i) Answer (a). Call the process isobaric cooling or isobaric contraction. The rubber wall is easy to stretch. The air inside is nearly at atmospheric pressure originally and stays at atmospheric pressure as the wall moves in, just maintaining equality of pressure outside and inside. The air is nearly an ideal gas to start with, and stays fairly ideal—fairly far from liquefaction—even at 100 K. The water vapor liquefies and then freezes, and the carbon dioxide turns to dry ice, but these are minor constituents of the air. Thus, as the absolute temperature drops to 1/3 of its original value and the volume will drop to 1/3 of what it was.

(ii) Answer (c). As noted above, the pressure stays nearly constant at 1 atm.

OQ19.11 Answer (c). For a quick approximation, multiply 93 m and 17 and

1 / (1 000 000 °C) and say 5°C for the temperature increase. To simplify, multiply 100 and 100 and 1 / 1 000 000 for an answer in meters: it is on the order of 1 cm.

OQ19.12 Answer (b). Around atmospheric pressure, 0°C is the only temperature at which liquid water and solid water can both exist.

OQ19.13 Answer (b). When a solid, containing a cavity, is heated, the cavity expands in the same way as it would if filled with the material making up the rest of the object.

OQ19.14 Answer (e).

$$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(-25°) + 32° = -13° \text{ F}$$

ANSWERS TO CONCEPTUAL QUESTIONS

CQ19.1 The coefficient of linear expansion must be greater for mercury than for glass, otherwise the interior of a glass thermomter would expand more and the mercury level would drop. See OQ19.7.

CQ19.2 (a) The copper's temperature drops and the water temperature rises until both temperatures are the same. (b) The water and copper are in thermal equilibrium when their temperatures are the same.

CQ19.3 (a) $PV = nRT$ predicts V going to zero as T goes to zero.

(b) The ideal-gas model does not apply when the material gets close to liquefaction and then turns into a liquid or solid. The molecules start to interact all the time, not just in brief collisions. The molecules start to take up a significant portion of the volume of the container.

CQ19.4 Air pressure decreases with altitude while the pressure inside the bags stays the same; thus, that inside pressure is greater than the outside pressure.

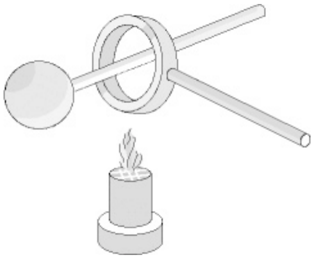
CQ19.5 (a) No. The thermometer will only measure the temperature of whatever is in contact with the thermometer. The thermometer would need to be brought to the surface in order to measure its temperature, since there is no atmosphere on the Moon to maintain a relatively consistent ambient temperature above the surface. (b) It would read the temperature of the glove, since it is in contact with the glove.

CQ19.6 The coefficient of expansion of metal is larger than that of glass. When hot water is run over the jar, both the glass and the lid expand, but at different rates. Since *all* dimensions expand, the inner diameter of the lid expands more than the top of the jar, and the lid will be easier to remove.

CQ19.7 (a) As the water rises in temperature, it expands or rises in pressure or both. The excess volume would spill out of the cooling system, or else the pressure would rise very high indeed.

(b) Modern cooling systems have an overflow reservoir to accept the excess volume when the coolant heats up and expands.

CQ19.8 (a) The sphere expands when heated, so that it no longer fits through the ring. With the sphere still hot, you can separate the sphere and ring by heating the ring. This more surprising result occurs because the thermal expansion of the ring is not like the inflation of a blood-pressure cuff. Rather, it is like a photographic enlargement; every linear dimension, including the hole diameter, increases by the same factor. The reason for this is that the atoms everywhere, including those around the inner circumference, push away from each other. The only way that the atoms can accommodate the greater distances is for the circumference—and corresponding diameter—to grow. This property was once used to fit metal rims to wooden wagon wheels. If the ring is heated and the sphere left at room temperature, the sphere would pass through the ring with more space to spare.



ANS. FIG. CQ19.8

(b) Heating the ring increases its diameter, the sphere can pass through it easily. The hole in the ring expands as if it were filled with the material of the ring.

CQ19.9 Two objects in thermal equilibrium need not be in contact. Consider the two objects that are in thermal equilibrium in Figure 16.1(c). The act of separating them by a small distance does not affect how the molecules are moving inside either object, so they will still be in thermal equilibrium.

CQ19.10 (a) One mole of H_2 has a mass of 2.016 0 g.

(b) One mole of He has a mass of 4.002 6 g.

(c) One mole of CO has a mass of 28.010 g.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 19.2 Thermometers and the Celsius Temperature Scale

Section 19.3 The Constant-Volume Gas Thermometer and the Absolute Temperature Scale

P19.1 (a) By Equation 19.2,

$$T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(41.5°\text{C}) + 32 = (74.7 + 32)°\text{F} = \boxed{107°\text{F}}$$

(b) $\boxed{\text{Yes. The normal body temperature is 98.6°F, so the patient has a high fever and needs immediate attention.}}$

P19.2 (a) Consider the freezing and boiling points of water in each scale: 0°C and 100°C; 32°F and 212°F. We see that there are 100 Celsius units for every 180 Fahrenheit units:

$$\frac{\Delta T_C}{\Delta T_F} = \frac{100°\text{C}}{180°\text{F}} \quad \rightarrow \quad \Delta T_C = \frac{5}{9}(\Delta T_F) = \frac{5}{9}(57.0)°\text{C} = \boxed{31.7°\text{C}}$$

(b) The Kelvin unit is the same size as the Celsius unit:

$$T = T_C + 273.15 \quad \rightarrow \quad \Delta T = \Delta T_C$$

$$\Delta T = \Delta T_\text{C}\left(\frac{1\text{ K}}{1\text{ °C}}\right) = 31.7\text{ K}\left(\frac{1\text{ K}}{1\text{ °C}}\right) = \boxed{31.7\text{ K}}$$

P19.3 (a) By Equation 19.2,

$$T_\text{F} = \frac{9}{5}T_\text{C} + 32 = \frac{9}{5}(-78.5) + 32 = \boxed{-109°\text{F}}$$

And, from Equation 19.1,

$$T = T_\text{C} + 273.15 = (-78.5 + 273.15)\text{ K} = \boxed{195\text{ K}}$$

(b) Again,

$$T_\text{F} = \frac{9}{5}T_\text{C} + 32 = \frac{9}{5}(37.0) + 32 = \boxed{98.6°\text{F}}$$

$$T = T_\text{C} + 273.15 = (37.0 + 273.15)\text{ K} = \boxed{310\text{ K}}$$

P19.4 (a) The relationship between the Kelvin and Celsius scales is given by Equation 19.1:

$$T = T_C + 273.15$$

Thus 20.3 K converts to

$$T_C = T - 273.15 = 20.3\text{ K} - 273.15\text{ K} = \boxed{-253°\text{C}}$$

(b) The relationship between the Celsius and Fahrenheit scales is, from Equation 19.2,

$$T_F = \frac{9}{5}T_C + 32°\text{F}$$

Thus –253°C converts to

$$T_F = \frac{9}{5}T_C + 32°\text{F} = \frac{9}{5}(-253°\text{C}) + 32°\text{F} = \boxed{-423°\text{F}}$$

P19.5 (a) By Equation 19.2,

$$T_F = \frac{9}{5}T_C + 32.0°\text{F} = \frac{9}{5}(-195.81°\text{C}) + 32.0 = \boxed{-320°\text{F}}$$

(b) Applying Equation 19.1,

$$T = T_C + 273.15 = -195.81°\text{C} + 273.15 = \boxed{77.3\text{ K}}$$

***P19.6** (a) To convert from Fahrenheit to Celsius, we use

$$T_C = \frac{5}{9}(T_F - 32.0)$$

The temperature at Furnace Creek Ranch in Death Valley is

$$T_C = \frac{5}{9}(T_F - 32.0) = \frac{5}{9}(134°\text{F} - 32.0) = \boxed{56.7°\text{C}}$$

and the temperature at Prospect Creek Camp in Alaska is

$$T_C = \frac{5}{9}(T_F - 32.0) = \frac{5}{9}(-79.8°\text{F} - 32.0) = \boxed{-62.1°\text{C}}$$

(b) We find the Kelvin temperature from Equation 19.1, $T = T_C + 273.15$. The record temperature on the Kelvin scale at Furnace Creek Ranch in Death Valley is

$$T = T_C + 273.15 = 56.7°\text{C} + 273.15 = \boxed{330\text{ K}}$$

and the temperature at Prospect Creek Camp in Alaska is

$$T = T_C + 273.15 = -62.11°\text{C} + 273.15 = \boxed{211\text{ K}}$$

P19.7 Since we have a linear graph, we know that the pressure is related to the temperature as $P = A + BT_C$, where A and B are constants. To find A and B, we use the given data:

$$0.900\text{ atm} = A + B(-78.5°\text{C}) \qquad \textbf{[1]}$$

and

$$1.635\text{ atm} = A + B(78.0°\text{C}) \qquad \textbf{[2]}$$

Solving Equations [1] and [2] simultaneously, we find:

$$A = 1.27\text{ atm} \quad \text{and} \quad B = 4.70\times10^{-3}\text{ atm/°C}$$

Therefore,

$$P = 1.27\text{ atm} + \left(4.70\times10^{-3}\text{ atm/°C}\right)T_C$$

(a) At absolute zero the gas exerts zero pressure ($P = 0$), so

$$T_C = \frac{-1.27\text{ atm}}{4.70\times10^{-3}\text{ atm/°C}} = \boxed{-270°\text{C}}$$

(b) At the freezing point of water, $T_C = 0$ and

$$P = 1.27\text{ atm} + 0 = \boxed{1.27\text{ atm}}$$

At the boiling point of water, $T_C = 100°\text{C}$, so

$$P = 1.27\text{ atm} + \left(4.70\times10^{-3}\text{ atm/°C}\right)(100°\text{C}) = \boxed{1.74\text{ atm}}$$

Section 19.4 Thermal Expansion of Solids and Liquids

P19.8 Each section can expand into the joint space to the north of it. We need think of only one section expanding. Using Equation 19.4,

$$\Delta L = L_i\alpha\,\Delta T = (25.0\text{ m})\left[12.0\times10^{-6}\ (°\text{C})^{-1}\right](40.0°\text{C})$$
$$= \boxed{1.20\text{ cm}}$$

: (a) By Equation 19.4,

$$\Delta L = \alpha L_i\Delta T = \left[9.00\times10^{-6}(°\text{C})^{-1}\right](30.0\text{ cm})(65.0°\text{C})$$
$$= \boxed{0.176\text{ mm}}$$

(b) The diameter is a linear dimension, so Equation 19.4 still applies:

$$\Delta L = \alpha L_i\Delta T = \left[9.00\times10^{-6}\ (°\text{C})^{-1}\right](1.50\text{ cm})(65.0°\text{C})$$
$$= 8.78\times10^{-4}\text{ cm} = \boxed{8.78\ \mu\text{m}}$$

(c) Using the volumetric coefficient of expansion β, and $V_i = \pi d^2L/4$,

$$\Delta V = \beta V_i\Delta T \approx 3\alpha V\Delta T$$

$$\Delta V = \beta V_i \Delta T \approx 3\alpha V_i \Delta T$$

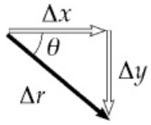
$$= 3\left[9.00\times10^{-6}\ (°\text{C})^{-1}\right]\left(\frac{30.0(\pi)(1.50)^2}{4}\ \text{cm}^3\right)(65.0°\text{C})$$

$$= \boxed{0.093\,0\ \text{cm}^3}$$

P19.10 The horizontal section expands according to $\Delta L = \alpha L_i \Delta T$.

$$\Delta x = \left[17\times10^{-6}\ (°\text{C})^{-1}\right](28.0\ \text{cm})(46.5°\text{C} - 18.0°\text{C})$$

$$= 1.36\times10^{-2}\ \text{cm}$$



ANS. FIG. P19.10

The vertical section expands similarly by

$$\Delta y = \left[17\times10^{-6}(°\text{C})^{-1}\right](134\ \text{cm})(28.5°\text{C}) = 6.49\times10^{-2}\ \text{cm}$$

The vector displacement of the pipe elbow has magnitude

$$\Delta r = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(0.136\ \text{mm})^2 + (0.649\ \text{mm})^2} = 0.663\ \text{mm}$$

and is directed to the right below the horizontal at angle

$$\theta = \tan^{-1}\left(\frac{\Delta y}{\Delta x}\right) = \tan^{-1}\left(\frac{0.649\ \text{mm}}{0.136\ \text{mm}}\right) = 78.2°$$

$$\boxed{\Delta r = 0.663\ \text{mm to the right at } 78.2°\text{ below the horizontal}}$$

P19.11 The wire is 35.0 m long when $T_C = -20.0°\text{C}$.

$$\Delta L = L_i \bar{\alpha}(T - T_i)$$

Since $\bar{\alpha} = \alpha(20.0°\text{C}) = 1.70\times10^{-5}\ (°\text{C})^{-1}$ for Cu,

$$\Delta L = (35.0\ \text{m})\left[1.70\times10^{-5}\ (°\text{C})^{-1}\right]\left[35.0°\text{C} - (-20.0°\text{C})\right]$$

$$= \boxed{+3.27\ \text{cm}}$$

***P19.12** For the dimensions to increase, $\Delta L = \alpha L_i \Delta T$:

$$1.00\times10^{-2}\ \text{cm} = \left[1.30\times10^{-4}\ (°\text{C})^{-1}\right](2.20\ \text{cm})(T - 20.0°\text{C})$$

$$T = \boxed{55.0°\text{C}}$$

***P19.13** By Equation 19.4,

$$\Delta L = \alpha L_i \Delta T = \left[11\times10^{-6}\ (^\circ\text{C})^{-1}\right](1\,300\text{ km})[35^\circ\text{C} - (-73^\circ\text{C})]$$
$$= \boxed{1.54\text{ km}}$$

The expansion can be compensated for by mounting the pipeline on rollers and placing Ω-shaped loops between straight sections. They bend as the steel changes length.

***P19.14** By Equation 19.4,

$$\Delta L = \alpha L_i \Delta T = \left[22\times10^{-6}\ (^\circ\text{C})^{-1}\right](2.40\text{ cm})(30.0^\circ\text{C})$$
$$= \boxed{1.58\times10^{-3}\text{ cm}}$$

***P19.15** (a) Following the logic in the textbook for obtaining Equation 19.6 from Equation 19.4, we can express an expansion in area as

$$\Delta A = 2\alpha A_i \Delta T$$
$$= 2\left[17.0\times10^{-6}(^\circ\text{C})^{-1}\right](0.080\,0\text{ m})^2(50.0^\circ\text{C})$$
$$= 1.09\times10^{-5}\text{ m}^2 = \boxed{0.109\text{ cm}^2}$$

(b) The length of each side of the hole has increased. Thus, this represents an $\boxed{\text{increase}}$ in the area of the hole.

***P19.16** By Equation 19.6,

$$\Delta V = (\beta - 3\alpha)V_i \Delta T$$
$$= \left[5.81\times10^{-4}(^\circ\text{C})^{-1} - 3\left(11.0\times10^{-6}(^\circ\text{C})^{-1}\right)\right]$$
$$\times(50.0\text{ gal})(20.0^\circ\text{C})$$
$$= \boxed{0.548\text{ gal}}$$

***P19.17** (a) By Equation 19.4, $L = L_i(1 + \alpha\Delta T)$, and

$$5.050\text{ cm} = 5.000\text{ cm}\left[1 + \left(24.0\times10^{-6}(^\circ\text{C})^{-1}\right)(T - 20.0^\circ\text{C})\right]$$

which gives $T = \boxed{437^\circ\text{C}}$

(b) We must get $L_{\text{Al}} = L_{\text{Brass}}$ for some ΔT, or

$$L_{i,\text{ Al}}(1 + \alpha_{\text{Al}}\Delta T) = L_{i,\text{ Brass}}(1 + \alpha_{\text{Brass}}\Delta T)$$
$$5.000\text{ cm}\left[1 + \left(24.0\times10^{-6}(^\circ\text{C})^{-1}\right)\Delta T\right]$$
$$= 5.050\text{ cm}\left[1 + \left(19.0\times10^{-6}(^\circ\text{C})^{-1}\right)\Delta T\right]$$

Solving for ΔT,

$$\Delta T = 2\ 080°\text{C}$$

so $\boxed{T = 2.1\times 10^{3}°\text{C}}$

(c) $\boxed{\text{No. Aluminum melts at 660°C (Table 17.2). Also, although it is not in Table 17.2, internet research shows that brass (an alloy of copper and zinc) melts at about 900°C.}}$

P19.18 We solve for the temperature *T* at which the brass ring would fit over the aluminum cylinder.

$$L_{\text{Al}}\left(1+\alpha_{\text{Al}}\Delta T\right) = L_{\text{Brass}}\left(1+\alpha_{\text{Brass}}\Delta T\right)$$

$$\Delta T = T - T_i = \frac{L_{\text{Al}} - L_{\text{Brass}}}{L_{\text{Brass}}\alpha_{\text{Brass}} - L_{\text{Al}}\alpha_{\text{Al}}}$$

$$\Delta T = \frac{10.02\text{ cm} - 10.00\text{ cm}}{(10.00\text{ cm})\left(19.0\times 10^{-6}\ (°\text{C})^{-1}\right) - (10.02\text{ cm})\left(24.0\times 10^{-6}\ (°\text{C})^{-1}\right)}$$

$$\Delta T = -396 = T - 20.0 \quad\rightarrow\quad T = -376°\text{C}$$

The situation is impossible because the $\boxed{\text{required } T = -376°\text{C is below absolute zero.}}$

P19.19 (a) The original volume of the acetone we take as precisely 100 mL. After it is finally cooled to 20.0°C, its volume is

$$V_f = V_i(1+\beta\Delta T) = (100\text{ ml})\left\{1+\left[1.50\times 10^{-4}(°\text{C})^{-1}\right](-15.0°\text{C})\right\}$$

$$= \boxed{99.8\text{ mL}}$$

(b) Initially, the volume of the acetone reaches the 100-mL mark on the flask, but the acetone cools and the flask warms to a temperature of 32.0 °C. Thus, the volume of the acetone decreases and the volume of the flask increases. $\boxed{\text{This means the acetone will be below the 100-mL mark on the flask.}}$

P19.20 (a) The material would expand by $\Delta L = \alpha L_i \Delta T$, or $\dfrac{\Delta L}{L_i} = \alpha\Delta T$, but instead feels stress

$$\frac{F}{A} = \frac{Y\Delta L}{L_i}$$

$$= Y\alpha\Delta T = \left(7.00\times 10^{9}\ \text{N/m}^2\right)\left[12.0\times 10^{-6}(\text{C°})^{-1}\right](30.0°\text{C})$$

$$= \boxed{2.52\times 10^{6}\ \text{N/m}^2}$$

(b) The stress is less than the compressive strength, so $\boxed{\text{the concrete will not fracture.}}$

P19.21 (a) The amount of turpentine that overflows equals the difference in the change in volume of the cylinder and the turpentine:

$$\Delta V = V_t\beta_t\Delta T - V_{\text{Al}}\beta_{\text{Al}}\Delta T = \left(\beta_t - 3\alpha_{\text{Al}}\right)V_i\Delta T$$

$$= \left[9.00\times 10^{-4}\,(^\circ\text{C})^{-1} - 3\left(24.0\times 10^{-6}\,(^\circ\text{C})^{-1}\right)\right] \times\left(2\ 000\ \text{cm}^3\right)\left(60.0^\circ\text{C}\right)$$

$$\Delta V = \boxed{99.4\ \text{cm}^3}\ \text{overflows.}$$

(b) Find the volume of the turpentine remaining in the cylinder at 80.0°C, which is the same as the volume of the aluminum cylinder at 80.0°C:

$$V_t = V_{\text{Al}} = V_{\text{Al}i} + \beta_{\text{Al}}V_{\text{Al}i}\Delta T = V_{\text{Al}i} + 3\alpha_{\text{Al}}V_{\text{Al}i}\Delta T$$

$$= V_{\text{Al}i}\left(1 + 3\alpha_{\text{Al}}\Delta T\right)$$

$$= \left(2\ 000\ \text{cm}^3\right)\left[1 + 3\left(24\times 10^{-6}\,(^\circ\text{C})^{-1}\right)\left(60.0^\circ\text{C}\right)\right]$$

$$= 2\ 008.64\ \text{cm}^3 = \boxed{2.01\ \text{L}}$$

(c) Find the volume of the turpentine in the cylinder after it cools back to 20.0°C:

$$V = V_{ti} + \beta_t V_{ti}\Delta T = V_{ti}\left(1 + \beta_t\Delta T\right)$$

$$= \left(2\ 008.64\ \text{cm}^3\right)\left[1 + \left(9\times 10^{-4}\,(^\circ\text{C})^{-1}\right)\left(-60.0^\circ\text{C}\right)\right]$$

$$= 1\ 900.17\ \text{cm}^3$$

Find the percentage of the cylinder that is empty at 20.0°C:

$$\frac{2\ 000\ \text{cm}^3 - 1\ 900.17\ \text{cm}^3}{2\ 000\ \text{cm}^3} = 4.99\%$$

Find the empty height of the cylinder above the turpentine:

$$\left(4.99\%\right)\left(20.0\ \text{cm}\right) = \boxed{0.998\ \text{cm}}$$

P19.22 We model the wire as contracting according to $\Delta L = \alpha L_i\Delta T$ and then stretching according to

$$\text{stress} = \frac{F}{A} = Y\frac{\Delta L}{L_i} = \frac{Y}{L_i}\alpha L_i\Delta T = Y\alpha\Delta T$$

(a) We find the tension from

$$\begin{aligned} F &= YA\alpha\Delta T \\ &= \left(20.0\times 10^{10}\ \text{N/m}^2\right)\left(4.00\times 10^{-6}\ \text{m}^2\right) \\ &\qquad \times\left[11\times 10^{-6}\ (°\text{C})^{-1}\right](45.0°\text{C}) \\ &= \boxed{396\ \text{N}} \end{aligned}$$

(b) $\Delta T = \dfrac{\text{stress}}{Y\alpha} = \dfrac{3.00\times 10^{8}\ \text{N/m}^2}{\left(20.0\times 10^{10}\ \text{N/m}^2\right)\left(11\times 10^{-6}/\text{C}°\right)} = 136°\text{C}$

To increase the stress the temperature must decrease to $35°\text{C} - 136°\text{C} = \boxed{-101°\text{C}}$.

(c) $\boxed{\text{The original length divides out, so the answers would not change.}}$

P19.23 (a) The density of a sample of lead of mass *m* = 20.0 kg, volume V_0, at temperature T_0 is

$$\rho_0 = \frac{m}{V_0} = 11.3\times 10^{3}\ \text{kg/m}^3$$

For a temperature change $\Delta T = T - T_0$, the same mass *m* occupies a larger volume $V = V_0(1+\beta\Delta T)$; therefore, the density is

$$\rho = \frac{m}{V_0(1+\beta\Delta T)} = \frac{\rho_0}{(1+\beta\Delta T)}$$

where $\beta = 3\alpha$, and $\alpha = 29\times 10^{-6}\ (°\text{C})^{-1}$.

For a temperature change of from 0.00°C to 90.0°C,

$$\begin{aligned} \rho &= \frac{\rho_0}{(1+\beta\Delta T)} = \frac{11.3\times 10^{3}\ \text{kg/m}^3}{1+3\left(29\times 10^{-6}\ \left({}^{\circ}\text{C}\right)^{-1}\right)\left(90.0\ {}^{\circ}\text{C}\right)} \\ &= \boxed{11.2\times 10^{3}\ \text{kg/m}^3} \end{aligned}$$

(b) The mass is still the same, $\boxed{20.0\ \text{kg}}$, because a temperature change would not change the mass.

P19.24 (a) The density of a solid substance of mass *m*, volume V_0, at temperature T_0 is

$$\rho_0 = \frac{m}{V_0}$$

For a temperature change $\Delta T = T - T_0$, the same mass *m* occupies a larger volume $V = V_0(1+\beta\Delta T)$; therefore, the density is

$$\rho = \frac{m}{V_0(1+\beta\Delta T)} = \boxed{\frac{\rho_0}{1+\beta\Delta T}}$$

(b) The mass is still the same, $\boxed{m}$, because a temperature change would not change the mass.

P19.25 From Equation 19.3, the difference in Celsius temperature in the underground tank and the tanker truck is

$$\Delta T_C = \frac{5}{9}(\Delta T_F) = \frac{5}{9}(95.0°\text{F} - 52.0°\text{F}) = 23.9°\text{C}$$

If $V_{52.0°\text{F}}$ is the volume of gasoline that fills the tank at 52.0°F, the volume this quantity of gas would occupy on the tanker truck at 95.0°F is

$$\begin{aligned} V_{95.0°\text{F}} &= V_{52.0°\text{F}} + \Delta V = V_{52.0°\text{F}} + \beta V_{52.0°\text{F}}\Delta T = V_{52.0°\text{F}}(1+\beta\Delta T) \\ &= (1.00\times 10^3\ \text{gal})\left\{1 + \left[9.6\times 10^{-4}\ (°\text{C})^{-1}\right](23.9°\text{C})\right\} \\ &= 1.02\times 10^3\ \text{gal} \end{aligned}$$

Section 19.5 Macroscopic Description of an Ideal Gas

P19.26 If the volume and the temperature are both constant, the ideal gas law gives

$$\frac{P_f \cancel{V_f}}{P_i \cancel{V_i}} = \frac{n_f \cancel{RT_f}}{n_i \cancel{RT_i}}$$

or $$n_f = \left(\frac{P_f}{P_i}\right) n_i = \left(\frac{5.00\ \text{atm}}{25.0\ \text{atm}}\right)(1.50\ \text{mol}) = 0.300\ \text{mol}$$

so the amount of gas to be withdrawn is

$$\Delta n = n_i - n_f = 1.50\ \text{mol} - 0.300\ \text{mol} = \boxed{1.20\ \text{mol}}$$

P19.27 The initial and final absolute temperatures are

$$T_i = T_{\text{C},i} + 273 = (25.0 + 273)\ \text{K} = 298\ \text{K}$$

and

$$T_f = T_{\text{C},f} + 273 = (75.0 + 273)\ \text{K} = 348\ \text{K}$$

The volume of the tank is assumed to be unchanged, or $V_f = V_i$. Also, since two-thirds of the gas is withdrawn, $n_f = n_i/3$. Thus, from the ideal gas law, we obtain

$$\frac{P_f \cancel{V_f}}{P_i \cancel{V_i}} = \frac{n_f \cancel{R} T_f}{n_i \cancel{R} T_i}$$

or

$$P_f = \left(\frac{n_f}{n_i}\right)\left(\frac{T_f}{T_i}\right)P_i = \left(\frac{1}{3}\right)\left(\frac{348\text{ K}}{298\text{ K}}\right)(11.0\text{ atm}) = \boxed{4.28\text{ atm}}$$

P19.28 When the tank has been prepared and is ready to use it contains 1.00 L of air and 4.00 L of water. Consider the air in the tank during one discharge process. We suppose that the process is slow enough that the temperature remains constant. Then as the pressure drops from 2.40 atm to 1.20 atm, the volume of the air doubles ($PV \approx$ constant) resulting in 1.00 L of water expelled and 3.00 L remaining. In the second discharge, the air volume doubles from 2.00 L to 4.00 L and 2.00 L of water is sprayed out. In the third discharge, only the last 1.00 L of water comes out.

> In each pump-up-and-discharge cycle, the volume of air in the tank doubles. Thus 1.00 L of water is driven out by the air injected at the first pumping, 2.00 L by the second, and only the remaining 1.00 L by the third. Each person could more efficiently use his device by starting with the tank half full of water, instead of 80% full.

P19.29 (a) From the ideal gas law,

$$n = \frac{PV}{RT} = \frac{(9.00\text{ atm})(1.013\times10^{5}\text{ Pa/atm})(8.00\times10^{-3}\text{ m}^3)}{(8.314\text{ N}\cdot\text{mol K})(293\text{ K})}$$

$$= \boxed{2.99\text{ mol}}$$

(b) The number of molecules is

$$N = nN_\text{A} = (2.99\text{ mol})(6.02\times10^{23}\text{ molecules/mol})$$

$$= \boxed{1.80\times10^{24}\text{ molecules}}$$

P19.30 (a) From $PV = nRT$, we obtain $n = \frac{PV}{RT}$. Then

$$m = nM = \frac{PVM}{RT}$$
$$= \frac{\left(1.013\times10^{5}\text{ Pa}\right)\left(0.100\text{ m}\right)^{3}\left(28.9\times10^{-3}\text{ kg/mol}\right)}{\left(8.314\text{ J/mol}\cdot\text{K}\right)\left(300\text{ K}\right)}$$
$$= \boxed{1.17\times10^{-3}\text{ kg}}$$

(b) $F_g = mg = \left(1.17\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = \boxed{11.5\text{ mN}}$

(c) $F = PA = \left(1.013\times10^{5}\text{ N/m}^2\right)\left(0.100\text{ m}\right)^2 = \boxed{1.01\text{ kN}}$

(d) The $\boxed{\text{molecules must be moving very fast}}$ to hit the walls hard.

P19.31 The equation of state of an ideal gas is $PV = nRT$, so we need to solve for the number of moles to find N.

$$n = \frac{PV}{RT} = \frac{\left(1.01\times10^{5}\text{ N/m}^2\right)\left[\left(10.0\text{ m}\right)\left(20.0\text{ m}\right)\left(30.0\text{ m}\right)\right]}{\left(8.314\text{ J/mol}\cdot\text{K}\right)\left(293\text{ K}\right)}$$
$$= 2.49\times10^{5}\text{ mol}$$

Then,

$$N = nN_A = \left(2.49\times10^{5}\text{ mol}\right)\left(6.022\times10^{23}\text{ molecules/mol}\right)$$
$$= \boxed{1.50\times10^{29}\text{ molecules}}$$

P19.32 From the ideal gas law, $PV = nRT$, and

$$\frac{m_f}{m_i} = \frac{n_f}{n_i} = \frac{P_f V_f}{RT_f}\frac{RT_i}{P_i V_i} = \frac{P_f}{P_i}$$

so $$m_f = m_i\left(\frac{P_f}{P_i}\right)$$

and $$|\Delta m| = m_i - m_f = m_i\left(\frac{P_i - P_f}{P_i}\right) = 12.0\text{ kg}\left(\frac{41.0\text{ atm} - 26.0\text{ atm}}{41.0\text{ atm}}\right)$$
$$= \boxed{4.39\text{ kg}}$$

P19.33 (a) From the ideal gas law, $PV = nRT$, so

$$n = \frac{PV}{RT} = \frac{\left(1.013\times10^{5}\text{ Pa}\right)\left(1.00\text{ m}^3\right)}{\left(8.314\text{ J/mol}\cdot\text{K}\right)\left(293\text{ K}\right)} = \boxed{41.6\text{ mol}}$$

(b) $m = nM = (41.6 \text{ mol})(28.9 \text{ g/mol}) = \boxed{1.20 \text{ kg}}$

(c) $\boxed{\text{This value agrees with the tabulated density}}$ of 1.20 kg/m^3 at 20.0°C.

***P19.34** One mole of helium contains Avogadro's number of molecules and has a mass of 4.00 g. Let us call m_0 the mass of one atom, and we have

$$N_\text{A} m_0 = 4.00 \text{ g/mol}$$

or

$$m_0 = \frac{4.00 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.64 \times 10^{-24} \text{ g/molecule}$$

$$= \boxed{6.64 \times 10^{-27} \text{ kg}}$$

***P19.35** The CO_2 is far from liquefaction, so after it comes out of solution it behaves as an ideal gas. Its molar mass is $M = 12.0 \text{ g/mol} + 2(16.0 \text{ g/mol}) = 44.0 \text{ g/mol}$. The quantity of gas in the cylinder is

$$n = \frac{m_\text{sample}}{M} = \frac{6.50 \text{ g}}{44.0 \text{ g/mol}} = 0.148 \text{ mol}$$

Then $PV = nRT$ gives

$$V = \frac{nRT}{P}$$

$$= \frac{0.148 \text{ mol}(8.314 \text{ J/mol} \cdot \text{K})(273.15 \text{ K} + 20°\text{C})}{1.013 \times 10^5 \text{ N/m}^2} \times \left(\frac{1 \text{ N} \cdot \text{m}}{1 \text{ J}}\right)\left(\frac{10^3 \text{ L}}{1 \text{ m}^3}\right)$$

$$= \boxed{3.55 \text{ L}}$$

P19.36 We use Equation 19.10, $PV = Nk_BT$:

$$N = \frac{PV}{k_\text{B}T} = \frac{(1.00 \times 10^{-9} \text{ Pa})(1.00 \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J/K})(300 \text{ K})} = \boxed{2.42 \times 10^{11} \text{ molecules}}$$

P19.37 (a) Initially, $P_iV_i = n_iRT_i$: $(1.00 \text{ atm})V_i = n_iR[(10.0°\text{C} + 273.15) \text{ K}]$ **[1]**

Finally, $P_fV_f = n_fRT_f$: $P_f(0.280V_i) = n_iR[(40.0°\text{C} + 273.15) \text{ K}]$ **[2]**

Dividing [2] by [1]: $\dfrac{0.280P_f}{1.00 \text{ atm}} = \dfrac{313.15 \text{ K}}{283.15 \text{ K}}$

giving $P_f = \boxed{3.95 \text{ atm} = 4.00 \times 10^5 \text{ Pa}}$

(b) After being driven, $P_d(1.02)(0.280V_i) = n_iR(85.0°\text{C} + 273.15)\ \text{K}$ **[3]**

Dividing [3] by [1]: $\dfrac{(1.02)(0.280)P_d}{1.00\ \text{atm}} = \dfrac{358.15\ \text{K}}{283.15\ \text{K}}$

$$P_d = 4.43\ \text{atm} = \boxed{4.49\times10^5\ \text{Pa}}$$

P19.38 The air in the tube is far from liquefaction, so it behaves as an ideal gas. At the ocean surface it is described by $P_tV_t = nRT$, where $P_t =$ 1 atm, $V_t = A$ (6.50 cm), and A is the cross-sectional area of the interior of the tube. At the bottom of the dive,

$$P_bV_b = nRT = P_bA(6.50\ \text{cm} - 2.70\ \text{cm})$$

By division,

$$\frac{P_b(3.80\ \text{cm})}{(1\ \text{atm})(6.50\ \text{cm})} = 1$$

$$P_b = (1.013\times10^5\ \text{N/m}^2)\left(\frac{6.50\ \text{cm}}{3.80\ \text{cm}}\right) = 1.73\times10^5\ \text{N/m}^2$$

The salt water enters the tube until the air pressure is equal to the water pressure at depth, which is described by

$$P_b = P_t + \rho gh$$

$$1.73\times10^5\ \text{N/m}^2 = 1.013\times10^5\ \text{N/m}^2 + (1\,030\ \text{kg/m}^3)(9.80\ \text{m/s}^2)h$$

solving for the depth h of the dive gives

$$h = \frac{7.20\times10^4\ \text{kg}\cdot\text{m}\cdot\text{m}^2\cdot\text{s}^2}{1.01\times10^4\ \text{s}^2\cdot\text{m}^2\cdot\text{kg}} = \boxed{7.13\ \text{m}}$$

P19.39 The density of the air inside the balloon, ρ_{in}, must be reduced until the buoyant force of the outside air is at least equal to the weight of the balloon plus the weight of the air inside it:

$$\sum F_y = 0: \quad B - W_{\text{air inside}} - W_{\text{balloon}} = 0$$

$$\rho_{\text{out}}gV - \rho_{\text{in}}gV - m_bg = 0 \quad \rightarrow \quad (\rho_{\text{out}} - \rho_{\text{in}})V = m_b$$

where $\rho_{\text{out}} = 1.244\ \text{kg/m}^3$, $V = 400\ \text{m}^3$, and $m_b = 200$ kg.

From $PV = nRT$, $\dfrac{n}{V} = \dfrac{P}{RT}$. This equation means that at constant pressure the density is inversely proportional to the temperature. Thus, the density of the hot air inside the balloon is

$$\rho_{\text{in}} = \rho_{\text{out}}\left(\frac{283\ \text{K}}{T_{\text{in}}}\right)$$

Substituting this result into the condition $(\rho_{\text{out}} - \rho_{\text{in}})V = m_b$ gives

$$\rho_{\text{out}}\left(1 - \frac{283\text{ K}}{T_{in}}\right) = \frac{m_b}{V} \quad \rightarrow \quad \frac{283\text{ K}}{T_{in}} = 1 - \frac{m_b}{\rho_{out}V}$$

$$\rightarrow \quad T_{in} = \frac{283\text{ K}}{\left(1 - \dfrac{m_b}{\rho_{out}V}\right)}$$

$$T_{in} = \frac{283\text{ K}}{\left(1 - \dfrac{200\text{ kg}}{(1.244\text{ kg/m}^3)(400\text{ m}^3)}\right)} = \boxed{473\text{ K}}$$

***P19.40** To compute the mass of air leaving the room, we begin with the ideal gas law:

$$P_0V = n_1RT_1 = \left(\frac{m_1}{M}\right)RT_1$$

As the temperature is increased at constant pressure,

$$P_0V = n_2RT_2 = \left(\frac{m_2}{M}\right)RT_2$$

Subtracting the two equations gives

$$\boxed{m_1 - m_2 = \frac{P_0VM}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)}$$

P19.41 At depth, $P = P_0 + \rho gh$ and $PV_i = nRT_i$

At the surface, $P_0V_f = nRT_f$: $\dfrac{P_0V_f}{(P_0 + \rho gh)V_i} = \dfrac{T_f}{T_i}$

Therefore, $V_f = V_i\left(\dfrac{T_f}{T_i}\right)\left(\dfrac{P_0 + \rho gh}{P_0}\right)$ and

$$V_f = 1.00\text{ cm}^3\left(\frac{293\text{ K}}{278\text{ K}}\right)$$
$$\times\left(\frac{(1.013\times10^5\text{ Pa}) + (1\,030\text{ kg/m}^3)(9.80\text{ m/s}^2)(25.0\text{ m})}{1.013\times10^5\text{ Pa}}\right)$$

$$V_f = \boxed{3.68\text{ cm}^3}$$

P19.42 My bedroom is 4 m long, 4 m wide, and 2.4 m high, enclosing air at 100 kPa and 20°C = 293 K. Think of the air as 80.0% N_2 and 20.0% O_2.

Avogadro's number of molecules has mass

$$(0.800)(28.0\ \text{g/mol})+(0.200)(32.0\ \text{g/mol})=0.028\ 8\ \text{kg/mol}$$

Then $PV=nRT=\left(\dfrac{m}{M}\right)RT$ gives

$$m=\frac{PVM}{RT}=\frac{(1.00\times10^5\ \text{N/m}^2)(38.4\ \text{m}^3)(0.028\ 8\ \text{kg/mol})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}$$

$$=45.4\ \text{kg}\ \boxed{\sim10^2\ \text{kg}}$$

P19.43 Pressure inside the cooker is due to the pressure of water vapor plus the air trapped inside. The pressure of the water vapor is

$$P_\text{v}=\frac{nRT}{V}=\left(\frac{9.00\ \text{g}}{18.0\ \text{g/mol}}\right)\left(\frac{8.314\ \text{J}}{\text{mol K}}\right)\left(\frac{773\ \text{K}}{2.00\times10^{-3}\ \text{m}^3}\right)$$

$$=1.61\ \text{MPa}$$

We find the pressure of the air at constant volume, assuming the initial temperature is 10°C:

$$\frac{P_{a2}}{P_{a1}}=\frac{T_2}{T_1}\quad\rightarrow\quad P_{a2}=P_{a1}\frac{T_2}{T_1}=(101\ \text{kPa})\frac{773\ \text{K}}{283\ \text{K}}$$

$$=276\ \text{kPa}=0.276\ \text{MPa}$$

The total pressure is

$$P=P_\text{v}+P_{a2}=1.61\ \text{MPa}+0.276\ \text{MPa}=\boxed{1.89\ \text{MPa}}$$

P19.44 If P_{gi} is the initial gauge pressure of the gas in the cylinder, the initial absolute pressure is $P_{\text{i,abs}}=P_{gi}+P_0$, where P_0 is the exterior pressure. Likewise, the final absolute pressure in the cylinder is $P_{\text{f,abs}}=P_{gf}+P_0$, where P_{gf} is the final gauge pressure. The initial and final masses of gas in the cylinder are $m_i=n_iM$ and $m_f=n_fM$, where n is the number of moles of gas present and M is the molecular weight of this gas. Thus, $m_f/m_i=n_f/n_i$.

We assume the cylinder is a rigid container whose volume does not vary with internal pressure. Also, since the temperature of the cylinder is constant, its volume does not expand or contract. Then, the ideal gas law (using absolute pressures) with both temperature and volume constant gives

$$\frac{P_{\text{f,abs}}\cancel{V}}{P_{\text{i,abs}}\cancel{V}}=\frac{n_f\cancel{RT}}{n_i\cancel{RT}}=\frac{m_f}{m_i}\qquad\text{or}\qquad m_f=m_i\left(\frac{P_{\text{f,abs}}}{P_{\text{i,abs}}}\right)$$

and in terms of gauge pressures,

$$\boxed{m_f = m_i\left(\frac{P_{gf} + P_0}{P_{gi} + P_0}\right)}$$

Additional Problems

P19.45 The astronauts exhale this much CO_2:

$$\begin{aligned} n &= \frac{m_{\text{sample}}}{M} \\ &= \left(\frac{1.09 \text{ kg}}{\text{astronaut}\cdot\text{day}}\right)\left(\frac{1\,000 \text{ g}}{1 \text{ kg}}\right) \\ &\qquad \times(3 \text{ astronauts})(7 \text{ days})\left(\frac{1 \text{ mol}}{44.0 \text{ g}}\right) = 520 \text{ mol} \end{aligned}$$

Then 520 mol of methane is generated. It is far from liquefaction and behaves as an ideal gas, so the pressure is

$$\begin{aligned} P &= \frac{nRT}{V} = \frac{(520 \text{ mol})(8.314 \text{ J/mol}\cdot\text{K})(273.15 \text{ K} - 45 \text{ K})}{150\times 10^{-3} \text{ m}^3} \\ &= \boxed{6.57\times 10^6 \text{ Pa}} \end{aligned}$$

P19.46 We must first convert both the initial and final temperatures to Celsius:

$$T_C = \frac{5}{9}(T_F - 32)$$

Thus,

$$T_{initial} = \frac{5}{9}\left(T_{F,initial} - 32\right) = \frac{5}{9}(15.000 - 32.000) = -9.444°\text{C}$$

$$T_{final} = \frac{5}{9}\left(T_{F,final} - 32\right) = \frac{5}{9}(90.000 - 32.000) = 32.222°\text{C}$$

The length of the steel beam after heating is L_f, and the linear expansion of the beam follows the equation: $\Delta L = L_f - L_i = \alpha L_i \Delta T$

Thus,

$$\begin{aligned} L_f &= \alpha L_i\left(T_f - T_i\right) + L_i \\ &= \left(11\times 10^{-6}°\text{C}^{-1}\right)(35.000 \text{ m})\left[32.222°\text{C} - (-9.444°\text{C})\right] \\ &\qquad + 35.000 \text{ m} \\ &= 0.016 \text{ m} + 35.000 \text{ m} = \boxed{35.016 \text{ m}} \end{aligned}$$

P19.47 (a) The diameter is a linear dimension, so we consider the linear expansion of steel:

$$\begin{aligned} d &= d_0\left[1+\alpha(\Delta T)\right] \\ &= (2.540\text{ cm})\left[1+\left(11\times10^{-6}\ (^\circ\text{C})^{-1}\right)(100.00^\circ\text{C}-25.00^\circ\text{C})\right] \\ &= \boxed{2.542\text{ cm}} \end{aligned}$$

(b) If the volume increases by 1%, then $\Delta V = (1.000\times10^{-2})\,V_0$. Then, using $\Delta V = \beta V_0(\Delta T)$, where $\beta = 3\alpha$ is the volume expansion coefficient, we find

$$\Delta T = \frac{\Delta V/V_0}{\beta} = \frac{1.000\times10^{-2}}{3\left[11.0\times10^{-6}\ (^\circ\text{C})^{-1}\right]} = \boxed{3.0\times10^2\ ^\circ\text{C}}$$

P19.48 The ideal gas law will be used to find the pressure in the tire at the higher temperature. However, one must always be careful to use absolute temperatures and absolute pressures in all ideal gas law calculations.

The initial absolute pressure is

$$P_i = P_{i,\text{gauge}} + P_{\text{atm}} = 2.50\text{ atm} + 1.00\text{ atm} = 3.50\text{ atm}$$

The initial absolute temperature is

$$T_i = T_{i,\text{C}} + 273.15 = (15.0 + 273.15)\text{ K} = 288.2\text{ K}$$

and the final absolute temperature is

$$T_f = T_{f,\text{C}} + 273.15 = (45 + 273.15)\text{ K} = 318.2\text{ K}$$

The ideal gas law, with volume and quantity of gas constant, gives the final absolute pressure as

$$\frac{P_f \cancel{V_f}}{P_i \cancel{V_i}} = \frac{\cancel{n_f}\cancel{R}T_f}{\cancel{n_i}\cancel{R}T_i}$$

$$\Rightarrow \quad P_f = \left(\frac{T_f}{T_i}\right)P_i = \left(\frac{318.2\text{ K}}{288.2\text{ K}}\right)(3.50\text{ atm}) = 3.86\text{ atm}$$

The final gauge pressure in the tire is

$$P_{f,\text{gauge}} = P_f - P_{\text{atm}} = 3.86\text{ atm} - 1.00\text{ atm} = \boxed{2.86\text{ atm}}$$

***P19.49** Some gas will pass through the porous plug from the reaction chamber 1 to the reservoir 2 as the reaction chamber is heated, but the net quantity of gas stays constant according to $n_{i1} + n_{i2} = n_{f1} + n_{f2}$.

Assuming the gas is ideal, we apply $n = \frac{PV}{RT}$ to each term:

$$\frac{P_i V_0}{(300\text{ K})R} + \frac{P_i(4V_0)}{(300\text{ K})R} = \frac{P_f V_0}{(673\text{ K})R} + \frac{P_f(4V_0)}{(300\text{ K})R}$$

$$1\text{ atm}\left(\frac{5}{300\text{ K}}\right) = P_f\left(\frac{1}{673\text{ K}} + \frac{4}{300\text{ K}}\right)$$

$$\boxed{P_f = 1.12\text{ atm}}$$

P19.50 Let us follow the cycle, assuming that the conditions for ideal gases apply. (That is, that the gas never comes near the conditions for which a phase transition would occur.)

We may use the ideal gas law:

$$PV = nRT$$

in which the pressure and temperature must be total pressure (in pascals or atm, depending on the units of *R* chosen), and absolute temperature (in K).

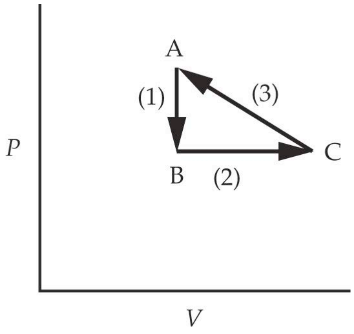
For **stage (1)** of the cycle, the process is:

$$PV = nRT \rightarrow V\Delta P = nR\Delta T$$

And, because only *T* and *P* vary:

$$\frac{\Delta T}{\Delta P} = \frac{V}{nR} = \text{const.}$$

Thus: $$\frac{T_f}{P_f} = \frac{T_i}{P_i} = \frac{V}{nR} = \text{const.}$$



ANS. FIG. P16.70

However, when we substitute into the temperature–pressure relation for **stage (1)**, we obtain:

$$\frac{T_f}{P_f}=\frac{T_i}{P_i} \quad\to\quad T_{\mathrm{B}}=T_f=\frac{P_f}{P_i}T_i=\frac{0.870\text{ atm}}{1.000\text{ atm}}\left(150°C+273.15\right)$$

$$=368.14\text{ K}=\boxed{95.0°\text{C}}$$

$\boxed{\text{$T$ falls below 100°C, so steam condenses and the expensive apparatus falls (assuming that the boiling point does not change significantly with the change in pressure).}}$

P19.51 We assume the dimensions of the capillary tube do not change.

For mercury, $\beta=1.82\times10^{-4}\,(°\text{C})^{-1}$

and for Pyrex glass, $\alpha=3.20\times10^{-6}\,(°\text{C})^{-1}$

The volume of the liquid increases as $\Delta V_\ell=V\beta\Delta T$.

The volume of the shell increases as $\Delta V_g=3\alpha V\Delta T$.

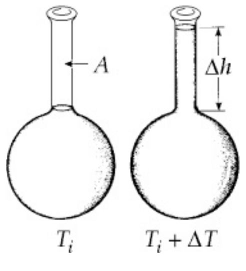
Therefore, the overflow in the capillary is $\Delta V_c=V\Delta T\left(\beta-3\alpha\right)$, and in the capillary $\Delta V_c=A\Delta h$.

$$\Delta V_c=A\Delta h=V\Delta T\left(\beta-3\alpha\right) \quad\to\quad \Delta h=\frac{\left(\beta-3\alpha\right)V\Delta T}{A}$$

$$\Delta h=\left(\frac{1}{\pi\left(\frac{0.004\ 00\times10^{-2}\text{ m}}{2}\right)^2}\right)\left[1.82\times10^{-4}\,(°\text{C})^{-1}-3\left(3.20\times10^{-6}\,(°\text{C})^{-1}\right)\right]$$

$$\times\left[\frac{4}{3}\pi\left(\frac{0.250\times10^{-2}\text{ m}}{2}\right)^3\right]\left(30.0^{\circ}\text{C}\right)$$

$$\Delta h=3.37\times10^{-2}\text{ m}=\boxed{3.37\text{ cm}}$$



ANS. FIG. P19.51

P19.52 We assume the dimensions of the capillary do not change. The volume of the liquid increases by $\Delta V_\ell = V\beta\Delta T$. The volume of the shell increases by $\Delta V_g = 3\alpha V\Delta T$. Therefore, the overflow in the capillary is $\Delta V_c = V\Delta T(\beta - 3\alpha)$; and in the capillary $\Delta V_c = A\Delta h$.

Therefore, $\boxed{\Delta h = (\beta - 3\alpha)\frac{V\Delta T}{A}}$.

P19.53 The fundamental frequency played by the cold-walled flute is

$$f_i = \frac{v}{\lambda_i} = \frac{v}{2L_i}$$

Assuming the change in the speed of sound as a function of temperature is negligible, when the instrument warms up

$$f_f = \frac{v}{\lambda_f} = \frac{v}{2L_f} = \frac{v}{2L_i(1+\alpha\Delta T)} = \frac{f_i}{1+\alpha\Delta T}$$

The final frequency is lower. The change in frequency is

$$\Delta f = f_f - f_i = f_i\left(\frac{1}{1+\alpha\Delta T} - 1\right)$$

$$\Delta f = \frac{(343\ \text{m/s})}{2(0.655\ \text{m})}\left(\frac{1}{1+(24.0\times 10^{-6}/\text{C}^\circ)(15.0^\circ\text{C})} - 1\right)$$

$$= \boxed{-0.0942\ \text{Hz}}$$

This change in frequency is imperceptibly small.

P19.54 Let L_0 represent the length of each bar at 0°C.

(a) In the diagram consider the right triangle that each invar bar makes with one half of the aluminum bar. We have

$$\sin\left(\frac{\theta}{2}\right) = \frac{L_0(1+\alpha_{\text{Al}}\Delta T)/2}{L_0} = \frac{L_0(1+\alpha_{\text{Al}}\Delta T)}{2L_0}.$$

Solving gives

$$\boxed{\theta = 2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2}\right)}$$

where T_C is the Celsius temperature.

(b) $\boxed{\text{Yes.}}$ If the temperature drops, the negative value of Celsius temperature describes the contraction. So the answer is accurate.

(c) $\boxed{\text{Yes.}}$ At $T_C = 0$ we have $\theta = 2\sin^{-1}(1/2) = 60.0°$, and this is accurate.

(d) From the same triangle we have

$$\sin\left(\frac{\theta}{2}\right) = \frac{L_0(1+\alpha_{\text{Al}}\Delta T)}{2L_0(1+\alpha_{\text{invar}}\Delta T)}$$

giving

$$\boxed{\theta = 2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2(1+\alpha_{\text{invar}}T_C)}\right)}$$

(e) The greatest angle is at 660°C,

$$\theta = 2\sin^{-1}\left(\frac{1+\alpha_{\text{Al}}T_C}{2(1+\alpha_{\text{invar}}T_C)}\right) = 2\sin^{-1}\left(\frac{1+(24\times10^{-6})660}{2(1+[0.9\times10^{-6}]660)}\right)$$

$$= 2\sin^{-1}\left(\frac{1.015\ 84}{2.0011\ 88}\right) = 2\sin^{-1}0.508 = \boxed{61.0°}$$

(f) The smallest angle is at –273°C,

$$\theta = 2\sin^{-1}\left(\frac{1+(24\times10^{-6})(-273)}{2(1+[0.9\times10^{-6}][-273])}\right)$$

$$= 2\sin^{-1}\left(\frac{0.9934}{1.9995}\right) = 2\sin^{-1}0.497 = \boxed{59.6°}$$

P19.55 The excess expansion of the brass is

$$\Delta L_{\text{rod}} - \Delta L_{\text{tape}} = \left(\alpha_{\text{brass}} - \alpha_{\text{steel}}\right)L_i\Delta T$$

$$\Delta(\Delta L) = (19.0 - 11.0)\times10^{-6}\ (°\text{C})^{-1}(0.950\text{ m})(35.0°\text{C})$$

$$\Delta(\Delta L) = 2.66\times10^{-4}\text{ m}$$

(a) The rod contracts more than the tape to a length reading

$$0.950\ 0\text{ m} - 0.000\ 266\text{ m} = \boxed{94.97\text{ cm}}$$

(b) $0.950\ 0\text{ m} + 0.000\ 266\text{ m} = \boxed{95.03\text{ cm}}$

P19.56 At 0°C, mass *m* of gasoline occupies volume $V_{0°\text{C}}$; the density of the gasoline is

$$\rho_{0°\text{C}} = \frac{m}{V_{0°\text{C}}} = 730\text{ kg/m}^3$$

At temperature ΔT above 0°C, the same mass of gasoline occupies a larger volume $V = V_{0°\text{C}}(1+\beta\Delta T)$: the density of the gasoline is

$$\rho = \frac{m}{V_{0°\text{C}}(1+\beta\Delta T)} = \frac{\rho_{0°\text{C}}}{1+\beta\Delta T},$$ which is slightly smaller than $\rho_{0°\text{C}}$.

For the same volume of gasoline, the difference in mass between gasoline at 0°C and gasoline at 20.0°C is

$$\Delta m = \rho_{0°\text{C}}V - \rho V = \rho_{0°\text{C}}V - \frac{\rho_{0°\text{C}}}{1+\beta\Delta T}V$$

$$\Delta m = \rho_{0°\text{C}}V\left(1 - \frac{1}{1+\beta\Delta T}\right)$$

$$\Delta m = \left[\left(730\ \text{kg/m}^3\right)\left(10.0\ \text{gal}\right)\left(\frac{0.003\,80\ \text{m}^3}{1.00\ \text{gal}}\right)\right]$$
$$\times\left(1 - \frac{1}{1+\left(9.60\times10^{-4}(°\text{C})^{-1}\right)(20.0°\text{C})}\right)$$

$$\Delta m = \boxed{0.523\ \text{kg}}$$

P19.57 (a) $\rho = \frac{m}{V}$ and $d\rho = -\frac{m}{V^2}dV$

For very small changes in *V* and ρ, this can be expressed as

$$\Delta\rho = -\frac{m}{V}\frac{\Delta V}{V} = -\rho\beta\Delta T \quad \rightarrow \quad \frac{\Delta\rho}{\rho} = -\beta\Delta T$$

(b) $\boxed{\text{As the temperature increases, the density decreases.}}$

(c) For water we have $\beta = -\frac{\Delta\rho}{\rho\Delta T} = -\frac{0.999\,7\ \text{g/cm}^3 - 1.000\,0\ \text{g/cm}^3}{\left(1.000\,0\ \text{g/cm}^3\right)(10.0°\text{C} - 4.0°\text{C})}$

$$= \boxed{5\times10^{-5}(°\text{C})^{-1}}$$

(d) $\beta = -\frac{\Delta\rho}{\rho\Delta T} = -\frac{1.000\,0\ \text{g/cm}^3 - 0.999\,9\ \text{g/cm}^3}{\left(1.000\,0\ \text{g/cm}^3\right)(4.00°\text{C} - 0.00°\text{C})}$

$$= \boxed{-2.5\times10^{-5}(°\text{C})^{-1}}$$

P19.58 (a) From *PV* = *nRT*, the volume is $V = \left(\frac{nR}{P}\right)T$.

Therefore, when pressure is held constant, $\frac{dV}{dT} = \frac{nR}{P} = \frac{V}{T}$.

Thus, $\beta \equiv \left(\dfrac{1}{V}\right)\dfrac{dV}{dT} = \left(\dfrac{1}{V}\right)\dfrac{V}{T}$ or $\beta = \boxed{\dfrac{1}{T}}$

(b) At T = 0°C = 273.15 K, this predicts $\beta = \dfrac{1}{273\text{ K}} = \boxed{3.66 \times 10^{-3}\text{ K}^{-1}}$.

Experimental values are:

(c) $\boxed{\beta_{\text{He}} = 3.665 \times 10^{-3}\text{ K}^{-1},\text{ this agrees within 0.06\% of the tabulated value.}}$

(d) $\boxed{\beta_{\text{air}} = 3.67 \times 10^{-3}\text{ K}^{-1},\text{ this agrees within 0.2\% of the tabulated value.}}$

P19.59 (a) Using the expression for the period T_P of a pendulum, we have

$$T_P = 2\pi\sqrt{\frac{L}{g}} \quad \rightarrow \quad dT_P = 2\pi\sqrt{\frac{1}{g}}\left(\frac{1}{2}\right)\frac{dL}{\sqrt{L}}$$

$$= 2\pi\sqrt{\frac{L}{g}}\left(\frac{1}{2}\right)\frac{dL}{L} = T_P\left(\frac{1}{2}\right)\frac{dL}{L}$$

$$\frac{dT_P}{T_P} = \left(\frac{1}{2}\right)\frac{dL}{L}$$

and $\Delta L = \alpha L_i \Delta T$, so, for temperature change dT,

$$dT_P = T_P \frac{1}{2}\frac{dL}{L} = T_P \frac{\alpha dT}{2}$$

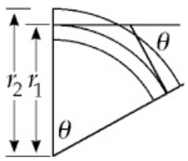
$$= 1.000\text{ s}\,\frac{\left(19.0 \times 10^{-6}\,(°\text{C})^{-1}\right)(10.0°\text{C})}{2} = \boxed{9.50 \times 10^{-5}\text{ s}}$$

(b) In one week, the time lost = 1 week (9.50×10^{-5} s lost per second)

$$\text{time lost} = (7.00\text{ d/week})\left(\frac{86\,400\text{ s}}{1.00\text{ d}}\right)\left(9.50 \times 10^{-5}\ \frac{\text{s lost}}{\text{s}}\right)$$

$$\text{time lost} = \boxed{57.5\text{ s lost}}$$

P19.60 The angle of bending θ, between tangents to the two ends of the strip, is equal to the angle the strip subtends at its center of curvature. (The angles are equal because their sides are perpendicular, right side to the right side and left side to left side.)



ANS. FIG. P19.60

(a) The definition of radian measure gives $L_i + \Delta L_1 = \theta r_1$ and $L_i + \Delta L_2 = \theta r_2$. By subtraction,

$$\Delta L_2 - \Delta L_1 = \theta(r_2 - r_1)$$

$$\alpha_2 L_i \Delta T - \alpha_1 L_i \Delta T = \theta \Delta r$$

$$\boxed{\theta = \frac{(\alpha_2 - \alpha_1) L_i \Delta T}{\Delta r}}$$

(b) In the expression from part (a), θ is directly proportional to ΔT and also to $(\alpha_2 - \alpha_1)$. Therefore, θ is zero when either of these quantities becomes zero.

(c) The material that expands more when heated contracts more when cooled, so $\boxed{\text{the bimetallic strip bends the other way}}$. It is fun to demonstrate this with liquid nitrogen.

P19.61 (a) From ANS. FIG. P19.61, we see that the change in area is

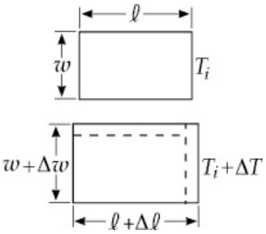
$$\Delta A = \ell \Delta w + w \Delta \ell + \Delta w \Delta \ell$$

Since $\Delta \ell$ and Δw are each small quantities, the product $\Delta w \Delta \ell$ will be very small. Therefore, we assume $\Delta w \Delta \ell \approx 0$.

Since $\Delta w = w\alpha\Delta T$ and $\Delta \ell = \ell \alpha \Delta T$,

we then have $\Delta A = \ell w \alpha \Delta T + \ell w \alpha \Delta T$,

and since $A = \ell w$, $\boxed{\Delta A = 2\alpha A \Delta T}$



ANS. FIG. P19.61

(b) The approximation assumes $\Delta w \Delta \ell \approx 0$, or $\alpha \Delta T \approx 0$. Another way of stating this is $\boxed{\alpha \Delta T << 1}$.

P19.62 Let ρ_0 represent the density of the liquid at 0°C. At temperature T_C, the volume of a sample has changed according to $\Delta V = \beta V \Delta T = \beta V T_C$, so the density has become

$$\rho = \frac{m}{V + \beta V T_C} = \rho_0 \frac{1}{1 + \beta T_C}$$

so $$\rho(1 + \beta T_C) = \rho_0$$

Now the pressure at the bottom of the U tube is the same, whichever column it supports:

$$P_0 + \rho_0 g h_0 = P_0 + \rho g h_t$$

Simplifying,

$$\rho_0 h_0 = \rho h_t$$

and substituting,

$$\rho(1+\beta T_C)h_0 = \rho h_t$$

$$\left(1+\beta T_C\right)h_0 = h_t \rightarrow \beta = \frac{1}{T_C}\left(\frac{h_t}{h_0}-1\right)$$

P19.63 (a) Yes, so long as the coefficients of expansion remain constant.

(b) The coefficient of linear expansion of copper, $17.0 \times 10^{-6}\ {}^\circ\text{C}^{-1}$, is greater than that of steel, $11.0 \times 10^{-6}\ {}^\circ\text{C}^{-1}$, so the copper rod should start with a smaller length. Since the difference between the lengths of the two rods is to remain constant, we require

$$\Delta L_{\text{Cu}} = \Delta L_S$$

$$\alpha_{\text{Cu}} L_{\text{Cu}} \Delta T = \alpha_S L_S \Delta T$$

$$\left(17.0\times 10^{-6}\ ({}^\circ\text{C})^{-1}\right)L_{\text{Cu}}\Delta T = \left(11.0\times 10^{-6}\ ({}^\circ\text{C})^{-1}\right)L_S\Delta T$$

which gives

$$17.0 L_{\text{Cu}} = 11.0 L_S$$

Now, with $L_{\text{Cu}} + 5.00\ \text{cm} = L_S$ at 0°C, we obtain by substitution,

$$L_{\text{Cu}} + 5.00\ \text{cm} = \left(\frac{17.0}{11.0}\right)L_{\text{Cu}}$$

or $$L_{\text{Cu}} = \left(\frac{11.0}{6.00}\right)(5.00\ \text{cm}) = 9.17\ \text{cm}$$

With $L_S - L_C = 5.00$ cm, the only possibility is $L_\text{S} = 14.17$ cm and $L_\text{C} = 9.17$ cm.

P19.64 (a) Particle in equilibrium model

(b) On the piston,

$$\sum F = F_{\text{gas}} - F_{\text{g}} - F_{\text{air}} = 0:$$

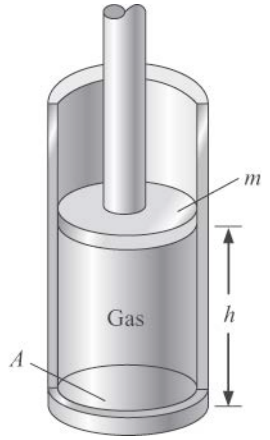
$$\sum F = PA - mg - P_0 A = 0$$

(c) In equilibrium, $P_{\text{gas}} = \dfrac{mg}{A} + P_0$.

Therefore, $\dfrac{nRT}{hA} = \dfrac{mg}{A} + P_0$,

or $$\boxed{h = \frac{nRT}{mg + P_0 A}},$$

where we have used $V = hA$ as the volume of the gas.



ANS. FIG. P19.64

P19.65 We compute the moment of inertia from

$$I = \int r^2 dm$$

and since $r(T) = r(T_i)(1 + \alpha\Delta T)$,

$$\frac{I(T)}{I(T_i)} = (1 + \alpha\Delta T)^2$$

Thus $$\frac{\Delta I}{I} = \frac{I(T) - I(T_i)}{I(T_i)} = (1 + \alpha\Delta T)^2 - 1.$$

(a) With $\alpha = 17.0 \times 10^{-6}\,(°\text{C})^{-1}$ and $\Delta T = 100°\text{C}$, we find for Cu:

$$\frac{\Delta I}{I} = \left[1 + \left(17.0 \times 10^{-6}\,(°\text{C})^{-1}\right)(100°\text{C})\right]^2 - 1 = \boxed{0.340\%}$$

(b) With $\alpha = 24.0 \times 10^{-6}\,(°\text{C})^{-1}$ and $\Delta T = 100°\text{C}$, we find for Al:

$$\frac{\Delta I}{I} = \left[1 + \left(24.0 \times 10^{-6}\,(°\text{C})^{-1}\right)(100°\text{C})\right]^2 - 1 = \boxed{0.481\%}$$

P19.66 (a) Let *m* represent the sample mass. The number of moles is $n = \frac{m}{M}$ and the density is $\rho = \frac{m}{V}$. So *PV* = *nRT* becomes $PV = \frac{m}{M}RT$ or $PM = \frac{m}{V}RT$.

Then, $\rho = \frac{m}{V} = \boxed{\frac{PM}{RT}}$

(b) $$\rho = \frac{PM}{RT} = \frac{\left(1.013 \times 10^5\ \text{N/m}^2\right)\left(0.032\,0\ \text{kg/mol}\right)}{\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(293\ \text{K}\right)} = \boxed{1.33\ \text{kg/m}^3}$$

P19.67 After expansion, the length of one of the spans is

$$L_f = L_i(1 + \alpha\Delta T) = (125\ \text{m})\left[1 + 12 \times 10^{-6}\,(°\text{C})^{-1}(20.0°\text{C})\right]$$
$$= 125.03\ \text{m}$$

L_f, *y*, and the original 125-m length of this span form a right triangle with *y* as the altitude. Using the Pythagorean theorem gives

$$(125.03\ \text{m})^2 = y^2 + (125\ \text{m})^2$$

yielding $y = \boxed{2.74\ \text{m}}$.

P19.68 Let $\ell = L/2$ represent the original length of one of the concrete slabs. After expansion, the length of each one of the spans is $\ell_f = \ell(1+\alpha\Delta T)$. Now, ℓ_f, *y*, and the original length ℓ of this span form a right triangle with *y* as the altitude. Using the Pythagorean theorem gives

$$\ell_f^2 = \ell^2 + y^2$$

or

$$y = \sqrt{\ell_f^2 - \ell^2} = \ell\sqrt{(1+\alpha\Delta T)^2 - 1} = (L/2)\sqrt{2\alpha\Delta T + (\alpha\Delta T)^2}$$

Since $\alpha\Delta T << 1$, we have $\boxed{y \approx L\sqrt{\alpha\Delta T/2}}$

P19.69 (a) Let *V′* represent the compressed volume at depth

$$B = \rho g V' \qquad P' = P_0 + \rho g d \qquad P'V' = P_0 V_i$$

$$B = \frac{\rho g P_0 V_i}{P'} = \boxed{\frac{\rho g P_0 V_i}{P_0 + \rho g d}}$$

(b) Since *d* is in the denominator, *B* must $\boxed{\text{decrease}}$ as the depth increases. (The volume of the balloon becomes smaller with increasing pressure.)

(c) To find the depth at which the buoyant force is half that at the surface, we write

$$\frac{1}{2} = \frac{B(d)}{B(0)} = \frac{\rho g P_0 V_i/(P_0 + \rho g d)}{\rho g P_0 V_i/P_0} = \frac{P_0}{P_0 + \rho g d}$$

Then, solve for *d* from $P_0 + \rho g d = 2P_0$:

$$d = \frac{P_0}{\rho g} = \frac{1.013\times 10^5\ \text{N/m}^2}{(1.00\times 10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = \boxed{10.3\ \text{m}}$$

P19.70 (a) $\boxed{\text{No torque acts on the disk so its angular momentum is constant. Yes: it increases. As the disk cools, its radius and, hence, its moment of inertia decrease. Conservation of angular momentum then requires that its angular speed increase.}}$

(b) $$I_i\omega_i = I_f\omega_f = \frac{1}{2}MR_i^2\omega_i = \frac{1}{2}MR_f^2\omega_f = \frac{1}{2}M\left[R_i + R_i\alpha\Delta T\right]^2\omega_f$$

$$= \frac{1}{2}MR_i^2\left[1 - \alpha|\Delta T|\right]^2\omega_f$$

$$\omega_f = \omega_i\left[1-\alpha|\Delta T|\right]^{-2} = \frac{25.0\ \text{rad/s}}{\left[1-\left(17\times10^{-6}\,(°\text{C})^{-1}\right)(830°\text{C})\right]^2} = \frac{25.0\ \text{rad/s}}{0.972}$$

$$= \boxed{25.7\ \text{rad/s}}$$

P19.71 Visualize the molecules of various species all moving randomly. The net force on any section of wall is the sum of the forces of all of the molecules pounding on it.

For each gas alone, $P_1 = \dfrac{N_1kT}{V}$ and $P_2 = \dfrac{N_2kT}{V}$ and $P_3 = \dfrac{N_3kT}{V}$, etc.

For all gases,

$$P_1V_1 + P_2V_2 + P_3V_3\ldots\ (N_1 + N_2 + N_3\ldots)kT \text{ and}$$
$$(N_1 + N_2 + N_3\ldots)kT = PV$$

Also, $V_1 = V_2 = V_3 = \ldots = V$; therefore, $\boxed{P = P_1 + P_2 + P_3\ldots}$

Challenge Problems

P19.72 (a) At 20.0°C, the unstretched lengths of the steel and copper wires are

$$L_s(20.0°\text{C}) = (2.000\ \text{m})\left[1+\left(11.0\times10^{-6}\ °\text{C}^{-1}\right)(-20.0°\text{C})\right]$$
$$= 1.999\,56\ \text{m}$$
$$L_c(20.0°\text{C}) = (2.000\ \text{m})\left[1+\left(17.0\times10^{-6}\ °\text{C}^{-1}\right)(-20.0°\text{C})\right]$$
$$= 1.999\,32\ \text{m}$$

Under a tension *F*, the length of the steel and copper wires are

$$L_s' = L_s\left[1+\frac{F}{YA}\right]_s \quad \text{and} \quad L_c' = L_c\left[1+\frac{F}{YA}\right]_c$$

where $L_s' + L_c' = 4.000$ m

Since the tension *F* must be the same in each wire, we solve for *F*:

$$F = \frac{(L_s' + L_c') - (L_s + L_c)}{L_s/Y_sA_s + L_c/Y_cA_c}$$

When the wires are stretched, their areas become

$$A_s = \pi\left(1.000\times10^{-3}\ \text{m}\right)^2\left[1+\left(11.0\times10^{-6}\ {}^\circ\text{C}^{-1}\right)(-20.0)\right]^2$$
$$= 3.140\times10^{-6}\ \text{m}^2$$
$$A_c = \pi\left(1.000\times10^{-3}\ \text{m}\right)^2\left[1+\left(17.0\times10^{-6}\ {}^\circ\text{C}^{-1}\right)(-20.0)\right]^2$$
$$= 3.139\times10^{-6}\ \text{m}^2$$

Recall $Y_s = 20.0\times10^{10}$ Pa and $Y_c = 11.0\times10^{10}$ Pa. Substituting into the equation for *F*, we obtain

$$F = \left[4.000\ \text{m} - \left(1.999\,56\ \text{m} + 1.999\,32\ \text{m}\right)\right]$$
$$\times\frac{1}{\dfrac{1.999\,56\ \text{m}}{\left(20.0\times10^{10}\ \text{Pa}\right)\left(3.140\times10^{-6}\right)\ \text{m}^2}+\dfrac{1.999\,32\ \text{m}}{\left(11.0\times10^{10}\ \text{Pa}\right)\left(3.139\times10^{-6}\right)\ \text{m}^2}}$$
$$F = \boxed{125\ \text{N}}$$

(b) To find the *x* coordinate of the junction,

$$L_s' = \left(1.999\,56\ \text{m}\right)\left[1+\frac{125\ \text{N}}{\left(20.0\times10^{10}\ \text{N/m}^2\right)\left(3.140\times10^{-6}\ \text{m}^2\right)}\right]$$
$$= 1.999958\ \text{m}$$

Thus the *x* coordinate is $-2.000 + 1.999958 = \boxed{-4.20\times10^{-5}\ \text{m}}$

P19.73 (a) We find the linear density from the volume density as the mass-per-volume multiplied by the volume-per-length, which is the cross-sectional area.

$$\mu = \frac{1}{4}\rho\left(\pi d^2\right) = \frac{1}{4}\pi(1.00\times10^{-3}\text{m})^2\left(7.86\times10^3\ \text{kg/m}^3\right)$$
$$= \boxed{6.17\times10^{-3}\ \text{kg/m}}$$

(b) Since $f_1 = \dfrac{v}{2L}$ and $v = \sqrt{\dfrac{T}{\mu}}$, then $f_1 = \dfrac{1}{2L}\sqrt{\dfrac{T}{\mu}}$, and we have for the tension

$$F = T = \mu\left(2Lf_1\right)^2 = \left(6.17\times10^{-3}\ \text{kg/m}\right)\left[2\left(0.800\ \text{m}\right)\left(200\ \text{s}^{-1}\right)\right]^2$$
$$= \boxed{632\ \text{N}}$$

(c) At 0°C, the length of the guitar string will be

$$L_{\text{actual}} = L_{0^\circ\text{C}}\left(1+\frac{F}{AY}\right) = 0.800\ \text{m}$$

Where $L_{0°\text{C}}$ is the unstressed length at the low temperature. We know the string's cross-sectional area

$$A = \left(\frac{\pi}{4}\right)(1.00\times10^{-3}\ \text{m})^2 = 7.85\times10^{-7}\ \text{m}^2$$

and modulus $Y = 20.0 \times 10^{10}\ \text{N/m}^2$

Therefore,

$$\frac{F}{AY} = \frac{632\ \text{N}}{\left(7.85\times10^{-7}\ \text{m}^2\right)\left(20.0\times10^{10}\ \text{N/m}^2\right)} = 4.02\times10^{-3}$$

and $L_{0°\text{C}} = \dfrac{0.800\ \text{m}}{1 + 4.02\times10^{-3}} = 0.796\ 8\ \text{m}$

Then at 30°C, the unstressed length is

$$L_{30°\text{C}} = (0.796\ 8\ \text{m})\left[1 + (30.0°\text{C})\left(11.0\times10^{-6}\ °\text{C}^{-1}\right)\right]$$
$$= 0.797\ 1\ \text{m}$$

With the same clamping arrangement,

$$0.800\ \text{m} = (0.797\ 1\ \text{m})\left[1 + \frac{F'}{A'Y}\right]$$

where F' and A' are the new tension and the new (expanded) cross-sectional area. Then

$$\frac{F'}{A'Y} = \frac{0.800\ 0}{0.797\ 1} - 1 = 3.693\times10^{-3}$$

and

$$F' = A'Y(3.693\times10^{-3})$$
$$F' = (7.85\times10^{-7}\ \text{m}^2)(20.0\times10^{10}\ \text{N/m}^2)(3.693\times10^{-3})(1+\alpha\Delta T)^2$$
$$F' = (580\ \text{N})(1 + 3.30\times10^{-4})^2 = \boxed{580\ \text{N}}$$

(d) Also the new frequency f_1' is given by $\dfrac{f_1'}{f_1} = \sqrt{\dfrac{F'}{F}}$,

so $f_1' = (200\ \text{Hz})\sqrt{\dfrac{580\ \text{N}}{632\ \text{N}}} = \boxed{192\ \text{Hz}}$

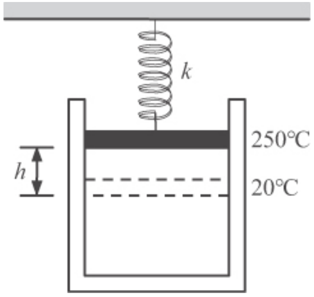
P19.74 (a) $\dfrac{P_0V}{T} = \dfrac{P'V'}{T'}$ because the amount of gas remains constant.

The volume increases by Ah when the piston rises:

$$V' = V + Ah$$

When the piston compresses the piston by *h*, so the spring force increases by *F* = *kx* = *kh*, increasing the external pressure on the piston by *kh/A*:

$$V' = V + Ah$$



ANS. FIG. P19.74

Using the particle in equilibrium model applied to the piston,

$$\left(P_0 + \frac{kh}{A}\right)(V + Ah) = P_0 V\left(\frac{T'}{T}\right)$$

$$\begin{aligned}&\left(1.013\times10^5\ \text{N/m}^2 + 2.00\times10^5\ \text{N/m}^3\,h\right)\\ &\qquad\times\left(5.00\times10^{-3}\ \text{m}^3 + \left(0.010\ 0\ \text{m}^2\right)h\right)\\ &\qquad = \left(1.013\times10^5\ \text{N/m}^2\right)\left(5.00\times10^{-3}\ \text{m}^3\right)\left(\frac{523\ \text{K}}{293\ \text{K}}\right)\end{aligned}$$

$$2\ 000h^2 + 2\ 013h - 397 = 0$$

Taking the positive root, $h = \dfrac{-2\ 013 + 2\ 689}{4\ 000} = \boxed{0.169\ \text{m}}$

(b) $P' = P + \dfrac{kh}{A} = 1.013\times10^5\ \text{Pa} + \dfrac{\left(2.00\times10^3\ \text{N/m}\right)(0.169\ \text{m})}{0.010\ 0\ \text{m}^2}$

$$P' = \boxed{1.35\times10^5\ \text{Pa}}$$

P19.75 Each half of the spherical container is a particle in equilibrium. Therefore, using the result of Problem 14.58,

$$\sum F = 0 \ \rightarrow\ F_{\text{from gas}} = F_{\text{holding hemispheres together}}$$

$$\rightarrow\ P\left(\pi r^2\right) = \frac{F}{A}A = \sigma(2\pi r t)\ \rightarrow\ t = \frac{Pr}{2\sigma}$$

where σ is the yield strength of the steel. Find the mass of the steel sphere:

$$m_{\text{St}} = \rho_{\text{St}}V = \rho_{\text{St}}\left(4\pi r^2 t\right) = \rho_{\text{St}}\left[4\pi r^2\left(\frac{Pr}{2\sigma}\right)\right] = 2\pi\rho_{\text{St}}\frac{Pr^3}{\sigma}$$

Find the pressure of the helium in the tank:

$$PV = nRT \quad \rightarrow \quad P = \frac{nRT}{V} = \frac{m_{\text{He}}}{M_{\text{He}}}\left(\frac{RT}{\frac{4}{3}\pi r^3}\right)$$

Substitute into the previous equation:

$$m_{\text{St}} = 2\pi\rho_{\text{St}}\frac{r^3}{\sigma}\left[\frac{m_{\text{He}}}{M_{\text{He}}}\left(\frac{RT}{\frac{4}{3}\pi r^3}\right)\right] = \frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}\frac{m_{\text{He}}}{M_{\text{He}}}RT$$

Find the buoyant force on the balloon:

$$B = \rho_{\text{air}} g V_{\text{balloon}} = \rho_{\text{air}} g \frac{PV}{P_0} = \rho_{\text{air}} g \frac{nRT}{P_0} = \rho_{\text{air}} g \frac{m_{\text{He}}}{M_{\text{He}}}\frac{RT}{P_0}$$

where we use the pressure of helium in the balloon $P = P_0$ = atmospheric pressure.

Find the net force on the balloon and tank:

$$\begin{aligned}\sum F &= B - m_{\text{He}}g - m_{\text{St}}g \\ &= \rho_{\text{air}} g \frac{m_{\text{He}}}{M_{\text{He}}}\frac{RT}{P_0} - m_{\text{He}}g - \frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}\frac{m_{\text{He}}g}{M_{\text{He}}}RT \\ &= m_{\text{He}}g\left(\rho_{\text{air}}\frac{RT}{P_0 M_{\text{He}}} - 1 - \frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}\frac{RT}{M_{\text{He}}}\right) \\ &= m_{\text{He}}g\left[\frac{RT}{M_{\text{He}}}\left(\frac{\rho_{\text{air}}}{P_0} - \frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}\right) - 1\right]\end{aligned}$$

Evaluate the brackets:

$$\begin{aligned}&\left[\frac{RT}{M_{\text{He}}}\left(\frac{\rho_{\text{air}}}{P_0} - \frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}\right) - 1\right] \\ &\quad = \left\{\left[\frac{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{4\times10^{-3}\ \text{kg/mol}}\right.\right. \\ &\qquad \left.\left.\left(\frac{1.20\ \text{kg/m}^3}{1.013\times10^5\ \text{Pa}} - \frac{3}{2}\left(\frac{7\,860\ \text{kg/m}^3}{5\times10^8\ \text{N/m}^2}\right)\right)\right] - 1\right\} \\ &\quad = \left\{\left[\left(6.09\times10^5\ \text{m}^2/\text{s}^2\right)\right.\right. \\ &\qquad \left.\left.\times\left(1.184\,6\times10^{-5}\ \text{s}^2/\text{m}^2 - 2.358\times10^{-5}\ \text{s}^2/\text{m}^2\right)\right] - 1\right\} \\ &\quad = -8.146\end{aligned}$$

Because the net force is negative, the balloon cannot lift the tank. If we can vary the strength of the steel, let's find out how strong the steel must be by evaluating σ to make the net force positive. We want the following to be true:

$$\frac{RT}{M_{\text{He}}}\left(\frac{\rho_{\text{air}}}{P_0}-\frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}\right)-1>0$$

Manipulating this inequality gives,

$$\frac{RT}{M_{\text{He}}}\left(\frac{\rho_{\text{air}}}{P_0}-\frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}\right)>1$$

$$\rightarrow\quad \frac{\rho_{\text{air}}}{P_0}-\frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}>\frac{M_{\text{He}}}{RT}\quad\rightarrow\quad -\frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}>\frac{M_{\text{He}}}{RT}-\frac{\rho_{\text{air}}}{P_0}$$

$$\rightarrow\quad \frac{3}{2}\frac{\rho_{\text{St}}}{\sigma}<-\frac{M_{\text{He}}}{RT}+\frac{\rho_{\text{air}}}{P_0}=\frac{-M_{\text{He}}P_0+\rho_{\text{air}}RT}{P_0RT}$$

$$\rightarrow\quad \frac{2}{3}\frac{\sigma}{\rho_{\text{St}}}>\frac{P_0RT}{-M_{\text{He}}P_0+\rho_{\text{air}}RT}$$

$$\rightarrow\quad \sigma>\frac{3}{2}\frac{\rho_{\text{St}}P_0RT}{-M_{\text{He}}P_0+\rho_{\text{air}}RT}$$

$$=\frac{3}{2}\left[\frac{(7\ 860\ \text{kg/m}^3)(1.013\times10^5\ \text{Pa})(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{-(4\times10^{-3}\ \text{kg/mol})(1.013\times10^5\ \text{Pa})+(1.20\ \text{kg/m}^3)(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})}\right]$$

$$\sigma=11.6\times10^8\ \text{N/m}^2=2.3\sigma_{\text{actual}}$$

$\boxed{\text{No, the steel would need to be 2.3 times stronger.}}$

P19.76 With piston alone, *T* = constant, so $PV=P_0V_0$ or $P(Ah_i)=P_0(Ah_0)$.

With *A* = constant, $P=P_0\left(\frac{h_0}{h_i}\right)$.

But, $P=P_0+\frac{m_pg}{A}$,

where m_p is the mass of the piston.

Thus $P_0+\frac{m_pg}{A}=P_0\left(\frac{h_0}{h_i}\right)$, which reduces to $h_i=\frac{h_0}{1+m_pg/P_0A}$.

With the dog of mass M on the piston, a very similar calculation (replacing m_p by $m_p + M$) gives

$$h' = \frac{h_0}{1 + \left(m_p + M\right)g / P_0 A}$$

Thus, when the dog steps on the piston, it moves downward by

$$\Delta h = h_i - h'$$

$$= \frac{50.0\text{ cm}}{1 + (20.0\text{ kg})(9.80\text{ m/s}^2)\big/\left[(1.013\times10^5\text{ Pa})\pi(0.400\text{ m})^2\right]}$$

$$- \frac{50.0\text{ cm}}{1 + (45.0\text{ kg})(9.80\text{ m/s}^2)\big/\left[(1.013\times10^5\text{ Pa})\pi(0.400\text{ m})^2\right]}$$

$$\Delta h = \boxed{2.38\text{ mm}}$$

(b) P = const, so $\dfrac{V}{T} = \dfrac{V'}{T_i}$ or $\dfrac{Ah_i}{T} = \dfrac{Ah'}{T_i}$,

giving

$$T = T_i\left(\frac{h_i}{h'}\right) = T_i \frac{1 + \left(m_p + M\right)g / P_0 A}{1 + m_p g / P_0 A}$$

$$= 293\text{ K}\ \frac{1 + (45.0\text{ kg})(9.80\text{ m/s}^2)\big/\left[(1.013\times10^5\text{ Pa})\pi(0.400\text{ m})^2\right]}{1 + (20.0\text{ kg})(9.80\text{ m/s}^2)\big/\left[(1.013\times10^5\text{ Pa})\pi(0.400\text{ m})^2\right]}$$

$$T = \boxed{294.4\text{ K} = 21.4\ ^\circ\text{C}}$$

P19.77 (a) $\dfrac{dL}{L} = \alpha dT$: $\displaystyle\int_{T_i}^{T_i} \alpha\, dT = \int_{L_i}^{L_i} \frac{dL}{L} \Rightarrow \ln\left(\frac{L_f}{L_i}\right) = \alpha\Delta T \Rightarrow \boxed{L_f = L_i e^{\alpha\Delta T}}$

(b) $L_f = (1.00\text{ m})e^{\left[(2.00\times10^{-5}\ ^\circ\text{C}^{-1})(100^\circ\text{C})\right]} = 1.002\ 002\text{ m}$

$L'_f = 1.00\text{ m}\left[1 + (2.00\times10^{-5}\ ^\circ\text{C}^{-1})(100^\circ\text{C})\right] = 1.002\ 000$:

$\dfrac{L_f - L'_f}{L_f} = 2.00\times10^{-6} = \boxed{2.00\times10^{-4}\%}$

(c) $L_f = (1.00\text{ m})e^{\left[(2.00\times10^{-2}\ ^\circ\text{C}^{-1})(100^\circ\text{C})\right]} = 7.389\text{ m}$

$L'_f = 1.00\text{ m}\left[1 + (0.020\ 0^\circ\text{C}^{-1})(100^\circ\text{C})\right] = 3.000\text{ m}$

$$\frac{L_f - L'_f}{L_f} = \boxed{59.4\%}$$

(d) P19.21 redone:

We start with

$$dV = \beta V dT \quad \rightarrow \quad \frac{dV}{V} = \beta dT \quad \rightarrow \quad V_f = V_i e^{\beta \Delta T}$$

where we assume β (and α) remains constant over the temperature range ΔT.

Thus, for the turpentine,

$$V_{\text{t, final}} = V_t e^{\beta_t \Delta T}$$

and for the aluminum cylinder, $V_{\text{Al, final}} = V_{\text{Al}} e^{3\alpha_{\text{Al}} \Delta T}$, where we assume α remains constant over the temperature range ΔT.

new (a):

$$\Delta V = V_t e^{\beta_t \Delta T} - V_{\text{Al}} e^{3\alpha_{\text{Al}} \Delta T}$$

$$\Delta V = V_i \left(e^{\beta_t \Delta T} - e^{3\alpha_{\text{Al}} \Delta T} \right)$$

$$= \left(2\ 000 \text{ cm}^3 \right) \left[e^{\left(9.00\times10^{-4}\,\text{C}^{-1}\right)(60.0°\text{C})} - e^{3\left(24.0\times10^{-6°}\,\text{C}^{-1}\right)(60.0°\text{C})} \right]$$

$$\Delta V = \boxed{\text{102 mL of turpentine spills,}}$$

new (b):

The volume of the turpentine remaining in the cylinder at 80.0°C is the same as the volume of the aluminum cylinder at 80.0°C:

$$V_{t,\text{remaining}} = V_{\text{Al, final}} = V_{\text{Al}} e^{3\alpha_{\text{Al}} \Delta T}$$

$$= \left(2\ 000 \text{ cm}^3 \right) e^{3\left(24.0\times10^{-6}(°\text{C})^{-1}\right)(60.0°\text{C})}$$

$$= 2\ 009 \text{ cm}^3$$

$$\boxed{\text{2.01 L remains in the cylinder at 80.0 °C}}$$

new (c):

The volume of turpentine at 80.0°C we found in part new (b), $V_{\text{t, remaining}} = V_{\text{Al}} e^{3\alpha_{\text{Al}} \Delta T}$, shrinks when the temperature changes by $-\Delta T$:

$$V_{\text{t, final}} = V_{\text{t, remaining}} e^{-\beta_t \Delta T} = V_{\text{Al}} e^{3\alpha_{\text{Al}} \Delta T} e^{-\beta_t \Delta T} = V_{\text{Al}} e^{\left(3\alpha_{\text{Al}} - \beta_t\right) \Delta T}$$

$$
\begin{aligned}
V_{\text{t, final}} &= V_{\text{Al}} e^{3\alpha_{\text{Al}}\Delta T} e^{-\beta_t \Delta T} = V_{\text{Al}} e^{(3\alpha_{\text{Al}} - \beta_t)\Delta T} \\
&= \left(2\ 000\ \text{cm}^3\right) e^{\left[3\left(24.0\times 10^{-6\circ}\,\text{C}^{-1}\right) - \left(9.00\times 10^{-4}\,\text{C}^{-1}\right)\right](60.0^\circ\text{C})} \\
V_{\text{t, final}} &\simeq 1\ 903\ \text{cm}^3
\end{aligned}
$$

Find the percentage of the cylinder that is empty at 20.0°C:

$$
\frac{V_{\text{Al}} - V_{\text{Al}} e^{(3\alpha_{\text{Al}} - \beta_t)\Delta T}}{V_{\text{Al}} e} = 1 - e^{(3\alpha_{\text{Al}} - \beta_t)\Delta T}
$$

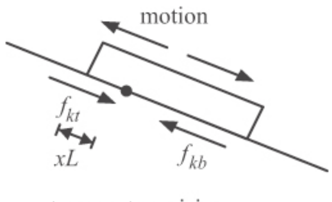
Find the empty height of the cylinder above the turpentine:

$$
\left(1 - e^{(3\alpha_{\text{Al}} - \beta_t)\Delta T}\right)(20.0\ \text{cm}) = 0.969\ \text{cm}
$$

$\boxed{\text{and the turpentine level at 20.0°C is 0.969 cm below the cylinder's rim.}}$

P19.78 (a) Let *xL* represent the distance of the stationary line below the top edge of the plate. The normal force on the lower part of the plate is $mg(1-x)\cos\theta$ and the force of kinetic friction on it is $\mu_k mg(1-x)\cos\theta$ up the roof. Again, $\mu_k mgx\cos\theta$ acts down the roof on the upper part of the plate. The near-equilibrium of the plate requires $\boxed{\sum F_x = 0}$.

$$
\begin{aligned}
&-\mu_k mgx\cos\theta + \mu_k mg(1-x)\cos\theta - mg\sin\theta = 0 \\
&-2\mu_k mgx\cos\theta = mg\sin\theta - \mu_k mg\cos\theta \\
&2\mu_k x = \mu_k - \tan\theta \\
&x = \frac{1}{2} - \frac{\tan\theta}{2\mu_k}
\end{aligned}
$$

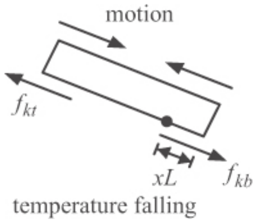


ANS. FIG. P19.78(a)

and the stationary line is indeed below the top edge by

$$
xL = \frac{L}{2}\left(1 - \frac{\tan\theta}{\mu_k}\right)
$$

(b) With the temperature falling, the plate contracts faster than the roof. The upper part slides down and feels an upward frictional force $\mu_k mg(1-x)\cos\theta$. The lower part slides up and feels downward frictional force $\mu_k mgx\cos\theta$. The equation $\sum F_x = 0$ is then the same as in part (a) and the stationary line is above the bottom edge by $xL = \frac{L}{2}\left(1-\frac{\tan\theta}{\mu_k}\right)$.



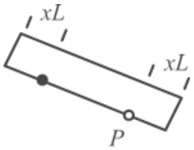
ANS. FIG. P19.78(b)

(c) Start thinking about the plate at dawn, as the temperature starts to rise. As in part (a), a line at distance *xL* below the top edge of the plate stays stationary relative to the roof as long as the temperature rises. The point *P* on the plate at distance *xL* above the bottom edge is destined to become the fixed point when the temperature starts falling. As the temperature rises, point *P* on the plate slides down the roof relative to the upper fixed line from (*L* – *xL* – *xL*) to $(L-xL-xL)(1+\alpha_2\Delta T)$, a change of $\Delta L_{\text{plate}} = (L-xL-xL)\alpha_2\Delta T$. The point on the roof originally under point *P* at the beginning of the expansion moves down not quite as much from (*L* – *xL* – *xL*) to $(L-xL-xL)(1+\alpha_1\Delta T)$ relative to the upper fixed line; a change of $\Delta L_{\text{roof}} = L(1-x-x)\alpha_1\Delta T$. When the temperature drops, point *P* remains stationary on the roof while the roof contracts, pulling point *P* back by approximately ΔL_{roof}. Therefore, relative to the upper fixed line, point *P* has moved down the roof $\Delta L_{\text{plate}} - \Delta L_{\text{roof}}$. Its displacement for the day is

$$\Delta L = \Delta L_{\text{plate}} - \Delta L_{\text{roof}} = (\alpha_2-\alpha_1)(L-xL-xL)\Delta T$$

$$= (\alpha_2-\alpha_1)\left[L-2\frac{L}{2}\left(1-\frac{\tan\theta}{\mu_k}\right)\right](T_h-T_c)$$

$$= (\alpha_2-\alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h-T_c)$$



ANS. FIG. P19.78(c)

At dawn the next day the point *P* is farther down the roof by the distance ΔL. It represents the displacement of every other point on the plate.

(d) $$(\alpha_2-\alpha_1)\left(\frac{L\tan\theta}{\mu_k}\right)(T_h-T_c)$$
$$=\left(24\times10^{-6}\ {}^\circ\text{C}^{-1}-15\times10^{-6}\ {}^\circ\text{C}^{-1}\right)\times\left[\frac{(1.20\text{ m})\tan 18.5^\circ}{0.42}\right](32.0^\circ\text{C})$$
$$=\boxed{0.275\text{ mm}}$$

(e) If $\alpha_2<\alpha_1$, the forces of friction reverse direction relative to parts (a) and (b) because the roof expands more than the plate as the temperature rises and less as the temperature falls. The diagram in part (a) then applies to temperature falling and the diagram in part (b) applies to temperature rising. A point on the plate *xL* from the top of the plate (which becomes the upper fixed line later when the plate contracts) moves upward from the lower fixed line by ΔL_{plate}, and when the temperature drops, the upper fixed line of the plate is carried down the roof by ΔL_{roof}, so the net change in the plate's position is $\Delta L_{\text{roof}}-\Delta L_{\text{plate}}$, same as before (up to a sign because now $\Delta L_{\text{roof}}>\Delta L_{\text{plate}}$).

The plate creeps down the roof each day by an amount given by the same expression (with α_2 and α_1 interchanged).

P19.79 See ANS. FIG. P19.79. Let 2θ represent the angle the curved rail subtends. We have

$$L_i+\Delta L=2\theta R=L_i(1+\alpha\Delta T)$$

and $$\sin\theta=\frac{L_i/2}{R}=\frac{L_i}{2R}.$$

Thus, $$\theta=\frac{L_i}{2R}(1+\alpha\Delta T)=(1+\alpha\Delta T)\sin\theta$$

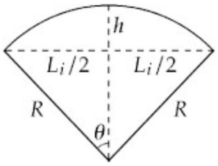
From Table 19.1, $\alpha = 11 \times 10^{-6}\left({}^{\circ}\mathrm{C}\right)^{-1}$, and ΔT = 25.0°C – 20.0°C = 5.00°C. We must solve the transcendental equation

$$\theta = \left(1 + \alpha\Delta T\right)\sin\theta = \left(1.000\,005\,5\right)\sin\theta$$

If your calculator is designed to solve such an equation, it may find the zero solution. Homing in on the nonzero solution gives, to five digits, θ = 0.018 165 rad = 1.040 8°.

Now, $$h = R - R\cos\theta = \frac{L_i\left(1-\cos\theta\right)}{2\sin\theta}$$

This yields $\boxed{h = 4.54\text{ m}}$, a remarkably large value compared to $\Delta L = 5.50$ cm.



ANS. FIG. P19.79

ANSWERS TO EVEN-NUMBERED PROBLEMS

P19.2 (a) 31.7° C; (b) 31.7 K

P19.4 (a) –253° C; (b) –423° F

P19.6 (a) 56.7°C and –62.1°C; (b) 330 K and 211 K

P19.8 1.20 cm

P19.10 $\Delta r = 0.663\text{ mm}$ to the right at 78.2° below the horizontal

P19.12 55.0°C

P19.14 1.58×10^{-3} cm

P19.16 0.548 gal

P19.18 Required $T = -376°$ C is below absolute zero.

P19.20 (a) $2.52 \times 10^{6}\text{ N/m}^2$; (b) the concrete will not fracture

P19.22 (a) 396 N; (b) –101° C; (c) The original length divides out, so the answers would not change

P19.24 (a) $\dfrac{\rho_0}{1+\beta\Delta T}$; (b) m

P19.26 1.20 mol

P19.28 In each pump-up-and-discharge cycle, the volume of air in the tank doubles. Thus 1.00 L of water is driven out by the air injected at the first pumping, 2.00 L by the second, and only the remaining 1.00 L by the third. Each person could more efficiently use his device by starting with the tank half full of water, instead of 80% full.

P19.30 (a) 1.17×10^{-3} kg; (b) 11.5 mN; (c) 1.01 kN; (d) molecules must be moving very fast

P19.32 4.39 kg

P19.34 6.64×10^{-27} kg

P19.36 2.42×10^{11} molecules

P19.38 7.13 m

P19.40 $m_1 - m_2 = \dfrac{P_0 VM}{R}\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)$

P19.42 $\sim 10^{2}$ kg

P19.44 $m_f = m_i\left(\dfrac{P_{gf} + P_0}{P_{gi} + P_0}\right)$

P19.46 35.016 m

P19.48 2.86 atm

P19.50 95.0°; T falls below 100°C, so steam condenses, and the expensive apparatus falls (assuming that the boiling point does not change significantly with the change in pressure).

P19.52 $\Delta h = (\beta - 3\alpha)\dfrac{V\Delta T}{A}$

P19.54 (a) $\theta = 2\sin^{-1}\left(\dfrac{1+\alpha_{\text{Al}}T_{\text{C}}}{2}\right)$; (b) Yes; (c) Yes,

(d) $\theta = 2\sin^{-1}\left(\dfrac{1+\alpha_{\text{Al}}T_{\text{C}}}{2\left(1+\alpha_{\text{invar}}T_{\text{C}}\right)}\right)$; (e) 61.0°; (f) 59.6°

P19.56 0.523 kg

P19.58 (a) $\beta = \dfrac{1}{T}$; (b) $3.66 \times 10^{-3}\ \text{K}^{-1}$; (c) $\beta_{\text{He}} = 3.665 \times 10^{-3}\ \text{K}^{-1}$, this agrees within 0.06% of the tabulated value; (d) $\beta_{\text{He}} = 3.67 \times 10^{-3}\ \text{K}^{-1}$, this agrees within 0.2% of the tabulated value

P19.60 $\theta = \dfrac{(\alpha_2 - \alpha_1)L_i\Delta T}{\Delta r}$; (b) In the expression from part (a), θ is directly proportional to ΔT and also to $(\alpha_2 - \alpha_1)$. Therefore, θ is zero when either of these quantities becomes zero; (c) the bimetallic strip bends the other way

P19.62 See P19.62 for the full solution.

P19.64 (a) Particle in equilibrium model; (b) On the piston, $\sum F = F_{\text{gas}} - F_{\text{g}} - F_{\text{air}} = 0 : \sum F = PA - mg - P_0A = 0$; (c) $h = \dfrac{nRT}{mg + P_0A}$

P19.66 (a) $\dfrac{PM}{RT}$; (b) $1.33\ \text{kg/m}^3$

P19.68 $y \approx L\sqrt{\alpha\Delta T/2}$

P19.70 (a) No torque acts on the disk so its angular momentum is constant. Yes: it increases. As the disk cools, its radius, and hence, its moment of inertia decrease. Conservation of angular momentum then requires that its angular speed increase; (b) 25.7 rad/s

P19.72 (a) 125 N; (b) -4.20×10^{-5} m

P19.74 (a) 0.169 m; (b) 1.35×10^{5} Pa

P19.76 (a) 2.38 mm; (b) 294.4 K = 21.4° C

P19.78 (a) $\sum F_x = 0$; (b) With the temperature falling, the plate contracts faster than the roof. The upper part slides down and feels an upward frictional force $\mu_k mg(1-x)\cos\theta$. The lower part slides up and feels downward frictional force $\mu_k mgx\cos\theta$. The equation $\sum F_x = 0$ is then the same as in part (a), and the stationary line is above the bottom edge by $xL = \frac{L}{2}\left(1-\frac{\tan\theta}{\mu_k}\right)$; (c) See P19.78(c) for the full explanation; (d) 0.275 mm; (e) The plate creeps down the roof each day by an amount given by the same expression (with α_2 and α_1 interchanged).

20

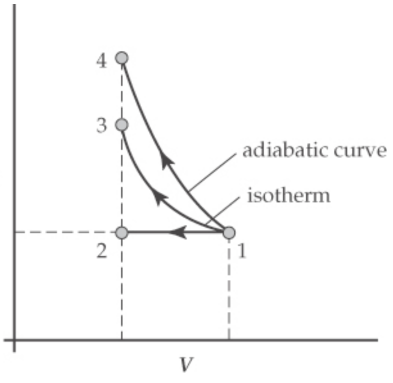
The First Law of Thermodynamics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ20.1 Answer (b). The work done on a gas equals the area under the process curve in a *PV* diagram. In an isobaric process, the pressure is constant, so $P_f = P_i$ and the work done is the area under curve 1–2 in ANS. FIG. OQ20.1. For an isothermal process, the ideal gas law gives $P_f V_f = P_i V_i$, so $P_f = (V_i / V_f) P_i = 2P_i$ and the work done is the area under curve 1–3 in ANS. FIG. OQ20.1. For an adiabatic process, $P_f V_f^\gamma = P_i V_i^\gamma =$ constant (see Ch. 21), so $P_f = (V_i/V_f)^\gamma P_i$ and $P_f = 2^\gamma P_i > 2P_i$ since $\gamma > 1$ for all ideal gases. The work done in an adiabatic process is the area under curve 1–4, which exceeds that done in either of the other processes.



ANS. FIG. OQ20.1

OQ20.2 Answer (d). The high specific heat will keep the end in the fire from warming up very fast. The low conductivity will make the handle end warm up only very slowly.

OQ20.3 Answer (a). Do a few trials with water at different original temperatures and choose the one where room temperature is halfway between the original and the final temperature of the water. Then you can reasonably assume that the contents of the calorimeter gained and lost equal quantities of heat to the surroundings, for net transfer zero. James Joule did it like this in his basement in London.

OQ20.4 Answer (c). Since less energy was required to produce a 5°C rise in the temperature of the ice than was required to produce a 5°C rise in temperature of an equal mass of water, we conclude that the specific heat of ice $\left[c = Q / m(\Delta T)\right]$ is less than that of water.

OQ20.5 Answer (e). The required energy input is

$$\begin{aligned} Q = mc(\Delta T) &= (5.00\text{ kg})(128\text{ J/kg}\cdot{}^\circ\text{C})(327^\circ\text{C} - 20.0^\circ\text{C}) \\ &= 1.96\times 10^5\text{ J} \end{aligned}$$

OQ20.6 Answer (c). With a specific heat half as large, the ΔT is twice as great in the ethyl alcohol.

OQ20.7 Answer (d). From the relation $Q = mc\Delta T,$ the change in temperature of a substance depends on the quantity of energy *Q* added to that substance, and its specific heat and mass: $\Delta T = Q/mc$. The masses of the substances are not given.

OQ20.8 Rankings (e) > (a) = (b) = (c) > (d). We think of the product $mc\Delta T$ in each case, with *c* = 1 for water and about 0.5 for beryllium: (a) 1 · 1 · 6 = 6, (b) 2 · 1 · 3 = 6, (c) 2 · 1 · 3 = 6, (d) 2(0.5)3 = 3, (e) > 6 because a large quantity of energy input is required to melt the ice.

OQ20.9 (i) Answer (d). (ii) Answer (d). Internal energy and temperature both increase by minuscule amounts due to the work input.

OQ20.10 Answer (b). The total change in internal energy is zero.

$$Q_{\text{Cu}} + Q_{\text{water}} + Q_{\text{Al}} = 0$$

$$\begin{aligned} &(100\text{ g})\left(0.092\frac{\text{cal}}{\text{g }^\circ\text{C}}\right)\left(T_f - 95.0^\circ\text{C}\right) \\ &\quad + (200\text{ g})\left(1.00\frac{\text{cal}}{\text{g }^\circ\text{C}}\right)\left(T_f - 15.0^\circ\text{C}\right) \\ &\quad + (280\text{ g})\left(0.215\frac{\text{cal}}{\text{g }^\circ\text{C}}\right)\left(T_f - 15.0^\circ\text{C}\right) = 0 \end{aligned}$$

$$9.20T_f - 874°\text{C} + 200T_f - 3\ 000°\text{C} + 60.2T_f - 903°\text{C} = 0$$

$$269.4T_f = 4\ 777°\text{C}$$

$$T_f = 17.7°\text{C}$$

OQ20.11 Answer (e). Twice the radius means four times the surface area. Twice the absolute temperature makes T^4 sixteen times larger in Stefan's law. The total effect is 4 × 16 = 64.

OQ20.12 Answer (d). During isothermal compression, the temperature remains unchanged. The internal energy of an ideal gas is proportional to its absolute temperature. As the gas is compressed, positive work is done on the gas but also energy is transferred from the gas by heat because the total change in internal energy is zero.

OQ20.13 Answer (c) only. By definition, in an adiabatic process, no energy is transferred to or from the gas by heat. In an expansion process, the gas does work on the environment. Since there is no energy input by heat, the first law of thermodynamics says that the internal energy of the ideal gas must decrease, meaning the temperature will decrease. Also, in an adiabatic process, PV^γ = constant, meaning that the pressure must decrease as the volume increases.

OQ20.14 Answer (b) only. In an isobaric process on an ideal gas, pressure is constant while the gas either expands or is compressed. Since the volume of the gas is changing, work is done either on or by the gas. Also, from the ideal gas law with pressure constant, $P\Delta V = nR\Delta T$; thus, the gas must undergo a change in temperature having the same sign as the change in volume. If $\Delta V > 0$, then both ΔT and the change in the internal energy of the gas are positive ($\Delta U > 0$). However, when $\Delta V > 0$, the work done *on the gas* is negative ($\Delta W < 0$), and the first law of thermodynamics says that there must be a positive transfer of energy by heat to the gas ($Q = \Delta U - W > 0$). When $\Delta V < 0$, a similar argument shows that $\Delta U < 0$, $W > 0$, and $Q = \Delta U - W < 0$. Thus, all of the other listed choices are false statements.

OQ20.15 Answer (d). The temperature of the ice must be raised to the melting point, $\Delta T = +20.0°\text{C}$, before it will start to melt. The total energy input required to melt the 1.00 kg of ice is

$$Q = mc_{\text{ice}}(\Delta T) + mL_f = (1.00\text{ kg})\big[(2\ 090\text{ J/kg}\cdot°\text{C})(20.0°\text{C}) + 3.33\times10^5\text{ J/kg}\big] = 3.75\times10^5\text{ J}$$

The time the heating element will need to supply this quantity of energy is

$$\Delta t = \frac{Q}{P} = \frac{3.75\times10^5\text{ J}}{1.00\times10^3\text{ J/s}} = (375\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = 6.25\text{ min}$$

ANSWERS TO CONCEPTUAL QUESTIONS

CQ20.1 Rubbing a surface results in friction converting kinetic energy to thermal energy. Metal, being a good thermal conductor, allows energy to transfer swiftly out of the rubbed area to the surrounding areas, resulting in a swift fall in temperature. Wood, being a poor conductor, permits a slower rate of transfer, so the temperature of the rubbed area does not fall as swiftly.

CQ20.2 Keep them dry. The air pockets in the pad conduct energy by heat, but only slowly. Wet pads would absorb some energy in warming up themselves, but the pot would still be hot and the water would quickly conduct a lot of energy right into you.

CQ20.3 Heat is a method of transferring energy, not energy contained in an object. Further, a low-temperature object with large mass, or an object made of a material with high specific heat, can contain more internal energy than a higher-temperature object.

CQ20.4 There are three properties to consider here: thermal conductivity, specific heat, and mass. With dry aluminum, the thermal conductivity of aluminum is much greater than that of (dry) skin. This means that the internal energy in the aluminum can more readily be transferred to the atmosphere than to your fingers. In essence, your skin acts as a thermal insulator. If the aluminum is wet, it can wet the outer layer of your skin to make it into a good thermal conductor; then more energy from the aluminum can transfer to you. Further, the water itself, with additional mass and with a relatively large specific heat compared to aluminum, can be a significant source of extra energy to burn you. In practical terms, when you let go of a hot, dry piece of aluminum foil, the energy transfer by heat immediately ends. When you let go of a hot *and* wet piece of aluminum foil, the hot water sticks to your skin, continuing the heat transfer, and resulting in more energy transfer by heat to you!

CQ20.5 If the system is isolated, no energy enters or leaves the system by heat, work, or other transfer processes. Within the system energy can change from one form to another, but since energy is conserved these transformations cannot affect the total amount of energy. The total energy is constant.

CQ20.6 (a) Warm a pot of coffee on a hot stove.

(b) Place an ice cube at 0ºC in warm water—the ice will absorb energy while melting, but not increase in temperature.

(c) Let a high-pressure gas at room temperature slowly expand by pushing on a piston. Energy comes out of the gas by work in a constant-temperature expansion as the same quantity of energy flows by heat in from the surroundings.

(d) Warm your hands by rubbing them together. Heat your tepid coffee in a microwave oven. Energy input by work, by electromagnetic radiation, or by other means, can all alike produce a temperature increase.

(e) Davy's experiment is an example of this process.

CQ20.7 (a) Yes, wrap the blanket around the ice chest. The environment is warmer than the ice, so the blanket prevents energy transfer by heat from the environment to the ice.

(b) Explain to your little sister that her body is warmer than the environment and requires energy transfer by heat into the air to remain at a fixed temperature. The blanket will prevent this conduction and cause her to feel warmer, not cool like the ice.

CQ20.8 Ice is a poor thermal conductor, and it has a high specific heat. The idea behind wetting fruit is that a coating of ice prevents the fruit from cooling below the freezing temperature even as the air outside is colder, and also to protect plants from frost. When frost melts it takes its heat from the fruit, and kills it. When ice melts it takes heat from the air, so it acts as insulation for the fruit.

CQ20.9 The person should add the cream immediately when the coffee is poured. Then the smaller temperature difference between coffee and environment will reduce the rate of energy transfer out of the cup during the several minutes.

CQ20.10 The sunlight hitting the peaks warms the air immediately around them. This air, which is slightly warmer and less dense than the surrounding air, rises, as it is buoyed up by cooler air from the valley below. The air from the valley flows up toward the sunny peaks, creating the morning breeze.

CQ20.11 Because water has a high specific heat, it can absorb or lose quite a bit of energy and not experience much change in temperature. The water would act as a means of preventing the temperature in the cellar from varying much so that stored goods would neither freeze nor become too warm.

CQ20.12 The steam locomotive engine is one perfect example of turning internal energy into mechanical energy. Liquid water is heated past the point of vaporization. Through a controlled mechanical process, the expanding water vapor is allowed to push a piston. The translational kinetic energy of the piston is usually turned into rotational kinetic energy of the drive wheel.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 20.1 Heat and Internal Energy

P20.1 (a) The energy equivalent of 540 Calories is found from

$$Q = 540\ \cancel{\text{Cal}}\left(\frac{10^3\ \cancel{\text{cal}}}{1\ \cancel{\text{Cal}}}\right)\left(\frac{4.186\ \text{J}}{1\ \cancel{\text{cal}}}\right) = \boxed{2.26\times 10^6\ \text{J}}$$

(b) The work done lifting her weight *mg* up one stair of height *h* is $W_1 = mgh$. Thus, the total work done in climbing *N* stairs is $W = Nmgh$, and we have $Q = Nmgh$, or

$$N = \frac{Q}{mgh} = \frac{2.26\times 10^6\ \text{J}}{(55.0\ \text{kg})(9.80\ \text{m/s}^2)(0.150\ \text{m})} \simeq \boxed{2.80\times 10^4\ \text{stairs}}$$

(c) If only 25% of the energy from the donut goes into mechanical energy, we have

$$N = \frac{0.25Q}{mgh} = 0.25\left(\frac{Q}{mgh}\right) = 0.25\left(2.80\times 10^4\ \text{stairs}\right)$$
$$= \boxed{6.99\times 10^3\ \text{stairs}}$$

Section 17.2 Specific Heat and Calorimetry

P20.2 The container is thermally insulated, so no energy is transferred by heat:

$$Q = 0$$

and $$\Delta E_{\text{int}} = Q + W_{\text{input}} = 0 + W_{\text{input}} = 2mgh$$

The work on the falling weights is equal to the work done on the water in the container by the rotating blades. This work results in an increase in internal energy of the water:

$$2mgh = \Delta E_{\text{int}} = m_{\text{water}} c\Delta T$$

$$\Delta T = \frac{2mgh}{m_{\text{water}}c} = \frac{2(1.50\ \text{kg})(9.80\ \text{m/s}^2)(3.00\ \text{m})}{(0.200\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})} = \frac{88.2\ \text{J}}{837\ \text{J/}^\circ\text{C}}$$
$$= \boxed{0.105^\circ\text{C}}$$

P20.3 The system is thermally isolated, so

$$Q_{\text{water}} + Q_{\text{Al}} + Q_{\text{Cu}} = 0$$

$$(0.250\text{ kg})\left(4\,186\frac{\text{J}}{\text{kg °C}}\right)\left(T_f - 20.0°\text{C}\right)$$

$$+(0.400\text{ kg})\left(900\frac{\text{J}}{\text{kg °C}}\right)\left(T_f - 26.0°\text{C}\right)$$

$$+(0.100\text{ kg})\left(387\frac{\text{J}}{\text{kg °C}}\right)\left(T_f - 100°\text{C}\right) = 0$$

$$1\,046.5T_f - 20\,930°\text{C} + 360T_f - 9\,360°\text{C} + 38.7T_f - 3\,870°\text{C} = 0$$

$$1\,445.2T_f = 34\,160°\text{C}$$

$$T_f = \boxed{23.6°\text{C}}$$

P20.4 As mass *m* of water drops from top to bottom of the falls, the gravitational potential energy given up (and hence, the kinetic energy gained) is *Q* = *mgh*. If all of this goes into raising the temperature, $Q = mc\Delta T$, and the rise in temperature will be

$$\Delta T = \frac{Q}{mc_{\text{water}}} = \frac{\cancel{m}gh}{\cancel{m}c_{\text{water}}} = \frac{(9.80\text{ m/s}^2)(807\text{ m})}{4\,186\text{ J/kg}\cdot°\text{C}} = 1.89°\text{C}$$

and the final temperature is

$$T_f = T_i + \Delta T = 15.0°\text{C} + 1.89°\text{C} = \boxed{16.9°\text{C}}$$

P20.5 When thermal equilibrium is reached, the water and aluminum will have a common temperature of $T_f = 65.0°C$. Assuming that the water-aluminum system is thermally isolated from the environment, $Q_{\text{cold}} = -Q_{\text{hot}}$:

$$m_w c_w\left(T_f - T_{i,w}\right) = -m_{\text{Al}}c_{\text{Al}}\left(T_f - T_{i,\text{Al}}\right)$$

or

$$m_w = \frac{-m_{\text{Al}}c_{\text{Al}}\left(T_f - T_{i,\text{Al}}\right)}{c_w\left(T_f - T_{i,w}\right)}$$

$$= \frac{-(1.85\text{ kg})(900\text{ J/kg}\cdot°\text{C})(65.0°\text{C} - 150°\text{C})}{(4\,186\text{ J/kg}\cdot°\text{C})(65.0°\text{C} - 25.0°\text{C})} = \boxed{0.845\text{ kg}}$$

P20.6 We find its specific heat from the definition, which is contained in the equation $Q = mc_{\text{silver}}\Delta T$ for energy input by heat to produce a temperature change. Solving, we have

$$c_{\text{silver}} = \frac{Q}{m\Delta T}$$

$$c_{\text{silver}} = \frac{1.23\times 10^{3}\text{ J}}{(0.525\text{ kg})(10.0°\text{C})} = \boxed{234\text{ J/kg}\cdot°\text{C}}$$

P20.7 We imagine the stone energy reservoir has a large area in contact with air and is always at nearly the same temperature as the air. Its overnight loss of energy is described by

$$P = \frac{Q}{\Delta t} = \frac{mc\Delta T}{\Delta t}$$

$$m = \frac{P\Delta t}{c\Delta T} = \frac{(-6\,000\text{ J/s})(14\text{ h})(3\,600\text{ s/h})}{(850\text{ J/kg}\cdot°\text{C})(18.0°\text{C} - 38.0°\text{C})}$$

$$= \frac{3.02\times 10^{8}\text{ J}}{(850\text{ J/kg}\cdot°\text{C})(20.0°\text{C})} = \boxed{1.78\times 10^{4}\text{ kg}}$$

***P20.8** From $Q = mc\Delta T$ we find

$$\Delta T = \frac{Q}{mc} = \frac{1\,200\text{ J}}{(0.050\,0\text{ kg})(387\text{ J/kg}\cdot°\text{C})} = 62.0°\text{C}$$

Thus, the final temperature is $25.0°\text{C} + 62.0°\text{C} = \boxed{87.0°\text{C}}$.

P20.9 Let us find the energy transferred in one minute:

$$Q = \left[m_{\text{cup}}c_{\text{cup}} + m_{\text{water}}c_{\text{water}}\right]\Delta T$$

$$Q = \left[(0.200\text{ kg})(900\text{ J/kg}\cdot°\text{C}) + (0.800\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})\right] \times(-1.50°\text{C}) = -5\,290\text{ J}$$

If this much energy is removed from the system each minute, the rate of removal is

$$P = \frac{|Q|}{\Delta t} = \frac{5\,290\text{ J}}{60.0\text{ s}} = 88.2\text{ J/s} = \boxed{88.2\text{ W}}$$

P20.10 We use $Q_{\text{cold}} = -Q_{\text{hot}}$ to find the equilibrium temperature:

$$m_{\text{Al}}c_{\text{Al}}(T_f - T_c) + m_c c_w (T_f - T_c) = -m_h c_w (T_f - T_h)$$

$$(m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_f - (m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c = -m_h c_w T_f + m_h c_w T_h$$

$$(m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w)T_f = (m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c + m_h c_w T_h$$

solving for the final temperature gives

$$T_f = \boxed{\frac{(m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c + m_h c_w T_h}{m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w}}$$

P20.11 We assume that the water-horseshoe system is thermally isolated (insulated) from the environment for the short time required for the horseshoe to cool off and the water to warm up. Then the total energy input from the surroundings is zero, as expressed by $Q_{\text{Fe}} + Q_{\text{water}} = 0$:

$$(mc\Delta T)_{\text{Fe}} + (mc\Delta T)_{\text{water}} = 0$$

$$m_{\text{Fe}}c_{\text{Fe}}(T - 600°\text{C}) + m_w c_w (T - 25.0°\text{C}) = 0$$

Note that the first term in this equation is a negative number of joules, representing energy lost by the originally hot subsystem, and the second term is a positive number with the same absolute value, representing energy gained by heat by the cold stuff. Solving for the final temperature gives

$$T = \frac{m_w c_w (25.0^\circ\text{C}) + m_{\text{Fe}}c_{\text{Fe}}(600^\circ\text{C})}{m_{\text{Fe}}c_{\text{Fe}} + m_w c_w}$$

Substituting $c_w = 4\,186$ J/kg · °C and $c_{\text{Fe}} = 448$ J/kg · °C and suppressing units, we obtain

$$T = \frac{(20.0)(4\,186)(25.0°\text{C}) + (1.50)(448)(600°\text{C})}{(1.50)(448) + (20.0\text{ kg})(4\,186)}$$

$$= \boxed{29.6°\text{C}}$$

P20.12 (a) The work that the bit does in deforming the block, breaking chips off, and giving them kinetic energy is not a final destination for energy. All of this work turns entirely into internal energy as soon as the chips stop their macroscopic motion. The amount of energy input to the steel is the work done by the bit:

$$W = \vec{\mathbf{F}} \cdot \Delta\vec{\mathbf{r}} = (3.20\text{ N})(40.0\text{ m/s})(15.0\text{ s})\cos 0.00° = 1\,920\text{ J}$$

To evaluate the temperature change produced by this energy we imagine injecting the same quantity of energy as heat from a stove. The bit, chips, and block all undergo the same temperature change. Any difference in temperature between one bit of steel and another would erase itself by causing an energy transfer by heat from the temporarily hotter to the colder region.

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc} = \frac{1\,920\text{ J}}{(0.267\text{ kg})(448\text{ J/kg}\cdot°\text{C})} = \boxed{16.1°\text{C}}$$

(b) See part (a). The same amount of work is done. $\boxed{16.1°\text{C}}$

(c) $\boxed{\text{It makes no difference whether the drill bit is dull or sharp, or how far into the block it cuts. The answers to (a) and (b) are the same because all of the work done by the bit on the block constitutes energy being transferred into the internal energy of the steel.}}$

P20.13 (a) To find the specific heat of the unknown sample, we start with $Q_{\text{cold}} = -Q_{\text{hot}}$ and substitute:

$$(m_w c_w + m_c c_c)(T_f - T_c) = -m_{\text{Cu}}c_{\text{Cu}}(T_f - T_{\text{Cu}}) - m_{\text{unk}}c_{\text{unk}}(T_f - T_{\text{unk}})$$

where w is for water, c the calorimeter, Cu the copper sample, and "unk" the unknown.

$$\begin{aligned}&\left[(0.250\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C}) + (0.100\text{ kg})(900\text{ J/kg}\cdot°\text{C})\right] \\ &\qquad\qquad (20.0°\text{C} - 10.0°\text{C}) \\ &= -(0.050\,0\text{ kg})(387\text{ J/kg}\cdot°\text{C})(20.0 - 80.0)°\text{C} \\ &\qquad - (0.070\,0\text{ kg})c_{\text{unk}}(20.0°\text{C} - 100°\text{C})\end{aligned}$$

$$1.020\,4 \times 10^4\text{ J} = (5.60\text{ kg}\cdot°\text{C})c_{\text{unk}}$$

$$c_{\text{unk}} = \boxed{1.82 \times 10^3\text{ J/kg}\cdot°\text{C}}$$

(b) $\boxed{\text{We cannot make definite identification. It might be beryllium.}}$

(c) $\boxed{\text{The material might be an unknown alloy or a material not listed in the table.}}$

P20.14 (a) Expressing the percentage change as $f = 0.60$, we have

$$(f)(mgh) = mc\Delta T \quad\rightarrow\quad \Delta T = \frac{fgh}{c}$$

$$\Delta T = \frac{(0.600)(9.80\text{ m/s}^2)(50.0\text{ m})}{387\text{ J/kg}\cdot°\text{C}} = 0.760°\text{C} = T - 25.0°\text{C}$$

which gives $\boxed{T = 25.8°\text{C}}$

(b) $\boxed{\text{As shown above, the symbolic result from part (a) shows no dependence on mass. Both the change in gravitational potential energy and the change in internal energy of the system depend on the mass, so the mass cancels.}}$

P20.15 (a) The gas comes to an equilibrium temperature according to

$$(mc\Delta T)_{\text{cold}} = -(mc\Delta T)_{\text{hot}}$$
$$n_1Mc\left(T_f - 300\text{ K}\right) + n_2Mc\left(T_f - 450\text{ K}\right) = 0$$

The molar mass *M* and specific heat divide out, and we can express *n* in terms of *P*, *V*, and *T*, using *PV* = *nRT*:

$$\frac{P_1V_1}{T_1}\cancel{Mc}\left(T_f - T_1\right) + \frac{P_2V_2}{T_2}\cancel{Mc}\left(T_f - T_2\right) = 0$$
$$\frac{P_1V_1}{T_1}T_f - \frac{P_1V_1}{T_1}T_1 + \frac{P_2V_2}{T_2}T_f - \frac{P_2V_2}{T_2}T_2 = 0$$
$$T_f\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right) = P_1V_1 + P_2V_2$$

$$T_f = \frac{P_1V_1 + P_2V_2}{\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)} = \frac{(1.75\text{ atm})(16.8\text{ L}) + (2.25\text{ atm})(22.4\text{ L})}{\left(\frac{(1.75\text{ atm})(16.8\text{ L})}{300\text{ K}} + \frac{(2.25\text{ atm})(22.4\text{ L})}{450\text{ K}}\right)}$$
$$= \boxed{380\text{ K}}$$

(b) The pressure of the whole sample in its final state is

$$P_f = (n_1 + n_2)\frac{R}{V_f}T_f = \left(\frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}\right)\left(\frac{R}{V_1 + V_2}\right)\frac{P_1V_1 + P_2V_2}{\left(\frac{P_1V_1}{T_1} + \frac{P_2V_2}{T_2}\right)}$$

$$P_f = \left(\frac{P_1V_1 + P_2V_2}{V_1 + V_2}\right) = \left(\frac{(1.75\text{ atm})(16.8\text{ L}) + (2.25\text{ atm})(22.4\text{ L})}{16.8\text{ L} + 22.4\text{ L}}\right)$$
$$= \boxed{2.04\text{ atm}}$$

Section 20.3 Latent Heat

***P20.16** To find the amount of steam to be condensed, we begin with

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

With the steam at 100°C, this becomes

$$(m_wc_w + m_cc_c)\left(T_f - T_i\right) = -m_s\left[-L_v + c_w\left(T_f - 100\right)\right]$$

Substituting numerical values,

$$\left[(0.250\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})+(0.050\,0\text{ kg})(387\text{ J/kg}\cdot{}^\circ\text{C})\right](50.0^\circ\text{C}-20.0^\circ\text{C})$$

$$=-m_s\left[-2.26\times10^6\text{ J/kg}+(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C}-100^\circ\text{C})\right]$$

Solving for the mass of steam gives

$$m_s=\frac{3.20\times10^4\text{ J}}{2.47\times10^6\text{ J/kg}}=0.012\,9\text{ kg}=\boxed{12.9\text{ g steam}}$$

***20.17** We assume that all work done against friction is used to melt the snow. Equation 8.2 for conservation of energy then gives

$$W_{\text{skier}}=Q_{\text{snow}}$$

or $f\cdot d=m_{\text{snow}}L_f$

where $f=\mu_k n=\mu_k(m_{\text{skier}}g)$

Substituting and solving for the distance gives

$$d=\frac{m_{\text{snow}}L_f}{\mu_k(m_{\text{skier}}g)}=\frac{(1.00\text{ kg})(3.33\times10^5\text{ J/kg})}{0.200(75.0\text{ kg})(9.80\text{ m/s}^2)}$$

$$=2.27\times10^3\text{ m}=\boxed{2.27\text{ km}}$$

P20.18 The energy input needed is the sum of the following terms:

$$\begin{aligned}Q_{\text{needed}}&=(\text{energy to reach melting point})+(\text{energy to melt})\\&\quad+(\text{energy to reach boiling point})\\&\quad+(\text{energy to vaporize})\\&\quad+(\text{energy to reach }110^\circ\text{C})\end{aligned}$$

Thus, we have

$$\begin{aligned}Q_{\text{needed}}&=(0.040\,0\text{ kg})\left[(2\,090\text{ J/kg}\cdot{}^\circ\text{C})(10.0^\circ\text{C})\right.\\&\quad+(3.33\times10^5\text{ J/kg})+(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C})\\&\quad\left.+(2.26\times10^6\text{ J/kg})+(2\,010\text{ J/kg}\cdot{}^\circ\text{C})(10.0^\circ\text{C})\right]\end{aligned}$$

$$Q_{\text{needed}}=\boxed{1.22\times10^5\text{ J}}$$

P20.19 Remember that energy must be supplied to melt the ice before its temperature will begin to rise. Then, assuming a thermally isolated system, $Q_{\text{cold}}=-Q_{\text{hot}}$, or

$$m_{\text{ice}}L_f+m_{\text{ice}}c_{\text{water}}(T_f-0^\circ\text{C})=-m_w c_{\text{water}}(T_f-25^\circ\text{C})$$

and

$$T_f = \frac{m_w c_{\text{water}}(25°\text{C}) - m_{\text{ice}} L_f}{(m_{\text{ice}} + m_w) c_{\text{water}}}$$

$$= \frac{(825\ \cancel{\text{g}})(4\,186\ \text{J/kg}\cdot°\text{C})(25°\text{C}) - (75\ \cancel{\text{g}})(3.33\times10^5\ \text{J/kg})}{(75\ \cancel{\text{g}} + 825\ \cancel{\text{g}})(4\,186\ \text{J/kg}\cdot°\text{C})}$$

yielding $\boxed{T_f = 16.3°\text{C}}$

P20.20 The bullet will not melt all the ice, so its final temperature is 0°C. Then, conservation of energy gives

$$\left(\frac{1}{2}mv^2 + mc|\Delta T|\right)_{\text{bullet}} = m_w L_f$$

where m_w is the mass of melted ice. Solving for m_w gives,

$$m_w = \left(\frac{3.00\times10^{-3}\ \text{kg}}{3.33\times10^5\ \text{J/kg}}\right) \times\left[(0.500)(240\ \text{m/s})^2 + (128\ \text{J/kg}\cdot°\text{C})(30.0°\text{C})\right]$$

$$m_w = \frac{86.4\ \text{J} + 11.5\ \text{J}}{333\,000\ \text{J/kg}} = \boxed{0.294\ \text{g}}$$

P20.21 (a) With 10.0 g of steam added to 50.0 g of ice, we first compute the energy required to melt all the ice:

$$Q_1 = (\text{energy to melt all the ice})$$
$$= (50.0\times10^{-3}\ \text{kg})(3.33\times10^5\ \text{J/kg}) = 1.67\times10^4\ \text{J}$$

Also, the energy required to raise the temperature of the melted ice to 100°C is

$$Q_2 = (\text{energy to raise temp of ice to }100°\text{C})$$
$$= (50.0\times10^{-3}\ \text{kg})(4\,186\ \text{J/kg}\cdot°\text{C})(100°\text{C}) = 2.09\times10^4\ \text{J}$$

Thus, the total energy to melt all of the ice and raise its temperature to 100°C is

$$Q_1 + Q_2 = 1.67\times10^4\ \text{J} + 2.09\times10^4\ \text{J} = 3.76\times10^4\ \text{J}$$

The energy available from the condensation of 10.0 g of steam is

$$Q_3 = (\text{energy available as steam condenses})$$
$$= (10.0\times10^{-3}\ \text{kg})(2.26\times10^6\ \text{J/kg}) = 2.26\times10^4\ \text{J}$$

Thus, we see that $Q_3 > Q_1$, but $Q_3 < Q_1 + Q_2$, which means that all of the ice will melt, $\boxed{\Delta m_{\text{ice}} = 50.0\text{ g}}$, but the final temperature of the mixture will be $T_f < 100^{\circ}\text{C}$. To find the final temperature T_f, we use $Q_{\text{cold}} = -Q_{\text{hot}}$, or

$$m_{\text{ice}}L_f + m_{\text{ice}}c_w\Delta T_{\text{ice}} = -m_{\text{steam}}L_v - m_{\text{steam}}c_w\Delta T_{\text{steam}}$$

Substituting numerical values,

$$\begin{aligned}&\left(50.0\times10^{-3}\text{ kg}\right)\left(3.33\times10^{5}\text{ J/kg}\right)\\ &\quad+\left(50.0\times10^{-3}\text{ kg}\right)\left(4\,186\text{ J/kg}\cdot{}^{\circ}\text{C}\right)\left(T_f-0^{\circ}\text{C}\right)\\ &\qquad=-\left(10.0\times10^{-3}\text{ kg}\right)\left(-2.26\times10^{6}\text{ J/kg}\right)\\ &\qquad\quad-\left(10.0\times10^{-3}\text{ kg}\right)\left(4\,186\text{ J/kg}\cdot{}^{\circ}\text{C}\right)\left(T_f-100^{\circ}\text{C}\right)\end{aligned}$$

From which we obtain

$$\boxed{T_f = 40.4^{\circ}\text{C}}$$

(b) Since the mass of steam is much smaller than in part (a), we know that the condensation of steam will not be sufficient to melt all of the ice and raise its temperature to 100°C. We do need to determine whether the condensation of steam can supply sufficient energy to melt all of the ice. Recall from part (a) that

$$Q_1 = \left(\text{energy to melt all the ice}\right) = 1.67\times10^{4}\text{ J}$$

The energy given up as the 1.00 g of steam condenses is

$$\begin{aligned}Q_2 &= \left\{\begin{matrix}\text{energy given up}\\ \text{as steam condenses}\end{matrix}\right\} = \left(10^{-3}\text{ kg}\right)\left(2.26\times10^{6}\text{ J/kg}\right)\\ &= 2.26\times10^{3}\text{ J}\end{aligned}$$

Also,

$$\begin{aligned}Q_3 &= \left\{\begin{matrix}\text{energy given up as condensed}\\ \text{steam cools to }0^{\circ}\text{C}\end{matrix}\right\}\\ &= \left(10^{-3}\text{ kg}\right)\left(4\,186\text{ J/kg}\cdot{}^{\circ}\text{C}\right)\left(100^{\circ}\text{C}\right) = 419\text{ J}\end{aligned}$$

Since $\boxed{Q_2 + Q_3 < Q_1}$, therefore all of the steam will cool to 0°C, and $\boxed{T_f = 0^{\circ}\text{C}}$ with some ice remaining. Let us now find the mass of ice which must melt to condense the steam and cool the condensate to 0°C. Again from $Q_{\text{cold}} = -Q_{\text{hot}}$,

$$m_{\text{ice}}L_f = Q_2 + Q_3 = 2.68\times10^{3}\text{ J}$$

Thus,

$$m_{\text{ice}} = \frac{2.68 \times 10^3 \text{ J}}{3.33 \times 10^5 \text{ J/kg}} = 8.04 \times 10^{-3} \text{ kg} = \boxed{8.04 \text{ g of ice melts}}$$

Therefore, there is 42.0 g of ice left over, also at 0ºC.

P20.22 The boiling point of nitrogen is 77.3 K. Using units of joules, we have

$$Q = m_{\text{Cu}} c_{\text{Cu}} \Delta T = m_{\text{N}_2} \left(L_{\text{vap}}\right)_{\text{N}_2}$$

Substituting numerical values,

$$(1.00 \text{ kg})(387 \text{ J/kg} \cdot {}^\circ\text{C})(293 - 77.3)^\circ\text{C} = m(2.01 \times 10^5 \text{ J/kg})$$

$$m = \boxed{0.415 \text{ kg}}$$

P20.23 (a) Since the heat required to melt 250 g of ice at 0°C *exceeds* the heat required to cool 600 g of water from 18°C to 0°C, the final temperature of the system (water + ice) must be $\boxed{0^\circ\text{C}}$.

(b) Let *m* represent the mass of ice that melts before the system reaches equilibrium at 0°C.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$mL_f = -m_w c_w \left(0^\circ\text{C} - T_i\right)$$

$$m(3.33 \times 10^5 \text{ J/kg}) = -(0.600 \text{ kg})(4\,186 \text{ J/kg} \cdot {}^\circ\text{C})(0^\circ\text{C} - 18.0^\circ\text{C})$$

$$m = 136 \text{ g, so the ice remaining} = 250 \text{ g} - 136 \text{ g} = \boxed{114 \text{ g}}$$

P20.24 (a) Let *n* represent the number of stops. Follow the energy:

$$nK = mc\Delta T$$

$$n\left[\frac{1}{2}(1\,500 \text{ kg})(25.0 \text{ m/s})^2\right] = (6.00 \text{ kg})(900 \text{ J/kg} \cdot {}^\circ\text{C})(660^\circ\text{C} - 20.0^\circ\text{C})$$

$$n = \frac{3.46 \times 10^6 \text{ J}}{4.69 \times 10^5 \text{ J}} = 7.37$$

Thus $\boxed{7}$ stops can happen before melting begins.

(b) As the car stops it transforms part of its kinetic energy into internal energy due to air resistance. As soon as the brakes rise above the air temperature they transfer energy by heat into the air, and transfer it very fast if they attain a high temperature.

Section 20.4 Work and Heat in Thermodynamic Processes

P20.25 For constant pressure, $W = -\int_{V_i}^{V_f} P dV = -P\Delta V = -P(V_f - V_i)$. Rather than evaluating the pressure numerically from atmospheric pressure plus the pressure due to the weight of the piston, we can just use the ideal gas law to write in the volumes, obtaining

$$W = -P\left(\frac{nRT_h}{P} - \frac{nRT_c}{P}\right) = -nR\left(T_h - T_c\right)$$

Therefore,

$$W = -nR\Delta T = -(0.200\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(280\text{ K}) = \boxed{-466\text{ J}}$$

P20.26 $$W = -\int_i^f P dV = -P\int_i^f dV = -P\Delta V = -nR\Delta T = \boxed{-nR\left(T_2 - T_1\right)}$$

The negative sign for work *on* the sample indicates that the expanding gas *does* positive work. The quantity of work is directly proportional to the quantity of gas and to the temperature change.

P20.27 During the warming process $P = \left(\frac{P_i}{V_i}\right)V$.

(a) $$W = -\int_i^f P dV = -\int_{V_i}^{3V_i}\left(\frac{P_i}{V_i}\right)V dV$$

$$W = -\left(\frac{P_i}{V_i}\right)\frac{V^2}{2}\bigg|_{V_i}^{3V_i} = -\frac{P_i}{2V_i}\left(9V_i^2 - V_i^2\right) = \boxed{-4P_iV_i}$$

(b) *PV* = *nRT* gives

$$\left[\left(\frac{P_i}{V_i}\right)V\right]V = nRT \to T = \left(\frac{P_i}{nRV_i}\right)V^2$$

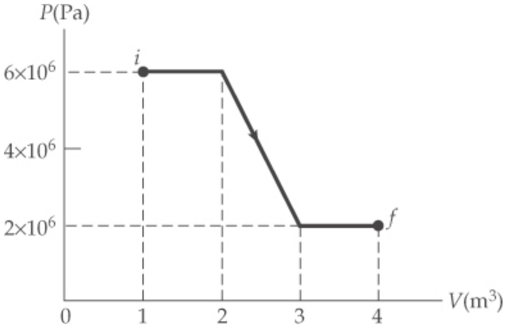
$$\boxed{\text{It is proportional to the square of the volume, according to } T = (P_i / nRV_i)V^2.}$$

P20.28 (a) $W = -\int P dV$

$$W = -\left(6.00\times10^6\ \text{Pa}\right)\left(2.00\ \text{m}^3 - 1.00\ \text{m}^3\right) + -\left(4.00\times10^6\ \text{Pa}\right)\left(3.00\ \text{m}^3 - 2.00\ \text{m}^3\right) + -\left(2.00\times10^6\ \text{Pa}\right)\left(4.00\ \text{m}^3 - 3.00\ \text{m}^3\right)$$

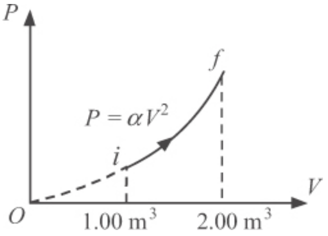
$$W_{i\to f} = \boxed{-12.0\ \text{MJ}}$$

(b) $W_{f\to i} = \boxed{+12.0\ \text{MJ}}$



ANS. FIG. P20.28

P20.29 The work done on the gas is the negative of the area under the curve $P = \alpha V^2$, from V_i to V_f. The work *on* the gas is negative, to mean that the expanding gas *does* positive work. We will find its amount by doing the integral



ANS. FIG. P20.29

$$W = -\int_i^f P dV$$

$$W = -\int_i^f \alpha V^2 dV = -\frac{1}{3}\alpha\left(V_f^3 - V_i^3\right)$$

$$V_f = 2V_i = 2\left(1.00\ \text{m}^3\right) = 2.00\ \text{m}^3$$

$$W = -\frac{1}{3}\left[\left(5.00\ \text{atm/m}^6\right)\left(1.013\times10^5\ \text{Pa/atm}\right)\right] \times\left[\left(2.00\ \text{m}^3\right)^3 - \left(1.00\ \text{m}^3\right)^3\right]$$

$$= \boxed{-1.18\ \text{MJ}}$$

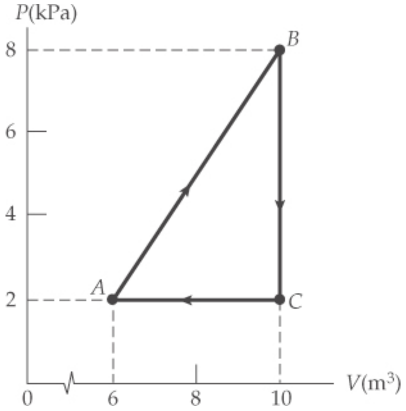
Section 20.5 The First Law of Thermodynamics

P20.30 (a) Refer to ANS. FIG. P20.30. From the first law, for a cyclic process, $Q = -W =$ Area of triangle, so

$$Q = \frac{1}{2}\left(4.00\ \text{m}^3\right)\left(6.00\ \text{kPa}\right)$$

$$= \boxed{12.0\ \text{kJ}}$$

(b) $Q = -W = \boxed{-12.0\ \text{kJ}}$



ANS. FIG. P20.30

P20.31 Refer to ANS. FIG. P20.30. We tabulate the signs for Q, W, and ΔE_{int} below:

	Q	W	ΔE_{int}	
BC	–	0	–	$(Q = \Delta E_{\text{int}}$ since $W_{BC} = 0)$
CA	–	+	–	$(\Delta E_{\text{int}} < 0$ and $W > 0$, so $Q < 0)$
AB	+	–	+	$(W < 0$, $\Delta E_{\text{int}} > 0$ since $\Delta E_{\text{int}} < 0$ for $B \to C \to A$; so $Q > 0)$

P20.32 From the first law of thermodynamics,

$$\Delta E_{\text{int}} = Q + W = 10.0\ \text{J} + 12.0\ \text{J} = +22.0\ \text{J}$$

The change in internal energy is a positive number, which would be consistent with an *increase* in temperature of the gas, but the problem statement indicates a *decrease* in temperature.

P20.33 From the first law of thermodynamics, $\Delta E_{\text{int}} = Q + W$, so

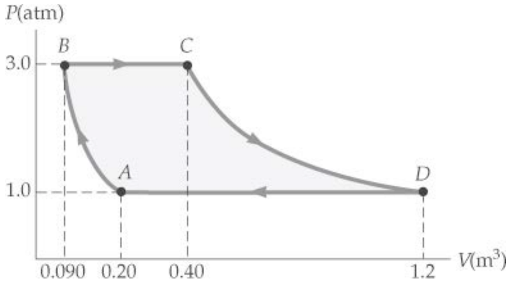
$$Q = \Delta E_{\text{int}} - W = -500\text{ J} - 220\text{ J} = \boxed{-720\text{ J}}$$

The negative sign indicates that positive energy is transferred *from* the system by heat.

P20.34 Because the gas goes through a cycle, the overall change in internal energy must be zero:

$$\Delta E_{\text{int}} = \Delta E_{\text{int},AB} + \Delta E_{\text{int},BC} + \Delta E_{\text{int},CD} + \Delta E_{\text{int},DA} = 0$$
$$\rightarrow \quad \Delta E_{\text{int},AB} = -\Delta E_{\text{int},BC} - \Delta E_{\text{int},CD} - \Delta E_{\text{int},DA}$$

Recognize that $\Delta E_{\text{int}} = 0$ for the isothermal process *CD* and substitute from the first law for the other internal energy changes:



ANS. FIG. P20.34

$$\begin{aligned}
\Delta E_{\text{int},AB} &= -\left(Q_{BC} + W_{BC}\right) - \left(Q_{DA} + W_{DA}\right) \\
&= -\left(Q_{BC} - P_B\Delta V_{BC}\right) - \left(Q_{DA} - P_D\Delta V_{DA}\right) \\
&= -\left(Q_{BC} + Q_{DA}\right) + \left(P_B\Delta V_{BC} + P_D\Delta V_{DA}\right) \\
&= -\left(345\text{ kJ} - 371\text{ kJ}\right) \\
&\quad + \left[\left(3.00\text{ atm}\right)\left(0.310\text{ m}^3\right) + \left(1.00\text{ atm}\right)\left(-1.00\text{ m}^3\right)\right] \\
&\qquad \times\left(\frac{1.013\times 10^5\text{ Pa}}{1\text{ atm}}\right) \\
&= \boxed{4.29\times 10^4\text{ J}}
\end{aligned}$$

Section 20.6 Some Applications of the First Law of Thermodynamics

P20.35 (a) Rearranging $PV = nRT$ we get $V_i = \dfrac{nRT}{P_i}$

The initial volume is

$$V_i = \frac{(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}{(0.400\text{ atm})\left(1.013\times10^5\text{ Pa/atm}\right)}\left(\frac{1\text{ Pa}}{\text{N/m}^2}\right) = 0.123\text{ m}^3$$

For isothermal compression, PV is constant, so $P_iV_i = P_fV_f$ and the final volume is

$$V_f = V_i\left(\frac{P_i}{P_f}\right) = \left(0.123\text{ m}^3\right)\left(\frac{0.400\text{ atm}}{1.20\text{ atm}}\right) = \boxed{0.041\ 0\text{ m}^3}$$

(b) $$W = -\int P\,dV = -nRT\ln\left(\frac{V_f}{V_i}\right) = -\left(4.99\times10^3\text{ J}\right)\ln\left(\frac{1}{3}\right) = \boxed{+5.48\text{ kJ}}$$

(c) The ideal gas keeps constant temperature so $\Delta E_{\text{int}} = 0 = Q + W$ and the heat is $Q = \boxed{-5.48\text{ kJ}}$.

P20.36 (a) We choose as a system the H_2O molecules that all participate in the phase change. For a constant-pressure process,

$$W = -P\Delta V = -P\left(V_s - V_w\right)$$

where V_s is the volume of the steam and V_w is the volume of the liquid water. We can find them respectively from

$$PV_s = nRT \quad \text{and} \quad V_w = \text{m/p} = nM/p.$$

Calculating each work term,

$$PV_s = (1.00\text{ mol})\left(8.314\frac{\text{J}}{\text{K}\cdot\text{mol}}\right)(373\text{ K}) = 3\ 101\text{ J}$$

$$PV_w = (1.00\text{ mol})(18.0\text{ g/mol})\left(\frac{1.013\times10^5\text{ N/m}^2}{1.00\times10^6\text{ g/m}^3}\right) = 1.82\text{ J}$$

Thus the work done is

$$W = -3\ 101\text{ J} + 1.82\text{ J} = \boxed{-3.10\text{ kJ}}$$

(b) The energy input by heat is

$$Q = L_v\Delta m = \left(18.0\text{ g}\right)\left(2.26\times10^6\text{ J/kg}\right) = 40.7\text{ kJ}$$

so the change in internal energy is

$$\Delta E_{\text{int}} = Q + W = 40.7\text{ kJ} - 3.10\text{ kJ} = \boxed{37.6\text{ kJ}}$$

P20.37 (a) We use the energy version of the nonisolated system model.

$$\Delta E_{\text{int}} = Q + W$$

where $W = -P\Delta V$ for a constant-pressure process so that

$$\Delta E_{\text{int}} = Q - P\Delta V$$
$$= 12.5\text{ kJ} - 2.50\text{ kPa}\left(3.00\text{ m}^3 - 1.00\text{ m}^3\right) = \boxed{7.50\text{ kJ}}$$

(b) Since pressure and quantity of gas are constant, we have from the equation of state

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

and

$$T_2 = \frac{V_2}{V_1}T_1 = \left(\frac{3.00\text{ m}^3}{1.00\text{ m}^3}\right)(300\text{ K}) = \boxed{900\text{ K}}$$

P20.38 (a) $W = -nRT\ln\left(\dfrac{V_f}{V_i}\right) = -P_fV_f\ln\left(\dfrac{V_f}{V_i}\right)$

Suppressing units,

$$V_i = V_f\exp\left(+\frac{W}{P_fV_f}\right) = (0.025\ 0)\exp\left[\frac{-3\ 000}{0.025\ 0\left(1.013\times10^5\right)}\right]$$
$$= \boxed{0.007\ 65\text{ m}^3}$$

(b) $T_f = \dfrac{P_fV_f}{nR} = \dfrac{1.013\times10^5\text{ Pa}\left(0.025\ 0\text{ m}^3\right)}{1.00\text{ mol}\left(8.314\text{ J/K}\cdot\text{mol}\right)} = \boxed{305\text{ K}}$

P20.39 (a) $W = -P\Delta V = -P\left[3\alpha V\Delta T\right]$

$$= -\left(1.013\times10^5\text{ N/m}^2\right)$$
$$\times\left[3\left(24.0\times10^{-6}\text{°C}^{-1}\right)\left(\frac{1.00\text{ kg}}{2.70\times10^3\text{ kg/m}^3}\right)(18.0\text{°C})\right]$$

$W = \boxed{-0.048\ 6\text{ J}}$

(b) $Q = cm\Delta T = (900\text{ J/kg}\cdot\text{°C})(1.00\text{ kg})(18.0\text{°C}) = \boxed{16.2\text{ kJ}}$

(c) $\Delta E_{\text{int}} = Q + W = 16.2\text{ kJ} - 48.6\text{ mJ} = \boxed{16.2\text{ kJ}}$

P20.40 From conservation of energy, $\Delta E_{\text{int},\,ABC} = \Delta E_{\text{int},\,AC}$.

(a) From the first law of thermodynamics, we have

$$\Delta E_{\text{int},\,ABC} = Q_{ABC} + W_{ABC}$$

Then,

$$Q_{ABC} = \Delta E_{\text{int},\,ABC} - W_{ABC} = 800\text{ J} + 500\text{ J} = \boxed{1\,300\text{ J}}$$

(b) $W_{CD} = -P_C \Delta V_{CD}$, $\Delta V_{AB} = -\Delta V_{CD}$, and $P_A = 5P_C$

Then, $W_{CD} = \frac{1}{5} P_A \Delta V_{AB} = -\frac{1}{5} W_{AB} = \boxed{100\text{ J}}$.

(+ means that work is done on the system)

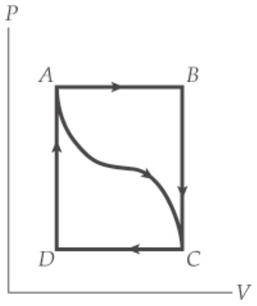
(c) $W_{CDA} = W_{CD}$ so that

$$Q_{CA} = \Delta E_{\text{int},\,CA} - W_{CDA} = -800\text{ J} - 100\text{ J} = \boxed{-900\text{ J}}$$

(– means that energy must be removed from the system by heat)

(d) $\Delta E_{\text{int},\,CD} = \Delta E_{\text{int},\,CDA} - \Delta E_{\text{int},\,DA} = -800\text{ J} - 500\text{ J} = -1\,300\text{ J}$

and $Q_{CD} = \Delta E_{\text{int},\,CD} - W_{CD} = -1\,300\text{ J} - 100\text{ J} = \boxed{-1\,400\text{ J}}$.



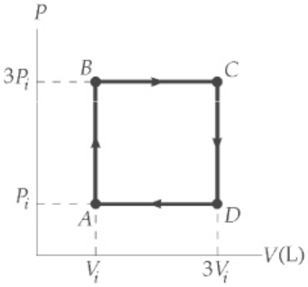
ANS. FIG. P20.40

P20.41 (a) The work done during each step of the cycle equals the negative of the area under that segment of the *PV* curve.

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$
$$W = 0 - 3P_i(3V_i - V_i) + 0 - P_i(V_i - 3V_i) + 0$$
$$W = -4P_iV_i = -4nRT_i$$
$$W = -4(1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(273\text{ K}) = \boxed{9.08\text{ kJ}}$$

(b) The initial and final values of *T* for the system are equal.

Therefore, $\Delta E_{\text{int}} = 0$ and $Q = -W = \boxed{9.08\text{ kJ}}$.



ANS. FIG. P20.41

P20.42 (a) The work done during each step of the cycle equals the negative of the area under that segment of the *PV* curve shown in ANS. FIG. P20.41.

$$W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$$

$$W = 0 - 3P_i\left(3V_i - V_i\right) + 0 - P_i\left(V_i - 3V_i\right) + 0 = \boxed{-4P_iV_i}$$

(b) The initial and final values of *T* for the system are equal.

Therefore, $\Delta E_{\text{int}} = 0$ and $Q = -W = \boxed{4P_iV_i}$.

Section 20.7 Energy Transfer Mechanisms in Thermal Processes

P20.43 (a) The rate of energy transfer by conduction through a material of area *A*, thickness *L*, with thermal conductivity *k*, and temperatures $T_h > T_c$ on opposite sides is $P = kA\,(T_h - T_c)/L$. For the given windowpane, this is

$$P = \left(0.8\ \text{W/m}\cdot{}^\circ\text{C}\right)\left[(1.0\text{ m})(2.0\text{ m})\right]\frac{\left(25.0^\circ\text{C} - 0^\circ\text{C}\right)}{0.620\times10^{-2}\text{ m}}$$

$$= \boxed{6.45\times10^{3}\text{ W}}$$

(b) The total energy lost per day is

$$E = P\cdot\Delta t = \left(6.45\times10^{3}\text{ J/s}\right)\left(8.64\times10^{4}\text{ s}\right) = \boxed{5.57\times10^{8}\text{ J}}$$

P20.44 The thermal conductivity of concrete is $k = 1.3\ \text{J/s} \cdot \text{m} \cdot {}^\circ\text{C}$, so the energy transfer rate through the slab is

$$P = kA\frac{(T_h - T_c)}{L} = (0.8\ \text{W/m}\cdot{}^\circ\text{C})(5.00\ \text{m}^2)\frac{(20^\circ\text{C})}{12.0\times 10^{-2}\ \text{m}}$$
$$= \boxed{667\ \text{W}}$$

P20.45 The net rate of energy transfer from his skin is

$$P_{\text{net}} = \sigma A e\left(T^4 - T_0^4\right)$$
$$= (5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.50\ \text{m}^2)$$
$$\times(0.900)\left[(308\ \text{K})^4 - (293\ \text{K})^4\right] = 125\ \text{W}$$

Note that the temperatures must be in kelvins. The energy loss in ten minutes is

$$T_{ER} = P_{\text{net}}\Delta t = (125\ \text{J/s})(600\ \text{s}) = \boxed{74.8\ \text{kJ}}$$

In the infrared, the person shines brighter than a hundred-watt light bulb.

P20.46 We find the power output of the Sun from Equation 20.19, Stefan's law:

$$P = \sigma A e T^4$$
$$= (5.669\,6\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)\left[4\pi(6.96\times 10^8\ \text{m})^2\right]$$
$$\times(0.986)(5\,800\ \text{K})^4$$
$$= \boxed{3.85\times 10^{26}\ \text{W}}$$

P20.47 From Stefan's law,

$$P = \sigma A e T^4$$
$$2.00\ \text{W} = (5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(0.250\times 10^{-6}\ \text{m}^2)(0.950)T^4$$
$$T = (1.49\times 10^{14}\ \text{K}^4)^{1/4} = \boxed{3.49\times 10^3\ \text{K}}$$

P20.48 We suppose the Earth below is an insulator. The square meter must radiate in the infrared as much energy as it absorbs, $P = \sigma A e T^4$. Assuming that $e = 1.00$ for blackbody blacktop:

$$1\,000\ \text{W} = (5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.00\ \text{m}^2)(1.00)T^4$$

$$T = (1.76\times 10^{10}\ \text{K}^4)^{1/4} = \boxed{364\ \text{K}}$$ (You *can* cook an egg on it.)

P20.49 (a) Because the bulb is evacuated, the filament loses energy by radiation but not by convection; we ignore energy loss by conduction. We convert the temperatures given in Celsius to Kelvin, with T_h = 2 100°C = 2 373 K and T_c = 2 000°C = 2 273 K. Then, from Stefan's law, the power ratio is

$$e\sigma A T_h^4 / e\sigma A T_c^4 = (2\ 373/2\ 273)^4 = \boxed{1.19}$$

(b) The radiating area is the lateral surface area of the cylindrical filament, $2\pi r\ell$. Now we want

$$e\sigma 2\pi r_h \ell T_h^4 = e\sigma 2\pi r_c \ell T_c^4$$

so $r_c / r_h = \boxed{1.19}$

***P20.50** We use Equation 20.16 for the rate of energy transfer by conduction:

$$P = kA\frac{(T_h - T_c)}{L} = (0.210\ \text{W/m}\cdot{}^\circ\text{C})(1.40\ \text{m}^2)\left(\frac{37.0^\circ\text{C} - 34.0^\circ\text{C}}{0.025\ 0\ \text{m}}\right)$$

$$= 35.3\ \text{W} = (35.3\ \text{J/s})\left(\frac{1\ \text{kcal}}{4\ 186\ \text{J}}\right)\left(\frac{3\ 600\ \text{s}}{1\ \text{h}}\right) = \boxed{30.3\ \text{kcal/h}}$$

Since this is much less than 240 kcal / h, blood flow is essential to cool the body.

***P20.51** When the temperature of the junction stabilizes, the energy transfer rate must be the same for each of the rods, or $P_{\text{Cu}} = P_{\text{Al}}$. The cross-sectional areas of the rods are equal, and if the temperature of the junction is 50.0°C, the temperature difference is $\Delta T = 50.0^\circ\text{C}$ for each rod. Thus,

$$P_{\text{Cu}} = k_{\text{Cu}} A\left(\frac{\Delta T}{L_{\text{Cu}}}\right) = k_{\text{Al}} A\left(\frac{\Delta T}{L_{\text{Al}}}\right) = P_{\text{Al}}$$

which gives

$$L_{\text{Al}} = \left(\frac{k_{\text{Al}}}{k_{\text{Cu}}}\right) L_{\text{Cu}} = \left(\frac{238\ \text{W/m}\cdot{}^\circ\text{C}}{397\ \text{W/m}\cdot{}^\circ\text{C}}\right)(15.0\ \text{cm}) = \boxed{9.00\ \text{cm}}$$

***P20.52** From $P = kA\frac{\Delta T}{L}$, we have

$$k = \frac{PL}{A\Delta T} = \frac{(10.0\ \text{W})(0.040\ 0\ \text{m})}{(1.20\ \text{m}^2)(15.0^\circ\text{C})} = \boxed{2.22\times 10^{-2}\ \text{W/m}\cdot{}^\circ\text{C}}$$

P20.53 (a) The *R*-value of the window is the sum of the *R*-values for the two 0.125-in window panes, which Table 20.4 lists as 0.890, plus the layer of air in between. Since the Table 20.4 lists the *R*-value for an air space of 3.50 in, the total *R*-value becomes

$$R = \left[0.890 + \left(\frac{0.250}{3.50}\right)1.01 + 0.890\right]\left(\frac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}}\right)$$
$$= \boxed{1.85\ \frac{\text{ft}^2\cdot{}^\circ\text{F}\cdot\text{h}}{\text{Btu}}}$$

(b) Since *A* and $(T_2 - T_1)$ are constants, heat flow is reduced by a factor of $\frac{1.85}{0.890} = \boxed{2.08}$.

P20.54 (a) $\boxed{\text{}}$ Intensity is defined as power per area perpendicular to the direction of energy flow. The direction of sunlight is along the line from the Sun to the object. The perpendicular area is the projected flat circular area enclosed by the *terminator*—the line that separates day from night on the object. The object radiates infrared light outward in all directions. The area perpendicular to this energy flow is its spherical surface area.

(b) The sphere of radius *R* absorbs sunlight over area πR^2. It radiates over area $4\pi R^2$. Then, in steady state,

$$P_{\text{in}} = P_{\text{out}}$$
$$e\left(1\,370\ \text{W/m}^2\right)\pi R^2 = e\sigma\left(4\pi R^2\right)T^4$$

The emissivity *e*, the radius *R*, and π all cancel. Therefore,

$$T = \left[\frac{1\,370\ \text{W/m}^2}{4\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{279\ \text{K}} = 6^\circ\text{C}$$

$\boxed{\text{It is chilly, well below temperatures we find comfortable.}}$

P20.55 Call the gold bar Object 1 and the silver bar Object 2. Each is a nonisolated system in steady state. When energy transfer by heat reaches a steady state, the flow rate through each will be the same, so that the junction can stay at constant temperature thereafter, with as much heat coming in through the gold as goes out through the silver.

$$P_1 = P_2 \qquad \text{or} \qquad \frac{k_1A_1\Delta T_1}{L_1} = \frac{k_2A_2\Delta T_2}{L_2}$$

In this case, $L_1 = L_2$ and $A_1 = A_2$, so $k_1\Delta T_1 = k_2\Delta T_2$.

Let T_3 be the temperature at the junction; then

$$k_1(80.0°\text{C} - T_3) = k_2(T_3 - 30.0°\text{C})$$

Rearranging, we find

$$T_3 = \frac{(80.0°\text{C})\,k_1 + (30.0°\text{C})k_2}{k_1 + k_2}$$

$$T_3 = \frac{(80.0°\text{C})(314\ \text{W/m}\cdot°\text{C}) + (30.0°\text{C})(427\ \text{W/m}\cdot°\text{C})}{314\ \text{W/m}\cdot°\text{C} + 427\ \text{W/m}\cdot°\text{C}}$$

$$= \boxed{51.2°\text{C}}$$

P20.56 (a) The heat leaving the box during the day is given by

$$P = kA\frac{(T_H - T_c)}{L} = \frac{Q}{\Delta t}$$

$$Q = \left(0.012\ 0\frac{\text{W}}{\text{m°C}}\right)\left(0.490\ \text{m}^2\right)\left(\frac{37.0°\text{C} - 23.0°\text{C}}{0.045\ 0\ \text{m}}\right) \times (12\ \text{h})\left(\frac{3\ 600\ \text{s}}{1\ \text{h}}\right)$$

$$= 7.90 \times 10^4\ \text{J}$$

The heat lost at night is

$$Q = \left(0.012\ 0\frac{\text{W}}{\text{m°C}}\right)\left(0.490\ \text{m}^2\right)\left(\frac{37.0°\text{C} - 16.0°\text{C}}{0.045\ 0\ \text{m}}\right) \times (12\ \text{h})\left(\frac{3\ 600\ \text{s}}{1\ \text{h}}\right)$$

$$= 1.19 \times 10^5\ \text{J}$$

The total heat is 1.19×10^5 J + 7.90×10^4 J = 1.98×10^5 J. It must be supplied by the solidifying wax: $Q = mL$

$$m = \frac{Q}{L} = \frac{1.98 \times 10^5\ \text{J}}{205 \times 10^3\ \text{J/kg}} = \boxed{0.964\ \text{kg or more}}$$

(b) $\boxed{\text{The test samples and the inner surface of the insulation can be prewarmed to 37.0°C as the box is assembled. Then nothing changes in temperature during the test period and the masses of the test samples and insulation make no difference.}}$

P20.57 (a) Suppose the pizza is 60 cm in diameter and $\ell = 2.0$ cm thick, sizzling at 100°C. It cannot transfer energy by conduction or convection. It radiates according to $P = \sigma A e T^4$. Here, *A* is its surface area, given by

$$A = 2\pi r^2 + 2\pi r\ell = 2\pi(0.30\text{ m})^2 + 2\pi(0.30\text{ m})(0.02\text{ m})$$
$$= 0.60\text{ m}^2$$

Suppose it is dark in the infrared, with emissivity about 0.8. Then

$$P = (5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4)(0.60\text{ m}^2)(0.80)(373\text{ K})^4$$
$$= 530\text{ W}\ \boxed{\sim 10^3\text{ W}}$$

(b) If the density of the pizza is half that of water, its mass is

$$m = \rho V = \rho\pi r^2\ell = (500\text{ kg/m}^3)\pi(0.30\text{ m})^2(0.02\text{ m}) = 2.8\text{ kg}$$

There's a lot of water in the cheese, but a lot of air in the crust, so we estimate a specific heat for the pizza between that of water and that of air. Suppose its specific heat is $c = 3\ 000\text{ J/kg}\cdot°\text{C}$. The drop in temperature of the pizza is described by

$$T_{ER} = mc(T_f - T_i)$$
$$P = \frac{dT_{ER}}{dt} = mc\frac{dT_f}{dt} - 0$$
$$\frac{dT_f}{dt} = \frac{P}{mc} = \frac{530\text{ J/s}}{(2.8\text{ kg})(3000\text{ J/kg}\cdot°\text{C})}$$
$$= 0.063\ °\text{C/s}\ \boxed{\sim 10^{-1}\text{ K/s}}$$

Additional Problems

***P20.58** (a) Along the direct path *IF* (ANS. FIG. P20.58), the work done on the gas is the negative of the area under the curve, or

$$W = -\Big[(1.00\text{ atm})(4.00\text{ L} - 2.00\text{ L}) + \frac{1}{2}(4.00\text{ atm} - 1.00\text{ atm})(4.00\text{ L} - 2.00\text{ L})\Big]$$
$$= (-5.00\text{ atm}\cdot\text{L})\left(\frac{1.013\times 10^5\text{ Pa}}{1\text{ atm}}\right)\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right) = -507\text{ J}$$

Thus,

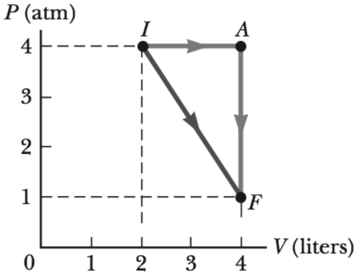
$$\Delta U = Q + W = 418\text{ J} - 507\text{ J} = \boxed{-88.5\text{ J}}$$

(b) Along path *IAF*, the work done on the gas is

$$W = -(4.00\text{ atm})(4.00\text{ L} - 2.00\text{ L})\left(\frac{1.013\times10^5\text{ Pa}}{1\text{ atm}}\right)\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right)$$
$$= -810\text{ J}$$

From the first law,

$$Q = \Delta U - W = -88.5\text{ J} - (-810\text{ J}) = \boxed{722\text{ J}}$$



ANS. FIG. P20.58

***P20.59** The constant pressure is

$$P = (1.50\text{ atm})\left(\frac{1.013\times10^5\text{ Pa}}{1\text{ atm}}\right) = 1.52\times10^5\text{ Pa}$$

and the work done on the gas is $W = -P(\Delta V)$.

(a) Here, $\Delta V = 4.00\text{ m}^3$ and

$$W = -P(\Delta V) = -(1.52\times10^5\text{ Pa})(4.00\text{ m}^3) = \boxed{-6.08\times10^5\text{ J}}$$

(b) In this case, $\Delta V = -3.00\text{ m}^3$, so

$$W = -P(\Delta V) = -(1.52\times10^5\text{ Pa})(-3.00\text{ m}^3) = \boxed{4.56\times10^5\text{ J}}$$

P20.60 The mass of nitrogen vaporized in a 4.00 h period is

$$m = \frac{Q}{L_f} = \frac{P\cdot(\Delta t)}{L_f} = \frac{(25.0\text{ J/s})(4.00\text{ h})(3\,600\text{ s/h})}{2.01\times10^5\text{ J/kg}} = \boxed{1.79\text{ kg}}$$

P20.61 (a) Before conduction has time to become important, the energy lost by the rod equals the energy gained by the helium. Therefore,

$$(mL_v)_{\text{He}} = (mc|\Delta T|)_{\text{Al}}$$

or $$(\rho V L_v)_{\text{He}} = (\rho V c|\Delta T|)_{\text{Al}},$$

so $$V_{\text{He}} = \frac{\left(\rho V c|\Delta T|\right)_{\text{Al}}}{\left(\rho L_v\right)_{\text{He}}}:$$

$$V_{\text{He}} = \frac{\left(2\ 700\ \text{kg/m}^3\right)\left(6.25\times10^{-5}\ \text{m}^3\right)}{\left(125\ \text{kg/m}^3\right)\left(2.09\times10^4\ \text{J/kg}\right)} \times\left[\left(900\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(295.8\ \text{K}\right)\left(\frac{1^\circ\text{C}}{1\ \text{K}}\right)\right]$$

$$V_{\text{He}} = \left(1.72\times10^{-2}\ \text{m}^3\right)\left(\frac{1\ \text{L}}{10^{-3}\ \text{m}^3}\right) = \boxed{17.2\ \text{liters}}$$

(b) The rate at which energy is supplied to the rod in order to maintain constant temperatures is given by

$$P = kA\left(\frac{dT}{dx}\right) = \left(3\ 100\ \text{W/m}\cdot\text{K}\right)\left(2.50\times10^{-4}\ \text{m}^2\right)\left(\frac{295.8\ \text{K}}{0.250\ \text{m}}\right)$$
$$= 917\ \text{W}$$

This power supplied to the helium will produce a “boil-off” rate of

$$\frac{P}{\rho L_v} = \frac{917\ \text{W}}{\left(125\ \text{kg/m}^3\right)\left(2.09\times10^4\ \text{J/kg}\right)} = 3.51\times10^{-4}\ \text{m}^3/\text{s}$$
$$= \boxed{0.351\ \text{L/s}}$$

P20.62 (a) Isolated system (momentum). The collision is a perfectly inelastic collision, where momentum is conserved, but kinetic energy is not (it is transformed into internal energy).

(b) Momentum is conserved; thus:

$$m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 = \left(m_1+m_2\right)\vec{\mathbf{v}} \rightarrow \vec{\mathbf{v}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1+m_2}$$

Substituting in numerically (positive to the right):

$$\vec{\mathbf{v}} = \frac{m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2}{m_1+m_2}$$
$$= \frac{\left(12.0\ \text{g}\right)\left(+300\ \text{m/s}\right)+\left(8.00\ \text{g}\right)\left(-400\ \text{m/s}\right)}{12.0\ \text{g}+8.00\ \text{g}}$$
$$= +20.0\ \text{m/s}$$

The final velocity is $\boxed{20.0\ \text{m/s to the right}}$.

(c) The initial kinetic energy is

$$K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_1v_1^2$$

$$K_i = \frac{1}{2}\left(12.0\times10^{-3}\text{ kg}\right)\left(300.0\text{ m/s}\right)^2$$

$$+\frac{1}{2}\left(8.00\times10^{-3}\text{ kg}\right)\left(-400.0\text{ m/s}\right)^2 = 1\,180\text{ J}$$

The final kinetic energy is:

$$K_f = \frac{1}{2}\left(m_1+m_2\right)v^2$$

$$K_f = \frac{1}{2}\left(12.0\times10^{-3}\text{ kg} + 8.00\times10^{-3}\text{ kg}\right)\left(20.0\text{ m/s}\right)^2 = 4.00\text{ J}$$

The amount of kinetic energy transformed into internal energy is

$$|\Delta K| = \left|K_f - K_i\right| = |4.00\text{ J} - 1180\text{ J}| = |-1176\text{ J}| = \boxed{1.18\times10^3\text{ J}}$$

(d) $\boxed{\text{No}}$. If this amount of heat is added to the mass of the bullets, the following amount will be needed to heat the bullets to their melting temperature:

$$\begin{aligned}Q &= mc\Delta T\\ &= \left(20.0\times10^{-3}\text{ kg}\right)\left(128\text{ J/kg}\cdot{}^\circ\text{C}\right)\left(327.3^\circ\text{C} - 30.0^\circ\text{C}\right)\\ &= 761\text{ J}\end{aligned}$$

At the beginning of the process, 1 176 joules are generated by the collision; therefore, the bullets will be heated to the melting point, with heat still available to start the melting process:

$$1\,176\text{ J} - 761\text{ J} = 415\text{ J}$$

Therefore, 415 J are available to melt the bullets. The amount of heat needed to melt all of the combined mass of the two bullets is:

$$Q = mL = \left(20.0\times10^{-3}\text{ kg}\right)\left(2.45\times10^4\text{ J/kg}\right) = 490\text{ J}$$

There are only 415 J available; so the lead does not entirely melt due to the collision.

(e) Because there is not enough energy available to melt all the mass of the bullets, the final temperature is the melting point of lead, $\boxed{327.3^\circ\text{C}}$.

(f) The total mass of the melted lead is:

$$Q = mL \to m = \frac{Q}{L} = \frac{415\ \text{J}}{\left(2.45 \times 10^4\ \text{J/kg}\right)} = 0.016\ 94\ \text{kg} = 16.9\ \text{g}$$

leaving behind 20.0 g – 16.9 g = 3.10 g of unmelted solid lead:

$$\boxed{\text{3.10 g of solid lead and 16.9 g of liquid lead}}$$

P20.63 $Q = mc\Delta T = (\rho V)c\Delta T$ so that when a constant temperature difference ΔT is maintained, the rate of adding energy to the liquid is $P = \frac{dQ}{dt} = \rho\left(\frac{dV}{dt}\right)c\Delta T = \rho Rc\Delta T$ and the specific heat of the liquid is

$$c = \frac{P}{\rho R\Delta T}$$

$$= \frac{200\ \text{W}}{900\ \text{kg/m}^3\left(2.00\ \text{L/min}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(3.50°\text{C}\right)}\left(\frac{1\ \text{L}}{10^{-3}\ \text{m}^3}\right)$$

$$= \boxed{1.90 \times 10^3\ \frac{\text{J}}{\text{kg}\cdot°\text{C}}}$$

P20.64 $Q = mc\Delta T = (\rho V)c\Delta T$ so that when a constant temperature difference ΔT is maintained, the rate of adding energy to the liquid is $\mathrm{P} = \frac{dQ}{dt} = \rho\left(\frac{dV}{dt}\right)c\Delta T = \rho Rc\Delta T$ and the specific heat of the liquid is $c = \boxed{\frac{\mathrm{P}}{\rho R\Delta T}}$.

P20.65 The disk is isolated, so angular momentum is conserved by the disk system. The initial moment of inertia of the disk is

$$\frac{1}{2}MR^2 = \frac{1}{2}\rho VR^2 = \frac{1}{2}\rho\left(\pi R^2 t\right)R^2$$

$$= \frac{1}{2}\left(8\ 920\ \text{kg/m}^3\right)\pi\left(28\ \text{m}\right)^4 1.2\ \text{m}$$

$$= 1.033 \times 10^{10}\ \text{kg}\cdot\text{m}^2$$

The rotation speeds up as the disk cools off, according to

$$I_i\omega_i = I_f\omega_f$$

$$\frac{1}{2}MR_i^2\omega_i = \frac{1}{2}MR_f^2\omega_f = \frac{1}{2}MR_i^2\left(1 - \alpha|\Delta T|\right)^2\omega_f$$

$$\begin{aligned}\omega_f &= \omega_i \frac{1}{\left(1-\alpha|\Delta T|\right)^2}\\ &= (25\ \text{rad/s})\frac{1}{\left[1-\left(17\times10^{-6}\ (^\circ\text{C})^{-1}\right)(830^\circ\text{C})\right]^2}\\ &= 25.720\ 7\ \text{rad/s}\end{aligned}$$

(a) The kinetic energy increases by

$$\begin{aligned}\frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2 = \frac{1}{2}(I_i\omega_i)\omega_f - \frac{1}{2}I_i\omega_i^2 &= \frac{1}{2}I_i\omega_i\left(\omega_f-\omega_i\right)\\ &= \frac{1}{2}\left[1.033\times10^{10}\ \text{kg}\cdot\text{m}^2(25\ \text{rad/s})\right]\\ &\qquad\times(0.720\,7\ \text{rad/s})\\ &= \boxed{9.31\times10^{10}\ \text{J}}\end{aligned}$$

(b) $$\begin{aligned}\Delta E_{\text{int}} &= mc\Delta T = 2.64\times10^7\ \text{kg}(387\ \text{J/kg}\cdot{}^\circ\text{C})(20^\circ\text{C}-850^\circ\text{C})\\ &= \boxed{-8.47\times10^{12}\ \text{J}}\end{aligned}$$

(c) Solve the appropriate reduction of Equation 8.2 for the energy radiated by the disk:

$$\Delta K + \Delta E_{\text{int}} = T_{\text{ER}}$$

$$\begin{aligned}T_{\text{ER}} &= \Delta K + \Delta E_{\text{int}} = 9.31\times10^{10}\ \text{J} - 8.47\times10^{12}\ \text{J}\\ &= \boxed{-8.38\times10^{12}\ \text{J}}\end{aligned}$$

P20.66 (a) First, energy must be removed from the liquid water to cool it to 0ºC. Next, energy must be removed from the water at 0ºC to freeze it, which corresponds to a liquid-to-solid phase transition. Finally, once all the water has frozen, additional energy must be removed from the ice to cool it from 0ºC to −8.00º C.

(b) The total energy that must be removed is

$$\begin{aligned}Q &= \left|Q_{\substack{\text{cool water}\\ \text{to }0^\circ\text{C}}}\right| + \left|Q_{\substack{\text{freeze}\\ \text{at }0^\circ\text{C}}}\right| + \left|Q_{\substack{\text{cool ice}\\ \text{to }-8.00^\circ\text{C}}}\right|\\ &= m_w c_w\left|0^\circ\text{C}-T_i\right| + m_w L_f + m_w c_{\text{ice}}\left|T_f - 0^\circ\text{C}\right|\end{aligned}$$

or

$$\begin{aligned} Q &= \left(75.0\times10^{-3}\text{ kg}\right)\left[\left(4\,186\text{ J/kg}\cdot{}^\circ\text{C}\right)\left|-20.0^\circ\text{C}\right| \right. \\ &\qquad \left. + 3.33\times10^{5}\text{ J/kg} + \left(2\,090\text{ J/kg}\cdot{}^\circ\text{C}\right)\left|-8.00^\circ\text{C}\right|\right] \\ &= 3.25\times10^{4}\text{ J} = \boxed{32.5\text{ kJ}} \end{aligned}$$

***P20.67** (a) The energy thus far gained by the copper equals the energy lost by the silver. Your down parka is an excellent insulator.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

or $$m_{\text{Cu}}c_{\text{Cu}}\left(T_f - T_i\right)_{\text{Cu}} = -m_{\text{Ag}}c_{\text{Ag}}\left(T_f - T_i\right)_{\text{Ag}}:$$

$$\begin{aligned} (9.00\text{ g})(387\text{ J/kg}\cdot{}^\circ\text{C})(16.0^\circ\text{C}) &= -(14.0\text{ g})(234\text{ J/kg}\cdot{}^\circ\text{C}) \\ &\qquad \times\left(T_f - 30.0^\circ\text{C}\right)_{\text{Ag}} \end{aligned}$$

$$\left(T_f - 30.0^\circ\text{C}\right)_{\text{Ag}} = -17.0^\circ\text{C}$$

so $T_{f,\,\text{Ag}} = \boxed{13.0^\circ\text{C}}$.

(b) Differentiating the energy gain-and-loss equation gives

$$m_{\text{Ag}}c_{\text{Ag}}\left(\frac{dT}{dt}\right)_{\text{Ag}} = -m_{\text{Cu}}c_{\text{Cu}}\left(\frac{dT}{dt}\right)_{\text{Cu}}$$

$$\begin{aligned} \left(\frac{dT}{dt}\right)_{\text{Ag}} &= -\frac{m_{\text{Cu}}c_{\text{Cu}}}{m_{\text{Ag}}c_{\text{Ag}}}\left(\frac{dT}{dt}\right)_{\text{Cu}} \\ &= -\frac{(9.00\text{ g})(387\text{ J/kg}\cdot{}^\circ\text{C})}{(14.0\text{ g})(234\text{ J/kg}\cdot{}^\circ\text{C})}\left(+0.500\,^\circ\text{C/s}\right) \end{aligned}$$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = \boxed{-0.532\,^\circ\text{C/s}}$$

(negative sign ⇒ decreasing temperature)

***P20.68** (a) The chemical energy input becomes partly work output and partly internal energy. The energy flow each second is described by

$$\begin{aligned} 400\text{ kcal/h} = 60\text{ J/s} + \frac{mL}{\Delta t} &= (400\text{ kcal/h})\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) \\ &= 465\text{ W} \end{aligned}$$

$$\frac{m}{\Delta t}L = 465\text{ W} - 60\text{ W} = 405\text{ J/s}$$

$$\frac{m}{\Delta t} = \left(\frac{405\text{ J/s}}{2.26\times10^{6}\text{ J/kg}}\right)\left(\frac{3\,600\text{ s}}{1\text{ h}}\right) = \boxed{0.645\text{ kg/h}}$$

(b) The rate of fat burning is ideally $\dfrac{400\ \text{kcal/h}}{9\ \text{kcal/g}} = 0.044\ 4\ \text{kg/h}$. The 0.044 4 kg/h of water produced by metabolism is this fraction of the water needed for cooling: $\dfrac{0.044\ 4\ \text{kg/h}}{0.645\ \text{kg/h}} = \boxed{0.068\ 9} = 6.89\%$.

Moral: drink plenty of water while you exercise.

***P20.69** (a) The rate of energy conversion is given by

$$Fv = (50.0\ \text{N})(40.0\ \text{m/s}) = \boxed{2\ 000\ \text{W}}$$

(b) Energy received by each object is (1 000 W)(10 s) = 10^4 J = 2 389 cal. The specific heat of iron is 0.107 cal / g · °C, so the heat capacity of each object is $5.00\times10^3 \times 0.107 = 535.0\ \text{cal/°C}$.

$$\Delta T = \frac{2\ 389\ \text{cal}}{535.0\ \text{cal/°C}} = \boxed{4.47\text{°C}}$$

***P20.70** We find the quantity of water vapor in one exhaled breath.

$PV = nRT$:

$$n = \frac{PV}{RT} = \frac{(3.20\times10^3\ \text{N/m}^2)(0.600\times10^{-3}\ \text{m}^3)}{(8.314\ \text{J/mol}\cdot\text{K})(273\ \text{K} + 37\text{°C})}$$
$$= 7.45\times10^{-4}\ \text{mol}$$

The molar mass of water (H_2O) is M = [2(1.00)+16.0] g / mol = 0.018 0 kg / mol. The mass of water vapor exhaled in one breath is

$$m_{\text{sample}} = nM = (7.45\times10^{-4}\ \text{mol})(0.0180\ \text{kg/mol})$$
$$= 1.34\times10^{-5}\ \text{kg}$$

The energy absorbed from your body as the water evaporates can be estimated as

$$Q = mL = 1.34\times10^{-5}\ \text{kg}(2.26\times10^6\ \text{J/kg}) = 30.3\ \text{J}$$

Your rate of energy loss is

$$P = \frac{Q}{\Delta t} = \left(\frac{30.3\ \text{J}}{\text{breath}}\right)\left(\frac{22.0\ \text{breath}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{11.1\ \text{W}}$$

Note that unlike a human, a dog does not perspire. Instead, the dog pants, and maximizes energy loss through the pathway considered here.

***P20.71** The energy conservation equation is $Q_{\text{cold}} = -Q_{\text{hot}}$, or

$$m_{\text{ice}}L_f + \left[\left(m_{\text{ice}} + m_w\right)c_w + m_{\text{cup}}c_{\text{Cu}}\right](12.0°\text{C} - 0°\text{C})$$
$$= -m_{\text{Pb}}c_{\text{Pb}}(12.0°\text{C} - 98.0°\text{C})$$

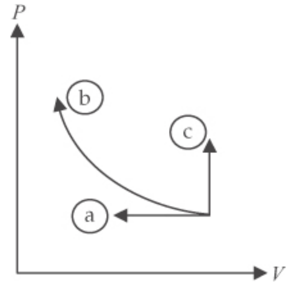
This gives

$$m_{\text{Pb}} = \left[\frac{1}{c_{\text{Pb}}(86.0°\text{C})}\right]\left\{m_{\text{ice}}L_f + \left[\left(m_{\text{ice}} + m_w\right)c_w + m_{\text{cup}}c_{\text{Cu}}\right](12.0°\text{C})\right\}$$
$$= \left[\frac{1}{(128\ \text{J/kg}\cdot°\text{C})(86.0°\text{C})}\right]\left\{(0.040\ 0\ \text{kg})(3.33\times10^5\ \text{J/kg})\right.$$
$$+\left[(0.240\ \text{kg})(4\ 186\ \text{J/kg}\cdot°\text{C})\right.$$
$$\left.\left.+(0.100\ \text{kg})(387\ \text{J/kg}\cdot°\text{C})\right](12.0°\text{C})\right\}$$
$$= \boxed{2.35\ \text{kg}}$$

P20.72 (a) Work done on the gas is the negative of the area under the *PV* curve:

$$W = -P_i\left(\frac{V_i}{2} - V_i\right) = \boxed{+\frac{P_iV_i}{2}}$$

Put the cylinder into a refrigerator at absolute temperature $T_i/2$. Let the piston move freely as the gas cools.



ANS. FIG. P20.72

(b) In this case the area under the curve is $W = -\int P\,dV$. Since the process is isothermal,

$$PV = P_iV_i = 4P_i\left(\frac{V_i}{4}\right) = nRT_i$$

and

$$W = -\int_{V_i}^{V_i/4}\left(\frac{dV}{V}\right)(P_iV_i) = -P_iV_i\ln\left(\frac{V_i/4}{V_i}\right) = P_iV_i\ln 4$$
$$= \boxed{+1.39P_iV_i}$$

With the gas in a constant-temperature bath at T_i, slowly push the piston in.

(c) The area under the curve is 0 and $\boxed{W = 0}$.

$\boxed{\text{Lock the piston in place and hold the cylinder over a hotplate at } 3T_i.}$

The student may be confused that the integral in part (c) is not explicitly covered in calculus class. Mathematicians ordinarily study integrals of functions, but the pressure is not a single-valued function of volume in a isovolumetric process. Our physics idea of an integral is more general. It still corresponds to the idea of area under the graph line.

P20.73 From Equation 8.2, for the isolated system of the meteorite and the Earth and for the time interval from when the meteorite is very far from Earth until just after it hits the Earth's surface,

$$\Delta K + \Delta U_g + \Delta E_{\text{int}} = 0 \quad \to \quad \Delta E_{\text{int}} = -\Delta K - \Delta U_g$$

The problem statement says that the internal energy increase of the system is shared equally by the meteorite and the Earth, so the change in internal energy for the meteorite *alone* is

$$\Delta E_{\text{int,meteorite}} = \frac{1}{2}\Delta E_{\text{int}} = -\frac{1}{2}\Delta K - \frac{1}{2}\Delta U_g$$

Substitute for the energies:

$$\begin{aligned}\Delta E_{\text{int,meteorite}} &= -\frac{1}{2}\left(0 - \frac{1}{2}mv_i^2\right) - \frac{1}{2}\left(-\frac{GM_E m}{R_E} - 0\right) \\ &= \frac{1}{4}mv_i^2 + \frac{GM_E m}{2R_E}\end{aligned}$$

Given the large amount of energy available for a meteorite falling to Earth, we expect the meteorite to both melt and vaporize as its temperature rises. Therefore, the internal energy change for the meteorite can be expressed as

$$\begin{aligned}\Delta E_{\text{int, meteorite}} &= mc\Delta T|_{\text{solid}} + L_f m + mc\Delta T|_{\text{liquid}} + L_v m + mc\Delta T|_{\text{gas}} \\ &= m\left(c\Delta T|_{\text{solid}} + L_f + c\Delta T|_{\text{liquid}} + L_v + c\Delta T|_{\text{gas}}\right)\end{aligned}$$

Setting the two expressions for the internal energy change of the meteorite equal gives us

$$\Delta T|_{\text{gas}} = \frac{\frac{1}{4}v_i^2 + \frac{GM_E}{2R_E} - c\Delta T|_{\text{solid}} - L_f - c\Delta T|_{\text{liquid}} - L_v}{c_{\text{gas}}}$$

Substitute numerical values:

$$\begin{aligned}\Delta T\big|_{\text{gas}} = {} & \left(\frac{1}{1170\ \text{J/kg}\cdot{}^\circ\text{C}}\right) \\ & \times\left[\frac{1}{4}\left(1.40\times10^{4}\ \text{m/s}\right)^{2}\right. \\ & + \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{2\left(6.37\times10^{6}\ \text{m}\right)} \\ & - \left(900\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(660^\circ\text{C} + 15.0^\circ\text{C}\right) \\ & - 3.97\times10^{5}\ \text{J/kg} \\ & - \left(1\,170\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(2\,450^\circ\text{C} - 660^\circ\text{C}\right) \\ & - 1.14\times10^{7}\ \text{J/kg} \\ = {} & 56\,247^\circ\text{C}\end{aligned}$$

This is the change in temperature from the boiling point of aluminum, so to find the final temperature, add 2 450°C and express to three significant figures:

$$T_f = 56\,247^\circ\text{C} + 2\,450^\circ\text{C} = \boxed{5.87\times10^{4}\,^\circ\text{C}}$$

P20.74 The time interval to boil the water is related to the solar power P absorbed by the water and the energy transfer T_{ER} required:

$$\Delta t = \frac{T_{\text{ER}}}{P}$$

The energy transfer required is

$$T_{\text{ER}} = mc\Delta T$$

The solar power is

$$P = fIA = fI\left(\pi\frac{d^2}{4}\right) = \frac{1}{4}\pi d^2 fI$$

Combining all three equations,

$$\begin{aligned}\Delta t &= \frac{mc\Delta T}{\left(\frac{1}{4}\pi d^2 fI\right)} = \frac{4mc\Delta T}{\pi d^2 fI} \\ &= \frac{4\left(1.50\ \text{kg}\right)\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(80.0^\circ\text{C}\right)}{\pi\left(0.600\ \text{m}\right)^2\left(0.400\right)\left(600\ \text{W/m}^2\right)} \\ &= 7.40\times10^{3}\ \text{s} = 2.06\ \text{h}\end{aligned}$$

If we include setup time and coffee brewing time, this time interval approaches 2.5 hours. In the morning, the solar intensity is not the maximum amount, which occurs later in the day. Therefore, the reduced intensity in the morning will increase the time interval further. Furthermore, we have not included the energy transfer necessary to raise the temperature of the container in which the water resides. These considerations will push the required time interval even higher, so that most of the morning is used in making coffee and there is no time left for a morning hike.

P20.75 (a) The power radiated by the quiet Sun is given by Stefan's law:

$$\begin{aligned} P &= \sigma A e T^4 \\ &= \left(5.67\times10^{-8}\ \text{W/m}^2\text{K}^4\right)\left[5.1\times10^{14}\ \text{m}^2\ \right](0.965)(5\,800\ \text{K})^4 \\ &= \boxed{3.16\times10^{22}\ \text{W}} \end{aligned}$$

(b) The power output of the patch of sunspot is

$$\begin{aligned} P &= \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right) \\ &\quad\times\Big\{\left[0.100\left(5.10\times10^{14}\ \text{m}^2\right)\right](0.965)(4\,800\ \text{K})^4 \\ &\quad+\left[0.900\left(5.10\times10^{14}\ \text{m}^2\right)\right](0.965)(5\,890\ \text{K})^4\Big\} \\ &= \boxed{3.17\times10^{22}\ \text{W}} \end{aligned}$$

(c) $\boxed{\text{This is larger than } 3.158\times10^{22}\ \text{W by } \dfrac{1.29\times10^{20}\ \text{W}}{3.16\times10^{22}\ \text{W}} = 0.408\%}$

(d) $T_{\text{avg}} = 0.100(4\,800\ \text{K}) + 0.900(5\,890\ \text{K}) = \boxed{5.78\times10^{3}\ \text{K}}$

P20.76 (a) The block starts with $K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(1.60\ \text{kg})(2.50\ \text{m/s})^2 = 5.00\ \text{J}$.

Write the appropriate reduction of Equation 8.2 for the isolated copper-ice system:

$$\Delta K + \Delta E_{\text{int}} = 0 \quad\rightarrow$$

$$\left(0 - \frac{1}{2}m_{\text{Cu}}v^2\right) + L_f\Delta m = 0 \quad\rightarrow$$

$$\Delta m = \frac{m_{\text{Cu}}v^2}{2L_f}$$

Substitute numerical values:

$$\Delta m = \frac{(1.60\ \text{kg})(2.50\ \text{m/s})^2}{2\left(3.33\times10^5\ \text{J/kg}\right)} = 1.50\times10^{-5}\ \text{kg} = \boxed{15.0\ \text{mg}}$$

(b) For the block as a system: *Q* = 0 (no energy transfers by heat since there is no temperature difference), $\Delta E_{\text{int}} = 0$ (no temperature or change of state).

For the block-ice system, $\Delta E_{\text{mech}} = -\Delta K = \boxed{-5.00\text{ J}}$.

(c) For the ice as a system: *Q* = 0 (no energy transfers by heat since there is no temperature difference), $\Delta E_{\text{int}} = \Delta m L_f = \boxed{5.00\text{ J}}$ (change of state—some ice melts).

(d) This is basically the same system as treated in part (a), treated in the same manner:

$$K_i = 5.00\text{ J} \quad\text{and}\quad m_{\text{ice}} = \boxed{15.0\text{ mg}}$$

(e) For the block of ice as the system: *Q* = 0 (no energy transfers by heat since there is no temperature difference), $\Delta E_{\text{int}} = \Delta m L_f = \boxed{5.00\text{ J}}$ (change of state—some ice melts).

For the block-ice system, $\Delta E_{\text{mech}} = -\Delta K = \boxed{-5.00\text{ J}}$.

(f) For the metal sheet as a system, *Q* = 0 (no temperature difference), $\boxed{\Delta E_{\text{int}} = 0}$ (no change in state or temperature).

(g) Write the appropriate reduction of Equation 8.2 for the isolated copper-copper system:

$$\Delta K + \Delta E_{\text{int}} = 0 \quad\rightarrow\quad \Delta E_{\text{int}} = -\Delta K$$

Because of the symmetry of the system, each copper slab possesses half of the internal energy change of the system:

$$\Delta E_{\text{int,copper}} = \frac{1}{2}\Delta E_{\text{int}} = -\frac{1}{2}\Delta K = -\frac{1}{2}\left(0 - \frac{1}{2}mv^2\right) = \frac{1}{4}mv^2$$

The internal energy change of the copper slab is related to its temperature change:

$$\Delta E_{\text{int,copper}} = mc\Delta T = \frac{1}{4}mv^2 \quad\rightarrow\quad \Delta T = \frac{v^2}{4c}$$

Substitute numerical values:

$$\Delta T = \frac{(2.50\text{ m/s})^2}{4(387\text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{4.04\times10^{-3}\ {}^\circ\text{C}}$$

(h) For the sliding slab, $\boxed{Q = 0}$ (no temperature difference),

$\boxed{\Delta E_{\text{int}} = 2.50\text{ J}}$ (friction transfers kinetic energy into internal energy).

For the two-slab system, $\boxed{\Delta E_{\text{mech}} = -5.00\text{ J}}$ ($\Delta K = -5.00$ J).

(i) For the stationary slab, $\boxed{Q = 0}$ (no temperature difference), $\boxed{\Delta E_{\text{int}} = 2.50 \text{ J}}$ (friction transfers kinetic energy into internal energy).

For each object in each situation, the general continuity equation for energy, in the form $\Delta K + \Delta E_{\text{int}} = Q$, correctly describes the relationship between energy transfers and changes in the object's energy content.

P20.77 From $Q = L_v \Delta m$, the rate of boiling is described by

$$Power = \frac{Q}{\Delta t} = \frac{L_v \Delta m}{\Delta t}$$

so that the mass flow rate of steam from the kettle is

$$\frac{\Delta m}{\Delta t} = \frac{Power}{L_v}$$

The symbols Δm for mass vaporized and *m* for mass leaving the kettle have the same meaning, but recall that *M* represents the molar mass. Even though it is on the point of liquefaction, we model the water vapor as an ideal gas. The volume flow rate $V / \Delta t$ of the fluid is the cross-sectional area of the spout multiplied by the speed of flow, forming the product *Av*.

$$P_0 V = nRT = \left(\frac{m}{M}\right) RT$$

$$\frac{P_0 V}{\Delta t} = \frac{m}{\Delta t}\left(\frac{RT}{M}\right)$$

$$P_0 A v = \frac{Power}{L_v}\left(\frac{RT}{M}\right)$$

$$v = \frac{(Power) RT}{M L_v P_0 A}$$

Suppressing units,

$$v = \frac{(1\ 000)(8.314)(373)}{(0.018\ 0)(2.26 \times 10^6)(1.013 \times 10^5)(2.00 \times 10^{-4})} = \boxed{3.76 \text{ m/s}}$$

P20.78 $A = A_{\text{end walls}} + A_{\text{ends of attic}} + A_{\text{side walls}} + A_{\text{roof}}$

$$A = 2(8.00 \text{ m} \times 5.00 \text{ m}) + 2\left[2 \times \frac{1}{2} \times 4.00 \text{ m} \times (4.00 \text{ m}) \tan 37.0°\right]$$

$$+ 2(10.0 \text{ m} \times 5.00 \text{ m}) + 2(10.0 \text{ m})\left(\frac{4.00 \text{ m}}{\cos 37.0°}\right)$$

$$A = 304\ \text{m}^2$$

$$P = \frac{kA\Delta T}{L} = \frac{\left(4.80\times10^{-4}\ \text{kW/m}\cdot{}^\circ\text{C}\right)\left(304\ \text{m}^2\right)(25.0^\circ\text{C})}{0.210\ \text{m}} = 17.4\ \text{kW}$$
$$= 4.15\ \text{kcal/s}$$

Thus, the energy transferred through the walls per day by heat is

$$\left(4.15\ \text{kcal/s}\right)\left(86\,400\ \text{s}\right) = 3.59\times10^5\ \text{kcal/day}$$

The gas needed to replace this transfer is $\dfrac{3.59\times10^5\ \text{kcal/day}}{9\,300\ \text{kcal/m}^3}$

$$= \boxed{38.6\ \text{m}^3/\text{day}}$$

P20.79 Energy goes in at a constant rate *P*. For the period from 50.0 min to 60.0 min, after the ice has melted,

$$P\Delta t = Q = mc\Delta T$$

$$P(10.0\ \text{min}) = \left(10\ \text{kg} + m_i\right)\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(2.00^\circ\text{C} - 0^\circ\text{C}\right)$$

$$P(10.0\ \text{min}) = 83.7\ \text{kJ} + \left(8.37\ \text{kJ/kg}\right)m_i \qquad \textbf{[1]}$$

For the period from 0 to 50.0 min, as the ice is melting,

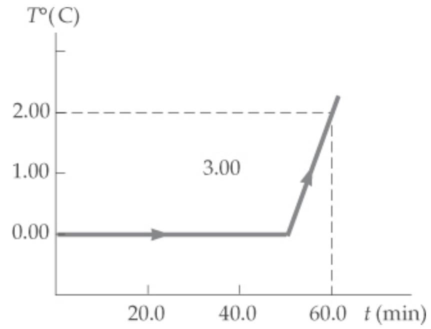
$$P\Delta t = Q = m_iL_f$$

$$P(50.0\ \text{min}) = m_i\left(3.33\times10^5\ \text{J/kg}\right)$$

Substitute $P = \dfrac{m_i\left(3.33\times10^5\ \text{J/kg}\right)}{50.0\ \text{min}}$ into equation [1] to find

$$\frac{m_i\left(3.33\times10^5\ \text{J/kg}\right)}{5.00} = 83.7\ \text{kJ} + \left(8.37\ \text{kJ/kg}\right)m_i$$

$$m_i = \frac{83.7\ \text{kJ}}{(66.6 - 8.37)\ \text{kJ/kg}} = \boxed{1.44\ \text{kg}}$$



ANS. FIG. P20.79

P20.80 (a) From conservation of energy, where the subscript *w* is for water and the subscript *c* is for the calorimeter,

$$Q_{\text{cold}} = -Q_{\text{hot}} \quad \text{or} \quad Q_{\text{Al}} = -\left(Q_w + Q_c\right)$$

$$m_{\text{Al}}c_{\text{Al}}\left(T_f - T_i\right)_{\text{Al}} = -\left(m_w c_w + m_c c_c\right)\left(T_f - T_i\right)_w$$

$$\left(0.200\text{ kg}\right)c_{\text{Al}}\left(+39.3°\text{C}\right)$$

$$= -\left[\left(0.400\text{ kg}\right)\left(4\,186\text{ J/kg}\cdot°\text{C}\right) + \left(0.040\,0\text{ kg}\right)\left(630\text{ J/kg}\cdot°\text{C}\right)\right]\left(-3.70°\text{C}\right)$$

$$c_{\text{Al}} = \frac{6.29\times10^3\text{ J}}{7.86\text{ kg}\cdot°\text{C}} = \boxed{800\text{ J/kg}\cdot°\text{C}}$$

(b) $$\frac{900-800}{900} = 11\%$$

$\boxed{\text{This differs from the tabulated value by 11\%, so the values agree within 15\%.}}$

Challenge Problems

P20.81 (a) The speed of rise of the piston is the same as the rate at which the height *h* of the steam above the water is increasing due to the boiling process. The volume of the gas is the area *A* of the cylinder times the height *h*:

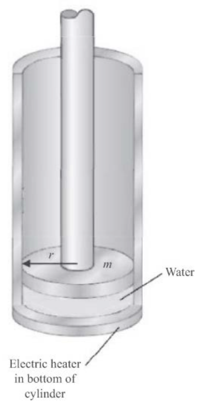
$$v = \frac{dh}{dt} = \frac{d}{dt}\left(\frac{V}{A}\right)$$

The volume of steam can be replaced using the ideal gas law, in which the pressure P and temperature T are constant:

$$v = \frac{1}{A}\frac{d}{dt}\left(\frac{nRT}{P}\right) = \frac{RT}{PA}\frac{dn}{dt}$$

The combination PA is the force applied by the gas on the piston. Assuming that the speed of the piston is constant, the piston is in equilibrium so this force is equal to the product of atmospheric pressure P_0 and the area of the piston plus the weight $m_p g$ of the piston.

$$F = P_0 A + m_p g$$



ANS. FIG. P20.81

The number of moles *n* is the ratio of the mass m_g of the gas to the molecular mass M_w:

$$n = \left(\frac{m_g}{M_w}\right)$$

which may both be substituted into our velocity equation:

$$v = \left(\frac{RT}{m_p g + P_0 A}\right)\frac{d}{dt}\left(\frac{m_g}{M_w}\right)$$

$$= \left[\frac{RT}{\left(m_p g + P_0 A\right)M_w}\right]\frac{dm_g}{dt}$$

The change in the mass of the steam is related to the latent heat of vaporization by Equation 20.7:

$$Q = \pm(\Delta m)L \qquad \rightarrow \frac{dm_g}{dt} = \frac{d}{dt}\left(\frac{Q}{L_v}\right)$$

$$v = \frac{RT}{\left(m_p g + P_0 A\right)M_w}\frac{d}{dt}\left(\frac{Q}{L_v}\right) = \frac{RT}{\left(m_p g + P_0 A\right)M_w L_v}\frac{dQ}{dt}$$

Finally, the rate at which energy is entering the cylinder is the power, (*Power*): (Notice that here we are careful to distinguish power from pressure *P* which normally would use the same symbols.)

$$v = \frac{RT(Power)}{\left(m_p g + P_0 A\right)M_w L_v}$$

Now we substitute numerical values, suppressing units:

$$v = \frac{(8.314)(373)(100)}{\left[(3.00)(9.80) + \left(1.013 \times 10^5\right)(\pi)(0.0750)^2\right](0.0180)\left(2.26 \times 10^6\right)}$$

$$= 4.19 \times 10^{-3}\ \text{m/s} = \boxed{4.19\ \text{mm/s}}$$

(b) Begin the same way as part (a):

$$v = \frac{dh}{dt} = \frac{d}{dt}\left(\frac{V}{A}\right) = \frac{1}{A}\frac{d}{dt}\left(\frac{nRT}{P}\right)$$

In this situation, however, the number of moles *n* is fixed and the temperature *T* changes:

$$v = \left(\frac{nR}{PA}\right)\frac{dT}{dt} = \frac{nR}{\left(m_p g + P_0 A\right)}$$

The temperature change is related to the energy input by means of Equation 20.4:

$$Q = mc\Delta T \to \frac{dT}{dt} = \frac{d}{dt}\left(\frac{Q}{m_g c}\right)$$

which may be substituted into our velocity equation:

$$v = \frac{nR}{\left(m_p g + P_0 A\right)}\frac{d}{dt}\left(\frac{Q}{m_g c}\right) = \frac{m_g R}{\left(m_p g + P_0 A\right)m_g c M_w}\frac{dQ}{dt}$$

$$v = \frac{R\left(Power\right)}{\left(m_p g + P_0 A\right)cM_w}$$

Substitute numerical values, suppressing units,

$$v = \frac{(8.314)(100)}{\left[(3.00)(9.80) + \left(1.013\times10^5\right)(\pi)(0.0750)^2\right](2\ 010)(0.018\ 0)}$$

$$= 0.012\ 6\ \text{m/s} = \boxed{12.6\ \text{mm/s}}$$

P20.82 (a) If the energy transfer *P* through one spherical surface within the shell were different from the energy transfer through another sphere, the temperature would be changing at a radius between the layers, so the steady state would not yet be established.

> The equation $dT/dr = P/4\pi kr^2$ represents the law of thermal conduction, incorporating the definition of thermal conductivity, applied to a spherical surface within the shell. The rate of energy transfer P must be the same for all radii so that each bit of material stays at a temperature that is constant in time.

(b) we separate the variables *T* and *r* in the thermal conduction equation and integrate the equation between points on the interior and exterior surfaces.

$$\int_5^{40} dT = \frac{P}{4\pi k}\int_{0.03}^{0.07}\frac{dr}{r^2}$$

where *T* is in degrees Celsius, *P* is in watts, and *r* is in meters.

(c) The integral yields

$$T\Big|_5^{40} = \frac{P}{4\pi k}\left(\frac{r^{-1}}{-1}\right)\Bigg|_{0.03}^{0.07}$$

$$40 - 5 = \frac{P}{4\pi(0.8)}\left(-\frac{1}{0.07} + \frac{1}{0.03}\right)$$

$$P = \boxed{18.5\ \text{W}}$$

(d) With *P* now known, we separate the variables again and integrate between a point on the interior surface and any point within the shell.

$$\int_5^T dT = \frac{P}{4\pi k}\int_{0.03}^{r}\frac{dr}{r^2}$$

(e) Integrating, we find

$$T\Big|_5^{T} = \frac{P}{4\pi k}\left(\frac{r^{-1}}{-1}\right)\Bigg|_{0.03}^{r} \quad \rightarrow \quad T - 5 = \frac{18.5}{4\pi(0.8)}\left(-\frac{1}{r} + \frac{1}{0.0300}\right)$$

$$\boxed{\begin{array}{l} T = 5 + 1.84\left(\dfrac{1}{0.030\,0} - \dfrac{1}{r}\right) \\ \text{Where } T \text{ is in degrees Celsius and } r \text{ is in meters} \end{array}}$$

(f) $T = 5 + 1.84\left(\frac{1}{0.0300} - \frac{1}{r}\right) = 5 + 1.84\left(\frac{1}{0.0300} - \frac{1}{0.0500}\right) = \boxed{29.5^\circ\text{C}}$

P20.83

$$\frac{L\rho A dx}{dt} = kA\left(\frac{\Delta T}{x}\right)$$

$$L\rho\int_{4.00}^{8.00} x dx = k\Delta T\int_0^{\Delta t} dt$$

$$L\rho\frac{x^2}{2}\Bigg|_{4.00}^{8.00} = k\Delta T\Delta t$$

$$\left(3.33\times10^5\ \text{J/kg}\right)\left(917\ \text{kg/m}^3\right)\left(\frac{\left(0.080\,0\ \text{m}\right)^2 - \left(0.040\,0\ \text{m}\right)^2}{2}\right) = \left(2.00\ \text{W/m}\cdot{}^\circ\text{C}\right)\left(10.0^\circ\text{C}\right)\Delta t$$

$$\Delta t = 3.66\times10^4\ \text{s} = \boxed{10.2\ \text{h}}$$

P20.84 (a) See ANS. FIG. P20.84. For a cylindrical shell of radius *r*, height *L*, and thickness *dr*, the equation for thermal conduction,

$$\frac{dQ}{dt} = -kA\frac{dT}{dx} \quad \text{becomes} \quad \frac{dQ}{dt} = -k\left(2\pi rL\right)\frac{dT}{dr}$$

Under equilibrium conditions, $\frac{dQ}{dt}$ is constant; therefore,

$$dT = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\left(\frac{dr}{r}\right) \quad \text{and} \quad \int_{T_a}^{T_b} dT = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\int_a^b \frac{dr}{r}$$

$$T_b - T_a = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\ln\left(\frac{b}{a}\right)$$

But $T_a > T_b$, so $\boxed{\frac{dQ}{dt} = 2\pi kL\left[\frac{\left(T_a - T_b\right)}{\ln\left(b/a\right)}\right]}$

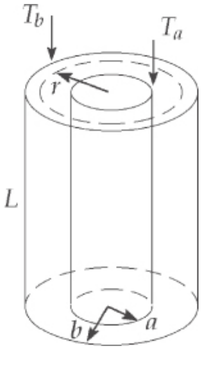
(b) From part (a), the rate of energy flow through the wall is

$$\frac{dQ}{dt} = \frac{2\pi kL\left(T_a - T_b\right)}{\ln\left(b/a\right)}$$

$$\frac{dQ}{dt} = \frac{2\pi\left(4.00\times10^{-5}\ \text{cal/s}\cdot\text{cm}\cdot{}^\circ\text{C}\right)\left(3\,500\ \text{cm}\right)\left(60.0^\circ\text{C}\right)}{\ln\left(256\ \text{cm}/250\ \text{cm}\right)}$$

$$\frac{dQ}{dt} = 2.23\times10^{3}\ \text{cal/s} = \boxed{9.32\ \text{kW}}$$

This is the rate of energy loss from the plane by heat, and consequently is the rate at which energy must be supplied in order to maintain a constant temperature.



ANS. FIG. P20.84(b)

ANSWERS TO EVEN-NUMBERED PROBLEMS

P20.2	0.105° C
P20.4	16.9° C
P20.6	0.234 kJ / kg · °C
P20.8	87.0°C
P20.10	$\dfrac{(m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c + m_h c_w T_h}{m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w}$
P20.12	(a) 16.1°C; (b) 16.1°C; (c) It makes no difference whether the drill bit is dull or sharp, or how far into the block it cuts. The answers to (a) and (b) are the same because all of the work done by the bit on the block constitutes energy being transferred into the internal energy of the steel.
P20.14	(a) T = 25.8°C; (b) The symbolic result from part (a) shows no dependence on mass. Both the change in gravitational potential energy and the change in internal energy of the system depend on the mass, so the mass cancels.
P20.16	12.9 g steam
P20.18	1.22×10^5 J
P20.20	0.294 g
P20.22	0.415 kg
P20.24	(a) 7; (b) As the car stops, it transforms part of its kinetic energy into internal energy due to air resistance. As soon as the brakes rise above the air temperature, they transfer energy by heat into the air and transfer it very fast if they attain a high temperature.
P20.26	$-nR(T_2 - T_1)$
P20.28	(a) –12.0 MJ; (b) + 12.0 MJ
P20.30	(a) 12.0 kJ; (b) –12.0 kJ
P20.32	From the first law of thermodynamics, $\Delta E_{\text{int}} = Q + W$ = 10.0 J + 12.0 J = +22.0 J. The change in internal energy is a positive number, which would be consistent with an *increase* in temperature of the gas, but the problem statement indicates a *decrease* in temperature.
P20.34	4.29×10^4 J
P20.36	(a) –3.10 kJ; (b) 37.6 kJ
P20.38	(a) 0.007 65 m^3; (b) 305 K

P20.40	(a) 1 300 J; (b) 100 J; (c) –900 J; (d) –1 400 J
P20.42	(a) $-4\,P_iV_i$; (b) $4\,P_iV_i$
P20.44	667 W
P20.46	3.85×10^{26} W
P20.48	364 K
P20.50	30.3 kcal / h
P20.52	2.22×10^{-2} W / m · °C
P20.54	(a) Intensity is defined as power per area perpendicular to the direction of energy flow. The direction of sunlight is along the line from the Sun to an object. The perpendicular area is the projected flat circular area enclosed by the *terminator*. The object radiates infrared light outward in all directions. The area perpendicular to this energy flow is its spherical surface area; (b) 279 K, it is chilly, well below room temperatures we find comfortable.
P20.56	(a) 0.964 kg or more; (b) The test samples and the inner surface of the insulation can be pre-warmed to 37.0° as the box is assembled. Then, nothing changes in temperature during the test period and the masses of the test samples and insulation make no difference.
P20.58	(a) –88.5 J; (b) 722 J
P20.60	1.79 kg
P20.62	(a) Isolated system (momentum). The collision is a perfectly inelastic collision, where momentum is conserved, but kinetic energy is not. (It is transformed to internal energy); (b) 20.0 m / s to the right; (c) 1.18×10^{3} J; (d) No; (e) 327.3°C; (f) 3.1 g of solid lead and 16.9 g of liquid lead
P20.64	$\dfrac{P}{\rho R \Delta T}$
P20.66	(a) First, energy must be removed from the liquid water to cool it to 0° C. Next, energy must be removed from the water at 0° C to freeze it, which corresponds to a liquid-to-solid phase transition. Finally, once all the water has frozen, additional energy must be removed from the ice to cool it from 0° to –8.00°C; (b) 32.5 kJ
P20.68	(a) 0.645 kg / h; (b) 0.068 9
P20.70	11.1 W
P20.72	(a) $+\dfrac{P_iV_i}{2}$; Put the cylinder into a refrigerator at absolute temperature $T/2$. Let the piston move freely as the gas cools; (b) $+1.39P_iV_i$; With the gas in a constant-temperature bath at T_f, slowly push the piston in; (c)

$W = 0$; Lock the piston in place and hold the cylinder over at hotplate at $3T_f$.

P20.74 Most of the morning is used in making coffee, and there is no time left for a morning hike.

P20.76 (a) 15.0 mg; (b) –5.00 J; (c) 5.00 J; (d) 15.0 mg; (e) 5.00 J; (f) $\Delta E_{\text{int}} = 0$; (g) 4.04×10^{-3} °C; (h) $Q = 0$; $\Delta E_{\text{int}} = 250$ J; $\Delta E_{\text{mech}} = -5.00$ J; (i) $Q = 0$; $\Delta E_{\text{int}} = 2.50$ J

P20.78 $38.6\ \text{m}^3/\text{day}$

P20.80 (a) 800 J/kg · °C; (b) This differs from the tabulated value by 11%, so the values agree within 15%.

P20.82 (a) The equation $dT/dr = P/4\pi kr^2$ represents the law of thermal conduction, incorporating the definition of thermal conductivity, applied to a spherical surface within the shell. The rate of energy transfer P must be the same for all radii so that each bit of material stays at a temperature that is constant in time; (b) See P20.70(b) for full proof; (c) 18.5 W; (d) See P20.70(d) for full proof; (e) $T = 5 + 1.84\left(\frac{1}{0.030\,0} - \frac{1}{r}\right)$, where T is in degrees Celsius and r is in meters; (f) 29.5° C

P20.84 (a) $\frac{dQ}{dt} = 2\pi kL\left[\frac{(T_a - T_b)}{\ln(b/a)}\right]$; (b) 9.32 kW

21

The Kinetic Theory of Gases

CHAPTER OUTLINE

21.1	Molecular Model of an Ideal Gas
21.2	Molar Specific Heat of an Ideal Gas
21.3	The Equipartition of Energy
21.4	Adiabatic Processes for an Ideal Gas
21.5	Distribution of Molecular Speeds

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ21.1 Answer (c). The molecular mass of nitrogen (N_2, 28 u) is smaller than the molecular mass of oxygen (O_2, 32 u), and the rms speed of a gas is $(3RT/M)^{1/2}$. Since the rms speeds are the same, the temperature of nitrogen is smaller than the temperature of oxygen. The average kinetic energy is proportional to the molecular mass and the square of the rms speed ($K = \frac{1}{2}mv_{\text{rms}}^2$), so the average kinetic energy of nitrogen is smaller.

OQ21.2 Answer (d). The rms speed of molecules in the gas is $v_{\text{rms}} = \sqrt{3RT/M}$. Thus, the ratio of the final speed to the original speed would be

$$\frac{(v_{\text{rms}})_f}{(v_{\text{rms}})_0} = \frac{\sqrt{3RT_f/M}}{\sqrt{3RT_0/M}} = \sqrt{\frac{T_f}{T_0}} = \sqrt{\frac{600\text{ K}}{200\text{ K}}} = \sqrt{3}$$

OQ21.3 Answer (b). The gases are the same so they have the same molecular mass, *M*. If the two samples have the same density, then their ratios of number of moles to volume, *n/V*, are the same because their densities, *(nM)/V*, are the same. The pressures are the same; thus, their temperatures are the same:

$$PV = nRT \to p = \frac{n}{V}RT = \text{ constant } \to T = \text{ constant}$$

Therefore the rms speed of their molecules, $(3RT/M)^{1/2}$, is the same.

OQ21.4 (i) Answer (b). The volume of the balloon will decrease because the gas cools.

(ii) Answer (c). The pressure inside the balloon is nearly equal to the constant exterior atmospheric pressure. Snap the mouth of the balloon over an absolute pressure gauge to demonstrate this fact. Then from $PV = nRT$, volume must decrease in proportion to the absolute temperature. Call the process isobaric contraction.

OQ21.5 Answer (d). At 200 K, $\frac{1}{2}m_0 v_{\text{rms0}}^2 = \frac{3}{2}k_B T_0$. At the higher temperature,

$$\frac{1}{2}m_0\left(2v_{\text{rms0}}\right)^2 = \frac{3}{2}k_B T$$

Then $T = 4T_0 = 4(200\text{ K}) = 800\text{ K}$.

OQ21.6 Answer (c) > (a) > (b) > (d). The average vector velocity is zero in a sample macroscopically at rest. As adjacent equations in the text note, the asymmetric distribution of molecular speeds makes the average speed greater than the most probable speed, and the rms speed greater still. The most probable speed is $(2RT/M)^{1/2}$, the average speed is $(8RT/\pi M)^{1/2} \cong (2.55RT/M)^{1/2}$, and the rms speed is $(3RT/M)^{1/2}$.

OQ21.7 (i) Statements (a) and (e) are correct statements that describe the temperature increase of a gas.

(ii) Statement (f) is a correct statement but does not apply to the situation. Statement (b) is true if the molecules have any size at all, but molecular collisions with other molecules have nothing to do with the temperature increase.

(iii) Statements (c) and (d) are incorrect. The molecular collisions are perfectly elastic. Temperature is determined by how fast molecules are moving through space, not by anything going on inside a molecule.

OQ21.8 (i) Answer (b). Average molecular kinetic energy, $3kT/2$, increases by a factor of 3.

(ii) Answer (c). The rms speed, $(3RT/M)^{1/2}$, increases by a factor of $\sqrt{3}$.

(iii) Answer (c). Average momentum change increases by $\sqrt{3}$: $\Delta p_{\text{avg}} = -2m_0 v_{\text{avg}}$.

(iv) Answer (c). Rate of collisions increases by a factor of $\sqrt{3}$: $\Delta t_{\text{avg}} = 2d / v_{\text{avg}}$.

(v) Answer (b). Pressure increases by a factor of 3. See Equation 21.15:

$$P = \frac{2}{3}\left(\frac{N_i}{V}\right)\left(\frac{1}{2}m_0\overline{v^2}\right) = \frac{2}{3}\left(\frac{N_i}{V}\right)\left(\overline{K}\right)$$

OQ21.9 Answer (c). The kinetic theory of gases assumes that the molecules do not interact with each other.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ21.1 As a parcel of air is pushed upward, it moves into a region of lower pressure, so it expands and does work on its surroundings. Its fund of internal energy drops, and so does its temperature. As mentioned in the question, the low thermal conductivity of air means that very little energy will be conducted by heat into the now-cool parcel from the denser but warmer air below it.

CQ21.2 A diatomic gas has more degrees of freedom—those of molecular vibration and rotation—than a monatomic gas. The energy content per mole is proportional to the number of degrees of freedom.

CQ21.3 Alcohol evaporates rapidly, so that high-speed molecules leave the liquid, reducing the average kinetic energy of the remaining molecules of the liquid and therefore reducing the temperature of the liquid. *Then*, because the alcohol is cool, energy transfers from the skin, reducing its temperature.

CQ21.4 As the balloon rises into the air, the air cannot be uniform in pressure because the lower layers support the weight of all the air above them. The rubber in a typical balloon is easy to stretch and stretches or contracts until interior and exterior pressures are nearly equal. So as the balloon rises it expands. This is an adiabatic expansion (see Section 21.4), with P decreasing as V increases (PV^γ = constant). If the rubber wall is very strong it will eventually contain the helium at higher pressure than the air outside but at the same density, so that the balloon will stop rising. More likely, the rubber will stretch and break, releasing the helium to keep rising and “boil out” of the Earth’s atmosphere.

CQ21.5 The dry air is more dense. Since the air and the water vapor are at the same temperature, the gases have the same average molecular kinetic energy. Imagine a controlled experiment in which equal-volume containers, one with humid air and one with dry air, are at the same pressure. The number of molecules must be the same for both containers ($PV = NkT$). The water molecule has a smaller molecular mass (18.0 u) than any of the gases that make up the air, so the humid air must have the smaller mass per unit volume.

CQ21.6 The helium must have the higher rms speed. According to Equation 21.22 for the rms speed, $(3RT/M)^{1/2}$, for the same temperature, the gas with the smaller mass per atom must have the higher average speed squared and thus the higher rms speed.

CQ21.7 The molecules of all different kinds collide with the walls of the container, so molecules of all different kinds exert partial pressures that contribute to the total pressure. The molecules can be so small that they collide with one another relatively rarely and each kind exerts partial pressure as if the other kinds of molecules were absent. If the molecules collide with one another often, the collisions exactly conserve momentum and so do not affect the net force on the walls. The partial pressure P_i of one of the gases can be expressed with Equation 21.15:

$$P_i = \frac{2}{3}\left(\frac{N_i}{V}\right)\left(\frac{1}{2}m_0\overline{v^2}\right) = \frac{2}{3}\left(\frac{N_i}{V}\right)\left(\overline{K}\right)$$

where N_i is the number of molecules of the ith gas and $\overline{K}$ is the average kinetic energy of the molecules. Let us add up these pressures for all the gases in the container:

$$P = \sum_i P_i = \sum_i \frac{2}{3}\left(\frac{N_i}{V}\right)\left(\overline{K}\right) = \frac{2}{3}\frac{\overline{K}}{V}\sum_i N_i = \frac{2}{3}\left(\frac{N}{V}\right)\left(\overline{K}\right)$$

where N is the total number of molecules of all types and we have used the fact that the average kinetic energies of all types of molecules are the same because all the gases have the same temperature. The final expression for the pressure is the same as that of a single gas with N molecules in the same volume V and at the given temperature.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 21.1 Molecular Model of an Ideal Gas

P21.1 (a) The volume is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.150\text{ m})^3 = 1.41\times 10^{-2}\text{ m}^3$. The quantity of gas can be obtained from *PV* = *nRT*:

$$n = \frac{PV}{RT} = \frac{\left(1.013\times 10^5\text{ N/m}^2\right)\left(1.41\times 10^{-2}\text{ m}^3\right)}{\left(8.314\text{ N}\cdot\text{m/mol}\cdot\text{K}\right)\left(293\text{ K}\right)} = 0.588\text{ mol}$$

The number of molecules is

$$N = nN_A = (0.588\text{ mol})\left(6.02\times 10^{23}\text{ molecules/mol}\right)$$

$$N = \boxed{3.54\times 10^{23}\text{ helium atoms}}$$

(b) The kinetic energy is given by $\bar{K} = \frac{1}{2}m_0\overline{v^2} = \frac{3}{2}k_BT$:

$$\bar{K} = \frac{3}{2}\left(1.38\times 10^{-23}\text{ J/K}\right)(293\text{ K}) = \boxed{6.07\times 10^{-21}\text{ J}}$$

(c) An atom of He has mass

$$m_0 = \frac{M}{N_A} = \frac{4.002\ 6\text{ g/mol}}{6.02\times 10^{23}\text{ molecules/mol}}$$
$$= 6.65\times 10^{-24}\text{ g} = 6.65\times 10^{-27}\text{ kg}$$

So the root-mean-square speed is given by

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{2\bar{K}}{m_0}} = \sqrt{\frac{2\times 6.07\times 10^{-27}\text{ J}}{6.65\times 10^{-27}\text{ kg}}} = \boxed{1.35\text{ km/s}}$$

P21.2 (a) Both kinds of molecules have the same average kinetic energy. It is

$$\bar{K} = \frac{3}{2}k_BT = \frac{3}{2}\left(1.38\times 10^{-23}\text{ J/K}\right)(423\text{ K}) = \boxed{8.76\times 10^{-21}\text{ J}}$$

(b) The root-mean square velocity can be calculated from the kinetic energy:

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{\frac{2\bar{K}}{m_0}}$$

so $$v_{\text{rms}} = \sqrt{\frac{1.75\times10^{-20}\ \text{J}}{m_0}} \qquad \textbf{[1]}$$

For helium,

$$m_0 = \frac{4.00\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 6.64\times10^{-24}\ \text{g/molecule}$$

$$m_0 = 6.64\times10^{-27}\ \text{kg/molecule}$$

Similarly for argon, $m_0 = \dfrac{39.9\ \text{g/mol}}{6.02\times10^{23}\ \text{molecules/mol}}$

$$= 6.63\times10^{-23}\ \text{g/molecule}$$

$$m_0 = 6.63\times10^{-26}\ \text{kg/molecule}$$

Substituting into [1] above,

we find for helium, $\boxed{v_{\text{rms}} = 1.62\ \text{km/s}}$

and for argon, $\boxed{v_{\text{rms}} = 514\ \text{m/s}}$

P21.3 (a) From Newton's second law, the average force is given by

$$\bar{F} = Nm\frac{\Delta v}{\Delta t} = 500\left(5.00\times10^{-3}\ \text{kg}\right) \times \frac{\left[8.00\sin 45.0^\circ - \left(-8.00\sin 45.0^\circ\right)\right]\ \text{m/s}}{30.0\ \text{s}}$$

$$= \boxed{0.943\ \text{N}}$$

(b) We find the pressure from

$$P = \frac{\bar{F}}{A} = \frac{0.943\ \text{N}}{0.600\ \text{m}^2} = 1.57\ \text{N/m}^2 = \boxed{1.57\ \text{Pa}}$$

P21.4 The equation of state for an ideal gas can be used with the given information to find the number of molecules in a specific volume.

$$PV = \left(\frac{N}{N_A}\right)RT \text{ means } N = \frac{PVN_A}{RT},$$

so that, suppressing units,

$$N = \frac{\left(1.00\times10^{-10}\right)(133)(1.00)\left(6.02\times10^{23}\right)}{(8.314)(300)}$$

$$= \boxed{3.21\times10^{12}\ \text{molecules}}$$

P21.5 The gas temperature must be that implied by $\frac{1}{2}m_0\overline{v^2} = \frac{3}{2}k_BT$ for a monatomic gas like helium.

$$T = \frac{2}{3}\left(\frac{\frac{1}{2}m_0\overline{v^2}}{k_B}\right) = \frac{2}{3}\left(\frac{3.60\times 10^{-22}\text{ J}}{1.38\times 10^{-23}\text{ J/K}}\right) = 17.4\text{ K}$$

Now *PV* = *nRT* gives

$$n = \frac{PV}{RT} = \frac{\left(1.20\times 10^{5}\text{ N/m}^2\right)\left(4.00\times 10^{-3}\text{ m}^3\right)}{(8.314\text{ J/mol}\cdot\text{K})(17.4\text{ K})} = \boxed{3.32\text{ mol}}$$

P21.6 $P = \frac{2}{3}\frac{N}{V}\overline{K}$ from the kinetic-theory account for pressure.

$$N = \frac{3}{2}\frac{PV}{\overline{K}}$$

$$n = \frac{N}{N_A} = \boxed{\frac{3}{2}\frac{PV}{\overline{K}N_A}}$$

P21.7 Use the equation describing the kinetic-theory account for pressure: $P = \frac{2N}{3V}\left(\frac{m_0\overline{v^2}}{2}\right)$. Then

$$\overline{K} = \frac{m_0\overline{v^2}}{2} = \frac{3PV}{2N},\ \text{where } N = nN_A$$

$$\overline{K} = \frac{3PV}{2nN_A} = \frac{3\left(8.00\text{ atm}\right)\left(1.013\times 10^{5}\text{ Pa/atm}\right)\left(5.00\times 10^{-3}\text{ m}^3\right)}{2\left(2\text{ mol}\right)\left(6.02\times 10^{23}\text{ molecules/mol}\right)}$$

$$\overline{K} = \boxed{5.05\times 10^{-21}\text{ J}}$$

P21.8 The molar mass of diatomic oxygen is 32.0 g. The rms speed of oxygen molecules is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$$

and

$$p_{\text{rms}} = mv_{\text{rms}} = \frac{M}{N_A}\sqrt{\frac{3RT}{M}} = \frac{1}{N_A}\sqrt{3RTM}$$

$$= \frac{1}{6.02\times 10^{23}}\sqrt{3\left(8.314\text{ J/mol}\cdot\text{K}\right)\left(350\text{ K}\right)\left(32.0\times 10^{-3}\text{ kg}\right)}$$

$$p_{\text{rms}} = \boxed{2.78\times 10^{-23}\text{ kg}\cdot\text{m/s}}$$

P21.9 We use 1 u = 1.66×10^{-24} g.

(a) For He, $m_0 = 4.00\text{ u}\left(\frac{1.66 \times 10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{6.64 \times 10^{-27}\text{ kg}}$

(b) For Fe, $m_0 = 55.9\text{ u}\left(\frac{1.66 \times 10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{9.28 \times 10^{-26}\text{ kg}}$

(c) For Pb, $m_0 = 207\text{ u}\left(\frac{1.66 \times 10^{-24}\text{ g}}{1\text{ u}}\right) = \boxed{3.44 \times 10^{-25}\text{ kg}}$

P21.10 The rms speed of molecules in a gas of molecular weight *M* and absolute temperature *T* is $v_{\text{rms}} = \sqrt{3RT/M}$. Thus, if v_{rms} = 625 m/s for molecules in oxygen (O_2), for which *M* = 32.0 g/mol = 32.0×10^{-3} kg/mol, the temperature of the gas is

$$T = \frac{Mv_{\text{rms}}^2}{3R} = \frac{(32.0 \times 10^{-3}\text{ kg/mol})(625\text{ m/s})^2}{3(8.31\text{ J/mol}\cdot\text{K})} = \boxed{501\text{ K}}$$

***P21.11** (a) From the ideal gas law,

$$PV = nRT = \frac{Nm_0v^2}{3}$$

The total translational kinetic energy is $\frac{Nm_0v^2}{2} = E_{\text{trans}}$:

$$E_{\text{trans}} = \frac{3}{2}PV = \frac{3}{2}(3.00 \times 1.013 \times 10^5\text{ Pa})(5.00 \times 10^{-3}\text{ m}^3)$$

$$= \boxed{2.28\text{ kJ}}$$

(b) $\frac{m_0v^2}{2} = \frac{3k_BT}{2} = \frac{3RT}{2N_A} = \frac{3(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}{2(6.02 \times 10^{23})} = \boxed{6.21 \times 10^{-21}\text{ J}}$

P21.12 (a) The volume occupied by this gas is

$$V = 7.00\text{ L}(10^3\text{ cm}^3/1\text{ L})(1\text{ m}^3/10^6\text{ cm}^3) = 7.00 \times 10^{-3}\text{ m}^3$$

Then, the ideal gas law gives

$$T = \frac{PV}{nR} = \frac{(1.60 \times 10^6\text{ Pa})(7.00 \times 10^{-3}\text{ m}^3)}{(3.50\text{ mol})(8.31\text{ J/mol}\cdot\text{K})} = \boxed{385\text{ K}}$$

(b) The average kinetic energy per molecule in this gas is

$$\overline{KE}_{\text{molecule}} = \frac{3}{2}k_BT = \frac{3}{2}(1.38 \times 10^{-23}\text{ J/K})(385\text{ K}) = \boxed{7.97 \times 10^{-21}\text{ J}}$$

(c) You would need to know the mass of the gas molecule to find its average speed, which in turn requires knowledge of $\boxed{\text{the molecular mass of the gas}}$.

P21.13 To find the pressure exerted by the nitrogen molecules, we first calculated the average force exerted by the molecules:

$$\bar{F} = Nm_0\frac{\Delta v}{\Delta t} = \frac{\left(5.00\times10^{23}\right)\left[\left(4.65\times10^{-26}\text{ kg}\right)2\left(300\text{ m/s}\right)\right]}{1.00\text{ s}} = 14.0\text{ N}$$

the pressure is then

$$P = \frac{\bar{F}}{A} = \frac{14.0\text{ N}}{8.00\times10^{-4}\text{ m}^2} = \boxed{17.4\text{ kPa}}$$

Section 21.2 Molar Specific Heat of an Ideal Gas

P21.14 $n = 1.00$ mol, $T_i = 300$ K

(a) Since V = constant, $W = \boxed{0}$.

(b) $\Delta E_{\text{int}} = Q + W = 209\text{ J} + 0 = \boxed{209\text{ J}}$

(c) $\Delta E_{\text{int}} = nC_V\Delta T = n\left(\frac{3}{2}R\right)\Delta T$

so $$\Delta T = \frac{2\Delta E_{\text{int}}}{3nR} = \frac{2(209\text{ J})}{3(1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = 16.8\text{ K}$$

$$T = T_i + \Delta T = 300\text{ K} + 16.8\text{ K} = \boxed{317\text{ K}}$$

P21.15 $Q = \left(nC_P\Delta T\right)_{\text{isobaric}} + \left(nC_V\Delta T\right)_{\text{isovolumetric}}$

In the isobaric process, V doubles so T must double, to $2T_i$.

In the isovolumetric process, P triples so T changes from $2T_i$ to $6T_i$.

$$Q = n\left(\frac{7}{2}R\right)\left(2T_i - T_i\right) + n\left(\frac{5}{2}R\right)\left(6T_i - 2T_i\right) = 13.5nRT_i$$

$$= \boxed{13.5PV}$$

P21.16 (a) Consider warming it at constant pressure. Oxygen and nitrogen are diatomic, so $C_P = \frac{7R}{2}$. Then,

$$Q = nC_P\Delta T = \frac{7}{2}nR\Delta T = \frac{7}{2}\left(\frac{PV}{T}\right)\Delta T$$

$$Q = \frac{7}{2}\frac{\left(1.013\times 10^5\ \text{N/m}^2\right)\left(100\ \text{m}^3\right)}{300\ \text{K}}\left(1.00\ \text{K}\right) = \boxed{118\ \text{kJ}}$$

(b) We use the definition of gravitational potential energy,

$$U_g = mgy$$

from which,

$$m = \frac{U_g}{gy} = \frac{1.18\times 10^5\ \text{J}}{\left(9.80\ \text{m/s}^2\right)\left(2.00\ \text{m}\right)} = \boxed{6.03\times 10^3\ \text{kg}}$$

P21.17 We use the tabulated values for C_P and C_V:

(a) Since this is a constant-pressure process, $Q = nC_P\Delta T$.

The temperature rises by ΔT = 420 K – 300 K = 120 K:

$$Q = nC_P\Delta T = \left(1.00\ \text{mol}\right)\left(28.8\ \text{J/mol}\cdot\text{K}\right)\left(420\ \text{K} - 300\ \text{K}\right)$$
$$= \boxed{3.46\ \text{kJ}}$$

(b) For any gas $\Delta E_{\text{int}} = nC_V\Delta T$, so

$$\Delta E_{\text{int}} = nC_V\Delta T = \left(1.00\ \text{mol}\right)\left(20.4\ \text{J/mol}\cdot\text{K}\right)\left(120\ \text{K}\right) = \boxed{2.45\ \text{kJ}}$$

(c) The first law says $\Delta E_{\text{int}} = Q + W$, so

$$W = -Q + \Delta E_{\text{int}} = -3.46\ \text{kJ} + 2.45\ \text{kJ} = \boxed{-1.01\ \text{kJ}}$$

P21.18 (a) Molar specific heat is $C_V = \frac{5}{2}R$.

Specific heat at constant volume per unit mass is given by

$$c_V = \frac{C_V}{M} = \frac{5}{2}R\left(\frac{1}{M}\right)$$
$$= \frac{5}{2}\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(\frac{1.00\ \text{mol}}{0.028\,9\ \text{kg}}\right)$$
$$= 719\ \text{J/kg}\cdot\text{K} = \boxed{0.719\ \text{kJ/kg}\cdot\text{K}}$$

(b) $m = Mn = M\left(\dfrac{PV}{RT}\right)$

$$m = \left(0.028\,9\ \text{kg/mol}\right)\left[\frac{\left(200\times10^{3}\ \text{Pa}\right)\left(0.350\ \text{m}^{3}\right)}{\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(300\ \text{K}\right)}\right] = \boxed{0.811\ \text{kg}}$$

(c) We consider a constant-volume process where no work is done.

$$\begin{aligned} Q &= mc_V\Delta T \\ &= \left(0.811\ \text{kg}\right)\left(0.719\ \text{kJ/kg}\cdot\text{K}\right)\left(700\ \text{K} - 300\ \text{K}\right) \\ &= \boxed{233\ \text{kJ}} \end{aligned}$$

(d) We now consider a constant-pressure process where the internal energy of the gas is increased and work is done.

$$Q = nC_P\Delta T = \frac{m}{M}\left(C_V + R\right)\Delta T = \frac{m}{M}\left(\frac{5}{2}R + R\right)\Delta T = \frac{m}{M}\left(\frac{7}{2}R\right)\Delta T$$

$$= m\left(\frac{7}{5}\right)\left(\frac{\frac{5}{2}R}{M}\right)\Delta T = m\left[\left(\frac{7}{5}\right)\left(\frac{C_V}{M}\right)\right]\Delta T$$

$$\rightarrow Q = \left(0.811\ \text{kg}\right)\left[\frac{7}{5}\left(0.719\ \text{kJ/kg}\cdot\text{K}\right)\right]\left(400\ \text{K}\right) = \boxed{327\ \text{kJ}}$$

***P21.19** $\Delta E_{\text{int}} = \dfrac{3}{2}nR\Delta T = \dfrac{3}{2}\left(3.00\ \text{mol}\right)\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(2.00\ \text{K}\right) = \boxed{74.8\ \text{J}}$

P21.20 Consider 800 cm^3 of tea (flavored water) at 90.0°C mixing with 200 cm^3 of diatomic ideal gas at 20.0°C:

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

or

$$m_{\text{air}}c_{P,\,\text{air}}\left(T_f - T_{i,\,\text{air}}\right) = -m_w c_w\left(\Delta T\right)_w$$

$$\left(\Delta T\right)_w = \frac{-m_{\text{air}}c_{P,\,\text{air}}\left(T_f - T_{i,\,\text{air}}\right)}{m_w c_w} = \frac{-\left(\rho V\right)_{\text{air}}c_{P,\,\text{air}}\left(90.0\text{°C} - 20.0\text{°C}\right)}{\left(\rho_w V_w\right)c_w}$$

where we have anticipated that the final temperature of the mixture will be close to 90.0°C.

The molar specific heat of air is $C_{P,\,\text{air}} = \dfrac{7}{2}R.$

So the specific heat per gram is

$$c_{P,\,\text{air}} = \frac{7}{2}\left(\frac{R}{M}\right) = \frac{7}{2}(8.314\ \text{J/mol}\cdot\text{K})\left(\frac{1.00\ \text{mol}}{28.9\ \text{g}}\right) = 1.01\ \text{J/g}\cdot{}^\circ\text{C}$$

and

$$(\Delta T)_w = -\frac{\left[(1.20\times10^{-3}\ \text{g/cm}^3)(200\ \text{cm}^3)\right](1.01\ \text{J/g}\cdot{}^\circ\text{C})(70.0{}^\circ\text{C})}{\left[(1.00\ \text{g/cm}^3)(800\ \text{cm}^3)\right](4.186\ \text{J/g}\cdot{}^\circ\text{C})}$$

or $(\Delta T)_w \approx -5.05\times10^{-3}{}^\circ\text{C}$

The change of temperature for the water is $\boxed{\text{between } 10^{-3}\ {}^\circ\text{C and } 10^{-2}\ {}^\circ\text{C}}$.

***P21.21** (a) The air is far from liquefaction so it behaves as an ideal gas. From $PV = nRT$ we have $PV = \frac{m}{M}RT$, or $PM = \frac{m}{V}RT = \rho RT$. For the samples of air in the balloon at 10.0°C (cold) and at the elevated temperature (hot) we have $PM = \rho_c RT_c$ and $PM = \rho_h RT_h$. Then $\rho_h T_h = \rho_c T_c$ and $\rho_h = \frac{\rho_c T_c}{T_h}$. For equilibrium of the balloon on the point of rising,

$$\sum F_y = ma_y: \qquad +B - F_{g\ \text{hot air}} - F_{g\ \text{cargo}} = 0$$

$$+\rho_c Vg - \rho_h Vg - mg = 0$$

$$+\rho_c V - \frac{\rho_c T_c}{T_h}V - m = 0$$

$$(1.25\ \text{kg/m}^3)(400\ \text{m}^3) - (1.25\ \text{kg/m}^3)\left(\frac{283\ \text{K}}{T_h}\right)(400\ \text{m}^3) - 200\ \text{kg} = 0$$

$$300\ \text{kg} = (500\ \text{kg})\left(\frac{283\ \text{K}}{T_h}\right)$$

$$T_h = \left(\frac{500}{300}\right)(283\ \text{K}) = 472\ \text{K}$$

The quantity of air that must be warmed is given by $PV = n_h RT_h$, or $n_h = \frac{PV}{RT_h}$. The heat input required is

$$\begin{aligned} Q &= nC_p\Delta T \\ &= -\frac{PV}{RT_h}\;\frac{7}{2}R\left(T_h - T_c\right) \\ &= \left(\frac{7}{2}\right)\frac{\left(1.013\times10^5\ \text{N/m}^2\right)\left(400\ \text{m}^3\right)\left(472\ \text{K} - 283\ \text{K}\right)}{472\ \text{K}} \\ &= \boxed{5.66\times10^7\ \text{J}} \end{aligned}$$

(b) *Q* = *mH*, so $m = \frac{Q}{H} = \frac{5.66\times10^7\ \text{J}}{5.03\times10^7\ \text{J/kg}} = \boxed{1.12\ \text{kg}}$

Section 21.3 The Equipartition of Energy

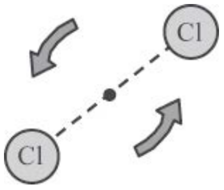
P21.22 (a) $E_{\text{int}} = Nf\left(\frac{k_BT}{2}\right) = f\left(\frac{nRT}{2}\right)$

(b) $C_V = \frac{1}{n}\left(\frac{dE_{\text{int}}}{dT}\right) = \frac{1}{2}fR$

(c) $C_P = C_V + R = \frac{1}{2}\left(f+2\right)R$

(d) $\gamma = \frac{C_P}{C_V} = \frac{f+2}{f}$

P21.23 The rotational kinetic energy of the molecule is given by $K_{\text{rot}} = \frac{1}{2}I\omega^2$. We determine the moment of inertia from $I = 2m_0r^2$, with $m_0 = 35.0\times1.67\times10^{-27}$ kg and $r = 10^{-10}$ m:



ANS. FIG. P21.23

$$\begin{aligned} I &= 2m_0r^2 = 2\left(35.0\times1.67\times10^{-27}\ \text{kg}\right)\left(10^{-10}\ \text{m}\right)^2 \\ &= 1.17\times10^{-45}\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

Then,

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(1.17\times10^{-45}\text{ kg}\cdot\text{m}^2\right)\left(2.00\times10^{12}\text{ s}^{-1}\right)^2$$
$$= \boxed{2.34\times10^{-21}\text{ J}}$$

P21.24 We must have the difference of molar specific heats given by Equation 21.31: $C_P - C_V = R$. The value of γ tells us that $C_P = 1.75C_V$, so

$$1.75C_V - C_V = R \quad\rightarrow\quad C_V = \frac{R}{0.75} = \frac{4}{3}R$$

$$\boxed{\begin{array}{l}\text{The maximum possible value of } \gamma = 1 + \dfrac{R}{C_V} = 1.67 \text{ occurs}\\ \text{for the lowest possible value for } C_V = \dfrac{3}{2}R. \text{ Therefore the}\\ \text{claim of } \gamma = 1.75 \text{ for the newly discovered gas cannot be}\\ \text{true.}\end{array}}$$

P21.25 The sample's total heat capacity at constant volume is nC_v. An ideal gas of diatomic molecules has three degrees of freedom for translation in the *x*, *y*, and *z* directions. If we take the *y* axis along the axis of a molecule, then outside forces cannot excite rotation about this axis, since they have no lever arms. Collisions will set the molecule spinning only about the *x* and *z* axes.

(a) If the molecules do not vibrate, they have five degrees of freedom. Random collisions put equal amounts of energy $\frac{1}{2}k_BT$ into all five kinds of motion. The average energy of one molecule is $\frac{5}{2}k_BT$. The internal energy of the two-mole sample is

$$N\left(\frac{5}{2}k_BT\right) = nN_A\left(\frac{5}{2}k_BT\right) = n\left(\frac{5}{2}R\right)T = nC_VT$$

The molar heat capacity is $C_V = \frac{5}{2}R$, and the sample's heat capacity is

$$nC_V = n\left(\frac{5}{2}R\right) = (2.00\text{ mol})\left[\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right]$$
$$\boxed{nC_V = 41.6\text{ J/K}}$$

(b) For the heat capacity at constant pressure, we have

$$nC_P = n(C_V + R) = n\left(\frac{5}{2}R + R\right) = \frac{7}{2}nR$$

$$= (2.00\text{ mol})\left[\frac{7}{2}(8.314\text{ J/mol}\cdot\text{K})\right]$$

$$\boxed{nC_P = 58.2\text{ J/K}}$$

(c) Vibration adds two more degrees of freedom for two more terms in the molecular energy, for kinetic and for elastic potential energy. We have

$$nC_V = n\left(\frac{7}{2}R\right) = \boxed{58.2\text{ J/K}}$$

and $nC_P = n\left(\frac{9}{2}R\right) = \boxed{74.8\text{ J/K}}$.

Section 21.4 Adiabatic Processes for an Ideal Gas

P21.26 (a) In an adiabatic process $P_iV_i^\gamma = P_fV_f^\gamma$:

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = (5.00\text{ atm})\left(\frac{12.0}{30.0}\right)^{1.40} = \boxed{1.39\text{ atm}}$$

(b) The initial temperature is

$$T_i = \frac{P_iV_i}{nR} = \frac{5.00(1.013\times10^5\text{ Pa})(12.0\times10^{-3}\text{ m}^3)}{(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = \boxed{366\text{ K}}$$

and similarly the final temperature is

$$T_f = \frac{P_fV_f}{nR} = \frac{1.39(1.013\times10^5\text{ Pa})(30.0\times10^{-3}\text{ m}^3)}{(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = \boxed{253\text{ K}}$$

(c) The process is adiabatic, so by definition, $\boxed{Q = 0}$.

(d) For any process, $\Delta E_{\text{int}} = nC_V\Delta T$,

and for this diatomic ideal gas, $C_V = \dfrac{R}{\gamma - 1} = \dfrac{5}{2}R$

Thus,

$$\begin{aligned}\Delta E_{\text{int}} &= nC_V\Delta T\\ &= (2.00\text{ mol})\left[\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right](253\text{ K} - 366\text{ K})\\ &= \boxed{-4.66\text{ kJ}}\end{aligned}$$

(e) $W = \Delta E_{\text{int}} - Q = -4.66\text{ kJ} - 0 = \boxed{-4.66\text{ kJ}}$

P21.27 (a) $P_iV_i^\gamma = P_fV_f^\gamma$ so $\dfrac{V_f}{V_i} = \left(\dfrac{P_i}{P_f}\right)^{1/\gamma} = \left(\dfrac{1.00}{20.0}\right)^{5/7} = \boxed{0.118}$.

(b) $\dfrac{T_f}{T_i} = \dfrac{P_fV_f}{P_iV_i} = \left(\dfrac{P_f}{P_i}\right)\left(\dfrac{V_f}{V_i}\right) = (20.0)(0.118) \quad \rightarrow \quad \dfrac{T_f}{T_i} = \boxed{2.35}$

(c) Since the process is adiabatic, $\boxed{Q = 0}$.

(d) Since $\gamma = 1.40 = \dfrac{C_P}{C_V} = \dfrac{R + C_V}{C_V}$, $C_V = \dfrac{5}{2}R$

and $\Delta T = 2.35T_i - T_i = 1.35T_i$, then

$$\begin{aligned}\Delta E_{\text{int}} &= nC_V\Delta T\\ &= (0.016\,0\text{ mol})\left(\frac{5}{2}\right)(8.314\text{ J/mol}\cdot\text{K})\left[1.35(300\text{ K})\right]\\ &= \boxed{135\text{ J}}\end{aligned}$$

(e) $W = -Q + \Delta E_{\text{int}} = 0 + 135\text{ J} = \boxed{+135\text{ J}}$

P21.28 (a) The work done *on* the gas is

$$W_{ab} = -\int_{V_a}^{V_b} P\,dV$$

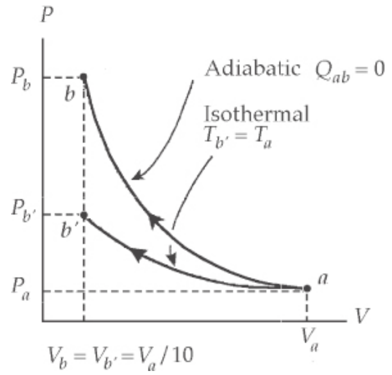
For the isothermal process,

$$W_{ab'} = -nRT_a\int_{V_a}^{V_{b'}}\left(\frac{1}{V}\right)dV$$

$$W_{ab'} = -nRT_a\ln\left(\frac{V_{b'}}{V_a}\right) = nRT\ln\left(\frac{V_a}{V_{b'}}\right)$$

Thus,

$$W_{ab'} = (5.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})\ln(10.0) = \boxed{28.0\text{ kJ}}$$



ANS. FIG. P21.28

(b) For the adiabatic process, we must first find the final temperature, T_b. Since air consists primarily of diatomic molecules, we shall use

$$\gamma_{\text{air}} = 1.40 \quad \text{and} \quad C_{V,\text{air}} = \frac{5R}{2} = \frac{5(8.314)}{2} = 20.8 \text{ J/mol} \cdot \text{K}$$

Then, for the adiabatic process,

$$T_b = T_a \left(\frac{V_a}{V_b} \right)^{\gamma - 1} = (293 \text{ K})(10.0)^{0.400} = 736 \text{ K}$$

Thus, the work done on the gas during the adiabatic process is

$$W_{ab} \left(-Q + \Delta E_{\text{int}} \right)_{ab} = \left(-0 + nC_V \Delta T \right)_{ab} = nC_V \left(T_b - T_a \right)$$

or $W_{ab} = (5.00 \text{ mol})(20.8 \text{ J/mol} \cdot \text{K})(736 \text{ K} - 293 \text{ K}) = \boxed{46.0 \text{ kJ}}$

(c) For the isothermal process, we have $P_{b'}V_{b'} = P_a V_a$.

Thus, $P_{b'} = P_a \left(\frac{V_a}{V_{b'}} \right) = (1.00 \text{ atm})(10.0) = \boxed{10.0 \text{ atm}}$.

(d) For the adiabatic process, we have $P_b V_b^\gamma = P_a V_a^\gamma$.

Thus, $P_b = P_a \left(\frac{V_a}{V_b} \right)^\gamma = (1.00 \text{ atm})(10.0)^{1.40} = \boxed{25.1 \text{ atm}}$.

P21.29 Combining PV^γ = constant with the ideal gas law gives one of the textbook equations describing adiabatic processes, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$.

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma - 1} = (300 \text{ K}) \left(\frac{1}{2} \right)^{(1.40-1)} = \boxed{227 \text{ K}}$$

P21.30 Use Equation 21.37 for an adiabatic process to find the temperature of the compressed fuel-air mixture at the end of the compression stroke, before ignition:

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

which gives

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} = (323\ \text{K})(14.5)^{1.40-1} = 941\ \text{K}$$

This is equivalent to 668°C, which is higher than the melting point of aluminum which is 660°C. Also, the temperature will rise much more when ignition occurs. The engine will melt when put into operation!Therefore, the claim of improved efficiency using an engine fabricated out of aluminum cannot be true.

P21.31 We suppose the air plus burnt gasoline behaves like a diatomic ideal gas. We find its final absolute pressure:

$$(21.0\ \text{atm})(50.0\ \text{cm}^3)^{7/5} = P_f(400\ \text{cm}^3)^{7/5}$$

$$P_f = (21.0\ \text{atm})\left(\frac{1}{8}\right)^{7/5} = 1.14\ \text{atm}$$

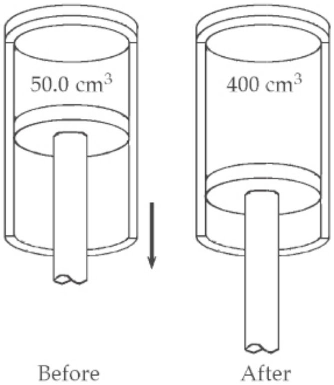
Now $Q = 0$ and $W = \Delta E_{\text{int}} = nC_V(T_f - T_i)$,

so $$W = \frac{5}{2}nRT_f - \frac{5}{2}nRT_i = \frac{5}{2}(P_f V_f - P_i V_i)$$

$$W = \frac{5}{2}[(1.14\ \text{atm})(400\ \text{cm}^3) - (21.0\ \text{atm})(50.0\ \text{cm}^3)]$$

$$= (-1\ 485\ \text{atm}\cdot\text{cm}^3)\left(\frac{1.013\times10^5\ \text{N/m}^2}{1\ \text{atm}}\right)(10^{-6}\ \text{m}^3/\text{cm}^3)$$

$$= -150\ \text{J}$$



ANS. FIG. P21.31

The output work is $-W = +150$ J

The time for this stroke is $\frac{1}{4}\left(\frac{1\text{ min}}{2\,500}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right) = 6.00\times10^{-3}$ s

$$P = \frac{-W}{\Delta t} = \frac{150\text{ J}}{6.00\times10^{-3}\text{ s}} = \boxed{25.0\text{ kW}}$$

P21.32 (a) $V_i = \pi\left(\frac{2.50\times10^{-2}\text{ m}}{2}\right)^2(0.500\text{ m}) = \boxed{2.45\times10^{-4}\text{ m}^3}$

(b) The quantity of air we find from $P_iV_i = nRT_i$:

$$n = \frac{P_iV_i}{RT_i} = \frac{(1.013\times10^5\text{ Pa})(2.45\times10^{-4}\text{ m}^3)}{(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})}$$

$$n = \boxed{9.97\times10^{-3}\text{ mol}}$$

(c) Absolute pressure = gauge pressure + external pressure:

$$P_f = 101.3\text{ kPa} + 800\text{ kPa} = 901.3\text{ kPa} = \boxed{9.01\times10^5\text{ Pa}}$$

(d) Adiabatic compression: $P_iV_i^\gamma = P_fV_f^\gamma$

$$V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = (2.45\times10^{-4}\text{ m}^3)\left(\frac{101.3}{901.3}\right)^{5/7}$$

$$V_f = \boxed{5.15\times10^{-5}\text{ m}^3}$$

(e) $P_fV_f = nRT_f$

$$T_f = T_i\frac{P_fV_f}{P_iV_i} = T_i\frac{P_f}{P_i}\left(\frac{P_i}{P_f}\right)^{1/\gamma} = T_i\left(\frac{P_i}{P_f}\right)^{(1/\gamma-1)}$$

$$T_f = 300\text{ K}\left(\frac{101.3}{901.3}\right)^{(5/7-1)} = \boxed{560\text{ K}}$$

(f) The work done on the gas in compressing it is $W = \Delta E_{\text{int}} = nC_V\Delta T$:

$$\Delta E_{\text{int}} = W = nC_V\Delta T$$

$$= (9.97\times10^{-3}\text{ mol})\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})(560\text{ K} - 300\text{ K})$$

$$\Delta E_{\text{int}} = \boxed{53.9\text{ J}}$$

(g) The pump wall has outer diameter 25.0 mm + 2.00 mm + 2.00 mm = 29.0 mm, and volume

$$\left[\pi\left(14.5\times10^{-3}\text{ m}\right)^2-\pi\left(12.5\times10^{-3}\text{ m}\right)^2\right]\times\left(4.00\times10^{-2}\text{ m}\right)=\boxed{6.79\times10^{-6}\text{ m}^3}$$

(h) The mass of the pump is given by

$$\rho V=\left(7.86\times10^{3}\text{ kg/m}^3\right)\left(6.79\times10^{-6}\text{ m}^3\right)=\boxed{53.3\text{ g}}$$

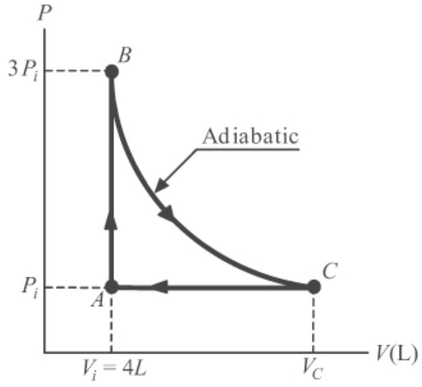
(i) Now imagine this energy being shared with the inner wall as the gas is held at constant volume. The overall warming process is described by

$$\Delta E_{\text{int}}=W=nC_V\Delta T+mc\Delta T\quad\rightarrow\quad\Delta T=\frac{W}{nC_V+mc}$$

Suppressing the units of *R*,

$$\Delta T=\frac{53.9\text{ J}}{\left(9.97\times10^{-3}\text{ mol}\right)\frac{5}{2}(8.314)+\left(0.053\,3\text{ kg}\right)\left(448\text{ J/kg}\cdot{}^\circ\text{C}\right)}$$
$$=2.24^\circ\text{C}=\boxed{2.24\text{ K}}$$

P21.33 (a) See ANS. FIG. P21.33(a) on the right.



ANS. FIG. P21.33(a)

(b) $P_BV_B^\gamma=P_CV_C^\gamma$

$3P_iV_i^\gamma=P_iV_C^\gamma$

$V_C=\left(3^{1/\gamma}\right)V_i=\left(3^{5/7}\right)V_i=2.19V_i$

$V_C=2.19(4.00\text{ L})=\boxed{8.77\text{ L}}$

(c) $P_BV_B=nRT_B=3P_iV_i=3nRT_i$

$T_B=3T_i=3(300\text{ K})=\boxed{900\text{ K}}$

(d) After one whole cycle, $T_A=T_i=\boxed{300\text{ K}}$.

(e) In *AB*, $Q_{AB}=nC_V\Delta V=n\left(\frac{5}{2}R\right)\left(3T_i-T_i\right)=(5.00)nRT_i$

$Q_{BC}=0$ as this process is adiabatic.

$P_CV_C=nRT_C=P_i\left(2.19V_i\right)=(2.19)nRT_i$

so $T_C = 2.19T_i$, and

$$Q_{CA} = nC_P\Delta T = n\left(\frac{7}{2}R\right)\left(T_i - 2.19T_i\right) = (-4.17)nRT_i$$

For the whole cycle,

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = (0.829)nRT_i$$

$$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$

$$W_{ABCA} = -Q_{ABCA} = -(0.829)nRT_i = -(0.829)P_iV_i$$

$$W_{ABCA} = -(0.829)\left(1.013\times10^5\ \text{Pa}\right)\left(4.00\times10^{-3}\ \text{m}^3\right) = \boxed{-336\ \text{J}}$$

P21.34 (a) Refer to ANS. FIG. P21.34(a).

(b) $P_BV_B^\gamma = P_CV_C^\gamma$

$$3P_iV_i^\gamma = P_iV_C^\gamma$$

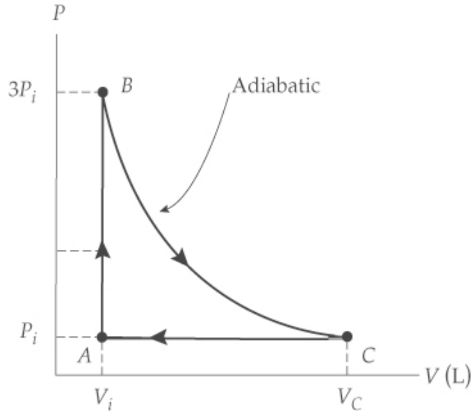
$$V_C = \boxed{\left(3^{1/\gamma}\right)V_i}$$

(c) $P_BV_B = nRT_B = 3P_iV_i = 3nRT_i$

$$T_B = \boxed{3T_i}$$

(d) After one whole cycle,

$T_A = \boxed{T_i}$.



ANS. FIG. P21.34(a)

(e) For *AB*,

$$Q_{AB} = nC_v\Delta T = n\frac{R}{\gamma-1}\Delta T = n\frac{R}{\gamma-1}\left(3T_\text{i} - T_\text{i}\right) = \frac{2nRT_\text{i}}{\gamma-1} = \frac{2P_\text{i}V_\text{i}}{\gamma-1}$$

$Q_{BC} = 0$ as this process is abiabatic.

$$P_CV_C = nRT_C = P_i\left(3^{1/\gamma}\right)V_i = \left(3^{1/\gamma}\right)nRT_i \text{ so } T_C = \left(3^{1/\gamma}\right)T_i$$

$$Q_{CA} = nC_P\Delta T = n\gamma C_V\left[T_i - \left(3^{1/\gamma}\right)T_i\right] = \gamma\frac{R}{\gamma-1}nT_i\left[1-\left(3^{1/\gamma}\right)\right]$$

$$= P_iV_i\gamma\left(\frac{1}{\gamma-1}\right)\left[1-\left(3^{1/\gamma}\right)\right]$$

For the whole cycle,

$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = \frac{2P_\text{i}V_\text{i}}{\gamma-1} + 0 + P_iV_i\gamma\left(\frac{1}{\gamma-1}\right)\left[1-3^{1/\gamma}\right]$$

$$Q_{ABCA} = P_iV_i\left[\frac{2}{\gamma-1}+\gamma\left(\frac{1}{\gamma-1}\right)\left[1-3^{1/\gamma}\right]\right]$$

$$= P_iV_i\left[\frac{2}{\gamma-1}+\left(\frac{\gamma-1+1}{\gamma-1}\right)\left[1-3^{1/\gamma}\right]\right]$$

$$Q_{ABCA} = P_iV_i\left[\frac{2}{\gamma-1}+\left(1-3^{1/\gamma}\right)+\left(\frac{1-3^{1/\gamma}}{\gamma-1}\right)\right]$$

$$= P_iV_i\left[\left(1-3^{1/\gamma}\right)+\left(\frac{3-3^{1/\gamma}}{\gamma-1}\right)\right]$$

$$\left(\Delta E_{\text{int}}\right)_{ABCA} = 0 = Q_{ABCA} + W_{ABCA}$$

$$W_{ABCA} = -Q_{ABCA} = \boxed{-P_iV_i\left[\left(\frac{1}{\gamma-1}\right)\left(1-3^{1/\gamma}\right)+\left(1-3^{1/\gamma}\right)\right]}$$

Section 21.5 Distribution of Molecular Speeds

***P21.35** The most probable speed is

$$v_{\text{mp}} = \sqrt{\frac{2k_BT}{m_0}} = \sqrt{\frac{2\left(1.38\times10^{-23}\text{ J/K}\right)(4.20\text{ K})}{6.64\times10^{-27}\text{ kg}}} = \boxed{132\text{ m/s}}$$

P21.36 (a) The average is

$$\bar{v} = \frac{\sum n_iv_i}{\sum n_i}$$

$$= \frac{1(2.00)+2(3.00)+3(5.00)+4(7.00)+3(9.00)+2(12.0)}{1+2+3+4+3+2}\text{ m/s}$$

$$\bar{v} = \boxed{6.80\text{ m/s}}$$

(b) To find the average squared speed we work out

$$\overline{v^2} = \frac{\sum n_iv_i^2}{\sum n_i}$$

$$\overline{v^2} = \left(\frac{1}{15}\right)\left[1\left(2.00^2\right)+2\left(3.00^2\right)+3\left(5.00^2\right)+4\left(7.00^2\right)+3\left(9.00^2\right)+2\left(12.0^2\right)\text{ m}^2/\text{s}^2\right]$$

$$\overline{v^2} = 54.9\text{ m}^2/\text{s}^2$$

Then the rms speed is

$$v_{\text{rms}} = \sqrt{\overline{v^2}} = \sqrt{54.9\ \text{m}^2/\text{s}^2} = \boxed{7.41\ \text{m/s}}$$

(c) More particles have $\boxed{v_{\text{mp}} = 7.00\ \text{m/s}}$ than any other speed.

P21.37 (a) The ratio of the number at higher energy to the number at lower energy is $e^{-\Delta E/k_BT}$, where ΔE is the energy difference. Here,

$$\Delta E = (10.2\ \text{eV})\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right) = 1.63\times10^{-18}\ \text{J}$$

and at 0°C,

$$k_BT = \left(1.38\times10^{-23}\ \text{J/K}\right)(273\ \text{K}) = 3.77\times10^{-21}\ \text{J}$$

Since this is much less than the excitation energy, nearly all the atoms will be in the ground state and the number excited is

$$\left(2.70\times10^{25}\right)\exp\left(\frac{-1.63\times10^{-18}\ \text{J}}{3.77\times10^{-21}\ \text{J}}\right) = \left(2.70\times10^{25}\right)e^{-433}$$

This number is much less than one, so
$\boxed{\text{almost all of the time no atom is excited.}}$

(b) At 10 000°C,

$$k_BT = \left(1.38\times10^{-23}\ \text{J/K}\right)(10\ 273\ \text{K}) = 1.42\times10^{-19}\ \text{J}$$

The number excited is

$$\left(2.70\times10^{25}\right)\exp\left(\frac{-1.63\times10^{-18}\ \text{J}}{1.42\times10^{-19}\ \text{J}}\right)$$

$$= \left(2.70\times10^{25}\right)e^{-11.5} = \boxed{2.70\times10^{20}}$$

P21.38 (a) $$\frac{V_{\text{rms, }35}}{V_{\text{rms, }37}} = \frac{\sqrt{3RT/M_{35}}}{\sqrt{3RT/M_{37}}} = \left(\frac{37.0\ \text{g/mol}}{35.0\ \text{g/mol}}\right)^{1/2} = \boxed{1.03}$$

(b) The lighter atom, $\boxed{^{35}\text{Cl}}$, moves faster.

P21.39 (a) From $v_{\text{avg}} = \sqrt{\dfrac{8k_BT}{\pi m_0}}$ we find the temperature as

$$T = \frac{\pi\left(6.64\times10^{-27}\ \text{kg}\right)\left(1.12\times10^{4}\ \text{m/s}\right)^2}{8\left(1.38\times10^{-23}\ \text{J/mol}\cdot\text{K}\right)} = \boxed{2.37\times10^{4}\ \text{K}}$$

(b) $$T = \frac{\pi\left(6.64\times 10^{-27}\ \text{kg}\right)\left(2.37\times 10^{3}\ \text{m/s}\right)^2}{8\left(1.38\times 10^{-23}\ \text{J/mol}\cdot\text{K}\right)} = \boxed{1.06\times 10^{3}\ \text{K}}$$

P21.40 For a molecule of diatomic nitrogen the mass is

$$m_0 = \frac{M}{N_A} = \frac{28.0\times 10^{-3}\ \text{kg/mol}}{6.02\times 10^{23}\ \text{molecules/mol}}$$
$$= 4.65\times 10^{-26}\ \text{kg/molecule}$$

(a) $$v_{\text{mp}} = \sqrt{\frac{2k_BT}{m_0}} = \sqrt{\frac{2(1.38\times 10^{-23}\ \text{J/molecule}\cdot\text{K})(900\ \text{K})}{4.65\times 10^{-26}\ \text{kg/molecule}}} = \boxed{731\ \text{m/s}}$$

(b) $$v_{\text{avg}} = \sqrt{\frac{8k_BT}{\pi m_0}} = \sqrt{\frac{8(1.38\times 10^{-23}\ \text{J/molecule}\cdot\text{K})(900\ \text{K})}{\pi\cdot 4.65\times 10^{-26}\ \text{kg/molecule}}} = \boxed{825\ \text{m/s}}$$

(c) $$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m_0}} = \sqrt{\frac{3(1.38\times 10^{-23}\ \text{J/molecule}\cdot\text{K})(900\ \text{K})}{4.65\times 10^{-26}\ \text{kg/molecule}}} = \boxed{895\ \text{m/s}}$$

(d) $\boxed{\text{The graph appears to be drawn correctly within about 10 m/s.}}$

P21.41 (a) From the Boltzmann distribution law, the number density of molecules with height *y* so that the gravitational potential energy of the molecule-Earth system is m_0gy is $n_0e^{-m_0gy/k_BT}$. These are the molecules with height *y*, so this is the number per volume at height *y* as a function of *y*.

(b) $$\frac{n(y)}{n_0} = e^{-m_0gy/k_BT} = e^{-Mgy/N_Ak_BT} = e^{-Mgy/RT}$$
$$= e^{-\left(28.9\times 10^{-3}\ \text{kg/mol}\right)\left(9.8\ \text{m/s}^2\right)\left(11\times 10^{3}\ \text{m}\right)/\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(293\ \text{K}\right)}$$
$$= e^{-1.279} = \boxed{0.278}$$

P21.42 In the Maxwell-Boltzmann speed distribution function take $\frac{dN_v}{dv} = 0$ to find

$$4\pi N\left(\frac{m_0}{2\pi k_BT}\right)^{3/2} \exp\left(-\frac{m_0v^2}{2k_BT}\right)\left(2v - \frac{2m_0v^3}{2k_BT}\right) = 0$$

and solve for *v* to find the most probable speed. Reject as solutions $v = 0$ and $v = \infty$. They describe minimally probable speeds.

Retain only $2-\frac{m_0 v^2}{k_B T}=0.$

Then, $v_{\text{mp}}=\sqrt{\frac{2k_B T}{m_0}}.$

P21.43 It is convenient in the following to define $a=\frac{m_0 g}{k_{\text{B}}T}$.

(a) We calculate

$$\int_0^\infty e^{-m_0 g y/k_B T}\,dy=\int_0^\infty e^{-ay}\,dy=\int_{y=0}^\infty e^{-ay}\left(-a\,dy\right)\left(-\frac{1}{a}\right)$$

$$=\left(-\frac{1}{a}\right)e^{-ay}\Big|_0^\infty=\left(-\frac{1}{a}\right)(0-1)=\frac{1}{a}$$

Using Table B.6 in the appendix,

$$\int_0^\infty ye^{-ay}dy=\frac{1!}{(a)^2}=\left(\frac{1}{a}\right)^2$$

Then,

$$y_{\text{avg}}=\frac{\int_0^\infty ye^{-ay}dy}{\int_0^\infty e^{-ay}dy}=\frac{(1/a)^2}{1/a}=\frac{1}{a}=\frac{k_B T}{m_0 g}$$

(b) $y_{\text{avg}}=\frac{k_B T}{(M/N_A)g}=\frac{RT}{Mg}=\frac{(8.314\text{ J/mol}\cdot\text{K})(283\text{ K})}{(28.9\times10^{-3}\text{ kg})(9.8\text{ m/s}^2)}=\boxed{8.31\text{ km}}$

Additional Problems

P21.44 (a) The average speed is given by $v_{\text{avg}} = \dfrac{\sum_i^N v_i}{N}$

which may be solved numerically for the values given. Suppressing units,

$$\begin{aligned} v_{\text{avg}} &= \frac{\sum_i^N v_i}{N} \\ &= \frac{\left[(3.00)+(4.00)+(5.80)+(2.50)+(3.60)+(1.90)+(3.80)+(6.60)\right]}{8} \\ &= \frac{31.2\ \text{km/s}}{8} = \boxed{3.90\ \text{km/s}} \end{aligned}$$

(b) The rms speed of the molecules is given by $v_{\text{rms}} = \sqrt{\dfrac{\sum_i^N v_i^2}{N}}$

which may be solved numerically for the values given. Suppressing units,

$$\begin{aligned} v_{\text{rms}} &= \sqrt{\frac{\sum_i^N v_i^2}{N}} \\ &= \sqrt{\frac{\left[(3.00)^2+(4.00)^2+(5.80)^2+(2.50)^2+(3.60)^2+(1.90)^2+(3.80)^2+(6.60)^2\right]}{8}} \\ &= \sqrt{\frac{\left[(9.00)+(16.00)+(33.64)+(6.25)+(12.96)+(3.61)+(14.44)+(43.56)\right]}{8}} \\ &= \sqrt{\frac{139.46\ \text{km}^2/\text{s}^2}{8}} = \sqrt{17.43\ \text{km}^2/\text{s}^2} = \boxed{4.18\ \text{km/s}} \end{aligned}$$

P21.45 (a) The total amount of oxygen in the tank is (using $PV = nRT$)

$$n = \frac{PV}{RT} = \frac{(125\ \text{atm})(6.88\ \text{L})}{(0.0821\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(21.0°\text{C}+273)} = 35.6\ \text{mol}$$

The rate at which the tank is being depleted (in moles / sec; again using *PV* = *nRT*) is

$$\begin{aligned}(\Delta n/\Delta t) &= \frac{P(\Delta V/\Delta t)}{RT}\\ &= \frac{(1\ \text{atm})(8.50\ \text{L/min})(1\,\text{min}/60\,\text{sec})}{(0.082\ 1\ \text{L}\cdot\text{atm/mol}\cdot\text{K})(21.0°\text{C}+273)}\\ &= 0.005\ 87\ \text{mol/s}\end{aligned}$$

The time interval to deplete the tank is the total molar mass divided by the molar rate:

$$\Delta t = n/(\Delta n\,/\,\Delta t) = \frac{35.6\ \text{mol}}{5.87\times10^{-3}\ \text{mol/s}}\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right) = \boxed{1.70\ \text{h}}$$

(b) Because the rms speed is dependent only on the molecular mass and the temperature, i.e.,

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

and because the masses of the molecule inside and at the outlet are the same, and the temperatures are the same (21.0°C), the rms speeds will be identical: the requested ratio is equal to $\boxed{1.00}$.

P21.46 (a) $n = \frac{PV}{RT} = \frac{(1.013\times10^5\ \text{Pa})(4.20\ \text{m})(3.00\ \text{m})(2.50\ \text{m})}{(8.314\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 1.31\times10^3\ \text{mol}$

$N = nN_A = (1.31\times10^3\ \text{mol})(6.02\times10^{23}\ \text{molecules/mol})$

$N = \boxed{7.89\times10^{26}\ \text{molecules}}$

(b) $m = nM = (1.31\times10^3\ \text{mol})(0.028\,9\ \text{kg/mol}) = \boxed{37.9\ \text{kg}}$

(c) $\frac{1}{2}m_0v^2 = \frac{3}{2}k_BT = \frac{3}{2}(1.38\times10^{-23}\ \text{J/k})(293\ \text{K}) = \boxed{6.07\times10^{-21}\ \text{J}}$

(d) For one molecule,

$$m_0 = \frac{M}{N_A} = \frac{0.028\,9\ \text{kg/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 4.80\times10^{-26}\ \text{kg/molecule}$$

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{2(6.07\times10^{-21}\ \text{J/molecule})}{4.80\times10^{-26}\ \text{kg/molecule}}} = \boxed{503\ \text{m/s}}$$

(e) $\boxed{0}$

(f) When the furnace operates, air expands and some of it leaves the room. The smaller mass of warmer air left in the room contains the same internal energy as the cooler air initially in the room.

P21.47 (a) The rms speed of molecules in a gas is related to the temperature by

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

which can be rearranged and solved numerically for the temperature:

$$T = \frac{mv_{\text{rms}}^2}{3k} = \frac{32\left(1.66\times10^{-27}\text{ kg}\right)\left(535\text{ m/s}\right)^2}{3\left(1.38\times10^{-23}\text{ J/K}\right)}$$

$$= \boxed{367\text{ K}}$$

(b) The rms speed is inversely related to the mass of the gas molecule (the mass is in the denominator of the square-root function above). The rms speed of nitrogen would be higher because the molar mass of nitrogen is less than that of oxygen.

(c) The rms speed of the nitrogen molecules is:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\text{ J/K}\right)(367\text{ K})}{28\left(1.66\times10^{-27}\text{ kg}\right)}} = \boxed{572\text{ m/s}}$$

P21.48 (a) The mean free path is given by:

$$\ell = \frac{1}{\sqrt{2}\pi d^2 N_V}$$

which can be solved numerically (noting that the pressure must be given as total pressure, not gauge pressure, and the temperature must be given in kelvins). Using *PV* = *nRT*:

$$N_V = N_A\left(\frac{n}{V}\right) = N_A\left(\frac{P}{RT}\right)$$

$$= \left(6.02\times10^{23}\text{ mol}^{-1}\right)\frac{(100\text{ atm} + 1\text{ atm})\left(1.013\times10^5\ \frac{\text{N/m}^2}{\text{atm}}\right)}{(8.314\text{ J/mol}\cdot\text{K})(25.0°\text{C} + 273.15°)}$$

$$= 2.48\times10^{27}\text{ molecules/m}^3$$

which can then be inserted into the mean free path equation:

$$\ell = \frac{1}{\sqrt{2}\pi d^2 N_V} = \frac{1}{\sqrt{2}\pi\left(2.00\times10^{-10}\text{ m}\right)^2\left(2.48\times10^{27}\text{ mol/m}^3\right)}$$

$$= \boxed{2.26\times10^{-9}\text{ m}}$$

(b) The average time interval between collisions is the inverse of the collision frequency:

$$t_{\text{collision}} = \frac{1}{f} = \frac{1}{\left(\frac{v_{\text{avg}}}{\ell}\right)} = \frac{\ell}{v_{\text{avg}}}$$

which can be solved numerically (where the value of v_{ave} is obtained from eqn. 21.26):

$$t_{\text{collision}} = \frac{\ell}{v_{\text{avg}}} = \frac{\ell}{\sqrt{\frac{8k_bT}{\pi m_0}}} = \frac{2.26\times10^{-9}\text{ m}}{\sqrt{\frac{8\left(1.38\times10^{-23}\text{ J/K}\right)\left(298.15\text{ K}\right)}{\pi\cdot 32\left(1.66\times10^{-27}\text{ kg}\right)}}}$$

$$= \frac{2.26\times10^{-9}\text{ m}}{\left(444.1\text{ m/s}\right)} = \boxed{5.09\times10^{-12}\text{ seconds}}$$

P21.49 For the system of the bullet, Equation 8.2 becomes

$$W_{\text{on bullet}} = \Delta K$$

For the system of the gas undergoing an adiabatic process, so that $Q = 0$, Equation 8.2 becomes

$$W_{\text{on gas}} = \Delta E_{\text{int}}$$

Recognizing that $W_{\text{on bullet}} = -W_{\text{on gas}}$, we see that

$$\Delta E_{\text{int}} = -\Delta K$$

Substituting for the internal energy, we find

$$nC_V\Delta T = -\Delta K$$

$$n\left(\frac{5}{2}R\right)\Delta T = -\Delta K \quad\rightarrow\quad \frac{5}{2}nR\left(T_f - T_i\right) = -\Delta K$$

Now use Equation 21.20 to substitute for T_f:

$$\frac{5}{2}nR\left[\left(\frac{V_i}{V_f}\right)^{\gamma-1}T_i - T_i\right] = -\Delta K$$

$$\frac{5}{2}nRT_i\left[\left(\frac{V_i}{V_f}\right)^{\gamma-1}-1\right]=-\Delta K$$

$$\frac{5}{2}P_iV_i\left[\left(\frac{V_i}{V_f}\right)^{\gamma-1}-1\right]=-\left(\frac{1}{2}mv^2-0\right)$$

$$P_i=\frac{mv^2}{5V_i\left[1-\left(\frac{V_i}{V_f}\right)^{\gamma-1}\right]}$$

The final volume of the gas is equal to the initial volume plus the volume of the rifle barrel:

$$V_f=V_i+Ah=12.0\ \text{cm}^3+\left(0.0300\ \text{cm}^2\right)(50.0\ \text{cm})=13.5\ \text{cm}^3$$

Substituting numerical values,

$$P_i=\frac{(0.001\,10\ \text{kg})(120\ \text{m/s})^2}{5\left(12\times10^{-6}\ \text{m}^3\right)\left[1-\left(\frac{12.0\ \text{cm}^3}{13.5\ \text{cm}^3}\right)^{0.400}\right]}$$

$$=5.74\times10^6\ \text{Pa}=\boxed{56.6\ \text{atm}}$$

P21.50 (a) For a pure metallic element, one atom is one molecule. Its energy can be represented as

$$\frac{1}{2}m_0v_x^2+\frac{1}{2}m_0v_y^2+\frac{1}{2}m_0v_z^2+\frac{1}{2}k_xx^2+\frac{1}{2}k_yy^2+\frac{1}{2}k_zz^2$$

Its average value is

$$\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT=3k_BT$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}}/n=3RT$, and the molar heat capacity at constant volume is $E_{\text{int}}/nT=3R$.

(b) We calculate the specific heat from

$$3(8.314\ \text{J/mol}\cdot\text{K})=\frac{3(8.314\ \text{J})}{\left(55.845\times10^{-3}\ \text{kg}\right)\cdot\text{K}}=\boxed{447\ \text{J/kg}\cdot\text{K}}$$

$\boxed{\text{This agrees with the tabulated value of 448 J/kg}\cdot{}^\circ\text{C within 0.3\%.}}$

(c) For gold,

$$3(8.314\ \text{J/mol}\cdot\text{K}) = \frac{3(8.314\ \text{J})}{(197\times10^{-3}\ \text{kg})\cdot\text{K}} = \boxed{127\ \text{J/kg}\cdot\text{K}}$$

$\boxed{\text{This agrees with the tabulated value of } 129\ \text{J/kg}\cdot{}^\circ\text{C within } 2\%.}$

P21.51 **(i)** (a) $P_f = \boxed{100\ \text{kPa}}$

(b) $$V_f = \frac{nRT_f}{P_f}$$

$$= \frac{2.00\ \text{mol}(8.314\ \text{J/mol}\cdot\text{K})(400\ \text{K})}{100\times10^3\ \text{Pa}} = 0.066\ 5\ \text{m}^3 = \boxed{66.5\ \text{L}}$$

(c) $T_f = \boxed{400\ \text{K}}$

(d) $$\Delta E_{\text{int}} = \frac{7}{2}nR\Delta T = \frac{7}{2}(2.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(100\ \text{K})$$

$$= \boxed{5.82\ \text{kJ}}$$

(e) $$Q = nC_P\Delta T = \frac{9}{2}(2.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(100\ \text{K})$$

$$= \boxed{7.48\ \text{kJ}}$$

(f) $$W = -P\Delta V = -nR\Delta T = -(2.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(100\ \text{K})$$

$$= \boxed{-1.66\ \text{kJ}}$$

(ii) (a) For an isovolumetric process:

$$\frac{P_f}{T_f} = \frac{P_i}{T_i}$$

$$\to P_f = P_i\frac{T_f}{T_i} = (1.00\times10^5\ \text{Pa})\left(\frac{400\ \text{K}}{300\ \text{K}}\right) = \boxed{133\ \text{kPa}}$$

(b) $$V_f = V_i = \frac{nRT_i}{P_i}$$

$$= \frac{(2.00\ \text{mol})(8.314\ \text{J/mol}\cdot\text{K})(300\ \text{K})}{100\times10^3\ \text{Pa}} = 0.049\ 9\ \text{m}^3$$

$$= \boxed{49.9\ \text{L}}$$

(c) $T_f = \boxed{400\ \text{K}}$

(d) $\Delta E_{\text{int}} = \frac{7}{2}nR\Delta T = \boxed{5.82\text{ kJ}}$ as in (**i**) part (d).

(e) $Q = nC_V\Delta T = \frac{7}{2}nR\Delta T = \frac{7}{2}(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(100\text{ K})$
$= \boxed{5.82\text{ kJ}}$

(f) $W = -\int P dV = \boxed{0}$ since V = constant

(**iii**) (a) $P_f = \boxed{120\text{ kPa}}$

(b) $V_f = V_i\left(\frac{P_i}{P_f}\right) = (49.9\text{ L})\left(\frac{100\text{ kPa}}{120\text{ kPa}}\right) = \boxed{41.6\text{ L}}$

(c) $T_f = T_i = \boxed{300\text{ K}}$

(d) $\Delta E_{\text{int}} = \frac{7}{2}nR\Delta T = \boxed{0}$ since T = constant

(e) From (f), $Q = \Delta E_{\text{int}} - W = 0 - 909\text{ J} = \boxed{-909\text{ J}}$

(f) $W = -\int P dV = -nRT_i\int_{V_i}^{V_f}\frac{dV}{V} = -nRT_i\ln\left(\frac{V_f}{V_i}\right) = -nRT_i\ln\left(\frac{P_i}{P_f}\right)$

$W = -(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(300\text{ K})\ln\left(\frac{100\text{ kPa}}{120\text{ kPa}}\right)$
$= \boxed{+909\text{ J}}$

(**iv**) (a) $P_f = \boxed{120\text{ kPa}}$

(b) $\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V} = \frac{\frac{7}{2}R + R}{\frac{7}{2}R} = \frac{\frac{9}{2}}{\frac{7}{2}} = \frac{9}{7}$

$P_f V_f^\gamma = P_i V_i^\gamma$, so

$V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = (49.9\text{ L})\left(\frac{100\text{ kPa}}{120\text{ kPa}}\right)^{7/9} = \boxed{43.3\text{ L}}$

(c) $PV = nRT \rightarrow \frac{T_i}{P_i V_i} = \frac{T_f}{P_f V_f}$

$$\rightarrow T_f = T_i\left(\frac{P_f V_f}{P_i V_i}\right) = (300\text{ K})\left(\frac{120\text{ kPa}}{100\text{ kPa}}\right)\left(\frac{43.3\text{ L}}{49.9\text{ L}}\right) = \boxed{312\text{ K}}$$

(d) $\Delta E_{\text{int}} = \frac{7}{2}nR\Delta T = \frac{7}{2}(2.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(12.4\text{ K})$

$= \boxed{+722\text{ J}}$

(e) $Q = \boxed{0}$

(f) $W = -Q + \Delta E_{\text{int}} = 0 + 722\text{ J} = \boxed{+722\text{ J}}$

P21.52 (a) The $\boxed{\text{pressure increases as volume decreases}}$ (and vice versa), so *dV*/*dP* is always negative.

In equation form, $\frac{dV}{dP} < 0$ and $-\left(\frac{1}{V}\right)\left(\frac{dV}{dP}\right) > 0$

(b) For an ideal gas, $V = \frac{nRT}{P}$ and $\kappa_1 = -\frac{1}{V}\frac{d}{dP}\left(\frac{nRT}{P}\right)$

For isothermal compression, *T* is constant and the derivative gives us

$$\kappa_1 = -\frac{nRT}{V}\left(\frac{-1}{P^2}\right) = \frac{1}{P}$$

(c) For an adiabatic compression, $PV^\gamma = C$ (where *C* is a constant) and we evaluate *dV*/*dP* as follows:

$$\kappa_2 = -\left(\frac{1}{V}\right)\frac{d}{dP}\left(\frac{C}{P}\right)^{1/\gamma} = \left(\frac{1}{V\gamma}\right)\frac{C^{1/\gamma}}{\left(P^{1/\gamma+1}\right)} = \frac{V}{V\gamma P} = \frac{1}{\gamma P}$$

(d) $\kappa_1 = \frac{1}{P} = \frac{1}{(2.00\text{ atm})} = \boxed{0.500\text{ atm}^{-1}}$

(e) For a monatomic ideal gas, $\gamma = \frac{C_P}{C_V} = \frac{5}{3}$, so

$$\kappa_2 = \frac{1}{\gamma P} = \frac{1}{\frac{5}{3}(2.00\text{ atm})} = \boxed{0.300\text{ atm}^{-1}}$$

P21.53 The pressure of the gas in the lungs of the diver must be the same as the absolute pressure of the water at this depth of 50.0 meters. This is:

$$P = P_0 + \rho g h$$

$$= 1.00\text{ atm} + \left(1.03\times10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(50.0\text{ m})$$

or $$P = 1.00\text{ atm} + 5.05\times 10^5\text{ Pa}\left(\frac{1.00\text{ atm}}{1.013\times 10^5\text{ Pa}}\right) = 5.98\text{ atm}$$

If the partial pressure due to the oxygen in the gas mixture is to be 1.00 atmosphere (or the fraction $\frac{1}{5.98}$ of the total pressure), oxygen molecules should make up only $\frac{1}{5.98}$ of the total number of molecules. This will be true if 1.00 mole of oxygen is used for every 4.98 mole of helium. The ratio by weight is then

$$\frac{(4.98\text{ mol He})(4.003\text{ g/mol He})g}{(1.00\text{ mol O}_2)(2\times 15.999\text{ g/mol O}_2)g} = \boxed{0.623}$$

P21.54 Sulfur dioxide is the gas with the greatest molecular mass of those listed. If the effective spring constants for various chemical bonds are comparable, SO_2 can then be expected to have low frequencies of atomic vibration. Vibration can be excited at lower temperature than for the other gases. Some vibration may be going on at 300 K. With more degrees of freedom for molecular motion, the material has higher specific heat.

P21.55 $$n = \frac{m}{M} = \frac{1.20\text{ kg}}{0.028\,9\text{ kg/mol}} = 41.5\text{ mol}$$

(a) $$V_i = \frac{nRT_i}{P_i} = \frac{(41.5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(298\text{ K})}{200\times 10^3\text{ Pa}} = \boxed{0.514\text{ m}^3}$$

(b) $$\frac{P_f}{P_i} = \frac{\sqrt{V_f}}{\sqrt{V_i}} \quad\text{so}\quad V_f = V_i\left(\frac{P_f}{P_i}\right)^2 = (0.514\text{ m}^3)\left(\frac{400}{200}\right)^2 = \boxed{2.06\text{ m}^3}$$

(c) $$T_f = \frac{P_fV_f}{nR} = \frac{(400\times 10^3\text{ Pa})(2.06\text{ m}^3)}{(41.5\text{ mol})(8.314\text{ J/mol}\cdot\text{K})} = \boxed{2.38\times 10^3\text{ K}}$$

(d) $$W = -\int_{V_i}^{V_f} P\,dV = -C\int_{V_i}^{V_f} V^{1/2}\,dV = -\left(\frac{P_i}{V_i^{1/2}}\right)\frac{2V^{3/2}}{3}\Bigg|_{V_i}^{V_f}$$

$$= -\frac{2}{3}\left(\frac{P_i}{V_i^{1/2}}\right)\left(V_f^{3/2} - V_i^{3/2}\right)$$

$$W = -\frac{2}{3}\left(\frac{200\times 10^3\text{ Pa}}{\sqrt{0.514\text{ m}^3}}\right)\left[(2.06\text{ m}^3)^{3/2} - (0.514\text{ m}^3)^{3/2}\right] = \boxed{-480\text{ kJ}}$$

(e) $\Delta E_{\text{int}} = nC_V\Delta T = (41.5\text{ mol})\left[\frac{5}{2}(8.314\text{ J/mol}\cdot\text{K})\right](2.38\times10^3 - 298)\text{ K}$

$= 1.80\times10^6\text{ J}$

$Q = \Delta E_{\text{int}} - W = 1.80\times10^6\text{ J} + 4.80\times10^5\text{ J} = 2.28\times10^6\text{ J} = \boxed{2.28\text{ MJ}}$

P21.56 (a) Begin with Equation 17.8 and substitute the definition of bulk modulus from Equation 12.8:

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1}{\rho}\left(-V\frac{dP}{dV}\right)}$$

Now substitute using Equation 21.37 with the constant on the right hand side represented by *K*:

$$v = \sqrt{\frac{-V}{\rho}\frac{d}{dV}\left(KV^{-\gamma}\right)} = \sqrt{\frac{-KV}{\rho}\left(-\gamma V^{-\gamma-1}\right)} = \sqrt{\frac{\gamma}{\rho}\left(KV^{-\gamma}\right)}$$

$$= \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma}{\rho}\frac{nRT}{V}} = \sqrt{\frac{\gamma(m/M)RT}{m}} = \sqrt{\frac{\gamma RT}{M}}$$

(b) $v = \sqrt{\frac{1.40(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})}{0.028\,9\text{ kg/mol}}} = \boxed{344\text{ m/s}}$

$\boxed{\text{This agrees within 0.2\% with the 343 m/s listed in Table 17.1.}}$

(c) We use $k_B = \frac{R}{N_A}$ and $M = m_0 N_A$: $v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{\gamma k_B N_A T}{m_0 N_A}} = \sqrt{\frac{\gamma k_B T}{m_0}}$

(d) The most probable molecular speed is $\sqrt{\frac{2k_BT}{m_0}}$, the average speed is $\sqrt{\frac{8k_BT}{\pi m_0}}$, and the rms speed is $\sqrt{\frac{3k_BT}{m_0}}$.

$\boxed{\text{The speed of sound is somewhat less than each measure of molecular speed. Sound propagation is orderly motion overlaid on the disorder of molecular motion.}}$

P21.57 (a) The average speed v_{avg} is just the weighted average of all the speeds.

$$v_{\text{avg}} = \frac{2(v)+3(2v)+5(3v)+4(4v)+3(5v)+2(6v)+1(7v)}{2+3+5+4+3+2+1}$$

$$= \boxed{3.65v}$$

(b) First find the average of the square of the speeds,

$$\begin{aligned}(v^2)_{\text{avg}} &= \frac{2(v)^2+3(2v)^2+5(3v)^2+4(4v)^2+3(5v)^2+2(6v)^2+1(7v)^2}{2+3+5+4+3+2+1}\\ &= 15.95v^2\end{aligned}$$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{(v_{avg})^2} = \boxed{3.99v}$

(c) The most probable speed is the one that most of the particles have; i.e., five particles have speed $\boxed{3.00v}$.

(d) $PV = \frac{1}{3}Nm_0v^2_{\text{av}}$

Therefore, $P = \frac{20}{3}\frac{\left[m_0(15.95)v^2\right]}{V} = \boxed{106\left(\frac{m_0v^2}{V}\right)}$

(e) The average kinetic energy for each particle is

$$\bar{K} = \frac{1}{2}m_0v^2_{\text{av}} = \frac{1}{2}m_0\left(15.95v^2\right) = \boxed{7.98m_0v^2}$$

P21.58 (a) For the adiabatic process $PV^\gamma = k$, a constant. The work is

$$W = -\int_i^f PdV = -k\int_{V_i}^{V_f}\frac{dV}{V^\gamma} = \left.\frac{-kV^{1-\gamma}}{1-\gamma}\right|_{V_i}^{V_f}$$

For k we can substitute $P_iV_i^\gamma$ and also $P_fV_f^\gamma$ to have

$$W = -\frac{P_fV_f^\gamma V_f^{1-\gamma} - P_iV_i^\gamma V_i^{1-\gamma}}{1-\gamma} = \frac{P_fV_f - P_iV_i}{\gamma-1}$$

(b) For an adiabatic process $\Delta E_{\text{int}} = Q + W$ and $Q = 0$. Therefore,

$$W = \Delta E_{\text{int}} = nC_V\Delta T = nC_V\left(T_f - T_i\right)$$

. (c) $\boxed{\text{The expressions are equal because } PV = nRT \text{ and } \gamma = (C_V+R)/C_V = 1 + R/C_V \text{ give } R = (\gamma-1)C_V\text{, so } PV = n(\gamma-1)C_VT \text{ and } PV/(\gamma-1) = nC_VT.}$

P21.59 (a) $\Delta E_{\text{int}} = Q + W = 0 + W \quad\rightarrow\quad W = nC_V\left(T_f - T_i\right)$

$$-2\ 500\text{ J} = (1\text{ mol})\left(\frac{3}{2}\right)(8.314\text{ J/mol}\cdot\text{K})\left(T_f - 500\text{ K}\right)$$

$T_f = \boxed{300\text{ K}}$

(b) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_i\left(\frac{nRT_i}{P_i}\right)^\gamma = P_f\left(\frac{nRT_f}{P_f}\right)^\gamma$$

$$T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$$

$$\frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right)^{\gamma/(\gamma-1)}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right)^{(5/3)(3/2)} = (3.60\text{ atm})\left(\frac{300}{500}\right)^{5/2} = \boxed{1.00\text{ atm}}$$

P21.60 (a) The process is adiabatic:

$$\Delta E_{\text{int}} = Q + W = 0 + W \quad\rightarrow\quad W = nC_V\left(T_f - T_i\right)$$

For an ideal gas,

$$W = nC_V\left(T_f - T_i\right) = n\left(\frac{3}{2}\right) R\left(T_f - T_i\right)$$

Solving for final temperature, we get

$$T_f = \boxed{T_i + \frac{2}{3}\frac{W}{nR}}$$

(b) $P_iV_i^\gamma = P_fV_f^\gamma$

$$P_i\left(\frac{nRT_i}{P_i}\right)^\gamma = P_f\left(\frac{nRT_f}{P_f}\right)^\gamma \quad\rightarrow\quad T_i^\gamma P_i^{1-\gamma} = T_f^\gamma P_f^{1-\gamma}$$

$$\rightarrow\quad \frac{T_i^{\gamma/(\gamma-1)}}{P_i} = \frac{T_f^{\gamma/(\gamma-1)}}{P_f} \quad\rightarrow\quad P_f = P_i\left(\frac{T_f}{T_i}\right)^{\gamma/(\gamma-1)}$$

where $\gamma = \frac{C_p}{C_V} = \frac{5}{3}$ for an ideal gas, and $\frac{\gamma}{\gamma-1} = \frac{\frac{5}{3}}{\frac{5}{3}-1} = \frac{\frac{5}{3}}{\frac{2}{3}} = \frac{5}{2}$.

Substituting this and the result from part (a) gives

$$P_f = P_i\left(\frac{T_i + \frac{2}{3}\frac{W}{nR}}{T_i}\right)^{5/2} = \boxed{P_i\left(1+\frac{2}{3}\frac{W}{nRT_i}\right)^{5/2}}$$

***P21.61** (a) Let *d* = 2*r* represent the diameter of the particle. Its mass is $m = \rho V = \rho\frac{4}{3}\pi r^3 = \rho\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 = \frac{\rho\pi d^3}{6}$. Then $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$ gives

$$\frac{\rho\pi d^3}{6}v_{\text{rms}}^2 = 3kT$$

so

$$v_{\text{rms}} = \left(\frac{18kT}{\rho\pi d^3}\right)^{1/2} = \left(\frac{18(1.38\times10^{-23}\ \text{J/K})(293\ \text{K})}{\pi(1\ 000\ \text{kg/m}^3)}\right)^{1/2} d^{-3/2}$$

$$= \boxed{(4.81\times10^{-12})d^{-3/2}}$$

where v_{rms} is in meters per second and *d* is in meters.

(b) $v = d/\Delta t \quad \rightarrow \quad (4.81\times10^{-12}\ \text{m}^{5/2}/\text{s})d^{-3/2} = d/\Delta t$

$$\Delta t = \frac{d}{(4.81\times10^{-12}\ \text{m}^{5/2}/\text{s})d^{-3/2}}$$

$$= \boxed{(2.08\times10^{11})d^{5/2}}$$

where Δt is in seconds and *d* is in meters.

(c) $$v_{\text{rms}} = \left(\frac{18kT}{\rho\pi d^3}\right)^{1/2} = \left(\frac{18(1.38\times10^{-23}\ \text{J/K})(293\ \text{K})}{(1\ 000\ \text{kg/m}^3)\pi(3\times10^{-6}\ \text{m})^3}\right)^{1/2}$$

$$= \boxed{0.926\ \text{mm/s}}$$

$$v = \frac{x}{\Delta t} \quad \rightarrow \quad \Delta t = \frac{x}{v} = \frac{3\times10^{-6}\ \text{m}}{9.26\times10^{-4}\ \text{m/s}} = \boxed{3.24\ \text{ms}}$$

(d) $$70\ \text{kg} = (1\ 000\ \text{kg/m}^3)\frac{\pi d^3}{6} \quad \rightarrow \quad d = 0.511\ \text{m}$$

$$v_{\text{rms}} = \left(\frac{18kT}{\rho\pi d^3}\right)^{1/2} = \left(\frac{18(1.38\times10^{-23}\ \text{J/K})(293\ \text{K})}{(1\ 000\ \text{kg/m}^3)\pi(0.511\ \text{m})^3}\right)^{1/2}$$

$$= \boxed{1.32\times10^{-11}\ \text{m/s}}$$

$$\Delta t = \frac{0.511\ \text{m}}{1.32\times 10^{-11}\ \text{m/s}} = \boxed{3.88\times 10^{10}\ \text{s}} = 1\,230\ \text{yr}$$

This motion is too slow to observe.

P21.62 (a) Maxwell's speed distribution function is

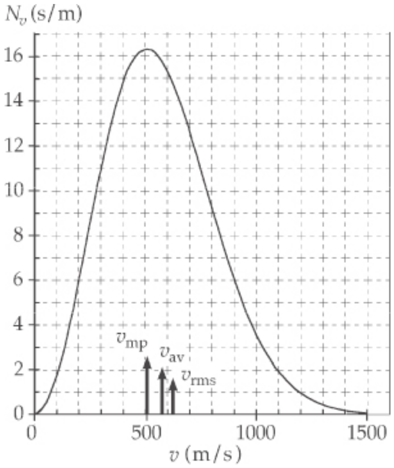
$$N_v = 4\pi N\left(\frac{m_0}{2\pi k_B T}\right)^{3/2} v^2 e^{-m_0 v^2/2k_B T}$$

With $N = 1.00\times 10^4$, $m_0 = \frac{M}{N_A} = \frac{0.032\ \text{kg}}{6.02\times 10^{23}} = 5.32\times 10^{-26}\ \text{kg}$,

T = 500 K, and $k_B = 1.38\times 10^{-23}\ \text{J/molecule}\cdot\text{K}$, this becomes

$$N_v = \left(1.71\times 10^{-4}\right)v^2 e^{-\left(3.85\times 10^{-6}\right)v^2}$$

ANS. FIG. P21.62(a) below is a plot of this function for the range $0 \le v \le 1\,500\ \text{m/s}$.



ANS. FIG. P21.62(a)

(b) The most probable speed occurs where N_v is a maximum.

From the graph, $\boxed{v_{\text{mp}} \approx 510\ \text{m/s}}$.

(c)
$$v_{\text{avg}} = \sqrt{\frac{8k_B T}{\pi m_0}}$$

$$= \sqrt{\frac{8\left(1.38\times 10^{-23}\ \text{J/molecule}\cdot\text{K}\right)\left(500\ \text{K}\right)}{\pi\left(5.32\times 10^{-26}\ \text{kg}\right)}} = \boxed{575\ \text{m/s}}$$

Also,

$$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m_0}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/molecule}\cdot\text{K}\right)(500\ \text{K})}{5.32\times10^{-26}\ \text{kg}}}$$
$$= \boxed{624\ \text{m/s}}$$

(d) The fraction of particles in the range $300\ \text{m/s} \le v \le 600\ \text{m/s}$

is

$$\frac{\int_{300}^{600} N_v\,dv}{N}$$

where $N = 10^4$ and the integral of N_v is read from the graph as the area under the curve. This is approximately the area of a large rectangle 11 s/m high and 300 m/s wide [corners at (300, 0), (300, 11), (600, 11), and (600, 0)], plus a smaller rectangle 5.5 s/m high and 100 m/s wide [corners at (500, 11), (500, 16.6), (600, 16.5), and (600, 11)], plus a triangle 5.5 s/m high with a 200 m/s base [vertices at (300, 11), (500, 16.5), and (500, 11)]:

$$(11)(300) + (5.5)(100) + (1/2)(5.5)(200) = 4\,400$$

and the fraction is 0.44 or $\boxed{44\%}$.

P21.63 For the system of the ball and the air, Equation 8.2 gives us,

$$\Delta K + \Delta E_{\text{int}} = 0$$

Substitute for the internal energy and solve for the temperature increase of the air:

$$\Delta K + nC_V\Delta T = 0 \rightarrow \Delta T = \frac{-\Delta K}{nC_V} = \frac{-\Delta K}{(m/M)\left(\frac{5}{2}R\right)} = -\frac{2M\Delta K}{5mR}$$

Express the mass *m* of the air in terms of the density and volume of the cylinder through which the ball passes, and evaluate the change in kinetic energy of the ball:

$$\Delta T = -\frac{2M\Delta K}{5\rho VR} = -\frac{2M\left(\frac{1}{2}m_{\text{ball}}v_f^2 - \frac{1}{2}m_{\text{ball}}v_i^2\right)}{5\rho\left(\pi r^2\ell\right)R}$$
$$= \frac{Mm_{\text{ball}}\left(v_i^2 - v_f^2\right)}{5\pi\rho r^2\ell R}$$

Substitute numerical values:

$$\Delta T = \frac{\left(28.9\times10^{-3}\ \text{kg/mol}\right)\left(0.142\ \text{kg}\right)\left[\left(47.2\ \text{m/s}\right)^2-\left(42.5\ \text{m/s}\right)^2\right]}{5\pi\left(1.20\ \text{kg/m}^3\right)\left(0.037\ 0\ \text{m}\right)^2\left(16.8\ \text{m}\right)\left(8.314\ \text{J/mol}\cdot{}^\circ\text{C}\right)}$$

$$= \boxed{0.480^\circ\text{C}}$$

P21.64 (a) The latent heat of evaporation per molecule is

$$2\ 430\ \text{J/g} = \left(2430\ \text{J/g}\right)\left(\frac{18.0\ \text{g}}{1\ \text{mol}}\right)\left(\frac{1\ \text{mol}}{6.02\times10^{23}\ \text{molecule}}\right)$$

$$= \boxed{7.27\times10^{-20}\ \text{J/molecule}}$$

If the molecule is about to break free, we assume that it possesses the energy as translational kinetic energy.

(b) Consider one gram of these molecules. From $K=\frac{1}{2}mv^2$ we obtain

$$v=\sqrt{\frac{2K}{m}}=\sqrt{\frac{2\left(2\ 430\ \text{J}\right)}{10^{-3}\ \text{kg}}}=2.20\times10^{3}\ \text{m/s}=\boxed{2.20\ \text{km/s}}$$

(c) The total translational kinetic energy of an ideal gas is $\frac{3}{2}nrT$, so we have

$$\left(2\ 430\ \text{J/g}\right)\left(\frac{18.0\ \text{g}}{1\ \text{mol}}\right)=\frac{3}{2}\left(1\ \text{mol}\right)\left(8.314\ \text{J/mol}\cdot\text{K}\right)T$$

which gives

$$T=\boxed{3.51\times10^{3}\ \text{K}}$$

(d) The evaporating particles emerge with much less kinetic energy, as negative work is performed on them by restraining forces as they leave the liquid. Much of the initial kinetic energy is used up in overcoming the latent heat of vaporization. There are also very few of these escaping at any moment in time.

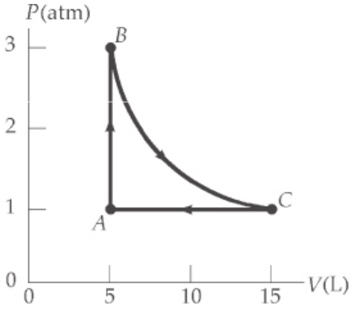
P21.65 (a) $$n=\frac{PV}{RT}=\frac{\left(1.013\times10^{5}\ \text{Pa}\right)\left(5.00\times10^{-3}\ \text{m}^3\right)}{\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(300\ \text{K}\right)}$$

$$=\boxed{0.203\ \text{mol}}$$

(b) $$T_B=T_A\left(\frac{P_B}{P_A}\right)=\left(300\ \text{K}\right)\left(\frac{3.00}{1.00}\right)=\boxed{900\ \text{K}}$$

(c) $T_C = T_B = \boxed{900\text{ K}}$

(d) $V_C = V_A\left(\frac{T_C}{T_A}\right) = (5.00\text{ L})\left(\frac{900}{300}\right) = \boxed{15.0\text{ L}}$



ANS. FIG. P21.65

(e) $\boxed{\text{}}$ $A \to B$: lock the piston in place and put the cylinder into an oven at 900 K, gradually heating the gas. $B \to C$: keep the sample in the oven while gradually letting the gas expand to lift a load on the piston as far as it can. $C \to A$: carry the cylinder back into the room at 300 K and let the gas gradually cool and contract without touching the piston.

(f) For *A*→*B*: $W = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V\Delta T = n\left(\frac{3}{2}\right)R\Delta T = Q + W$$

$$\Delta E_{\text{int}} = (0.203\text{ mol})\left(\frac{3}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(600\text{ K}) = \boxed{1.52\text{ kJ}}$$

$$Q = \Delta E_{\text{int}} - W = \boxed{1.52\text{ kJ}}$$

For *B*→*C*: $\Delta E_{\text{int}} = \boxed{0}$, because $\Delta T = 0$;

$$W = -nRT_B\ln\left(\frac{V_C}{V_B}\right)$$

$$W = -(0.203\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(900\text{ K})\ln(3.00)$$

$$= \boxed{-1.67\text{ kJ}}$$

$$Q = \Delta E_{\text{int}} - W = \boxed{1.67\text{ kJ}}$$

For *C*→*A*: $\Delta E_{\text{int}} = nC_V\Delta T$

$$\Delta E_{\text{int}} = (0.203\text{ mol})\left(\frac{3}{2}\right)(8.314\text{ J/mol}\cdot\text{K})(-600\text{ K})\text{J}$$

$$= \boxed{-1.52\text{ kJ}}$$

$$W = -P\Delta V = -nR\Delta T$$

$$= -(0.203\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(-600\text{ K})$$

$$= \boxed{1.01\text{ kJ}}$$

$$Q = \Delta E_{\text{int}} - W = -1.52\text{ kJ} - 1.01\text{ kJ} = \boxed{-2.53\text{ kJ}}$$

(g) We add the amounts of energy for each process to find them for the whole cycle.

$$Q_{ABCA} = +1.52\text{ kJ} + 1.67\text{ kJ} - 2.53\text{ kJ} = \boxed{0.656\text{ kJ}}$$

$$W_{ABCA} = 0 - 1.67\text{ kJ} + 1.01\text{ kJ} = \boxed{-0.656\text{ kJ}}$$

$$(\Delta E_{\text{int}})_{ABCA} = Q_{ABCA} + W_{ABCA} = +1.52\text{ kJ} + 0 - 1.52\text{ kJ} = \boxed{0}$$

For any cyclic process, $\Delta E_{\text{int}} = 0$.

P21.66 (a) The effect of large centripetal acceleration is like the effect of a very high gravitational field on an atmosphere. The result is:

$\boxed{\text{The larger-mass molecules settle to the outside}}$ while the region at smaller *r* has a higher concentration of low-mass molecules.

(b) Consider a single kind of molecule, all of mass m_0. To cause the centripetal acceleration of the molecules between *r* and *r* + *dr*, the inward force must increase with increasing distance from the center according to $\sum F_r = m_0 a_r$. Taking the positive direction toward the center of the centrifuge, we have

$$(P + dP)A - PA = n_V(m_0 A\,dr)(r\omega^2)$$

where $n_V = n_V(r) = N/V$, the number of molecules per unit volume, is an implicit function of *r*, and *A* is the area of any cylindrical shell of thickness *dr* and radius *r*. The equation reduces to

$$dP = n_V m_0 \omega^2 r\,dr \qquad \textbf{[1]}$$

But also within any small cylindrical shell,

$$PV = Nk_BT \rightarrow P = \left(\frac{N}{V}\right)k_BT$$

$$\rightarrow dP = d\left(\frac{N}{V}\right)k_BT = d\left(n_V\right)k_BT = dn_Vk_BT$$

Therefore, equation [1] becomes

$$dn_Vk_BT = n_Vm_0\omega^2rdr \quad \rightarrow \quad \frac{dn_V}{n_V} = \frac{m_0\omega^2}{k_BT}rdr$$

giving $\int_{n_0}^{n} \frac{dn_V}{n_V} = \frac{m_0\omega^2}{k_BT}\int_0^r rdr$, where $n_V\left(r = 0\right) = n_0$.

Integrating, we find

$$\ln\left(n_V\right)\Big|_{n_0}^{n_V} = \frac{m_0\omega^2}{k_BT}\left(\frac{r^2}{2}\right)\Bigg|_0^r \quad \rightarrow \quad \ln\left(\frac{n_V}{n_0}\right) = \frac{m_0\omega^2}{2k_BT}r^2$$

and solving for $n \equiv n_V$, we have $\boxed{n = n_0e^{m_0r^2\omega^2/2k_BT}}$.

P21.67 $N_v\left(v\right) = 4\pi N\left(\frac{m_0}{2\pi k_BT}\right)^{3/2} v^2 \exp\left(\frac{-m_0v^2}{2k_BT}\right)$, where exp(*x*) represents e^x

Note that $v_{\text{mp}} = \left(\frac{2k_BT}{m_0}\right)^{1/2}$.

Thus, $N_v\left(v\right) = 4\pi N\left(\frac{m_0}{2\pi k_BT}\right)^{3/2} v^2e^{\left(-v^2/v_{\text{mp}}^2\right)}$

and $\frac{N_v\left(v\right)}{N_v\left(v_{\text{mp}}\right)} = \left(\frac{v}{v_{\text{mp}}}\right)^2 e^{\left(1-v^2/v_{\text{mp}}^2\right)}$.

For $v = \frac{v_{\text{mp}}}{50}$,

$$\frac{N_v\left(v\right)}{N_v\left(v_{\text{mp}}\right)} = \left(\frac{1}{50}\right)^2 e^{\left[1-\left(1/50\right)^2\right]} = 1.09 \times 10^{-3}$$

The other values are computed similarly, with the following results:

	$\dfrac{v}{v_{\text{mp}}}$	$\dfrac{N_v(v)}{N_v(v_{\text{mp}})}$
(a)	$\dfrac{1}{50}$	1.09×10^{-3}
(b)	$\dfrac{1}{10}$	2.69×10^{-2}
(c)	$\dfrac{1}{2}$	0.529
(d)	1	1.00
(e)	2	0.199
(f)	10	1.01×10^{-41}
(g)	50	$1.25 \times 10^{-1\,082}$

To find the last value, we note:

$$(50)^2 e^{1-2\,500} = 2\,500e^{-2\,499}$$

$$10^{\log 2\,500} e^{(\ln 10)(-2\,499/\ln 10)} = 10^{\log 2\,500} 10^{-2\,499/\ln 10} = 10^{\log 2\,500 - 2\,499/\ln 10}$$

$$= 10^{-1\,081.904} = 10^{0.096} \times 10^{-1\,082}$$

P21.68 (a) The energy of one molecule can be represented as

$$\frac{1}{2}m_0 v_x^2 + \frac{1}{2}m_0 v_y^2 + \frac{1}{2}m_0 v_z^2 + \frac{1}{2}I\omega_x^2 + \frac{1}{2}I\omega_z^2$$

Its average value is

$$\frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T + \frac{1}{2}k_B T = \frac{5}{2}k_B T$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}} / n = \dfrac{5}{2}RT$.

And the molar heat capacity at constant volume is

$E_{\text{int}} / nT = \boxed{\dfrac{5}{2}R}$.

(b) The energy of one molecule can be represented as

$$\frac{1}{2}m_0v_x^2+\frac{1}{2}m_0v_y^2+\frac{1}{2}m_0v_z^2+\frac{1}{2}I\omega_x^2+\frac{1}{2}I\omega_z^2+\frac{1}{2}I\omega_y^2$$

Its average value is

$$\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT=3k_BT$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}}/n = 3RT$.

And the molar heat capacity at constant volume is $E_{\text{int}}/nT = \boxed{3R}$.

(c) Let the modes of vibration be denoted by 1 and 2. The energy of one molecule can be represented as

$$\frac{1}{2}m_0\left(v_x^2+_0 v_y^2+v_z^2\right)+\frac{1}{2}I\omega_x^2+\frac{1}{2}I\omega_z^2 +\left(\frac{1}{2}\mu v_{\text{rel}}^2+\frac{1}{2}kx^2\right)_1+\left(\frac{1}{2}\mu v_{\text{rel}}^2+\frac{1}{2}kx^2\right)_2$$

Its average value is

$$\frac{3}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT=\frac{9}{2}k_BT$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}}/n=\frac{9}{2}RT$.

And the molar heat capacity at constant volume is $E_{\text{int}}/nT=\boxed{\frac{9}{2}R}$.

(d) The energy of one molecule can be represented as

$$\frac{1}{2}m_0\left(v_x^2+_0 v_y^2+v_z^2\right)+\frac{1}{2}I\omega_x^2+\frac{1}{2}I\omega_z^2+\frac{1}{2}I\omega_y^2 +\left(\frac{1}{2}\mu v_{\text{rel}}^2+\frac{1}{2}kx^2\right)_1+\left(\frac{1}{2}\mu v_{\text{rel}}^2+\frac{1}{2}kx^2\right)_2$$

Its average value is

$$\frac{3}{2}k_BT+\frac{3}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT+\frac{1}{2}k_BT=5k_BT$$

The energy of one mole is obtained by multiplying by Avogadro's number, $E_{\text{int}}/\text{n} = 5RT$.

And the molar heat capacity at constant volume is $E_{\text{int}}/\text{n}T = \boxed{5R}$.

(e) Measure the constant-volume specific heat of the gas as a function of temperature and look for plateaus on the graph. If the first jump goes from $\frac{3}{2}R$ to $\frac{5}{2}R$, the molecules can be diagnosed as linear. If the first jump goes from $\frac{3}{2}R$ to $3R$, the molecules must be nonlinear. The tabulated data at one temperature are insufficient for the determination. At room temperature some of the heavier molecules appear to be vibrating.

P21.69 (a) First find $\overline{v^2}$ as $\overline{v^2} = \frac{1}{N}\int_0^\infty v^2 N_v\, dv$. Let $a = \frac{m_0}{2k_BT}$.

Then, $$\overline{v^2} = \frac{\left[4N\pi^{-1/2}a^{3/2}\right]}{N}\int_0^\infty v^4 e^{-av^2dv} = \left[4a^{3/2}\pi^{-1/2}\right]\frac{3}{8a^2}\sqrt{\frac{\pi}{a}} = \frac{3k_BT}{m}$$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{\overline{v^2}} = \boxed{\sqrt{\frac{3k_BT}{m_0}}}$

(b) To find the average speed, we have

$$v_{\text{avg}} = \frac{1}{N}\int_0^\infty vN_v\, dv = \frac{\left(4Na^{3/2}\pi^{-1/2}\right)}{N}\int_0^\infty v^3 e^{-av^2}\, dv = \frac{4a^{3/2}\pi^{-1/2}}{2a^2}$$

$$= \boxed{\sqrt{\frac{8k_BT}{\pi m_0}}}$$

P21.70 We want to evaluate $\frac{dP}{dV}$ for the function implied by *PV* = *nRT* = constant, and also for the different function implied by $PV^\gamma =$ constant. We can use implicit differentiation:

From *PV* = constant $$P\frac{dV}{dV} + V\frac{dP}{dV} = 0 \rightarrow \left(\frac{dP}{dV}\right)_{\text{isotherm}} = -\frac{P}{V}$$

From PV^γ = constant $$P\gamma V^{\gamma-1} + V^\gamma\frac{dP}{dV} = 0 \rightarrow \left(\frac{dP}{dV}\right)_{\text{adiabat}} = -\frac{\gamma P}{V}$$

Therefore, $$\left(\frac{dP}{dV}\right)_{\text{adiabat}} = \gamma\left(\frac{dP}{dV}\right)_{\text{isotherm}}$$

The theorem is proved.

P21.71 (a) The number of molecules in the pot is given by

$$(10\,000\text{ g})\left(\frac{1.00\text{ mol}}{18.0\text{ g}}\right)\left(\frac{6.02\times10^{23}\text{ molecules}}{1.00\text{ mol}}\right)$$

$$=\boxed{3.34\times10^{26}\text{ molecules}}$$

(b) Each day, $\frac{1}{10}$ of the original molecules are left in the pot. Let us find out how many days are required for there to be one molecule left. We solve the following equation for n_d, the number of days:

$$\text{Number left }=1=\left(\tfrac{1}{10}\right)^{n_d}N \quad\rightarrow\quad \frac{1}{N}=\left(\tfrac{1}{10}\right)^{n_d}$$

where *N* is the original number of molecules. Take the logarithm of both sides:

$$\log\frac{1}{N}=-\log N=\log\left(\tfrac{1}{10}\right)^{n_d}=n_d\log\left(\tfrac{1}{10}\right)=-n_d\log 10=-n_d$$

$$\rightarrow\quad n_d=\log N$$

Substitute the numerical value for *N*:

$$n_d=\log\left(3.34\times10^{26}\right)=26.5$$

$$\boxed{\text{Therefore, the last molecule is ladled out after the 26th day and so during the 27th day.}}$$

(c) The soup is this fraction of the hydrosphere: $\left(\dfrac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)$

Therefore, today's soup likely contains this fraction of the original molecules. The number of original molecules likely in the pot again today is

$$\left(\frac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)\left(3.34\times10^{26}\text{ molecules}\right)$$

$$=\boxed{2.53\times10^{6}\text{ molecules}}$$

P21.72 (a) Consider the molecule-Earth system to be isolated. Treat an escaping molecule as going from $r=R_E$, $v=v_0$, to $r=\infty$, $v=0$:

$$\Delta K+\Delta U=0$$

$$\left(0-\frac{1}{2}m_0v^2\right)+\left[0-\left(-\frac{Gm_0M}{R_E}\right)\right]=0 \quad\rightarrow\frac{1}{2}m_0v^2=\frac{Gm_0M}{R_E}$$

Since the free-fall acceleration at the surface is, $g = \dfrac{GM}{R_E^2}$, this can also be written as: $\dfrac{1}{2}m_0v^2 = \dfrac{Gm_0M}{R_E} = \boxed{m_0gR_E}$

(b) For O_2, the mass of one molecule is

$$m_0 = \frac{0.0320 \text{ kg/mol}}{6.02\times10^{23} \text{ molecules/mol}} = 5.32\times10^{-26} \text{ kg/molecule}$$

Then, if $m_0gR_E = 10\left(\dfrac{3k_BT}{2}\right)$, the temperature is

$$T = \frac{m_0gR_E}{15k_B} = \frac{(5.32\times10^{-26}\text{ kg})(9.80 \text{ m/s}^2)(6.37\times10^6 \text{ m})}{15(1.38\times10^{-23} \text{ J/mol}\cdot\text{K})}$$

$$= \boxed{1.60\times10^4 \text{ K}}$$

P21.73 (a) For sodium atoms (with a molar mass *M* = 23.0 g / mol):

$$\frac{1}{2}m_0v^2 = \frac{3}{2}k_BT$$

$$\frac{1}{2}\left(\frac{M}{N_A}\right)v^2 = \frac{3}{2}k_BT$$

$$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.314 \text{ J/mol}\cdot\text{K})(2.40\times10^{-4} \text{ K})}{23.0\times10^{-3} \text{ kg}}}$$

$$= \boxed{0.510 \text{ m/s}}$$

(b) $\Delta t = \dfrac{d}{v_{\text{rms}}} = \dfrac{0.010\ 0 \text{ m}}{0.510 \text{ m/s}} = 19.6 \text{ ms} \approx \boxed{20 \text{ ms}}$

Challenge Problems

P21.74 (a) The average value of a collection of particle speeds is

$$v_{\text{avg}} = \frac{\sum_i^N v_i}{N}$$

Use this equation to find the average for the two speeds given in the problem:

$$v_{\text{avg}} = \frac{v_1 + v_2}{2} = \frac{av_{\text{avg}} + (2-a)v_{\text{avg}}}{2} = v_{\text{avg}}$$

(b) The rms average value of a collection of particle speeds is

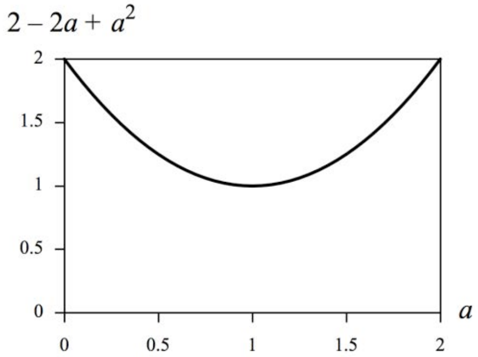
$$v_{\text{rms}} = \frac{\sum_{i}^{N} v_i^2}{N}$$

Use this equation to find the square of the rms average for the two speeds given in the problem:

$$\begin{aligned} v_{\text{rms}}^2 = \frac{v_1^2 + v_2^2}{2} &= \frac{\left(av_{\text{avg}}\right)^2 + \left[\left(2-a\right)v_{\text{avg}}\right]^2}{2} \\ &= \frac{v_{\text{avg}}^2}{2}\left[a^2 + \left(4 - 4a + a^2\right)\right] \\ &= v_{\text{avg}}^2\left(2 - 2a + a^2\right) \end{aligned}$$

$$v_{\text{rms}}^2 = v_{\text{avg}}^2\left(2 - 2a + a^2\right) \qquad \textbf{[1]}$$

(c) The graph of $(2 - 2a + a^2)$ versus a appears below, over the range of possible values $0 \le a \le 2$.



ANS. FIG. P21.74(c)

Because the factor $(2 - 2a + a^2)$ is generally larger than 1, equation [1] tells us that $v_{\text{rms}} > v_{\text{avg}}$ except at one point in the graph.

(d) From the graph, we see that that $v_{\text{rms}} = v_{\text{avg}}$ when the factor $(2 - 2a + a^2) = 1$, which occurs at $\boxed{a = 1}$.

P21.75 Let the subscripts "1" and "2" refer to the hot and cold compartments, respectively. The pressure is higher in the hot compartment, therefore the hot compartment expands and the cold compartment contracts. Because the walls of the cylinder are insulating, the total internal energy of the system must remain constant:

$$\Delta E_{\text{int}} = 0:$$

$$\Delta E_{\text{int},1} + \Delta E_{\text{int},2} = 0$$

$$nC_V\Delta T_1 + nC_V\Delta T_2 = 0$$

$$\left(T_{1f} - T_{1i}\right) + \left(T_{2f} - T_{2i}\right) = 0$$

$$T_{1f} + T_{2f} = T_{1i} + T_{2i} = 550\text{ K} + 250\text{ K} = 800\text{ K} \qquad [1]$$

Consider the adiabatic changes of the gases.

$$P_{1i}V_{1i}^{\gamma} = P_{1f}V_{1f}^{\gamma} \quad \text{and} \quad P_{2i}V_{2i}^{\gamma} = P_{2f}V_{2f}^{\gamma}$$

or

$$\frac{P_{1i}V_{1i}^{\gamma}}{P_{2i}V_{2i}^{\gamma}} = \frac{P_{1f}V_{1f}^{\gamma}}{P_{2f}V_{2f}^{\gamma}}$$

The initial volumes are equal: $V_{1i} = V_{2i}$. Applying the particle in equilibrium model to the piston, the final force on each side of the piston must be the same, and the areas on each side are equal, therefore $P_{1f} = P_{2f}$. The equation simplifies to

$$\frac{P_{1i}}{P_{2i}} = \left(\frac{V_{1f}}{V_{2f}}\right)^{\gamma}$$

Using the ideal gas law:

$$\frac{nRT_{1i}/V_{1i}}{nRT_{2i}/V_{2i}} = \left(\frac{nRT_{1f}/P_{1f}}{nRT_{2f}/P_{2f}}\right)^{\gamma}$$

Simplifying, this gives

$$\frac{T_{1i}}{T_{2i}} = \left(\frac{T_{1f}}{T_{2f}}\right)^{\gamma}$$

since $V_{1i} = V_{2i}$ and $P_{1f} = P_{2f}$, substituting values, we get

$$\frac{T_{1f}}{T_{2f}} = \left(\frac{T_{1i}}{T_{2i}}\right)^{1/\gamma} = \left(\frac{550\text{ K}}{250\text{ K}}\right)^{1/1.4} = 1.756 \qquad [2]$$

Solving equations [1] and [2] simultaneously gives

$$\boxed{T_{1f} = 510\text{ K},\ T_{2f} = 290\text{ K}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P21.2	(a) 8.76×10^{-21} J; (b) For helium, $v_{\text{rms}} = 1.62$ km/s and for argon, $v_{\text{rms}} = 514$ m/s
P21.4	3.21×10^{21} molecules
P121.6	$\frac{3}{2}\frac{PV}{\bar{K}N_A}$
P21.8	2.78×10^{-23} kg · m/s
P21.10	501 K
P21.12	(a) 385 K; (b) 7.97×10^{-21} J; (c) the molecular mass of the gas
P21.14	(a) $W = 0$; (b) $\Delta E_{\text{int}} = 209$ J; (c) 317 K
P21.16	(a) 118 kJ; (b) 6.03×10^{3} kg
P21.18	(a) 0.719 kJ/kg · K; (b) 0.811 kg; (c) 233 kJ; (d) 327 kJ
P21.20	Between 10^{-3} °C and 10^{-2} °C
P21.22	(a) See P21.22(a) for full explanation; (b) See P21.22(b) for full explanation; (c) See P21.22(c) for full explanation
P21.24	The maximum possible value of $\gamma = 1 + \frac{R}{C_V} = 1.67$ occurs for the lowest possible value for $C_V = \frac{3}{2}R$. Therefore the claim of $\gamma = 1.75$ for the newly discovered gas cannot be true.
P21.26	(a) 1.39 atm; (b) 366 K and 253 K; (c) $Q = 0$; (d) –4.66 kJ; (e) –4.66 kJ
P21.28	(a) 28.0 kJ; (b) 46.0 kJ; (c) 10.0 atm; (d) 25.1 atm
P21.30	The compressed gas would reach a temperature of 941 K, exceeding the melting point of aluminum. Therefore, the claim of improved efficiency using an engine fabricated out of aluminum cannot be true.
P21.32	(a) 2.45×10^{-4} m^3; (b) 9.97×10^{-3} mol; (c) 9.01×10^{5} Pa; (d) 5.15×10^{-5} m^3; (e) 560 K; (f) 53.9 J; (g) 6.79×10^{-6} m^3; (h) 53.3 g; (i) 2.24 K
P21.34	(a) See ANS. FIG. P21.34(a); (b) $\left(3^{1/\gamma}\right)V_i$; (c) $3T_i$; (d) T_i; (e) $-P_iV_i\left[\left(\frac{1}{\gamma-1}\right)\left(1-3^{1/\gamma}\right)+\left(1-3^{1/\gamma}\right)\right]$
P21.36	(a) 6.80 m/s; (b) 7.41 m/s; (c) 7.00 m/s
P21.38	(a) 1.03; (b) ^{35}Cl

P21.40 (a) 731 m / s; (b) 825 m / s; (c) 895 m / s; (d) The graph appears to be drawn correctly within about 10 m / s.

P21.42 See P21.42 for the full explanation.

P21.44 (a) 3.90 km / s; (b) 4.18 km / s

P21.46 (a) 7.89×10^{26} molecules; (b) 37.9 kg; (c) 6.07×10^{-21} J; (d) 503 m / s; (e) 0; (f) When the furnace operates, air expands and some of it leaves the room. The smaller mass of warmer air left in the room contains the same internal energy as the cooler air initially in the room.

P21.48 (a) 2.26×10^{-9} m; (b) 5.09×10^{-12} seconds

P21.50 (a) See P21.50(a) for the full explanation; (b) 447 J / kg · °C. This agrees with the tabulated value of 448 J / kg · °C within 0.3%; (c) 127 J / kg · °C. This agrees with the tabulated value of 129 J / kg · °C within 2%

P21.52 (a) pressure increases as volume decreases; (b) See P21.52(b) for full answer; (c) See P21.52(c) for full answer; (d) 0.500 atm^{-1}; (e) 0.300 atm^{-1}

P21.54 Sulfur dioxide is the gas with the greatest molecular mass of those listed. If the effective spring constants for various chemical bonds are comparable, SO_2 can then be expected to have low frequencies of atomic vibration. Vibration can be excited at lower temperature than for other gases. Some vibration may be going on at 300 K. With more degrees of freedom for molecular motion, the material has higher specific heat.

P21.56 (a) See P21.56(a) for full explanation; (b) This agrees within 0.2% with the 343 m / s listed in the Table 17.1; (c) See P21.56(c) for full answer; (d) The speed of sound is somewhat less than each measure of molecular speed. Sound propagation is orderly motion overlaid on the disorder of molecular motion.

P21.58 (a) See P21.58(a) for full explanation; (b) See P21.58(b) for full explanation; (c) The expressions are equal because $PV = nRT$ and $\gamma = (C_V + R)/C_V = 1 + R/C_V$ give $R = (\gamma - 1)C_V$, so $PV = n(\gamma - 1)C_V T$ and $PV/(\gamma - 1) = nC_V T$.

P21.60 (a) $T_i + \frac{2}{3}\frac{W}{nR}$; (b) $P_i\left(1 + \frac{2}{3}\frac{W}{nRT_i}\right)^{5/2}$

P21.62 (a) See ANS. FIG. P21.62(a); (b) $v_{\text{mp}} \approx 510$ m / s; (c) 575 m / s, 624 m / s; (d) 44%

P21.64 (a) 7.27×10^{-20} J / molecule; (b) 2.20 km / s; (c) 3.51×10^{3} K; (d) The evaporating particles emerge with much less kinetic energy, as negative work is performed on them by restraining forces as they leave the liquid. Much of the initial kinetic energy is used up in overcoming the latent heat of vaporization. There are also very few of these escaping at any moment in time.

P21.66 (a) The larger-mass molecules settle to the outside; (b) $n = n_0 e^{m_0 r^2 \omega^2 / 2k_B T}$

P21.68 (a) $\frac{5}{2}R$; (b) 3R; (c) $\frac{9}{2}R$; (d) 5R; (e) Measure the constant-volume specific heat of the gas as a function of temperature and look for plateaus on the graph. If the first jump goes from $\frac{3}{2}R$ to $\frac{5}{2}R$, the molecules can be diagnosed as linear. If the first jump goes from $\frac{3}{2}R$ to 3R, the molecules must be nonlinear. The tabulated data at one temperature are insufficient for the determination. At room temperature some of the heavier molecules appear to be vibrating.

P21.70 See P21.70 for full explanation.

P21.72 (a) $m_0 g R_E$; (b) 1.60×10^4 K

P21.74 (a) See P21.74(a) for full explanation; (b) See P21.74(b) for full explanation; (c) See ANS FIG P21. 74(c); (d) $a = 1$

22

Heat Engines, Entropy, and the Second Law of Thermodynamics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ22.1 Answer (d). The second law says that you must put in some work to pump heat from a lower-temperature to a higher-temperature location. But it can be very little work if the two temperatures are very nearly equal.

OQ22.2 Answer (d). Heat input will not *necessarily* produce an entropy increase, because a heat input could go on simultaneously with a larger work output, to carry the gas to a lower-temperature, lower-entropy final state. Work input will not *necessarily* produce an entropy increase, because work input could go on simultaneously with heat output to carry the gas to a lower-volume, lower-entropy final state. Either temperature increase at constant volume, or volume increase at constant temperature, or simultaneous increases in both temperature and volume, will necessarily end in a higher-entropy final state.

OQ22.3 Answer (c). The coefficient of performance of this refrigerator is

$$\text{COP} = \frac{|Q_c|}{W} = \frac{115 \text{ kJ}}{18.0 \text{ kJ}} = 6.39$$

OQ22.4 Answer (c). Choice (c) is a statement of the first law of thermodynamics, *not* the second law. Choices (a), (b), (d), and (e) are alternative statements of the second law, (a) being the Kelvin-Planck formulation, (b) the Carnot statement, (d) the Clausius statement, and (e) summarizes the primary consequence of all these various statements.

OQ22.5 (i) Answer (b). (ii) Answer (a). (iii) Answer (b). (iv) Answer (a). (v) Answer (c). (vi) Answer (a). For any cyclic process the total input energy must be equal to the total output energy. This is a consequence of the first law of thermodynamics. It is satisfied by processes (ii), (iv), (v), and (vi) but not by processes (i) and (iii). The second law says that a cyclic process that takes in energy by heat must put out some of the energy by heat. This is not satisfied for process (v).

OQ22.6 Answer (a). The air conditioner operates on a cyclic process so the change in the internal energy of the refrigerant is zero. Then, the conservation of energy gives the thermal energy exhausted to the room as $Q_h = Q_c + W_{\text{eng}}$, where Q_c is the thermal energy the air conditioner removes from the room and W_{eng} is the work done to operate the device. Since $W_{\text{eng}} > 0$, the air conditioner is returning more thermal energy to the room than it is removing, so the average temperature in the room will increase.

OQ22.7 Answer (c). The maximum theoretical efficiency (the Carnot efficiency) of a device operating between absolute temperatures $T_c < T_h$ is $e_c = 1 - T_c / T_h$. For the given steam turbine, this is

$$e_c = 1 - \frac{3.0 \times 10^2 \text{ K}}{450 \text{ K}} = 0.33 \quad \text{or} \quad 33\%.$$

OQ22.8 Answer (d). The whole Universe must have an entropy change of zero or more. The environment around the system comprises the rest of the Universe, and must have an entropy change of +8.0 J/K, or more.

OQ22.9 Answer: $E > D > C > B > A$. Recall that for and ideal gas, $PV = nRT$, and $C_v = 3R/2$ and $C_p = 5R/2$.

Process A: isobaric, volume V goes to $0.5V$, so temperature T goes to $0.5T$, $dQ = nC_p dT$, so $dS = nC_p dT/T$; therefore $\Delta S = -\frac{5}{2} nR \ln 2$.

Process *B*: isothermal, volume *V* goes to 0.5*V*, so temperature *T* is constant and pressure *P* goes to 2*P*, $dQ = pdV = nRT(dV/V)$, so $dS = nR(dV/V)$; therefore $\Delta S = -nR \ln 2$.

Process *C*: adiabatic, $Q = 0$; therefore $\Delta S = 0$.

Process *D*: isovolumetric, pressure *P* goes to 2*P*, so temperature *T* goes to 2*T*, $dQ = nC_v dT$, so $dS = n\,C_v\,dT/T$; therefore $\Delta S = \frac{3}{2} nR \ln 2$.

Process *E*: isobaric, volume *V* goes to 2*V*; therefore $\Delta S = \frac{5}{2} nR \ln 2$.

OQ22.10 Answer (b). From conservation of energy, the energy input to the engine must be

$$Q_h = W_{\text{eng}} + Q_c = 15.0 \text{ kJ} + 37.0 \text{ kJ} = 52.0 \text{ kJ}$$

so the efficiency is

$$e = \frac{W_{\text{eng}}}{Q_c} = \frac{15.0 \text{ kJ}}{52.0 \text{ kJ}} = 0.288 \quad \text{or} \quad 28.8\%.$$

OQ22.11 Answer (b). In the reversible adiabatic expansion *OA,* the gas does work against a piston, takes in no energy by heat, and so drops in internal energy and in temperature. In the free adiabatic expansion *OB*, there is no piston, no work output, constant internal energy, and constant temperature for the ideal gas. Point *A* is at a lower temperature than *O* and point *C* is at an even lower temperature. The only point that could possibly have the same temperature as *O* is point *B*.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ22.1 (a) The reduced flow rate of ‘cooling water’ reduces the amount of heat exhaust Q_c that the plant can put out each second. Even with constant efficiency, the rate at which the turbines can take in heat is reduced and so is the rate at which they can put out work to the generators. If anything, the efficiency will drop, because the smaller amount of water carrying the heat exhaust will tend to run hotter. The steam going through the turbines will undergo a smaller temperature change. Thus there are two reasons for the work output to drop.

(b) The engineer’s version of events, as seen from inside the plant, is complete and correct. Hot steam pushes hard on the front of a turbine blade. Still-warm steam pushes less hard on the back of the blade, which turns in response to the pressure difference. Higher temperature at the heat exhaust port in the lake works its way back to a corresponding higher temperature of the steam leaving a turbine blade, a smaller temperature drop across the blade, and a lower work output.

CQ22.2 One: Energy flows by heat from a hot bowl of chili into the cooler surrounding air. Heat lost by the hot stuff is equal to heat gained by the cold stuff, but the entropy decrease of the hot stuff is less than the entropy increase of the cold stuff.

Two: As you inflate a soft car tire at a service station, air from a tank at high pressure expands to fill a larger volume. That air increases in entropy and the surrounding atmosphere undergoes no significant entropy change.

Three: The brakes of your car get warm as you come to a stop. The shoes and drums increase in entropy and nothing loses energy by heat, so nothing decreases in entropy.

CQ22.3 No. The first law of thermodynamics is a statement about energy conservation, while the second is a statement about stable thermal equilibrium. They are by no means mutually exclusive. For the particular case of a cycling heat engine, the first law implies $|Q_h| = W_{\text{eng}} + |Q_c|$, and the second law implies $|Q_c| > 0$.

CQ22.4 Take an automobile as an example. According to the first law or the idea of energy conservation, it must take in all the energy it puts out. Its energy source is chemical energy in gasoline. During the combustion process, some of that energy goes into moving the pistons and eventually into the mechanical motion of the car. The chemical potential energy turning into internal energy can be modeled as energy input by heat. The second law says that not all of the energy input can become output mechanical energy. Much of the input energy must and does become energy output by heat, which, through the cooling system, is dissipated into the atmosphere. Moreover, there are numerous places where friction, both mechanical and fluid, turns mechanical energy into internal energy. In even the most efficient internal combustion engine cars, less than 30% of the energy from the fuel actually goes into moving the car. The rest ends up as useless internal energy in the atmosphere.

CQ22.5 Either statement can be considered an instructive analogy. We choose to take the first view. All processes require energy, either as energy content or as energy input. The kinetic energy that it possessed at its formation continues to make the Earth go around. Energy released by nuclear reactions in the core of the Sun drives weather on the Earth and essentially all processes in the biosphere.

The energy intensity of sunlight controls how lush a forest or jungle can be and how warm a planet is. Continuous energy input is not required for the motion of the planet. Continuous energy input is required for life because energy tends to be continuously degraded, as heat flows into lower-temperature sinks. The continuously increasing entropy of the Universe is the index to energy-transfers completed.

CQ22.6 (a) A slice of hot pizza cools off. Road friction brings a skidding car to a stop. A cup falls to the floor and shatters. Your cat dies. Any process is irreversible if it looks funny or frightening when shown in a videotape running backwards. (b) The free flight of a projectile is nearly reversible.

CQ22.7 (a) When the two sides of the semiconductor are at different temperatures, an electric potential (voltage) is generated across the material, which can drive electric current through an external circuit. The two cups at 50°C contain the same amount of internal energy as the pair of hot and cold cups. But no energy flows by heat through the converter bridging between them and no voltage is generated across the semiconductors.

(b) A heat engine must put out exhaust energy by heat. The cold cup provides a sink to absorb output or wasted energy by heat, which has nowhere to go between two cups of equally warm water.

CQ22.8 A higher steam temperature means that more energy can be extracted from the steam. For a constant temperature heat sink at T_c, and steam at T_h, the efficiency of the power plant goes as $\frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$ and is maximized for a high T_h.

CQ22.9 (a) For an expanding ideal gas at constant temperature, the internal energy stays constant. The gas must absorb by heat the same amount of energy that it puts out by work. Then its entropy change is $\Delta S = \frac{\Delta Q}{T} = nR\ln\left(\frac{V_2}{V_1}\right)$.

(b) For a reversible adiabatic expansion, $\Delta Q = 0$ and $\Delta S = 0$. An ideal gas undergoing an irreversible adiabatic expansion can have any positive value for ΔS up to the value given in part (a).

CQ22.10 No. Your roommate creates “order” locally, but as she works, she transfers energy by heat to the room, causing the net entropy to increase. An analogy used by Carnot is instructive: A waterfall continuously converts mechanical energy into internal energy. It continuously creates entropy as the motion of the falling water turns into molecular motion at the bottom of the falls. We humans put turbines into the waterfall, diverting some of the energy stream to our use. Water flows spontaneously from high to low elevation and energy spontaneously flows by heat from high to low temperature. Into the great flow of solar radiation from Sun to Earth, living things put themselves. They live on energy flow, more than just on energy. A basking snake diverts energy from a high-temperature source (the Sun) through itself temporarily, before the energy inevitably is radiated from the body of the snake to a low-temperature sink (outer space). A tree builds cellulose molecules and we build libraries and babies who look like their grandmothers, all out of a thin diverted stream in the universal flow of energy. We do not violate the second law, for we build local reductions in the entropy of one thing within the inexorable increase in the total entropy of the Universe.

CQ22.11 No. An engine with no thermal pollution would absorb energy from a reservoir and convert it completely into work; this is a clear violation of the second law of thermodynamics.

CQ22.12 (a) Shaking opens up spaces between jellybeans. The smaller ones more often can fall down into spaces below them. (b) The accumulation of larger candies on top and smaller ones on the bottom implies a small decrease in one contribution to the total entropy, but the second law is not violated. The total entropy increases as the system warms up, its increase in internal energy coming from the work put into shaking the box and also from a small decrease in gravitational potential energy as the beans settle compactly together.

CQ22.13 First, the efficiency of the automobile engine cannot exceed the Carnot efficiency: it is limited by the temperature of burning fuel and the temperature of the environment into which the exhaust is dumped. Second, the engine block cannot be allowed to go over a certain temperature. Third, any practical engine has friction, incomplete burning of fuel, and limits set by timing and energy transfer by heat.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 22.1 Heat Engines and the Second Law of Thermodynamics

P22.1 (a) We have $e=\dfrac{W_{\text{eng}}}{|Q_h|}=1-\dfrac{|Q_c|}{|Q_h|} \quad\rightarrow\quad \dfrac{|Q_c|}{|Q_h|}=1-e \quad\rightarrow\quad |Q_h|=\dfrac{|Q_c|}{1-e}$

With $|Q_c|=8\ 000\ \text{J}$, we have $|Q_h|=\dfrac{|Q_c|}{1-e}=\dfrac{8\ 000\ \text{J}}{1-0.250}=\boxed{10.7\ \text{kJ}}$

(b) The work per cycle is

$$W_{\text{eng}}=|Q_h|-|Q_c|=2\ 667\ \text{J}$$

From the definition of output power,

$$P=\frac{W_{\text{eng}}}{\Delta t}$$

we have the time for one cycle:

$$\Delta t=\frac{W_{\text{eng}}}{P}=\frac{2\ 667\ \text{J}}{5\ 000\ \text{J/s}}=\boxed{0.533\ \text{s}}$$

P22.2 (a) The efficiency of a heat engine is $e=W_{\text{env}}/|Q_h|$, where W_{env} is the work done by the engine and $|Q_h|$ is the energy absorbed from the higher temperature reservoir. Thus, if $W_{\text{env}}=|Q_h|/4$, the efficiency is $e=1/4=\boxed{0.25\ \text{or}\ 25\%}$.

(b) From conservation of energy, the energy exhausted to the lower temperature reservoir is $|Q_c|=|Q_h|-W_{\text{env}}$. Therefore, if $W_{\text{env}}=|Q_h|/4$, we have $|Q_c|=3|Q_h|/4$ or $\boxed{|Q_c|/|Q_h|=3/4}$.

P22.3 (a) The efficiency of the engine is

$$e=\frac{W_{\text{eng}}}{|Q_h|}=\frac{25.0\ \text{J}}{360\ \text{J}}=\boxed{0.069\ 4}\ \text{or}\ \boxed{6.94\%}$$

(b) The energy expelled to the cold reservoir during each cycle is

$$|Q_c|=|Q_h|-W_{\text{eng}}=360\ \text{J}-25.0\ \text{J}=\boxed{335\ \text{J}}$$

P22.4 The engine's output work we identify with the kinetic energy of the bullet:

$$W_{\text{eng}} = K = \frac{1}{2}mv^2 = \frac{1}{2}(0.002\ 4\text{ kg})(320\text{ m/s})^2 = 123\text{ J}$$

$$e = \frac{W_{\text{eng}}}{Q_h}$$

$$Q_h = \frac{W_{\text{eng}}}{e} = \frac{123\text{ J}}{0.011} = 1.12\times 10^4\text{ J}$$

$$Q_h = W_{\text{eng}} + |Q_c|$$

The energy exhaust is

$$|Q_c| = Q_h - W_{\text{eng}} = 1.12\times 10^4\text{ J} - 123\text{ J} = 1.10\times 10^4\text{ J}$$

$$Q = mc\Delta T$$

$$\Delta T = \frac{Q}{mc} = \frac{1.10\times 10^4\text{ J}\cdot\text{kg°C}}{(1.80\text{ kg})(448\text{ J})} = \boxed{13.7\text{°C}}$$

P22.5 (a) The engine's efficiency is given by

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{|Q_h| - |Q_c|}{|Q_h|} = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{1.20\text{ kJ}}{1.70\text{ kJ}}$$

$$= \boxed{0.294\text{ (or 29.4\%)}}$$

(b) During each cycle, the work done by the engine is

$$W_{\text{eng}} = |Q_h| - |Q_c| = 1.70\text{ kJ} - 1.20\text{ kJ} = \boxed{5.00\times 10^2\text{ J}}$$

(c) The power transferred out of the engine is

$$P = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.00\times 10^2\text{ J}}{0.300\text{ s}} = 1.67\times 10^3\text{ W} = \boxed{1.67\text{ kW}}$$

P22.6 (a) The input energy each hour is

$$(7.89\times 10^3\text{ J/revolution})(2\ 500\text{ rev/min})\left(\frac{60\text{ min}}{1\text{ h}}\right)$$
$$= 1.18\times 10^9\text{ J/h}$$

implying fuel input $(1.18\times 10^9\text{ J/h})\left(\frac{1\text{ L}}{4.03\times 10^7\text{ J}}\right) = \boxed{29.4\text{ L/h}}$.

(b) $Q_h = W_{\text{eng}} + |Q_c|$. For a continuous-transfer process we may divide by time to have

$$\frac{Q_h}{\Delta t} = \frac{W_{\text{eng}}}{\Delta t} + \frac{|Q_c|}{\Delta t}$$

$$\text{Useful power output} = \frac{W_{\text{eng}}}{\Delta t} = \frac{Q_h}{\Delta t} - \frac{|Q_c|}{\Delta t}$$

$$= \left(\frac{7.89\times10^3\ \text{J}}{\text{revolution}} - \frac{4.58\times10^3\ \text{J}}{\text{revolution}}\right)\left(\frac{2\ 500\ \text{rev}}{1\ \text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

$$= 1.38\times10^5\ \text{W}$$

$$P_{\text{eng}} = \left(1.38\times10^5\ \text{W}\right)\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = \boxed{185\ \text{hp}}$$

(c) $P_{\text{eng}} = \tau\omega \Rightarrow \tau = \dfrac{P_{\text{eng}}}{\omega} = \left(\dfrac{1.38\times10^5\ \text{J/s}}{2\ 500\ \text{rev/60 s}}\right)\left(\dfrac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{527\ \text{N}\cdot\text{m}}$

(d) $\dfrac{|Q_c|}{\Delta t} = \left(\dfrac{4.58\times10^3\ \text{J}}{\text{revolution}}\right)\left(\dfrac{2\ 500\ \text{rev}}{60\ \text{s}}\right) = \boxed{1.91\times10^5\ \text{W}}$

P22.7 The energy to melt a mass Δm_{Hg} of Hg is $|Q_c| = m_{\text{Hg}}L_f$. The energy absorbed to freeze Δm_{Al} of aluminum is $|Q_h| = m_{\text{Al}}L_f$. The efficiency is

$$e = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{\Delta m_{\text{Hg}}L_{\text{Hg}}}{\Delta m_{\text{Al}}L_{\text{Al}}} = 1 - \frac{(15.0\ \text{g})(1.18\times10^4\ \text{J/kg})}{(1.00\ \text{g})(3.97\times10^5\ \text{J/kg})}$$

$$= 0.554 = \boxed{55.4\%}$$

Section 22.2 Heat Pumps and Refrigerators

P22.8 $\text{COP(refrigerator)} = \dfrac{|Q_c|}{W}$

(a) If $|Q_c| = 120\ \text{J}$ and COP = 5.00, then $W = \boxed{24.0\ \text{J}}$.

(b) $|Q_h| = |Q_c| + W = 120\ \text{J} + 24\ \text{J} = \boxed{144\ \text{J}}$

P22.9 (a) The work done on the refrigerant in each cycle is

$$W = Q_H - Q_L = 625\ \text{kJ} - 550\ \text{kJ} = \boxed{75.0\ \text{kJ}}$$

(b) The coefficient of performance of a refrigerator is:

$$\text{COP} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

Solving numerically:

$$\text{COP} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L} = \frac{550\text{ kJ}}{625\text{ kJ} - 550\text{ kJ}} = \boxed{7.33}$$

P22.10 (a) The coefficient of performance of a heat pump is $\text{COP} = |Q_h|/W$, where $|Q_h|$ is the thermal energy delivered to the warm space and W is the work input required to operate the heat pump. Therefore,

$$|Q_h| = W \cdot \text{COP} = (P \cdot \Delta t) \cdot \text{COP}$$

$$= \left[\left(7.03 \times 10^3\ \frac{\text{J}}{\cancel{\text{s}}}\right)(8.00\ \cancel{\text{h}})\left(\frac{3\,600\ \cancel{\text{s}}}{1\ \cancel{\text{h}}}\right)\right]3.80 = \boxed{7.69 \times 10^8\text{ J}}$$

(b) The energy extracted from the cold space (outside air) is

$$|Q_c| = |Q_h| - W = |Q_h| - \frac{|Q_h|}{\text{COP}} = |Q_h|\left(1 - \frac{1}{\text{COP}}\right)$$

or $$|Q_c| = (7.69 \times 10^8\text{ J})\left(1 - \frac{1}{3.80}\right) = \boxed{5.67 \times 10^8\text{ J}}$$

***P22.11** $\text{COP} = 3.00 = \dfrac{Q_c}{W}$. Therefore, $W = \dfrac{Q_c}{3.00}$.

The heat removed each minute is

$$\frac{Q_C}{t} = (0.030\,0\text{ kg})(4\,186\text{ J/kg}\,°\text{C})(22.0°\text{C})$$
$$+ (0.030\,0\text{ kg})(3.33 \times 10^5\text{ J/kg})$$
$$+ (0.030\,0\text{ kg})(2\,090\text{ J/kg}\,°\text{C})(20.0°\text{C})$$
$$= 1.40 \times 10^4\text{ J/min} = 233\text{ J/s}$$

Thus, the work done per second $= P = \dfrac{233\text{ J/s}}{3.00} = \boxed{77.8\text{ W}}$.

P22.12 (a) The coefficient of performance of a heat pump is

$$\text{COP}_{h.p.} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L}$$

Because work (energy) is power times time $(W = P\Delta t)$, the equation above may be rearranged to obtain the heat added to the home:

$$Q_H = \text{COP} \cdot W = \text{COP} \cdot P\Delta t$$

$$= (4.20)(1.75 \times 10^3 \text{ J/s})(3600 \text{ s}) = \boxed{2.65 \times 10^7 \text{ J}}$$

(b) The coefficient of performance of a refrigerator or air conditioner is

$$\text{COP}_{\text{refr.}} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$$

and can be written in terms of the coefficient of performance of a heat pump because:

$$W = Q_H - Q_L;$$

$$\text{COP}_{\text{refr.}} = \frac{Q_L}{W} = \frac{Q_H - W}{Q_H - Q_L} = \frac{Q_H}{Q_H - Q_L} + \frac{W}{Q_H - Q_L}$$

Where the first term on the far right is identically the coefficient of performance of the heat pump, and the second term is identically one (because $W = Q_H - Q_L$). Thus,

$$\text{COP}_{\text{refr.}} = \frac{Q_H}{Q_H - Q_L} + \frac{W}{Q_H - Q_L} = \text{COP}_{\text{h.p.}} - 1 = (4.20) - 1$$

$$= \boxed{3.20}$$

P22.13 (a) The energy use by the freezer each day is

$$W = P \cdot \Delta t = \left(457 \ \frac{\cancel{\text{kWh}}}{\cancel{\text{y}}}\right)\left(\frac{3.60 \times 10^6 \text{ J}}{1 \ \cancel{\text{kWh}}}\right)\left(\frac{1 \ \cancel{\text{y}}}{365 \text{ d}}\right) \cdot (1 \text{ d})$$

$$= \boxed{4.51 \times 10^6 \text{ J}}$$

(b) From the definition of the coefficient of performance for a refrigerator, $(\text{COP})_R = |Q_c|/W$, the thermal energy removed from the cold space each day is

$$|Q_c| = (\text{COP})_R \cdot W = 6.30(4.51 \times 10^6 \text{ J}) = \boxed{2.84 \times 10^7 \text{ J}}$$

(c) The water must be cooled 20.0°C before it will start to freeze, so the thermal energy that must be removed from mass *m* of water to freeze it is $|Q_c| = mc_w|\Delta T| + mL_f$. The mass of water that can be frozen each day is then

$$m = \frac{|Q_c|}{c_w|\Delta T| + L_f} = \frac{2.84\times10^7\ \text{J}}{(4186\ \text{J/kg}\cdot{}^\circ\text{C})(20.0^\circ\text{C}) + 3.33\times10^5\ \text{J/kg}}$$

$$= \boxed{68.1\ \text{kg}}$$

Section 22.3 Reversible and Irreversible Processes

Section 22.4 The Carnot Engine

P22.14 The maximum possible efficiency for a heat engine operating between reservoirs with absolute temperatures of T_c = 25° + 273 = 298 K and T_h = 375° + 273 = 648 K is the Carnot efficiency:

$$e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{298\ \text{K}}{648\ \text{K}} = \boxed{0.540\ (\text{or } 54.0\%)}$$

P22.15 We use the Carnot expression for maximum possible efficiency, and the definition of efficiency to find the useful output. The engine is a steam turbine in an electric generating station with

$$T_c = 430^\circ\text{C} = 703\ \text{K} \qquad \text{and} \qquad T_h = 1\,870^\circ\text{C} = 2\,143\ \text{K}$$

(a) $$e_C = \frac{\Delta T}{T_h} = \frac{1\,440\ \text{K}}{2\,143\ \text{K}} = 0.672 = \boxed{67.2\%}$$

(b) $$e = W_{\text{eng}}/|Q_h| = 0.420 \qquad \text{and} \qquad |Q_h| = 1.40\times10^5\ \text{J}$$

for one second of operation, so

$$W_{\text{eng}} = 0.420|Q_h| = 5.88\times10^4\ \text{J}$$

and the power is

$$P = \frac{W_{\text{eng}}}{\Delta t} = \frac{5.88\times10^4\ \text{J}}{1\ \text{s}} = \boxed{58.8\ \text{kW}}$$

P22.16 The efficiency of a Carnot engine operating between these temperatures is

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{273\ \text{K}}{293\ \text{K}} = 0.068\,3 = 6.83\%$$

Therefore, there is no way that the inventor's engine can have an efficiency of 0.110 = 11.0%.

P22.17 $$e = \frac{W_{\text{eng}}}{|Q_h|} = e_c = 1 - \frac{T_c}{T_h} \quad \rightarrow \quad \frac{W_{\text{eng}} / \Delta t}{|Q_h| / \Delta t} = \frac{P}{|Q_h|/\Delta t} = 1 - \frac{T_c}{T_h}$$

(a) $$|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{P\Delta t}{1 - (T_c / T_h)} = \frac{(1.50 \times 10^5 \text{ W})(3\ 600 \text{ s})}{1 - (293 \text{ K} / 773 \text{ K})} = \boxed{8.70 \times 10^8 \text{ J}}$$

(b) $$|Q_c| = |Q_h| - W_{\text{eng}} = |Q_h| - P\Delta t$$
$$= 8.70 \times 10^8 \text{ J} - (1.50 \times 10^5 \text{ W})(3\ 600 \text{ s})$$
$$= \boxed{3.30 \times 10^8 \text{ J}}$$

P22.18 $$e = \frac{W_{\text{eng}}}{|Q_h|} = e_c = 1 - \frac{T_c}{T_h} \quad \rightarrow \quad \frac{W_{\text{eng}} / \Delta t}{|Q_h| / \Delta t} = \frac{P}{|Q_h|/\Delta t} = 1 - \frac{T_c}{T_h}$$

(a) $$|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{P\Delta t}{1 - (T_c / T_h)} = \boxed{P\Delta t\left(\frac{T_h}{T_h - T_c}\right)}$$

(b) $$|Q_c| = |Q_h| - W_{\text{eng}} = |Q_h| - P\Delta t = P\Delta t\left(\frac{T_h}{T_h - T_c}\right) - P\Delta t$$
$$= P\Delta t\left(\frac{T_h}{T_h - T_c} - 1\right) = P\Delta t\left(\frac{T_h - (T_h - T_c)}{T_h - T_c}\right) = \boxed{P\Delta t\left(\frac{T_c}{T_h - T_c}\right)}$$

P22.19 $$(\text{COP})_{\text{refrig}} = \frac{T_c}{\Delta T} = \frac{270 \text{ K}}{30.0 \text{ K}} = \boxed{9.00}$$

P22.20 (a) For a complete cycle, $\Delta E_{\text{int}} = 0$ and

$$W = |Q_h| - |Q_c| = |Q_c|\left[\frac{|(Q_h)|}{|Q_c|} - 1\right]$$

The text shows that for a Carnot cycle (and only for a reversible cycle), $\frac{|Q_h|}{|Q_c|} = \frac{T_h}{T_c}$. Therefore, $W = \frac{T_h - T_c}{T_c}|Q_c|$.

(b) We have the definition of the coefficient of performance for a refrigerator, $\text{COP} = \frac{|Q_c|}{W}$. Using the result from part (a), this becomes $\text{COP} = \frac{T_c}{T_h - T_c}$.

P22.21 $$(\text{COP})_{\text{heat pump}} = \frac{|Q_c| + W}{W} = \frac{T_h}{\Delta T} = \frac{295\text{ K}}{25\text{ K}} = \boxed{11.8}$$

P22.22 $$(\text{COP})_{\text{Carnot refrig}} = \frac{T_c}{\Delta T} = \frac{4.00\text{ K}}{289\text{ K}} = 0.013\ 8 = \frac{|Q_c|}{W}$$

$\therefore W = \boxed{72.2\text{ J}}$ per 1 J of energy removed by heat.

P22.23 We wish to evaluate $\text{COP} = |Q_c|/W$ for a refrigerator, which is a Carnot engine run in reverse. For a Carnot engine,

$$\left.\begin{aligned} |Q_h| &= |Q_c| + W \\ e &= \frac{W}{|Q_h|} = \frac{W}{|Q_c| + W} \end{aligned}\right\} \rightarrow \frac{1}{e} = \frac{|Q_c| + W}{W} = \frac{|Q_c|}{W} + 1$$

which gives

$$\text{COP} = \frac{|Q_c|}{W} = \frac{1}{e} - 1$$

Therefore,

$$\text{COP} = \frac{1}{e} - 1 = \frac{1}{0.350} - 1 = \boxed{1.86}.$$

***P22.24** The Carnot summer efficiency is

$$e_{c,s} = 1 - \frac{T_c}{T_h} = 1 - \frac{(273\text{ K} + 20.0°\text{C})}{(273\text{ K} + 350°\text{C})} = 0.530$$

And in winter,

$$e_{c,w} = 1 - \frac{283}{623} = 0.546$$

Then the actual winter efficiency is

$$0.320\left(\frac{0.546}{0.530}\right) = \boxed{0.330} \quad \text{or} \quad \boxed{33.0\%}$$

P22.25 (a) The absolute temperature of the cold reservoir is $T_c = 20.0° + 273 = 293$ K. If the Carnot efficiency is to be $e_C = 0.650$, it is necessary that

$$1 - \frac{T_c}{T_h} = 0.650 \quad \text{or} \quad \frac{T_c}{T_h} = 0.350 \quad \text{and} \quad T_h = \frac{T_c}{0.35}$$

Thus,

$$T_h = \frac{293\text{ K}}{0.35} = 837\text{ K} \quad \text{or} \quad T_h = 837 - 273 = \boxed{564°\text{C}}$$

(b) $\boxed{\text{No. A real engine will always have an efficiency } \textit{less} \text{ than the Carnot efficiency because it operates in an irreversible manner.}}$

P22.26 (a) $e_C = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{350\text{ K}}{500\text{ K}} = \boxed{0.300}$

(b) We differentiate $e_C = 1 - T_c / T_h$ to find

$$\frac{de_c}{dT_h} = 0 - T_c(-1)T_h^{-2} = \frac{T_c}{T_h^2} = \frac{350\text{ K}}{(500\text{ K})^2} = \boxed{1.40 \times 10^{-3}\text{ K}^{-1}}$$

(c) We differentiate $e_C = 1 - T_c / T_h$ to find

$$\frac{de_c}{dT_c} = 0 - \frac{1}{T_h} = -\frac{1}{500\text{ K}} = \boxed{-2.00 \times 10^{-3}\text{ K}^{-1}}$$

(d) $\boxed{\text{No. The derivative in part (c) depends only on } T_h.}$

P22.27 Isothermal expansion at $T_h = 523$ K

Isothermal compression at $T_c = 323$ K

Gas absorbs 1 200 J during expansion.

(a) For a Carnot cycle, $e_c = 1 - \dfrac{T_c}{T_h}$

For any engine, $e = \dfrac{W_{\text{eng}}}{|Q_h|} = 1 - \dfrac{|Q_c|}{|Q_h|}$

Therefore, for a Carnot engine, $1 - \dfrac{T_c}{T_h} = 1 - \dfrac{|Q_c|}{|Q_h|}$

Then we have

$$|Q_c| = |Q_h|\left(\frac{T_c}{T_h}\right) = (1\ 200\text{ J})\left(\frac{323\text{ K}}{523\text{ K}}\right) = \boxed{741\text{ J}}$$

(b) The work we can calculate as

$$W_{\text{eng}} = |Q_h| - |Q_c| = (1\ 200\text{ J} - 741\text{ J}) = \boxed{459\text{ J}}$$

P22.28 (a) $e_{\max} = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{278\text{ K}}{293\text{ K}} = 5.12 \times 10^{-2} = \boxed{5.12\%}$

(b) $P = \dfrac{W_{\text{eng}}}{\Delta t} = 75.0 \times 10^6\text{ J/s}$

Therefore, $W_{\text{eng}} = (75.0\times10^{6}\ \text{J/s})(3\ 600\ \text{s/h}) = 2.70\times10^{11}\ \text{J/h}$.

From $e = \dfrac{W_{\text{eng}}}{|Q_h|}$ we find

$$|Q_h| = \frac{W_{\text{eng}}}{e} = \frac{2.70\times10^{11}\ \text{J/h}}{5.12\times10^{-2}} = 5.27\times10^{12}\ \text{J/h} = \boxed{5.27\ \text{TJ/h}}$$

(c) $\boxed{\text{As fossil-fuel prices rise, this way to use solar energy will become a good buy.}}$

***P22.29** (a) With reservoirs at absolute temperatures of T_c = 80.0°C + 273 = 353 K and T_h = 350°C + 273 = 623 K, the Carnot efficiency is

$$e_C = 1-\frac{T_c}{T_h} = 1-\frac{353\ \text{K}}{623\ \text{K}} = \boxed{0.433} \quad (\text{or } 43.3\%)$$

so the maximum power output is

$$P_{\max} = \frac{W_{\text{eng}}}{\Delta t} = \frac{e_C|Q_h|}{\Delta t} = \frac{0.433(21.0\ \text{kJ})}{1.00\ \text{s}} = \boxed{9.10\ \text{kW}}$$

(b) From $e = 1-\dfrac{|Q_c|}{|Q_h|}$, the energy expelled by heat each cycle is

$$|Q_c| = |Q_h|(1-e) = (21.0\ \text{kJ})(1-0.433) = \boxed{11.9\ \text{kJ}}$$

P22.30 (a) $e = \dfrac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \dfrac{e_1Q_{1h} + e_2Q_{2h}}{Q_{1h}}$

Now, $Q_{2h} = Q_{1c} = Q_{1h} - W_{\text{eng1}} = Q_{1h} - e_1Q_{1h}$,

so

$$e = \frac{e_1Q_{1h} + e_2(Q_{1h} - e_1Q_{1h})}{Q_{1h}} = \boxed{e_1 + e_2 - e_1e_2}$$

(b) $$e = e_1 + e_2 - e_1e_2 = 1-\frac{T_i}{T_h} + 1-\frac{T_c}{T_i} - \left(1-\frac{T_i}{T_h}\right)\left(1-\frac{T_c}{T_i}\right)$$

$$= 2-\frac{T_i}{T_h}-\frac{T_c}{T_i}-1+\frac{T_i}{T_h}+\frac{T_c}{T_i}-\frac{T_c}{T_h} = \boxed{1-\frac{T_c}{T_h}}$$

(c) $\boxed{\text{The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency. No improvement in net efficiency has resulted.}}$

(d) With $W_{\text{eng2}} = W_{\text{eng1}}$, $e = \dfrac{W_{\text{eng1}} + W_{\text{eng2}}}{Q_{1h}} = \dfrac{2W_{\text{eng1}}}{Q_{1h}} = 2e_1$

$$1 - \frac{T_c}{T_h} = 2\left(1 - \frac{T_i}{T_h}\right)$$

$$0 - \frac{T_c}{T_h} = 1 - \frac{2T_i}{T_h}$$

$$2T_i = T_h + T_c$$

$$\boxed{T_i = \frac{1}{2}\left(T_h + T_c\right)}$$

(e) $e_1 = e_2 = 1 - \dfrac{T_i}{T_h} = 1 - \dfrac{T_c}{T_i}$

$$T_i^2 = T_c T_h$$

$$\boxed{T_i = \left(T_h T_c\right)^{1/2}}$$

P22.31 (a) In an adiabatic process, $P_f V_f^\gamma = P_i V_i^\gamma$. Also, $\left(\dfrac{P_f V_f}{T_f}\right)^\gamma = \left(\dfrac{P_i V_i}{T_i}\right)^\gamma$.

Dividing the second equation by the first yields $T_f = T_i\left(\dfrac{P_f}{P_i}\right)^{(\gamma-1)/\gamma}$.

Since $\gamma = \dfrac{5}{3}$ for argon, $\dfrac{\gamma - 1}{\gamma} = \dfrac{2}{5} = 0.400$ and we have

$$T_f = \left(1\,073\text{ K}\right)\left(\frac{300 \times 10^3\text{ Pa}}{1.50 \times 10^6\text{ Pa}}\right)^{0.400} = \boxed{564\text{ K}}$$

(b) $\Delta E_{\text{int}} = nC_V\Delta T = Q - W_{\text{eng}} = 0 - W_{\text{eng}}$, so $W_{\text{eng}} = -nC_V\Delta T$,

and the power output is (suppressing the units of *R*)

$$P = \frac{W_{\text{eng}}}{\Delta t} = \frac{-nC_V\Delta T}{\Delta t}$$

$$= \frac{\left(-80.0\text{ kg}\right)\left(1\text{ mol}/0.039\,9\text{ kg}\right)\left(\frac{3}{2}\right)(8.314)\left(564\text{ K} - 1\,073\text{ K}\right)}{60.0\text{ s}}$$

$$P = 2.12 \times 10^5\text{ W} = \boxed{212\text{ kW}}$$

(c) $e_C = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{564\text{ K}}{1\,073\text{ K}} = 0.475$ or $\boxed{47.5\%}$

P22.32 (a) First, consider the adiabatic process $D \to A$:

$P_D V_D^\gamma = P_A V_A^\gamma$ so

$$P_D = P_A \left(\frac{V_A}{V_D}\right)^\gamma = (1\,400\text{ kPa})\left(\frac{10.0\text{ L}}{15.0\text{ L}}\right)^{5/3} = \boxed{712\text{ kPa}}$$

Also, $\left(\frac{nRT_D}{V_D}\right)V_D^\gamma = \left(\frac{nRT_A}{V_A}\right)V_A^\gamma$,

or $T_D = T_A\left(\frac{V_A}{V_D}\right)^{\gamma-1} = (720\text{ K})\left(\frac{10.0}{15.0}\right)^{2/3} = \boxed{549\text{ K}}$.

Now, consider the isothermal process $C \to D$: $T_C = T_D = \boxed{549\text{ K}}$

$$P_C = P_D\left(\frac{V_D}{V_C}\right) = \left[P_A\left(\frac{V_A}{V_D}\right)^\gamma\right]\left(\frac{V_D}{V_C}\right) = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$$

$$P_C = \frac{(1\,400\text{ kPa})(10.0\text{ L})^{5/3}}{(24.0\text{ L})(15.0\text{ L})^{2/3}} = \boxed{445\text{ kPa}}$$

Next, consider the adiabatic process $B \to C$: $P_B V_B^\gamma = P_C V_C^\gamma$

But, $P_C = \frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}$ from above. Also considering the isothermal process, $P_B = P_A\left(\frac{V_A}{V_B}\right)$.

Hence, $P_A\left(\frac{V_A}{V_B}\right)V_B^\gamma = \left(\frac{P_A V_A^\gamma}{V_C V_D^{\gamma-1}}\right)V_C^\gamma$, which reduces to

$$V_B = \frac{V_A V_C}{V_D} = \frac{(10.0\text{ L})(24.0\text{ L})}{15.0\text{ L}} = \boxed{16.0\text{ L}}$$

Finally, $P_B = P_A\left(\frac{V_A}{V_B}\right) = (1\,400\text{ kPa})\left(\frac{10.0\text{ L}}{16.0\text{ L}}\right) = \boxed{875\text{ kPa}}$.

State	*P* (kPa)	*V* (L)	*T* (K)
A	1 400	10.0	720
B	875	16.0	720
C	445	24.0	549
D	712	15.0	549

TABLE P22.32(a)

(b) For the isothermal process $A \to B$: $\Delta E_{\text{int}} = nC_V\Delta T = \boxed{0}$ so

$$Q = -W = nRT\ln\left(\frac{V_B}{V_A}\right)$$

$$= (2.34\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(720\text{ K})\ln\left(\frac{16.0\text{ L}}{10.0\text{ L}}\right)$$

$$= \boxed{+6.58\text{ kJ}}$$

For the adiabatic process $B \to C$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V\left(T_C - T_B\right)$$

$$= (2.34\text{ mol})\left[\frac{3}{2}(8.314\text{ J/mol}\cdot\text{K})\right](549\text{ K} - 720\text{ K})$$

$$= \boxed{-4.98\text{ kJ}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + (-4.98\text{ kJ}) = \boxed{-4.98\text{ kJ}}$.

For the isothermal process $C \to D$: $\Delta E_{\text{int}} = nC_V\Delta T = \boxed{0}$ and

$$Q = -W = nRT\ln\left(\frac{V_D}{V_C}\right)$$

$$= (2.34\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(549\text{ K})\ln\left(\frac{15.0\text{ L}}{24.0\text{ L}}\right)$$

$$= \boxed{-5.02\text{ kJ}}$$

Finally, for the adiabatic process $D \to A$: $Q = \boxed{0}$

$$\Delta E_{\text{int}} = nC_V\left(T_A - T_D\right)$$

$$= (2.34\text{ mol})\left[\frac{3}{2}(8.314\text{ J/mol}\cdot\text{K})\right](720\text{ K} - 549\text{ K})$$

$$= \boxed{+4.98\text{ kJ}}$$

and $W = -Q + \Delta E_{\text{int}} = 0 + 4.98\text{ kJ} = \boxed{+4.98\text{ kJ}}$

Process	**Q (kJ)**	**W (kJ)**	**ΔE_{int} (kJ)**
$A \to B$	+6.58	–6.58	0
$B \to C$	0	–4.98	–4.98
$C \to D$	–5.02	+5.02	0
$D \to A$	0	+4.98	+4.98
ABCDA	+1.56	–1.56	0

TABLE P22.32(b)

The work done *by* the engine is the negative of the work input. The output work W_{eng} is given by the work column in TABLE P22.32(b) with all signs reversed.

(c) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{-W_{ABCD}}{Q_{A\to B}} = \frac{1.56\text{ kJ}}{6.58\text{ kJ}} = 0.237$ or $\boxed{23.7\%}$

(d) $e_c = 1 - \frac{T_c}{T_h} = 1 - \frac{549\text{ K}}{720\text{ K}} = 0.237$ or $\boxed{23.7\%}$

P22.33 (a) "The actual efficiency is two thirds the Carnot efficiency" reads as an equation

$$\frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{|Q_c| + W_{\text{eng}}} = \frac{2}{3}\left(1 - \frac{T_c}{T_h}\right) = \frac{2}{3}\frac{T_h - T_c}{T_h}$$

All the *T*'s represent absolute temperatures. Then

$$\frac{|Q_c| + W_{\text{eng}}}{W_{\text{eng}}} = \frac{1.5\ T_h}{T_h - T_c} \quad \to \quad \frac{|Q_c|}{W_{\text{eng}}} = \frac{1.5\ T_h}{T_h - T_c} - 1 = \frac{1.5\ T_h - T_h + T_c}{T_h - T_c}$$

$$|Q_c| = W_{\text{eng}}\frac{0.5\ T_h + T_c}{T_h - T_c} \to \frac{|Q_c|}{\Delta t} = \frac{W_{\text{eng}}}{\Delta t}\frac{0.5\ T_h + T_c}{T_h - T_c}$$

$$\boxed{\frac{Q_c}{\Delta t} = 1.40\left(\frac{0.5T_h + 383}{T_h - 383}\right), \text{ where } Q_c/\Delta t \text{ is in megawatts and } T \text{ is in kelvins.}}$$

(b) $\boxed{\text{The exhaust power decreases as the firebox temperature increases.}}$

(c) $$\frac{|Q_c|}{\Delta t} = (1.40\text{ MW})\left(\frac{0.5\,T_h + 383\text{ K}}{T_h - 383\text{ K}}\right)$$

$$= (1.40\text{ MW})\left(\frac{0.5(1\,073\text{ K}) + 383\text{ K}}{1\,073\text{ K} - 383\text{ K}}\right) = \boxed{1.87\text{ MW}}$$

(d) We require

$$\frac{|Q_c|}{\Delta t} = \frac{1}{2}(1.87\text{ MW}) = (1.40\text{ MW})\left(\frac{0.5\,T_h + 383\text{ K}}{T_h - 383\text{ K}}\right)$$

$$\frac{0.5\,T_h + 383\text{ K}}{T_h - 383\text{ K}} = 0.666$$

$$0.5\,T_h + 383\text{ K} = 0.666T_h - 255\text{ K}$$

$$T_h = 638\text{ K}/0.166 = \boxed{3.84 \times 10^3\text{ K}}$$

(e) The minimum possible heat exhaust power is approached as the firebox temperature goes to infinity, and it is $|Q_c|/\Delta t =$ 1.40 MW(0.5/1) = 0.700 MW. The heat exhaust power cannot be as small as (1/4)(1.87 MW) = 0.466 MW. $\boxed{\text{So no answer exists. The energy exhaust cannot be that small.}}$

P22.34 We determine the power required from

$$\frac{|Q_c|}{W} = \text{COP}_c\ (\text{refrigerator}) = \frac{T_c}{T_h - T_c} = \frac{|Q_c|/\Delta t}{W/\Delta t}$$

$$\frac{0.150\text{ W}}{W/\Delta t} = \frac{260\text{ K}}{40.0\text{ K}}$$

$$P = \frac{W}{\Delta t} = (0.150\text{ W})\left(\frac{40.0\text{ K}}{260\text{ K}}\right) = \boxed{23.1\text{ mW}}$$

P22.35 The coefficient of performance of the device is

$$\text{COP} = 0.100\ \text{COP}_{\text{Carnot cycle}}$$

or

$$\frac{|Q_h|}{W} = 0.100\left(\frac{|Q_h|}{W}\right)_{\text{Carnot cycle}} = 0.100\left(\frac{1}{\text{Carnot efficiency}}\right)$$

$$\frac{|Q_h|}{W} = 0.100\left(\frac{T_h}{T_h - T_c}\right) = 0.100\left(\frac{293\text{ K}}{293\text{ K} - 268\text{ K}}\right) = \boxed{1.17}$$

Thus, 1.17 joules of energy enter the room by heat for each joule of work done.

Section 22.5 Gasoline and Diesel Engines

P22.36 Compression ratio = 6.00, $\gamma = 1.40$

(a) Efficiency of an Otto engine: $e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma - 1}$

$$e = 1 - \left(\frac{1}{6.00}\right)^{0.400} = \boxed{51.2\%}$$

(b) If actual efficiency e' = 15.0%, the fraction of fuel wasted is (assuming complete combustion of the air-fuel mixture) $e - e' = \boxed{36.2\%}$.

P22.37 (a) For adiabatic expansion, $P_i V_i^\gamma = P_f V_f^\gamma$. Therefore,

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = (3.00 \times 10^6 \text{ Pa})\left(\frac{50.0 \text{ cm}^3}{300 \text{ cm}^3}\right)^{1.40}$$

$$= \boxed{2.44 \times 10^5 \text{ Pa}}$$

(b) Since $Q = 0$, we have $W_{\text{eng}} = Q - \Delta E = -nC_V \Delta T = -nC_V(T_f - T_i)$.

From $\gamma = \frac{C_P}{C_V} = \frac{C_V + R}{C_V}$, we get $(\gamma - 1)C_V = R$, so that

$$C_V = \frac{R}{1.40 - 1} = 2.50\, R$$

The work done by the gas in expanding is then

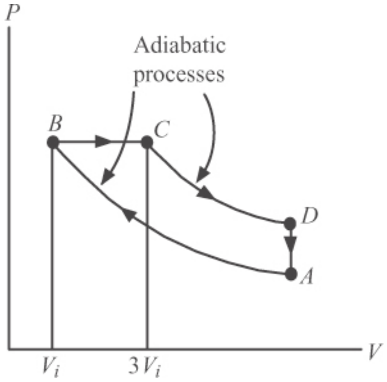
$$\begin{aligned} W_{\text{eng}} &= n(2.50\, R)(T_i - T_f) = 2.50 P_i V_i - 2.50 P_f V_f \\ &= 2.50\Big[\big(3.00 \times 10^6 \text{ Pa}\big)\big(50.0 \times 10^{-6} \text{ m}^3\big) \\ &\qquad - \big(2.44 \times 10^5 \text{ Pa}\big)\big(300 \times 10^{-6} \text{ m}^3\big)\Big] \\ &= \boxed{192 \text{ J}} \end{aligned}$$

P22.38 The energy transferred by heat over the paths *CD* and *BA* is zero since they are adiabatic.

Over path *BC*: $Q_{BC} = nC_P\,(T_C - T_B) > 0$

Over path *DA*: $Q_{DA} = nC_V\,(T_A - T_D) < 0$

Therefore, $|Q_c| = |Q_{DA}|$ and $Q_h = Q_{BC}$.



ANS. FIG. P22.38

The efficiency is then

$$e = 1 - \frac{|Q_c|}{Q_h} = 1 - \frac{(T_D - T_A)C_V}{(T_C - T_B)C_P} = 1 - \frac{1}{\gamma}\left(\frac{T_D - T_A}{T_C - T_B}\right)$$

Section 22.6 Entropy

P22.39 Each marble is returned to the bag before the next is drawn, so the probability of drawing a red one is the same as drawing a green one.

(a)

Result	Possible Combinations	Total
All red	RRR	1
2R, 1G	RRG, RGR, GRR	3
1R, 2G	RGG, GRG, GGR	3
All green	GGG	1

TABLE P22.39(a)

(b)

Result	Possible Combinations	Total
All red	RRRRR	1
4R, 1G	RRRRG, RRRGR, RRGRR, RGRRR, GRRRR	5
3R, 2G	RRRGG, RRGRG, RGRRG, GRRRG, RRGGR, RGRGR, GRRGR, RGGRR, GRGRR, GGRRR	10
2R, 3G	GGGRR, GGRGR, GRGGR, RGGGR, GGRRG, GRGRG, RGGRG, GRRGG, RGRGG, RRGGG	10
1R, 4G	RGGGG, GRGGG, GGRGG, GGGRG, GGGGR	5
All green	GGGGG	1

TABLE P22.39(b)

P22.40 (a) The table is shown in TABLE P22.40 below.

(b) On the basis of the table, the most probable recorded result of a toss is $\boxed{\text{2 heads and 2 tails}}$.

Result	**Possible Combinations**	**Total**
All heads	HHHH	1
3H, 1T	THHH, HTHH, HHTH, HHHT	4
2H, 2T	TTHH, THTH, THHT, HTTH, HTHT, HHTT	6
1H, 3T	HTTT, THTT, TTHT, TTTH	4
All tails	TTTT	1

TABLE P22.40

P22.41 (a) A 12 can only be obtained $\boxed{\text{one}}$ way, as 6 + 6.

(b) A 7 can be obtained $\boxed{\text{six}}$ ways: 6 + 1, 5 + 2, 4 + 3, 3 + 4, 2 + 5, and 1 + 6.

Section 22.7 Changes in Entropy for Thermodynamic Systems

Section 22.8 Entropy and the Second Law

P22.42 For a freezing process,

$$\Delta S = \frac{\Delta Q}{T} = \frac{-(0.500\text{ kg})(3.33\times 10^5\text{ J/kg})}{273\text{ K}} = \boxed{-610\text{ J/K}}$$

P22.43 The hot water has negative energy input by heat, given by $Q = mc\Delta T$. The surrounding room has positive energy input of this same number of joules, which we can write as $Q_{\text{room}} = (mc|\Delta T|)_{\text{water}}$. Imagine the room absorbing this energy reversibly by heat, from a stove at 20.001°C. Then its entropy increase is Q_{room}/T:

$$\Delta S = \frac{Q_r}{T} = \frac{mc_w|\Delta T|}{T} = \frac{(0.125\text{ kg})(4186\text{ J/kg}\cdot{}^\circ\text{C})(80^\circ\text{C})}{293\text{ K}}$$
$$= \boxed{143\text{ J/K}}$$

***P22.44** $c_{\text{iron}} = 448\ \text{J/kg}\cdot{}^\circ\text{C};\ c_{\text{water}} = 4\,186\ \text{J/kg}\cdot{}^\circ\text{C}$

From $Q_{\text{cold}} = -Q_{\text{hot}}$:

$$(4.00\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(T_f - 10.0^\circ\text{C}) = -(1.00\ \text{kg})(448\ \text{J/kg}\cdot{}^\circ\text{C})(T_f - 900^\circ\text{C})$$

which yields $T_f = 33.2^\circ\text{C} = 306.3\ \text{K}$. Then,

$$\Delta S = \int_{283\ \text{K}}^{306.3\ \text{K}} \frac{c_{\text{water}} m_{\text{water}} dT}{T} + \int_{1\,173\ \text{K}}^{306.3\ \text{K}} \frac{c_{\text{iron}} m_{\text{iron}} dT}{T}$$

$$\Delta S = c_{\text{water}} m_{\text{water}} \ln\left(\frac{306.3\ \text{K}}{283\ \text{K}}\right) + c_{\text{iron}} m_{\text{iron}} \ln\left(\frac{306.3\ \text{K}}{1\,173\ \text{K}}\right)$$

$$\Delta S = (4\,186\ \text{J/kg}\cdot\text{K})(4.00\ \text{kg})(0.078\,7) + (448\ \text{J/kg}\cdot\text{K})(1.00\ \text{kg})(-1.34)$$

$$\Delta S = \boxed{717\ \text{J/K}}$$

***P22.45** The car ends up in the same thermodynamic state as it started, so it undergoes zero changes in entropy. The original kinetic energy of the car is transferred by heat to the surrounding air, adding to the internal energy of the air. Its change in entropy is

$$\Delta S = \frac{\frac{1}{2}mv^2}{T} = \frac{\frac{1}{2}(1\,500\ \text{kg})(20.0\ \text{m/s})^2}{293\ \text{K}} = \boxed{1.02\ \text{kJ/K}}$$

P22.46 The total momentum before collision is zero, so the combined mass must be at rest after the collision. The energy dissipated by heat equals the total initial kinetic energy,

$$Q = 2\left(\frac{1}{2}mv^2\right) = (2000\ \text{kg})(20.0\ \text{m/s})^2 = 8.00\times 10^5\ \text{J} = 800\ \text{kJ}$$

With the environment at an absolute temperature of $T = 23 + 273 = 296$ K, the change in entropy is

$$\Delta S = \frac{\Delta Q_r}{T} = \frac{800\ \text{kJ}}{296\ \text{K}} = \boxed{2.70\ \text{kJ/K}}$$

P22.47 The potential energy lost by the log is eventually transferred by heat into thermal energy of the environment, so $Q = mgh$, and the change in entropy is

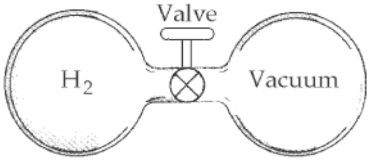
$$\Delta S = \frac{Q}{T} = \frac{mgh}{T} = \frac{(70.0\ \text{kg})(9.80\ \text{m/s}^2)(25.0\ \text{m})}{300\ \text{K}} = \boxed{57.2\ \text{J/K}}$$

P22.48 (a) This is a free expansion process. From Equation 22.17,

$$\Delta S = nR\ln\left(\frac{V_f}{V_i}\right) = (1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})\ln\left(\frac{2}{1}\right)$$

$$= \boxed{5.76\text{ J/K}}$$

(b) The gas is expanding into an evacuated region. Therefore, $W = 0$. It expands so fast that energy has no time to flow by heat: $Q = 0$. But $\Delta E_{\text{int}} = Q + W$, so in this case $\Delta E_{\text{int}} = 0$. For an ideal gas, the internal energy is a function of the temperature and no other variables, so with $\Delta E_{\text{int}} = 0$, there is $\boxed{\text{no change in temperature}}$.



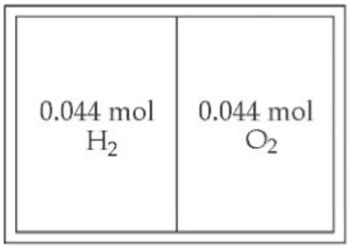
ANS. FIG. P22.48

P22.49 Each gas expands into the other half of the container as though the other gas were not there; therefore, consider each gas to undergo a free expansion process in which its volume doubles. From Equation 22.17, the entropy change is twice that for a single gas:

$$\Delta S = 2\left[nR\ln\left(\frac{V_f}{V_i}\right)\right]$$

$$= 2\big[(0.044\ 0)(8.314\text{ J/mol}\cdot\text{K})(\ln 2)\big]$$

$$= \boxed{0.507\text{ J/K}}$$



ANS. FIG. P22.49

***P22.50** We take data from Tables 20.1 and 20.2, and we assume a constant specific heat for each phase. As the ice is warmed from –12.0°C to 0°C, its entropy increases by

$$\Delta S=\int_i^f \frac{dQ}{T}=\int_{261\text{ K}}^{273\text{ K}}\frac{mc_{\text{ice}}dT}{T}=mc_{\text{ice}}\int_{261\text{ K}}^{273\text{ K}}T^{-1}\,dT=mc_{\text{ice}}\ln T\Big|_{261\text{ K}}^{273\text{ K}}$$

$$\Delta S=(0.027\ 9\text{ kg})(2\ 090\text{ J/kg}\cdot{}^\circ\text{C})(\ln 273\text{ K}-\ln 261\text{ K})$$

$$=(0.027\ 9\text{ kg})(2\ 090\text{ J/kg}\cdot{}^\circ\text{C})\left[\ln\left(\frac{273\text{ K}}{261\text{ K}}\right)\right]$$

$$\Delta S=2.62\text{ J/K}$$

As the ice melts its entropy change is

$$\Delta S=\frac{Q}{T}=\frac{mL_f}{T}=\frac{(0.027\ 9\text{ kg})(3.33\times10^5\text{ J/kg})}{273\text{ K}}=34.0\text{ J/K}$$

As liquid water warms from 273 K to 373 K,

$$\Delta S=\int_i^f\frac{mc_{\text{liquid}}dT}{T}=mc_{\text{liquid}}\ln\left(\frac{T_f}{T_i}\right)$$

$$=(0.027\ 9\text{ kg})(4\ 186\text{ J/kg}\cdot{}^\circ\text{C})\ln\left(\frac{373\text{ K}}{273\text{ K}}\right)=36.5\text{ J/K}$$

As the water boils and the steam warms,

$$\Delta S=\frac{mL_v}{T}+mc_{\text{steam}}\ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S=\frac{(0.027\ 9\text{ kg})(2.26\times10^6\text{ J/kg})}{373\text{ K}}+(0.027\ 9\text{ kg})(2\ 010\text{ J/kg}\cdot{}^\circ\text{C})\ln\left(\frac{388\text{ K}}{373\text{ K}}\right)$$

$$=169\text{ J/K}+2.21\text{ J/K}$$

The total entropy change is

$$\Delta S_{\text{tot}}=(2.62+34.0+36.5+169+2.21)\text{ J/K}=\boxed{244\text{ J/K}}$$

For steam at constant pressure, the molar specific heat in Table 20.1 implies a specific heat of $(35.4\text{ J/mol}\cdot\text{K})\left(\frac{1\text{ mol}}{0.018\text{ kg}}\right)=1\ 970\text{ J/kg}\cdot\text{K}$, nearly agreeing with 2 010 J/kg · K.

***P22.51** The change in entropy is given by

$$\Delta S=\int_i^f\frac{dQ}{T}=\int_{T_i}^{T_f}mc\frac{dT}{T}$$

Here *T* means the absolute temperature. We would ordinarily think of *dT* as the change in the Celsius temperature, but one Celsius degree of temperature change is the same size as one kelvin of change, so *dT* is also the change in absolute *T*.

$$\Delta S = mc\ln T\Big|_{T_i}^{T_f} = mc\ln\left(\frac{T_f}{T_i}\right)$$

$$= (0.250\text{ kg})(4\,186\text{ J/kg}\cdot\text{K})\ln\left(\frac{353\text{ K}}{293\text{ K}}\right) = \boxed{195\text{ J/K}}$$

P22.52 Sitting here writing, I convert chemical energy from molecules in food, into internal energy that leaves my body by heat into the room-temperature surroundings. My rate of energy output is equal to my metabolic rate,

$$2\,500\text{ kcal/d} = \left(\frac{2\,500\times10^3\text{ cal}}{86\,400\text{ s}}\right)\left(\frac{4.186\text{ J}}{1\text{ cal}}\right) = 120\text{ W}$$

My body is in steady state, changing little in entropy, as the environment increases in entropy at the rate

$$\frac{\Delta S}{\Delta t} = \frac{Q/T}{\Delta t} = \frac{Q/\Delta t}{T} = \frac{120\text{ W}}{293\text{ K}} = 0.4\text{ W/K} \sim \boxed{1\text{ W/K}}$$

When using powerful appliances or an automobile, my personal contribution to entropy production is much greater than the above estimate, based only on metabolism.

P22.53 The change in entropy of a reservoir is $\Delta S = Q_r/T$, where Q_r is the energy absorbed ($Q_r > 0$) or expelled ($Q_r < 0$) by the reservoir, and *T* is the absolute temperature of the reservoir.

(a) For the hot reservoir: $\Delta S_h = \dfrac{-2.50\times10^3\text{ J}}{725\text{ K}} = \boxed{-3.45\text{ J/K}}$

(b) For the cold reservoir: $\Delta S_c = \dfrac{+2.50\times10^3\text{ J}}{310\text{ K}} = \boxed{+8.06\text{ J/K}}$

(c) For the Universe:

$$\Delta S_U = \Delta S_h + \Delta S_c = -3.45\text{ J/K} + 8.06\text{ J/K} = \boxed{+4.61\text{ J/K}}$$

P22.54 The change in entropy of a reservoir is $\Delta S = Q_r/T$, where Q_r is the energy absorbed ($Q_r > 0$) or expelled ($Q_r < 0$) by the reservoir, and *T* is the absolute temperature of the reservoir.

(a) Energy is transferred from the hot reservoir by heat: $|Q_h| = -Q_r$

and $\boxed{\Delta S_h = -\frac{Q}{T_h}}$.

(b) Energy is transferred to the cold reservoir by heat: $|Q_c| = +Q$, and

$\boxed{\Delta S_c = \frac{Q}{T_c}}$.

(c) For the Universe, $\Delta S_U = \Delta S_h + \Delta S_c = \boxed{Q\left(\frac{1}{T_c} - \frac{1}{T_h}\right)}$.

P22.55 $$\Delta S = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = \left(\frac{1\,000\text{ J}}{290\text{ K}} - \frac{1\,000\text{ J}}{5\,800\text{ K}}\right) = \boxed{3.28\text{ J/K}}$$

***P22.56** Define T_1 = Temp Cream = 5.00°C = 278 K.

Define T_2 = Temp Coffee = 60.0°C = 333 K.

The final temperature of the mixture is

$$T_f = \frac{(20.0\text{ g})T_1 + (200\text{ g})T_2}{220\text{ g}} = 55.0°\text{C} = 328\text{ K}$$

The entropy change due to this mixing is

$$\begin{aligned}\Delta S &= (20.0\text{ g})\int_{T_1}^{T_f}\frac{c_V dT}{T} + (200\text{ g})\int_{T_2}^{T_f}\frac{c_V dT}{T}\\ &= (84.0\text{ J/K})\ln\left(\frac{T_f}{T_1}\right) + (840\text{ J/K})\ln\left(\frac{T_f}{T_2}\right)\\ &= (84.0\text{ K/J})\ln\left(\frac{328\text{ K}}{278\text{ K}}\right) + (840\text{ J/K})\ln\left(\frac{328\text{ K}}{333\text{ K}}\right)\\ \Delta S &= \boxed{+1.18\text{ J/K}}\end{aligned}$$

***P22.57** We first determine the energy that must be extracted from tap water at 10.0°C to produce ice at –20.0°C:

$$|Q_c| = mc\Delta T + mL + mc\Delta T$$

$$\begin{aligned}|Q_c| &= (0.500\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(10.0°\text{C})\\ &\quad + (0.500\text{ kg})(3.33\times 10^5\text{ J/kg})\\ &\quad + (0.500\text{ kg})(2\,090\text{ J/kg}\cdot°\text{C})(20.0°\text{C})\\ &= 2.08\times 10^5\text{ J}\end{aligned}$$

The work required to accomplish this is then found from

$$\frac{|Q_c|}{W} = \text{COP}_c(\text{refrigerator}) = \frac{T_c}{T_h - T_c}$$

or

$$W = \frac{|Q_c|(T_h - T_c)}{T_c} = \frac{(2.08\times10^5\ \text{J})[20.0°\text{C} - (-20.0°\text{C})]}{273\ \text{K} - 20.0°\text{C}}$$

$$= \boxed{32.9\ \text{kJ}}$$

P22.58 We are given $T_c = 273$ K.

(a) For steam at 100°C, $T_h = 373$ K and

$$e = 1 - \frac{T_c}{T_h} = 1 - \frac{273\ \text{K}}{373\ \text{K}} = \boxed{0.268}$$

(b) For superheated steam at 200°C, $T_h = 473$ K and

$$e = 1 - \frac{T_c}{T_h} = 1 - \frac{273\ \text{K}}{473\ \text{K}} = \boxed{0.423}$$

P22.59 $|Q_h| = 3W$, and for an engine, $|Q_h| = W + |Q_c| = 3W \quad \rightarrow \quad |Q_c| = 2W$.

(a) $$e = \frac{W}{|Q_h|} = \frac{W}{3W} = \boxed{\frac{1}{3}}$$

(b) $$\frac{|Q_c|}{|Q_h|} = \frac{2W}{3W} = \boxed{\frac{2}{3}}$$

P22.60 The conversion of gravitational potential energy into kinetic energy as the water falls is reversible. But the subsequent conversion into internal energy is not. We imagine arriving at the same final state by adding energy by heat, in amount *mgy*, to the water from a stove at a temperature infinitesimally above 20.0°C. Then,

$$\frac{\Delta S}{\Delta t} = \int \frac{dQ_r/\Delta t}{T} = \frac{Q/\Delta t}{T} = \frac{mgy/\Delta t}{T}$$

$$= \frac{(5\ 000\ \text{m}^3/\text{s})(1\ 000\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(50.0\ \text{m})}{293\ \text{K}}$$

$$= \boxed{8.36\times10^6\ \text{J/K}\cdot\text{s}}$$

P22.61 The maximum (Carnot) or ideal efficiency is

$$e_{\text{ideal}} = 1 - \frac{T_c}{T_h}$$

(where we note that the temperatures must be given in kelvin.) Thus,

$$e_{\text{ideal}} = 1 - \frac{T_c}{T_h} = 1 - \frac{(185°C + 273°)}{(545°C + 273°)} = 1 - \frac{458.15K}{818.15K}$$

$$= \boxed{0.440 = 44.0\%}$$

P22.62 (a) $\left(10.0\ \frac{\text{Btu/h}}{\text{W}}\right)\left(\frac{1\,055\ \text{J}}{1\ \text{Btu}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\ \text{W}}{1\ \text{J/s}}\right) = \boxed{2.93}$

(b) The energy extracted by heat from the cold side divided by required work input is by definition the coefficient of performance for a refrigerator: $\boxed{(\text{COP})_{\text{refrigerator}}}$

(c) With an EER of 5,

$$5\ \frac{\text{Btu}}{\text{h}\cdot\text{W}} = \frac{10\,000\ \text{Btu/h}}{P}$$

which gives

$$P = \frac{10\,000\ \text{Btu/h}}{5\ \text{Btu/h}\cdot\text{W}} = 2\ 000\ \text{W} = 2.00\ \text{kW}$$

Energy purchased is $P\Delta t = (2.00\ \text{kW})(1\ 500\ \text{h}) = 3.00 \times 10^3\ \text{kWh}$.

Cost = $(3.00 \times 10^3\ \text{kWh})(0.170\ \$/\text{kWh}) = \$510$:

$\boxed{\text{With EER 5, \$510}}$

With EER 10,

$$10\ \frac{\text{Btu}}{\text{h}\cdot\text{W}} = \frac{10\,000\ \text{Btu/h}}{P}$$

$$\rightarrow P = \frac{10\,000\ \text{Btu/h}}{10\ \text{Btu/h}\cdot\text{W}} = 1\ 000\ \text{W} = 1.00\ \text{kW}$$

Energy purchased is $P\Delta t = (1.00\ \text{kW})(1\ 500\ \text{h}) = 1.50 \times 10^3\ \text{kWh}$

Cost = $(1.50 \times 10^3\ \text{kWh})(0.170\ \$/\text{kWh}) = \$255$:

$\boxed{\text{With EER 10, \$255}}$

Thus, the cost for air conditioning is half as much for an air conditioner with EER 10 compared with an air conditioner with EER 5.

P22.63 (a) $P_{\text{electric}} = \frac{H_{ET}}{\Delta t}$ so if all the electric energy is converted into internal energy, the steady-state condition of the house is described by $H_{ET} = |Q|$.

Therefore, $P_{\text{electric}} = \frac{Q}{\Delta t} = \boxed{5.00\text{ kW}}$.

(b) For a heat pump, $(\text{COP})_{\text{Carnot}} = \frac{T_h}{\Delta T} = \frac{295\text{ K}}{27.0\text{ K}} = 10.93$

Actual $\text{COP} = 0.6(10.93) = 6.56 = \frac{|Q_h|}{W} = \frac{|Q_h|/\Delta t}{W/\Delta t}$

Therefore, to bring 5 000 W of energy into the house only requires input power

$$P_{\text{heat pump}} = \frac{W}{\Delta t} = \frac{|Q_h|/\Delta t}{\text{COP}} = \frac{5\ 000\text{ W}}{6.56} = \boxed{763\text{ W}}$$

***P22.64** (a) The energy transferred to the gas by heat is

$$Q = mc\Delta T = (1.00\text{ mol})(20.79\text{ J/mol}\cdot\text{K})(120\text{ K})$$
$$= 2.49\times 10^3\text{ J} = \boxed{2.49\text{ kJ}}$$

(b) Treating the neon as a monatomic ideal gas, Equation 21.25 gives the change in internal energy as $\Delta U = \frac{3}{2}nR\Delta T$, or

$$\Delta U = \frac{3}{2}(1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(120\text{ K})$$
$$= 1.50\times 10^3\text{ J} = \boxed{1.50\text{ kJ}}$$

(c) From the first law, the work done on the gas is

$$W = \Delta U - Q = 1.50\times 10^3\text{ J} - 2.49\times 10^3\text{ J} = \boxed{-990\text{ J}}$$

P22.65 Energy transfer by heat for infinitesimal temperature change *dT* is *dQ* = *nCdT*, where *C* is the molar specific heat for either constant volume ($C_V = 5R/2$) or pressure ($C_P = 7R/2$) for air, a diatomic gas. The corresponding entropy change is

$$dS = \frac{dQ_r}{T} = \frac{nCdT}{T} \quad \rightarrow \quad \Delta S = \int_{T_i}^{T_f} \frac{nCdT}{T} = nC\ln\frac{T_f}{T_i},$$

with $T_i = 25.0 + 273 = 298\text{ K}$ and $T_f = -18.0 + 273 = 255\text{ K}$.

(a) $\Delta S = nC_V \ln\frac{T_f}{T_i} = n\frac{5}{2}R \ln\left(\frac{255\text{ K}}{298\text{ K}}\right) = \boxed{-0.390nR}$

(b) $\Delta S = nC_P \ln\frac{T_f}{T_i} = n\frac{7}{2}R \ln\left(\frac{255\text{ K}}{298\text{ K}}\right) = \boxed{-0.545nR}$

P22.66 (a) The coefficient of performance of an air conditioner is defined as

$$(\text{COP})_{\text{ac}} = \frac{|Q_c|}{W} = \frac{|Q_c|}{|Q_h| - |Q_c|} = \frac{1}{|Q_h|/|Q_c| - 1}$$

But when a device operates on the Carnot cycle, $|Q_h|/|Q_c| = T_h/T_c$. Thus, the coefficient of performance for a Carnot heat pump would be

$$(\text{COP})_{\text{ac}} = \frac{1}{T_h/T_c - 1} = \boxed{\frac{T_c}{T_h - T_c}}$$

(b) From the result of part (a) above, we observe that the COP of a Carnot air conditioner would increase if the temperature difference $T_h - T_c$ becomes $\boxed{\text{smaller}}$.

(c) If $T_c = 20° + 273 = 293$ K and $T_h = 40° + 273 = 313$ K, the COP of a Carnot heat pump would be

$$(\text{COP})_{\text{ac},C} = \frac{T_c}{T_h - T_c} = \frac{293\text{ K}}{313\text{ K} - 293\text{ K}} = \boxed{14.6}$$

P22.67 (a) For the constant volume process *AB*,

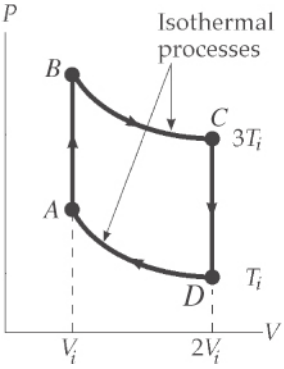
$$Q_{\text{AB}} = \Delta E_{\text{int},\,AB} = \frac{3}{2}nR\Delta T = \frac{3}{2}nR(3T_i - T_i) = \boxed{3nRT_i}$$

(b) For an isothermal process, $Q = nRT \ln\left(\frac{V_2}{V_1}\right)$.

Therefore, for process *BC*, $Q_{BC} = \boxed{3nRT_i \ln 2}$.

(c) For the constant volume process *CD*,

$$Q_{CD} = \Delta E_{\text{int},\,CD} = \frac{3}{2}nR\Delta T = \frac{3}{2}nR(T_i - 3T_i) = \boxed{-3nRT_i}$$



ANS. FIG. P22.67

(d) For an isothermal process *DA*, $Q_{DA} = nRT_i \ln\frac{1}{2} = \boxed{-nRT_i \ln 2}$.

(e) $Q_h = Q_{AB} + Q_{BC} = 3nRT_i + 3nRT_i \ln 2 = \boxed{3nRT_i(1+\ln 2)}$

(f) Since the change in temperature for the complete cycle is zero,

$$\Delta E_{\text{int}} = 0 \quad \text{and} \quad W_{\text{eng}} = Q$$

and work done by the engine is

$$\begin{aligned} W &= Q = Q_{\text{AB}} + Q_{\text{BC}} + Q_{\text{CD}} + Q_{\text{DA}} \\ &= 3nRT_i + 3nRT_i \ln 2 - 3nRT_i - nRT_i \ln 2 \\ W &= \boxed{2nRT_i \ln 2} \end{aligned}$$

(g) The efficiency is

$$e_{\text{c}} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{Q}{|Q_h|} = \frac{2\ln 2}{3(1+\ln 2)} = \boxed{0.273}$$

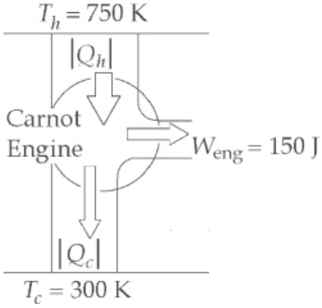
P22.68 For the Carnot engine,

$$e_{\text{c}} = 1 - \frac{T_c}{T_h} = 1 - \frac{300\text{ K}}{750\text{ K}} = 0.600$$

Also, $e_{\text{c}} = \frac{W_{\text{eng}}}{|Q_h|}$,

so $|Q_h| = \frac{W_{\text{eng}}}{e_{\text{c}}} = \frac{150\text{ J}}{0.600} = 250\text{ J}$

and $|Q_c| = |Q_h| - W_{\text{eng}} = 250\text{ J} - 150\text{ J} = 100\text{ J}$.



ANS. FIG. P22.68

(a) $|Q_h| = \frac{W_{\text{eng}}}{e_{\text{S}}} = \frac{150\text{ J}}{0.700} = \boxed{214\text{ J}}$

$$|Q_c| = |Q_h| - W_{\text{eng}} = \frac{W_{\text{eng}}}{e_{\text{S}}} - 150\text{ J} = \boxed{64.3\text{ J}}$$

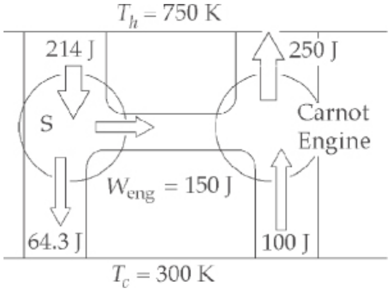
(b) When engine S delivers 150 J of work to the Carnot engine, the Carnot engine transfers 250 J to the firebox while engine S takes 214 J from the firebox:

$$|Q_{h,\text{net}}| = -\frac{W_{\text{eng}}}{e_{\text{S}}} + 250\text{ J} = \boxed{35.7\text{ J}}$$

and the Carnot engine removes 100 J from the environment while engine S returns 64.3 J:

$$|Q_{c,\text{net}}| = |64.3\text{ J} - 100\text{ J}| = \boxed{35.7\text{ J}}$$

$$\boxed{\text{The total energy the firebox puts out equals the total energy transferred to the environment.}}$$



ANS. FIG. P22.68(a–b)

(c) $\boxed{\text{The net flow of energy by heat from the cold to the hot reservoir without work input is impossible.}}$

(d) For engine S: $|Q_{c,\text{S}}| = |Q_{h,\text{S}}| - W_{\text{eng S}} = \dfrac{W_{\text{eng S}}}{e_{\text{S}}} - W_{\text{eng S}}$

so work output is $W_{\text{eng S}} = \dfrac{|Q_{c,\text{S}}|}{\frac{1}{e_{\text{S}}} - 1} = \dfrac{100\text{ J}}{\frac{1}{0.700} - 1} = \boxed{233\text{ J}}$

and energy input to engine S is

$$|Q_{h,\text{S}}| = |Q_{c,\text{S}}| + W_{\text{eng S}} = 233\text{ J} + 100\text{ J} = \boxed{333\text{ J}}$$

(e) Engine S contributes 150 J out of 233 J to running the Carnot engine:

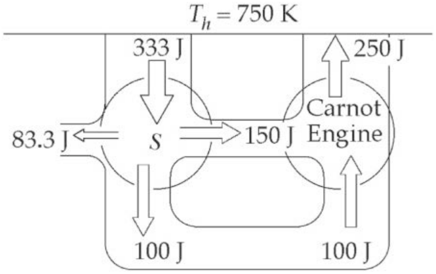
$$|Q_{h,\text{net}}| = |Q_{h,\text{S}}| - 250\text{ J} = 333\text{ J} - 250\text{ J} = \boxed{83.3\text{ J}}$$

This is the net energy lost by the firebox.

(f) The remaining work output is

$$W_{\text{net}} = W_{\text{eng S}} - 250\text{ J} = 233\text{ J} - 150\text{ J} = \boxed{83.3\text{ J}}$$

(g) $\left|Q_{c,\text{net}}\right| = \boxed{0}$



ANS. FIG. P22.68(e–g)

(h) $\boxed{\text{The output of 83.3 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible.}}$

(i) Both engines operate in cycles, so $\Delta S_{\text{S}} = \Delta S_{\text{Carnot}} = 0$.

For the reservoirs, $\Delta S_h = -\dfrac{|Q_h|}{T_h}$ and $\Delta S_c = +\dfrac{|Q_c|}{T_c}$.

Thus,

$$\Delta S_{\text{total}} = \Delta S_{\text{S}} + \Delta S_{\text{Carnot}} + \Delta S_h + \Delta S_c = 0 + 0 - \frac{83.3\text{ J}}{750\text{ K}} + \frac{0}{300\text{ K}}$$
$$= \boxed{-0.111\text{ J/K}}$$

(j) $\boxed{\text{A decrease in total entropy is impossible.}}$

P22.69 (a) Let state *i* represent the gas before its compression and state *f* afterwards, $V_f = \dfrac{V_i}{8}$. For a diatomic ideal gas, $C_V = \dfrac{5}{2}R$, $C_p = \dfrac{7}{2}R$, and $\gamma = \dfrac{C_p}{C_V} = 1.40$. Next,

$$P_i V_i^\gamma = P_f V_f^\gamma$$

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^\gamma = P_i 8^{1.40} = 18.4 P_i$$

$$P_i V_i = nRT_i$$

$$P_f V_f = \frac{18.4 P_i V_i}{8} = 2.30 P_i V_i = 2.30 nRT_i = nRT_f$$

so $T_f = 2.30T_i$

$$\Delta E_{\text{int}} = nC_V\Delta T = n\frac{5}{2}R\left(T_f - T_i\right) = \frac{5}{2}nR\left(1.30T_i\right) = \frac{5}{2}\left(1.30P_iV_i\right)$$

$$= \frac{5}{2}(1.30)\left(1.013\times10^5\ \text{N/m}^2\right)\left(0.120\times10^{-3}\ \text{m}^3\right) = 39.4\ \text{J}$$

Since the process is adiabatic, $Q = 0$ and $\Delta E_{\text{int}} = Q + W$ gives

$$W = \boxed{39.4\ \text{J}}$$

(b) The moment of inertia of the wheel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\left(5.10\ \text{kg}\right)\left(0.085\ 0\ \text{m}\right)^2 = 0.018\ 4\ \text{kg}\cdot\text{m}^2$$

We want the flywheel to do work 39.4 J, so the work on the flywheel should be –39.4 J:

$$K_{\text{rot}\,i} + W = K_{\text{rot}\,f}$$

$$\frac{1}{2}I\omega_i^2 - 39.4\ \text{J} = 0$$

$$\omega_i = \left[\frac{2\left(39.4\ \text{J}\right)}{0.018\ 4\ \text{kg}\cdot\text{m}^2}\right]^{1/2} = \boxed{65.4\ \text{rad/s} = 625\ \text{rev/min}}$$

(c) Now we want $W = 0.05K_{\text{rot}\,i}$:

$$39.4\ \text{J} = 0.05\left[\frac{1}{2}\left(0.018\ 4\ \text{kg}\cdot\text{m}^2\right)\omega_i^2\right]$$

$$\omega_i = \left(\frac{2\left(789\ \text{J}\right)}{0.018\ 4\ \text{kg}\cdot\text{m}^2}\right)^{1/2} = \boxed{293\ \text{rad/s} = 2.79\times10^3\ \text{rev/min}}$$

P22.70 Like a refrigerator, an air conditioner has as its purpose the removal of energy by heat from the cold reservoir.

Its ideal COP is $\text{COP}_{\text{Carnot}} = \dfrac{T_c}{T_h - T_c} = \dfrac{280\ \text{K}}{20.0\ \text{K}} = 14.0.$

(a) Its actual COP is

$$0.400(14.0) = 5.60 = \frac{|Q_c|}{|Q_h| - |Q_c|} = \frac{|Q_c/\Delta t|}{|Q_h/\Delta t| - |Q_c/\Delta t|}$$

$$5.60\left|\frac{Q_h}{\Delta t}\right| - 5.60\left|\frac{Q_c}{\Delta t}\right| = \left|\frac{Q_c}{\Delta t}\right|$$

$$5.60(10.0\text{ kW}) = 6.60\left|\frac{Q_c}{\Delta t}\right| \quad \text{and} \quad \left|\frac{Q_c}{\Delta t}\right| = \boxed{8.48\text{ kW}}$$

(b) $|Q_h| = W_{\text{eng}} + |Q_c|$:

$$\frac{W_{\text{eng}}}{\Delta t} = \left|\frac{Q_h}{\Delta t}\right| - \left|\frac{Q_c}{\Delta t}\right| = 10.0\text{ kW} - 8.48\text{ kW} = \boxed{1.52\text{ kW}}$$

(c) The air conditioner operates in a cycle, so the entropy of the working fluid does not change. The hot reservoir increases in entropy by

$$\frac{|Q_h|}{T_h} = \frac{(10.0\times 10^3\text{ J/s})(3\,600\text{ s})}{300\text{ K}} = 1.20\times 10^5\text{ J/K}$$

The cold room decreases in entropy by

$$\Delta S = -\frac{|Q_c|}{T_c} = -\frac{(8.48\times 10^3\text{ J/s})(3\,600\text{ s})}{280\text{ K}}$$
$$= -1.09\times 10^5\text{ J/K}$$

The net entropy change is positive, as it must be:

$$+1.20\times 10^5\text{ J/K} - 1.09\times 10^5\text{ J/K} = \boxed{1.09\times 10^4\text{ J/K}}$$

(d) The new ideal COP is $\text{COP}_{\text{Carnot}} = \frac{T_c}{T_h - T_c} = \frac{280\text{ K}}{25\text{ K}} = 11.2.$

We suppose the actual COP is 0.400(11.2) = 4.48.

As a fraction of the original 5.60, this is $\frac{4.48}{5.60} = 0.800$, so the fractional change is to $\boxed{\text{drop by 20.0\%}}$.

P22.71 $e_C = 1 - \frac{T_c}{T_h} = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}/\Delta t}{|Q_h|/\Delta t}: \quad \frac{|Q_h|}{\Delta t} = \frac{P}{(1 - T_c/T_h)} = \frac{PT_h}{T_h - T_c}$

$|Q_h| = W_{\text{eng}} + |Q_c|: \quad \frac{|Q_c|}{\Delta t} = \frac{|Q_h|}{\Delta t} - \frac{W_{\text{eng}}}{\Delta t}$

$$\frac{|Q_c|}{\Delta t} = \frac{PT_h}{T_h - T_c} - P = \frac{PT_c}{T_h - T_c}$$

$|Q_c| = mc\Delta T: \quad \frac{|Q_c|}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)c\Delta T = \frac{PT_c}{T_h - T_c}$

$$\frac{\Delta m}{\Delta t}=\frac{PT_c}{(T_h-T_c)c\Delta T}$$

$$\frac{\Delta m}{\Delta t}=\frac{(1.00\times10^9\ \text{W})(300\ \text{K})}{(200\ \text{K})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(6.00^\circ\text{C})}=\boxed{5.97\times10^4\ \text{kg/s}}$$

P22.72 $e_{\text{C}}=1-\dfrac{T_c}{T_h}=\dfrac{W_{\text{eng}}}{|Q_h|}=\dfrac{W_{\text{eng}}/\Delta t}{|Q_h|/\Delta t}$: $\quad \dfrac{|Q_h|}{\Delta t}=\dfrac{P}{1-(T_c/T_h)}=\dfrac{PT_h}{T_h-T_c}$

$$\frac{|Q_c|}{\Delta t}=\left(\frac{|Q_h|}{\Delta t}\right)-P=\frac{PT_c}{T_h-T_c}$$

But $|Q_c|=mc\Delta T$, where *c* is the specific heat of water.

Therefore, $\dfrac{|Q_c|}{\Delta t}=\left(\dfrac{\Delta m}{\Delta t}\right)c\Delta T=\dfrac{PT_c}{T_h-T_c}$

and $\dfrac{\Delta m}{\Delta t}=\boxed{\dfrac{PT_c}{(T_h-T_c)c\Delta T}}$.

P22.73 (a) For the isothermal process *AB*, the work on the gas is

$$W_{AB}=-P_AV_A\ln\left(\frac{V_B}{V_A}\right)$$

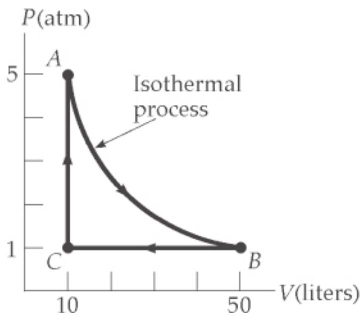
$$W_{AB}=-5(1.013\times10^5\ \text{Pa})(10.0\times10^{-3}\ \text{m}^3)\ln\left(\frac{50.0\ \text{L}}{10.0\ \text{L}}\right)$$

$$W_{AB}=-8.15\times10^3\ \text{J}$$

where we have used 1.00 atm = 1.013 × 10^5 Pa and 1.00 L = 1.00 × 10^{-3} m^3.

$$W_{BC}=-P_B\Delta V=-(1.013\times10^5\ \text{Pa})[(10.0-50.0)\times10^{-3}]\text{m}^3$$
$$=+4.05\times10^3\ \text{J}$$

$$W_{CA}=0\quad\text{and}\quad W_{\text{eng}}=-W_{AB}-W_{BC}=4.10\times10^3\ \text{J}=\boxed{4.10\ \text{kJ}}$$



ANS. FIG. P22.73

(b) Since *AB* is an isothermal process, $\Delta E_{\text{int},AB} = 0$

and $Q_{AB} = -W_{AB} = 8.15 \times 10^3$ J.

For an ideal monatomic gas, $C_V = \frac{3R}{2}$ and $C_P = \frac{5R}{2}$.

$$T_B = T_A = \frac{P_B V_B}{nR} = \frac{(1.013\times10^5\ \text{Pa})(50.0\times10^{-3}\ \text{m}^3)}{R} = \frac{5.06\times10^3}{R}$$

Also, $T_C = \frac{P_C V_C}{nR} = \frac{(1.013\times10^5\ \text{Pa})(10.0\times10^{-3}\ \text{m}^3)}{R} = \frac{1.01\times10^3}{R}$

$$Q_{CA} = nC_V\Delta T = 1.00\left(\frac{3}{2}R\right)\left(\frac{5.06\times10^3 - 1.01\times10^3}{R}\right) = 6.08\ \text{kJ}$$

so the total energy absorbed by heat is

$$Q_{AB} + Q_{CA} = 8.15\ \text{kJ} + 6.08\ \text{kJ} = \boxed{1.42\times10^4\ \text{J}}$$

(c) $Q_{BC} = nC_P\Delta T = \frac{5}{2}(nR\Delta T) = \frac{5}{2}P_B\Delta V_{BC}$

$$|Q_{BC}| = \left|\frac{5}{2}(1.013\times10^5)\left[(10.0-50.0)\times10^{-3}\right]\right| = \left|-1.01\times10^4\ \text{J}\right|$$

$= \boxed{1.01\times10^4\ \text{J}}$

(d) $e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{Q_{AB}+Q_{CA}} = \frac{4.10\times10^3\ \text{J}}{1.42\times10^4\ \text{J}} = 0.289$ or $\boxed{28.9\%}$

(e) A Carnot engine operating between $T_{\text{hot}} = T_A = 5\,060/R$ and $T_{\text{cold}} = T_C = 1\,010/R$ has $e_C = 1 - T_c/T_h = 1 - 1/5 =$ $\boxed{\text{80.0\%. The efficiency of the cycle is much lower than that of a Carnot engine operating between the same temperature extremes.}}$

P22.74 (a) The ideal gas at constant temperature keeps constant internal energy. As it puts out energy by work in expanding, it must take in an equal amount of energy by heat. Thus its entropy increases. Let P_i, V_i, and T_i represent the state of the gas before the isothermal expansion. Let P_C, V_C, and T_i represent the state after this process, so that $P_iV_i = P_CV_C$. Let P_i, $3V_i$, and T_f represent the state after the adiabatic compression.

Then $P_C V_C^\gamma = P_i(3V_i)^\gamma$

Substituting $P_C = \dfrac{P_i V_i}{V_C}$

gives $P_i V_i V_C^{\gamma-1} = P_i\left(3^\gamma V_i^\gamma\right)$

Then $V_C^{\gamma-1} = 3^\gamma V_i^{\gamma-1}$ and $\dfrac{V_C}{V_i} = 3^{\gamma/(\gamma-1)}$

The work output in the isothermal expansion is

$$W = \int_i^C P\,dV = nRT_i \int_i^C V^{-1}\,dV$$

$$= nRT_i \ln\left(\frac{V_C}{V_i}\right) = nRT_i \ln\left(3^{\gamma/(\gamma-1)}\right) = nRT_i\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

This is also the input heat, so the entropy change is

$$\Delta S = \frac{Q}{T} = nR\left(\frac{\gamma}{\gamma-1}\right)\ln 3$$

Since $C_P = \gamma C_V = C_V + R,$

we have $(\gamma-1)C_V = R,\ C_V = \dfrac{R}{\gamma-1}$

and $C_P = \dfrac{\gamma R}{\gamma-1}.$

Then the result is $\boxed{\Delta S = nC_P \ln 3}$.

(b) The pair of processes considered here carries the gas from the initial state in P22.77 to the final state here. Entropy is a function of state. Entropy change does not depend on path. Therefore the entropy change in P22.77 equals $\Delta S_{\text{isothermal}} + \Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}} = 0$, the answers to P22.77 and P22.74(a) must be the same.

P22.75 We recognize that $T_c = T_1$ and $T_h = T_2$, and $Q_c = 350$ J and $Q_h = -1\,000$ J.

$$\Delta S_{\text{hot}} = \frac{Q_h}{T_h} = -\frac{|Q_h|}{T_2} = \frac{-1000\text{ J}}{600\text{ K}}$$

$$\Delta S_{\text{cold}} = \frac{Q_c}{T_c} = \frac{|Q_c|}{T_1} = \frac{+750\text{ J}}{350\text{ K}}$$

(a) $\Delta S_U = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = -\dfrac{1\ 000\text{ J}}{600\text{ K}} - \dfrac{750\text{ J}}{350\text{ K}} = \boxed{0.476\ \text{J/K}}$

(b) $e_C = 1 - \dfrac{T_1}{T_2} = 1 - \dfrac{350\text{ K}}{600\text{ K}} = 0.417$

$W_{\text{eng}} = e_C\left|Q_h\right| = 0.417\left(1\ 000\text{ J}\right) = \boxed{417\text{ J}}$

(c)
$$\begin{aligned}
\Delta W &= W_C - W_{\text{real}} \\
&= e_C\left|Q_h\right| - \left(\left|Q_h\right| - \left|Q_c\right|\right) \\
&= \left(1 - \frac{T_c}{T_h}\right)\left|Q_h\right| - \left(\left|Q_h\right| - \left|Q_c\right|\right) \\
&= \left(\left|Q_h\right| - \frac{T_c}{T_h}\left|Q_h\right|\right) - \left(\left|Q_h\right| - \left|Q_c\right|\right) \\
&= \left|Q_c\right| - \frac{T_c}{T_h}\left|Q_h\right| = T_c\left(\frac{\left|Q_c\right|}{T_c} - \frac{\left|Q_h\right|}{T_h}\right) \\
&= T_c\Delta S_U = T_1\Delta S_U
\end{aligned}$$

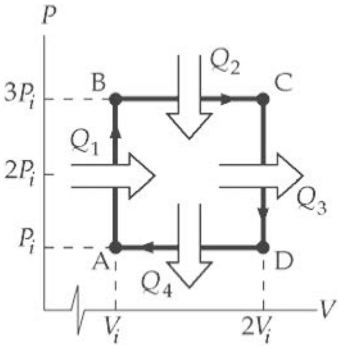
P22.76 At point *A*, $P_iV_i = nRT_i$ and $n = 1.00$ mol.

At point *B*, $3P_iV_i = nRT_B$ so $T_B = 3T_i$.

At point *C*, $\left(3P_i\right)\left(2V_i\right) = nRT_C$ and $T_C = 6T_i$.

At point *D*, $P_i\left(2V_i\right) = nRT_D$ so $T_D = 2T_i$.

The heat for each step in the cycle is found using $C_V = \dfrac{3R}{2}$ and $C_P = \dfrac{5R}{2}$:



ANS. FIG. P22.76

$$\begin{aligned}
Q_{AB} &= nC_V\left(3T_i - T_i\right) = 3nRT_i \\
Q_{BC} &= nC_P\left(6T_i - 3T_i\right) = 7.50nRT_i \\
Q_{CD} &= nC_V\left(2T_i - 6T_i\right) = -6nRT_i \\
Q_{DA} &= nC_P\left(T_i - 2T_i\right) = -2.50nRT_i
\end{aligned}$$

(a) Therefore, $Q_{\text{entering}} = \left|Q_h\right| = Q_{AB} + Q_{BC} = 3nRT_i + 7.5nRT_i = \boxed{10.5nRT_i}$.

(b) $Q_{\text{leaving}} = \left|Q_c\right| = \left|Q_{CD} + Q_{DA}\right| = \left|-6nRT_i - 2.50nRT_i\right| = \boxed{8.50nRT_i}$

(c) Actual efficiency: $e = \dfrac{\left|Q_h\right| - \left|Q_c\right|}{\left|Q_h\right|} = \dfrac{10.5nRT_i - 8.5nRT_i}{10.5nRT_i} = \boxed{0.190}$

(d) Carnot efficiency: $e_C = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{T_i}{6T_i} = \boxed{0.833}$

$\boxed{\text{The Carnot efficiency is much higher.}}$

P22.77 $$\Delta S = \int_i^f \frac{dQ}{T} = \int_i^f \frac{nC_P dT}{T} = nC_P \int_i^f T^{-1} dT = nC_P \ln T\Big|_{T_i}^{T_f} = nC_P\left(\ln T_f - \ln T_i\right)$$

$$= nC_P \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = nC_P \ln\left(\frac{PV_f}{nR}\frac{nR}{PV_i}\right) = nC_P \ln\left[\frac{P(3V_i)}{nR}\frac{nR}{PV_i}\right] = \boxed{nC_P \ln 3}$$

P22.78 (a) water: $T_{\text{water}} = 35.0°\text{F} \rightarrow \frac{5}{9}(35.0 - 32.0)°\text{C}$

$\rightarrow (1.67 + 273.15)\text{ K} = 274.82\text{ K}$

body: $T_{\text{body}} = 98.6°\text{F} \rightarrow \frac{5}{9}(98.6 - 32.0)°\text{C}$

$\rightarrow (37.0 + 273.15)\text{ K} = 310.15\text{ K}$

$$\Delta S_{\text{cold water}} = \int \frac{dQ}{T} = m_{\text{w}} c \times \int_{T_{\text{water}}}^{T_{\text{body}}} \frac{dT}{T} = m_{\text{w}} c \times \ln\left(\frac{T_{\text{body}}}{T_{\text{water}}}\right)$$

$$\Delta S_{\text{body}} = -\frac{|Q|}{T_{\text{body}}} = -\frac{m_{\text{w}} c\left(T_{\text{body}} - T_{\text{water}}\right)}{T_{\text{body}}}$$

$$\Delta S_{\text{system}} = \Delta S_{\text{cold water}} + \Delta S_{\text{body}}$$

$$= (0.454\text{ kg})(4\,186\text{ J/kg}\cdot\text{K}) \times \ln\left(\frac{310.15}{274.82}\right)$$

$$-(0.454\text{ kg})(4\,186\text{ J/kg}\cdot\text{K})\frac{(310.15 - 274.82)}{310.15} = \boxed{13.4\text{ J/K}}$$

(b) Conservation of energy, $Q_{\text{hot}} = -Q_{\text{cold}}$, gives

$$m_w \cancel{c}\left(T_F - T_{\text{water}}\right) = -m_{\text{Ath}} \cancel{c}\left(T_F - T_{\text{body}}\right)$$

$$m_w\left(T_F - T_{\text{water}}\right) = -m_{\text{Ath}}\left(T_F - T_{\text{body}}\right)$$

$$m_w T_F - m_w T_{\text{water}} = -m_{\text{Ath}} T_F + m_{\text{Ath}} T_{\text{body}}$$

$$\left(m_w + m_{\text{Ath}}\right) T_F = m_w T_{\text{water}} + m_{\text{Ath}} T_{\text{body}}$$

Solving for T_F,

$$T_F = \frac{m_w T_{\text{water}} + m_{\text{Ath}} T_{\text{body}}}{m_w + m_{\text{Ath}}}$$

$$= \frac{(0.454\text{ kg})(274.82\text{ K}) + (70.0\text{ kg})(310.15\text{ K})}{0.454\text{ kg} + 70.0\text{ kg}}$$

$$= 309.92\text{ K} = \boxed{310\text{ K}}$$

(c) $\Delta S = \Delta S'_{\text{ice water}} + \Delta S'_{\text{body}}$

$$= m_{\text{w}} c \times \ln\left(\frac{T_F}{T_{\text{water}}}\right) + m_{\text{Ath}} c \times \ln\left(\frac{T_F}{T_{\text{body}}}\right)$$

$$= (0.454\text{ kg})(4\ 186\text{ J/kg}\cdot\text{K})\ln\left(\frac{309.92}{274.82}\right)$$

$$+ (70.0\text{ kg})(4\ 186\text{ J/kg}\cdot\text{K})\ln\left(\frac{309.92}{310.15}\right)$$

$$= \boxed{11.1\text{ J/K}}$$

(d) $\boxed{\text{Smaller by less than 1\%}}$

P22.79 (a) $W = \int_{V_i}^{V_f} P\,dV = nRT \int_{V_i}^{2V_i} \frac{dV}{V} = (1.00)RT \ln\left(\frac{2V_i}{V_i}\right) = \boxed{RT \ln 2}$

(b) $\boxed{\text{Yes.}}$

(c) $\boxed{\text{No. The second law refers to an engine operating in a cycle, whereas this problem involves only a single process.}}$

P22.80 When energy enters a substance by heat, we describe the process with Equation 20.4, $Q = mc\Delta T$. This is a reversible process; if energy leaves the substance, the temperature drops down again. Therefore, the entropy change for one of the samples of water is

$$\Delta S = \int \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{mc\,dT}{T} = mc \ln\left(\frac{T_f}{T_i}\right)$$

Consequently, the entropy change for both samples of water is

$$\Delta S_{\text{total}} = \Delta S_{\text{hot}} + \Delta S_{\text{cold}}$$

$$= mc \ln\left(\frac{T_f}{T_{hi}}\right) + mc \ln\left(\frac{T_f}{T_{ci}}\right) = mc \ln\left[\left(\frac{T_f}{T_{hi}}\right)\left(\frac{T_f}{T_{ci}}\right)\right]$$

$$(1.00\text{ kg})(4\ 186\text{ J/kg}\cdot{}^\circ\text{C})\ln\left[\left(\frac{293\text{ K}}{303\text{ K}}\right)\left(\frac{293\text{ K}}{283\text{ K}}\right)\right] = 4.88\text{ J/K}$$

This is *not* zero. While the statements about energy transfer by heat are true, the mixing process is irreversible. After the water has come to equilibrium, it will not spontaneously separate again into warm and cool water. Therefore, there is an entropy increase of the mixture during this irreversible process.

Challenge Problems

P22.81 (a) Given: $P_A = \boxed{25.0\text{ atm}}$ and $P_C = \boxed{1.00\text{ atm}}$

Use the equation of state for an ideal gas:

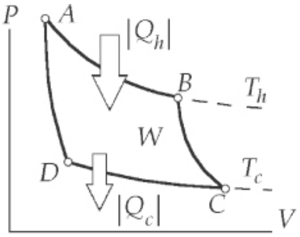
$$V = \frac{nRT}{P}$$

$$V_A = \frac{1.00(8.314\text{ J/mol}\cdot\text{K})(600\text{ K})}{25.0(1.013\times10^5\text{ Pa})} = \boxed{1.97\times10^{-3}\text{ m}^3}$$

$$V_C = \frac{1.00(8.314\text{ J/mol}\cdot\text{K})(400\text{ K})}{1.013\times10^5\text{ Pa}} = \boxed{32.8\times10^{-3}\text{ m}^3}$$

Since AB is isothermal, $P_AV_A = P_BV_B$,

and since BC is adiabatic, $P_BV_B^\gamma = P_CV_C^\gamma$.



ANS. FIG. P22.81

Combining these expressions,

$$V_B = \left[\left(\frac{P_C}{P_A}\right)\frac{V_C^\gamma}{V_A}\right]^{1/(\gamma-1)}$$

$$= \left[\left(\frac{1.00\text{ atm}}{25.0\text{ atm}}\right)\left(\frac{(32.8\times10^{-3}\text{ m}^3)^{1.40}}{1.97\times10^{-3}\text{ m}^3}\right)\right]^{(1/0.400)}$$

$$= \boxed{11.9\times10^{-3}\text{ m}^3}$$

Similarly,

$$V_D = \left[\left(\frac{P_A}{P_C}\right)\frac{V_A^\gamma}{V_C}\right]^{1/(\gamma-1)}$$

$$= \left[\left(\frac{25.0\text{ atm}}{1.00\text{ atm}}\right)\left(\frac{\left(1.97\times10^{-3}\text{ m}^3\right)^{1.40}}{32.8\times10^{-3}\text{ m}^3}\right)\right]^{(1/0.400)}$$

$$= \boxed{5.44\times10^{-3}\text{ m}^3}$$

Since *AB* is isothermal, $P_AV_A = P_BV_B$

and $P_B = P_A\left(\frac{V_A}{V_B}\right) = (25.0\text{ atm})\left(\frac{1.97\times10^{-3}\text{ m}^3}{11.9\times10^{-3}\text{ m}^3}\right) = \boxed{4.14\text{ atm}}$

Also, *CD* is an isothermal and

$$P_D = P_C\left(\frac{V_C}{V_D}\right) = (1.00\text{ atm})\left(\frac{32.8\times10^{-3}\text{ m}^3}{5.44\times10^{-3}\text{ m}^3}\right) = \boxed{6.03\text{ atm}}$$

(b) Energy is added by heat to the gas during the process *AB*. For the isothermal process, $\Delta E_{\text{int}} = 0$, and the first law gives

$$Q_{AB} = -W_{AB} = nRT_h\ln\left(\frac{V_B}{V_A}\right)$$

or

$$|Q_h| = Q_{AB} = (1.00\text{ mol})(8.314\text{ J/mol}\cdot\text{K})(600\text{ K})\ln\left(\frac{11.9\text{ atm}}{1.97\text{ atm}}\right)$$

$$= 8.97\text{ kJ}$$

Then, from $e = \frac{W_{\text{eng}}}{|Q_h|}$, the net work done per cycle is

$$W_{\text{eng}} = e_c|Q_h| = 0.333(8.97\text{ kJ}) = \boxed{2.99\text{ kJ}}$$

P22.82 The quantity of gas is

$$n = \frac{P_AV_A}{RT_A} = \frac{\left(100\times10^3\text{ Pa}\right)\left(500\times10^{-6}\text{ m}^3\right)}{(8.314\text{ J/mol}\cdot\text{K})(293\text{ K})} = 0.020\ 5\text{ mol}$$

(a) In process *A*→*B*,

$$P_B = P_A\left(\frac{V_A}{V_B}\right)^\gamma = \left(100\times10^3\text{ Pa}\right)(8.00)^{1.40} = 1.84\times10^6\text{ Pa}$$

$$T_B = \frac{P_B V_B}{nR} = \frac{\left(1.84\times10^{6}\ \text{Pa}\right)\left(500\times10^{-6}\ \text{m}^3/8.00\right)}{\left(0.020\ 5\ \text{mol}\right)\left(8.314\ \text{J/mol}\cdot\text{K}\right)} = 673\ \text{K}$$

$$V_A/V_B = 8.00 \quad \to \quad V_B = V_A/8.00 = 500/8\ =\ 62.5\ \text{cm}^3$$

State C:

$$V_C = V_B$$

$$P_C = \frac{nRT_C}{V_C} = \frac{\left(0.020\ 5\ \text{mol}\right)\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(1\ 023\ \text{K}\right)}{62.5\times10^{-6}\ \text{m}^3}$$

$$= 2.79\times10^{6}\ \text{Pa}$$

State D:

$$V_D = V_A$$

In process *C*→*D*:

$$P_D = P_C\left(\frac{V_C}{V_D}\right)^{\gamma} = \left(2.79\times10^{6}\ \text{Pa}\right)\left(\frac{1}{8.00}\right)^{1.40} = 1.52\times10^{5}\ \text{Pa}$$

$$T_D = \frac{P_D V_D}{nR} = \frac{\left(1.52\times10^{5}\ \text{Pa}\right)\left(500\times10^{-6}\ \text{m}^3\right)}{\left(0.020\ 5\ \text{mol}\right)\left(8.314\ \text{J/mol}\cdot\text{K}\right)} = 445\ \text{K}$$

TABLE P22.82(a) tabulates these results:

	T (*K*)	*P* (kPa)	*V* (cm^3)
A	293	100	500
B	673	1.84×10^3	62.5
C	1023	2.79×10^3	62.5
D	445	152	500

TABLE P22.82(a)

(b) In the adiabatic process *A*→*B*, *Q* = 0,

$$\Delta E_{\text{int},\,A\to B} = \frac{5}{2}nR\left(T_B - T_A\right)$$

$$= \frac{5}{2}\left(0.020\ 5\ \text{mol}\right)\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(673\ \text{K} - 293\ \text{K}\right)$$

$$= 162\ \text{J}$$

and $\Delta E_{\text{int},\,AB} = 162\ \text{J} = Q - W_{\text{out}} = 0 - W_{\text{out}} \quad \to \quad W_{AB} = -162\ \text{J}$

In the isovolumetric process *B*→*C*, *W* = 0

$$\begin{aligned}\Delta E_{\text{int, }B\to C} &= \frac{5}{2}nR\left(T_C - T_B\right)\\ &= \frac{5}{2}(0.020\ 5\text{ mol})\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(1\ 023\text{ K} - 673\text{ K}\right)\\ &= 149\text{ J}\\ \Delta E_{\text{int, }B\to C} &= 149\text{ J} = Q - W_{\text{out}} = Q - 0 \quad\rightarrow\quad Q_{BC} = 149\text{ J}\end{aligned}$$

In the adiabatic process *C*→*D*, *Q* = 0

$$\begin{aligned}\Delta E_{\text{int, }C\to D} &= \frac{5}{2}nR\left(T_D - T_C\right)\\ &= \frac{5}{2}(0.020\ 5\text{ mol})\left(8.314\ \text{J/mol}\cdot\text{K}\right)\left(445\text{ K} - 1\ 023\text{ K}\right)\\ &= -246\text{ J}\\ \Delta E_{\text{int, }C\to D} &= -246\text{ J} = Q - W_{\text{out}} = 0 - W_{\text{out}} \quad\rightarrow\quad W_{CD} = 246\text{ J}\end{aligned}$$

For the entire cycle, $\Delta E_{\text{int, net}} = \frac{5}{2}nR\Delta T = 0$:

$$\begin{aligned}W_{\text{eng}} &= -162\text{ J} + 0 + 246.3\text{ J} + 0 = 84.3\text{ J}\\ \Delta E_{\text{int}} &= Q_{\text{net}} + W_{\text{eng}} = 0 \quad\rightarrow\quad Q_{\text{net}} = -W_{\text{eng}} = 84.3\text{ J}\end{aligned}$$

TABLE P22.82(b) tabulates these results:

	Q	W_{eng}	ΔE_{int}
A→*B*	0	–162	162
B→*C*	149	0	149
C→*D*	0	246	–246
D→*A*	–65.0	0	–65.0
ABCD	84.3	84.3	0

TABLE P22.82(b)

(c) From B→C, the input energy is $Q_h = \boxed{149\text{ J}}$.

(d) From D→A, the energy exhaust is $|Q_c| = \boxed{65.0\text{ J}}$.

(e) From ABCDA, $W_{\text{eng}} = \boxed{84.3\text{ J}}$.

(f) The efficiency is: $e = \frac{W_{\text{eng}}}{Q_h} = \frac{84.3\ \text{J}}{149\ \text{J}} = \boxed{0.565}$

(g) Let *f* represent the angular speed of the crankshaft. Then $\frac{f}{2}$ is the frequency at which we obtain work in the amount of $84.3\ \text{J/cycle}$:

$$1\ 000\ \text{J/s} = \left(\frac{f}{2}\right)(84.3\ \text{J/cycle})$$

$$f = \frac{2\ 000\ \text{J/s}}{84.3\ \text{J/cycle}} = 23.7\ \text{rev/s} = \boxed{1.42 \times 10^3\ \text{rev/min}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P22.2 (a) 0.25 or 25%; (b) $|Q_C|/|Q_h| = 3/4$

P22.4 13.7°C

P22.6 (a) 29.4 L/h; (b) 185 hp; (c) 527 N · m; (d) 1.91×10^5 W

P22.8 (a) 24.0 J; (b) 144 J

P22.10 (a) 7.69×10^8 J; (b) 5.67×10^8 J

P22.12 (a) 2.65×10^7 J; (b) 3.20

P22.14 0.540 or 54.0%

P22.16 The efficiency of a Carnot engine operating between these temperatures is 6.83%. Therefore, there is no way that the inventor's engine can have an efficiency of 0.110 = 11.0%.

P22.18 (a) $P\Delta t\left(\frac{T_h}{T_h - T_c}\right)$; (b) $P\Delta t\left(\frac{T_c}{T_h - T_c}\right)$

P22.20 (a) See P22.20(a) for the full solution; (b) See P22.20(b) for the full solution.

P22.22 72.2 J

P22.24 0.330 or 33.0%

P22.26 (a) 0.300; (b) 1.40×10^{-3} K^{-1}; (c) -2.00×10^{-3} K^{-1}; (d) No. The derivative in part (c) depends only on T_h

P22.28 (a) 5.12%; (b) 5.27 TJ/h; (c) As fossil-fuel prices rise, this way to use solar energy will become a good buy.

P22.30 (a) See P22.30(a) for full explanation; (b) $1 - \frac{T_c}{T_h}$; (c) The combination of reversible engines is itself a reversible engine so it has the Carnot efficiency. No improvement in net efficiency has resulted; (d) $T_i = \frac{1}{2}(T_h + T_c)$;
(e) $T_i = (T_h T_c)^{1/2}$

P22.32 (a) See TABLE P22.32(a); (b) See TABLE P22.32(b); (c) 23.7%; (d) 23.7%

P22.34 23.1 mW

P22.36 (a) 51.2%; (b) 36.2%

P22.38 See P22.38 for the full derivation

P22.40 (a) See TABLE P22.40; (b) 2 heads and 2 tails

P22.42 $-610\ \mathrm{J/K}$

P22.44 $717\ \mathrm{J/K}$

P22.46 $2.70\ \mathrm{kJ/K}$

P22.48 (a) $5.76\ \mathrm{J/K}$; (b) no change in temperature

P22.50 $244\ \mathrm{J/K}$

P22.52 $1\ \mathrm{W/K}$

P22.54 (a) $\Delta S_h = -\dfrac{Q}{T_h}$; (b) $\Delta S_c = -\dfrac{Q}{T_c}$; (c) $Q\left(\dfrac{1}{T_c} - \dfrac{1}{T_h}\right)$

P22.56 $+1.18\ \mathrm{J/K}$

P22.58 (a) 0.268; (b) 0.423

P22.60 $8.36 \times 10^6\ \mathrm{J/K \cdot s}$

P22.62 (a) 2.93; (b) $(\mathrm{COP})_{\text{refrigerator}}$; (c) with EER 5, \$510, with EER 10, \$255; Thus, the cost for air conditioning is half as much for an air conditioner with EER 10 compared with an air conditioner with EER 5.

P22.64 (a) 2.49 kJ; (b) 1.50 kJ; (c) –990 J

P22.66 (a) $\dfrac{T_c}{T_h - T_c}$; (b) smaller; (c) 14.6

P22.68 (a) 214 J and 64.3 J; (b) 35.7 J and 35.7 J. The total energy the firebox puts out equals to the total energy transferred to the environment; (c) The net flow of energy by heat from the cold to the hot reservoir without work input is possible; (d) $W_{\text{eng S}}$ = 233 J, $\left|Q_{h,\text{S}}\right|$ = 333 J;

(e) 83.3 J; (f) 83.3 J; (g) 0; (h) The output of 83.8 J of energy from the heat engine by work in a cyclic process without any exhaust by heat is impossible; (i) $-0.111\ \mathrm{J/K}$; (j) A decrease in total entropy is impossible.

P22.70 (a) 8.48 kW; (b) 1.52 kW; (c) $1.09 \times 10^4\ \mathrm{J/K}$; (d) drop by 20.0%

P22.72 $\dfrac{PT_c}{\left(T_h - T_c\right)c\Delta T}$

P22.74 (a) $\Delta S = nC_p \ln 3$; (b) The pair of processes considered here carries the gas from the initial state in P22.77 to the final state here. Entropy is a function of state. Entropy change does not depend on path. Therefore, the entropy change in P22.77 equals $\Delta S_{\text{isothermal}} + \Delta S_{\text{adiabatic}}$ in this problem. Since $\Delta S_{\text{adiabatic}} = 0$, the answers to P22.77 and P22.74(a) must be the same.

P22.76 (a) 10.5 nRT_i; (b) 8.50 nRT_i; (c) 0.190; (d) 0.833; The Carnot efficiency is much higher.

P22.78 (a) 13.4 J/K; (b) 310 K; (c) 11.1 J/K; (d) smaller by less than 1%

P22.80 The computed change in entropy is 4.88 J/K, which is *not* zero. While the statements about energy transfer by heat are true, the mixing process is irreversible. After the water has come to equilibrium, it will not spontaneously separate again into warm and cool water. Therefore, there is an entropy increase of the mixture during the irreversible process.

P22.82 (a) See TABLE P22.82(a); (b) See TABLE P22.82(b); (c) 149 J; (d) 65.0 J; (e) 84.3 J; (f) 0.565; (g) 1.42×10^3 rev/min

23

Electric Fields

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ23.1 (i) Answer (c). The electron and proton have equal-magnitude charges.

(ii) Answer (b). The proton's mass is 1836 times larger than the electron's.

OQ23.2 Answer (e). The outer regions of the atoms in your body and the atoms making up the ground both contain negatively charged electrons. When your body is in close proximity to the ground, these negatively charged regions exert repulsive forces on each other. Since the atoms in the solid ground are rigidly locked in position and cannot move away from your body, this repulsive force prevents your body from penetrating the ground.

OQ23.3 Answer (b). To balance the weight of the ball, the magnitude of the upward electric force must equal the magnitude of the downward gravitational force, or $qE = mg$, which gives

$$E = \frac{mg}{q} = \frac{\left(5.0\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{4.0\times10^{-6}\text{ C}} = 1.2\times10^{4}\text{ N/C}$$

OQ23.4 Answer (a). The electric force is opposite to the field direction, so it is opposite to the velocity of the electron. From Newton's second law, the acceleration the electron will be

$$a_x = \frac{F_x}{m} = \frac{qE_x}{m} = \frac{\left(-1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{3}\text{ N/C}\right)}{9.11\times10^{-31}\text{ kg}}$$
$$= -1.76\times10^{14}\text{ m/s}^2$$

The kinematics equation $v_x^2 = v_{0x}^2 + 2a_x(\Delta x)$, with $v_x = 0$, gives the stopping distance as

$$\Delta x = \frac{-v_{0x}^2}{2a_x} = \frac{-\left(3.00\times10^{6}\text{ m/s}\right)^2}{2\left(-1.76\times10^{14}\text{ m/s}^2\right)} = 2.56\times10^{-2}\text{ m} = 2.56\text{ cm}$$

OQ23.5 Answer (d). The displacement from the –4.00 nC charge at point (0, 1.00) m to the point (4.00, –2.00) m has components $r_x = \left(x_f - x_i\right) = +4.00\text{ m}$ and $r_y = \left(y_f - y_i\right) = -3.00\text{ m}$, so the magnitude of this displacement is $r = \sqrt{r_x^2 + r_y^2} = 5.00\text{ m}$ and its direction is $\theta = \tan^{-1}\left(\frac{r_y}{r_x}\right) = -36.9°$. The x component of the electric field at point (4.00, –2.00) m is then

$$E_x = E\cos\theta = \frac{k_e q}{r^2}\cos\theta$$
$$= \frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(-4.00\times10^{-9}\text{ C}\right)}{(5.00\text{ m})^2}\cos\left(-36.9°\right)$$
$$= -1.15\text{ N/C}$$

OQ23.6 Answer (a). The equal-magnitude radially directed field contributions add to zero.

OQ23.7 Answer (b). When a charged insulator is brought near a metallic object, free charges within the metal move around, causing the metallic object to become polarized. Within the metallic object, the center of charge for the type of charge opposite to that on the insulator will be located closer to the charged insulator than will the center of charge for the same type of charge as that on the insulator.

This causes the attractive force between the charged insulator and the opposite type of charge in the metal to exceed the magnitude of the repulsive force between the insulator and the same type of charge in the metal. Thus, the net electric force between the insulator and an the metallic object is one of attraction.

OQ23.8 Answer (e). The magnitude of the electric field at distance *r* from a point charge *q* is $E = k_e q / r^2$, so

$$E = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)}{\left(5.11\times10^{-11}\ \mathrm{m}\right)^2}$$

$$= 5.51\times10^{11}\ \mathrm{N/C}\ \sim\ 10^{12}\ \mathrm{N/C}$$

making (e) the best choice for this question.

OQ23.9 (i) Answer (d). Suppose the positive charge has the large value 1 μC. The object has lost some of its conduction electrons, in number

$$10^{-6}\ \mathrm{C}\ (1\ \mathrm{e}/1.60\times10^{-19}\ \mathrm{C}) = 6.25\times10^{12}$$

and in mass

$$6.25\times10^{12}\,(9.11\times10^{-31}\ \mathrm{kg}) = 5.69\times10^{-18}\ \mathrm{kg}.$$

This is on the order of 10^{14} times smaller than the ~1 g mass of the coin, so it is an immeasurably small change.

(ii) Answer (b). The coin gains extra electrons, gaining mass on the order of 10^{-14} times its original mass for the charge –1 μC.

OQ23.10 Answer (c). Each charge produces a field as if it were alone in the Universe.

OQ23.11 (i) Answer (d). The charge at the upper left creates at the field point an electric field to the left, with magnitude we call E_1. The charge at lower right creates a downward electric field with an equal magnitude E_1. These two charges together create a field $\sqrt{2}E_1$ downward and to the left (at 45°). The positive charge has twice the charge but is $\sqrt{2}$ times farther from the field point, so it creates a field $2E_1\big/\left(\sqrt{2}\right)^2 = E_1$ upward and to the right. The fields from the two charges are opposite in direction, and the field from the negative charges is stronger, so the net field is then $\left(\sqrt{2}-1\right)E_1$, which is downward and to the left (at 45°).

(ii) Answer (a). With the positive charge removed, the magnitude of the field becomes $\sqrt{2}E_1$, larger than before.

OQ23.12 Answer (a). The magnitude of the electric force between charges Q_1 and Q_2, separated by distance r_i is $F = k_e Q_1 Q_2 / r^2$. If changes are made so $Q_1 \rightarrow Q_1/3$ and $r \rightarrow 2r$, the magnitude of the new force F' will be

$$F' = k_e \frac{(Q_1/3)Q_2}{(2r)^2} = \frac{1}{3(4)} k_e \frac{Q_1Q_2}{r^2} = \frac{1}{12} k_e \frac{Q_1Q_2}{r^2} = \frac{1}{12} F$$

OQ23.13 Answer (c). The charges nearer the center of the disk produce electric fields that make smaller angles with the central axis of the disk; therefore, these fields have smaller components perpendicular to the axis that cancel each other and larger components parallel to the axis which reinforce each other.

OQ23.14 Answer (b). A negative charge experiences a force opposite to the direction of the electric field.

OQ23.15 Answer (a). The magnitude of the electric force between two protons separated by distance r is $F = k_e e^2 / r^2$, so the distance of separation must be

$$r = \sqrt{\frac{k_e e^2}{F}} = \sqrt{\frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2)(1.60 \times 10^{-19} \text{ C})^2}{2.30 \times 10^{-26} \text{ N}}} = 0.100 \text{ m}$$

ANSWERS TO CONCEPTUAL QUESTIONS

CQ23.1 No. Life would be no different if electrons were positively charged and protons were negatively charged. Opposite charges would still attract, and like charges would repel. The naming of positive and negative charge is merely a convention.

CQ23.2 The dry paper is initially neutral. The comb attracts the paper because its electric field causes the molecules of the paper to become polarized—the paper as a whole cannot be polarized because it is an insulator. Each molecule is polarized so that its unlike-charged side is closer to the charged comb than its like-charged side, so the molecule experiences a net attractive force toward the comb. Once the paper comes in contact with the comb, like charge can be transferred from the comb to the paper, and if enough of this charge is transferred, the like-charged paper is then repelled by the like-charged comb.

CQ23.3 The answer depends on whether the person is initially (a) uncharged or (b) charged.

(a) No. If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall.

(b) If the person carries a (small) charge q, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

CQ23.4 All of the constituents of air are nonpolar except for water. The polar water molecules in the air quite readily "steal" charge from a charged object, as any physics teacher trying to perform electrostatics demonstrations in humid weather well knows. As a result—it is difficult to accumulate large amounts of excess charge on an object in a humid climate. During a North American winter, the cold, dry air allows accumulation of significant excess charge, giving the potential (pun intended) for a shocking (pun also intended) introduction to static electricity sparks.

CQ23.5 No. Object A might have a charge opposite in sign to that of B, but it also might be neutral. In this latter case, object B causes object A (or the molecules of A if its material is an insulator) to be polarized, pulling unlike charge to the near face of A and pushing an equal amount of like charge to the far face. Then the force of attraction exerted by B on the induced unlike charge on the near side of A is slightly larger than the force of repulsion exerted by B on the induced like charge on the far side of A. Therefore, the net force on A is toward B.

CQ23.6 (a) Yes. The positive charges create electric fields that extend in all directions from those charges. The total field at point A is the vector sum of the individual fields produced by the charges at that point.

(b) No, because there are no field lines emanating from or converging on point A.

(c) No. There must be a charged object present to experience a force.

CQ23.7 The charge on the ground is negative because electric field lines produced by negative charge point toward their source.

CQ23.8 Conducting shoes are worn to avoid the build up of a static charge on them as the wearer walks. Rubber-soled shoes acquire a charge by friction with the floor and could discharge with a spark, possibly causing an explosive burning situation, where the burning is enhanced by the oxygen.

CQ23.9 (a) No. The balloon induces polarization of the molecules in the wall, so that a layer of positive charge exists near the balloon. This is just like the situation in Figure 23.4a, except that the signs of the charges are reversed. The attraction between these charges and the negative charges on the balloon is stronger than the repulsion between the negative charges on the balloon and the negative charges in the polarized molecules (because they are farther from the balloon), so that there is a net attractive force toward the wall.

(b) Polar water molecules in the air surrounding the balloon are attracted to the excess electrons on the balloon. The water molecules can pick up and transfer electrons from the balloon, reducing the charge on the balloon and eventually causing the attractive force to be insufficient to support the weight of the balloon.

CQ23.10 (a) Yes. (b) The situation is similar to that of magnetic bar magnets, which can attract or repel each other depending on their orientation.

CQ23.11 Electrons have been removed from the glass object. Negative charge has been removed from the initially neutral rod, resulting in a net positive charge on the rod. The protons cannot be removed from the rod; protons are not mobile because they are within the nuclei of the atoms of the rod.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 23.1 Properties of Electric Charges

P23.1 (a) The charge due to loss of one electron is

$$0-1\left(-1.60\times10^{-19}\ \text{C}\right)=\boxed{+1.60\times10^{-19}\ \text{C}}$$

The mass of an average neutral hydrogen atom is 1.007 9 u. Losing one electron reduces its mass by a negligible amount, to

$$1.007\,9\left(1.660\times10^{-27}\ \text{kg}\right)-9.11\times10^{-31}\ \text{kg}=\boxed{1.67\times10^{-27}\ \text{kg}}$$

(b) By similar logic, charge $=\boxed{+1.60\times10^{-19}\ \text{C}}$

$$\text{mass}=22.99\left(1.66\times10^{-27}\ \text{kg}\right)-9.11\times10^{-31}\ \text{kg}=\boxed{3.82\times10^{-26}\ \text{kg}}$$

(c) Gain of one electron: charge of $\text{Cl}^-=\boxed{1.60\times10^{-19}\ \text{C}}$

$$\text{mass}=35.453\left(1.66\times10^{-27}\ \text{kg}\right)+9.11\times10^{-31}\ \text{kg}=\boxed{5.89\times10^{-26}\ \text{kg}}$$

(d) Loss of two electrons: charge of $\text{Ca}^{++}=-2\left(-1.60\times10^{-19}\ \text{C}\right)=$ $\boxed{+3.20\times10^{-19}\ \text{C}}$

$$\text{mass}=40.078\left(1.66\times10^{-27}\ \text{kg}\right)-2\left(9.11\times10^{-31}\ \text{kg}\right)$$

$$=\boxed{6.65\times10^{-26}\ \text{kg}}$$

(e) Gain of three electrons: charge of $\mathrm{N}^{3-} = 3\left(-1.60\times10^{-19}\ \mathrm{C}\right) =$ $\boxed{-4.80\times10^{-19}\ \mathrm{C}}$

$$\text{mass} = 14.007\left(1.66\times10^{-27}\ \mathrm{kg}\right) + 3\left(9.11\times10^{-31}\ \mathrm{kg}\right)$$

$$= \boxed{2.33\times10^{-26}\ \mathrm{kg}}$$

(f) Loss of four electrons: charge of $\mathrm{N}^{4+} = 4\left(1.60\times10^{-19}\ \mathrm{C}\right) =$ $\boxed{+6.40\times10^{-19}\ \mathrm{C}}$

$$\text{mass} = 14.007\left(1.66\times10^{-27}\ \mathrm{kg}\right) - 4\left(9.11\times10^{-31}\ \mathrm{kg}\right)$$

$$= \boxed{2.32\times10^{-26}\ \mathrm{kg}}$$

(g) We think of a nitrogen nucleus as a seven-times ionized nitrogen atom. Charge $= 7\left(1.60\times10^{-19}\ \mathrm{C}\right) = \boxed{1.12\times10^{-18}\ \mathrm{C}}$

$$\text{mass} = 14.007\left(1.66\times10^{-27}\ \mathrm{kg}\right) - 7\left(9.11\times10^{-31}\ \mathrm{kg}\right)$$

$$= \boxed{2.32\times10^{-26}\ \mathrm{kg}}$$

(h) Gain of one electron: charge $= \boxed{-1.60\times10^{-19}\ \mathrm{C}}$

$$\text{mass} = \left[2\left(1.007\ 9\right) + 15.999\right]1.66\times10^{-27}\ \mathrm{kg} + 9.11\times10^{-31}\ \mathrm{kg}$$

$$= \boxed{2.99\times10^{-26}\ \mathrm{kg}}$$

P23.2 (a) $$N = \left(\frac{10.0\ \text{grams}}{107.87\ \text{grams/mol}}\right)\left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(47\ \frac{\text{electrons}}{\text{atom}}\right)$$

$$= \boxed{2.62\times10^{24}}$$

(b) $$\#\ \text{electrons added} = \frac{Q}{e} = \frac{1.00\times10^{-3}\ \text{C added}}{1.60\times10^{-19}\ \text{C/electron}}$$

$$= 6.25\times10^{15}\ \text{electrons added}$$

Thus,

$$\left(6.25\times10^{15}\ \text{added}\right)\left(\frac{1}{2.62\times10^{24}\ \text{present}}\right) = \left(\frac{2.38\ \text{added}}{10^{9}\ \text{present}}\right)$$

$\rightarrow$ 2.38 electrons for every 10^9 already present

Section 23.2 Charging Objects by Induction

Section 23.3 Coulomb's Law

***P23.3** The force on one proton is $\vec{\mathbf{F}} = \dfrac{k_e q_1 q_2}{r^2}$ away from the other proton. Its magnitude is

$$\left(8.99\times10^{9}\ \mathrm{N\cdot m/C^2}\right)\left(\frac{1.60\times10^{-19}\ \mathrm{C}}{2\times10^{-15}\ \mathrm{m}}\right)^2 = \boxed{57.5\ \mathrm{N}}$$

***P23.4** In the first situation, $\vec{\mathbf{F}}_{A\ \text{on}\ B,1} = \dfrac{k_e|q_A||q_B|}{r_1^2}\hat{\mathbf{i}}$. In the second situation, $|q_A|$ and $|q_B|$ are the same.

$$\vec{\mathbf{F}}_{B\ \text{on}\ A,2} = -\vec{\mathbf{F}}_{A\ \text{on}\ B} = \frac{k_e|q_A||q_B|}{r_2^2}\left(-\hat{\mathbf{i}}\right)$$

$$\frac{F_2}{F_1} = \frac{k_e|q_A||q_B|}{r_2^2}\frac{r_1^2}{k_e|q_A||q_B|}$$

$$F_2 = \frac{F_1 r_1^2}{r_2^2} = (2.62\ \mu\mathrm{N})\left(\frac{13.7\ \mathrm{mm}}{17.7\ \mathrm{mm}}\right)^2 = 1.57\ \mu\mathrm{N}$$

Then $\vec{\mathbf{F}}_{B\ \text{on}\ A,2} = \boxed{1.57\ \mu\mathrm{N}\ \text{to the left}}$.

***P23.5** The electric force is given by

$$F = k_e\frac{q_1q_2}{(r_{12})^2} = \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{(+40\ \mathrm{C})(-40\ \mathrm{C})}{(2\ 000\ \mathrm{m})^2}$$

$$= -3.60\times10^{6}\ \mathrm{N}\ (\text{attractive}) = \boxed{3.60\times10^{6}\ \mathrm{N}\ \ \text{downward}}$$

P23.6 (a) The two ions are both singly charged, $|q| = 1e$, one positive and one negative. Thus,

$$|F| = \frac{k_e|q_1||q_2|}{r^2} = \frac{k_e e^2}{r^2}$$

$$= \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)^2}{\left(0.500\times10^{-9}\ \mathrm{m}\right)^2}$$

$$= \boxed{9.21\times10^{-10}\ \mathrm{N}}$$

(b) No. The electric force depends only on the magnitudes of the two charges and the distance between them.

P23.7 The end charges, of charge magnitude e, are distance $r = 2.17\ \mu$m apart. The spring stretches by $x = 0.010\,0r$, and the effective spring force balances the electrostatic attraction of the end charges:

$$kx = k_e\frac{e^2}{r^2} \rightarrow k = k_e\frac{e^2}{xr^2} = k_e\frac{e^2}{(0.010\ 0r)r^2} = k_e\frac{e^2}{(0.010\ 0)r^3}$$

$$k = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/C^2\right)\frac{\left(1.60\times10^{-19}\ \text{C}\right)^2}{(0.010\ 0)\left(2.17\times10^{-6}\ \text{m}\right)^3}$$

$$= \boxed{2.25\times10^{-9}\ \text{N/m}}$$

P23.8 Suppose each person has mass 70 kg. In terms of elementary charges, each person consists of precisely equal numbers of protons and electrons and a nearly equal number of neutrons. The electrons comprise very little of the mass, so for each person we find the total number of protons and neutrons, taken together:

$$(70\text{ kg})\left(\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\right) = 4\times10^{28}\text{ u}$$

Of these, nearly one half, 2×10^{28}, are protons, and 1% of this is 2×10^{26}, constituting a charge of $(2\times10^{26})(1.60\times10^{-19}\text{ C}) = 3\times10^{7}$ C.

Thus, Feynman’s force has magnitude

$$F = \frac{k_e q_1 q_1}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3\times10^7\ \text{C})^2}{(0.5\text{ m})^2} \sim \boxed{10^{26}\ \text{N}}$$

where we have used a half-meter arm’s length. According to the particle in a gravitational field model, if the Earth were in an externally-produced uniform gravitational field of magnitude 9.80 m/s^2, it would weigh $F_g = mg = (6\times10^{24}\text{ kg})(10\text{ m/s}^2) \sim 10^{26}$ N.

Thus, the forces are of the same order of magnitude.

P23.9 (a) $|F| = \dfrac{k_e|q_1||q_2|}{r^2}$

$$F = \frac{k_e e^2}{r^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.50\times10^{-9}\ \text{C}\right)\left(4.20\times10^{-9}\ \text{C}\right)}{(1.80\text{ m})^2}$$

$$= \boxed{8.74\times10^{-8}\ \text{N}}$$

(b) The charges are like charges. $\boxed{\text{The force is repulsive.}}$

P23.10 (a) $$F_e = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(3.80\times10^{-10}\ \text{m}\right)^2}$$

$$= \boxed{1.59\times10^{-9}\ \text{N}}\ \text{(repulsion)}$$

(b) $$F_g = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.67\times10^{-27}\ \text{kg}\right)^2}{\left(3.80\times10^{-10}\ \text{m}\right)^2}$$

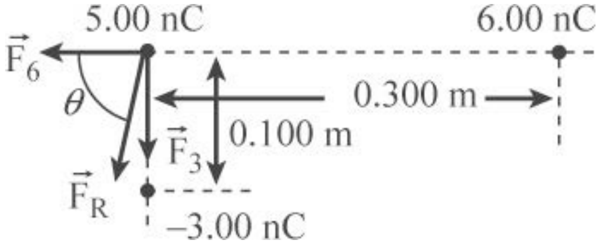
$$= \boxed{1.29\times10^{-45}\ \text{N}}$$

The electric force is $\boxed{\text{larger by } 1.24\times10^{36} \text{ times}}$.

(c) If $k_e\frac{q_1q_2}{r^2} = G\frac{m_1m_2}{r^2}$ with $q_1 = q_2 = q$ and $m_1 = m_2 = m$, then

$$\frac{q}{m} = \sqrt{\frac{G}{k_e}} = \sqrt{\frac{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{8.61\times10^{-11}\ \text{C/kg}}$$

P23.11 The particle at the origin carries a positive charge of 5.00 nC. The electric force between this particle and the –3.00-nC particle located on the –y axis will be attractive and point toward the –y direction and is shown with $\vec{\mathbf{F}}_3$ in the diagram, while the electric force between this particle and the 6.00-nC particle located on the x axis will be repulsive and point toward the –x direction, shown with $\vec{\mathbf{F}}_6$ in the diagram. The resultant force should point toward the third quadrant, as shown in the diagram with $\vec{\mathbf{F}}_R$. Although the charge on the x axis is greater in magnitude, its distance from the origin is three times larger than the –3.00-nC charge. We expect the resultant force to make a small angle with the –y axis and be approximately equal in magnitude with F_3.



ANS. FIG. P23.11

From the diagram in ANS. FIG. P23.11, the two forces are perpendicular, and the components of the resultant force are

$$F_x = -F_6 = -\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00\times10^{-9}\ \text{C}\right)\left(5.00\times10^{-9}\ \text{C}\right)}{\left(0.300\ \text{m}\right)^2}$$

$$= -3.00\times10^{-6}\ \text{N}\quad\text{(to the left)}$$

$$F_y = -F_3 = -\left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{(3.00\times10^{-9}\ \mathrm{C})(5.00\times10^{-9}\ \mathrm{C})}{(0.100\ \mathrm{m})^2}$$

$$= -1.35\times10^{-5}\ \mathrm{N}\quad(\text{downward})$$

(a) The forces are perpendicular, so the magnitude of the resultant is

$$F_R = \sqrt{(F_6)^2+(F_3)^2} = \boxed{1.38\times10^{-5}\ \mathrm{N}}$$

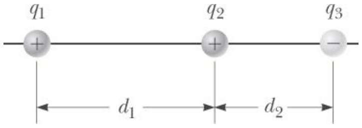
(b) The magnitude of the angle of the resultant is

$$\theta = \tan^{-1}\left(\frac{F_3}{F_6}\right) = 77.5°$$

The resultant force is in the third quadrant, so the direction is

$$\boxed{77.5°\ \text{below}\ -x\ \text{axis}}$$

P23.12 The forces are as shown in ANS. FIG. P23.12.



ANS. FIG. P23.12

$$F_1 = \frac{k_e q_1 q_2}{r_{12}^2} = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{(6.00\times10^{-6}\ \mathrm{C})(1.50\times10^{-6}\ \mathrm{C})}{(3.00\times10^{-2}\ \mathrm{m})^2}$$

$$= 89.9\ \mathrm{N}$$

$$F_2 = \frac{k_e q_1 |q_3|}{r_{13}^2} = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{(6.00\times10^{-6}\ \mathrm{C})(2.00\times10^{-6}\ \mathrm{C})}{(5.00\times10^{-2}\ \mathrm{m})^2}$$

$$= 43.2\ \mathrm{N}$$

$$F_3 = \frac{k_e q_2 |q_3|}{r_{23}^2} = \left(8.99\times10^{9}\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{(1.50\times10^{-6}\ \mathrm{C})(2.00\times10^{-6}\ \mathrm{C})}{(2.00\times10^{-2}\ \mathrm{m})^2}$$

$$= 67.4\ \mathrm{N}$$

(a) The net force on the 6 μC charge is

$$F_{(6\mu\mathrm{C})} = F_1 - F_2 = \boxed{46.7\ \mathrm{N}\ \text{to the left}}$$

(b) The net force on the 1.5 μC charge is

$$F_{(1.5\mu\mathrm{C})} = F_1 + F_3 = \boxed{157\ \mathrm{N}\ \text{to the right}}$$

(c) The net force on the –2 μC charge is

$$F_{(-2\mu\mathrm{C})} = F_2 + F_3 = \boxed{111\ \mathrm{N}\ \text{to the left}}$$

P23.13 (a) Let the third bead have charge *Q* and be located distance *x* from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e(3q)Q}{x^2}\hat{\mathbf{i}} + \frac{k_e(q)Q}{(d-x)^2}\left(-\hat{\mathbf{i}}\right), \text{ where } d = 1.50 \text{ m}$$

The net force will be zero if $\frac{3}{x^2} = \frac{1}{(d-x)^2}$, or $d - x = \frac{x}{\sqrt{3}}$.

This gives an equilibrium position of the third bead of

$$x = 0.634d = 0.634(1.50 \text{ m}) = \boxed{0.951 \text{ m}}$$

(b) $\boxed{\text{Yes, if the third bead has positive charge.}}$ The equilibrium would be stable because if charge *Q* were displaced either to the left or right on the rod, the new net force would be opposite to the direction *Q* has been displaced, causing it to be pushed back to its equilibrium position.

P23.14 (a) Let the third bead have charge *Q* and be located distance *x* from the left end of the rod. This bead will experience a net force given by

$$\vec{\mathbf{F}} = \frac{k_e q_1 Q}{x^2}\hat{\mathbf{i}} + \frac{k_e q_2 Q}{(d-x)^2}\left(-\hat{\mathbf{i}}\right)$$

The net force will be zero if $\frac{q_1}{x^2} = \frac{q_2}{(d-x)^2}$:

$$\frac{q_1}{x^2} = \frac{q_2}{(d-x)^2} \quad\rightarrow\quad (d-x)^2 = x^2\left(\frac{q_2}{q_1}\right) \quad\rightarrow\quad d - x = x\sqrt{\frac{q_2}{q_1}}$$

because *d* > *x*. Thus,

$$d - x = x\sqrt{\frac{q_2}{q_1}} \quad\rightarrow\quad d = x + x\frac{\sqrt{q_2}}{\sqrt{q_1}} = x\left(\frac{\sqrt{q_1}+\sqrt{q_2}}{\sqrt{q_1}}\right)$$

$$\rightarrow\quad x = \boxed{\frac{\sqrt{q_1}}{\sqrt{q_1}+\sqrt{q_2}}d}$$

(b) $\boxed{\text{Yes, if the third bead has positive charge.}}$ The equilibrium would be stable because if charge *Q* were displaced either to the left or right on the rod, the new net force would be opposite to the direction *Q* has been displaced, causing it to be pushed back to its equilibrium position.

P23.15 The force exerted on the 7.00-μC charge by the 2.00-μC charge is

$$\vec{\mathbf{F}}_1 = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$$

$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.00\times10^{-6}\ \text{C})(2.00\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2} \times(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}})$$

$$\vec{\mathbf{F}}_1 = (0.252\,\hat{\mathbf{i}} + 0.436\,\hat{\mathbf{j}})\ \text{N}$$

Similarly, the force on the 7.00-μC charge by the –4.00-μC charge is

$$\mathbf{F}_2 = k_e \frac{q_1 q_3}{r^2}\hat{\mathbf{r}}$$

$$= -\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.00\times10^{-6}\ \text{C})(-4.00\times10^{-6}\ \text{C})}{(0.500\ \text{m})^2} \times(\cos 60°\hat{\mathbf{i}} - \sin 60°\hat{\mathbf{j}})$$

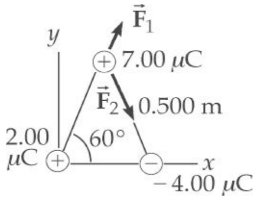
$$\vec{\mathbf{F}}_2 = (0.503\hat{\mathbf{i}} - 0.872\,\hat{\mathbf{j}})\ \text{N}$$

Thus, the total force on the 7.00-μC charge is

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = (0.755\,\hat{\mathbf{i}} - 0.436\,\hat{\mathbf{j}})\ \text{N}$$

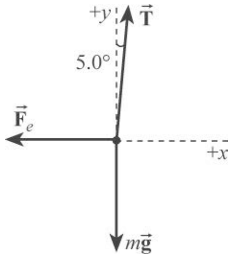
We can also write the total force as:

$$\vec{\mathbf{F}} = (0.755\ \text{N})\hat{\mathbf{i}} - (0.436\ \text{N})\hat{\mathbf{j}} = \boxed{0.872\ \text{N at an angle of } 330°}$$



ANS. FIG. P23.15

P23.16 Consider the free-body diagram of one of the spheres shown in ANS. FIG. P23.16. Here, T is the tension in the string and F_e is the repulsive electrical force exerted by the other sphere.



ANS. FIG. P23.16

$$\sum F_y = 0 \ \Rightarrow\ T\cos 5.0° = mg$$

or $$T = \frac{mg}{\cos 5.0°}$$

$$\sum F_x = 0 \ \Rightarrow\ F_e = T\sin 5.0° = mg\tan 5.0°$$

At equilibrium, the distance separating the two spheres is $r = 2L\sin 5.0°$.

Thus, $F_e = mg\tan 5.0°$ becomes $\dfrac{k_e q^2}{(2L\sin 5.0°)^2} = mg\tan 5.0°$, which yields

$$L = \sqrt{\frac{k_e q^2}{mg\tan 5.0°(2\sin 5.0°)^2}}$$

$$= \sqrt{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(7.20\times10^{-9}\ \text{C})^2}{(0.200\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan 5.0°(2\sin 5.0°)^2}} = \boxed{0.299\ \text{m}}$$

P23.17 (a) $F = \dfrac{k_e e^2}{r^2} = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\dfrac{(1.60\times10^{-19}\ \text{C})^2}{(0.529\times10^{-10}\ \text{m})^2} = \boxed{8.22\times10^{-8}\ \text{N}}$

toward the other particle.

(b) We have $F = \dfrac{mv^2}{r}$ from which

$$v = \sqrt{\frac{Fr}{m}} = \sqrt{\frac{(8.22\times10^{-8}\ \text{N})(0.529\times10^{-10}\ \text{m})}{9.11\times10^{-31}\ \text{kg}}}$$

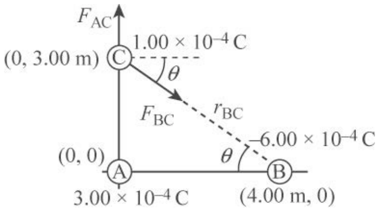
$$= \boxed{2.19\times10^6\ \text{m/s}}$$

P23.18 Charge C is attracted to charge B and repelled by charge A, as shown in ANS. FIG. P23.18. In the sketch,

$$r_{\text{BC}} = \sqrt{(4.00\ \text{m})^2 + (3.00\ \text{m})^2} = 5.00\ \text{m}$$

and

$$\theta = \tan^{-1}\left(\frac{3.00\ \text{m}}{4.00\ \text{m}}\right) = 36.9°$$



ANS. FIG. P23.18

(a) $(F_{\text{AC}})_x = \boxed{0}$

(b) $(F_{\text{AC}})_y = |F_{\text{AC}}| = k_e \dfrac{|q_{\text{A}}||q_{\text{C}}|}{r_{\text{AC}}^2}$

$$(F_{\text{AC}})_y = (8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(3.00\times10^{-4}\ \text{C})(1.00\times10^{-4}\ \text{C})}{(3.00\ \text{m})^2}$$

$$= \boxed{30.0\ \text{N}}$$

(c) $$|F_{\mathrm{BC}}| = k_e \frac{|q_{\mathrm{B}}||q_{\mathrm{C}}|}{r_{\mathrm{BC}}^2}$$

$$= \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\frac{\left(6.00\times10^{-4}\ \mathrm{C}\right)\left(1.00\times10^{-4}\ \mathrm{C}\right)}{\left(5.00\ \mathrm{m}\right)^2}$$

$$= \boxed{21.6\ \mathrm{N}}$$

(d) $$\left(F_{\mathrm{BC}}\right)_x = |F_{\mathrm{BC}}|\cos\theta = \left(21.6\ \mathrm{N}\right)\cos\left(36.9^\circ\right) = \boxed{17.3\ \mathrm{N}}$$

(e) $$\left(F_{\mathrm{BC}}\right)_y = -|F_{\mathrm{BC}}|\sin\theta = -\left(21.6\ \mathrm{N}\right)\sin\left(36.9^\circ\right) = \boxed{-13.0\ \mathrm{N}}$$

(f) $$\left(F_R\right)_x = \left(F_{\mathrm{AC}}\right)_x + \left(F_{\mathrm{BC}}\right)_x = 0 + 17.3\ \mathrm{N} = \boxed{17.3\ \mathrm{N}}$$

(g) $$\left(F_R\right)_y = \left(F_{\mathrm{AC}}\right)_y + \left(F_{\mathrm{BC}}\right)_y = 30.0 - 13.0\ \mathrm{N} = \boxed{17.0\ \mathrm{N}}$$

(h) $$F_R = \sqrt{\left(F_R\right)_x^2 + \left(F_R\right)_y^2} = \sqrt{\left(17.3\ \mathrm{N}\right)^2 + \left(17.0\ \mathrm{N}\right)^2} = 24.3\ \mathrm{N}$$

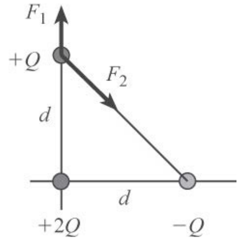
Both components are positive, placing the force in the first quadrant:

$$\varphi = \tan^{-1}\left[\frac{\left(F_R\right)_y}{\left(F_R\right)_x}\right] = \tan^{-1}\left(\frac{17.0\ \mathrm{N}}{17.3\ \mathrm{N}}\right) = 44.5^\circ$$

Therefore, $\vec{\mathbf{F}}_R = \boxed{24.3\ \mathrm{N\ at\ 44.5^\circ\ above\ the\ +}x\ \mathrm{direction}}$.

P23.19 The force due to the first charge is given by

$$\vec{\mathbf{F}}_1 = \frac{k_e Q(2Q)}{d^2}\hat{\mathbf{j}} = \frac{k_e Q^2}{d^2}\left[2\hat{\mathbf{j}}\right]$$



ANS. FIG. P23.19

and the force due to the second charge is given by

$$\vec{\mathbf{F}}_2 = \frac{k_e Q(Q)}{\left(d^2+d^2\right)}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{\sqrt{2}}\right] = \frac{k_e Q^2}{d^2}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{2\sqrt{2}}\right]$$

thus the total force on the point charge +Q located at $x = 0$ and $y = d$ is

$$\vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{k_e Q^2}{d^2}\left[2\hat{\mathbf{j}}\right] + \frac{k_e Q^2}{d^2}\left[\frac{\hat{\mathbf{i}}-\hat{\mathbf{j}}}{2\sqrt{2}}\right] = \boxed{k_e\frac{Q^2}{d^2}\left[\frac{1}{2\sqrt{2}}\hat{\mathbf{i}} + \left(2-\frac{1}{2\sqrt{2}}\right)\hat{\mathbf{j}}\right]}$$

P23.20 Each charge exerts a force of magnitude $\dfrac{k_e qQ}{\left(d/2\right)^2 + x^2}$ on the negative charge –*Q*: the top charge exerts its force directed upward and to the left, and bottom charge exerts its force directed downward and to the left, each at angle $\theta = \tan^{-1}\left(\frac{d}{2x}\right)$, respectively, above and below the *x* axis. The two positive charges together exert a net force:

$$\begin{aligned}\vec{\mathbf{F}} &= -2\frac{k_e qQ}{(d/2)^2 + x^2}\cos\theta\hat{\mathbf{i}}\\ &= -2\left[\frac{k_e qQ}{\left(d^2/4 + x^2\right)}\right]\left[\frac{x}{\left(d^2/4 + x^2\right)^{1/2}}\right]\hat{\mathbf{i}}\\ &= \left[\frac{-2xk_e qQ}{\left(d^2/4 + x^2\right)^{3/2}}\right]\hat{\mathbf{i}} = m\vec{\mathbf{a}}\end{aligned}$$

or for $x \ll \frac{d}{2}$, $\vec{\mathbf{a}} \approx -\left(\frac{2k_e qQ}{md^3/8}\right)\vec{\mathbf{x}} \quad \rightarrow \quad \vec{\mathbf{a}} \approx -\left(\frac{16k_e qQ}{md^3}\right)\vec{\mathbf{x}}$

(a) $\boxed{\text{The acceleration of the charge is equal to a negative constant times its displacement from equilibrium, as in } \vec{\mathbf{a}} = -\omega^2\vec{\mathbf{x}}, \text{ so we have Simple Harmonic Motion with } \omega^2 = \frac{16k_e qQ}{md^3}.}$

(b) $\omega^2 = \left(\frac{2\pi}{T}\right)^2 = \frac{16k_e qQ}{md^3} \rightarrow T = \frac{2\pi}{\omega} = \boxed{\frac{\pi}{2}\sqrt{\frac{md^3}{k_e qQ}}}$, where *m* is the mass of the object with charge –*Q*.

(c) $v_{\max} = \omega A = \boxed{4a\sqrt{\frac{k_e qQ}{md^3}}}$

P23.21 (a) The force is one of $\boxed{\text{attraction}}$. The distance *r* in Coulomb's law is the distance between the centers of the spheres. The magnitude of the force is

$$\begin{aligned}F &= \frac{k_e q_1 q_2}{r^2}\\ &= \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(12.0\times10^{-9}\ \text{C}\right)\left(18.0\times10^{-9}\ \text{C}\right)}{(0.300\ \text{m})^2}\\ &= \boxed{2.16\times10^{-5}\ \text{N}}\end{aligned}$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of $\boxed{\text{repulsion}}$, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2}$$

$$= \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(3.00\times10^{-9}\ \text{C}\right)\left(3.00\times10^{-9}\ \text{C}\right)}{\left(0.300\ \text{m}\right)^2}$$

$$= \boxed{8.99\times10^{-7}\ \text{N}}$$

P23.22 Each of the dust particles is a particle in equilibrium. Express this mathematically for one of the particles:

$$\sum\vec{\mathbf{F}} = 0 \quad\rightarrow\quad F_e - F_g = 0 \quad\rightarrow\quad F_e = F_g$$

where we have recognized that the gravitational force is attractive and the electric force is repulsive, so the forces on one particle are in opposite directions. Substitute for the forces from Coulomb's law and Newton's law of universal gravitation, and solve for q, the unknown charge on each dust particle:

$$k_e\frac{q^2}{r^2} = G\frac{m^2}{r^2} \quad\rightarrow\quad q = \sqrt{\frac{G}{k_e}}\ m$$

Substitute numerical values:

$$q = \sqrt{\frac{6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}{8.987\,6\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}}\left(1.00\times10^{-9}\ \text{kg}\right)$$

$$= 8.61\times10^{-20}\ \text{C}$$

This is about half of the smallest possible free charge, the charge of the electron. No such free charge exists. Therefore, the forces cannot balance. Even if the charge on each dust particle is due to one electron, the net force will be repulsive and the particles will move apart.

Section 23.4 Analysis Model: Particle in a Field (Electric)

***P23.23** For equilibrium, $\vec{\mathbf{F}}_e = -\vec{\mathbf{F}}_g$ or $q\vec{\mathbf{E}} = -mg\left(-\hat{\mathbf{j}}\right)$. Thus,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}}.$$

(a) For an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{(9.11\times10^{-31}\ \text{kg})(9.80\ \text{m/s}^2)}{-1.60\times10^{-19}\ \text{C}}\hat{\mathbf{j}}$$

$$= \boxed{-(5.58\times10^{-11}\ \text{N/C})\hat{\mathbf{j}}}$$

(b) For a proton, which is 1 836 times more massive than an electron,

$$\vec{\mathbf{E}} = \frac{mg}{q}\hat{\mathbf{j}} = \frac{(1.67\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)}{-1.60\times10^{-19}\ \text{C}}\hat{\mathbf{j}}$$

$$= \boxed{(1.02\times10^{-7}\ \text{N/C})\hat{\mathbf{j}}}$$

P23.24 In order for the object to “float” in the electric field, the electric force exerted on the object by the field must be directed upward and have a magnitude equal to the weight of the object. Thus, $F_e = qE = mg$, and the magnitude of the electric field must be

$$E = \frac{mg}{|q|} = \frac{(3.80\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{18.0\times10^{-6}\ \text{C}} = \boxed{2.07\times10^{3}\ \text{N/C}}$$

The electric force on a negatively charged object is in the direction opposite to that of the electric field. Since the electric force must be directed upward, the electric field must be directed $\boxed{\text{downward}}$.

P23.25 We sum the electric fields from each of the other charges using Equation 23.7 for the definition of the electric field.

The field at charge q is given by

$$\vec{\mathbf{E}} = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \frac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \frac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3$$

(a) Substituting for each of the charges gives

$$\vec{\mathbf{E}} = \frac{k_e(2q)}{a^2}\hat{\mathbf{i}} + \frac{k_e(3q)}{2a^2}\left(\hat{\mathbf{i}}\cos 45.0° + \hat{\mathbf{j}}\sin 45.0°\right) + \frac{k_e(4q)}{a^2}\hat{\mathbf{j}}$$

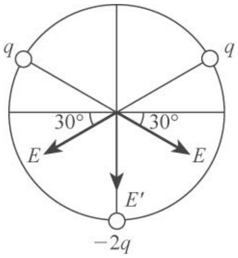
$$= \frac{k_e q}{a^2}\left[\left(2 + \frac{3}{2}\cos 45.0°\right)\hat{\mathbf{i}} + \left(\frac{3}{2}\sin 45.0° + 4\right)\hat{\mathbf{j}}\right]$$

$$= \boxed{\frac{k_e q}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

(b) The electric force on charge q is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \boxed{\frac{k_e q^2}{a^2}\left(3.06\hat{\mathbf{i}} + 5.06\hat{\mathbf{j}}\right)}$$

P23.26 Call the fields $E = \frac{k_e q}{r^2}$ and $E' = \frac{k_e(2q)}{r^2} = 2E$ (see ANS. FIG. P23.26).



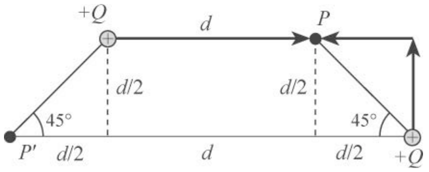
ANS. FIG. P23.26

The total field at the center of the circle has components

$$\begin{aligned}\vec{\mathbf{E}} &= (E\cos 30.0° - E\cos 30.0°)\hat{\mathbf{i}} - (E' + 2E\sin 30.0°)\hat{\mathbf{j}} \\ &= -(E' + 2E\sin 30.0°)\hat{\mathbf{j}} = -(2E + 2E\sin 30.0°)\hat{\mathbf{j}} \\ &= -2E(1 + \sin 30.0°)\hat{\mathbf{j}} \\ &= -2\frac{k_e q}{r^2}(1 + \sin 30.0°)\hat{\mathbf{j}} = -2\frac{k_e q}{r^2}(1.50)\hat{\mathbf{j}} = \boxed{-k_e \frac{3q}{r^2}\hat{\mathbf{j}}}\end{aligned}$$

P23.27 (a) See ANS. FIG. P23.27(a). The distance from the +*Q* charge on the upper left is *d*, and the distance from the +*Q* charge on the lower right to point *P* is

$$\sqrt{(d/2)^2 + (d/2)^2}$$

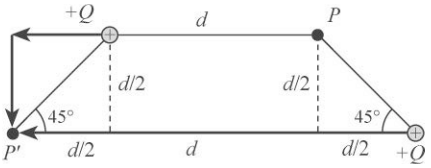


ANS. FIG. P23.27(a)

The total electric field at point *P* is then

$$\begin{aligned}\vec{\mathbf{E}}_P &= k_e \frac{Q}{d^2}\hat{\mathbf{i}} + k_e \frac{Q}{\left[(d/2)^2 + (d/2)^2\right]}\left(\frac{-\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) \\ &= k_e\left[\frac{Q}{d^2}\hat{\mathbf{i}} + \frac{Q}{d^2/2}\left(\frac{-\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right)\right] \\ &= \boxed{k_e \frac{Q}{d^2}\left[\left(1 - \sqrt{2}\right)\hat{\mathbf{i}} + \sqrt{2}\hat{\mathbf{j}}\right]}\end{aligned}$$

(b) See ANS. FIG. P23.27(b). The distance from the +Q charge on the lower right to point P’ is 2*d*, and the distance from the +*Q* charge on the upper right to point *P′* is



ANS. FIG. P23.27(b)

$$\sqrt{(d/2)^2 + (d/2)^2}$$

The total electric field at point *P’* is then

$$\vec{\mathbf{E}}_{P'} = k_e \frac{Q}{\left[(d/2)^2 + (d/2)^2\right]}\left(\frac{-\hat{\mathbf{i}} - \hat{\mathbf{j}}}{\sqrt{2}}\right) + k_e \frac{Q}{(2d)^2}\left(-\hat{\mathbf{i}}\right)$$

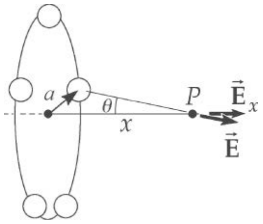
$$\vec{\mathbf{E}}_{P'} = -k_e\left[\frac{Q}{d^2/2}\left(\frac{\hat{\mathbf{i}} + \hat{\mathbf{j}}}{\sqrt{2}}\right) + \frac{Q}{4d^2}\left(-\hat{\mathbf{i}}\right)\right]$$

$$= -k_e \frac{Q}{4d^2}\left[\frac{8}{\sqrt{2}}\left(\hat{\mathbf{i}} + \hat{\mathbf{j}}\right) + \left(\hat{\mathbf{i}}\right)\right]$$

$$\vec{\mathbf{E}}_{P'} = \boxed{-k_e \frac{Q}{4d^2}\left[\left(1 + 4\sqrt{2}\right)\hat{\mathbf{i}} + 4\sqrt{2}\hat{\mathbf{j}}\right]}$$

P23.28 (a) One of the charges creates at *P* a field

$$\vec{\mathbf{E}} = E_x\hat{\mathbf{i}} = \frac{\left(k_e Q/n\right)}{a^2 + x^2}\hat{\mathbf{i}}$$



ANS. FIG. P23.28

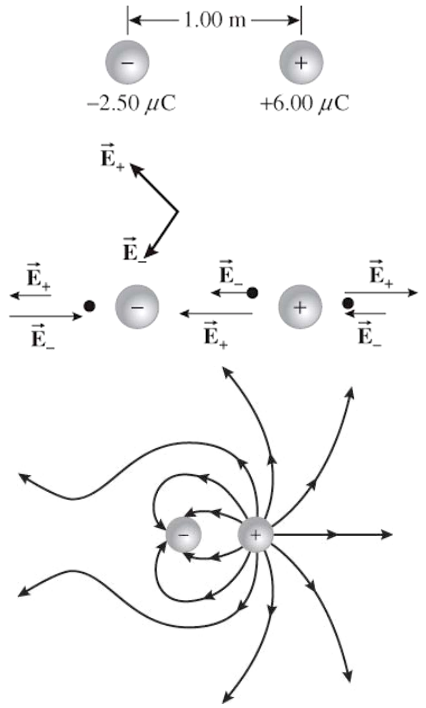
at an angle θ to the *x* axis as shown in ANS. FIG. P23.28. When all the charges produce the field, for $n > 1$, by symmetry the components perpendicular to the *x* axis add to zero.

The total field is then

$$\vec{\mathbf{E}} = nE_x\hat{\mathbf{i}} = n\left(\frac{k_e(Q/n)\hat{\mathbf{i}}}{a^2 + x^2}\cos\theta\right) = \boxed{\frac{k_e Q x\hat{\mathbf{i}}}{\left(a^2 + x^2\right)^{3/2}}}$$

(b) A circle of charge corresponds to letting *n* grow beyond all bounds, but the result does not depend on *n*. Because of the symmetrical arrangement of the charges, smearing the charge around the circle does not change its amount or its distance from the field point, so it does not change the field.

P23.29 The field of the positively-charged object is everywhere pointing radially away from its location. The object with negative charge creates everywhere a field pointing toward its different location. These two fields are directed along different lines at any point in the plane except for points along the extended line joining the particles; so the two fields cannot be oppositely-directed to add to zero except at some location along this line, which we take as the *x* axis. Observing the middle panel of ANS. FIG. P23.29, we see that at points to the left of the negatively-charged object, this particle creates field pointing to the right and the positive object creates field to the left. At some point along this segment the fields will add to zero. At locations in between the objects, both create fields pointing toward the left, so the total field is not zero. At points to the right of the positive 6-μC object, its field is directed to the right and is stronger than the leftward field of the –2.5-μC object, so the two fields cannot be equal in magnitude to add to zero. We have argued that only at a certain point straight to the left of both charges can the fields they separately produce be opposite in direction and equal in strength to add to zero.



ANS. FIG. P23.29

Let *x* represent the distance from the negatively-charged particle (charge q_-) to the zero-field point to its left. Then 1.00 m + *x* is the distance from the positive particle (of charge q_+) to this point. Each field is separately described by

$$\vec{\mathbf{E}} = k_e q \hat{\mathbf{r}} / x^2$$

so the equality in magnitude required for the two oppositely-directed vector fields to add to zero is described by

$$\frac{k_e |q_-|}{x^2} = \frac{k_e |q_+|}{(1\text{ m} + x)^2}$$

It is convenient to solve by taking the square root of both sides and cross-multiplying to clear of fractions:

$$|q_-|^{1/2}(1\text{ m} + x) = q_+^{1/2} x$$

$$1\text{ m} + x = \left(\frac{6.00}{2.50}\right)^{1/2} x = 1.55x$$

$$1\text{ m} = 0.549x$$

and $x = \boxed{1.82\text{ m}}$ to the left of the negatively-charged object.

P23.30 (a) Let s = 0.500 m be length of a side of the triangle. Call $q_1 = 7.00\ \mu\text{C}$ and $q_2 = |-4.00\ \mu\text{C}| = 4.00\ \mu\text{C}$. The electric field at the position of the 2.00-μC charge is the sum of the fields from the other two charges:

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q_1}{r_1^2}\hat{\mathbf{r}}_1 + k_e \frac{q_2}{r_2^2}\hat{\mathbf{r}}_2$$

substituting,

$$\vec{\mathbf{E}} = k_e \frac{q_1}{s^2}\left(-\cos 60.0^\circ\,\hat{\mathbf{i}} - \sin 60.0^\circ\,\hat{\mathbf{j}}\right) + k_e \frac{q_2}{s^2}\hat{\mathbf{i}}$$

$$= \frac{k_e}{s^2}\left[\left(q_2 - q_1\cos 60.0^\circ\right)\hat{\mathbf{i}} - q_1 \sin 60.0^\circ\,\hat{\mathbf{j}}\right]$$

substituting numerical values,

$$\vec{\mathbf{E}} = \left[\frac{8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}{(0.500\text{ m})^2}\right] \times\left[4.00\times 10^{-6}\ \text{C} - \left(7.00\times 10^{-6}\ \text{C}\right)\cos 60.0^\circ\right]\hat{\mathbf{i}} - \left(7.00\times 10^{-6}\ \text{C}\right)\sin 60.0^\circ\,\hat{\mathbf{j}}$$

$$\vec{\mathbf{E}} = \left(1.80\times 10^{4}\ \text{N/C}\right)\hat{\mathbf{i}} - \left(2.18\times 10^{5}\ \text{N/C}\right)\hat{\mathbf{j}}$$

$$= \boxed{\left(18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right)\ \text{kN/C}}$$

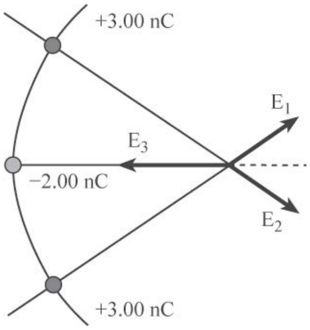
(b) The force on this charge is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(2.00\times 10^{-6}\ \text{C}\right)\left(18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right)\ \text{kN/C}$$

$$= \boxed{\left(0.0360\hat{\mathbf{i}} - 0.436\hat{\mathbf{j}}\right)\ \text{N}}$$

P23.31 Call $Q = 3.00$ nC and $q = |-2.00 \text{ nC}| = 2.00$ nC, and $r = 4.00 \text{ cm} = 0.040\,0$ m. Then,

$$E_1 = E_2 = \frac{k_e Q}{r^2} \quad \text{and} \quad E_3 = \frac{k_e q}{r^2}$$



ANS. FIG. P23.31

Then,

$$E_y = 0$$

$$E_x = E_{total} = 2\frac{k_e Q}{r^2}\cos 30.0° - \frac{k_e q}{r^2}$$

$$E_x = \frac{k_e}{r^2}\left(2Q\cos 30.0° - q\right)$$

$$E_x = \left[\frac{8.99\times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}{(0.040\,0 \text{ m})^2}\right]$$

$$\times\left[2\left(3.00\times 10^{-9} \text{ C}\right)\cos 30.0° - 2.00\times 10^{-9} \text{ C}\right]$$

$$= 1.80\times 10^4 \text{ N/C}$$

(a) $\boxed{1.80\times 10^4 \text{ N/C to the right}}$

(b) The electric force on a point charge placed at point *P* is

$$F = qE = \left(-5.00\times 10^{-9} \text{ C}\right)E = \boxed{-8.98\times 10^{-5} \text{ N (to the left)}}$$

P23.32 The first charge creates at the origin a field $\frac{k_e Q}{a^2}$ to the right. Both charges are on the *x* axis, so the total field cannot have a vertical component, but it can be either to the right or to the left. If the total field at the origin is to the right, then *q* must be negative:

ANS. FIG. P23.32

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\hat{\mathbf{i}} \quad \rightarrow \quad q = -9Q$$

In the alternative, if the total field at the origin is to the left,

$$\frac{k_e Q}{a^2}\hat{\mathbf{i}} + \frac{k_e q}{9a^2}\left(-\hat{\mathbf{i}}\right) = \frac{2k_e Q}{a^2}\left(-\hat{\mathbf{i}}\right) \quad \rightarrow \quad q = +27Q$$

$\boxed{\text{The field at the origin can be to the right, if the unknown charge is } -9Q, \text{ or the field can be to the left, if and only if the unknown charge is } +27Q.}$

***23.33** From the free-body diagram shown in ANS. FIG. P23.33,

$$\sum F_y = 0: \qquad T\cos 15.0° = 1.96\times 10^{-2}\text{ N}$$

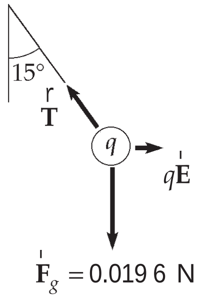
So $T = 2.03\times 10^{-2}$ N.

From $\sum F_x = 0$, we have $qE = T\sin 15.0°$,

or

$$q = \frac{T\sin 15.0°}{E} = \frac{\left(2.03\times 10^{-2}\text{ N}\right)\sin 15.0°}{1.00\times 10^{3}\text{ N/C}}$$

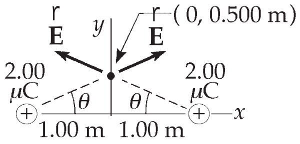
$$= 5.25\times 10^{-6}\text{ C} = \boxed{5.25\ \mu\text{C}}$$



ANS. FIG. P23.33

***P23.34** (a) The distance from each charge to the point at $y = 0.500$ m is

$$d = \sqrt{(1.00\text{ m})^2 + (0.500\text{ m})^2} = 1.12\text{ m}$$



ANS. FIG. P23.34

the magnitude of the electric field from each charge at that point is then given by

$$E = \frac{k_e q}{r^2} = \frac{\left(8.99\times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times 10^{-6}\text{ C}\right)}{(1.12\text{ m})^2} = 14\,400\text{ N/C}$$

The x components of the two fields cancel and the y components add, giving

$$E_x = 0 \text{ and } \quad E_y = 2\left(14\,400\text{ N/C}\right)\sin 26.6° = 1.29\times 10^{4}\text{ N/C}$$

so $\boxed{\vec{\mathbf{E}} = 1.29\times 10^{4}\,\hat{\mathbf{j}}\text{ N/C}}$.

(b) The electric force at this point is given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(-3.00\times 10^{-6}\text{ C}\right)\left(1.29\times 10^{4}\text{ N/C}\hat{\mathbf{j}}\right)$$

$$= \boxed{-3.86\times 10^{-2}\,\hat{\mathbf{j}}\text{ N}}$$

***P23.35** (a) The electric field at the origin due to each of the charges is given by

$$\vec{\mathbf{E}}_1 = \frac{k_e|q_1|}{r_1^2}\left(-\hat{\mathbf{j}}\right)$$

$$= \frac{\left(8.99\times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times 10^{-9}\text{ C}\right)}{(0.100\text{ m})^2}\left(-\hat{\mathbf{j}}\right)$$

$$= -\left(2.70\times 10^{3}\text{ N/C}\right)\hat{\mathbf{j}}$$

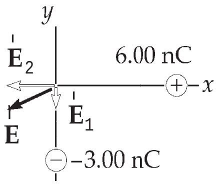
$$\vec{\mathbf{E}}_2 = \frac{k_e|q_2|}{r_2^2}\left(-\hat{\mathbf{i}}\right)$$

$$= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.00\times10^{-9}\ \text{C}\right)}{(0.300\ \text{m})^2}\left(-\hat{\mathbf{i}}\right)$$

$$= -\left(5.99\times10^2\ \text{N/C}\right)\hat{\mathbf{i}}$$

and their sum is

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_1 = \boxed{-\left(5.99\times10^2\ \text{N/C}\right)\hat{\mathbf{i}} - \left(2.70\times10^3\ \text{N/C}\right)\hat{\mathbf{j}}}$$



ANS. FIG. P23.35

(b) The vector electric force is

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(5.00\times10^{-9}\ \text{C}\right)\left(-599\hat{\mathbf{i}} - 2\ 700\hat{\mathbf{j}}\right)\ \text{N/C}$$

$$\vec{\mathbf{F}} = \left(-3.00\times10^{-6}\hat{\mathbf{i}} - 13.5\times10^{-6}\hat{\mathbf{j}}\right)\ \text{N} = \boxed{\left(-3.00\hat{\mathbf{i}} - 13.5\hat{\mathbf{j}}\right)\ \mu\text{N}}$$

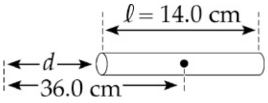
***P23.36** The electric field at any point x is

$$E = \frac{k_e q}{(x-a)^2} - \frac{k_e q}{[x-(-a)]^2} = \frac{k_e q(4ax)}{\left(x^2-a^2\right)^2}$$

When x is much, much greater than a, we find $E \approx \boxed{\dfrac{4a\left(k_e q\right)}{x^3}}$.

Section 23.5 Electric Field of a Continuous Charge Distribution

P23.37 (a) From Example 23.7, the magnitude of the electric field produced by the rod is



ANS. FIG. P23.37

$$|E| = \frac{k_e\lambda\ell}{d(\ell+d)} = \frac{k_e(Q/\ell)\ell}{d(\ell+d)} = \frac{k_e Q}{d(\ell+d)}$$

$$= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(22.0\times10^{-6}\ \text{C}\right)}{(0.290\ \text{m})(0.140\ \text{m}+0.290\ \text{m})}$$

$$E = \boxed{1.59\times10^6\ \text{N/C}}$$

(b) The charge is negative, so the electric field is directed towards its source, $\boxed{\text{to the right}}$.

P23.38 The electric field for the disk is given by

$$E = 2\pi k_e \sigma \left(1 - \frac{x}{\sqrt{x^2 + R^2}}\right)$$

in the positive *x* direction (away from the disk). Substituting,

$$E = 2\pi\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.90\times10^{-3}\ \text{C/m}^2\right)$$
$$\times\left(1 - \frac{x}{\sqrt{x^2 + (0.350)^2}}\right)$$
$$= \left(4.46\times10^8\ \text{N/C}\right)\left(1 - \frac{x}{\sqrt{x^2 + 0.123}}\right)$$

(a) At *x* = 0.050 0 m,

$$E = 3.83\times10^8\ \text{N/C} = \boxed{383\ \text{MN/C}}$$

(b) At *x* = 0.100 m,

$$E = 3.24\times10^8\ \text{N/C} = \boxed{324\ \text{MN/C}}$$

(c) At *x* = 0.500 m,

$$E = 8.07\times10^7\ \text{N/C} = \boxed{80.7\ \text{MN/C}}$$

(d) At *x* = 2.000 m,

$$E = 6.68\times10^8\ \text{N/C} = \boxed{6.68\ \text{MN/C}}$$

P23.39 We may particularize the result of Example 23.8 to

$$|E| = \frac{k_e x Q}{\left(x^2 + a^2\right)^{3/2}} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(75.0\times10^{-6}\ \text{C/m}^2\right)x}{\left(x^2 + 0.100^2\right)^{3/2}}$$
$$= \frac{6.74\times10^5 x}{\left(x^2 + 0.010\,0\right)^{3/2}}$$

where we choose the *x* axis along the axis of the ring. The field is parallel to the axis, directed away from the center of the ring above and below it.

(a) At *x* = 0.010 0 m, $\vec{\mathbf{E}} = 6.64\times10^6\hat{\mathbf{i}}\ \text{N/C} = \boxed{6.64\hat{\mathbf{i}}\ \text{MN/C}}$

(b) At $x = 0.0500$ m, $\vec{\mathbf{E}} = 2.41\times10^{7}\,\hat{\mathbf{i}}\text{ N/C} = \boxed{24.1\hat{\mathbf{i}}\text{ MN/C}}$

(c) At $x = 0.300$ m, $\vec{\mathbf{E}} = 6.40\times10^{6}\,\hat{\mathbf{i}}\text{ N/C} = \boxed{6.40\hat{\mathbf{i}}\text{ MN/C}}$

(d) At $x = 1.00$ m, $\vec{\mathbf{E}} = 6.64\times10^{5}\,\hat{\mathbf{i}}\text{ N/C} = \boxed{0.664\hat{\mathbf{i}}\text{ MN/C}}$

P23.40 The electric field at a distance x is $E_x = 2\pi k_e\sigma\left[1 - \dfrac{x}{\sqrt{x^2+R^2}}\right]$

This is equivalent to $E_x = 2\pi k_e\sigma\left[1 - \dfrac{1}{\sqrt{1+R^2/x^2}}\right]$

For large x, $\dfrac{R^2}{x^2} << 1$ and $\sqrt{1+\dfrac{R^2}{x^2}} \approx 1 + \dfrac{R^2}{2x^2}$

so
$$E_x = 2\pi k_e\sigma\left(1 - \frac{1}{\left[1+R^2/(2x^2)\right]}\right) = 2\pi k_e\sigma\frac{\left(1+R^2/(2x^2)-1\right)}{\left[1+R^2/(2x^2)\right]}$$

Substitute $\sigma = \dfrac{Q}{\pi R^2}$,

$$E_x = \frac{k_eQ\left(1/x^2\right)}{\left[1+R^2/(2x^2)\right]} = \frac{k_eQ}{x^2+R^2/2}$$

But for $x >> R$, $\dfrac{1}{x^2+R^2/2} \approx \dfrac{1}{x^2}$, so

$$\boxed{E_x \approx \frac{k_eQ}{x^2}\text{ for a disk at large distances}}$$

P23.41 (a) From Example 23.9,

$$E = 2\pi k_e\sigma\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

here,

$$\sigma = \frac{Q}{\pi R^2} = \frac{5.20\times10^{-6}}{\pi(0.0300)^2} = 1.84\times10^{-3}\text{ C/m}^2$$

the electric field is then

$$E = 2\pi k_e\sigma\left(1-\frac{x}{\sqrt{x^2+R^2}}\right)$$

$$E = 2\pi\left(8.99\times10^9\right)\left(1.84\times10^{-3}\right)$$

$$\times\left(1-\frac{0.00300}{\sqrt{(0.00300)^2+(0.0300)^2}}\right)$$

$$E = \left(1.04\times10^8\text{ N/C}\right)\left(1-\frac{0.00300}{\sqrt{(0.00300)^2+(0.0300)^2}}\right)$$

$$=\boxed{9.36\times10^7\text{ N/C}}$$

(b) The near-field approximation gives:

$$E = 2\pi k_e\sigma = \boxed{1.04\times10^8\text{ N/C (about 11\% high)}}$$

(c) The electric field at this point is

$$E = \left(1.04\times10^8\text{ N/C}\right)\left(1-\frac{0.300}{\sqrt{(0.300)^2+(0.0300)^2}}\right)$$

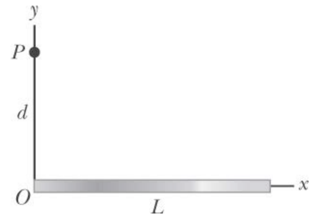
$$=\boxed{5.16\times10^5\text{ N/C}}$$

(d) With this approximation, suppressing units,

$$E = k_e\frac{Q}{r^2} = \left(8.99\times10^9\right)\left[\frac{5.20\times10^{-6}}{(0.30)^2}\right]$$

$$=\boxed{5.19\times10^5\text{ N/C (about 0.6\% high)}}$$

P23.42 (a) The electric field at point *P* due to each element of length *dx* is $dE = \frac{k_e dq}{x^2+d^2}$ and is directed along the line joining the element to point *P*. The charge element *dq* = *Qdx* / L. The *x* and *y* components are



ANS. FIG. P23.42

$$E_x = \int dE_x = \int dE\sin\theta$$

where $\sin\theta = \frac{x}{\sqrt{d^2+x^2}}$

and

$$E_y = \int dE_y = \int dE\cos\theta \quad \text{where} \quad \cos\theta = \frac{d}{\sqrt{d^2+x^2}}$$

Therefore,

$$E_x = -k_e\frac{Q}{L}\int_0^L \frac{x\,dx}{\left(d^2+x^2\right)^{3/2}} = -k_e\frac{Q}{L}\left[\frac{-1}{\left(d^2+x^2\right)^{1/2}}\right]_0^L$$

$$E_x = -k_e\frac{Q}{L}\left[\frac{-1}{\left(d^2+L^2\right)^{1/2}} - \frac{-1}{\left(d^2+0\right)^{1/2}}\right]$$

$$E_x = \boxed{-k_e\frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2+L^2\right)^{1/2}}\right]}$$

and

$$E_y = k_e\frac{Qd}{L}\int_0^L \frac{dx}{\left(d^2+x^2\right)^{3/2}} = k_e\frac{Qd}{L}\left[\frac{x}{d^2\left(d^2+x^2\right)^{1/2}}\right]_0^L$$

$$E_y = k_e\frac{Q}{\cancel{L}d}\left[\frac{\cancel{L}}{\left(d^2+L^2\right)^{1/2}} - 0\right] \quad \rightarrow \quad E_y = \boxed{k_e\frac{Q}{d}\frac{1}{\left(d^2+L^2\right)^{1/2}}}$$

(b) When *d* >> *L*,

$$E_x = -k_e\frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2+L^2\right)^{1/2}}\right] \rightarrow -k_e\frac{Q}{L}\left[\frac{1}{d} - \frac{1}{\left(d^2\right)^{1/2}}\right] \rightarrow \boxed{E_x \approx 0}$$

and

$$E_y = k_e\frac{Q}{d}\frac{1}{\left(d^2+L^2\right)^{1/2}} \rightarrow k_e\frac{Q}{d}\frac{1}{\left(d^2\right)^{1/2}} \rightarrow \boxed{E_y \approx k_e\frac{Q}{d^2}}$$

which is the field of a point charge Q at a distance d along the y axis above the charge.

P23.43 (a) Magnitude $|E| = \int \frac{k_e dq}{x^2}$, where $dq = \lambda_0 dx$

$$E = k_e\lambda_0\int_{x_0}^{\infty}\frac{dx}{x^2} = k_e\lambda_0\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} = \boxed{\frac{k_e\lambda_0}{x_0}}$$

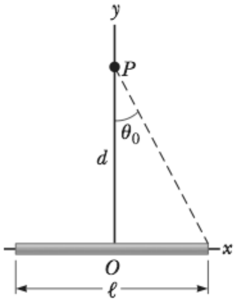
(b) The charge is positive, so the electric field points away from its source, $\boxed{\text{to the left}}$.

P23.44 (a) The electric field at point *P,* due to each element of length *dx*, is $dE = \dfrac{k_e dq}{x^2 + d^2}$ and is directed along the line joining the element to point *P*. By symmetry,

$$E_x = \int dE_x = 0$$

and since $dq = \lambda dx$,

$$E = E_y = \int dE_y = \int dE \cos\theta$$



ANS. FIG. P23.44

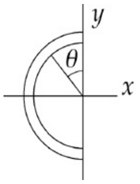
where $\cos\theta = \dfrac{y}{\sqrt{x^2 + d^2}}$.

Therefore, $E = 2k_e\lambda d \int_0^{\ell/2} \dfrac{dx}{\left(x^2 + d^2\right)^{3/2}} = \boxed{\dfrac{2k_e\lambda \sin\theta_0}{d}}$

with $\sin\theta_0 = \dfrac{\ell/2}{\sqrt{\left(\ell/2\right)^2 + d^2}}$.

(b) For a bar of infinite length, $\theta_0 = 90°$ and $E_y = \boxed{\dfrac{2k_e\lambda}{d}}$.

P23.45 Due to symmetry, $E_y = \int dE_y = 0$, and $E_x = -\int dE \sin\theta = -k_e \int \dfrac{dq \sin\theta}{r^2}$ where $dq = \lambda ds = \lambda r d\theta$; the component E_x is negative because charge $q = -7.50\ \mu\text{C}$, causing the net electric field to be directed to the left.



ANS. FIG. P23.45

$$E_x = -\frac{k_e\lambda}{r}\int_0^{\pi} \sin\theta d\theta = -\frac{k_e\lambda}{r}\left(-\cos\theta\right)\Big|_0^{\pi} = -\frac{2k_e\lambda}{r}$$

where $\lambda = \dfrac{|q|}{L}$ and $r = \dfrac{L}{\pi}$. Thus,

$$E_x = -\frac{2k_e|q|\pi}{L^2} = -\frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(7.50\times10^{-6}\ \text{C}\right)\pi}{\left(0.140\ \text{m}\right)^2}$$

$$E_x = -2.16\times10^7\ \text{N/C}$$

(a) magnitude $E = \boxed{2.16\times10^7\ \text{N/C}}$

(b) $\boxed{\text{to the left}}$

P23.46 (a) We define *x* = 0 at the point where we are to find the field. One ring, with thickness *dx*, has charge $\frac{Qdx}{h}$ and produces, at the chosen point, a field

$$d\vec{\mathbf{E}} = \frac{k_e x}{\left(x^2 + R^2\right)^{3/2}} \frac{Qdx}{h} \hat{\mathbf{i}}$$

The total field is

$$\vec{\mathbf{E}} = \int_{\text{all charge}} d\mathbf{E} = \int_{d}^{d+h} \frac{k_e Qxdx}{h\left(x^2 + R^2\right)^{3/2}} \hat{\mathbf{i}}$$

$$= \frac{k_e Q\hat{\mathbf{i}}}{2h} \int_{x=d}^{d+h} \left(x^2 + R^2\right)^{-3/2} 2x\,dx$$

integrating,

$$\vec{\mathbf{E}} = \frac{k_e Q\hat{\mathbf{i}}}{2h} \frac{\left(x^2 + R^2\right)^{-1/2}}{(-1/2)}\Bigg|_{x=d}^{d+h}$$

$$= \boxed{\frac{k_e Q\hat{\mathbf{i}}}{h}\left[\frac{1}{\left(d^2 + R^2\right)^{1/2}} - \frac{1}{\left((d+h)^2 + R^2\right)^{1/2}}\right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness *dx*, charge $\frac{Qdx}{h}$, and charge-per-area $\sigma = \frac{Qdx}{\pi R^2 h}$. One disk produces a field

$$d\vec{\mathbf{E}} = \frac{2\pi k_e Qdx}{\pi R^2 h}\left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right)\hat{\mathbf{i}}$$

So, $\vec{\mathbf{E}} = \int_{\text{all charge}} d\vec{\mathbf{E}} = \int_{x=d}^{d+h} \frac{2k_e Qdx}{R^2 h}\left(1 - \frac{x}{\left(x^2 + R^2\right)^{1/2}}\right)\hat{\mathbf{i}}$

$$\vec{\mathbf{E}} = \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[\int_{d}^{d+h} dx - \frac{1}{2}\int_{x=d}^{d+h}\left(x^2 + R^2\right)^{-1/2} 2x\,dx\right]$$

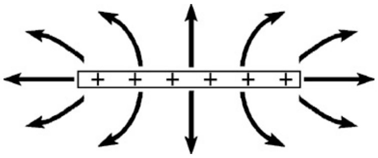
$$= \frac{2k_e Q\hat{\mathbf{i}}}{R^2 h}\left[x\Big|_{d}^{d+h} - \frac{1}{2}\frac{\left(x^2 + R^2\right)^{1/2}}{1/2}\Bigg|_{d}^{d+h}\right]$$

$$\vec{\mathbf{E}} = \frac{2k_eQ\hat{\mathbf{i}}}{R^2h}\left[d+h-d-\left((d+h)^2+R^2\right)^{1/2}+\left(d^2+R^2\right)^{1/2}\right]$$

$$\vec{\mathbf{E}} = \boxed{\frac{2k_eQ\hat{\mathbf{i}}}{R^2h}\left[h+\left(d^2+R^2\right)^{1/2}-\left((d+h)^2+R^2\right)^{1/2}\right]}$$

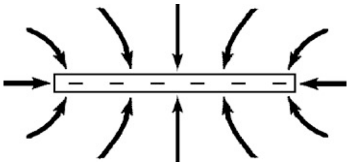
Section 23.6 Electric Field Lines

P23.47 The field lines are shown in ANS. FIG. P23.47, where we've followed the rules for drawing field lines where field lines point toward negative charge, meeting the rod perpendicularly and ending there.



ANS. FIG. P23.47

P23.48 For the positively charged disk, a side view of the field lines, pointing into the disk, is shown in ANS. FIG. P23.48.



ANS. FIG. P23.48

P23.49 Field lines emerge from positive charge and enter negative charge.

(a) The number of field lines emerging from positive q_2 and entering negative charge q_1 is proportional to their charges:

$$\frac{q_1}{q_2} = \frac{-6}{18} = \boxed{-\frac{1}{3}}$$

(b) From above, $\boxed{q_1 \text{ is negative, } q_2 \text{ is positive}}$.

P23.50 (a) The electric field has the general appearance shown in ANS. FIG. P23.50 below.

(b) It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.

In addition to the center of the triangle, the electric field lines in the second panel of ANS. FIG. P23.50 indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.

(c) You may need to review vector addition in Chapter 1. The electric field at point *P* can be found by adding the electric field vectors due to each of the two lower point charges: $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$.

The electric field from a point charge is

$$\vec{\mathbf{E}} = k_e \frac{q}{r^2}\hat{\mathbf{r}}.$$

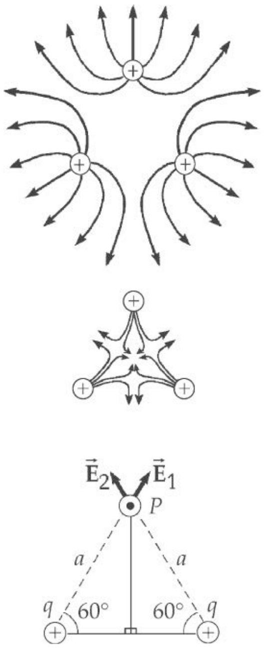
As shown in the bottom panel of ANS. FIG. P23.50,

$$\vec{\mathbf{E}}_1 = k_e \frac{q}{a^2}$$

to the right and upward at 60°, and

$$\vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}$$

to the left and upward at 60°. So,



ANS. FIG. P23.50

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right) + \left(-\cos 60°\hat{\mathbf{i}} + \sin 60°\hat{\mathbf{j}}\right)\right]$$

$$= k_e \frac{q}{a^2}\left[2\left(\sin 60°\hat{\mathbf{j}}\right)\right] = \boxed{1.73 k_e \frac{q}{a^2}\hat{\mathbf{j}}}$$

Section 23.7 Motion of a Charged Particle in a Uniform Electric Field

P23.51 (a) We obtain the acceleration of the proton from the particle under a net force model, with *F* = *qE* representing the electric force:

$$a = \frac{F}{m} = \frac{qE}{m} = \frac{\left(1.602 \times 10^{-19}\text{ C}\right)\left(640\text{ N/C}\right)}{1.67 \times 10^{-27}\text{ kg}} = \boxed{6.14 \times 10^{10}\text{ m/s}^2}$$

(b) The particle under constant acceleration model gives us $v_f = v_i + at$, from which we obtain

$$t = \frac{v_f - 0}{a} = \frac{1.20 \times 10^6\text{ m/s}}{6.14 \times 10^{10}\text{ m/s}^2} = \boxed{19.5\ \mu\text{s}}$$

(c) Again, from the particle under constant acceleration model,

$$\Delta x = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}(6.14 \times 10^{10}\text{ m/s}^2)(19.5 \times 10^{-6}\text{ s})^2$$

$$= \boxed{11.7\text{ m}}$$

(d) The final kinetic energy of the proton is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67\times10^{-27}\text{ kg})(1.20\times10^{6}\text{ m/s})^2 = \boxed{1.20\times10^{-15}\text{ J}}$$

P23.52 (a) $|a| = \dfrac{qE}{m} = \dfrac{\left(1.602\times10^{-19}\text{ C}\right)\left(6.00\times10^{5}\text{ N/C}\right)}{1.67\times10^{-27}\text{ kg}} = 5.76\times10^{13}\text{ m/s},$

so $\vec{\mathbf{a}} = \boxed{-5.76\times10^{13}\hat{\mathbf{i}}\text{ m/s}^2}$.

(b) $v_f^2 = v_i^2 + 2a\left(x_f - x_i\right)$

$0 = v_i^2 + 2\left(-5.76\times10^{13}\text{ m/s}^2\right)\left(0.070\,0\text{ m}\right)$

$\vec{\mathbf{v}}_i = \boxed{2.84\times10^{6}\hat{\mathbf{i}}\text{ m/s}}$

(c) $v_f = v_i + at$

$0 = 2.84\times10^{6}\text{ m/s} + \left(-5.76\times10^{13}\text{ m/s}^2\right)t \quad\rightarrow\quad t = \boxed{4.93\times10^{-8}\text{ s}}$

P23.53 We use $v_f = v_i + at$, where $v_i = 0$, $t = 48.0\times10^{-9}$ s, and $a = F/m = eE/m$.

For the electron, $m = m_e = 9.11\times10^{-31}\text{ kg}$

and for the proton, $m = m_p = 1.67\times10^{-27}\text{ kg}$

The electric force on both particles is given by

$$F = eE = \left(1.60\times10^{-19}\text{ C}\right)\left(5.20\times10^{2}\text{ N/C}\right) = 8.32\times10^{-17}\text{ N}$$

Then, for the electron,

$$v_{fe} = v_{ie} + at = 0 + \left(\frac{eE}{m_e}\right)t = \left(\frac{8.32\times10^{-17}\text{ N}}{9.11\times10^{-31}\text{ kg}}\right)\left(48.0\times10^{-9}\text{ s}\right)$$

$$= \boxed{4.38\times10^{6}\text{ m/s}}$$

and for the proton,

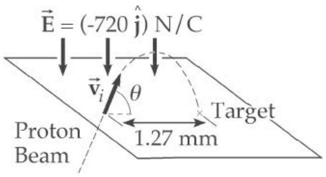
$$v_{fp} = v_{ip} + at = 0 + \left(\frac{eE}{m_p}\right)t = \left(\frac{8.32\times10^{-17}\text{ N}}{1.67\times10^{-27}\text{ kg}}\right)\left(48.0\times10^{-9}\text{ s}\right)$$

$$= \boxed{2.39\times10^{3}\text{ m/s}}$$

P23.54 (a) $\boxed{\text{Particle under constant velocity}}$

(b) $\boxed{\text{Particle under constant acceleration}}$

(c) The vertical acceleration caused by the



ANS. FIG. P23.54

electric force is constant and downward; therefore, $\boxed{\text{the proton moves in a parabolic path just like a projectile in a gravitational field.}}$

(d) We may neglect the effect of the acceleration of gravity on the proton because the magnitude of the vertical acceleration caused by the electric force is

$$a_y = \frac{eE}{m_p} = \frac{(1.60\times 10^{-19}\ \text{C})(720\ \text{N/C})}{1.67\times 10^{-27}\ \text{kg}} = 6.90\times 10^{10}\ \text{m/s}^2$$

which is much greater than that of gravity.

Replacing acceleration *g* in Equation 4.13 with eE/m_P, we have

$$R = \frac{v_i^2 \sin 2\theta}{eE/m_p} = \boxed{\frac{m_p v_i^2 \sin 2\theta}{eE}}$$

(e) $R = \dfrac{m_p v_i^2 \sin 2\theta}{eE} = \dfrac{(1.67\times 10^{-27}\ \text{kg})(9.55\times 10^3\ \text{m/s})^2 \sin 2\theta}{(1.60\times 10^{-19}\ \text{C})(720\ \text{N/C})}$

$= 1.27\times 10^{-3}\ \text{m}$

which gives $\sin 2\theta = 0.961$, or

$$\theta = \boxed{36.9^\circ} \quad \text{or} \quad 90.0^\circ - \theta = \boxed{53.1^\circ}$$

(f) $\Delta t = \dfrac{R}{v_{ix}} = \dfrac{R}{v_i \cos\theta}$

If $\theta = 36.9^\circ$, $\Delta t = \boxed{166\ \text{ns}}$. If $\theta = 53.1^\circ$, $\Delta t = \boxed{221\ \text{ns}}$.

P23.55 The work done on the charge is $W = \vec{\mathbf{F}}\cdot\vec{\mathbf{d}} = q\vec{\mathbf{E}}\cdot\vec{\mathbf{d}}$ and the kinetic energy changes according to $W = K_f - K_i = 0 - K$.

Assuming $\vec{\mathbf{v}}$ is in the $+x$ direction, we have $(-e)\vec{\mathbf{E}}\cdot d\hat{\mathbf{i}} = -K$.

Then, $e\vec{\mathbf{E}}\cdot\left(d\hat{\mathbf{i}}\right) = K$, and

$$\vec{\mathbf{E}} = \frac{K}{ed}\hat{\mathbf{i}}$$

(a) $E = \boxed{\dfrac{K}{ed}}$

(b) Because a negative charge experiences an electric force opposite to the direction of an electric field, the required electric field will be $\boxed{\text{in the direction of motion}}$.

P23.56 (a) The positive charge experiences a constant downward force (in the direction of the electric field):

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} = \left(1.00\times10^{-6}\ \text{C}\right)\left(2\,000\ \text{N/C}\right)\left(-\hat{\mathbf{j}}\right) = 2.00\times10^{-3}\left(-\hat{\mathbf{j}}\right)\ \text{N}$$

and moves with acceleration:

$$\vec{\mathbf{a}} = \frac{\sum\vec{\mathbf{F}}}{m} = \frac{\left(2.00\times10^{-3}\ \text{N}\right)\left(-\hat{\mathbf{j}}\right)}{2.00\times10^{-16}\ \text{kg}} = 1.00\times10^{13}\left(-\hat{\mathbf{j}}\right)\ \text{m/s}^2$$

Note that the gravitational acceleration is on the order of a trillion times smaller than the electrical acceleration of the particle. Thus, its trajectory is $\boxed{\text{a parabola}}$ opening downward.

(b) The maximum height the charge attains above the bottom negative plate is described by

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

Solving for the height gives

$$y_f - y_i = \frac{v_{yf}^2 - v_{yi}^2}{2a_y} = \frac{0 - \left[\left(1.00\times10^5\ \text{m/s}\right)\sin 37.0^\circ\right]^2}{2\left(1.00\times10^{13}\ \text{m/s}^2\right)}$$

$$= 1.81\times10^{-4}\ \text{m} = 0.181\ \text{mm}$$

Since this height is less than the 1.00 cm separation of the plates, the charge passes through its highest point and returns to strike $\boxed{\text{the negative plate}}$.

(c) The particle's *x*-component of velocity is constant at

$$(1.00\times10^5\ \text{m/s})\cos 37^\circ = 7.99\times10^4\ \text{m/s}$$

Starting at time $t = 0$, we find the time t when the particle returns to the negative plate from

$$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$$

Substituting numerical values,

$$0 = 0 + \left[\left(1.00\times10^5\ \text{m/s}\right)\sin 37.0^\circ\right]t + \frac{1}{2}\left(-1.00\times10^{13}\ \text{m/s}^2\right)t^2$$

since $t > 0$, the only valid solution to this quadratic equation is $t = 1.20\times10^{-8}$ s. The particle's range is then

$$x_f = x_i + v_x t = 0 + \left(7.99\times10^4\ \text{m/s}\right)\left(1.20\times10^{-8}\ \text{s}\right)$$

$$= 9.61\times10^{-4}\ \text{m}$$

$$\boxed{\text{The particle strikes the negative plate after moving a horizontal distance of 0.961 mm.}}$$

P23.57 $\vec{\mathbf{E}}$ is directed along the *y* direction; therefore, $a_x = 0$ and $x = v_{xi}t$.

(a) $$t = \frac{x}{v_{xi}} = \frac{0.050\,0\text{ m}}{4.50\times10^{5}\text{ s}} = 1.11\times10^{-7}\text{ s} = \boxed{111\text{ ns}}$$

(b) $$a_y = \frac{qE}{m} = \frac{(1.602\times10^{-19}\text{ C})(9.60\times10^{3}\text{ N/C})}{1.67\times10^{-27}\text{ kg}} = 9.21\times10^{11}\text{ m/s}^2$$

$$y_f - y_i = v_{yi}t + \frac{1}{2}a_yt^2:$$

$$y_f = \frac{1}{2}(9.21\times10^{11}\text{ m/s}^2)(1.11\times10^{-7}\text{ s})^2$$

$$= 5.68\times10^{-3}\text{ m} = \boxed{5.67\text{ mm}}$$

(c) $$v_x = 4.50\times10^{5}\text{ m/s}$$

$$v_{yf} = v_{yi} + a_yt = (9.21\times10^{11}\text{ m/s}^2)(1.11\times10^{-7}\text{ s}) = 1.02\times10^{5}\text{ m/s}$$

$$\vec{\mathbf{v}} = \boxed{(450\hat{\mathbf{i}} + 102\hat{\mathbf{j}})\text{ km/s}}$$

Additional Problems

P23.58 (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$$Q = \sigma A$$
$$= (15.0\times10^{-9}\text{ C/m}^2)2\pi(0.025\,0\text{ m})[0.025\,0\text{ m} + 0.060\,0\text{ m}]$$
$$= \boxed{2.00\times10^{-10}\text{ C}}$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$Q = \sigma A = (15.0\times10^{-9}\text{ C/m}^2)[2\pi(0.025\,0\text{ m})(0.060\,0\text{ m})]$$
$$= \boxed{1.41\times10^{-10}\text{ C}}$$

(c) $$Q = \rho V = \rho\pi r^2L = (500\times10^{-9}\text{ C/m}^3)[\pi(0.025\,0\text{ m})^2(0.060\,0\text{ m})]$$
$$= \boxed{5.89\times10^{-11}\text{ C}}$$

P23.59 The electric field is given by the sum of the fields due to each of the *n* particles:

$$\vec{\mathbf{E}} = \sum \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{k_e q}{a^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(2a)^2}\left(-\hat{\mathbf{i}}\right) + \frac{k_e q}{(3a)^2}\left(-\hat{\mathbf{i}}\right) + \cdots$$

$$= \frac{-k_e q\hat{\mathbf{i}}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right)$$

$$= \boxed{-\frac{\pi^2 k_e q}{6a^2}\hat{\mathbf{i}}}$$

P23.60 The positive charge, call it *q*, is 50.0 cm – 20.9 cm = 29.1 cm from charge *Q*. The force on *q* from the –3.00 nC charge balances the force on *q* from the –*Q* charge:

$$\frac{k_e(3.00\text{ nC})q}{(0.209\text{ m})^2} = \frac{k_e Q q}{(0.291\text{ m})^2}$$

which then gives

$$Q = (3.00\text{ nC})\left(\frac{0.291\text{ m}}{0.209\text{ m}}\right)^2 = \boxed{5.82\text{ nC}}$$

P23.61 (a) Take up the incline as the positive x direction. Newton's second law along the incline gives

$$\sum F_x = -mg\sin\theta + |Q|E = 0$$

solving for the electric field gives

$$E = \boxed{\frac{mg}{|Q|}\sin\theta}$$

(b) The electric force must be up the incline, so the electric field must point down the incline because the charge is negative.

$$E = \frac{mg}{|Q|}\sin\theta = \frac{(5.40\times10^{-3})(9.80)}{|7.00\times10^{-6}|}\sin 25.0°$$

$$= \boxed{3.19\times10^3\text{ N/C, down the incline}}$$

P23.62 The downward electric force on the 0.800 μC charge is balanced by the upward spring force:

$$\frac{k_e q_1 q_2}{r^2} = kx$$

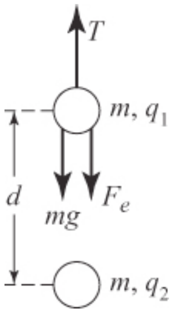
solving for the spring constant gives

$$\begin{aligned} k &= \frac{k_e q_1 q_2}{x r^2} \\ &= \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(0.800\times10^{-6}\ \mathrm{C}\right)\left(0.600\times10^{-6}\ \mathrm{C}\right)}{(0.0350\ \mathrm{m})(0.0500\ \mathrm{m})^2} \\ &= \boxed{49.3\ \mathrm{N/m}} \end{aligned}$$

P23.63 We integrate the expression for the incremental electric field to obtain

$$\begin{aligned} \vec{\mathbf{E}} &= \int d\vec{\mathbf{E}} = \int_{x_0}^{\infty}\left[\frac{k_e\lambda_0 x_0\,dx\left(-\hat{\mathbf{i}}\right)}{x^3}\right] = -k_e\lambda_0 x_0\hat{\mathbf{i}}\int_{x_0}^{\infty} x^{-3}\,dx \\ &= -k_e\lambda_0 x_0\hat{\mathbf{i}}\left(-\left.\frac{1}{2x^2}\right|_{x_0}^{\infty}\right) \\ &= \boxed{\frac{k_e\lambda_0}{2x_0}\left(-\hat{\mathbf{i}}\right)} \end{aligned}$$

***P23.64** (a) The gravitational force exerted on the upper sphere by the lower one is negligible in comparison to the gravitational force exerted by the Earth and the downward electrical force exerted by the lower sphere. Therefore,



ANS. FIG. P23.64

$$\sum F_y = 0 \quad \rightarrow \quad T - mg - F_e = 0$$

or $$T = mg + \frac{k_e|q_1||q_2|}{d^2}$$

substituting numerical values,

$$\begin{aligned} T &= \left(7.50\times10^{-3}\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right) \\ &\quad + \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(32.0\times10^{-9}\ \mathrm{C}\right)\left(58.0\times10^{-9}\ \mathrm{C}\right)}{\left(2.00\times10^{-2}\ \mathrm{m}\right)^2} \\ &= \boxed{0.115\ \mathrm{N}} \end{aligned}$$

(b) Once again, from the particle under a net force model,

$$\sum F_y = 0 \quad \rightarrow \quad T - mg - F_e = 0$$

or $$\frac{k_e|q_1||q_2|}{d^2} = T - mg$$

solving for the distance d then gives

$$d=\sqrt{\frac{k_e|q_1||q_2|}{T-mg}}$$

substituting numerical values, with T = 0.180 N,

$$d=\sqrt{\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(32.0\times10^{-9}\ \text{C}\right)\left(58.0\times10^{-9}\ \text{C}\right)}{0.180\ \text{N}-\left(7.50\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}}$$

$$=1.25\times10^{-2}\ \text{m}=\boxed{1.25\ \text{cm}}$$

P23.65 The proton moves with acceleration

$$|a_p|=\frac{qE}{m}=\frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(640\ \text{N/C}\right)}{1.673\times10^{-27}\ \text{kg}}=6.13\times10^{10}\ \text{m/s}^2$$

while the electron has acceleration

$$|a_e|=\frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(640\ \text{N/C}\right)}{9.110\times10^{-31}\ \text{kg}}=1.12\times10^{14}\ \text{m/s}^2=1\,836a_p$$

(a) We want to find the distance traveled by the proton (i.e., $d=\frac{1}{2}a_pt^2$), knowing:

$$4.00\ \text{cm}=\frac{1}{2}a_pt^2+\frac{1}{2}a_et^2=\left(1\,837\right)\left(\frac{1}{2}a_pt^2\right)$$

Thus,

$$d=\frac{1}{2}a_pt^2=\frac{4.00\ \text{cm}}{1\,837}=\boxed{2.18\times10^{-5}\ \text{m}}$$

(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{\text{Na}}=\frac{1}{2}a_{\text{Na}}t^2$). This is found from:

$$4.00\ \text{cm}=\frac{1}{2}a_{\text{Na}}t^2+\frac{1}{2}a_{\text{Cl}}t^2:$$

$$4.00\ \text{cm}=\frac{1}{2}\left(\frac{eE}{22.99\ \text{u}}\right)t^2+\frac{1}{2}\left(\frac{eE}{35.45\ \text{u}}\right)t^2$$

This may be written as

$$4.00\ \text{cm}=\frac{1}{2}a_{\text{Na}}t^2+\frac{1}{2}\left(0.649a_{\text{Na}}\right)t^2=1.65\left(\frac{1}{2}a_{\text{Na}}t^2\right)$$

so $$d_{\text{Na}}=\frac{1}{2}a_{\text{Na}}t^2=\frac{4.00\ \text{cm}}{1.65}=\boxed{2.43\ \text{cm}}$$

P23.66 We find the equal-magnitude charges on both spheres:

$$F = k_e \frac{q_1 q_2}{r^2} = k_e \frac{q^2}{r^2}$$

so $$q = r\sqrt{\frac{F}{k_e}} = (1.00\text{ m})\sqrt{\frac{1.00\times 10^4\text{ N}}{8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times 10^{-3}\text{ C}$$

The number of electrons transferred is then

$$N_{\text{xfer}} = \frac{1.05\times 10^{-3}\text{ C}}{1.60\times 10^{-19}\text{ C}/e^-} = 6.59\times 10^{15}\text{ electrons}$$

The whole number of electrons in each sphere is

$$N_{\text{tot}} = \left(\frac{10.0\text{ g}}{107.87\text{ g/mol}}\right)\left(6.02\times 10^{23}\text{ atoms/mol}\right)\left(47\ e^-/\text{atom}\right)$$
$$= 2.62\times 10^{24}\ e^-$$

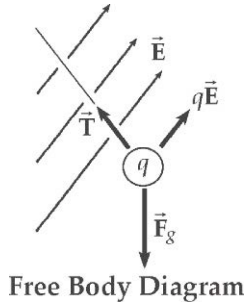
The fraction transferred is then

$$f = \frac{N_{\text{xfer}}}{N_{\text{tot}}} = \left(\frac{6.59\times 10^{15}}{2.62\times 10^{24}}\right) = \boxed{2.51\times 10^{-9}}$$

or 2.51 charges in every billion.

P23.67 ANS. FIG. P23.67 shows the free-body diagram for Newton's second law gives

$$\sum\vec{\mathbf{F}} = \vec{\mathbf{T}} + q\vec{\mathbf{E}} + \vec{\mathbf{F}}_g = 0$$



ANS. FIG. P23.67

We are given

$$E_x = 3.00\times 10^5\text{ N/C}$$

and $$E_y = 5.00\times 10^5\text{ N/C}$$

Applying Newton's second law or the first condition for equilibrium in the *x* and *y* directions,

$$\sum F_x = qE_x - T\sin 37.0° = 0 \qquad \textbf{[1]}$$

$$\sum F_y = qE_y + T\cos 37.0° - mg = 0 \qquad \textbf{[2]}$$

(a) We solve for *T* from equation [1]:

$$T = \frac{qE_x}{\sin 37.0°}$$

and substitute into equation [2] to obtain

$$q = \frac{mg}{E_y + \dfrac{E_x}{\tan 37.0^\circ}}$$

$$= \frac{\left(1.00 \times 10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{5.00 \times 10^5\ \text{N/C} + \left(\dfrac{3.00 \times 10^5\ \text{N/C}}{\tan 37.0^\circ}\right)}$$

$$q = \boxed{1.09 \times 10^{-8}\ \text{C}}$$

(b) Using the above result for q in equation [1], we find that the tension is

$$T = \frac{qE_x}{\sin 37.0^\circ} = \frac{\left(1.09 \times 10^{-8}\ \text{C}\right)\left(3.00 \times 10^5\ \text{N/C}\right)}{\sin 37.0^\circ}$$

$$= \boxed{5.44 \times 10^{-3}\ \text{N}}$$

P23.68 This is the general version of the preceding problem . The known quantities are *A*, *B*, *m*, *g*, and *θ*. The unknowns are *q* and *T*.

Refer to ANS. FIG. P23.67 above. The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 51.

Again, from Newton's second law,

$$\sum F_x = -T\sin\theta + qA = 0 \qquad \textbf{[1]}$$

and

$$\sum F_y = +T\cos\theta + qB - mg = 0 \qquad \textbf{[2]}$$

(a) Substituting $T = \dfrac{qA}{\sin\theta}$ into equation [2], we obtain

$$\frac{qA\cos\theta}{\sin\theta} + qB = mg$$

Isolating *q* on the left,

$$\boxed{q = \frac{mg}{\left(A\cot\theta + B\right)}}$$

(b) Substituting this value into equation [1], we obtain

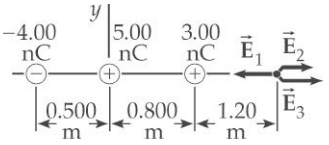
$$\boxed{T = \frac{mgA}{\left(A\cos\theta + B\sin\theta\right)}}$$

If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 51. If you find this problem more difficult than problem 51, the little list at the first step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the analysis step, and for recognizing when we have an answer.

P23.69 (a) Refer to ANS. FIG. P23.69(a). The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{\left(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-4.00\times 10^{-9}\ \text{C}\right)}{\left(2.50\ \text{m}\right)^2}\hat{\mathbf{i}}$$

$$= -5.75\hat{\mathbf{i}}\ \text{N/C}$$



ANS. FIG. P23.69(a)

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge, are

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \frac{\left(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times 10^{-9}\ \text{C}\right)}{\left(2.00\ \text{m}\right)^2}\hat{\mathbf{i}}$$

$$= 11.2\ \text{N/C}\ \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_3 = \frac{\left(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times 10^{-9}\ \text{C}\right)}{\left(1.20\ \text{m}\right)^2}\hat{\mathbf{i}} = 18.7\ \text{N/C}\ \hat{\mathbf{i}}$$

$$\vec{\mathbf{E}}_R = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3 = \boxed{24.2\ \text{N/C in } +x \text{ direction}}$$

(b) In this case, referring to ANS. FIG. P23.69(b),

$$\vec{\mathbf{E}}_1 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \left(-8.46\ \text{N/C}\right)\left(0.243\hat{\mathbf{i}} + 0.970\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{E}}_2 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \left(11.2\ \text{N/C}\right)\left(+\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{E}}_3 = \frac{k_e q}{r^2}\hat{\mathbf{r}} = \left(5.81\ \text{N/C}\right)\left(-0.371\hat{\mathbf{i}} + 0.928\hat{\mathbf{j}}\right)$$

The components of the resultant electric field are

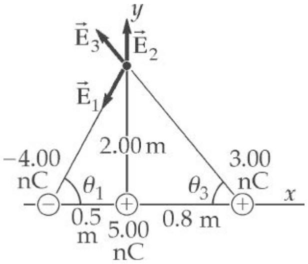
$$E_x = E_{1x} + E_{3x} = -4.21\hat{\mathbf{i}}\ \text{N/C} \quad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\hat{\mathbf{j}}\ \text{N/C}$$

then, the magnitude of the resultant electric field is

$$E_R = \boxed{9.42\ \text{N/C}}$$

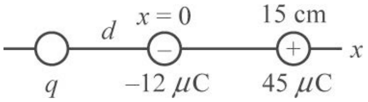
and is directed at

$$\theta = \tan^{-1}\left(\frac{|E_y|}{|E_x|}\right) = \tan^{-1}\left(\frac{8.43\ \text{N/C}}{4.21\ \text{N/C}}\right) = \boxed{63.4°\ \text{above}\ -x\ \text{axis}}$$



ANS. FIG. P23.69(b)

P23.70 (a) The two given charges exert equal-size forces of attraction on each other. If a third charge, positive or negative, were placed between them they could not be in equilibrium. If the third charge were at a point *x* > 15.0 cm, it would exert a stronger force on the 45.0-μC charge than on the –12.0-μC charge, and could not produce equilibrium for both. Thus the third charge must be at *x* = –*d* < 0.



ANS. FIG. P23.70

$\boxed{\text{It is possible in just one way.}}$

(b) The equilibrium of the third charge requires

$$\frac{k_e q(12.0\ \mu\text{C})}{d^2} = \frac{k_e q(45.0\ \mu\text{C})}{(15.0\ \text{cm} + d)^2} \rightarrow \left(\frac{15.0\ \text{cm} + d}{d}\right)^2 = \frac{45.0}{12.0} = 3.75$$

Solving,

$$15.0\ \text{cm} + d = 1.94d \qquad \rightarrow \qquad d = 16.0\ \text{cm}$$

The third charge is at $\boxed{x = -16.0\ \text{cm}}$.

(c) The equilibrium of the –12.0-μC charge requires

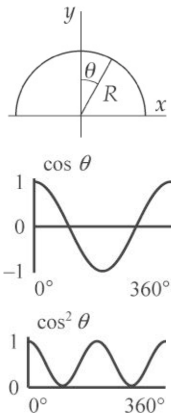
$$\frac{k_e q(12.0\ \mu\text{C})}{(16.0\ \text{cm})^2} = \frac{k_e(45.0\ \mu\text{C})(12.0\ \mu\text{C})}{(15.0\ \text{cm})^2}$$

solving,

$$q = \boxed{+51.3\ \mu\text{C}}$$

All six individual forces are now equal in magnitude, so we have equilibrium as required, and this is the only solution.

P23.71 To find the force on the test charge at point *P*, we first determine the charge per unit length on the semicircle:



ANS. FIG. P23.71

$$Q = \int \lambda\, d\ell = \int_{-90.0^\circ}^{90.0^\circ} \lambda_0 \cos\theta R\, d\theta = \lambda_0 R \sin\theta \Big|_{-90.0^\circ}^{90.0^\circ}$$

$$= \lambda_0 R\left[1-(-1)\right] = 2\lambda_0 R$$

or $Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\ \text{m}$,

which gives $\lambda_0 = 10.0\ \mu\text{C/m}$.

The force on the charge from each incremental section of the semicircle is

$$dF_y = \frac{k_e q(\lambda d\ell)\cos\theta}{R^2} = \frac{k_e q\left(\lambda_0 \cos^2\theta R d\theta\right)}{R^2}$$

Integrating,

$$F_y = \int_{-90.0^\circ}^{90.0^\circ} \frac{k_e q \lambda_0}{R}\cos^2\theta\, d\theta = \frac{k_e q\lambda_0}{R}\int_{-\pi/2}^{\pi/2}\left(\frac{1}{2}+\frac{1}{2}\cos 2\theta\right)d\theta$$

$$F_y = \frac{k_e q\lambda_0}{R}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\Bigg|_{-\pi/2}^{\pi/2} = \frac{k_e q\lambda_0}{R}\left[\left(\frac{\pi}{4}+0\right)-\left(\frac{-\pi}{4}+0\right)\right]$$

$$F_y = \frac{k_e q\lambda_0}{R}\left(\frac{\pi}{2}\right)$$

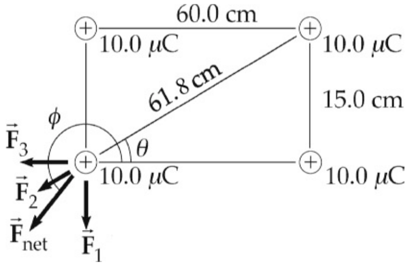
$$F_y = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times10^{-6}\ \text{C}\right)\left(10.0\times10^{-6}\ \text{C/m}\right)\left(\frac{\pi}{2}\right)}{(0.600\ \text{m})}$$

$$F_y = 0.706\ \text{N, downward} = \boxed{-0.706\hat{\mathbf{i}}\ \text{N}}$$

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

P23.72 The magnitude of the electric force is given by $F = \dfrac{k_e q_1 q_2}{r^2}$. The angle θ in ANS. FIG. P23.72 is found from

$$\theta = \tan^{-1}\left(\frac{15.0}{60.0}\right) = 14.0^\circ$$



ANS. FIG. P23.72

$$F_1 = \frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times 10^{-6}\ \text{C}\right)^2}{\left(0.150\ \text{m}\right)^2}$$

$$= 40.0\ \text{N}$$

$$F_2 = \frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times 10^{-6}\ \text{C}\right)^2}{\left(0.618\ \text{m}\right)^2} = 2.35\ \text{N}$$

$$F_3 = \frac{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(10.0\times 10^{-6}\ \text{C}\right)^2}{\left(0.600\ \text{m}\right)^2} = 2.50\ \text{N}$$

$$F_x = -F_3 - F_2\cos 14.0° = -2.50 - 2.35\cos 14.0° = -4.78\ \text{N}$$

$$F_y = -F_1 - F_2\sin 14.0° = -40.0 - 2.35\sin 14.0° = -40.5\ \text{N}$$

(a) $F_{\text{net}} = \sqrt{F_x^2 + F_y^2} = \sqrt{(-4.78\ \text{N})^2 + (-40.5\ \text{N})^2} = \boxed{40.8\ \text{N}}$

(b) $\tan\phi = \dfrac{F_y}{F_x} = \dfrac{-40.5\ \text{N}}{-4.78\ \text{N}} \rightarrow \phi = \boxed{263°}$

P23.73 We model the spheres as particles. They have different charges. They exert on each other forces of equal magnitude. They have equal masses, so their strings make equal angles θ with the vertical. We define r as the distance between the centers of the two spheres. We find r from

$$\sin\theta = \frac{r/2}{40.0\ \text{cm}}$$

from which we obtain

$$r = (80.0\ \text{cm})\sin\theta$$

Now let T represent the string tension. We have, from the particle under a net force model,

$$\sum F_x = 0:\quad \frac{k_e q_1 q_2}{r^2} - T\sin\theta = 0 \quad\rightarrow\quad \frac{k_e q_1 q_2}{r^2} = T\sin\theta \qquad \textbf{[1]}$$

$$\sum F_y = 0:\quad T\cos\theta - mg = 0 \quad\rightarrow\quad mg = T\cos\theta \qquad \textbf{[2]}$$

Dividing equation [1] by [2] to eliminate T gives

$$\frac{k_e q_1 q_2}{r^2 mg} = \tan\theta = \frac{r/2}{\sqrt{(40.0\ \text{cm})^2 - r^2/4}}$$

Clearing the fractions,

$$k_e q_1 q_2 \sqrt{(80.0\ \text{cm})^2 - r^2} = mgr^3$$

Substituting numerical values gives

$$\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(200\times10^{-9}\ \mathrm{C}\right)\left(300\times10^{-9}\ \mathrm{C}\right)$$
$$\times\sqrt{(0.800\ \mathrm{m})^2-r^2}=\left(2.40\times10^{-3}\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right)r^3$$

Suppressing units,

$$(0.800)^2-r^2=1\,901\,r^6$$

Instead of attempting to solve this equation, we instead home in on a solution by trying values, tabulated below:

r	$0.640-r^2-1901\,r^6$
0	+0.64
0.5	–29.3
0.2	+0.48
0.3	–0.84
0.24	+0.22
0.27	–0.17
0.258	+0.013
0.259	–0.001

Thus the distance to three digits is 0.259 m = $\boxed{2.59\text{ cm.}}$

P23.74 Use Figure 23.24 for guidance on the physical setup of this problem. Let the electron enter at the origin of coordinates at the left end and just under the upper plate, which we choose to be negative so that the electron accelerates downward. The electron is a particle under constant velocity in the horizontal direction:

$$x_f=v_{xi}t$$

The electron is a particle under constant acceleration in the vertical direction:

$$y_f=\frac{1}{2}a_yt^2$$

Eliminate *t* between the equations:

$$y_f=\frac{1}{2}a_y\left(\frac{x_f}{v_{xi}}\right)^2\quad\to\quad y_f=\left(\frac{a_y}{2v_{xi}{}^2}\right)x_f{}^2$$

Substitute for the acceleration of the particle in terms of the electric force:

$$y_f = \left(\frac{-eE}{2v_{xi}^{\ 2}\ m_e}\right)x_f^{\ 2}$$

Substitute numerical values, letting the final horizontal position be at the right end of the plates:

$$y_f = \left[\frac{-\left(1.60\times10^{-19}\ \text{C}\right)\left(200\ \text{N/C}\right)}{2\left(3.00\times10^{6}\ \text{m/s}\right)^2\left(9.11\times10^{-31}\ \text{kg}\right)}\right]\left(0.200\ \text{m}\right)^2$$

$$= -0.078\ 1\ \ \text{m}$$

Therefore, when the electron leaves the plates, its final position is well below that of the lower plate, which is at position $y = -1.50$ cm = -0.015 m. Consequently, because we have let the electron enter the field at as high a position as possible, the electron will strike the lower plate long before it reaches the end, regardless of where it enters the field.

P23.75 Charge Q resides on each of the blocks, which repel as point charges:

$$F = \frac{k_e Q^2}{L^2} = k\left(L - L_i\right)$$

Solving for Q, we find

$$Q = L\sqrt{\frac{k\left(L-L_i\right)}{k_e}} = \left(0.500\ \text{m}\right)\sqrt{\frac{\left(100\ \text{N/m}\right)\left(0.500\ \text{m} - 0.400\ \text{m}\right)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}}$$

$$= \boxed{1.67\times10^{-5}\,\text{C}}$$

P23.76 Charge Q resides on each of the blocks, which repel as point charges:

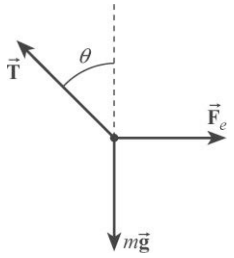
$$F = \frac{k_e Q^2}{L^2} = k\left(L - L_i\right)$$

Solving for Q, we find

$$Q = \boxed{L\sqrt{\frac{k\left(L-L_i\right)}{k_e}}}$$

P23.77 Consider the free-body diagram of the rightmost charge given in ANS. FIG. P23.77. Newton’s second law then gives

$$\sum F_y = 0 \ \Rightarrow\ T\cos\theta = mg \quad \text{or} \quad T = \frac{mg}{\cos\theta}$$



ANS. FIG. P23.77

and

$$\sum F_x = 0$$

$$\Rightarrow\ F_e = T\sin\theta = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta$$

But,

$$F_e = \frac{k_e q^2}{r_1^2} + \frac{k_e q^2}{r_2^2} = \frac{k_e q^2}{(L\sin\theta)^2} + \frac{k_e q^2}{(2L\sin\theta)^2} = \frac{5k_e q^2}{4L^2\sin^2\theta}$$

Thus,

$$\frac{5k_e q^2}{4L^2\sin^2\theta} = mg\tan\theta \text{ or } q = \sqrt{\frac{4L^2 mg\sin^2\theta\tan\theta}{5k_e}}$$

If θ = 45°, m = 0.100 kg, and L = 0.300 m, then

$$q = \sqrt{\frac{4(0.300\text{ m})^2(0.100\text{ kg})(9.80\text{ m/s}^2)\sin^2(45.0°)\tan(45.0°)}{5(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}}$$

or $q = 1.98\times10^{-6}\text{ C} = \boxed{1.98\ \mu\text{C}}$

P23.78 From Example 23.8, the electric field due to a uniformly charged ring is given by

$$E = \frac{k_e Qx}{(x^2+a^2)^{3/2}}$$

For a maximum, we differentiate *E* with respect to *x* and set the result equal to zero:

$$\frac{dE}{dx} = Qk_e\left[\frac{1}{(x^2+a^2)^{3/2}} - \frac{3x^2}{(x^2+a^2)^{5/2}}\right] = 0$$

solving for *x* gives

$$x^2 + a^2 - 3x^2 = 0 \text{ or } x = \frac{a}{\sqrt{2}}$$

Substituting into the expression for *E* gives

$$E = \frac{k_e Qa}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}} = \frac{k_e Q}{3\frac{\sqrt{3}}{2}a^2}$$

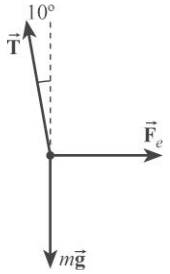
$$= \boxed{\frac{2k_e Q}{3\sqrt{3}a^2}} = \boxed{\frac{Q}{6\sqrt{3}\pi\epsilon_0 a^2}}$$

P23.79 The charges are *q* and *2q*. The magnitude of the repulsive force that one charge exerts on the other is

$$F_e = 2k_e \frac{q^2}{r^2}$$

From Figure P23.79 in the textbook, observe that the distance separating the two spheres is

$$r = d + 2L\sin 10°$$



ANS. FIG. P23.79

From the free-body diagram of one sphere given in ANS. FIG. P23.79, observe that

$$\sum F_y = 0 \Rightarrow T\cos 10° = mg \quad \text{or} \quad T = mg / \cos 10°$$

and

$$\sum F_x = 0 \Rightarrow F_e = T\sin 10° = \left(\frac{mg}{\cos 10°}\right)\sin 10° = mg\tan 10°$$

Thus,

$$2k_e \frac{q^2}{r^2} = mg\tan 10° \quad \rightarrow \quad 2k_e \frac{q^2}{\left(d + 2L\sin 10°\right)^2} = mg\tan 10°$$

or

$$q = \sqrt{\frac{mg\left(d + 2L\sin\theta\right)^2 \tan 10°}{2k_e}}$$

$$= \sqrt{\frac{(0.015\ \text{kg})(9.80\ \text{m/s}^2)\left[0.0300\ \text{m} + 2(0.0500\ \text{m})\sin 10°\right]^2 \tan 10°}{2\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}}$$

$$= 5.69\times 10^{-8}\ \text{C}$$

giving $\boxed{1.14\times 10^{-7}\ \text{C on one sphere and } 5.69\times 10^{-8}\ \text{C on the other.}}$

P23.80 (a) The bowl exerts a normal force on each bead, directed along the radius line at angle θ above the horizontal. Consider the free-body diagram shown in ANS. FIG. P23.80 for the bead on the left side of the bowl:

$$\sum F_y = n\sin\theta - mg = 0 \quad \rightarrow \quad n = \frac{mg}{\sin\theta}$$

Also,

$$\sum F_x = -F_e + n\cos\theta = 0$$

which gives

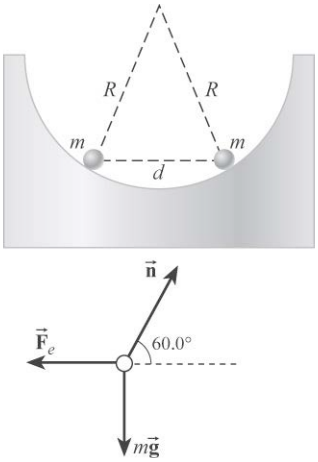
$$F_e = n\cos\theta = \left(\frac{mg}{\sin\theta}\right)\cos\theta = \frac{mg}{\tan\theta}$$

The electric force is

$$F_e = \frac{k_e q^2}{d^2}$$

And from ANS. FIG. P23.80,

$$\tan\theta = \frac{\sqrt{R^2-(d/2)^2}}{(d/2)} = \frac{\sqrt{4R^2-d^2}}{d}$$



ANS. FIG. P23.80

Therefore,

$$F_e = \frac{k_e q^2}{d^2} = \frac{mg}{\tan\theta} = \frac{mg}{\sqrt{4R^2-d^2}/d} \quad \rightarrow \quad q = \boxed{\left(\frac{mgd^3}{k_e\sqrt{4R^2-d^2}}\right)^{1/2}}$$

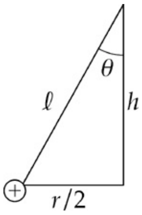
(b) As $d \rightarrow 2R$, $\sqrt{4R^2-d^2} \rightarrow 0$; therefore, $\boxed{q \rightarrow \infty}$.

P23.81 (a) From the 2*Q* charge we have

$$F_e - T_2\sin\theta_2 = 0 \text{ and } mg - T_2\cos\theta_2 = 0$$

Combining these we find

$$\frac{F_e}{mg} = \frac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$$



ANS. FIG. P23.81

From the *Q* charge we have

$$F_e = T_1\sin\theta_1 = 0 \text{ and } mg - T_1\cos\theta_1 = 0$$

Combining these we find

$$\frac{F_e}{mg} = \frac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1 \text{ or } \boxed{\theta_2 = \theta_1}$$

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_eQ^2}{r^2}$ · If we assume θ is small then $\tan\theta \approx \dfrac{r/2}{\ell}$.

Substitute expressions for F_e and tan θ into either equation found in part (a) and solve for *r*. $\dfrac{F_e}{mg} = \tan\theta$, then $\dfrac{2k_eQ^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell}$ and solving for *r* we find $r \approx \left(\dfrac{4k_eQ^2\ell}{mg}\right)^{1/3}$.

P23.82 The field on the axis of the ring is calculated in Example 19.6 in the chapter text as

$$E = E_x = \frac{k_e x Q}{\left(x^2 + a^2\right)^{3/2}}$$

The force experienced by a charge $-q$ placed along the axis of the ring is

$$F = -k_e Q q\left[\frac{x}{\left(x^2 + a^2\right)^{3/2}}\right]$$

and when $x \ll a,$ this becomes

$$F = -\left(\frac{k_e Q q}{a^3}\right)x$$

This expression for the force is in the form of Hooke's law, with an effective spring constant of

$$k = \frac{k_e Q q}{a^3}$$

Since $\omega = 2\pi f = \sqrt{\frac{k}{m}}$, we have

$$f = \boxed{\frac{1}{2\pi}\sqrt{\frac{k_e Q q}{m a^3}}}$$

P23.83 (a) The total non-contact force on the cork ball is:

$$F = qE + mg = m\left(g + \frac{qE}{m}\right)$$

which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$\begin{aligned} T &= 2\pi\sqrt{\frac{L}{g + qE/m}} \\ &= 2\pi\sqrt{\frac{0.500\text{ m}}{9.80\text{ m/s}^2 + \left[\frac{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N/C}\right)}{1.00\times10^{-3}\text{ kg}}\right]}} \\ &= \boxed{0.307\text{ s}} \end{aligned}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get

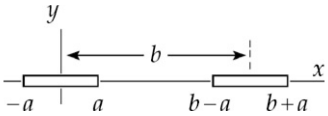
$$T = 2\pi\sqrt{\frac{L}{qE/m}}$$

$$T = 2\pi\sqrt{\frac{0.500\text{ m}}{\left[\frac{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N/C}\right)}{1.00\times10^{-3}\text{ kg}}\right]}} = 0.314\text{ s}$$

(a 2.28% difference).

Challenge Problems

P23.84 According to the result of Example 23.7 in the textbook, the left-hand rod creates this field at a distance *d* from its right-hand end:



ANS FIG. P23.84

$$E = \frac{k_eQ}{d(2a+d)}$$

The force per unit length exerted by the left-hand rod on the right-hand rod is then

$$dF = \frac{k_eQQ}{2a}\frac{dx}{d(d+2a)}$$

Integrating,

$$F = \frac{k_eQ^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x+2a)} = \frac{k_eQ^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a+x}{x}\right)\Bigg|_{b-2a}^{b}$$

$$= \frac{+k_eQ^2}{4a^2}\left(-\ln\frac{2a+b}{b}+\ln\frac{b}{b-2a}\right) = \frac{k_eQ^2}{4a^2}\ln\frac{b^2}{(b-2a)(b+2a)}$$

$$= \boxed{\left(\frac{k_eQ^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2-4a^2}\right)}$$

P23.85 First, we use unit vectors to find the total electric field at point *A* produced by the 7 other charges.

source charge	vector field components	equivalent field
(1) lower left, front:	$\vec{\mathbf{E}}_1 = \frac{k_eq}{r_1^2}\hat{\mathbf{r}}_1 = \frac{k_eq}{s^2+s^2}\frac{\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_eq}{s^2}\left(\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$

(2) lower right, front:	$\vec{\mathbf{E}}_2 = \frac{k_e q}{r_2^2}\hat{\mathbf{r}}_2 = \frac{k_e q}{s^2}\hat{\mathbf{k}}$	$\frac{k_e q}{s^2}\hat{\mathbf{k}}$
(3) lower right, back:	$\vec{\mathbf{E}}_3 = \frac{k_e q}{r_3^2}\hat{\mathbf{r}}_3 = \frac{k_e q}{s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{k}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{k}}\right)$
(4) lower left, back:	$\vec{\mathbf{E}}_4 = \frac{k_e q}{r_4^2}\hat{\mathbf{r}}_4 = \frac{k_e q}{s^2+s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}}{\sqrt{3}}$	$\left(\frac{1}{3\sqrt{3}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$
(5) upper right, back	$\vec{\mathbf{E}}_5 = \frac{k_e q}{r_5^2}\hat{\mathbf{r}}_5 = \frac{k_e q}{s^2}\hat{\mathbf{i}}$	$\frac{k_e q}{s^2}\hat{\mathbf{i}}$
(6) upper left, back	$\vec{\mathbf{E}}_6 = \frac{k_e q}{r_6^2}\hat{\mathbf{r}}_6 = \frac{k_e q}{s^2+s^2}\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}}{\sqrt{2}}$	$\left(\frac{1}{2\sqrt{2}}\right)\frac{k_e q}{s^2}\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}\right)$
(7) upper right, front	$\vec{\mathbf{E}}_7 = \frac{k_e q}{r_7^2}\hat{\mathbf{r}}_7 = \frac{k_e q}{s^2}\hat{\mathbf{j}}$	$\frac{k_e q}{s^2}\hat{\mathbf{j}}$
total field $\vec{\mathbf{E}}_{\text{total}} = \frac{k_e q}{s^2}\left[1+\frac{2}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$		

Notice that because of symmetry, the components of the field have the same magnitude.

(a) At point *A*,

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}}_{\text{total}} = \frac{k_e q^2}{s^2}\left[1+\frac{2}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}\right]\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$$

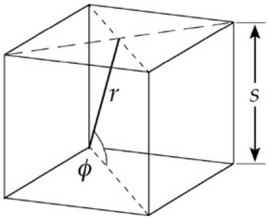
$$= \frac{k_e q^2}{s^2}(1.90)\left(\hat{\mathbf{i}}+\hat{\mathbf{j}}+\hat{\mathbf{k}}\right)$$

$$\rightarrow \boxed{F_x = F_y = F_z = 1.90 k_e \frac{q^2}{s^2}}$$

(b) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\frac{k_e q^2}{s^2}}$

(c) $\boxed{\text{away from the origin}}$

P23.86 (a) Zero contribution from the same face due to symmetry, opposite face contributes



ANS. FIG. P23.86

$$E=4\left(\frac{k_e q}{r^2}\sin\phi\right)$$

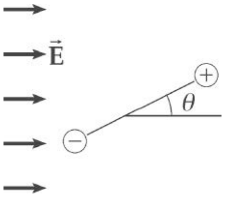
where

$$r=\sqrt{\left(\frac{s}{2}\right)^2+\left(\frac{s}{2}\right)^2+s^2}=\sqrt{1.5}s=1.22s$$

$$\sin\phi=\frac{s}{r},\qquad E=4\frac{k_e qs}{r^3}=\frac{4}{(1.22)^3}\frac{k_e q}{s^2}=\boxed{2.18\frac{k_e q}{s^2}}$$

(b) At the top face, $\boxed{\text{the electric field is in the } \hat{\mathbf{k}} \text{ direction.}}$

P23.87 (a) The electrostatic forces exerted on the two charges result in a net torque



ANS. FIG. P23.87

$$\tau=-2Fa\sin\theta=-2Eqa\sin\theta$$

For small θ, $\sin\theta\approx\theta$ and using $p = 2qa$, we have

$$\tau=-Ep\theta$$

The torque produces an angular acceleration given by

$$\tau=I\alpha=I\frac{d^2\theta}{dt^2}$$

where the moment of inertia of the dipole is $I = 2ma^2$

Combining the two expressions for torque, we have

$$\frac{d^2\theta}{dt^2}=-\left(\frac{Ep}{I}\right)\theta$$

This equation can be written in the form $\frac{d^2\theta}{dt^2}=-\omega^2\theta$ which is the standard equation characterizing simple harmonic motion, with

$$\omega^2=\frac{Ep}{I}=\frac{E(2qa)}{2ma^2}=\frac{qE}{ma}$$

The frequency of oscillation is $f=\omega/2\pi$, so

$$f=\boxed{\frac{1}{2\pi}\sqrt{\frac{qE}{ma}}}$$

(b) If the masses are unequal, the dipole will oscillate about its center of mass (CM). Assume mass m_2 is greater than mass m_1, and treat the *center* of the dipole as being at the origin of an x axis, so that mass m_1 is at $x = -a$ and mass m_2 is at $x = +a$. The coordinate of the CM of the dipole is then

$$x_{\text{cm}} = \frac{m_2 a - m_1 a}{m_1 + m_2} = a\left(\frac{m_2 - m_1}{m_1 + m_2}\right)$$

relative to the center of the dipole. Notice that the moment of inertia of the dipole about its *center* is

$$I = m_1 a^2 + m_2 a^2$$

but its center is a distance x_{cm} from its CM. By the parallel-axis theorem, the moment of inertia of the dipole about its *center* is related to its moment about its CM thus:

$$I = m_1 a^2 + m_2 a^2 = I_{\text{CM}} + (m_1 + m_2) x_{\text{cm}}{}^2$$

therefore,

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - (m_1 + m_2) x_{\text{cm}}{}^2$$

The moment of inertia of the dipole about its CM is then

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - (m_1 + m_2) a^2 \left(\frac{m_2 - m_1}{m_1 + m_2}\right)^2$$

$$I_{\text{CM}} = m_1 a^2 + m_2 a^2 - a^2 \frac{(m_2 - m_1)^2}{(m_1 + m_2)}$$

$$I_{\text{CM}} = \frac{(m_1 + m_2)(m_1 a^2 + m_2 a^2) - (m_2{}^2 a^2 - 2m_1 m_2 a^2 + m_1{}^2 a^2)}{(m_1 + m_2)}$$

$$I_{\text{CM}} = \frac{(m_1{}^2 a^2 + 2m_1 m_2 a^2 + m_2{}^2 a^2) - (m_2{}^2 a^2 - 2m_1 m_2 a^2 + m_1{}^2 a^2)}{(m_1 + m_2)}$$

$$I_{\text{CM}} = \frac{4m_1 m_2 a^2}{(m_1 + m_2)}$$

Therefore, from part (a),

$$\omega^2 = \frac{Ep}{I_{\text{CM}}} = \frac{E(2qa)}{\left[\dfrac{4m_1 m_2 a^2}{(m_1 + m_2)}\right]} = \frac{qE(m_1 + m_2)}{2m_1 m_2 a} = (2\pi f)^2$$

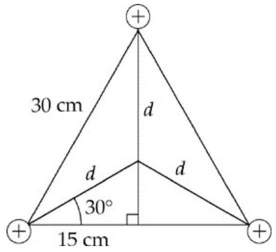
and

$$f = \frac{1}{2\pi}\sqrt{\frac{qE(m_1 + m_2)}{2m_1 m_2 a}}$$

P23.88 From ANS. FIG. P23.88(a) we have

$$d\cos 30.0° = 15.0\text{ cm}$$

or $$d = \frac{15.0\text{ cm}}{\cos 30.0°}.$$



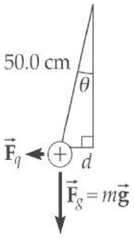
ANS. FIG. P23.88(a)

From ANS. FIG. P23.88(b) we have

$$\theta = \sin^{-1}\left(\frac{d}{50.0\text{ cm}}\right)$$

$$\theta = \sin^{-1}\left(\frac{15.0\text{ cm}}{(50.0\text{ cm})(\cos 30.0°)}\right) = 20.3°$$

$$\frac{F_q}{mg} = \tan\theta \quad\text{or}\quad F_q = mg\tan 20.3° \qquad [1]$$

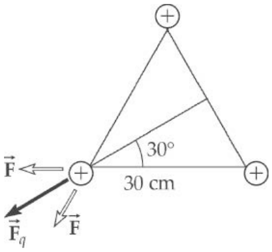


ANS. FIG. P23.88(b)

From ANS. FIG. P23.88(c) we have

$$F_q = 2F\cos 30.0°$$

$$F_q = 2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0° \qquad [2]$$



ANS. FIG. P23.88(c)

Combining equations [1] and [2],

$$2\left[\frac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0° = mg\tan 20.3°$$

$$q^2 = \frac{mg(0.300\text{ m})^2\tan 20.3°}{2k_e\cos 30.0°}$$

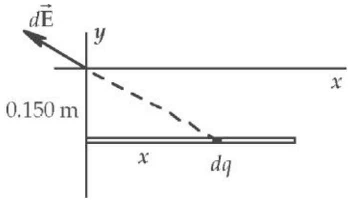
$$q^2 = \frac{(2.00\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)(0.300\text{ m})^2\tan 20.3°}{2(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\cos 30.0°}$$

$$q = \sqrt{4.20\times 10^{-14}\text{ C}^2} = 2.05\times 10^{-7}\text{ C} = \boxed{0.205\ \mu\text{C}}$$

P23.89

$$d\vec{\mathbf{E}} = \frac{k_e dq}{x^2 + (0.150\text{ m})^2}\left(\frac{-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right)$$

$$= \frac{k_e\lambda\left(-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$



ANS. FIG. P23.89

$$\vec{\mathbf{E}} = \int_{\text{all charge}} d\vec{\mathbf{E}} = k_e\lambda\int_{x=0}^{0.400\text{ m}} \frac{\left(-x\hat{\mathbf{i}} + 0.150\text{ m}\hat{\mathbf{j}}\right)dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$

$$\vec{\mathbf{E}} = k_e\lambda\left[\left.\frac{+\hat{\mathbf{i}}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\hat{\mathbf{j}}x}{(0.150\text{ m})^2\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\vec{\mathbf{E}} = \left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(35.0\times 10^{-9}\text{ C/m}\right)$$
$$\times\left[\hat{\mathbf{i}}(2.34 - 6.67)\text{ m}^{-1} + \hat{\mathbf{j}}(6.24 - 0)\text{ m}^{-1}\right]$$

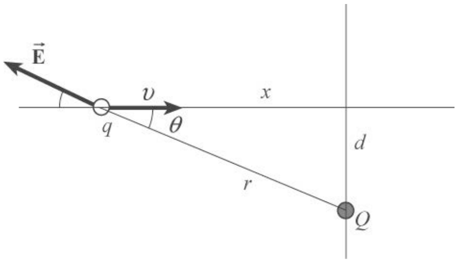
$$\vec{\mathbf{E}} = \left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)\times 10^3\text{ N/C} = \boxed{\left(-1.36\hat{\mathbf{i}} + 1.96\hat{\mathbf{j}}\right)\text{ kN/C}}$$

P23.90 We work under the assumption that v_x has the nearly constant value v. Initially, with the particle nearly at infinity, $v_x = v$ and $v_y = 0$. As the moving charge travels toward and passes the fixed charge Q, the velocity component v_y increases according to

$$m\frac{dv_y}{dt} = F_y$$

or

$$m\frac{dv_y}{dx}\frac{dx}{dt} = qE_y$$



ANS. FIG. P23.90

Now $\dfrac{dx}{dt} = v_x$ has the nearly constant value v; therefore, we have

$$dv_y = \frac{q}{mv}E_y dx \quad\rightarrow\quad v_y = \int_0^{v_y} dv_y = \frac{q}{mv}\int_{-\infty}^{\infty} E_y dx$$

The radially outward component of the electric field varies along the x axis. We assume that the distance r between charges is does not depend significantly on y.

From the figure, we see that $E = \frac{k_e Q}{r^2}$, $r \approx \sqrt{d^2 + x^2}$, $E_y = E\sin\theta$, and $\sin\theta \approx \frac{d}{\sqrt{d^2 + x^2}}$. We evaluate the integral from above:

$$\begin{aligned}\int_{-\infty}^{\infty} E_y\,dx &= \int_{-\infty}^{\infty} E\sin\theta\,dx \approx \int_{-\infty}^{\infty} \frac{k_e Q}{\left(d^2 + x^2\right)}\frac{d}{\sqrt{d^2 + x^2}}\,dx \\ &= k_e Q d\int_{-\infty}^{\infty} \frac{dx}{\left(d^2 + x^2\right)^{3/2}} \\ &= \left(k_e Q d\right)\left(\frac{x}{d^2\left(d^2 + x^2\right)^{1/2}}\right)\Bigg|_{-\infty}^{\infty} = \frac{k_e Q d}{d^2}\left[1 - (-1)\right] = \frac{2k_e Q}{d}\end{aligned}$$

So, the v_y is

$$v_y = \frac{q}{mv}\int_{-\infty}^{\infty} E_y\,dx = \frac{q}{mv}\left(\frac{2k_e Q}{d}\right) = \frac{2k_e qQ}{mvd}$$

The angle of deflection is described by

$$\tan\theta = \frac{v_y}{v_x} \approx \frac{v_y}{v} = \frac{2k_e qQ}{mv^2 d} \quad \rightarrow \quad \theta = \boxed{\tan^{-1}\frac{2k_e qQ}{mv^2 d}}$$

P23.91 (a) The two charges create fields of equal magnitude, both with outward components along the *x* axis and with upward and downward *y* components that add to zero. The net field is then

$$\begin{aligned}\vec{\mathbf{E}} &= \frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} + \frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} = 2\frac{k_e q}{r^2}\frac{x}{r}\hat{\mathbf{i}} \\ &= \frac{2(8.99\times 10^9)\left(52\times 10^{-9}\right)x\ \hat{\mathbf{i}}}{\left[(0.25)^2 + x^2\right]^{3/2}}\end{aligned}$$

$$\boxed{\vec{\mathbf{E}} = \frac{935x}{\left(0.0625 + x^2\right)^{3/2}}\hat{\mathbf{i}} \text{ where } \vec{\mathbf{E}} \text{ is in newtons per coulomb and } x \text{ is in meters.}}$$

(b) At $x = 0.36$ m,

$$\vec{\mathbf{E}} = \frac{935(0.36)\ \hat{\mathbf{i}}}{(0.0625 + (0.36)^2)^{3/2}} = \boxed{4.00 \text{ kN/C}\ \hat{\mathbf{i}}}$$

(c) We solve $1\,000 = (935\,x)(0.0625 + x^2)^{-3/2}$ by tabulating values for the field function:

x	$(935\,x)(0.0625 + x^2)^{-3/2}$
0	0
0.01	597
0.02	1 185
0.1	4 789
0.2	5 698
0.36	4 000
0.9	1 032
1	854
∞	0

We see that there are two points where E = 1 000 N / C. We home in on them to determine their coordinates as (to three digits)

$\boxed{x = 0.016\,8 \text{ m} \text{ and } x = 0.916 \text{ m.}}$

(d) The table in part (c) shows that

$\boxed{\text{nowhere is the field so large as 16 000 N/C.}}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P23.2 (a) 2.62×10^{24}; (b) 2.38 electrons for every 10^9 already present

P23.4 $1.57\,\mu$N to the left

P23.6 (a) 9.21×10^{-10} N; (b) No. The electric force depends only on the magnitudes of the two charges and the distance between them.

P23.8 $\sim 10^{26}$ N

P23.10 (a) 1.59×10^{-9} N; (b) 1.29×10^{-45} N, larger by 1.24×10^{36} times; (c) 8.61×10^{-11} C / kg

P23.12 (a) 46.7 N to the left; (b) 157 N to the right; (c) 111 N to the left

P23.14 (a) $\dfrac{\sqrt{q_1}}{\sqrt{q_1} + \sqrt{q_2}}d$; (b) Yes, if the third bead has a positive charge.

P23.16 0.229 m

P23.18 (a) 0; (b) 30.0 N; (c) 21.6 N; (d) 17.3 N; (e) –13.0 N; (f) 17.3 N; (g) 17.0 N; (h) 24.3 N at 44.5° above the +*x* direction

P23.20 (a) The acceleration of the charge is equal to a negative constant times its displacement from equilibrium, as in $\vec{\mathbf{a}} = -\omega^2\vec{\mathbf{x}}$, so we have Simple Harmonic Motion with $\omega^2 = \dfrac{16k_e qQ}{md^3}$; (b) $\dfrac{\pi}{2}\sqrt{\dfrac{md^3}{k_e qQ}}$; (c) $4a\sqrt{\dfrac{k_e qQ}{md^3}}$

P23.22 The unknown charge on each dust particle is about half of the smallest possible free charge, the charge of the electron. No such free charge exists. Therefore, the forces cannot balance.

P23.24 2.07×10^3 N / C; down

P23.26 $-k_e \dfrac{3q}{r^2}\hat{\mathbf{j}}$

P23.28 (a) $\dfrac{k_e Q x\hat{\mathbf{i}}}{\left(a^2 + x^2\right)^{3/2}}$; (b) A circle of charge corresponds to letting *n* grow beyond all bounds, but the result does not depend on *n*. Because of the symmetrical arrangement of the charges, smearing the charge around the circle does not change its amount or its distance from the field point, so it does not change the field.

P23.30 (a) $\left(18.0\hat{\mathbf{i}} - 218\hat{\mathbf{j}}\right)$ kN / C; (b) $\left(0.0360\hat{\mathbf{i}} - 0.436\hat{\mathbf{j}}\right)$ N

P23.32 The field at the origin can be to the right, if the unknown charge is –9*Q*, or the field can be to the left, if and only if the unknown charge is +27*Q*.

P23.34 (a) $1.29 \times 10^4 \hat{\mathbf{j}}$ N/C; (b) $-3.86 \times 10^{-2} \hat{\mathbf{j}}$ N

P23.36 $\dfrac{4a\left(k_e q\right)}{x^3}$

P23.38 (a) 383 MN/C; (b) 324 MN/C; (c) 80.7 MN/C; (d) 6.68 MN/C

P23.40 $E_x \approx \dfrac{k_e Q}{x^2}$ for a disk at large distances

P23.42 (a) $-k_e \dfrac{Q}{L}\left[\dfrac{1}{d} - \dfrac{1}{\left(d^2 + L^2\right)^{1/2}}\right]$ and $k_e \dfrac{Q}{d} \dfrac{1}{\left(d^2 + L^2\right)^{1/2}}$; (b) $E_x \approx 0$ and

$E_y \approx k_e \dfrac{Q}{d^2}$ which is the field of a point charge *Q* at a distance *d* along the *y* axis above the charge.

P23.44 (a) $\dfrac{2k_e \lambda \sin\theta_0}{d}$; (b) $\dfrac{2k_e \lambda}{d}$

P23.46 (a) $\dfrac{k_e Q \hat{\mathbf{i}}}{h}\left[\dfrac{1}{\left(d^2 + R^2\right)^{1/2}} - \dfrac{1}{\left(\left(d + h\right)^2 + R^2\right)^{1/2}}\right]$

(b) $\dfrac{2k_e Q \hat{\mathbf{i}}}{R^2 h}\left[h + \left(d^2 + R^2\right)^{1/2} - \left(\left(d + h\right)^2 + R^2\right)^{1/2}\right]$

P23.48 See ANS. FIG. P23.48.

P23.50 (a) See ANS. FIG. P23.50; (b) At the center; (c) $1.73 k_e \dfrac{q}{a^2} \hat{\mathbf{j}}$

P23.52 (a) $-5.76 \times 10^{13} \hat{\mathbf{i}}$ m/s^2; (b) $\vec{v}_i = 2.84 \times 10^6 \hat{\mathbf{i}}$ m/s; (c) 4.93 × 10^{-8} s

P23.54 (a) Particle under constant velocity; (b) Particle under constant acceleration; (c) the proton moves in a parabolic path just like a projectile in a gravitational field; (d) $\dfrac{m_p v_i^2 \sin 2\theta}{eE}$; (e) 36.9° or 53.1°; (f) 166 ns or 221 ns

P23.56 (a) a parabola; (b) the negative plate; (c) The particle strikes the negative plate after moving a horizontal distance of 0.961 mm.

P23.58 (a) 2.00 × 10^{-10} C; (b) 1.41 × 10^{-10} C; (c) 5.89 × 10^{-11} C

P23.60	5.81 nC
P23.62	49.3 N / m
P23.64	(a) 0.115 N; (b) 1.25 cm
P23.66	2.51×10^{-9}
P23.68	(a) $q = \dfrac{mg}{(A\cot\theta + B)}$; (b) $T = \dfrac{mgA}{(A\cos\theta + B\sin\theta)}$
P23.70	(a) It is possible in just one way; (b) $x = -16.0$ cm; (c) $+51.3\mu$C
P23.72	(a) 40.9 N; (b) 263°
P23.74	See P23.74 for complete solution
P23.76	$L\sqrt{\dfrac{k(L-L_i)}{k_e}}$
P23.78	$\dfrac{2k_eQ}{3\sqrt{3}a^2} = \dfrac{Q}{6\sqrt{3}\pi \in_0 a^2}$
P23.80	(a) $\left(\dfrac{mgd^3}{k_e\sqrt{4R^2 - d^2}}\right)^{1/2}$; (b) $q \to \infty$
P23.82	$\dfrac{1}{2\pi}\sqrt{\dfrac{k_eQq}{ma^3}}$
P23.84	See P23.84 for full solution
P23.86	(a) $2.18\dfrac{k_eq}{s^2}$; (b) the direction is the $\hat{\mathbf{k}}$ direction
P23.88	$0.205\mu C$
P23.90	$\theta = \tan^{-1}\left(\dfrac{2k_eqQ}{mv^2d}\right)$

24

Gauss's Law

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ24.1 (i) Answer (a). The field is cylindrically radial to the filament, and is nowhere zero at any face of the gaussian surface.

(ii) Answer (b). The flux is zero through the two faces pierced by the filament because the field is parallel to those surfaces.

OQ24.2 Answer (c). The outer wall of the conducting shell will become polarized to cancel out the external field. The interior field is the same as before.

OQ24.3 Answer (e). The symmetry of a charge distribution and of its field is the same. Gauss's law applies to these charge distributions because (a) has cylindrical symmetry, (b) has translational symmetry, (c) has spherical symmetry, and (d) has spherical symmetry.

OQ24.4 (i) Answer (c). Equal amounts of flux pass through each of the six faces of the cube.

(ii) Answer (b). Move the charge to very close below the center of one face, so that half the flux passes through that face and half the flux passes through the other five faces.

OQ24.5 Answer (b). The electric flux through a closed surface equals q/ϵ_0, where *q* is the total charge contained within the surface:

$$\begin{aligned} q/\epsilon_0 &= \left[(3.00 - 2.00 - 7.00 + 1.00)\times 10^{-9}\text{ C}\right] \\ &\qquad /\left(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right) \\ &= -5.65\times 10^{-2}\text{ N}\cdot\text{m}^2/\text{C} \end{aligned}$$

OQ24.6 **(i)** Answer (e). The shell becomes polarized.

(ii) Answer (a). The net charge on the shell's inner and outer surfaces is zero.

(iii) Answer (c). The charge has been transferred to the outer surface of the conductor.

(iv) Answer (c). The charge has been transferred to the outer surface of the conductor.

(v) Answer (a). The charge has been transferred to the outer surface of the conductor.

OQ24.7 **(i)** Answer (c). Because the charge distributions are spherically symmetric, both spheres create equal fields at exterior points, like particles at the centers of the spheres.

(ii) Answer (e). The field within the conductor is zero. The field a distance *r* from the center of the insulator is proportional to *r*, so it is 4/5 of its value at the surface.

OQ24.8 Answer (c). The electric field inside a conductor is zero.

OQ24.9 (a) The ranking is A > B > D > C. Let *q* represent the charge of the insulating sphere. The field at A is $(4/5)^3 q/[4\pi(4\text{ cm})^2\epsilon_0]$. The field at B is $q/[4\pi(8\text{ cm})^2\epsilon_0]$. The field at C is zero. The field at D is $q/[4\pi(16\text{ cm})^2\epsilon_0]$.

(b) The ranking is B = D > A > C. The flux through the 4-cm sphere is $(4/5)^3 q/\epsilon_0$. The flux through the 8-cm sphere and through the 16-cm sphere is q/ϵ_0 because they enclose the same amount of charge. The flux through the 12-cm sphere is 0 because the field is zero inside the conductor.

OQ24.10 **(i)** Answer (a). The field is perpendicular to the sheet, and is nowhere zero at any face of the gaussian surface.

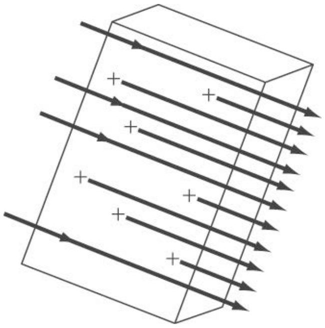
(ii) Answer (c). The flux is nonzero through the top and bottom faces because the field is perpendicular to them, and zero through the other four faces because the field is parallel to them.

OQ24.11 The ranking is C > A = B > D. The total flux is proportional to the enclosed charge: $3Q > Q = Q > 0$.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ24.1 (a) If the volume charge density is nonzero, the field cannot be uniform in magnitude. Consider a gaussian surface in the shape of a rectangular box with two faces perpendicular to the direction of the field. It encloses some charge, so the net flux out of the box is nonzero. The field must be stronger on one side than on the other. The field cannot be uniform in magnitude.

(b) Now the volume contains no charge. The net flux out of the box is zero. The flux entering is equal to the flux exiting. The field must be uniform in magnitude along any line in the direction of the field. It can vary between points in a plane perpendicular to the field lines.



ANS. FIG. CQ24.1

CQ24.2 The electric flux through a closed surface is proportional to the total charge contained within the surface: (a) the flux is doubled because the charge is doubled, (b) the flux remains the same because the charge is the same, (c) the flux remains the same because the charge is the same, (d) the flux remains the same because the charge is the same, (e) the flux becomes zero because the charge inside the surface is zero.

CQ24.3 The net flux through any gaussian surface is zero. We can argue it two ways. Any surface contains zero charge, so Gauss's law says the total flux is zero. The field is uniform, so the field lines entering one side of the closed surface come out the other side and the net flux is zero.

CQ24.4 Gauss's law cannot be used to find the electric field at different points on a surface if the field is not constant over that surface. If the symmetry of an electric field allows us to say that $\int E\cos\theta\, dA = E\int\cos\theta\, dA$, where E is an unknown *constant* on the surface, then we can use Gauss's law. When electric field is a general unknown function $E(x, y, z)$, there can be no such simplification.

CQ24.5 The electric flux is independent of the size and shape of the closed surface that contains the charge because all the field lines from the enclosed charge pass through the surface.

CQ24.6 The surface must enclose a positive total charge. Field lines emerge from positive charge and disappear into negative charge.

CQ24.7 (a) No. If the person is uncharged, the electric field inside the sphere is zero. The interior wall of the shell carries no charge. The person is not harmed by touching this wall.

(b) If the person carries a (small) charge q, the electric field inside the sphere is no longer zero. Charge $-q$ is induced on the inner wall of the sphere. The person will get a (small) shock when touching the sphere, as all the charge on his body jumps to the metal.

CQ24.8 The sphere with large charge creates a strong field to polarize the other sphere. That means it pushes the excess like charge over to the far side, leaving charge of the opposite sign on the near side. This patch of opposite charge is smaller in amount but located in a stronger external field, so it can feel a force of attraction that is larger than the repelling force felt by the larger charge in the weaker field on the other side.

CQ24.9 There is zero force. The huge charged sheet creates a uniform field. The field can polarize the neutral sheet, creating in effect a film of opposite charge on the near face and a film with an equal amount of like charge on the far face of the neutral sheet. Since the field is uniform, the films of charge feel equal-magnitude forces of attraction and repulsion to the charged sheet. The forces add to zero.

CQ24.10 Inject some charge at arbitrary places within a conducting object. Every bit of the charge repels every other bit, so each bit runs away as far as it can, stopping only when it reaches the outer surface of the conductor.

CQ24.11 (a) The luminous flux on a given area is less when the sun is low in the sky, because the angle between the rays of the sun and the local area vector, $d\vec{\mathbf{A}}$, is greater than zero. The cosine of this angle is reduced.

(b) The decreased flux results, on the average, in colder weather.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 24.1 Electric Flux

P24.1 For a uniform electric field passing through a plane surface, $\Phi_E = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta$, where θ is the angle between the electric field and the normal to the surface.

(a) The electric field is perpendicular to the surface, so $\theta = 0°$:

$$\Phi_E = \left(6.20 \times 10^5 \text{ N/C}\right)\left(3.20 \text{ m}^2\right)\cos 0°$$

$$\Phi_E = \boxed{1.98 \times 10^6 \text{ N} \cdot \text{m}^2/\text{C}}$$

(b) The electric field is parallel to the surface: $\theta = 90°$, so $\cos\theta = 0$, and the flux is $\boxed{\text{zero.}}$

P24.2 The electric flux through the bottom of the car is given by

$$\Phi_E = EA\cos\theta = \left(2.00 \times 10^4 \text{ N/C}\right)(3.00 \text{ m})(6.00 \text{ m})\cos 10.0°$$

$$= \boxed{355 \text{ kN} \cdot \text{m}^2/\text{C}}$$

P24.3 For a uniform field the flux is $\Phi = \vec{\mathbf{E}} \cdot \vec{\mathbf{A}} = EA\cos\theta$.

The maximum value of the flux occurs when $\theta = 0$, or when the field is in the same direction as the area vector, which is defined as having the direction of the perpendicular to the area. Therefore, we can calculate the field strength at this point as

$$E = \frac{\Phi_{\text{max}}}{A} = \frac{\Phi_{\text{max}}}{\pi r^2}$$

$$E = \frac{5.20 \times 10^5 \text{ N} \cdot \text{m}^2/\text{C}}{\pi (0.200 \text{ m})^2} = 4.14 \times 10^6 \text{ N/C} = \boxed{4.14 \text{ MN/C}}$$

P24.4 (a) For the vertical rectangular surface, the area (shown as A' in ANS FIG. P24.4) is

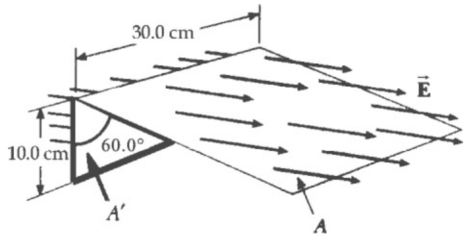
$$A' = (10.0 \text{ cm})(30.0 \text{ cm}) = 300 \text{ cm}^2 = 0.030\,0 \text{ m}^2$$

Since the electric field is perpendicular to the surface and is directed inward, $\theta = 180°$ and

$$\Phi_{E,A'} = EA'\cos\theta$$

$$\Phi_{E,A'} = \left(7.80 \times 10^4 \text{ N/C}\right)\left(0.030\,0 \text{ m}^2\right)\cos 180°$$

$$\Phi_{E,A'} = \boxed{-2.34 \text{ kN} \cdot \text{m}^2/\text{C}}$$



ANS. FIG. P24.4

(b) To find the area *A* of the slanted surface, we note that the side for which dimensions are not given has length (10.0 cm) $= w\cos 60.0°$, so that

$$A = (30.0\text{ cm})(w) = (30.0\text{ cm})\left(\frac{10.0\text{ cm}}{\cos 60.0°}\right) = 600\text{ cm}^2$$
$$= 0.060\,0\text{ m}^2$$

The flux through this surface is then

$$\Phi_{E,A} = EA\cos\theta = (7.80\times10^4)(A)\cos 60.0°$$
$$= (7.80\times10^4\text{ N/C})(0.060\,0\text{ m}^2)\cos 60.0°$$
$$= \boxed{+2.34\text{ kN}\cdot\text{m}^2/\text{C}}$$

(c) The bottom and the two triangular sides all lie *parallel* to $\vec{\mathbf{E}}$, so $\Phi_E = 0$ for each of these. Thus,

$$\Phi_{E,\text{ total}} = -2.34\text{ kN}\cdot\text{m}^2/\text{C} + 2.34\text{ kN}\cdot\text{m}^2/\text{C} + 0 + 0 + 0 = \boxed{0}$$

P24.5 For a uniform electric field passing through a plane surface, $\Phi_E = \vec{\mathbf{E}}\cdot\vec{\mathbf{A}} = EA\cos\theta$, where θ is the angle between the electric field and the normal to the surface.

(a) The electric field is perpendicular to the surface, so $\theta = 0°$:

$$\Phi_E = (3.50\times10^3\text{ N/C})[(0.350\text{ m})(0.700\text{ m})]\cos 0°$$
$$= \boxed{858\text{ N}\cdot\text{m}^2/\text{C}}$$

(b) The electric field is parallel to the surface: $\theta = 90°$, so $\cos\theta = 0$, and the flux is $\boxed{\text{zero}}$.

(c) For the specified plane,

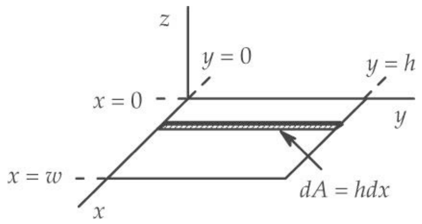
$$\Phi_E = (3.50\times10^3\text{ N/C})[(0.350\text{ m})(0.700\text{ m})]\cos 40.0°$$
$$= \boxed{657\text{ N}\cdot\text{m}^2/\text{C}}$$

P24.6 We are given an electric field in the general form

$$\vec{\mathbf{E}} = ay\hat{\mathbf{i}} + bz\hat{\mathbf{j}} + cx\hat{\mathbf{k}}$$

In the *xy* plane, $z = 0$ so that the electric field reduces to

$$\vec{\mathbf{E}} = ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}$$



ANS. FIG. P24.6

To obtain the flux, we integrate (see ANS. FIG. P24.6 for the definition of *dA*):

$$\Phi_E = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \int \left(ay\hat{\mathbf{i}} + cx\hat{\mathbf{k}}\right) \cdot \hat{\mathbf{k}}\, dA$$

$$\Phi_E = ch \int_{x=0}^{w} x\, dx = ch \left.\frac{x^2}{2}\right|_{x=0}^{w} = \boxed{\frac{chw^2}{2}}$$

Where the $\hat{\mathbf{k}}$ term was eliminated since $\hat{\mathbf{k}} \cdot \hat{\mathbf{k}} = 0$.

Section 24.2 Gauss's Law

P24.7 The electric flux through the hole is given by Gauss's Law (Equation 24.6) as

$$\begin{aligned}\Phi_{E,\text{ hole}} &= \vec{\mathbf{E}} \cdot \vec{\mathbf{A}}_{\text{hole}} = \left(\frac{k_e Q}{R^2}\right)\left(\pi r^2\right) \\ &= \left(\frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(10.0 \times 10^{-6} \text{ C}\right)}{(0.100 \text{ m})^2}\right) \\ &\qquad \times \pi\left(1.00 \times 10^{-3} \text{ m}\right)^2 \\ &= \boxed{28.2 \text{ N} \cdot \text{m}^2/\text{C}}\end{aligned}$$

P24.8 The gaussian surface encloses the +1.00-nC and –3.00-nC charges, but not the +2.00-nC charge. The electric flux is therefore

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{\left(1.00 \times 10^{-9} \text{ C} - 3.00 \times 10^{-9} \text{ C}\right)}{8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2} = \boxed{-226 \text{ N} \cdot \text{m}^2/\text{C}}$$

P24.9 The total charge within the closed surface is

$$5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C} = -61.0\ \mu\text{C}$$

(a) So, from Equation 24.6, the total electric flux is

$$\Phi_E = \frac{q}{\epsilon_0} = \frac{-61.0\times 10^{-6}\ \text{C}}{\left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{-6.89\ \text{MN}\cdot\text{m}^2/\text{C}}$$

(b) $\boxed{\text{Since the net electric flux is negative, more lines enter than leave the surface.}}$

P24.10 (a) From $E = \dfrac{k_e Q}{r^2}$,

$$Q = \frac{Er^2}{k_e} = \frac{\left(8.90\times 10^2\ \text{N/C}\right)\left(0.750\ \text{m}\right)^2}{\left(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)} = 5.57\times 10^{-8}\ \text{C}$$

But Q is negative since $\vec{\mathbf{E}}$ points inward, so

$$Q = -5.57\times 10^{-8}\ \text{C} = \boxed{-55.7\ \text{nC}}$$

(b) The $\boxed{\text{negative}}$ charge has a $\boxed{\text{spherically symmetric}}$ charge distribution, concentric with the spherical shell.

P24.11 The electric flux through each of the surfaces is given by $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0}$.

Flux through S_1: $\Phi_E = \dfrac{-2Q+Q}{\epsilon_0} = \boxed{-\dfrac{Q}{\epsilon_0}}$

Flux through S_2: $\Phi_E = \dfrac{+Q-Q}{\epsilon_0} = \boxed{0}$

Flux through S_3: $\Phi_E = \dfrac{-2Q+Q-Q}{\epsilon_0} = \boxed{-\dfrac{2Q}{\epsilon_0}}$

Flux through S_4: $\Phi_E = \boxed{0}$

P24.12 The total flux through the surface of the cube is

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{170\times 10^{-6}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.92\times 10^7\ \text{N}\cdot\text{m}^2/\text{C}$$

(a) By symmetry, the flux through each face of the cube is the same.

$$\left(\Phi_E\right)_{\text{one face}} = \frac{1}{6}\Phi_E = \frac{1}{6}\frac{q_{\text{in}}}{\epsilon_0}$$

$$(\Phi_E)_{\text{one face}} = \frac{1}{6}\left(\frac{170\times 10^{-6}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right)$$

$$= \boxed{3.20\times 10^{6}\ \text{N}\cdot\text{m}^2/\text{C}}$$

(b) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} = \left(\dfrac{170\times 10^{-6}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}\right) = \boxed{1.92\times 10^{7}\ \text{N}\cdot\text{m}^2/\text{C}}$

(c) The answer to part (a) would change because the charge could now be at different distances from each face of the cube. The answer to part (b) would be unchanged because the flux through the entire closed surface depends only on the total charge inside the surface.

P24.13 Consider as a gaussian surface a box with horizontal area *A*, lying between 500 and 600 m elevation. From Gauss's Law,

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}:$$

$$(+120\ \text{N/C})A + (-100\ \text{N/C})A = \frac{\rho A(100\ \text{m})}{\epsilon_0}$$

$$\rho = \frac{(20.0\ \text{N/C})(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{100\ \text{m}} = \boxed{1.77\times 10^{-12}\ \text{C/m}^3}$$

The charge is $\boxed{\text{positive}}$, to produce the net outward flux of electric field.

P24.14 (a) The total electric flux through the surface of the shell is

$$\Phi_{E,\,\text{shell}} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{12.0\times 10^{-6}}{8.85\times 10^{-12}} = 1.36\times 10^{6}\ \text{N}\cdot\text{m}^2/\text{C}$$

$$= \boxed{1.36\ \text{MN}\cdot\text{m}^2/\text{C}}$$

(b) Through any hemispherical urface of the shell, by symmetry,

$$\Phi_{E,\,\text{half shell}} = \frac{1}{2}(1.36\times 10^{6}\ \text{N}\cdot\text{m}^2/\text{C}) = 6.78\times 10^{5}\ \text{N}\cdot\text{m}^2/\text{C}$$

$$= \boxed{678\ \text{kN}\cdot\text{m}^2/\text{C}}$$

(c) $\boxed{\text{No}}$, the same number of field lines will pass through each surface, no matter how the radius changes.

P24.15 (a) The gaussian surface encloses a charge of +3.00 nC.

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{3.00\times 10^{-9}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 339\ \text{N}\cdot\text{m}^2/\text{C}$$

(b) $\boxed{\text{No. The electric field is not uniform on this surface. Gauss's law is only practical to use when all portions of the surface satisfy one or more of the conditions listed in Section 24.3.}}$

P24.16 (a) One-half of the total flux created by the charge *q* goes through the plane. Thus,

$$\Phi_{E,\text{ plane}} = \frac{1}{2}\Phi_{E,\text{ total}} = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \boxed{\frac{q}{2\epsilon_0}}$$

(b) The square looks like an infinite plane to a charge *very close* to the surface. Hence,

$$\Phi_{E,\text{ square}} \approx \Phi_{E,\text{ plane}} = \boxed{\frac{q}{2\epsilon_0}}$$

(c) $\boxed{\text{The plane and the square look the same to the charge.}}$

P24.17 (a) If $R \le d$, the sentence encloses no charge and $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \boxed{0}$.

(b) If $R > d$, the length of line falling within the sphere is $2\sqrt{R^2 - d^2}$

so $\Phi_E = \boxed{\frac{2\lambda\sqrt{R^2 - d^2}}{\epsilon_0}}$

P24.18 (a) $\boxed{\text{The net flux is zero through the sphere because the number of field lines entering the sphere equals the number of lines leaving the sphere.}}$

(b) The electric field through the curved side of the cylinder is zero because the field lines are parallel to that surface and do not pass through it. The electric field lines pass outward through the ends of the cylinder, so both have a positive flux. Because the field is uniform, the flux is $\pi R^2 E$ for each end.

$\boxed{\text{The net flux is } 2\pi R^2 E \text{ through the cylinder.}}$

(c) The net flux is positive, so the charge in the cylinder is positive. To be a uniform field, the field lines must originate from a plane of charge. $\boxed{\text{The net charge inside the cylinder is positive and is distributed on a plane parallel to the ends of the cylinder.}}$

P24.19 The total charge is $Q - 6|q|$. The total outward flux from the cube is $\frac{Q-6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \frac{Q-6|q|}{6\epsilon_0}$$

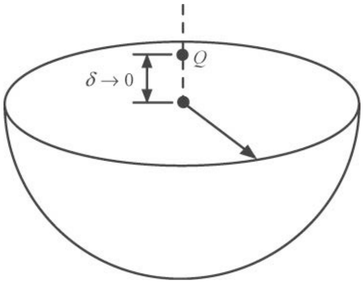
$$(\Phi_E)_{\text{one face}} = \frac{Q-6|q|}{6\epsilon_0} = \frac{(5.00-6.00)\times 10^{-6}\ \text{C}\cdot\text{N}\cdot\text{m}^2}{6\times 8.85\times 10^{-12}\ \text{C}^2}$$

$$= \boxed{-18.8\ \text{kN}\cdot\text{m}^2/\text{C}}$$

P24.20 The total charge is $Q - 6|q|$. The total outward flux from the cube is $\frac{Q-6|q|}{\epsilon_0}$, of which one-sixth goes through each face:

$$(\Phi_E)_{\text{one face}} = \boxed{\frac{Q-6|q|}{6\epsilon_0}}$$

P24.21 (a) With δ very small, all points on the hemisphere are nearly at a distance *R* from the charge, so the field everywhere on the curved surface is $\frac{k_e Q}{R^2}$ radially outward (normal to the surface). Therefore, the flux is this field strength times the area of half a sphere:



ANS. FIG. P24.21

$$\Phi_{\text{curved}} = \int \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = E_{\text{local}}A_{\text{hemisphere}}$$

$$\Phi_{\text{curved}} = \left(k_e\frac{Q}{R^2}\right)\left(\frac{1}{2}4\pi R^2\right) = \frac{1}{4\pi\epsilon_0}Q(2\pi) = \boxed{\frac{+Q}{2\epsilon_0}}$$

(b) The closed surface encloses zero charge so Gauss's law gives

$$\Phi_{\text{curved}} + \Phi_{\text{flat}} = 0 \quad \text{or} \quad \Phi_{\text{flat}} = -\Phi_{\text{curved}} = \boxed{\frac{-Q}{2\epsilon_0}}$$

P24.22 For uniform electric field lines passing through a flat surface, the electric flux is $\Phi_E = EA\cos\theta$, where θ is the angle between the electric field vector and the normal to the surface.

(a) $(\Phi_E)_{\text{face 1}} = \boxed{EA\cos\theta}$

(b) The normal points to the right; the angle between the electric field and the normal is 90° + θ:

$$\left(\Phi_E\right)_{\text{face 2}} = EA\cos\left(90° + \theta\right) = \boxed{-EA\sin\theta}$$

(c) The normal points downward in the figure, the angle between the electric field and the normal is 180° – θ:

$$\left(\Phi_E\right)_{\text{face 3}} = EA\cos\left(180° - \theta\right) = \boxed{-EA\cos\theta}$$

(d) The normal points to the left; the angle between the electric field and the normal is 90° – θ:

$$\left(\Phi_E\right)_{\text{face 4}} = EA\cos\left(90° - \theta\right) = \boxed{EA\sin\theta}$$

(e) The normal points in or out of the page; the angle between the electric field and the normal is 90°:

$$\left(\Phi_E\right)_{\text{top or bottom}} = EA\cos\left(90°\right) = \boxed{0}$$

(f) $\Phi_E = \sum\left(\Phi_E\right)_{\text{faces}} = EA\cos\theta - EA\sin\theta - EA\cos\theta + EA\sin\theta + 0 + 0 = \boxed{0}$

(g) $\Phi_E = \dfrac{q_{\text{in}}}{\epsilon_0} \to q_{\text{in}} = \boxed{0}$

Section 24.3 Application of Gauss's Law to Various Charge Distributions

***P24.23** The distance between centers is $2 \times 5.90 \times 10^{-15}$ m. Each produces a field as if it were a point charge at its center, and each feels a force as if all its charge were a point at its center.

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{(46)^2\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(2 \times 5.90 \times 10^{-15}\ \text{m}\right)^2}$$

$$= 3.50 \times 10^3\ \text{N} = \boxed{3.50\ \text{kN}}$$

P24.24 Note that the electric field in each case is directed radially inward, toward the filament. We use $E = \dfrac{2k_e\lambda}{r}$ and substitute numerical values.

(a) At r = 10.0 cm = 0.100 m,

$$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0 \times 10^{-6}\ \text{C/m}\right)}{0.100\ \text{m}}$$

$$= \boxed{16.2\ \text{MN/C}}$$

(b) At r = 20.0 cm = 0.200 m,

$$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C/m}\right)}{0.200\ \text{m}}$$
$$= \boxed{8.09\ \text{MN/C}}$$

(c) At r = 100 cm = 1.00 m,

$$E = \frac{2k_e\lambda}{r} = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(90.0\times10^{-6}\ \text{C/m}\right)}{1.00\ \text{m}}$$
$$= \boxed{1.62\ \text{MN/C}}$$

P24.25 The charge per unit area of the plastic sheet must be sufficiently large to result in an upward electric force on the Styrofoam that cancels the downward gravitational force:

$$mg = qE = q\left(\frac{\sigma}{2\in_0}\right) = q\left(\frac{Q/A}{2\in_0}\right)$$

Solving for the charge per unit area gives

$$\frac{Q}{A} = \frac{2\in_0 mg}{q}$$
$$= \frac{2\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(10.0\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{\left|-0.700\times10^{-6}\ \text{C}\right|}$$
$$= \boxed{2.48\ \mu\text{C/m}^2}$$

P24.26 The charge distributed through the nucleus creates a field at the surface equal to that of a point charge at its center: $E = \dfrac{k_e q}{r^2}$.

$$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(82\times1.60\times10^{-19}\ \text{C}\right)}{\left[(208)^{1/3}\left(1.20\times10^{-15}\ \text{m}\right)\right]^2}$$

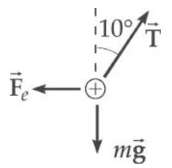
$$E = \boxed{2.33\times10^{21}\ \text{N/C}}\ \text{away from the nucleus}$$

P24.27 For a large uniformly charged sheet, $\vec{\mathbf{E}}$ will be perpendicular to the sheet, and will have a magnitude of

$$E = \frac{\sigma}{2\in_0} = 2\pi k_e\sigma$$
$$= (2\pi)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(9.00\times10^{-6}\ \text{C/m}^2\right)$$

so $\vec{\mathbf{E}} = 5.08\times10^5\ \text{N/C}\ \hat{\mathbf{j}}$

P24.28 Consider two balloons of diameter 0.200 m, each with mass 1.00 g, hanging apart with a 0.050 0 m separation on the ends of strings making angles of 10.0° with the vertical.



ANS. FIG. P24.28

(a) $$\sum F_y = T\cos 10° - mg = 0 \Rightarrow T = \frac{mg}{\cos 10°}$$

$$\sum F_x = T\sin 10° - F_e = 0 \Rightarrow F_e = T\sin 10°$$

so $$F_e = \left(\frac{mg}{\cos 10.0°}\right)\sin 10.0° = mg\tan 10,0°$$

$$= (0.001\ 00\ \text{kg})(9.80\ \text{m/s}^2)\tan 10.0°$$

$$F_e \approx 2\times 10^{-3}\ \text{N}\ \boxed{\sim 10^{-3}\ \text{N or 1 mN}}$$

(b) The charge on each balloon can be found from $F_e = \dfrac{k_e q^2}{r^2}$:

$$q = \sqrt{\frac{F_e r^2}{k_e}} \approx \sqrt{\frac{(2\times 10^{-3}\ \text{N})(0.25\ \text{m})^2}{8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}}$$

$$\approx 1.2\times 10^{-7}\ \text{C}\ \boxed{\sim 10^{-7}\ \text{C or 100 nC}}$$

(c) $$E = \frac{k_e q}{r^2} \approx \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.2\times 10^{-7}\ \text{C})}{(0.25\ \text{m})^2} \approx 1.7\times 10^4\ \text{N/C}$$

$$\boxed{\sim 10\,\text{kN/C}}$$

(d) The electric flux created by each balloon is

$$\Phi_E = \frac{q}{\epsilon_0} \approx \frac{1.2\times 10^{-7}\ \text{C}}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = 1.4\times 10^4\ \text{N}\cdot\text{m}^2/\text{C}$$

$$\boxed{\sim 10\,\text{kN}\cdot\text{m}^2/\text{C}}$$

P24.29 (a) Consider the spherical symmetry of the situation. A gaussian sphere concentric wth the shell, with radius 10.0 cm, encloses 0 charge. Then at the surface of this sphere, inside the charged shell, we have $\vec{\mathbf{E}} = \boxed{0}$.

(b) For a gaussian sphere of radius 20.0 cm, we apply $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \dfrac{q_{in}}{\epsilon_0}$.

The field is radially outward, and $4\pi r^2 E = q/\epsilon_0$:

$$E = \frac{k_e q}{r^2} = \frac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(32.0\times 10^{-6}\ \text{C})}{(0.200\ \text{m})^2}$$

$$= 7.19\times 10^6\ \text{N/C}$$

so $\vec{\mathbf{E}} = \boxed{7.19\ \mathrm{MN/C}\text{ radially outward}}$

P24.30 (a) The charge per unit area of the wall is

$$\sigma = \left(8.60\times10^{-6}\ \mathrm{C/cm^2}\right)\left(\frac{100\ \mathrm{cm}}{\mathrm{m}}\right)^2 = 8.60\times10^{-2}\ \mathrm{C/m^2}$$

The electric field at a distance of 2.00 cm is then

$$E = \frac{\sigma}{2\in_0} = \frac{8.60\times10^{-2}\ \mathrm{C/m^2}}{2\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)}$$
$$= \boxed{4.86\times10^{9}\ \mathrm{N/C}\text{ away from the wall}}$$

(b) $\boxed{\text{So long as the distance from the wall is small compared to the width and height of the wall, the distance does not affect the field.}}$

P24.31 The approximation in this case is that the filament length is so large when compared to the cylinder length that the "infinite line" of charge can be assumed.

(a) We have

$$E = \frac{2k_e\lambda}{r}$$

where the linear charge density is

$$\lambda = \frac{2.00\ \times 10^{-6}\ \mathrm{C}}{7.00\ \mathrm{m}} = 2.86\times10^{-7}\ \mathrm{C/m}$$

so

$$E = \frac{(2)(8.99\times10^{9}\ \mathrm{N\cdot m^2/C})(2.86\times10^{-7}\ \mathrm{C/m})}{0.100\ \mathrm{m}}$$
$$= \boxed{51.4\ \mathrm{kN/C}\text{ radially outward}}$$

(b) We can find the flux by multiplying the field and the lateral surface area of the cylinder:

$$\Phi_E = 2\pi rLE = 2\pi rL\left(\frac{2k_e\lambda}{r}\right) = 4\pi k_e\lambda L$$

so

$$\Phi_E = 4\pi(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(2.86\times10^{-7}\ \mathrm{C/m})(0.020\,0\ \mathrm{m})$$
$$= \boxed{6.46\times10^{2}\ \mathrm{N\cdot m^2/C}}$$

P24.32 (a) The area of each face is $A = 1.00\ \text{m}^2$.

For the left face, the angle between the electric field and the normal is 0°:

$$\begin{aligned}(\Phi_E)_{\text{left face}} &= EA\cos\theta = (20.0\ \text{N/C})(1.00\ \text{m}^2)\cos 0° \\ &= 20.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the right face, the angle between the electric field and the normal is 180°:

$$\begin{aligned}(\Phi_E)_{\text{right face}} &= EA\cos\theta = (35.0\ \text{N/C})(1.00\ \text{m}^2)\cos 180° \\ &= -35.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the top face, the angle between the electric field and the normal is 180°:

$$\begin{aligned}(\Phi_E)_{\text{top face}} &= EA\cos\theta = (25.0\ \text{N/C})(1.00\ \text{m}^2)\cos 180° \\ &= -25.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the bottom face, the angle between the electric field and the normal is 0°:

$$\begin{aligned}(\Phi_E)_{\text{bottom face}} &= EA\cos\theta = (15.0\ \text{N/C})(1.00\ \text{m}^2)\cos 0° \\ &= 15.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the front face, the angle between the electric field and the normal is 0°:

$$\begin{aligned}(\Phi_E)_{\text{front face}} &= EA\cos\theta = (20.0\ \text{N/C})(1.00\ \text{m}^2)\cos 0° \\ &= 20.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

For the back face, the angle between the electric field and the normal is 0°:

$$\begin{aligned}(\Phi_E)_{\text{back face}} &= EA\cos\theta = (20.0\ \text{N/C})(1.00\ \text{m}^2)\cos 0° \\ &= 20.0\ \text{N}\cdot\text{m}^2/\text{C}\end{aligned}$$

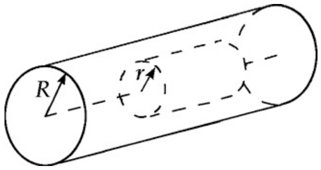
The total flux is then

$$\begin{aligned}\Phi_E &= (20.0 - 35.0 - 25.0 + 15.0 + 20.0 + 20.0)\ \text{N}\cdot\text{m}^2/\text{C} \\ &= \boxed{15.0\ \text{N}\cdot\text{m}^2/\text{C}}\end{aligned}$$

(b) $$\begin{aligned}\Phi_E &= \frac{q_{\text{in}}}{\epsilon_0} \rightarrow q_{\text{in}} = \epsilon_0\,\Phi_E = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(15.0\ \text{N}\cdot\text{m}^2/\text{C}) \\ &= \boxed{1.33\times10^{-10}\ \text{C}}\end{aligned}$$

(c) $\boxed{\text{No; fields on the faces would not be uniform.}}$

P24.33 If ρ is positive, the field must be radially outward. Choose as the gaussian surface a cylinder of length L and radius r, contained inside the charged rod. Its volume is $\pi r^2 L$ and it encloses charge $\rho \pi r^2 L$. Because the charge distribution is long, no electric flux passes through the circular end caps; $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EdA\cos 90.0° = 0$. The curved surface has $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EdA\cos 0°$, and E must be the same strength everywhere over the curved surface.



ANS. FIG. P24.33

Gauss's law, $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \dfrac{q}{\epsilon_0}$, becomes $E \displaystyle\int_{\substack{\text{Curved}\\\text{Surface}}} dA = \dfrac{\rho \pi r^2 L}{\epsilon_0}$.

Now the lateral surface area of the cylinder is $2\pi rL$:

$$E(2\pi r)L = \frac{\rho \pi r^2 L}{\epsilon_0}$$

Thus, $\vec{\mathbf{E}} = \boxed{\dfrac{\rho r}{2\epsilon_0} \text{ radially away from the cylinder axis}}$.

P24.34 (a) The electric field is given by

$$E = \frac{2k_e \lambda}{r} = \frac{2k_e (Q/\ell)}{r}$$

Solving for the charge Q gives

$$Q = \frac{Er\ell}{2k_e} = \frac{(3.60 \times 10^4 \text{ N/C})(0.190 \text{ m})(2.40 \text{ m})}{2(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C})} =$$

$$Q = +9.13 \times 10^{-7} \text{ C} = \boxed{+913 \text{ nC}}$$

(b) Since the charge is uniformly distributed on the surface of the cylindrical shell, a gaussian surface in the shape of a cylinder of 4.00 cm in radius encloses no charge, and $\vec{\mathbf{E}} = \boxed{0}$.

P24.35 (a) At the center of the sphere, the total charge is zero, so

$$E = \frac{k_e Q r}{a^3} = \boxed{0}$$

(b) At a distance of 10.0 cm = 0.100 m from the center,

$$E = \frac{k_e Q r}{a^3} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C})(26.0 \times 10^{-6} \text{ C})(0.100 \text{ m})}{(0.400 \text{ m})^3}$$

$$= \boxed{365 \text{ kN/C}}$$

(c) At a distance of 40.0 cm = 0.400 m from the center, all of the charge is enclosed, so

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}\right)\left(26.0\times10^{-6}\ \text{C}\right)}{(0.400\ \text{m})^2}$$
$$= \boxed{1.46\ \text{MN/C}}$$

(d) At a distance of 60.0 cm = 0.600 m from the center,

$$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}\right)\left(26.0\times10^{-6}\ \text{C}\right)}{(0.600\ \text{m})^2}$$
$$= \boxed{649\ \text{kN/C}}$$

The direction for each electric field is $\boxed{\text{radially outward}}$.

P24.36 The volume of the spherical shell is

$$\frac{4}{3}\pi\left[(0.25\ \text{m})^3 - (0.20\ \text{m})^3\right] = 3.19\times10^{-2}\ \text{m}^3$$

Its charge is

$$\rho V = \left(-1.33\times10^{-6}\ \text{C/m}^3\right)\left(3.19\times10^{-2}\ \text{m}^3\right) = -4.25\times10^{-8}\ \text{C}$$

The net charge inside a sphere containing the proton's path as its equator is

$$-60\times10^{-9}\ \text{C} - 4.25\times10^{-8}\ \text{C} = -1.02\times10^{-7}\ \text{C}$$

The electric field is radially inward with magnitude

$$E = \frac{k_e|q|}{r^2} = \frac{|q|}{\epsilon_0\, 4\pi r^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.02\times10^{-7}\ \text{C}\right)}{(0.250\ \text{m})^2}$$
$$= 1.47\times10^{4}\ \text{N/C}$$

For the proton, Newton's second law gives

$$\sum F = ma: \qquad eE = \frac{mv^2}{r}$$

solving for the proton's speed then gives

$$v = \left(\frac{eEr}{m}\right)^{1/2} = \left[\frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.47\times10^{4}\ \text{N/C}\right)(0.250\ \text{m})}{1.67\times10^{-27}\ \text{kg}}\right]^{1/2}$$
$$= \boxed{5.94\times10^{5}\ \text{m/s}}$$

Section 24.4 Conductors in Electrostatic Equilibrium

P24.37 $\oint E\,dA = E(2\pi r l) = \dfrac{q_{\text{in}}}{\epsilon_0}$ $\quad E = \dfrac{q_{\text{in}}/l}{2\pi\epsilon_0 r} = \dfrac{\lambda}{2\pi\epsilon_0 r}$ for the field outside the metal rod.

(a) At r = 3.00 cm, $\quad \vec{\mathbf{E}} = \boxed{0}$

(b) At r = 10.0 cm,

$$\vec{\mathbf{E}} = \frac{30.0\times10^{-9}\text{ C}}{2\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(0.100\text{ m})}$$

$$= \boxed{5\,400\text{ N/C, outward}}$$

(c) At r = 100 cm,

$$\vec{\mathbf{E}} = \frac{30.0\times10^{-9}\text{ C}}{2\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(1.00\text{ m})}$$

$$= \boxed{540\text{ N/C, outward}}$$

P24.38 Let's calculate the electric field just outside the surface:

$$E = k_e\frac{q}{r^2} = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{40.0\times10^{-9}\text{ C}}{(0.15\text{ m})^2}\right]$$

$$= 1.60\times10^4\text{ N} = 16.0\text{ kN/C}$$

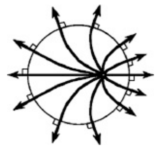
This should be the value of the electric field at the peak of the curve in Figure P24.38. We see, however, that the peak in the figure occurs at about 6.5 kN/C. Therefore, it is not possible that this figure represents the electric field for the given situation.

P24.39 The surface area is $A = 4\pi a^2$. The field is then

$$E = \frac{k_e Q}{a^2} = \frac{Q}{4\pi\epsilon_0 a^2} = \frac{Q}{A\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$$

It is not equal to $\sigma/2\epsilon_0$. At a point just outside, the uniformly charged surface looks just like a uniform flat sheet of charge. The distance to the field point is negligible compared to the radius of curvature of the surface.

P24.40 An approximate sketch is given at the right. Note that the electric field lines should be perpendicular to the conductor both inside and outside.



ANS. FIG. P24.40

P24.41 The fields are equal. The equation $E = \frac{\sigma_{\text{conductor}}}{\epsilon_0}$ suggested in the chapter for the field outside the aluminum looks different from the equation $E = \frac{\sigma_{\text{insulator}}}{2\epsilon_0}$ for the field around glass. But its charge will spread out to cover both sides of the aluminum plate, so the density is $\sigma_{\text{conductor}} = \frac{Q}{2A}$. The glass carries charge only on area *A*, with $\sigma_{\text{insulator}} = \frac{Q}{A}$. The two fields are $\frac{Q}{2A\epsilon_0}$, the same in magnitude, and both are perpendicular to the plates, vertically upward if *Q* is positive.

P24.42 (a) Let a flat box have face area *A* perpendicular to its thickness *dx*. The flux at *x* = 0.3 m is into the box is

$$\Phi_E = -EA = -(6\ 000\ \text{N/C}\cdot\text{m}^2)(0.3\ \text{m})^2 A = -(540\ \text{N/C})A$$

The flux at *x* = 0.3 m + *dx* is out of the box is

$$\begin{aligned}\Phi_E &= +EA = +(6\ 000\ \text{N/C}\cdot\text{m}^2)(0.3\ \text{m} + dx)^2 A\\ &= +(540\ \text{N/C})A + (3\ 600\ \text{N/C}\cdot\text{m})\,dx\,A\end{aligned}$$

(The term in $(dx)^2$ is negligible.) The charge in the box is $\rho A\,dx$ where ρ is the unknown. Applying Gauss's law, $\Phi_E = \frac{q_{\text{in}}}{\epsilon_0}$, we obtain

$$\begin{aligned}&-(540\ \text{N/C})A + (540\ \text{N/C})A\\ &\qquad + (3\,600\ \text{N/C}\cdot\text{m})\,dx\,A = \rho A\,dx/\epsilon_0\end{aligned}$$

Solving for ρ gives

$$\begin{aligned}\rho &= (3\,600\ \text{N/C}\cdot\text{m})\epsilon_0\\ &= (3\,600\ \text{N/C}\cdot\text{m})(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)\\ &= \boxed{31.9\ \text{nC/m}^3}\end{aligned}$$

(b) $\boxed{\text{No; then the field would have to be zero.}}$

P24.43 The charge divides equally between the identical spheres, with charge *Q*/2 on each. Then, they repel like point charges at their centers:

$$\begin{aligned}F &= \frac{k_e(Q/2)(Q/2)}{(L+R+R)^2} = \frac{k_eQ^2}{4(L+2R)^2}\\ &= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(60.0\times10^{-6}\ \text{C})^2}{4(2.01\ \text{m})^2} = \boxed{2.00\ \text{N}}\end{aligned}$$

P24.44 (a) $E = \dfrac{\sigma}{\epsilon_0}$, so

$$\sigma = \left(8.00 \times 10^4 \text{ N/C}\right)\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)$$
$$= 7.08 \times 10^{-7} \text{ C/m}^2$$

$\sigma = \boxed{708 \text{ nC/m}^2}$, positive on one face and negative on the other.

(b) $\sigma = \dfrac{Q}{A}$, so

$$Q = \sigma A = \left(7.08 \times 10^{-7} \text{ C/m}^2\right)\left(0.500 \text{ m}\right)^2$$
$$= 1.77 \times 10^{-7} \text{ C} = \boxed{177 \text{ nC}}$$

positive on one face and negative on the other.

P24.45 (a) Inside surface: consider a cylindrical gaussian surface of arbitrary length ℓ within the metal. Since *E* inside the conducting shell is zero, the total charge inside the gaussian surface must be zero:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0} \quad \rightarrow \quad 0 = \frac{\left(\lambda + \lambda_{\text{inner}}\right)\ell}{\epsilon_0}$$

so $\lambda_{\text{inner}} = \boxed{-\lambda}$.

(b) Outside surface: consider a cylindrical gaussian surface of arbitrary length ℓ outside the metal. The total charge within the gaussian surface is

$$q_{\text{wire}} + q_{\text{cylinder}} = q_{\text{wire}} + \left(q_{\text{inner surface}} + q_{\text{outer surface}}\right)$$
$$\lambda\ell + 2\lambda\ell = \lambda\ell + \left(-\lambda\ell + \lambda_{\text{outer}}\ell\right) \quad \rightarrow \quad \lambda_{\text{outer}} = \boxed{3\lambda}$$

(c) Gauss's law:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E 2\pi r\ell = \frac{3\lambda\ell}{\epsilon_0} \quad \rightarrow \quad E = 2\frac{3\lambda}{4\pi\epsilon_0 r} = \boxed{6k_e\frac{\lambda}{r}, \text{ radially outward}}$$

P24.46 (a) We ignore "edge" effects and assume that the total charge distributes itself uniformly over each side of the plate, with one half the total charge on each side. The charge density on each of the surfaces (upper and lower) of the plate is:

$$\sigma = \frac{1}{2}\left(\frac{q}{A}\right) = \frac{1}{2}\frac{\left(4.00\times 10^{-8}\ \text{C}\right)}{\left(0.500\ \text{m}\right)^2} = 8.00\times 10^{-8}\ \text{C/m}^2$$

$$= \boxed{80.0\ \text{nC/m}^2}$$

(b) Just above the plate,

$$\vec{\mathbf{E}} = \left(\frac{\sigma}{\epsilon_0}\right)\hat{\mathbf{k}} = \left(\frac{8.00\times 10^{-8}\ \text{C/m}^2}{8.85\times 10^{-12}\ \text{C}^2\ /\ \text{N}\cdot\text{m}^2}\right)\hat{\mathbf{k}} = \boxed{\left(9.04\ \text{kN/C}\right)\hat{\mathbf{k}}}$$

(c) Just below the plate, $\vec{\mathbf{E}} = \boxed{\left(-9.04\ \text{kN/C}\right)\hat{\mathbf{k}}}$.

***P24.47** (a) $\vec{\mathbf{E}} = \boxed{0}$

(b) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00\times 10^{-6}\ \text{C}\right)}{\left(0.030\,0\ \text{m}\right)^2} = 7.99\times 10^{7}\ \ \text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{79.9\ \ \text{MN/C radially outward}}$$

(c) $\vec{\mathbf{E}} = \boxed{0}$

(d) $$E = \frac{k_e Q}{r^2} = \frac{\left(8.99\times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00\times 10^{-6}\ \text{C}\right)}{\left(0.070\,0\ \text{m}\right)^2} = 7.34\times 10^{6}\ \ \text{N/C}$$

$$\vec{\mathbf{E}} = \boxed{7.34\ \ \text{MN/C radially outward}}$$

Additional Problems

P24.48 The electric field makes an angle of 70.0° to the normal. The square has side $d = 5.00$ cm.

$$\Phi_E = EA\cos\theta = Ed^2\cos\theta$$

$$\rightarrow E = \frac{\Phi_E}{d^2\cos\theta} = \frac{6.00\ \text{N}\cdot\text{m}^2/\text{C}}{\left(0.150\ \text{m}\right)^2\cos 70.0°} = \boxed{780\ \text{N/C}}$$

P24.49 The electric field makes an angle of 60.0° with to the normal. The square has side d = 5.00 cm.

$$\Phi_E = EA\cos\theta = \left(3.50\times 10^{2}\text{ N/C}\right)\left(5.00\times 10^{-2}\text{ m}\right)^{2}\cos 60.0°$$
$$= \boxed{0.438\text{ N}\cdot\text{m}^{2}/\text{C}}$$

P24.50 (a) The field is zero within the metal of the shell. The exterior electric field lines end at equally spaced points on the outer surface because the surface of the conductor is an equipotential surface. The charge on the outer surface is distributed uniformly. Its amount is given by

$$EA = Q/\epsilon_0$$

Solving for the charge Q gives

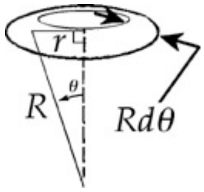
$$Q = -(890\text{ N/C})\ 4\pi\ (0.750\text{ m})^{2}\left(8.85\times 10^{-12}\text{ C}^{2}/\text{N}\cdot\text{m}^{2}\right)$$
$$= -55.7\text{ nC}$$

$$\boxed{\text{The charge on the exterior surface is } -55.7\text{ nC distributed uniformly.}}$$

(b) For the net charge of the shell to be zero, the shell must carry +55.7 nC on its inner surface, induced there by –55.7 nC in the cavity within the shell. The charge in the cavity could have any distribution and give any corresponding distribution to the charge on the inner surface of the shell. $\boxed{\text{The charge on the interior surface is +55.7 nC. It can have any distribution.}}$ For example, a large positive charge might be within the cavity close to its topmost point, and a slightly larger negative charge near its easternmost point. The inner surface of the shell would then have plenty of negative charge near the top and even more positive charge centered on the eastern side.

(c) See the comments in (b). $\boxed{\text{The charge within the shell is –55.7 nC. It can have any distribution.}}$ For example, the charge could be distributed on the surface of an insulator of arbitrary shape.

P24.51 The $\vec{\mathbf{E}}$ field due to the point charge is uniform and points radially outward, so $\Phi_E = EA$. The arc length of a small ring-shaped element of the sphere is $ds = Rd\theta$, and its circumference is $2\pi r = 2\pi R\sin\theta$.



ANS. FIG. P24.51

The area of the circular cap is

$$A=\int 2\pi r\,ds=\int_0^\theta (2\pi R\sin\theta)R\,d\theta = 2\pi R^2\int_0^\theta \sin\theta\,d\theta$$
$$A=2\pi R^2(-\cos\theta)\Big|_0^\theta = 2\pi R^2(1-\cos\theta)$$

The flux is then

$$\Phi_E = EA = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{R^2}\cdot(2\pi R^2)(1-\cos\theta)$$
$$=\left(\frac{Q}{2\epsilon_0}\right)(1-\cos\theta)$$
$$\Phi_E=\left[\frac{50.0\times10^{-6}\text{ C}}{2(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)}\right](1-\cos 45.0°)$$
$$=\boxed{8.27\times10^5\text{ N}\cdot\text{m}^2/\text{C}}$$

P24.52 Refer to ANS. FIG. P24.51 above. The $\vec{\mathbf{E}}$ field due to the point charge is uniform and points radially outward, so $\Phi_E = EA$. The arc length of a small ring-shaped element of the sphere is *ds* = *Rdθ*, and its circumference is $2\pi r = 2\pi R\sin\theta$.

The area of the circular cap is

$$A=\int 2\pi r\,ds=\int_0^\theta (2\pi R\sin\theta)R\,d\theta = 2\pi R^2\int_0^\theta \sin\theta\,d\theta$$
$$A=2\pi R^2(-\cos\theta)\Big|_0^\theta = 2\pi R^2(1-\cos\theta)$$

The flux is then

$$\Phi_E = EA = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{Q}{R^2}\cdot(2\pi R^2)(1-\cos\theta)$$
$$=\boxed{\left(\frac{Q}{2\epsilon_0}\right)(1-\cos\theta)}$$

***P24.53** Please review Example 23.9 in your textbook, emphazising the Finalize section. There, it is shown that the electric field due to a nonconducting plane sheet of charge has a constant magnitude given by $E_z = \frac{|\sigma_\text{sheet}|}{2\epsilon_0}$, where σ_sheet is the uniform charge per unit area on the sheet. This field is everywhere perpendicular to the *xy* plane, is directed away from the sheet if it has a positive charge density, and is directed toward the sheet if it has a negative charge density.

In this problem, we have two plane sheets of charge, both parallel to the *xy* plane and separated by a distance of z_0. The upper sheet has charge density $\sigma_{\text{sheet}} = -2\sigma$, while the lower sheet has $\sigma_{\text{sheet}} = +\sigma$. Taking upward as the positive *z*-direction, the fields due to each of the sheets in the three regions of interest are:

	Lower sheet (at $z = 0$)	**Upper sheet (at $z = z_0$)**
Region	Electric Field	Electric Field
$z < 0$	$E_z = -\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = -\dfrac{\sigma}{2\epsilon_0}$	$E_z = +\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{\epsilon_0}$
$0 < z < z_0$	$E_z = +\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{2\epsilon_0}$	$E_z = +\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{\epsilon_0}$
$z > z_0$	$E_z = +\dfrac{\lvert+\sigma\rvert}{2\epsilon_0} = +\dfrac{\sigma}{2\epsilon_0}$	$E_z = -\dfrac{\lvert-2\sigma\rvert}{2\epsilon_0} = -\dfrac{\sigma}{\epsilon_0}$

When both plane sheets of charge are present, the resultant electric field in each region is the vector sum of the fields due to the individual sheets for that region.

(a) For $z < 0$,

$$E_z = E_{z,\text{ lower}} + E_{z,\text{ upper}} = -\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{\epsilon_0} = \boxed{+\frac{\sigma}{2\epsilon_0}}$$

(b) For $0 < z < z_0$,

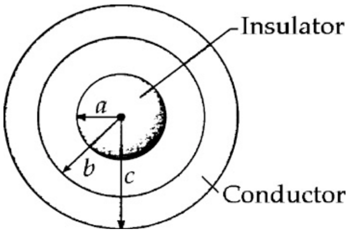
$$E_z = E_{z,\text{ lower}} + E_{z,\text{ upper}} = +\frac{\sigma}{2\epsilon_0} + \frac{\sigma}{\epsilon_0} = \boxed{+\frac{3\sigma}{2\epsilon_0}}$$

(c) For $z > z_0$,

$$E_z = E_{z,\text{ lower}} + E_{z,\text{ upper}} = +\frac{\sigma}{2\epsilon_0} - \frac{\sigma}{\epsilon_0} = \boxed{-\frac{\sigma}{2\epsilon_0}}$$

P24.54 Choose as each gaussian surface a concentric sphere of radius *r*. The electric field will be perpendicular to its surface, and will be uniform in strength over its surface. The density of charge in the insulating sphere is

$$\rho = Q / \left(\tfrac{4}{3}\pi a^3\right)$$



ANS. FIG. P24.54

(a) The sphere of radius *r* < *a* encloses charge

$$q_{\text{in}} = \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{Q}{\frac{4}{3}\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) = \boxed{Q\left(\frac{r}{R}\right)^3}$$

(b) Applying Gauss's law to this sphere reveals the magnitude of the field at its surface.

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0}\left(\frac{r}{a}\right)^3 \rightarrow E = \frac{1}{4\pi \epsilon_0}\frac{Qr}{a^3} = \boxed{k_e \frac{Qr}{a^3}}$$

(c) For a sphere of radius *r* with $a < r < b$, the whole insulating sphere is enclosed, so the charge within is *Q*: $q_{\text{in}} = \boxed{Q}$.

(d) Gauss's law for this sphere becomes:

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{Q}{\epsilon_0} \rightarrow E = \frac{1}{4\pi \epsilon_0}\frac{Q}{r^2} = \boxed{k_e \frac{Q}{r^2}}$$

(e) For $b \le r \le c$, $\boxed{E = 0}$ because there is no electric field inside a conductor.

(f) For $b \le r \le c$, we know *E* = 0. Assume the inner surface of the hollow sphere holds charge Q_{inner}. By Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$0 = \frac{Q + Q_{\text{inner}}}{\epsilon_0} \rightarrow Q_{\text{inner}} = \boxed{-Q}$$

(g) The total charge on the hollow sphere is zero; therefore, charge on the outer surface is opposite to that on the inner surface:

$$Q_{\text{outer}} = -Q_{\text{inner}} = \boxed{+Q}$$

(h) A surface of area *A* holding charge *Q* has surface charge $\sigma = q/A$. The solid, insulating sphere has small surface charge because its total charge *Q* is uniformly distributed throughout its volume. The $\boxed{\text{inner surface of radius } b}$ has smaller surface area, and therefore larger surface charge, than the outer surface of radius *c*.

P24.55 The electric field has these values (consult the solution to P24.54(a)–(e) for details). Suppressing units,

For $0 < r < a$, $$E = k_e \frac{Qr}{a^3} = \left(8.99\times10^9\right)\frac{3.00\times10^{-6}}{(0.050\ 0)^3}r$$

For $a < r < b$, $$E = k_e \frac{Q}{r^2} = \left(8.99\times10^9\right)\frac{3.00\times10^{-6}}{r^2}$$

For $b < r < c$, $E = 0$ (inside conductor)

For $r > c$, from Gauss's law (suppressing units):

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0} \to E\left(4\pi r^2\right) = \frac{Q+q}{\epsilon_0}$$

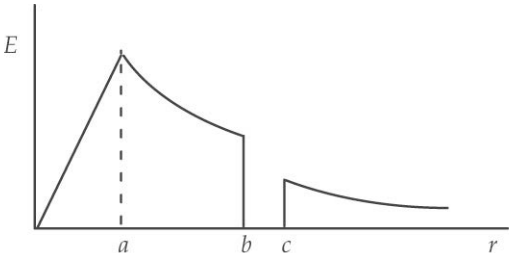
$$\to E = \frac{1}{4\pi\epsilon_0}\frac{Q+q}{r^2} = k_e\frac{Q+q}{r^2}$$

$$= \left(8.99\times10^9\right)\frac{3.00\times10^{-6} - 1.00\times10^{-6}}{r^2}$$

$$E = \left(8.99\times10^9\right)\frac{2.00\times10^{-6}}{r^2}$$

where *r* is in meters and *E* in N/C. The field is radially outward.

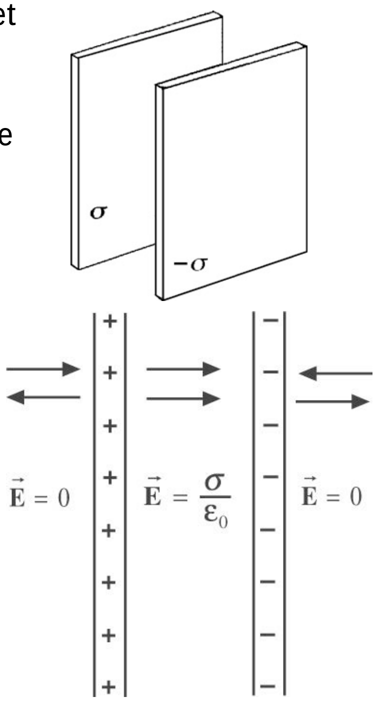
The graph appears in ANS. FIG. P24.55 below, with $a = 0.050\ 0$ m, $b = 0.100$ m, and $c = 0.150$ m.



ANS. FIG. P24.55

P24.56 Consider the field due to a single sheet and let E_+ and E_- represent the fields due to the positive and negative sheets. The field at any distance from each sheet has a magnitude given by the textbook equation

$$|E_+| = |E_-| = \frac{\sigma}{2\epsilon_0}$$



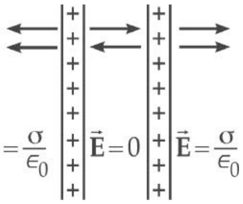
ANS. FIG. P24.56(a-c)

(a) To the left of the positive sheet, E_+ is directed toward the left and E_- toward the right and the net field over this region is $\vec{\mathbf{E}} = \boxed{0}$.

(b) In the region between the sheets, E_+ and E_- are both directed toward the right and the net field is

$$\vec{\mathbf{E}} = \boxed{\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

(c) To the right of the negative sheet, E_+ and E_- are again oppositely directed and $\vec{\mathbf{E}} = \boxed{0}$.



ANS. FIG. P24.56(d)

(d) Now, both sheets are positively charged. We find that

(1) To the left of both sheets, both fields are directed toward the left:

$$\vec{\mathbf{E}} = \boxed{2\frac{\sigma}{\epsilon_0} \text{ to the left}}$$

(2) Between the sheets, the fields cancel because they are opposite to each other: $\vec{\mathbf{E}} = \boxed{0}$.

(3) To the right of both sheets, both fields are directed toward the right:

$$\vec{\mathbf{E}} = \boxed{2\frac{\sigma}{\epsilon_0} \text{ to the right}}$$

P24.57 We have

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = E\left(4\pi r^2\right) = \frac{q_{\text{in}}}{\epsilon_0}$$

(a) Solving for the charge Q on the insulating sphere, we write, for the region $a < r < b$,

$$\begin{aligned} Q &= \epsilon_0 E\left(4\pi r^2\right) \\ &= \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(-3.60\times10^3\ \text{N/C}\right)4\pi(0.100\ \text{m})^2 \\ &= -4.00\times10^{-9}\ \text{C} = \boxed{-4.00\ \text{nC}} \end{aligned}$$

(b) We take Q' to be the net charge on the hollow sphere. For $r > c$,

$$\begin{aligned} Q + Q' &= \epsilon_0 E\left(4\pi r^2\right) \\ &= \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.00\times10^2\ \text{N/C}\right) \\ &\qquad \times 4\pi(0.500\ \text{m})^2 \\ &= 5.56\times10^{-9}\ \text{C} \end{aligned}$$

so

$$Q' = +9.56\times10^{-9}\ \text{C} = \boxed{+9.56\ \text{nC}}$$

(c) For $b < r < c$, $E = 0$; therefore, $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = q_{\text{in}}/\epsilon_0 = 0$ implies $q_{\text{in}} = Q + Q_1 = 0$, where Q_1 is the total charge on the inner surface of the hollow sphere. Thus, $Q_1 = -Q = \boxed{+4.00\ \text{nC}}$.

(d) Let Q_2 be the total charge on the outer surface of the hollow sphere; then,

$$Q' = Q_1 + Q_2 \rightarrow Q_2 = Q' - Q_1 = 9.56\ \text{nC} - 4.00\ \text{nC} = \boxed{+5.56\ \text{nC}}$$

P24.58 The charge density is determined by $Q = \frac{4}{3}\pi a^3 \rho$. Solving gives

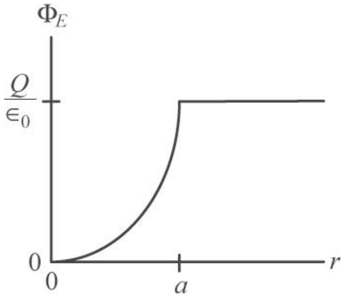
$$\rho = \frac{3Q}{4\pi a^3}$$

(a) The flux is that created by the enclosed charge within radius r:

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{4\pi r^3 \rho}{3\epsilon_0} = \frac{4\pi r^3 3Q}{3\epsilon_0 4\pi a^3} = \boxed{\frac{Qr^3}{\epsilon_0 a^3}}$$

(b) $\Phi_E = \boxed{\dfrac{Q}{\epsilon_0}}$. Note that the answers to parts (a) and (b) agree at $r = a$.

(c) ANS. FIG. P24.58(c) plots the flux vs. *r*.



ANS. FIG. P24.58(c)

P24.59 Consider the charge distribution to be an unbroken charged spherical shell with uniform charge density σ and a circular disk with charge per area $-\sigma$. The total field is that due to the whole sphere,

$$E_{\text{sphere}} = \frac{Q}{4\pi \in_0 R^2} = \frac{4\pi R^2 \sigma}{4\pi \in_0 R^2} = \frac{\sigma}{\in_0} \text{ outward}$$

plus the field of the disk

$$E_{\text{disk}} = -\frac{\sigma}{2\in_0} = \frac{\sigma}{2\in_0} \text{ radially inward}$$

The total field is

$$E_{\text{sphere}} + E_{\text{disk}} = \frac{\sigma}{\in_0} - \frac{\sigma}{2\in_0} = \boxed{\frac{\sigma}{2\in_0} \text{ radially outward}}$$

P24.60 The cylindrical symmetry of the charge distribution implies that the field direction is radially outward perpendicular to the axis. The field strength depends on *r* but not on the other cylindrical coordinates θ or *z*. Choose a gaussian cylinder of radius *r* and length *L*; the electric field is normal to this surface. Recalling that $k_e = \frac{1}{4\pi\in_0} \to \frac{1}{\in_0} = 4\pi k_e$, we have $\Phi_E = \frac{q_{\text{in}}}{\in_0} = 4\pi k_e q_{\text{in}}$.

(a) If $r < a$, we have

$$\Phi_E = 4\pi k_e q_{\text{in}}$$

$$E(2\pi rL) = (4\pi k_e)\lambda L \to E = \boxed{2k_e \frac{\lambda}{r}, \text{ outward}}$$

(b) If $a < r < b$, we have

$$\Phi_E = 4\pi k_e q_{\text{in}}$$
$$E(2\pi rL) = (4\pi k_e)\left[\lambda L + \rho\pi\left(r^2 - a^2\right)L\right] \rightarrow$$
$$E = \boxed{\frac{2k_e}{r}\left[\lambda + \rho\pi\left(r^2 - a^2\right)\right], \text{ outward}}$$

(c) If $r > b$, we have

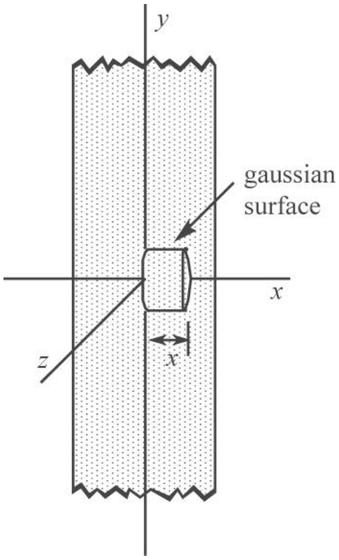
$$\Phi_E = 4\pi k_e q_{\text{in}}$$
$$E(2\pi rL) = (4\pi k_e)\left[\lambda L + \rho\pi\left(b^2 - a^2\right)L\right]$$
$$E = \boxed{\frac{2k_e}{r}\left[\lambda + \rho\pi\left(b^2 - a^2\right)\right], \text{ outward}}$$

Challenge Problems

P24.61 (a) Consider a cylindrical shaped gaussian surface perpendicular to the *yz* plane with its left end in the *yz* plane and its right end at distance *x*, as shown in ANS. FIG. P24.61.

Use Gauss's law: $\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$

By symmetry, the electric field is zero in the *yz* plane and is perpendicular to $d\vec{\mathbf{A}}$ over the wall of the gaussian cylinder. Therefore, the only contribution to the integral is over the end cap.



ANS. FIG. P24.61

For $x > \frac{d}{2}$,

$$dq = \rho dV = \rho A dx = CAx^2 dx$$

$$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\epsilon_0}\int dq$$

$$EA = \frac{CA}{\epsilon_0}\int_0^{d/2} x^2\, dx = \frac{1}{3}\left(\frac{CA}{\epsilon_0}\right)\left(\frac{d^3}{8}\right)$$

Then

$$E = \frac{Cd^3}{24 \in_0}$$

or $\boxed{\vec{\mathbf{E}} = \frac{Cd^3}{24 \in_0}\hat{\mathbf{i}} \text{ for } x > \frac{d}{2}; \qquad \vec{\mathbf{E}} = -\frac{Cd^3}{24 \in_0}\hat{\mathbf{i}} \text{ for } x < -\frac{d}{2}}$

(b) For $-\frac{d}{2} < x < \frac{d}{2}$,

$$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{1}{\in_0}\int dq = \frac{CA}{\in_0}\int_0^x x^2\,dx = \frac{CAx^3}{3 \in_0}$$

$$\boxed{\vec{\mathbf{E}} = \frac{Cx^3}{3 \in_0}\hat{\mathbf{i}} \text{ for } x > 0; \qquad \vec{\mathbf{E}} = -\frac{Cx^3}{3 \in_0}\hat{\mathbf{i}} \text{ for } x < 0}$$

P24.62 First, consider the field at distance $r < R$ from the center of a uniform sphere of positive charge ($Q = +e$) with radius R. From Gauss's law,

$$\int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\in_0}$$

$$\left(4\pi r^2\right)E = \frac{q_{\text{in}}}{\in_0} = \frac{1}{\in_0}\rho V = \frac{1}{\in_0}\left(\frac{+e}{\frac{4}{3}\pi R^3}\right)\frac{4}{3}\pi r^3$$

$$\rightarrow \left(4\pi r^2\right)E = \left(\frac{e}{\in_0 R^3}\right)r^3$$

$$\rightarrow E = \left(\frac{e}{4\pi \in_0 R^3}\right)r, \text{ directed outward}$$

(a) The force exerted on a point charge $q = -e$ located at distance r from the center is then

$$F = qE = -e\left(\frac{e}{4\pi \in_0 R^3}\right)r = -\left(\frac{e^2}{4\pi \in_0 R^3}\right)r = \boxed{-Kr}$$

(b) From (a),

$$K = \frac{e^2}{4\pi \in_0 R^3} = \boxed{\frac{k_e e^2}{R^3}}$$

(c) $F_r = m_e a_r = -\left(\dfrac{k_e e^2}{R^3}\right) r$, so $a_r = -\left(\dfrac{k_e e^2}{m_e R^3}\right) r = -\omega^2 r$

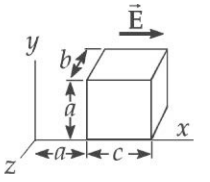
Thus, the motion is simple harmonic with frequency

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{k_e e^2}{m_e R^3}}}$$

(d) $f = 2.47\times 10^{15}\ \text{Hz} = \dfrac{1}{2\pi}\sqrt{\dfrac{(8.99\times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times 10^{-19}\ \text{C})^2}{(9.11\times 10^{-31}\ \text{kg})R^3}}$

which yields $R^3 = 1.05\times 10^{-30}\ \text{m}^3$, or $R = \boxed{1.02\times 10^{-10}\ \text{m}}$

P24.63 (a) The electric field throughout the region is directed along *x*; therefore, $\vec{\mathbf{E}}$ will be perpendicular to normal *dA* over the four faces of the surface which are perpendicular to the *yz* plane, and *E* will be parallel to normal *dA* over the two faces which are parallel to the *yz* plane. Therefore,



ANS. FIG. P24.63

$$\Phi_E = -\left(E_x\big|_{x=a}\right)A + \left(E_x\big|_{x=a+c}\right)A$$

$$\Phi_E = -\left(3 + 2a^2\right)ab + \left[3 + 2(a+c)^2\right]ab$$

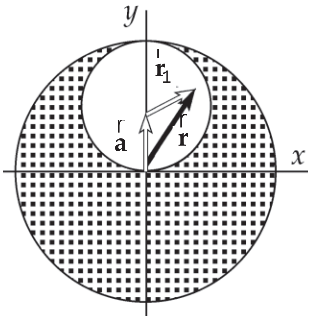
$$\Phi_E = 2abc(2a + c)$$

Substituting the given values for *a*, *b*, and *c*, and noting that the units of electric flux are N · m/C, we find

$$\Phi_E = \boxed{0.269\ \text{N}\cdot\text{m}^2/\text{C}}$$

(b) $\Phi_E = \dfrac{q_{in}}{\epsilon_0} \rightarrow q_{in} = \epsilon_0\,\Phi_E = \boxed{2.38\times 10^{-12}\ \text{C}}$

P24.64 The resultant field within the cavity is the superposition of two fields, one $\vec{\mathbf{E}}_+$ due to a uniform sphere of positive charge of radius 2*a*, and the other $\vec{\mathbf{E}}_-$ due to a sphere of negative charge of radius *a* centered within the cavity.



ANS. FIG. P24.64

$$\frac{4}{3}\left(\frac{\pi r^3 \rho}{\epsilon_0}\right) = 4\pi r^2 E_+$$

so $$\vec{\mathbf{E}}_+ = \frac{\rho r}{3\epsilon_0}\hat{\mathbf{r}} = \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0}$$

$$-\frac{4}{3}\left(\frac{\pi r_1^3\rho}{\epsilon_0}\right) = 4\pi r_1^2 E_-$$

so $$\vec{\mathbf{E}}_- = \frac{\rho r_1}{3\epsilon_0}(-\hat{\mathbf{r}}_1) = \frac{-\rho}{3\epsilon_0}\vec{\mathbf{r}}_1$$

Substituting $\vec{\mathbf{r}} = \vec{\mathbf{a}} + \vec{\mathbf{r}}_1$ gives

$$\vec{\mathbf{E}}_- = \frac{-\rho(\vec{\mathbf{r}} - \vec{\mathbf{a}})}{3\epsilon_0}$$

Adding the fields gives

$$\vec{\mathbf{E}} = \vec{\mathbf{E}}_+ + \vec{\mathbf{E}}_- = \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0} - \frac{\rho\vec{\mathbf{r}}}{3\epsilon_0} + \frac{\rho\vec{\mathbf{a}}}{3\epsilon_0} = \frac{\rho\vec{\mathbf{a}}}{3\epsilon_0} = 0\hat{\mathbf{i}} + \frac{\rho a}{3\epsilon_0}\hat{\mathbf{j}}$$

Thus, $\boxed{E_x = 0}$ and $\boxed{E_y = \frac{\rho a}{3\epsilon_0}}$ at all points within the cavity.

P24.65 By symmetry, the electric field is radial and, therefore, uniform over the gaussian surface:

$$\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E\left(4\pi r^2\right) = \frac{1}{\epsilon_0}\int_0^r \rho\,dV = \frac{1}{\epsilon_0}\int_0^r\left(\frac{a}{r}\right)4\pi r^2\,dr = \frac{a}{\epsilon_0}\int_0^r 4\pi r\,dr$$

$$E\left(4\pi r^2\right) = \frac{2\pi a}{\epsilon_0}r^2$$

$E = \frac{a}{2\epsilon_0}$, $\boxed{\text{radially outward}}$ (if *a* is positive)

P24.66 (a) We call the constant *A'*, reserving the symbol *A* to denote area. The whole charge of the ball is

$$Q = \int_{\text{ball}} dQ = \int_{\text{ball}} \rho\,dV = \int_{r=0}^{R} A'r^2\,4\pi r^2\,dr = 4\pi A'\left.\frac{r^5}{5}\right|_0^R = \frac{4\pi A'R^5}{5}$$

To find the electric field, consider as gaussian surface a concentric sphere of radius *r* outside the ball of charge:

In this case, $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{Q}{\epsilon_0}$ reads $EA\cos 0° = \frac{Q}{\epsilon_0}$

Solving, $E(4\pi r^2) = \dfrac{4\pi A' R^5}{5\in_0}$

and the electric field is $E = \boxed{\dfrac{A'R^5}{5\in_0 r^2}}$

(b) Let the gaussian sphere lie inside the ball of charge:

$$\underset{\substack{\text{spherical surface,}\\ \text{radius } r}}{\oint} \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \underset{\substack{\text{spherical volume,}\\ \text{radius } r}}{\int} dQ / \in_0$$

Now the integrals become

$$E(\cos 0)\oint dA = \int \frac{\rho dV}{\in_0} \quad \text{or} \quad EA = \int_0^r \frac{A'r^2\left(4\pi r^2\right)dr}{\in_0}$$

Performing the integration,

$$E(4\pi r^2) = \left(\frac{A'4\pi}{\in_0}\right)\left(\frac{r^5}{5}\right)\Bigg|_0^r = \frac{A'4\pi r^5}{5\in_0}$$

and the field is $E = \boxed{\dfrac{A'r^3}{5\in_0}}$

P24.67 In this case the charge density is *not uniform*, and Gauss's law is written as $\oint \vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \dfrac{1}{\in_0}\int \rho\, dV$. We use a gaussian surface which is a cylinder of radius *r*, length ℓ, and is coaxial with the charge distribution.

(a) When *r* < *R*, this becomes $E\left(2\pi r\ell\right) = \dfrac{\rho_0}{\in_0}\displaystyle\int_0^r\left(a - \frac{r}{b}\right)dV$. The element of volume is a cylindrical shell of radius *r*, length ℓ, and thickness *dr* so that $dV = 2\pi r\ell dr$.

$E\left(2\pi r\ell\right) = \left(\dfrac{2\pi r^2\ell\rho_0}{\in_0}\right)\left(\dfrac{a}{2} - \dfrac{r}{3b}\right)$ so inside the cylinder,

$$E = \boxed{\frac{\rho_0 r}{2\in_0}\left(a - \frac{2r}{3b}\right)}$$

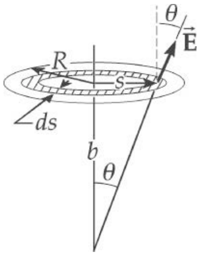
(b) When *r* > *R*, Gauss's law becomes $E\left(2\pi r\ell\right) = \dfrac{\rho_0}{\in_0}\displaystyle\int_0^R\left(a - \frac{r}{b}\right)\left(2\pi r\ell dr\right)$

or outside the cylinder, $E = \boxed{\dfrac{\rho_0 R^2}{2\in_0 r}\left(a - \dfrac{2R}{3b}\right)}$

P24.68 The total flux through a surface enclosing the charge Q is $\frac{Q}{\epsilon_0}$. The flux through the disk is

$$\Phi_{\text{disk}} = \int \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}}$$

where the integration covers the area of the disk. We must evaluate this integral and set it equal to $\frac{1}{4}\frac{Q}{\epsilon_0}$ to find how b and R are related. In the figure, take $d\vec{\mathbf{A}}$ to be the area of an annular ring of radius s and width ds. The flux through $d\vec{\mathbf{A}}$ is $\vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = EdA\cos\theta = E(2\pi s ds)\cos\theta$.



ANS. FIG. P24.68

The magnitude of the electric field has the same value at all points within the annular ring,

$$E = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{Q}{s^2+b^2} \quad \text{and} \quad \cos\theta = \frac{b}{r} = \frac{b}{\left(s^2+b^2\right)^{1/2}}$$

Integrating from $s = 0$ to $s = R$ to get the flux through the entire disk,

$$\Phi_{E,\text{ disk}} = \frac{Qb}{2\epsilon_0}\int_0^R \frac{sds}{\left(s^2+b^2\right)^{3/2}} = \frac{Qb}{2\epsilon_0}\left[-\left(s^2+b^2\right)^{-1/2}\right]\Bigg|_0^R$$

$$= \frac{Q}{2\epsilon_0}\left[1 - \frac{b}{\left(R^2+b^2\right)^{1/2}}\right]$$

The flux through the disk equals $\frac{Q}{4\epsilon_0}$ provided that $\frac{b}{\left(R^2+b^2\right)^{1/2}} = \frac{1}{2}$.

This is satisfied if $\boxed{R = \sqrt{3}b}$.

P24.69 (a) The slab has left-to-right symmetry, so its field must be equal in strength at x and at $-x$. The field points everywhere away from the central plane. Take as gaussian surface a rectangular box of thickness $2x$ and height and width L, centered on the $x = 0$ plane. The gaussian surface, shown shaded in the second panel of ANS. FIG. P24.69, lies inside the slab. The charge the surface contains is $\rho V = \rho(2xL^2)$. The total flux leaving it is EL^2 through the right face, EL^2 through the left face, and zero through each of the other four sides.

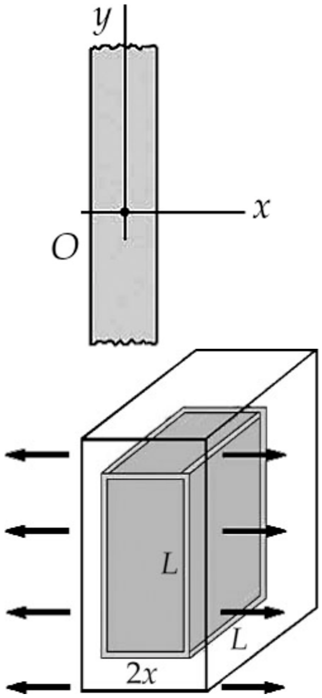
Thus Gauss's law,

$$\oint \vec{\mathbf{E}} \cdot d\vec{\mathbf{A}} = \frac{q}{\epsilon_0}$$

becomes

$$2EL^2 = \frac{\rho 2xL^2}{\epsilon_0}$$

so the field is $E = \boxed{\dfrac{\rho x}{\epsilon_0}}$



ANS. FIG. P24.69

(b) The electron experiences a force opposite to $\vec{\mathbf{E}}$. When displaced to $x > 0$, it experiences a restoring force to the left. For the electron, Newton's second law gives

$$\sum \vec{\mathbf{F}} = m_e \vec{\mathbf{a}}:$$

$$q\vec{\mathbf{E}} = m_e \vec{\mathbf{a}} \quad \text{or} \quad \frac{-e\rho x\hat{\mathbf{i}}}{\epsilon_0} = m_e \vec{\mathbf{a}}$$

Solving for the acceleration,

$$\vec{\mathbf{a}} = -\left(\frac{e\rho}{m_e \epsilon_0}\right) x\hat{\mathbf{i}} \quad \text{or} \quad \vec{\mathbf{a}} = -\omega^2 x\hat{\mathbf{i}}$$

That is, its acceleration is proportional to its displacement and oppositely directed, as is required for simple harmonic motion.

Solving for the frequency, $\omega^2 = \dfrac{e\rho}{m_e \epsilon_0}$ and

$$f = \frac{\omega}{2\pi} = \boxed{\frac{1}{2\pi}\sqrt{\frac{e\rho}{m_e \epsilon_0}}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P24.2	$355\ \text{kN}\cdot\text{m}^2/\text{C}$
P24.4	(a) $-2.34\ \text{kN}\cdot\text{m}^2/\text{C}$; (b) $+2.34\ \text{kN}\cdot\text{m}^2/\text{C}$; (c) 0
P24.6	$chw^2/2$
P24.8	$-226\ \text{N}\cdot\text{m}^2/\text{C}$
P24.10	(a) –55.7 nC; (b) negative, spherically symmetric
P24.12	(a) $3.20\times10^6\ \text{N}\cdot\text{m}^2/\text{C}$; (b) $1.92\times10^7\ \text{N}\cdot\text{m}^2/\text{C}$; (c) The answer to part (a) would change because the charge could now be at different distances from each face of the cube. The answer to part (b) would be unchanged because the flux through the entire closed surface depends only on the total charge inside the surface.
P24.14	(a) $1.36\ \text{MN}\cdot\text{m}^2/\text{C}$; (b) $678\ \text{kN}\cdot\text{m}^2/\text{C}$; (c) no
P24.16	(a) $\dfrac{q}{2\epsilon_0}$; (b) $\dfrac{q}{2\epsilon_0}$; (c) The plane and the square look the same to the charge.
P24.18	(a) The net flux is zero through the sphere because the number of field lines entering the sphere equals the number of lines leaving the sphere; (b) The net flux is $2\pi R^2E$ through the cylinder; (c) The net charge inside the cylinder is positive and is distributed on a plane parallel to the ends of the cylinder.
P24.20	$\dfrac{Q-6\lvert q\rvert}{6\epsilon_0}$
P24.22	(a) $EA\cos\theta$; (b) $-EA\sin\theta$; (c) $-EA\cos\theta$; (d) $EA\sin\theta$; (e) 0; (f) 0; (g) 0
P24.24	(a) 16.2 MN/C; (b) 8.09 MN/C; (c) 1.62 MN/C
P24.26	2.33×10^{21} N/C
P24.28	(a) $\sim10^{-3}$ N or 1 mN; (b) $\sim10^{-7}$ C or 100 nC; (c) ~10 kN/C; (d) $\sim10\ \text{kN}\cdot\text{m}^2/\text{C}$
P24.30	(a) 4.86×10^9 N/C away from the wall; (b) So long as the distance from the wall is small compared to the width and height of the wall, the distance does not affect the field.
P24.32	(a) $15.0\ \text{N}\cdot\text{m}^2/\text{C}$; (b) 1.33×10^{-10} C; (c) No; fields on the faces would not be uniform.
P24.34	(a) +913 nC; (b) 0

P24.36 5.94×10^5 m/s

P24.38 The electric field just outside the surface occurs at 16.0 kN/C. The peak in the figure occurs at about 6.5 kN/C. Therefore, it is not possible that this figure represents the electric field for the given situation.

P24.40 See ANS. FIG. P24.40.

P24.42 (a) 31.9 nC/m^3; (b) No; then the field would have to be zero.

P24.44 (a) 708 nC/m^2; (b) 177 nC

P24.46 (a) 80.0 nC/m^2; (b) $(9.04\ \text{kN/C})\hat{\mathbf{k}}$; (c) $(-9.04\ \text{kN/C})\hat{\mathbf{k}}$

P24.48 780 N/C

P24.50 (a) The charge on the exterior surface is –55.7 nC distributed uniformly; (b) The charge on the interior surface is +55.7 nC. It can have any distribution; (c) The charge within the shell is –55.7 nC. It can have any distribution.

P24.52 $\dfrac{Q}{2\in_0}(1-\cos\theta)$

P24.54 (a) $Q\left(\dfrac{r}{R}\right)^3$; (b) $k_e\dfrac{Qr}{a^3}$; (c) Q; (d) $k_e\dfrac{Q}{r^2}$; (e) $E = 0$; (f) $-Q$; (g) $+Q$; (h) inner surface of radius b

P24.56 (a) 0; (b) $\dfrac{\sigma}{\in_0}$ to the right; (c) 0; (d) (1) $2\dfrac{\sigma}{\in_0}$ to the left; (2) 0; (3) $2\dfrac{\sigma}{\in_0}$ to the right

P24.58 (a) $\dfrac{Qr^3}{\in_0 a^3}$; (b) $\dfrac{Q}{\in_0}$; (c) See ANS. FIG. P24.58(c).

P24.60 (a) $2k_e\dfrac{\lambda}{r}$, outward; (b) $\dfrac{2k_e}{r}\left[\lambda + \rho\pi\left(r^2 - a^2\right)\right]$, outward;
(c) $\dfrac{2k_e}{r}\left[\lambda + \rho\pi\left(b^2 - a^2\right)\right]$, outward

P24.62 (a) $-Kr$; (b) $\dfrac{k_e e^2}{R^3}$; (c) $\dfrac{1}{2\pi}\sqrt{\dfrac{k_e e^2}{m_e R^3}}$; (d) 1.02×10^{-10} m

P24.64 $E_x = 0$ and $E_y = \dfrac{\rho a}{3\in_0}$

P24.66 (a) $AR^5/5\in_0 r^2$; (b) $AR^5/5\in_0$

P24.68 $R = \sqrt{3}b$

25

Electric Potential

CHAPTER OUTLINE

25.1	Electric Potential and Potential Difference
25.2	Potential Difference in a Uniform Electric Field
25.3	Electric Potential and Potential Energy Due to Point Charges
25.4	Obtaining the Value of the Electric Field from the Electric Potential
25.5	Electric Potential Due to Continuous Charge Distributions
25.6	Electric Potential Due to a Charged Conductor
25.7	The Millikan Oil-Drop Experiment
25.8	Applications of Electrostatics

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ25.1 Answer (b). Taken without reference to any other point, the potential could have any value.

OQ25.2 Answer (d). The potential is decreasing toward the bottom of the page, so the electric field is downward.

OQ25.3 **(i)** Answer (c). The two spheres come to the same potential, so *Q*/*R* is the same for both. If charge *q* moves from A to B, we find the charge on B:

$$\frac{Q_A}{R_A}=\frac{Q_B}{R_B}\rightarrow\frac{450\text{ nC}-q}{1.00\text{ cm}}=\frac{q}{2.00\text{ cm}}\rightarrow q=\frac{900\text{ nC}}{3}=300\text{ nC}$$

Sphere A has charge 450 nC – 300 nC = 150 nC.

(ii) Answer (a). Contact between conductors allows all charge to flow to the exterior surface of sphere B.

OQ25.4 Answer (d).

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{\left(1.90\times 10^2\text{ V} - 1.20\times 10^2\text{ V}\right)}{\left(5.00\text{ m} - 3.00\text{ m}\right)} = -35.0\text{ N/C}$$

OQ25.5 Ranking a > b = d > c. The potential energy of a system of two charges is $U = k_e q_1 q_2 / r$. The potential energies are: (a) $U = 2k_e Q^2/r$, (b) $U = k_e Q^2/r$, (c) $U = -k_e Q^2/2r$, (d) $U = k_e Q^2/r$.

OQ25.6 (**i**) Answer (a). The particle feels an electric force in the negative x direction. An outside agent pushes it uphill against this force, increasing the potential energy.

(**ii**) Answer (c). The potential decreases in the direction of the electric field.

OQ25.7 Ranking $D > C > B > A$. Let L be length of a side of the square. The potentials are:

$$V_A = \frac{k_e Q}{L} + \frac{2k_e Q}{\sqrt{2}L} = \left(1+\sqrt{2}\right)\frac{k_e Q}{L}$$

$$V_B = \frac{2k_e Q}{L} + \frac{k_e Q}{\sqrt{2}L} = \left(2+\frac{1}{\sqrt{2}}\right)\frac{k_e Q}{L}$$

$$V_C = \frac{k_e Q}{\sqrt{2}L/2} + \frac{2k_e Q}{\sqrt{2}L/2} = 3\sqrt{2}\frac{k_e Q}{L}$$

$$V_D = \frac{k_e Q}{L/2} + \frac{2k_e Q}{L/2} = 6\frac{k_e Q}{L}$$

OQ25.8 Answer (a). The change in kinetic energy is the negative of the change in electric potential energy:

$$\Delta K = -q\Delta V = -(-e)V = e\left(1.00\times 10^4\text{ V}\right) = 1.00\times 10^4\text{ eV}$$

OQ25.9 Ranking c > a > d > b. We add the electric potential energies of all possible pairs. They are:

(a) $3\frac{k_e Q^2}{d}$

(b) $-2\frac{k_e Q^2}{d} + \frac{k_e Q^2}{d} = -\frac{k_e Q^2}{d}$

(c) $4\frac{k_e Q^2}{d} + 2\frac{k_e Q^2}{\sqrt{2}d} = \left(4+\sqrt{2}\right)\frac{k_e Q^2}{d}$

(d) $2\frac{k_e Q^2}{d} + \frac{k_e Q^2}{\sqrt{2}d} - 2\frac{k_e Q^2}{d} - \frac{k_e Q^2}{\sqrt{2}d} = 0$

OQ25.10 Answer (b). All charges are the same distance from the center. The potentials from the +1.50-*μ*C, –1.00-*μ*C, and –0.500-*μ*C charges cancel.

OQ25.11 Answer (b). The work done on the proton equals the negative of the change in electric potential energy:

$$W = -q\Delta V \rightarrow q\Delta V = -W = -qEs\cos\theta$$

$$= -e\left(8.50\times10^{2}\ \mathrm{N/C}\right)\left(2.50\ \mathrm{m}\right)(1) = -3.40\times10^{-16}\ \mathrm{J}$$

OQ25.12 (**i**) Answer (b). At points off the x axis the electric field has a nonzero y component. At points on the negative x axis the field is to the right and positive. At points to the right of x = 0.500 m the field is to the left and nonzero. The field is zero at one point between x = 0.250 m and x = 0.500 m.

(**ii**) Answer (c). The electric potential is negative at this and at all points because both charges are negative.

(**iii**) Answer (d). The potential cannot be zero at a finite distance because both charges are negative.

OQ25.13 Answer (b). The same charges at the same distance away create the same contribution to the total potential.

OQ25.14 The ranking is e > d > a = c > b. The change in kinetic energy is the negative of the change in electric potential energy, so we work out $-q\Delta V = -q\left(V_f - V_i\right)$ in each case.

(a) $-(-e)(60\ \mathrm{V} - 40\ \mathrm{V}) = +20\ \mathrm{eV}$

(b) $-(-e)(20\ \mathrm{V} - 40\ \mathrm{V}) = -20\ \mathrm{eV}$

(c) $-(e)(20\ \mathrm{V} - 40\ \mathrm{V}) = +20\ \mathrm{eV}$

(d) $-(e)(10\ \mathrm{V} - 40\ \mathrm{V}) = +30\ \mathrm{eV}$

(e) $-(-2e)(60\ \mathrm{V} - 40\ \mathrm{V}) = +40\ \mathrm{eV}$

OQ25.15 Answer (b). The change in kinetic energy is the negative of the change in electric potential energy:

$$\Delta K = -q\Delta V \rightarrow K_B - K_A = q\left(V_A - V_B\right)$$

$$\frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 + (2e)\left(V_A - V_B\right)$$

Solving for the speed gives

$$v_B = \sqrt{v_A^2 + \frac{4e\left(V_A - V_B\right)}{m}}$$

$$= \sqrt{\left(6.20\times10^{5}\mathrm{m/s}\right)^2 + \frac{4\left(1.60\times10^{-19}\mathrm{C}\right)\left(1.50\times10^{3}\mathrm{V} - 4.00\times10^{3}\mathrm{V}\right)}{6.63\times10^{-27}\mathrm{kg}}}$$

$$= 3.78\times10^{5}\ \mathrm{m/s}$$

ANSWERS TO CONCEPTUAL QUESTIONS

CQ25.1 The main factor is the radius of the dome. One often overlooked aspect is also the humidity of the air—drier air has a larger dielectric breakdown strength, resulting in a higher attainable -electric potential. If other grounded objects are nearby, the maximum potential might be reduced.

CQ25.2 (a) The proton accelerates in the direction of the electric field, (b) its kinetic energy increases as (c) the electric potential energy of the system decreases.

CQ25.3 To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system by an outside agent. Hence energy is stored, and potential energy is positive. As charges with opposite signs move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.

CQ25.4 (a) The grounding wire can be touched equally well to any point on the sphere. Electrons will drain away into the ground.

(b) The sphere will be left positively charged. The ground, wire, and sphere are all conducting. They together form an equipotential volume at zero volts during the contact. However close the grounding wire is to the negative charge, electrons have no difficulty in moving within the metal through the grounding wire to ground. The ground can act as an infinite source or sink of electrons. In this case, it is an electron sink.

CQ25.5 When one object B with electric charge is immersed in the electric field of another charge or charges A, the system possesses electric potential energy. The energy can be measured by seeing how much work the field does on the charge B as it moves to a reference location. We choose not to visualize A's effect on B as an action-at-a-distance, but as the result of a two-step process: Charge A creates electric potential throughout the surrounding space. Then the potential acts on B to inject the system with energy.

CQ25.6 (a) The electric field is cylindrically radial. The equipotential surfaces are nesting coaxial cylinders around an infinite line of charge.

(b) The electric field is spherically radial. The equipotential surfaces are nesting concentric spheres around a uniformly charged sphere.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 25.1 Electric Potential and Potential Difference

Section 25.2 Potential Difference in a Uniform Electric Field

***P25.1** (a) From Equation 25.6,

$$E = \frac{|\Delta V|}{d} = \frac{600\ \text{J/C}}{5.33\times 10^{-3}\ \text{m}} = \boxed{1.13\times 10^{5}\ \text{N/C}}$$

(b) The force on an electron is given by

$$F = |q|E = \left(1.60\times 10^{-19}\ \text{C}\right)\left(1.13\times 10^{5}\ \text{N/C}\right) = \boxed{1.80\times 10^{-14}\ \text{N}}$$

(c) Because the electron is repelled by the negative plate, the force used to move the electron must be applied in the direction of the electron's displacement. The work done to move the electron is

$$W = F\cdot s\cos\theta = \left(1.80\times 10^{-14}\ \text{N}\right)\left[(5.33-2.00)\times 10^{-3}\ \text{m}\right]\cos 0°$$

$$= \boxed{4.37\times 10^{-17}\ \text{J}}$$

***P25.2** (a) We follow the path from (0, 0) to (20.0 cm, 0) to (20.0 cm, 50.0 cm).

$$\Delta U = -\,(\text{work done})$$

$$\Delta U = -\,[\text{work from origin to (20.0 cm, 0)}]$$
$$-\,[\text{work from (20.0 cm, 0) to (20.0 cm, 50.0 cm)}]$$

Note that the last term is equal to 0 because the force is perpendicular to the displacement.

$$\Delta U = -\left(qE_x\right)\Delta x = -\left(12.0\times 10^{-6}\ \text{C}\right)\left(250\ \text{V/m}\right)\left(0.200\ \text{m}\right)$$

$$= \boxed{-6.00\times 10^{-4}\ \text{J}}$$

(b) $$\Delta V = \frac{\Delta U}{q} = -\frac{6.00\times 10^{-4}\ \text{J}}{12.0\times 10^{-6}\ \text{C}} = -50.0\ \text{J/C} = \boxed{-50.0\ \text{V}}$$

P25.3 (a) Energy of the proton-field system is conserved as the proton moves from high to low potential, which can be defined for this problem as moving from 120 V down to 0 V.

$$K_i + U_i = K_f + U_f: \qquad 0 + qV = \frac{1}{2}mv_p^2 + 0$$

$$\left(1.60\times 10^{-19}\ \text{C}\right)\left(120\ \text{V}\right)\left(\frac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right) = \frac{1}{2}\left(1.67\times 10^{-27}\ \text{kg}\right)v_p^2$$

$$v_p = \boxed{1.52\times 10^{5}\ \text{m/s}}$$

(b) The electron will gain speed in moving the other way, from $V_i = 0$ to $V_f = 120$ V: $K_i + U_i = K_f + U_f$

$$0 + 0 = \frac{1}{2}mv_e^2 + qV$$

$$0 = \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)v_e^2 + \left(-1.60\times10^{-19}\text{ C}\right)\left(120\text{ J/C}\right)$$

$$v_e = \boxed{6.49\times10^{6}\text{ m/s}}$$

P25.4 The potential difference is

$$\Delta V = V_f - V_i = -5.00\text{ V} - 9.00\text{ V} = -14.0\text{ V}$$

and the total charge to be moved is

$$Q = -N_Ae = -\left(6.02\times10^{23}\right)\left(1.60\times10^{-19}\text{ C}\right) = -9.63\times10^{4}\text{ C}$$

Now, from $\Delta V = \frac{W}{Q}$, we obtain

$$W = Q\Delta V = (-9.63\times10^{4}\text{ C})(-14.0\text{ J/C}) = \boxed{1.35\text{ MJ}}$$

P25.5 The electric field is uniform. By Equation 25.3,

$$V_B - V_A = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} = -\int_A^C \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}} - \int_C^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$$

$$V_B - V_A = \left(-E\cos180°\right)\int_{-0.300}^{0.500} dy - \left(E\cos90.0°\right)\int_{-0.200}^{0.400} dx$$

$$V_B - V_A = \left(325\text{ V/m}\right)\left(0.800\text{ m}\right) = \boxed{+260\text{ V}}$$

P25.6 Assume the opposite. Then at some point *A* on some equipotential surface the electric field has a nonzero component E_p in the plane of the surface. Let a test charge start from point *A* and move some distance on the surface in the direction of the field component. Then $\Delta V = -\int_A^B \vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ is nonzero. The electric potential charges across the surface and it is not an equipotential surface. The contradiction shows that our assumption is false, that $E_p = 0$, and that the field is perpendicular to the equipotential surface.

P25.7 We use the energy version of the isolated system model to equate the energy of the electron-field system when the electron is at $x = 0$ to the energy when the electron is at $x = 2.00$ cm. The unknown will be the difference in potential $V_f - V_i$. Thus, $K_i + U_i = K_f + U_f$ becomes

$$\frac{1}{2}mv_i^2 + qV_i = \frac{1}{2}mv_f^2 + qV_f$$

or $$\frac{1}{2}m\left(v_i^2 - v_f^2\right) = q\left(V_f - V_i\right),$$

so $$V_f - V_i = \Delta V = \frac{m\left(v_i^2 - v_f^2\right)}{2q}.$$

(a) Noting that the electron's charge is negative, and evaluating the potential difference, we have

$$\Delta V = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left[(3.70\times10^{6}\text{ m/s})^2 - (1.40\times10^{5}\text{ m/s})^2\right]}{2\left(-1.60\times10^{-19}\text{ C}\right)}$$

$$= \boxed{-38.9\text{ V}}$$

(b) The negative sign means that the 2.00-cm location is lower in potential than the origin:

$$\boxed{\text{The origin is at the higher potential.}}$$

P25.8 (a) The electron-electric field is an isolated system:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_e v_i^2 + (-e)V_i = 0 + (-e)V_f$$

$$e\left(V_f - V_i\right) = -\frac{1}{2}m_e v_i^2$$

The potential difference is then

$$\Delta V_e = -\frac{m_e v_i^2}{2e} = -\frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(2.85\times10^{7}\text{ m/s}\right)^2}{2\left(1.60\times10^{-19}\text{ C}\right)}$$

$$= -2.31\times10^{3}\text{ V} = \boxed{-2.31\text{ kV}}$$

(b) From (a), we see that the stopping potential is proportional to the kinetic energy of the particle.

$$\boxed{\begin{array}{l}\text{Because a proton is more massive than an electron, a proton}\\ \text{traveling at the same speed as an electron has more initial kinetic}\\ \text{energy and requires a greater magnitude stopping potential.}\end{array}}$$

(c) The proton-electric field is an isolated system:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_p v_i^2 + eV_i = 0 + eV_f$$

$$e\left(V_f - V_i\right) = \frac{1}{2}m_p v_i^2$$

The potential difference is

$$\Delta V_p = \frac{m_p v_i^2}{2e}$$

Therefore, from (a),

$$\frac{\Delta V_p}{\Delta V_e} = \frac{m_p v_i^2/2e}{-m_e v_i^2/2e} \quad \rightarrow \quad \boxed{\Delta V_p/\Delta V_e = -m_p/m_e}$$

P25.9 Arbitrarily take $V = 0$ at point P. Then the potential at the original position of the charge is (by Equation 25.3)

$$\Delta V = V - 0 = V = -\vec{\mathbf{E}}\cdot\vec{\mathbf{s}} = -EL\cos\theta \quad \text{(relative to } P\text{)}$$

At the final point a,

$$V = -EL \qquad \text{(relative to } P\text{)}$$

Because the table is frictionless and the particle-field system is isolated, we have

$$\left(K + U\right)_i = \left(K + U\right)_f$$

or

$$0 - qEL\cos\theta = \frac{1}{2}mv^2 - qEL$$

solving for the speed gives

$$v = \sqrt{\frac{2qEL\left(1-\cos\theta\right)}{m}}$$

$$= \sqrt{\frac{2\left(2.00\times10^{-6}\text{ C}\right)\left(300\text{ N/C}\right)\left(1.50\text{ m}\right)\left(1-\cos60.0^\circ\right)}{0.010\,0\text{ kg}}}$$

$$= \boxed{0.300\text{ m/s}}$$

P25.10 (a) The system consisting of the mass-spring-electric field is $\boxed{\text{isolated.}}$

(b) The system has both $\boxed{\text{electric potential energy and elastic potential energy}}$: U_e and U_{sp}.

(c) Taking the electric potential to be zero at the initial configuration, after the block has stretched the spring a distance *x*, the final electric potential is (from equation 25.3)

$$\Delta V = V = -\vec{\mathbf{E}}\cdot\vec{\mathbf{s}} = -Ex$$

By energy conservation within the system,

$$\left(K + U_{sp} + U_e\right)_i = \left(K + U_{sp} + U_e\right)_f$$

$$0 + 0 + 0 = 0 + \frac{1}{2}kx^2 + QV$$

$$0 = \frac{1}{2}kx^2 + Q(-Ex) \quad \rightarrow \quad x = \boxed{\frac{2QE}{k}}$$

(d) Particle in equilibrium

(e) $\sum F = 0 \quad \rightarrow \quad -kx_0 + QE = 0 \quad \rightarrow \quad x_0 = \boxed{\frac{QE}{k}}$

(f) The particle is no longer in equilibrium; therefore, the force equation becomes

$$\sum F = ma \quad \rightarrow \quad -kx + QE = m\frac{d^2x}{dt^2}$$

$$-k\left(x - \frac{QE}{k}\right) = m\frac{d^2x}{dt^2}$$

Defining $x' = x - x_0$, we have $\frac{d^2x'}{dt^2} = \frac{d^2(x - x_0)}{dt^2} = \frac{d^2x}{dt^2}$.

Substitute $x' = x - x_0$ into the force equation:

$$-k\left(x - \frac{QE}{k}\right) = m\frac{d^2x}{dt^2} \quad \rightarrow \quad -kx' = m\frac{d^2x'}{dt^2}$$

$$\rightarrow \quad \boxed{\frac{d^2x'}{dt^2} = -\frac{kx'}{m}}$$

(g) The result of part (f) is the equation for simple harmonic motion $a_{x'} = -\omega^2 x'$ with

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \quad \rightarrow \quad T = \frac{2\pi}{\omega} = \boxed{2\pi\sqrt{\frac{m}{k}}}$$

(h) The period does not depend on the electric field. The electric field just shifts the equilibrium point for the spring, just like a gravitational field does for an object hanging from a vertical spring.

P25.11 Arbitrarily take $V = 0$ at the initial point. Then at distance d downfield, where L is the rod length, $V = -Ed$ and $U_e = -\lambda LEd$.

(a) The rod-field system is isolated:

$$K_i + U_i = K_f + U_f$$

$$0 + 0 = \frac{1}{2}m_{\text{rod}}v^2 - qV$$

$$0 = \frac{1}{2}\mu Lv^2 - \lambda LEd$$

$$\frac{1}{2}\mu Lv^2 = \lambda LEd$$

Solving for the speed gives

$$v = \sqrt{\frac{2\lambda Ed}{\mu}} = \sqrt{\frac{2\left(40.0\times10^{-6}\text{ C/m}\right)\left(100\text{ N/C}\right)\left(2.00\text{ m}\right)}{\left(0.100\text{ kg/m}\right)}}$$

$$= \boxed{0.400\text{ m/s}}$$

(b) $\boxed{\text{The same.}}$ Each bit of the rod feels a force of the same size as before.

Section 25.3 Electric Potential and Potential Energy Due to Point Charges

P25.12 (a) At a distance of 0.250 cm from an electron, the electric potential is

$$V = k_e\frac{q}{r} = \left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-1.60\times10^{-19}\text{ C}}{0.250\times10^{-2}\text{ m}}\right)$$

$$= \boxed{-5.76\times10^{-7}\text{ V}}$$

(b) The difference in potential between the two points is given by

$$|\Delta V| = \left|k_e\frac{q}{r_2} - k_e\frac{q}{r_1}\right| = k_eq\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Substituting numerical values,

$$|\Delta V| = \left|\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\text{ C}\right)\times\left(\frac{1}{0.250\times10^{-2}\text{ m}} - \frac{1}{0.750\times10^{-2}\text{ m}}\right)\right|$$

$$|\Delta V| = \boxed{3.84\times10^{-7}\text{ V}}$$

(c) $\boxed{\text{Because the charge of the proton has the same magnitude as that of the electron, only the sign of the answer to part (a) would change.}}$

P25.13 The total electric potential is the sum of the potentials from the individual charges,

$$V = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

(a) The 4.50-μC and –2.24-μC charges are distances 1.25 cm and 1.80 cm, respectively, from the origin. The electric potential is then

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{4.50\times10^{-6}\ \text{C}}{1.25\times10^{-2}\ \text{m}} + \frac{-2.24\times10^{-6}\ \text{C}}{1.80\times10^{-2}\ \text{m}}\right]$$

$$V = \boxed{2.12\times10^6\ \text{V}}$$

(b) The distance of the 4.50-μC charge to the point is

$$r_1 = \sqrt{(0.0150\ \text{m})^2 + (0.0125\ \text{m})^2} = 0.0195\ \text{m},$$

and the distance of the –2.24-μC charge to the point is

$$r_2 = \sqrt{(0.0150\ \text{m})^2 + (0.0180\ \text{m})^2} = 0.0234\ \text{m}$$

The potential is

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{4.50\times10^{-6}\ \text{C}}{r_1} + \frac{-2.24\times10^{-6}\ \text{C}}{r_2}\right]$$

$$V = \boxed{1.21\times10^6\ \text{V}}$$

P25.14 The potential due to the two charges is given by $V = k_e \sum_i \frac{q_i}{r_i}$.

(a) The electric potential at point *A* is

$$V = k_e \sum_i \frac{q_i}{r_i} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)$$

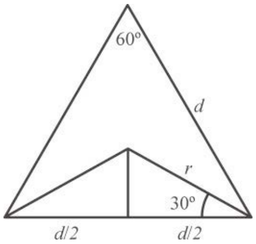
$$\times\left(\frac{-15.0\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}} + \frac{27.0\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}}\right)$$

$$= \boxed{5.39\ \text{kV}}$$

(b) The electric potential at point *B* is

$$V = k_e \sum_i \frac{q_i}{r_i} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right) \times \left(\frac{-15.0\times10^{-9}\ \text{C}}{1.00\times10^{-2}\ \text{m}} + \frac{27.0\times10^{-9}\ \text{C}}{1.00\times10^{-2}\ \text{m}}\right) = \boxed{10.8\ \text{kV}}$$

P25.15 By symmetry, a line from the center to each vertex forms a 30° angle with each side of the triangle. The figure shows the relationship between the length *d* of a side of the equilateral triangle and the distance *r* from a vertex to the center:



ANS. FIG. P25.15

$$r\cos 30.0° = d/2$$
$$\rightarrow r = d/\left(2\cos 30.0°\right)$$

The electric potential at the center is

$$V = k_e \sum_i \frac{q_i}{r_i}$$
$$= k_e\left(\frac{Q}{d/\left(2\cos 30.0°\right)} + \frac{Q}{d/\left(2\cos 30.0°\right)} + \frac{2Q}{d/\left(2\cos 30.0°\right)}\right)$$
$$V = (4)\left(2\cos 30.0°\, k_e \frac{Q}{d}\right) = \boxed{6.93 k_e \frac{Q}{d}}$$

***P25.16** (a) From Equation 25.12, the electric potential due to the two charges is

$$V = k_e \sum_i \frac{q_i}{r_i} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right) \times \left(\frac{5.00\times10^{-9}\ \text{C}}{0.175\ \text{m}} + \frac{-3.00\times10^{-9}\ \text{C}}{0.175\ \text{m}}\right) = \boxed{103\ \text{V}}$$

(b) The potential energy of the pair of charges is

$$U = \frac{k_e q_1 q_2}{r_{12}} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right) \times \frac{\left(5.00\times10^{-9}\ \text{C}\right)\left(-3.00\times10^{-9}\ \text{C}\right)}{0.350\ \text{m}} = \boxed{-3.85\times10^{-7}\ \text{J}}$$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges by an infinite distance (that is, to bring them to a state of zero potential energy).

***P25.17** (a) In an empty universe, the 20.0-nC charge can be placed at its location with no energy investment. At a distance of 4.00 cm, it creates a potential

$$V_1 = \frac{k_e q_1}{r} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(20.0\times10^{-9}\ \text{C}\right)}{0.040\ 0\ \text{m}} = 4.50\ \text{kV}$$

To place the 10.0-nC charge there we must put in energy

$$U_{12} = q_2V_1 = \left(10.0\times10^{-9}\ \text{C}\right)\left(4.50\times10^3\ \text{V}\right) = 4.50\times10^{-5}\ \text{J}$$

Next, to bring up the –20.0-nC charge requires energy

$$\begin{aligned} U_{23}+U_{13} &= q_3V_2+q_3V_1 = q_3\left(V_2+V_1\right) \\ &= \left(-20.0\times10^{-9}\ \text{C}\right)\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right) \\ &\qquad\times\left(\frac{10.0\times10^{-9}\ \text{C}}{0.040\ 0\ \text{m}}+\frac{20.0\times10^{-9}\ \text{C}}{0.080\ 0\ \text{m}}\right) \\ &= -4.50\times10^{-5}\ \text{J}-4.50\times10^{-5}\ \text{J} \end{aligned}$$

The total energy of the three charges is

$$U_{12}+U_{23}+U_{13} = \boxed{-4.50\times10^{-5}\ \text{J}}$$

(b) The three fixed charges create this potential at the location where the fourth is released:

$$\begin{aligned} V &= V_1+V_2+V_3 \\ &= \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right) \\ &\quad\times\left(\frac{20.0\times10^{-9}\ \text{C}}{\sqrt{(0.040\ 0\ \text{m})^2+(0.030\ 0\ \text{m})^2}}\right. \\ &\qquad\left.+\frac{10.0\times10^{-9}\ \text{C}}{0.030\ 0\ \text{m}}-\frac{20.0\times10^{-9}\ \text{C}}{\sqrt{(0.040\ 0\ \text{m})^2+(0.030\ 0\ \text{m})^2}}\right) \\ V &= 3.00\times10^3\ \text{V} \end{aligned}$$

Energy of the system of four charged objects is conserved as the fourth charge flies away:

$$\left(\frac{1}{2}mv^2+qV\right)_i = \left(\frac{1}{2}mv^2+qV\right)_f$$

$$0+\left(40.0\times10^{-9}\ \text{C}\right)\left(3.00\times10^3\ \text{V}\right) = \frac{1}{2}\left(2.00\times10^{-13}\ \text{kg}\right)v^2+0$$

$$v = \sqrt{\frac{2\left(1.20\times10^{-4}\ \text{J}\right)}{2\times10^{-13}\ \text{kg}}} = \boxed{3.46\times10^4\ \text{m/s}}$$

P25.18 (a) $V_A = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{Q}{d} + \frac{2Q}{d\sqrt{2}}\right) = k_e \frac{Q}{d}\left(1+\sqrt{2}\right)$

$$V = \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{5.00\times10^{-9}\ \mathrm{C}}{2.00\times10^{-2}\ \mathrm{m}}\right)\left(1+\sqrt{2}\right) = \boxed{5.43\ \mathrm{kV}}$$

(b) $V_B = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{Q}{d\sqrt{2}} + \frac{2Q}{d}\right) = k_e \frac{Q}{d}\left(\frac{1}{\sqrt{2}}+2\right)$

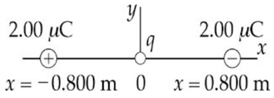
$$V_B = \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{5.00\times10^{-9}\ \mathrm{C}}{2.00\times10^{-2}\ \mathrm{m}}\right)\left(\frac{1}{\sqrt{2}}+2\right) = \boxed{6.08\ \mathrm{kV}}$$

(c) $V_B - V_A = k_e\frac{Q}{d}\left(\frac{1}{\sqrt{2}}+2\right) - k_e\frac{Q}{d}\left(1+\sqrt{2}\right) = k_e\frac{Q}{d}\left(\frac{1}{\sqrt{2}}+1-\sqrt{2}\right)$

$$V_B - V_A = \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{5.00\times10^{-9}\ \mathrm{C}}{2.00\times10^{-2}\ \mathrm{m}}\right)\left(\frac{1}{\sqrt{2}}+1-\sqrt{2}\right)$$

$$= \boxed{658\ \mathrm{V}}$$

P25.19 (a) Since the charges are equal and placed symmetrically, $\boxed{F = 0}$.



ANS. FIG. P25.19

(b) Since $F = qE = 0$, $\boxed{E = 0}$.

(c) $V = 2k_e\frac{q}{r}$

$$= 2\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{2.00\times10^{-6}\ \mathrm{C}}{0.800\ \mathrm{m}}\right)$$

$$V = 4.50\times10^{4}\ \mathrm{V} = \boxed{45.0\ \mathrm{kV}}$$

P25.20 At a distance r from a charged particle, the voltage is $V = \frac{k_eQ}{r}$ and the field magnitude is $E = \frac{k_e|Q|}{r^2}$.

(a) $r = \frac{|V|}{|E|} = \frac{3.00\times10^{3}\ \mathrm{V}}{5.00\times10^{2}\ \mathrm{V/m}} = \boxed{6.00\ \mathrm{m}}$

(b) $V = -3\,000\ \mathrm{V} = \frac{Q}{4\pi\epsilon_0\left(6.00\ \mathrm{m}\right)}$

Then,

$$Q = \left(6.00\ \mathrm{m}\right)\left(-3\,000\ \mathrm{V}\right)\left(4\pi\epsilon_0\right) = \boxed{-2.00\ \mu\mathrm{C}}$$

P25.21 (a) Each charge is a distance $\sqrt{a^2 + a^2}/2 = a/\sqrt{2}$ from the center.

$$V = k_e \sum_i \frac{q_i}{r_i} = 4k_e\left(\frac{Q}{a/\sqrt{2}}\right) = \boxed{4\sqrt{2}k_e\frac{Q}{a}}$$

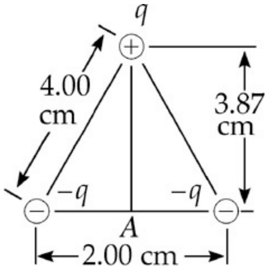
(b) The potential at infinity is zero. The work done by an external agent is

$$W = q\Delta V = q\left(V_f - V_i\right) = q\left(4\sqrt{2}k_e\frac{Q}{a} - 0\right) = \boxed{4\sqrt{2}k_e\frac{qQ}{a}}$$

P25.22 The charges at the base vertices are $d/2 = 0.010\ 0$ m from point A, and the charge at the top vertex is

$$\sqrt{(2d)^2 - \left(\frac{d}{2}\right)^2} = \frac{\sqrt{15}}{2}d$$

from point A.



ANS. FIG. P25.22

$$V = \sum_i k_e\frac{q_i}{r_i}$$

$$= k_e\left(\frac{-q}{d/2} + \frac{-q}{d/2} + \frac{q}{d\sqrt{15}/2}\right) = k_e\frac{q}{d}\left(-4 + \frac{2}{\sqrt{15}}\right)$$

$$V = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{7.00\times10^{-6}\ \text{C}}{0.020\ 0\ \text{m}}\right)\left(-4 + \frac{2}{\sqrt{15}}\right)$$

$$= \boxed{-1.10\times10^7\ \text{V}}$$

P25.23 (a) $E_x = \frac{k_e q_1}{x^2} + \frac{k_e q_2}{(x - 2.00)^2} = 0$ becomes $E_x = k_e\left(\frac{+q}{x^2} + \frac{-2q}{(x - 2.00)^2}\right) = 0$

Dividing by k_e, $2qx^2 = q(x - 2.00)^2$

or $x^2 + 4.00x - 4.00 = 0.$

Therefore $E = 0$ when $x = \frac{-4.00 \pm \sqrt{16.0 + 16.0}}{2} = -\boxed{-4.83\ \text{m}}.$

(Note that the positive root does not correspond to a physically valid situation.)

(b) Assuming $0 < x < 2.00$ m, the potential is zero when

$$V = \frac{k_e q_1}{x} + \frac{k_e q_2}{2.00 - x} = 0 \quad \text{or} \quad \frac{V}{k_e} = \left[\frac{(q)}{x} + \frac{(-2q)}{2.00 - x}\right] = 0$$

giving $q(2.00 - x) = 2qx$ or $x = \frac{2.00}{3} = \boxed{0.667\text{ m}}$

For $x > 2.00$ m, the potential is zero when

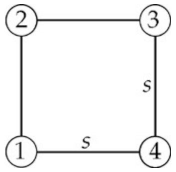
$$\frac{V}{k_e} = \left[\frac{(q)}{x} + \frac{(-2q)}{x - 2.00}\right] = 0 \quad\text{or}\quad q(x - 2.00) = 2qx$$

This has no positive x solution. Physically, the total potential cannot be zero for any point where $x > 2.00$ m because that point is closer to charge $-2q$, so its potential is always more negative than the potential from charge q is positive. For $x < 0$, the potential is zero when $\frac{V}{k_e} = \left[\frac{(q)}{|x|} + \frac{(-2q)}{|2.00 + x|}\right] = 0$, giving

$$\frac{q}{|x|} < \frac{2q}{2.00 + |x|} \quad\text{or}\quad q(2.00 + |x|) = 2q|x|$$

which has the solution $|x| = 2.00$ correspond to $x = \boxed{-2.00\text{ m}}$.

P25.24 The work required equals the sum of the potential energies for all pairs of charges. No energy is involved in placing q_4 at a given position in empty space. When q_3 is brought from far away and placed close to q_4, the system potential energy can be expressed as q_3V_4, where V_4 is the potential at the position of q_3 established by charge q_4. When q_2 is brought into the system, it interacts with two other charges, so we have two additional terms q_2V_3 and q_2V_4 in the total potential energy. Finally, when we bring the fourth charge q_1 into the system, it interacts with three other charges, giving us three more energy terms. Thus, the complete expression for the energy is:



ANS. FIG. P25.24

$$U = U_1 + U_2 + U_3 + U_4$$

$$U = 0 + U_{12} + (U_{13} + U_{23}) + (U_{14} + U_{24} + U_{34})$$

$$U = 0 + \frac{k_eQ^2}{s} + \frac{k_eQ^2}{s}\left(\frac{1}{\sqrt{2}} + 1\right) + \frac{k_eQ^2}{s}\left(1 + \frac{1}{\sqrt{2}} + 1\right)$$

$$U = \frac{k_eQ^2}{s}\left(4 + \frac{2}{\sqrt{2}}\right) = \boxed{5.41\frac{k_eQ^2}{s}}$$

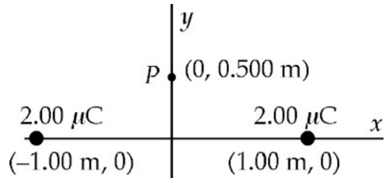
We can visualize the term $\left(4 + \frac{2}{\sqrt{2}}\right)$ as arising directly from the 4 side pairs and 2 face diagonal pairs.

P25.25 (a) The electric potential at $y = 0.500$ m on the y axis is given by

$$V = \frac{k_e q_1}{r_1} + \frac{k_e q_2}{r_2} = 2\left(\frac{k_e q}{r}\right)$$

$$V = 2\left(\frac{\left(8.99\times 10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(2.00\times 10^{-6}\ \mathrm{C}\right)}{\sqrt{\left(1.00\ \mathrm{m}\right)^2 + \left(0.500\ \mathrm{m}\right)^2}}\right)$$

$$V = 3.22\times 10^{4}\ \mathrm{V} = \boxed{32.2\ \mathrm{kV}}$$



ANS. FIG. P25.25

(b) The change in potential energy of the system when a third charge is brought to this point is

$$U = qV = \left(-3.00\times 10^{-6}\ \mathrm{C}\right)\left(3.22\times 10^{4}\ \mathrm{J/C}\right) = \boxed{-9.65\times 10^{-2}\ \mathrm{J}}$$

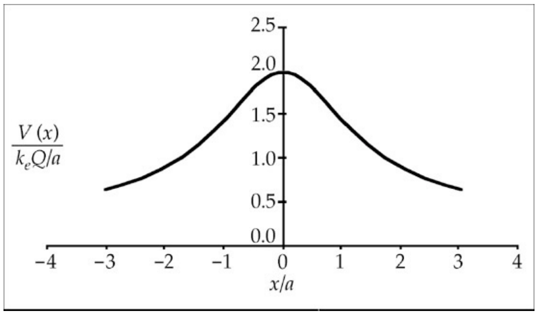
P25.26 (a) The potential due to the two charges along the x axis is

$$V(x) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{\sqrt{x^2 + a^2}} + \frac{k_e(+Q)}{\sqrt{x^2 + (-a)^2}}$$

$$V(x) = \frac{2k_e Q}{\sqrt{x^2 + a^2}} = \frac{k_e Q}{a}\left(\frac{2}{\sqrt{(x/a)^2 + 1}}\right)$$

$$\frac{V(x)}{(k_e Q/a)} = \boxed{\frac{2}{\sqrt{(x/a)^2 + 1}}}$$

ANS. FIG. P25.26(a) shows the plot of this function for $|x/a| < 3$.



ANS. FIG. P25.26(a)

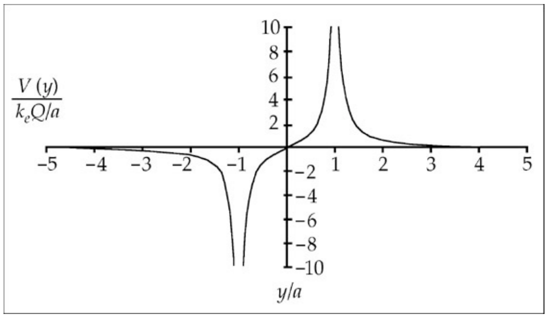
(b) The potential due to the two charges along the y axis is

$$V(y) = \frac{k_e Q_1}{r_1} + \frac{k_e Q_2}{r_2} = \frac{k_e(+Q)}{|y-a|} + \frac{k_e(-Q)}{|y+a|}$$

$$V(y) = \frac{k_e Q}{a}\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)$$

$$\frac{V(y)}{(k_e Q/a)} = \boxed{\left(\frac{1}{|y/a-1|} - \frac{1}{|y/a+1|}\right)}$$

ANS. FIG. P25.26(b) shows the plot of this function for $|y/a| < 4$.



ANS. FIG. P25.26(b)

P25.27 The total change in potential energy is the sum of the change in potential energy of the $q_1 - q_4$, $q_2 - q_4$, and $q_3 - q_4$ particle systems:

$$U_e = q_4 V_1 + q_4 V_2 + q_4 V_3 = q_4 k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$U_e = \left(10.0\times10^{-6}\ \text{C}\right)^2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)$$

$$\times\left(\frac{1}{0.600\ \text{m}} + \frac{1}{0.150\ \text{m}} + \frac{1}{\sqrt{(0.600\ \text{m})^2 + (0.150\ \text{m})^2}}\right)$$

$$U_e = \boxed{8.95\ \text{J}}$$

P25.28 (a) Each charge separately creates positive potential everywhere. The total potential produced by the three charges together is then the sum of three positive terms. There is $\boxed{\text{no point}}$, located at a finite distance from the charges, at which this total potential is zero.

(b) $V = \frac{k_e q}{a} + \frac{k_e q}{a} = \boxed{\frac{2k_e q}{a}}$

P25.29 Each charge creates equal potential at the center. The total potential is

$$V = 5\left[\frac{k_e(-q)}{R}\right] = \boxed{-\frac{5k_eq}{R}}$$

P25.30 The original electrical potential energy is

$$U_e = qV = q\frac{k_eq}{d}$$

In the final configuration we have mechanical equilibrium. The spring and electrostatic forces on each charge are

$$F_{\text{spring}} + F_{\text{charge}} = -k(2d) + q\frac{k_eq}{(3d)^2} = 0$$

Then $k = \dfrac{k_eq^2}{18d^3}$

In the final configuration the total potential energy is

$$\frac{1}{2}kx^2 + qV = \frac{1}{2}\frac{k_eq^2}{18d^3}(2d)^2 + q\frac{k_eq}{3d} = \frac{4}{9}\frac{k_eq^2}{d}$$

The missing energy must have become internal energy, as the system is isolated:

$$\Delta U + \Delta E_{\text{int}} = 0$$

$$\frac{4k_eq^2}{9d} - \frac{k_eq^2}{d} + \Delta E_{\text{int}} = 0$$

The increase in internal energy of the system is then

$$\Delta E_{\text{int}} = \boxed{\frac{5k_eq^2}{9d}}$$

P25.31 Consider the two spheres as a system.

(a) Conservation of momentum:

$$0 = m_1v_1\hat{\mathbf{i}} + m_2v_2\left(-\hat{\mathbf{i}}\right) \quad \text{or} \quad v_2 = \frac{m_1v_1}{m_2}$$

By conservation of energy,

$$0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$$

and $\dfrac{k_eq_1q_2}{r_1 + r_2} - \dfrac{k_eq_1q_2}{d} = \dfrac{1}{2}m_1v_1^2 + \dfrac{1}{2}\dfrac{m_1^2v_1^2}{m_2}$, which yields

$$v_1 = \sqrt{\frac{2m_2k_eq_1q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}$$

suppressing units,

$$v_1 = \sqrt{\frac{2(0.700)(8.99\times10^{9})(2\times10^{-6})(3\times10^{-6})}{(0.100)(0.800)}\left(\frac{1}{8\times10^{-3}} - \frac{1}{1.00}\right)}$$

$$= \boxed{10.8\ \text{m/s}}$$

$$v_2 = \frac{m_1v_1}{m_2} = \frac{(0.100\ \text{kg})(10.8\ \text{m/s})}{0.700\ \text{kg}} = \boxed{1.55\ \text{m/s}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P25.32 Consider the two spheres as a system.

(a) Conservation of momentum:

$$0 = m_1v_1\hat{\mathbf{i}} + m_2v_2\left(-\hat{\mathbf{i}}\right)$$

or $\quad v_2 = \dfrac{m_1v_1}{m_2}.$

By conservation of energy,

$$0 = \frac{k_e(-q_1)q_2}{d} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{k_e(-q_1)q_2}{r_1 + r_2}$$

and $\quad \dfrac{k_eq_1q_2}{r_1 + r_2} - \dfrac{k_eq_1q_2}{d} = \dfrac{1}{2}m_1v_1^2 + \dfrac{1}{2}\dfrac{m_1^2v_1^2}{m_2}.$

$$v_1 = \boxed{\sqrt{\frac{2m_2k_eq_1q_2}{m_1(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

$$v_2 = \left(\frac{m_1}{m_2}\right)v_1 = \boxed{\sqrt{\frac{2m_1k_eq_1q_2}{m_2(m_1 + m_2)}\left(\frac{1}{r_1 + r_2} - \frac{1}{d}\right)}}$$

(b) If the spheres are metal, electrons will move around on them with negligible energy loss to place the centers of excess charge on the insides of the spheres. Then just before they touch, the effective distance between charges will be less than $r_1 + r_2$ and the spheres will really be moving $\boxed{\text{faster than calculated in (a)}}$.

P25.33 A cube has 12 edges and 6 faces. Consequently, there are 12 edge pairs separated by s, 2 × 6 = 12 face diagonal pairs separated by $\sqrt{2}s$, and 4 interior diagonal pairs separated by $\sqrt{3}s$.

$$U = \frac{k_e q^2}{s}\left[12 + \frac{12}{\sqrt{2}} + \frac{4}{\sqrt{3}}\right] = \boxed{22.8\frac{k_e q^2}{s}}$$

P25.34 Each charge moves off on its diagonal line. All charges have equal speeds.

$$\sum(K+U)_i = \sum(K+U)_f$$

$$0 + \frac{4k_e q^2}{L} + \frac{2k_e q^2}{\sqrt{2}L} = 4\left(\frac{1}{2}mv^2\right) + \frac{4k_e q^2}{2L} + \frac{2k_e q^2}{2\sqrt{2}L}$$

$$\left(2 + \frac{1}{\sqrt{2}}\right)\frac{k_e q^2}{L} = 2mv^2$$

Solving for the speed gives

$$v = \boxed{\sqrt{\left(1 + \frac{1}{\sqrt{8}}\right)\frac{k_e q^2}{mL}}}$$

P25.35 Using conservation of energy for the alpha particle-nucleus system,

we have $K_f + U_f = K_i + U_i$.

But $U_i = \dfrac{k_e q_\alpha q_{\text{gold}}}{r_i}$ and $r_i \approx \infty$. Thus, $U_i = 0$.

Also, $K_f = 0$ ($v_f = 0$ at turning point),

so $U_f = K_i$

or $\dfrac{k_e q_\alpha q_{\text{gold}}}{r_{\min}} = \dfrac{1}{2}m_\alpha v_\alpha^2$

$$r_{\min} = \frac{2k_e q_\alpha q_{\text{gold}}}{m_\alpha v_\alpha^2}$$

$$= \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.00\times10^7\ \text{m/s}\right)^2}$$

$$= 2.74\times10^{-14}\ \text{m} = \boxed{27.4\ \text{fm}}$$

Section 25.4 Obtaining the Value of the Electric Field from the Electric Potential

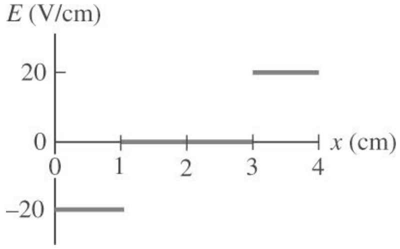
P25.36 $E_x = -\frac{\partial V}{\partial x} \quad \rightarrow \quad E_x = -\frac{\Delta V}{\Delta x} = -\text{ (slope of line)}$

The sign indicates the direction of the x component of the field.

$x = 0$ to 1 cm: $E_x = -\frac{\Delta V}{\Delta x} = -\frac{20\text{ V} - 0}{1\text{ cm}} = -20\text{ V/cm}$

$x = 1$ to 3 cm: $E_x = -\frac{\Delta V}{\Delta x} = -\frac{0}{2\text{ cm}} = 0\text{ V/m}$

$x = 3$ to 4 cm: $E_x = -\frac{\Delta V}{\Delta x} = -\frac{0 - 20\text{ V}}{1\text{ cm}} = +20\text{ V/cm}$



ANS. FIG. P25.36

P25.37 $V = a + bx = 10.0\text{ V} + (-7.00\text{ V/m})x$

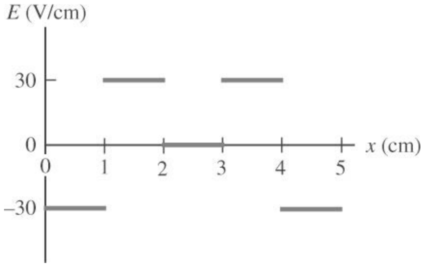
(a) At $x = 0$, $V = \boxed{10.0\text{ V}}$

At $x = 3.00$ m, $V = \boxed{-11.0\text{ V}}$

At $x = 6.00$ m, $V = \boxed{-32.0\text{ V}}$

(b) $E = -\frac{dV}{dx} = -b = -(-7.00\text{ V/m}) = \boxed{7.00\text{ N/C in the } +x\text{ direction}}$

P25.38 $E_x = -\frac{\partial V}{\partial x} \quad \rightarrow \quad E_x = -\frac{\Delta V}{\Delta x} = -\text{ (slope of line)}$



ANS. FIG. P25.38

The sign indicates the direction of the *x* component of the field.

$x = 0$ to 1 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{30\text{ V} - 0}{1\text{ cm}} = -30\text{ V/cm}$$

$x = 1$ to 2 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{0 - 30\text{ V}}{2\text{ cm}} = 30\text{ V/m}$$

$x = 2$ to 3 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{0}{1\text{ cm}} = 0\text{ V/cm}$$

$x = 3$ to 4 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{-30\text{ V} - 0}{1\text{ cm}} = +30\text{ V/cm}$$

$x = 4$ to 5 cm: $$E_x = -\frac{\Delta V}{\Delta x} = -\frac{0 - (-30\text{ V})}{1\text{ cm}} = -30\text{ V/cm}$$

P25.39 (a) $V = 5x - 3x^2y + 2yz^2$, where *x*, *y* and *z* are in meters and *V* is in volts.

$$E_x = -\frac{\partial V}{\partial x} = -5 + 6xy$$
$$E_y = -\frac{\partial V}{\partial y} = +3x^2 - 2z^2$$
$$E_z = -\frac{\partial V}{\partial z} = -4yz$$

which gives

$$\boxed{\vec{\mathbf{E}} = (-5 + 6xy)\hat{\mathbf{i}} + (3x^2 - 2z^2)\hat{\mathbf{j}} - 4yz\hat{\mathbf{k}}}$$

(b) Evaluate *E* at (1.00, 0, −2.00) m, suppressing units,

$$E_x = -5 + 6(1.00)(0) = -5.00$$
$$E_y = 3(1.00)^2 - 2(-2.00)^2 = -5.00$$
$$E_z = -4(0)(-2.00) = 0$$

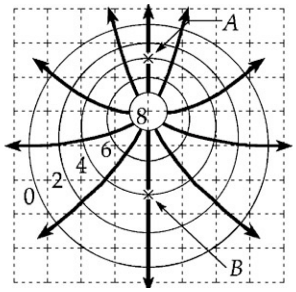
which gives

$$E = \sqrt{E_x^2 + E_y^2 + E_z^2} = \sqrt{(-5.00)^2 + (-5.00)^2 + 0^2} = \boxed{7.07\text{ N/C}}$$

P25.40 (a) $\boxed{E_A > E_B \text{ since } E = \frac{\Delta V}{\Delta s}}$

(b) $$E_B = -\frac{\Delta V}{\Delta s} = -\frac{(6-2)\text{ V}}{2\text{ cm}} = \boxed{200\text{ N/C}}\text{ down}$$

(c) ANS. FIG. P25.40 shows a sketch of the field lines.



ANS. FIG. P25.40

P25.41 (a) For $r < R$, $V = \frac{k_eQ}{R}$

$$E_r = -\frac{dV}{dr} = \boxed{0}$$

(b) For $r \geq R$, $V = \frac{k_eQ}{r}$

$$E_r = -\frac{dV}{dr} = -\left(-\frac{k_eQ}{r^2}\right) = \boxed{\frac{k_eQ}{r^2}}$$

P25.42 For a general expression for the potential on the y-axis, replace the a with y. The y component of the electric field is

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left[\frac{k_eQ}{\ell}\ln\left(\frac{\ell + \sqrt{\ell^2 + y^2}}{y}\right)\right]$$

$$E_y = \frac{k_eQ}{\ell y}\left[1 - \frac{y^2}{\ell^2 + y^2 + \ell\sqrt{\ell^2 + y^2}}\right] = \boxed{\frac{k_eQ}{y\sqrt{\ell^2 + y^2}}}$$

Section 25.5 Electric Potential Due to Continuous Charge Distributions

P25.43 The potential difference between the two points is

$$\Delta V = V_{2R} - V_0 = \frac{k_eQ}{\sqrt{R^2 + (2R)^2}} - \frac{k_eQ}{R} = \frac{k_eQ}{R}\left(\frac{1}{\sqrt{5}} - 1\right)$$

$$= \boxed{-0.553\frac{k_eQ}{R}}$$

P25.44 $V = \int dV = \dfrac{1}{4\pi \in_0} \int \dfrac{dq}{r}$

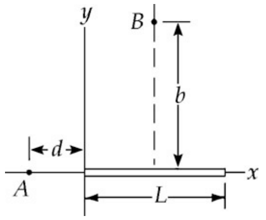
All bits of charge are at the same distance from *O*. So

$$V = \frac{1}{4\pi \in_0}\left(\frac{Q}{R}\right) = \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(\frac{-7.50\times10^{-6}\ \mathrm{C}}{0.140\ \mathrm{m}/\pi}\right)$$

$$= \boxed{-1.51\ \mathrm{MV}}$$

P25.45 (a) As a linear charge density, λ has units of C/m. So $\alpha = \lambda/x$ must have units of C/m^2:

$$[\alpha] = \left[\frac{\lambda}{x}\right] = \frac{\mathrm{C}}{\mathrm{m}}\cdot\left(\frac{1}{\mathrm{m}}\right) = \boxed{\frac{\mathrm{C}}{\mathrm{m}^2}}$$



ANS. FIG. P25.45

(b) Consider a small segment of the rod at location *x* and of length *dx*. The amount of charge on it is $\lambda\, dx = (\alpha x)\, dx$. Its distance from *A* is *d* + *x*, so its contribution to the electric potential at *A* is

$$dV = k_e \frac{dq}{r} = k_e \frac{\alpha x\, dx}{d+x}$$

Relative to *V* = 0 infinitely far away, to find the potential at *A* we must integrate these contributions for the whole rod, from *x* = 0 to *x* = *L*. Then $V = \int_{\text{all } q} dV = \int_0^L \frac{k_e \alpha x}{d+x}\, dx.$

To perform the integral, make a change of variables to

u = *d* + *x*, *du* = *dx*, *u*(at *x* = 0) = *d*, and *u*(at *x* = *L*) = *d* + *L*:

$$V = \int_d^{d+L} \frac{k_e\alpha(u-d)}{u}\, du = k_e\alpha\int_d^{d+L} du - k_e\alpha d\int_d^{d+L}\left(\frac{1}{u}\right) du$$

$$V = k_e\alpha u\Big|_d^{d+L} - k_e\alpha d \ln\, u\Big|_d^{d+L}$$

$$= k_e\alpha(d+L-d) - k_e\alpha d\left[\ln(d+L) - \ln\, d\right]$$

$$V = \boxed{k_e\alpha\left[L - d\ln\left(1+\frac{L}{d}\right)\right]}$$

P25.46

$$V = \int \frac{k_e dq}{r} = k_e \int \frac{\alpha x dx}{\sqrt{b^2 + \left(L/2 - x\right)^2}}$$

Let $z = \frac{L}{2} - x$. Then $x = \frac{L}{2} - z$, and $dx = -dz$.

$$V = k_e\alpha \int \frac{\left(L/2 - z\right)\left(-dz\right)}{\sqrt{b^2 + z^2}} = -\frac{k_e\alpha L}{2} \int \frac{dz}{\sqrt{b^2 + z^2}} + k_e\alpha \int \frac{z dz}{\sqrt{b^2 + z^2}}$$

$$= -\frac{k_e\alpha L}{2} \ln\left(z + \sqrt{z^2 + b^2}\right) + k_e\alpha\sqrt{z^2 + b^2}$$

$$V = -\frac{k_e\alpha L}{2} \ln\left[\left(\frac{L}{2} - x\right) + \sqrt{\left(\frac{L}{2} - x\right)^2 + b^2}\right]\Bigg|_0^L + k_e\alpha\sqrt{\left(\frac{L}{2} - x\right)^2 + b^2}\Bigg|_0^L$$

$$V = -\frac{k_e\alpha L}{2} \ln\left[\frac{L/2 - L + \sqrt{\left(L/2\right)^2 + b^2}}{L/2 + \sqrt{\left(L/2\right)^2 + b^2}}\right]$$

$$+ k_e\alpha\left[\sqrt{\left(\frac{L}{2} - L\right)^2 + b^2} - \sqrt{\left(\frac{L}{2}\right)^2 + b^2}\right]$$

$$V = \boxed{-\frac{k_e\alpha L}{2} \ln\left[\frac{\sqrt{b^2 + \left(L^2/4\right)} - L/2}{\sqrt{b^2 + \left(L^2/4\right)} + L/2}\right]}$$

P25.47

$$V = k_e \int_{\text{all charge}} \frac{dq}{r} = k_e \int_{-3R}^{-R} \frac{\lambda dx}{-x} + k_e \int_{\text{semicircle}} \frac{\lambda ds}{R} + k_e \int_R^{3R} \frac{\lambda dx}{x}$$

$$V = -k_e\lambda \ln\left(-x\right)\Big|_{-3R}^{-R} + \frac{k_e\lambda}{R}\pi R + k_e\lambda \ln x\Big|_R^{3R}$$

$$V = k_e\lambda \ln\frac{3R}{R} + k_e\lambda\pi + k_e\lambda \ln 3 = \boxed{k_e\lambda\left(\pi + 2\ln 3\right)}$$

Section 25.6 Electric Potential Due to a Charged Conductor

P25.48 $\boxed{\text{No.}}$ A conductor of any shape forms an equipotential surface. If the conductor is a sphere of radius R, and if it holds charge Q, the electric field at its surface is $E = k_eQ/R^2$ and the –potential of the surface is $V = k_eQ/R$; thus, if we know E and R, we can find V. However, if the surface varies in shape, there is no clear way to relate electric field at a point on the surface to the potential of the surface.

P25.49 Substituting given values into $V = \frac{k_e Q}{r}$, with $Q = Nq$:

$$7.50 \times 10^3 \text{ V} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)Q}{0.300 \text{ m}}$$

Substituting $q = 2.50 \times 10^{-7}$ C,

$$N = \frac{2.50 \times 10^{-7} \text{ C}}{1.60 \times 10^{-19} \text{ C}/e^-} = \boxed{1.56 \times 10^{12} \text{ electrons}}$$

P25.50 For points on the surface and outside, the sphere of charge behaves like a charged particle at its center, both for creating field and potential.

(a) Inside a conductor when charges are not moving, the electric field is zero and the potential is uniform, the same as on the surface, and $E = \boxed{0}$.

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right)\left(26.0 \times 10^{-6} \text{ C}\right)}{0.140 \text{ m}} = \boxed{1.67 \text{ MV}}$$

(b) $$E = \frac{k_e q}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right)\left(26.0 \times 10^{-6} \text{ C}\right)}{\left(0.200 \text{ m}\right)^2}$$

$= \boxed{5.84 \text{ MN/C}}$ away

$$V = \frac{k_e q}{R} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right)\left(26.0 \times 10^{-6} \text{ C}\right)}{0.200 \text{ m}} = \boxed{1.17 \text{ MV}}$$

(c) $$E = \frac{k_e q}{R^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right)\left(26.0 \times 10^{-6} \text{ C}\right)}{\left(0.140 \text{ m}\right)^2}$$

$= \boxed{11.9 \text{ MN/C}}$ away

$$V = \frac{k_e q}{R} = \boxed{1.67 \text{ MV}}$$

P25.51 (a) Both spheres must be at the same potential according to $\frac{k_e q_1}{r_1} = \frac{k_e q_2}{r_2}$, where also $q_1 + q_2 = 1.20 \times 10^{-6}$ C.

Then $q_1 = \frac{q_2 r_1}{r_2}$ and

$$\frac{q_2 r_1}{r_2} + q_2 = 1.20 \times 10^{-6} \text{ C}$$

$$q_2 = \frac{1.20 \times 10^{-6}\ \text{C}}{1 + 6.00\ \text{cm}/2.00\ \text{cm}} = 0.300 \times 10^{-6}\ \text{C}$$

on the smaller sphere.

$$q_1 = 1.20 \times 10^{-6}\ \text{C} - 0.300 \times 10^{-6}\ \text{C} = 0.900 \times 10^{-6}\ \text{C}$$

$$V = \frac{k_e q_1}{r_1} = \frac{\left(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2\right)\left(0.900 \times 10^{-6}\ \text{C}\right)}{6.00 \times 10^{-2}\ \text{m}}$$

$$= \boxed{1.35 \times 10^5\ \text{V}}$$

(b) Outside the larger sphere,

$$\vec{\mathbf{E}}_1 = \frac{k_e q_1}{r_1^2}\hat{\mathbf{r}} = \frac{V_1}{r_1}\hat{\mathbf{r}} = \frac{1.35 \times 10^5\ \text{V}}{0.060\ 0\ \text{m}}\hat{\mathbf{r}} = \boxed{2.25 \times 10^6\ \text{V/m away}}$$

Outside the smaller sphere,

$$\vec{\mathbf{E}}_2 = \frac{1.35 \times 10^5\ \text{V}}{0.020\ 0\ \text{m}}\hat{\mathbf{r}} = \boxed{6.74 \times 10^6\ \text{V/m away}}$$

The smaller sphere carries less charge but creates a much stronger electric field than the larger sphere.

Section 25.8 Applications of Electrostatics

P25.52 From the maximum allowed electric field, we can find the charge and potential that would create this situation. Since we are only given the diameter of the dome, we will assume that the conductor is spherical, which allows us to use the electric field and potential equations for a spherical conductor.

(a) $$E_{\text{max}} = 3.00 \times 10^6\ \text{V/m} = \frac{k_e Q}{r^2} = \frac{k_e Q}{r}\left(\frac{1}{r}\right) = V_{\text{max}}\left(\frac{1}{r}\right)$$

$$V_{\text{max}} = E_{\text{max}} r = \left(3.00 \times 10^6\ \text{V/m}\right)(0.150\ \text{m}) = \boxed{450\ \text{kV}}$$

(b) $$\frac{k_e Q_{\text{max}}}{r^2} = E_{\text{max}} \quad \left\{\text{or } \frac{k_e Q_{\text{max}}}{r} = V_{\text{max}}\right\}$$

$$Q_{\text{max}} = \frac{E_{\text{max}} r^2}{k_e} = \frac{\left(3.00 \times 10^6\ \text{V/m}\right)(0.150\ \text{m})^2}{8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2} = \boxed{7.51\ \mu\text{C}}$$

Additional Problems

P25.53 From Equation 25.13, solve for the separation distance of the electron and proton:

$$U = k_e \frac{q_1 q_2}{r_{12}} \quad \rightarrow \quad r_{12} = k_e \frac{q_1 q_2}{U} = -k_e \frac{e^2}{U}$$

The separation distance r_{12} between the electron and proton is the same as the radius r of the orbit of the electron. Substitute numerical values:

$$r = -\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(1.6 \times 10^{-19} \text{ C}\right)^2}{-13.6 \text{ eV}}\left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}\right)$$
$$= 1.06 \times 10^{-10} \text{ m}$$

Set this equal to $r = n^2(0.052\,9 \text{ nm})$ and solve for n:

$$r = n^2\left(0.052\;9 \text{ nm}\right) = 1.06 \times 10^{-10} \text{ m} = 0.106 \text{ nm}$$

Which gives $n = 1.42$. Because n is not an integer, this is not possible. Therefore, the energy given cannot be possible for an allowed state of the atom.

P25.54 (a) The field within the conducting Earth is zero. The field is downward, so the Earth is negatively charged. Treat the surface of Earth at this location as a charged conducting plane: thus, use

$$E = \sigma / \epsilon_0$$

which gives

$$\sigma = E\epsilon_0 = (120 \text{ N/C})\left(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)$$
$$= \boxed{1.06 \text{ nC/m}^2\text{, negative}}$$

(b) $$Q = \sigma A = \sigma 4\pi r^2 = (-1.06 \times 10^{-9} \text{ C/m}^2)\left(4\pi\right)(6.37 \times 10^6 \text{ m})^2$$
$$= \boxed{-542 \text{ kC}}$$

(c) The Earth acts like a conducting sphere:

$$V = \frac{k_e Q}{R} = \frac{\left(8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)(-5.42 \times 10^5 \text{ C})}{6.37 \times 10^6 \text{ m}} = \boxed{-764 \text{ MV}}$$

(d) Electric potential decreases in the direction of the electric field; therefore, the potential is greater at greater heights:

$$V_{head} - V_{feet} = Ed = (120 \text{ N/C})(1.75 \text{ m}) = 210 \text{ V}.$$

$$\rightarrow \boxed{\text{The person's head is higher in potential by 210 V.}}$$

(e) Like charges repel:

$$F_E = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)(5.42\times10^5\ \mathrm{C})^2(0.273)}{(3.84\times10^8\ \mathrm{m})^2}$$

$$F_E = 4.88\times10^3\ \mathrm{N} = \boxed{4.88\times10^3\ \mathrm{N}\ \text{away from Earth}}$$

(f) The gravitational force is

$$F_G = \frac{GM_E M_M}{r^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)(5.98\times10^{24}\ \mathrm{kg})(7.36\times10^{22}\ \mathrm{kg})}{(3.84\times10^8\ \mathrm{m})^2}$$

$$F_G = 1.99\times10^{20}\ \mathrm{N}$$

Comparing the two forces,

$$\frac{F_G}{F_E} = \frac{1.99\times10^{20}\ \mathrm{N}}{4.88\times10^3\ \mathrm{N}} = 4.08\times10^{16}$$

$$\boxed{\text{The gravitational force is in the opposite direction and } 4.08\times10^{16} \text{ times larger. Electrical forces are negligible in accounting for planetary motion.}}$$

P25.55 Assume the particles move along the x direction.

(a) Momentum is constant within the isolated system throughout the process. We equate it at the large-separation initial point and the point c of closest approach.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1c} + m_2\vec{\mathbf{v}}_{2c}$$

$$m_1 v\hat{\mathbf{i}} + 0 = m_1\vec{\mathbf{v}}_c + m_2\vec{\mathbf{v}}_c$$

$$\vec{\mathbf{v}}_c = \frac{m_1 v}{m_1+m_2}\hat{\mathbf{i}} = \frac{\left(2.00\times10^{-3}\ \mathrm{kg}\right)(21.0\ \mathrm{m/s})}{7.00\times10^{-3}\ \mathrm{kg}}\hat{\mathbf{i}} = \boxed{6.00\hat{\mathbf{i}}\ \mathrm{m/s}}$$

(b) Energy is conserved within the isolated system. Compare energy terms between the large-separation initial point and the point of closest approach:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 + 0 = \frac{1}{2}(m_1+m_2)v_c^2 + \frac{k_e q_1 q_2}{r_c}$$

$$\frac{1}{2}m_1 v^2 + 0 = \frac{1}{2}(m_1+m_2)\left(\frac{m_1 v}{m_1+m_2}\right)^2 + \frac{k_e q_1 q_2}{r_c}$$

$$\rightarrow m_1v^2 + 0 = \frac{m_1^2v^2}{m_1 + m_2} + 2\frac{k_eq_1q_2}{r_c}$$

$$\rightarrow (m_1 + m_2)m_1v^2 - m_1^2v^2 = 2\frac{k_eq_1q_2(m_1 + m_2)}{r_c}$$

$$m_1m_2v^2 = 2\frac{k_eq_1q_2(m_1 + m_2)}{r_c}$$

$$r_c = \frac{2k_eq_1q_2(m_1 + m_2)}{m_1m_2v^2}$$

$$= \frac{2(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(15.0\times10^{-6}\ \text{C})(8.50\times10^{-6}\ \text{C})(7.00\times10^{-3}\ \text{kg})}{(2.00\times10^{-3}\ \text{kg})(5.00\times10^{-3}\ \text{kg})(21.0\ \text{m/s})^2}$$

$$= \boxed{3.64\ \text{m}}$$

(c) The overall elastic collision is described by conservation of momentum:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$m_1v\hat{\mathbf{i}} + 0 = m_1v_{1f}\hat{\mathbf{i}} + m_2v_{2f}\hat{\mathbf{i}}$$

and by the relative velocity equation:

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

$$v - 0 = v_{2f} - v_{1f} \rightarrow v_{2f} = v + v_{1f}$$

We substitute the expression for v_{2f} into the momentum equation:

$$m_1v = m_1v_{1f} + m_2v_{2f}$$

$$m_1v = m_1v_{1f} + m_2(v + v_{1f})$$

$$m_1v = m_1v_{1f} + m_2v + m_2v_{1f}$$

$$m_1v - m_2v = m_1v_{1f} + m_2v_{1f}$$

$$(m_1 - m_2)v = (m_1 + m_2)v_{1f}$$

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v = \left(\frac{2.00\,\text{g} - 5.00\,\text{g}}{2.00\,\text{g} + 5.00\,\text{g}}\right)(21.0\ \text{m/s})$$

$$= -9.00\ \text{m/s}$$

Therefore, the velocity of the particle of mass m_1 is $\boxed{-9.00\ \hat{\mathbf{i}}\ \text{m/s}}$.

(d) Substitute the expression for v_{1f} back into $v_{2f} = v + v_{1f}$:

$$v_{2f} = v + v_{1f} = (21.0\ \text{m/s}) + (-9.00\ \text{m/s}) = 12.0\ \text{m/s}$$

Therefore, the velocity of the particle of mass m_2 is $\boxed{12.0\ \hat{\mathbf{i}}\ \text{m/s}}$.

P25.56 Assume the particles move along the *x* direction.

(a) Momentum is constant within the isolated system throughout the process. We equate it at the large-separation initial point and the point *c* of closest approach.

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$m_1 v\hat{\mathbf{i}} + 0 = m_1 v_c\hat{\mathbf{i}} + m_2 v_c\hat{\mathbf{i}} \quad \rightarrow \quad v_c = \boxed{\frac{m_1 v}{m_1 + m_2}}$$

(b) Energy is conserved within the isolated system. Compare energy terms between the large-separation initial point and the point of closest approach:

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m_1 v_{1i}^2 + \frac{1}{2}m_2 v_{2i}^2 + 0 = \frac{1}{2}(m_1 + m_2)v_c^2 + \frac{k_e q_1 q_2}{r_c}$$

$$\frac{1}{2}m_1 v^2 + 0 = \frac{1}{2}(m_1 + m_2)\left(\frac{m_1 v}{m_1 + m_2}\right)^2 + \frac{k_e q_1 q_2}{r_c}$$

$$\rightarrow m_1 v^2 + 0 = \frac{m_1{}^2 v^2}{m_1 + m_2} + 2\frac{k_e q_1 q_2}{r_c}$$

$$\rightarrow (m_1 + m_2)m_1 v^2 - m_1{}^2 v^2 = 2\frac{k_e q_1 q_2 (m_1 + m_2)}{r_c}$$

$$m_1 m_2 v^2 = 2\frac{k_e q_1 q_2 (m_1 + m_2)}{r_c} \rightarrow r_c = \boxed{\frac{2k_e q_1 q_2 (m_1 + m_2)}{m_1 m_2 v^2}}$$

(c) The overall elastic collision is described by conservation of momentum:

$$m_1\vec{\mathbf{v}}_{1i} + m_2\vec{\mathbf{v}}_{2i} = m_1\vec{\mathbf{v}}_{1f} + m_2\vec{\mathbf{v}}_{2f}$$

$$m_1 v\hat{\mathbf{i}} + 0 = m_1 v_{1f}\hat{\mathbf{i}} + m_2 v_{2f}\hat{\mathbf{i}} \quad \rightarrow \quad m_1 v = m_1 v_{1f} + m_2 v_{2f}$$

and by the relative velocity equation:

$$v_{1i} - v_{2i} = v_{2f} - v_{1f}$$

$$v - 0 = v_{2f} - v_{1f} \quad \rightarrow \quad v_{2f} = v + v_{1f}$$

We substitute the expression for v_{2f} into the momentum equation:

$$m_1 v = m_1 v_{1f} + m_2 v_{2f}$$

$$m_1 v = m_1 v_{1f} + m_2(v + v_{1f})$$

$$m_1 v = m_1 v_{1f} + m_2 v + m_2 v_{1f}$$

$$m_1v - m_2v = m_1v_{1f} + m_2v_{1f}$$

$$(m_1 - m_2)v = (m_1 + m_2)v_{1f} \to v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v$$

Therefore, the velocity of the particle of mass m_1 is $\boxed{\left(\frac{m_1 - m_2}{m_1 + m_2}\right)v\hat{\mathbf{i}}}$.

(d) Substitute the expression for v_{1f} back into $v_{2f} = v + v_{1f}$:

$$v_{2f} = v + v_{1f} = v + \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v$$

$$= \left[\frac{(m_1 + m_2) + (m_1 - m_2)}{m_1 + m_2}\right]v = \left(\frac{2m_1}{m_1 + m_2}\right)v$$

Therefore, the velocity of the particle of mass m_2 is $\boxed{\left(\frac{2m_1}{m_1 + m_2}\right)v\hat{\mathbf{i}}}$.

P25.57 The two spheres of charge have together electric potential energy

$$U = qV = k_e\frac{q_1q_2}{r_{12}}$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(38)(54)(1.60 \times 10^{-19}\ \text{C})^2}{(5.50 + 6.20) \times 10^{-15}\ \text{m}}$$

$$= 4.04 \times 10^{-11}\ \text{J} = \boxed{253\ \text{MeV}}$$

P25.58 (a) To make a spark 5 mm long in dry air between flat metal plates requires potential difference

$$\Delta V = Ed = (3 \times 10^6\ \text{V/m})(5 \times 10^{-3}\ \text{m}) = 1.5 \times 10^4\ \text{V}\ \boxed{\sim 10^4\ \text{V}}$$

(b) The area of your skin is perhaps 1.5 m^2, so model your body as a sphere with this surface area. Its radius is given by $1.5\ \text{m}^2 = 4\pi r^2$, $r = 0.35$ m. We require that you are at the potential found in part (a), with $V = \frac{k_eq}{r}$. Then,

$$q = \frac{Vr}{k_e} = \frac{1.5 \times 10^4\ \text{V}(0.35\ \text{m})}{8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}\left(\frac{\text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$$

$$q = 5.8 \times 10^{-7}\ \text{C}\ \boxed{\sim 10^{-6}\ \text{C}}$$

P25.59 We have $V_1 = k_eQ/R = 200$ V and $V_2 = k_eQ/(R + 10 \text{ cm}) = 150$ V.

(a) $$\frac{V_1}{V_2} = \frac{R + 10 \text{ cm}}{R} = \frac{200}{150} \rightarrow 150(R + 10 \text{ cm}) = 200R \rightarrow R = \boxed{30.0 \text{ cm}}$$

(b) From $V_1 = k_e\dfrac{Q}{R}$, we have

$$Q = \frac{V_1R}{k_e} = \frac{(200 \text{ V})(0.300 \text{ m})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 6.67 \times 10^{-9} \text{ C} = \boxed{6.67 \text{ nC}}$$

(c) We have $V = k_eQ/R = 210$ V and $E = k_eQ/(R + 10 \text{ cm})^2 = 400$ V/m. Therefore,

$$\frac{V}{E} = \frac{(R + 10 \text{ cm})^2}{R} = \frac{210}{400} = \frac{21}{40} \rightarrow 40(R + 0.100)^2 = 21R$$

where R is in meters.

Thus, we have

$$40R^2 + 8R + 0.4 = 21R \rightarrow 40R^2 - 13R + 0.4 = 0$$

There are two possibilities, according to

$$R = \frac{+13 \pm \sqrt{13^2 - 4(40)(0.4)}}{80} = \text{ either } 0.291 \text{ m or } 0.034\,4 \text{ m}$$

$$= \boxed{29.1 \text{ cm or } 3.44 \text{ cm}}$$

(d) If the radius is 29.1 cm,

$$Q = \frac{VR}{k_e} = \frac{(210 \text{ V})(0.291 \text{ m})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 6.79 \times 10^{-9} \text{ C} = \boxed{6.79 \text{ nC}}$$

If the radius is 3.44 cm,

$$Q = \frac{VR}{k_e} = \frac{(210 \text{ V})(0.0344 \text{ m})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = 8.04 \times 10^{-10} \text{ C} = \boxed{804 \text{ pC}}$$

(e) $\boxed{\text{No; two answers exist for each part.}}$

P25.60 (a) The exact potential is

$$+\frac{k_eq}{r + a} - \frac{k_eq}{r - a} = +\frac{k_eq}{3a + a} - \frac{k_eq}{3a - a} = \frac{k_eq}{4a} - \frac{2k_eq}{4a} = \boxed{-\frac{k_eq}{4a}}$$

(b) The approximate expression $-2k_eqa/x^2$ gives

$$-2k_eqa/(3a)^2 = -k_eq/4.5a$$

Compare the exact to the approximate solution:

$$\frac{1/4-1/4.5}{1/4}=\frac{0.5}{4.5}=0.111\cdot$$

$$\boxed{\text{The approximate expression } -2k_eqa/x^2 \text{ gives } -k_eq/4.5a, \text{ which is different by only 11.1\%.}}$$

P25.61 $W=\int_0^Q V\,dq$, where $V=\dfrac{k_eq}{R}$.

Therefore, $W=\dfrac{k_eQ^2}{2R}=\dfrac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(125\times10^{-6}\ \text{C}\right)^2}{2\left(0.100\ \text{m}\right)}=\boxed{702\ \text{J}}$.

P25.62 $W=\int_0^Q V\,dq$, where $V=\dfrac{k_eq}{R}$. Therefore, $W=\boxed{\dfrac{k_eQ^2}{2R}}$.

P25.63 For a charge at (x = –1 m, y = 0), the radial distance away is given by $\sqrt{(x+1)^2+y^2}$. So the first term will be the potential it creates if

$$(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)Q_1=36\ \text{V}\cdot\text{m}\quad\rightarrow\quad Q_1=4.00\ \text{nC}$$

The second term is the potential of a charge at (x = 0, y = 2 m) with

$$(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)Q_2=-45\ \text{V}\cdot\text{m}\quad\rightarrow\quad Q_2=-5.01\ \text{nC}$$

Thus we have $\boxed{4.00\ \text{nC at }(-1.00\ \text{m}, 0)\text{ and }-5.01\ \text{nC at }(0, 2.00\ \text{m}).}$

P25.64 From Example 25.5, the potential along the x axis of a ring of charge of radius R is

$$V=\frac{k_eQ}{\sqrt{R^2+x^2}}$$

Therefore, the potential at the center of the ring is

$$V=\frac{k_eQ}{\sqrt{R^2+(0)^2}}=\frac{k_eQ}{R}$$

When we place the point charge Q at the center of the ring, the electric potential energy of the charge–ring system is

$$U=QV=Q\left(\frac{k_eQ}{R}\right)=\frac{k_eQ^2}{R}$$

Now, apply Equation 8.2 to the isolated system of the point charge and the ring with initial configuration being that with the point charge at the center of the ring and the final configuration having the point charge infinitely far away and moving with its highest speed:

$$\Delta K + \Delta U = 0 \quad \rightarrow \quad \left(\frac{1}{2}mv_{\max}^2 - 0\right) + \left(0 - \frac{k_eQ^2}{R}\right) = 0$$

Solve for the maximum speed:

$$v_{\max} = \left(\frac{2k_eQ^2}{mR}\right)^{1/2}$$

Substitute numerical values:

$$v_{\max} = \left(\frac{2(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(50.0 \times 10^{-6}\ \text{C})^2}{(0.100\ \text{kg})(0.500\ \text{m})}\right)^{1/2}$$

$$= 30.0\ \text{m/s}$$

Therefore, even if the charge were to accelerate to infinity, it would only achieve a maximum speed of 30.0 m/s, so it cannot strike the wall of your laboratory at 40.0 m/s.

P25.65 In Equation 25.3, $V_2 - V_1 = \Delta V = -\int_1^2 \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}}$, think about stepping from distance r_1 out to the larger distance r_2 away from the charged line. Then $d\vec{\mathbf{s}} = dr\hat{\mathbf{r}}$, and we can make *r* the variable of integration:

$$V_2 - V_1 = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi \epsilon_0 r}\,\hat{\mathbf{r}} \cdot dr\ \hat{\mathbf{r}} \quad \text{with} \quad \hat{\mathbf{r}} \cdot \hat{\mathbf{r}} = 1 \cdot 1\cos 0^\circ = 1$$

The potential difference is

$$V_2 - V_1 = -\frac{\lambda}{2\pi \epsilon_0}\int_{r_1}^{r_2} \frac{dr}{r} = -\frac{\lambda}{2\pi \epsilon_0}\ln r\Big|_{r_1}^{r_2}$$

and
$$V_2 - V_1 = -\frac{\lambda}{2\pi \epsilon_0}(\ln r_2 - \ln r_1) = \boxed{-\frac{\lambda}{2\pi \epsilon_0}\ln\frac{r_2}{r_1}}$$

P25.66 (a) Modeling the filament as a single charged particle, we obtain

$$V = \frac{k_eQ}{r} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-9}\ \text{C})}{2.00\ \text{m}} = \boxed{7.19\ \text{V}}$$

(b) Modeling the filament as two charged particles, we obtain

$$V = \frac{k_eQ_1}{r_1} + \frac{k_eQ_2}{r_2} = k_e\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2}\right)$$

$$= (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(\frac{0.800 \times 10^{-9}\ \text{C}}{1.50\ \text{m}} + \frac{0.800 \times 10^{-9}\ \text{C}}{2.50\ \text{m}}\right)$$

$$= \boxed{7.67\ \text{V}}$$

(c) Modeling the filament as four charged particles, we obtain

$$\begin{aligned} V &= k_e\left(\frac{Q_1}{r_1}+\frac{Q_2}{r_2}+\frac{Q_3}{r_3}+\frac{Q_4}{r_4}\right) \\ &= \left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right) \\ &\qquad \times\left(\frac{0.400\times10^{-9}\ \mathrm{C}}{1.25\ \mathrm{m}}+\frac{0.400\times10^{-9}\ \mathrm{C}}{1.75\ \mathrm{m}}\right. \\ &\qquad\qquad \left.+\frac{0.400\times10^{-9}\ \mathrm{C}}{2.25\ \mathrm{m}}+\frac{0.400\times10^{-9}\ \mathrm{C}}{2.75\ \mathrm{m}}\right) \\ &= \boxed{7.84\ \mathrm{V}} \end{aligned}$$

(d) We represent the exact result as

$$\begin{aligned} V &= \frac{k_eQ}{\ell}\ln\left(\frac{\ell+a}{a}\right) \\ &= \left[\frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-9}\ \mathrm{C}\right)}{2.00\ \mathrm{m}}\right]\ln\left(\frac{3}{1}\right) \\ &= 7.901\,2\ \mathrm{V} \end{aligned}$$

Modeling the line as a set of points works nicely. The exact result, represented as 7.90 V, is approximated to within 0.8% by the four-particle version.

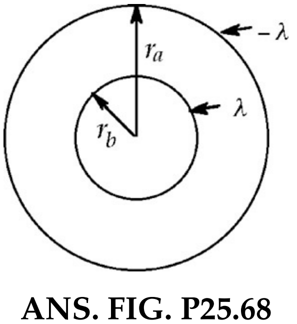
P25.67 We obtain the electric potential at *P* by integrating:

$$\begin{aligned} V &= k_e\int_a^{a+L}\frac{\lambda\,dx}{\sqrt{x^2+b^2}} = k_e\lambda\ln\left[x+\sqrt{\left(x^2+b^2\right)}\right]_a^{a+L} \\ &= \boxed{k_e\lambda\ln\left[\frac{a+L+\sqrt{(a+L)^2+b^2}}{a+\sqrt{a^2+b^2}}\right]} \end{aligned}$$

P25.68 (a) $V_B-V_A=-\int_A^B\vec{\mathbf{E}}\cdot d\vec{\mathbf{s}}$ and the field at distance *r* from a uniformly charged rod (where *r* > radius of charged rod) is

$$E=\frac{\lambda}{2\pi\epsilon_0 r}=\frac{2k_e\lambda}{r}$$

In this case, the field between the central wire and the coaxial cylinder is directed



ANS. FIG. P25.68

perpendicular to the line of charge so that

$$V_B - V_A = -\int_{r_a}^{r_b} \frac{2k_e\lambda}{r}dr = 2k_e\lambda\ln\left(\frac{r_a}{r_b}\right)$$

or $\boxed{\Delta V = 2k_e\lambda\ln\left(\frac{r_a}{r_b}\right)}$.

(b) From part (a), when the outer cylinder is considered to be at zero potential, the potential at a distance *r* from the axis is

$$V = 2k_e\lambda\ln\left(\frac{r_a}{r}\right)$$

The field at *r* is given by

$$E = -\frac{\partial V}{\partial r} = -2k_e\lambda\left(\frac{r}{r_a}\right)\left(-\frac{r_a}{r^2}\right) = \frac{2k_e\lambda}{r}$$

But, from part (a), $2k_e\lambda = \dfrac{\Delta V}{\ln(r_a/r_b)}$.

Therefore, $\boxed{E = \dfrac{\Delta V}{\ln(r_a/r_b)}\left(\dfrac{1}{r}\right)}$.

P25.69 (a) The positive plate by itself creates a field

$$E = \frac{\sigma}{2\epsilon_0} = \frac{36.0\times10^{-9}\ \mathrm{C/m^2}}{2(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2})} = 2.03\ \mathrm{kN/C}$$

away from the positive plate. The negative plate by itself creates the same size field and between the plates it is in the same direction. Together the plates create a uniform field $\boxed{4.07\ \mathrm{kN/C}}$ in the space between.

(b) Take $V = 0$ at the negative plate. The potential at the positive plate is then

$$\Delta V = V - 0 = -\int_{x_i}^{x_f} E_x dx = -\int_0^{12.0\ \mathrm{cm}} (-4.07\ \mathrm{kN/C})dx$$

The potential difference between the plates is

$$V = (4.07\times10^3\ \mathrm{N/C})(0.120\ \mathrm{m}) = \boxed{488\ \mathrm{V}}$$

(c) The positive proton starts from rest and accelerates from higher to lower potential. Taking $V_i = 488$ V and $V_f = 0$, by energy conservation, we find the proton's final kinetic energy.

$$(K+qV)_i = (K+qV)_f \rightarrow K_f = qV_i$$

$$\left(\frac{1}{2}mv^2 + qV\right)_i = \left(\frac{1}{2}mv^2 + qV\right)_f$$

$$qV_i = (1.60\times10^{-19}\text{ C})(488\text{ V}) = \frac{1}{2}mv_f^2 = \boxed{7.81\times10^{-17}\text{ J}}$$

(d) From the kinetic energy of part (c),

$$K = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(7.81\times10^{-17}\text{ J})}{1.67\times10^{-27}\text{ kg}}} = 3.06\times10^5\text{ m/s} = \boxed{306\text{ km/s}}$$

(e) Using the constant-acceleration equation, $v_f^2 = v_i^2 + 2a(x_f - x_i)$,

$$a = \frac{v_f^2 - v_i^2}{2(x_f - x_i)} = \frac{(3.06\times10^5\text{ m/s})^2 - 0}{2(0.120\text{ m})}$$

$$= \boxed{3.90\times10^{11}\text{ m/s}^2}\text{ toward the negative plate}$$

(f) The net force on the proton is given by Newton's second law:

$$\sum F = ma = (1.67\times10^{-27}\text{ kg})(3.90\times10^{11}\text{ m/s}^2)$$

$$= \boxed{6.51\times10^{-16}\text{ N}}\text{ toward the negative plate}$$

(g) The magnitude of the electric field is

$$E = \frac{F}{q} = \frac{6.51\times10^{-16}\text{ N}}{1.60\times10^{-19}\text{ C}} = \boxed{4.07\text{ kN/C}}$$

(h) $\boxed{\text{They are the same.}}$

P25.70 (a) Inside the sphere, $\boxed{E_x = E_y = E_z = 0}$.

(b) Outside,

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}\left(V_0 - E_0 z + E_0 a^3 z(x^2+y^2+z^2)^{-3/2}\right)$$

$$= -\left[0 + 0 + E_0 a^3 z\left(-\frac{3}{2}\right)(x^2+y^2+z^2)^{-5/2}(2x)\right]$$

$$E_x = \boxed{3E_0 a^3 xz(x^2+y^2+z^2)^{-5/2}}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}\left(V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-3/2}\right)$$

$$= -E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2} 2y$$

$$E_y = \boxed{3E_0 a^3 yz\left(x^2+y^2+z^2\right)^{-5/2}}$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}\left[V_0 - E_0 z + E_0 a^3 z\left(x^2+y^2+z^2\right)^{-3/2}\right]$$

$$= E_0 - E_0 a^3 z\left(-\frac{3}{2}\right)\left(x^2+y^2+z^2\right)^{-5/2}(2z) - E_0 a^3\left(x^2+y^2+z^2\right)^{-3/2}$$

$$E_z = \boxed{E_0 + E_0 a^3\left(2z^2 - x^2 - y^2\right)\left(x^2+y^2+z^2\right)^{-5/2}}$$

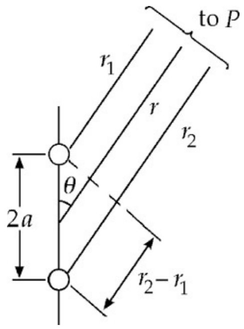
Challenge Problems

P25.71 (a) The total potential is

$$V = \frac{k_e q}{r_1} - \frac{k_e q}{r_2} = \frac{k_e q}{r_1 r_2}\left(r_2 - r_1\right)$$

From the figure, for $r >> a$, $r_2 - r_1 \approx 2a\cos\theta$. Note that r_1 is approximately equal to r_2. Then

$$V \approx \frac{k_e q}{r_1 r_2} 2a\cos\theta \approx \frac{k_e p\cos\theta}{r^2}$$



ANS. FIG. P25.71

(b) $$E_r = -\frac{\partial V}{\partial r} = \boxed{\frac{2k_e p\cos\theta}{r^3}}$$

In spherical coordinates, the θ component of the gradient is $-\frac{1}{r}\left(\frac{\partial}{\partial\theta}\right)$. Therefore,

$$E_\theta = -\frac{1}{r}\left(\frac{\partial V}{\partial\theta}\right) = \boxed{\frac{k_e p\sin\theta}{r^3}}$$

(c) For $r >> a$, $\theta = 90°$: $E_r(90°) = 0$, $E_\theta(90°) = \frac{k_e p}{r^3}$

For $r >> a$, $\theta = 0°$: $E_r(0°) = \frac{2k_e p}{r^3}$, $E_\theta(0°) = 0$

$\boxed{\text{Yes,}}$ these results are reasonable.

(d) $\boxed{\text{No,}}$ because as $r \to 0$, $E \to \infty$. The magnitude of the electric field between the charges of the dipole is not infinite.

(e) Substituting $r_1 \approx r_2 \approx r = (x^2 + y^2)^{1/2}$ and $\cos\theta = \dfrac{y}{\left(x^2+y^2\right)^{1/2}}$ into

$$V = \frac{k_e p\cos\theta}{r^2} \text{ gives } \boxed{V = \frac{k_e p y}{\left(x^2+y^2\right)^{3/2}}}.$$

(f) $$\boxed{E_x = -\frac{\partial V}{\partial x} = \frac{3k_e p x y}{\left(x^2+y^2\right)^{5/2}}} \text{ and } \boxed{E_y = -\frac{\partial V}{\partial y} = \frac{k_e p\left(2y^2 - x^2\right)}{\left(x^2+y^2\right)^{5/2}}}$$

P25.72 Following the problem's suggestion, we use $dU = Vdq$, where the potential is given by $V = \frac{k_e q}{r}$. The element of charge in a shell is $dq = \rho$ (volume element) or $dq = \rho\left(4\pi r^2 dr\right)$ and the charge q in a sphere of radius r is

$$q = 4\pi\rho\int_0^r r^2 dr = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substituting this into the expression for dU, we have

$$dU = \left(\frac{k_e q}{r}\right)dq = k_e\rho\left(\frac{4\pi r^3}{3}\right)\left(\frac{1}{r}\right)\rho\left(4\pi r^2 dr\right) = k_e\left(\frac{16\pi^2}{3}\right)\rho^2 r^4 dr$$

$$U = \int dU = k_e\left(\frac{16\pi^2}{3}\right)\rho^2\int_0^R r^4 dr = k_e\left(\frac{16\pi^2}{15}\right)\rho^2 R^5$$

But the *total* charge, $Q = \rho\frac{4}{3}\pi R^3$. Therefore, $\boxed{U = \frac{3}{5}\frac{k_e Q^2}{R}}$.

P25.73 For an element of area which is a ring of radius r and width dr, the incremental potential is given by $dV = \frac{k_e dq}{\sqrt{r^2 + x^2}}$, where

$$dq = \sigma dA = Cr\left(2\pi r dr\right)$$

The electric potential is then given by

$$V = C\left(2\pi k_e\right)\int_0^R \frac{r^2 dr}{\sqrt{r^2 + x^2}}$$

From a table of integrals,

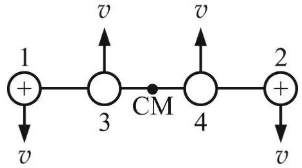
$$\int \frac{r^2 dr}{\sqrt{r^2+x^2}} = \frac{r}{2}\sqrt{r^2+x^2} - \frac{x^2}{2}\ln\left(r+\sqrt{r^2+x^2}\right)$$

The potential then becomes, after substituting and rearranging,

$$V = C\left(2\pi k_e\right)\int_0^R \frac{r^2 dr}{\sqrt{r^2+x^2}}$$

$$= \boxed{\pi k_e C\left[R\sqrt{R^2+x^2} + x^2\ln\left(\frac{x}{R+\sqrt{R^2+x^2}}\right)\right]}$$

P25.74 Take the illustration presented with the problem as an initial picture. No external horizontal forces act on the set of four balls, so its center of mass stays fixed at the location of the center of the square. As the charged balls 1 and 2 swing out and away from each other, balls 3 and 4 move up with equal *y*-components of velocity. The maximum-kinetic-energy point is illustrated. System energy is conserved because it is isolated:



ANS. FIG. P25.74

$$K_i + U_i = K_f + U_f$$

$$0 + U_i = K_f + U_f$$

$$\rightarrow U_i = K_f + U_f$$

$$\frac{k_e q^2}{a} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{k_e q^2}{3a}$$

$$\frac{2k_e q^2}{3a} = 2mv^2 \quad \rightarrow \quad \boxed{v = \sqrt{\frac{k_e q^2}{3am}}}$$

P25.75 (a) Take the origin at the point where we will find the potential. One ring, of width *dx*, has charge $\frac{Qdx}{h}$ and, according to Example 25.5, creates potential

$$dV = \frac{k_e Q dx}{h\sqrt{x^2+R^2}}$$

The whole stack of rings creates potential

$$V = \int_{\text{all charge}} dV = \int_d^{d+h} \frac{k_e Q dx}{h\sqrt{x^2+R^2}} = \frac{k_e Q}{h}\ln\left(x+\sqrt{x^2+R^2}\right)\Big|_d^{d+h}$$

$$= \boxed{\frac{k_e Q}{h}\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)}$$

(b) A disk of thickness dx has charge $\frac{Q\,dx}{h}$ and charge-per-area $\frac{Q\,dx}{\pi R^2 h}$. According to Example 25.6, it creates potential

$$dV = 2\pi k_e \frac{Q\,dx}{\pi R^2 h}\left(\sqrt{x^2+R^2} - x\right)$$

Integrating,

$$V = \int_d^{d+h} \frac{2k_eQ}{R^2h}\left(\sqrt{x^2+R^2}\,dx - x\,dx\right)$$

$$= \frac{2k_eQ}{R^2h}\left[\frac{1}{2}x\sqrt{x^2+R^2} + \frac{R^2}{2}\ln\left(x+\sqrt{x^2+R^2}\right) - \frac{x^2}{2}\right]_d^{d+h}$$

$$V = \boxed{\begin{aligned}&\frac{k_eQ}{R^2h}\Bigg[(d+h)\sqrt{(d+h)^2+R^2} - d\sqrt{d^2+R^2}\\ &\qquad - 2dh - h^2 + R^2\ln\left(\frac{d+h+\sqrt{(d+h)^2+R^2}}{d+\sqrt{d^2+R^2}}\right)\Bigg]\end{aligned}}$$

P25.76 The plates create a uniform electric field to the right in the picture, with magnitude

$$\frac{V_0 - (-V_0)}{d} = \frac{2V_0}{d}$$

Assume the ball swings a small distance x to the right so that the thread is at angle θ from the vertical. The ball moves to a place where the voltage created by the plates is lower by

$$-Ex = -\frac{2V_0}{d}x$$

Because its ground connection maintains the ball at $V = 0$, charge q flows from ground onto the ball, so that

$$-\frac{2V_0x}{d} + \frac{k_eq}{R} = 0 \quad\rightarrow\quad q = \frac{2V_0xR}{k_ed}$$

Then the ball feels an electric force

$$F = qE = \frac{4V_0^2xR}{k_ed^2}$$

to the right. For equilibrium, the electric force must be balanced by the horizontal component of string tension according to

$$T\sin\theta = qE = \frac{4V_0^2 xR}{k_e d^2}$$

and the weight of the ball must be balanced by the vertical component of string tension according to $T\cos\theta = mg$. Dividing the expression for the horizontal component by that for the vertical component, we find that

$$\tan\theta = \frac{4V_0^2 xR}{k_e d^2 mg}$$

For very small angles, we can approximate $\tan\theta \sim \sin\theta = \frac{x}{L}$, so the above expression becomes

$$\frac{x}{L} = \frac{4V_0^2 xR}{k_e d^2 mg} \quad \rightarrow \quad V_0 = \left(\frac{k_e d^2 mg}{4RL}\right)^{1/2} \text{ for small } x$$

If V_0 is less than this value, the only equilibrium position of the ball is hanging straight down. If V_0 exceeds this value, the ball will swing over to one plate or the other.

P25.77 For the given charge distribution,

$$V(x,\ y,\ z) = \frac{k_e(q)}{r_1} + \frac{k_e(-2q)}{r_2}$$

where $r_1 = \sqrt{(x+R)^2 + y^2 + z^2}$

and $r_2 = \sqrt{x^2 + y^2 + z^2}$

The surface on which $V(x, y, z) = 0$ is given by

$$k_e q\left(\frac{1}{r_1} - \frac{2}{r_2}\right) = 0 \qquad \text{or} \qquad 2r_1 = r_2$$

This gives:

$$4(x+R)^2 + 4y^2 + 4z^2 = x^2 + y^2 + z^2$$

which may be written in the form:

$$x^2 + y^2 + z^2 + \left(\frac{8}{3}R\right)x + (0)y + (0)z + \left(\frac{4}{3}R^2\right) = 0 \qquad \textbf{[1]}$$

The general equation for a sphere of radius a centered at (x_0, y_0, z_0) is:

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 - a^2 = 0$$

or

$$x^2 + y^2 + z^2 + (-2x_0)x + (-2y_0)y + (-2z_0)z + (x_0^2 + y_0^2 + z_0^2 - a^2) = 0 \qquad \textbf{[2]}$$

Comparing equations [1] and [2], it is seen that the equipotential surface for which $V = 0$ is indeed a sphere and that:

$$-2x_0 = \frac{8}{3}R;\ -2y_0 = 0;\ -2z_0 = 0;\ x_0^2 + y_0^2 + z_0^2 - a^2 = \frac{4}{3}R^2$$

Thus, $x_0 = -\frac{4}{3}R$, $y_0 = z_0 = 0$, and $a^2 = \left(\frac{16}{9} - \frac{4}{3}\right)R^2 = \frac{4}{9}R^2$

The equipotential surface is therefore a sphere centered at $\boxed{\left(-\frac{4}{3}R, 0, 0\right)}$, having a radius $\boxed{\frac{2}{3}R}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P25.2 (a) -6.00×10^{-4} J; (b) -50.0 V

P25.4 1.35 MJ

P25.6 See P25.6 for full explanation.

P25.8 (a) -2.31 kV; (b) Because a proton is more massive than an electron, a proton traveling at the same speed as an electron has more initial kinetic energy and requires a greater magnitude stopping potential; (c) $\Delta V_p / \Delta V_e = -m_p / m_e$

P25.10 (a) isolated; (b) electric potential energy and elastic potential energy; (c) $\frac{2QE}{k}$; (d) Particle in equilibrium; (e) $\frac{QE}{k}$; (f) $\frac{d^2x'}{dt^2} = -\frac{kx'}{m}$; (g) $2\pi\sqrt{\frac{m}{k}}$; (h) The period does not depend on the electric field. The electric field just shifts the equilibrium point for the spring, just like a gravitational field does for an object hanging from a vertical spring.

P25.12 (a) -5.76×10^{-7} V; (b) 3.84×10^{-7} V; (c) Because the charge of the proton has the same magnitude as that of the electron, only the sign of the answer to part (a) would change.

P25.14 (a) 5.39 kV; (b) 10.8 kV

P25.16 (a) 103 V; (b) -3.85×10^{-7} J, positive work must be done

P25.18 (a) 5.43 kV; (b) 6.08 kV; (c) 658 V

P25.20 (a) 6.00 m; (b) -2.00 μC

P25.22 -11.0×10^{7} V

P25.24 $5.41\frac{k_e Q^2}{s}$

P25.26 (a) $\frac{2}{\sqrt{(x/a)^2 + 1}}$; (b) See ANS. FIG. P25.26(b).

P25.28 (a) no point; (b) $\frac{2k_e q}{a}$

P25.30 $\Delta E_{\text{int}} = \frac{5k_e q^2}{9d}$

P25.32 (a) $v_1 = \sqrt{\frac{2m_2k_eq_1q_2}{m_1(m_1+m_2)}\left(\frac{1}{r_1+r_2}-\frac{1}{d}\right)}$

and $v_2 = \sqrt{\frac{2m_1k_eq_1q_2}{m_2(m_1+m_2)}\left(\frac{1}{r_1+r_2}-\frac{1}{d}\right)}$; (b) faster than calculated in (a)

P25.34 $v = \sqrt{\left(1+\frac{1}{\sqrt{8}}\right)\frac{k_eq^2}{mL}}$

P25.36 See ANS. FIG. P25.36.

P25.38 See ANS. FIG. P25.38.

P25.40 (a) $E_A > E_B$ since $E = \frac{\Delta V}{\Delta s}$; (b) 200 N/C; (c) See ANS. FIG. P25.40.

P25.42 $E_y = \frac{k_eQ}{y\sqrt{\ell^2+y^2}}$

P25.44 –1.51 MV

P25.46 $-\frac{k_e\alpha L}{2}\ln\left[\frac{\sqrt{b^2+(L^2/4)}-L/2}{\sqrt{b^2+(L^2/4)}+L/2}\right]$

P25.48 No. A conductor of any shape forms an equipotential surface. However, if the surface varies in shape, there is no clear way to relate electric field at a point on the surface to the potential of the surface.

P25.50 (a) 0, 1.67 MV; (b) 5.84 MN/C away, 1.17 MV; (c) 11.9 MN/C away, 1.67 MV

P25.52 (a) 450 kV; 7.51 μC

P25.54 (a) 1.06 nC/m^2, negative; (b) –542 kC; (c) –764 MV; (d) The person's head is higher in potential by 210 V; (e) 4.88 × 10^3 N away from Earth; (f) The gravitational force is in the opposite direction and 4.08 × 10^{16} times larger. Electrical forces are negligible in accounting for planetary motion.

P25.56 (a) $\frac{m_1v}{m_1+m_2}$; (b) $\frac{2k_eq_1q_2(m_1+m_2)}{m_1m_2v^2}$; (c) $\left(\frac{m_1-m_2}{m_1+m_2}\right)v\hat{\mathbf{i}}$; (d) $\left(\frac{2m_1}{m_1+m_2}\right)v\hat{\mathbf{i}}$

P25.58 (a) ~10^4 V; (b) ~10^{-6} C

P25.60 (a) $-\frac{k_eq}{4a}$; (b) The approximate expression $-2k_eqa/x^2$ gives $-k_eq/4.5$, which is different by only 11.1%.

P25.62 $\frac{k_e Q^2}{2R}$

P25.64 Even if the charge were to accelerate to infinity, it would only achieve a maximum speed of 30.0 m/s, so it cannot strike the wall of your laboratory at 40.0 m/s.

P25.66 (a) 7.19 V; (b) 7.67 V; (c) 7.84 V; (d) The exact result, represented as 7.90 V, is approximated to within 0.8% by the four-particle version.

P25.68 (a) $\Delta V = 2k_e \lambda \ln\left(\frac{r_a}{r_b}\right)$; (b) $E = \frac{\Delta V}{\ln(r_a/r_b)}\left(\frac{1}{r}\right)$

P25.70 (a) $E_x = E_y = E_z = 0$; (b) $E_x = 3E_0 a^3 xz\left(x^2 + y^2 + z^2\right)^{-5/2}$,

$E_y = 3E_0 a^3 yz\left(x^2 + y^2 + z^2\right)^{-5/2}$, $E_z = E_0 + E_0 a^3\left(2z^2 - x^2 - y^2\right)\left(x^2 + y^2 + z^2\right)^{-5/2}$

P25.72 $U = \frac{3}{5}\frac{k_e Q^2}{R}$

P25.74 $v = \sqrt{\frac{k_e q^2}{3am}}$

P25.76 See P25.76 for full explanation.

26

Capacitance and Dielectrics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ26.1 (i) Answer (a). Because $C = \kappa \in_0 A/d$ and the dielectric constant κ increases.

(ii) Answer (a). Because ΔV is constant, and C increases, so $Q = C\Delta V$ increases.

(iii) Answer (c).

(iv) Answer (a). Because ΔV is constant, and C increases, $U_E = \frac{1}{2}C(\Delta V)^2$ increases.

OQ26.2 Answer (b). The capacitance of a metal sphere is proportional to its radius ($C = Q/V = R/k_e$), and its volume is proportional to radius cubed; therefore, the capacitance of a metal sphere is proportional to the cube root of the volume: $3^{1/3}$.

OQ26.3 Answer (a).

$$\begin{aligned} C &= \frac{\kappa \in_0 A}{d} \\ &= \frac{(1.00\times 10^{2})(8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(1.00\times 10^{-4}\ \mathrm{m^2})}{1.00\times 10^{-3}\ \mathrm{m}} \\ &= 8.85\times 10^{-11}\ \mathrm{F} \quad \text{or} \quad 88.5\ \mathrm{pF} \end{aligned}$$

OQ26.4 Answer (c). The voltage remains constant, but C decreases by a factor of 2 because $C = \kappa \in_0 A/d \to \kappa \in_0 A/(2d) = C/2$. Therefore,

$$U_E = \frac{1}{2}C(\Delta V)^2 \quad \to \quad \left(\frac{1}{2}\right)\left(\frac{1}{2}C\right)(\Delta V)^2 = \frac{1}{2}U_E$$

OQ26.5 Answer (b). Choice (a) is not true because $1/C_{\mathrm{eq}}$ is always larger than $1/C_1 + 1/C_2 + 1/C_3$. Choice (c) is not true because capacitors in series carry the same charge Q, and the voltage across capacitance C_i *is* $\Delta V_i = Q/C_i$. Choices (d) and (e) are not true because capacitors in series carry the same charge.

OQ26.6 Answer (b). Let C = the capacitance of an individual capacitor, and C_S represent the equivalent capacitance of the group in series. While being charged in parallel, each capacitor receives charge

$$Q = C\Delta V_{\mathrm{charge}} = (5.00\times 10^{-4}\ \mathrm{F})(800\ \mathrm{V}) = 0.400\ \mathrm{C}$$

While being discharged in series,

$$\Delta V_{\mathrm{discharge}} = \frac{Q}{C_s} = \frac{Q}{C/10} = \frac{0.400\ \mathrm{C}}{5.00\times 10^{-5}\ \mathrm{F}} = 8.00\ \mathrm{kV}$$

(or 10 times the original voltage).

OQ26.7 (i) Answer (b), because $Q = C\Delta V$.

(ii) Answer (a), because $U_E = \frac{1}{2}C(\Delta V)^2$.

OQ26.8 Answer (d). Let C_2 be the capacitance of the large capacitor and C_1 that of the small one. The equivalent capacitance is

$$C_{\mathrm{eq}} = \frac{1}{1/C_1 + 1/C_2} = \frac{1}{\left(\dfrac{C_1 + C_2}{C_1C_2}\right)} = C_1\left(\frac{C_2}{C_2 + C_1}\right)$$

This is slightly less than C_1.

OQ26.9 Answer (a). Charge Q remains fixed, but the capacitance doubles: $C = \kappa \in_0 A/d \rightarrow (2\kappa) \in_0 A/d = 2C$. Therefore, $\Delta V = Q/C \rightarrow Q/(2C) = \Delta V/2$.

OQ26.10 **(i)** Answer (c). For capacitors in parallel, choices (a), (b), (d), and (e) are not true because the potential difference ΔV is the same, and the charge across capacitance C_i is $Q_i = C_i \Delta V$.

(ii) Answer (e). Although the charges on capacitors in series are the same, the equivalent capacitance is less than the capacitance of any of the capacitors in the group, because $1/C_{eq}$ is always larger than $1/C_1 + 1/C_2 + 1/C_3$; therefore, choices (a) and (c) are not true. Choices (b), (c), and (d) are not true because the charge Q is the same, and choice (c) is also not true because the potential difference across capacitance C_i is $\Delta V_i = Q/C_i$.

OQ26.11 Answer (b). The charge stays constant but C decreases by a factor of 2 because $C = \kappa \in_0 A/d \rightarrow \kappa \in_0 A/(2d) = C/2$. Therefore,

$$U_E = \frac{Q^2}{2C} \quad \rightarrow \quad \frac{Q^2}{2\left(\frac{1}{2}C\right)} = 2U_E$$

OQ26.12 We find the capacitance, voltage, charge, and energy for each capacitor.

(a)	$C = 20\ \mu\text{F}$	$\Delta V = 4\ \text{V}$	$Q = C\Delta V = 80\ \mu\text{C}$	$U_E = \frac{1}{2}Q\Delta V = 160\ \mu\text{J}$
(b)	$C = 30\ \mu\text{F}$	$\Delta V = Q/C = 3\ \text{V}$	$Q = 90\ \mu\text{C}$	$U_E = 135\ \mu\text{J}$
(c)	$C = Q/\Delta V = 40\ \mu\text{F}$	$\Delta V = 2\ \text{V}$	$Q = 80\ \mu\text{C}$	$U_E = 80\ \mu\text{J}$
(d)	$C = 10\ \mu\text{F}$	$\Delta V = (2U/C)^{1/2} = 5\ \text{V}$	$Q = 50\ \mu\text{C}$	$U_E = 125\ \mu\text{J}$
(e)	$C = 2U_E/(\Delta V)^2 = 5\ \mu\text{F}$	$\Delta V = 10\ \text{V}$	$Q = 50\ \mu\text{C}$	$U_E = 250\ \mu\text{J}$

(i) The ranking by capacitance is c > b > a > d > e.

(ii) The ranking by voltage ΔV is e > d > a > b > c.

(iii) The ranking by charge Q is b > a = c > d = e.

(iv) The ranking by energy U_E is e > a > b > d > c.

OQ26.13 (a) False. (b) True. The equation $C = Q/\Delta V$ implies that as charge Q approaches zero, the voltage ΔV also approaches zero so that their ratio remains constant.

OQ26.14 (i) Answer (b). Because $C = \kappa \in_0 A/d$ and the plate separation d increases.

(ii) Answer (c).

(iii) Answer (c). Because $E = Q/\kappa \in_0 A$ remains the same.

(iv) Answer (a). Because $\Delta V = Ed$ and d increases.

ANSWERS TO CONCEPTUAL QUESTIONS

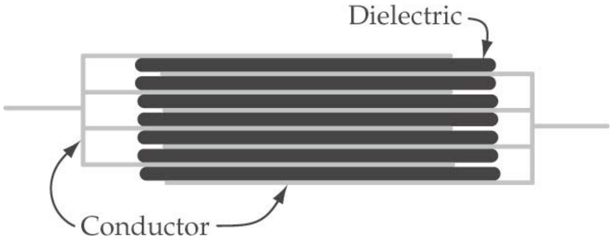
CQ26.1 (a) The capacitor may be charged!

(b) Discharge the capacitor by connecting its terminals together.

CQ26.2 Put a material with higher dielectric strength between the plates, or evacuate the space between the plates. At very high voltages, you may want to cool off the plates or choose to make them of a different chemically stable material, because atoms in the plates themselves can ionize, showing *thermionic emission* under high electric fields.

CQ26.3 The primary choice would be the dielectric. You would want to choose a dielectric that has a large dielectric constant and dielectric strength, such as strontium titanate, where $\kappa \approx 233$ (Table 26.1). A convenient choice could be thick plastic or Mylar. Secondly, geometry would be a factor. To maximize capacitance, one would want the individual plates as close as possible, since the capacitance is proportional to the inverse of the plate separation—hence the need for a dielectric with a high dielectric strength. Also, one would want to build, instead of a single parallel plate capacitor, several capacitors in parallel. This could be achieved through "stacking" the plates of the capacitor. For example, you can alternately lay down sheets of a conducting material, such as aluminum foil, sandwiched between sheets of insulating dielectric. Making sure that none of the conducting sheets are in contact with their immediate neighbors, connect every other plate together. ANS. FIG. CQ26.3 illustrates this idea.

This technique is often used when "home-brewing" signal capacitors for radio applications, as they can withstand huge potential differences without flashover (without either discharge between plates around the dielectric or dielectric breakdown). One variation on this technique is to sandwich together flexible materials such as aluminum roof flashing and thick plastic, so the whole product can be rolled up into a "capacitor burrito" and placed in an insulating tube, such as a PVC pipe, and then filled with motor oil (again to prevent flashover).



ANS. FIG. CQ26.3

CQ26.4 The dielectric decreases the electric field between the plates, causing the potential difference to decrease for the same amount of charge. More charge may be placed on the capacitor before the capacitor experiences dielectric breakdown (resulting in charge jumping from one plate to the other, and in a path being burned through the dielectric) because the electric forces between charges on opposite plates are smaller. The capacitor can have a higher maximum operating voltage, allowing it to hold more charge.

CQ26.5 The work done, $W = Q\Delta V$, is the work done by an external agent, like a battery, to move a charge through a potential difference, ΔV. To determine the energy in a charged capacitor, we must add the work done to move bits of charge from one plate to the other. Initially, there is no potential difference between the plates of an uncharged capacitor. As more charge is transferred from one plate to the other, the potential difference increases, meaning that more work is needed to transfer each additional bit of charge. The total work is given by $W = \frac{1}{2}Q\Delta V$. Another explanation is that the charge Q is moved through an average potential difference $\frac{1}{2}\Delta V$, requiring total work $W = \frac{1}{2}Q\Delta V$.

***CQ26.6** The potential difference must decrease. Since there is no external power supply, the charge on the capacitor, Q, will remain constant—that is, assuming that the resistance of the meter is sufficiently large. Adding a dielectric *increases* the capacitance, which must therefore *decrease* the potential difference between the plates.

CQ26.7 A capacitor stores energy in the electric field between the plates. This is most easily seen when using a "dissectible" capacitor. If the capacitor is charged, carefully pull it apart into its component pieces. One will find that very little residual charge remains on each plate. When reassembled, the capacitor is suddenly "recharged"—by induction—due to the electric field set up and "stored" in the dielectric. This proves to be an instructive classroom demonstration, especially when you ask a student to reconstruct the capacitor without supplying him/her with any rubber gloves or other insulating material. (Of course, this is *after* they sign a liability waiver.)

CQ26.8 The work you do to pull the plates apart becomes additional electric potential energy stored in the capacitor. The charge is constant and the capacitance decreases but the potential difference increases to drive up the potential energy $\frac{1}{2}Q\Delta V$. The electric field between the plates is constant in strength but fills more volume as you pull the plates apart.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 20.1 Definition of Capacitance

P26.1 (a) From Equation 26.1 for the definition of capacitance, $C = \frac{Q}{\Delta V}$, we have

$$\Delta V = \frac{Q}{C} = \frac{27.0\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{9.00\ \text{V}}$$

(b) Similarly,

$$\Delta V = \frac{Q}{C} = \frac{36.0\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{12.0\ \text{V}}$$

P26.2 (a) $C = \frac{Q}{\Delta V} = \frac{10.0 \times 10^{-6}\ \text{C}}{10.0\ \text{V}} = 1.00 \times 10^{-6}\ \text{F} = \boxed{1.00\ \mu\text{F}}$

(b) $\Delta V = \frac{Q}{C} = \frac{100 \times 10^{-6}\ \text{C}}{1.00 \times 10^{-6}\ \text{F}} = \boxed{100\ \text{V}}$

P26.3 (a) $Q = C\Delta V = \left(4.00\times10^{-6}\text{ F}\right)\left(12.0\text{ V}\right) = 4.80\times10^{-5}\text{ C} = \boxed{48.0\ \mu\text{C}}$

(b) $Q = C\Delta V = \left(4.00\times10^{-6}\text{ F}\right)\left(1.50\text{ V}\right) = 6.00\times10^{-6}\text{ C} = \boxed{6.00\ \mu\text{C}}$

Section 26.2 Calculating Capacitance

P26.4 (a) For a spherical capacitor with inner radius *a* and outer radius *b*,

$$C = \frac{ab}{k_e(b-a)} = \frac{(0.070\,0\text{ m})(0.140\text{ m})}{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)(0.140\text{ m}-0.070\,0\text{ m})}$$

$$= \boxed{15.6\text{ pF}}$$

(b) $\Delta V = \dfrac{Q}{C} = \dfrac{4.00\times10^{-6}\text{ C}}{1.56\times10^{-11}\text{ F}} = 2.57\times10^{5}\text{ V} = \boxed{257\text{ kV}}$

P26.5 (a) The capacitance of a cylindrical capacitor is

$$C = \frac{\ell}{2k_e\ln(b/a)}$$

$$= \frac{50.0\text{ m}}{2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\ln(7.27\text{ mm}/2.58\text{ mm})}$$

$$= \boxed{2.68\text{ nF}}$$

(b) Method 1: $\Delta V = 2k_e\lambda\ln\left(\dfrac{b}{a}\right)$

$$\lambda = \frac{Q}{\ell} = \frac{8.10\times10^{-6}\text{ C}}{50.0\text{ m}} = 1.62\times10^{-7}\text{ C/m}$$

$$\Delta V = 2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.62\times10^{-7}\text{ C/m}\right)\ln\left(\frac{7.27\text{ mm}}{2.58\text{ mm}}\right)$$

$$= \boxed{3.02\text{ kV}}$$

Method 2: $\Delta V = \dfrac{Q}{C} = \dfrac{8.10\times10^{-6}\text{ C}}{2.68\times10^{-9}\text{ F}} = \boxed{3.02\text{ kV}}$

P26.6 (a) $C = \dfrac{\kappa\epsilon_0 A}{d} = \dfrac{(1.00)\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(1.00\times10^{3}\text{ m}\right)^2}{800\text{ m}}$

$= \boxed{11.1\text{ nF}}$

(b) The potential between ground and cloud is

$$\Delta V = Ed = \left(3.00\times10^{6}\text{ N/C}\right)\left(800\text{ m}\right) = 2.40\times10^{9}\text{ V}$$

$$Q = C(\Delta V) = \left(11.1\times10^{-9}\text{ C/V}\right)\left(2.40\times10^{9}\text{ V}\right) = \boxed{26.6\text{ C}}$$

P26.7 We have $Q = C\Delta V$ and $C = \epsilon_0 A/d$. Thus, $Q = \epsilon_0 A\Delta V/d$

The surface charge density on each plate has the same magnitude, given by

$$\sigma = \frac{Q}{A} = \frac{\epsilon_0\,\Delta V}{d}$$

Thus,

$$d = \frac{\epsilon_0\,\Delta V}{Q/A} = \frac{\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(150\text{ V})}{\left(30.0\times10^{-9}\text{ C/cm}^2\right)}$$

$$d = \left(4.43\times10^{-2}\,\frac{\text{V}\cdot\text{C}\cdot\text{cm}^2}{\text{N}\cdot\text{m}^2}\right)\frac{\left(1\text{ m}^2\right)}{\left(10^4\text{ cm}^2\right)}\frac{\text{J}}{\text{V}\cdot\text{C}}\frac{\text{N}\cdot\text{m}}{\text{J}} = \boxed{4.43\ \mu\text{m}}$$

P26.8 (a) $$C = \frac{\kappa\,\epsilon_0\,A}{d} = \frac{(1.00)\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(2.30\times10^{-4}\text{ m}^2\right)}{1.50\times10^{-3}\text{ m}}$$

$$= 1.36\times10^{-12}\text{ F} = \boxed{1.36\text{ pF}}$$

(b) $$Q = C\Delta V = \left(1.36\text{ pF}\right)\left(12.0\text{ V}\right) = \boxed{16.3\text{ pC}}$$

(c) $$E = \frac{\Delta V}{d} = \frac{12.0\text{ V}}{1.50\times10^{-3}\text{ m}} = \boxed{8.00\times10^{3}\text{ V/m}}$$

P26.9 (a) The potential difference between two points in a uniform electric field is $\Delta V = Ed$, so

$$E = \frac{\Delta V}{d} = \frac{20.0\text{ V}}{1.80\times10^{-3}\text{ m}} = \boxed{1.11\times10^{4}\text{ V/m}}$$

(b) The electric field between capacitor plates is $E = \dfrac{\sigma}{\epsilon_0}$, so $\sigma = \epsilon_0 E$:

$$\sigma = \left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(1.11\times10^{4}\text{ V/m}\right) = 9.83\times10^{-8}\text{ C/m}^2$$

$$= \boxed{98.3\text{ nC/m}^2}$$

(c) For a parallel-plate capacitor, $C = \dfrac{\epsilon_0 A}{d}$:

$$C = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\times10^{-4}\ \text{m}^2\right)}{1.80\times10^{-3}\ \text{m}}$$

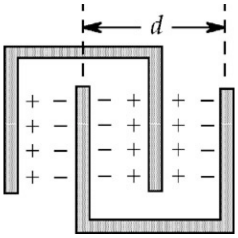
$$= 3.74\times10^{-12}\ \text{F} = \boxed{3.74\ \text{pF}}$$

(d) The charge on each plate is $Q = C\Delta V$:

$$Q = \left(3.74\times10^{-12}\ \text{F}\right)\left(20.0\ \text{V}\right) = \boxed{74.7\ \text{pC}}$$

P26.10 With $\theta = \pi$, the plates are out of mesh and the overlap area is zero. With $\theta = 0$, the overlap area is that of a semi-circle, $\dfrac{\pi R^2}{2}$. By proportion, the effective area of a single sheet of charge is

$$\frac{(\pi-\theta)R^2}{2}$$



ANS. FIG. P26.10

When there are two plates in each comb, the number of adjoining sheets of positive and negative charge is 3, as shown in the sketch. When there are *N* plates on each comb, the number of parallel capacitors is 2*N* – 1 and the total capacitance is

$$C = (2N-1)\frac{\epsilon_0 A_{\text{effective}}}{\text{distance}} = \frac{(2N-1)\epsilon_0(\pi-\theta)R^2/2}{d/2}$$

$$= \boxed{\frac{(2N-1)\epsilon_0(\pi-\theta)R^2}{d}}$$

P26.11 (a) The electric field outside a spherical charge distribution of radius *R* is $E = k_e q/r^2$. Therefore,

$$q = \frac{Er^2}{k_e} = \frac{\left(4.90\times10^4\ \text{N/C}\right)\left(0.210\ \text{m}\right)^2}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = 0.240\ \mu\text{C}$$

Then

$$\sigma = \frac{q}{A} = \frac{0.240\times10^{-6}\ \text{C}}{4\pi\left(0.120\ \text{m}\right)^2} = \boxed{1.33\ \mu\text{C/m}^2}$$

(b) For an isolated charged sphere of radius *R*,

$$C = 4\pi\epsilon_0 r = 4\pi\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.120\ \text{m}\right) = \boxed{13.3\ \text{pF}}$$

P26.12 $\sum F_y = 0: \quad T\cos\theta - mg = 0$

$\sum F_x = 0: \quad T\sin\theta - Eq = 0$

Dividing, $\tan\theta = \dfrac{Eq}{mg}$,

so $E = \dfrac{mg}{q}\tan\theta$

and $\Delta V = Ed = \boxed{\dfrac{mgd\tan\theta}{q}}$.

Section 26.3 Combinations of Capacitors

P26.13 (a) Capacitors in parallel add. Thus, the equivalent capacitor has a value of

$$C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$$

(b) The potential difference across each branch is the same and equal to the voltage of the battery.

$$\Delta V = \boxed{9.00\ \text{V}}$$

(c) $Q_5 = C\Delta V = (5.00\ \mu\text{F})(9.00\ \text{V}) = \boxed{45.0\ \mu\text{C}}$

$Q_{12} = C\Delta V = (12.0\ \mu\text{F})(9.00\ \text{V}) = \boxed{108\ \mu\text{C}}$

P26.14 (a) In series capacitors add as

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5.00\ \mu\text{F}} + \frac{1}{12.0\ \mu\text{F}}$$

$$C_{eq} = \boxed{3.53\ \mu\text{F}}$$

(c) We must answer part (c) first before we can answer part (b). The charge on the equivalent capacitor is

$$Q_{eq} = C_{eq}\Delta V = (3.53\ \mu\text{F})(9.00\ \text{V}) = 31.8\ \mu\text{C}$$

Each of the series capacitors has this same charge on it.

So $Q_1 = Q_2 = \boxed{31.8\ \mu\text{C}}$.

(b) The potential difference across each is

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{6.35\ \text{V}}$$

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\ \mu\text{C}}{12.0\ \mu\text{F}} = \boxed{2.65\ \text{V}}$$

P26.15 (a) When connected in series, the equivalent capacitance is $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$, or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{4.20\ \mu\text{F}} + \frac{1}{8.50\ \mu\text{F}} \rightarrow C_{eq} = \boxed{2.81\ \mu\text{F}}$$

(b) When connected in parallel, the equivalent capacitance is

$$C_{eq} = C_1 + C_2 = 4.20\ \mu\text{F} + 8.50\ \mu\text{F} = \boxed{12.70\ \mu\text{F}}$$

P26.16 (a) When connected in series, the equivalent capacitance is $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$, or

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.50\ \mu\text{F}} + \frac{1}{6.25\ \mu\text{F}} \rightarrow C_{eq} = 1.79\ \mu\text{F}$$

Capacitors in series carry the same charge as their equivalent capacitance:

$$Q = C_{eq}(\Delta V) = (1.79\ \mu\text{F})(6.00\ \text{V}) = \boxed{10.7\ \mu\text{C}}\ \text{on each capacitor}$$

(b) When connected in parallel, each capacitor has the same potential difference across it. The charge stored on each capacitor is then

For $C_1 = 2.50\ \mu\text{F}$: $Q_1 = C_1(\Delta V) = (2.50\ \mu\text{F})(6.00\ \text{V}) = \boxed{15.0\ \mu\text{C}}$

For $C_2 = 6.25\ \mu\text{F}$: $Q_2 = C_2(\Delta V) = (6.25\ \mu\text{F})(6.00\ \text{V}) = \boxed{37.5\ \mu\text{C}}$

P26.17 (a) In $\boxed{\text{series}}$, to reduce the effective capacitance:

$$\frac{1}{32.0\ \mu\text{F}} = \frac{1}{34.8\ \mu\text{F}} + \frac{1}{C_s} \rightarrow \frac{1}{C_s} = \frac{1}{32.0\ \mu\text{F}} - \frac{1}{34.8\ \mu\text{F}}$$

$$\rightarrow C_s = \boxed{398\ \mu\text{F}}$$

(b) In $\boxed{\text{parallel}}$, to increase the total capacitance:

$$29.8\ \mu\text{F} + C_p = 32.0\ \mu\text{F}$$

$$C_p = \boxed{2.20\ \mu\text{F}}$$

P26.18 The capacitance of the combination of extra capacitors must be $\frac{7}{3}C - C = \frac{4}{3}C$. The possible combinations are: one capacitor: C; two capacitors: $2C$ or $\frac{1}{2}C$; three capacitors: $3C$, $\frac{1}{3}C$, $\frac{2}{3}C$ or $\frac{3}{2}C$. None of these is $\frac{4}{3}C$, so the desired capacitance cannot be achieved.

P26.19 (a) The equivalent capacitance of the series combination in the upper branch is

$$\frac{1}{C_{\text{upper}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3.00\ \mu\text{F}} + \frac{1}{6.00\ \mu\text{F}} \to C_{\text{upper}} = 2.00\ \mu\text{F}$$

Likewise, the equivalent capacitance of the series combination in the lower branch is

$$\frac{1}{C_{\text{lower}}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{2.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} \to C_{\text{lower}} = 1.33\ \mu\text{F}$$

These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is

$$C_{\text{eq}} = C_{\text{upper}} + C_{\text{lower}} = 2.00\ \mu\text{F} + 1.33\ \mu\text{F} = \boxed{3.33\ \mu\text{F}}$$

(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. Each of the capacitors in series combination holds the same charge as that on the equivalent capacitor. For the upper branch:

$$Q_3 = Q_6 = Q_{\text{upper}} = C_{\text{upper}}(\Delta V) = (2.00\ \mu\text{F})(90.0\ \text{V}) = 180\ \mu\text{C}\ \text{s}$$

so, $\boxed{180\ \mu\text{C on the 3.00-}\mu\text{F and the 6.00-}\mu\text{F capacitors}}$

For the lower branch:

$$Q_2 = Q_4 = Q_{\text{lower}} = C_{\text{lower}}(\Delta V) = (1.33\ \mu\text{F})(90.0\ \text{V}) = 120\ \mu\text{C}$$

so, $\boxed{120\ \mu\text{C on the 2.00-}\mu\text{F and 4.00-}\mu\text{F capacitors}}$

(c) The potential difference across each of the capacitors in the circuit is:

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\ \text{V}}$$

$$\boxed{60.0\ \text{V across the 3.00-}\mu\text{F and the 2.00-}\mu\text{F capacitors}}$$

$$\Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$$\Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\ \text{V}}$$

$$\boxed{30.0\ \text{V across the 6.00-}\mu\text{F and the 4.00-}\mu\text{F capacitors}}$$

P26.20 (a) Capacitors 2 and 3 are in parallel and present equivalent capacitance $6C$. This is in series with capacitor 1, so the battery sees capacitance $\left[\frac{1}{3C} + \frac{1}{6C}\right]^{-1} = \boxed{2C}$.

(b) If they were initially uncharged, C_1 stores the same charge as C_2 and C_3 together. With greater capacitance, C_3 stores more charge than C_2. Then $\boxed{Q_1 > Q_3 > Q_2}$.

(c) The $(C_2 \parallel C_3)$ equivalent capacitor stores the same charge as C_1. Since it has greater capacitance, $\Delta V = \frac{Q}{C}$ implies that it has smaller potential difference across it than C_1. In parallel with each other, C_2 and C_3 have equal voltages: $\boxed{\Delta V_1 > \Delta V_2 = \Delta V_3}$.

(d) If C_3 is increased, the overall equivalent capacitance increases. More charge moves through the battery and Q increases. As ΔV_1 increases, ΔV_2 must decrease so Q_2 decreases. Then Q_3 must increase even more: $\boxed{Q_3 \text{ and } Q_1 \text{ increase; } Q_2 \text{ decreases}}$.

P26.21 Call C the capacitance of one capacitor and n the number of capacitors. The equivalent capacitance for n capacitors in parallel is

$$C_p = C_1 + C_2 + \cdots + C_n = nC$$

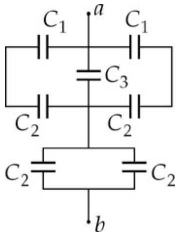
The relationship for *n* capacitors in series is

$$\frac{1}{C_s}=\frac{1}{C_1}+\frac{1}{C_2}+\cdots+\frac{1}{C_n}=\frac{n}{C}$$

Therefore,

$$\frac{C_p}{C_s}=\frac{nC}{C/n}=n^2 \quad \text{or} \quad n=\sqrt{C_p/C_s}=\sqrt{100}=\boxed{10}$$

P26.22 (a) In the upper section, each C_1-C_2 pair, on either side of C_3, are in series:



ANS. FIG. P26.22

$$C_s=\left(\frac{1}{5.00}+\frac{1}{10.0}\right)^{-1}=3.33\ \mu\text{F}$$

and both C_1-C_2 pairs are in parallel to C_3:

$$C_{\text{upper}}=2(3.33)+2.00=8.67\ \mu\text{F}$$

In the lower section, the C_2-C_2 pair are in parallel:

$$C_{\text{lower}}=2(10.0)=20.0\ \mu\text{F}$$

The upper section is in series with the lower section:

$$C_{\text{eq}}=\left(\frac{1}{8.67}+\frac{1}{20.0}\right)^{-1}=\boxed{6.05\ \mu\text{F}}$$

(b) Capacitors in series carry the same charge as their equivalent capacitor; therefore, the upper section, equivalent to a 8.67-μF capacitor, and the lower section, equivalent to a 20.0-μF capacitor, carry the same charge as a 6.05-μF capacitor:

$$Q_{\text{upper}}=Q_{\text{eq}}=C_{\text{eq}}\Delta V=(6.05\ \mu\text{F})(60.0\ \text{V})\simeq 363\ \mu\text{C}$$

The upper section is equivalent to capacitor C_3 and two 3.33-μF capacitors in parallel, and the voltage across each is the same as that across a 8.67-μF capacitor:

$$\Delta V_{\text{upper}}=\frac{Q_{\text{eq}}}{C_{\text{eq}}}=\frac{363\ \mu\text{C}}{8.67\ \mu\text{F}}\simeq 41.9\ \text{V}$$

Therefore, the charge on C_3 is

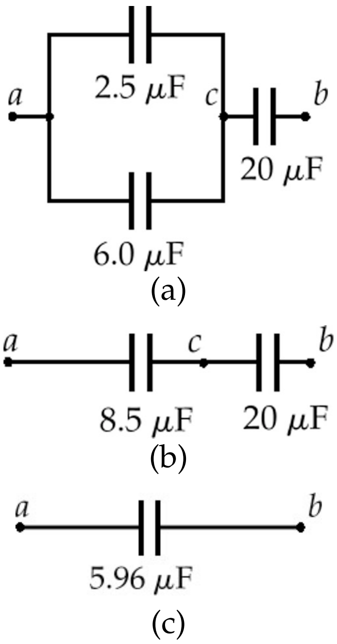
$$Q_3=C_3\Delta V_3\simeq(2.00\ \mu\text{F})(41.9\ \text{V})=\boxed{83.7\ \mu\text{C}}$$

P26.23 (a) We simplify the circuit of Figure P26.23 in three steps as shown in ANS. FIG. P26.23 panels (a), (b), and (c). First, the 15.0-μF and 3.00-μF capacitors in series are equivalent to

$$\frac{1}{(1/15.0\ \mu\text{F}) + (1/3.00\ \mu\text{F})} = 2.50\ \mu\text{F}$$

Next, the 2.50-μF capacitor combines in parallel with the 6.00-μF capacitor, creating an equivalent capacitance of 8.50 μF. At last, this 8.50-μF equivalent capacitor and the 20.0-μF capacitor are in series, equivalent to

$$\frac{1}{(1/8.50\ \mu\text{F}) + (1/20.00\ \mu\text{F})} = \boxed{5.96\ \mu\text{F}}$$



ANS. FIG. P26.23

(b) We find the charge on each capacitor and the voltage across each by working backwards through solution figures (c)–(a), alternately applying $Q = C\Delta V$ and $\Delta V = Q/C$ to every capacitor, real or equivalent. For the 5.96-μF capacitor, we have

$$Q = C\Delta V = (5.96\ \mu\text{F})(15.0\ \text{V})$$
$$= \boxed{89.5\ \mu\text{C}}$$

Thus, if *a* is higher in potential than *b*, just 89.5 μC flows between the wires and the plates to charge the capacitors in each picture. In (b) we have, for the 8.5-μF capacitor,

$$\Delta V_{ac} = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{8.50\ \mu\text{F}} = 10.5\ \text{V}$$

and for the 20.0-μF capacitor in (b), (a), and the original circuit, we have $Q_{20} = 89.5\ \mu$C. Then

$$\Delta V_{cb} = \frac{Q}{C} = \frac{89.5\ \mu\text{C}}{20.0\ \mu\text{F}} = 4.47\ \text{V}$$

Next, the circuit in diagram (a) is equivalent to that in (b), so $\Delta V_{cb} = 4.47$ V and $\Delta V_{ac} = 10.5$ V.

For the 2.50-μF capacitor, $\Delta V\ = 10.5$ V and

$$Q = C\Delta V = (2.50\ \mu\text{F})(10.5\ \text{V}) = \boxed{26.3\ \mu\text{C}}$$

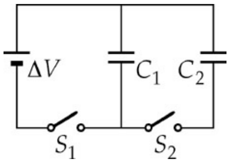
For the 6.00-μF capacitor, $\Delta V = 10.5$ V and

$$Q_6 = C\Delta V = (6.00\ \mu\text{F})(10.5\ \text{V}) = \boxed{63.2\ \mu\text{C}}$$

Now, 26.3 μC having flowed in the upper parallel branch in (a), back in the original circuit we have $Q_{15} = 26.3\ \mu$C and $Q_3 = 26.3\ \mu$C.

P26.24 (a) $C = \dfrac{Q}{\Delta V}$. When S_1 is closed, the charge on C_1 will be

$$Q = C\Delta V = (6.00\ \mu\text{F})(20.0\ \text{V}) = \boxed{120\ \mu\text{C}}$$



ANS. FIG. P26.24

(b) When S_1 is opened and S_2 is closed, the total charge will remain constant and be shared by the two capacitors. We let primed symbols represent the new charges on the capacitors, in $Q_1' = 120\ \mu\text{C} - Q_2'$. The potential differences across the two capacitors will be equal.

$$\Delta V' = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \quad \text{or} \quad \frac{120\ \mu\text{C} - Q_2'}{6.00\ \mu\text{F}} = \frac{Q_2'}{3.00\ \mu\text{F}}$$

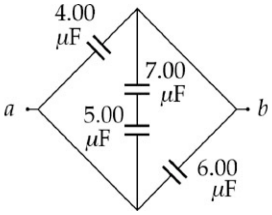
Then we do the algebra to find

$$Q_2' = \frac{360}{9.00}\ \mu\text{C} = \boxed{40.0\ \mu\text{C}}$$

and $Q_1' = 120\ \mu\text{C} - 40.0\ \mu\text{C} = \boxed{80.0\ \mu\text{C}}$.

P26.25 $$C_s = \left(\frac{1}{5.00} + \frac{1}{7.00}\right)^{-1} = 2.92\ \mu\text{F}$$

$$C_p = 2.92 + 4.00 + 6.00 = \boxed{12.9\ \mu\text{F}}$$



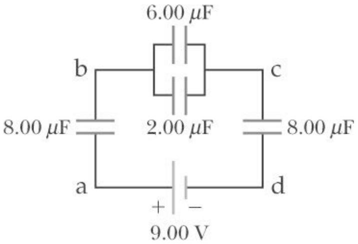
ANS. FIG. P26.25

P26.26 (a) First, we replace the parallel combination between points b and c by its equivalent capacitance,

$C_{bc} = 2.00\ \mu\text{F} + 6.00\ \mu\text{F} = 8.00\ \mu\text{F}$.

Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore

$$\frac{1}{C_{eq}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}} = \frac{3}{8.00\ \mu\text{F}}$$



ANS. FIG. P26.26

giving

$$C_{eq} = \frac{8.00\ \mu F}{3} = \boxed{2.67\ \mu F}$$

(b) The charge on each capacitor in the series is the same as the charge on the equivalent capacitor:

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq}(\Delta V_{ad}) = (2.67\ \mu F)(9.00\ V) = 24.0\ \mu C$$

Then, note that $\Delta V_{bc} = \frac{Q_{bc}}{C_{bc}} = \frac{24.0\ \mu C}{8.00\ \mu F} = 3.00\ V$. The charge on each capacitor in the original circuit is:

On the 8.00 μF between a and b:

$$Q_8 = Q_{ab} = \boxed{24.0\ \mu C}$$

On the 8.00 μF between c and d:

$$Q_8 = Q_{cd} = \boxed{24.0\ \mu C}$$

On the 2.00 μF between b and c:

$$Q_2 = C_2(\Delta V_{bc}) = (2.00\ \mu F)(3.00\ V) = \boxed{6.00\ \mu C}$$

On the 6.00 μF between b and c:

$$Q_6 = C_6(\Delta V_{bc}) = (6.00\ \mu F)(3.00\ V) = \boxed{18.0\ \mu C}$$

(c) We earlier found that $\Delta V_{bc} = 3.00\ V$. The two 8.00 μF capacitors have the same voltage: $\Delta V_8 = \Delta V_8 = \frac{Q}{C} = \frac{24.0\ \mu C}{8.00\ \mu F} = 3.00\ V$, so we conclude that the potential difference across each capacitor is the same: $\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00\ V}$.

P26.27 $C_p = C_1 + C_2$ and $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$. Substitute $C_2 = C_p - C_1$:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$$

Simplifying,

$$C_1^2 - C_1C_p + C_pC_s = 0$$

$$C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_pC_s}}{2} = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$$

We choose arbitrarily the + sign. (This choice can be arbitrary, since with the case of the minus sign, we would get the same two answers with their names interchanged.)

$$\begin{aligned} C_1 &= \frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_pC_s} \\ &= \frac{1}{2}(9.00\text{ pF}) + \sqrt{\frac{1}{4}(9.00\text{ pF})^2 - (9.00\text{ pF})(2.00\text{ pF})} \\ &= \boxed{6.00\text{ pF}} \end{aligned}$$

$$\begin{aligned} C_2 &= C_p - C_1 = \frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s} \\ &= \frac{1}{2}(9.00\text{ pF}) - 1.50\text{ pF} = \boxed{3.00\text{ pF}} \end{aligned}$$

P26.28 $C_p = C_1 + C_2$ and $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$.

Substitute

$$C_2 = C_p - C_1: \quad \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_p - C_1} = \frac{C_p - C_1 + C_1}{C_1(C_p - C_1)}$$

Simplifying,

$$C_1^2 - C_1C_p + C_pC_s = 0$$

and
$$C_1 = \frac{C_p \pm \sqrt{C_p^2 - 4C_pC_s}}{2} = \boxed{\frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$$

where the positive sign was arbitrarily chosen (choosing the negative sign gives the same values for the capacitances, with the names reversed). Then, from $C_2 = C_p - C_1$, we obtain

$$C_2 = \boxed{\frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$$

P26.29 For C_1 connected by itself, $C_1\Delta V = 30.8\ \mu\text{C}$ where ΔV is the battery voltage: $\Delta V = \dfrac{30.8\ \mu\text{C}}{C_1}$.

For C_1 and C_2 in series:

$$\left(\frac{1}{1/C_1 + 1/C_2}\right)\Delta V = 23.1\ \mu\text{C}$$

substituting, $\frac{30.8\ \mu\text{C}}{C_1} = \frac{23.1\ \mu\text{C}}{C_1} + \frac{23.1\ \mu\text{C}}{C_2}$ which gives $C_1 = 0.333C_2$

For C_1 and C_3 in series:

$$\left(\frac{1}{1/C_1 + 1/C_3}\right)\Delta V = 25.2\ \mu\text{C}$$

$$\frac{30.8\ \mu\text{C}}{C_1} = \frac{25.2\ \mu\text{C}}{C_1} + \frac{25.2\ \mu\text{C}}{C_3} \qquad \text{which gives} \quad C_1 = 0.222C_3$$

For all three:

$$Q = \left(\frac{1}{1/C_1 + 1/C_2 + 1/C_3}\right)\Delta V = \frac{C_1\Delta V}{1 + C_1/C_2 + C_1/C_3}$$

$$= \frac{30.8\ \mu\text{C}}{1 + 0.333 + 0.222} = \boxed{19.8\ \mu\text{C}}$$

This is the charge on each one of the three.

Section 26.4 Energy Stored in a Charged Capacitor

P26.30 From $U_E = \frac{1}{2}C\Delta V^2$, we have

$$\Delta V = \sqrt{\frac{2U_E}{C}} = \sqrt{\frac{2(300\ \text{J})}{30.0\times10^{-6}\ \text{F}}} = \boxed{4.47\times10^{3}\ \text{V}}$$

P26.31 The energy stored in the capacitor is given by

$$U_E = \frac{Q^2}{2C} = \frac{1}{2}Q\Delta V = \frac{1}{2}\left(54.0\times10^{-6}\ \text{C}\right)\left(12.0\ \text{V}\right) = \boxed{3.24\times10^{-4}\ \text{J}}$$

P26.32 (a) $U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(12.0\ \text{V})^2 = \boxed{216\ \mu\text{J}}$

(b) $U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{54.0\ \mu\text{J}}$

P26.33 (a) $Q = C\Delta V = \left(150\times10^{-12}\ \text{F}\right)\left(10\times10^{3}\ \text{V}\right) = 1.50\times10^{-6}\ \text{C} = \boxed{1.50\ \mu\text{C}}$

(b) From $U_E = \frac{1}{2}C(\Delta V)^2$,

$$\Delta V = \sqrt{\frac{2U_E}{C}} = \sqrt{\frac{2\left(250\times10^{-6}\ \text{J}\right)}{150\times10^{-12}\ \text{F}}} = 1.83\times10^{3}\ \text{V} = \boxed{1.83\ \text{kV}}$$

P26.34 (a) The equivalent capacitance of a series combination of C_1 and C_2 is

$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}=\frac{1}{18.0\ \mu\text{F}}+\frac{1}{36.0\ \mu\text{F}}\rightarrow C_{eq}=\boxed{12.0\ \mu\text{F}}$$

(b) This series combination is connected to a 12.0-V battery, the total stored energy is

$$U_{E,\,eq}=\frac{1}{2}C_{eq}(\Delta V)^2=\frac{1}{2}(12.0\times10^{-6}\ \text{F})(12.0\ \text{V})^2=\boxed{8.64\times10^{-4}\ \text{J}}$$

(c) Capacitors in series carry the same charge as their equivalent capacitor. The charge stored on each of the two capacitors in the series combination is

$$Q_1=Q_2=Q_{total}=C_{eq}(\Delta V)=(12.0\ \mu\text{F})(12.0\ \text{V})$$
$$=144\ \mu\text{C}=1.44\times10^{-4}\ \text{C}$$

and the energy stored in each of the individual capacitors is:

18.0 μF capacitor:

$$U_{E1}=\frac{Q_1^2}{2C_1}=\frac{(1.44\times10^{-4}\ \text{C})^2}{2(18.0\times10^{-6}\ \text{F})}=\boxed{5.76\times10^{-4}\ \text{J}}$$

36.0 μF capacitor:

$$U_{E2}=\frac{Q_2^2}{2C_2}=\frac{(1.44\times10^{-4}\ \text{C})^2}{2(36.0\times10^{-6}\ \text{F})}=\boxed{2.88\times10^{-4}\ \text{J}}$$

(d) $\boxed{U_{E1}+U_{E2}=5.76\times10^{-4}\ \text{J}+2.88\times10^{-4}\ \text{J}=8.64\times10^{-4}\ \text{J}=U_{E,\,eq},}$ which is one reason why the 12.0 μF capacitor is considered to be equivalent to the two capacitors.

(e) The total energy of the equivalent capacitance will always equal the sum of the energies stored in the individual capacitors.

(f) If C_1 and C_2 were connected in parallel rather than in series, the equivalent capacitance would be $C_{eq}=C_1+C_2=18.0\ \mu\text{F}+36.0\ \mu\text{F}=54.0\ \mu\text{F}$. If the total energy stored in this parallel combination is to be the same as stored in the original series combination, it is necessary that

$$\frac{1}{2}C_{eq}(\Delta V)^2=U_{E,\,eq}$$

From which we obtain

$$\Delta V = \sqrt{\frac{2U_{E,\,\text{eq}}}{C_{\text{eq}}}} = \sqrt{\frac{2\left(8.64\times10^{-4}\ \text{J}\right)}{54.0\times10^{-6}\ \text{F}}} = \boxed{5.66\ \text{V}}$$

(g) Because the potential difference is the same across the two capacitors when connected in parallel, and $U_E = \frac{1}{2}C(\Delta V)^2$, $\boxed{\text{the larger capacitor } C_2 \text{ stores more energy.}}$

P26.35 (a) Because the capacitors are connected in parallel, their voltage remains the same:

$$\begin{aligned} U_E &= \frac{1}{2}C(\Delta V)^2 + \frac{1}{2}C(\Delta V)^2 = C(\Delta V)^2 \\ &= \left(10.0\times10^{-6}\ \mu\text{F}\right)(50.0\ \text{V})^2 \\ &= \boxed{2.50\times10^{-2}\ \text{J}} \end{aligned}$$

(b) Because $C = \frac{\kappa\epsilon_0 A}{d}$ and $d \to 2d$, the altered capacitor has new capacitance to $C' = \frac{C}{2}$. The total charge is the same as before:

$$Q_{\text{initial}} = Q_{\text{final}}$$

$$C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V')$$

$$2C(\Delta V) = \frac{3}{2}C(\Delta V') \quad \rightarrow \quad \Delta V' = \frac{4}{3}\Delta V = \frac{4}{3}(50.0\ \text{V}) = \boxed{66.7\ \text{V}}$$

(c) New $U_E' = \frac{1}{2}C(\Delta V')^2 + \frac{1}{2}\left(\frac{1}{2}C\right)(\Delta V')^2 = \frac{3}{4}C(\Delta V')^2 = \frac{3}{4}C\left(\frac{4\Delta V}{3}\right)^2$

$$U_E' = \frac{4}{3}C(\Delta V)^2 = \frac{4}{3}U_E = \frac{4}{3}\left(2.50\times10^{-2}\ \text{J}\right) = \boxed{3.30\times10^{-2}\ \text{J}}$$

(d) $\boxed{\text{Positive work is done by the agent pulling the plates apart.}}$

P26.36 Before the capacitors are connected, each has voltage ΔV and charge Q.

(a) Connecting plates of like sign places the capacitors in parallel, so the voltage on each capacitor remains the same.

$$U_{E,\,\text{total}} = \frac{1}{2}C(\Delta V)^2 + \frac{1}{2}C(\Delta V)^2 = \boxed{C(\Delta V)^2}$$

(b) Because $C = \frac{\epsilon_0 A}{d}$, the altered capacitor has new capacitance $C' = \frac{\epsilon_0 A}{2d} = \frac{C}{2}$, and this change in capacitance results in a new potential difference $\Delta V'$ across the parallel capacitors. We can solve for the new potential difference because the total charge remains the same:

$$2Q = C(\Delta V) + C(\Delta V) = C(\Delta V') + \frac{C}{2}(\Delta V') \quad \rightarrow \quad \boxed{\Delta V' = \frac{4\Delta V}{3}}$$

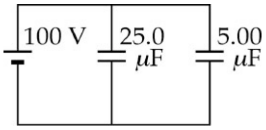
(c) Each capacitor has potential difference $\Delta V'$:

$$U'_{E,\text{ total}} = \frac{1}{2}C(\Delta V')^2 + \frac{1}{2}C'(\Delta V')^2 = \frac{1}{2}C\left(\frac{4\Delta V}{3}\right)^2 + \frac{1}{2}\left(\frac{C}{2}\right)\left(\frac{4\Delta V}{3}\right)^2$$

$$= \frac{12}{9}C(\Delta V)^2 = \boxed{4C\frac{(\Delta V)^2}{3}}$$

(d) $\boxed{\text{Positive work is done by the agent pulling the plates apart.}}$

P26.37 (a) The circuit diagram for capacitors connected in parallel is shown in ANS. FIG. P26.37(a).



ANS. FIG. P26.37(a)

(b) $U_E = \frac{1}{2}C(\Delta V)^2$, and

$$C_p = C_1 + C_2 = 25.0\ \mu\text{F} + 5.00\ \mu\text{F}$$
$$= 30.0\ \mu\text{F}$$

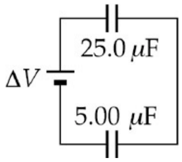
$$U_E = \frac{1}{2}(30.0\times10^{-6})(100)^2 = \boxed{0.150\ \text{J}}$$

(c) $C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{25.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}}\right)^{-1} = 4.17\ \mu\text{F}$

$$U_E = \frac{1}{2}C(\Delta V)^2$$

$$\Delta V = \sqrt{\frac{2U_E}{C}} = \sqrt{\frac{2(0.150\ \text{J})}{4.17\times10^{-6}\ \text{F}}} = \boxed{268\ \text{V}}$$

(d) The circuit diagram for capacitors connected in series is shown in ANS. FIG. P26.37(d).



ANS. FIG. P26.37(d)

P26.38 To prove this, we follow the hint, and calculate the work done in separating the plates, which equals the potential energy stored in the charged capacitor:

$$U_E = \frac{1}{2}\frac{Q^2}{C} = \int F dx$$

Now from the fundamental theorem of calculus, $dU_E = Fdx$

and $$F = \frac{d}{dx}U_E = \frac{d}{dx}\left(\frac{Q^2}{2C}\right) = \frac{1}{2}\frac{d}{dx}\left(\frac{Q^2}{A\in_0/x}\right).$$

Performing the differentiation,

$$F = \frac{1}{2}\frac{d}{dx}\left(\frac{Q^2x}{A\in_0}\right) = \boxed{\frac{Q^2}{2\in_0 A}}$$

P26.39 The energy transferred is

$$T_{ET} = \frac{1}{2}Q\Delta V = \frac{1}{2}(50.0\ \text{C})(1.00\times10^8\ \text{V}) = 2.50\times10^9\ \text{J}$$

and 1% of this (or $\Delta E_{\text{int}} = 2.50\times10^7$ J) is absorbed by the tree. If *m* is the amount of water boiled away, then

$$\begin{aligned}\Delta E_{\text{int}} &= m(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 30.0^\circ\text{C}) \\ &\qquad + m(2.26\times10^6\ \text{J/kg}) \\ &= 2.50\times10^7\ \text{J}\end{aligned}$$

giving $m = \boxed{9.79\ \text{kg}}$.

P26.40 (a) According to Equation 26.2, we may think of a sphere of radius *R* that holds charge *Q* as having a capacitance $C = \frac{R}{k_e}$. The energy stored is

$$U_E = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\left(\frac{R}{k_e}\right)\left(\frac{k_eQ}{R}\right)^2 = \boxed{\frac{k_eQ^2}{2R}}$$

(b) The total energy is

$$U_E = U_{E1} + U_{E2} = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2} = \frac{1}{2}\frac{q_1^2}{R_1/k_e} + \frac{1}{2}\frac{(Q-q_1)^2}{R_2/k_e}$$

$$= \boxed{\frac{k_eq_1^2}{2R_1} + \frac{k_e(Q-q_1)^2}{2R_2}}$$

(c) For a minimum we set $\frac{dU_E}{dq_1} = 0$:

$$\frac{2k_e q_1}{2R_1} + \frac{2k_e(Q - q_1)}{2R_2}(-1) = 0$$

which gives

$$R_2 q_1 = R_1 Q - R_1 q_1 \quad \rightarrow \quad q_1 = \boxed{\frac{R_1 Q}{R_1 + R_2}}$$

(d) $q_2 = Q - q_1 = \boxed{\frac{R_2 Q}{R_1 + R_2}}$

(e) $V_1 = \frac{k_e q_1}{R_1} = \frac{k_e R_1 Q}{R_1(R_1 + R_2)} \rightarrow \boxed{V_1 = \frac{k_e Q}{R_1 + R_2}}$, and

$$V_2 = \frac{k_e q_2}{R_2} = \frac{k_e R_2 Q}{R_2(R_1 + R_2)} \rightarrow \boxed{V_2 = \frac{k_e Q}{R_1 + R_2}}$$

(f) $V_1 - V_2 = \boxed{0}$

P26.41 Originally, the capacitance of each pair of plates is $C = \frac{\epsilon_0 A}{d}$, but after the switch is closed and the distance *d* is changed to *d′* = 0.500*d*, the plates have new capacitance

$$C' = \frac{\epsilon_0 A}{d'} = \frac{\epsilon_0 A}{d/2} = \frac{2\epsilon_0 A}{d} = 2C$$

The capacitors are identical and in series, so each has half the total voltage $(\Delta V) = 100$ V.

(a) The plates are in series, so each collects the same charge:

$$Q = C'(\Delta V) = 2C(\Delta V) = 2(2.00\ \mu\text{C})(100\ \text{V}) = \boxed{400\ \mu\text{C}}$$

(b) Each plate contributes half of the total electric field between the plates, $\frac{E}{2} = \frac{\sigma}{2\epsilon_0} = \frac{Q}{2\epsilon_0 A}$, where $Q = 2C(\Delta V)$ is the magnitude of the charge on a plate, from (a) above. The electric force that each plate exerts on the charge of its neighboring plate is

$$F = Q\frac{E}{2} = \frac{Q^2}{2\epsilon_0 A} = \frac{[2C(\Delta V)]^2}{2\epsilon_0 A} = \frac{2C^2(\Delta V)^2}{(\epsilon_0 A/d)d} = \frac{2C(\Delta V)^2}{d}$$

and this force is balanced by the spring force $F = kx$ on each plate.

Each spring stretches by distance $x = \frac{d}{4}$, so we obtain

$$\frac{2C(\Delta V)^2}{d} = k\frac{d}{4}$$

and solving for the force constant gives

$$k = \frac{8C(\Delta V)^2}{d^2} = \frac{8(2.00 \times 10^{-6}\ \text{F})(100\ \text{V})^2}{(8.00 \times 10^{-3}\ \text{m})^2} = \boxed{2.50\ \text{kN/m}}$$

Section 26.5 Capacitors with Dielectrics

P26.42 (a) Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them.

(b) Suppose the plastic has $\kappa \approx 3$, $E_{\text{max}} \sim 10^7\ \text{V/m}$, and thickness $1\ \text{mil} = \frac{2.54\ \text{cm}}{1\,000}$.

Then, $$C = \frac{\kappa \in_0 A}{d} \sim \frac{3(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2)(0.400\ \text{m}^2)}{2.54 \times 10^{-5}\ \text{m}} \sim \boxed{10^{-6}\ \text{F}}$$

(c) $\Delta V_{\text{max}} = E_{\text{max}} d \sim (10^7\ \text{V/m})(2.54 \times 10^{-5}\ \text{m}) \sim \boxed{10^2\ \text{V}}$

P26.43 $Q_{\text{max}} = C\Delta V_{\text{max}}$, but $\Delta V_{\text{max}} = E_{\text{max}} d$.

Also, $C = \frac{\kappa \in_0 A}{d}$.

Thus, $Q_{\text{max}} = \frac{\kappa \in_0 A}{d}(E_{\text{max}} d) = \kappa \in_0 A E_{\text{max}}$.

(a) With air between the plates, from Table 26.1, the dielectric constant is $\kappa = 1.00$, and the dielectric strength is $E_{\text{max}} = 3.00 \times 10^6\ \text{V/m}$. Therefore,

$$\begin{aligned} Q_{\text{max}} &= \kappa \in_0 A E_{\text{max}} \\ &= (8.85 \times 10^{-12}\ \text{F/m})(5.00 \times 10^{-4}\ \text{m}^2)(3.00 \times 10^6\ \text{V/m}) \\ &= \boxed{13.3\ \text{nC}} \end{aligned}$$

(b) With polystyrene between the plates, from Table 26.1, $\kappa = 2.56$ and $E_{\text{max}} = 24.0 \times 10^6$ V/m.

$$\begin{aligned} Q_{\text{max}} &= \kappa \epsilon_0 A E_{\text{max}} \\ &= 2.56\left(8.85 \times 10^{-12} \text{ F/m}\right)\left(5.00 \times 10^{-4} \text{ m}^2\right) \\ &\qquad \times \left(24.0 \times 10^6 \text{ V/m}\right) \\ &= \boxed{272 \text{ nC}} \end{aligned}$$

P26.44 (a) Note that the charge on the plates remains constant at the original value, Q_0, as the dielectric is inserted. Thus, the change in the potential difference, $\Delta V = Q/C$, is due to a change in capacitance alone. The ratio of the final and initial capacitances is

$$\frac{C_f}{C_i} = \frac{\kappa \epsilon_0 A/d}{\epsilon_0 A/d} = \kappa$$

and $$\frac{C_f}{C_i} = \frac{Q_0/(\Delta V)_f}{Q_0/(\Delta V)_i} = \frac{(\Delta V)_i}{(\Delta V)_f} = \frac{85.0 \text{ V}}{25.0 \text{ V}} = 3.40$$

Thus, the dielectric constant of the inserted material is $\boxed{\kappa = 3.40}$.

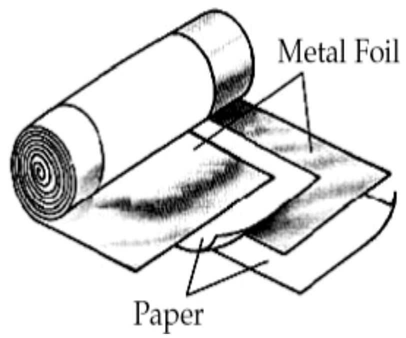
(b) The material is probably $\boxed{\text{nylon}}$ (see Table 26.1).

(c) The presence of a dielectric weakens the field between plates, and the weaker field, for the same charge on the plates, results in a smaller potential difference. If the dielectric only partially filled the space between the plates, the field is weakened only within the dielectric and not in the remaining air-filled space, so the potential difference would not be as small. $\boxed{\text{The voltage would lie somewhere between 25.0 V and 85.0 V.}}$

P26.45 (a) $$C = \frac{\kappa \epsilon_0 A}{d} = \frac{2.10\left(8.85 \times 10^{-12} \text{ F/m}\right)\left(1.75 \times 10^{-4} \text{ m}^2\right)}{4.00 \times 10^{-5} \text{ m}} = 8.13 \times 10^{-11} \text{F}$$
$$= \boxed{81.3 \text{ pF}}$$

(b) $$\Delta V_{\text{max}} = E_{\text{max}} d = \left(60.0 \times 10^6 \text{ V/m}\right)\left(4.00 \times 10^{-5} \text{ m}\right) = \boxed{2.40 \text{ kV}}$$

P26.46 ANS. FIG. P26.46 exaggerates how the strips can be offset to avoid contact between the two foils. It shows how a second paper strip can be used to roll the capacitor into a convenient cylindrical shape with electrical contacts at the two ends. We suppose that the overlapping width of the two metallic strips is still $w = 7.00$ cm. Then for the area of the plates we have $A = \ell w$ in $C = \kappa\epsilon_0 A/d = \kappa\epsilon_0 \ell w/d$. Solving the equation gives



ANS. FIG. P26.46

$$\ell = \frac{Cd}{\kappa\epsilon_0 w} = \frac{(9.50\times10^{-8}\ \text{F})(2.50\times10^{-5}\ \text{m})}{3.70(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(0.070\ 0\ \text{m})} = \boxed{1.04\ \text{m}}$$

P26.47 Originally, $C_i = \dfrac{\epsilon_0 A}{d} = \dfrac{Q}{(\Delta V)_i}$.

(a) The charge is the same before and after immersion, with value

$$Q = C_i(\Delta V)_i = \frac{\epsilon_0 A(\Delta V)_i}{d}$$

$$Q = \frac{(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(25.0\times10^{-4}\ \text{m}^2)(250\ \text{V})}{1.50\times10^{-2}\ \text{m}}$$

$$= \boxed{369\ \text{pC}}$$

(b) Finally,

$$C_f = \frac{\kappa\epsilon_0 A}{d} = \frac{Q}{(\Delta V)_f}:$$

$$C_f = \frac{(80)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(25.0\times10^{-4}\ \text{m}^2)}{1.50\times10^{-2}\ \text{m}}$$

$$= \boxed{1.20\times10^{-10}\ \text{F}}$$

and $(\Delta V)_f = \dfrac{Q}{C_f} = \dfrac{C_i(\Delta V)_i}{C_f} = \dfrac{(\epsilon_0 A/d)}{(\kappa\epsilon_0 A/d)}(\Delta V)_i = \dfrac{(\Delta V)_i}{\kappa} = \dfrac{250\ \text{V}}{80}$

$$= \boxed{3.10\ \text{V}}$$

(c) Originally, $U_i = \dfrac{1}{2}C_i(\Delta V)_i^2 = \dfrac{\epsilon_0 A(\Delta V)_i^2}{2d}$.

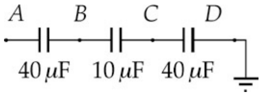
Finally, $U_f = \dfrac{1}{2}C_f(\Delta V)_f^2 = \dfrac{1}{2}\left(\dfrac{\kappa\epsilon_0 A}{d}\right)\left(\dfrac{(\Delta V)_i}{\kappa}\right)^2 = \dfrac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa}$, where, from Table 26.1, $\kappa = 80$ for distilled water. So,

$$\begin{aligned}\Delta U &= U_f - U_i \\ &= \frac{\epsilon_0 A(\Delta V)_i^2}{2d\kappa} - \frac{\epsilon_0 A(\Delta V)_i^2}{2d} \\ &= \frac{\epsilon_0 A(\Delta V)_i^2}{2d}\left(\frac{1}{\kappa}-1\right) = \frac{\epsilon_0 A(\Delta V)_i^2(1-\kappa)}{2d\kappa}\end{aligned}$$

$$\Delta U = \frac{\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(25.0\times10^{-4}\ \mathrm{m^2}\right)(250\ \mathrm{V})^2(1-80)}{2\left(1.50\times10^{-2}\ \mathrm{m}\right)(80)}$$

$$= -4.55\times10^{-8}\ \mathrm{J} = \boxed{-45.5\ \mathrm{nJ}}$$

P26.48 The given combination of capacitors is equivalent to the circuit diagram shown in ANS. FIG. P26.48.



ANS. FIG. P26.48

Put charge Q on point A. Then,

$$Q = (40.0\ \mu\mathrm{F})\Delta V_{AB} = (10.0\ \mu\mathrm{F})\Delta V_{BC} = (40.0\ \mu\mathrm{F})\Delta V_{CD}$$

So, $\Delta V_{BC} = 4\Delta V_{AB} = 4\Delta V_{CD}$, and the center capacitor will break down first, at $\Delta V_{BC} = 15.0$ V. When this occurs,

$$\Delta V_{AB} = \Delta V_{CD} = \frac{1}{4}(\Delta V_{BC}) = 3.75\ \mathrm{V}$$

and $V_{AD} = V_{AB} + V_{BC} + V_{CD} = 3.75\ \mathrm{V} + 15.0\ \mathrm{V} + 3.75\ \mathrm{V} = \boxed{22.5\ \mathrm{V}}$.

P26.49 (a) We use the equation $U_E = Q^2/2C$ to find the potential energy of the capacitor. As we will see, the potential difference ΔV changes as the dielectric is withdrawn. The initial and final energies are $U_{E,i} = \frac{Q^2}{2C_i}$ and $U_{E,f} = \frac{Q^2}{2C_f}$. But the initial capacitance (with the dielectric) is $C_i = \kappa C_f$. Therefore, $U_{E,f} = \kappa\frac{Q^2}{2C_i} = \kappa U_{E,i}$. Since the work done by the external force in removing the dielectric equals the change in potential energy, we have

$$W = U_f - U_i = \kappa U_i - U_i = (\kappa-1)U_i = (\kappa-1)\frac{Q^2}{2C_i}$$

To express this relation in terms of potential difference ΔV_i, we substitute $Q = C_i(\Delta V_i)$, and evaluate:

$$W = \frac{1}{2}C_i(\Delta V_i)^2(\kappa - 1) = \frac{1}{2}(2.00\times10^{-9}\text{ F})(100\text{ V})^2(5.00 - 1.00)$$
$$= 4.00\times10^{-5}\text{ J} = \boxed{40.0\ \mu\text{J}}$$

The positive result confirms that the final energy of the capacitor is greater than the initial energy. The extra energy comes from the work done *on* the system by the external force that pulled out the dielectric.

(b) The final potential difference across the capacitor is $\Delta V_f = \dfrac{Q}{C_f}$.

Substituting $C_f = \dfrac{C_i}{\kappa}$ and $Q = C_i(\Delta V_i)$ gives

$$\Delta V_f = k\Delta V_i = 5.00(100\text{ V}) = \boxed{500\text{ V}}$$

Even though the capacitor is isolated and its charge remains constant, the potential difference across the plates does increase in this case.

Section 26.6 Electric Dipole in an Electric Field

P26.50 (a) The displacement from negative to positive charge is

$$2\vec{\mathbf{a}} = \left(-1.20\hat{\mathbf{i}} + 1.10\hat{\mathbf{j}}\right)\text{ mm} - \left(1.40\hat{\mathbf{i}} - 1.30\hat{\mathbf{j}}\right)\text{ mm}$$
$$= \left(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\times10^{-3}\text{ m}$$

The electric dipole moment is $\vec{\mathbf{p}} = 2\vec{\mathbf{a}}q$

$$\vec{\mathbf{p}} = \left(3.50\times10^{-9}\text{ C}\right)\left(-2.60\hat{\mathbf{i}} + 2.40\hat{\mathbf{j}}\right)\times10^{-3}\text{ m}$$
$$= \boxed{\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times10^{-12}\text{ C}\cdot\text{m}}$$

(b) The torque exerted by the field on the dipole is

$$\vec{\tau} = \vec{\mathbf{p}}\times\vec{\mathbf{E}}$$
$$= \left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times10^{-12}\text{ C}\cdot\text{m}\right]\times\left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right)\times10^{3}\text{ N/C}\right]$$
$$= \left(+44.6\hat{\mathbf{k}} - 65.5\hat{\mathbf{k}}\right)\times10^{-9}\text{ N}\cdot\text{m} = \boxed{-2.09\times10^{-8}\hat{\mathbf{k}}\text{ N}\cdot\text{m}}$$

(c) Relative to zero energy when it is perpendicular to the field, the dipole has potential energy

$$\begin{aligned} U &= -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}} \\ &= -\left[\left(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}\right)\times 10^{-12}\ \text{C}\cdot\text{m}\right] \\ &\qquad\qquad \cdot\left[\left(7.80\hat{\mathbf{i}} - 4.90\hat{\mathbf{j}}\right)\times 10^{3}\ \text{N/C}\right] \\ &= \left(71.0 + 41.2\right)\times 10^{-9}\ \text{J} = \boxed{112\ \text{nJ}} \end{aligned}$$

(d) For convenience we compute the magnitudes

$$\left|\vec{\mathbf{p}}\right| = \sqrt{\left(9.10\right)^2 + \left(8.40\right)^2}\times 10^{-12}\ \text{C}\cdot\text{m} = 12.4\times 10^{-12}\ \text{C}\cdot\text{m}$$

and $\left|\vec{\mathbf{E}}\right| = \sqrt{\left(7.80\right)^2 + \left(4.90\right)^2}\times 10^{3}\ \text{N/C} = 9.21\times 10^{3}\ \text{N/C}$

The maximum potential energy occurs when the dipole moment is opposite in direction to the field, and is

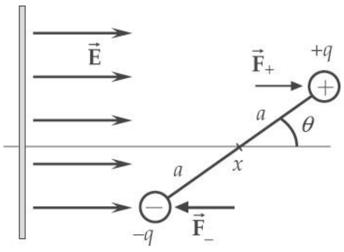
$$U_{\text{max}} = -\vec{\mathbf{p}}\cdot\vec{\mathbf{E}} = -\left|\vec{\mathbf{p}}\right|\left|\vec{\mathbf{E}}\right|(-1) = \left|\vec{\mathbf{p}}\right|\left|\vec{\mathbf{E}}\right| = 114\ \text{nJ}$$

The minimum potential energy configuration is the stable equilibrium position with the dipole aligned with the field. The value is $U_{\text{min}} = -114\ \text{nJ}$

Then the difference, representing the range of potential energies available to the dipole, is $U_{\text{max}} - U_{\text{min}} = \boxed{228\ \text{nJ}}$.

P26.51 (a) The electric field produced by the line of charge has radial symmetry about the *y* axis. According to Equation 24.7 in Example 24.4, the electric field to the right of the *y* axis is

$$\vec{\mathbf{E}} = E(r)\hat{\mathbf{i}} = 2k_e\frac{\lambda}{r}\hat{\mathbf{i}}$$



ANS. FIG. P26.51

Let $x = 25.0$ cm represent the coordinate of the center of the dipole charge, and let $2a = 2.00$ cm represent the distance between the charges. Then $r_- = x - a\cos\theta$ is the coordinate of the negative charge and $r_+ = x + a\cos\theta$ is the coordinate of the positive charge.

The force on the positive charge is

$$\vec{\mathbf{F}}_+ = qE\left(r_+\right)\hat{\mathbf{i}} = q\left(2k_e\frac{\lambda}{r_+}\hat{\mathbf{i}}\right) = 2k_e\frac{q\lambda}{x + a\cos\theta}\hat{\mathbf{i}}$$

and the force on the negative charge is

$$\vec{\mathbf{F}}_- = -qE(r_-)\hat{\mathbf{i}} = -q\left(2k_e\frac{\lambda}{r_-}\hat{\mathbf{i}}\right) = -2k_e\frac{q\lambda}{x - a\cos\theta}\hat{\mathbf{i}}$$

The force on the dipole is

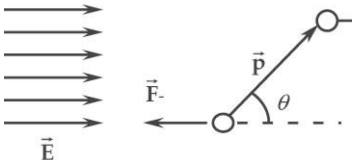
$$\begin{aligned}\vec{\mathbf{F}} = \vec{\mathbf{F}}_+ + \vec{\mathbf{F}}_- &= \left(2k_e\frac{q\lambda}{x + a\cos\theta} - 2k_e\frac{q\lambda}{x - a\cos\theta}\right)\hat{\mathbf{i}}\\ &= 2k_eq\lambda\left(\frac{1}{x + a\cos\theta} - \frac{1}{x - a\cos\theta}\right)\hat{\mathbf{i}}\\ &= 2k_eq\lambda\left[\frac{(x - a\cos\theta) - (x + a\cos\theta)}{x^2 + (a\cos\theta)^2}\right]\hat{\mathbf{i}}\\ &= -\left[\frac{4k_eaq\lambda\cos\theta}{x^2 + (a\cos\theta)^2}\right]\hat{\mathbf{i}}\end{aligned}$$

Substituting numerical values and suppressing units,

$$\begin{aligned}\vec{\mathbf{F}} &= -\frac{4(8.99\times10^9)(0.010\ 0)(10.0\times10^{-6})(2.00\times10^{-6})\cos35.0°}{(0.250)^2 + [(0.010\ 0)(\cos35.0°)]^2}\hat{\mathbf{i}}\\ &= \boxed{-9.42\times10^{-2}\hat{\mathbf{i}}\ \mathrm{N}}\end{aligned}$$

P26.52 Let x represent the coordinate of the negative charge. Then $x + 2a\cos\theta$ is the coordinate of the positive charge. The force on the negative charge is $\vec{\mathbf{F}}_- = -qE(x)\hat{\mathbf{i}}$. The force on the positive charge is

$$\vec{\mathbf{F}}_+ = +qE(x + 2a\cos\theta)\hat{\mathbf{i}} \approx q\left[E(x) + \left(\frac{dE}{dx}\right)(2a\cos\theta)\right]\hat{\mathbf{i}}$$



ANS. FIG. P26.52

The force on the dipole is altogether

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_- + \vec{\mathbf{F}}_+ = q\frac{dE}{dx}(2a\cos\theta)\hat{\mathbf{i}} = \boxed{p\frac{dE}{dx}\cos\theta\hat{\mathbf{i}}}$$

Section 26.7 An Atomic Description of Dielectrics

P26.53 (a) Consider a gaussian surface in the form of a cylindrical pillbox with ends of area $A' << A$ parallel to the sheet. The side wall of the cylinder passes no flux of electric field since this surface is everywhere parallel to the field. Gauss's law becomes

$EA' + EA' = \frac{Q}{\in A}A'$, so $\boxed{E = \frac{Q}{2\in A}}$ directed away from the positive sheet.

(b) In the space between the sheets, each creates field $\frac{Q}{2\in A}$ away from the positive and toward the negative sheet. Together, they create a field of

$$\boxed{E = \frac{Q}{\in A}}$$

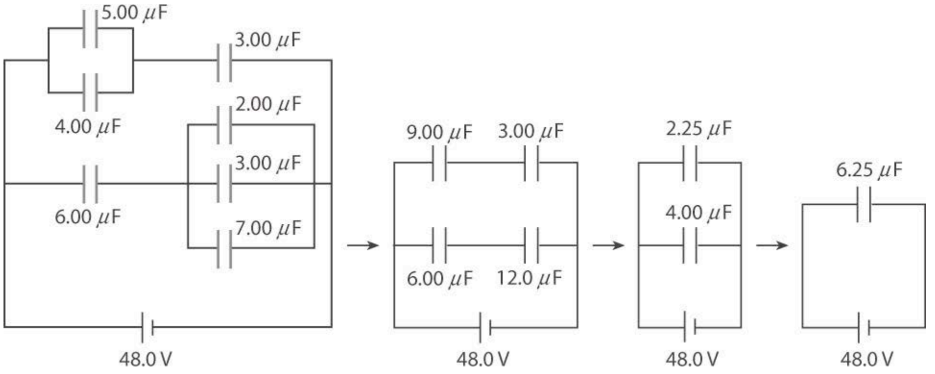
(c) Assume that the field is in the positive x-direction. Then, the potential of the positive plate relative to the negative plate is

$$\Delta V = -\int_{-\text{plate}}^{+\text{plate}} \vec{E}\cdot d\vec{s} = -\int_{-\text{plate}}^{+\text{plate}} \frac{Q}{\in A}\hat{i}\cdot\left(-\hat{i}dx\right) = \boxed{+\frac{Qd}{\in A}}$$

(d) Capacitance is defined by: $C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\in A} = \boxed{\frac{\in A}{d} = \frac{\kappa \in_0 A}{d}}$.

Additional Problems

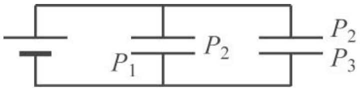
P26.54 The stages for the reduction of this circuit are shown in ANS. FIG. P26.54 below.



ANS. FIG. P26.54

Thus, $C_{eq} = \boxed{6.25\ \mu F}$

P26.55 (a) Each face of P_2 carries charge, so the three-plate system is equivalent to what is shown in ANS. FIG. P26.55 below.



ANS. FIG. P26.55

Each capacitor by itself has capacitance

$$C = \frac{\kappa \in_0 A}{d} = \frac{1\left(8.85\times10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)\left(7.50\times10^{-4}\ \mathrm{m^2}\right)}{1.19\times10^{-3}\ \mathrm{m}}$$

$$= 5.58\ \mathrm{pF}$$

Then equivalent capacitance $= 5.58\ \mathrm{pF} + 5.58\ \mathrm{pF} = \boxed{11.2\ \mathrm{pF}}$.

(b) $Q = C\Delta V + C\Delta V = \left(11.2\times10^{-12}\ \mathrm{F}\right)\left(12\ \mathrm{V}\right) = \boxed{134\ \mathrm{pC}}$

(c) Now P_3 has charge on two surfaces and in effect three capacitors are in parallel:

$$C = 3\left(5.58\ \mathrm{pF}\right) = \boxed{16.7\ \mathrm{pF}}$$

(d) Only one face of P_4 carries charge:

$$Q = C\Delta V = \left(5.58\times10^{-12}\ \mathrm{F}\right)\left(12\ \mathrm{V}\right) = \boxed{66.9\ \mathrm{pC}}$$

P26.56 The upper pair of capacitors, 3-μF and 6-μF, are in series. Their equivalent capacitance is

$$\left(\frac{1}{3.00}+\frac{1}{6.00}\right)^{-1} = 2.00\ \mu\mathrm{F}$$

The lower pair of capacitors, 2-μF and 4-μF, are in series. Their equivalent capacitance is

$$\left(\frac{1}{2.00}+\frac{1}{4.00}\right)^{-1} = 1.33\ \mu\mathrm{F}$$

The upper pair are in parallel to the lower pair, so the total capacitance is

$$C_{eq} = 2.00\mu\mathrm{F} + 1.33\mu\mathrm{F} = 3.33\mu\mathrm{F}$$

(a) The total energy stored in the full circuit is then

$$\left(\text{Energy stored}\right)_{\text{total}} = \frac{1}{2}C_{eq}\left(\Delta V\right)^2 = \frac{1}{2}\left(3.33\times10^{-6}\ \mathrm{F}\right)\left(90.0\ \mathrm{V}\right)^2$$

$$= 1.35\times10^{-2}\ \mathrm{J} = 13.5\times10^{-3}\ \mathrm{J} = \boxed{13.5\ \mathrm{mJ}}$$

(b) Refer to P26.19 for the calculation of the charges used below. The energy stored in each individual capacitor is

For 2.00 μF:

$$(\text{Energy stored})_2 = \frac{Q_2^2}{2C_2} = \frac{(120\times10^{-6}\text{ C})^2}{2(2.00\times10^{-6}\text{ F})} = 3.60\times10^{-3}\text{ J}$$

$$= \boxed{3.60\text{ mJ}}$$

For 3.00 μF:

$$(\text{Energy stored})_3 = \frac{Q_3^2}{2C_3} = \frac{(180\times10^{-6}\text{ C})^2}{2(3.00\times10^{-6}\text{ F})} = 5.40\times10^{-3}\text{ J}$$

$$= \boxed{5.40\text{ mJ}}$$

For 4.00 μF:

$$(\text{Energy stored})_4 = \frac{Q_4^2}{2C_4} = \frac{(120\times10^{-6}\text{ C})^2}{2(4.00\times10^{-6}\text{ F})} = 1.80\times10^{-3}\text{ J}$$

$$= \boxed{1.80\text{ mJ}}$$

For 6.00 μF:

$$(\text{Energy stored})_6 = \frac{Q_6^2}{2C_6} = \frac{(180\times10^{-6}\text{ C})^2}{2(6.00\times10^{-6}\text{ F})} = 2.70\times10^{-3}\text{ J}$$

$$= \boxed{2.70\text{ mJ}}$$

(c) Energy stored $= (3.60 + 5.40 + 1.80 + 2.70)$ mJ $= 13.5$ mJ $=$ $(\text{Energy stored})_{\text{total}}$

$$\boxed{\text{The total energy stored by the system equals the sum of the energies stored in the individual capacitors.}}$$

***P26.57** From Equation 26.13,

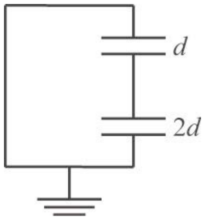
$$u_E = \frac{U_E}{V} = \frac{1}{2}\epsilon_0 E^2$$

Solving for the volume gives

$$V = \frac{U_E}{\frac{1}{2}\epsilon_0 E^2} = \frac{1.00\times10^{-7}\text{ J}}{\frac{1}{2}(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(3\,000\text{ V/m})^2}$$

$$= \boxed{2.51\times10^{-3}\text{ m}^3} = (2.51\times10^{-3}\text{ m}^3)\left(\frac{1\,000\text{ L}}{\text{m}^3}\right) = \boxed{2.51\text{ L}}$$

P26.58 Imagine the center plate is split along its midplane and pulled apart. We have two capacitors in parallel, supporting the same ΔV and carrying total charge Q. The upper capacitor has capacitance $C_1 = \frac{\epsilon_0 A}{d}$ and the lower $C_2 = \frac{\epsilon_0 A}{2d}$. Charge flows from ground onto each of the outside plates so that



ANS. FIG. P26.58

$$Q_1 + Q_2 = Q \quad \text{and} \quad \Delta V_1 = \Delta V_2 = \Delta V.$$

Then $$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_1 d}{\epsilon_0 A} = \frac{Q_2 2d}{\epsilon_0 A} \quad \rightarrow \quad Q_1 = 2Q_2 \quad \rightarrow \quad 2Q_2 + Q_2 = Q.$$

(a) $Q_2 = \frac{Q}{3}$. $\boxed{\text{On the lower plate the charge is } -\frac{Q}{3}}$.

$Q_1 = \frac{2Q}{3}$. $\boxed{\text{On the upper plate the charge is } -\frac{2Q}{3}}$

(b) $\Delta V = \frac{Q_1}{C_1} = \boxed{\frac{2Qd}{3\epsilon_0 A}}$

P26.59 The dielectric strength is $E_{\max} = 2.00 \times 10^8 \text{ V/m} = \frac{\Delta V_{\max}}{d}$,

so we have for the distance between plates $d = \frac{\Delta V_{\max}}{E_{\max}}$.

Now to also satisfy $C = \frac{\kappa \epsilon_0 A}{d} = 0.250 \times 10^{-6}$ F with $\kappa = 3.00$, we combine by substitution to solve for the plate area:

$$A = \frac{Cd}{\kappa \epsilon_0} = \frac{C\Delta V_{\max}}{\kappa \epsilon_0 E_{\max}} = \frac{(0.250 \times 10^{-6} \text{ F})(4\,000 \text{ V})}{(3.00)(8.85 \times 10^{-12} \text{ F/m})(2.00 \times 10^8 \text{ V/m})}$$

$$= \boxed{0.188 \text{ m}^2}$$

P26.60 We can use the energy U_C stored in the capacitor to find the potential difference across the plates:

$$U_C = \frac{1}{2}C(\Delta V)^2 \quad \rightarrow \quad \Delta V = \sqrt{\frac{2U_C}{C}}$$

When the particle moves between the plates, the change in potential energy of the charge-field system is

$$\Delta U_{\text{system}} = q\Delta V = -q\sqrt{\frac{2U_C}{C}}$$

where we have noted that the potential difference is negative from the positive plate to the negative plate. Apply the isolated system (energy) model to the charge-field system:

$$\Delta K + \Delta U_{\text{system}} = 0 \;\rightarrow\; \Delta K = -\Delta U_{\text{system}} = q\sqrt{\frac{2U_C}{C}}$$

Substitute numerical values:

$$\Delta K = \left(-3.00 \times 10^{-6}\ \text{C}\right)\sqrt{\frac{2\left(0.050\ 0\ \text{J}\right)}{10.0 \times 10^{-6}\ \text{F}}} = -3.00 \times 10^{-4}\ \text{J}$$

This decrease in kinetic energy of the particle is more than the energy with which it began. Therefore, the particle does not arrive at the negative plate but rather turns around and moves back to the positive plate.

***P26.61** (a) $V = \dfrac{m}{\rho} = \dfrac{1.00\times 10^{-12}\ \text{kg}}{1\ 100\ \text{kg/m}^3} = \boxed{9.09\times 10^{-16}\ \text{m}^3}$

Since $V = \dfrac{4\pi r^3}{3}$, the radius is $r = \left[\dfrac{3V}{4\pi}\right]^{1/3}$, and the surface area is

$$A = 4\pi r^2 = 4\pi\left[\frac{3V}{4\pi}\right]^{2/3} = 4\pi\left[\frac{3\left(9.09\times 10^{-16}\ \text{m}^3\right)}{4\pi}\right]^{2/3}$$

$$= \boxed{4.54\times 10^{-10}\ \text{m}^2}$$

(b) $C = \dfrac{\kappa\epsilon_0 A}{d} = \dfrac{(5.00)\left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.54\times 10^{-10}\ \text{m}^2\right)}{100\times 10^{-9}\ \text{m}}$

$$= \boxed{2.01\times 10^{-13}\ \text{F}}$$

(c) $Q = C(\Delta V) = \left(2.01\times 10^{-13}\ \text{F}\right)\left(100\times 10^{-3}\ \text{V}\right) = \boxed{2.01\times 10^{-14}\ \text{C}}$,

and the number of electronic charges is

$$n = \frac{Q}{e} = \frac{2.01\times 10^{-14}\ \text{C}}{1.60\times 10^{-19}\ \text{C}} = \boxed{1.26\times 10^{5}}$$

P26.62 (a) With the liquid filling the space between the plates to height *fd*, the top of the fluid at the air-fluid interface develops an induced dipole layer of charge so that it acts as a thin plate with opposite charge on its upper and lower sides; thus, the partially filled capacitor behaves as two capacitors in series connected at the interface. The upper and lower capacitors have separate capacitances:

$$C_{\text{up}} = \frac{1 \in_0 A}{d(1-f)} \quad \text{and} \quad C_{\text{down}} = \frac{6.5 \in_0 A}{fd}$$

The equivalent series capacitance is

$$C_f = \frac{1}{\dfrac{d(1-f)}{\in_0 A} + \dfrac{fd}{6.5 \in_0 A}} = \frac{6.5 \in_0 A}{6.5d - 6.5df + fd}$$

$$= \left(\frac{\in_0 A}{d}\right)\left(\frac{6.5}{6.5 - 5.5f}\right)$$

$$= \boxed{25.0\ \mu\text{F}(1 - 0.846f)^{-1}}$$

(b) For $f = 0$, the capacitor is empty so we can expect capacitance $\boxed{25.0\ \mu\text{F}}$. For $f = 0$,

$$C_f = 25.0\ \mu\text{F}(1 - 0.846f)^{-1} = 25.0\ \mu\text{F}(1-0)^{-1} = 25.0\ \mu\text{F}$$

and $\boxed{\text{the general expression agrees}}$.

(c) For $f = 1$, we expect $6.5(25.0\ \mu\text{F}) = 162\ \mu\text{F}$. For $f = 1$,

$$C_f = 25.0\ \mu\text{F}(1 - 0.846f)^{-1} = 25.0\ \mu\text{F}(1 - 0.846)^{-1} = \boxed{162\ \mu\text{F}}$$

and $\boxed{\text{the general expression agrees}}$.

P26.63 The initial charge on the larger capacitor is

$$Q = C\Delta V = (10.0\ \mu\text{F})(15.0\ \text{V}) = 150\ \mu\text{C}$$

An additional charge q is pushed through the 50.0-V battery, giving the smaller capacitor charge q and the larger charge $150\ \mu\text{C} + q$.

Then $$50.0\ \text{V} = \frac{q}{5.00\ \mu\text{F}} + \frac{150\ \mu\text{C} + q}{10.0\ \mu\text{F}}.$$

$$500\ \mu\text{C} = 2q + 150\ \mu\text{C} + q$$
$$q = 117\ \mu\text{C}$$

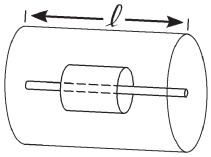
So across the 5.00-μF capacitor,

$$\Delta V = \frac{q}{C} = \frac{117\ \mu\text{C}}{5.00\ \mu\text{F}} = \boxed{23.3\ \text{V}}$$

Across the 10.0-μF capacitor,

$$\Delta V = \frac{150\ \mu\text{C} + 117\ \mu\text{C}}{10.0\ \mu\text{F}} = \boxed{26.7\ \text{V}}$$

***P26.64** From Gauss's Law, for the electric field inside the cylinder, $2\pi r\ell E = \dfrac{q_{\text{in}}}{\epsilon_0}$.



ANS. FIG. P26.64

so $\qquad E = \dfrac{\lambda}{2\pi r\,\epsilon_0}$.

$$\Delta V = -\int_{r_1}^{r_2} \vec{\mathbf{E}}\cdot d\vec{\mathbf{r}} = \int_{r_1}^{r_2} \frac{\lambda}{2\pi r\,\epsilon_0}dr = \frac{\lambda}{2\pi\epsilon_0}\ln\left(\frac{r_1}{r_2}\right)$$

Recognizing that $\dfrac{\lambda_{\text{max}}}{2\pi\epsilon_0} = E_{\text{max}}r_{\text{inner}}$, we obtain

$$\Delta V = \left(1.20\times10^{6}\ \text{V/m}\right)\left(0.100\times10^{-3}\ \text{m}\right)\ln\left(\frac{25.0\ \text{m}}{0.200\ \text{m}}\right)$$

$$\Delta V_{\text{max}} = \boxed{579\ \text{V}}$$

P26.65 Where the metal block and the plates overlap, the electric field between the plates is zero. The plates do not lose charge in the overlapping region, but opposite charge induced on the surfaces of the inserted portion of the block cancels the field from charge on the plates. The unfilled portion of the capacitor has capacitance

$$C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0\,\ell(\ell - x)}{d}$$

The effective charge on this portion (the charge producing the remaining electric field between the plates) is proportional to the unblocked area:

$$Q = \frac{(\ell - x)Q_0}{\ell}$$

(a) The stored energy is

$$U = \frac{Q^2}{2C} = \frac{\left[(\ell - x)Q_0/\ell\right]^2}{2\epsilon_0\,\ell(\ell - x)/d} = \boxed{\frac{Q_0^2 d(\ell - x)}{2\epsilon_0\,\ell^3}}$$

(b) $F = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{Q_0^2(\ell - x)d}{2\epsilon_0 \ell^3}\right) = +\frac{Q_0^2 d}{2\epsilon_0 \ell^3}$

$\vec{\mathbf{F}} = \boxed{\frac{Q_0^2 d}{2\epsilon_0 \ell^3} \text{ to the right}}$ (into the capacitor: the block is pulled in)

(c) Stress $= \frac{F}{\ell d} = \boxed{\frac{Q_0^2}{2\epsilon_0 \ell^4}}$

(d) The energy density is

$$u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0\left(\frac{\sigma}{\epsilon_0}\right)^2 = \frac{1}{2\epsilon_0}\left(\frac{Q}{A}\right)^2 = \frac{1}{2\epsilon_0}\left[\frac{\cancel{(\ell - x)}Q_0/\ell}{\ell\cancel{(\ell - x)}}\right]^2$$

$$= \boxed{\frac{Q_0^2}{2\epsilon_0 \ell^4}}$$

(e) $\boxed{\text{They are precisely the same.}}$

P26.66 (a) Put charge *Q* on the sphere of radius *a* and –*Q* on the other sphere. Relative to *V* = 0 at infinity, because *d* is larger compared to *a* and to *b*.

The potential at the surface of *a* is approximately $V_a = \frac{k_e Q}{a} - \frac{k_e Q}{d}$

and the potential of *b* is approximately $V_b = \frac{-k_e Q}{b} + \frac{k_e Q}{d}$.

The difference in potential is $V_a - V_b = \frac{k_e Q}{a} + \frac{k_e Q}{b} - \frac{k_e Q}{d} - \frac{k_e Q}{d}$

and $C = \frac{Q}{V_a - V_b} = \frac{4\pi\epsilon_0}{\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right) - \left(\frac{2}{d}\right)}$

(b) As $d \to \infty$, $\frac{1}{d}$ becomes negligible compared to $\frac{1}{a}$ and $\frac{1}{b}$. Then,

$$C = \frac{4\pi\epsilon_0}{\left(\frac{1}{a}\right) + \left(\frac{1}{b}\right)} \quad \text{and} \quad \frac{1}{C} = \boxed{\frac{1}{4\pi\epsilon_0 a} + \frac{1}{4\pi\epsilon_0 b}}$$

as for two spheres in series.

P26.67 Call the unknown capacitance C_u. The charge remains the same:

$$Q = C_u(\Delta V_i) = (C_u + C)(\Delta V_f)$$

$$\boxed{C_u = \frac{C(\Delta V_f)}{(\Delta V_i) - (\Delta V_f)}} = \frac{(10.0\ \mu\text{F})(30.0\ \text{V})}{(100\ \text{V} - 30.0\ \text{V})} = \boxed{4.29\ \mu\text{F}}$$

P26.68 (a) $C_0 = \dfrac{\epsilon_0 A}{d} = \dfrac{Q_0}{\Delta V_0}$ for a capacitor with air or vacuum beteen its plates. When the dielectric is inserted at constant voltage,

$$C = \kappa C_0 = \frac{Q}{\Delta V_0}$$

The original energy is

$$U_{E0} = \frac{C_0(\Delta V_0)^2}{2}$$

and the final energy is

$$U_E = \frac{C(\Delta V_0)^2}{2} = \frac{\kappa C_0(\Delta V_0^2)}{2}$$

therefore,

$$\frac{U_E}{U_{E0}} = \kappa$$

(b) The electric field between the plates polarizes molecules within the dielectric; therefore the field does work on charge within the molecules to create electric dipoles. The extra energy comes from (part of the) electrical work done by the battery in separating that charge.

(c) The charge on the plates increases because the voltage remains the same:

$$Q_0 = C_0 \Delta V_0$$

and $Q = C\Delta V_0 = \kappa C_0 \Delta V_0$

so the charge increases according to $\boxed{\dfrac{Q}{Q_0} = \kappa}$.

P26.69 Initially (capacitors charged in parallel),

$$q_1 = C_1(\Delta V) = (6.00\ \mu\text{F})(250\ \text{V}) = 1\,500\ \mu\text{C}$$

$$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(250\ \text{V}) = 500\ \mu\text{C}$$

After reconnection (positive plate to negative plate),

$$q'_{\text{total}} = q_1 - q_2 = 1\,000\ \mu\text{C}$$

and $$\Delta V' = \frac{q'_{\text{total}}}{C_{\text{total}}} = \frac{1\,000\ \mu\text{C}}{8.00\ \mu\text{F}} = 125\ \text{V}$$

Therefore,

$$q'_1 = C_1(\Delta V') = (6.00\ \mu\text{F})(125\ \text{V}) = \boxed{750\ \mu\text{C}}$$

$$q'_2 = C_2(\Delta V') = (2.00\ \mu\text{F})(125\ \text{V}) = \boxed{250\ \mu\text{C}}$$

P26.70 The condition that we are testing is that the capacitance increases by less than 10%, or,

$$\frac{C'}{C} < 1.10$$

Substituting the expressions for C and C' from Example 26.1, we have

$$\frac{C'}{C} = \frac{\dfrac{\ell}{2k_e \ln(b/1.10a)}}{\dfrac{\ell}{2k_e \ln(b/a)}} = \frac{\ln(b/a)}{\ln(b/1.10a)} < 1.10$$

This becomes

$$\ln\left(\frac{b}{a}\right) < 1.10\ln\left(\frac{b}{1.10a}\right) = 1.10\ln\left(\frac{b}{a}\right) + 1.10\ln\left(\frac{1}{1.10}\right)$$

$$= 1.10\ln\left(\frac{b}{a}\right) - 1.10\ln(1.10)$$

We can rewrite this as

$$-0.10\ln\left(\frac{b}{a}\right) < -1.10\ln(1.10)$$

$$\ln\left(\frac{b}{a}\right) > 11.0\ln(1.10) = \ln(1.10)^{11.0}$$

where we have reversed the direction of the inequality because we multiplied the whole expression by –1 to remove the negative signs.

Comparing the arguments of the logarithms on both sides of the inequality, we see that

$$\frac{b}{a} > (1.10)^{11.0} = 2.85$$

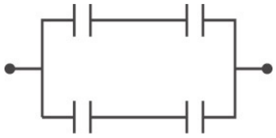
Thus, if $b > 2.85a$, the increase in capacitance is less than 10% and it is more effective to increase ℓ.

P26.71 Placing two identical capacitor is series will split the voltage evenly between them, giving each a voltage of 45 V, but the total capacitance will be half of what is needed. To double the capacitance, another pair of series capacitors must be placed in parallel with the first pair, as shown in ANS. FIG. P26.71A. The equivalent capacitance is

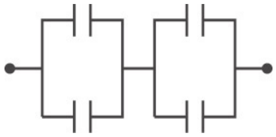
$$\left(\frac{1}{100\ \mu\text{F}} + \frac{1}{100\ \mu\text{F}}\right)^{-1} + \left(\frac{1}{100\ \mu\text{F}} + \frac{1}{100\ \mu\text{F}}\right)^{-1} = 100\ \mu\text{F}$$

Another possibility shown in ANS. FIG. P26.71B: two capacitors in parallel, connected in series to another pair of capacitors in parallel; the voltage across each parallel section is then 45 V. The equivalent capacitance is

$$\frac{1}{(100\ \mu\text{F} + 100\ \mu\text{F})^{-1} + (100\ \mu\text{F} + 100\ \mu\text{F})^{-1}} = 100\ \mu\text{F}$$



ANS. FIG. P26.71A



ANS. FIG. P26.71B

(a) $\boxed{\text{One capacitor cannot be used by itself — it would burn out. She can use two capacitors in series, connected in parallel to another two capacitors in series. Another possibility is two capacitors in parallel, connected in series to another two capacitors in parallel. In either case, one capacitor will be left over.}}$

(b) $\boxed{\text{Each of the four capacitors will be exposed to a maximum voltage of 45 V.}}$

Challenge Problems

P26.72 From Example 26.1, when there is a vacuum between the conductors, the voltage between them is

$$\Delta V = |V_b - V_a| = 2k_e\lambda \ln\left(\frac{b}{a}\right) = \frac{\lambda}{2\pi \in_0}\ln\left(\frac{b}{a}\right)$$

With a dielectric, a factor $1/\kappa$ must be included, and the equation becomes

$$\Delta V = \frac{\lambda}{2\pi\kappa \in_0}\ln\left(\frac{b}{a}\right)$$

The electric field is

$$E = \frac{\lambda}{2\pi\kappa \in_0 r}$$

So when $E = E_{\max}$ at $r = a$,

$$\frac{\lambda_{\max}}{2\pi\kappa \in_0} = E_{\max}a \quad \text{and} \quad \Delta V_{\max} = \frac{\lambda_{\max}}{2\pi\kappa \in_0}\ln\left(\frac{b}{a}\right) = E_{\max}a\ \ln\left(\frac{b}{a}\right)$$

Thus,

$$\Delta V_{\max} = \left(18.0\times 10^6\ \text{V/m}\right)\left(0.800\times 10^{-3}\ \text{m}\right)\ln\left(\frac{3.00\ \text{mm}}{0.800\ \text{mm}}\right)$$

$$= \boxed{19.0\ \text{kV}}$$

P26.73 According to the suggestion, the combination of capacitors shown is equivalent to

Then, from ANS. FIG. P26.73,

$$\frac{1}{C} = \frac{1}{C_0} + \frac{1}{C + C_0} + \frac{1}{C_0}$$

$$= \frac{C + C_0 + C_0 + C + C_0}{C_0\left(C + C_0\right)}$$

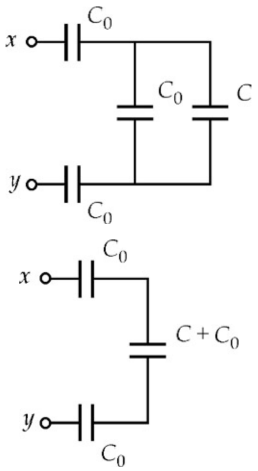
$$C_0C + C_0^2 = 2C^2 + 3C_0C$$

$$2C^2 + 2C_0C - C_0^2 = 0$$

$$C = \frac{-2C_0 \pm \sqrt{4C_0^2 + 4\left(2C_0^2\right)}}{4}$$

Only the positive root is physical:

$$\boxed{C = \frac{C_0}{2}\left(\sqrt{3} - 1\right)}$$



ANS. FIG. P26.73

P26.74 Let charge λ per length be on one wire and $-\lambda$ be on the other. The electric field due to the charge on the positive wire is perpendicular to the wire, radial, and of magnitude

$$E_+ = \frac{\lambda}{2\pi \in_0 r}$$

The potential difference between the surfaces of the wires due to the presence of this charge is

$$\Delta V_1 = -\int_{-\text{wire}}^{+\text{wire}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{r}} = -\frac{\lambda}{2\pi \in_0} \int_{D-r}^{r} \frac{dr}{r} = \frac{\lambda}{2\pi \in_0} \ln\left(\frac{D-r}{r}\right)$$

The presence of the linear charge density $-\lambda$ on the negative wire makes an identical contribution to the potential difference between the wires. Therefore, the total potential difference is

$$\Delta V = 2(\Delta V_1) = \frac{\lambda}{\pi \in_0} \ln\left(\frac{D-r}{r}\right)$$

With *D* much larger than *r* we have nearly $\Delta V = \dfrac{\lambda}{\pi \in_0} \ln\left(\dfrac{D}{r}\right)$

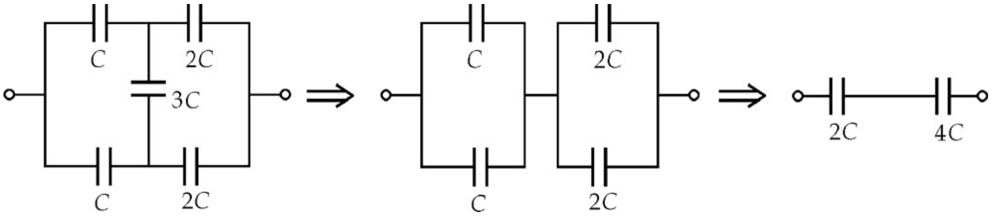
and the capacitance of this system of two wires, each of length ℓ, is

$$C = \frac{Q}{\Delta V} = \frac{\lambda \ell}{\Delta V} = \frac{\lambda \ell}{(\lambda/\pi \in_0)\ln[D/r]} = \frac{\pi \in_0 \ell}{\ln[D/r]}$$

The capacitance per unit length is $\boxed{\dfrac{C}{\ell} = \dfrac{\pi \in_0}{\ln[D/r]}}$.

P26.75 By symmetry, the potential difference across 3*C* is zero, so the circuit reduces to (see ANS. FIG. P26.75):

$$C_{eq} = \left(\frac{1}{2C} + \frac{1}{4C}\right)^{-1} = \frac{8}{6}C = \boxed{\frac{4}{3}C}$$



ANS. FIG. P26.75

P26.76 (a) Consider a strip of width *dx* and length *W* at position *x* from the front left corner. The capacitance of the lower portion of this strip is $\dfrac{\kappa_1 \in_0 W\,dx}{t\,x/L}$. The capacitance of the upper portion is $\dfrac{\kappa_2 \in_0 W\,dx}{t\,(1-x/L)}$.

The series combination of the two elements has capacitance

$$\frac{1}{\dfrac{\text{tx}}{\kappa_1 \in_0 \text{WL dx}}+\dfrac{\text{t(L}-\text{x)}}{\kappa_2 \in_0 \text{W Ldx}}}=\frac{\kappa_1\kappa_2 \in_0 \text{W Ldx}}{\kappa_2\text{tx}+\kappa_1\text{tL}-\kappa_1\text{tx}}$$

The whole capacitance is a combination of elements in parallel:

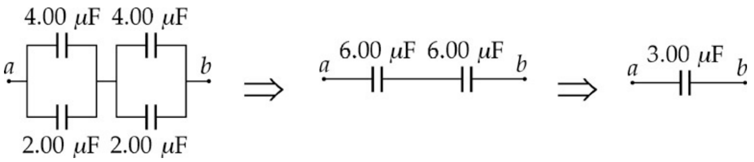
$$\begin{aligned}
\text{C} &= \int_0^{\text{L}} \frac{\kappa_1\kappa_2 \in_0 \text{W Ldx}}{(\kappa_2-\kappa_1)\text{tx}+\kappa_1\text{tL}} \\
&= \frac{1}{(\kappa_2-\kappa_1)\text{t}}\int_0^{\text{L}} \frac{\kappa_1\kappa_2 \in_0 \text{W L}(\kappa_2-\kappa_1)\text{tdx}}{(\kappa_2-\kappa_1)\text{tx}+\kappa_1\text{tL}} \\
&= \frac{\kappa_1\kappa_2 \in_0 \text{W L}}{(\kappa_2-\kappa_1)\text{t}}\ln\left[(\kappa_2-\kappa_1)\text{tx}+\kappa_1\text{tL}\right]_0^{\text{L}} \\
&= \frac{\kappa_1\kappa_2 \in_0 \text{WL}}{(\kappa_2-\kappa_1)\text{t}}\ln\left[\frac{(\kappa_2-\kappa_1)\text{tL}+\kappa_1\text{tL}}{0+\kappa_1\text{tL}}\right] \\
&= \frac{\kappa_1\kappa_2 \in_0 \text{WL}}{(\kappa_2-\kappa_1)\text{t}}\ln\left[\frac{\kappa_2}{\kappa_1}\right]=\frac{\kappa_1\kappa_2 \in_0 \text{WL}}{(-1)(\kappa_2-\kappa_1)\text{t}}\ln\left[\left(\frac{\kappa_2}{\kappa_1}\right)^{-1}\right] \\
&= \boxed{\frac{\kappa_1\kappa_2 \in_0 \text{WL}}{(\kappa_1-\kappa_2)\text{t}}\ln\left[\frac{\kappa_1}{\kappa_2}\right]}
\end{aligned}$$

(b) The capacitor physically has the same capacitance if it is turned upside down, so the answer should be the same with κ_1 and κ_2 interchanged. We have proven that it has this property in the solution to part (a).

(c) Let $\kappa_1 = \kappa_2\,(1 + x)$. Then $\text{C}=\dfrac{\kappa_2(1+\text{x})\kappa_2 \in_0 \text{WL}}{\kappa_2\text{xt}}\ln[1+\text{x}]$.

As x approaches zero we have $\text{C}=\dfrac{\kappa(1+0)\in_0 \text{WL}}{\text{xt}}\text{x}=\dfrac{\kappa \in_0 \text{WL}}{\text{t}}$ as was to be shown.

P26.77 Assume a potential difference across a and b, and notice that the potential difference across 8.00 μF the capacitor must be zero by symmetry. Then the equivalent capacitance can be determined from the circuit shown in ANS. FIG. P26.77, and is $\text{C}_{\text{ab}} = \boxed{3.00\ \mu\text{F}}$.



ANS. FIG. P26.77

P26.78 (a) The portion of the device containing the dielectric has plate area ℓx and capacitance $C_1 = \frac{\kappa \epsilon_0 \ell x}{d}$. The unfilled part has area $\ell(\ell - x)$ and capacitance $C_2 = \frac{\epsilon_0 \ell(\ell - x)}{d}$. The total capacitance is $C_1 + C_2 = \boxed{\frac{\epsilon_0 \ell}{d}\left[\ell + x(\kappa - 1)\right]}$.

(b) The stored energy is $U = \frac{1}{2}\frac{Q^2}{C} = \boxed{\frac{Q^2 d}{2\epsilon_0 \ell\left[\ell + x(\kappa - 1)\right]}}$.

(c) $\vec{\mathbf{F}} = -\left(\frac{dU}{dx}\right)\hat{\mathbf{i}} = \boxed{\frac{Q^2 d(\kappa - 1)}{2\epsilon_0 \ell\left[\ell + x(\kappa - 1)\right]^2}\hat{\mathbf{i}}}$. When $x = 0$, the original value of the force is $\frac{Q^2 d(\kappa - 1)}{2\epsilon_0 \ell^3}\hat{\mathbf{i}}$. As the dielectric slides in, the charges on the plates redistribute themselves. The force decreases to its final value, when $x = \ell$, of $\frac{Q^2 d(\kappa - 1)}{2\epsilon_0 \ell^3 \kappa^2}\hat{\mathbf{i}}$.

(d) At $x = \frac{\ell}{2}$, $\vec{\mathbf{F}} = \frac{2Q^2 d(\kappa - 1)}{\epsilon_0 \ell^3 (\kappa + 1)^2}\hat{\mathbf{i}}$.

For the constant charge on the capacitor and the initial voltage we have the relationship

$$Q = C_0 \Delta V = \frac{\epsilon_0 \ell^2 \Delta V}{d}$$

Then the force is $\vec{\mathbf{F}} = \frac{2\epsilon_0 \ell(\Delta V)^2(\kappa - 1)}{d(\kappa + 1)^2}\hat{\mathbf{i}}$.

$$\vec{\mathbf{F}} = \frac{2\left(8.85 \times 10^{-12}\ \mathrm{C^2/N\cdot m^2}\right)(0.050\ 0\ \mathrm{m})\left(2.00 \times 10^3\ \mathrm{V}\right)^2(4.50 - 1)}{(0.002\ 00\ \mathrm{m})(4.50 + 1)^2}\hat{\mathbf{i}}$$

$$= \boxed{205\hat{\mathbf{i}}\ \mu\mathrm{N}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P26.2 (a) 1.00 μF; (b) 100 V

P26.4 (a) 15.6 pF; (b) 257 kV

P26.6 (a) 11.1 nF; (b) 26.6 C

P26.8 (a) 1.36 pF; (b) 16.3 pC; (c) 8.00×10^3 V/m

P26.10 $\dfrac{(2N-1)\epsilon_0(\pi-\theta)R^2}{d}$

P26.12 $\dfrac{mgd\tan\theta}{q}$

P26.14 (a) 3.53 μF; (b) 6.35 V and 2.65 V; (c) 31.8 μC

P26.16 (a) 10.7 μC; (b) 15.0 μC and 37.5 μC

P26.18 None of the possible combinations of the extra capacitors is $\frac{4}{3}C$, so the desired capacitance cannot be achieved.

P26.20 (a) 2*C*; (b) $Q_1 > Q_3 > Q_2$; (c) $\Delta V_1 > \Delta V_2 > \Delta V_3$; (d) Q_3 and Q_1 increase; Q_2 decreases

P26.22 (a) 6.05 μF; (b) 83.7 μC

P26.24 120 μC; (b) 40.0 μC and 80.0 μC

P26.26 (a) 2.67 μF; (b) 24.0 μC, 24.0 μC, 6.00 μC, 18.0 μC; (c) 3.00 V

P26.28 $C_1 = \frac{1}{2}C_p + \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$ and $C_2 = \frac{1}{2}C_p - \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$

P26.30 4.47×10^3 V

P26.32 (a) 216 μJ; (b) 54.0 μJ

P26.34 (a) 12.0 μF; (b) 8.64×10^{-4} J; (c) $U_1 = 5.76 \times 10^{-4}$ J and $U_2 = 2.88 \times 10^{-4}$ J; (d) $U_1 + U_2 = 5.76\times10^{-4}\,\text{J} + 2.88\times10^{-4}\,\text{J} = 8.64\times10^{-4}\,\text{J} = U_{\text{eq}}$, which is one reason why the 12.0 μF capacitor is considered to be equivalent to the two capacitors; (e) The total energy of the equivalent capacitance will always equal the sum of the energies stored in the individual capacitors; (f) 5.66 V; (g) The larger capacitor C_2 stores more energy.

P26.36 (a) $C(\Delta V)^2$; (b) $\Delta V' = \dfrac{4\Delta V}{3}$; (c) $4C\dfrac{(\Delta V)^2}{3}$; (d) Positive work is done by the agent pulling the plates apart.

P26.38 $\dfrac{Q^2}{2\epsilon_0 A}$

P26.40 (a) $\dfrac{k_e Q^2}{2R}$; (b) $\dfrac{k_e q_1^2}{2R_1} + \dfrac{k_e (Q - q_1)^2}{2R_2}$; (c) $\dfrac{R_1 Q}{R_1 + R_2}$; (d) $\dfrac{R_2 Q}{R_1 + R_2}$;

(e) $V_1 = \dfrac{k_e Q}{R_1 + R_2}$ and $V_2 = \dfrac{k_e Q}{R_1 + R_2}$; (f) 0

P26.42 (a) Consider two sheets of aluminum foil, each 40 cm by 100 cm, with one sheet of plastic between them; (b) 10^{-6} F; (c) 10^2 V

P26.44 (a) $\kappa = 3.40$; (b) nylon; (c) The voltage would lie somewhere between 25.0 V and 85.0 V.

P26.46 1.04 m

P26.48 22.5 V

P26.50 (a) $(-9.10\hat{\mathbf{i}} + 8.40\hat{\mathbf{j}}) \times 10^{-12}$ C·m; (b) $-2.09 \times 10^{-8}\hat{\mathbf{k}}$ N·m; (c) 112 nJ; (d) 228 nJ

P26.52 $p\dfrac{dE}{dx}\cos\theta\,\hat{\mathbf{i}}$

P26.54 6.25 μF

P26.56 (a) 13.5 mJ; (b) 3.60 mJ, 5.40 mJ, 1.80 mJ, 2.70 mJ; (c) The total energy stored by the system equals the sum of the energies stored in the individual capacitors.

P26.58 (a) On the lower plate the charge is $-\dfrac{Q}{3}$, and on the upper plate the charge is $-\dfrac{2Q}{3}$; (b) $\dfrac{2Qd}{3\epsilon_0 A}$

P26.60 The decrease in kinetic energy of the particle is more than the energy with which it began. Therefore, the particle does not arrive at the negative plate but rather turns around and moves back to the positive plate.

P26.62 (a) $2.50\,\mu\text{F}(1 - 0.846\,f)^{-1}$; (b) $25.0\,\mu\text{F}$, the general expression agrees; (c) 162 μF; The general expression agrees.

P26.64 579 V

P26.66 (a) See P26.66(a) for full explanation; (b) $\dfrac{1}{4\pi \in_0 a}+\dfrac{1}{4\pi \in_0 b}$

P26.68 (a) See P26.68(a) for full explanation; (b) The electric field between the plates polarizes molecules within the dielectric; therefore the field does work on charge within the molecules to create electric dipoles. The extra energy comes from (part of the) electrical work done by the battery in separating that charge; (c) $\dfrac{Q}{Q_0}=\kappa$

P26.70 See P26.70 for full mathematical verification.

P26.72 19.0 kV

P26.74 $\dfrac{C}{\ell}=\dfrac{\pi \in_0}{\mathrm{1n}\left[D/r\right]}$

P26.76 (a) $\dfrac{\kappa_1\kappa_2 \in_0 WL}{\left(\kappa_1-\kappa_2\right)t}\ln\left[\dfrac{\kappa_1}{\kappa_2}\right]$; (b) The capacitor physically has the same capacitance if it is turned upside down, so the answer should be the same with κ_1 and κ_2 interchanged. We have proven that it has this property in the solution to part (a); (c) See P26.76(c) for full explanation.

P26.78 (a) $\dfrac{\in_0 \ell}{d}\left[\ell+x(\kappa-1)\right]$; (b) $\dfrac{Q^2 d}{2\in_0 \ell\left[\ell+x(\kappa-1)\right]}$; (c) $\dfrac{Q^2 d(\kappa-1)}{2\in_0 \ell\left[\ell+x(\kappa-1)\right]^2}\hat{\mathbf{i}}$;

(d) $205\hat{\mathbf{i}}\ \mu\mathrm{N}$

27

Current and Resistance

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ27.1 Answer (d). One ampere–hour is (1 C/s)(3 600 s) = 3 600 coulombs. The ampere–hour rating is the quantity of charge that the battery can lift though its nominal potential difference.

OQ27.2 (**i**) Answer (e). We require $\rho L/A_A = 3\rho L/A_B$. Then $A_A/A_B = 1/3$.

(**ii**) Answer (d). $\pi r_A{}^2/\pi r_B{}^2 = 1/3$ gives $r_A/r_B = 1/\sqrt{3}$.

OQ27.3 The ranking is c > a > b > d > e. Because

(a) $I = \Delta V / R$, so the current becomes 3 times larger.

(b) $P = I\Delta V = I^2R$, so the current is $\sqrt{3}$ times larger.

(c) R is 1/4 as large, so the current is 4 times larger.

(d) R is 2 times larger, so the current is 1/2 as large.

(e) R increases by a small percentage, so the current has a small decrease.

OQ27.4 (**i**) Answer (a). The cross-sectional area decreases, so the current density increases, thus the drift speed must increase.

(ii) Answer (a). The cross-sectional area decreases, so the resistance per unit length, $R/L = \rho/A$, increases.

OQ27.5 Answer (c). $I = \Delta V/R = 1.00\text{ V}/10.0\ \Omega = 0.100\text{ A} = 0.100\text{ C/s}$. Because current is constant, $I = dq/dt = \Delta q/\Delta t$, and we find that

$$\Delta q = I\Delta t = (0.100\text{ C/s})(20.0\text{ s}) = 2.00\text{ C}$$

OQ27.6 Answer (c). The resistances are: $R_1 = \rho L/A = \rho L/\pi r^2$, $R_2 = \rho L/\pi(2r)^2 = (1/4)\rho L/\pi r^2$, $R_3 = \rho(2L)/\pi(3r)^2 = (2/9)\rho L/\pi r^2$.

OQ27.7 Answer (a). The new cross-sectional area is three times the original. Originally, $R = \dfrac{\rho L}{A}$. Finally, $R_f = \dfrac{\rho(L/3)}{3A} = \dfrac{\rho L}{9A} = \dfrac{R}{9}$.

OQ27.8 Answer (b). Using $R_0 = 10.0\ \Omega$ at $T = 20.0$ °C, we have $R = R_0(1+\alpha\Delta T)$ or

$$\alpha = \frac{R/R_0 - 1}{\Delta T} = \frac{10.6/10.0 - 1}{(90.0°\text{C} - 20.0°\text{C})} = 8.57\times10^{-4}\,°\text{C}^{-1}$$

At $T = -20.0$°C, we have

$$\begin{aligned} R &= R_0(1+\alpha\Delta T) \\ &= (10.0\ \Omega)\left[1 + 8.57\times10^{-4}\,°\text{C}^{-1}(-20.0°\text{C} - 20.0°\text{C})\right] = 9.66\ \Omega \end{aligned}$$

OQ27.9 Answer (a). $R = V/I = 2\text{ V}/2\text{ A} = 1\ \Omega$.

OQ27.10 Answer (c). Compare resistances:

$$\frac{R_A}{R_B} = \frac{\rho L_A/\pi(d_A/2)^2}{\rho L_B/\pi(d_B/2)^2} = \frac{L_A}{L_B}\frac{d_B{}^2}{d_A{}^2} = \frac{(2L_B)}{L_B}\frac{d_B{}^2}{(2d_B)^2} = \frac{2}{4} = \frac{1}{2}$$

Compare powers: $\dfrac{P_A}{P_B} = \dfrac{\Delta V^2/R_A}{\Delta V^2/R_B} = \dfrac{R_B}{R_A} = 2$.

OQ27.11 Answer (e). $R_A = \dfrac{\rho_A L}{A} = \dfrac{(2\rho_B)L}{A} = 2R_B$. Therefore,

$$\frac{P_A}{P_B} = \frac{\Delta V^2/R_A}{\Delta V^2/R_B} = \frac{R_B}{R_A} = \frac{1}{2}$$

OQ27.12 **(i)** Answer (a). $P = \Delta V^2/R$, and ΔV is the same for both bulbs, so the 25 W bulb must have higher resistance so that it will have lower power.

(ii) Answer (b). ΔV is the same for both bulbs, so the 100 W bulb must have lower resistance so that it will have more current.

OQ27.13 Answer (d). Because wire B has twice the radius, it has four times the cross-sectional area of wire A. For wire A, $R_A = R = \rho L/A$. For wire B, $R_B = \rho(2L)/(4A) = (1/2)\rho L/A = R/2$.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ27.1 Choose the voltage of the power supply you will use to drive the heater. Next calculate the required resistance R as $\frac{\Delta V^2}{P}$. Knowing the resistivity ρ of the material, choose a combination of wire length and cross-sectional area to make $\left(\frac{\ell}{A}\right) = \left(\frac{R}{\rho}\right)$. You will have to pay for less material if you make both ℓ and A smaller, but if you go too far the wire will have too little surface area to radiate away the energy; then the resistor will melt.

CQ27.2 Geometry and resistivity. In turn, the resistivity of the material depends on the temperature.

CQ27.3 The conductor does not follow Ohm's law, and must have a resistivity that is current-dependent, or more likely temperature-dependent.

CQ27.4 In a normal metal, suppose that we could proceed to a limit of zero resistance by lengthening the average time between collisions. The classical model of conduction then suggests that a constant applied voltage would cause constant acceleration of the free electrons. The drift speed and the current would increase steadily in time.

It is not the situation envisioned in the question, but we can actually switch to zero resistance by substituting a superconducting wire for the normal metal. In this case, the drift velocity of electrons is established by vibrations of atoms in the crystal lattice; the maximum current is limited; and it becomes impossible to establish a potential difference across the superconductor.

CQ27.5 The resistance of copper *increases* with temperature, while the resistance of silicon *decreases* with increasing temperature. The conduction electrons are scattered more by vibrating atoms when copper heats up. Silicon's charge carrier density increases as temperature increases and more atomic electrons are promoted to become conduction electrons.

CQ27.6 The amplitude of atomic vibrations increases with temperature. Atoms can then scatter electrons more efficiently.

CQ27.7 Because there are so many electrons in a conductor (approximately 10^{28} electrons/m^3) the average velocity of charges is very slow. When you connect a wire to a potential difference, you establish an electric field everywhere in the wire nearly instantaneously, to make electrons start drifting everywhere all at once.

CQ27.8 Voltage is a measure of potential difference, not of current. "Surge" implies a flow—and only charge, in coulombs, can flow through a system. It would also be correct to say that the victim carried a certain current, in amperes.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 21.1 Electric Current

***P27.1** The drift speed of electrons in the line is

$$v_d = \frac{I}{nqa} = \frac{I}{n|e|\left(\pi d^2/4\right)}$$

The time to travel the 200-km length of the line is then

$$\Delta t = \frac{L}{v_d} = \frac{Ln|e|\left(\pi d^2\right)}{4I}$$

Substituting numerical values,

$$\Delta t = \frac{\left(200\times 10^{3}\text{ m}\right)\left(8.50\times 10^{28}\text{ m}^{-3}\right)\left(1.60\times 10^{-19}\text{ C}\right)\pi\left(0.02\text{ m}\right)^2}{4\left(1\,000\text{ A}\right)}$$

$$= \left(8.55\times 10^{8}\text{ s}\right)\left(\frac{1\text{ yr}}{3.156\times 10^{7}\text{ s}}\right) = \boxed{27.1\text{ yr}}$$

***P27.2** The period of revolution for the sphere is $T = \dfrac{2\pi}{\omega}$, and the average current represented by this revolving charge is $I = \dfrac{q}{T} = \boxed{\dfrac{q\omega}{2\pi}}$.

P27.3 We use $I = nqAv_d$, where n is the number of charge carriers per unit volume, and is identical to the number of atoms per unit volume. We assume a contribution of 1 free electron per atom in the relationship above. For aluminum, which has a molar mass of 27, we know that Avogadro's number of atoms, N_A, has a mass of 27.0 g. Thus, the mass per atom is

$$m = \frac{27.0\text{ g}}{N_A} = \frac{27.0\text{ g}}{6.02\times 10^{23}} = 4.49\times 10^{-23}\text{ g/atom}$$

Thus,

$$n = \frac{\rho}{m} = \frac{\text{density of aluminum}}{\text{mass per atom}} = \frac{2.70\ \text{g/cm}^3}{4.49 \times 10^{-23}\ \text{g/atom}}$$

$$n = 6.02 \times 10^{22}\ \text{atoms/cm}^3 = 6.02 \times 10^{28}\ \text{atoms/m}^3$$

Therefore,

$$v_d = \frac{I}{nqA} = \frac{5.00\ \text{A}}{\left(6.02 \times 10^{28}\ \text{m}^{-3}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)\left(4.00 \times 10^{-6}\ \text{m}^2\right)}$$

$$= 1.30 \times 10^{-4}\ \text{m/s}$$

or, $v_d = \boxed{0.130\ \text{mm/s}}$.

P27.4 The period of the electron in its orbit is $T = 2\pi r/v$, and the current represented by the orbiting electron is

$$I = \frac{\Delta Q}{\Delta t} = \frac{|e|}{T} = \frac{v|e|}{2\pi r}$$

$$= \frac{\left(2.19 \times 10^{6}\ \text{m/s}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)}{2\pi\left(5.29 \times 10^{-11}\ \text{m}\right)}$$

$$= 1.05 \times 10^{-3}\ \text{C/s} = \boxed{1.05\ \text{mA}}$$

P27.5 If N is the number of protons, each with charge e, that hit the target in time Δt, the average current in the beam is $I = \Delta Q / \Delta t = Ne / \Delta t$, giving

$$N = \frac{I(\Delta t)}{e} = \frac{\left(125 \times 10^{-6}\ \text{C/s}\right)(23.0\ \text{s})}{1.60 \times 10^{-19}\ \text{C/proton}} = \boxed{1.80 \times 10^{16}\ \text{protons}}$$

P27.6 (a) From Example 27.1 in the textbook, the density of charge carriers (electrons) in a copper wire is $n = 8.46 \times 10^{28}$ electrons/m^3. With $A = \pi r^2$ and $|q| = e$, the drift speed of electrons in this wire is

$$v_d = \frac{I}{n|q|A} = \frac{I}{ne\left(\pi r^2\right)}$$

$$= \frac{3.70\ \text{C/s}}{\left(8.46 \times 10^{28}\ \text{m}^{-3}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)\pi\left(1.25 \times 10^{-3}\ \text{m}\right)^2}$$

$$= \boxed{5.57 \times 10^{-5}\ \text{m/s}}$$

(b) $\boxed{\text{The drift speed is smaller because more electrons are being conducted.}}$ To create the same current, therefore, the drift speed need not be as great.

P27.7 From $I = \frac{dQ}{dt}$, we have *dQ* = *I dt.*

From this, we derive the general integral: $Q = \int dQ = \int I\ dt$

In all three cases, define an end-time, *T*: $Q = \int_0^T I_0 e^{-t/\tau} dt$

Integrating from time *t* = 0 to time *t* = *T*: $Q = \int_0^T \left(-I_0\tau\right) e^{-t/\tau}\left(-\frac{dt}{\tau}\right)$

We perform the integral and set *Q* = 0 at *t* = 0 to obtain

$$Q = -I_0\tau\left(e^{-T/\tau} - e^0\right) = I_0\tau\left(1 - e^{-T/\tau}\right)$$

(a) If $T = \tau$: $Q(\tau) = I_0\tau\left(1 - e^{-1}\right) = \boxed{(0.632)I_0\tau}$

(b) If $T = 10\tau$: $Q(10\tau) = I_0\tau\left(1 - e^{-10}\right) = \boxed{(0.999\,95)I_0\tau}$

(c) If $T = \infty$: $Q(\infty) = I_0\tau\left(1 - e^{-\infty}\right) = \boxed{I_0\tau}$

P27.8 (a) $J = \frac{I}{A} = \frac{5.00\ \text{A}}{\pi\left(4.00 \times 10^{-3}\ \text{m}\right)^2} = \boxed{99.5\ \text{kA/m}^2}$

(b) $\boxed{\text{Current is the same.}}$

(c) The cross-sectional area is greater; therefore $\boxed{\text{the current density is smaller.}}$

(d) $A_2 = 4A_1$ or $\pi r_2^2 = 4\pi r_1^2$ so $r_2 = 2r_1 = \boxed{0.800\ \text{cm}}$.

(e) $\boxed{I = 5.00\ \text{A}}$

(f) $J_2 = \frac{1}{4}J_1 = \frac{1}{4}\left(9.95 \times 10^4\ \text{A/m}^2\right) = \boxed{2.49 \times 10^4\ \text{A/m}^2}$

P27.9 We are given $q = 4t^3 + 5t + 6$. The area is

$$A = \left(2.00\ \text{cm}^2\right)\left(\frac{1.00\ \text{m}}{100\ \text{cm}}\right)^2 = 2.00 \times 10^{-4}\ \text{m}^2$$

(a) $I(1.00\ \text{s}) = \left.\frac{dq}{dt}\right|_{t=1.00\ \text{s}} = \left.\left(12t^2 + 5\right)\right|_{t=1.00\ \text{s}} = \boxed{17.0\ \text{A}}$

(b) $J = \frac{I}{A} = \frac{17.0\ \text{A}}{2.00 \times 10^{-4}\ \text{m}^2} = \boxed{85.0\ \text{kA/m}^2}$

P27.10 (a) We obtain the speed of each deuteron from $K = \frac{1}{2}mv^2$:

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(2.00\times10^6\right)\left(1.60\times10^{-19}\text{ J}\right)}{2\left(1.67\times10^{-27}\text{ kg}\right)}} = 1.38\times10^7\text{ m/s}$$

The time between deuterons passing a stationary point is *t* in $I = \frac{q}{t}$, so

$$t = \frac{q}{I} = \frac{1.60\times10^{-19}\text{ C}}{10.0\times10^{-6}\text{ C/s}} = 1.60\times10^{-14}\text{ s}$$

So the distance between individual deuterons is

$$vt = (1.38\times10^7\text{ m/s})(1.60\times10^{-14}\text{ s}) = \boxed{2.21\times10^{-7}\text{ m}}$$

(b) One nucleus will put its nearest neighbor at potential

$$V = \frac{k_e q}{r} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)}{2.21\times10^{-7}\text{ m}}$$
$$= 6.49\times10^{-3}\text{ V}$$

This is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.

P27.11 (a) $J = \frac{I}{A} = \frac{8.00\times10^{-6}\text{ A}}{\pi\left(1.00\times10^{-3}\text{ m}\right)^2} = \boxed{2.55\text{ A/m}^2}$

(b) From $J = nev_d$, we have

$$n = \frac{J}{ev_d} = \frac{2.55\text{ A/m}^2}{\left(1.60\times10^{-19}\text{ C}\right)\left(3.00\times10^8\text{ m/s}\right)} = \boxed{5.31\times10^{10}\text{ m}^{-3}}$$

(c) From $I = \frac{\Delta Q}{\Delta t}$, we have

$$\Delta t = \frac{\Delta Q}{I} = \frac{N_A e}{I} = \frac{\left(6.02\times10^{23}\right)\left(1.60\times10^{-19}\text{ C}\right)}{8.00\times10^{-6}\text{ A}}$$
$$= \boxed{1.20\times10^{10}\text{ s}}$$

(This is about 382 years!)

P27.12 To find the total charge passing a point in a given amount of time, we use $I = \frac{dq}{dt}$, from which we can write

$$q = \int dq = \int I\,dt = \int_0^{1/240\text{ s}} (100\text{ A})\sin\left(\frac{120\pi t}{\text{s}}\right)dt$$

$$q = \frac{-100\text{ C}}{120\pi}\left[\cos\left(\frac{\pi}{2}\right) - \cos 0\right] = \frac{+100\text{ C}}{120\pi} = \boxed{0.265\text{ C}}$$

P27.13 The molar mass of silver = 107.9 g/mole and the volume *V* is

$$V = (\text{area})(\text{thickness}) = (700\times10^{-4}\text{ m}^2)(0.133\times10^{-3}\text{ m})$$
$$= 9.31\times10^{-6}\text{ m}^3$$

The mass of silver deposited is

$$m_{\text{Ag}} = \rho V = (10.5\times10^3\text{ kg/m}^3)(9.31\times10^{-6}\text{ m}^3)$$
$$= 9.78\times10^{-2}\text{ kg}$$

And the number of silver atoms deposited is

$$N = (9.78\times10^{-2}\text{ kg})\left(\frac{6.02\times10^{23}\text{ atoms}}{107.9\text{ g}}\right)\left(\frac{1\,000\text{ g}}{1\text{ kg}}\right)$$
$$= 5.45\times10^{23}\text{ atoms}$$

The current is then

$$I = \frac{\Delta V}{R} = \frac{12.0\text{ V}}{1.80\ \Omega} = 6.67\text{ A} = 6.67\text{ C/s}$$

The time interval required for the silver coating is

$$\Delta t = \frac{\Delta Q}{I} = \frac{Ne}{I} = \frac{(5.45\times10^{23})(1.60\times10^{-19}\text{ C})}{6.67\text{ C/s}}$$
$$= 1.31\times10^4\text{ s} = \boxed{3.64\text{ h}}$$

Section 27.2 Resistance

P27.14 From Equation 27.7, we obtain

$$I = \frac{\Delta V}{R} = \frac{120\text{ V}}{240\ \Omega} = 0.500\text{ A} = \boxed{500\text{ mA}}$$

***P27.15** From Ohm's law, $R = \Delta V / I$, and from Equation 27.10,

$$R = \rho\ell / A = \rho\ell / \left(\pi d^2 / 4\right)$$

Solving for the resistivity gives

$$\rho = \left(\frac{\pi d^2}{4\ell}\right) R = \left(\frac{\pi d^2}{4\ell}\right)\left(\frac{\Delta V}{I}\right) = \left[\frac{\pi\left(2.00\times10^{-3}\text{ m}\right)^2}{4\left(50.0\text{ m}\right)}\right]\left(\frac{9.11\text{ V}}{36.0\text{ A}}\right)$$

$$= 1.59\times10^{-8}\ \Omega\cdot\text{m}$$

Then, from Table 27.2, we see that the wire is made of $\boxed{\text{silver}}$.

P27.16 $\Delta V = IR$ and $R = \dfrac{\rho\ell}{A}$. The area is

$$A = \left(0.600\text{ mm}^2\right)\left(\frac{1.00\text{ m}}{1\,000\text{ mm}}\right)^2 = 6.00\times10^{-7}\text{ m}^2$$

From the potential difference, we can solve for the current, which gives

$$\Delta V = \frac{I\rho\ell}{A} \quad\rightarrow\quad I = \frac{\Delta VA}{\rho\ell} = \frac{\left(0.900\text{ V}\right)\left(6.00\times10^{-7}\text{ m}^2\right)}{\left(5.60\times10^{-8}\ \Omega\cdot\text{m}\right)\left(1.50\text{ m}\right)}$$

$$I = \boxed{6.43\text{ A}}$$

P27.17 From the definition of resistance,

$$R = \frac{\Delta V}{I} = \frac{120\text{ V}}{13.5\text{ A}} = \boxed{8.89\ \Omega}$$

P27.18 Using $R = \dfrac{\rho L}{A}$ and data from Table 27.2, we have

$$\rho_{\text{Cu}}\frac{L_{\text{Cu}}}{\pi r_{\text{Cu}}^2} = \rho_{\text{Al}}\frac{L_{\text{Al}}}{\pi r_{\text{Al}}^2} \rightarrow \frac{r_{\text{Al}}^2}{r_{\text{Cu}}^2} = \frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}$$

which yields

$$\frac{r_{\text{Al}}}{r_{\text{Cu}}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} = \sqrt{\frac{2.82\times10^{-8}\ \Omega\cdot\text{m}}{1.70\times10^{-8}\ \Omega\cdot\text{m}}} = \boxed{1.29}$$

P27.19 (a) Given total mass $m = \rho_m V = \rho_m A\ell \quad\rightarrow\quad A = \dfrac{m}{\rho_m\ell}$, where $\rho_m \equiv$ mass density.

Taking $\rho \equiv$ resistivity, $R = \dfrac{\rho\ell}{A} = \dfrac{\rho\ell}{m/\rho_m\ell} = \dfrac{\rho\rho_m\ell^2}{m}$.

Thus,

$$\ell = \sqrt{\frac{mR}{\rho\rho_m}} = \sqrt{\frac{\left(1.00\times10^{-3}\text{ kg}\right)\left(0.500\ \Omega\right)}{\left(1.70\times10^{-8}\ \Omega\cdot\text{m}\right)\left(8.92\times10^{3}\text{ kg/m}^3\right)}}$$

$$= \boxed{1.82\text{ m}}$$

(b) $V = \dfrac{m}{\rho_m}$, or $\pi r^2\ell = \dfrac{m}{\rho_m}$

Thus,

$$r = \sqrt{\frac{m}{\pi\rho_m\ell}} = \sqrt{\frac{1.00\times10^{-3}\text{ kg}}{\pi\left(8.92\times10^{3}\text{ kg/m}^3\right)\left(1.82\text{ m}\right)}} = 1.40\times10^{-4}\text{ m}$$

The diameter is twice this distance: diameter $= \boxed{280\ \mu\text{m}}$

P27.20 (a) Given total mass $m = \rho_m V = \rho_m A\ell \quad\rightarrow\quad A = \dfrac{m}{\rho_m\ell}$, where

$\rho_m \equiv$ mass density.

Taking $\rho \equiv$ resistivity, $R = \dfrac{\rho\ell}{A} = \dfrac{\rho\ell}{m/\rho_m\ell} = \dfrac{\rho\rho_m\ell^2}{m}$.

Thus, $\ell = \boxed{\sqrt{\dfrac{mR}{\rho\rho_m}}}$.

(b) Volume $V = \dfrac{m}{\rho_m}$, or

$$\frac{1}{4}\pi d^2\ell = \frac{m}{\rho_m}$$

$$d = \sqrt{\frac{4}{\pi}}\left(\frac{m}{\rho_m}\frac{1}{\ell}\right)^{1/2} = \sqrt{\frac{4}{\pi}}\left(\frac{m}{\rho_m}\sqrt{\frac{\rho\rho_m}{mR}}\right)^{1/2} = \sqrt{\frac{4}{\pi}}\left(\sqrt{\frac{m^2}{\rho_m^2}\frac{\rho\rho_m}{mR}}\right)^{1/2}$$

$$= \boxed{\sqrt{\frac{4}{\pi}}\left(\frac{\rho m}{\rho_m R}\right)^{1/4}}$$

P27.21 (a) From the definition of resistance,

$$R = \frac{\Delta V}{I} = \frac{120\text{ V}}{9.25\text{ A}} = \boxed{13.0\ \Omega}$$

(b) The resistivity of Nichrome (from Table 27.2) is $1.50\times10^{-6}\ \Omega\cdot\text{m}$.

We find the length of wire from

$$R = \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi r^2}$$

solving for the length ℓ gives

$$\ell = \frac{R\pi r^2}{\rho} = \frac{(13.0\ \Omega)\pi\left(2.50\times10^{-3}\ \text{m}\right)^2}{\left(1.50\times10^{-6}\ \Omega\cdot\text{m}\right)} = \boxed{170\ \text{m}}$$

Section 27.3 A Model for Electrical Conduction

***P27.22** (a) n is $\boxed{\text{unaffected}}$.

(b) $|J| = \frac{I}{A} \propto I$ so it $\boxed{\text{doubles}}$.

(c) $J = nev_d$ so v_d $\boxed{\text{doubles}}$.

(d) $\tau = \frac{m\sigma}{nq^2}$ is $\boxed{\text{unchanged}}$ as long as σ does not change due to a temperature change in the conductor.

***P27.23** $J = \sigma E$ so $\sigma = \frac{J}{E} = \frac{6.00\times10^{-13}\ \text{A/m}^2}{100\ \text{V/m}} = \boxed{6.00\times10^{-15}\ (\Omega\cdot\text{m})^{-1}}$.

P27.24 (a) From Appendix C, the molar mass of iron is

$$M_{\text{Fe}} = 55.85\ \text{g/mol} = \left(55.85\ \text{g/mol}\right)\left(1\ \text{kg}/10^3\ \text{g}\right)$$
$$= \boxed{5.58\times10^{-2}\ \text{kg/mol}}$$

(b) From Table 14.1, the density of iron is $\rho_{\text{Fe}} = 7.86\times10^3\ \text{kg/m}^3$, so the molar density is

$$\left(\text{molar density}\right)_{\text{Fe}} = \frac{\rho_{\text{Fe}}}{M_{\text{Fe}}} = \frac{7.86\times10^3\ \text{kg/m}^3}{5.58\times10^{-2}\ \text{kg/mol}}$$
$$= \boxed{1.41\times10^5\ \text{mol/m}^3}$$

(c) The density of iron atoms is

$$\text{density of atoms} = N_A\left(\text{molar density}\right)$$
$$= \left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(1.41\times10^5\ \frac{\text{mol}}{\text{m}^3}\right)$$
$$= \boxed{8.49\times10^{28}\ \frac{\text{atoms}}{\text{m}^3}}$$

(d) With two conduction electrons per iron atom, the density of charge carriers is

$$\begin{aligned} n &= (\text{charge carriers/atom})(\text{density of atoms}) \\ &= \left(2\ \frac{\text{electrons}}{\text{atom}}\right)\left(8.49\times10^{28}\ \frac{\text{atoms}}{\text{m}^3}\right) \\ &= \boxed{1.70\times10^{29}\ \text{electrons/m}^3} \end{aligned}$$

(e) With a current of I = 30.0 A and cross-sectional area $A = 5.00 \times 10^{-6}\ \text{m}^2$, the drift speed of the conduction electrons in this wire is

$$\begin{aligned} v_d &= \frac{I}{nqA} = \frac{30.0\ \text{C/s}}{(1.70\times10^{29}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(5.00\times10^{-6}\ \text{m}^3)} \\ &= \boxed{2.21\times10^{-4}\ \text{m/s}} \end{aligned}$$

P27.25 From Equations 27.16 and 27.13, the resistivity and drift velocity can be related to the electric field within the copper wire:

$$\rho = \frac{m}{ne^2\tau} \rightarrow \tau = \frac{m}{\rho ne^2}$$

and

$$v_d = \frac{eE}{m}\tau = \frac{eE}{m}\frac{m}{\rho ne^2} = \frac{E}{\rho ne} \qquad \rightarrow \qquad E = \rho n e v_d$$

where n is the electron density. From Example 27.1,

$$n = \frac{N_A\rho_{Cu}}{M} = \frac{(6.02\times10^{23}\ \text{mol}^{-1})(8\ 920\ \text{kg/m}^3)}{0.063\ 5\ \text{kg/mol}} = 8.46\times10^{28}\ \text{m}^{-3}$$

The electric field is then

$$\begin{aligned} E &= \rho n e v_d \\ E &= (1.7\times10^{-8}\ \Omega\cdot\text{m})(8.46\times10^{28}\ \text{m}^{-3}) \\ &\qquad \times(1.60\times10^{-19}\ \text{C})(7.84\times10^{-4}\ \text{m/s}) \\ &= \boxed{0.18\ \text{V/m}} \end{aligned}$$

Section 27.4 Resistance and Temperature

P27.26 $R = R_0[1+\alpha(\Delta T)]$ gives

$$140\ \Omega = (19.0\ \Omega)\left[1+\left(4.50\times10^{-3}/^\circ\text{C}\right)\Delta T\right]$$

Solving,

$$\Delta T = 1.42\times10^3\ ^\circ\text{C} = T - 20.0^\circ\text{C}$$

And the final temperature is $\boxed{T = 1.44\times10^3\ ^\circ\text{C}}$

P27.27 If we ignore thermal expansion, the change in the material's resistivity with temperature $\rho = \rho_0[1+\alpha\Delta T]$ implies that the change in resistance is $R - R_0 = R_0\alpha\Delta T$. The fractional change in resistance is defined by $f = (R - R_0)/R_0$. Therefore,

$$f = \frac{R_0\alpha\Delta T}{R_0} = \alpha\Delta T = \left(5.00\times10^{-3}\ ^\circ\text{C}^{-1}\right)(50.0^\circ\text{C} - 25.0^\circ\text{C}) = \boxed{0.12}$$

***P27.28** At the low temperature T_C we write

$$R_C = \frac{\Delta V}{I_C} = R_0\left[1+\alpha(T_C - T_0)\right]$$

where $T_0 = 20.0^\circ\text{C}$. At the high temperature T_h,

$$R_h = \frac{\Delta V}{I_h} = \frac{\Delta V}{1\ \text{A}} = R_0\left[1+\alpha(T_h - T_0)\right]$$

Then,

$$\frac{(\Delta V)/(1.00\ \text{A})}{(\Delta V)/I_C} = \frac{1+\left(3.90\times10^{-3}\ (^\circ\text{C})^{-1}\right)(58.0^\circ\text{C} - 20.0^\circ\text{C})}{1+\left(3.90\times10^{-3}\ (^\circ\text{C})^{-1}\right)(-88.0^\circ\text{C} - 20.0^\circ\text{C})}$$

and $I_C = (1.00\ \text{A})\left(\dfrac{1.15}{0.579}\right) = \boxed{1.98\ \text{A}}$.

P27.29 We use Equation 27.20 and refer to Table 27.2:

$$\begin{aligned} R &= R_0\left[1+\alpha(T - T_0)\right] \\ &= (6.00\ \Omega)\left[1+\left(3.8\times10^{-3}\ (^\circ\text{C})^{-1}\right)(34.0^\circ\text{C} - 20.0^\circ\text{C})\right] \\ &= \boxed{6.32\ \Omega} \end{aligned}$$

P27.30 (a) From $R = \rho L/A$, the initial resistance of the mercury is

$$R_i = \frac{\rho L_i}{A_i} = \frac{\rho L_i}{\pi d_i^2/4} = \frac{\left(9.58\times10^{-7}\ \Omega\cdot\text{m}\right)(1.000\ 0\ \text{m})}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^2/4} = \boxed{1.22\ \Omega}$$

(b) Since the volume of mercury is constant, $V = A_f \cdot L_f = A_i \cdot L_i$ gives the final cross-sectional area as $A_f = A_i \cdot \left(L_i / L_f\right)$. Thus, the final resistance is given by $R_f = \dfrac{\rho L_f}{A_f} = \dfrac{\rho L_f^2}{A_i \cdot L_i}$. The fractional change in the resistance is then

$$\frac{\Delta R}{R} = \frac{R_f - R_i}{R_i} = \frac{R_f}{R_i} - 1 = \frac{\rho L_f^2/\left(A_i \cdot L_i\right)}{\rho L_i / A_i} - 1 = \left(\frac{L_f}{L_i}\right)^2 - 1$$

$$\frac{\Delta R}{R} = \left(\frac{100.040\ 0\,\text{cm}}{100.000\ 0\,\text{cm}}\right)^2 - 1 = \boxed{8.00 \times 10^{-4}\ \text{increase}}$$

***P27.31** (a) The resistance at 20.0°C is

$$R_0 = \frac{\rho \ell}{A} = \frac{\left(1.7 \times 10^{-8}\ \Omega \cdot \text{m}\right)(34.5\ \text{m})}{\pi\left(0.25 \times 10^{-3}\ \text{m}\right)^2} = 2.99\ \Omega$$

and the current is

$$I = \frac{\Delta V}{R_0} = \frac{9.00\ \text{V}}{3.00\ \Omega} = \boxed{3.01\ \text{A}}$$

(b) At 30.0°C, from Equation 27.20,

$$\begin{aligned} R &= R_0\left[1 + \alpha(\Delta T)\right] \\ &= (2.99\ \Omega)\left[1 + \left(3.9 \times 10^{-3}\ (°\text{C})^{-1}\right)(30.0°\text{C} - 20.0°\text{C})\right] = 3.10\ \Omega \end{aligned}$$

The current is then

$$I = \frac{\Delta V}{R_0} = \frac{9.00\ \text{V}}{3.10\ \Omega} = \boxed{2.90\ \text{A}}$$

P27.32 (a) We require two conditions:

$$R = \frac{\rho_1 \ell_1}{\pi r^2} + \frac{\rho_2 \ell_2}{\pi r^2} \qquad \textbf{[1]}$$

where carbon = 1 and Nichrome = 2, and for any ΔT

$$R = \frac{\rho_1 \ell_1}{\pi r^2}\left(1 + \alpha_1 \Delta T\right) + \frac{\rho_2 \ell_2}{\pi r^2}\left(1 + \alpha_2 \Delta T\right) \qquad \textbf{[2]}$$

Setting equations [1] and [2] equal to each other, we have

$$\frac{\rho_1 \ell_1}{\pi r^2} + \frac{\rho_2 \ell_2}{\pi r^2} = \frac{\rho_1 \ell_1}{\pi r^2}\left(1 + \alpha_1 \Delta T\right) + \frac{\rho_2 \ell_2}{\pi r^2}\left(1 + \alpha_2 \Delta T\right)$$

simplifying,

$$\cancel{\frac{\rho_1\ell_1}{\pi r^2}}+\cancel{\frac{\rho_2\ell_2}{\pi r^2}}=\cancel{\frac{\rho_1\ell_1}{\pi r^2}}+\frac{\rho_1\ell_1}{\pi r^2}\alpha_1\Delta T+\cancel{\frac{\rho_2\ell_2}{\pi r^2}}+\frac{\rho_2\ell_2}{\pi r^2}\alpha_2\Delta T$$

or $\frac{\rho_2\ell_2}{\pi r^2}\alpha_2\Delta T=-\frac{\rho_1\ell_1}{\pi r^2}\alpha_1\Delta T$, which gives

$$\rho_2\ell_2\alpha_2=-\rho_1\ell_1\alpha_1 \qquad \textbf{[3]}$$

The two equations [1] and [3] are just sufficient to determine ℓ_1 and ℓ_2. $\boxed{\text{The design goal can be met.}}$

(b) From Table 27.2, $\alpha_1=-0.5\times10^{-3}\,(°\text{C})^{-1}$ and $\alpha_2=0.4\times10^{-3}\,(°\text{C})^{-1}$. Use equation [3] to solve for ℓ_2 in terms of ℓ_1:

$$\ell_2=-\frac{\rho_1}{\rho_2}\frac{\alpha_1}{\alpha_2}\ell_1$$

then substitute this into equation [1]:

$$R=\frac{\rho_1\ell_1}{\pi r^2}+\frac{\cancel{\rho_2}}{\pi r^2}\left(-\frac{\rho_1}{\cancel{\rho_2}}\frac{\alpha_1}{\alpha_2}\ell_1\right)=\frac{\rho_1}{\pi r^2}\left(1-\frac{\alpha_1}{\alpha_2}\right)\ell_1$$

$$10.0\ \Omega=\frac{\left(3.5\times10^{-5}\,\Omega\cdot\text{m}\right)}{\pi(1.50\times10^{-3}\,\text{m})^2}\left(1-\frac{-0.5\times10^{-3}}{0.4\times10^{-3}}\right)\ell_1$$

$$\rightarrow \ell_1=0.898\ \text{m}$$

and so

$$\ell_2=-\frac{\rho_1}{\rho_2}\frac{\alpha_1}{\alpha_2}\ell_1=-\frac{\left(3.5\times10^{-5}\,\Omega\cdot\text{m}\right)}{\left(1.50\times10^{-6}\,\Omega\cdot\text{m}\right)}\left(\frac{-0.5\times10^{-3}}{0.4\times10^{-3}}\right)\ell_1=26.2\ \text{m}$$

Therefore, $\boxed{\ell_1=0.898\ \text{m and }\ell_2=26.2\ \text{m.}}$

P27.33 (a) The resistivity is computed from $\rho=\rho_0\left[1+\alpha\left(T-T_0\right)\right]$:

$$\rho=\left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)\left[1+\left(3.90\times10^{-3}\ °\text{C}^{-1}\right)\left(30.0°\text{C}\right)\right]$$
$$=\boxed{3.15\times10^{-8}\ \Omega\cdot\text{m}}$$

(b) The current density is

$$J=\sigma E=\frac{E}{\rho}=\left(\frac{0.200\ \text{V/m}}{3.15\times10^{-8}\ \Omega\cdot\text{m}}\right)\left(\frac{1\ \Omega\cdot\text{A}}{\text{V}}\right)=\boxed{6.35\times10^{6}\ \text{A/m}^2}$$

(c) The current density is related to the current by $J=\frac{I}{A}=\frac{I}{\pi r^2}$.

$$I=J\left(\pi r^2\right)=\left(6.35\times10^{6}\ \text{A/m}^2\right)\left[\pi\left(5.00\times10^{-5}\ \text{m}\right)^2\right]=\boxed{49.9\ \text{mA}}$$

(d) The mass density gives the number-density of free electrons; we assume that each atom donates one conduction electron:

$$n=\left(\frac{2.70\times10^{3}\,\text{kg}}{\text{m}^{3}}\right)\left(\frac{1\text{ mol}}{26.98\text{ g}}\right)\left(\frac{10^{3}\,\text{g}}{\text{kg}}\right)\left(\frac{6.02\times10^{23}\text{ free e}^{-}}{1\text{ mol}}\right)$$

$$=6.02\times10^{28}\ \text{e}^{-}/\text{m}^{3}$$

Now $J = nqv_d$ gives the drift speed as

$$v_d=\frac{J}{nq}=\frac{6.35\times10^{6}\ \text{A/m}^{2}}{\left(6.02\times10^{28}\ \text{e}^{-}/\text{m}^{3}\right)\left(-1.60\times10^{-19}\ \text{C/e}^{-}\right)}$$

$$=\boxed{-6.59\times10^{-4}\ \text{m/s}}$$

The sign indicates that the electrons drift opposite to the field and current.

(e) The applied voltage is $\Delta V = E\ell = (0.200\ \text{V/m})(2.00\ \text{m}) = \boxed{0.400\ \text{V}}$.

P27.34 For aluminum,

$$\alpha_E = 3.90\times10^{-3}\ {}^\circ\text{C}^{-1}\quad\text{(Table 27.2)}$$

and $\alpha = 24.0\times10^{-6}\ {}^\circ\text{C}^{-1}$ (Table 19.1)

The resistance is then

$$R=\frac{\rho\ell}{A}=\frac{\rho_0(1+\alpha_E\Delta T)\ell(1+\alpha\Delta T)}{A(1+\alpha\Delta T)^2}=R_0\frac{(1+\alpha_E\Delta T)}{(1+\alpha\Delta T)}$$

$$=(1.23\ \Omega)\left[\frac{1+\left(3.90\times10^{-3}\ ({}^\circ\text{C})^{-1}\right)(120^\circ\text{C}-20.0^\circ\text{C})}{1+\left(24.0\times10^{-6}\ ({}^\circ\text{C})^{-1}\right)(120^\circ\text{C}-20.0^\circ\text{C})}\right]$$

$$=\boxed{1.71\ \Omega}$$

P27.35 Room temperature is $T_0 = 20.0^\circ$. From Equation 27.19,

$$\rho_{\text{Al}}=(\rho_0)_{\text{Al}}\left[1+\alpha_{\text{Al}}(T-T_0)\right]=3(\rho_0)_{\text{Cu}}$$

Then, substituting numerical values from Table 27.2 gives

$$T-T_0=\frac{1}{\alpha_{\text{Al}}}\left[\frac{3(\rho_0)_{\text{Cu}}}{(\rho_0)_{\text{Al}}}-1\right]$$

$$=\frac{1}{3.9\times10^{-3}\ ({}^\circ\text{C})^{-1}}\left[\frac{3\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)}{2.82\times10^{-8}\ \Omega\cdot\text{m}}-1\right]$$

and solving for the temperature gives

$$T - 20.0\ °\text{C} = 207°\text{C}$$

$$T = \boxed{227°\text{C}}$$

where we have assumed three significant figures throughout.

Section 27.6 Electrical Power

***P27.36** (a) $P = (\Delta V)I = (300\times 10^3\ \text{J/C})(1.00\times 10^3\ \text{C/s}) = \boxed{3.00\times 10^8\ \text{W}}$

A large electric generating station, fed by a trainload of coal each day, converts energy faster.

(b) $I = \dfrac{P}{A} = \dfrac{P}{\pi r^2}$

$$P = I(\pi r^2) = (1\ 370\ \text{W/m}^2)\left[\pi(6.37\times 10^6\ \text{m})^2\right] = \boxed{1.75\times 10^{17}\ \text{W}}$$

Terrestrial solar power is immense compared to lightning and compared to all human energy conversions.

***P27.37** $P = 0.800(1\ 500\ \text{hp})(746\ \text{W/hp}) = 8.95\times 10^5\ \text{W}$

Then, from $P = I\Delta V$,

$$I = \frac{P}{\Delta V} = \frac{8.95\times 10^5\ \text{W}}{2\ 000\ \text{V}} = \boxed{448\ \text{A}}$$

P27.38 From Equation 27.21,

$$P = I\Delta V = 500\times 10^{-6}\ \text{A}(15\times 10^3\ \text{V}) = \boxed{7.50\ \text{W}}$$

P27.39 (a) From Equation 27.21,

$$P = I\Delta V \rightarrow I = P/\Delta V = (1.00\times 10^3\ \text{W})/(120\ \text{V}) = \boxed{8.33\ \text{A}}$$

(b) From Equation 27.23,

$$P = \Delta V^2/R \rightarrow R = \Delta V^2/P = (120\ \text{V})^2/(1.00\times 10^3\ \text{W}) = \boxed{14.4\ \Omega}$$

P27.40 From Equation 27.21,

$$P = I\Delta V = (0.200\times 10^{-3}\ \text{A})(75.0\times 10^{-3}\ \text{V})$$

$$= 15.0\times 10^{-6}\ \text{W} = \boxed{15.0\ \mu\text{W}}$$

P27.41 From Equation 27.21,

$$P = I\Delta V = \left(350\times 10^{-3}\text{ A}\right)\left(6.00\text{ V}\right) = \boxed{2.10\text{ W}}$$

P27.42 If the tank has good insulation, essentially all of the energy electrically transmitted to the heating element becomes internal energy in the water: $\Delta E_{\text{(internal)}} = E_{\text{(electrical)}}$. Our symbol $E_{\text{(electrical)}}$ represents the same thing as the textbook's T_{ET}, namely electrically transmitted energy.

Since $\Delta E_{\text{(internal)}} = mc\Delta T$ and $E_{\text{(electrical)}} = P\Delta t = (\Delta V)^2\Delta t/R$

where $c = 4\,186\text{ J/kg}\cdot{}^\circ\text{C}$

the resistance is

$$R = \frac{(\Delta V)^2\Delta t}{cm\Delta T} = \frac{(240\text{ V})^2(1\,500\text{ s})}{\left(4\,186\text{ J/kg}\cdot{}^\circ\text{C}\right)\left(109\text{ kg}\right)\left(29.0^\circ\text{C}\right)} = \boxed{6.53\,\Omega}$$

P27.43 From $P = (\Delta V)^2/R$, we find that

$$R = \frac{\left(\Delta V_i\right)^2}{P} = \frac{(120\text{ V})^2}{100\text{ W}} = 144\,\Omega$$

The final current is

$$I_f = \frac{\Delta V_f}{R} = \frac{140\text{ V}}{144\,\Omega} = 0.972\text{ A}$$

The power during the surge is

$$P = \frac{\left(\Delta V_f\right)^2}{R} = \frac{(140\text{ V})^2}{144\,\Omega} = 136\text{ V}$$

So the percentage increase is

$$\frac{136\text{ W} - 100\text{ W}}{100\text{ W}} = 0.361 = \boxed{36.1\%}$$

P27.44 You pay the electric company for energy transferred in the amount $E = P\Delta t$.

(a) $$P\Delta t = (40\text{ W})(2\text{ weeks})\left(\frac{7\text{ d}}{1\text{ week}}\right)\left(\frac{24\text{ h}}{1\text{ d}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.110\ \$}{\text{kWh}}\right)$$

$$= \boxed{\$1.48}$$

(b) $$P\Delta t = (970\text{ W})(3\text{ min})\left(\frac{1\text{ h}}{60\text{ min}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{0.110\ \$}{\text{kWh}}\right) = \boxed{\$0.005\,34}$$

(c) $$P\Delta t = (5\ 200\ \text{W})(40\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{\text{k}}{1\ 000}\right)\left(\frac{0.110\ \$}{\text{kWh}}\right) = \boxed{\$0.381}$$

P27.45 (a) The total energy stored in the battery is

$$\begin{aligned}\Delta U_E &= q(\Delta V) = It(\Delta V)\\ &= (55.0\ \text{A}\cdot\text{h})(12.0\ \text{V})\left(\frac{1\ \text{C}}{1\ \text{A}\cdot\text{s}}\right)\left(\frac{1\ \text{J}}{1\ \text{V}\cdot\text{C}}\right)\left(\frac{1\ \text{W}\cdot\text{s}}{1\ \text{J}}\right)\\ &= 660\ \text{W}\cdot\text{h} = \boxed{0.660\ \text{kWh}}\end{aligned}$$

(b) The value of the electricity is

$$\text{Cost} = (0.660\ \text{kWh})\left(\frac{\$0.110}{1\ \text{kWh}}\right) = \boxed{\$0.072\ 6}$$

P27.46 (a) The resistance of 1.00 m of 12-gauge copper wire is

$$\begin{aligned}R &= \frac{\rho\ell}{A} = \frac{\rho\ell}{\pi(d/2)^2} = \frac{4\rho\ell}{\pi d^2} = \frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(1.00\ \text{m})}{\pi(0.205\times10^{-2}\ \text{m})^2}\\ &= 5.2\times10^{-3}\ \Omega\end{aligned}$$

The rate of internal energy production is

$$P = I\Delta V = I^2R = (20.0\ \text{A})^2(5.2\times10^{-3}\ \Omega) = \boxed{2.1\ \text{W}}$$

(b) $$R = \frac{4\rho\ell}{\pi d^2} = \frac{4(2.82\times10^{-8}\ \Omega\cdot\text{m})(1.00\ \text{m})}{\pi(0.205\times10^{-2}\ \text{m})^2} = 8.54\times10^{-3}\ \Omega$$

$$P = I\Delta V = I^2R = (20.0\ \text{A})^2(8.54\times10^{-3}\ \Omega) = \boxed{3.42\ \text{W}}$$

(c) $\boxed{\text{It would not be as safe. If surrounded by thermal insulation, it would get much hotter than a copper wire.}}$

P27.47 The power of the lamp is $P = I\Delta V = U/\Delta t$, where *U* is the energy transformed. Then the energy you buy, in standard units, is

$$\begin{aligned}U &= \Delta V I \Delta t\\ &= (110\ \text{V})(1.70\ \text{A})(1\ \text{day})\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{3\ 600\ \text{s}}{\text{h}}\right)\left(\frac{1\ \text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{1\ \text{C}}{\text{A}\cdot\text{s}}\right)\\ &= 16.2\ \text{MJ}\end{aligned}$$

In kilowatt hours, the energy is

$$\begin{aligned} U &= \Delta V I \Delta t \\ &= (110\ \text{V})(1.70\ \text{A})(1\ \text{day})\left(\frac{24\ \text{h}}{1\ \text{day}}\right)\left(\frac{1\ \text{J}}{\text{V}\cdot\text{C}}\right)\left(\frac{1\ \text{C}}{\text{A}\cdot\text{s}}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right) \\ &= 4.49\ \text{kWh} \end{aligned}$$

So operating the lamp costs (4.49 kWh)($0.110/kWh) = $\boxed{\$0.494\ /\text{day}}$.

P27.48 The energy taken in by electric transmission for the fluorescent bulb is

$$P\Delta t = 11\ \text{J/s}(100\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 3.96\times 10^{6}\ \text{J}$$

$$\text{cost} = 3.96\times 10^{6}\ \text{J}\left(\frac{\$0.110}{\text{kWh}}\right)\left(\frac{\text{k}}{1\,000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\,600\ \text{s}}\right) = \$0.121$$

For the incandescent bulb,

$$P\Delta t = 40\ \text{W}(100\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 1.44\times 10^{7}\ \text{J}$$

$$\text{cost} = 1.44\times 10^{7}\ \text{J}\left(\frac{\$0.110}{3.6\times 10^{6}\ \text{J}}\right) = \$0.440$$

$$\text{savings} = \$0.440 - \$0.121 = \boxed{\$0.319}$$

P27.49 First, we compute the resistance of the wire:

$$R = \frac{\rho\ell}{A} = \frac{(1.50\times 10^{-6}\ \Omega\cdot\text{m})25.0\ \text{m}}{\pi(0.200\times 10^{-3}\ \text{m})^{2}} = 298\ \Omega$$

The potential drop across the wire is then

$$\Delta V = IR = (0.500\ \text{A})(298\ \Omega) = 149\ \text{V}$$

(a) The magnitude of the electric field in the wire is

$$E = \frac{\Delta V}{\ell} = \frac{149\ \text{V}}{25.0\ \text{m}} = \boxed{5.97\ \text{V/m}}$$

(b) The power delivered to the wire is

$$P = (\Delta V)I = (149\ \text{V})(0.500\ \text{A}) = \boxed{74.6\ \text{W}}$$

(c) We use Equation 27.20 and Table 27.2:

$$\begin{aligned} R &= R_0\left[1+\alpha(T-T_0)\right] = (298\ \Omega)\left[1+(0.400\times 10^{-3}/^\circ\text{C})320^\circ\text{C}\right] \\ &= 337\ \Omega \end{aligned}$$

To find the power delivered, we first compute the current flowing through the wire:

$$I = \frac{\Delta V}{R} = \frac{149\text{ V}}{337\ \Omega} = 0.443\text{ A}$$

then,

$$P = (\Delta V)I = (149\text{ V})(0.443\text{ A}) = \boxed{66.1\text{ W}}$$

P27.50 The battery takes in energy by electric transmission:

$$P\Delta t = (\Delta V)I(\Delta t) = (2.3\text{ J/C})(13.5\times10^{-3}\text{ C/s})(4.2\text{ h})\left(\frac{3\,600\text{ s}}{1\text{ h}}\right)$$
$$= 469\text{ J}$$

It puts out energy by electric transmission:

$$(\Delta V)I(\Delta t) = (1.6\text{ J/C})(18\times10^{-3}\text{ C/s})(2.4\text{ h})\left(\frac{3\,600\text{ s}}{1\text{ h}}\right) = 249\text{ J}$$

(a) $\text{efficiency} = \frac{\text{useful output}}{\text{total input}} = \frac{249\text{ J}}{469\text{ J}} = \boxed{0.530}$

(b) The only place for the missing energy to go is into internal energy:

$$469\text{ J} = 249\text{ J} + \Delta E_{\text{int}}$$
$$\Delta E_{\text{int}} = \boxed{221\text{ J}}$$

(c) We imagine toasting the battery over a fire with 221 J of heat input:

$$Q = mc\Delta T$$
$$\Delta T = \frac{Q}{mc} = \frac{221\text{ J}}{(0.015\text{ kg})(975\text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{15.1^\circ\text{C}}$$

P27.51 We compute the resistance of the wire from

$$P = \frac{(\Delta V)^2}{R} \rightarrow R = \frac{(\Delta V)^2}{P} = \frac{(110\text{ V})^2}{500\text{ W}} = 24.2\ \Omega$$

(a) Then, Equation 27.10, $R = \frac{\rho}{A}\ell$, gives us the length of wire used:

$$\ell = \frac{RA}{\rho} = \frac{(24.2\ \Omega)\pi(2.50\times10^{-4}\text{ m})^2}{1.50\times10^{-6}\ \Omega\cdot\text{m}} = \boxed{3.17\text{ m}}$$

(b) From Equation 27.20, the resistance of the wire at this temperature is

$$\begin{aligned} R &= R_0[1+\alpha\Delta T] = 24.2\ \Omega\left[1+\left(0.400\times10^{-3}\right)\left(1\,200-20\right)\right] \\ &= 35.6\ \Omega \end{aligned}$$

The power delivered to the coil is then

$$P = \frac{(\Delta V)^2}{R} = \frac{(110\ \text{V})^2}{35.6\ \Omega} = \boxed{340\ \text{W}}$$

P27.52 We find the energy transferred into a number *N* of these clocks in one year:

$$\begin{aligned} T_{\text{ET}} &= P_{\text{total}}\Delta t = NP_{\text{one clock}}\Delta t \\ &= \left(270\times10^{6}\ \text{clocks}\right)\left(2.50\ \text{W/clock}\right) \\ &\qquad \times\left(365\ \text{d/yr}\right)\left(24\ \text{h/d}\right)\left(1\ \text{kW/1000 W}\right) \\ &= 5.91\times10^{9}\ \text{kWh} \end{aligned}$$

Divide this energy into the total cost claimed by the politician to find the cost of the electricity:

$$\text{cost} = \frac{\$100\times10^{6}}{5.91\times10^{9}\ \text{kWh}} = \$0.017\ /\ \text{kWh}$$

This is significantly lower than the average cost of electricity in the United States. While the situation is not actually impossible, the politician would have a better argument by using the actual average cost of electricity in the United States, which would raise his estimate of the total cost to operate the clocks to about $650 million every year.

P27.53 At operating temperature,

(a) $P = I\Delta V = (1.53\ \text{A})(120\ \text{V}) = \boxed{184\ \text{W}}$

(b) Use the change in resistance to find the final operating temperature of the toaster.

$$R = R_0(1+\alpha\Delta T)$$

$$\frac{120\ \text{V}}{1.53\ \text{A}} = \left(\frac{120\ \text{V}}{1.80\ \text{A}}\right)\left[1+\left(0.400\times10^{-3}\ (^\circ\text{C})^{-1}\right)\Delta T\right]$$

which gives

$$\Delta T = 441^\circ\text{C}$$

and $T = 20.0^\circ\text{C} + 441^\circ\text{C} = \boxed{461^\circ\text{C}}$

P27.54 Consider a 400-W blow dryer used for ten minutes daily for a year. The energy transferred to the dryer is

$$P\Delta t = (400\ \text{J/s})(600\ \text{s/d})(365\ \text{d})$$
$$\approx 9\times 10^{7}\ \text{J}\left(\frac{1\ \text{kWh}}{3.6\times 10^{6}\ \text{J}}\right)\approx 20\ \text{kWh}$$

We suppose that electrically transmitted energy costs on the order of ten cents per kilowatt-hour. Then the cost of using the dryer for a year is on the order of

$$\text{Cost} \approx (20\ \text{kWh})(\$0.10/\text{kWh}) = \$2\ \boxed{\sim \$1}$$

P27.55 We first compute the power delivered to the resistor:

$$P = I\Delta V = (2.00\ \text{A})(120\ \text{V}) = 240\ \text{W}$$

The change in internal energy of the water as it is heated from 23.0°C to 100°C is

$$\Delta E_{\text{int}} = (0.500\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(77.0^\circ\text{C}) = 161\ \text{kJ}$$

The time interval required to heat the water is then

$$\Delta t = \frac{\Delta E_{\text{int}}}{P} = \frac{1.61\times 10^{5}\ \text{J}}{240\ \text{W}} = \boxed{672\ \text{s}}$$

P27.56 (a) We know that

$$\text{efficiency} = \frac{\text{mechanical power output}}{\text{total power input}}$$
$$= 0.900 = \frac{(2.50\ \text{hp})(746\ \text{W/1 hp})}{(120\ \text{V})\,I}$$

from which, we calculate the current as

$$I = \frac{1\,860\ \text{J/s}}{0.9(120\ \text{V})} = \frac{2\,070\ \text{J/s}}{120\ \text{V}} = \boxed{17.3\ \text{A}}$$

(b) The energy delivered to the motor in 3.00 h is

$$\text{energy input} = P_{\text{input}}\Delta t = (2\,070\ \text{J/s})[3.00(3\,600\ \text{s})]$$
$$= 2.24\times 10^{7}\ \text{J} = \boxed{22.4\ \text{MJ}}$$

(c) At $0.110/kWh, the cost of running the motor for 3.00 h is

$$\text{cost} = (2.24\times 10^{7}\ \text{J})\left(\frac{\$0.110}{1\ \text{kWh}}\right)\left(\frac{\text{k}}{10^{3}}\frac{\text{J}}{\text{W s}}\frac{\text{h}}{3\,600\ \text{s}}\right) = \boxed{\$0.684}$$

Additional Problems

***P27.57** From Equation 27.22, $P = \frac{(\Delta V)^2}{R}$, we find that the total resistance needed in the wire is

$$R = \frac{(\Delta V)^2}{P} = \frac{(20\ \text{V})^2}{48\ \text{W}} = 8.3\ \Omega$$

We then solve for the length of the wire from Equation 27.10:

$$\ell = \frac{RA}{\rho} = \frac{(8.3\ \Omega)(4.0\times10^{-6}\ \text{m}^2)}{3.0\times10^{-8}\ \Omega\cdot\text{m}} = 1.1\times10^3\ \text{m} = \boxed{1.1\ \text{km}}$$

P27.58 At $T_0 = 20.0°$, $R = R_0$. Then, from Equation 27.20,

$$R = R_0\left[1+\alpha(T-T_0)\right] = 2R_0$$

Solving for the change in temperature gives

$$T - T_0 = \frac{1}{\alpha} = \frac{1}{3.9\times10^{-3}\ (°\text{C})^{-1}}$$

$$T - 20.0°\text{C} = 256°\text{C} \quad\rightarrow\quad T = \boxed{276°\text{C}}$$

P27.59 We find the amount of current each headlight draws:

$$P = I\Delta V \rightarrow I = \frac{P}{\Delta V} = \frac{36.0\ \text{W}}{12.0\ \text{V}} = 3.00\ \text{A}$$

For two headlights, the total current from battery is 6.00 A. The battery rating is the total amount of charge the battery can deliver, without being recharged, over a time interval Δt at a rate (current) I:

$$\Delta Q = I\Delta t = 90.0\ \text{A}\cdot\text{h}$$

The total time interval to discharge the battery is then

$$\Delta t = \frac{\Delta Q}{I} = \frac{90.0\ \text{A}\cdot\text{h}}{6.00\ \text{A}} = \boxed{15.0\ \text{h}}$$

P27.60 (a) $P = I\Delta V = \frac{(\Delta V)^2}{R} \rightarrow R = \frac{(\Delta V)^2}{P}$

Lightbulb A: $R = \frac{(\Delta V)^2}{P} = \frac{(120\ \text{V})^2}{25.0\ \text{W}} = \boxed{576\ \Omega}$

Lightbulb B: $R = \frac{(\Delta V)^2}{P} = \frac{(120\ \text{V})^2}{100\ \text{W}} = \boxed{144\ \Omega}$

(b) $I = \frac{Q}{\Delta t} = \frac{P}{\Delta V} \rightarrow \Delta t = \frac{Q\Delta V}{P} = \frac{(1.00\ \text{C})(120\ \text{V})}{25.0\ \text{W}} = \boxed{4.80\ \text{s}}$

(c) $\boxed{\text{The charge is the same. It is at a location that is lower in potential.}}$

(d) $P = \dfrac{\Delta U}{\Delta t} \to \Delta t = \dfrac{\Delta U}{P} = \dfrac{1.00\ \text{J}}{25.0\ \text{W}} = \boxed{0.040\ 0\ \text{s}}$

(e) $\boxed{\begin{array}{l}\text{Because of energy conservation, the energy entering and leaving}\\ \text{the lightbulb is the same. Energy enters the lightbulb by electric}\\ \text{transmission and leaves by heat and electromagnetic radiation.}\end{array}}$

(f) $\Delta U = P\Delta t = (25.0\ \text{J/s})(86\ 400\ \text{s/d})(30.0\ \text{d}) = 64.8 \times 10^6\ \text{J}$

$$\text{Cost} = (64.8 \times 10^6\ \text{J})\left(\frac{\$0.110\ 0}{\text{kWh}}\right)\left(\frac{\text{k}}{1\ 000}\right)\left(\frac{\text{W}\cdot\text{s}}{\text{J}}\right)\left(\frac{\text{h}}{3\ 600\ \text{s}}\right) = \boxed{\$1.98}$$

P27.61 The resistance of one wire is $\left(\dfrac{0.500\ \Omega}{\text{mi}}\right)(100\ \text{mi}) = 50.0\ \Omega$.

The whole wire is at nominal 700 kV away from ground potential, but the potential difference between its two ends is

$$IR = (1\ 000\ \text{A})(50.0\ \Omega) = 50.0\ \text{kV}$$

Then it radiates as heat power

$$P = I\Delta V = (1\ 000\ \text{A})(50.0 \times 10^3\ \text{V}) = \boxed{50.0\ \text{MW}}$$

P27.62 (a) From $\rho = \dfrac{RA}{\ell} = \dfrac{(\Delta V)}{I}\dfrac{A}{\ell}$ we compute

ℓ (m)	R (Ω)	ρ ($\Omega\cdot$m)
0.540	7.25	9.80×10^{-7}
1.028	14.1	9.98×10^{-7}
1.543	21.1	1.00×10^{-6}

(b) $\bar{\rho} = \boxed{9.93 \times 10^{-7}\ \Omega\cdot\text{m}}$

(c) $\boxed{\begin{array}{l}\text{The average value is within 1\% of the tabulated value of}\\ 1.00 \times 10^{-6}\ \Omega\cdot\text{m given in Table 27.2.}\end{array}}$

P27.63 The original stored energy is $U_{E,i} = \frac{1}{2}Q\Delta V_i = \frac{1}{2}\frac{Q^2}{C}$.

(a) When the switch is closed, charge *Q* distributes itself over the plates of *C* and 3*C* in parallel, presenting equivalent capacitance 4*C*. Then the final potential difference is $\boxed{\Delta V_f = \frac{Q}{4C}}$ for both.

(b) The smaller capacitor then carries charge $C\Delta V_f = \frac{Q}{4C}C = \boxed{\frac{Q}{4}}$.

The larger capacitor carries charge $3C\frac{Q}{4C} = \boxed{\frac{3Q}{4}}$.

(c) The smaller capacitor stores final energy $\frac{1}{2}C(\Delta V_f)^2 = \frac{1}{2}C\left(\frac{Q}{4C}\right)^2 =$ $\boxed{\frac{Q^2}{32C}}$. The larger capacitor possesses energy $\frac{1}{2}3C\left(\frac{Q}{4C}\right)^2 = \boxed{\frac{3Q^2}{32C}}$.

(d) The total final energy is $\frac{Q^2}{32C} + \frac{3Q^2}{32C} = \frac{Q^2}{8C}$. The loss of potential energy is the energy appearing as internal energy in the resistor:

$$\frac{Q^2}{2C} = \frac{Q^2}{8C} + \Delta E_{\text{int}} \quad \text{so} \quad \Delta E_{\text{int}} = \boxed{\frac{3Q^2}{8C}}$$

P27.64 (a) The heater should put out constant power

$$P = \frac{Q}{\Delta t} = \frac{mc(T_f - T_i)}{\Delta t}$$
$$= \frac{(0.250\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 20^\circ\text{C})}{(4\text{ min})}\left(\frac{1\text{ min}}{60\text{ s}}\right)$$
$$= 349\text{ J/s}$$

Then its resistance should be described by

$$P = \frac{(\Delta V)^2}{R} \rightarrow R = \frac{(\Delta V)^2}{P} = \frac{(120\text{ J/C})^2}{349\text{ J/s}} = 41.3\ \Omega$$

Its resistivity at 100 °C is given by

$$\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right] = \left(1.50\times10^{-6}\ \Omega\cdot\text{m}\right)\left[1+0.4\times10^{-3}(80)\right]$$
$$= 1.55\times10^{-6}\ \Omega\cdot\text{m}$$

Then for a wire of circular cross section, from Equation 27.10,

$$R = \rho\frac{\ell}{A} = \rho\frac{\ell}{\pi r^2} = \rho\frac{\ell 4}{\pi d^2}$$

$$41.3\ \Omega = \left(1.55\times10^{-6}\ \Omega\cdot\text{m}\right)\frac{4\ell}{\pi d^2}$$

$$\frac{\ell}{d^2} = 2.09\times10^{+7}/\text{m} \quad \text{or} \quad d^2 = \left(4.77\times10^{-8}\ \text{m}\right)\ell$$

One possible choice is $\ell = 0.900$ m and $d = 2.07 \times 10^{-4}$ m. If ℓ and d are made too small, the surface area will be inadequate to transfer heat into the water fast enough to prevent overheating of the filament. To make the volume less than 0.5 cm^3, we want ℓ and d less than those described by $\frac{\pi d^2}{4}\ell = 0.5\times10^{-6}\ \text{m}^3$. Substituting $d^2 = \left(4.77\times10^{-8}\ \text{m}\right)\ell$ gives

$\frac{\pi}{4}\left(4.77\times10^{-8}\ \text{m}\right)\ell^2 = 0.5\times10^{-6}\ \text{m}^3$, $\ell = 3.65$ m and $d = 4.18 \times 10^{-4}$ m. Thus our answer is:

$$\boxed{\text{Any diameter } d \text{ and length } \ell \text{ related by } d^2 = (4.77\times10^{-8})\ell, \text{ where } d \text{ and } \ell \text{ are in meters.}}$$

(b) $\boxed{\text{Yes; for } V = 0.500\ \text{cm}^3 \text{ of Nichrome, } \ell = 3.65\ \text{m and } d = 0.418\ \text{mm.}}$

***P27.65** The power the beam delivers to the target is

$$P = I\Delta V = \left(25.0\times10^{-3}\ \text{A}\right)\left(4.00\times10^{6}\ \text{V}\right) = 1.00\times10^{5}\ \text{W}$$

The mass of cooling water that must flow through the tube each second if the rise in the water temperature is not to exceed 50°C is found from

$$Q = P\Delta t = (\Delta m)c\Delta T$$

Therefore,

$$\frac{\Delta m}{\Delta t} = \frac{P}{c\Delta T} = \frac{1.00\times10^{5}\ \text{J/s}}{\left(4\ 186\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(50.0^\circ\text{C}\right)} = \boxed{0.478\ \text{kg/s}}$$

P27.66 (a) Since $\mathsf{P} = \mathsf{I}\Delta\mathsf{V}$, we have

$$\mathsf{I} = \frac{\mathsf{P}}{\Delta\mathsf{V}} = \frac{8.00\times10^3\ \mathrm{W}}{12.0\ \mathrm{V}} = \boxed{667\ \mathrm{A}}$$

(b) From $\mathsf{P} = \mathsf{U}/\Delta\mathsf{t}$, the time the car runs is

$$\Delta\mathsf{t} = \frac{\Delta\mathsf{U}}{\mathsf{P}} = \frac{2.00\times10^7\ \mathrm{J}}{8.00\times10^3\ \mathrm{W}} = 2.50\times10^3\ \mathrm{s}$$

So it moves a distance of

$$\Delta\mathsf{x} = \mathsf{v}\Delta\mathsf{t} = (20.0\ \mathrm{m/s})(2.50\times10^3\ \mathrm{s}) = \boxed{50.0\ \mathrm{km}}$$

P27.67 (a) Assuming the change in *V* is uniform:

$$\mathsf{E}_x = -\frac{d\mathsf{V}(x)}{dx} \rightarrow \mathsf{E}_x = -\frac{\Delta\mathsf{V}}{\Delta x} = -\frac{(0-4.00\ \mathrm{V})}{(0.500\ \mathrm{m}-0)} = +8.00\ \mathrm{V/m}$$

or $\boxed{8.00\ \mathrm{V/m}\text{ in the positive } x \text{ direction.}}$

(b) From Equation 27.10, we have

$$\mathsf{R} = \frac{\rho\ell}{\mathsf{A}} = \frac{(4.00\times10^{-8}\ \Omega\cdot\mathrm{m})(0.500\ \mathrm{m})}{\pi(1.00\times10^{-4}\ \mathrm{m})^2} = \boxed{0.637\ \Omega}$$

(c) From Equation 27.7,

$$\mathsf{I} = \frac{\Delta\mathsf{V}}{\mathsf{R}} = \frac{4.00\ \mathrm{V}}{0.637\ \Omega} = \boxed{6.28\ \mathrm{A}}$$

(d) From Equation 27.5, the current density is given by

$$\mathsf{J} = \frac{\mathsf{I}}{\mathsf{A}} = \frac{6.28\ \mathrm{A}}{\pi(1.00\times10^{-4}\ \mathrm{m})^2} = 2.00\times10^8\ \mathrm{A/m^2} = \boxed{200\ \mathrm{MA/m^2}}$$

The field and the current are both in the *x* direction.

(e) We intend to derive the equivalent of Equation 27.6. We start with the definition of current density, $J = I/A$, and, using Equations 27.7 and 27.10, note that the current is given by

$$\mathsf{I} = \frac{\Delta\mathsf{V}}{\mathsf{R}} = \frac{\mathsf{E}\ell}{\mathsf{R}} = \frac{\mathsf{EA}}{\rho}$$

Then,

$$\mathsf{J} = \frac{\mathsf{I}}{\mathsf{A}} = \frac{\mathsf{EA}/\rho}{\mathsf{A}} = \frac{\mathsf{E}}{\rho}$$

so

$$E = \rho J = \left(4.00\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.00\times10^{8}\ \text{A/m}^2\right) = \boxed{8.00\ \text{V/m}}$$

P27.68 (a) Assuming the change in V is uniform:

$$E_x = -\frac{dV(x)}{dx} \rightarrow E_x = -\frac{\Delta V}{\Delta x} = -\frac{0-V}{L-0} = +\frac{V}{L}$$

Therefore, the electric field is $\boxed{V/L \text{ in the positive } x \text{ direction.}}$

(b) From Equation 27.10, we have

$$R = \frac{\rho\ell}{A} = \frac{\rho L}{\pi d^2/4} = \boxed{4\rho L/\pi d^2}$$

(c) From Equation 27.7,

$$I = \Delta V/R = \boxed{V\pi d^2/4\rho L}$$

(d) From Equation 27.5, the current density is given by

$$J = \frac{I}{A} = \frac{V\pi d^2/4\rho L}{\pi d^2/4} = \boxed{V/\rho L \text{ in the positive } x \text{ direction}}$$

The field and the current both have the same direction.

(e) We intend to derive the equivalent of Equation 27.6. We start with the definition of current density, $J = I/A$, and, using Equations 27.7 and 27.10, note that the current is given by

$$I = \frac{\Delta V}{R} = \frac{E\ell}{R} = \frac{EA}{\rho}$$

Then,

$$J = \frac{I}{A} = \frac{EA/\rho}{A} = \frac{E}{\rho}$$

so $$E = \rho J = \rho\left(\frac{V}{\rho L}\right) = \frac{V}{L}$$

P27.69 Since there are 2 wires, the total length is $\ell = 100$ m. The resistance of the wires is

$$R = \left(\frac{0.108\ \Omega}{300\ \text{m}}\right)(100\ \text{m}) = 0.036\,0\ \Omega$$

(a) We find the potential difference at the customer's house from

$$(\Delta V)_{\text{home}} = (\Delta V)_{\text{line}} - IR = 120\ \text{V} - (110\ \text{A})(0.036\,0\ \Omega) = \boxed{116\ \text{V}}$$

(b) The power delivered to the customer is

$$\mathsf{P} = \mathsf{I}(\Delta\mathsf{V}) = (110\ \text{A})(116\ \text{V}) = \boxed{12.8\ \text{kW}}$$

(c) The power dissipated in the wires, or the energy produced in the wires, is

$$\mathsf{P}_{\text{wires}} = \mathsf{I}^2\mathsf{R} = (110\ \text{A})^2(0.036\,0\ \Omega) = \boxed{436\ \text{W}}$$

P27.70 The original resistance is $R_i = \rho L_i/A_i$. The new length is

$$L = L_i + \delta L_i = L_i(1 + \delta)$$

(a) Constancy of volume implies $AL = A_iL_i$ so

$$\mathsf{A} = \frac{\mathsf{A}_i\mathsf{L}_i}{\mathsf{L}} = \frac{\mathsf{A}_i\mathsf{L}_i}{\mathsf{L}_i(1+\delta)} = \frac{\mathsf{A}_i}{(1+\delta)}$$

The new resistance is

$$\mathsf{R} = \frac{\rho\mathsf{L}}{\mathsf{A}} = \frac{\rho\mathsf{L}_i(1+\delta)}{\mathsf{A}_i/(1+\delta)} = \mathsf{R}_i(1+\delta)^2 = \mathsf{R}_i(1+2\delta+\delta^2)$$

(b) The result is exact if the assumptions are precisely true. Our derivation contains no approximation steps where delta is assumed to be small.

P27.71 (a) A thin cylindrical shell of radius r, thickness dr, and length L contributes resistance

$$\mathsf{dR} = \frac{\rho\,\mathsf{d}\ell}{\mathsf{A}} = \frac{\rho\,\mathsf{dr}}{(2\pi\mathsf{r})\mathsf{L}} = \left(\frac{\rho}{2\pi\mathsf{L}}\right)\frac{\mathsf{dr}}{\mathsf{r}}$$

The resistance of the whole annulus is the series summation of the contributions of the thin shells:

$$\mathsf{R} = \frac{\rho}{2\pi\mathsf{L}}\int_{\mathsf{r}_c}^{\mathsf{r}_b}\frac{\mathsf{dr}}{\mathsf{r}} = \boxed{\frac{\rho}{2\pi\mathsf{L}}\ln\left(\frac{\mathsf{r}_b}{\mathsf{r}_a}\right)}$$

(b) In this equation $\dfrac{\Delta\mathsf{V}}{\mathsf{I}} = \dfrac{\rho}{2\pi\mathsf{L}}\ln\left(\dfrac{\mathsf{r}_b}{\mathsf{r}_a}\right)$.

Solving, we get $\boxed{\rho = \dfrac{2\pi\mathsf{L}\Delta\mathsf{V}}{\mathsf{I}\ln(\mathsf{r}_b/\mathsf{r}_a)}}$.

P27.72 The value of 11.4 A is what results from substituting the given voltage and resistance into Equation 27.7. However, the resistance measured for a lightbulb with an ohmmeter is not the resistance at which it operates because of the change in resistivity with temperature. The higher resistance of the filament at the operating temperature brings the current down significantly.

P27.73 Let α be the temperature coefficient at 20.0°C, and α' be the temperature coefficient at 0°C. Then $\rho = \rho_0\left[1+\alpha(T-20.0°\text{C})\right]$ and $\rho = \rho'\left[1+\alpha'(T-0°\text{C})\right]$ must both give the correct resistivity at any temperature T. That is, we must have:

$$\rho_0\left[1+\alpha(T-20.0°\text{C})\right] = \rho'\left[1+\alpha'(T-0°\text{C})\right] \qquad \textbf{[1]}$$

Setting $T = 0$ in equation [1] yields:

$$\rho' = \rho_0\left[1-\alpha(20.0°\text{C})\right]$$

and setting $T = 20.0°\text{C}$ in equation [1] gives:

$$\rho_0 = \rho'\left[1+\alpha'(20.0°\text{C})\right]$$

Substitute ρ' from the first of these results into the second to obtain:

$$\rho_0 = \rho_0\left[1-\alpha(20.0°\text{C})\right]\left[1+\alpha'(20.0°\text{C})\right]$$

Therefore,

$$1+\alpha'(20.0°\text{C}) = \frac{1}{1-\alpha(20.0°\text{C})}$$

which simplifies:

$$\alpha'(20.0°\text{C}) = \frac{1}{1-\alpha(20.0°\text{C})} - 1 = \frac{1-\left[1-\alpha(20.0°\text{C})\right]}{1-\alpha(20.0°\text{C})}$$

$$\alpha'(20.0°\text{C}) = \frac{\alpha(20.0°\text{C})}{1-\alpha(20.0°\text{C})} \rightarrow \alpha' = \frac{\alpha}{1-\alpha(20.0°\text{C})}$$

Therefore,

$$\alpha' = \frac{\alpha}{\left[1-\alpha(20.0°\text{C})\right]} = \frac{3.8\times10^{-3}\ (°\text{C})^{-1}}{\left[1-\left(3.8\times10^{-3}\ (°\text{C})^{-1}\right)(20.0°\text{C})\right]}$$

$$= \boxed{4.1\times10^{-3}\ (°\text{C})^{-1}}$$

P27.74 (a) We begin from $\Delta V = -E\cdot\ell$ or $dV = -E\cdot dx$. Then,

$$\Delta V = -IR = -E\cdot\ell$$

and the current is

$$I = \frac{dq}{dt} = \frac{E\cdot\ell}{R} = \frac{A}{\rho\ell}E\cdot\ell = \frac{A}{\rho}E = -\sigma A\frac{dV}{dx} = \boxed{\sigma A\left|\frac{dV}{dx}\right|}$$

(b) Current flows in the direction of decreasing voltage. Energy flows by heat in the direction of decreasing temperature.

P27.75 We begin with

$$R = \frac{\rho\ell}{A} = \frac{\rho_0\left[1+\alpha\left(T-T_0\right)\right]\ell_0\left[1+\alpha'\left(T-T_0\right)\right]}{A_0\left[1+\alpha'\left(T-T_0\right)\right]^2}$$

$$= \frac{\rho_0\ell_0}{A_0}\frac{1+\alpha\left(T-T_0\right)}{1+\alpha'\left(T-T_0\right)}$$

For copper (for $T_0 = 20.0°\text{C}$): $\rho_0 = 1.700\times10^{-8}\ \Omega\cdot\text{m}$, $\alpha = 3.900\times10^{-3}\ °\text{C}^{-1}$, and $\alpha' = 17.00\times10^{-6}\ °\text{C}^{-1}$. Then,

$$R = \frac{\rho_0\ell_0}{A_0}\frac{1+\alpha\left(T-T_0\right)}{1+\alpha'\left(T-T_0\right)}$$

$$R = \frac{\left(1.700\times10^{-8}\right)\left(2.000\right)}{\pi\left(0.1000\times10^{-3}\right)^2}\left[\frac{1+\left(3.900\times10^{-3}\ °\text{C}^{-1}\right)\left(80.00°\text{C}\right)}{1+\left(17.00\times10^{-6}\ °\text{C}^{-1}\right)\left(80.00°\text{C}\right)}\right]$$

$$R = \boxed{1.418\ \Omega}$$

P27.76 The wire has length ℓ, and radius r; its cross-sectional area is A (πr^2, if circular), which is proportional to r^2. Because both ℓ and r change with a temperature variation ΔT according to $L = L_0\left(1+\alpha'\Delta T\right)$, the cross-sectional area changes according to $A = A_0\left(1+\alpha'\Delta T\right)^2$.

Calling $R_0 = \frac{\rho_0\ell_0}{A_0}$ at temperature T_0, we have

$$R_0 = \frac{\rho_0\ell_0}{A_0} \rightarrow R = \frac{\rho_0\left[1+\alpha\left(T-T_0\right)\right]\ell_0\left[1+\alpha'\left(T-T_0\right)\right]}{A_0\left[1+\alpha'\left(T-T_0\right)\right]^2}$$

$$= \frac{\rho\ell_0}{A_0}\frac{\left[1+\alpha\left(T-T_0\right)\right]}{\left[1+\alpha'\left(T-T_0\right)\right]}x$$

which gives

$$R = \boxed{R_0 \frac{1+\alpha(T-T_0)}{1+\alpha'(T-T_0)}}$$

P27.77 (a) Think of the device as two capacitors in parallel. The one on the left has $\kappa_1 = 1$, $A_1 = \left(\frac{\ell}{2} + x\right)\ell$. The equivalent capacitance is

$$\frac{\kappa_1 \in_0 A_1}{d} + \frac{\kappa_2 \in_0 A_2}{d} = \frac{\in_0 \ell}{d}\left(\frac{\ell}{2} + x\right) + \frac{\kappa \in_0 \ell}{d}\left(\frac{\ell}{2} - x\right)$$

$$= \boxed{\frac{\in_0 \ell}{2d}(\ell + 2x + \kappa\ell - 2\kappa x)}$$

(b) The charge on the capacitor is $Q = C\Delta V$

$$Q = \frac{\in_0 \ell\Delta V}{2d}(\ell + 2x + \kappa\ell - 2\kappa x)$$

The current is

$$I = \frac{dQ}{dt} = \frac{dQ}{dx}\frac{dx}{dt} = \frac{\in_0 \ell\Delta V}{2d}(0 + 2 + 0 - 2\kappa)v = -\frac{\in_0 \ell\Delta Vv}{d}(\kappa - 1)$$

The negative value indicates that the current drains charge from the capacitor. Positive current is $\boxed{\text{clockwise } \frac{\in_0 \ell\Delta Vv}{d}(\kappa - 1)}$.

P27.78 (a) The resistance of the dielectric block is $R = \frac{\rho\ell}{A} = \frac{d}{\sigma A}$.

The capacitance of the capacitor is $C = \frac{\kappa \in_0 A}{d}$.

Then $RC = \frac{d}{\sigma A}\frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0}{\sigma}$ is a characteristic of the material only.

(b) The resistance between the plates of the capacitor is

$$R = \frac{\kappa \in_0}{\sigma C} = \frac{\rho\kappa \in_0}{C}$$

$$= \frac{(75 \times 10^{16}\ \Omega \cdot \text{m})(3.78)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{14.0 \times 10^{-9}\ \text{F}}$$

$$= \boxed{1.79 \times 10^{15}\ \Omega}$$

P27.79 The volume of the gram of gold is given by $\rho = \dfrac{m}{V}$.

$$V = \frac{m}{\rho} = \frac{10^{-3}\text{ kg}}{19.3\times10^{3}\text{ kg/m}^3} = 5.18\times10^{-8}\text{ m}^3 = A\left(2.40\times10^{3}\text{ m}\right)$$

Then, $A = 2.16\times10^{-11}\text{ m}^2$ and the resistance is

$$R = \frac{\rho\ell}{A} = \frac{\left(2.44\times10^{-8}\ \Omega\cdot\text{m}\right)\left(2.4\times10^{3}\text{ m}\right)}{2.16\times10^{-11}\text{ m}^2} = \boxed{2.71\times10^{6}\ \Omega}$$

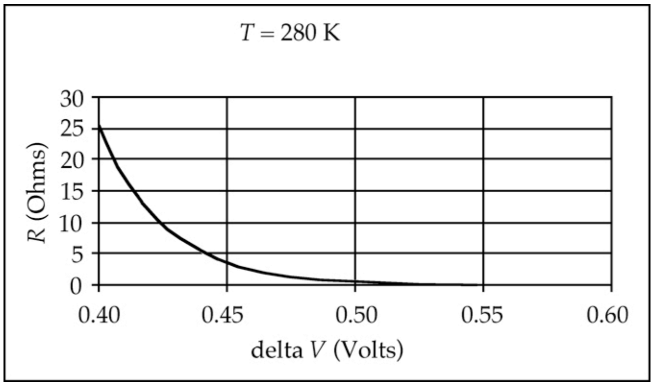
P27.80 Evaluate $I = I_0\left[\exp\left(\dfrac{e\Delta V}{k_BT}\right) - 1\right]$ and $R = \dfrac{\Delta V}{I}$ with

$I_0 = 1.00\times10^{-9}$ A, $e = 1.60\times10^{-19}$ C, and $k_B = 1.38\times10^{-23}$ J/K.

Parts (a) and (b): The following includes a partial table of calculated values and a graph for each of the specified temperatures.

(i) For $T = 280$ K:

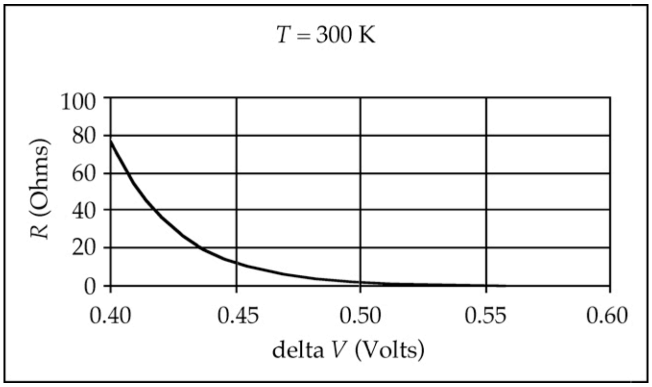
ΔV (V)	I (A)	R (Ω)
0.400	0.015 6	25.6
0.440	0.081 8	5.38
0.480	0.429	1.12
0.520	2.25	0.232
0.560	11.8	0.047 6
0.600	61.6	0.009 7



ANS. FIG. P27.80(i)

(ii) For $T = 300$ K:

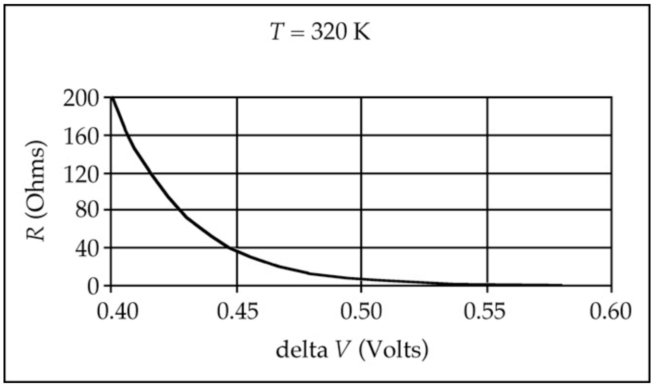
ΔV (V)	I (A)	R (Ω)
0.400	0.005	77.3
0.440	0.024	18.1
0.480	0.114	4.22
0.520	0.534	0.973
0.560	2.51	0.223
0.600	11.8	0.051



ANS. FIG. P27.80(ii)

(iii) For $T = 320$ K:

ΔV (V)	I (A)	R (Ω)
0.400	0.002 0	203
0.440	0.008 4	52.5
0.480	0.035 7	13.4
0.520	0.152	3.42
0.560	0.648	0.864
0.600	2.76	0.217



ANS. FIG. P27.80(iii)

P27.81 To find the final operating temperature, we begin with

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right]$$

and solve for the temperature *T*:

$$T = T_0 + \frac{1}{\alpha}\left[\frac{R}{R_0}-1\right] = T_0 + \frac{1}{\alpha}\left[\frac{I_0}{I}-1\right]$$

In this case, $I = \dfrac{I_0}{10}$, so

$$T = T_0 + \frac{1}{\alpha}(9) = 20° + \frac{9}{0.004\ 50/°\text{C}} = \boxed{2\ 020°\text{C}}$$

Challenge Problems

P27.82 (a) We are given $\alpha = \dfrac{1}{\rho}\dfrac{d\rho}{dT}$

Separating variables, $\int_{\rho_0}^{\rho}\dfrac{d\rho}{\rho} = \int_{T_0}^{T}\alpha\, dT$. We integrate, on both sides, from the physical situation at temperature T_0 to that at temperature *T*.

Integrating both sides, $\ln(\rho/\rho_0) = \alpha(T - T_0)$

Thus $\boxed{\rho = \rho_0 e^{\alpha(T-T_0)}}$

(b) From the series expansion $e^x \approx 1 + x$, with *x* much less than 1,

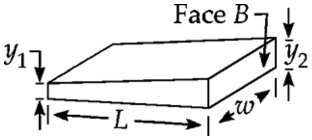
$$\boxed{\rho \approx \rho_0\left[1+\alpha\left(T-T_0\right)\right]}$$

P27.83 A spherical layer within the shell, with radius *r* and thickness *dr*, has resistance

$$dR = \frac{\rho dr}{4\pi r^2}$$

The whole resistance is the absolute value of the quantity

$$R = \int_a^b dR = \int_{r_a}^{r_b}\frac{\rho dr}{4\pi r^2} = \frac{\rho}{4\pi}\left.\frac{r^{-1}}{-1}\right|_{r_a}^{r_b} = -\frac{\rho}{4\pi}\left(-\frac{1}{r_a}+\frac{1}{r_b}\right) = \frac{\rho}{4\pi}\left(\frac{1}{r_a}-\frac{1}{r_b}\right)$$



ANS. FIG. P27.84

P27.84 Refer to ANS. FIG. P27.84. The current flows generally parallel to L. Consider a slice of the material perpendicular to this current, of thickness dx, and at distance x from face A. Then the other dimensions of the slice are w and y, where by proportion $\frac{y-y_1}{x} = \frac{y_2-y_1}{L}$

so $y = y_1 + (y_2 - y_1)\frac{x}{L}$. The bit of resistance which this slice contributes is

$$dR = \frac{\rho dx}{A} = \frac{\rho dx}{wy} = \frac{\rho dx}{w\left(y_1 + \left(y_2 - y_1\right)\left(x/L\right)\right)}$$

The whole resistance is that of all the slices:

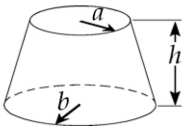
$$R = \int_{x=0}^{L} dR = \int_0^L \frac{\rho dx}{w\left(y_1 + \left(y_2 - y_1\right)\left(x/L\right)\right)}$$

$$= \frac{\rho}{w}\frac{L}{y_2 - y_1}\int_{x=0}^{L}\frac{\left(\left(y_2 - y_1\right)/L\right)dx}{y_1 + \left(y_2 - y_1\right)\left(x/L\right)}$$

With $u = y_1 + \left(y_2 - y_1\right)\frac{x}{L}$ this is of the form $\int\frac{du}{u}$, so

$$R = \frac{\rho L}{w\left(y_2 - y_1\right)}\ln\left[y_1 + \left(y_2 - y_1\right)(x/L)\right]_{x=0}^{L}$$

$$= \frac{\rho L}{w\left(y_2 - y_1\right)}\left(\ln y_2 - \ln y_1\right) = \boxed{\frac{\rho L}{w\left(y_2 - y_1\right)}\ln\frac{y_2}{y_1}}$$



ANS. FIG. P27.85

P27.85 From the geometry of the longitudinal section of the resistor shown in ANS. FIG. P27.85, we see that

$$\frac{(b-r)}{y} = \frac{(b-a)}{h}$$

From this, the radius at a distance y from the base is $r = (a-b)\frac{y}{h} + b$. For a disk-shaped element of volume $dR = \frac{\rho dy}{\pi r^2}$:

$$R = \frac{\rho}{\pi}\int_0^h \frac{dy}{\left[(a-b)(y/h) + b\right]^2}$$

Using the integral formula $\int\frac{du}{(au+b)^2} = -\frac{1}{a(au+b)}$, $\boxed{R = \frac{\rho}{\pi}\frac{h}{ab}}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P27.2	$\frac{q\omega}{2\pi}$
P27.4	1.05 mA
P27.6	(a) 5.57×10^{-5} m/s; (b) The drift speed is smaller because more electrons are being conducted.
P27.8	(a) 99.5 kA/m^2; (b) The current is the same; (c) The current density is smaller; (d) 0.800 cm; (e) I = 5.00 A; (f) 2.49×10^4 A/m^2
P27.10	(a) 2.21×10^{-7} m; (b) The potential of the nearest neighbor is very small compared to the 2 MV accelerating potential, so repulsion within the beam is a small effect.
P27.12	0.256 C
P27.14	500 mA
P27.16	6.43 A
P27.18	1.29
P27.20	(a) $\sqrt{\frac{mR}{\rho\rho_m}}$; (b) $\sqrt{\frac{4}{\pi}}\left(\frac{\rho m}{\rho_m R}\right)^{1/4}$
P27.22	(a) unaffected; (b) doubles; (c) doubles; (d) unchanged
P27.24	(a) 5.58×10^{-2} kg/mol; (b) 1.41×10^5 mol/m^3; (c) 8.49×10^{28} $\frac{\text{atoms}}{\text{m}^3}$; (d) 1.70×10^{29} electrons/m^3; (e) 2.21×10^{-4} m/s
P27.26	$T = 1.44 \times 10^3$ °C
P27.28	1.98 A
P27.30	(a) 1.22 Ω; (b) 8.00×10^{-4} increase
P27.32	(a) The design goal can be met; (b) $\ell_1 = 0.898$ m and $\ell_2 = 26.2$ m
P27.34	1.71 Ω
P27.36	(a) 3.00×10^8 W; (b) 1.75×10^{17} W
P27.38	7.50 W
P27.40	15.0 μW
P27.42	6.53 Ω
P27.44	(a) \$1.48; (b) \$0.005 34; (c) \$0.381

P27.46 (a) 2.1 W; (b) 3.42 W; (c) It would not be as safe. If surrounded by thermal insulation, it would get much hotter than a copper wire.

P27.48 \$0.319

P27.50 (a) 0.530; (b) 221 J; (c) 15.1°C

P27.52 See P27.52 for full explanation.

P27.54 ~ \$1

P27.56 (a) 17.3 A; (b) 22.4 MJ; (c) \$0.684

P27.58 276°C

P27.60 (a) Lightbulb A = 576 Ω and Lightbulb B = 144 Ω; (b) 4.80 s; (c) The charge is the same. It is at a location that is lower in potential;
(d) 0.040 0 s; (e) The energy is the same. Energy enters the lightbulb by electric transmission and leaves by heat and electromagnetic radiation;
(f) \$1.98

P27.62 (a) See the table in P27.62(a); (b) $9.93\times10^{-7}\ \Omega\cdot\text{m}$; (c) The average value is within 1% of the tabulated value of $1.00\times10^{-6}\ \Omega\cdot\text{m}$ given in Table 27.2.

P27.64 (a) Any diameter d and length ℓ related by $d^2 = \left(4.77\times10^{-8}\right)\ell$, where d and ℓ are in meters; (b) Yes; for $V = 0.500\ \text{cm}^3$ of Nichrome, $\ell = 3.65$ m and $d = 0.418$ mm.

P27.66 (a) 667 A; (b) 50.0 km

P27.68 (a) V/L in the positive x direction; (b) $4\rho L/\pi d^2$; (c) $V\pi d^2/4\rho L$;

(d) $V/\rho L$ in the positive x direction; (e) $\rho J = \rho\left(\dfrac{V}{\rho L}\right) = \dfrac{V}{L} = E$

P27.70 See P27.70(a) for the full explanation; (b) The result is exact if the assumptions are precisely true. Our derivation contains no approximation steps where delta is assumed to be small.

P27.72 The value of 11.4 A is what results from substituting the given voltage and resistance into Equation 27.7. However, the resistance measured for a lightbulb with an ohmmeter is not the resistance at which it operates because of the change in resistivity with temperature. The higher resistance of the filament at the operating temperature brings the current down significantly.

P27.74 (a) $\sigma A\left|\dfrac{dV}{dx}\right|$; (b) Current flows in the direction of decreasing voltage.

Energy flows by heat in the direction of decreasing temperature.

P27.76 $R = R_0 \dfrac{1+\alpha\left(T-T_0\right)}{1+\alpha'\left(T-T_0\right)}$

P27.78 (a) See P27.78 for full explanation; (b) $1.79 \times 10^{15}\ \Omega$

P27.80 (a) See Table P27.80 (i), (ii), and (iii); (b) See ANS. FIG. P27.80 (i), (ii), and (iii).

P27.82 (a) $\rho = \rho_0 e^{\alpha\left(T-T_0\right)}$; (b) $\rho \approx \rho_0\left[1+\alpha\left(T-T_0\right)\right]$

P27.84 $\dfrac{\rho L}{w\left(y_2-y_1\right)}\ln\left(\dfrac{y_2}{y_1}\right)$

28

Direct-Current Circuits

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ28.1 Answer (a). When the breaker trips to off, current does not go through the device.

OQ28.2 (**i**) Answer (d). The terminal potential difference is $\Delta V = \mathcal{E} - Ir$, where current *I* within the battery is considered positive when it flows from the negative to the positive terminal. When *I* = 0, $\Delta V = \mathcal{E}$

(**ii**) Answer (b). When the battery is absorbing electrical energy, the current within the battery flows from the positive to the negative terminal; in this case, *I* is considered negative, making $\Delta V = \mathcal{E} - Ir = \mathcal{E} + |I|r > \mathcal{E}$.

OQ28.3 Answer (c). In a series connection, the same current exists in each element. The potential difference across a resistor in this series connection is directly proportional to the resistance of that resistor, $\Delta V = IR$, and independent of its location within the series connection.

OQ28.4 Answer (b). because the appliances are connected in parallel, the total power used is proportion to the total current:

$$\sum P_i = \sum I_i \Delta V = \Delta V \sum I_i \to \sum I_i = \frac{\sum P_i}{\Delta V}$$

or

$$\sum I_i = \frac{P_{\text{heater}} + P_{\text{toaster}} + P_{\text{oven}}}{\Delta V}$$

$$= \frac{\left(1.30\times10^3 + 1.00\times10^3 + 1.54\times10^3\right)\text{ W}}{120\text{ V}} = \boxed{32.0\text{ A}}$$

OQ28.5 Answer (b). When the two identical resistors are in series, the current supplied by the battery is $I = \Delta V/2R$, and the total power delivered is $P_s = (\Delta V)I = (\Delta V)^2/2R$. With the resistors connected in parallel, the potential difference across each resistor is ΔV and the power delivered to each resistor is $P_1 = (\Delta V)^2/R$. Thus, the total power delivered in this case is

$$P_p = 2P_1 = 2\frac{(\Delta V)^2}{R} = 4\left[\frac{(\Delta V)^2}{2R}\right] = 4P_s = 4(8.0\text{ W}) = 32\text{ W}$$

OQ28.6 Answer (a), (d). According to the relationship for resistors in series,

$$R_{eq} = R_1 + R_2 + \cdots$$

the sum R_{eq} is always larger than any of the resistances R_1, R_2, etc.

OQ28.7 Answer (d). The equivalent resistance for the series combination of five identical resistors is $R_{eq} = 5R$, and the equivalent capacitance of five identical capacitors in parallel is $C_{eq} = 5C$. The time constant for the circuit is therefore $\tau = R_{eq}C_{eq} = (5R)(5C) = 25RC$.

OQ28.8 Answers (b) and (d). The current is the same in each series resistor, as described by Kirchhoff's junction rule. The potential difference in each resistor is different because $\Delta V = IR$ and each R is different.

OQ28.9 Answer (a). The potential is the same across each parallel resistor, but the current and power in each resistor is different because $I = \Delta V/R$ and $P = I\Delta V$ and each R is different.

OQ28.10 Answer (b) and (c). The same potential difference exists across all elements connected in parallel with each other, while the current through each element is inversely proportional to the resistance of that element $(I = \Delta V/R)$.

OQ28.11 Answer (b). Each headlight's terminals are connected to the positive and negative terminals of the battery so that each headlight can operate if the other is burned out.

OQ28.12 **(i)** The ranking of potentials are: $a > d > b = c > e$. For both batteries to be delivering electric energy, currents are in the direction a to b, and d to c, and so current flows downward through e. Point e is at zero potential. Points b and c are at the same higher potential, d (equal to 9 V) is still higher, and a (equal to 12 V) is highest of all.

(ii) The ranking of magnitudes of current are: $e > a = b > c = d$. The current through e must be the sum of the other two currents. The change in potential from a to b is greater than the change in potential from d to c, so the current from a to b must be greater.

OQ28.13 Answer (b). According to the relationship for resistors in parallel,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

the larger the sum on the right-hand side of the equation, $1/R_1 + 1/R_2 + \cdots$, the smaller the equivalent resistance R_{eq}; therefore, R_{eq} is always smaller than any of the resistances R_1, R_2, etc.

OQ28.14 Answers: **(i)** (b) **(ii)** (a) **(iii)** (a) **(iv)** (b) **(v)** (a) **(vi)** (a). Closing the switch lights lamp C. The action increases the battery current so it decreases the terminal voltage of the battery. Lamps A and B are in series, so they carry the same current, but when the terminal voltage of the battery drops, the total voltage drops across lamps A and B combined, thus reducing the potential difference across each. Total power delivered to the lamps increases because the current through the battery increases.

OQ28.15 Answers: (**i**) (a) (**ii**) (d) (**iii**) (a) (**iv**) (a) (**v**) d (**vi**) (a). Closing the switch removes lamp C from the circuit, decreasing the resistance seen by the battery, and so increasing the current in the battery. Lamps A and B are in series, so the potential difference across each is proportional to the current. Total power delivered to the lamps increases because the current through the battery increases.

ANSWERS TO CONCEPTUAL QUESTIONS

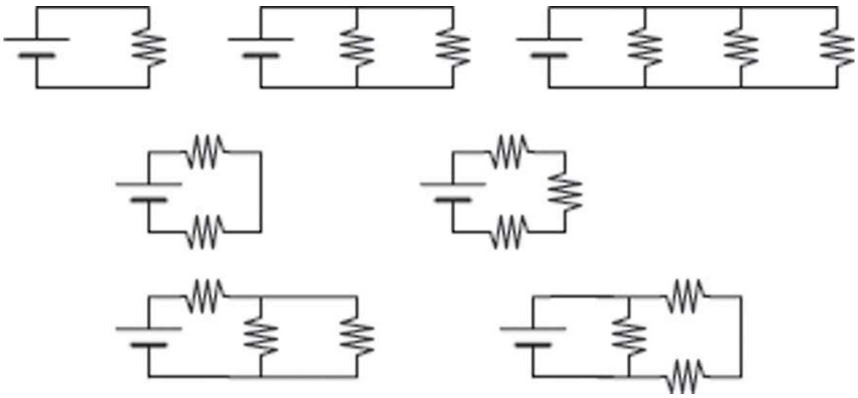
CQ28.1 (a) No. As is the case with the bird in CQ28.3, the resistance of a small length of wire is small, so the potential change along that length is small.

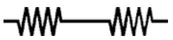
(b) No! When she eventually touches the ground, she will act as a connection to ground, resulting in perhaps several thousand volts across her.

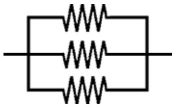
CQ28.2 Answer their question with a challenge. If the student is just looking at a diagram, provide the materials to build the circuit. If you are looking at a circuit where the second bulb really is fainter, get the student to unscrew them both and interchange them. But check that the student's understanding of potential has not been impaired: if you add wires to bypass and short out the first bulb, the second gets brighter.

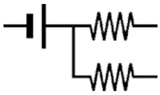
CQ28.3 Because the resistance of a short length of wire is small, the change in potential along that length is small; therefore, there is essentially zero *difference* in potential between the bird's feet. Then negligible current goes through the bird. The resistance through the bird's body between its feet is much larger than the resistance through the wire between the same two points.

CQ28.4



CQ28.5 Two runs in series:  = one run down one slope followed by a second run down a second slope.

Three runs in parallel:  = parallel runs down the same hill so that the change in elevation is the same for each.

Junction of one lift and two runs: .

Gustav Robert Kirchhoff, Professor of Physics at Heidelberg and Berlin, was master of the obvious. A junction rule: The number of skiers coming into any junction must be equal to the number of skiers leaving. A loop rule: the total change in altitude must be zero for any skier completing a closed path.

CQ28.6 The bulb will light up for a while immediately after the switch is closed. As the capacitor charges, the bulb gets progressively dimmer. When the capacitor is fully charged the current in the circuit is zero and the bulb does not glow at all. If the value of *RC* is small, this whole process might occupy a very short time interval.

CQ28.7 (a) The hospital maintenance worker is right. A hospital room is full of electrical grounds, including the bed frame. If your grandmother touched the faulty knob and the bed frame at the same time, she could receive quite a jolt, as there would be a potential difference of 120 V across her. If the 120 V is DC, the shock could send her into ventricular fibrillation, and the hospital staff could use the defibrillator you read about in Chapter 26. If the 120 V is AC, which is most likely, the current could produce external and internal burns along the path of conduction.

(b) Likely no one got a shock from the radio back at home because her bedroom contained no electrical grounds—no conductors connected to zero volts. Just like the bird in CQ28.3, granny could touch the "hot" knob without getting a shock so long as there was no path to ground to supply a potential difference across her. A new appliance in the bedroom or a flood could make the radio lethal. Repair it or discard it. Enjoy the news from Lake Wobegon on the new plastic radio.

CQ28.8 (a) Both 120-V and 240-V lines can deliver injurious or lethal shocks, but there is a somewhat better safety factor with the lower voltage. To say it a different way, the insulation on a 120-V line can be thinner.

(b) On the other hand, a 240-V device carries less current to operate a device with the same power, so the conductor itself can be thinner. Finally, the last step-down transformer can also be somewhat smaller if it has to go down only to 240 volts from the high voltage of the main power line.

CQ28.9 No. If there is one battery in a circuit, the current inside it will be from its negative terminal to its positive terminal. Whenever a battery is delivering energy to a circuit, it will carry current in this direction. On the other hand, when another source of emf is charging the battery in question, it will have a current pushed through it from its positive terminal to its negative terminal.

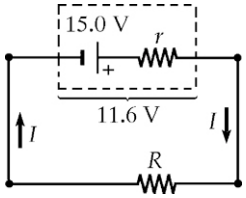
CQ28.10 In Figure 20.13, temperature is similar to electric potential, and temperature difference $\Delta T = T_h - T_c$ is similar to voltage ΔV. Energy transfer is similar to electric current. The upper picture is similar to a series circuit, where the resistors (rods) carry the same current (energy transfer by conduction), and the sum of the voltages (temperature differences) across the rods equals the total voltage (total temperature difference) across both resistors (rods). The lower picture is similar to a parallel circuit, where the resistors (rods) have the same voltage (temperature difference) but carry different currents (energy transfer by conduction).

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 28.1 Electromotive Force

P28.1 (a) Combining Joule's law, $P = I\Delta V$, and the definition of resistance, $\Delta V = IR$, gives

$$R = \frac{(\Delta V)^2}{P} = \frac{(11.6\ \text{V})^2}{20.0\ \text{W}} = \boxed{6.73\ \Omega}$$

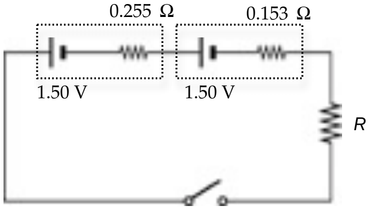


ANS. FIG. P28.1

(b) The electromotive force of the battery must equal the voltage drops across the resistances: $\mathcal{E} = IR + Ir$, where $I = \Delta V/R$.

$$r = \frac{(\mathcal{E} - IR)}{I} = \frac{(\mathcal{E} - \Delta V)R}{\Delta V}$$
$$= \frac{(15.0\ \text{V} - 11.6\ \text{V})(6.73\ \Omega)}{11.6\ \text{V}} = \boxed{1.97\ \Omega}$$

P28.2 The total resistance is $R = \dfrac{3.00\ \text{V}}{0.600\ \text{A}} = 5.00\ \Omega.$



ANS. FIG. P28.2

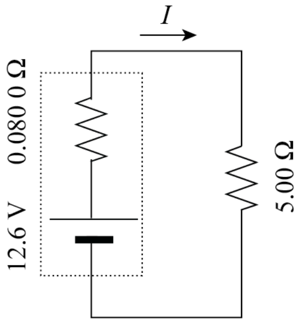
(a) $R_{\text{lamp}} = R - r_{\text{batteries}} = 5.00\ \Omega - 0.408\ \Omega = \boxed{4.59\ \Omega}$

(b) $\dfrac{P_{\text{batteries}}}{P_{\text{total}}} = \dfrac{(0.408\ \Omega)I^2}{(5.00\ \Omega)I^2} = 0.081\ 6 = \boxed{8.16\%}$

P28.3 (a) Here $\mathcal{E} = I(R + r)$,

so

$$I = \frac{\mathcal{E}}{R + r} = \frac{12.6\ \text{V}}{(5.00\ \Omega + 0.080\ 0\ \Omega)}$$
$$= 2.48\ \text{A}.$$



ANS. FIG. P28.3(a)

Then,

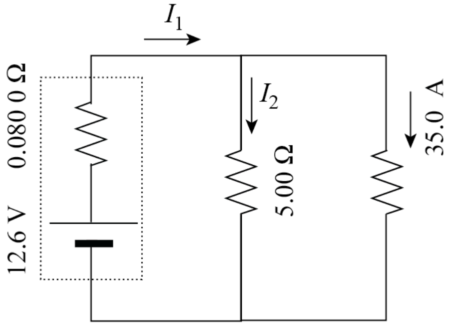
$$\Delta V = IR = (2.48\ \text{A})(5.00\ \Omega)$$
$$= \boxed{12.4\ \text{V}}$$

(b) Let I_1 and I_2 be the currents flowing through the battery and the headlights, respectively.

Then,

$$I_1 = I_2 + 35.0\text{ A}$$

and $\mathcal{E} - I_1R - I_2R = 0$



ANS. FIG. P28.3(b)

so

$$\mathcal{E} = (I_2 + 35.0\text{ A})(0.080\,0\ \Omega) + I_2(5.00\ \Omega) = 12.6\text{ V}$$

giving $I_2 = 1.93$ A.

Thus,

$$\Delta V_2 = (1.93\text{ A})(5.00\ \Omega) = \boxed{9.65\text{ V}}$$

P28.4 (a) At maximum power transfer, $r = R$. Equal powers are delivered to r and R. The efficiency is $\boxed{50.0\%}$.

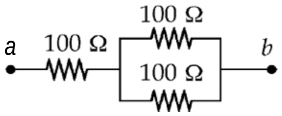
(b) For maximum fractional energy transfer to R, we want zero energy absorbed by r, so we want $r = \boxed{0}$.

(c) $\boxed{\text{High efficiency.}}$ The electric company's economic interest is to minimize internal energy production in its power lines, so that it can sell a large fraction of the energy output of its generators to the customers.

(d) $\boxed{\text{High power transfer.}}$ Energy by electric transmission is so cheap compared to the sound system that she does not spend extra money to buy an efficient amplifier.

Section 28.2 Resistors in Series and Parallel

P28.5 (a) Since all the current in the circuit must pass through the series 100-Ω resistor,



ANS. FIG. P28.5

$$P_{\max} = I_{\max}^2 R$$

so $$I_{\max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{25.0\text{ W}}{100\ \Omega}} = 0.500\text{ A.}$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$$

$$\Delta V_{\max} = R_{eq} I_{\max} = \boxed{75.0\text{ V}}$$

(b) From *a* to *b* in the circuit, the power delivered is

$P_{\text{series}} = \boxed{25.0\text{ W}}$ for the first resistor, and

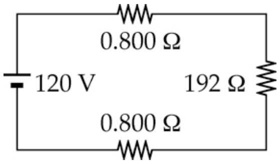
$$P_{\text{parallel}} = I^2R = (0.250\text{ A})^2(100\ \Omega) = \boxed{6.25\text{ W}}$$

for each of the two parallel resistors.

(c) $P = I\Delta V = (0.500\text{ A})(75.0\text{ V}) = \boxed{37.5\text{ W}}$

P28.6 (a) The 120-V potential difference is applied across the series combination of the two conductors in the extension cord and the lightbulb. The potential difference across the lightbulb is less than 120 V, and its power is less than 75 W.

(b) See the circuit diagram in ANS. FIG. P28.6; the 192-Ω resistor is the lightbulb (see below).



ANS. FIG. P28.6

(c) First, find the operating resistance of the lightbulb:

$$P = \frac{(\Delta V)^2}{R}$$

or $$R = \frac{(\Delta V)^2}{P} = \frac{(120\text{ V})^2}{75.0\text{ W}} = 192\ \Omega$$

From the circuit, the total resistance is 193.6 Ω. The current is

$$I = \frac{120\text{ V}}{193.6\ \Omega} = 0.620\text{ A}$$

so the power delivered to the lightbulb is

$$P = I^2\Delta R = (0.620\text{ A})^2(192\ \Omega) = \boxed{73.8\text{ W}}$$

P28.7 The equivalent resistance of the parallel combination of three identical resistors is

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{3}{R} \quad \text{or} \quad R_p = \frac{R}{3}$$

The total resistance of the series combination between points *a* and *b* is then

$$R_{ab} = R + R_p + R = 2R + \frac{R}{3} = \boxed{\frac{7}{3}R}$$

P28.8 (a) By Ohm’s law, the current in *A* is $\boxed{I_A = \mathcal{E}/R}$. The equivalent resistance of the series combination of bulbs *B* and *C* is 2*R*. Thus, the current in each of these bulbs is $\boxed{I_B = I_C = \mathcal{E}/2R}$.

(b) $\boxed{\text{B and C have the same brightness because they carry the same current.}}$

(c) $\boxed{\text{A is brighter than B or C because it carries twice as much current.}}$

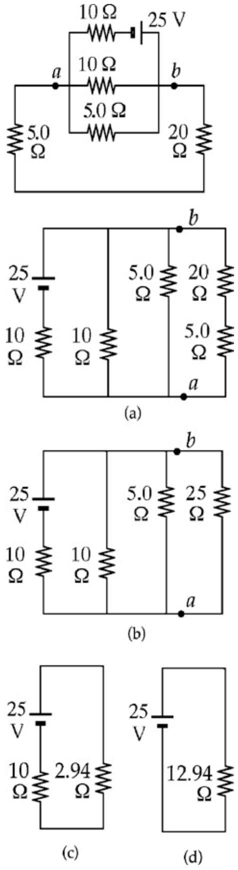
P28.9 If we turn the given diagram on its side and change the lengths of the wires, we find that it is the same as ANS. FIG. P28.9(a). The 20.0-Ω and 5.00-Ω resistors are in series, so the first reduction is shown in ANS. FIG. P28.9(b). In addition, since the 10.0-Ω, 5.00-Ω, and 25.0-Ω resistors are then in parallel, we can solve for their equivalent resistance as:

$$\frac{1}{R_{eq}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} + \frac{1}{25.0\ \Omega} \rightarrow R_{eq} = 2.94\ \Omega$$

This is shown in ANS. FIG. P28.9(c), which in turn reduces to the circuit shown in ANS. FIG. P28.9(d), from which we see that the total resistance of the circuit is 12.94 Ω.

Next, we work backwards through the diagrams applying $I = \frac{\Delta V}{R}$ and $\Delta V = IR$ alternately to every resistor, real and equivalent. The total 12.94-Ω resistor is connected across 25.0 V, so the current through the battery in every diagram is

$$I = \frac{\Delta V}{R} = \frac{25.0\ \text{V}}{12.94\ \Omega} = 1.93\ \text{A}$$



ANS. FIG. P28.9

In ANS. FIG. P28.9(c), this 1.93 A goes through the 2.94-Ω equivalent resistor to give a potential difference of:

$$\Delta V = IR = (1.93\ \text{A})(2.94\ \Omega) = 5.68\ \text{V}$$

From ANS. FIG. P28.9(b), we see that this potential difference is the same as the potential difference ΔV_{ab} across the 10-Ω resistor and the 5.00-Ω resistor.

Thus we have first found the answer to part (b), which is

$$\Delta V_{ab} = \boxed{5.68\ \text{V}}$$

Since the current through the 20.0-Ω resistor is also the current through the 25.0-Ω line *ab*,

$$I = \frac{\Delta V_{ab}}{R_{ab}} = \frac{5.68\ \text{V}}{25.0\ \Omega} = 0.227\ \text{A} = \boxed{227\ \text{mA}}$$

P28.10 (a) Connect two 50-Ω resistors in parallel to get 25 Ω. Then connect that parallel combination in series with a 20-Ω for a total resistance of 45 Ω.

(b) Connect two 50-Ω resistors in parallel to get 25 Ω. Also, connect two 20-Ω resistors in parallel to get 10 Ω. Then, connect these two combinations in series with each other to obtain 35 Ω.

P28.11 <u>When *S* is open</u>, R_1, R_2, and R_3 are in series with the battery. Thus,

$$R_1 + R_2 + R_3 = \frac{6\ \text{V}}{10^{-3}\ \text{A}} = 6\ \text{k}\Omega \qquad \textbf{[1]}$$

<u>When *S* is closed in position *a*</u>, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. Thus,

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{6\ \text{V}}{1.2\times 10^{-3}\ \text{A}} = 5\ \text{k}\Omega \qquad \textbf{[2]}$$

<u>When *S* is closed in position *b*</u>, R_1 and R_2 are in series with the battery and R_3 is shorted. Thus,

$$R_1 + R_2 = \frac{6\ \text{V}}{2\times 10^{-3}\ \text{A}} = 3\ \text{k}\Omega \qquad \textbf{[3]}$$

Subtracting [3] from [1] gives $R_3 = 3\ \text{k}\Omega$.

Subtracting [2] from [1] gives $R_2 = 2\ \text{k}\Omega$.

Then, from [3], $R_1 = 1\ \text{k}\Omega$.

Answers: (a) $\boxed{R_1 = 1.00\ \text{k}\Omega}$ (b) $\boxed{R_2 = 2.00\ \text{k}\Omega}$ (c) $\boxed{R_3 = 3.00\ \text{k}\Omega}$

P28.12 <u>When *S* is open</u>, R_1, R_2, and R_3 are in series with the battery. Thus,

$$R_1 + R_2 + R_3 = \frac{\varepsilon}{I_0} \qquad \textbf{[1]}$$

When S is closed in position *a*, the parallel combination of the two R_2's is in series with R_1, R_3, and the battery. Thus,

$$R_1 + \frac{1}{2}R_2 + R_3 = \frac{\mathcal{E}}{I_a} \qquad \textbf{[2]}$$

When *S* is closed in position *b*, R_1 and R_2 are in series with the battery. R_3 is shorted. Thus:

$$R_1 + R_2 = \frac{\mathcal{E}}{I_b} \qquad \textbf{[3]}$$

Subtracting [3] from [1] gives

$$(R_1 + R_2 + R_3) - (R_1 + R_2) = \frac{\mathcal{E}}{I_0} - \frac{\mathcal{E}}{I_b}$$

$$R_3 = \mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_b}\right)$$

Subtracting [2] from [1] gives

$$(R_1 + R_2 + R_3) - \left(R_1 + \frac{1}{2}R_2 + R_3\right) = \frac{\mathcal{E}}{I_0} - \frac{\mathcal{E}}{I_a}$$

$$\frac{1}{2}R_2 = \mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right)$$

$$R_2 = 2\mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right)$$

Then, from [3],

$$R_1 + R_2 = \frac{\mathcal{E}}{I_b}$$

$$R_1 = \frac{\mathcal{E}}{I_b} - R_2$$

$$R_1 = \frac{\mathcal{E}}{I_b} - 2\mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right)$$

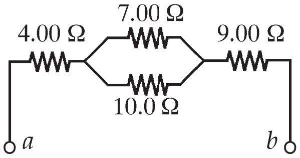
$$R_1 = \mathcal{E}\left(-\frac{2}{I_0} + \frac{2}{I_a} + \frac{1}{I_b}\right)$$

Answers: (a) $\boxed{R_1 = \mathcal{E}\left(-\frac{2}{I_0} + \frac{2}{I_a} + \frac{1}{I_b}\right)}$ (b) $\boxed{R_2 = 2\mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right)}$

(c) $\boxed{R_3 = \mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_b}\right)}$

***P28.13** (a) The equivalent resistance of the two parallel resistors is

$$R_p = \frac{1}{(1/7.00\ \Omega) + (1/10.0\ \Omega)} = 4.12\ \Omega$$



ANS. FIG. P28.13

Thus,

$$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00$$
$$= \boxed{17.1\ \Omega}$$

(b) $\Delta V = IR$

$$34.0\ \text{V} = I(17.1\ \Omega)$$

$$I = \boxed{1.99\ \text{A}} \text{ for the } 4.00\text{-}\Omega \text{ and } 9.00\text{-}\Omega \text{ resistors.}$$

Applying $\Delta V = IR$,

$$(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$$

$$8.18\ \text{V} = I(7.00\ \Omega)$$

so $I = \boxed{1.17\ \text{A}}$ for the $7.00\text{-}\Omega$ resistor. Finally,

$$8.18\ \text{V} = I(10.0\ \Omega)$$

so $I = \boxed{0.818\ \text{A}}$ for the $10.0\text{-}\Omega$ resistor.

P28.14 (a) The resistance between *a* and *b* $\boxed{\text{decreases.}}$ The original resistance is

$$R + \frac{1}{\frac{1}{90+10} + \frac{1}{10+90}} = R + 50\ \Omega$$

Closing the switch changes the resistance to

$$R + \frac{1}{\frac{1}{90} + \frac{1}{10}} + \frac{1}{\frac{1}{10} + \frac{1}{90}} = R + 18\ \Omega$$

(b) We require $R + 18\ \Omega = 0.50(R + 50\ \Omega)$, so $R = \boxed{14.0\ \Omega}$.

P28.15 Denoting the two resistors as x and y, and suppressing units,

$$x + y = 690, \text{ and } \frac{1}{150} = \frac{1}{x} + \frac{1}{y}$$

$$\frac{1}{150} = \frac{1}{x} + \frac{1}{690 - x} = \frac{(690 - x) + x}{x(690 - x)}$$

$$x^2 - 690x + 103\,500 = 0$$

$$x = \frac{690 \pm \sqrt{(690)^2 - 414\,000}}{2}$$

$$x = \boxed{470\ \Omega} \quad y = \boxed{220\ \Omega}$$

P28.16 (a) The resistors 2, 3, and 4 can be combined to a single 2*R* resistor. This is in series with resistor 1, with resistance *R*, so the equivalent resistance of the whole circuit is 3*R*. In series, potential difference is shared in proportion to the resistance, so resistor 1 gets $\frac{1}{3}$ of the battery voltage and the 2-3-4 parallel combination gets $\frac{2}{3}$ of the battery voltage. This is the potential difference across resistor 4, but resistors 2 and 3 must share this voltage. $\frac{1}{3}$ goes to 2 and $\frac{2}{3}$ to 3. The ranking by potential difference is

$$\Delta V_4 > \Delta V_3 > \Delta V_1 > \Delta V_2$$

Based on the reasoning above the potential differences are

$$\boxed{\Delta V_1 = \frac{\mathcal{E}}{3},\ \Delta V_2 = \frac{2\mathcal{E}}{9},\ \Delta V_3 = \frac{4\mathcal{E}}{9},\ \Delta V_4 = \frac{2\mathcal{E}}{3}}$$

(b) All the current goes through resistor 1, so it gets the most. The current then splits at the parallel combination. Resistor 4 gets more than half, because the resistance in that branch is less than in the other branch. Resistors 2 and 3 have equal currents because they are in series. The ranking by current is

$$I_1 > I_4 > I_2 = I_3$$

Resistor 1 has a current of *I*. Because the resistance of 2 and 3 in series is twice that of resistor 4, twice as much current goes through 4 as through 2 and 3. The current through the resistors are

$$\boxed{I_1 = I,\ I_2 = I_3 = \frac{I}{3},\ I_4 = \frac{2I}{3}}$$

(c) Increasing resistor 3 increases the equivalent resistance of the entire circuit. The current in the circuit, which is the current through resistor 1, decreases. This decreases the potential difference across resistor 1, increasing the potential difference across the parallel combination. With a larger potential difference the current through resistor 4 is increased. With more current through 4, and less in the circuit to start with, the current through resistors 2 and 3 must decrease. To summarize,

$$\boxed{I_4 \text{ increases and } I_1,\ I_2, \text{ and } I_3 \text{ decrease}}$$

(d) If resistor 3 has an infinite resistance it blocks any current from passing through that branch, and the circuit effectively is just resistor 1 and resistor 4 in series with the battery. The circuit now has an equivalent resistance of 4*R*. The current in the circuit drops to $\frac{3}{4}$ of the original current because the resistance has increased by $\frac{4}{3}$. All this current passes through resistors 1 and 4, and none passes through 2 or 3. Therefore,

$$\boxed{I_1 = \frac{3I}{4},\ I_2 = I_3 = 0,\ I_4 = \frac{3I}{4}}$$

P28.17 (a) The parallel combination of the 6.0 Ω and 12 Ω resistors has an equivalent resistance of

$$\frac{1}{R_{p1}} = \frac{1}{6.0\ \Omega} + \frac{1}{12\ \Omega} = \frac{2+1}{12\ \Omega}$$

or $R_{p1} = \frac{12\ \Omega}{3} = 4.0\ \Omega$

Similarly, the equivalent resistance of the 4.0 Ω and 8.0 Ω parallel combination is

$$\frac{1}{R_{p2}} = \frac{1}{4.0\ \Omega} + \frac{1}{8.0\ \Omega} = \frac{2+1}{8.0\ \Omega}$$

or $R_{p2} = \frac{8\ \Omega}{3}$

The total resistance of the series combination between points *a* and *b* is then

$$R_{ab} = R_{p1} + 5.0\ \Omega + R_{p2} = 4.0\ \Omega + 5.0\ \Omega + \frac{8.0}{3}\ \Omega$$

$$= \frac{35}{3}\ \Omega = \boxed{11.7\ \Omega}$$

(b) If $\Delta V_{ab} = 35\text{ V}$, the total current from *a* to *b* is $I_{ab} = \Delta V_{ab}/R_{ab} = 35\text{ V}/(35\ \Omega/3) = 3.0\text{ A}$ and the potential differences across the two parallel combinations are

$$\Delta V_{p1} = I_{ab}R_{p1} = (3.0\text{ A})(4.0\ \Omega) = 12\text{ V}$$

and $$\Delta V_{p2} = I_{ab}R_{p2} = (3.0\text{ A})\left(\frac{8.0}{3}\ \Omega\right) = 8.0\text{ V}$$

so the individual currents through the various resistors are:

$$I_{12} = \Delta V_{p1}/12\ \Omega = \boxed{1.0\text{ A}}$$

$$I_6 = \Delta V_{p1}/6.0\ \Omega = \boxed{2.0\text{ A}}$$

$$I_5 = I_{ab} = \boxed{3.0\text{ A}}$$

$$I_8 = \Delta V_{p2}/8.0\ \Omega = \boxed{1.0\text{ A}}$$

and $$I_4 = \Delta V_{p2}/4.0\ \Omega = \boxed{2.0\text{ A}}$$

P28.18 We assume that the metal wand makes low-resistance contact with the person's hand and that the resistance through the person's body is negligible compared to the resistance R_{shoes} of the shoe soles. The equivalent resistance seen by the power supply is $1.00\text{ M}\Omega + R_{\text{shoes}}$. The current through both resistors is $\dfrac{50.0\text{ V}}{1.00\text{ M}\Omega + R_{\text{shoes}}}$. The voltmeter displays

$$\Delta V = I(1.00\text{ M}\Omega)$$

$$\Delta V = \frac{50.0\text{ V}}{1.00\text{ M}\Omega + R_{\text{shoes}}} = 1.00\text{ M}\Omega$$

(a) We solve to obtain

$$50.0\text{ V}(1.00\text{ M}\Omega) = \Delta V(1.00\text{ M}\Omega) + \Delta V(R_{\text{shoes}})$$

$$R_{\text{shoes}} = \frac{(1.00\text{ M}\Omega)(50.0 - \Delta V)}{\Delta V}$$

or

$$R_{\text{shoes}} = \frac{50.0 - \Delta V}{\Delta V}$$

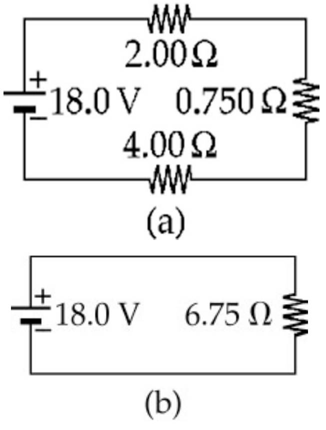
where resistance is measured in MΩ.

(b) With $R_{\text{shoes}} \to 0$, the current through the person's body is

$$\frac{50.0\ \text{V}}{1.00\ \text{M}\Omega} = 50.0\ \mu\text{A} \quad \boxed{\text{The current will never exceed 50 }\mu\text{A.}}$$

P28.19 To find the current in each resistor, we find the resistance seen by the battery. The given circuit reduces as shown in ANS. FIG. P28.19(a), since

$$\frac{1}{(1/1.00\ \Omega) + (1/3.00\ \Omega\ )} = 0.750\ \Omega$$



ANS. FIG. P28.19

In ANS. FIG. P28.19(b),

$$I = 18.0\ \text{V}/6.75\ \Omega = 2.67\ \text{A}$$

This is also the current in ANS. FIG. P28.19(a), so the 2.00-Ω and 4.00-Ω resistors convert powers

$$P_2 = I\Delta V = I^2R = (2.67\ \text{A})^2(2.00\ \Omega) = \boxed{14.2\ \text{W}}$$

and $P_4 = I^2R = (2.67\ \text{A})^2(4.00\ \Omega) = \boxed{28.4\ \text{W}}$

The voltage across the 0.750-Ω resistor in ANS. FIG. P28.19(a), and across both the 3.00-Ω and the 1.00-Ω resistors in Figure P28.19, is

$$\Delta V = IR = (2.67\ \text{A})(0.750\ \Omega) = \boxed{2.00\ \text{V}}$$

Then for the 3.00-Ω resistor,

$$I = \frac{\Delta V}{R} = \frac{2.00\ \text{V}}{3.00\ \Omega}$$

and the power is

$$P_3 = I\Delta V = \left(\frac{2.00\ \text{V}}{3.00\ \Omega}\right)(2.00\ \text{V}) = \boxed{1.33\ \text{W}}$$

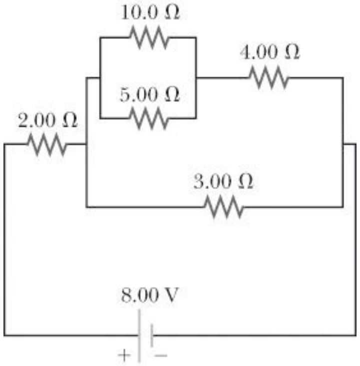
For the 1.00-Ω resistor,

$$I = \frac{2.00\ \text{V}}{1.00\ \Omega} \quad \text{and} \quad P_1 = \left(\frac{2.00\ \text{V}}{1.00\ \Omega}\right)(2.00\ \text{V}) = \boxed{4.00\ \text{W}}$$

P28.20 The resistance of the combination of extra resistors must be $\frac{7}{3}R - R = \frac{4}{3}R$. The possible combinations are: one resistor: R; two resistors: $2R$, $\frac{1}{2}R$; three resistors: $3R$, $\frac{1}{3}R$, $\frac{2}{3}R$, $\frac{3}{2}R$. None of these is $\frac{4}{3}R$, so the desired resistance cannot be achieved.

P28.21 (a) The equivalent resistance of this first parallel combination is

$$\frac{1}{R_{p1}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} \quad \text{or} \quad R_{p1} = 3.33\ \Omega$$



ANS. FIG. P28.21

For this series combination,

$$R_{\text{upper}} = R_{p1} + 4.00\ \Omega = 7.33\ \Omega$$

For the second parallel combination,

$$\frac{1}{R_{p2}} = \frac{1}{R_{\text{upper}}} + \frac{1}{3.00\ \Omega} = \frac{1}{7.33\ \Omega} + \frac{1}{3.00\ \Omega} \quad \text{or} \quad R_{p2} = 2.13\ \Omega$$

For the second series combination (and hence the entire resistor network)

$$R_{\text{total}} = 2.00\ \Omega + R_{p2} = 2.00\ \Omega + 2.13\ \Omega = 4.13\ \Omega$$

The total current supplied by the battery is

$$I_{\text{total}} = \frac{\Delta V}{R_{\text{total}}} = \frac{8.00\ \text{V}}{4.13\ \Omega} = 1.94\ \text{A}$$

The potential drop across the 2.00 Ω resistor is

$$\Delta V_2 = R_2 I_{\text{total}} = (2.00\ \Omega)(1.94\ \text{A}) = 3.88\ \text{V}$$

The potential drop across the second parallel combination must be

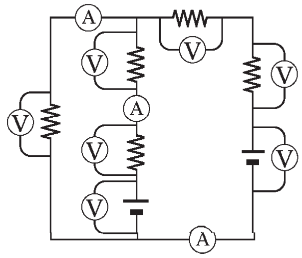
$$\Delta V_{p2} = \Delta V - \Delta V_2 = 8.00\ \text{V} - 3.88\ \text{V} = \boxed{4.12\ \text{V}}$$

(b) So the current through the 3.00 Ω resistor is

$$I_{\text{total}} = \frac{\Delta V_{p2}}{R_3} = \frac{4.12\ \text{V}}{3.00\ \Omega} = \boxed{1.38\ \text{A}}$$

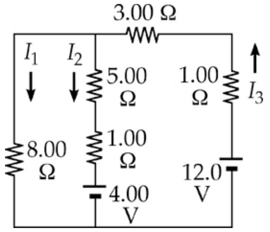
Section 28.3 Kirchhoff ′s Rules

***P28.22** We need one voltmeter across each resistor and each battery. These are shown with Ⓥ in ANS. FIG. P28.22. From Kirchhoff's junction rule, we need one ammeter in each segment of the circuit. Ammeters are shown with Ⓐ in ANS. FIG. P28.22. ANS. FIG. P28.22 is the complete answer to this problem.



ANS. FIG. P28.22

P28.23 We name currents I_1, I_2, and I_3 as shown in ANS. FIG. P28.23. From Kirchhoff's current rule, $I_3 = I_1 + I_2$.



ANS. FIG. P28.23

Applying Kirchhoff's voltage rule to the loop containing I_2 and I_3,

$$12.0\text{ V} - (4.00\ \Omega)I_3 - (6.00\ \Omega)I_2 - 4.00\text{ V} = 0$$

$$8.00 = (4.00)I_3 + (6.00)I_2$$

Applying Kirchhoff's voltage rule to the loop containing I_1 and I_2,

$$-(6.00\ \Omega)I_2 - 4.00\text{ V} + (8.00\ \Omega)I_1 = 0$$

or $$(8.00\ \Omega)I_1 = 4.00 + (6.00\ \Omega)I_2$$

Solving the above linear system (by substituting $I_1 + I_2$ for I_3), we proceed to the pair of simultaneous equations:

$$\begin{cases} 8 = 4I_1 + 4I_2 + 6I_2 \\ 8I_1 = 4 + 6I_2 \end{cases} \quad \text{or} \quad \begin{cases} 8 = 4I_1 + 10I_2 \\ I_2 = \dfrac{4}{3}I_1 - \dfrac{2}{3} \end{cases}$$

and to the single equation

$$8 = 4I_1 + 10\left(\frac{4}{3}I_1 - \frac{2}{3}\right) = \frac{52}{3}I_1 - \frac{20}{3}$$

which gives

$$I_1 = \frac{3}{52}\left(8 + \frac{20}{3}\right) = 0.846\text{ A}$$

Then $$I_2 = I_2 = \frac{4}{3}(0.846) - \frac{2}{3} = 0.462$$

and $\quad I_3 = I_1 + I_2 = 1.31\text{ A}$

give $\quad \boxed{I_1 = 846\text{ mA},\ I_2 = 462\text{ mA},\ I_3 = 1.31\text{ A}}$

(a) The results are: $\boxed{\text{0.846 A down in the 8.00-}\Omega\text{ resistor; 0.462 A down in the middle branch; 1.31 A up in the right-hand branch}}$.

(b) For 4.00-V battery:

$$\Delta U = P\Delta t = (\Delta V)I\Delta t = (4.00\text{ V})(-0.462\text{ A})(120\text{ s}) = -222\text{ J}$$

For 12.0-V battery:

$$\Delta U = (12.0\text{ V})(1.31\text{ A})(120\text{ s}) = 1.88\text{ kJ}$$

The results are: $\boxed{\text{−222 J by the 4.00-V battery and 1.88 kJ by the 12.0-V battery}}$.

(c) To the 8.00-Ω resistor:

$$\Delta U = I^2R\Delta t = (0.846\text{ A})^2(8.00\ \Omega)(120\text{ s}) = \boxed{687\text{ J}}$$

To the 5.00-Ω resistor:

$$\Delta U = (0.462\text{ A})^2(5.00\ \Omega)(120\text{ s}) = \boxed{128\text{ J}}$$

To the 1.00-Ω resistor in the center branch:

$$(0.462\text{ A})^2(1.00\ \Omega)(120\text{ s}) = \boxed{25.6\text{ J}}$$

To the 3.00-Ω resistor:

$$(1.31\text{ A})^2(3.00\ \Omega)(120\text{ s}) = \boxed{616\text{ J}}$$

To the 1.00-Ω resistor in the right-hand branch:

$$(1.31\text{ A})^2(1.00\ \Omega)(120\text{ s}) = \boxed{205\text{ J}}$$

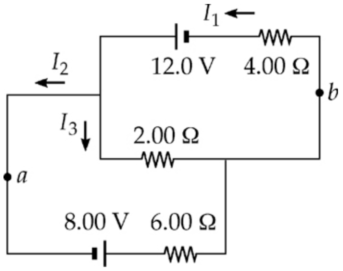
(d) $\boxed{\text{Chemical energy in the 12.0-V battery is transformed into internal energy in the resistors. The 4.00-V battery is being charged, so its chemical potential energy is increasing at the expense of some of the chemical potential energy in the 12.0-V battery.}}$

(e) Either sum the results in part (b): $-222\text{ J} + 1.88\text{ kJ} = 1.66\text{ kJ}$,

or in part (c): $687\text{ J} + 128\text{ J} + 25.6\text{ J} + 616\text{ J} + 205\text{ J} = 1.66\text{ kJ}$

The total amount of energy transformed is $\boxed{1.66\text{ kJ}}$.

P28.24 We name the currents I_1, I_2, and I_3 and arbitrarily choose current directions as labeled in ANS. FIG. P28.24.



ANS FIG. P28.24

(a) From the point rule for the junction below point *b*,

$$-I_1 + I_2 + I_3 = 0 \qquad \textbf{[1]}$$

Traversing the top loop counterclockwise gives the voltage loop equation

$$+12.0\text{ V} - (2.00\ )\,I_3 - (4.00\ )\,I_1 = 0 \qquad \textbf{[2]}$$

Traversing the bottom loop CCW,

$$+8.00\text{ V} - (6.00\ )\,I_2 + (2.00\ )I_3 = 0 \qquad \textbf{[3]}$$

Solving for I_1 from equation [2],

$$I_1 = \frac{12.0\text{ V} - (2.00\ \Omega)I_3}{4.00\ \Omega}$$

Solving for I_2 from equation [3],

$$I_2 = \frac{8.00\text{ V} + (2.00\ \Omega)I_3}{6.00\ \Omega}$$

Substituting both of these values into equation [1], we find

$$-(3.00\text{ V} - 0.500I_3) + 1.33\text{ V} + 0.333I_3 + I_3 = 0$$

so $\quad -1.67\text{ V} + 1.833I_3 = 0$

and the current in the 2.00-Ω resistor is $\boxed{I_3 = 909\text{ mA}}$

(b) Through the center wire,

$$V_a - (0.909\text{ A})(2.00\ \Omega) = V_b$$

Therefore,

$$V_b - V_a = \boxed{-1.82\text{ V}}, \text{ with } V_a > V_b$$

P28.25 (a) Let I_6 represent the current in the ammeter and the top 6-Ω resistor. The bottom 6-Ω resistor has the same potential difference across it, so it carries an equal current.

We assume both I_6 in the upper branch and I_6 in the lower branch flow to the right. We assume current I_{10} flows to the left through the 10-Ω resistor. For the top loop we have

$$6.00 - 10.0 I_{10} - 6.00 I_6 = 0 \quad\rightarrow\quad I_{10} = 0.6 - 0.6\, I_6 \qquad \textbf{[1]}$$

We assume current I_5 flows to the left through the 5-Ω resistor. For the bottom loop,

$$4.50 - 5.00 I_5 - 6.00 I_6 = 0 \quad\rightarrow\quad I_5 = 0.9 - 1.2\, I_6 \qquad \textbf{[2]}$$

For the junctions on the left side, taken together,

$$+I_{10} + I_5 - I_6 - I_6 = 0 \qquad \textbf{[3]}$$

Substituting I_{10} and I_5 into [3], we have

$$(0.6 - 0.6\, I_6) + (0.9 - 1.2\, I_6) - 2\, I_6 = 0 \rightarrow I_6 = 1.5/3.8 = \boxed{0.395\text{ A}}$$

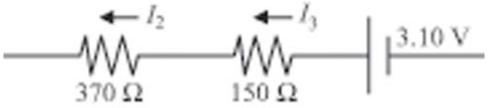
(b) The loop theorem for the little loop containing the voltmeter gives

$$+\,6.00\text{ V} - \Delta V - 4.50\text{ V} = 0 \quad\rightarrow\quad \Delta V = \boxed{1.50\text{ V}}$$

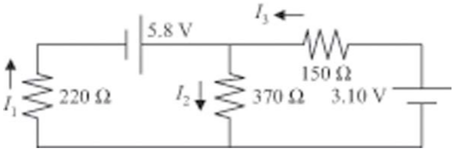
P28.26 (a) The first equation represents Kirchhoff's loop theorem. We choose to think of it as describing a clockwise trip around the left-hand loop in a circuit; see ANS. FIG. P28.26(a).

ANS. FIG. P28.26(a)

For the right-hand loop see ANS. FIG. P28.26(b). The junctions must be between the 5.80-V emf and the 370-Ω resistor and between the 370-Ω resistor and the 150-Ω resistor. Then we have ANS. FIG. P28.26(c). This is consistent with the third equation,



ANS. FIG. P28.26(b)



ANS. FIG. P28.26(c)

$$I_1 + I_3 - I_2 = 0$$
$$I_2 = I_1 + I_3$$

(b) Suppressing units, we substitute:

$$-220I_1 + 5.80 - 370I_1 - 370I_3 = 0$$
$$+370I_1 + 370I_3 + 150I_3 - 3.10 = 0$$

$$\text{Next, } I_3 = \frac{5.80 - 590I_1}{370}$$

$$370I_1 + \frac{520}{370}\left(5.80 - 590I_1\right) - 3.1 = 0$$
$$370I_1 + 8.15 - 829I_1 - 3.10 = 0$$

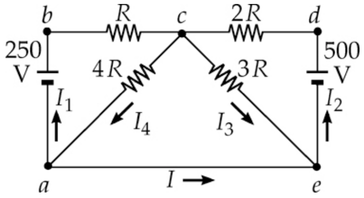
$$I_1 = \frac{5.05\text{ V}}{459\ \Omega} = \boxed{\begin{array}{l}\text{11.0 mA in the 220-}\Omega\text{ resistor and out of}\\ \text{the positive pole of the 5.80-V battery}\end{array}}$$

$$I_3 = \frac{5.80 - 590\left(0.011\,0\right)}{370} = -1.87\text{ mA}$$

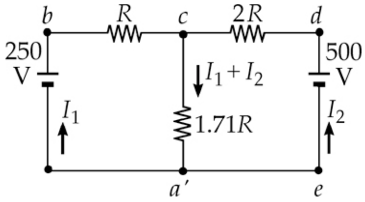
$$\boxed{\begin{array}{l}\text{The current is 1.87 mA in the 150-}\Omega\text{ resistor and out of the}\\ \text{negative pole of the 3.10-V battery.}\end{array}}$$

$$I_2 = 11.0 - 1.87 = \boxed{\text{9.13 mA in the 370-}\Omega\text{ resistor}}$$

P28.27 Label the currents in the branches as shown in ANS. FIG. P28.27(a). Reduce the circuit by combining the two parallel resistors as shown in ANS. FIG. P28.27(b).



ANS. FIG. P28.27(a)



ANS. FIG. P28.27(b)

Apply Kirchhoff's loop rule to both loops in ANS. FIG. P28.27(b) to obtain:

$$\left(2.71R\right)I_1 + \left(1.71R\right)I_2 = 250\text{ V}$$

$$\left(1.71R\right)I_1 + \left(3.71R\right)I_2 = 500\text{ V}$$

With $R = 1\,000\ \Omega$, simultaneous solution of these equations yields:

$$I_1 = 10.0\text{ mA}$$

$$I_2 = 130.0\text{ mA}$$

From ANS. FIG. P28.27(b), $V_c - V_a = \left(I_1 + I_2\right)\left(1.71R\right) = 240\text{ V}$.

Thus, from ANS. FIG. P28.27(a), $I_4 = \frac{V_c - V_a}{4R} = \frac{240\text{ V}}{4\,000\ \Omega} = 60.0\text{ mA}$.

Finally, applying Kirchhoff's point rule at point a in ANS. FIG. P28.27(a) gives:

$$I = I_4 - I_1 = 60.0\text{ mA} - 10.0\text{ mA} = +50.0\text{ mA}$$

or $I = \boxed{50.0 \text{ mA from point a to point e}}$.

P28.28 Using Kirchhoff's rules and suppressing units,

$$12.0 - (0.01)I_1 - (0.06)I_3 = 0 \qquad \textbf{[1]}$$

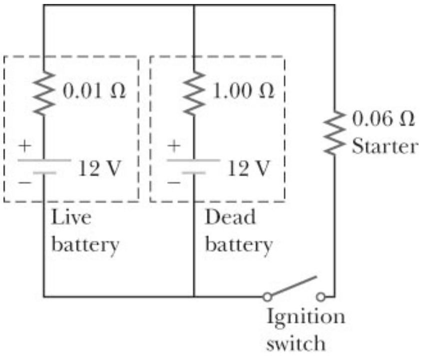
$$12.0 + (1.00)I_2 - (0.06)I_3 = 0 \qquad \textbf{[2]}$$

and $I_1 = I_2 + I_3$. **[3]**

Substitute [3] into [1]:

$$12.0 - (0.01)(I_2 + I_3) - (0.06)I_3 = 0$$

$$12.0 - (0.01)I_2 - (0.07)I_3 = 0 \qquad \textbf{[4]}$$



ANS. FIG. P28.28

Solving [4] and [2] simultaneously gives

(a) $I_3 = 172\text{ A} = \boxed{172 \text{ A downward}}$ in the starter.

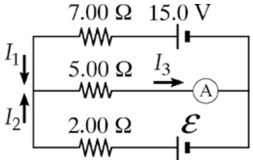
(b) $I_2 = -1.70\text{ A} = \boxed{1.70 \text{ A upward}}$ in the dead battery.

(c) $\boxed{\text{No, the current in the dead battery is upward in Figure P28.28, so it is not being charged. The dead battery is providing a small amount of power to operate the starter, so it is not really "dead."}}$

P28.29 (a) For the upper loop:

$$+15.0 \text{ V} - (7.00\ \Omega)I_1 - (2.00 \text{ A})(5.00\ \Omega) = 0$$

$5.00 = 7.00I_1$ so $\boxed{I_1 = 0.714 \text{ A}}$



ANS. FIG. P28.29

(b) For the center-left junction:

$$I_3 = I_1 + I_2 = 2.00 \text{ A}$$

where I_3 is the current through the ammeter (assumed to travel to the right):

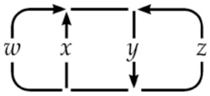
$$0.714 + I_2 = 2.00 \quad \text{so} \quad \boxed{I_2 = 1.29\ \text{A}}$$

(c) For the lower loop:

$$+\varepsilon - (2.00\ \Omega)(1.29\ \text{A}) - (5.00\ \Omega)(2.00\ \text{A}) = 0 \rightarrow \boxed{\varepsilon = 12.6\ \text{V}}$$

P28.30 Name the currents as shown in ANS. FIG. P28.30. Then

$$y = w + x + z$$



ANS. FIG. P28.30

The loop equations are (suppressing units):

$$\left.\begin{aligned} -200\text{w} - 40.0 + 80.0\text{x} &= 0 \\ -80.0\text{x} + 40.0 + 360 - 20.0\text{y} &= 0 \\ +360 - 20.0\text{y} - 70.0\text{z} + 80.0 &= 0 \end{aligned}\right\} \rightarrow \left\{\begin{aligned} \text{x} &= 2.50\text{w} + 0.500 \\ 20.0 &= 4.00\text{x} + 1.00\text{y} \\ 22.0 &= 1.00\text{y} + 3.50\text{z} \end{aligned}\right.$$

Use $y = w + x + z$ to eliminate y by substitution:

$$\left.\begin{aligned} &\text{x} = 2.50\text{w} + 0.500 \\ &20.0 = 4.00\text{x} + 1.00\text{y} \rightarrow 20.0 = 4.00\text{x} + 1.00(\text{w} + \text{x} + \text{z}) \\ &22.0 = 1.00\text{y} + 3.50\text{z} \rightarrow 22.0 = 1.00(\text{w} + \text{x} + \text{z}) + 3.50\text{z} \end{aligned}\right\}$$

$$\rightarrow \left\{\begin{aligned} \text{x} &= 2.50\text{w} + 0.500 \\ 20.0 &= 5.00\text{x} + 1.00\text{w} + 1.00\text{z} \\ 22.0 &= 1.00\text{w} + 1.00\text{x} + 4.50\text{z} \end{aligned}\right.$$

Eliminate x:

$$\left.\begin{aligned} 20.0 &= 5.00(2.50\text{w} + 0.500) + 1.00\text{w} + 1.00\text{z} \\ 22.0 &= 1.00\text{w} + 1.00(2.50\text{w} + 0.500) + 4.50\text{z} \end{aligned}\right\}$$

$$\rightarrow \left\{\begin{aligned} 17.5 &= 13.5\text{w} + 1.00\text{z} \\ 21.5 &= 3.50\text{w} + 4.50\text{z} \end{aligned}\right.$$

Eliminate $z = 17.5 - 13.5w$ to obtain

$$\begin{aligned} &21.5 = 3.50\text{w} + 4.50(17.5 - 13.5\text{w}) \\ &21.5 = 3.50\text{w} + 4.50(17.5) - 4.50(13.5\text{w}) \\ &\rightarrow 57.25 = 57.25\text{w} \rightarrow \text{w} = 1.00 \end{aligned}$$

(a) $\text{w} = \boxed{1.00\ \text{A upward in the 200-}\Omega\ \text{resistor}}$

$$z = 17.5 - 13.5w = 17.5 - 13.5(1.00)$$

$$= \boxed{4.00\text{ A upward in the }70.0\text{-}\Omega\text{ resisor}}$$

$$x = 2.50w + 0.500 = 2.50(1.00) + 0.500$$

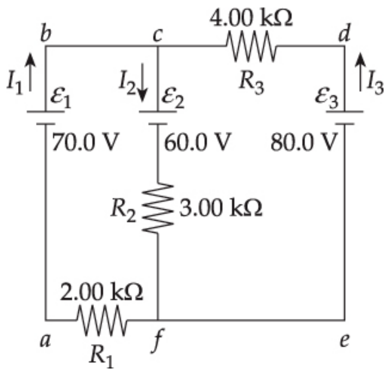
$$= \boxed{3.00\text{ A upward in the }80.0\text{-}\Omega\text{ resistor}}$$

$$y = w + x + z = 1.00 + 3.00 + 4.00$$

$$= \boxed{8.00\text{ A downward in the }20.0\text{-}\Omega\text{ resistor}}$$

(b) For the 200-Ω resistor, $\Delta V = IR = (1.00\text{ A})(200\ \Omega) = \boxed{200\text{ V}}$

***P28.31** (a) We name the currents I_1, I_2, and I_3 as shown in ANS. FIG. P28.31. Applying Kirchhoff's loop rule to loop *abcfa* gives



ANS. FIG. P28.31

$$+\mathcal{E}_1 - \mathcal{E}_2 - R_2I_2 - R_1I_1 = 0$$

or,

$$70.0\text{ V} - 60.0\text{ V} - (3.00\text{ k}\Omega)I_2 - (2.00\text{ k}\Omega)I_1 = 0$$

which gives

$$3I_2 + 2I_1 = 10.0\text{ mA}$$

or $$I_1 = 5.00\text{ mA} - 1.50I_2 \tag*{[1]}$$

Applying the loop rule to loop *edcfe* yields

$$+\mathcal{E}_3 - R_3I_3 - \mathcal{E}_2 - R_2I_2 = 0$$

which gives

$$80.0\text{ V} - (4.00\text{ k}\Omega)I_3 - 60.0\text{ V} - (3.00\text{ k}\Omega)I_2 = 0$$

or $$3I_2 + 4I_3 = 20.0\text{ mA}$$

and $$I_3 = 5.00\text{ mA} - 0.750I_2 \tag*{[2]}$$

Finally, applying Kirchhoff's junction rule at junction *c* gives

$$I_2 = I_1 + I_3 \tag*{[3]}$$

Substituting equations [1] and [2] into [3] yields

$$I_2 = 5.00\text{ mA} - 1.50I_2 + 5.00\text{ mA} - 0.750I_2$$

or $$3.25I_2 = 10.0\text{ mA}$$

This gives $I_2 = \dfrac{10.0\text{ mA}}{3.25} = \boxed{3.08\text{ mA}}$. Then, equation [1] yields

$$I_1 = 5.00\text{ mA} - 1.50 I_2 = 5.00\text{ mA} - 1.50(3.08\text{ mA}) = \boxed{0.385\text{ mA}}$$

and from equation [2],

$$\begin{aligned} I_3 &= 5.00\text{ mA} - 0.750 I_2 = 5.00\text{ mA} - 0.750(3.08\text{ mA}) \\ &= \boxed{2.69\text{ mA}} \end{aligned}$$

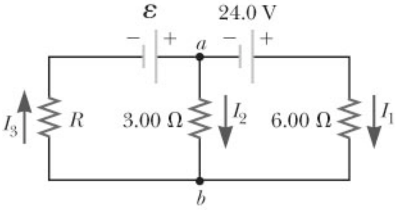
(b) Start at point c and go to point f, recording changes in potential to obtain

$$\begin{aligned} V_f - V_c &= -\mathcal{E}_2 - R_2 I_2 \\ &= -60.0\text{ V} - \left(3.00\times 10^3\ \Omega\right)\left(3.08\times 10^{-3}\text{ A}\right) = -69.2\text{ V} \end{aligned}$$

or $|\Delta V|_{cf} = \boxed{69.2\text{ V and point c is at the higher potential}}$.

P28.32 Following the path of I_1 from a to b, and recording changes in potential gives

$$V_b - V_a = +24.0\text{ V} - (6.00\ \Omega)(3.00\text{ A}) = +6.00\text{ V}$$



ANS. FIG. P28.32

Now, following the path of I_2 from a to b, and recording changes in potential gives

$$V_b - V_a = -(3.00\ \Omega) I_2 = +6.00\text{ V} \quad \rightarrow \quad I_2 = -2.00\text{ A}$$

which means the initial choice of the direction of I_2 in Figure P28.32 was incorrect. Applying Kirchhoff's junction rule at point a gives

$$I_3 = I_1 + I_2 = 3.00\text{ A} + (-2.00\text{ A}) = 1.00\text{ A}$$

The results are:

(a) $\boxed{I_2 \text{ is directed from b toward a and has a magnitude of 2.00 A.}}$

(b) $\boxed{I_3 = 1.00\text{ A}}$ and flows in the direction shown in Figure P28.32.

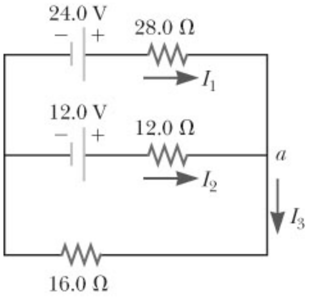
(c) $\boxed{\text{No. Neither of the equations used to find } I_2 \text{ and } I_3 \text{ contained } \mathcal{E} \text{ and } R\text{. The third equation that we could generate from Kirchhoff's rules contains both the unknowns. Therefore, we have only one equation with two unknowns.}}$

P28.33 (a) Applying Kirchhoff's junction rule at point *a* gives

$$I_3 = I_1 + I_2 \qquad \textbf{[1]}$$

Using the loop rule on the lower loop yields

$$+12.0 - 12.0I_2 - 16.0I_3 = 0 \quad \text{or} \quad I_2 = 1.00 - \frac{4.00I_3}{3.00} \qquad \textbf{[2]}$$



ANS. FIG. P28.33

Applying the loop rule to loop forming the outer perimeter of the circuit gives

$$+24.0 - 28.0I_1 - 16.0I_3 = 0 \quad \text{or} \quad I_1 = \frac{24.0 - 16.0I_3}{28.0} \qquad \textbf{[3]}$$

Substituting equations [2] and [3] into [1] yields

$$I_3 = \frac{24.0 - 16.0I_3}{28.0} + 1.00 - \frac{4.00I_3}{3.00}$$

and multiplying by 84 to eliminate fractions:

$$84.0I_3 = 72.0 - 48.0I_3 + 84.0 - 112I_3$$

$$244I_3 = 156$$

$$\boxed{I_3 = 0.639 \text{ A}}$$

Then, equation [2] gives $\boxed{I_2 = 0.148 \text{ A}}$ and equation [3] yields $\boxed{I_1 = 0.492 \text{ A}}$.

(b) The power delivered to each of the resistors in this circuit is:

$$P_{28.0\ \Omega} = I_1^2 R_{28.0\ \Omega} = (0.492 \text{ A})^2 (28.0\ \Omega) = \boxed{6.77 \text{ W}}$$

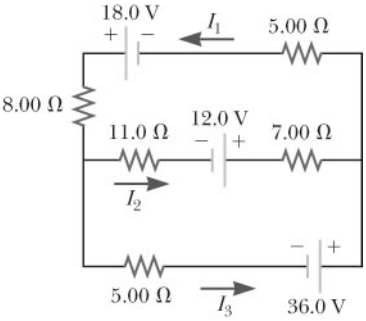
$$P_{12.0\ \Omega} = I_2^2 R_{12.0\ \Omega} = (0.148\ \text{A})^2(12.0\ \Omega) = \boxed{0.261\ \text{W}}$$

$$P_{16.0\ \Omega} = I_3^2 R_{16.0\ \Omega} = (0.639\ \text{A})^2(16.0\ \Omega) = \boxed{6.54\ \text{W}}$$

P28.34 (a) Going counterclockwise around the upper loop and suppressing units, Kirchhoff's loop rule gives

$$-11.0I_2 + 12.0 - 7.00I_2 - 5.00I_1 + 18.0 - 8.00I_1 = 0$$

or $\boxed{13.0I_1 + 18.0I_2 = 30.0}$. **[1]**



ANS. FIG. P28.34

(b) Going counterclockwise around the lower loop:

$$-5.00I_3 + 36.0 + 7.00I_2 - 12.0 + 11.0I_2 = 0$$

or $\boxed{18.0I_2 - 5.00I_3 = -24.0}$. **[2]**

(c) Applying the junction rule at the node in the left end of the circuit gives $\boxed{I_1 - I_2 - I_3 = 0}$ **[3]**

(d) Solving equation [3] for I_3 yields $\boxed{I_3 = I_1 - I_2}$ **[4]**

(e) Substituting equation [4] into [2] gives

$$5.00(I_1 - I_2) - 18.0I_2 = 24.0$$

or $\boxed{5.00I_1 - 23.0I_2 = 24.0}$. **[5]**

(f) Solving equation [5] for I_1 yields $I_1 = (24.0 + 23.0I_2)/5$. Substituting this into equation [1] gives

$$13.0I_1 + 18.0I_2 = 30.0$$

$$13.0\frac{(24.0 + 23.0I_2)}{5.00} + 18.0I_2 = 30.0$$

$$13.0(24.0 + 23.0I_2) + 5.00(18.0I_2) = 5.00(30.0)$$

$$389I_2 = -162 \quad \rightarrow \quad I_2 = -162/389 \quad \rightarrow \quad \boxed{I_2 = -0.416\ \text{A}}$$

Then, from equation [2], $I_1 = (30 - 18I_2)/13$ which yields

$$\boxed{I_1 = 2.88\ \text{A}}$$

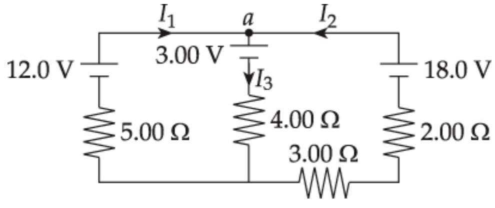
(g) Equation [4] gives

$$I_3 = I_1 - I_2 = 2.88\ \text{A} - (-0.416\ \text{A}) \quad \rightarrow \quad \boxed{I_3 = 3.30\ \text{A}}$$

(h) The negative sign in the answer for I_2 means that this current flows in the opposite direction to that shown in the circuit diagram and assumed during the solution. That is, the actual current in the middle branch of the circuit flows from right to left and has a magnitude of 0.416 A.

***P28.35** Refer to ANS. FIG. P28.35. Applying Kirchhoff's junction rule at junction *a* gives

$$I_3 = I_1 + I_2 \qquad \textbf{[1]}$$



ANS. FIG. P28.35

Using Kirchhoff's loop rule on the leftmost loop yields

$$-3.00\ \text{V} - (4.00\ \Omega)I_3 - (5.00\ \Omega)I_1 + 12.0\ \text{V} = 0$$

so
$$I_1 = \frac{9.00\ \text{A} - 4.00I_3}{5.00} = 1.80\ \text{A} - 0.800I_3 \qquad \textbf{[2]}$$

For the rightmost loop,

$$-3.00\ \text{V} - (4.00\ \Omega)I_3 - (3.00\ \Omega + 2.00\ \Omega)I_2 + 18.0\ \text{V} = 0$$

and
$$I_2 = \frac{15.0\ \text{A} - 4.00I_3}{5.00} = 3.00\ \text{A} - 0.800I_3 \qquad \textbf{[3]}$$

Substituting equations [2] and [3] into [1] and simplifying gives $2.60I_3 = 4.80$ A and $I_3 = 1.846$ A. Then equations [2] and [3] yield $I_1 = 0.323$ A and $I_2 = 1.523$ A.

Therefore, the potential differences across the resistors are

$$\Delta V_2 = I_2(2.00\ \Omega) = (1.523\ \text{A})(2.00\ \Omega) = \boxed{3.05\ \text{V}}$$

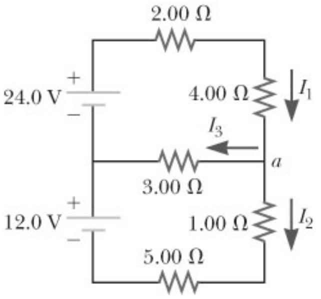
$$\Delta V_3 = I_2(3.00\ \Omega) = (1.523\ \text{A})(3.00\ \Omega) = \boxed{4.57\ \text{V}}$$

$$\Delta V_4 = I_3(4.00\ \Omega) = (1.846\ \text{A})(4.00\ \Omega) = \boxed{7.38\ \text{V}}$$

$$\Delta V_5 = I_1(5.00\ \Omega) = (0.323\ \text{A})(5.00\ \Omega) = \boxed{1.62\ \text{V}}$$

P28.36 (a) $\boxed{\text{No.}}$ Some simplification could be made by recognizing that the 2.0 Ω and 4.0 Ω resistors are in series, adding to give a total of 6.0 Ω; and the 5.0 Ω and 1.0 Ω resistors form a series combination with a total resistance of 6.0 Ω.

$\boxed{\text{The circuit cannot be simplified any further, and Kirchhoff's rules must be used to analyze it.}}$



ANS. FIG. P28.36

(b) Applying Kirchhoff's junction rule at junction *a* gives

$$I_1 = I_2 + I_3 \qquad \textbf{[1]}$$

Using Kirchhoff's loop rule on the upper loop yields

$$+24.0\text{ V} - (2.00 + 4.0)I_1 - (3.00)I_3 = 0$$

or $I_3 = 8.00\text{ A} - 2.00 I_1$ **[2]**

and for the lower loop,

$$+12.0\text{ V} + (3.00)I_3 - (1.00 + 5.00)I_2 = 0$$

Using equation [2], this reduces to

$$I_2 = \frac{12.0\text{ V} + 3.00(8.00\text{ A} - 2.00 I_1)}{6.00}$$

giving

$$I_2 = 6.00\text{ A} - 1.00 I_1 \qquad \textbf{[3]}$$

Substituting equations [2] and [3] into [1] gives $\boxed{I_1 = 3.50\text{ A}}$

(c) Then, equation [3] gives $\boxed{I_2 = 2.50\text{ A}}$, and

(d) Equation [2] yields $\boxed{I_3 = 1.00\text{ A}}$

Section 28.4 *RC* Circuits

P28.37 (a) The time constant of the circuit is

$$\tau = RC = (100\ \Omega)(20.0\times10^{-6}\ \text{F}) = 2.00\times10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$$

(b) The maximum charge on the capacitor is given by Equation 28.13:

$$Q_{\text{max}} = C\mathcal{E} = (20.0\times10^{-6}\ \text{F})(9.00\ \text{V}) = \boxed{1.80\times10^{-4}\ \text{C}}$$

(c) We use $q(t) = Q_{\text{max}}(1 - e^{-t/RC})$, when $t = RC$. Then,

$$q(t) = Q_{\text{max}}(1 - e^{-RC/RC}) = Q_{\text{max}}(1 - e^{-1}) = (1.80\times10^{-4}\ \text{C})(1 - e^{-1})$$
$$= \boxed{1.14\times10^{-4}\ \text{C}}$$

P28.38 (a) The time constant is

$$RC = (1.00\times10^{6}\ \Omega)(5.00\times10^{-6}\ \text{F}) = \boxed{5.00\ \text{s}}$$

(b) After a long time interval, the capacitor is "charged to thirty volts," separating charges of

$$Q = C\mathcal{E} = (5.00\times10^{-6}\ \text{C})(30.0\ \text{V}) = \boxed{150\ \mu\text{C}}$$

(c) $$I(t) = \frac{\mathcal{E}}{R}e^{-t/RC} = \left(\frac{30.0\ \text{V}}{1.00\times10^{6}\ \Omega}\right)\exp\left[\frac{-10.0\ \text{s}}{(1.00\times10^{6}\ \Omega)(5.00\times10^{-6}\ \text{F})}\right]$$
$$= \boxed{4.06\ \mu\text{A}}$$

P28.39 (a) From $I(t) = -I_0 e^{-t/RC}$,

$$I_0 = \frac{Q}{RC} = \frac{5.10\times10^{-6}\ \text{C}}{(1\,300\ \Omega)(2.00\times10^{-9}\ \text{F})} = 1.96\ \text{A}$$

$$I(t) = -(1.96\ \text{A})\exp\left[\frac{-9.00\times10^{-6}\ \text{s}}{(1\,300\ \Omega)(2.00\times10^{-9}\ \text{F})}\right] = \boxed{-61.6\ \text{mA}}$$

(b) $$q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\frac{-8.00\times10^{-6}\ \text{s}}{(1\,300\ \Omega)(2.00\times10^{-9}\ \text{F})}\right]$$
$$= \boxed{0.235\ \mu\text{C}}$$

(c) The magnitude of the maximum current is $I_0 = \boxed{1.96\ \text{A}}$.

P28.40 The potential difference across the capacitor is

$$\Delta V(t) = \Delta V_{\text{max}}(1 - e^{-t/RC})$$

Using 1 farad = 1 s/Ω,

$$4.00\text{ V} = (10.0\text{ V})\left[1 - e^{-(3.00\text{ s})/\left[R\left(10.0\times10^{-6}\text{ s}/\Omega\right)\right]}\right]$$

Therefore,

$$0.400 = 1.00 - e^{-\left(3.00\times10^{5}\,\Omega\right)/R}$$

or $$e^{-\left(3.00\times10^{5}\,\Omega\right)/R} = 0.600.$$

Taking the natural logarithm of both sides,

$$-\frac{3.00\times10^{5}\ \Omega}{R} = \ln(0.600)$$

and $$R = -\frac{3.00\times10^{5}\ \Omega}{\ln(0.600)} = +5.87\times10^{5}\ \Omega = \boxed{587\text{ k}\Omega}.$$

P28.41 (a) Before the switch is closed, the two resistors are in series. The time constant is

$$\tau = (R_1 + R_2)C = \left(1.50\times10^{5}\ \Omega\right)\left(10.0\times10^{-6}\text{ F}\right) = \boxed{1.50\text{ s}}$$

(b) After the switch is closed, the capacitor discharges through resistor R_2. The time constant is

$$\tau = \left(1.00\times10^{5}\ \Omega\right)\left(10.0\times10^{-6}\text{ F}\right) = \boxed{1.00\text{ s}}$$

(c) Before the switch is closed, there is no current in the circuit because the capacitor is fully charged, and the voltage across the capacitor is $\mathcal{E}$. After the switch is closed, current flows clockwise from the battery to resistor R_1 and down through the switch, and current from the capacitor flows counterclockwise and down through the switch to resistor R_2; the result is that the total current through the switch is $I_1 + I_2$.

Going clockwise around the left loop,

$$\mathcal{E} - I_1R_1 = 0 \rightarrow I_1 = \frac{\mathcal{E}}{R_1}$$

so the battery carries current $$I_1 = \frac{10.0\text{ V}}{50.0\times10^{3}\ \Omega} = 200\ \mu\text{A}.$$

Going counterclockwise around the right loop,

$$\frac{q}{C} - I_2R_2 = 0 \rightarrow I_2 = \frac{q}{R_2C} = \frac{\mathcal{E}}{R_2}e^{-t/(R_2C)}$$

so the 100-kΩ resistor carries current of magnitude

$$I_2 = \frac{\mathcal{E}}{R_2} e^{-t/RC} = \left(\frac{10.0\text{ V}}{100 \times 10^3\ \Omega}\right) e^{-t/1.00\text{ s}}$$

and the switch carries downward current

$$I_1 + I_2 = \boxed{200\ \mu\text{A} + (100\ \mu\text{A}) e^{-t/1.00\text{ s}}}$$

P28.42 (a) Before the switch is closed, the two resistors are in series. The time constant is

$$\tau = \boxed{(R_1 + R_2)C}$$

(b) After the switch is closed, the capacitor discharges through resistor R_2. The time constant is

$$\tau = \boxed{R_2 C}$$

(c) Before the switch is closed, there is no current in the circuit because the capacitor is fully charged, and the voltage across the capacitor is $\mathcal{E}$. After the switch is closed, current flows clockwise from the battery to resistor R_1 and down through the switch, and current from the capacitor flows counterclockwise and down through the switch to resistor R_2; the result is that the total current through the switch is $I_1 + I_2$. Going clockwise around the left loop,

$$\mathcal{E} - I_1 R_1 = 0 \rightarrow I_1 = \frac{\mathcal{E}}{R_1} \text{ is the current in the battery.}$$

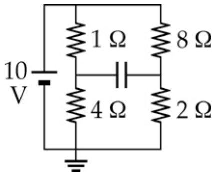
Going counterclockwise around the right loop,

$$\frac{q}{C} - I_2 R_2 = 0 \rightarrow I_2 = \frac{q}{R_2 C} = \frac{\mathcal{E}}{R_2} e^{-t/(R_2 C)}$$

is the magnitude of the current in R_2. The total current through the switch is

$$I_1 + I_2 = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} e^{-t/(R_2 C)} = \boxed{\mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2} e^{-t/(R_2 C)}\right)}$$

P28.43 (a) Call the potential at the left junction V_L and at the right V_R. After a "long" time, the capacitor is fully charged.



ANS. FIG. P28.43(a)

$$I_L = \frac{10.0\text{ V}}{5.00\ \Omega} = 2.00\text{ A}$$

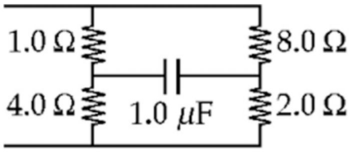
$$V_L = 10.0\text{ V} - (2.00\text{ A})(1.00\ \Omega) = 8.00\text{ V}$$

$$I_R = \frac{10.0\text{ V}}{10.0\ \Omega} = 1.00\text{ A}$$

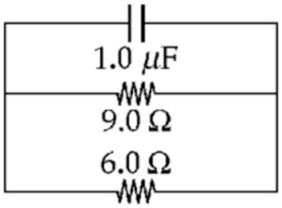
$$V_R = (10.0\text{ V}) - (8.00\ \Omega)(1.00\text{ A}) = 2.00\text{ V}$$

Therefore, $\Delta V = V_L - V_R = 8.00 - 2.00 = \boxed{6.00\text{ V}}$

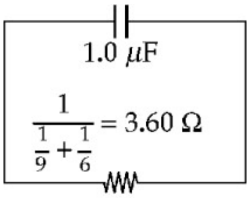
(b) We suppose the battery is pulled out leaving an open circuit. We are left with ANS. FIG. P28.43(b), which can be reduced to the equivalent circuits shown in ANS. FIG. P28.43(c) and ANS. FIG. P28.43(d). From ANS. FIG. P28.43(d), we can see that the capacitor discharges through a 3.60-Ω equivalent resistance.



ANS. FIG. P28.43(b)



ANS. FIG. P28.43(c)



ANS. FIG. P28.43(d)

According to $q = Qe^{-t/RC}$,

we calculate that $qC = QCe^{-t/RC}$

and $\Delta V = \Delta V_i e^{-t/RC}$.

We proceed to solve for *t:*

$$\frac{\Delta V}{\Delta V_i} = e^{-t/RC} \quad \text{or} \quad \frac{\Delta V_i}{\Delta V} = e^{+t/RC}$$

Take natural logarithms of both sides:

$$\ln\left(\frac{\Delta V_i}{\Delta V}\right) = t/RC$$

so $t = RC\ln\left(\frac{\Delta V_i}{\Delta V}\right)$

$$= (3.60\ \Omega)(1.00\times10^{-6}\text{ F})\ln\left(\frac{\Delta V_i}{0.100\Delta V_i}\right) = (3.60\times10^{-6}\text{ s})\ln 10$$

$$= \boxed{8.29\ \mu\text{s}}$$

P28.44 We are to calculate

$$\int_0^\infty e^{-2t/RC}\,dt = -\frac{RC}{2}\int_0^\infty e^{-2t/RC}\left(-\frac{2dt}{RC}\right)$$

$$= -\frac{RC}{2}e^{-2t/RC}\Big|_0^\infty = -\frac{RC}{2}\left[e^{-\infty} - e^0\right]$$

$$= -\frac{RC}{2}[0-1] = \boxed{+\frac{RC}{2}}$$

P28.45 (a) The charge remaining on the capacitor after time t is $q = Qe^{-t/\tau}$.

Thus, if $q = 0.750Q$, then

$$0.750Q = Qe^{-t/\tau}$$

$$e^{-t/\tau} = 0.750$$

$$t = -\tau\ln(0.750) = -(1.50\text{ s})\ln(0.750) = \boxed{0.432\text{ s}}$$

(b) $\tau = RC$, so

$$C = \frac{\tau}{R} = \frac{1.50\text{ s}}{250\times10^3\ \Omega} = 6.00\times10^{-6}\text{ F} = \boxed{6.00\ \mu\text{F}}$$

Section 28.5 Household Wiring and Electrical Safety

P28.46 (a) $P = I\Delta V$: So for the heater, $I = \dfrac{P}{\Delta V} = \dfrac{1\,500\text{ W}}{120\text{ V}} = \boxed{12.5\text{ A}}$.

For the toaster, $I = \dfrac{750\text{ W}}{120\text{ V}} = \boxed{6.25\text{ A}}$.

And for the grill, $I = \dfrac{1\,000\text{ W}}{120\text{ V}} = \boxed{8.33\text{ A}}$.

(b) The total current drawn is 12.5 A + 6.25 A + 8.33 A = 27.1 A.

$\boxed{\text{The current draw is greater than 25.0 amps, so this circuit will trip the circuit breaker.}}$

***P28.47** From $P = (\Delta V)^2/R$, the resistance of the element is

$$R = \frac{(\Delta V)^2}{P} = \frac{(240\text{ V})^2}{3\ 000\text{ W}} = 19.2\ \Omega$$

When the element is connected to a 120-V source, we find that

(a) $I = \dfrac{\Delta V}{R} = \dfrac{120\text{ V}}{19.2\ \Omega} = \boxed{6.25\text{ A}}$

(b) $P = I\Delta V = (6.25\text{ A})(120\text{ V}) = \boxed{750\text{ W}}$

P28.48 (a) Suppose that the insulation between either of your fingers and the conductor adjacent is a chunk of rubber with contact area 4 mm^2 and thickness 1 mm. Its resistance is

$$R = \frac{\rho\ell}{A} \approx \frac{(10^{13}\ \Omega\cdot\text{m})(10^{-3}\text{ m})}{4\times10^{-6}\text{ m}^2} \approx 2\times10^{15}\ \Omega$$

The current will be driven by 120 V through total resistance (series)

$$2\times10^{15}\ \Omega+10^{4}\ \Omega+2\times10^{15}\ \Omega\approx5\times10^{15}\ \Omega$$

It is: $I=\frac{\Delta V}{R}\sim\frac{120\ \text{V}}{5\times10^{15}\ \Omega}\boxed{\sim10^{-14}\ \text{A}}$

(b) The resistors form a voltage divider, with the center of your hand at potential $\frac{V_h}{2}$, where V_h is the potential of the "hot" wire. The potential difference between your finger and thumb is

$$\Delta V=IR\sim\left(10^{-14}\ \text{A}\right)\left(10^{4}\ \Omega\right)\sim10^{-10}\ \text{V}$$

So the points where the rubber meets your fingers are at potentials of

$$\boxed{\sim\frac{V_h}{2}+10^{-10}\ \text{V}}\quad\text{and}\quad\boxed{\sim\frac{V_h}{2}-10^{-10}\ \text{V}}$$

Additional Problems

P28.49 (a) With the lightbulbs in series, the equivalent resistance is $R_{eq}=3R$, and the current is given by $I=\frac{\mathcal{E}}{3R}$. Then,

$$P_{\text{series}}=\mathcal{E}I=\boxed{\frac{\mathcal{E}^2}{3R}}$$

(b) With the lightbulbs in parallel, the equivalent resistance is

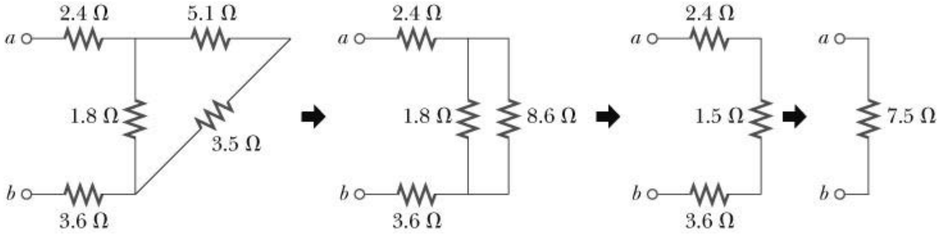
$$R_{eq}=\frac{1}{(1/R)+(1/R)+(1/R)}=\frac{R}{3}$$

the current is given by $I=\frac{3\mathcal{E}}{R}$. Then,

$$P_{\text{parallel}}=\mathcal{E}I=\boxed{\frac{3\mathcal{E}^2}{R}}$$

(c) Nine times more power is converted in the $\boxed{\text{parallel}}$ connection.

P28.50 The resistive network between *a* an *b* reduces, in the stages shown in ANS. FIG. P28.50, to an equivalent resistance of $R_{eq} = \boxed{7.49\ \Omega}$.



ANS. FIG. P28.50

P28.51 The set of four batteries boosts the electric potential of each bit of charge that goes through them by 4 × 1.50 V = 6.00 V. The chemical energy they store is

$$\Delta U = q\Delta V = (240\ \text{C})(6.00\ \text{J/C}) = 1\ 440\ \text{J}$$

The radio draws current

$$I = \frac{\Delta V}{R} = \frac{6.00\ \text{V}}{200\ \Omega} = 0.030\ 0\ \text{A}.$$

So, its power is

$$P = I\Delta V = (0.030\ 0\ \text{A})(6.00\ \text{V}) = 0.180\ \text{J/s}$$

Then for the time the energy lasts, we have $P = \frac{E}{\Delta t}$:

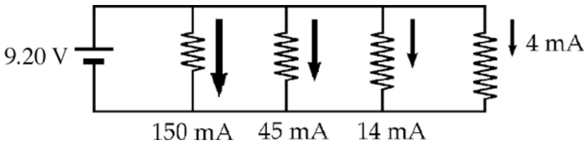
$$\Delta t = \frac{E}{P} = \frac{1\ 440\ \text{J}}{0.180\ \text{J/s}} = 8.00 \times 10^3\ \text{s}$$

We could also compute this from $I = \frac{Q}{\Delta t}$:

$$\Delta t = \frac{Q}{I} = \frac{240\ \text{C}}{0.030\ 0\ \text{A}} = 8.00 \times 10^3\ \text{s} = \boxed{2.22\ \text{h}}$$

P28.52 The battery current is

$$(150 + 45.0 + 14.0 + 4.00)\ \text{mA} = 213\ \text{mA}$$



ANS. FIG. P28.52

(a) The resistor with highest resistance is that carrying 4.00 mA. Doubling its resistance will reduce the current it carries to

2.00 mA. Then the total current is (150 + 45 + 14 + 2) mA = 211 mA, nearly the same as before. The ratio is $\frac{211}{213} = \boxed{0.991}$.

(b) The resistor with least resistance carries 150 mA. Doubling its resistance changes this current to 75 mA and changes the total to (75 + 45 + 14 + 4) mA = 138 mA. The ratio is $\frac{138}{213} = \boxed{0.648}$.

(c) The energy flows are precisely analogous to the currents in parts (a) and (b). The ceiling has the smallest R value of the thermal resistors in parallel, so increasing its thermal resistance will produce the biggest reduction in the total energy flow.

P28.53 Several seconds is many time constants, so the capacitor is fully charged and (d) the current in its branch is $\boxed{\text{zero.}}$

For the center loop, Kirchhoff's loop rule gives

$$+8 + (3\ \Omega)\,I_2 - (5\ \Omega)I_1 = 0$$

or $I_1 = 1.6 + 0.6I_2$ **[1]**

For the right-hand loop, Kirchhoff's loop rule gives

$$+4\text{ V} - (3\ \Omega)\,I_2 - (5\ \Omega)\,I_3 = 0$$

or $I_3 = 0.8 - 0.6I_2$ **[2]**

For the top junction, Kirchhoff's junction rule gives

$+I_1 + I_2 - I_3 = 0$ **[3]**

Now we eliminate I_1 and I_3 by substituting [1] and [2] into [3]. Suppressing units,

$$1.6 + 0.6I_2 + I_2 - 0.8 + 0.6I_2 = 0 \rightarrow I_2 = -0.8/2.2 = -0.3636$$

(b) $\boxed{\text{The current in } 3\ \Omega \text{ is } 0.364\text{ A down.}}$

(a) Now, from [2], we find $I_3 = 0.8 - 0.6(-0.364) = \boxed{1.02\text{ A down in 4 V and in } 5\ \Omega.}$

(c) From [1] we have $I_1 = 1.6 + 0.6(-0.364) = \boxed{1.38\text{ A up in the 8 V battery}}$.

(e) For the left loop $+3\text{ V} - (Q/6\ \mu\text{F}) + 8\text{ V} = 0$, so $Q = (6\ \mu\text{F})(11\text{ V}) = \boxed{66.0\ \mu\text{C}}$

P28.54 The current in the battery is $\dfrac{15\text{ V}}{10\ \Omega + \dfrac{1}{1/5\ \Omega + 1/8\ \Omega}} = 1.15\text{ A}$.

The voltage across 5 Ω is 15 V – (10 Ω) (1.15 A) = 3.53 V.

(a) The current in it is 3.53 V/5 Ω = $\boxed{0.706\text{ A.}}$

(b) $P = (3.53\text{ V})(0.706\text{ A}) = \boxed{2.49\text{ W.}}$

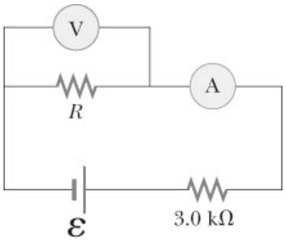
(c) Only the circuit in Figure P28.54(c) requires the use of Kirchhoff's rules for solution. In the other circuits the 5-Ω and 8-Ω resistors are still in parallel with each other.

(d) The power is lowest in Figure P28.54(c). The circuits in Figures P28.54(b) and P28.54(d) have in effect 30-V batteries driving the current. The power is lowest in Figure P28.54(c) because the current in the 10-W resistor is lowest because the battery voltage driving the current is lowest.

P28.55 (a) $R = \dfrac{\Delta V}{I} = \dfrac{6.00\text{ V}}{3.00\times10^{-3}\text{ A}} = 2.00\times10^{3}\ \Omega = \boxed{2.00\text{ k}\Omega}$

(b) The resistance in the circuit consists of a series combination with an equivalent resistance of $R_{\text{eq}} = 5.00\ \Omega$. The emf of the battery is then

$$\mathcal{E} = IR_{\text{eq}} = (3.00\times10^{-3}\text{ A})(5.00\times10^{3}\ \Omega) = \boxed{15.0\text{ V}}$$



ANS. FIG. P28.55

(c) $\Delta V_3 = IR_3 = (3.00\times10^{-3}\text{ A})(3.00\times10^{3}\ \Omega) = \boxed{9.00\text{ V}}$

P28.56 The equivalent resistance is $R_{eq} = R + R_p$, where R_p is the total resistance of the three parallel branches;

$$R_p = \left(\frac{1}{120\ \Omega} + \frac{1}{40.0\ \Omega} + \frac{1}{R + 5.00\ \Omega}\right)^{-1}$$
$$= \left(\frac{1}{30.0\ \Omega} + \frac{1}{R + 5.00\ \Omega}\right)^{-1}$$
$$= \frac{(30.0\ \Omega)(R + 5.00\ \Omega)}{R + 35.0\ \Omega}$$

Thus,

$$75.0\ \Omega = R + \frac{(30.0\ \Omega)(R + 5.00\ \Omega)}{R + 35.0\ \Omega} = \frac{R^2 + (65.0\ \Omega)R + 150\ \Omega^2}{R + 35.0\ \Omega}$$

which reduces to

$$R^2 - (10.0\ \Omega)R - 2\,475\ \Omega^2 = 0$$

or $$(R - 55\ \Omega)(R + 45\ \Omega) = 0$$

Only the positive solution is physically acceptable, so $R = \boxed{55.0\ \Omega}$.

P28.57 (a) Using Kirchhoff's loop rule for the closed loop,

$$+12.0 - 2.00I - 4.00I = 0$$

so $I = 2.00\text{ A}$

Then,

$$V_b - V_a = +4.00\text{ V} - (2.00\text{ A})(4.00\ \Omega) - (0)(10.0\ \Omega) = -4.00\text{ V}$$

Thus, $|\Delta V_{ab}| = \boxed{4.00\text{ V}}$

(b) $V_b - V_a = -4.00\text{ V} \rightarrow V_a = V_b + 4.00\text{ V}$; thus,

$\boxed{\text{a is at the higher potential}}$.

P28.58 Find an expression for the power delivered to the load resistance R:

$$P = I^2R = \left(\frac{\mathcal{E}}{r + R}\right)^2 R \quad \rightarrow \quad (r + R)^2 = \frac{\mathcal{E}^2}{P}R = aR$$

where $a = \dfrac{\mathcal{E}^2}{P}$

Carry out the squaring process:

$$r^2 + 2rR + R^2 = aR$$
$$R^2 + (2r - a)R + r^2 = 0$$
$$R^2 + bR + r^2 = 0$$

where $\quad b = 2r - a = 2r - \dfrac{\mathcal{E}^2}{P}.$

Solve the quadratic equation:

$$R = \frac{-b \pm \sqrt{b^2 - 4r^2}}{2}$$

Evaluate *b*:

$$b = 2(1.20\ \Omega) - \frac{(9.20\ \text{V})^2}{21.2\ \text{W}} = -1.59\ \Omega$$

Substitute numerical values into the expression for *R*:

$$R = \frac{-(-1.59\ \Omega) \pm \sqrt{(-1.59\ \Omega)^2 - 4(1.20\ \Omega)^2}}{2}$$

$$= \frac{1.59\ \Omega \pm \sqrt{-3.22\ \Omega^2}}{2}$$

There is no real solution to this expression for *R*. Therefore, no load resistor can extract 21.2 W from this battery.

P28.59 The charging circuit is shown in the left-hand panel of ANS. FIG. P28.59. Kirchhoff's loop rule gives

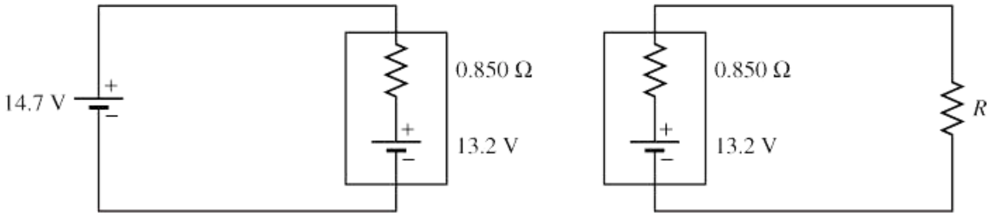
$$+14.7\ \text{V} - 13.2\ \text{V} - I\,(0.850\ \Omega) = 0$$

so the charging current is

$$I = 1.5\ \text{V}/0.850\ \Omega = 1.76\ \text{A}.$$

The charge passing through the battery as it charges is

$$q = I\Delta t = (1.76\ \text{A})(1.80\ \text{h}) = 3.18\ \text{A}\cdot\text{h} = 11.4\ \text{kC}$$



ANS. FIG. P28.59

We can think of this charge as indexing a certain number of chemical reactions, producing a certain quantity of high-energy molecules in the battery. When the battery returns to its original state in discharging, we assume that the same number of reverse reactions uses up all of the high-energy chemical. In our model, the same charge passes through the battery in discharging, in the opposite direction.

The discharge current is then

$$I = \frac{q}{\Delta t} = \frac{3.18\ \text{A}\cdot\text{h}}{7.30\ \text{h}} = 0.435\ \text{A}$$

In the discharge circuit , shown in the right-hand panel of ANS. FIG. P28.59, the loop rule gives

$$13.2\ \text{V} - (0.435\ \text{A})(0.850\ \Omega) - (0.435\ \text{A})R = 0$$

so the load resistance R is 12.8 V/0.435 A = 29.5 Ω. Now we can get around to thinking about energy. The energy output of the 14.7-V power supply is

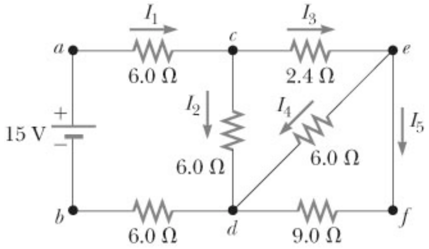
$$q\Delta V = (3.18\ \text{A}\cdot\text{h})(14.7\ \text{V}) = 46.7\ \text{W}\cdot\text{h} = 168\ \text{kJ}$$

The energy delivered to the load during discharge is

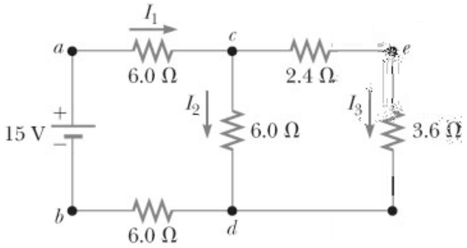
$$q\Delta V = qIR = (3.18\ \text{A}\cdot\text{h})(0.435\ \text{A})(29.5\ \Omega) = 40.8\ \text{W}\cdot\text{h} = 147\ \text{kJ}$$

The storage efficiency is $\frac{40.8\ \text{W}\cdot\text{h}}{46.7\ \text{W}\cdot\text{h}} = 0.873 = \boxed{87.3\%}$.

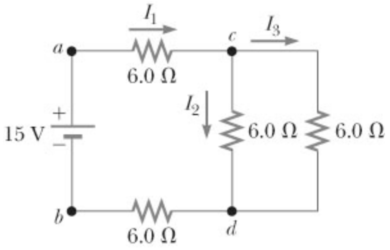
P28.60 (a) The resistors combine to an equivalent resistance of $R_{eq} = \boxed{15.0\ \Omega}$ as shown in ANS. FIGs P28.60(a-e).



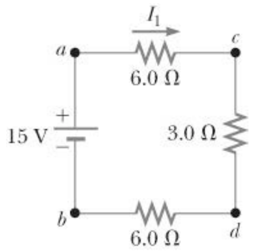
ANS. FIG. P28.60(a)



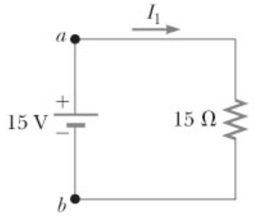
ANS. FIG. P28.60(b)



ANS. FIG. P28.60(c)



ANS. FIG. P28.60(d)



ANS. FIG. P28.60(e)

From ANS. FIG. P28.60(e),

$$I_1 = \frac{\Delta V_{ab}}{R_{eq}} = \frac{15.0\text{ V}}{15.0\ \Omega} = 1.00\text{ A}$$

Then, from ANS. FIG. P28.60(d),

$$\Delta V_{ac} = \Delta V_{db} = I_1(6.00\ \Omega) = 6.00\text{ V}$$

and $\Delta V_{cd} = I_1(3.00\ \Omega) = 3.00\text{ V}$

From ANS. FIG. P28.60(c),

$$I_2 = I_3 = \frac{\Delta V_{cd}}{6.00\ \Omega} = \frac{3.00\text{ V}}{6.00\ \Omega} = 0.500\text{ A}$$

From ANS. FIG. P28.60(b),

$$\Delta V_{ed} = I_3(3.60\ \Omega) = 1.80\text{ V}$$

Then, from ANS. FIG. P28.60(a),

$$I_4 = \frac{\Delta V_{ed}}{6.00\ \Omega} = \frac{1.80\text{ V}}{6.00\ \Omega} = 0.300\text{ A}$$

and $I_5 = \dfrac{\Delta V_{fd}}{9.00\ \Omega} = \dfrac{\Delta V_{ed}}{9.00\ \Omega} = \dfrac{1.80\text{ V}}{9.00\ \Omega} = 0.200\text{ A}$

From ANS. FIG. P28.60(b),

$$\Delta V_{ce} = I_3(2.40\ \Omega) = 1.20\text{ V}.$$

The collected results are:

(b) $\boxed{\Delta V_{ac} = \Delta V_{db} = 6.00\text{ V},\ \Delta V_{ce} = 1.20\text{ V},\ \Delta V_{fd} = \Delta V_{ed} = 1.80\text{ V},\ \Delta V_{cd} = 3.00\text{ V}}$

(c) $\boxed{I_1 = 1.00\text{ A}, I_2 = 0.500\text{ A}, I_3 = 0.500\text{ A}, I_4 = 0.300\text{ A}, I_5 = 0.200\text{ A}}$

(d) The power dissipated in each resistor is found from $P = (\Delta V)^2/R$ with the following results:

$$P_{ac} = \frac{(\Delta V)_{ac}^2}{R_{ac}} = \frac{(6.00\text{ V})^2}{6.00\ \Omega} = \boxed{6.00\text{ W}}$$

$$P_{ce} = \frac{(\Delta V)_{ce}^2}{R_{ce}} = \frac{(1.20\text{ V})^2}{2.40\ \Omega} = \boxed{0.600\text{ W}}$$

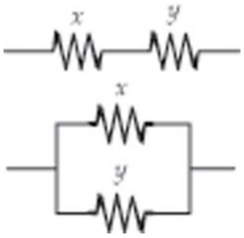
$$P_{ed} = \frac{(\Delta V)_{ed}^2}{R_{ed}} = \frac{(1.80\text{ V})^2}{6.00\ \Omega} = \boxed{0.540\text{ W}}$$

$$P_{fd} = \frac{(\Delta V)_{fd}^2}{R_{fd}} = \frac{(1.80\text{ V})^2}{9.00\ \Omega} = \boxed{0.360\text{ W}}$$

$$P_{cd} = \frac{(\Delta V)_{cd}^2}{R_{cd}} = \frac{(3.00\text{ V})^2}{6.00\ \Omega} = \boxed{1.50\text{ W}}$$

$$P_{db} = \frac{(\Delta V)_{db}^2}{R_{db}} = \frac{(6.00\text{ V})^2}{6.00\ \Omega} = \boxed{6.00\text{ W}}$$

P28.61 Let the two resistances be x and y.



ANS. FIG. P28.61

Then,

$$R_s = x + y = \frac{P_s}{I^2} = \frac{225\text{ W}}{(5.00\text{ A})^2} = 9.00\ \Omega \to y = 9.00\ \Omega - x$$

and $$R_p = \frac{xy}{x+y} = \frac{P_p}{I^2} = \frac{50.0\text{ W}}{(5.00\text{ A})^2} = 2.00\ \Omega$$

so $$\frac{x(9.00\ \Omega - x)}{x + (9.00\ \Omega - x)} = 2.00\ \Omega$$

$$x^2 - 9.00x + 18.0 = 0$$

Factoring the second equation,

$$(x-6.00)(x-3.00)=0$$

so $x = 6.00\ \Omega$ or $x = 3.00\ \Omega$

Then, $y = 9.00\ \Omega - x$ gives

$$y = 3.00\ \Omega \quad \text{or} \quad y = 6.00\ \Omega$$

There is only one physical answer: The two resistances are $\boxed{6.00\ \Omega}$ and $\boxed{3.00\ \Omega}$.

P28.62 Refer to ANS. FIG. P28.61 above. Let the two resistances be x and y.

Then, $R_s = x + y = \frac{P_s}{I^2}$ and $R_p = \frac{xy}{x+y} = \frac{P_p}{I^2}$.

From the first equation, $y = \frac{P_s}{I^2} - x$, and the second

becomes $\frac{x\left(P_s/I^2 - x\right)}{x + \left(P_s/I^2 - x\right)} = \frac{P_p}{I^2}$ or $x^2 - \left(\frac{P_s}{I^2}\right)x + \frac{P_sP_p}{I^4} = 0$.

Using the quadratic formula, $x = \frac{P_s \pm \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$.

Then, $y = \frac{P_s}{I^2} - x$ gives $y = \frac{P_s \mp \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$.

The two resistances are $\boxed{\frac{P_s + \sqrt{P_s^2 - 4P_sP_p}}{2I^2}}$ and $\boxed{\frac{P_s - \sqrt{P_s^2 - 4P_sP_p}}{2I^2}}$.

P28.63 (a) The equivalent capacitance of this parallel combination is

$$C_{eq} = C_1 + C_2 = 3.00\ \mu\text{F} + 2.00\ \mu\text{F} = 5.00\ \mu\text{F}$$

When fully charged by a 12.0-V battery, the total stored charge before the switch is closed is

$$Q_0 = C_{eq}(\Delta V) = (5.00\ \mu\text{F})(12.0\ \text{V}) = 60.0\ \mu\text{C}$$

Once the switch is closed, the time constant of the resulting RC circuit is

$$\tau = RC_{eq} = \left(5.00\times10^2\ \Omega\right)(5.00\ \mu\text{F}) = 2.50\times10^{-3}\ \text{s} = 2.50\ \text{ms}$$

Thus, at $t = 1.00$ ms after closing the switch, the remaining total stored charge is

$$q = Q_0e^{-t/\tau} = (60.0\ \mu\text{C})e^{-1.00\ \text{ms}/2.50\ \text{ms}} = (60.0\ \mu\text{C})e^{-0.400} = 40.2\ \mu\text{C}$$

The potential difference across the parallel combination of capacitors is then

$$\Delta V = \frac{q}{C_{eq}} = \frac{40.2\ \mu\text{C}}{5.00\ \mu\text{F}} = 8.04\ \text{V}$$

and the charge remaining on the 3.00 μF capacitor will be

$$q_3 = C_3(\Delta V) = (3.00\ \mu\text{F})(8.04\ \text{V}) = \boxed{24.1\ \mu\text{C}}$$

(b) The charge remaining on the 2.00 μF at this time is

$$q_2 = q - q_3 = 40.2\ \mu\text{C} - 24.1\ \mu\text{C} = \boxed{16.1\ \mu\text{C}}$$

or, alternately,

$$q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(8.04\ \text{V}) = \boxed{16.1\ \mu\text{C}}$$

(c) Since the resistor is in parallel with the capacitors, it has the same potential difference across it as do the capacitors at all times. Thus, Ohm's law gives

$$I = \frac{\Delta V}{R} = \frac{8.04\ \text{V}}{5.00 \times 10^2\ \Omega} = 1.61 \times 10^{-2}\ \text{A} = \boxed{16.1\ \text{mA}}$$

P28.64 (a) Around the circuit,

$$\varepsilon - I\left(\sum R\right) - (\varepsilon_1 + \varepsilon_2) = 0$$

Substituting numerical values,

$$40.0\ \text{V} - (4.00\ \text{A})\left[(2.00 + 0.300 + 0.300 + R)\Omega\right] - (6.00 + 6.00)\ \text{V} = 0$$

so $R = \boxed{4.40\ \Omega}$

(b) Inside the supply,

$$P = I^2R = (4.00\ \text{A})^2(2.00\ \Omega) = \boxed{32.0\ \text{W}}$$

(c) Inside both batteries together,

$$P = I^2R = (4.00\ \text{A})^2(0.600\ \Omega) = \boxed{9.60\ \text{W}}$$

(d) For the limiting resistor,

$$P = (4.00\ \text{A})^2(4.40\ \Omega) = \boxed{70.4\ \text{W}}$$

(e) $P = I(\varepsilon_1 + \varepsilon_2) = (4.00\ \text{A})\left[(6.00 + 6.00)\ \text{V}\right] = \boxed{48.0\ \text{W}}$

P28.65 The total resistance in the circuit is

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{2.00\ \text{k}\Omega} + \frac{1}{3.00\ \text{k}\Omega}\right)^{-1} = 1.20\ \text{k}\Omega$$

and the total capacitance is

$$C = C_1 + C_2 = 2.00\ \mu\text{F} + 3.00\ \mu\text{F} = 5.00\ \mu\text{F}$$

Thus, $Q_{\text{max}} = C\mathcal{E} = (5.0\ \mu\text{F})(120\ \text{V}) = 600\ \mu\text{C}$

and $\tau = RC = (1.2\times 10^3\ \Omega)(5.0\times 10^{-6}\ \text{F}) = 6.0\times 10^{-3}\ \text{s} = \dfrac{6.0\ \text{s}}{1\,000}$

The total stored charge at any time *t* is then

$$q = q_1 + q_2 = Q_{\text{max}}\left(1 - e^{-t/\tau}\right)$$

or $q_1 + q_2 = (600\ \mu\text{C})\left(1 - e^{-1\,000t/6.0\ \text{s}}\right)$ **[1]**

Since the capacitors are in parallel with each other, the same potential difference exists across both at any time.

Therefore,

$$(\Delta V)_C = \frac{q_1}{C_1} = \frac{q_2}{C_2} \quad \to \quad q_2 = \left(\frac{C_2}{C_1}\right)q_1 = 1.5q_1$$ **[2]**

(a) Substituting equation [2] into [1] gives

$$2.5q_1 = (600\ \mu\text{C})\left(1 - e^{-1\,000t/6.0\ \text{s}}\right)$$

$$q_1 = \left(\frac{600\ \mu\text{C}}{2.5}\right)\left(1 - e^{-t/(6.0\ \text{s}/1\,000)}\right)$$

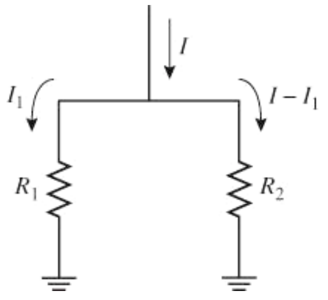
$$q_1 = 240\ \mu\text{C}\left(1 - e^{-t/6\ \text{ms}}\right)$$

or $\boxed{q = 240\left(1 - e^{-t/6}\right)}$, $\boxed{\text{where } q \text{ is in microcoulombs and } t \text{ is in milliseconds}}$.

(b) and from equation [2],

$$q_2 = 1.5q_1 = 1.5\left[240\ \mu\text{C}\left(1 - e^{-t/6\ \text{ms}}\right)\right] = 360\ \mu\text{C}\left(1 - e^{-t/6\ \text{ms}}\right)$$

or, $q = \boxed{360\left(1 - e^{-t/6}\right)}$, $\boxed{\text{where } q \text{ is in microcoulombs and } t \text{ is in milliseconds}}$.



ANS. FIG. P28.66

P28.66 (a) In the diagram we could show the two resistors connected top end to top end and bottom end to bottom end with wires; we represent this connection instead by showing the bottom ends of both resistors connected to ground. The ground represents a conductor that is always at zero volts, and also can carry any current. Think of I, R_1, and R_2 as known quantities. We represent the current in R_1 as the unknown I_1. Then the current in the second resistor must by $I - I_1$. The total potential difference clockwise around the little loop containing both resistors must be zero:

$$-(I - I_1)R_2 + I_1R_1 + 0$$

We can already solve for I_1 in terms of the total current:

$$-IR_2 + I_1R_2 + I_1R_1 = 0 \quad \rightarrow \quad I_1 = \boxed{IR_2/(R_1 + R_2)}$$

Then the current in the second resistor is

$$I_2 = I - I_1 = I - IR_2/(R_1 + R_2) = I(R_1 + R_2 - R_2)/(R_1 + R_2)$$

$$I_2 = \boxed{IR_1/(R_1 + R_2)}$$

(b) Continue to think of I, R_1, and R_2 as known quantities and I_1 as an unknown. The power being converted by both resistors together is $P = I_1^2R_1 + (I - I_1)^2R_2$. Because the current is squared, the power would be extra large if all of the current went through either one of the resistors with zero current in the other. The minimum power condition must be with a more equitable division of current, and we find it by taking the derivative of P with respect to I_1 and setting the derivative equal to zero:

$$dP/dI_1 = 2\,I_1R_1 + 2(I - I_1)(0 - 1)R_2 = 0$$

Again we can solve directly for the real value of I_1 in

$$I_1R_1 - IR_2 + I_1R_2 = 0 \qquad \text{as} \qquad I_1 = IR_2/(R_1 + R_2)$$

So then again

$$I_2 = I - I_1 = IR_1/(R_1 + R_2)$$

This power-minimizing division of current is the same as the voltage-equalizing division of current that we found in part (a), so the proof is complete.

P28.67 (a) The charge on the capacitor at this instant is

$$q = C\Delta V\left(1 - e^{-t/RC}\right)$$

$$q = \left(1.00\times 10^{-6}\text{ F}\right)\left(10.0\text{ V}\right)\left[1 - e^{-10.0\text{ s}/\left[\left(2.00\times 10^{6}\ \Omega\right)\left(1.00\times 10^{-6}\text{ F}\right)\right]}\right]$$

$$= \boxed{9.93\ \mu\text{C}}$$

(b) The current in the resistor is given by

$$I = \frac{dq}{dt} = \left(\frac{\Delta V}{R}\right)e^{-t/RC}$$

$$I = \left(\frac{10.0\text{ V}}{2.00\times 10^{6}\ \Omega}\right)e^{-5.00} = 3.37\times 10^{-8}\text{ A} = \boxed{33.7\text{ nA}}$$

(c) Since the energy stored in the capacitor is $U = q^2/2C$, the rate of storing energy is

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{q^2}{C}\right) = \left(\frac{q}{C}\right)\frac{dq}{dt} = \left(\frac{q}{C}\right)I$$

$$\frac{dU}{dt} = \left(\frac{9.93\times 10^{-6}\text{ C}}{1.00\times 10^{-6}\text{ C/V}}\right)\left(3.37\times 10^{-8}\text{ A}\right)$$

$$= 3.34\times 10^{-7}\text{ W} = \boxed{334\text{ nW}}$$

(d) $$P_{\text{battery}} = I\mathcal{E} = \left(3.37\times 10^{-8}\text{ A}\right)\left(10.0\text{ V}\right) = 3.37\times 10^{-7}\text{ W} = \boxed{337\text{ nW}}$$

The battery power could also be computed as the sum of the instantaneous powers delivered to the resistor and to the capacitor:

$$I^2R + \frac{dU}{dt} = \left(33.7\times 10^{-9}\text{ A}\right)\left(2.00\times 10^{6}\ \Omega\right) + 334\text{ nW} = 337\text{ nW}$$

P28.68 The battery supplies energy at a changing rate

$$\frac{dE}{dt} = P = \mathcal{E}I = \mathcal{E}\left(\frac{\mathcal{E}}{R}e^{-t/RC}\right)$$

Then the total energy put out by the battery is

$$\int dE = \int_{t=0}^{\infty}\frac{\mathcal{E}^2}{R}\exp\left(-\frac{t}{RC}\right)d$$

$$\int dE = \frac{\mathcal{E}^2}{R}(-RC)\int_0^{\infty}\exp\left(-\frac{t}{RC}\right)\left(-\frac{dt}{RC}\right)$$

$$= -\mathcal{E}^2C\exp\left(-\frac{t}{RC}\right)\Bigg|_0^{\infty} = -\mathcal{E}^2C\left[0-1\right] = \mathcal{E}^2C$$

The power delivered to the resistor is

$$\frac{dE}{dt} = P = \Delta V_R I = I^2 R = R\frac{\mathcal{E}^2}{R^2}\exp\left(-\frac{2t}{RC}\right)$$

So the total internal energy appearing in the resistor is

$$\int dE = \int_0^\infty \frac{\mathcal{E}^2}{R}\exp\left(-\frac{2t}{RC}\right)dt$$

$$\int dE = \frac{\mathcal{E}^2}{R}\left(-\frac{RC}{2}\right)\int_0^\infty \exp\left(-\frac{2t}{RC}\right)\left(-\frac{2dt}{RC}\right)$$

$$= -\frac{\mathcal{E}^2 C}{2}\exp\left(-\frac{2t}{RC}\right)\Bigg|_0^\infty = -\frac{\mathcal{E}^2 C}{2}[0-1] = \frac{\mathcal{E}^2 C}{2}$$

The energy finally stored in the capacitor is $U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}C\mathcal{E}^2$.

Thus, energy of the circuit is conserved, $\mathcal{E}^2 C = \frac{1}{2}\mathcal{E}^2 C + \frac{1}{2}\mathcal{E}^2 C$, and resistor and capacitor share equally in the energy from the battery.

P28.69 (a) We find the resistance intrinsic to the vacuum cleaner:

$$P = I\Delta V = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\text{ V})^2}{535\text{ W}} = 26.9\ \Omega$$

with the inexpensive cord, the equivalent resistance is

$$R_{\text{Tot}} = R + 2r$$

$$= 26.9\ \Omega + 2(0.9\ \Omega) = 28.7\ \Omega$$

so the current throughout the circuit is

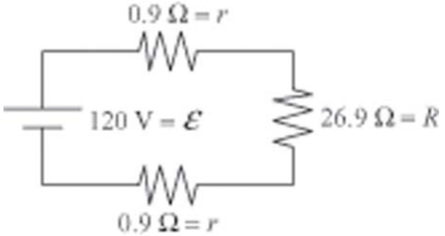
$$I = \frac{\Delta V}{R_{\text{Tot}}} = \frac{120\text{ V}}{28.7\ \Omega} = 4.18\text{ A}$$

and the cleaner power is

$$P_{\text{cleaner}} = I(\Delta V)_{\text{cleaner}} = I^2 R = (4.18\text{ A})^2(26.9\ \Omega) = \boxed{470\text{ W}}$$

In symbols,

$$R_{\text{tot}} = R + 2r,\ I = \frac{\Delta V}{R+2r} \text{ and } P_{\text{cleaner}} = I^2 R = \frac{(\Delta V)^2 R}{(R+2r)^2}$$



ANS. FIG. P28.69

(b) Using $P_{\text{cleaner}} = I^2R = \frac{(\Delta V)^2 R}{(R+2r)^2}$, we find that

$$R + 2r = \left(\frac{(\Delta V)^2 R}{P_{\text{cleaner}}}\right)^{1/2}$$

solving for *r* gives

$$r = \frac{\Delta V}{2}\left(\frac{R}{P_{\text{cleaner}}}\right)^{1/2} - \frac{R}{2} = \frac{120\text{ V}}{2}\left(\frac{26.9\ \Omega}{525\text{ W}}\right)^{1/2} - \frac{26.9\ \Omega}{2}$$

$$= 0.128\ \Omega = \frac{\rho\ell}{A} = \frac{\rho\ell 4}{\pi d^2}$$

then,

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(15\text{ m})}{\pi(0.128\ \Omega)}\right)^{1/2}$$

$$= \boxed{1.60\text{ mm or more}}$$

(c) To move from 525 W to 532 W will require a lot more copper:

$$r = \frac{\Delta V}{2}\left(\frac{R}{P_{\text{cleaner}}}\right)^{1/2} - \frac{R}{2} = \frac{120\text{ V}}{2}\left(\frac{26.9\ \Omega}{532\text{ W}}\right)^{1/2} - \frac{26.9\ \Omega}{2}$$

$$= 0.037\,9\ \Omega$$

$$d = \left(\frac{4\rho\ell}{\pi r}\right)^{1/2} = \left(\frac{4(1.7\times10^{-8}\ \Omega\cdot\text{m})(15\text{ m})}{\pi(0.037\,9\ \Omega)}\right)^{1/2}$$

$$= \boxed{2.93\text{ mm or more}}$$

P28.70 (a) When the capacitor is fully charged, no current exists in its branch. The current in the left resistors is $I_L = 5.00\text{ V}/83.0\ \Omega$. The current in the right resistors is $I_R = 5.00\text{ V}/(2.00\ \Omega + R)$.

Relative to the positive side of the battery, the left capacitor plate is at voltage

$$V_L = 5.00\text{ V} - (3.00\ \Omega)\left(\frac{5.00\text{ V}}{83.0\ \Omega}\right) = (5.00\text{ V})\left(1 - \frac{3.00}{83.0}\right)$$

and the right plate is at voltage

$$V_R = 5.00\text{ V} - \frac{(2.00\ \Omega)(5.00\text{ V})}{2.00\ \Omega + R} = (5.00\text{ V})\left(1 - \frac{2.00}{2.00 + R}\right)$$

where R is in ohms. The voltage across the capacitor is

$$\Delta V = V_L - V_R = (5.00\text{ V})\left(1 - \frac{3.00}{83.0}\right) - (5.00\text{ V})\left(1 - \frac{2.00}{2.00 + R}\right)$$

$$\Delta V = (5.00\text{ V})\left(\frac{2.00}{2.00 + R} - \frac{3.00}{83.0}\right)$$

The charge on the capacitor is

$$q = C\Delta V = (3.00\ \mu\text{C})(5.00\text{ V})\left(\frac{2.00}{2.00 + R} - \frac{3.00}{83.0}\right)$$

$$q = (15.0\ \mu\text{C})\left(\frac{2.00}{2.00 + R} - \frac{3.00}{83.0}\right)$$

$$\boxed{q = \frac{30.0}{2.00 + R} - 0.542, \text{ where } q \text{ is in microcoulombs and } R \text{ is in ohms.}}$$

(b) With $R = 10.0\ \Omega$,

$$q = \frac{30.0}{2.00 + R} - 0.542 = \frac{30.0}{2.00 + 10.0} - 0.542 = \boxed{1.96\ \mu\text{C}}$$

(c) $\boxed{\text{Yes.}}$ Setting $q = 0$, and solving for R,

$$q = (15.0\ \mu\text{C})\left[\frac{2.00}{2.00 + R} - \frac{3.00}{83.0}\right] = 0$$

$$R = \frac{2.00(83.0)}{3.00} - 2.00 = \boxed{53.3\ \Omega}$$

(d) By inspection, the maximum charge occurs for $R = 0$. It is

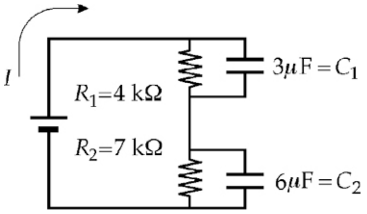
$$q = (15.0\ \mu\text{C})\left[\frac{2.00}{2.00+0} - \frac{3.00}{83.0}\right] = \boxed{14.5\ \mu\text{C}}$$

(e) $\boxed{\text{Yes. Taking } R = \infty \text{ corresponds to disconnecting the wire}}$ to remove the branch containing R:

$$|q| = (15.0\ \mu\text{C})\left|\frac{2.00}{2.00+\infty} - \frac{3.00}{83.0}\right| = (15.0\ \mu\text{C})\frac{3.00}{83.0} = \boxed{0.542\ \mu\text{C}}$$

P28.71 (a) With the switch closed, current exists in a simple series circuit as shown. The capacitors carry no current. For R_2 we have

$$P = I^2R_2 \text{ and } I = \sqrt{\frac{P}{R_2}} = \sqrt{\frac{2.40\ \text{V}\cdot\text{A}}{7\ 000\ \text{V/A}}} = 18.5\ \text{mA}$$



ANS. FIG. P28.71(a)

The potential difference across R_1 and C_1 is

$$\Delta V = IR_1 = (1.85\times10^{-2}\ \text{A})(4\ 000\ \text{V/A}) = 74.1\ \text{V}$$

The charge on C_1 is

$$Q = C_1\Delta V = (3.00\times10^{-6}\ \text{C/V})(74.1\ \text{V}) = \boxed{222\ \mu\text{C}}$$

The potential difference across R_2 and C_2 is

$$\Delta V = IR_2 = (1.85\times10^{-2}\ \text{A})(7\ 000\ \Omega) = 130\ \text{V}$$

The charge on C_2

$$Q = C_2\Delta V = (6.00\times10^{-6}\ \text{C/V})(130\ \text{V}) = 778\ \mu\text{C}$$

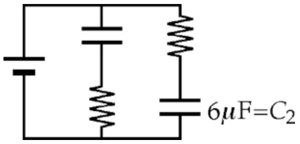
The battery emf is

$$IR_{\text{eq}} = I(R_1 + R_2) = (1.85\times10^{-2}\ \text{A})(4\ 000\ \Omega + 7\ 000\ \Omega) = 204\ \text{V}$$

(b) In equilibrium after the switch has been opened, no current exists. The potential difference across each resistor is zero. The full 204 V appears across both capacitors. The new charge on C_2 is

$$Q = C_2\Delta V = (6.00\times10^{-6}\ \text{C/V})(204\ \text{V}) = 1\ 222\ \mu\text{C}$$

for a change of $1\,222\ \mu\text{C} - 778\ \mu\text{C} = \boxed{444\ \mu\text{C}}$.



ANS. FIG. P28.71(b)

P28.72 (a) First determine the resistance of each light bulb. From $P = \frac{(\Delta V)^2}{R}$, we have

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\ \text{V})^2}{60.0\ \text{W}} = 240\ \Omega$$

We obtain the equivalent resistance R_{eq} of the network of light bulbs by identifying series and parallel equivalent resistances:

$$R_{\text{eq}} = R_1 + \frac{1}{(1/R_2) + (1/R_3)} = 240\ \Omega + 120\ \Omega = 360\ \Omega$$

The total power dissipated in the 360 Ω is

$$P = \frac{(\Delta V)^2}{R_{\text{eq}}} = \frac{(120\ \text{V})^2}{360\ \Omega} = \boxed{40.0\ \text{W}}$$

(b) The current through the network is given by $\Delta V = IR_{\text{eq}}$:

$$I = \frac{120\ \text{V}}{360\ \Omega} = \frac{1}{3}\ \text{A}$$

The potential difference across R_1 is

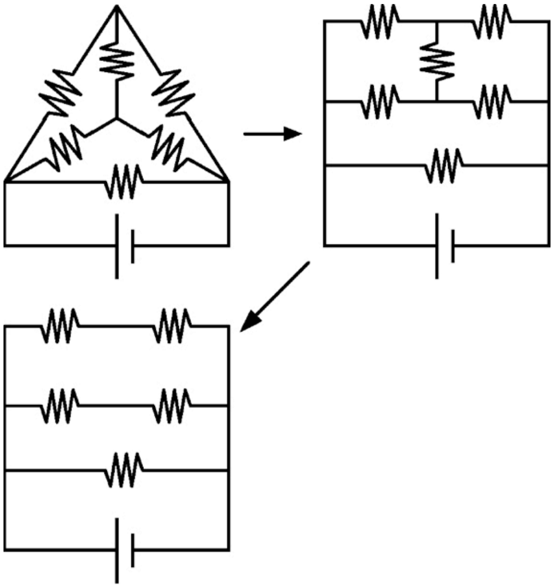
$$\Delta V_1 = IR_1 = \left(\frac{1}{3}\ \text{A}\right)(240\ \Omega) = \boxed{80.0\ \text{V}}$$

The potential difference ΔV_{23} across the parallel combination of R_2 and R_3 is

$$\Delta V_{23} = IR_{23} = \left(\frac{1}{3}\ \text{A}\right)\left(\frac{1}{(1/240\ \Omega) + (1/240\ \Omega)}\right) = \boxed{40.0\ \text{V}}$$

P28.73 (a) First let us flatten the circuit on a 2-D plane as shown in ANS. FIG. P28.73; then reorganize it to a format easier to read. Notice that the two resistors shown in the top horizontal branch carry the same current as the resistors in the horizontal branch second from the top. The center junctions in these two branches are at the same potential. The vertical resistor between these two junctions has no potential difference across it and carries no current. This middle resistor can be removed without affecting the circuit. The remaining resistors over the three parallel branches have equivalent resistance

$$R_{eq} = \left(\frac{1}{20} + \frac{1}{20} + \frac{1}{10}\right)^{-1} = \boxed{5.00\ \Omega}$$



ANS. FIG. P28.73

(b) So the current through the battery is

$$\frac{\Delta V}{R_{eq}} = \frac{12.0\ \text{V}}{5.00\ \Omega} = \boxed{2.40\ \text{A}}$$

P28.74 (a) The emf of the battery is $\boxed{9.30\ \text{V}}$.

(b) Its internal resistance is given by

$$\Delta V = 9.30\ \text{V} - (3.70\ \text{A})r = 0 \quad \rightarrow \quad r = \boxed{2.51\ \Omega}$$

(c) The batteries are in series: Total emf = 2(9.30 V) = $\boxed{18.6\ \text{V}}$.

(d) The batteries are in series, so their total internal resistance is $2r = 5.03\ \Omega$. The maximum current is given by

$$I = \frac{\Delta V}{R} = \frac{18.6\ \text{V}}{5.03\ \Omega} = \boxed{3.70\ \text{A}}$$

(e) For the circuit the total series resistance is

$$R_{eq} = 2r + 12.0\ \Omega = 17.0\ \Omega$$

and

$$I = \frac{\Delta V}{R} = \frac{18.6\text{ V}}{17.0\ \Omega} = \boxed{1.09\text{ A}}$$

(f) $P = I^2R = (1.09\text{ A})^2(12.0\ \Omega) = \boxed{14.3\text{ W}}$

(g) The two 12.0-Ω resistors in parallel are equivalent to one 6.00-Ω, Resistor, and this is in series with the internal resistances of the batteries: $R_{\text{eq}} = 6.00\ \Omega + 2r = 11.0\ \Omega$. Therefore, the current in the batteries is

$$I = \frac{\Delta V}{R} = \frac{18.6\text{ V}}{11.0\ \Omega} = 1.69\text{ A}$$

and the terminal voltage across both batteries is

$$\Delta V = \mathcal{E} - I(2r) = 18.6\text{ V} - (1.69\text{ A})(5.03\ \Omega) = 10.1\text{ V}$$

The power delivered to each resistor is

$$P = \frac{(\Delta V)^2}{R} = \frac{(10.1\text{ V})^2}{12.0\ \Omega} = \boxed{8.54\text{ W}}$$

(h) $\boxed{\text{Because of the internal resistance of the batteries, the terminal voltage of the pair of batteries is not the same in both cases.}}$

P28.75 (a) After steady-state conditions have been reached, there is no DC current through the capacitor.

Thus, for R_3 : $\boxed{I_{R_3} = 0\ (\text{steady-state})}$

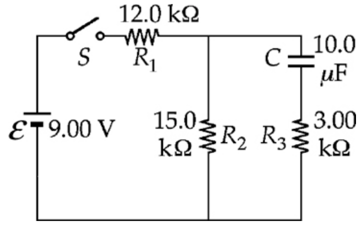
For the other two resistors, the steady-state current is simply determined by the 9.00-V emf across the 12-kΩ and 15-kΩ resistors in series:

For R_1 and R_2:

$$I_{(R_1+R_2)} = \frac{\mathcal{E}}{R_1 + R_2} = \frac{9.00\text{ V}}{(12.0\text{ k}\Omega + 15.0\text{ k}\Omega)}$$
$$= \boxed{333\ \mu\text{A}\ (\text{steady-state})}$$

(b) After the transient currents have ceased, the potential difference across C is the same as the potential difference across $R_2 (= IR_2)$ because there is no voltage drop across R_3. Therefore, the charge Q on C is

$$Q = C(\Delta V)_{R_2} = C(IR_2) = (10.0\ \mu\text{F})(333\ \mu\text{A})(15.0\text{ k}\Omega)$$
$$= \boxed{50.0\ \mu\text{C}}$$



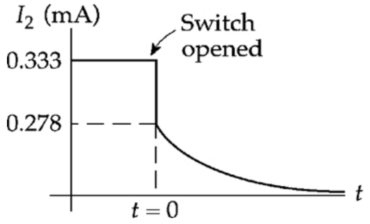
ANS. FIG. P28.75(b)

(c) When the switch is opened, the branch containing R_1 is no longer part of the circuit. The capacitor discharges through $(R_2 + R_3)$ with a time constant of

$$(R_2 + R_3)C = (15.0\text{ k}\Omega + 3.00\text{ k}\Omega)(10.0\ \mu\text{F}) = 0.180\text{ s}$$

The initial current I_i in this discharge circuit is determined by the initial potential difference across the capacitor applied to $(R_2 + R_3)$ in series:

$$I_i = \frac{(\Delta V)_C}{(R_2 + R_3)} = \frac{IR_2}{(R_2 + R_3)} = \frac{(333\ \mu\text{A})(15.0\text{ k}\Omega)}{(15.0\text{ k}\Omega + 3.00\text{ k}\Omega)} = 278\ \mu\text{A}$$



ANS. FIG. P28.75(c)

Thus, when the switch is opened, the current through R_2 changes instantaneously from 333 μA (downward) to 278 μA (downward) as shown in the graph. Thereafter, it decays according to

$$I_{R_2} = I_i e^{-t/(R_2+R_3)C} = \boxed{(278\ \mu\text{A})e^{-t/(0.180\text{ s})}\ \ (\text{for } t > 0)}$$

(d) The charge q on the capacitor decays from Q_i to $\frac{Q_i}{5}$ according to

$$q = Q_i e^{-t/(R_2+R_3)C}$$

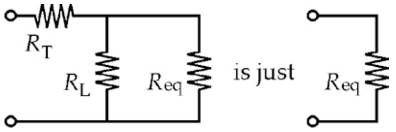
$$\frac{Q_i}{5} = Q_i e^{(-t/0.180\text{ s})}$$

$$5 = e^{t/0.180\text{ s}}$$

$$\ln 5 = \frac{t}{180\text{ ms}}$$

$$t = (0.180\text{ s})(\ln 5) = \boxed{290\text{ ms}}$$

P28.76 From the hint, the equivalent resistance of



That is, $$R_T + \frac{1}{1/R_L + 1/R_{eq}} = R_{eq}$$

$$R_T + \frac{R_L R_{eq}}{R_L + R_{eq}} = R_{eq}$$

$$R_T R_L + R_T R_{eq} + R_L R_{eq} = R_L R_{eq} + R_{eq}^2$$

$$R_{eq}^2 - R_T R_{eq} - R_T R_L = 0$$

$$R_{eq} = \frac{R_T \pm \sqrt{R_T^2 - 4(1)(-R_T R_L)}}{2(1)}$$

Only the + sign is physical:

$$R_{eq} = \frac{1}{2}\left(\sqrt{4R_T R_L + R_T^2} + R_T\right)$$

For example, if $R_T = 1\ \Omega$ and $R_L = 20\ \Omega$, then $R_{eq} = 5\ \Omega$.

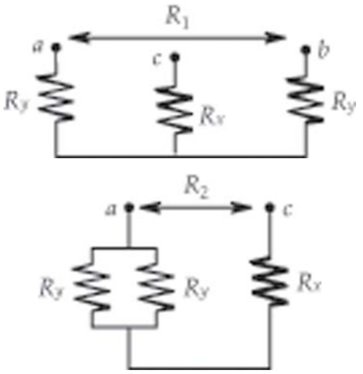
P28.77 (a) For the first measurement, the equivalent circuit is as shown in the top panel of ANS. FIG. P28.77.

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

so $$R_y = \frac{1}{2}R_1. \qquad \textbf{[1]}$$

For the second measurement, the equivalent circuit is shown in the bottom panel of ANS. FIG. P28.77. Thus,

$$R_{ac} = R_2 = \frac{1}{2}R_y + R_x \qquad \textbf{[2]}$$



ANS. FIG. P28.77

Substitute [1] into [2] to obtain:

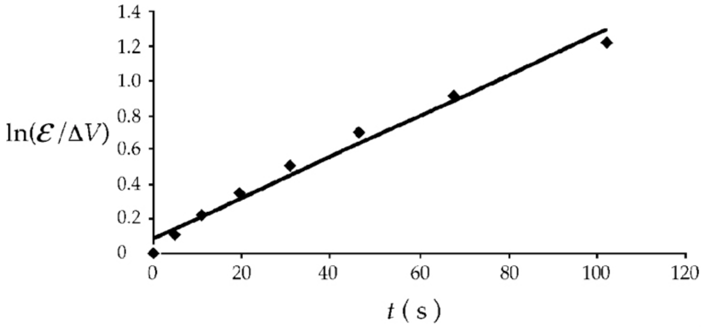
$$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x, \quad \text{or} \quad \boxed{R_x = R_2 - \frac{1}{4}R_1}$$

(b) If $R_1 = 13.0\ \Omega$ and $R_2 = 6.00\ \Omega$, then $\boxed{R_x = 2.75\ \Omega}$.

$\boxed{\text{The antenna is inadequately grounded}}$ since this exceeds the limit of 2.00 Ω.

P28.78 $\Delta V = \mathcal{E} e^{-t/RC}$ so $\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right) = \left(\dfrac{1}{RC}\right)t.$

A plot of $\ln\left(\dfrac{\mathcal{E}}{\Delta V}\right)$ versus *t* should be a straight line with slope equal to $\dfrac{1}{RC}$, as shown in ANS. FIG. P28.78.



ANS. FIG. P28.78

Using the given data values:

(a) A least-square fit to this data yields the graph shown in ANS. FIG. P28.78.

$$\sum x_i = 282, \quad \sum x_i^2 = 1.86 \times 10^4,$$

$$\sum x_i y_i = 244, \ \sum y_i = 4.03, \quad N = 8$$

$$\text{Slope} = \frac{N\left(\sum x_i y_i\right) - \left(\sum x_i\right)\left(\sum y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.011\,8$$

t (s)	Δ*V* (*V*)	ln(ℰ/Δ*V*)
0	6.19	0
4.87	5.55	0.109
11.1	4.93	0.228
19.4	4.34	0.355
30.8	3.72	0.509
46.6	3.09	0.695
67.3	2.47	0.919
102.2	1.83	1.219

$$\text{Intercept} = \frac{\left(\sum x_i^2\right)\left(\sum y_i\right) - \left(\sum x_i\right)\left(\sum x_i y_i\right)}{N\left(\sum x_i^2\right) - \left(\sum x_i\right)^2} = 0.088\,2$$

The equation of the best fit line is: $\boxed{\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = (0.011\,8)t + 0.088\,2}$

(b) Thus, the time constant is

$$\tau = RC = \frac{1}{\text{slope}} = \frac{1}{0.011\,8} = \boxed{84.7\text{ s}}$$

and the capacitance is

$$C = \frac{\tau}{R} = \frac{84.7\text{ s}}{10.0 \times 10^6\ \Omega} = \boxed{8.47\ \mu\text{F}}$$

P28.79 A certain quantity of energy $\Delta E_{\text{int}} = P\Delta t$ is required to raise the temperature of the water to 100°C in time interval Δt. For the power delivered to the heaters we have $P = I\Delta V = \frac{(\Delta V)^2}{R}$ where ΔV is a constant. Thus, comparing coils 1 and 2, we have for the energy $\Delta E_{\text{int}} = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 2\Delta t}{R_2}$. Therefore, $R_2 = 2R_1$.

(a) When connected in parallel, the coils present equivalent resistance

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R_1} + \frac{1}{2R_1} = \frac{3}{2R_1} \quad \to \quad R_p = \frac{2}{3}R_1.$$

Now,

$$\Delta E_{\text{int}} = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 \Delta t_p}{\frac{2}{3}R_1} \quad \to \quad \Delta t_p = \boxed{\frac{2}{3}\Delta t}$$

(b) For the series connection, $R_s = R_1 + R_2 = R_1 + 2R_1 = 3R_1$ and

$$\Delta E_{\text{int}} = \frac{(\Delta V)^2 \Delta t}{R_1} = \frac{(\Delta V)^2 \Delta t_s}{3R_1} \quad \to \quad \Delta t_s = \boxed{3\Delta t}$$

P28.80 When connected in series, the equivalent resistance is $R_{eq} = R_1 + R_2 + \cdots + R_n = nR$. Thus, the current is $I_s = (\Delta V)/R_{eq}$, and the power consumed by the series configuration is

$$P_s = I_s \Delta V = \frac{(\Delta V)^2}{R_{eq}} = \frac{(\Delta V)^2}{nR}$$

For the parallel connection, the power consumed by each individual resistor is $P_1 = \frac{(\Delta V)^2}{R}$, and the total power consumption is

$$P_p = nP_1 = \frac{n(\Delta V)^2}{R}$$

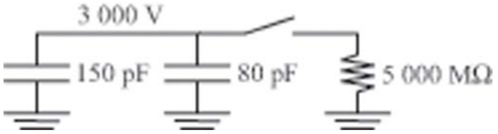
Therefore, $\frac{P_s}{P_p} = \frac{(\Delta V)^2}{nR} \cdot \frac{R}{n(\Delta V)^2} = \frac{1}{n^2}$ or $\boxed{P_s = \frac{1}{n^2} P_p}$

P28.81 We model the person's body and street shoes as shown in ANS. FIG. P28.81. For the discharge to reach 100 V,

$$q(t) = Qe^{-t/RC} = C\Delta V(t) = C\Delta V_0 e^{-t/RC}$$

$$\frac{\Delta V}{\Delta V_0} = e^{-t/RC} \quad \rightarrow \quad \frac{\Delta V_0}{\Delta V} = e^{+t/RC}$$

$$\rightarrow \quad \frac{t}{RC} = \ln\left(\frac{\Delta V_0}{\Delta V}\right)$$



ANS. FIG. P28.81

The equivalent capacitance for parallel capacitors is

150 pF + 80.0 pF = 230 pF.

(a) For R = 5.00 MΩ, a change from 3 000 V to 100 V requires that

$$t = RC\ln\left(\frac{\Delta V_0}{\Delta V}\right) = \left(5\ 000 \times 10^6\ \Omega\right)\left(230 \times 10^{-12}\ \text{F}\right)\ln\left(\frac{3\ 000\ \text{V}}{100\ \text{V}}\right)$$

$$= \boxed{3.91\ \text{s}}$$

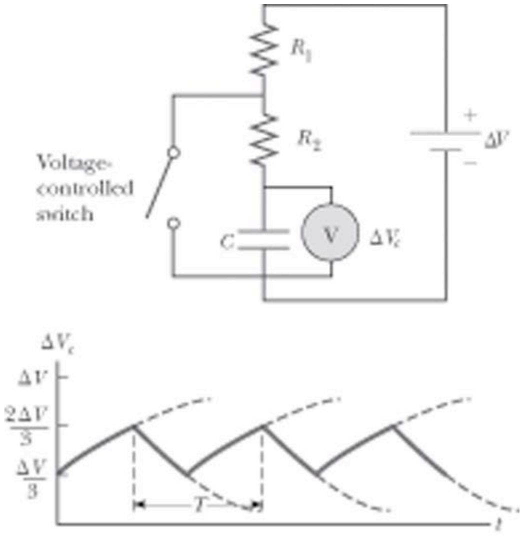
(b) For R = 1.00 MΩ, the same change requires that

$$t = RC\ln\left(\frac{\Delta V_0}{\Delta V}\right) = \left(1.00 \times 10^6\ \Omega\right)\left(230 \times 10^{-12}\ \text{F}\right)\ln\left(\frac{3\ 000\ \text{V}}{100\ \text{V}}\right)$$

$$= 7.82 \times 10^{-4}\ \text{s} = \boxed{782\ \mu\text{s}}$$

Challenge Problems

P28.82 Start at the point when the voltage has just reached $\frac{2}{3}\Delta V$ and the switch has just closed. The voltage is $\frac{2}{3}\Delta V$ and is decaying towards 0 V with a time constant R_2C.



ANS. FIG. P28.82

$$\Delta V_C(t)=\left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$$

We want to know when $\Delta V_C(t)$ will reach $\frac{1}{3}\Delta V$.

Therefore, $\frac{1}{3}\Delta V=\left[\frac{2}{3}\Delta V\right]e^{-t/R_2C}$

$$e^{-t/R_2C}=\frac{1}{2}$$

$$t_1=R_2C\ln 2$$

After the switch opens, the voltage is $\frac{1}{3}\Delta V$, increasing toward ΔV with time constant $(R_1+R_2)C$:

$$\Delta V_C(t)=\Delta V-\left[\frac{2}{3}\Delta V\right]e^{-t/(R_1+R_2)C}$$

When $\Delta V_C(t)=\frac{2}{3}\Delta V$,

$$\frac{2}{3}\Delta V=\Delta V-\frac{2}{3}\Delta V e^{-t/(R_1+R_2)C} \quad \text{or} \quad e^{-t/(R_1+R_2)C}=\frac{1}{2}$$

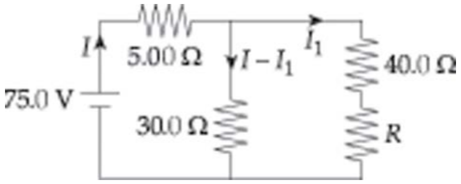
so $t_2=(R_1+R_2)C\ln 2$ and $T=t_1+t_2=\boxed{(R_1+2R_2)C\ln 2}$

P28.83 Assume a set of currents as shown in the circuit diagram in ANS. FIG. P28.83. Applying Kirchhoff's loop rule to the leftmost loop and suppressing units gives

$$+75.0-(5.00)I-(30.0)(I-I_1)=0$$

$$75.0-35.0I+30.0I_1=0$$

or $7I-6I_1=15$ **[1]**



ANS. FIG. P28.83

For the rightmost loop, the loop rule gives, suppressing units,

$$-(40.0 + R)I_1 + (30.0)(I - I_1) = 0$$
$$-(70.0 + R)I_1 + 30.0\,I = 0$$

or $$I = \left(\frac{7}{3} + \frac{R}{30}\right)I_1 \qquad \textbf{[2]}$$

Substituting equation [2] into [1] and simplifying gives

$$(310 + 7R)\,I_1 = 450 \qquad \textbf{[3]}$$

Also, it is known that $P_R = I_1^2 R = 20.0\text{ W}$,

so $$R = \frac{20.0\text{ W}}{I_1^2} \qquad \textbf{[4]}$$

Substituting equation [4] into [3] yields

$$310\,I_1 + \frac{140}{I_1} = 450$$

or $$310\,I_1^2 - 450\,I_1 + 140 = 0$$

Using the quadratic formula,

$$I_1 = \frac{-(-450) \pm \sqrt{(-450)^2 - 4(310)(140)}}{2(310)} = \frac{450 \pm 170}{620}$$

yielding $I_1 = 1.00$ A and $I_1 = 0.452$ A. Then, from $R = \dfrac{20.0\text{ W}}{I_1^2}$, we find two possible values for the resistance R. These are:

$$\boxed{R = 20.0\ \Omega \quad \text{or} \quad R = 98.1\ \Omega}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P28.2 (a) 4.59 Ω; (b) 8.16%

P28.4 (a) 50%; (b) 0; (c) High efficiency; (d) High power transfer

P28.6 (a) The 120-V potential difference is applied across the series combination of the two conductors in the extension cord and the lightbulb. The potential difference across the lightbulb is less than 120 V, and its power is less than 75 W; (b) See ANS. FIG. P28.6; (c) 73.8 W

P28.8 (a) $I_A = \mathcal{E}/R,\ I_B = I_C = \mathcal{E}/2R;$ (b) B and C have the same brightness because they carry the same current; (c) A is brighter than B or C because it carries twice as much current.

P28.10 (a) Connect two 50-Ω resistors in parallel to get 25 Ω. Then connect that parallel combination in series with a 20 Ω for a total resistance of 45 Ω; (b) Connect two 50-Ω resistors in parallel to get 2 Ω. Also, connect two 20 Ω resistors in parallel to get 10 Ω. Then connect these two combinations in a series with each other to obtain 35 Ω.

P28.12 (a) $R_1 = \mathcal{E}\left(-\frac{2}{I_0} + \frac{2}{I_a} + \frac{1}{I_b}\right);$ (b) $R_2 = 2\mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_a}\right);$ (c) $R_3 = \mathcal{E}\left(\frac{1}{I_0} - \frac{1}{I_b}\right)$

P28.14 (a) decreases; (b) 14 Ω

P28.16 (a) $\Delta V_1 = \frac{\mathcal{E}}{3},\ \Delta V_2 = \frac{2\mathcal{E}}{9},\ \Delta V_3 = \frac{4\mathcal{E}}{9},\ \Delta V_4 = \frac{2\mathcal{E}}{3};$

(b) $I_1 = I,\ I_2 = I_3 = \frac{I}{3},\ I_4 = \frac{2I}{3}$; (c) I_4 increases and I_1, I_2, and I_3 decrease;

(d) $I_1 = \frac{3I}{4},\ I_2 = I_3 = 0,\ I_4 = \frac{3I}{4}$

P28.18 (a) See P28.18(a) for the full solution; (b) The current never exceeds 50 μA.

P28.20 None of these is $\frac{4}{3}R,$ so the desired resistance cannot be achieved.

P28.22 (a) See ANS. FIG. P28.22

P28.24 (a) I_3 = 909 mA; (b) –1.82 V

P28.26 (a) See ANS. FIG. P28.26; (b) 11.0 mA in the 220-Ω resistor and out of the positive pole of the 5.80-V battery; The current is 1.87 mA in the 150-Ω resistor and out of the negative pole of the 3.10-V battery; 9.13 mA in the 370-Ω resistor

P28.28 (a) 172 A downward; (b) 1.70 A downward; (c) No, the current in the dead battery is upward in Figure P28.28, so it is not being charged. The dead battery is providing a small amount of power to operate the starter, so it is not really "dead."

P28.30 (a) w = 1.00 A upward in 200 Ω; z = 4.00 A upward in 70.0 Ω; x = 3.00 A upward in 80.0 Ω; y = 8.00 A downward in 20.0 Ω; (b) 200 V

P28.32 (a) I_2 is directed from b toward a and has a magnitude of 2.00 A; (b) I_3 = 1.00 A; (c) No. Neither of the equations used to find I_2 and I_3 contained $\mathcal{E}$ and R. The third equation that we could generate from Kirchhoff's rules contains both the unknowns. Therefore, we have only one equation with two unknowns.

P28.34 (a) $13.0I_1 + 18.0I_2 = 30.0$; (b) $18.0I_2 - 5.00I_3 = -24.0$; (c) $I_1 - I_2 - I_3 = 0$; (d) $I_3 = I_1 - I_2$; (e) $5.00I_1 - 23.0I_2 = 24.0$; (f) $I_2 = -0.416$ A and I_1 = 2.88 A; (g) I_3 = 3.30 A; (h) The negative sign in the answer for I_2 means that this current flows in the opposite direction to that shown in the circuit diagram and assumed during the solution. That is, the actual current in the middle branch of the circuit flows from right to left and has a magnitude of 0.416 A.

P28.36 (a) No. The circuit cannot be simplified further, and Kirchhoff's rules must be used to analyze it; (b) I_1 = 3.50 A; (c) I_2 = 2.50 A; (d) I_3 = 1.00 A

P28.38 (a) 5.00 s; (b) 150 μC; (c) 4.06 μA

P28.40 587 kΩ

P28.42 (a) $(R_1 + R_2)C$; (b) R_2C; (c) $\mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}e^{-t/(R_2C)}\right)$

P28.44 $+\frac{RC}{2}$

P28.46 (a) For the heater, 12.5 A; For the toaster, 6.25 A; For the grill, 8.33 A; (b) The current draw is greater than 25.0 amps, so this circuit will trip the circuit breaker.

P28.48 (a) $\sim 10^{-14}$; (b) $\sim \frac{V_h}{2} + 10^{-10}$ V and $\sim \frac{V_h}{2} - 10^{-10}$ V

P28.50 7.49 Ω

P28.52 (a) 0.991; (b) 0.648; (c) The energy flows are precisely analogous to the currents in parts (a) and (b). The ceiling has the smallest R value of the thermal resistors in parallel, so increasing its thermal resistance will produce the biggest reduction in the total energy flow.

P28.54 (a) 0.706 A; (b) 2.49 W; (c) Only the circuit in Figure P28.54(c) requires the use of Kirchhoff's rules for solution. In the other circuits, the 5-Ω and 8-Ω resistors are still in parallel with each other; (c) The power is lowest in Figure P28.54(c). The circuits in Figures P28.54(b) and P28.54(d) have in effect 30-V batteries driving the current.

P28.56 55.0 Ω

P28.58 See P28.58 for full explanation.

P28.60 (a) 15.0 Ω; (b) $\Delta V_{ac} = \Delta V_{db} = 6.00$ V, $\Delta V_{ce} = 1.20$ V, $\Delta V_{fd} = \Delta V_{ed} = 1.80$ V, $\Delta V_{cd} = 3.00$ V; (c) $I_1 = 1.00$ A, $I_2 = 0.500$ A, $I_3 = 0.500$ A, $I_4 = 0.300$ A, $I_5 = 0.200$ A; (d) $P_{ac} = 6.00$ W, $P_{ce} = 0.600$ W, $P_{ed} = 0.540$ W, $P_{fd} = 0.360$ W, $P_{cd} = 1.50$ W, $P_{db} = 6.00$ W

P28.62 $\frac{P_s + \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$ and $\frac{P_s - \sqrt{P_s^2 - 4P_sP_p}}{2I^2}$

P28.64 (a) 4.40 Ω; (b) 32.0 W; (c) 9.60 W; (d) 70.4 W; (e) 48.0 W

P28.66 (a) $I_1 = \frac{IR_2}{R_1 + R_2}$ and $\frac{IR_1}{R_1 + R_2} = I_2$; (b) See P28.66(b) for full proof.

P28.68 See P28.68 for full explanation.

P28.70 (a) $q = \frac{30.0}{2.00 + R} - 0.542$, where q is in microcoulombs and R is in ohms; (b) 1.96 μC; (c) Yes; 53.3 Ω; (d) 14.5 μC; (e) Yes. Taking $R = \infty$ corresponds to disconnecting the wire; 0.542 μC

P28.72 (a) 40.0 W; (b) 80.0 V and 40.0 V

P28.74 (a) 9.30 V; (b) 2.51 Ω; (c) 18.6 V; (d) 3.70 A; (e) 1.09 A; (f) 14.3 W; (g) 8.54 W; (h) Because of the internal resistance of the batteries, the terminal voltage of the pair of batteries is not the same in both cases.

P28.76 See P28.76 for full explanation.

P28.78 (a) $\ln\left(\frac{\mathcal{E}}{\Delta V}\right) = (0.011\,8)t + 0.088\,2$; (b) The time constant is 84.7 s and the capacitance is 8.47 μF.

P28.80 $P_s = \frac{1}{n^2} P_p$

P28.82 $(R_1 + 2R_2)C \ln 2$

29

Magnetic Fields

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ29.1 Answers (c) and (e). The magnitude of the magnetic force experienced by a charged particle in a magnetic field is given by $F_B = |q| vB \sin\theta$, where v is the speed of the particle and θ is the angle between the direction of the particle's velocity and the direction of the magnetic field. If either $v = 0$ [choice (e)] or $\sin\theta = 0$ [choice (c)], this force has zero magnitude.

OQ29.2 The ranking is (c) > (a) = (d) > (e) > (b). We consider the quantity $F_B = |qvB \sin\theta|$, in units of e (m/s)(T). (a) $\theta = 90°$ and $F_B = (1 \times 10^6)(10^{-3})(1) = 1\,000$. (b) $\theta = 0°$ and $F_B = (1 \times 10^6)(10^{-3})(0) = 0$. (c) $\theta = 90°$ and $F_B = (2 \times 10^6)(10^{-3})(1) = 2\,000$. For (d) $\theta = 90°$ and $F_B = (1 \times 10^6)(1 \times 10^{-3})(1) = 1\,000$ (e) $\theta = 45°$ and $F_B = (1 \times 10^6)(10^{-3})(0.707) = 707$.

OQ29.3 Answer (c). It is not necessarily zero. If the magnetic field is parallel or antiparallel to the velocity of the charged particle, then the particle will experience no magnetic force.

OQ29.4 Answer (c). Use the right-hand rule for the cross produce to determine the direction of the magnetic force, $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. When the proton first enters the field, it experiences a force directed upward, toward the top of the page. This will deflect the proton upward, and as the proton's velocity changes direction, the force changes direction always staying perpendicular to the velocity. The force, being perpendicular to the motion, causes the particle to follow a circular path, with no change in speed, as long as it is in the field. After completing a half circle, the proton will exit the field traveling toward the left.

OQ29.5 Answer (c). $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ and $\hat{\mathbf{i}} \times (-\hat{\mathbf{k}}) = \hat{\mathbf{j}}$.

OQ29.6 Answer (c). The magnetic force must balance the weight of the rod. From Equation 29.10,

$$\left|\vec{\mathbf{F}}_B\right| = \left|I\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right| \rightarrow F_B = ILB\sin\theta$$

For maximum current, $\theta = 90°$, and we have $ILB\sin 90° = mg$, from which we obtain

$$I = \frac{mg}{LB} = \frac{(0.050\ 0\ \text{kg})(9.80\ \text{m/s}^2)}{(1.00\ \text{m})(0.100\ \text{T})} = 4.90\ \text{A}$$

OQ29.7 (**i**) Answer (b). The magnitude of the magnetic force experienced by the electron is given by $F_B = |q|vB\sin\theta = evB$ because $|q| = |-e| = e$, and the angle between the electron's velocity and the magnetic field is $\theta = 90°$. We see that force is proportion to speed.

(**ii**) Answer (a). According to Equation 29.3, $r = mv/qB$; thus, electron A has a smaller radius of curvature.

OQ29.8 (**i**) Answer (c).

(**ii**) Answer (c). $F_E = |q|E$ and $F_B = |q|vB\sin\theta$.

(**iii**) Answer (c). $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$ and $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$.

(**iv**) Answer (a). $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$ and $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$.

(**v**) Answer (d). But $F_B = |q|vB\sin\theta$ is zero if $\theta = \pm 90°$.

(**vi**) Answer (b). $F_B = |q|vB\sin\theta$ is non-zero unless $\theta = \pm 90°$.

(**vii**) Answer (b). Because $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is perpendicular to the particle's velocity.

(**viii**) Answer (b). $F_B = |q|vB\sin\theta$.

OQ29.9 Answer (c). The magnitude of the magnetic force experienced by the electron is given by $F_B = |q|vB\sin\theta$, where the angle between the electron's velocity and the magnetic field is $\theta = 55.0°$, and the magnitude of the electron's (negative) charge is $|q| = |-e| = e$. The magnitude of the force is

$$\begin{aligned} F_B &= |q|vB\sin\theta \\ &= \left(1.60\times10^{-19}\text{ C}\right)\left(2.50\times10^{6}\text{ m/s}\right)\left(3.00\times10^{-5}\text{ T}\right)\sin 55.0° \\ &= 9.83\times10^{-18}\text{ N} \end{aligned}$$

Use the right-hand rule for the cross produce to determine the direction of the magnetic force, $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$. The force is upward on a positive charge but downward on a negative charge.

OQ29.10 Answers (d) and (e). The force that a magnetic field exerts on a moving charge is always perpendicular to both the direction of the field and the direction of the particle's motion. Since the force is perpendicular to the direction of motion, it does no work on the particle and hence does not alter its speed. Because the speed is unchanged, both the kinetic energy and the magnitude of the linear momentum will be constant.

OQ29.11 Answer (d). The electrons will feel a constant electric force and a magnetic force that will change in direction and in magnitude as their speed changes.

OQ29.12 (a) Yes, as described by $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$. (b) No, because, as described by

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}}, \text{ when } v = 0,\ F_B = 0.$$

(c) Yes. $\vec{\mathbf{F}} = q\vec{\mathbf{E}}$ does not depend upon velocity. (d) Yes, because the velocity and magnetic field are perpendicular. (e) No, because the wire is uncharged. (f) Yes, because the current and magnetic field are perpendicular. (g) Yes. (h) Yes.

OQ22.13 Ranking $A_A > A_C > A_B$. The torque exerted on a single turn coil carrying current I by a magnetic field B is $\tau = BIA\sin\theta$. The normal perpendicular to the plane of each coil is also perpendicular to the direction of the magnetic field (i.e., $\theta = 90°$). Since B and I are the same for all three coils, the torques exerted on them are proportional to the area A enclosed by each of the coils. Coil A is rectangular with the largest area $A_A = (1\text{ m})(2\text{ m}) = 2\text{ m}^2$. Coil C is triangular with area $A_C = \frac{1}{2}(1\text{ m})(3\text{ m}) = 1.5\text{ m}^2$. By inspection of the figure, coil B encloses the smallest area.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ29.1 No. Changing the velocity of a particle requires an accelerating force. The magnetic force is proportional to the speed of the particle. If the particle is not moving, there can be no magnetic force on it.

CQ29.2 If you can hook a spring balance to the particle and measure the force on it in a known electric field, then $q = F/E$ will tell you its charge. You cannot hook a spring balance to an electron. Measuring the acceleration of small particles by observing their deflection in known electric and magnetic fields can tell you the charge-to-mass ratio, but not separately the charge or mass. Both an acceleration produced by an electric field and an acceleration caused by a magnetic field depend on the properties of the particle only by being proportional to the ratio q/m.

CQ29.3 Yes. If the magnetic field is perpendicular to the plane of the loop, then it exerts no torque on the loop.

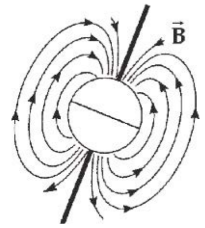
CQ29.4 Send the particle through the uniform field and look at its path. If the path of the particle is parabolic, then the field must be electric, as the electric field exerts a constant force on a charged particle, independent of its velocity. If you shoot a proton through an electric field, it will feel a constant force in the same direction as the electric field—it's similar to throwing a ball through a gravitational field.

If the path of the particle is helical or circular, then the field is magnetic.

If the path of the particle is straight, then observe the speed of the particle. If the particle accelerates, then the field is electric, as a constant force on a proton with or against its motion will make its speed change. If the speed remains constant, then the field is magnetic.

CQ29.5 If the current loop feels a torque, it must be caused by a magnetic field. If the current loop feels no torque, try a different orientation—the torque is zero if the field is along the axis of the loop.

CQ29.6 The Earth's magnetic field exerts force on a charged incoming cosmic ray, tending to make it spiral around a magnetic field line. If the particle energy is low enough, the spiral will be tight enough that the particle will first hit some matter as it follows a field line down into the atmosphere or to the surface at a high geographic latitude.



ANS. FIG. P29.6

CQ29.7 If they are projected in the same direction into the same magnetic field, the charges are of opposite sign.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 29.1 Analysis Model: Particle in a Field (Magnetic)

***P29.1** Gravitational force:

$$F_g = mg = \left(9.11\times10^{-31}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)$$
$$= \boxed{8.93\times10^{-30}\text{ N down}}$$

Electric force:

$$F_e = qE = \left(-1.60\times10^{-19}\text{ C}\right)\left(100\text{ N/C down}\right)$$
$$= \boxed{1.60\times10^{-17}\text{ N up}}$$

Magnetic force:

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = \left(-1.60\times10^{-19}\text{ C}\right)\left(6.00\times10^{6}\text{ m/s }\hat{\mathbf{E}}\right)$$
$$\times\left(50.0\times10^{-6}\text{ N}\cdot\text{s/C}\cdot\text{m }\hat{\mathbf{N}}\right)$$
$$= -4.80\times10^{-17}\text{ N up} = \boxed{4.80\times10^{-17}\text{ N down}}$$

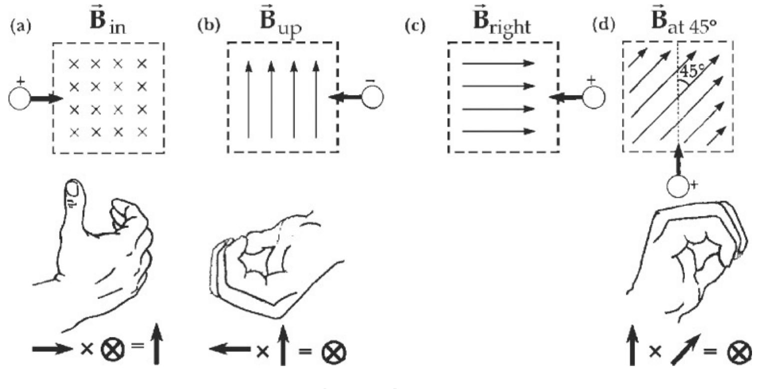
P29.2 See ANS. FIG. P29.2 for right-hand rule diagrams for each of the situations.

(a) up

(b) out of the page, since the charge is negative.

(c) no deflection

(d) into the page



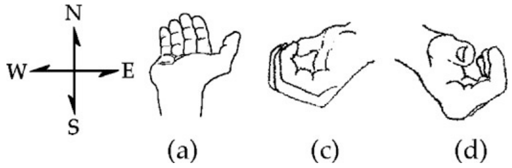
ANS. FIG. P29.2

P29.3 To find the direction of the magnetic field, we use $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Since the particle is positively charged, we can use the right hand rule. In this case, we start with the fingers of the right hand in the direction of $\vec{\mathbf{v}}$ and the thumb pointing in the direction of $\vec{\mathbf{F}}$. As we start closing the hand, our fingers point in the direction of $\vec{\mathbf{B}}$ after they have moved 90°. The results are

(a) $\boxed{\text{into the page}}$ (b) $\boxed{\text{toward the right}}$

(c) $\boxed{\text{toward the bottom of the page}}$

P29.4 At the equator, the Earth's magnetic field is horizontally north. Because an electron has negative charge, $\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$ is opposite in direction to $\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Figures are drawn looking down.



ANS. FIG. P29.4

(a) Down × North = East, so the force is directed $\boxed{\text{West}}$.

(b) North × North = sin 0° = 0: $\boxed{\text{Zero deflection}}$.

(c) West × North = Down, so the force is directed $\boxed{\text{Up}}$.

(d) Southeast × North = Up, so the force is $\boxed{\text{Down}}$.

P29.5 We use $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$. Consider a three-dimensional coordinate system with the xy plane in the plane of this page, the $+x$ direction toward the right edge of the page and the $+y$ direction toward the top of the page. Then, the z axis is perpendicular to the page with the $+z$ direction being upward, out of the page. The magnetic field is directed in the $+x$ direction, toward the right.

(a) When a proton (positively charged) moves in the $+y$ direction, the right-hand rule gives the direction of the magnetic force as into the page or in the $\boxed{-z \text{ direction}}$.

(b) With velocity in the $-y$ direction, the right-hand rule gives the direction of the force on the proton as out of the page, in $\boxed{\text{the } +z \text{ direction}}$.

(c) When the proton moves in the $+x$ direction, parallel to the magnetic field, the magnitude of the magnetic force it experiences is $F = qvB\sin(0°) = 0$. $\boxed{\text{The magnetic force is zero in this case.}}$

P29.6 The magnitude of the force on a moving charge in a magnetic field is $F_B = qvB\sin\theta$, so

$$\theta = \sin^{-1}\left[\frac{F_B}{qvB}\right]$$

$$\theta = \sin^{-1}\left[\frac{8.20\times 10^{-13}\ \text{N}}{\left(1.60\times 10^{-19}\ \text{C}\right)\left(4.00\times 10^{6}\ \text{m/s}\right)\left(1.70\ \text{T}\right)}\right]$$

$$= \boxed{48.9° \text{ or } 131°}$$

P29.7 We first find the speed of the electron from the isolated system model:

$$\left(\Delta K + \Delta U\right)_i = \left(\Delta K + \Delta U\right)_f \rightarrow \frac{1}{2}mv^2 = e\Delta V:$$

$$v = \sqrt{\frac{2e\Delta V}{m}} = \sqrt{\frac{2\left(1.60\times 10^{-19}\ \text{C}\right)\left(2\ 400\ \text{J/C}\right)}{9.11\times 10^{-31}\ \text{kg}}} = 2.90\times 10^{7}\ \text{m/s}$$

(a) $F_{B,\,\max} = qvB = \left(1.60\times 10^{-19}\ \text{C}\right)\left(2.90\times 10^{7}\ \text{m/s}\right)\left(1.70\ \text{T}\right)$

$$= \boxed{7.90\times 10^{-12}\ \text{N}}$$

(b) $F_{B,\,\min} = \boxed{0}$ occurs when $\vec{\mathbf{v}}$ is either parallel to or anti-parallel to $\vec{\mathbf{B}}$.

P29.8 The force on a charged particle is proportional to the vector product of the velocity and the magnetic field:

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = (1.60\times 10^{-19}\ \text{C})\left[(2\hat{\mathbf{i}} - 4\hat{\mathbf{j}} + \hat{\mathbf{k}})(\text{m/s})\times(\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - \hat{\mathbf{k}})\ \text{T}\right]$$

Since 1 C · m · T/s = 1 N, we can write this in determinant form as:

$$\vec{\mathbf{F}}_B = (1.60\times 10^{-19}\ \text{N})\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ 2 & -4 & 1\\ 1 & 2 & -1\end{vmatrix}$$

Expanding the determinant as described in Equation 11.8, we have

$$\vec{\mathbf{F}}_{B,x} = (1.60\times 10^{-19}\ \text{N})\left[(-4)(-1) - (1)(2)\right]\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}}_{B,y} = (1.60\times 10^{-19}\ \text{N})\left[(1)(1) - (2)(-1)\right]\hat{\mathbf{j}}$$

$$\vec{\mathbf{F}}_{B,z} = (1.60\times 10^{-19}\ \text{N})\left[(2)(2) - (1)(-4)\right]\hat{\mathbf{k}}$$

Again in unit-vector notation,

$$\begin{aligned}\vec{\mathbf{F}}_B &= (1.60\times10^{-19}\text{ N})(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} + 8\hat{\mathbf{k}})\\ &= \left(3.20\hat{\mathbf{i}} + 4.80\hat{\mathbf{j}} + 12.8\hat{\mathbf{k}}\right)\times10^{-19}\text{ N}\end{aligned}$$

$$\left|\vec{\mathbf{F}}_B\right| = \left(\sqrt{3.20^2 + 4.80^2 + 12.8^2}\right)\times 10^{-19}\text{ N} = \boxed{13.2\times10^{-19}\text{ N}}$$

P29.9 (a) The magnetic force is given by

$$\begin{aligned}F &= qvB\sin\theta\\ &= \left(1.60\times10^{-19}\text{ C}\right)\left(5.02\times10^{6}\text{ m/s}\right)(0.180\text{ T})\sin(60.0°)\\ &= \boxed{1.25\times10^{-13}\text{ N}}\end{aligned}$$

(b) From Newton's second law,

$$a = \frac{F}{m} = \frac{1.25\times10^{-13}\text{ N}}{1.67\times10^{-27}\text{ kg}} = \boxed{7.50\times10^{13}\text{ m/s}^2}$$

P29.10 (a) The proton experiences maximum force when it moves perpendicular to the magnetic field, and the magnitude of this maximum force is

$$\begin{aligned}F_{\text{max}} &= qvB\sin 90°\\ &= \left(1.60\times10^{-19}\text{ C}\right)\left(6.00\times10^{6}\text{ m/s}\right)(1.50\text{ T})(1)\\ &= \boxed{1.44\times10^{-12}\text{ N}}\end{aligned}$$

(b) From Newton's second law,

$$a_{\text{max}} = \frac{F_{\text{max}}}{m_p} = \frac{1.44\times10^{-12}\text{ N}}{1.67\times10^{-27}\text{ kg}} = \boxed{8.62\times10^{14}\text{ m/s}^2}$$

(c) Since the magnitude of the charge of an electron is the same as that of a proton, $\boxed{\text{a force would be exerted on the electron that had the same magnitude as the force on a proton, but in the opposite direction because of its negative charge.}}$

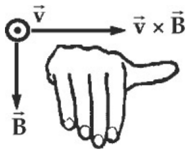
(d) $\boxed{\text{The acceleration of the electron would be much greater than that of the proton because the mass of the electron is much smaller.}}$

P29.11 $F = ma = \left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{13}\text{ m/s}^2\right) = 3.34\times10^{-14}\text{ N} = qvB\sin 90°$

$$\begin{aligned}B &= \frac{F}{qv} = \frac{3.34\times10^{-14}\text{ N}}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{7}\text{ m/s}\right)} = 2.09\times10^{-2}\text{ T} = 20.9\times10^{-3}\text{ T}\\ &= 20.9\text{ mT}\end{aligned}$$

From ANS. FIG. P29.11, the right-hand rule shows that B must be in the $-y$ direction to yield a force in the $+x$ direction when v is in the z direction. Therefore,

$$\boxed{\vec{\mathbf{B}} = -20.9\,\hat{\mathbf{j}}\text{ mT}}$$



ANS. FIG. P29.11

P29.12 The problem implies that the particle undergoes a deflection perpendicular to its motion as if the force direction remained constant. Treat this as a projectile motion problem where the particle travels in the horizontal direction but is displaced vertically 0.150 m at a constant acceleration.

We find the acceleration from

$$\Delta y = \frac{1}{2}a_y\Delta t^2 \rightarrow a_y = \frac{2\Delta y}{\Delta t^2} = \frac{2(0.150\text{ m})}{(1.00\text{ s})^2} = 0.300\text{ m/s}^2$$

Then, from Newton's second law,

$$F_y = ma_y = qvB$$

$$q = \frac{ma_y}{vB} = \frac{(1.50\times10^{-3}\text{ kg})(0.300\text{ m/s}^2)}{(1.50\times10^{4}\text{ m/s})(0.150\times10^{-3}\text{ T})}$$

$$= 2.00\times10^{-4}\text{ C} = 200.\times10^{-6}\text{ C} = \boxed{200\ \mu\text{C}}$$

Section 29.2 Motion of a Charged Particle in a Uniform Magnetic Field

P29.13 (a) The magnetic force acting on the electron provides the centripetal acceleration, holding the electron in the circular path. Therefore, $F = |q|vB\sin 90° = m_e v^2/r$, or

$$r = \frac{m_e v}{eB} = \frac{(9.11\times10^{-31}\text{ kg})(1.50\times10^{7}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(2.00\times10^{-3}\text{ T})}$$

$$= 0.042\,7\text{ m} = \boxed{4.27\text{ cm}}$$

(b) The time to complete one revolution around the orbit (i.e., the period) is

$$T = \frac{\text{distance traveled}}{\text{constant speed}} = \frac{2\pi r}{v} = \frac{2\pi(0.042\,7\text{ m})}{1.50\times10^{7}\text{ m/s}} = \boxed{1.79\times10^{-8}\text{ s}}$$

P29.14 Find the initial horizontal velocity component of an electron in the beam:

$$\frac{1}{2}mv_{xi}^2 = |q|\Delta V$$

$$v_{xi} = v = \sqrt{\frac{2|q|\Delta V}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})\left(2\,500\text{ V}\right)}{9.11\times10^{-31}\text{ kg}}}$$

$$= 2.96\times10^{7}\text{ m/s}$$

Gravitational deflection: The electron's horizontal component of velocity does not change, so its time of flight to the screen is

$$\Delta t = \frac{\Delta x}{v} = \frac{0.350\text{ m}}{2.96\times10^{7}\text{ m/s}} = 1.18\times10^{-8}\text{ s}$$

Its vertical deflection is downward:

$$y = \frac{1}{2}g(\Delta t)^2 = \frac{1}{2}\left(9.80\text{ m/s}^2\right)\left(1.18\times10^{-8}\text{ s}\right)^2 = 6.84\times10^{-16}\text{ m}$$

which is unobservably small.

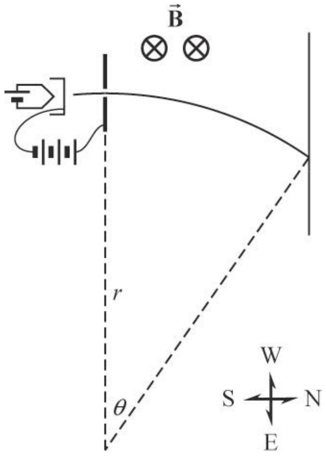
(a) $\boxed{6.84\times10^{-16}\text{ m}}$

(b) $\boxed{\text{down}}$

Magnetic deflection: Use the cross product to find the initial direction of the magnetic force on an electron:

velocity (north) × magnetic field (down) = –west = east.

Because the direction of the magnetic force direction is always perpendicular to the velocity, the electron is deflected so that it curves toward the east in a circular path with radius *r*—see ANS. FIG. 29.14(a):



ANS. FIG. P29.14(a)

$$r = \frac{mv}{|q|B} = \frac{m}{|q|B}\sqrt{\frac{2|q|\Delta V}{m}}$$

$$= \frac{1}{B}\sqrt{\frac{2m\Delta V}{|q|}} = \frac{1}{20.0\times10^{-6}\text{ T}}\sqrt{\frac{2\left(9.11\times10^{-31}\text{ kg}\right)\left(2\,500\text{ V}\right)}{\left(1.60\times10^{-19}\text{ C}\right)}}$$

$$\simeq 8.44\text{ m}$$

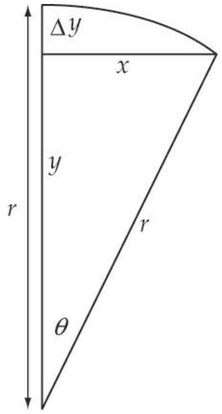
The path of the beam to the screen

subtends at the center of curvature an angle θ, as shown in ANS. FIG. 29.14(b):

$$\theta = \sin^{-1}\left(\frac{x}{r}\right) = \sin^{-1}\left(\frac{0.350\text{ m}}{8.44\text{ m}}\right) = 2.38°$$

The deflection to the east is

$$\begin{aligned}\Delta y &= r(1-\cos\theta)\\ &= (8.44\text{ m})(1-\cos 2.38°)\\ &= 0.007\ 26\text{ m} = 7.26\text{ mm}\end{aligned}$$



ANS. FIG. P29.14(b)

(c) $\boxed{7.26\text{ mm}}$

(d) $\boxed{\text{east}}$

The speed of an electron in the beam remains constant, but its velocity direction changes as it travels along the path, and the force direction changes because it is always perpendicular to the velocity; therefore an electron does not move as a projectile with constant vector acceleration perpendicular to a constant northward component of velocity.

(e) $\boxed{\text{The beam moves on an arc of a circle rather than on a parabola.}}$

However, an electron's northward velocity component stays nearly constant, changing from $v_x = v$ to $v_x = v\cos 2.38°$. The relative change is

$$\frac{\Delta v_x}{v_x} = \frac{v\cos 2.38° - v}{v} = (1-\cos 2.38°) = 0.000\ 863 \simeq 0.000\ 9$$

that is,

(f) $\boxed{\begin{array}{l}\text{Its northward velocity component stays constant within 0.09\%.}\\ \text{It is a good approximation to think of it as moving on a parabola}\\ \text{as it really moves on a circle.}\end{array}}$

P29.15 An electric field changes the speed of each particle according to $(K + U)_i = (K + U)_f$. Therefore, noting that the particles start from rest, we can write

$$q\Delta V = \frac{1}{2}mv^2$$

After they are fired, the particles have the magnetic field change their direction as described by $\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}$:

$$qvB\sin 90° = \frac{mv^2}{r} \quad \text{thus} \quad r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

For the protons, $r_p = \frac{1}{B}\sqrt{\frac{2m_p\Delta V}{e}}$

(a) For the deuterons,

$$r_d = \frac{1}{B}\sqrt{\frac{2(2m_p)\Delta V}{e}} = \boxed{\sqrt{2}r_p}$$

(b) For the alpha particles,

$$r_\alpha = \frac{1}{B}\sqrt{\frac{2(4m_p)\Delta V}{2e}} = \boxed{\sqrt{2}r_p}$$

P29.16 (a) The magnetic force provides the centripetal force to keep the particle moving on a circle:

$$\sum F = ma \quad \rightarrow \quad qvB\sin 90.0° = \frac{mv^2}{R} \qquad \textbf{[1]}$$

and the kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 \qquad \textbf{[2]}$$

Both equations have the same term mv^2 in common:

From [1], $mv^2 = qvBR$, and from [2], $mv^2 = 2K$.

Setting these equal to each other gives

$$mv^2 = qvBR = 2K \quad \rightarrow \quad \boxed{v = \frac{2K}{qBR}}$$

(b) From [1], we have $m = \frac{qBR}{v}$. Using our result from (a), we get

$$m = \frac{qBR}{v} = qBR\left(\frac{qBR}{2K}\right) = \boxed{\frac{q^2B^2R^2}{2K}}$$

P29.17 For each electron, $|q|vB\sin 90.0° = \frac{mv^2}{r}$ and $v = \frac{eBr}{m}$.

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$$K = \frac{1}{2}m_e v_{1i}^2 + 0 = \frac{1}{2}m_e v_{1f}^2 + \frac{1}{2}m_e v_{2f}^2$$

$$K = \frac{1}{2}m_e\left(\frac{e^2B^2R_1^2}{m_e^2}\right) + \frac{1}{2}m\left(\frac{e^2B^2R_2^2}{m_e^2}\right) = \frac{e^2B^2}{2m_e}\left(R_1^2 + R_2^2\right)$$

$$K = \frac{\left(1.60\times10^{-19}\text{ C}\right)^2\left(0.044\ 0\text{ T}\right)^2}{2\left(9.11\times10^{-31}\text{ kg}\right)} \times\left[\left(0.010\ 0\text{ m}\right)^2 + \left(0.024\ 0\text{ m}\right)^2\right]$$

$$= 1.84\times10^{-14}\text{ J} = \boxed{115\text{ keV}}$$

P29.18 For each electron, $|q|vB\sin 90.0° = \frac{mv^2}{r}$ and $v = \frac{eBr}{m}$.

The electrons have no internal structure to absorb energy, so the collision must be perfectly elastic:

$$K = \frac{1}{2}m_e v_{1i}^2 + 0 = \frac{1}{2}m_e v_{1f}^2 + \frac{1}{2}m_e v_{2f}^2$$

$$K = \frac{1}{2}m_e\left(\frac{e^2B^2r_1^2}{m_e^{\ 2}}\right) + \frac{1}{2}m_e\left(\frac{e^2B^2r_2^2}{m_e^{\ 2}}\right) = \boxed{\frac{e^2B^2}{2m_e}\left(r_1^2 + r_2^2\right)}$$

P29.19 (a) We begin with $qvB = \frac{mv^2}{R}$, or $qRB = mv$.

But, $L = mvR = qR^2B$.

Therefore,

$$R = \sqrt{\frac{L}{qB}} = \sqrt{\frac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{-3}\text{ T}\right)}} = 0.050\ 0\text{ m}$$

$$= \boxed{5.00\text{ cm}}$$

(b) Thus,

$$v = \frac{L}{mR} = \frac{4.00\times10^{-25}\text{ J}\cdot\text{s}}{\left(9.11\times10^{-31}\text{ kg}\right)\left(0.050\ 0\text{ m}\right)} = \boxed{8.78\times10^6\text{ m/s}}$$

P29.20 (a) We must use a right-handed coordinate system, so treat north as the positive *x* direction, up as the positive *y* direction, and east as the positive *z* direction. The ball's initial velocity is north, and is given by

$$\vec{\mathbf{v}}_i = v_{xi}\hat{\mathbf{i}} + v_{yi}\hat{\mathbf{j}} = v\hat{\mathbf{i}}$$

and the magnetic field is west,

$$\vec{\mathbf{B}} = -B\hat{\mathbf{k}}$$

The trajectory of the ball is that of an object moving under the influence of gravity: projectile motion. The ball's final velocity is

$$\vec{\mathbf{v}}_f = v_{xf}\hat{\mathbf{i}} + v_{yf}\hat{\mathbf{j}} = v\hat{\mathbf{i}} + v_{yf}\hat{\mathbf{j}}$$

where $v = 20.0$ m/s, because under gravity, the horizontal component of velocity does not change.

We find the final y component of velocity of the ball after it falls a distance h and just before it hits the ground:

$$v_{yf}^2 = v_{yi}^2 + 2a_y\left(y_f - y_i\right)$$

Substituting and solving,

$$v_{yf}^2 = 0 + 2\left(-g\right)\left(-h\right) \quad \rightarrow \quad v_{yf} = -\sqrt{2gh}$$

The force on the ball just before it hits the ground is

$$\begin{aligned}\vec{\mathbf{F}}_B &= Q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = Q\left(v\hat{\mathbf{i}} + v_{yf}\hat{\mathbf{j}}\right) \times \left(-B\hat{\mathbf{k}}\right) = Q\left(v\hat{\mathbf{i}} - \sqrt{2gh}\hat{\mathbf{j}}\right) \times \left(-B\hat{\mathbf{k}}\right)\\ &= -QBv\left(\hat{\mathbf{i}} \times \hat{\mathbf{k}}\right) + QB\sqrt{2gh}\left(\hat{\mathbf{j}} \times \hat{\mathbf{k}}\right) = -QBv\left(-\hat{\mathbf{j}}\right) + QB\sqrt{2gh}\left(\hat{\mathbf{i}}\right)\\ &= QB\left[\sqrt{2gh}\hat{\mathbf{i}} + v\hat{\mathbf{j}}\right]\\ &= \left(5.00 \times 10^{-6}\ \mathrm{C}\right)\left(0.010\,0\ \mathrm{T}\right)\\ &\qquad \times \left[\sqrt{2\left(9.80\ \mathrm{m/s^2}\right)\left(20.0\ \mathrm{m}\right)}\hat{\mathbf{i}} + \left(20.0\ \mathrm{m/s}\right)\hat{\mathbf{j}}\right]\\ &= \boxed{\left(0.990 \times 10^{-6}\hat{\mathbf{i}} + 1.00 \times 10^{-6}\hat{\mathbf{j}}\right)\mathrm{N}}\end{aligned}$$

(b) We find the time interval the ball takes to reach the ground under the acceleration due to gravity:

$$\Delta y = h = \frac{1}{2}g\Delta t^2 \rightarrow \Delta t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\left(20.0\ \mathrm{m}\right)}{9.80\ \mathrm{m/s^2}}} = 2.02\ \mathrm{s}$$

We can estimate an extreme upper bound in the change in the ball's horizontal velocity caused by the magnetic force by assuming the average horizontal component of the force to be half its final maximum horizontal value of 0.990×10^{-6} N. For such an average horizontal component over the entire fall, the change in the horizontal velocity would be less than

$$\begin{aligned}\Delta v_x &= a_x\Delta t = \frac{F_x}{m}\Delta t = \frac{0.5\left(0.990 \times 10^{-6}\ \mathrm{N}\right)}{0.0300\ \mathrm{kg}}\left(2.02\ \mathrm{s}\right)\\ &= 3.33 \times 10^{-5}\ \mathrm{m/s}\end{aligned}$$

Compare this to the initial value of 20.0 m/s:

$$\frac{20.0\text{ m/s}}{3.33\times10^{-5}\text{ m/s}}\approx10^{6}$$

$$\boxed{\text{Yes. In the vertical direction, the gravitational force on the ball is 0.294 N, five orders of magnitude larger than the magnetic force. In the horizontal direction, the change in the horizontal component of velocity due to the magnetic force is six orders of magnitude smaller than the horizontal velocity component.}}$$

P29.21 By conservation of energy for the proton-electric field system in the process that set the proton moving, its kinetic energy is

$$E=\frac{1}{2}mv^2=e\Delta V$$

so its speed is

$$v=\sqrt{\frac{2e\Delta V}{m}}$$

Now Newton's second law for its circular motion in the magnetic field gives

$$\sum F=ma \text{ which becomes } \frac{mv^2}{R}=evB\sin 90^\circ.$$

so $$B=\frac{mv}{eR}=\frac{m}{eR}\sqrt{\frac{2e\Delta V}{m}}=\frac{1}{R}\sqrt{\frac{2m\Delta V}{e}}.$$

and

$$B=\left(\frac{1}{5.80\times10^{10}\text{ m}}\right)\sqrt{\frac{2\left(1.67\times10^{-27}\text{ kg}\right)\left(10.0\times10^{6}\text{ V}\right)}{1.60\times10^{-19}\text{ C}}}$$

$$=\boxed{7.88\times10^{-12}\text{ T}}$$

P29.22 (a) The boundary between a region of strong magnetic field and a region of zero field cannot be perfectly sharp, but we ignore the thickness of the transition zone. In the field the electron moves on an arc of a circle:

$$\sum F=ma: \quad |q|vB\sin 90^\circ=\frac{mv^2}{r}$$

$$\frac{v}{r}=\omega=\frac{|q|B}{m}=\frac{\left(1.60\times10^{-19}\text{ C}\right)\left(10^{-3}\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}{9.11\times10^{-31}\text{ kg}}$$

$$=1.76\times10^{8}\text{ rad/s}$$

The time for one half revolution is, from $\Delta\theta = \omega\Delta t$,

$$\Delta t = \frac{\Delta\theta}{\omega} = \frac{\pi \text{ rad}}{1.76 \times 10^{8} \text{ rad/s}} = \boxed{1.79 \times 10^{-8} \text{ s}}$$

(b) The maximum depth of penetration is the radius of the path. The magnetic force cannot alter the kinetic energy of the electron.

Then,

$$v = \omega r = \left(1.76 \times 10^{8} \text{ s}^{-1}\right)\left(0.020\ 0 \text{ m}\right) = 3.51 \times 10^{6} \text{ m/s}$$

and

$$K = \frac{1}{2}mv^{2} = \frac{1}{2}\left(9.11 \times 10^{-31} \text{ kg}\right)\left(3.51 \times 10^{6} \text{ m/s}\right)^{2}$$

$$= 5.62 \times 10^{-18} \text{ J} = \frac{5.62 \times 10^{-18} \text{ J}}{1.60 \times 10^{-19} \text{ J/eV}} = \boxed{35.1 \text{ eV}}$$

P29.23 To find the ratio of the masses, we first use conservation of energy to find the velocity of each particle after it has been accelerated by the potential drop:

$$\frac{1}{2}mv^{2} = q(\Delta V) \quad \text{so} \quad v = \sqrt{\frac{2q(\Delta V)}{m}}$$

The radius of the particles' orbits is given by

$$r = \frac{mv}{qB} = \frac{m\sqrt{2q(\Delta V)/m}}{qB}$$

Squaring gives, for the first particle,

$$r^{2} = \frac{m}{q} \cdot \frac{2(\Delta V)}{B^{2}}$$

and, for the second particle,

$$(r')^{2} = \frac{m'}{q'} \cdot \frac{2(\Delta V)}{B^{2}}$$

Solving for the masses gives

$$m = \frac{qB^{2}r^{2}}{2(\Delta V)} \quad \text{and} \quad (m') = \frac{(q')B^{2}(r')^{2}}{2(\Delta V)}$$

so

$$\frac{m'}{m} = \frac{q'}{q} \cdot \frac{(r')^{2}}{r^{2}} = \left(\frac{2e}{e}\right)\left(\frac{2R}{R}\right)^{2} = \boxed{8}$$

Section 29.3 Applications Involving Charged Particles Moving in a Magnetic Field

P29.24 (a) The name "cyclotron frequency" refers to the angular frequency or angular speed

$$\omega = \frac{qB}{m}$$

For protons,

$$\omega = \frac{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(0.450\ \text{T}\right)}{1.67 \times 10^{-27}\ \text{kg}} = \boxed{4.31 \times 10^{7}\ \text{rad/s}}$$

(b) The path radius is $R = \frac{mv}{Bq}$.

Just before the protons escape, their speed is

$$v = \frac{BqR}{m} = \frac{\left(0.450\ \text{T}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)\left(1.20\ \text{m}\right)}{1.67 \times 10^{-27}\ \text{kg}} = \boxed{5.17 \times 10^{7}\ \text{m/s}}$$

P29.25 In the velocity selector,

$$v = \frac{E}{B} = \frac{2\,500\ \text{V/m}}{0.035\,0\ \text{T}} = 7.14 \times 10^{4}\ \text{m/s}$$

In the deflection chamber,

$$r = \frac{mv}{qB} = \frac{\left(2.18 \times 10^{-26}\ \text{kg}\right)\left(7.14 \times 10^{4}\ \text{m/s}\right)}{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(0.035\,0\ \text{T}\right)} = \boxed{0.278\ \text{m}}$$

P29.26 We first determine the velocity of the particles from

$$K = \frac{1}{2}mv^2 = q\left(\Delta V\right)$$

so $$v = \sqrt{\frac{2q\left(\Delta V\right)}{m}}$$

Then, from

$$\left|\vec{\mathbf{F}}_B\right| = \left|q\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = \frac{mv^2}{r}$$

we solve for the radius:

$$r = \frac{mv}{qB} = \frac{m}{q}\sqrt{\frac{2q\left(\Delta V\right)/m}{B}} = \frac{1}{B}\sqrt{\frac{2m\left(\Delta V\right)}{q}}$$

(a) Substituting numerical values for uranium-238,

$$r_{238} = \left(\frac{1}{1.20\text{ T}}\right)\sqrt{\frac{2\left[238\left(1.66\times10^{-27}\text{ kg}\right)\right]\left(2\,000\text{ V}\right)}{1.60\times10^{-19}\text{ C}}}$$

$$= 8.28\times10^{-2}\text{ m} = \boxed{8.28\text{ cm}}$$

(b) For uranium-235 ions,

$$r_{235} = \left(\frac{1}{1.20\text{ T}}\right)\sqrt{\frac{2\left[235\left(1.66\times10^{-27}\text{ kg}\right)\right]\left(2\,000\text{ V}\right)}{1.60\times10^{-19}\text{ C}}}$$

$$= 8.23\times10^{-2}\text{ m} = \boxed{8.23\text{ cm}}$$

(c) From $r = \frac{1}{B}\sqrt{\frac{2m(\Delta V)}{q}}$, we see for two different masses m_A and m_B of the same charge q, the ratio of the path radii is $\frac{r_B}{r_A} = \sqrt{\frac{m_B}{m_A}}$.

(d) The ratio of the path radii is independent of ΔV.

(e) The ratio of the path radii is independent of B.

P29.27 Note that the "cyclotron frequency" is an angular speed. The motion of the proton is described by

$$\sum F = ma:$$

$$|q|vB\sin 90° = \frac{mv^2}{r}$$

$$|q|B = m\frac{v}{r} = m\omega$$

(a) $$\omega = \frac{|q|B}{m} = \frac{\left(1.60\times10^{-19}\text{ C}\right)\left(0.800\text{ N}\cdot\text{s/C}\cdot\text{m}\right)}{\left(1.67\times10^{-27}\text{ kg}\right)}\left(\frac{\text{kg}\cdot\text{m}}{\text{N}\cdot\text{s}^2}\right)$$

$$= \boxed{7.66\times10^{7}\text{ rad/s}}$$

(b) $$v = \omega r = \left(7.66\times10^{7}\text{ rad/s}\right)\left(0.350\text{ m}\right)\left(\frac{1}{1\text{ rad}}\right) = \boxed{2.68\times10^{7}\text{ m/s}}$$

(c) $$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(1.67\times10^{-27}\text{ kg}\right)\left(2.68\times10^{7}\text{ m/s}\right)^2\left(\frac{1\text{ eV}}{1.6\times10^{-19}\text{ J}}\right)$$

$$= \boxed{3.76\times10^{6}\text{ eV}}$$

(d) The kinetic energy of the proton changes by $\Delta K = e\Delta V = e(600\text{ V}) =$ 600 eV twice during each revolution, so the number of revolutions is

$$\frac{3.76\times10^{6}\text{ eV}}{2(600\text{ eV})} = \boxed{3.13\times10^{3}\text{ revolutions}}$$

(e) From $\theta = \omega\Delta t$,

$$\Delta t = \frac{\theta}{\omega} = \frac{3.13\times10^{3}\text{ rev}}{7.66\times10^{7}\text{ rad/s}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{2.57\times10^{-4}\text{ s}}$$

P29.28 (a) The path radius is $r = mv/qB$, which we can write in terms of the (kinetic) energy E of the particle:

$$E = K = \frac{1}{2}mv^2 \qquad \rightarrow \qquad v = \left(\frac{2E}{m}\right)^{1/2}$$

so $$r = \frac{mv}{qB} = \frac{m}{qB}\left(\frac{2E}{m}\right)^{1/2} = \frac{m}{qB}\left(\frac{2}{m}\right)^{1/2}E^{1/2} = \frac{m^{1/2}2^{1/2}}{qB}E^{1/2}$$

Differentiating, we get,

$$\frac{dr}{dt} = \frac{m^{1/2}2^{1/2}}{qB}\frac{d\left(E^{1/2}\right)}{dt} = \frac{m^{1/2}2^{1/2}}{qB}\left[\frac{1}{2}\left(E^{-1/2}\right)\frac{dE}{dt}\right]$$

$$= \frac{m^{1/2}2^{1/2}}{qB}\frac{1}{2}\left[\left(\frac{1}{2}mv^2\right)^{-1/2}\right]\frac{dE}{dt}$$

$$= \frac{m^{1/2}2^{1/2}}{qB}\frac{1}{2}\left[\frac{2^{1/2}}{m^{1/2}v}\right]\frac{dE}{dt} = \frac{1}{qBv}\frac{dE}{dt}$$

From the relation $r = mv/qB$, we have $v = qBr/m$, which we substitute:

$$\frac{dr}{dt} = \frac{1}{qBv}\frac{dE}{dt} = \frac{1}{qB}\frac{m}{qBr}\frac{dE}{dt} = \frac{m}{q^2B^2}\frac{1}{r}\frac{dE}{dt}$$

From the relation for the particle's average rate of increase in energy (given in the problem), we have

$$\frac{dr}{dt} = \frac{m}{q^2B^2}\frac{1}{r}\left(\frac{q^2B\Delta V}{\pi m}\right) = \frac{1}{r}\frac{\Delta V}{\pi B}$$

(b) $\boxed{\text{The dashed red line in Figure 29.16a spirals around many times, with its turns relatively far apart on the inside and closer together on the outside. This demonstrates the } 1/r \text{ behavior of the rate of change in radius exhibited by the result in part (a).}}$

(c) $\frac{dr}{dt} = \frac{1}{r}\frac{\Delta V}{\pi B} = \frac{1}{0.350\text{ m}}\frac{600\text{ V}}{\pi\,(0.800\text{ T})} = \boxed{682\text{ m/s}}$

(d) We use the approximation

$$\Delta r \simeq \frac{dr}{dt}\Delta t = \frac{dr}{dt}T = \left(\frac{1}{r}\frac{\Delta V}{\pi B}\right)\left(\frac{2\pi m}{qB}\right) = \frac{2\Delta Vm}{rqB^2}$$

$$= \frac{2(600\text{ V})(1.67\times10^{-27}\text{ kg})}{(0.350\text{ m})(1.60\times10^{-19}\text{ C})(0.800\text{ T})^2}$$

$$= 5.59\times10^{-5}\text{ m} = \boxed{55.9\ \mu\text{m}}$$

P29.29 Fro the electron to travel undeflected, we require $F_B = F_e$, so

$$qvB = qE$$

where $v = \sqrt{\frac{2K}{m}}$ and K is kinetic energy of the electron. Then,

$$E = vB = \sqrt{\frac{2K}{m}}B = \sqrt{\frac{2(750\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}}(0.015\,0\text{ T})$$

$$= \boxed{244\text{ kV/m}}$$

P29.30 (a) Yes: The constituent of the beam is present in all kinds of atoms.

(b) Yes: Everything in the beam has a single charge-to-mass ratio.

(c) In a charged macroscopic object most of the atoms are uncharged. A molecule never has all of its atoms ionized. Any atom other than hydrogen contains neutrons and so has more mass per charge if it is ionized than hydrogen does. The greatest charge-to-mass ratio Thomson could expect was then for ionized hydrogen,

$$1.6\times10^{-19}\text{ C}/1.67\times10^{-27}\text{ kg}$$

smaller than the value e/m he measured,

$$1.6\times10^{-19}\text{ C}/9.11\times10^{-31}\text{ kg}$$

by 1 836 times. The particles in his beam could not be whole atoms, but rather must be much smaller in mass.

(d) With kinetic energy 100 eV, an electron has speed given by

$$\frac{1}{2}mv^2 = 100\text{ eV}$$

from which we obtain

$$v = \sqrt{\frac{2(100\text{ eV})(1.6\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 5.93\times10^{6}\text{ m/s}$$

The time interval to travel 40.0 cm is

$$\Delta t = \frac{\Delta x}{v} = \frac{0.400\text{ m}}{5.93\times10^{6}\text{ m/s}} = 6.75\times10^{-8}\text{ s}$$

If it is fired horizontally it will fall vertically by

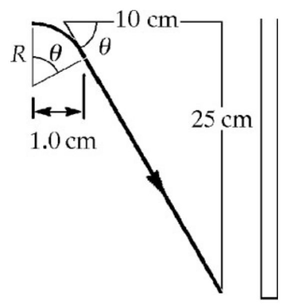
$$y = \frac{1}{2}gt^2 = \frac{1}{2}(9.80\text{ m/s}^2)(6.75\times10^{-8}\text{ s})^2 = 2.24\times10^{-14}\text{ m}$$

an immeasurably small amount. An electron with higher energy falls by a smaller amount.

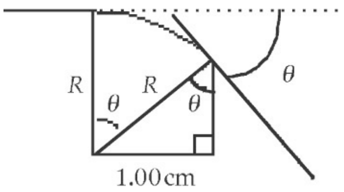
No. The particles move with speed on the order of ten million meters per second, so they fall by an immeasurably small amount over a distance of less than 1 m.

P29.31 From the large triangle in ANS. FIG. P29.31(a):

$$\theta = \tan^{-1}\left(\frac{25.0}{10.0}\right) = 68.2°$$



ANS. FIG. P29.31(a)



ANS. FIG. P29.31(b)

The electron beam, at the point where it enters the magnetic field region, travels to the right, but the beam, at the point it where emerges from the magnetic field region, has been deflected from its original direction by angle θ. Because the radius R is always perpendicular to the path, the radii drawn to these points form the same angle θ with each other. The length of the hypotenuse of the small right triangle appearing in ANS. FIG. P29.31(a) – shown in close-up in ANS. FIG. P29.31(b) – equals the radius R, and the base of the triangle equals the width of the magnetic field region, 1.00 cm. Therefore,

$$R = \frac{1.00\text{ cm}}{\sin 68.2°} = 1.08\text{ cm}$$

Ignoring relativistic correction, the kinetic energy of the electrons is

$$\frac{1}{2}mv^2 = q\Delta V$$

so $$v = \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{2\left(1.60\times10^{-19}\text{ C}\right)\left(50.0\times10^{3}\text{ V}\right)\Delta V}{9.11\times10^{-31}\text{ kg}}}$$
$$= 1.33\times10^{8}\text{ m/s}$$

From Newton's second law, $\frac{mv^2}{R} = qvB$, we find the magnetic field:

$$B = \frac{mv}{|q|R} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(1.33\times10^{8}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.08\times10^{-2}\text{ m}\right)} = \boxed{70.0\text{ mT}}$$

Section 29.4 Magnetic Force on a Current-Carrying Conductor

P29.32 (a) The magnitude of the magnetic force is given by

$$F = ILB\sin\theta = (3.00\text{ A})(0.140\text{ m})(0.280\text{ T})\sin 90° = \boxed{0.118\text{ N}}$$

(b) Neither the direction of the magnetic field nor that of the current is given. Both must be known in order to determine the direction of the magnetic force. In this problem, you can only say that the force is perpendicular to both the wire and the field.

P29.33 (a) From $F = BIL\sin\theta$, the magnetic field is

$$B = \frac{F/L}{I\sin\theta} = \frac{0.120\text{ N/m}}{(15.0\text{ A})\sin 90°} = \boxed{8.00\times10^{-3}\text{ T}}$$

(b) The magnetic field must be in the $+z$ direction to produce a force in the $-y$ direction when the current is in the $+x$ direction.

P29.34 (a) $F_B = ILB\sin\theta = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 60.0° = \boxed{4.73\text{ N}}$

(b) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 90.0° = \boxed{5.46\text{ N}}$

(c) $F_B = (5.00\text{ A})(2.80\text{ m})(0.390\text{ T})\sin 120° = \boxed{4.73\text{ N}}$

P22.35 The vector magnetic force on the wire is

$$\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}} = (2.40\text{ A})(0.750\text{ m})\hat{\mathbf{i}}\times(1.60\text{ T})\hat{\mathbf{k}} = \boxed{\left(-2.88\hat{\mathbf{j}}\right)\text{ N}}$$

P22.36 At all points on the wire, the magnetic force is upward and the gravitational force is downward. For the entire length L of the wire, apply the particle in equilibrium model, assuming that the wire is levitated as claimed, and then solve for the required magnetic field B:

$$\sum F = F_B - F_g = 0 \quad \rightarrow \quad mg = ILB \quad \rightarrow \quad B = \frac{mg}{IL}$$

Express the mass of the wire in terms of the density of copper and its volume and the current in terms of the power delivered to the wire of resistance R:

$$B = \frac{(\rho_{\text{Cu}}V)g}{\left(\sqrt{P/R}\right)L} = \frac{\rho_{\text{Cu}}Vg}{L}\sqrt{\frac{R}{P}}$$

Substitute for the volume of the wire and its resistance in terms of its length L and area A:

$$B = \frac{\rho_{\text{Cu}}(AL)g}{L}\sqrt{\frac{\rho L/A}{P}} = \rho_{\text{Cu}}g\sqrt{\frac{\rho LA}{P}}$$

where ρ is the resistivity of copper. Express the length L of the wire in terms of the radius of the Earth and the area A of the wire in terms of its radius:

$$B = \rho_{\text{Cu}}g\sqrt{\frac{\rho(2\pi R_E)(\pi r^2)}{P}} = \pi\rho_{\text{Cu}}gr\sqrt{\frac{2\rho R_E}{P}}$$

Substitute numerical values:

$$B = \pi\left(8.92\times10^3 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)\left(1.00\times10^{-3} \text{ m}\right) \times\sqrt{\frac{2\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)\left(6.37\times10^6 \text{ m}\right)}{100\times10^6 \text{ W}}}$$

$$= 1.28\times10^{-2} \text{ T}$$

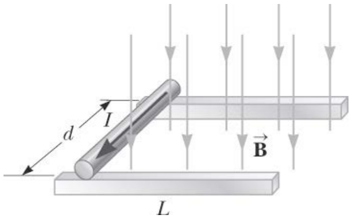
This field magnitude is far larger than that of the Earth, which is about 30 μT at the equator. Therefore, this wire could not be levitated in the Earth's magnetic field as described.

P29.37 Refer to ANS. FIG. P29.37. The rod feels force

$$\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{L}}\times\vec{\mathbf{B}}\right) = Id\left(\hat{\mathbf{k}}\right)\times B\left(-\hat{\mathbf{j}}\right) = IdB\left(\hat{\mathbf{i}}\right)$$

From the work-energy theorem, we have

$$\left(K_{\text{trans}} + K_{\text{rot}}\right)_i + \Delta E = \left(K_{\text{trans}} + K_{\text{rot}}\right)_f$$



ANS. FIG. P29.37

$$0 + 0 + F_B L\cos\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

or $$IdBL\cos 0° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

and $$IdBL = \frac{3}{4}mv^2$$

$$v = \sqrt{\frac{4IdBL}{3m}} = \sqrt{\frac{4(48.0\text{ A})(0.120\text{ m})(0.240\text{ T})(0.450\text{ m})}{3(0.720\text{ kg})}}$$

$$= \boxed{1.07\text{ m/s}}$$

P29.38 Refer to ANS. FIG. P29.37 above. The rod feels force

$$\vec{\mathbf{F}}_B = I\left(\vec{\mathbf{d}} \times \vec{\mathbf{B}}\right) = Id\left(\hat{\mathbf{k}}\right) \times B\left(-\hat{\mathbf{j}}\right) = IdB\left(\hat{\mathbf{i}}\right)$$

From the work-energy theorem, we have

$$\left(K_{\text{trans}} + K_{\text{rot}}\right)_i + \Delta E = \left(K_{\text{trans}} + K_{\text{rot}}\right)_f$$

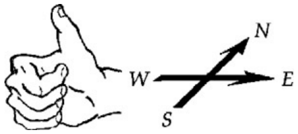
$$0 + 0 + F_B L\cos\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$(IBL)d\cos 0° = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v}{R}\right)^2$$

Solving for the velocity gives

$$v = \boxed{\sqrt{\frac{4IdBL}{3m}}}$$

P29.39 (a) The magnetic force must be upward to lift the wire. For current in the south direction, the magnetic field must be $\boxed{\text{east}}$ to produce an upward force, as shown by the right-hand rule in the figure.



ANS. FIG. P29.39

(b) $F_B = ILB\sin\theta$ with $F_B = F_g = mg$

$$mg = ILB\sin\theta \quad \text{so} \quad \frac{m}{L}g = IB\sin\theta \quad \rightarrow \quad B = \frac{m}{L}\frac{g}{I\sin\theta}$$

$$B = \frac{m}{L}\frac{g}{I\sin\theta} = \left(\frac{0.500\times10^{-3}\text{ kg}}{1.00\times10^{-2}\text{ m}}\right)\left(\frac{9.80\text{ m/s}^2}{(2.00\text{ A})\sin 90.0°}\right) = \boxed{0.245\text{ T}}$$

P29.40 (a) $\boxed{\text{The magnetic force and the gravitational force both act on the wire.}}$

(b) $\boxed{\text{When the magnetic force is upward and balances the downward gravitational force, the net force on the wire is zero, and the wire can move upward at constant velocity.}}$

(c) The minimum magnetic filed would be perpendicular to the current in the wire so that the magnetic force is a maximum. For the magnetic force to be directed upward when the current is toward the left, $\vec{\mathbf{B}}$ must be directed out of the page. Then,

$$F_B = ILB_{\text{min}} \sin 90° = mg$$

from which we obtain

$$B_{\text{min}} = \frac{mg}{IL} = \frac{(0.015\ 0\ \text{kg})(9.80\ \text{m/s}^2)}{(5.00\ \text{A})(0.150\ \text{m})}$$
$$= \boxed{0.196\ \text{T, out of the page}}$$

(d) If the field exceeds 0.200 T, the upward magnetic force exceeds the downward gravitational force, so the wire accelerates upward.

P29.41 (a) The magnitude of the force is

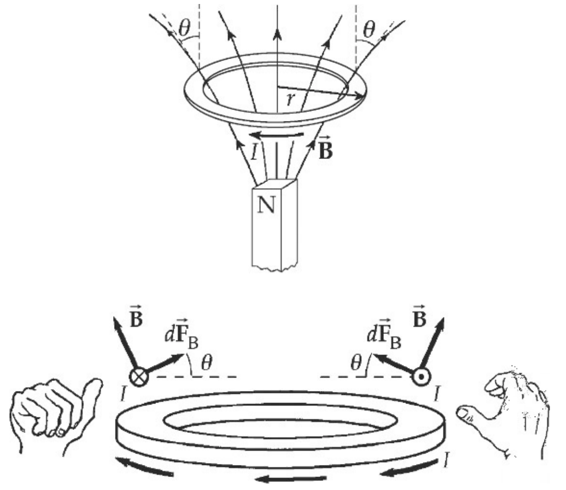
$$F = ILB \sin\theta$$
$$= (2.20\times 10^3\ \text{A})(58.0\ \text{m})(5.00\times 10^{-5}\ \text{N})\sin 65.0°$$
$$= \boxed{5.78\ \text{N}}$$

(b) By the right-hand rule, the direction of the magnetic force is $\boxed{\text{into the page}}$.

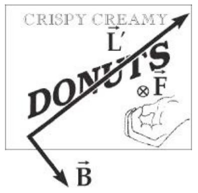
P29.42 (a) Refer to ANS. FIG. P29.42. The magnetic field is perpendicular to all line elements $d\vec{\mathbf{s}}$ on the ring, so the magnetic force $d\vec{\mathbf{F}} = Id\vec{\mathbf{s}} \times \vec{\mathbf{B}}$ on each element has magnitude $I\left|d\vec{\mathbf{s}} \times \vec{\mathbf{B}}\right| = IdsB$ and is radially inward and upward, at angle θ above the radial line. The radially inward components $IdsB \cos\theta$ tend to squeeze the ring but all cancel out because forces on opposite sides of the ring cancel in pairs. The upward components $IdsB \sin\theta$ all add to $I(2\pi r)B\sin\theta$.

(a) magnitude: $\boxed{2\pi r I B \sin\theta}$

(b) direction: $\boxed{\text{up, away from magnet}}$



ANS. FIG. P29.42



ANS. FIG. P29.43

P29.43 Take the *x* axis east, the *y* axis up, and the *z* axis south. The field is

$$\vec{\mathbf{B}} = (52.0\ \mu\mathrm{T})\cos 60.0°\left(-\hat{\mathbf{k}}\right) + (52.0\ \mu\mathrm{T})\sin 60.0°\left(-\hat{\mathbf{j}}\right)$$

The current then has equivalent length:

$$\vec{\mathbf{L}}' = 1.40\ \mathrm{m}\left(-\hat{\mathbf{k}}\right) + 0.850\ \mathrm{m}\left(\hat{\mathbf{j}}\right)$$

The magnetic force is then

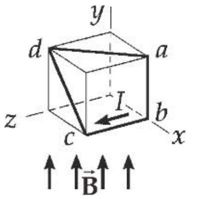
$$\vec{\mathbf{F}}_B = I\vec{\mathbf{L}}' \times \vec{\mathbf{B}} = (0.035\,0\ \mathrm{A})\left(0.850\hat{\mathbf{j}} - 1.40\hat{\mathbf{k}}\right)\ \mathrm{m} \times \left(-45.0\hat{\mathbf{j}} - 26.0\hat{\mathbf{k}}\right)10^{-6}\ \mathrm{T}$$

$$\vec{\mathbf{F}}_B = 3.50 \times 10^{-8}\ \mathrm{N}\left(-22.1\hat{\mathbf{i}} - 63.0\hat{\mathbf{i}}\right) = 2.98 \times 10^{-6}\ \mathrm{N}\left(-\hat{\mathbf{i}}\right) = \boxed{2.98\ \mu\mathrm{N\ west}}$$

P29.44 For each segment, $I = 5.00$ A and $\vec{\mathbf{B}} = 0.020\,0\hat{\mathbf{j}}$ T.

	Segment	$\vec{\ell}$	$\vec{\mathbf{F}}_B = I\left(\vec{\ell} \times \vec{\mathbf{B}}\right)$
(a)	*ab*	$-0.400\ \mathrm{m}\ \hat{\mathbf{j}}$	$\boxed{0}$
(b)	*bc*	$0.400\ \mathrm{m}\ \hat{\mathbf{k}}$	$\boxed{-40.0\hat{\mathbf{i}}\ \mathrm{mN}}$
(c)	*cd*	$-0.400\ \mathrm{m}\ \hat{\mathbf{i}} + 0.400\ \mathrm{m}\ \hat{\mathbf{j}}$	$\boxed{-40.0\hat{\mathbf{k}}\ \mathrm{mN}}$
(d)	*da*	$0.400\ \mathrm{m}\ \hat{\mathbf{i}} - 0.400\ \mathrm{m}\ \hat{\mathbf{k}}$	$\boxed{\left(40.0\hat{\mathbf{i}} + 40.0\hat{\mathbf{k}}\right)\ \mathrm{mN}}$

(e) $\boxed{\text{The forces on the four segments must add to zero, so the force on the fourth segment must be the negative of the resultant of the forces on the other three.}}$



ANS. FIG. P29.44

Section 29.5 Torque on a Current Loop in a Uniform Magnetic Field

***P29.45** (a) From Equation 29.17, we have

$$\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$$

so $\tau = \left(0.10\ \text{A}\cdot\text{m}^2\right)\left(0.080\ \text{T}\right)\sin 30° = \boxed{4.0\ \text{mN}\cdot\text{m}}$.

(b) The potential energy of a system of a magnetic moment in a magnetic field is given by Equation 29.18:

$$U = -\vec{\mu}\cdot\vec{\mathbf{B}} = \mu B\cos\phi = \left(0.10\ \text{A}\cdot\text{m}^2\right)\left(0.080\ \text{T}\right)\cos 30°$$

$$= \boxed{-6.9\ \text{mJ}}$$

***P29.46** The torque on a current loop in a magnetic field is $\tau = BIAN\sin\theta$, and maximum torque occurs when the field is directed parallel to the plane of the loop ($\theta = 90°$). Thus,

$$\tau_{\max} = \left(0.500\ \text{T}\right)\left(25.0\times10^{-3}\ \text{A}\right) \times\left[\pi\left(5.00\times10^{-2}\ \text{m}\right)^2\right]\left(50.0\right)\sin 90.0°$$

$$= \boxed{4.91\times10^{-3}\ \text{N}\cdot\text{m}}$$

P29.47 (a) The field exerts torque on the needle tending to align it with the field, so the minimum energy orientation of the needle is:

$\boxed{\text{pointing north at } 48.0° \text{ below the horizontal}}$

where its energy is

$$U_{\min} = -\mu B\cos 0° = -\left(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right)$$

$$= -5.34\times10^{-7}\ \text{J}$$

(b) It has maximum energy when pointing in the opposite direction, $\boxed{\text{south at } 48.0° \text{ above the horizontal}}$

where its energy is

$$\begin{aligned} U_{\max} &= -\mu B \cos 180° = +\left(9.70\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(55.0\times10^{-6}\ \text{T}\right) \\ &= +5.34\times10^{-7}\ \text{J} \end{aligned}$$

(c) From $U_{\min} + W = U_{\max}$, we have

$$\begin{aligned} W = U_{\max} - U_{\min} &= +5.34\times10^{-7}\ \text{J} - \left(-5.34\times10^{-7}\ \text{J}\right) \\ &= \boxed{1.07\ \mu\text{J}} \end{aligned}$$

P29.48 (a) From the circumference of the loop, $2\pi r = 2.00$ m, we find its radius to be $r = 0.318$ m. The magnitude of the magnetic moment is then

$$\mu = IA = \left(17.0\times10^{-3}\ \text{A}\right)\left[\pi(0.318)^2\ \text{m}^2\right] = \boxed{5.41\ \text{mA}\cdot\text{m}^2}$$

(b) The torque on the loop is given by Equation 29.17, $\vec{\boldsymbol{\tau}} = \vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}$, and its magnitude is

$$\tau = \left(5.41\times10^{-3}\ \text{A}\cdot\text{m}^2\right)\left(0.800\ \text{T}\right) = \boxed{4.33\ \text{mN}\cdot\text{m}}$$

***P29.49** The area of the elliptical loop is given by $A = \pi ab$, where $a = 0.200$ m and $b = 0.150$ m. Since the field is parallel to the plane of the loop, $\theta = 90°$ and the magnitude of the torque is

$$\begin{aligned} \tau &= NBIA\sin\theta \\ &= 8\left(2.00\times10^{-4}\ \text{T}\right)\left(6.00\ \text{A}\right)\left[\pi\left(0.200\ \text{m}\right)\left(0.150\ \text{m}\right)\right]\sin 90.0° \\ &= \boxed{9.05\times10^{-4}\ \text{N}\cdot\text{m}} \end{aligned}$$

The torque is directed to make the left-hand side of the loop move toward you and the right-hand side move away.

P29.50 (a) $\left|\vec{\boldsymbol{\tau}}\right| = \left|\vec{\boldsymbol{\mu}} \times \vec{\mathbf{B}}\right| = NIAB\sin\theta$

$$\begin{aligned} \tau_{\max} &= 80\left(10.0\times10^{-3}\ \text{A}\right)\left(0.025\ 0\ \text{m}\right)\left(0.040\ 0\ \text{m}\right)\left(0.800\ \text{T}\right)\sin 90.0° \\ &= \boxed{6.40\times10^{-4}\ \text{N}\cdot\text{m}} \end{aligned}$$

(b)
$$\begin{aligned} P_{\max} &= \tau_{\max}\omega = \left(6.40\times10^{-4}\ \text{N}\cdot\text{m}\right)\left(3\ 600\ \text{rev/min}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) \\ &= \boxed{0.241\ \text{W}} \end{aligned}$$

(c) In one half revolution the work is

$$W = U_{\max} - U_{\min} = -\mu B\cos 180° - \left(-\mu B\cos 0°\right) = 2\mu B$$
$$= 2NIAB = 2\left(6.40\times 10^{-4}\ \text{N}\cdot\text{m}\right) = 1.28\times 10^{-3}\ \text{J}$$

In one full revolution, $W = 2\left(1.28\times 10^{-3}\ \text{J}\right) = \boxed{2.56\times 10^{-3}\ \text{J}}$.

(d) The time for one revolution is $\Delta t = \dfrac{60\ \text{s}}{3600\ \text{rev}} = \dfrac{1}{60}\ \text{s}$.

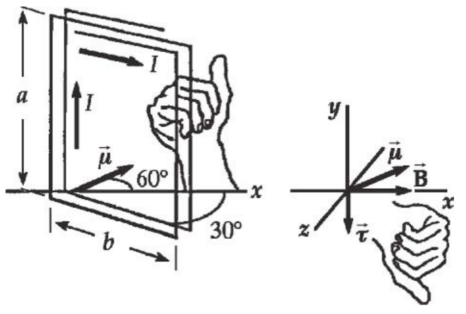
$$P_{\text{avg}} = \frac{W}{\Delta t} = \frac{2.56\times 10^{-3}\ \text{J}}{\left(1/60\right)\ \text{s}} = \boxed{0.154\ \text{W}}$$

The peak power in (b) is greater by the factor $\dfrac{\pi}{2}$.

P29.51 (a) $\tau = NBAI\sin\phi$

$$\tau = 100\left(0.800\ \text{T}\right)\left(0.400\times 0.300\ \text{m}^2\right)\times\left(1.20\ \text{A}\right)\sin 60°$$
$$\tau = \boxed{9.98\ \text{N}\cdot\text{m}}$$

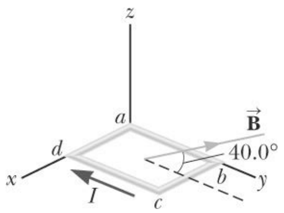
(b) Note that ϕ is the angle between the magnetic moment and the $\vec{\mathbf{B}}$ field. The loop will rotate so as to align the magnetic moment with the $\vec{\mathbf{B}}$ field, $\boxed{\text{clockwise as seen looking down from a position on the positive } y \text{ axis.}}$



ANS. FIG. P29.51

P29.52 (a) The current in segment ab is in the $+y$ direction. Thus, by the right-hand rule, the magnetic force on it is in the $\boxed{+x \text{ direction}}$.

(b) Imagine the force on segment ab being concentrated at its center. Then, with a pivot at point a (a point on the x axis), this force would tend to rotate segment ab in a clockwise direction about the z axis, so the direction of this $\boxed{\text{torque is in the } -z \text{ direction}}$.



ANS. FIG. P29.52

(c) The current in segment *cd* is in the –*y* direction, and the right-hand rule gives the direction of the magnetic force as the $\boxed{-x \text{ direction}}$.

(d) With a pivot at point *d* (a point on the *x* axis), the force on segment *cd* (to the left, in –*x* direction) would tend to rotate it counterclockwise about the *z* axis, and the direction of this $\boxed{\text{torque is in the } +z \text{ direction}}$.

(e) $\boxed{\text{No.}}$

(f) $\boxed{\begin{array}{l}\text{Both the forces and the torques are equal in magnitude and}\\ \text{opposite in direction, so they sum to zero and cannot affect}\\ \text{the motion of the loop.}\end{array}}$

(g) The magnetic force is perpendicular to both the direction of the current in *bc* (the +*x* direction) and the magnetic field. As given by the right-hand rule, this places it $\boxed{\text{in the } yz \text{ plane at } 130° \text{ counterclockwise from the } +y \text{ axis.}}$

(h) The force acting on segment *bc* tends to rotate it counterclockwise about the *x* axis, so the torque is in $\boxed{\text{the } +x \text{ direction}}$.

(i) $\boxed{\text{Zero.}}$ There is no torque about the *x* axis because the lever arm of the force on segment *ad* is zero.

(j) From the answers to (b), (d), (f), and (h), the loop tends to rotate $\boxed{\text{counterclockwise}}$ about the *x* axis.

(k) $\mu = IAN = (0.900\text{ A})\big[(0.500\text{ m})(0.300\text{ m})\big](1) = \boxed{0.135\text{ A}\cdot\text{m}^2}$

(l) The magnetic moment vector is perpendicular to the plane of the loop (the *xy* plane), and is therefore parallel to the *z* axis. Because the current flows clockwise around the loop, the magnetic moment vector is directed downward, in the negative *z* direction. This means that the angle between it and the direction of the magnetic field is $\theta = 90.0° + 40.0° = \boxed{130°}$.

(m) $\tau = \mu B\sin\theta = (0.135\text{ A}\cdot\text{m}^2)(1.50\text{ T})\sin(130°) = \boxed{0.155\text{ N}\cdot\text{m}}$

P29.53 (a) From Equation 29.17, $\vec{\tau} = \vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}$, so the maximum magnitude of the torque on the loop is

$$\tau = \left|\vec{\boldsymbol{\mu}}\times\vec{\mathbf{B}}\right| = \mu B\sin\theta = NIAB\sin\theta$$

$$\begin{aligned}\tau_{\max} &= NIAB\sin 90.0° \\ &= 1(5.00\text{ A})\left[\pi(0.050\,0\text{ m})^2\right](3.00\times10^{-3}\text{ T}) \\ &= \boxed{118\ \mu\text{N}\cdot\text{m}}\end{aligned}$$

(b) The potential energy is given by

$$U = -\vec{\mu}\cdot\vec{\mathbf{B}}$$

so $-\mu B \le U \le +\mu B$

Now, since

$$\begin{aligned}\mu B &= (NIA)B \\ &= 1(5.00\text{ A})\left[\pi(0.050\,0\text{ m})^2\right](3.00\times10^{-3}\text{ T}) \\ &= 118\ \mu\text{J}\end{aligned}$$

the range of the potential energy is: $\boxed{-118\ \mu\text{J} \le U \le +118\ \mu\text{J}}$.

Section 29.6 The Hall Effect

P29.54 (a) $\Delta V_H = \dfrac{IB}{nqt}$ so $\dfrac{nqt}{I} = \dfrac{B}{\Delta V_H} = \dfrac{0.080\,0\text{ T}}{0.700\times10^{-6}\text{ V}} = 1.14\times10^{5}\text{ T/V}$

Then, the unknown field is

$$\begin{aligned}B &= \left(\frac{nqt}{I}\right)(\Delta V_H) \\ &= (1.14\times10^{5}\text{ T/V})(0.330\times10^{-6}\text{ V}) = 0.037\,7\text{ T} = \boxed{37.7\text{ mT}}\end{aligned}$$

(b) $\dfrac{nqt}{I} = 1.14\times10^{5}\text{ T/V}$ so

$$\begin{aligned}n &= (1.14\times10^{5}\text{ T/V})\frac{I}{qt} \\ &= (1.14\times10^{5}\text{ T/V})\left[\frac{0.120\text{ A}}{(1.60\times10^{-19}\text{ C})(2.00\times10^{-3}\text{ m})}\right] \\ &= \boxed{4.29\times10^{25}\text{ m}^{-3}}\end{aligned}$$

P29.55 The magnetic field can be found from the Hall effect voltage, Equation 29.22:

$$\Delta V_H = \frac{IB}{nqt}$$

Solving for the magnetic field gives

$$B = \frac{nqt(\Delta V_H)}{I}$$

$$= \frac{(8.46\times10^{28}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(0.500\times10^{-2}\ \text{m})(5.10\times10^{-12}\ \text{V})}{8.00\ \text{A}}$$

$$B = 4.31\times10^{-5}\ \text{T} = \boxed{43.1\ \mu\text{T}}$$

Additional Problems

P29.56 From $\sum F = ma$, we have

$$qvB\sin 90.0° = \frac{mv^2}{r}$$

therefore, the angular frequency for each ion is

$$\frac{v}{r} = \omega = \frac{qB}{m} = 2\pi f$$

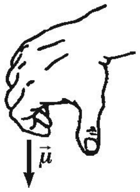
and

$$\Delta\omega = \omega_{12} - \omega_{14} = qB\left(\frac{1}{m_{12}} - \frac{1}{m_{14}}\right)$$

$$= \frac{(1.60\times10^{-19}\ \text{C})(2.40\ \text{T})}{(1.66\times10^{-27}\ \text{kg/u})}\left(\frac{1}{12.0\ \text{u}} - \frac{1}{14.0\ \text{u}}\right)$$

$$\Delta\omega = 2.75\times10^{6}\ \text{s}^{-1} = \boxed{2.75\ \text{Mrad/s}}$$

P29.57 (a) The current carried by the electron is $I = \frac{ev}{2\pi r}$, and the magnetic moment is given by



ANS. FIG. P29.57

$$\mu = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2$$

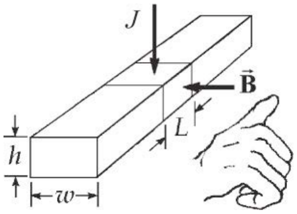
$$= \boxed{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2}$$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

(b) Because the electron is (–), its [conventional] current is clockwise, as seen from above, and μ points $\boxed{\text{downward}}$.

P29.58 (a) Define vector $\vec{\mathbf{h}}$ to have the downward direction of the current, and vector $\vec{\mathbf{L}}$ to be along the pipe into the page as shown. The $\boxed{\text{electric current experiences a magnetic force}}$:

$$I\left(\vec{\mathbf{h}}\times\vec{\mathbf{B}}\right) \text{ in the direction of } \vec{\mathbf{L}}.$$



ANS. FIG. P29.58

(b) The sodium, consisting of ions and electrons, flows along the pipe transporting no net charge. But inside the section of length L, electrons drift upward to constitute downward electric current $J\times(\text{area}) = JLw$.

The current then feels a magnetic force $I\left|\vec{\mathbf{h}}\times\vec{\mathbf{B}}\right| = JLwhB\sin 90°$.

This force along the pipe axis will make the fluid move, exerting pressure

$$\frac{F}{\text{area}} = \frac{JLwhB}{hw} = \boxed{JLB}$$

(c) $\boxed{\text{Charge moves within the fluid inside the length } L\text{, but charge does not accumulate: the fluid is not charged after it leaves the pump.}}$

(d) $\boxed{\text{It is not current-carrying,}}$ and

(e) $\boxed{\text{it is not magnetized.}}$

P29.59 (a) The net force is the Lorentz force given by

$$\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

$$\vec{\mathbf{F}} = \left(3.20 \times 10^{-19}\right)$$

$$\left[\left(4\hat{\mathbf{i}} - 1\hat{\mathbf{j}} - 2\hat{\mathbf{k}}\right) + \left(2\hat{\mathbf{i}} + 3\hat{\mathbf{j}} - 1\hat{\mathbf{k}}\right) \times \left(2\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 1\hat{\mathbf{k}}\right)\right] \text{ N}$$

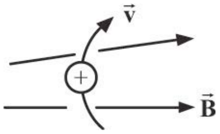
Carrying out the indicated operations, we find:

$$\vec{\mathbf{F}} = \boxed{\left(3.52\hat{\mathbf{i}} - 1.60\hat{\mathbf{j}}\right) \times 10^{-18} \text{ N}}$$

(b) $\theta = \cos^{-1}\left(\dfrac{F_x}{F}\right) = \cos^{-1}\left(\dfrac{3.52}{\sqrt{(3.52)^2 + (1.60)^2}}\right) = \boxed{24.4°}$

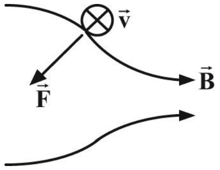
below the $+x$ axis.

P29.60 (a) At the moment shown in Figure 29.11, the particle must be moving upward in order for the magnetic force on it to be into the page, toward the center of this turn of its spiral path. Throughout its motion it circulates clockwise.



ANS. FIG. P29.60(a)

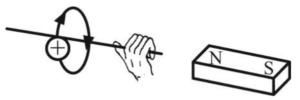
(b) After the particle has passed the middle of the bottle and moves into the region of increasing magnetic field, the magnetic force on it has a component to the left (as well as a radially inward component) as shown. This force in the –*x* direction slows and reverses the particle's motion along the axis.



ANS. FIG. P29.60(b)

(c) The magnetic force is perpendicular to the velocity and does no work on the particle. The particle keeps constant kinetic energy. As its axial velocity component decreases, its tangential velocity component increases.

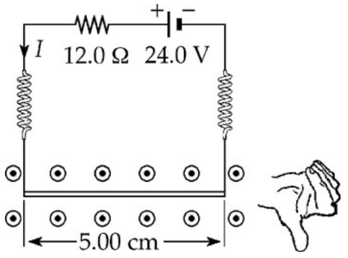
(d) The orbiting particle constitutes a loop of current in the *yz* plane and therefore a magnetic dipole moment $IA = \frac{q}{T}A$ in the –*x* direction. It is like a little bar magnet with its N pole on the left.



ANS. FIG. P29.60(d)

P29.61 Let Δx_1 be the elongation due to the weight of the wire and let Δx_2 be the additional elongation of the springs when the magnetic field is turned on. Then $F_{\text{magnetic}} = 2k\Delta x_2$ where *k* is the force constant of the spring and can be determined from $k = \frac{mg}{2\Delta x_1}$. (The factor 2 is included in the two previous equations since there are 2 springs in parallel.) Combining these two equations, we find

$$F_{\text{magnetic}} = 2\left(\frac{mg}{2\Delta x_1}\right)\Delta x_2 = \frac{mg\Delta x_2}{\Delta x_1}\text{; but } \left|\vec{\mathbf{F}}_B\right| = I\left|\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right| = ILB$$



ANS. FIG. P29.61

Therefore, where $I = \frac{24.0\ \text{V}}{12.0\ \Omega} = 2.00\ \text{A}$,

$$B = \frac{mg\Delta x_2}{IL\Delta x_1} = \frac{(0.100\ \text{kg})(9.80\ \text{m/s}^2)(3.00 \times 10^{-3}\ \text{m})}{(2.00\ \text{A})(0.050\,0\ \text{m})(5.00 \times 10^{-3}\ \text{m})}$$

$$= \boxed{0.588\ \text{T}}$$

P29.62 (a) The particle moves in an arc of a circle with radius

$$r = \frac{mv}{qB} = \frac{1.67 \times 10^{-27}\ \text{kg}\ \ 3 \times 10^{7}\ \text{m/s}\ \ \text{C m}}{1.6 \times 10^{-19}\ \text{C}\ \ 25 \times 10^{-6}\ \text{N s}} = \boxed{12.5\ \text{km}}$$

(b) $\boxed{\text{It will not arrive at the center, but will perform a hairpin turn and go back parallel to its original direction.}}$

P29.63 Let v_i represent the original speed of the alpha particle. Let v_α and v_p represent the particles′ speeds after the collision. We have conservation of momentum

$$4m_p v_i = 4m_p v_\alpha + m_p v_p \quad \rightarrow \quad 4v_i = 4v_\alpha + v_p$$

and the relative velocity equation

$$v_{1i} - v_{2i} = v_{2f} - v_{1f} \quad \rightarrow \quad v_i - 0 = v_p - v_\alpha$$

Eliminating v_i,

$$4v_p - 4v_\alpha = 4v_\alpha + v_p \quad \rightarrow \quad 3v_p = 8v_\alpha \quad \rightarrow \quad v_\alpha = \frac{3}{8}v_p$$

For the proton′s motion in the magnetic field,

$$\sum F = ma \quad \rightarrow \quad ev_p B\sin 90° = \frac{m_p v_p^2}{R} \quad \rightarrow \quad \frac{eBR}{m_p} = v_p$$

For the alpha particle,

$$2ev_\alpha B\sin 90° = \frac{4m_p v_\alpha^2}{r_\alpha}$$

and the radius of the alpha particle′s trajectory is given by

$$r_\alpha = \frac{2m_p v_\alpha}{eB} = \frac{2m_p}{eB}\frac{3}{8}v_p = \frac{2m_p}{eB}\frac{3}{8}\frac{eBR}{m_p} = \boxed{\frac{3}{4}R}$$

P29.64 (a) If $\vec{\mathbf{B}} = B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}$, then

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = e\left(v_i\hat{\mathbf{i}}\right) \times \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}} + B_z\hat{\mathbf{k}}\right) = 0 + ev_iB_y\hat{\mathbf{k}} - ev_iB_z\hat{\mathbf{j}}$$

Since the force actually experienced is $\vec{\mathbf{F}}_B = F_i\hat{\mathbf{j}}$, observe that

$\boxed{B_x \text{ could have any value}}$, $\boxed{B_y = 0}$, and $\boxed{B_z = -\frac{F_i}{ev_i}}$.

(b) If $\vec{\mathbf{v}} = -v_i\hat{\mathbf{i}}$, then

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = e\left(-v_i\hat{\mathbf{i}}\right) \times \left(B_x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \frac{F_i}{ev_i}\hat{\mathbf{k}}\right) = \boxed{-F_i\hat{\mathbf{j}}}$$

(c) If $q = -e$ and $\vec{\mathbf{v}} = -v_i\hat{\mathbf{i}}$, then

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}} = -e\left(-v_i\hat{\mathbf{i}}\right) \times \left(B_x\hat{\mathbf{i}} + 0\hat{\mathbf{j}} - \frac{F_i}{ev_i}\hat{\mathbf{k}}\right) = \boxed{+F_i\hat{\mathbf{j}}}$$

Reversing either the velocity or the sign of the charge reverses the force.

P29.65 From the particle in equilibrium model,

$$\sum F_y = 0: \quad +n - mg = 0$$

$$\sum F_x = 0: \quad -f_k + F_B = -\mu_k n + IBd\sin 90.0° = 0$$

Solving for the magnetic field gives

$$B = \frac{\mu_k mg}{Id} = \frac{0.100(0.200\text{ kg})(9.80\text{ m/s}^2)}{(10.0\text{ A})(0.500\text{ m})} = \boxed{39.2\text{ mT}}$$

P29.66 From the particle in equilibrium model,

$$\sum F_y = 0: \quad +n - mg = 0$$

$$\sum F_x = 0: \quad -f_k + F_B = -\mu_k n + IBd\sin 90.0° = 0$$

Solving for the magnetic field gives

$$B = \boxed{\frac{\mu_k mg}{Id}}$$

P29.67 (a) The field should be in the $+z$-direction, perpendicular to the final as well as to the initial velocity, and with $\hat{\mathbf{i}} \times \hat{\mathbf{k}} = -\hat{\mathbf{j}}$ as the direction of the initial force.

(b) $$r = \frac{mv}{qB} = \frac{(1.67\times10^{-27}\text{ kg})(20\times10^{6}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.3\text{ N}\cdot\text{s/C}\cdot\text{m})} = \boxed{0.696\text{ m}}$$

(c) The path is a quarter circle, of length

$$s = \theta r = \left(\frac{\pi}{2}\right)(0.696\text{ m}) = \boxed{1.09\text{ m}}$$

(d) $$\Delta t = \frac{1.09\text{ m}}{20.0\times10^{6}\text{ m/s}} = \boxed{54.7\text{ ns}}$$

P29.68 Suppose the input power is 120 W = (120 V)I, which gives a current of

$$\boxed{I \sim 1\text{ A} = 10^{0}\text{ A}}$$

Also suppose

$$\omega = 2\,000\text{ rev/min}\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) \sim 200\text{ rad/s}$$

and the output power is

$$20\text{ W} = \tau\omega = \tau(200\text{ rad/s})$$

The torque is then $\boxed{\tau \sim 10^{-1}\text{ N}\cdot\text{m}}$

Suppose the area is about (3 cm) × (4 cm), or $\boxed{A \sim 10^{-3}\ \text{m}^2}$

Suppose that the field is $\boxed{B \sim 10^{-1}\ \text{T}}$

Then, the number of turns in the coil may be found from

$$\tau \cong NIAB:$$

$$0.1\ \text{N}\cdot\text{m} \sim N(1\ \text{C/s})(10^{-3}\ \text{m}^2)(10^{-1}\ \text{N}\cdot\text{s/C}\cdot\text{m})$$

giving $\boxed{N \sim 10^3}$

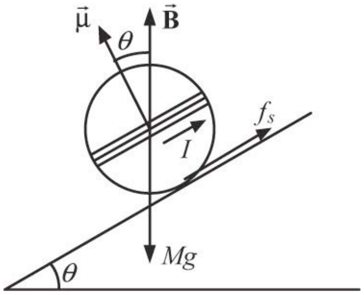
The results are:

(a) $\boxed{B \sim 10^{-1}\ \text{T}}$ (b) $\boxed{\tau \sim 10^{-1}\ \text{N}\cdot\text{m}}$ (c) $\boxed{I \sim 1\ \text{A} = 10^0\ \text{A}}$

(d) $\boxed{A \sim 10^{-3}\ \text{m}^2}$ (e) $\boxed{N \sim 10^3}$

P29.69 The sphere is in translational equilibrium; thus

$$f_s - Mg\sin\theta = 0 \qquad \textbf{[1]}$$



ANS. FIG. P29.69

The sphere is also in rotational equilibrium. If torques are taken about the center of the sphere, the magnetic field produces a clockwise torque of magnitude $\mu B\sin\theta$, and the frictional force a counterclockwise torque of magnitude f_sR, where R is the radius of the sphere. Thus,

$$f_sR - \mu B\sin\theta = 0 \qquad \textbf{[2]}$$

From [1], we obtain $f_s = Mg\sin\theta$. Substituting this into [2], the sin θ term will cancel—see part (b) below. One obtains

$$\mu B = MgR \qquad \textbf{[3]}$$

Now, $\mu = NI\pi R^2$. Thus [3] gives

(a) $$I = \frac{Mg}{\pi NBR} = \frac{(0.080\ 0\ \text{kg})(9.80\ \text{m/s}^2)}{\pi(5)(0.350\ \text{T})(0.200\ \text{m})} =$$

$\boxed{0.713\ \text{A counterclockwise as seen from above}}$

(b) Substitute [1] into [2] and use $\mu = NIA = NI\pi R^2$:

$$f_sR - \mu B\sin\theta = 0$$

$$(Mg\cancel{\sin\theta})R = \mu B\cancel{\sin\theta}$$

$$MgR = \mu B = (NI\pi R^2)B$$

solving for the current gives

$$I = \frac{Mg}{\pi NBR}$$

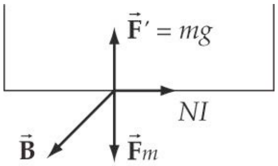
The current is clearly independent of θ.

P29.70 The radius of the circular path followed by the particle is

$$r = \frac{mv}{qB} = \frac{(2.00 \times 10^{-13}\ \text{kg})(2.00 \times 10^{5}\ \text{m/s})}{(1.00 \times 10^{-6}\ \text{C})(0.400\ \text{T})} = 0.100\ \text{m}$$

This is exactly equal to the length h of the field region. Therefore, the particle will not exit the field at the top, but rather will complete a semicircle in the magnetic field region and will exit at the bottom, traveling in the opposite direction with the same speed.

P29.71 (a) When switch S is closed, a total current NI (current I in a total of N conductors) flows toward the right through the lower side of the coil. This results in a downward force of magnitude $F_m = B(NI)w$ being exerted on the coil by the magnetic field, with the requirement that the balance exert a upward force $F' = mg$ on the coil to bring the system back into balance.



ANS. FIG. P29.71

For the system to be restored to balance, it is necessary that

$$F_m = F' \quad \text{or} \quad B(NI)w = mg, \quad \text{giving} \quad B = \boxed{mg/NIw}$$

(b) The magnetic field exerts forces of equal magnitude and opposite directions on the two sides of the coils, so the forces cancel each other and do not affect the balance of the system. Hence, the vertical dimension of the coil is not needed.

(c) $$B = \frac{mg}{NIw} = \frac{(20.0\times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{(50)(0.300\ \text{A})(5.00\times 10^{-2}\ \text{m})} = \boxed{0.261\ \text{T}}$$

P29.72 (a) The magnetic force acting on ions in the blood stream will deflect positive charges toward point A and negative charges toward point B. This separation of charges produces an electric field directed from A toward B. At equilibrium, the electric force caused by this field must balance the magnetic force, so

$$qvB = qE = q\left(\frac{\Delta V}{d}\right)$$

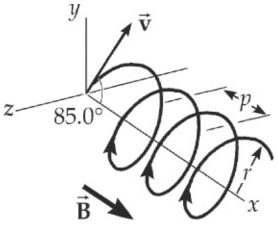
which gives

$$v = \frac{\Delta V}{Bd} = \frac{160 \times 10^{-6}\ \text{V}}{(0.040\ 0\ \text{T})(3.00 \times 10^{-3}\ \text{m})} = \boxed{1.33\ \text{m/s}}$$

(b) Positive ions carried by the blood flow experience an upward force resulting in the upper wall of the blood vessel at electrode A becoming positively charged and the lower wall of the blood vessel at electrode B becoming negatively charged.

(c) No. Negative ions moving in the direction of v would be deflected toward point B, giving A a higher potential than B. Positive ions moving in the direction of v would be deflected toward A, again giving A a higher potential than B. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

P29.73 Let v_x and $v_\perp$ be the components of the velocity of the positron parallel to and perpendicular to the direction of the magnetic field.



ANS. FIG. P29.73

(a) The pitch of trajectory is the distance moved along *x* by the positron during each period, *T* (determined by the cyclotron frequency):

$$p = v_x T = (v\cos 85.0°)\left(\frac{2\pi m}{Bq}\right)$$

$$p = \frac{(5.00 \times 10^{6})(\cos 85.0°)(2\pi)(9.11 \times 10^{-31})}{0.150(1.60 \times 10^{-19})} = \boxed{1.04 \times 10^{-4}\ \text{m}}$$

(b) The equation about circular motion in a magnetic field still applies to the radius of the spiral:

$$r = \frac{mv_\perp}{Bq} = \frac{mv\sin 85.0°}{Bq}$$

$$r = \frac{\left(9.11\times10^{-31}\right)\left(5.00\times10^{6}\right)\left(\sin 85.0°\right)}{\left(0.150\right)\left(1.60\times10^{-19}\right)} = \boxed{1.89\times10^{-4}\ \text{m}}$$

P29.74 (a) The torque on the dipole $\vec{\tau} = \vec{\mu}\times\vec{\mathbf{B}}$ has magnitude $\mu B \sin\theta \approx \mu B\theta$, proportional to the angular displacement if the angle is small. It is a restoring torque, tending to turn the dipole toward its equilibrium orientation. Then the statement that its motion is simple harmonic is true for small angular displacements.

(b) The statement is true only for small angular displacements for which $\sin\theta \approx \theta$.

(c) $\tau = I\alpha$ becomes

$$-\mu B\theta = I\, d^2\theta/dt^2 \quad\rightarrow\quad d^2\theta/dt^2 = -(\mu B/I)\theta = -\omega^2\theta$$

where $\omega = (\mu B/I)^{1/2}$ is the angular frequency and

$$f = \omega/2\pi = \frac{1}{2\pi}\sqrt{\frac{\mu B}{I}}$$

is the frequency in hertz.

(d) The equilibrium orientation of the needle shows the direction of the field. In a stronger field, the frequency is higher. The frequency is easy to measure precisely over a wide range of values.

(e) From part (c), we see that the frequency is proportional to the square root of the magnetic field strength:

$$\frac{f_2}{f_1} = \sqrt{\frac{B_2}{B_1}} \rightarrow \frac{B_2}{B_1} = \left(\frac{f_2}{f_1}\right)^2$$

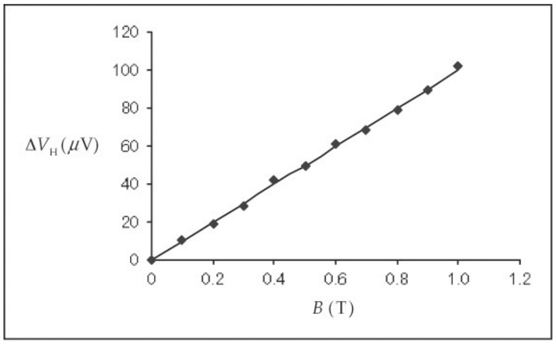
Therefore,

$$B_2 = B_1\left(\frac{f_2}{f_1}\right)^2 = \left(39.2\times10^{-6}\ \text{T}\right)\left(\frac{4.90\ \text{Hz}}{0.680\ \text{Hz}}\right)^2$$

$$= 2.04\times10^{-3}\ \text{T} = \boxed{2.04\ \text{mT}}$$

P29.75 (a) See the graph in ANS. FIG. P29.75. The Hall voltage is directly proportional to the magnetic field. A least-square fit to the data gives the equation of the best fitting line as:

$$\boxed{\begin{array}{c}\Delta V_H = (1.00\times10^{-4})B \\ \text{where } \Delta V_H \text{ is in volts and } B \text{ is in teslas.}\end{array}}$$



ANS. FIG. P29.75

(b) Comparing the equation of the line which fits the data to

$$\Delta V_H = \left(\frac{1}{nqt}\right)B$$

observe that the slope: $\frac{I}{nqt} = 1.00 \times 10^{-4}$, or

$$t = \frac{I}{nq\left(1.00 \times 10^{-4}\right)}$$

Then, if $I = 0.200$ A, $q = 1.60 \times 10^{-19}$ C, and $n = 1.00 \times 10^{26}$ m^{-3}, the thickness of the sample is

$$t = \frac{0.200\text{ A}}{\left(1.00\times 10^{26}\text{ m}^{-3}\right)\left(1.60\times 10^{-19}\text{ C}\right)\left(1.00\times 10^{-4}\text{ V/T}\right)}$$

$$= 1.25\times 10^{-4}\text{ m} = \boxed{0.125\text{ mm}}$$

P29.76 Call the length of the rod L and the tension in each wire alone $\frac{T}{2}$.

Then, at equilibrium:

$$\sum F_x = T\sin\theta - ILB\sin 90.0° = 0 \text{ or } T\sin\theta = ILB$$

$$\sum F_y = T\cos\theta - mg = 0 \qquad \text{or } T\cos\theta = mg$$

combining the equations gives

$$\tan\theta = \frac{ILB}{mg} = \frac{IB}{(m/L)g}$$

solving for the magnetic field,

$$B = \frac{(m/L)g}{I}\tan\theta = \boxed{\frac{\lambda g}{I}\tan\theta}$$

Challenge Problems

P29.77 $|\tau| = IAB$ where the effective current due to the orbiting electrons is $I = \frac{\Delta q}{\Delta t} = \frac{q}{T}$ and the period of the motion is $T = \frac{2\pi R}{v}$.

The electron's speed in its orbit is found by requiring $\frac{k_e q^2}{R^2} = \frac{mv^2}{R}$ or

$$v = q\sqrt{\frac{k_e}{mR}}$$

Substituting this expression for v into the equation for T, we find

$$\begin{aligned} T &= 2\pi\sqrt{\frac{mR^3}{q^2 k_e}} \\ &= 2\pi\sqrt{\frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.29\times10^{-11}\ \text{m}\right)^3}{\left(1.60\times10^{-19}\ \text{C}\right)^2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}} \\ &= 1.52\times10^{-16}\ \text{s} \end{aligned}$$

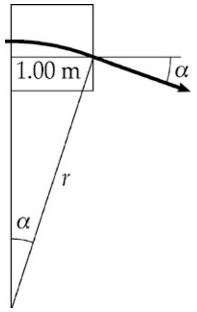
Therefore,

$$\begin{aligned} |\tau| &= \left(\frac{q}{T}\right)AB = \left(\frac{1.60\times10^{-19}\ \text{C}}{1.52\times10^{-16}\ \text{s}}\right)\left[\pi\left(5.29\times10^{-11}\ \text{m}\right)^2\right](0.400\ \text{T}) \\ &= \boxed{3.70\times10^{-24}\ \text{N}\cdot\text{m}} \end{aligned}$$

P29.78 The magnetic force on each proton, $\vec{\mathbf{F}}_B = q\vec{\mathbf{v}}\times\vec{\mathbf{B}} = qvB\sin 90°$ downward and perpendicular to the velocity vector, causes centripetal acceleration, guiding it into a circular path of radius r, with

$$qvB = \frac{mv^2}{r}$$

and $$r = \frac{mv}{qB}$$



ANS. FIG. P29.78

We compute this radius by first finding the proton's speed from $K = \frac{1}{2}mv^2$:

$$\begin{aligned} v &= \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(5.00\times10^{6}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)}{1.67\times10^{-27}\ \text{kg}}} \\ &= 3.10\times10^{7}\ \text{m/s} \end{aligned}$$

Now, $$r = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.10\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.050\,0\text{ N}\cdot\text{s/C}\cdot\text{m}\right)} = 6.46\text{ m.}$$

(a) From ANS. FIG. P29.78 observe that

$$\sin\alpha = \frac{1.00\text{ m}}{r} = \frac{1\text{ m}}{6.46\text{ m}}$$

$$\boxed{\alpha = 8.90°}$$

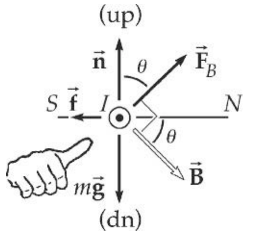
(b) The magnitude of the proton momentum stays constant, and its final y component is

$$-\left(1.67\times10^{-27}\text{ kg}\right)\left(3.10\times10^{7}\text{ m/s}\right)\sin 8.90°$$

$$= \boxed{-8.00\times10^{-21}\text{ kg}\cdot\text{m/s}}$$

P29.79 A key to solving this problem is that reducing the normal force will reduce the friction force:

$F_B = BIL$ or $B = \frac{F_B}{IL}$.



ANS. FIG. P29.79

When the wire is just able to move,

$$\sum F_y = n + F_B\cos\theta - mg = 0$$

so $$n = mg - F_B\cos\theta$$

and $$f = \mu\left(mg - F_B\cos\theta\right)$$

Also, $$\sum F_x = F_B\sin\theta - f = 0$$

so $$F_B\sin\theta = f : F_B\sin\theta = \mu\left(mg - F_B\cos\theta\right) \text{ and } F_B = \frac{\mu mg}{\sin\theta + \mu\cos\theta}$$

We minimize B by minimizing F_B:

$$\frac{dF_B}{d\theta} = -\left(\mu mg\right)\frac{\cos\theta - \mu\sin\theta}{\left(\sin\theta + \mu\cos\theta\right)^2} = 0 \Rightarrow \mu\sin\theta = \cos\theta$$

Thus, $\theta = \tan^{-1}\left(\frac{1}{\mu}\right) = \tan^{-1}\left(5.00\right) = 78.7°$ for the smallest field, and

$$B = \frac{F_B}{IL} = \left(\frac{\mu g}{I}\right)\frac{\left(m/L\right)}{\sin\theta + \mu\cos\theta}$$

$$B_{\text{min}} = \left[\frac{(0.200)\left(9.80\text{ m/s}^2\right)}{1.50\text{ A}}\right]\frac{0.100\text{ kg/m}}{\sin 78.7° + (0.200)\cos 78.7°}$$

$$= 0.128\text{ T}$$

The answers are

(a) magnitude: $\boxed{0.128\text{ T}}$ and

(b) direction: $\boxed{78.7° \text{ below the horizontal}}$

P29.80 (a) The kinetic energy of the proton in joules is

$$K = \frac{1}{2}mv^2 = 6.00\text{ MeV} = \left(6.00\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)$$
$$= 9.60\times10^{-13}\text{ J}$$

From which we find the proton's velocity to be

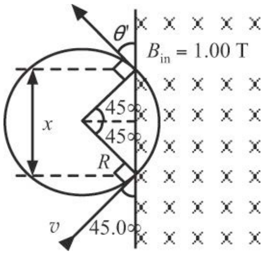
$$v = \sqrt{\frac{2\left(9.60\times10^{-13}\text{ J}\right)}{1.67\times10^{-27}\text{ kg}}} = 3.39\times10^{7}\text{ m/s}$$

We can find the radius of the proton's orbit from

$$F_B = qvB = \frac{mv^2}{R}$$

so $$R = \frac{mv}{qB} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.39\times10^{7}\text{ m/s}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\text{ T}\right)} = 0.354\text{ m}$$

Then, from the diagram, $x = 2R\sin 45.0° = 2\left(0.354\text{ m}\right)\sin 45.0° =$ $\boxed{0.501\text{ m}}$.



ANS. FIG. P29.80

(b) From ANS. FIG. P29.80, observe that $\theta' = \boxed{45.0°}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P29.2 (a) up; (b) out of the page, since the charge is negative; (c) no deflection; (d) into the page

P29.4 (a) west; (b) zero deflection; (c) up; (d) down

P29.6 48.9° or 131°

P29.8 13.2×10^{-19} N

P29.10 (a) 1.44×10^{-12} N; (b) 8.62×10^{14} m/s^2; (c) A force would be exerted on the electron that had the same magnitude as the force on a proton but in the opposite direction because of its negative charge; (d) The acceleration of the electron would be much greater than that of the proton because the mass of the electron is much smaller.

P29.12 200 μC

P29.14 (a) 6.84×10^{-16} m; (b) down; (c) 7.26 mm; (d) east; (e) The beam moves on an arc of a circle rather than on a parabola; (f) Its northward velocity component stays constant within 0.09%. It is a good approximation to think of it as moving on a parabola as it really moves on a circle.

P29.16 (a) $v = \frac{2K}{qBR}$; (b) $\frac{q^2B^2R^2}{2K}$

P29.18 $\frac{e^2B^2}{2m_e}\left(r_1^2 + r_2^2\right)$

P29.20 (a) $\left(0.990\times 10^{-6}\,\hat{\mathbf{i}} + 1.00\times 10^{-6}\,\hat{\mathbf{j}}\right)$ N; (b) Yes. In the vertical direction, the gravitational force on the ball is 0.294 N, five orders of magnitude larger than the magnetic force. In the horizontal direction, the change in the horizontal component of velocity due to the magnetic force is six orders of magnitude smaller than the horizontal velocity component.

P29.22 1.79×10^{-8} s; (b) 35.1 eV

P29.24 4.31×10^{7} rad/s; (b) 5.17×10^{7} m/s

P29.26 (a) 8.28 cm; (b) 8.23 cm; (c) From $r = \frac{1}{B}\sqrt{\frac{2m(\Delta V)}{q}}$, we see for two different masses m_A and m_B of the same charge q, the ratio of the path radii is $\frac{r_B}{r_A} = \sqrt{\frac{m_B}{m_B}}$; (d) The ratio of the path radii is independent of ΔV; (e) The ratio of the path radii is independent of B.

P29.28 (a) See P29.28 for full explanation; (b) The dashed red line in Figure P29.16(a) spirals around many times, with it turns relatively far apart on the inside and closer together on the outside. This demonstrates the $1/r$ behavior of the rate of change in radius exhibited by the result in part (a); (c) 682 m/s; (d) 55.9 μm

P29.30 (a) Yes. The constituent of the beam is present in all kinds of atoms; (b) Yes. Everything in the beam has single charge-to-mass ratio; (c) In a charged macroscopic object most of the atoms are uncharged. A molecule never has all of its atoms ionized. Any atoms other than hydrogen contain neutrons and so has more mass per charge if it is ionized than hydrogen does. The greatest charge-to-mass ratio Thomson could expect was then for ionized hydrogen, 1.6×10^{-19} C/1.67×10^{-27} kg, smaller than the value e/m he measured, 1.6×10^{-19} C/9.11×10^{-31} kg, by 1 836 times. The particles in his beam could not be whole atoms but rather must be much smaller in mass; (d) No. The particles move with speed on the order of ten million meters per second, so they fall by an immeasurably small amount over a distance of less than 1 m.

P29.32 (a) 0.118 N; (b) Neither the direction of the magnetic field nor that of the current is given. Both must be known in order to determine the direction of the magnetic force.

P29.34 (a) 4.73 N; (b) 5.46 N; (c) 4.73 N

P29.36 See P29.36 for full explanation.

P29.38 $\sqrt{\frac{4IdBL}{3m}}$

P29.40 The magnetic force and the gravitational force both act on the wire; (b) When the magnetic force is upward and balances the downward gravitational force, the net force on the wire is zero, and the wire can move can move upward at constant velocity; (c) 0.196 T, out of the page; (d) If the field exceeds 0.20 T, the upward magnetic force exceeds the downward gravitational force, so the wire accelerates upward.

P29.42 (a) $2\pi rIB \sin\theta$; (b) up, away from magnet

P29.44 (a) 0; (b) $-40.0\hat{\mathbf{i}}$ mN; (c) $-40.0\hat{\mathbf{k}}$ mN; (d) $\left(40.0\hat{\mathbf{i}} + 40.0\hat{\mathbf{k}}\right)$ mN; (e) The forces on the four segments must add to zero, so the force on the fourth segment must be the negative of the resultant of the forces on the other three.

P29.46 $4.91 \times 10^{-3}\ \mathrm{N \cdot m}$

P29.48 (a) $5.41\ \mathrm{mA \cdot m^2}$; (b) $4.33\ \mathrm{mN \cdot m}$

P29.50 (a) $6.40 \times 10^{-4}\ \text{N} \cdot \text{m}$; (b) 0.241 W; (c) 2.56×10^{-3} J; (d) 0.154 W

P29.52 (a) $+x$ direction; (b) torque is in the $-z$ direction; (c) $-x$ direction; (d) torque is in the $+z$ direction; (e) No; (f) Both the forces and the torques are equal in magnitude and opposite in direction, so they sum to zero and cannot affect the motion of the loop; (g) in the yz plane at 130° counterclockwise from the $+y$ axis; (h) the $+x$ direction; (i) zero; (j) counterclockwise; (k) $0.135\ \text{A} \cdot \text{m}^2$; (l) 130°; (m) $0.155\ \text{N} \cdot \text{m}$

P29.54 (a) 37.7 mT; (b) $4.29 \times 10^{25}\ \text{m}^{-3}$

P29.56 2.75 Mrad/s

P29.58 (a) The electric current experiences a magnetic force; (b) JLB; (c) Charge moves within the fluid inside the length L, but charge does not accumulate: the fluid is not charged after it leaves the pump; (d) It is not current-carrying; (e) It is not magnetized.

P29.60 (a–d) See P29.60 for full explanation.

P29.62 (a) 12.5 km; (b) It will not arrive at the center but will perform a hairpin turn and go back parallel to its original direction.

P29.64 (a) B_x could have any value, $B_y = 0$, $B_z = -\dfrac{F_i}{ev_i}$; (b) $-F_i\hat{\mathbf{j}}$; (c) $+F_i\hat{\mathbf{j}}$

P29.66 $\dfrac{\mu_k mg}{Id}$

P29.68 (a) $B \sim 10^{-1}$ T; (b) $\tau \sim 10^{-1}\ \text{N} \cdot \text{m}$; (c) $I \sim 1\ \text{A} = 10^{0}$ A; (d) $A \sim 10^{-3}\ \text{m}^2$; (e) $N \sim 10^{3}$

P29.70 The particle will not exit the field at the top but rather will complete a semicircle in the magnetic field region and will exit at the bottom, traveling in the opposite direction with the same speed.

P29.72 (a) 1.33 m/s; (b) Positive ions carried by the blood flow experience an upward force resulting in the upper wall of the blood vessel at electrode A becoming positively charged and the lower wall of the blood vessel at electrode B becoming negatively charged; (b) No. Negative ions moving in the direction of v would be deflected toward point B, giving A a higher potential than B. Positive ions moving in the direction of v would be deflected toward A, again giving A a higher potential than B. Therefore, the sign of the potential difference does not depend on whether the ions in the blood are positively or negatively charged.

P29.74 (a) See P29.74(a) for full explanation; (b) The statement is true only for small angular displacements for which sin $\theta \approx \theta$; (c) See P29.74(c) for full explanation; (d) The equilibrium orientation of the needle shows the direction of the field. In a stronger field, the frequency is higher. The frequency is easy to measure precisely over a wide range of values; (e) 2.04 mT

P29.76 $\frac{\lambda g}{I}\tan\theta$

P29.78 (a) $\alpha = 8.90°$; (b) $-8.00\times10^{-21}\ \mathrm{kg\cdot m/s}$

P29.80 (a) 0.501 m; (b) 45.0°

30

Sources of the Magnetic Field

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ30.1 **(i)** Answer (b). The field is proportional to the current. **(ii)** Answer (d). The field is inversely proportional to the length of the solenoid. **(iii)** Answer (b). The field is proportional to the number of turns. **(iv)** Answer (c). The field does not depend on the radius of the solenoid. All the questions can be answered by referring to Equation 30.17, $\mathrm{B} = \frac{\mu_0 \mathrm{NI}}{\ell}$.

OQ30.2 Answer (c). Newton's third law describes the relationship.

OQ30.3 (a) No. At least two would be of like sign, so they would repel. (b) Yes, if all are alike in sign. (c) Yes, if all carry current in the same direction. (d) No. If one current-carrying wire repelled the other two, those two would attract each other.

OQ30.4 Answer (a). The contribution made to the magnetic field at point *P* by the lower wire is directed out of the page, while the contribution due to the upper wire is directed into the page. Since point *P* is equidistant from the two wires, and the wires carry the same magnitude currents, these two oppositely directed contributions to the magnetic field have equal magnitudes and cancel each other.

OQ30.5 Answer (a) and (c). The magnetic field due to the current in the vertical wire is directed into the page on the right side of the wire and out of the page on the left side. The field due to the current in the horizontal wire is out of the page above this wire and into the page below the wire. Thus, the two contributions to the total magnetic field have the same directions at points B (both out of the page) and D (both contributions into the page), while the two contributions have opposite directions at points A and C. The magnitude of the total magnetic field will be greatest at points B and D where the two contributions are in the same direction, and smallest at points A and C where the two contributions are in opposite directions and tend to cancel.

OQ30.6 **(i)** Answer (b). Magnetic field lines lie in horizontal planes and go around the wire clockwise as seen from above. East of the wire the field points horizontally south.

(ii) Answer (b). The direction of the magnetic field at a given point is determined by the direction of the conventional current that creates it.

***OQ30.7** **(i)** Answer (d). **(ii)** Answer (c). Current on each side of the frame produces magnetic field lines that wrap around the tubes. The field lines pass into the plane enclosed by the frame (away from you) and then return to pass back through the plane outside the frame (toward you).

OQ30.8 Answer (a). According to the right-hand rule, the magnetic field at point P due to the current in the wire is directed out of the page, and the magnitude of this field is given by Equation 30.14: $B = \mu_0 I/2\pi r$.

OQ30.9 Answers (c) and (d). Any point in region I is closer to the upper wire, which carries the larger current. At all points in this region, the outward directed field due the upper wire will have a greater magnitude than will the inward directed field due to the lower wire. Thus, the resultant field in region I will be nonzero and out of the page, meaning that choice (d) is a true statement and choice (a) is false. In region II, the field due to each wire is directed into the page, so their magnitudes add and the resultant field cannot be zero at any point in this region. This means that choice (b) is false. In region III, the field due to the upper wire is directed into the page while that due to the lower wire is out of the page. Since points in this region are closer to the wire carrying the smaller current, there are points in this region where the magnitudes of the oppositely directed fields due to the two wires will have equal magnitudes, canceling each other and producing a zero resultant field. Thus, choice (c) is true and choice (e) is false.

OQ30.10 Answer (b). Wires carrying currents in opposite directions repel. In regions II and III, the field due to the upper wire is directed into the page. The lower wire, with its current to the left, experiences a downward force in the field of the upper wire.

OQ30.11 Answers (b) and (c). In each case, electric charge is moving.

OQ30.12 Answer (a). The adjacent wires carry currents in the same direction.

OQ30.13 Answer (c). Conceptually, for there to be magnetic flux through a coil, magnetic field lines must pass through the area enclosed by the coil. The magnetic field lines do not pass through the areas of the coils in the *xy* and *xz* planes, but they do through the area of the coil in the *yz* plane. Mathematically, the magnetic flux is $\Phi_B = BA\cos\theta$, where θ is the angle between the normal to the area enclosed by the coil and the magnetic field. The flux is maximum when the field is perpendicular to the area of the coil. The flux is zero when there is no component of magnetic field perpendicular to the loop—that is, when the plane of the loop contains the *x* axis.

OQ30.14 The ranking is e > c > b > a > d. Express the fields in units of μ_0 (ampere/cm):

(a) for a long, straight wire,

$$\mu_0 I/2\pi r = \mu_0\left[3/2\pi(2)\right] = \mu_0\,[0.75/\pi]\,(\text{ampere/cm})$$

(b) for a circular coil,

$$N\mu_0 I/2r = \mu_0\left[(10)(0.3)/2(2)\right] = \mu_0\,[0.75]\ (\text{ampere/cm})$$

(c) for a solenoid,

$$N\mu_0 I/\ell = \mu_0\left[(1\ 000)(0.3)/200\right] = \mu_0\,[1.5]\ (\text{ampere/cm})$$

which is also

$$\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left[1.5\ \text{A}/(0.01\ \text{m})\right] = 0.19\times10^{-3}\ \text{T} = 0.19\ \text{mT}$$

(d) The field is zero at the center of a current-carrying wire.

(e) 1 mT is larger than 0.19 mT, so it is largest of all.

OQ30.15 The ranking is C > A > B. The magnetic field inside a solenoid, carrying current *I*, with *N* turns and length *L*, is $B = \mu_0 nI = \mu_0\left(\frac{N}{L}\right)I$.

Thus, $B_A = \frac{\mu_0 N_A I}{L_A}$, $B_B = \frac{\mu_0 N_A I}{2L_A} = \frac{1}{2}B_A$, and $B_C = \frac{\mu_0 (2N_A) I}{L_A/2} = 4B_A$.

ANSWERS TO CONCEPTUAL QUESTIONS

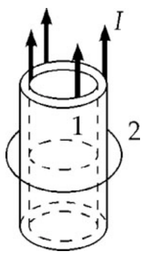
CQ30.1 No. The magnetic field created by a single current loop resembles that of a bar magnet – strongest inside the loop, and decreasing in strength as you move away from the loop. Neither is the field uniform in direction – the magnetic field lines loop through the loop.

CQ30.2 Yes. Either pole of the magnet creates a field that turns the atoms of the domains inside the iron to align their magnetic moments with the external field. Then the nonuniform field exerts a net force on each domain toward the direction in which the field is getting stronger.

A magnet on a refrigerator door goes through the same steps to exert a strong normal force on the door. Then the magnet is supported by a frictional force.

CQ30.3 The Biot-Savart law considers the contribution of each element of current in a conductor to determine the magnetic field, while for Ampère's law, one need only know the current passing through a given surface. Given situations of high degrees of symmetry, Ampère's law is more convenient to use, even though both laws are equally valid in all situations.

CQ30.4 Apply Ampère's law to the circular path labeled 1 in the picture. Because the current has a cylindrical symmetry about its central axis, the line integral reduces to the magnitude of the magnetic field times the circumference of the path, but this is equal to zero because there is no current inside this path; therefore, the magnetic field inside the tube must be zero. On the other hand, the current through path 2 is the current carried by the conductor; then the line integral is not equal to zero, so the magnetic field outside the tube is nonzero.



ANS. FIG. CQ30.4

CQ30.5 Magnetic field lines come out of north magnetic poles. The Earth's north magnetic pole is off the coast of Antarctica, near the south geographic pole. Straight up.

CQ30.6 Ampère's law is valid for all closed paths surrounding a conductor, but not always convenient. There are many paths along which the integral is cumbersome to calculate, although not impossible. Consider a circular path around but *not* coaxial with a long, straight current-carrying wire. Ampère's law is useful in calculating $\vec{\mathbf{B}}$ if the current in a conductor has sufficient symmetry that the line integral can be reduced to the magnitude of $\vec{\mathbf{B}}$ times an integral.

CQ30.7 Magnetic domain alignment within the magnet creates an external magnetic field, which in turn induces domain alignment within the first piece of iron, creating another external magnetic field. The field of the first piece of iron in turn can align domains in another iron sample. A nonuniform magnetic field exerts a net force of attraction on the magnetic dipoles of the domains aligned with the field.

CQ30.8 The shock misaligns the domains. Heating will also decrease magnetism (see Curie Temperature).

CQ30.9 Zero in each case. The fields have no component perpendicular to the area.

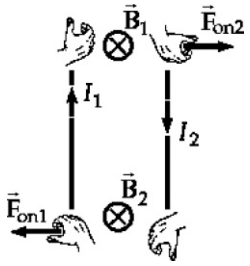
CQ30.10 (a) The third magnet from the top repels the second one with a force equal to the weight of the top two. The yellow magnet repels the blue one with a force equal to the weight of the blue one.

(b) The rods (or a pencil) prevent motion to the side and prevents the magnets from rotating under their mutual torques. Its constraint changes unstable equilibrium into stable.

(c) Most likely, the disks are magnetized perpendicular to their flat faces, making one face a north pole and the other a south pole. One disk has its north pole on the top side and the adjacent magnets have their north poles on their bottom sides.

(d) If the blue magnet were inverted, it and the yellow one would stick firmly together. The pair would still produce an external field and would float together above the red magnets.

CQ30.11 In the figure, the magnetic field created by wire 1 at the position of wire 2 is into the paper. Hence, the magnetic force on wire 2 is in direction (current down) × (field into the paper) = (force to the right), away from wire 1. Now wire 2 creates a magnetic field into the page at the location of wire 1, so wire 1 feels force (current up) × (field into the paper) = (force to the left), away from wire 2.



ANS. FIG. CQ30.11

CQ30.12 (a) The field can be uniform in magnitude. Gauss's law for magnetism implies that magnetic field lines never start or stop. If the field is uniform in direction, the lines are parallel and their density stays constant along any one bundle of lines. Therefore, the magnitude of the field has the same value at all points along a line in the direction of the field. (b) The magnitude of the field could vary over a plane perpendicular to the lines, or it could be constant throughout the volume.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 30.1 The Biot–Savart Law

***P30.1** (a) Each coil separately produces field given by $B = \dfrac{N\mu_0 I R^2}{2\left(R^2 + x^2\right)^{3/2}}$ at the point halfway between them. Together they produce field

$$\begin{aligned} 2B &= \frac{N\mu_0 I R^2}{\left(R^2 + x^2\right)^{3/2}} = 4.50\times 10^{-5}\ \text{T} \\ &= \frac{50\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)I(0.012\ \text{m})^2}{\left[(0.012\ \text{m})^2 + (0.011\ \text{m})^2\right]^{3/2}} \\ &= \frac{9.05\times 10^{-9}\ \text{T}\cdot\text{m}^3/\text{A}}{4.31\times 10^{-6}\ \text{m}^3} I \end{aligned}$$

$$\rightarrow I = \frac{4.50\times 10^{-5}\ \text{T A}}{2.10\times 10^{-3}\ \text{T}} = \boxed{21.5\ \text{mA}}$$

(b) $\Delta V = IR = (0.021\,5\ \text{A})(210\ \Omega) = \boxed{4.51\ \text{V}}$

(c) $P = (\Delta V)I = (4.51\ \text{V})(0.021\,5\ \text{A}) = \boxed{96.7\ \text{mW}}$

P30.2 Imagine grasping the conductor with the right hand so the fingers curl around the conductor in the direction of the magnetic field. The thumb then points along the conductor in the direction of the current. The results are

(a) $\boxed{\text{toward the left}}$ (b) $\boxed{\text{out of the page}}$ (c) $\boxed{\text{lower left to upper right}}$

P30.3 The magnetic field is given by

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi(0.250\ \text{m})} = \boxed{1.60\times 10^{-6}\ \text{T}}$$

P30.4 Model the tornado as a long, straight, vertical conductor and imagine grasping it with the right hand so the fingers point northward on the western side of the tornado (that is, at the observatory's location). The thumb is directed downward, meaning that the conventional current is downward. The magnitude of the current is found from $B = \mu_0 I/2\pi r\,a$ as

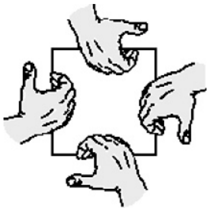
$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(9.00\times 10^3\ \text{m}\right)\left(1.50\times 10^{-8}\ \text{T}\right)}{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}} = 675\ \text{A}$$

Thus, the current is $\boxed{\text{675 A, downward}}$.

P30.5 (a) Use Equation 30.4 for the field produced by each side of the square.

$$B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

where $\theta_1 = 45.0°$, $\theta_2 = -45.0°$, and $a = \frac{\ell}{2}$



ANS. FIG. P30.5

Each side produces a field into the page. The four sides altogether produce

$$B_{\text{center}} = 4B = 4\frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$$

$$= \frac{\mu_0 I}{\pi \ell/2}[\sin 45.0° - \sin(-45.0°)]$$

$$= \frac{2\mu_0 I}{\pi \ell}\left[\frac{2}{\sqrt{2}}\right] = \frac{2\sqrt{2}\mu_0 I}{\pi \ell}$$

$$B = \frac{2\sqrt{2}(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})}{\pi(0.400\ \text{m})}$$

$$= 2\sqrt{2} \times 10^{-5}\ \text{T} = \boxed{28.3\ \mu\text{T into the page}}$$

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(10.0\ \text{A})}{4(0.400\ \text{m})}$$

$$= \boxed{24.7\ \mu\text{T into the page}}$$

P30.6 Treat the magnetic field as that produced in the center of a ring of radius *R* carrying current *I*: from Equation 30.8, the field is $B = \frac{\mu_0 I}{2R}$. The current due to the electron is

$$I = \frac{\Delta q}{\Delta t} = \frac{e}{2\pi R/v} = \frac{ev}{2\pi R}$$

so the magnetic field is

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0}{2R}\left(\frac{ev}{2\pi R}\right) = \frac{\mu_0}{4\pi}\frac{ev}{R^2}$$

$$= \left(\frac{4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}}{4\pi}\right)\frac{(1.60\times 10^{-19}\ \text{C})(2.19\times 10^{6}\ \text{m/s})}{(5.29\times 10^{-11}\ \text{m})^2}$$

$$= \boxed{12.5\ \text{T}}$$

P30.7 We can think of the total magnetic field as the superposition of the field due to the long straight wire, having magnitude $\frac{\mu_0 I}{2\pi R}$ and directed into the page, and the field due to the circular loop, having magnitude $\frac{\mu_0 I}{2R}$ and directed into the page. The resultant magnetic field is:

$$\vec{\mathbf{B}} = \left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R} = \left(1+\frac{1}{\pi}\right)\frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(1.00\ \text{A})}{2(0.150\ \text{m})}$$

$$= 5.52\times 10^{-6} = \boxed{5.52\ \mu\text{T into the page}}$$

P30.8 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\frac{\mu_0 I}{2\pi R}$ and directed into the page) and the field due to the circular loop (having magnitude $\frac{\mu_0 I}{2R}$ and directed into the page). The resultant magnetic field is:

$$\boxed{\vec{\mathbf{B}} = \left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R}\ \text{(directed into the page)}}$$

P30.9 Wire 1 creates at the origin magnetic field:

$\vec{\mathbf{B}}_1 = \frac{\mu_0 I}{2\pi r}$ right hand rule $= \frac{\mu_0 I_1}{2\pi a}$  $= \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}}$

(a) If the total field at the origin is $\frac{2\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} + \vec{\mathbf{B}}_2$ then the second wire must create field according to $\vec{\mathbf{B}}_2 = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} = \frac{\mu_0 I_2}{2\pi(2a)}$ .

Then $I_2 = \boxed{2I_1 \text{ out of the paper}}$.

(b) The other possibility is $\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{2\mu_0 I_1}{2\pi a}\left(-\hat{\mathbf{j}}\right) = \frac{\mu_0 I_1}{2\pi a}\hat{\mathbf{j}} + \vec{\mathbf{B}}_2$. Then,

$$\vec{\mathbf{B}}_2 = \frac{3\mu_0 I_1}{2\pi a}\left(-\hat{\mathbf{j}}\right) = \frac{\mu_0 I_2}{2\pi(2a)}$$  and $I_2 = \boxed{6I_1 \text{ into the paper}}$.

P30.10 The vertical section of wire constitutes one half of an infinitely long, straight wire at distance *x* from *P*, so it creates a field equal to

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right)$$

Hold your right hand with extended thumb in the direction of the current; the field is away from you, into the paper.

For each bit of the horizontal section of wire $d\vec{\mathbf{s}}$ is to the left and $\hat{\mathbf{r}}$ is to the right, so $d\vec{\mathbf{s}} \times \hat{\mathbf{r}} = 0$. The horizontal current produces zero field at *P*. Thus,

$$B = \boxed{\frac{\mu_0 I}{4\pi x} \text{ into the paper}}$$

P30.11 Every element of current creates magnetic field in the same direction, into the page, at the center of the arc. The upper straight portion creates one-half of the field that an infinitely long straight wire would create. The curved portion creates one-quarter of the field that a circular loop produces at its center. The lower straight segment also creates field $\frac{1}{2}\frac{\mu_0 I}{2\pi r}$.

The total field is

$$\vec{\mathbf{B}} = \left(\frac{1}{2}\frac{\mu_0 I}{2\pi r} + \frac{1}{4}\frac{\mu_0 I}{2r} + \frac{1}{2}\frac{\mu_0 I}{2\pi r}\right) \text{ into the page}$$

$$= \boxed{\frac{\mu_0 I}{2r}\left(\frac{1}{\pi} + \frac{1}{4}\right) \text{ into the plane of the paper}}$$

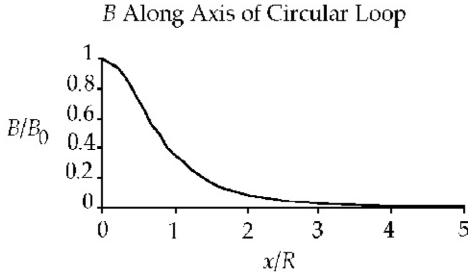
$$= \left(\frac{0.284\,15\mu_0 I}{r}\right) \text{ into the page}$$

P30.12 Along the axis of a circular loop of radius *R*,

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

or $$\frac{B}{B_0} = \left[\frac{1}{(x/R)^2 + 1}\right]^{3/2},$$

where $$B_0 \equiv \frac{\mu_0 I}{2R}.$$



ANS. FIG. P30.12

x/R	*B/B*₀
0.00	1.00
1.00	0.354
2.00	0.089 4
3.00	0.031 6
4.00	0.014 3
5.00	0.007 54

P30.13 We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\vec{s}$ is at 0° or 180° to $\hat{r}$, so $d\vec{s} \times \hat{r} = 0$. Thus, only the curved section of wire contributes to $\vec{B}$ at *P*. Hence, $d\vec{s}$ is tangent to the arc and $\hat{r}$ is radially inward; so $d\vec{s} \times \hat{r} = |ds| 1 \sin 90° \otimes = |ds| \otimes$. All points along the curve are the same distance *r* = 0.600 m from the field point, so

$$B = \int_{\text{all current}} |d\vec{B}| = \int \frac{\mu_0}{4\pi} \frac{I |d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int |ds| = \frac{\mu_0}{4\pi} \frac{I}{r^2} s$$

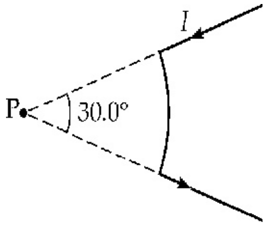
where s is the arc length of the curved wire,

$$s = r\theta = (0.600 \text{ m})(30.0°)\left(\frac{2\pi}{360°}\right) = 0.314 \text{ m}$$

Then,

$$B = \left(10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{(3.00\ \text{A})}{(0.600\ \text{m})^2}(0.314\ \text{m})$$

$$B = \boxed{262\ \text{nT into the page}}$$



ANS. FIG. P30.13

P30.14 (a) Above the pair of wires, the field out of the page of the 50.0-A current will be stronger than the $\left(-\hat{\mathbf{k}}\right)$ field of the 30.0-A current, so they cannot add to zero. Between the wires, both produce fields into the page. They can only add to zero below the wires, at coordinate $y = -|y|$. Here the total field is

$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}$ ☞ $+\frac{\mu_0 I}{2\pi r}$ ☜:

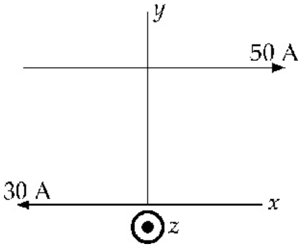
$$0 = \frac{\mu_0}{2\pi}\left[\frac{50.0\ \text{A}}{(|y| + 0.280\ \text{m})}\left(-\hat{\mathbf{k}}\right) + \frac{30.0\ \text{A}}{|y|}\left(\hat{\mathbf{k}}\right)\right]$$

$$50.0|y| = 30.0(|y| + 0.280\ \text{m})$$

$$50.0(-y) = 30.0(0.280\ \text{m} - y)$$

$$-20.0y = 30.0(0.280\ \text{m})$$

$$\boxed{y = -0.420\ \text{m}}$$



ANS. FIG. P30.14

(b) At $y = 0.100$ m the total field is

$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}$ ☞ $+\frac{\mu_0 I}{2\pi r}$ ☜:

$$\vec{\mathbf{B}} = \left(\frac{4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}}{2\pi}\right) \times \left(\frac{50.0\ \mathrm{A}}{(0.280 - 0.100)\ \mathrm{m}}\left(-\hat{\mathbf{k}}\right) + \frac{30.0\ \mathrm{A}}{0.100\ \mathrm{m}}\left(-\hat{\mathbf{k}}\right)\right)$$
$$= 1.16 \times 10^{-4}\ \mathrm{T}\left(-\hat{\mathbf{k}}\right)$$

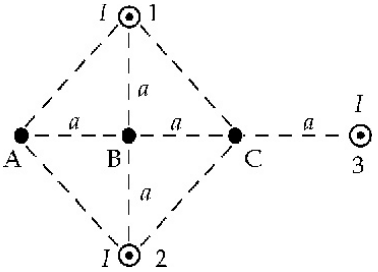
The force on the particle is

$$\vec{\mathbf{F}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$
$$= \left(-2 \times 10^{-6}\ \mathrm{C}\right)\left(150 \times 10^{6}\ \mathrm{m/s}\right)\left(\hat{\mathbf{i}}\right) \times \left(1.16 \times 10^{-4}\ \mathrm{N \cdot s/C \cdot m}\right)\left(-\hat{\mathbf{k}}\right)$$
$$= \boxed{3.47 \times 10^{-2}\ \mathrm{N}\left(-\hat{\mathbf{j}}\right)}$$

(c) We require $\vec{\mathbf{F}}_e = 3.47 \times 10^{-2}\ \mathrm{N}\left(+\hat{\mathbf{j}}\right) = q\vec{\mathbf{E}} = \left(-2 \times 10^{-6}\ \mathrm{C}\right)\vec{\mathbf{E}}$,

so $\vec{\mathbf{E}} = \boxed{-1.73 \times 10^{4}\hat{\mathbf{j}}\ \mathrm{N/C}}$.

P30.15 Label the wires 1, 2, and 3 as shown in ANS. FIG. P30.15(a) and let the magnetic field created by the currents in these wires be $\vec{\mathbf{B}}_1$, $\vec{\mathbf{B}}_2$, and $\vec{\mathbf{B}}_3$, respectively.

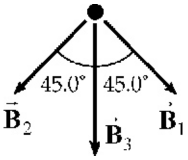


ANS. FIG. P30.15(a)

(a) At point *A*:

$$B_1 = B_2 = \frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$$

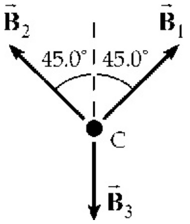
and $B_3 = \dfrac{\mu_0 I}{2\pi(3a)}$.



ANS. FIG. P30.15(b)

The directions of these fields are shown in ANS. FIG. P30.15(b). Observe that the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel while their vertical components both add onto $\vec{\mathbf{B}}_3$. Therefore, the net field at point *A* is:

$$B_A = B_1\cos 45.0° + B_2\cos 45.0° + B_3$$
$$= \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° + \frac{1}{3}\right]$$



ANS. FIG. P30.15(c)

$$B_A = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(2.00\ \mathrm{A})}{2\pi(1.00\times10^{-2}\ \mathrm{m})}\left[\frac{2}{\sqrt{2}}\cos 45° + \frac{1}{3}\right]$$

$$B_A = \boxed{53.3\ \mu\mathrm{T}\text{ toward the bottom of the page}}$$

(b) At point *B*: $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel, leaving

$$B_B = B_3 = \frac{\mu_0 I}{2\pi(2a)}$$

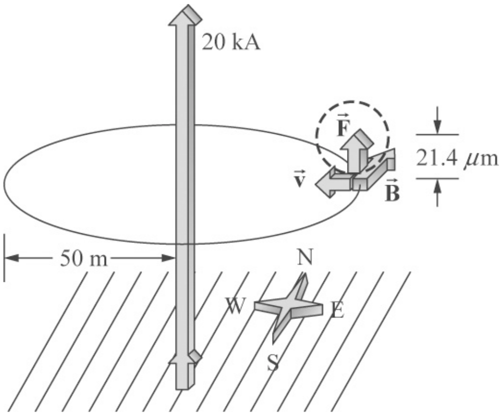
$$B_B = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(2.00\ \mathrm{A})}{2\pi(2)(1.00\times10^{-2}\ \mathrm{m})}$$

$$= \boxed{20.0\ \mu\mathrm{T}\text{ toward the bottom of the page}}$$

(c) At point *C*: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi(a\sqrt{2})}$ and $B_3 = \dfrac{\mu_0 I}{2\pi a}$ with the directions shown in ANS. FIG. P30.15(c). Again, the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel. The vertical components both oppose $\vec{\mathbf{B}}_3$ giving

$$B_C = 2\left[\frac{\mu_0 I}{2\pi(a\sqrt{2})}\cos 45.0°\right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° - 1\right] = \boxed{0}$$

P30.16 (a) ANS. FIG. P30.16 shows the various vectors.



ANS. FIG. P30.16

(b) The upward lightning current creates field lines in counterclockwise horizontal circles.

$$\begin{aligned} B &= \frac{\mu_0 I}{2\pi r}\ [\text{righthand rule}] \\ &= \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(20.0\times10^{3}\ \text{A}\right)}{2\pi\left(50.0\ \text{m}\right)}\ \text{north} \\ &= 8.00\times10^{-5}\ \text{T north} \end{aligned}$$

The force on the electron is

$$\begin{aligned} \vec{\mathbf{F}} &= q\vec{\mathbf{v}}\times\vec{\mathbf{B}} \\ &= \left(-1.6\times10^{-19}\ \text{C}\right)(300\ \text{m/s west})\times\left(8.00\times10^{-5}\ \text{T north}\right) \\ &= -\left(3.84\times10^{-21}\ \text{N down}\right) = \boxed{3.84\times10^{-21}\ \text{N up}} \end{aligned}$$

(c) From Equation 29.3,

$$r = \frac{mv}{qB} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(300\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(8.00\times10^{-5}\ \text{T}\right)} = \boxed{2.14\times10^{-5}\ \text{m}}.$$

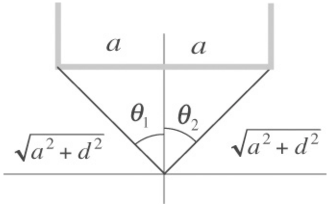
(d) $\boxed{\text{This distance is negligible compared to 50 m, so the electron does}}$ $\boxed{\text{move in a uniform field.}}$

(e) Use Equation 29.4, $\omega = qB/m$, which is equal to $2\pi N/\Delta t$, where N is the number of revolutions:

$$\begin{aligned} N &= \frac{qB\Delta t}{2\pi m} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(8.00\times10^{-5}\ \text{T}\right)\left(60.0\times10^{-6}\ \text{s}\right)}{2\pi\left(9.11\times10^{-31}\ \text{kg}\right)} \\ &= \boxed{134\ \text{revolutions}} \end{aligned}$$

P30.17 Apply the Equation 30.4, $B = \frac{\mu_0 I}{4\pi d}\left(\sin\theta_1 - \sin\theta_2\right)$, to each of the wires.

For the horizontal wire (*H*), $\sin\theta_1 = -\frac{a}{\sqrt{d^2+a^2}}$ and $\sin\theta_2 = \frac{a}{\sqrt{d^2+a^2}}$ because θ_1 measures to the wire's end point on the –*x*-axis and θ_2 measures to the wire's end point on the +*x*-axis. For the left vertical wire (*VL*) and the right vertical wire (*VR*), $\sin\theta_1 = \frac{d}{\sqrt{d^2+a^2}}$ and sin θ_2 = 1 because both angles measure to the wire's end points on the +*y*-axis.



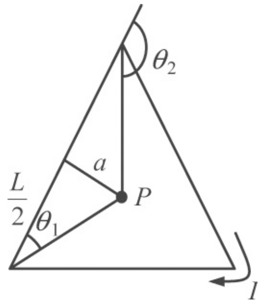
ANS. FIG. P30.17

Take out of the page as the positive direction, and into the page as the negative direction. The field at the origin is

$$
\begin{aligned}
B_O &= |B_{VL}| - |B_H| + |B_{VR}| \\
&= \frac{\mu_0 I}{4\pi a}\left(1 - \frac{d}{\sqrt{d^2 + a^2}}\right) - \frac{\mu_0 I}{4\pi d}\left[\frac{a}{\sqrt{d^2 + a^2}} - \left(-\frac{a}{\sqrt{d^2 + a^2}}\right)\right] \\
&\qquad + \frac{\mu_0 I}{4\pi a}\left(1 - \frac{d}{\sqrt{d^2 + a^2}}\right) \\
&= \frac{\mu_0 I}{4\pi a}\left(2 - \frac{2d}{\sqrt{d^2 + a^2}}\right) - \frac{\mu_0 I}{4\pi d}\left(\frac{2a}{\sqrt{d^2 + a^2}}\right) \\
&= \frac{\mu_0 I}{2\pi ad}\left(d - \frac{d^2}{\sqrt{d^2 + a^2}} - \frac{a^2}{\sqrt{d^2 + a^2}}\right) \\
&= \frac{\mu_0 I}{2\pi ad}\left(d - \frac{d^2 + a^2}{\sqrt{d^2 + a^2}}\right) = \frac{\mu_0 I}{2\pi ad}\left(d - \sqrt{a^2 + d^2}\right) \\
&= -\frac{\mu_0 I}{2\pi ad}\left(\sqrt{a^2 + d^2} - d\right)
\end{aligned}
$$

The field is negative: magnetic field at the origin is $\boxed{\frac{\mu_0 I}{2\pi ad}\left(\sqrt{a^2 + d^2} - d\right)}$ into the page.

P30.18 (a) We use Equation 30.4 in the chapter text for the field created by a straight wire of limited length. The sines of the angles appearing in that equation are equal to the cosines of the complementary angles shown in our diagram. For the distance a from the wire to the field point we have $\tan 30° = \frac{a}{L/2}$, $a = 0.288\,7L$. One wire contributes to the field at P



ANS. FIG. P30.18(a)

$$
\begin{aligned}
B &= \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) = \frac{\mu_0 I}{4\pi(0.288\,7L)}(\cos 30° - \cos 150°) \\
&= \frac{\mu_0 I(1.732)}{4\pi(0.288\,7L)} = \frac{1.50\mu_0 I}{\pi L}
\end{aligned}
$$

Each side contributes the same amount of field in the same direction, which is perpendicularly into the paper in the picture. So the total field is $3\left(\frac{1.50\mu_0 I}{\pi L}\right)=\boxed{\frac{4.50\mu_0 I}{\pi L}}$.

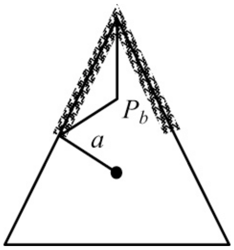
(b) As we showed in part (a), one whole side of the triangle creates field at the center $\frac{\mu_0 I(1.732)}{4\pi a}$. Now one-half of one nearby side of the triangle will be half as far away from point P_b and have a geometrically similar situation. Then it creates at P_b field

$$\frac{\mu_0 I(1.732)}{4\pi(a/2)}=\frac{2\mu_0 I(1.732)}{4\pi a}$$

The two half-sides shown crosshatched in the picture create at P_b field

$$2\left(\frac{2\mu_0 I(1.732)}{4\pi a}\right)=\frac{4\mu_0 I(1.732)}{4\pi(0.288\,7L)}=\frac{6\mu_0 I}{\pi L}$$

The rest of the triangle will contribute somewhat more field in the same direction, so we already have a proof that the field at P_b is $\boxed{\text{stronger}}$.



ANS. FIG. P30.18(b)

P30.19 Assume that the wire on the right is wire 1 and that on the left is wire 2. Also, choose the positive direction for the magnetic field to be out of the page and negative into the page.

(a) At the point half way between the two wires,

$$B_{net}=-B_1-B_2=-\left[\frac{\mu_0 I_1}{2\pi r_1}+\frac{\mu_0 I_2}{2\pi r_2}\right]=-\frac{\mu_0}{2\pi r}(I_1+I_2)$$

$$=-\frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(5.00\times10^{-2}\ \text{m})}(10.0\ \text{A})=-4.00\times10^{-5}\ \text{T}$$

or $B_{net}=\boxed{40.0\ \mu\text{T into the page}}$

(b) At point P_1,

$$B_{\text{net}} = +B_1 - B_2 = \frac{\mu_0}{2\pi}\left[+\frac{I_1}{r_1} - \frac{I_2}{r_2}\right]$$

$$B_{\text{net}} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi}\left[\frac{5.00\ \text{A}}{0.100\ \text{m}} - \frac{5.00\ \text{A}}{0.200\ \text{m}}\right]$$

$$= \boxed{5.00\ \mu\text{T out of page}}$$

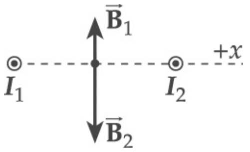
(c) At point P_2,

$$B_{\text{net}} = -B_1 + B_2 = \frac{\mu_0}{2\pi}\left[-\frac{I_1}{r_1} + \frac{I_2}{r_2}\right]$$

$$B_{\text{net}} = \frac{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}{2\pi}\left[-\frac{5.00\ \text{A}}{0.300\ \text{m}} + \frac{5.00\ \text{A}}{0.200\ \text{m}}\right]$$

$$= \boxed{1.67\ \mu\text{T out of page}}$$

P30.20 Call the wire carrying a current of 3.00 A wire 1 and the other wire 2. Also, choose the line running from wire 1 to wire 2 as the positive *x* direction.



ANS. FIG. P30.20(a)

(a) At the point midway between the wires, the field due to each wire is parallel to the *y*-axis and the net field is

$$B_{\text{net}} = +B_{1y} - B_{2y} = \mu_0(I_1 - I_2)/2\pi r$$

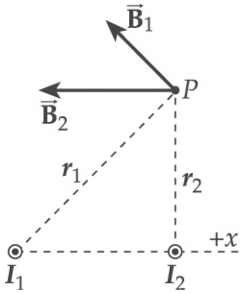
Thus,

$$B_{\text{net}} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}{2\pi(0.100\ \text{m})}(3.00\ \text{A} - 5.00\ \text{A}) = -4.00\times10^{-6}\ \text{T}$$

or $B_{\text{net}} = \boxed{4.00\ \mu\text{T toward the bottom of the page}}$

(b) Refer to ANS. FIG. P30.20(b). At point P, $r_1 = (0.200\ \text{m})\sqrt{2}$ and B_1 is directed at $\theta_1 = +135°$. The magnitude of B_1 is

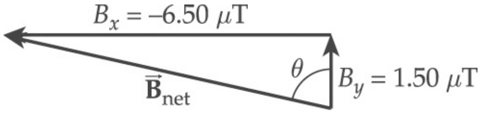
$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(3.00\ \text{A})}{2\pi(0.200\sqrt{2}\ \text{m})} = 2.12\ \mu\text{T}$$



ANS. FIG. P30.20(b)

The contribution from wire 2 is in the –*x* direction and has magnitude

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(5.00\ \mathrm{A})}{2\pi(0.200\ \mathrm{m})} = 5.00\ \mu\mathrm{T}$$



ANS. FIG. P30.20(c)

Therefore, the components of the net field at point *P* are:

$$B_x = B_1 \cos 135° + B_2 \cos 180°$$

$$= (2.12\ \mu\mathrm{T})\cos 135° + (5.00\ \mu\mathrm{T})\cos 180° = -6.50\ \mu\mathrm{T}$$

and

$$B_y = B_1 \sin 135° + B_2 \sin 180° = (2.12\ \mu\mathrm{T})\sin 135° + 0 = +1.50\ \mu\mathrm{T}$$

Therefore,

$$B_{\text{net}} = \sqrt{B_x^2 + B_y^2} = 6.67\ \mu\mathrm{T}$$

at $\theta = \tan^{-1}\left(\frac{|B_x|}{B_y}\right) = \tan^{-1}\left(\frac{6.50\ \mu\mathrm{T}}{1.50\ \mu\mathrm{T}}\right) = 77.0°$

in ANS. FIG. P30.20(c), which is 77.0° + 90.0° = 167.0° from the positive *x* axis. Therefore,

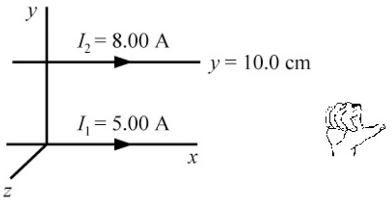
$$\vec{\mathbf{B}}_{\text{net}} = \boxed{6.67\ \mu\mathrm{T\ at\ 167.0°\ from\ the\ positive\ } x \mathrm{\ axis}}.$$

Section 30.2 The Magnetic Force Between Two Parallel Conductors

P30.21 Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

(a) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}\hat{\mathbf{k}} = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(5.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}\hat{\mathbf{k}}$$

$$\vec{\mathbf{B}} = \boxed{1.00 \times 10^{-5}\ \mathrm{T\ out\ of\ the\ page}}$$



ANS. FIG. P30.21(a)

(b) $$\vec{\mathbf{F}}_B = I_2\vec{\ell} \times \vec{\mathbf{B}} = (8.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\hat{\mathbf{i}} \times (1.00 \times 10^{-5}\ \mathrm{T})\hat{\mathbf{k}}\right]$$
$$= (8.00 \times 10^{-5}\ \mathrm{N})(-\hat{\mathbf{j}})$$

ANS. FIG. P30.21(b)

$$\vec{\mathbf{F}}_B = \boxed{8.00 \times 10^{-5}\ \mathrm{N\ toward\ the\ first\ wire}}$$

(c) $$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}(-\hat{\mathbf{k}}) = \frac{(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A})(8.00\ \mathrm{A})}{2\pi(0.100\ \mathrm{m})}(-\hat{\mathbf{k}})$$
$$= (1.60 \times 10^{-5}\ \mathrm{T})(-\hat{\mathbf{k}})$$

ANS. FIG. P30.21(c)

$$\vec{\mathbf{B}} = \boxed{1.60 \times 10^{-5}\ \mathrm{T\ into\ the\ page}}$$

(d) $$\vec{\mathbf{F}}_B = I_1\vec{\ell} \times \vec{\mathbf{B}} = (5.00\ \mathrm{A})\left[(1.00\ \mathrm{m})\hat{\mathbf{i}} \times (1.60 \times 10^{-5}\ \mathrm{T})(-\hat{\mathbf{k}})\right]$$
$$= (8.00 \times 10^{-5}\ \mathrm{N})(+\hat{\mathbf{j}})$$

ANS. FIG. P30.21(d)

$$\vec{\mathbf{F}}_B = \boxed{8.00\times 10^{-5}\text{ N towards the second wire}}$$

P30.22 (a) The force per unit length that parallel conductors exert on each other is, from Equation 30.12, $F/\ell = \mu_0 I_1 I_2/2\pi d$. Thus, if $F/\ell = 2.00\times 10^{-4}\text{ N/m}$, I_1 = 5.00 A, and d = 4.00 cm, the current in the second wire must be

$$I_2 = \frac{2\pi d}{\mu_0 I_1}\left(\frac{F}{\ell}\right)$$

$$= \left[\frac{2\pi\left(4.00\times 10^{-2}\text{ m}\right)}{\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)\left(5.00\text{ A}\right)}\right]\left(2.00\times 10^{-4}\text{ N/m}\right)$$

$$= \boxed{8.00\text{ A}}$$

(b) Since parallel conductors carrying currents in the same direction attract each other (see Section 30.2 in the textbook), the currents in these conductors which repel each other must be $\boxed{\text{opposite directions.}}$

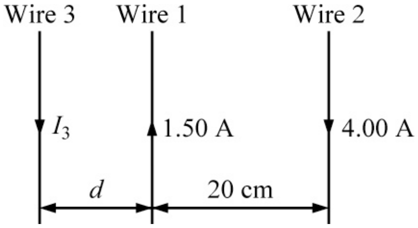
(c) From Equation 30.12, the force is directly proportional to the product of the currents. The result of reversing the direction of either of the currents and doubling the magnitude would be that the $\boxed{\text{force of interaction would be attractive and the magnitude}}$ $\boxed{\text{of the force would double}}$.

P30.23 (a) From Equation 30.12, the force per unit length that one wire exerts on the other is $F/\ell = \mu_0 I_1 I_2/2\pi d$, where d is the distance separating the two wires. In this case, the value of this force is

$$\frac{F}{\ell} = \frac{\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)\left(3.00\text{ A}\right)^2}{2\pi\left(6.00\times 10^{-2}\text{ m}\right)} = \boxed{3.00\times 10^{-5}\text{ N/m}}$$

(b) We can answer this question by consulting Section 30.2 in the textbook, or we can reason it out. Imagine these two wires lying side by side on a table with the two currents flowing toward you, wire 1 on the left and wire 2 on the right. The right-hand rule that relates current to field direction shows the magnetic field due to wire 1 at the location of wire 2 is directed vertically upward. Then, the right-hand rule that relates current and field to force gives the direction of the force experienced by wire 2, with its current flowing through this field, as being to the left, back toward wire 1. Thus, the force one wire exerts on the other is an $\boxed{\text{attractive}}$ force.

P30.24 Carrying oppositely directed currents, wires 1 and 2 repel each other. If wire 3 were between them, it would have to repel either 1 or 2, so the force on that wire could not be zero. If wire 3 were to the right of wire 2, it would feel a larger force exerted by 2 than that exerted by 1, so the total force on 3 could not be zero. Therefore wire 3 must be to the left of both other wires as shown. It must carry downward current so that it can attract wire 2. We answer part (b) first.



ANS. FIG. P30.24

(b) For the equilibrium of wire 3 we have

$$F_{1\text{ on }3} = F_{2\text{ on }3}: \qquad \frac{\mu_0(1.50\text{ A})I_3}{2\pi d} = \frac{\mu_0(4.00\text{ A})I_3}{2\pi(20.0\text{ cm} + d)}$$

$$1.50(20.0\text{ cm} + d) = 4.00d$$

$$d = \frac{30.0\text{ cm}}{2.50} = \boxed{12.0\text{ cm to the left of wire 1}}$$

(a) Thus the situation is possible in just $\boxed{\text{one way.}}$

(c) For the equilibrium of wire 1,

$$\frac{\mu_0 I_3(1.50\text{ A})}{2\pi(12.0\text{ cm})} = \frac{\mu_0(4.00\text{ A})(1.50\text{ A})}{2\pi(20.0\text{ cm})}$$

$$I_3 = \frac{12}{20}(4.00\text{ A}) = \boxed{2.40\text{ A down}}$$

We know that wire 2 must be in equilibrium because the forces on it are equal in magnitude to the forces that it exerts on wires 1 and 3, which are equal because they both balance the equal-magnitude forces that 1 exerts on 3 and that 3 exerts on 1.

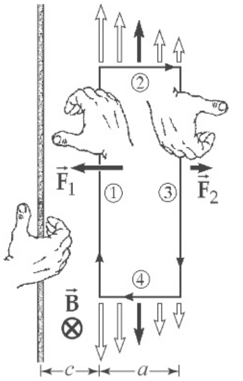
P30.25 To the right of the long, straight wire, current I_1 creates a magnetic field into the page. By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (from Equation 30.12):

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{-a}{c(c+a)}\right]\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \frac{\left(4\pi\times10^{-7}\ \text{N/A}^2\right)(5.00\ \text{A})(10.0\ \text{A})(0.450\ \text{m})}{2\pi}$$

$$\times\left(\frac{-0.150\ \text{m}}{(0.100\ \text{m})(0.250\ \text{m})}\right)\hat{\mathbf{i}}$$

$$\vec{\mathbf{F}} = \left(-2.70\times10^{-5}\hat{\mathbf{i}}\right)\ \text{N} = \left(-27.0\times10^{-6}\hat{\mathbf{i}}\right)\ \text{N} = \boxed{-27.0\hat{\mathbf{i}}\ \mu\text{N}}$$



ANS. FIG. P30.25

P30.26 See ANS. FIG. P30.25. By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (from Equation 30.12)

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{\mathbf{i}} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\hat{\mathbf{i}}$$

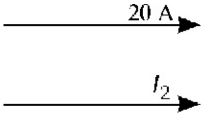
$$\vec{\mathbf{F}} = \boxed{\frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{a}{c(c+a)}\right]\text{ to the left}}$$

P30.27 To attract, both currents ($I_1 = 20.0$ A, and I_2) must be to the right. The attraction is described by (from Equation 30.12)

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

So $$I_2 = \frac{F}{\ell}\frac{2\pi a}{\mu_0 I_1}$$

$$= \left(320\times10^{-6}\ \text{N/m}\right)\left(\frac{2\pi(0.500\ \text{m})}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(20.0\ \text{A})}\right) = 40.0\ \text{A}$$



ANS. FIG. P30.27

The zero-field point must lie between the two wires: this point cannot be above the upper wire or below the lower wire because the fields in these regions have the same direction, out of the page above the upper wire, and into the page below the lower wire. Let y represent the coordinate of the zero-field point above the lower wire; then, $r_1 = (0.500\text{ m}) - y$ and $r_2 = y$ represent the respective distances of currents I_1 and I_2 to the zero-field point. Taking the positive direction to be out of the page, at the zero-field point,

$$B = -B_1 + B_2$$

$$0 = -\frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2}$$

Eliminating and solving for r_1,

$$\frac{I_1}{r_1} = \frac{I_2}{r_2} \quad \rightarrow \quad r_1 = r_2\frac{I_1}{I_2} \quad \rightarrow \quad (0.500\text{ m}) - y = y\frac{I_1}{I_2}$$

Then,

$$(0.500\text{ m}) = y\left(\frac{I_1}{I_2} + 1\right)$$

$$y = \frac{(0.500\text{ m})}{\left(\frac{I_1}{I_2} + 1\right)} = \frac{(0.500\text{ m})}{\left(\frac{20.0\text{ A}}{40.0\text{ A}} + 1\right)} = \boxed{0.333\text{ m}}$$

P30.28 From Equation 30.12, we find the separation distance between the wires as

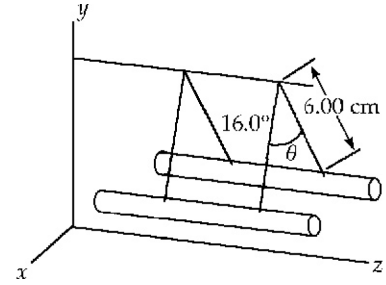
$$\frac{F_B}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} \qquad \rightarrow \qquad a = \frac{\mu_0 I_1 I_2 \ell}{2\pi F_B}$$

Substituting numerical values,

$$a = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(10.0\text{ A})(10.0\text{ A})(0.500\text{ m})}{2\pi(1.00\text{ N})}$$

$$= 1.00 \times 10^{-5}\text{ m} = 10.0\ \mu\text{m}$$

This is the required center-to-center separation distance of the wires, but the wires cannot be this close together. Their minimum possible center-to-center separation distance occurs if the wires are touching, but this value is $2r = 2(250\ \mu\text{m}) = 500\ \mu\text{m}$, which is much larger than the required value above. We could try to obtain this force between wires of smaller diameter, but these wires would have higher resistance and less surface area for radiating energy. It is likely that the wires would melt very shortly after the current begins.



ANS. FIG. P30.29

P30.29 This is almost a standard equilibrium problem involving tension, weight, and a horizontal repulsive force; however, here we must consider the magnetic force per unit length and the weight per unit length. The tension makes an angle $\theta/2 = 8.00°$ with the vertical. The mass per unit length is $\lambda = mg/L$. The separation between the wires is $a = 2\ell \sin\theta/2$.

(a) Because the wires repel, the currents are in $\boxed{\text{opposite directions}}$.

(b) For balance, the ratio of the horizontal tension component $T \sin \theta/2$ to the vertical tension component $T \cos \theta/2$ is equal to the ratio of the horizontal magnetic force per unit length F_B/L to the vertical weight per unit length F_g/L:

$$\frac{T\sin\theta/2}{T\cos\theta/2} = \frac{F_B/L}{F_g/L}$$

But,

$$F_B/L = IB\sin 90.0° = IB = I\frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I^2}{2\pi a}$$

$$F_g/L = \lambda g$$

Rearranging and substituting gives

$$\tan\theta/2 = \frac{\mu_0 I^2/2\pi a}{\lambda g} = \frac{\mu_0 I^2}{2\pi(2\ell\sin\theta/2)\lambda g}$$

Solving,

$$I^2 = \frac{4\pi\ell\lambda g}{\mu_0}(\tan\theta/2)(\sin\theta/2)$$

$$I^2 = \left[\frac{4\pi(0.060\ 0\ \text{m})(40.0\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})}\right] \times(\tan 8.00°)(\sin 8.00°)$$

$$I = \boxed{67.8\ \text{A}}$$

(c) $\boxed{\text{Smaller. A smaller gravitational force would be pulling down on the wires, requiring less magnetic force to raise the wires to the same angle and therefore less current.}}$

Section 30.3 Ampère's Law

P30.30 From $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 I$, $I = \dfrac{2\pi r B}{\mu_0} = \dfrac{2\pi\left(1.00 \times 10^{-3}\text{ m}\right)(0.100\text{ T})}{4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}} = \boxed{500\text{ A}}$.

P30.31 (a) From Ampère's law, the magnetic field at point *a* is given by $B_a = \dfrac{\mu_0 I_a}{2\pi r_a}$, where I_a is the net current through the area of the circle of radius r_a. In this case, I_a = 1.00 A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}\right)(1.00\text{ A})}{2\pi\left(1.00 \times 10^{-3}\text{ m}\right)}$$

$$= \boxed{200\ \mu\text{T toward top of page}}$$

(b) Similarly at point *b*: $B_b = \dfrac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current through the area of the circle having radius r_b. Taking out of the page as positive, I_b = 1.00 A – 3.00 A = –2.00 A, or I_b = 2.00 A into the page. Therefore,

$$B_b = \frac{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}\right)(2.00\text{ A})}{2\pi\left(3.00 \times 10^{-3}\text{ m}\right)}$$

$$= \boxed{133\ \mu\text{T toward bottom of page}}$$

P30.32 (a) $B_{\text{inner}} = \dfrac{\mu_0 N I}{2\pi r} = \dfrac{\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)(900)\left(14.0\times 10^3\text{ A}\right)}{2\pi(0.700\text{ m})} = \boxed{3.60\text{ T}}$

(b) $B_{\text{outer}} = \dfrac{\mu_0 N I}{2\pi r} = \dfrac{\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)(900)\left(14.0\times 10^3\text{ A}\right)}{2\pi(1.30\text{ m})} = \boxed{1.94\text{ T}}$

P30.33 Let the current *I* be to the right, in the positive *x* direction. The proton travels to the left, and is a distance *d* above the wire. Take up as the positive *y* direction. At the proton's location, the current creates a field $B = \dfrac{\mu_0 I}{2\pi d}$ in the positive *z* direction. The weight of the proton and the magnetic force are in balance:

$$mg\left(-\hat{\mathbf{j}}\right) + qv\left(-\hat{\mathbf{i}}\right) \times \frac{\mu_0 I}{2\pi d}\left(\hat{\mathbf{k}}\right) = 0$$

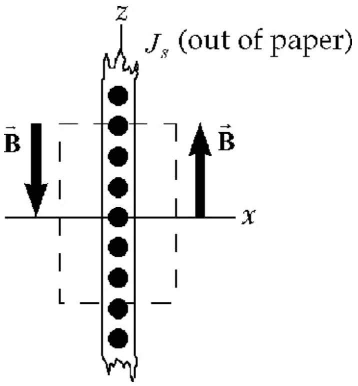
$$mg\left(-\hat{\mathbf{j}}\right) + \frac{qv\mu_0 I}{2\pi d}\left(\hat{\mathbf{j}}\right) = 0$$

$$d = \frac{qv\mu_0 I}{2\pi mg}$$

$$= \frac{(1.60\times10^{-19}\ \text{C})(2.30\times10^{4}\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.20\times10^{-6}\ \text{A})}{2\pi(1.67\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)}$$

$$= \boxed{5.40\ \text{cm}}$$

P30.34 We may regard the sheet as being composed of filaments of current $J_s d\vec{s}$ directed out of the page. According to the Biot-Savart law, the field contribution at a point has the direction $d\vec{s}\times\hat{\mathbf{r}}$, where $\hat{\mathbf{r}}$ points from the current filament to the point. Consider the field contributions at an arbitrary point *P* to the right of the sheet. Draw a line normal to the sheet that passes through *P*. Consider the contributions to the field at *P* from two filaments that lie along the same vertical line and are equidistant from the normal (and *P*). The upper filament contributes +*z* and +*x* field components, but the lower filament +*z* and –*x* field components. The resulting field from both filaments points in the +*z*-direction. By similar reasoning, the magnetic field at any point on the left side of the sheet points in the –*z* direction. These same arguments hold for any point within the sheet. Also, the same reasoning shows that for any pair of filaments that lie on the same vertical line, the magnetic field at a point midway between them is zero. Thus, the field has no horizontal component within the sheet.



ANS. FIG. P30.34

Therefore, each filament of current creates a contribution to the total field that is parallel to the sheet and perpendicular to the current direction. They create field lines straight up to the right of the sheet and straight down to the left of the sheet.

From Ampère's law applied to the suggested rectangle,

$$\oint \vec{\mathbf{B}}\cdot d\vec{s} = \mu_0 I: \quad B\cdot 2\ell + 0 = \mu_0 J_s \ell$$

Therefore the field is uniform in space, with the magnitude

$$\boxed{B = \frac{\mu_0 J_s}{2}}$$

P30.35 (a) In $B = \dfrac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400\ \text{cm}}$.

(b) $\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r_1}\hat{\mathbf{k}} + \frac{\mu_0 I}{2\pi r_2}\left(-\hat{\mathbf{k}}\right)$

so $B = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(2.00\ \text{A}\right)}{2\pi}\left(\frac{1}{0.398\,5\ \text{m}} - \frac{1}{0.401\,5\ \text{m}}\right)$

$= \boxed{7.50\ \text{nT}}$

(c) Call *r* the distance from cord center to field point and $2d = 3.00$ mm the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\frac{2d}{r^2 - d^2}$$

$$7.50 \times 10^{-10}\ \text{T}$$
$$= \left(2.00 \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(2.00\ \text{A}\right)\frac{\left(3.00 \times 10^{-3}\ \text{m}\right)}{r^2 - \left(2.25 \times 10^{-6}\ \text{m}\right)^2}$$

so $r = \boxed{1.26\ \text{m}}$.

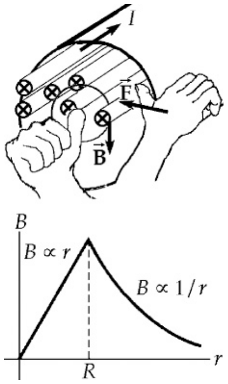
The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

P30.36 By Ampère's law, the field at the position of the wire at distance *r* from the center is due to the fraction of the other 99 wires that lie within the radius *r*.

$$\oint \vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I:$$

$$B\cdot 2\pi r = \mu_0\left[99I\left(\frac{\pi r^2}{\pi R^2}\right)\right] \quad \rightarrow \quad B = \frac{\mu_0(99I)}{2\pi r}\left(\frac{r^2}{R^2}\right) = \frac{\mu_0(99I)}{2\pi R}\left(\frac{r}{R}\right)$$



ANS. FIG. P30.36

The field is proportional to r, as shown in ANS. FIG. P30.36. This field points tangent to a circle of radius r and exerts a force $\vec{\mathbf{F}} = I\vec{\ell}\times\vec{\mathbf{B}}$ on the wire toward the center of the bundle. The magnitude of the force is

$$\frac{F}{\ell} = IB\sin\theta = I\left[\frac{\mu_0(99)I}{2\pi R}\left(\frac{r}{R}\right)\right]\sin 90° = \frac{\mu_0(99)I^2}{2\pi R}\left(\frac{r}{R}\right)$$

$$= \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(99)(2.00\text{ A})^2}{2\pi\left(0.500\times10^{-2}\text{ m}\right)}(0.400)$$

$$= 6.34\times10^{-3}\text{ N/m}$$

(a) $\boxed{6.34\times10^{-3}\text{ N/m}}$

(b) Referring to the figure, the field is clockwise, so at the position of the wire, the field is downward, and the force is $\boxed{\text{inward toward the center of the bundle}}$.

(c) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is $\boxed{\text{greatest at the outer surface}}$.

P30.37 We assume the current is vertically upward.

(a) Consider a circle of radius r slightly less than R. It encloses no current, so from

$$\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{inside}}\text{ gives } B(2\pi r) = 0,$$

we conclude that the magnetic field is $\boxed{\text{zero}}$.

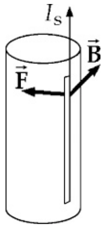
ANS. FIG. P30.37(a)

(b) Now let the r be barely larger than R. Ampère's law becomes

$$B(2\pi R) = \mu_0 I,$$

so $\boxed{B = \dfrac{\mu_0 I}{2\pi R}\quad\text{tangent to the wall}}$.

By the right-hand rule, the field direction is counterclockwise ↺ (as seen from above).



ANS. FIG. P30.37(b)

(c) Consider a strip of the wall of horizontal width ds and length ℓ. Its width is so small compared to $2\pi R$ that the field at its location would be essentially unchanged if the current in the strip were turned off.

The current it carries is $I_s = \dfrac{I\,ds}{2\pi R}$ up.

The force on it is

$$d\vec{\mathbf{F}} = I_s\vec{\ell}\times\vec{\mathbf{B}} = \frac{Ids}{2\pi R}\left(\ell\frac{\mu_0 I}{2\pi R}\right)\left[\widehat{\textbf{up}}\times\widehat{\textbf{into page}}\right]$$

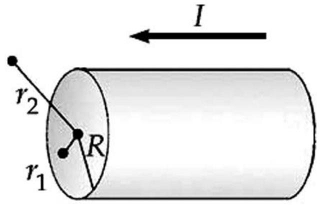
$$= \frac{\mu_0 I^2\ell ds}{(2\pi R)^2}\widehat{\textbf{radially inward}}$$

The pressure on the strip, and therefore, everywhere on the cylinder, is

$$P = \frac{dF}{dA} = \frac{\mu_0 I^2\ell ds}{(2\pi R)^2}\frac{1}{\ell ds} = \boxed{\frac{\mu_0 I^2}{(2\pi R)^2}\text{ inward}}$$

The pinch effect makes an effective demonstration when an aluminum can crushes itself as it carries a large current along its length.

P30.38 Take a circle of radius r_1 or r_2 to apply $\oint\vec{\mathbf{B}}\cdot d\vec{\mathbf{s}} = \mu_0 I$, where for nonuniform current density $I = \int J dA$. In this case $\vec{\mathbf{B}}$ is parallel to $d\vec{\mathbf{s}}$ and the direction of J is straight through the area element dA, so Ampère's law gives



ANS. FIG. P30.38

$$\oint B ds = \mu_0\int J dA$$

(a) For $r_1 < R$,

$$2\pi r_1 B = \mu_0\int_0^{r_1} br(2\pi r dr) = \mu_0 2\pi b\left[\frac{r_1^3}{3} - 0\right]$$

and $B = \boxed{\frac{1}{3}\left(\mu_0 b r_1^2\right)\text{ (inside)}}$

(b) For $r_2 > R$,

$$2\pi r_2 B = \mu_0\int_0^{R} br(2\pi r dr)$$

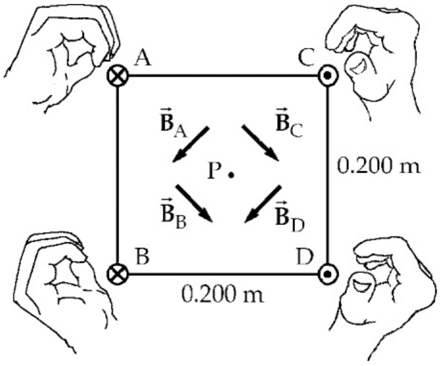
and $B = \boxed{\frac{\mu_0 b R^3}{3r_2}\text{ (outside)}}$

P30.39 Each wire is distant from P by ($\ell = 0.200$ m)

$$r = \sqrt{\ell^2 + \ell^2}/2 = \ell/\sqrt{2}$$

and each wire produces a field at P of equal magnitude

$$B = \frac{\mu_0 I}{2\pi r}$$



ANS. FIG. P30.39

Carrying currents into the page, A produces at P a field to the left and downward at –135°, while B creates a field to the right and downward at –45°. Carrying currents out of the page, C produces a field downward and to the right at –45°, while D's contribution is downward and to the left. All horizontal components cancel; thus, all remaining components are vertically downward. The magnitude of the resulting field is

$$B_P = 4B\cos 45.0° = 4\frac{\mu_0 I}{2\pi r}\cos 45.0° = 4\frac{\mu_0 I}{2\pi\left(\ell/\sqrt{2}\right)}\frac{1}{\sqrt{2}} = \frac{2\mu_0 I}{\pi\ell}$$

$$= \frac{2\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(5.00\ \text{A})}{\pi(0.200\ \text{m})} = 2.00\times 10^{-5}\ \text{T}$$

The magnetic field is $\boxed{20.0\ \mu\text{T toward the bottom of the page}}$.

Section 30.4 The Magnetic Field of a Solenoid

P30.40 The magnetic field inside of a solenoid is $B = \mu_0 nI = \mu_0(N/L)I$. Thus, the number of turns on this solenoid must be

$$N = \frac{BL}{\mu_0 I} = \frac{(9.00\ \text{T})(0.500\ \text{m})}{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(75.0\ \text{A})} = \boxed{4.77\times 10^4\ \text{turns}}$$

P30.41 The magnetic field at the center of a solenoid is $B = \mu_0 \frac{N}{\ell} I$, so

$$I = \frac{B}{\mu_0 n} = \frac{\left(1.00\times 10^{-4}\ \text{T}\right)(0.400\ \text{m})}{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1\,000)} = \boxed{31.8\ \text{mA}}$$

P30.42 In the expression $B = N\mu_0 I/\ell$ for the field within a solenoid with radius much less than 20 cm, all we want to do is increase N.

(a) Make the wire as long and thin as possible without melting when it carries the 5-A current. Then the solenoid can have many turns.

(b) As small in radius as possible with your experiment fitting inside. Then with a smaller circumference the wire can form a solenoid with more turns.

P30.43 (a) The field produced by the solenoid in its interior is given by

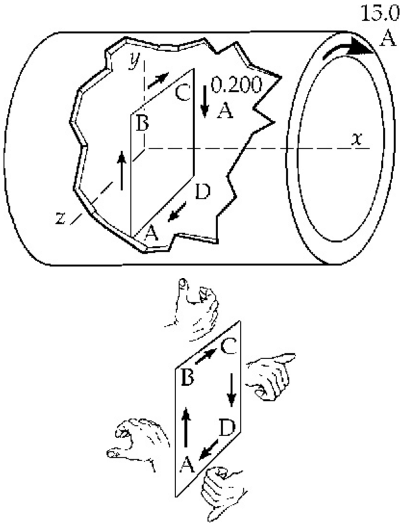
$$\vec{\mathbf{B}} = \mu_0 n I\left(-\hat{\mathbf{i}}\right) = \left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)\left(\frac{30.0}{10^{-2}\ \mathrm{m}}\right)\left(15.0\ \mathrm{A}\right)\left(-\hat{\mathbf{i}}\right)$$

$$\vec{\mathbf{B}} = -\left(5.65\times10^{-2}\ \mathrm{T}\right)\hat{\mathbf{i}}$$

The force exerted on side AB of the square current loop is

$$\left(\vec{\mathbf{F}}_B\right)_{AB} = I\vec{\mathbf{L}}\times\vec{\mathbf{B}} = \left(0.200\ \mathrm{A}\right)$$
$$\times\left[\left(2.00\times10^{-2}\ \mathrm{m}\right)\hat{\mathbf{j}}\times\left(5.65\times10^{-2}\ \mathrm{T}\right)\left(-\hat{\mathbf{i}}\right)\right]$$

$$\left(\vec{\mathbf{F}}_B\right)_{AB} = \left(2.26\times10^{-4}\ \mathrm{N}\right)\hat{\mathbf{k}}$$



ANS. FIG. P30.43

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of

$\boxed{226\ \mu\mathrm{N}\text{ directed away from the center of the loop}}$.

(b) From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid is $\boxed{\text{zero}}$. More formally, the magnetic dipole moment of the square loop is given by

$$\vec{\mu} = I\vec{\mathbf{A}} = \left(0.200\ \mathrm{A}\right)\left(2.00\times10^{-2}\ \mathrm{m}\right)^2\left(-\hat{\mathbf{i}}\right) = -80.0\ \mu\mathrm{A\cdot m^2}\ \hat{\mathbf{i}}$$

The torque exerted on the loop is then

$$\vec{\tau} = \vec{\mu}\times\vec{\mathbf{B}} = \left(-80.0\ \mu\mathrm{A\cdot m^2}\ \hat{\mathbf{i}}\right)\times\left(-5.65\times10^{-2}\ \mathrm{T}\ \hat{\mathbf{i}}\right) = \boxed{0}$$

P30.44 The number of turns is $N = \dfrac{75.0\text{ cm}}{0.100\text{ cm}} = 750.$ We assume that the solenoid is long enough to qualify as a long solenoid. Then the field within it (not close to the ends) is $B = \dfrac{N\mu_0 I}{\ell}$, so

$$I = \frac{B\ell}{N\mu_0} = \frac{\left(8.00\times10^{-3}\text{ T}\right)\left(0.750\text{ m}\right)}{750\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)} = 6.37\text{ A}$$

The resistance of the wire is

$$R = \frac{\rho\ell_{\text{wire}}}{A} = \frac{\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)2\pi\left(0.050\ 0\text{ m}\right)750}{\pi\left(0.050\ 0\times10^{-2}\text{ m}\right)^2} = 5.10\ \Omega$$

The power delivered is

$$P = I\Delta V = I^2R = \left(6.37\text{ A}\right)^2\left(5.10\ \Omega\right) = \boxed{207\text{ W}}$$

The power required would be smaller if wire were wrapped in several layers.

P30.45 (a) From $R = \rho L/A$, the required length of wire to be used is $L = \dfrac{R\cdot A}{\rho}$. The total number of turns on the solenoid (that is, the number of times this length of wire will go around a 1.00 cm radius cylinder) is

$$N = \frac{L}{2\pi r} = \frac{R\cdot A}{2\pi r\rho} = \frac{\left(5.00\ \Omega\right)\left[\pi\left(0.500\times10^{-3}\text{ m}\right)^2/4\right]}{2\pi\left(1.00\times10^{-2}\text{ m}\right)\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)}$$

$$= \boxed{9.2\times10^2\text{ turns}}$$

(b) From $B = \mu_0 nI$, the number of turns per unit length on the solenoid is

$$n = \frac{B}{\mu_0 I} = \frac{4.00\times10^{-2}\text{ T}}{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(4.00\text{ A}\right)} = 7.96\times10^3\text{ turns/m}$$

Thus, the required length of the solenoid is

$$L = \frac{N}{n} = \frac{9.2\times10^2\text{ turns}}{7.96\times10^3\text{ turns/m}} = 0.12\text{ m} = \boxed{12\text{ cm}}$$

Section 30.5 Gauss's Law in Magnetism

P30.46 (a) The magnetic flux through the flat surface S_1 is

$$\left(\Phi_B\right)_{\text{flat}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = B\pi R^2 \cos(180-\theta) = \boxed{-B\pi R^2 \cos\theta}$$

(b) The net flux out of the closed surface is zero:

$$\left(\Phi_B\right)_{\text{flat}} + \left(\Phi_B\right)_{\text{curved}} = 0$$

Therefore,

$$\left(\Phi_B\right)_{\text{curved}} = \boxed{B\pi R^2 \cos\theta}$$

P30.47 The flux is defined as $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$

(a) The flux through the shaded face is $\Phi_B = B_x A_x + B_y A_y + B_z A_z$. The shaded square's area is in the *yz* plane, so it counts as an *x* component of area. Here $A_y = A_z = 0$. Then,

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = \left(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right) \text{ T} \cdot \left(2.50 \times 10^{-2} \text{ m}\right)^2 \hat{\mathbf{i}}$$

$$\Phi_B = 3.12 \times 10^{-3} \text{ T} \cdot \text{m}^2 = 3.12 \times 10^{-3} \text{ Wb} = \boxed{3.12 \text{ mWb}}$$

(b) For a closed surface, $\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = 0$, so $\left(\Phi_B\right)_{\text{total}} = \oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \boxed{0}$

P30.48 (a) $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA$ where *A* is the cross-sectional area of the solenoid. Then,

$$\begin{aligned}\Phi_B &= \left(\frac{\mu_0 N I}{\ell}\right)\left(\pi r^2\right) \\ &= \frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(300)(12.0 \text{ A})}{0.300 \text{ m}}\left[\pi\left(0.012\ 5 \text{ m}\right)^2\right] \\ &= 7.40 \times 10^{-7} \text{ Wb} = \boxed{7.40\ \mu\text{Wb}}\end{aligned}$$

(b) $\Phi_B = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = BA = \left(\dfrac{\mu_0 N I}{\ell}\right)\left[\pi\left(r_2^2 - r_1^2\right)\right]$

$$\Phi_B = \left[\frac{\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)(300)(12.0 \text{ A})}{(0.300 \text{ m})}\right] \times \pi\left[(8.00)^2 - (4.00)^2\right]\left(10^{-3} \text{ m}\right)^2$$

$$= \boxed{2.27\ \mu\text{Wb}}$$

Section 30.6 Magnetism in Matter

P30.49 (a) The Bohr magneton is

$$\mu_B = \left(9.27\times10^{-24}\,\frac{\text{J}}{\text{T}}\right)\left(\frac{\text{N}\cdot\text{m}}{1\ \text{J}}\right)\left(\frac{1\ \text{T}}{\text{N}\cdot\text{s/C}\cdot\text{m}}\right)\left(\frac{1\ \text{A}}{\text{C/s}}\right)$$

$$= 9.27\times10^{-24}\,\text{A}\cdot\text{m}^2$$

The number of unpaired electrons is

$$N = \frac{8.00\times10^{22}\ \text{A}\cdot\text{m}^2}{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2} = \boxed{8.63\times10^{45}\ \text{e}^-}$$

(b) Each iron atom has two unpaired electrons, so the number of iron atoms required is

$$\frac{1}{2}N = \frac{1}{2}(8.63\times10^{45}) = 4.31\times10^{45}\ \text{iron atoms}$$

Thus,

$$M_{\text{Fe}} = \frac{(4.31\times10^{45}\ \text{atoms})(7\,900\ \text{kg/m}^3)}{8.50\times10^{28}\ \text{atoms/m}^3} = \boxed{4.01\times10^{20}\ \text{kg}}$$

P30.50 The magnetic moment of one electron is taken as one Bohr magneton μ_B. Let x represent the number of electrons per atom contributing and n the number of atoms per unit volume. Then $nx\mu_B$ is the magnetic moment per volume and the magnetic field (in the absence of any currents in wires) is $B = \mu_0 nx\mu_B = 2.00$ T.. Then

$$x = \frac{B}{\mu_0\mu_B n}$$

$$= \frac{2.00\ \text{T}}{\left(8.50\times10^{28}\ \text{m}^{-3}\right)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(9.27\times10^{-24}\ \text{N}\cdot\text{m/T}\right)}$$

$$= \boxed{2.02}$$

Additional Problems

P30.51 The magnetic field inside of a solenoid is $B = \mu_0 nI = \mu_0(N/L)I$. Thus, the current in this solenoid must be

$$I = \frac{BL}{\mu_0 N} = \frac{\left(2.00\times10^{-3}\ \text{T}\right)\left(6.00\times10^{-2}\ \text{m}\right)}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(30.0)} = \boxed{3.18\ \text{A}}$$

***P30.52** Call the wire along the x-axis wire 1 and the other wire 2. Also, choose the positive direction for the magnetic fields at point P to be out of the page. At point P,

$$B_{\text{net}} = +B_1 - B_2 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$$

Substituting numerical values,

$$B_{\text{net}} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2\pi}\left(\frac{7.00\ \text{A}}{3.00\ \text{m}} - \frac{6.00\ \text{A}}{4.00\ \text{m}}\right) = +1.67\times10^{-7}\ \text{T}$$

$$\vec{\mathbf{B}}_{\text{net}} = \boxed{0.167\ \mu\text{T out of the page}}$$

P30.53 (a) Suppose you have two 100-W headlights running from a 12-V battery, with the whole $I = \frac{P}{\Delta V} = \frac{200\ \text{W}}{12\ \text{V}} = 17\ \text{A}$ current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so $\mu \approx \mu_0$. Model the current as being from a long, straight wire. Then,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(17\ \text{A})}{2\pi(0.6\ \text{m})}\boxed{\sim 10^{-5}\ \text{T}}$$

(b) If the local geomagnetic field is 5×10^{-5} T, this is $\boxed{\sim 10^{-1}\ \text{times as large,}}$ enough to affect the compass noticeably.

P30.54 Use Equation 30.7 to find the field at a distance from a current loop equal to the radius of the loop:

$$B = \frac{\mu_0 I a^2}{2\left(a^2 + x^2\right)^{3/2}} = \frac{\mu_0 I a^2}{2\left(a^2 + a^2\right)^{3/2}} = \frac{\mu_0 I a^2}{2\left(2a^2\right)^{3/2}}$$

$$= \frac{\mu_0 I a^2}{2^{5/2} a^3} = \frac{\mu_0 I}{2^{5/2} a}$$

Solve for the current:

$$I = \frac{2^{5/2} a B}{\mu_0}$$

Let a be the radius of the Earth and substitute numerical values:

$$I = \frac{2^{5/2} R_E B}{\mu_0} = \frac{2^{5/2}\left(6.37\times10^{6}\ \text{m}\right)\left(7.00\times10^{-5}\ \text{T}\right)}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}$$

$$= 2.01\times10^{9}\ \text{A}$$

This current would instantly vaporize any wire of reasonable size. For example, if we imagine a 1.00-m segment of copper wire 10 cm in diameter, a *huge* wire, this current delivers over a terawatt of power to this short segment! Furthermore, the power delivered to such a wire wrapped around the Earth is on the order of 10^{20} W, which is larger than all of the solar power delivered to the Earth by the Sun.

P30.55 On the axis of a current loop, the magnetic field is given by

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

where in this case $I = \frac{q}{(2\pi/\omega)}$. The magnetic field is directed away from the center, with a magnitude of

$$\begin{aligned} B &= \frac{\mu_0 \omega R^2 q}{4\pi\left(x^2 + R^2\right)^{3/2}} \\ &= \frac{\mu_0 (20.0 \text{ rad/s})(0.100 \text{ m})^2 \left(10.0 \times 10^{-6} \text{ C}\right)}{4\pi\left[(0.050\,0 \text{ m})^2 + (0.100 \text{ m})^2\right]^{3/2}} \\ &= 1.43 \times 10^{-10} \text{ T} = \boxed{143 \text{ pT}} \end{aligned}$$

P30.56 On the axis of a current loop, the magnetic field is given by

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

where in this case $I = \frac{q}{(2\pi/\omega)}$. Therefore,

$$B = \frac{\mu_0 \omega R^2 q}{4\pi\left(x^2 + R^2\right)^{3/2}}$$

when $x = \frac{R}{2}$, then

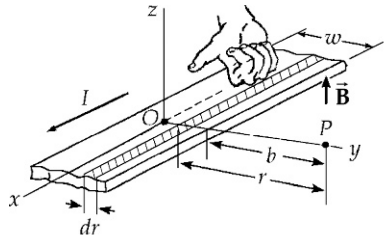
$$B = \frac{\mu_0 \omega R^2 q}{4\pi\left(\frac{5}{4} R^2\right)^{3/2}} = \boxed{\frac{\mu_0 q \omega}{2.5\sqrt{5}\pi R}}$$

P30.57 Consider a longitudinal filament of the strip of width *dr* as shown in the sketch. The contribution to the field at point *P* due to the current *dI* in the element *dr* is

$$dB = \frac{\mu_0 dI}{2\pi r}$$

where $dI = I\left(\frac{dr}{w}\right)$. Then,

$$\vec{\mathbf{B}} = \int d\vec{\mathbf{B}} = \int_b^{b+w} \frac{\mu_0 I}{2\pi w}\frac{dr}{r}\hat{\mathbf{k}} = \boxed{\frac{\mu_0 I}{2\pi w}\ln\left(1+\frac{w}{b}\right)\hat{\mathbf{k}}}$$



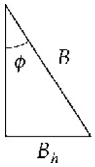
ANS. FIG. P30.57

P30.58 (a) The horizontal component of Earth's magnetic field is given by

$$B_h = B_{\text{coil}} = \frac{\mu_0 NI}{2R} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5)(0.600\ \text{A})}{0.300\ \text{m}}$$
$$= \boxed{12.6\ \mu\text{T}}$$

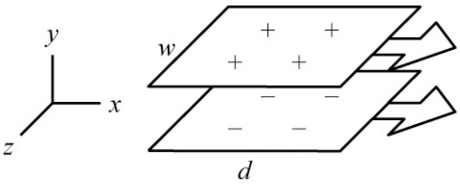
(b) Refer to ANS. FIG. P30.58. We obtain the total magnetic field from

$$B_h = B\sin\phi \to B = \frac{B_h}{\sin\phi} = \frac{12.6\ \mu\text{T}}{\sin 13.0^\circ} = \boxed{56.0\ \mu\text{T}}$$



ANS. FIG. P30.58

P30.59 In ANS FIG. P30.59(a), the upper sheet acts as conventional current to the right. Consider a patch of the sheet of width w parallel to the z axis and length d parallel to the x axis. The charge on it, $\Delta q = \sigma wd$, passes a point in time interval $\Delta t = d/v$, so the current it constitutes is $\Delta q/\Delta t = \sigma wd/(d/v) = \sigma wv$ and the linear current density is $J_s = \sigma wv/w = \sigma v$.



ANS. FIG. P30.59(a)

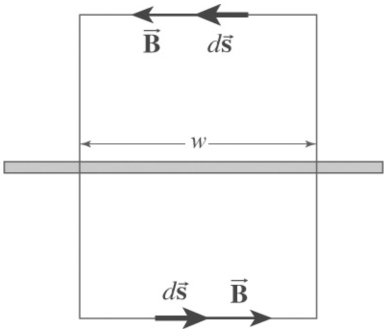
We may use Ampere's law to find the magnitude of the magnetic field produced by a sheet because of the translational symmetry along the z axis. In ANS. FIG. P30.59(b), we look at the upper sheet as it approaches us: the upper sheet (and z-axis) lies in a horizontal plane and the conventional current is out of the page. Choose a closed rectangular path of width w centered about the upper sheet. Because the current it out on the page, we expect the field to point to the right below the sheet and to the left above the sheet.

For the loop, the term $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ is non-zero along the sides parallel to the sheet and zero along the sides perpendicular to the sheet. From Ampere's law, we find the magnitude of the magnetic field on either side of the sheet:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I$$

$$B(2w) = \mu_0 (J_s w)$$

$$B = \frac{\mu_0 J_s}{2} = \frac{\mu_0 \sigma v}{2}$$



ANS. FIG. P30.59(b)

Therefore, the upper sheet creates field $\vec{\mathbf{B}} = \frac{\mu_0 J_s}{2}\hat{\mathbf{k}}$ above it and $\frac{\mu_0 J_s}{2}(-\hat{\mathbf{k}})$ below it. Similarly, the lower sheet in its motion toward the right constitutes conventional current toward the left. It creates magnetic field $\frac{1}{2}\mu_0 \sigma v(-\hat{\mathbf{k}})$ above it and $\frac{1}{2}\mu_0 \sigma v\hat{\mathbf{k}}$ below it.

(a) Between the plates, their fields add to

$$\mu_0 \sigma v(-\hat{\mathbf{k}}) = \boxed{\mu_0 \sigma v \text{ into the page}}.$$

(b) Above both sheets and below both, their equal-magnitude fields add to $\boxed{\text{zero}}$.

(c) The upper plate exerts no force on itself. The field of the lower plate, $\frac{1}{2}\mu_0\sigma v(-\hat{\mathbf{k}})$ will exert a force on the current in the *w* by *d* section, given by

$$\vec{\mathbf{F}}_{\mathbf{B}} = I\vec{\ell}\times\vec{\mathbf{B}} = \sigma wvd\hat{\mathbf{i}}\times\frac{1}{2}\mu_0\sigma v(-\hat{\mathbf{k}}) = \frac{1}{2}\mu_0\sigma^2v^2wd\hat{\mathbf{j}}$$

The force per area is

$$\frac{\vec{\mathbf{F}}_{\mathbf{B}}}{wd} = \frac{1}{2}\frac{\mu_0\sigma^2v^2wd}{wd}\hat{\mathbf{j}}$$

$$= \boxed{\frac{1}{2}\mu_0\sigma^2v^2 \text{ up toward the top of the page}}$$

(d) The electrical force on our section of the upper plate is

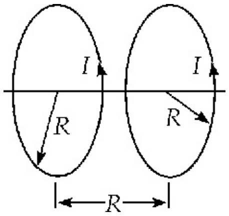
$$q\vec{\mathbf{E}}_{\text{lower}} = (\sigma wd)\frac{\sigma}{2\epsilon_0}(-\hat{\mathbf{j}}) = \frac{\sigma^2wd}{2\epsilon_0}(-\hat{\mathbf{j}})$$

The electrical force per area is $\frac{\sigma^2wd}{2\epsilon_0 wd}$ down $= \frac{\sigma^2}{2\epsilon_0}$ down. To have $\frac{1}{2}\mu_0\sigma^2v^2 = \frac{\sigma^2}{2\epsilon_0}$ we require

$$\boxed{v = \frac{1}{\sqrt{\mu_0\epsilon_0}}\text{. We will find out in Chapter 34 that this speed is the speed of light. We will find out in Chapter 39 that this speed is not possible for the capacitor plates.}}$$

P30.60 (a) Use Equation 30.7 twice for the field created by a current loop

$$B_x = \frac{\mu_0 IR^2}{2(x^2+R^2)^{3/2}}$$



ANS. FIG. P30.60

If each coil has *N* turns, the field is just *N* times larger.

$$B = B_{x1} + B_{x2} = \frac{N\mu_0 IR^2}{2}\left[\frac{1}{\left(x^2+R^2\right)^{3/2}} + \frac{1}{\left[(R-x)^2+R^2\right]^{3/2}}\right]$$

$$\boxed{B = \frac{N\mu_0 IR^2}{2}\left[\frac{1}{\left(R^2+x^2\right)^{3/2}} + \frac{1}{\left(2R^2+x^2-2xR\right)^{3/2}}\right]}$$

(b) $$\frac{dB}{dx} = \frac{N\mu_0 IR^2}{2}\left[-\frac{3}{2}(2x)\left(x^2+R^2\right)^{-5/2} - \frac{3}{2}\left(2R^2+x^2-2xR\right)^{-5/2}(2x-2R)\right]$$

Substituting $x = \frac{R}{2}$ and canceling terms, $\boxed{\frac{dB}{dx} = 0}$.

$$\frac{d^2B}{dx^2} = \frac{-3N\mu_0 IR^2}{2}\left[\left(x^2+R^2\right)^{-5/2} - 5x^2\left(x^2+R^2\right)^{-7/2} + \left(2R^2+x^2-2xR\right)^{-5/2} - 5(x-R)^2\left(2R^2+x^2-2xR\right)^{-7/2}\right]$$

Again substituting $x = \frac{R}{2}$ and canceling terms, $\boxed{\frac{d^2B}{dx^2} = 0}$.

P30.61 We have a pair of Helmholtz coils whose separation distance is equal to their radius *R*. To find the magnetic field halfway between the coils on their common axis, we use Equation 30.7 to find the field produced on the axis of a loop the distance $x = R/2$ from its center:

$$B = 2\frac{\mu_0 IR^2}{2\left[(R/2)^2+R^2\right]^{3/2}} = \frac{\mu_0 IR^2}{\left[\frac{1}{4}+1\right]^{3/2}R^3} = \frac{\mu_0 I}{1.40R} \text{ for 1 turn}$$

For *N* turns in each coil,

$$B = \frac{\mu_0 NI}{1.40R} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)100(10.0\text{ A})}{1.40(0.500\text{ m})}$$

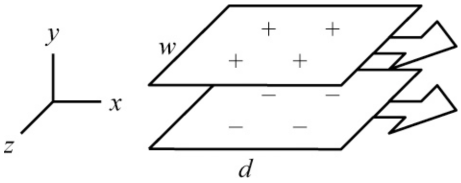
$$= 1.80\times10^{-3}\text{ T} = \boxed{1.80\text{ mT}}$$

P30.62 Model the two wires as straight parallel wires (!). From the treatment of this situation in the chapter text (refer to Equation 30.12), we have

(a) $$F_B = \frac{\mu_0 I^2\ell}{2\pi a}$$

$$F_B = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(140\text{ A})^2\left[(2\pi)(0.100\text{ m})\right]}{2\pi\left(1.00\times10^{-3}\text{ m}\right)}$$

$$= \boxed{2.46\text{ N upward}}$$



ANS. FIG. P30.62

(b) Equation 30.7, $B_x = \dfrac{\mu_0 I a^2}{2\left(a^2+x^2\right)^{3/2}}$ is the expression for the magnetic field produced a distance *x* above the center of a loop. The magnetic field at the center of the loop or on its axis is much weaker than the magnetic field just outside the wire. The wire has negligible curvature on the scale of 1 mm, so we model the lower loop as a long straight wire to find the field it creates at the location of the upper wire.

(c) The acceleration of the upper loop is found from Newton's second law:

$$\sum F = m_{\text{loop}} a_{\text{loop}} = F_B - m_{\text{loop}} g:$$

$$a_{\text{loop}} = \frac{F_B - m_{\text{loop}} g}{m_{\text{loop}}} = \frac{2.46\text{ N} - (0.021\,0\text{ kg})(9.80\text{ m/s}^2)}{(0.021\,0\text{ kg})}$$

$$= \boxed{107\ \text{m/s}^2\quad\text{upward}}$$

P30.63 In the textbook Figure P30.63, wire 1 carries current along the *x* axis and wire 2 carries current along the *y* axis.

Choosing out of the page as the positive field direction, the field at point *P* is

$$B = B_1 - B_2 = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$$

$$= \frac{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2\pi}\left(\frac{5.00\text{ A}}{0.400\text{ m}} - \frac{3.00\text{ A}}{0.300\text{ m}}\right) = 5.00\times 10^{-7}\ \text{T}$$

The result is positive; therefore, the field at *P* is

(a) $\boxed{0.500\ \mu\text{T}}$

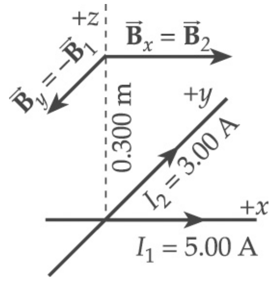
(b) $\boxed{\text{out of the page}}$

(c) At 30.0 cm above the intersection of the wires, the field components are as shown in ANS. FIG. P30.63, where

$$B_y = -B_1 = -\frac{\mu_0 I_1}{2\pi r}$$

$$= -\frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})}{2\pi(0.300\ \text{m})} = -3.33\times 10^{-6}\ \text{T}$$

and $B_x = B_2 = \dfrac{\mu_0 I_2}{2\pi r} = \dfrac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(3.00\ \text{A})}{2\pi(0.300\ \text{m})} = 2.00\times 10^{-6}\ \text{T}$



ANS. FIG. P30.63

The resultant field is

$$B = \sqrt{B_x^2 + B_y^2} = 3.89\times 10^{-6}\ \text{T} \quad \text{at} \quad \theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = -59.0°$$

or $\vec{\mathbf{B}} = \boxed{3.89\ \mu\text{T in the } xy \text{ plane and at } 59.0° \text{ clockwise}}$

$\boxed{\text{from the } +x \text{ direction}}$

P30.64 (a) The magnetic field at the center of a circular current loop of radius *R* and carrying current *I* is $B = \mu_0 I / 2R$. The direction of the field at this center is given by the right-hand rule. Taking out of the page (toward the reader) as positive, the net magnetic field at the common center of these coplanar loops is

$$B_{\text{net}} = B_2 - B_1 = \frac{\mu_0 I_2}{2r_2} - \frac{\mu_0 I_1}{2r_1}$$

$$= \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})}{2}\left(\frac{3.00\ \text{A}}{9.00\times 10^{-2}\ \text{m}} - \frac{5.00\ \text{A}}{12.0\times 10^{-2}\ \text{m}}\right)$$

$$= -5.24\times 10^{-6}\ \text{T} \rightarrow |B_{\text{net}}| = \boxed{5.24\ \mu\text{T}}$$

(b) By our convention above (out of the page is positive), the result of part (a) tells us that the net magnetic field is $\boxed{\text{into the page}}$.

(c) To have $B_{\text{net}} = 0$, it is necessary that $I_2/r_2 = I_1/r_1$, or

$$r_2 = \left(\frac{I_2}{I_1}\right)r_1 = \left(\frac{3.00\text{ A}}{5.00\text{ A}}\right)(12.0\text{ cm}) = \boxed{7.20\text{ cm}}$$

P30.65 (a) In $d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi r^2} I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}$, the moving charge constitutes a bit of current as in $I = nqvA$. For a positive charge the direction of $d\vec{\mathbf{s}}$ is the direction of $\vec{\mathbf{v}}$, so $d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi r^2} nqA(ds)\vec{\mathbf{v}} \times \hat{\mathbf{r}}$. Next, A (ds) is the volume occupied by the moving charge, and nA (ds) = 1 for just one charge. Then,

$$\boxed{\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{q\vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}}$$

(b) The magnitude of the field is

$$B = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(1.60 \times 10^{-19}\text{ C})(2.00 \times 10^{7}\text{ m/s})}{4\pi(1.00 \times 10^{-3}\text{ m})^2}\sin 90.0°$$
$$= \boxed{3.20 \times 10^{-13}\text{ T}}$$

(c) The magnetic force on a second proton moving in the opposite direction is

$$F_B = q\left|\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right| = (1.60 \times 10^{-19}\text{ C})(2.00 \times 10^{7}\text{ m/s}) \times (3.20 \times 10^{-13}\text{ T})\sin 90.0°$$

$F_B = \boxed{1.02 \times 10^{-24}\text{ N}}$ directed away from the first proton

(d) The electric force on a second proton moving in the opposite direction is

$$F_e = qE = \frac{k_e q_1 q_2}{r^2} = \frac{(8.99 \times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19}\text{ C})^2}{(1.00 \times 10^{-3})^2}$$

$F_e = \boxed{2.30 \times 10^{-22}\text{ N}}$ directed away from the first proton

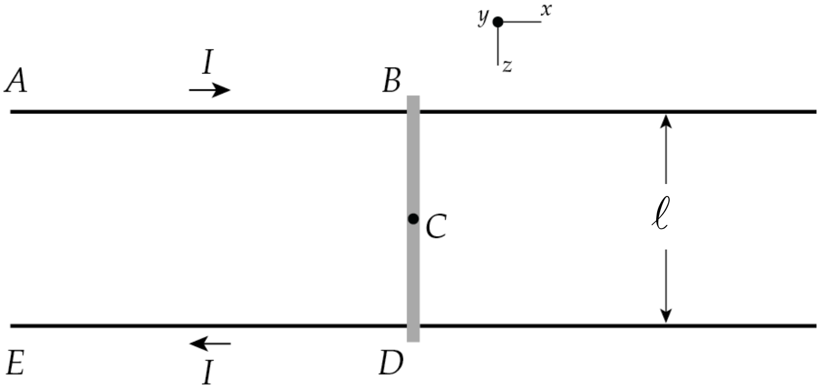
P30.66 (a) $B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(24.0\text{ A})}{2\pi(0.017\,5\text{ m})} = \boxed{2.74 \times 10^{-4}\text{ T}}$

(b) Because current is diverted through the bar, only half of each rail carries current, so the field produced by each rail is half what an infinitely long wire produces.

Therefore, at point *C*, conductor *AB* produces a field

$$\frac{1}{2}\left(2.74\times10^{-4}\ \mathrm{T}\right)\left(-\hat{\mathbf{j}}\right),$$

conductor *DE* produces a field of $\frac{1}{2}\left(2.74\times10^{-4}\ \mathrm{T}\right)\left(-\hat{\mathbf{j}}\right)$, *BD* produces no field, and *AE* produces negligible field. The total field at *C* is $\boxed{2.74\times10^{-4}\ \mathrm{T}\left(-\hat{\mathbf{j}}\right)}$.



ANS. FIG. P30.66

(c) $\boxed{\text{Under the assumption that the rails are infinitely long, the length of rail to the left of the bar does not depend on the location of the bar.}}$

The force on the bar is

$$\vec{\mathbf{F}}_B = I\vec{\ell}\times\vec{\mathbf{B}} = (24.0\ \mathrm{A})\left(0.035\,0\ \mathrm{m}\hat{\mathbf{k}}\right)\times\left[5\left(2.74\times10^{-4}\ \mathrm{T}\right)\left(-\hat{\mathbf{j}}\right)\right]$$

$$= 1.15\times10^{-3}\,\hat{\mathbf{i}}\ \mathrm{N}$$

The field has magnitude

(d) $\boxed{1.15\times10^{-3}\ \mathrm{N}}$ in the

(e) $\boxed{+x\text{ direction.}}$

(f) The bar is already so far from *AE* that it moves through nearly constant magnetic field.

$\boxed{\text{Yes, length of the bar, current, and field are constant, so force is constant.}}$

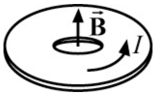
(g) The acceleration is $\vec{\mathbf{a}} = \dfrac{\sum \vec{\mathbf{F}}}{m} = \dfrac{\left(1.15\times10^{-3}\ \text{N}\right)\hat{\mathbf{i}}}{3.00\times10^{-3}\ \text{kg}} = \left(0.384\ \text{m/s}^2\right)\hat{\mathbf{i}}$:

$$v_f^2 = v_i^2 + 2ax = 0 + 2\left(0.384\ \text{m/s}^2\right)\left(1.30\ \text{m}\right)$$

so $\vec{\mathbf{v}}_f = \boxed{\left(0.999\ \text{m/s}\right)\hat{\mathbf{i}}}$.

P30.67 Each turn creates a field of $\dfrac{\mu_0 I}{2R}$ at the center of the coil. In all, they create the field

$$B = \frac{\mu_0 I}{2}\left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_{50}}\right)$$



ANS. FIG. P30.67

Using a spreadsheet to calculate the sum, we have

$$B = \frac{\mu_0 I}{2}\left(\frac{1}{5.05} + \frac{1}{5.15} + \cdots + \frac{1}{9.95}\right)\left(\frac{1}{10^{-2}\ \text{m}}\right)$$
$$= \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)I}{2}\left(6.931347\ldots\right)\left(100\ \text{m}^{-1}\right)$$

Therefore, $\boxed{B = 4.36\times10^{-4}\, I\text{, where } B \text{ is in teslas and } I \text{ is in amperes.}}$

P30.68 The central wire creates field $\vec{\mathbf{B}} = \dfrac{\mu_0 I_1}{2\pi R}$ counterclockwise. The curved portions of the loop feel no force since $\vec{\ell}\times\vec{\mathbf{B}} = 0$ there. The straight portions both feel $I\vec{\ell}\times\vec{\mathbf{B}}$ forces to the right, amounting to

$$\vec{\mathbf{F}}_B = I_2 2L \frac{\mu_0 I_1}{2\pi R} = \boxed{\frac{\mu_0 I_1 I_2 L}{\pi R}\ \text{to the right}}$$

Challenge Problems

P30.69 (a) Let the axis of the solenoid lie along the *y* axis from $y = -\ell$ to $y = 0$. We will determine the field at position $y = x$: this point will be inside the solenoid if $-\ell < x < 0$ and outside if $x < -\ell$ or $x > 0$. We think of solenoid as formed of rings, each of thickness *dy*. Now *I* is the symbol for the current in each turn of wire and the number of turns per length is $\left(\frac{N}{\ell}\right)$. So the number of turns in the ring is $\left(\frac{N}{\ell}\right)dy$ and the current in the ring is $I_{\text{ring}} = I\left(\frac{N}{\ell}\right)dy$. Now, we use Equation 30.7 for the field created by one ring:

$$B_{\text{ring}} = \frac{\mu_0 I_{\text{ring}} a^2}{2\left[(x-y)^2 + a^2\right]^{3/2}}$$

where $x - y$ is the distance from the center of the ring, at location *y*, to the field point (note that *y* is negative, so $x - y = x + |y|$). Each ring creates a field in the same direction, along the *y* axis, so the whole field of the solenoid is

$$B = \sum_{\text{all rings}} B_{\text{ring}} = \sum \frac{\mu_0 I_{\text{ring}} a^2}{2\left[(x-y)^2 + a^2\right]^{3/2}} \to \int_{-\ell}^{0} \frac{\mu_0 I (N/\ell) a^2 dy}{2\left[(x-y)^2 + a^2\right]^{3/2}}$$

$$= \frac{\mu_0 I N a^2}{2\ell} \int_{-\ell}^{0} \frac{dy}{\left[(x-y)^2 + a^2\right]^{3/2}}$$

To perform the integral we change variables to $u = x - y$ and $dy = -du$. Then,

$$B = -\frac{\mu_0 I N a^2}{2\ell} \int_{x+\ell}^{x} \frac{du}{\left(u^2 + a^2\right)^{3/2}}$$

and then using the table of integrals in the appendix,

$$B = -\frac{\mu_0 I N a^2}{2\ell} \left.\frac{u}{a^2\sqrt{u^2 + a^2}}\right|_{x+\ell}^{x}$$

$$= -\frac{\mu_0 I N}{2\ell}\left[\frac{x}{\sqrt{x^2 + a^2}} - \frac{x+\ell}{\sqrt{(x+\ell)^2 + a^2}}\right]$$

$$= \boxed{\frac{\mu_0 I N}{2\ell}\left[\frac{x+\ell}{\sqrt{(x+\ell)^2 + a^2}} - \frac{x}{\sqrt{x^2 + a^2}}\right]}$$

(b) If ℓ is much larger than a and $x = 0$, we have

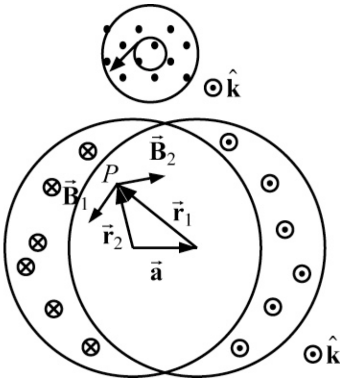
$$B \cong \frac{\mu_0 IN}{2\ell}\left[\frac{\ell}{\sqrt{\ell^2}} + 0\right] = \frac{\mu_0 IN}{2\ell}$$

This is just half the magnitude of the field deep within the solenoid. We would get the same result by substituting $x = -\ell$ to describe the other end.

P30.70 Consider first a solid cylindrical rod of radius R carrying current toward you, uniformly distributed over its cross-sectional area. To find the field at distance r from its center we consider a circular loop of radius r:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{\text{inside}}$$

$$B2\pi r = \mu_0 \pi r^2 J \qquad B = \frac{\mu_0 Jr}{2} \qquad \vec{\mathbf{B}} = \frac{\mu_0 J}{2}\hat{\mathbf{k}} \times \mathbf{r}$$



ANS. FIG. P30.70

Now the total field at P inside the saddle coils is the field due to a solid rod carrying current toward you, centered at the head of vector $\vec{\mathbf{d}}$, plus the field of a solid rod centered at the tail of vector $\vec{\mathbf{d}}$ carrying current away from you.

$$\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}} \times \vec{\mathbf{r}}_1 + \frac{\mu_0 J}{2}\left(-\hat{\mathbf{k}}\right) \times \vec{\mathbf{r}}_2$$

Now note $\vec{\mathbf{d}} + \vec{\mathbf{r}}_1 = \vec{\mathbf{r}}_2$. Then,

$$\vec{\mathbf{B}}_1 + \vec{\mathbf{B}}_2 = \frac{\mu_0 J}{2}\hat{\mathbf{k}} \times \vec{\mathbf{r}}_1 - \frac{\mu_0 J}{2}\hat{\mathbf{k}} \times \left(\vec{\mathbf{d}} + \vec{\mathbf{r}}_1\right) = \frac{\mu_0 J}{2}\vec{\mathbf{d}} \times \hat{\mathbf{k}}$$

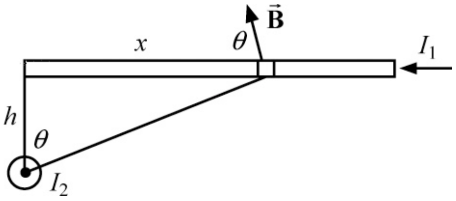
$$= \frac{\mu_0 Jd}{2} \text{ down in the diagram}$$

P30.71 At a point at distance *x* from the left end of the bar, current I_2 creates magnetic field $\vec{\mathbf{B}} = \frac{\mu_0 I_2}{2\pi\sqrt{h^2 + x^2}}$ to the left and above the horizontal at angle θ where $\tan\theta = \frac{x}{h}$. This field exerts force on an element of the rod of length *dx*

$$d\vec{\mathbf{F}} = I_1 \vec{\ell} \times \vec{\mathbf{B}} = I_1 \frac{\mu_0 I_2 dx}{2\pi\sqrt{h^2 + x^2}} \underset{\text{right hand rule}}{\sin\theta}$$

$$= \frac{\mu_0 I_1 I_2 dx}{2\pi\sqrt{h^2 + x^2}} \frac{x}{\sqrt{h^2 + x^2}} \text{ into the page}$$

$$d\vec{\mathbf{F}} = \frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2 + x^2\right)}\left(-\hat{\mathbf{k}}\right)$$



ANS. FIG. P30.71

The whole force is the sum of the forces on all of the elements of the bar:

$$\vec{\mathbf{F}} = \int_{x=0}^{\ell} \frac{\mu_0 I_1 I_2 x dx}{2\pi\left(h^2 + x^2\right)}\left(-\hat{\mathbf{k}}\right) = \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\int_0^{\ell} \frac{2x dx}{h^2 + x^2}$$

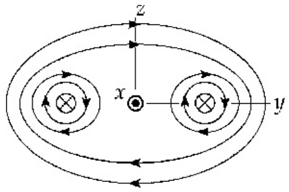
$$= \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi} \ln\left(h^2 + x^2\right)\Big|_0^{\ell}$$

$$= \frac{\mu_0 I_1 I_2\left(-\hat{\mathbf{k}}\right)}{4\pi}\left[\ln\left(h^2 + \ell^2\right) - \ln h^2\right]$$

$$= \frac{\left(10^{-7}\text{ N}\right)(100\text{ A})(200\text{ A})\left(-\hat{\mathbf{k}}\right)}{\text{A}^2} \ln\left[\frac{(0.500\text{ cm})^2 + (10.0\text{ cm})^2}{(0.500\text{ cm})^2}\right]$$

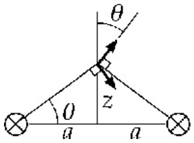
$$= 2\times 10^{-3}\text{ N}\left(-\hat{\mathbf{k}}\right)\ln 401 = \boxed{1.20\times 10^{-2}\text{ N}\left(-\hat{\mathbf{k}}\right)}$$

P30.72 (a) See ANS. FIG. P30.72(a).



(currents are into the paper)

ANS. FIG. P30.72(a)



at a distance *z* above the plane of the conductors

ANS. FIG. P30.72(b)

(b) By symmetry, the contribution of each wire to the magnetic field at the origin is the same, but the directions of the fields are opposite, so the total field is $\boxed{\text{zero}}$. We can see this from cancellation of the separate fields in ANS. FIG. P30.72(a).

(d) We choose to do part (d) first. At a point on the z axis, the contribution from each wire has magnitude $B = \dfrac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}$ and is perpendicular to the line from this point to the wire as shown in ANS. FIG. P30.72(b). Combining fields, the vertical components cancel while the horizontal components add, yielding

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}\sin\theta\right) = \frac{\mu_0 I}{\pi\sqrt{a^2+z^2}}\left(\frac{z}{\sqrt{a^2+z^2}}\right) = \frac{\mu_0 I z}{\pi\left(a^2+z^2\right)}$$

$$B_y = \frac{\left(4\pi\times10^{-7}\right)(8.00)\,z}{\pi\left[(0.0300)^2+z^2\right]} \quad \text{so}$$

$$\boxed{\vec{\mathbf{B}} = \frac{32\times10^{-7}\,z}{9\times10^{-4}+z^2}\,\hat{\mathbf{j}}, \text{ where } \vec{\mathbf{B}} \text{ is in teslas and } z \text{ is in meters.}}$$

(c) From part (d), taking the limit $z \to \infty$ gives $1/z \to 0$; so, the field is $\boxed{\text{zero}}$, as we should expect.

(e) The condition for a maximum is:

$$\frac{dB_y}{dz} = \frac{-\mu_0 I z(2z)}{\pi\left(a^2+z^2\right)^2} + \frac{\mu_0 I}{\pi\left(a^2+z^2\right)} = 0 \quad \text{or} \quad \frac{\mu_0 I}{\pi}\frac{\left(a^2-z^2\right)}{\left(a^2+z^2\right)^2} = 0$$

Thus, along the z axis, the field is a maximum at $\boxed{d = a = 3.00\text{ cm}}$.

(f) Using the equation derived in part (d), the value of the maximum field is

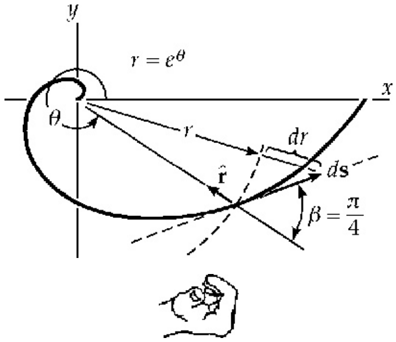
$$\vec{\mathbf{B}} = \frac{(32\times10^{-7})(0.030\ 0)}{9\times10^{-4}+(0.030\ 0)^2}\hat{\mathbf{j}}\ \text{T} = 5.33\times10^{-5}\ \text{T} = \boxed{53.3\hat{\mathbf{j}}\ \mu\text{T}}$$

P30.73 (a) From the shape of the wire,

$$r = f(\theta) = e^{\theta} \quad \rightarrow \quad \frac{dr}{d\theta} = e^{\theta} = r$$

and so we have

$$\tan\beta = \frac{r}{dr/d\theta} = \frac{r}{r} = 1 \rightarrow \beta = 45° = \pi/4$$



ANS. FIG. P30.73

(b) At the origin, there is no contribution from the straight portion of the wire since $d\vec{\mathbf{s}}\times\hat{\mathbf{r}} = 0$. For the field contribution from the spiral, refer to the figure. The direction of $d\vec{\mathbf{s}}\times\hat{\mathbf{r}}$ is out of the page. The magnitude $|d\vec{\mathbf{s}}\times\hat{\mathbf{r}}| = \sin(3\pi/4)$ because the angle between $d\vec{\mathbf{s}}$ and $\hat{\mathbf{r}}$ is always $180° - 45° = 135° = 3\pi/4$.

Also, from the figure,

$$dr = ds\sin\pi/4 = ds/\sqrt{2} \rightarrow ds = \sqrt{2}dr$$

The contribution to the magnetic field is then

$$dB = |d\vec{\mathbf{B}}| = \frac{\mu_0 I}{(4\pi)}\left|\frac{(d\vec{\mathbf{s}}\times\hat{\mathbf{r}})}{r^2}\right| = \frac{\mu_0 I}{(4\pi)}\frac{|d\vec{\mathbf{s}}|\sin\theta|\hat{\mathbf{r}}|}{r^2}$$

$$= \frac{\mu_0 I}{(4\pi)}\frac{\sqrt{2}dr}{r^2}\left[\sin\left(\frac{3\pi}{4}\right)\right]$$

The total magnetic field is

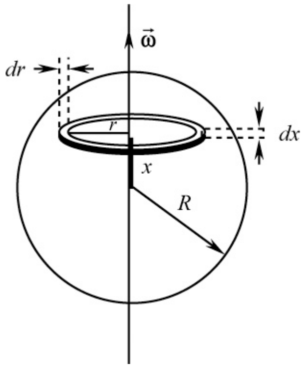
$$B=\frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{\cancel{\sqrt{2}}dr}{r^2}\left[\frac{1}{\cancel{\sqrt{2}}}\right]\frac{1}{r^2}=\frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}r^{-2}dr=-\frac{\mu_0 I}{4\pi}\left(r^{-1}\right)\Big|_{\theta=0}^{2\pi}$$

Substitute $r=e^{\theta}$: $B=-\frac{\mu_0 I}{4\pi}\left[e^{-\theta}\right]_0^{2\pi}=-\frac{\mu_0 I}{4\pi}\left[e^{-2\pi}-e^{0}\right]=\boxed{\frac{\mu_0 I}{4\pi}\left(1-e^{-2\pi}\right)}$

$\boxed{\text{out of the page.}}$

P30.74 (a) Consider the sphere as being built up of little spinning ring elements of radius r, thickness dr, and height dx, centered on the rotation axis. Each ring holds charge dQ:

$$dQ=\rho dV=\rho\left(2\pi rdr\right)\left(dx\right)$$



ANS. FIG. P30.74

Each ring, with angular speed ω, takes a period $T=\omega/2\pi$ to complete one rotation. Thus, each ring carries current

$$dI=\frac{dQ}{T}=\frac{\omega}{2\pi}\left[\rho\left(2\pi rdr\right)\left(dx\right)\right]=\rho\omega rdrdx$$

The contribution of each ring element to the magnetic field at a point on the rotation axis a distance x from the center of the sphere is given by Equation 30.7:

$$dB=\frac{\mu_0 r^2 dI}{2\left(x^2+r^2\right)^{3/2}}$$

Combining the above terms, the field contribution is of a ring element is

$$dB=\frac{\mu_0\rho\omega r^3 drdx}{2\left(x^2+r^2\right)^{3/2}}$$

The contributions of all rings gives

$$B=\int_{x=-R}^{+R}\int_{r=0}^{\sqrt{R^2-x^2}}\frac{\mu_0\rho\omega}{2}\frac{r^3drdx}{\left(x^2+r^2\right)^{3/2}}$$

To evaluate the integral, let $v = r^2 + x^2$, $dv = 2rdr$, and $r^2 = v - x^2$.

$$B=\int_{x=-R}^{+R}\int_{v=x^2}^{R^2}\frac{\mu_0\rho\omega}{2}\frac{\left(v-x^2\right)dv}{2v^{3/2}}dx$$

$$=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[\int_{v=x^2}^{R^2}v^{-1/2}dv-x^2\int_{v=x^2}^{R^2}v^{-3/2}dv\right]dx$$

$$B=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[2v^{1/2}\Big|_{x^2}^{R^2}+\left(2x^2\right)v^{-1/2}\Big|_{x^2}^{R^2}\right]dx$$

$$=\frac{\mu_0\rho\omega}{4}\int_{x=-R}^{R}\left[2\left(R-|x|\right)+2x^2\left(\frac{1}{R}-\frac{1}{|x|}\right)\right]dx$$

$$B=\frac{\mu_0\rho\omega}{4}\int_{-R}^{R}\left[2\frac{x^2}{R}-4|x|+2R\right]dx$$

$$=\frac{2\mu_0\rho\omega}{4}\int_{0}^{R}\left[2\frac{x^2}{R}-4x+2R\right]dx$$

$$B=\frac{2\mu_0\rho\omega}{4}\left(\frac{2R^3}{3R}-\frac{4R^2}{2}+2R^2\right)=\boxed{\frac{\mu_0\rho\omega R^2}{3}}$$

(b) From part (a), the current associated with each rotating ring of charge is

$$dI=\rho\omega rdrdx$$

The magnetic moment contributed by this ring is

$$d\mu=A\left(dI\right)=\left(\pi r^2\right)\left(\rho\omega rdrdx\right)=\pi\omega\rho r^3drdx$$

The total magnetic moment is

$$\mu=\pi\omega\rho\int_{x=-R}^{+R}\left[\int_{r=0}^{\sqrt{R^2-x^2}}r^3\,dr\right]dx=\pi\omega\rho\int_{x=-R}^{+R}\frac{\left(\sqrt{R^2-x^2}\right)^4}{4}dx$$

$$=\pi\omega\rho\int_{x=-R}^{+R}\frac{\left(R^2-x^2\right)^2}{4}dx$$

$$\mu = \frac{\pi\omega\rho}{4}\int_{x=-R}^{+R}\left(R^4 - 2R^2x^2 + x^4\right)dx$$

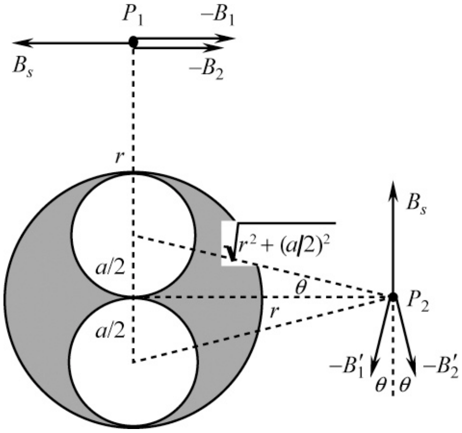
$$= \frac{\pi\omega\rho}{4}\left[R^4(2R) - 2R^2\left(\frac{2R^3}{3}\right) + \frac{2R^5}{5}\right]$$

$$\mu = \frac{\pi\omega\rho}{4}R^5\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \frac{\pi\omega\rho R^5}{4}\left(\frac{16}{15}\right) = \boxed{\frac{4\pi\omega\rho R^5}{15}}$$

P30.75 Note that the current *I* exists in the conductor with a current density $J = \frac{I}{A}$, where

$$A = \pi\left[a^2 - \frac{a^2}{4} - \frac{a^2}{4}\right] = \frac{\pi a^2}{2}$$

Therefore $J = \frac{2I}{\pi a^2}$.



ANS. FIG. P30.75

To find the field at either point P_1 or P_2, find B_s which would exist if the conductor were solid, using Ampère's law. Next, find B_1 and B_2 that *would* be due to the conductors of radius $\frac{a}{2}$ that *could* occupy the void where the holes exist. Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

(a) At point P_1,

$$B_s = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r},\ B_1 = \frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r - (a/2)\right)},\ \text{and}\ B_2 = \frac{\mu_0 J\pi(a/2)^2}{2\pi\left(r + (a/2)\right)}$$

$$B = B_s - B_1 - B_2$$

$$= \frac{\mu J \pi a^2}{2\pi}\left[\frac{1}{r} - \frac{1}{4\left(r - (a/2)\right)} - \frac{1}{4\left(r + (a/2)\right)}\right]$$

$$B = \frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2 - a^2 - 2r^2}{4r\left(r^2 - (a^2/4)\right)}\right]$$

$$= \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right] \text{ directed to the left}}$$

(b) At point P_2,

$$B_s = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r} \text{ and } B_1' = B_2' = \frac{\mu_0 J \pi (a/2)^2}{2\pi\sqrt{r^2 + (a/2)^2}}$$

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$B = B_s - B_1'\cos\theta - B_2'\cos\theta$$

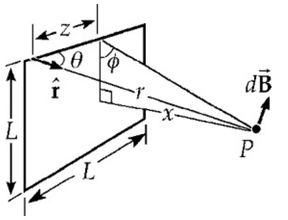
$$= \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r} - 2\left(\frac{\mu_0 J \pi a^2/4}{2\pi\sqrt{r^2 + \left(a^2/4\right)}}\right)\frac{r}{\sqrt{r^2 + \left(a^2/4\right)}}$$

$$B = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}\left[1 - \frac{r^2}{2\left(r^2 + \left(a^2/4\right)\right)}\right] = \frac{\mu_0(2I)}{2\pi r}\left[1 - \frac{2r^2}{4r^2 + a^2}\right]$$

$$= \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right] \text{ directed toward the top of the page}}$$

P30.76 By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B_P = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $\frac{L}{2}$. In order to calculate B_0, we use the Biot-Savart law and consider the plane of the square to be the yz plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by the integral form of the Biot-Savart law as

$$\vec{\mathbf{B}}_0 = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{\ell} \times \hat{\mathbf{r}}}{r^2}$$



ANS. FIG. P30.76

From ANS. FIG. P30.76 we see that

$$r = \sqrt{x^2 + L^2/4 + z^2} \quad \text{and} \quad \left|d\vec{\ell} \times \hat{\mathbf{r}}\right| = dz\sin\theta = dz\sqrt{\frac{L^2/4 + x^2}{L^2/4 + x^2 + z^2}}$$

By symmetry all components of the field $\vec{\mathbf{B}}$ at *P* cancel except the components along *x* (perpendicular to the plane of the square); and

$$B_{0x} = B_0\cos\phi \quad \text{where} \quad \cos\phi = \frac{L/2}{\sqrt{L^2/4 + x^2}}$$

Therefore,

$$\left|\vec{\mathbf{B}}_0\right| = B_{0x} = \frac{\mu_0 I}{4\pi}\int_0^{L/2}\frac{\sin\theta\cos\phi\, dz}{r^2}$$

and at *P*, $B_P = 8B_{0x}$.

Using the expressions given above for $\sin\theta$, $\cos\phi$, and *r*, we find

$$B_P = 8\left(\frac{\mu_0 I}{4\pi}\right)\int_0^{L/2}\frac{1}{L^2/4 + x^2 + z^2}\sqrt{\frac{L^2/4 + x^2}{L^2/4 + x^2 + z^2}}\frac{L/2}{\sqrt{L^2/4 + x^2}}dz$$

$$= \frac{\mu_0 IL}{\pi}\int_0^{L/2}\frac{dz}{\left(L^2/4 + x^2 + z^2\right)^{3/2}}$$

$$= \frac{\mu_0 IL}{8\pi}\frac{1}{\left(L^2/4 + x^2\right)}\frac{z}{\sqrt{L^2/4 + x^2 + z^2}}\Bigg|_0^{L/2}$$

$$= \frac{\mu_0 IL}{\pi}\frac{1}{\left(L^2/4 + x^2\right)}\left[\frac{L/2}{\sqrt{L^2/4 + x^2 + L^2/4}} - 0\right]$$

Therefore,

$$B_P = \frac{\mu_0 IL^2}{2\pi\left(x^2 + L^2/4\right)\sqrt{x^2 + L^2/2}}$$

P30.77 (a) From Equation 30.9, the magnetic field produced by one loop at the center of the second loop is given by

$$B = \frac{\mu_0 I R^2}{2x^3} = \frac{\mu_0 I\left(\pi R^2\right)}{2\pi x^3} = \frac{\mu_0 \mu}{2\pi x^3}$$

where the magnetic moment of either loop is $\mu = I\left(\pi R^2\right)$. Therefore,

$$\left|F_x\right| = \mu\frac{dB}{dx} = \mu\frac{d}{dx}\left(\frac{\mu_0\mu}{2\pi x^3}\right) = \mu\left(\frac{\mu_0\mu}{2\pi}\right)\left(\frac{3}{x^4}\right)$$

$$= \frac{3\mu_0\left(I\pi R^2\right)^2}{2\pi x^4} = \boxed{\frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4}}$$

(b) $$\left|F_x\right| = \frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4} = \frac{3\pi}{2}\frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(10.0\text{ A}\right)^2\left(5.00\times10^{-3}\text{ m}\right)^4}{\left(5.00\times10^{-2}\text{ m}\right)^4}$$

$$= \boxed{5.92\times10^{-8}\text{ N}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P30.2	(a) toward the left; (b) out of the page; (c) lower left to upper right
P30.4	675 A, downward
P30.6	12.5 T
P30.8	$\vec{\mathbf{B}} = \left(1+\frac{1}{\pi}\right)\frac{\mu_0 I}{2R}$ (directed into the page)
P30.10	$\frac{\mu_0 I}{4\pi x}$ into the paper
P30.12	See ANS. FIG. P30.12
P30.14	(a) at $y = -0.420$ m; (b) 3.47×10^{-2} N$\left(-\hat{\mathbf{j}}\right)$; (c) $-1.73\times10^{4}\hat{\mathbf{j}}$ N/C
P30.16	(a) See ANS. FIG. P30.16; (b) 3.84×10^{-21} N up; (c) 2.14×10^{-5} m; (d) This distance is negligible compared to 50 m, so the electron does move in a uniform field; (e) 134 revolutions
P30.18	(a) $\frac{4.50\mu_0 I}{\pi L}$; (b) stronger
P30.20	(a) 4.00 μT toward the bottom of the page; (b) 6.67 μT at 167.0° from the positive x axis
P30.22	(a) 8.00 A; (b) opposite directions; (c) force of interaction would be attractive and the magnitude of the force would double
P30.24	(a) The situation is possible in just one way; (b) 12.0 cm to the left of wire 1; (c) 2.40 A down
P30.26	$\frac{\mu_0 I_1 I_2 \ell}{2\pi}\left[\frac{a}{c(c+a)}\right]$ to the left
P30.28	This is the required center-to-center separation distance of the wires, but the wires cannot be this close together. Their minimum possible center-to-center separation distance occurs if the wires are touching, but this value is $2r = 2(25.0\ \mu\text{m}) = 50.0\ \mu\text{m}$, which is much larger than the required value above. We could try to obtain this force between wires of smaller diameter, but these wires would have higher resistance and less surface area for radiating energy. It is likely that the wires would melt very shortly after the current begins.
P30.30	500 A
P30.32	(a) 3.60 T; (b) 1.94 T

P30.34 $\dfrac{\mu_0 J_s}{2}$

P30.36 (a) 6.34×10^{-3} N/m; (b) inward toward the center of the bundle; (c) greatest at the outer surface

P30.38 (a) $\dfrac{\mu_0 b r_1^2}{3}$ (for $r_1 < R$ or inside the cylinder);

(b) $\dfrac{\mu_0 b R^3}{3r_2}$ (for $r_2 > R$ or outside the cylinder)

P30.40 4.77×10^4 turns

P30.42 (a) Make the wire as long and thin as possible without melting when it carries the 5-A current; (b) As small in radius as possible with your experiment fitting inside. Then with a smaller circumference, the wire can form a solenoid with more turns.

P30.44 207 W

P30.46 (a) $-B\pi R^2 \cos\theta$; (b) $B\pi R^2 \cos\theta$

P30.48 (a) 7.40 μWb; (b) 2.27 μWb

P30.50 2.02

P30.52 0.167 μT out of the page

P30.54 This current would instantly vaporize any wire of reasonable size. For example, if we imagine a 1.00-m segment of copper wire 10 cm in diameter, a *huge* wire, this current delivers over a terawatt of power to this short segment! Furthermore, the power delivered to such a wire wrapped around the Earth is on the order of 10^{20} W, which is larger than all of the solar power delivered to the Earth by the Sun.

P30.56 $\dfrac{\mu_0 q\omega}{2.5\sqrt{5}\pi R}$

P30.58 (a) 12.6 μT; (b) 56.0 μT

P30.60 (a) $B = B_{x1} + B_{x2} = \dfrac{N\mu_0 I R^2}{2}\left[\dfrac{1}{\left(x^2+R^2\right)^{3/2}} + \dfrac{1}{\left[\left(R-x\right)^2+R^2\right]^{3/2}}\right]$; (b) See P30.60(b) for full explanation.

P30.62 (a) 2.46 N upward; (b) Equation 30.7 is the expression for the magnetic field produced a distance x above the center of a loop. The magnetic field at the center of the loop or on its axis is much weaker than the magnetic field just outside the wire. The wire has negligible curvature on the scale of 1 mm, so we model the lower loop as a long straight wire to find the field it creates at the location of the upper wire; (c) 107 m/s^2 upward

P30.64 (a) 5.24 μT; (b) into the page; (c) 7.20 cm

P30.66 (a) 2.74×10^{-4} T; (b) $2.74\times10^{-4}\text{ T}\left(-\hat{\mathbf{j}}\right)$; (c) Under the assumption that the rails are infinitely long, the length of rail to the left of the bar does not depend on the location of the bar; (d) 1.15×10^{-3} N; (e) $+x$ direction; (f) Yes, length of the bar, current, and field are constant, so force is constant; (g) $\left(0.999\text{ m/s}\right)\hat{\mathbf{i}}$

P30.68 $\dfrac{\mu_0 I_1 I_2 L}{\pi R}$ to the right

P30.70 See P30.70 for full explanation.

P30.72 (a) See ANS FIG P30.72(a); (b) zero; (c) zero; (d) $\vec{\mathbf{B}} = \dfrac{32\times10^{-7} z}{9\times10^{-4} + z^2}\hat{\mathbf{j}}$, where $\vec{\mathbf{B}}$ is in teslas and z is in meters; (e) $d = a = 3.00$ cm; (f) $53.3\hat{\mathbf{j}}$ μT

P30.74 (a) $\dfrac{\mu_0 \rho\omega R^2}{3}$; (b) $\dfrac{4\pi\omega\rho R^5}{15}$

P30.76 See P30.76 for full explanation.

31

Faraday's Law and Inductance

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ31.1 The ranking is E > A > B = D = 0 > C. The emf is given by the negative of the time derivative of the magnetic flux. We pick out the steepest downward slope at instant E as marking the moment of largest emf. Next comes A. At B and at D the graph line is horizontal so the emf is zero. At C the emf has its greatest negative value.

OQ31.2 **(i)** Answer (c). **(ii)** Answers (a) and (b). The magnetic flux is $\Phi_B = BA\cos\theta$. Therefore the flux is a maximum when $\vec{\mathbf{B}}$ is perpendicular to the loop of wire and zero when there is no component of magnetic field perpendicular to the loop. The flux is zero when the loop is turned so that the field lies in the plane of its area.

OQ31.3 Answer (b). With the current in the long wire flowing in the direction shown in Figure OQ31.3, the magnetic flux through the rectangular loop is directed into the page. If this current is decreasing in time, the *change* in the flux is directed opposite to the flux itself (or out of the page). The induced current will then flow clockwise around the loop, producing a flux directed into the page through the loop and opposing the change in flux due to the decreasing current in the long wire.

OQ31.4 Answer (a). Treating the original flux as positive (i.e., choosing the normal to have the same direction as the original field), the flux changes from

$$\Phi_{Bi} = B_i A\cos\theta_i = B_i A\cos 0° = B_i A$$

to $$\Phi_{Bf} = B_f A\cos\theta_f = B_f A\cos 180° = -B_f A.$$

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\left[\frac{\left(-B_f A\right)-\left(B_i A\right)}{\Delta t}\right] = \frac{2\left(B_f + B_i\right)A}{\Delta t}$$

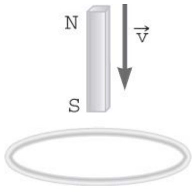
$$= 2\left[\frac{(0.060\text{ T})+(0.040\text{ T})}{0.50\text{ s}}\right]\left[\pi(0.040\text{ m})^2\right] = 2.0\times10^{-3}\text{ V}$$

$$= 2.0\text{ mV}$$

OQ31.5 Answers (c) and (d). The magnetic flux through the coil is constant in time, so the induced emf is zero, but positive test charges in the leading and trailing sides of the square experience a $\vec{\mathbf{F}} = q\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$ force that is in direction (velocity to the right) × (field perpendicularly into the page away from you) = (force toward the top of the square). The charges migrate upward to give positive charge to the top of the square until there is a downward electric field large enough to prevent more charge separation.

OQ31.6 Answers (b) and (d). By the magnetic force law $\vec{\mathbf{F}} = q\left(\vec{\mathbf{v}}\times\vec{\mathbf{B}}\right)$: the positive charges in the moving bar will feel a magnetic force in direction (velocity to the right) × (field perpendicularly out of the page) = (force downward toward the bottom end of the bar). These charges will move downward and therefore clockwise in the circuit. The current induced in the bar experiences a force in the magnetic field that tends to slow the bar: (current downward) × (field perpendicularly out of the page) = (force to the left); therefore, an external force is required to keep the bar moving at constant speed to the right.

OQ31.7 Answer (a). As the bar magnet approaches the loop from above, with its south end downward as shown in the figure, the magnetic flux through the area enclosed by the loop is directed upward and increasing in magnitude. To oppose this increasing upward flux, the induced current in the loop will flow clockwise, as seen from above, producing a flux directed downward through the area enclosed

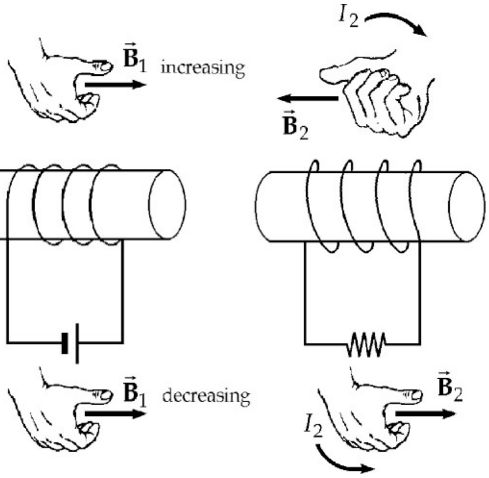


ANS. FIG. OQ31.7

by the loop. After the bar magnet has passed through the plane of the loop, and is departing with its north end upward, a decreasing flux is directed upward through the loop. To oppose this decreasing upward flux, the induced current in the loop flows counterclockwise as seen from above, producing flux directed upward through the area enclosed by the loop. From this analysis, we see that (a) is the only true statement among the listed choices.

OQ31.8 Answer (b). The maximum induced emf in a generator is proportional to the rate of rotation. The rate of change of flux of the external magnetic field through the turns of the coil is doubled, so the maximum induced emf is doubled.

OQ31.9 **(i)** Answer (b). The battery makes counterclockwise current I_1 in the primary coil, so its magnetic field $\vec{\mathbf{B}}_1$ is to the right and increasing just after the switch is closed. The secondary coil will oppose the change with a leftward field $\vec{\mathbf{B}}_2$, which comes from an induced clockwise current I_2 that goes to the right in the resistor. The upper pair of hands in ANS. FIG. OQ31.9 represent this effect.

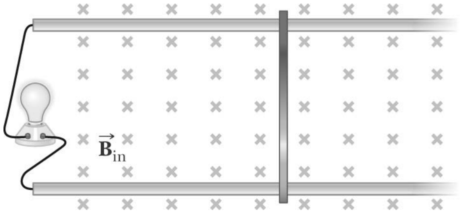


ANS. FIG. OQ31.9

(ii) Answer (c). At steady state the primary magnetic field is unchanging, so no emf is induced in the secondary.

(iii) Answer (a). The primary's field is to the right and decreasing as the switch is opened. The secondary coil opposes this decrease by making its own field to the right, carrying counterclockwise current to the left in the resistor. The lower pair of hands shown in ANS. FIG. OQ31.9 represent this chain of events.

OQ31.10 Answers (a), (b), (c), and (d). With the magnetic field perpendicular to the plane of the page in the figure, the flux through the closed loop to the left of the bar is given by $\Phi_B = BA$, where *B* is the magnitude of the field and A is the area enclosed by the loop. Any action which produces a change in this product, BA, will induce a current in the loop and cause the bulb to light. Such actions include increasing or decreasing the magnitude of the field B, and moving the bar to the right or left and changing the enclosed area A. Thus, the bulb will light during all of the actions in choices (a), (b), (c), and (d).



ANS. FIG. OQ31.10

ANS. FIG. OQ31.11

OQ31.11 Answers (b) and (d). A current flowing counterclockwise in the outer loop of the figure produces a magnetic flux through the inner loop that is directed out of the page. If this current is increasing in time, the *change* in the flux is in the same direction as the flux itself (or out of the page). The induced current in the inner loop will then flow clockwise around the loop, producing a flux through the loop directed into the page, opposing the change in flux due to the increasing current in the outer loop. The flux through the inner loop is given by $\Phi_B = BA$, where B is the magnitude of the field and A is the area enclosed by the loop. The magnitude of the flux, and thus the magnitude of the rate of change of the flux, depends on the size of the area A.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ31.1 Recall that the net work done by a conservative force on an object is path independent; thus, if an object moves so that it starts and ends at the same place, the net conservative work done on it is zero. A positive electric charge carried around a circular electric field line in the direction of the field gains energy from the field every step of the way. It can be a test charge imagined to exist in vacuum or it can be an actual free charge participating in a current driven by an induced emf. By doing net work on an object carried around a closed path to its starting point, the magnetically-induced electric field exerts by definition a nonconservative force. We can get a larger and larger voltage just by looping a wire around into a coil with more and more turns.

CQ31.2 The spacecraft is traveling through the magnetic field of the Earth. The magnetic flux through the coil must be changing to produce an emf, and thus a current. The orientation of the coil could be changing relative to the external magnetic field, or the field is changing through the coil because it is not uniform, or both.

CQ31.3 As water falls, it gains speed and kinetic energy. It then pushes against turbine blades, transferring its energy to the rotor coils of a large AC generator. The rotor of the generator turns within a strong magnetic field. Because the rotor is spinning, the magnetic flux through its coils changes in time as $\Phi_B = BA\cos\omega t$. Generated in the rotor is an induced emf of $\mathcal{E} = \frac{-Nd\Phi_B}{dt}$. This induced emf is the voltage driving the current in our electric power lines.

CQ31.4 Let us assume the north pole of the magnet faces the ring. As the bar magnet falls toward the conducting ring, a magnetic field is induced in the ring pointing upward. This upward directed field will oppose the motion of the magnet, preventing it from moving as a freely-falling body. Try it for yourself to show that an upward force also acts on the falling magnet if the south end faces the ring.

CQ31.5 To produce an emf, the magnetic flux through the loop must change. The flux cannot change if the orientation of the loop remains fixed in space because the magnetic field is uniform and constant. The flux does change if the loop is rotated so that the angle between the normal to the surface and the direction of the magnetic field changes.

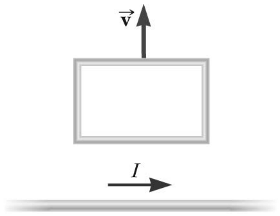
CQ31.6 Yes. The induced eddy currents on the surface of the aluminum will slow the descent of the aluminum. In a strong field the piece may fall very slowly.

CQ31.7 Magnetic flux measures the "flow" of the magnetic field through a given area of a loop—even though the field does not actually flow. By changing the size of the loop, or the orientation of the loop and the field, one can change the magnetic flux through the loop, but the magnetic field will not change.

CQ31.8 The increasing counterclockwise current in the solenoid coil produces an upward magnetic field that increases rapidly. The increasing upward flux of this field through the ring induces an emf to produce clockwise current in the ring. The magnetic field of the solenoid has a radially outward component at each point on the ring. This field component exerts upward force on the current in the ring there. The whole ring feels a total upward force larger than its weight.

CQ31.9 Oscillating current in the solenoid produces an always-changing magnetic field. Vertical flux through the ring, alternately increasing and decreasing, produces current in it with a direction that is alternately clockwise and counterclockwise. The current through the ring's resistance converts electrically transmitted energy into internal energy at the rate I^2R.

CQ31.10 (a) Counterclockwise. With the current in the long wire flowing in the direction shown in the figure, the magnetic flux through the rectangular loop is directed out of the page. As the loop moves away from the wire, the magnetic field through the loop becomes weaker, so the magnetic flux through the loop is decreasing in time, and the *change* in the flux is directed opposite to the flux itself (or into the page). The induced current will then flow counterclockwise around the loop, producing a flux directed out of the page through the loop and opposing the change in flux due to the decreasing flux through the loop.



ANS. FIG. CQ31.10

(b) Clockwise. In this case, as the loop moves toward from the wire, the magnetic field through the loop becomes stronger, so the magnetic flux through the loop is increasing in time, and the *change* in the flux has the same direction as the flux itself (or out of the page). The induced current will then flow clockwise around the loop, producing a flux directed into the page through the loop and opposing the change in flux due to the increasing flux through the loop.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 31.1 Faraday's Law of Induction

***P31.1** From Equation 31.1, the induced emf is given by

$$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{\Delta\left(\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}\right)}{\Delta t}$$

$$= \frac{(2.50\ \text{T} - 0.500\ \text{T})\left(8.00\times10^{-4}\ \text{m}^2\right)}{1.00\ \text{s}}\left(\frac{1\ \text{N}\cdot\text{s}}{1\ \text{T}\cdot\text{C}\cdot\text{m}}\right)\left(\frac{1\ \text{V}\cdot\text{C}}{1\ \text{N}\cdot\text{m}}\right)$$

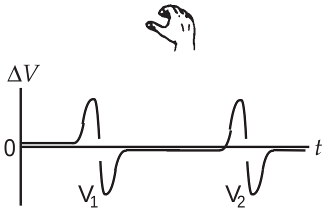
$$= 1.60\ \text{mV}$$

We then find the current induced in the loop from

$$I_{\text{loop}} = \frac{\mathcal{E}}{R} = \frac{1.60\ \text{mV}}{2.00\ \Omega} = \boxed{0.800\ \text{mA}}$$

***P31.2** (a) Each coil has a pulse of voltage tending to produce counterclockwise current as the projectile approaches, and then a pulse of clockwise voltage as the projectile recedes.

(b) $v = \frac{d}{t} = \frac{1.50\ \text{m}}{2.40\times10^{-3}\ \text{s}} = \boxed{625\ \text{m/s}}$



ANS. FIG. P31.2

P31.3 (a) From Faraday's law,

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -N\left(\frac{\Delta B}{\Delta t}\right)A\cos\theta$$

$$|\mathcal{E}| = \left|-(1)\left(\frac{B_f - B_i}{\Delta t}\right)\left(\pi r^2\right)\cos\theta\right|$$

$$= \left(\frac{1.50\ \text{T} - 0}{0.120\ \text{s}}\right)\left[\pi(0.001\ 60\ \text{m})^2\right](1)$$

$$= (12.5\ \text{T/s})\left[\pi(0.001\ 60\ \text{m})^2\right] = 1.01\times10^{-4}\ \text{T}$$

$$= \boxed{\begin{array}{l}101\ \mu\text{V tending to produce clockwise current}\\ \text{as seen from above}\end{array}}$$

(b) In case (a), the rate of change of the magnetic field was +12.5 T/s. In this case, the rate of change of the magnetic field is (–0.5 T – 1.5 T)/ 0.08 s = –25.0 T/s: it is twice as large in magnitude and in the opposite sense from the rate of change in case (a), so the emf is also

$\boxed{\text{twice as large in magnitude and in the opposite sense}}$.

P31.4 From Equation 31.2,

$$\mathcal{E} = -N\frac{\Delta(BA\cos\theta)}{\Delta t} = -NB\pi r^2\left(\frac{\cos\theta_f - \cos\theta_i}{\Delta t}\right)$$

$$= -25.0\left(50.0\times10^{-6}\text{ T}\right)\left[\pi(0.500\text{ m})^2\right]\left(\frac{\cos180° - \cos0°}{0.200\text{ s}}\right)$$

$$\mathcal{E} = \boxed{+9.82\text{ mV}}$$

P31.5 With the field directed perpendicular to the plane of the coil, the flux through the coil is $\Phi_B = BA\cos 0° = BA$. For a single loop,

$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = \frac{B(\Delta A)}{\Delta t}$$

$$= \frac{(0.150\text{ T})\left[\pi(0.120\text{ m})^2 - 0\right]}{0.200\text{ s}} = 3.39\times10^{-2}\text{ V} = \boxed{33.9\text{ mV}}$$

P31.6 With the field directed perpendicular to the plane of the coil, the flux through the coil is $\Phi_B = BA\cos 0° = BA$. As the magnitude of the field increases, the magnitude of the induced emf in the coil is

$$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = \left(\frac{\Delta B}{\Delta t}\right)A = (0.050\,0\text{ T/s})\left[\pi(0.120\text{ m})^2\right]$$

$$= 2.26\times10^{-3}\text{ V} = \boxed{2.26\text{ mV}}$$

P31.7 The angle between the normal to the coil and the magnetic field is 90.0° – 28.0° = 62.0°. For a loop of *N* turns,

$$\mathcal{E} = -N\frac{d\Phi_B}{dt} = -N\frac{d}{dt}(BA\cos\theta)$$

$$\mathcal{E} = -NB\cos\theta\left(\frac{\Delta A}{\Delta t}\right)$$

$$= -200\left(50.0\times10^{-6}\text{ T}\right)(\cos62.0°)\left(\frac{39.0\times10^{-4}\text{ m}^2}{1.80\text{ s}}\right)$$

$$= \boxed{-10.2\ \mu\text{V}}$$

P31.8 For a loop of *N* turns, the induced voltage is

$$\mathcal{E} = -N\frac{d\left(\vec{\mathbf{B}}\cdot\vec{\mathbf{A}}\right)}{dt} = -N\left(\frac{0 - B_iA\cos\theta}{\Delta t}\right)$$

$$= \frac{+200(1.60\text{ T})\left(0.200\text{ m}^2\right)\cos0°}{20.0\times10^{-3}\text{ s}} = 3\ 200\text{ V}$$

The induced current is then

$$I = \frac{\mathcal{E}}{R} = \frac{3\ 200\ \text{V}}{20.0\ \Omega} = \boxed{160\ \text{A}}$$

P31.9 Faradays law gives

$$|\mathcal{E}| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = N\left(\frac{dB}{dt}\right)A = N\left[\frac{d}{dt}\left(0.010\,0t + 0.040\,0t^2\right)\right]A$$

or $$|\mathcal{E}| = N\left(0.010\,0 + 0.080\,0t\right)A$$

where $|\mathcal{E}|$ is in volts, A is in meters squared, and t is in seconds. At $t = 5.00$ s, suppressing units,

$$|\mathcal{E}| = 30.0\left[0.010\ 0 + 0.080\ 0(5.00)\right]\left[\pi(0.040\ 0)^2\right]$$
$$= 6.18\times10^{-2} = \boxed{61.8\ \text{mV}}$$

P31.10 We have a stationary loop in an oscillating magnetic field that varies sinusoidally in time: $B = B_{\max}\sin\omega t,$ where $B_{\max} = 1.00\times10^{-8}$ T, $\omega = 2\pi f,$ and $f = 60.0$ Hz. The loop consists of a single band ($N = 1$) around the perimeter of a red blood cell with diameter $d = 8.00 \times 10^{-6}$ m and area $A = \pi d^2/4$. The induced emf is then

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -N\left(\frac{dB}{dt}\right)A$$
$$= -N\frac{d}{dt}\left(B_{\max}\sin\omega t\right)A = -\omega NAB_{\max}\cos\omega t$$

Comparing this expression to $\mathcal{E} = \mathcal{E}_{\max}\cos\omega t,$ we see that $\mathcal{E}_{\max} = \omega NAB_{\max}$. Therefore,

$$\mathcal{E}_{\max} = \omega NAB_{\max}$$
$$= \left[2\pi(60.0\ \text{Hz})\right](1)\left[\frac{\pi\left(8.00\times10^{-6}\ \text{m}\right)^2}{4}\right]\left(1.00\times10^{-3}\ \text{T}\right)$$
$$= \boxed{1.89\times10^{-11}\ \text{V}}$$

P31.11 The symbol for the radius of the ring is r_1, and we use R to represent its resistance. The emf induced in the ring is

$$\mathcal{E} = -\frac{d}{dt}(BA\cos\theta) = -\frac{d}{dt}(0.500\mu_0 nIA\ \cos 0°) = -0.500\mu_0 nA\frac{dI}{dt}$$

Note that A must be interpreted as the area $A = \pi r_2^2$ of the solenoid, where the field is strong:

$$\mathcal{E} = -0.500(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1\ 000\ \text{turns/m}) \times [\pi(0.030\ 0\ \text{m})^2](270\ \text{A/s})$$

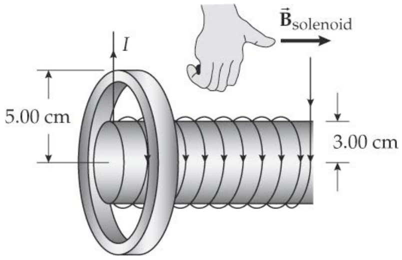
$$\mathcal{E} = \left(-4.80 \times 10^{-4}\,\frac{\text{T}\cdot\text{m}^2}{\text{s}}\right)\left(\frac{1\ \text{N}\cdot\text{s}}{\text{C}\cdot\text{m}\cdot\text{T}}\right)\left(\frac{1\ \text{V}\cdot\text{C}}{\text{N}\cdot\text{m}}\right) = -4.80 \times 10^{-4}\ \text{V}$$

(a) The negative sign means that the current in the ring is counterclockwise, opposite to the current in the solenoid. Its magnitude is

$$I_{\text{ring}} = \frac{|\mathcal{E}|}{R} = \frac{0.000\ 480\ \text{V}}{0.000\ 300\ \Omega} = \boxed{1.60\ \text{A}}$$

(b) $$B_{\text{ring}} = \frac{\mu_0 I}{2r_1} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.60\ \text{A})}{2(0.050\ 0\ \text{m})} = 2.01 \times 10^{-5}\ \text{T} = \boxed{20.1\ \mu\text{T}}$$

(c) The solenoid's field points to the right through the ring, and is increasing, so to oppose the increasing field, B_{ring} points to the $\boxed{\text{left}}$.



ANS. FIG. P31.11

P31.12 See ANS. FIG. P31.11. The emf induced in the ring is

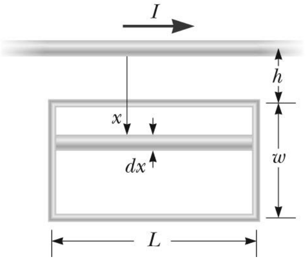
$$|\mathcal{E}| = \frac{d(BA)}{dt} = \frac{1}{2}\frac{d}{dt}(\mu_0 n I)A = \frac{1}{2}\mu_0 n \frac{dI}{dt}\pi r_2^2 = \frac{1}{2}\mu_0 n \pi r_2^2 \frac{\Delta I}{\Delta t}$$

(a) $I_{\text{ring}} = \dfrac{\mathcal{E}}{R} = \boxed{\dfrac{\mu_0 n \pi r_2^2}{2R}\dfrac{\Delta I}{\Delta t}}$, counterclockwise as viewed from the left end.

(b) $$B = \frac{\mu_0 I}{2r_1} = \boxed{\frac{\mu_0^2 n \pi r_2^2}{4 r_1 R}\frac{\Delta I}{\Delta t}}$$

(c) The solenoid's field points to the right through the ring, and is increasing, so to oppose the increasing field, B_{ring} points to the $\boxed{\text{left}}$.

P31.13 (a) At a distance x from the long, straight wire, the magnetic field is $B = \dfrac{\mu_0 I}{2\pi x}$. The flux through a small rectangular element of length L and width dx within the loop is



ANS. FIG. P31.13

$$d\Phi_B = \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \frac{\mu_0 I}{2\pi x} L dx:$$

$$\Phi_B = \int_h^{h+w} \frac{\mu_0 IL}{2\pi} \frac{dx}{x} = \boxed{\frac{\mu_0 IL}{2\pi} \ln\left(\frac{h+w}{h}\right)}$$

(b) $$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 IL}{2\pi} \ln\left(\frac{h+w}{h}\right)\right] = -\left[\frac{\mu_0 L}{2\pi} \ln\left(\frac{h+w}{h}\right)\right]\frac{dI}{dt}$$

where $\dfrac{dI}{dt} = \dfrac{d}{dt}(a+bt) = b$:

$$\mathcal{E} = -\frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(1.00\text{ m})}{2\pi} \times \ln\left(\frac{0.010\,0\text{ m} + 0.100\text{ m}}{0.010\,0\text{ m}}\right)(10.0\text{ A/s})$$
$$= -4.80\times10^{-6}\text{ V}$$

Therefore, the emf induced in the loop is $\boxed{4.80\ \mu\text{V}}$.

(c) The long, straight wire produces magnetic flux into the page through the rectangle, shown in ANS. FIG. P31.13. As the magnetic flux increases, the rectangle produces its own magnetic field out of the page to oppose the increase in flux. The induced current creates this opposing field by traveling $\boxed{\text{counterclockwise}}$ around the loop.

P31.14 The magnetic field lines are confined to the interior of the solenoid, so even though the coil has a larger area, the flux through the coil is the same as the flux through the solenoid:

$$\Phi_B = (\mu_0 n I) A_{\text{solenoid}}$$

$$\begin{aligned}\mathcal{E} &= -N\frac{d\Phi_B}{dt} = -N\mu_0 n\left(\pi r_{\text{solenoid}}^2\right)\frac{dI}{dt}\\ &= -(15)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.00\times10^{3}\ \text{m}^{-1}\right)\\ &\qquad\qquad\times\pi(0.020\ 0\ \text{m})^2(600)\cos(120t)\\ &= -0.014\ 2\cos(120t)\end{aligned}$$

$$\boxed{\begin{array}{l}\mathcal{E} = -\left(1.42\times10^{-2}\right)\cos(120t),\\ \text{where } t \text{ is in seconds and } \mathcal{E} \text{ is in V.}\end{array}}$$

P31.15 The initial magnetic field inside the solenoid is

$$\begin{aligned}B &= \mu_0 nI = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{100}{0.200\ \text{m}}\right)(3.00\ \text{A})\\ &= 1.88\times10^{-3}\ \text{T}\end{aligned}$$

(a) $\Phi_B = BA\cos\theta = \left(1.88\times10^{-3}\ \text{T}\right)\left(1.00\times10^{-2}\ \text{m}\right)^2\cos 0°$

$$= \boxed{1.88\times10^{-7}\ \text{T}\cdot\text{m}^2}$$

(b) When the current is zero, the flux through the loop is $\Phi_B = 0$ and the average induced emf has been

$$|\mathcal{E}| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{\left|0-1.88\times10^{-7}\ \text{T}\cdot\text{m}^2\right|}{3.00\ \text{s}} = \boxed{6.28\times10^{-8}\ \text{V}}$$

P31.16 The solenoid creates a magnetic field

$$B = \mu_0 nI = (4\pi\times10^{-7}\ \text{N/A}^2)(400\ \text{turns/m})(30.0\ \text{A})(1-e^{-1.60\,t})$$

$$B = (1.51\times10^{-2}\ \text{N/m}\cdot\text{A})(1-e^{-1.60\,t})$$

The magnetic flux through one turn of the flat coil is $\Phi_B = \int B\,dA\cos\theta$, but since $dA\cos\theta$ refers to the area perpendicular to the flux, and the magnetic field is uniform over the area A of the flat coil, this integral simplifies to

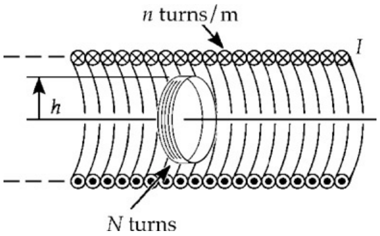
$$\begin{aligned}\Phi_B &= B\int dA = B\left(\pi R^2\right)\\ &= \left(1.51\times10^{-2}\ \text{N/m}\cdot\text{A}\right)\left(1-e^{-1.60t}\right)\left[\pi(0.060\ 0\ \text{m})^2\right]\\ &= \left(1.71\times10^{-4}\ \text{N/m}\cdot\text{A}\right)\left(1-e^{-1.60t}\right)\end{aligned}$$

The emf generated in the *N*-turn coil is $\mathcal{E} = -N\, d\Phi_B/dt$. Because *t* has the standard unit of seconds, the factor 1.60 must have the unit s^{-1}.

$$\mathcal{E} = -(250)\left(1.71\times10^{-4}\,\frac{\mathrm{N\cdot m}}{\mathrm{A}}\right)\frac{d\left(1-e^{-1.60\,t}\right)}{dt}$$

$$= -\left(0.042\,6\,\frac{\mathrm{N\cdot m}}{\mathrm{A}}\right)(1.60\ \mathrm{s}^{-1})e^{t-1.60}$$

$$\boxed{\mathcal{E} = 68.2e^{-1.60t},\ \text{where } t \text{ is in seconds and } \mathcal{E} \text{ is in mV.}}$$



ANS. FIG. P31.16

P31.17 Faraday's law, $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$, becomes here

$$\mathcal{E} = -N\frac{d}{dt}\left(BA\cos\theta\right) = -NA\cos\theta\frac{dB}{dt}$$

The magnitude of the emf is

$$|\mathcal{E}| = NA\cos\theta\left(\frac{\Delta B}{\Delta t}\right)$$

The area is

$$A = \frac{|\mathcal{E}|}{N\cos\theta\left(\dfrac{\Delta B}{\Delta t}\right)}$$

$$A = \frac{80.0\times10^{-3}\ \mathrm{V}}{50\left(\cos 30.0^\circ\right)\left(\dfrac{600\times10^{-6}\ \mathrm{T} - 200\times10^{-6}\ \mathrm{T}}{0.400\ \mathrm{s}}\right)} = 1.85\ \mathrm{m}^2$$

Each side of the coil has length $d = \sqrt{A}$, so the total length of the wire is

$$L = N(4d) = 4N\sqrt{A} = (4)(50)\sqrt{1.85\ \mathrm{m}^2} = \boxed{272\ \mathrm{m}}$$

P31.18 (a) Suppose, first, that the central wire is long and straight. The enclosed current of unknown amplitude creates a circular magnetic field around it, with the magnitude of the field given by Ampère's law.

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I: \qquad B = \frac{\mu_0 I_{\max} \sin \omega t}{2\pi R}$$

at the location of the Rogowski coil, which we assume is centered on the wire. This field passes perpendicularly through each turn of the toroid, producing flux

$$\vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = \frac{\mu_0 I_{\max} A}{2\pi R} \sin \omega t$$

The toroid has $2\pi Rn$ turns. As the magnetic field varies, the emf induced in it is

$$\begin{aligned} \mathcal{E} &= -N \frac{d}{dt} \vec{\mathbf{B}} \cdot \vec{\mathbf{A}} = -2\pi Rn \frac{\mu_0 I_{\max} A}{2\pi R} \frac{d}{dt} \sin \omega t \\ &= -\mu_0 I_{\max} nA\omega \cos \omega t \end{aligned}$$

This is an alternating voltage with amplitude $\mathcal{E}_{\max} = \mu_0 nA\omega I_{\max}$. Measuring the amplitude determines the size $I_{\max}$ of the central current. Our assumptions that the central wire is long and straight and passes perpendicularly through the center of the Rogowski coil are all unnecessary.

(b) If the wire is not centered, the coil will respond to stronger magnetic fields on one side, but to correspondingly weaker fields on the opposite side. $\boxed{\text{The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampère's law says that this line integral depends only on the amount of current the coil encloses.}}$ It does not depend on the shape or location of the current within the coil, or on any currents outside the coil.

P31.17 In a toroid, all the flux is confined to the inside of the toroid. From Equation 30.16, the field inside the toroid at a distance r from its center is

$$B = \frac{\mu_0 N I}{2\pi r}$$

The magnetic flux is then

$$\begin{aligned} \Phi_B &= \int B\,dA = \frac{\mu_0 N I_{\max}}{2\pi} \sin \omega t \int \frac{a\,dr}{r} \\ &= \frac{\mu_0 N I_{\max}}{2\pi} a \sin \omega t \ln\left(\frac{b+R}{R}\right) \end{aligned}$$

and the induced emf is

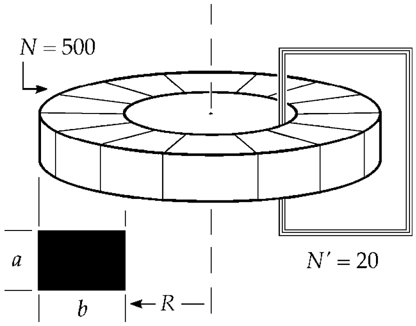
$$\mathcal{E} = N'\frac{d\Phi_B}{dt} = N'\left(\frac{\mu_0 N I_{\max}}{2\pi}\right)\omega a \ln\left(\frac{b+R}{R}\right)\cos\omega t$$

Substituting numerical values and suppressing units,

$$\mathcal{E} = 20\frac{\left(4\pi\times10^{-7}\right)(500)(50.0)}{2\pi}$$

$$\times\left[2\pi(60.0)\right](0.020\,0)\ln\left(\frac{0.030\,0+0.040\,0}{0.040\,0}\right)\cos\omega t$$

$$\boxed{\mathcal{E} = 0.422\cos\omega t \text{ where } \mathcal{E} \text{ is in volts and t is in seconds.}}$$



ANS. FIG. P31.19

P31.20 In Figure P31.20, the original magnetic field points into the page and is increasing. The induced emf in the upper loop attempts to generate a counterclockwise current in order to produce a magnetic field out of the page that opposes the increasing external magnetic flux. The induced emf in the lower loop also must attempt to generate a counterclockwise current in order to produce a magnetic field out of the page that opposes the increasing external magnetic flux. Because of the crossing over between the two loops, the emf generated in the loops will be in opposite directions. Therefore, the magnitude of the net emf generated is

$$\mathcal{E}_{\text{net}} = \mathcal{E}_2 - \mathcal{E}_1 = A_2\frac{dB}{dt} - A_1\frac{dB}{dt} = \left(\pi r_2^2 - \pi r_1^2\right)\frac{dB}{dt}$$

$$= \pi\frac{dB}{dt}\left(r_2^2 - r_1^2\right)$$

where the upper loop is loop 1 and the lower one is loop 2.

(a) The induced current will be the ratio of the net emf to the total resistance of the loops:

$$I = \frac{\mathcal{E}_{\text{net}}}{R} = \frac{\pi \frac{dB}{dt}\left(r_2^2 - r_1^2\right)}{\left(\frac{R}{\ell}\right)\ell_{\text{total}}} = \frac{\pi \frac{dB}{dt}\left(r_2^2 - r_1^2\right)}{\left(\frac{R}{\ell}\right)\left(2\pi r_2 + 2\pi r_1\right)}$$

$$= \frac{\frac{dB}{dt}\left(r_2^2 - r_1^2\right)}{2\left(\frac{R}{\ell}\right)\left(r_2 + r_1\right)} = \frac{\frac{dB}{dt}\left(r_2 - r_1\right)\left(r_2 + r_1\right)}{2\left(\frac{R}{\ell}\right)\left(r_2 + r_1\right)}$$

$$= \frac{\frac{dB}{dt}\left(r_2 - r_1\right)}{2\left(\frac{R}{\ell}\right)}$$

Substitute numerical values:

$$I = \frac{(2.00\ \text{T/s})(0.090\,0\ \text{m} - 0.050\,0\ \text{m})}{2(3.00\ \Omega/\text{m})} = \boxed{0.013\,3\ \text{A}}$$

(b) The emf in each loop is trying to push charge in opposite directions through the wire, but the emf in the lower loop is larger because its area is larger (changing flux is proportional to the area of the loop), so the lower loop "wins": the $\boxed{\text{current is counterclockwise in the lower loop and clockwise in the upper loop}}$.

Section 31.2 Motional emf

Section 31.3 Lenz's Law

***P31.21** The angular speed of the rotor blades is

$$\omega = (2.00\ \text{rev/s})(2\pi\ \text{rad/rev}) = 4.00\pi\ \text{rad/s}$$

Thus, the motional emf is then

$$\mathcal{E} = \frac{1}{2}B\omega\ell^2 = \frac{1}{2}\left(50.0\times10^{-6}\ \text{T}\right)(4.00\pi\ \text{rad/s})(3.00\ \text{m})^2$$

$$= \boxed{2.83\ \text{mV}}$$

P31.22 (a) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\hat{\mathbf{i}}$ and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_{\text{induced}} = B_{\text{induced}}\hat{\mathbf{i}}$ (to the right) and the current in the resistor is directed from a to b, $\boxed{\text{to the right}}$.

(b) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\left(-\hat{\mathbf{i}}\right)$ increases; therefore, the induced field $\vec{\mathbf{B}}_{\text{induced}} = B_{\text{induced}}\left(+\hat{\mathbf{i}}\right)$ is to the right, and the current in the resistor is directed from *a* to *b*, $\boxed{\text{out of the page}}$ in the textbook picture.

(c) $\vec{\mathbf{B}}_{\text{ext}} = B_{\text{ext}}\left(-\hat{\mathbf{k}}\right)$ into the paper and B_{ext} decreases; therefore, the induced field is $\vec{\mathbf{B}}_{\text{induced}} = B_{\text{induced}}\left(-\hat{\mathbf{k}}\right)$ into the paper, and the current in the resistor is directed from *a* to *b*, $\boxed{\text{to the right}}$.

P31.23 The motional emf induced in a conductor is proportional to the component of the magnetic field perpendicular to the conductor and to its velocity.

$$\begin{aligned}\mathcal{E} &= B\ell v = \left(35.0\times 10^{-6}\ \text{T}\right)(15.0\ \text{m})(25.0\ \text{m/s}) \\ &= 1.31\times 10^{-2}\ \text{V} = \boxed{13.1\ \text{mV}}\end{aligned}$$

P31.24 (a) The potential difference is equal to the motional emf and is given by

$$\begin{aligned}\mathcal{E} &= B\ell v = \left(1.20\times 10^{-6}\ \text{T}\right)(14.0\ \text{m})(70.0\ \text{m/s}) \\ &= 1.18\times 10^{-3}\ \text{V} = \boxed{11.8\ \text{mV}}\end{aligned}$$

(b) A free positive test charge in the wing feels a magnetic force in direction $\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ = (north) × (down) = (west): it migrates west. $\boxed{\text{The wingtip on the pilot's left is positive.}}$

(c) $\boxed{\text{No change}}$. A positive test charge in the wing feels a magnetic force in direction $\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ = (east) × (down) = (north): it migrates north. The left wingtip is north of the pilot.

(d) $\boxed{\text{No. If you try to connect the wings to a circuit containing the light bulb, you must run an extra insulated wire along the wing. In a uniform field the total emf generated in the one-turn coil is zero.}}$

P31.25 (a) The motional emf induced in a conductor is proportional to the component of the magnetic field perpendicular to the conductor and to its velocity; in this case, the vertical component of the Earth's magnetic field is perpendicular to both. Thus, the magnitude of the motional emf induced in the wire is

$$\begin{aligned}\mathcal{E} &= B_{\perp}\ell v = \left[\left(50.0\times 10^{-6}\ \text{T}\right)\sin 53.0^{\circ}\right](2.00\ \text{m})(0.500\ \text{m/s}) \\ &= 3.99\times 10^{-5}\ \text{V} = \boxed{39.9\ \mu\text{V}}\end{aligned}$$

(b) Imagine holding your right hand horizontal with the fingers pointing north (the direction of the wire's velocity), such that when you close your hand the fingers curl downward (in the direction of $B_\perp$). Your thumb will then be pointing westward. By the right-hand rule, the magnetic force on charges in the wire would tend to move positive charges westward.

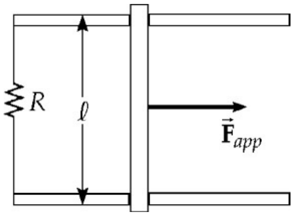
$\boxed{\text{The west end is positive.}}$

***P31.26** See ANS. FIG. P31.26. The current is given by

$$I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$$

Solving for the velocity gives

$$v = \frac{IR}{B\ell} = \frac{(0.500\ \text{A})(6.00\ \Omega)}{(2.50\ \text{T})(1.20\ \text{m})} = \boxed{1.00\ \text{m/s}}$$



ANS. FIG. P31.26

P31.27 (a) Refer to ANS. FIG. P31.26 above. At constant speed, the net force on the moving bar equals zero, or

$$\left|\vec{\mathbf{F}}_{\text{app}}\right| = I\left|\vec{\mathbf{L}} \times \vec{\mathbf{B}}\right|$$

where the current in the bar is $I = \mathcal{E}/R$ and the motional emf is $\mathcal{E} = B\ell v$. Therefore,

$$F_B = \frac{B\ell v}{R}(\ell B) = \frac{B^2\ell^2 v}{R} = \frac{(2.50\ \text{T})^2(1.20\ \text{m})^2(2.00\ \text{m/s})}{6.00\ \Omega}$$
$$= 3.00\ \text{N}$$

The applied force is $\boxed{3.00\ \text{N to the right}}$.

(b) $P = I^2R = \dfrac{B^2\ell^2v^2}{R} = 6.00\ \text{W}$ or $P = Fv = \boxed{6.00\ \text{W}}$

***P31.28** With v representing the initial speed of the bar, let u represent its speed at any later time. The motional emf induced in the bar is $\mathcal{E} = B\ell u$. The induced current is $I = \frac{\mathcal{E}}{R} = \frac{B\ell u}{R}$. The magnetic force on the bar is backward $F = -I\ell B = -\frac{B^2\ell^2 u}{R} = \frac{m\,du}{dt}$.

<u>Method one:</u> To find u as a function of time, we separate variables thus:

$$-\frac{B^2\ell^2}{Rm}dt = \frac{du}{u}$$

$$\int_0^t -\frac{B^2\ell^2}{Rm}dt = \int_v^u \frac{du}{u}$$

$$-\frac{B^2\ell^2}{Rm}(t-0) = \ln u - \ln v = \ln\frac{u}{v}$$

$$e^{-B^2\ell^2 t/Rm} = \frac{u}{v}$$

$$u = ve^{-B^2\ell^2 t/Rm} = \frac{dx}{dt}$$

The distance traveled is given by

$$\int_0^{x_{max}} dx = \int_0^{\infty} ve^{-B^2\ell^2 t/Rm}\,dt = v\left(-\frac{Rm}{B^2\ell^2}\right)\int_0^{\infty} e^{-B^2\ell^2 t/Rm}\left(-\frac{-B^2\ell^2 dt}{Rm}\right)$$

$$x_{max} - 0 = -\frac{Rmv}{B^2\ell^2}\left[e^{-\infty} - e^{-0}\right] = \boxed{\frac{Rmv}{B^2\ell^2}}$$

<u>Method two:</u> Newton's second law is

$$-\frac{B^2\ell^2 u}{R} = -\frac{B^2\ell^2}{R}\frac{dx}{dt} = m\frac{du}{dt}$$

$$m\,du = -\frac{B^2\ell^2}{R}dx$$

Direct integration from the initial to the stopping point gives

$$\int_v^0 m\,du = \int_0^{x_{max}} -\frac{B^2\ell^2}{R}dx$$

$$m(0-v) = -\frac{B^2\ell^2}{R}(x_{max} - 0)$$

$$x_{max} = \frac{mvR}{B^2\ell^2}$$

***P31.29** The magnetic force on the rod is given by

$$F_B = I\ell B$$

and the motional emf by

$$\mathcal{E} = B\ell v$$

The current is given by $I = \dfrac{\mathcal{E}}{R} = \dfrac{B\ell v}{R}$, so $B = \dfrac{IR}{\ell v}$.

(a) $F_B = \dfrac{I^2\ell R}{\ell v}$ and $I = \sqrt{\dfrac{F_B v}{R}} = \sqrt{\dfrac{(1.00\text{ N})(2.00\text{ m/s})}{8.00\ \Omega}} = \boxed{0.500\text{ A}}$

(b) The rate at which energy is delivered to the resistor is the power delivered, given by

$$P = I^2R = (0.500\text{ A})^2(8.00\ \Omega) = \boxed{2.00\text{ W}}$$

(c) For constant force, $P = \vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = (1.00\text{ N})(2.00\text{ m/s}) = \boxed{2.00\text{ W}}$.

P31.30 To maximize the motional emf, the automobile must be moving east or west. Only the component of the magnetic field to the north generates an emf in the moving antenna. Therefore, the maximum motional emf is

$$\mathcal{E}_{\text{max}} = B\ell v\cos\theta$$

Let's solve for the unknown speed of the car:

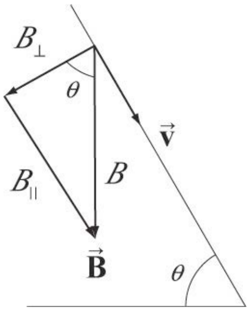
$$v = \frac{\mathcal{E}_{\text{max}}}{B\ell\cos\theta}$$

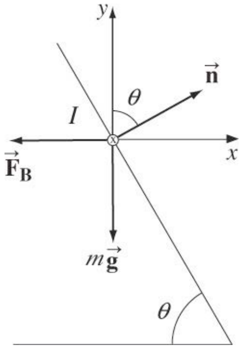
Substitute numerical values:

$$v = \frac{4.50\times10^{-3}\text{ V}}{(50.0\times10^{-6}\text{ T})(1.20\text{ m})\cos 65.0°} = 177\text{ m/s}$$

This is equivalent to about 640 km/h or 400 mi/h, much faster than the car could drive on the curvy road and much faster than any standard automobile could drive in general.

P31.31 The motional emf induced in a conductor is proportional to the component of the magnetic field perpendicular to the conductor and to its velocity. The total field is perpendicular to the conductor, but not to its velocity. As shown in the left figure, the component of the field perpendicular to the velocity is $B_\perp = B\cos\theta$. The motion of the bar down the rails produces an induced emf $\mathcal{E} = B_\perp\ell v = B\ell v\cos\theta$ that pushes charge into the page. The induced emf produces a current $I = \mathcal{E}/R = B\ell v\cos\theta/R$, where we assume that significant resistance is present only in the resistor. Because current in the bar travels into the page, and the field is downward, a magnetic force acts on the bar to the left: its magnitude is $F = I\ell B\sin 90.0° = I\ell B = B^2\ell^2 v\cos\theta/R$.





ANS. FIG. P31.31(a) **ANS. FIG. P31.31(b)**

In the free-body diagram shown in ANS. FIG. P31.31(b), it is convenient to use a coordinate system with axes vertical and horizontal. The force relationships are

$$\sum F_x = -F + n\sin\theta = 0 \quad \rightarrow \quad n\sin\theta = F = B^2\ell^2 v\cos\theta/R$$

$$\sum F_y = -mg + n\cos\theta = 0 \quad \rightarrow \quad n\cos\theta = mg$$

Dividing the first by the second equation, we get

$$\frac{\cancel{n}\sin\theta}{\cancel{n}\cos\theta} = \frac{B^2\ell^2 v\cos\theta/R}{mg} \quad \rightarrow \quad v = \frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}$$

Substituting numerical values,

$$v = \frac{(0.200\text{ kg})(9.80\text{ m/s}^2)(1.00\ \Omega)\sin 25.0°}{(0.500\text{ T})^2(1.20\text{ m})^2\cos^2 25.0°} = \boxed{2.80\text{ m/s}}$$

P31.32 Refer to ANS. FIG. P31.31 above. The motional emf induced in a conductor is proportional to the component of the magnetic field perpendicular to the conductor and to its velocity. The total field is perpendicular to the conductor, but not to its velocity. As shown in the left figure, the component of the field perpendicular to the velocity is $B_\perp = B\cos\theta$. The motion of the bar down the rails produces an induced emf $\mathcal{E} = B_\perp \ell v = B\ell v\cos\theta$ that pushes charge into the page. The induced emf produces a current $I = \mathcal{E}/R = B\ell v\cos\theta/R$, where we assume that significant resistance is present only in the resistor. Because current in the bar travels into the page, and the field is downward, a magnetic force acts on the bar to the left: its magnitude is $F = I\ell B\sin 90.0° = I\ell B = B^2\ell^2 v\cos\theta/R$. In the free-body diagram shown in ANS. FIG. P31.31(b), it is convenient to use a coordinate system with axes vertical and horizontal. The force relationships are

$$\sum F_x = -F + n\sin\theta = 0 \quad \rightarrow \quad n\sin\theta = F = B^2\ell^2 v\cos\theta/R$$

$$\sum F_y = -mg + n\cos\theta = 0 \quad \rightarrow \quad n\cos\theta = mg$$

Dividing the first by the second equation, we get

$$\frac{\cancel{m}\sin\theta}{\cancel{m}\cos\theta} = \frac{B^2\ell^2 v\cos\theta/R}{mg} \quad \rightarrow \quad v = \boxed{\frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}}$$

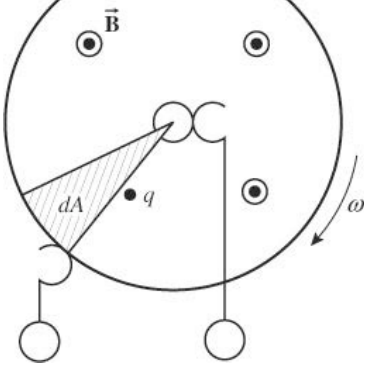
P31.33 From Example 31.4, the magnitude of the emf is

$$|\mathcal{E}| = B\left(\frac{1}{2}r^2\omega\right)$$

$$= (0.9\ \mathrm{N\cdot s/C\cdot m})\left[\frac{1}{2}(0.4\ \mathrm{m})^2(3\,200\ \mathrm{rev/min})\right]\left(\frac{2\pi\ \mathrm{rad/rev}}{60\ \mathrm{s/min}}\right)$$

$$|\mathcal{E}| = \boxed{24.1\ \mathrm{V}}$$

A free positive charge q, represented in our version of the diagram, turning with the disk, feels a magnetic force $q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ ☜ radially inward. Thus the outer contact is $\boxed{\text{negative}}$.



ANS. FIG. P31.33

P31.34 (a) The motional emf induced in the bar must be $\mathcal{E} = IR$, where I is the current in this series circuit. Since $\mathcal{E} = B\ell v$, the speed of the moving bar must be

$$v = \frac{\mathcal{E}}{B\ell} = \frac{IR}{B\ell} = \frac{(8.50\times10^{-3}\ \mathrm{A})(9.00\ \Omega)}{(0.300\ \mathrm{T})(0.350\ \mathrm{m})} = \boxed{0.729\ \mathrm{m/s}}$$

(b) The flux through the closed loop formed by the rails, the bar, and the resistor is directed into the page and is increasing in magnitude. To oppose this change in flux, the current must flow in a manner so as to produce flux out of the page through the area enclosed by the loop. This means the current will flow $\boxed{\text{counterclockwise}}$.

(c) The rate at which energy is delivered to the resistor is

$$\mathrm{P} = \mathrm{I}^2\mathrm{R} = \left(8.50\times10^{-3}\ \mathrm{A}\right)^2(9.00\ \Omega)$$
$$= 6.50\times10^{-4}\ \mathrm{W} = \boxed{0.650\ \mathrm{mW}}$$

(d) $\boxed{\text{Work is being done by the external force, which is transformed into internal energy in the resistor.}}$

P31.35 The speed of waves on the wire is

$$\mathrm{v} = \sqrt{\frac{\mathrm{T}}{\mu}} = \sqrt{\frac{\mathrm{mg}}{\mu}} = \sqrt{\frac{267\ \mathrm{N}}{3.00\times10^{-3}\ \mathrm{kg/m}}} = 298\ \mathrm{m/s}$$

In the simplest standing-wave vibration state,

$$\mathrm{d}_{\mathrm{NN}} = 0.64\ \mathrm{m} = \frac{\lambda}{2} \rightarrow \lambda = 1.28\ \mathrm{m}$$

and $$\mathrm{f} = \frac{\mathrm{v}}{\lambda} = \frac{298\ \mathrm{m/s}}{1.28\ \mathrm{m}} = 233\ \mathrm{Hz}$$

(a) The changing flux of magnetic field through the circuit containing the wire will drive current to the left in the wire as it moves up and to the right as it moves down. The emf will have this same frequency of $\boxed{233\ \mathrm{Hz}}$.

(b) The vertical coordinate of the center of the wire is described by

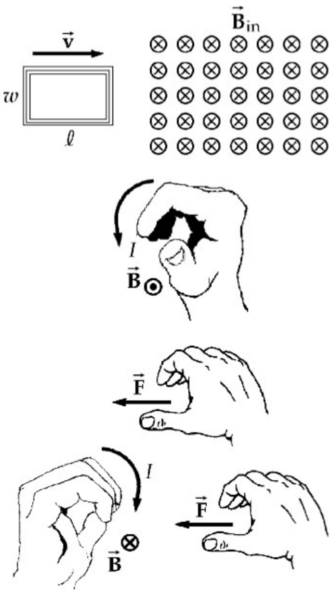
$$\mathrm{x} = \mathrm{A}\cos\omega\mathrm{t} = \mathrm{A}\cos 2\pi\mathrm{ft}$$

Its velocity is $\mathrm{v} = \frac{\mathrm{dx}}{\mathrm{dt}} = -2\pi\mathrm{fA}\sin 2\pi\mathrm{ft}$.

Its maximum speed is $\mathrm{v}_{\max} = 2\pi\mathrm{fA}$.

The induced emf is $\mathcal{E} = -\mathrm{B}\ell\mathrm{v}$, with amplitude

$$\mathcal{E}_{\max} = \mathrm{B}\ell\mathrm{v}_{\max} = \mathrm{B}\ell 2\pi\mathrm{fA}$$
$$= \left(4.50\times10^{-3}\ \mathrm{T}\right)(0.0200\ \mathrm{m})2\pi(233\ \mathrm{Hz})(0.015\,0\ \mathrm{m})$$
$$= 1.98\times10^{-3}\ \mathrm{V} = \boxed{1.98\ \mathrm{mV}}$$



ANS. FIG. P31.36

P31.36 (a) The force on the side of the coil entering the field (consisting of *N* wires) is

$$F = N(ILB) = N(IwB)$$

The induced emf in the coil is

$$|\mathcal{E}| = N\frac{d\Phi_B}{dt} = N\frac{d(Bwx)}{dt} = NBwv$$

so the current is $I = \frac{|\mathcal{E}|}{R} = \frac{NBwv}{R}$ counterclockwise.

The force on the leading side of the coil is then:

$$F = N\left(\frac{NBwv}{R}\right)wB$$

$$= \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left}}$$

(b) Once the coil is entirely inside the field,

$$\Phi_B = NBA = \text{constant}$$

so $\mathcal{E} = 0$, $I = 0$, and $F = \boxed{0}$.

(c) As the coil starts to leave the field, the flux *decreases* at the rate *Bwv*, so the magnitude of the current is the same as in part (a), but now the current is clockwise. Thus, the force exerted on the trailing side of the coil is:

$$F = \boxed{\frac{N^2B^2w^2v}{R} \text{ to the left again}}$$

P31.37 The emfs induced in the rods are proportional to the lengths of the sections of the rods between the rails. The emfs are $\mathcal{E}_1 = B\ell v_1$ with positive end downward, and $\mathcal{E}_2 = B\ell v_2$ with positive end upward, where $\ell = d = 10.0$ cm is the distance between the rails.

We apply Kirchhoff's laws. We assume current I_1 travels downward in the left rod, current I_2 travels upward in the right rod, and current I_3 travels upward in the resisitor R_3.

For the left loop, $+B\ell v_1 - I_1R_1 - I_3R_3 = 0$ **[1]**

For the right loop, $+B\ell v_2 - I_2R_2 + I_3R_3 = 0$ **[2]**

At the top junction, $I_1 = I_2 + I_3$ **[3]**

Substituting [3] into [1] gives

$$B\ell v_1 - I_1R_1 - I_3R_3 = 0$$

$$B\ell v_1 - (I_2 + I_3)R_1 - I_3R_3 = 0$$

$$I_2R_1 + I_3(R_1 + R_3) = B\ell v_1 \qquad \textbf{[4]}$$

Now using [2] and [4] to solve for I_2,

$$I_2 = \frac{B\ell v_2 + I_3R_3}{R_2} = \frac{B\ell v_1 - I_3(R_1 + R_3)}{R_1}$$

then equating gives

$$(B\ell v_2 + I_3R_3)R_1 = \left[B\ell v_1 - I_3(R_1 + R_3)\right]R_2$$

$$I_3\left[R_3R_1 + (R_1 + R_3)R_2\right] = B\ell v_1R_2 - B\ell v_2R_1$$

Solving for I3 gives

$$I_3 = B\ell\frac{(v_1R_2 - v_2R_1)}{R_1R_2 + R_1R_3 + R_2R_3}$$

Substituting numerical values, and noting that

$$\begin{aligned}R_1R_2 + R_1R_3 + R_2R_3 &= (10.0\ \Omega)(15.0\ \Omega)\\ &\quad + (10.0\ \Omega)(5.00\ \Omega) + (15.0\ \Omega)(5.00\ \Omega)\\ &= 275\ \Omega^2\end{aligned}$$

we obtain

$$\begin{aligned}I_3 &= (0.010\,0\ \text{T})(0.100\ \text{m})\\ &\quad\times\frac{\left[(4.00\ \text{m/s})(15.0\ \Omega) - (2.00\ \text{m/s})(10.0\ \Omega)\right]}{275\ \Omega^2}\\ &= 1.45\times10^{-4}\ \text{A}\end{aligned}$$

Therefore, $I_3 = \boxed{145\ \mu\text{A upward in the picture}}$, as was originally chosen.

P31.38 (a) The induced emf is $\mathcal{E} = B\ell v$, where *B* is the magnitude of the component of the magnetic field perpendicular to the tether, which, in this case, is the vertical component of the Earth's magnetic field at this location:

$$B_{\text{vertical}} = B_\perp = \frac{\mathcal{E}}{\ell v} = \frac{1.17\ \text{V}}{(25.0\ \text{m})(7.80\times10^3\ \text{m/s})}$$

$$= 6.00\times10^{-6}\ \text{T} = \boxed{6.00\ \mu\text{T}}$$

(b) Yes. The magnitude and direction of the Earth's field varies from one location to the other, so the induced voltage in the wire changes. Furthermore, the voltage will change if the tether cord or its velocity changes their orientations relative to the Earth's field.

(c) Either the long dimension of the tether or the velocity vector could be parallel to the magnetic field at some instant.

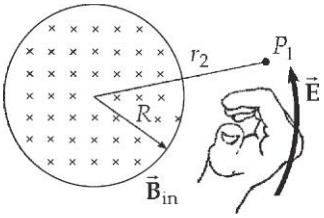
Section 31.4 Induced emf and Electric Fields

P31.39 Point P_1 lies outside the region of the uniform magnetic field. The rate of change of the field, in teslas per second, is

$$\frac{dB}{dt} = \frac{d}{dt}\left(2.00t^3 - 4.00t^2 + 0.800\right) = 6.00t^2 - 8.00t$$

where *t* is in seconds. At *t* = 2.00 s, we see that the field is increasing:

$$\frac{dB}{dt} = 6.00(2.00)^2 - 8.00(2.00) = 8.00\ \text{T/s}$$



ANS. FIG. P31.39

The magnetic flux is increasing into the page; therefore, by the right-hand rule (see figure), the induced electric field lines are counterclockwise. [Also, if a conductor of radius r_1 were placed concentric with the field region, by Lenz's law, the induced current would be counterclockwise. Therefore, the direction of the induced electric field lines are counterclockwise.] The electric field at point P_1 is tangent to the electric field line passing through it.

(a) The magnitude of the electric field is (refer to Section 31.4 and Equation 31.8)

$$|E| = \frac{r}{2}\frac{dB}{dt} = \frac{r}{2}\left(6.00t^2 - 8.00t\right)$$
$$= \frac{0.050\,0}{2}\left[6.00(2.00)^2 - 8.00(2.00)\right] = 0.200\ \text{N/C}$$

The magnitude of the force on the electron is

$$F = qE = eE = \left(1.60\times10^{-19}\ \text{C}\right)\left(0.200\ \text{N/C}\right) = \boxed{3.20\times10^{-20}\ \text{N}}$$

(b) Because the electron holds a negative charge, the direction of the force is opposite to the field direction. The force is $\boxed{\text{tangent to the electric field line passing through at point } P_1 \text{ and clockwise.}}$

(c) The force is zero when the rate of change of the magnetic field is zero:

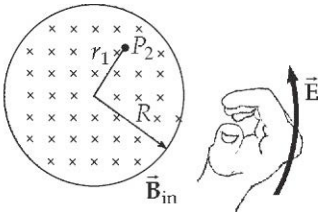
$$\frac{dB}{dt} = 6.00t^2 - 8.00t = 0 \rightarrow t = \boxed{0} \text{ or } t = \frac{8.00}{6.00} = \boxed{1.33\ \text{s}}$$

P31.40 Point P_2 lies inside the region of the uniform magnetic field. The rate of change of the field, in teslas per second, is

$$\frac{dB}{dt} = \frac{d}{dt}\left(0.030\ 0t^2 + 1.40\right) = 0.060\ 0t$$

where t is in seconds. At $t = 3.00$ s, we see that the field is increasing:

$$\frac{dB}{dt} = 0.060\ 0(3.00) = 0.180\ \text{T/s}$$



ANS. FIG. P31.40

The magnetic flux is increasing into the page; therefore, by the right-hand rule (see figure), the induced electric field lines are counterclockwise. The electric field at point P_2 is tangent to the electric field line passing through it.

(a) The situation is similar to that of Example 31.7.

$$\oint \vec{\mathbf{E}}\cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$$E2\pi r = -\frac{d\Phi_B}{dt} = -\frac{d\left(B\pi R^2\right)}{dt} = -\pi R^2\frac{dB}{dt}$$

$$E = -\frac{R^2}{2r}(0.060\ 0t)$$

For $r = r_2 = 0.020\ 0$ m,

$$|E| = \frac{R^2}{2r}(0.060\ 0t)$$
$$= \frac{(0.025\ 0\ \text{m})^2}{2(0.020\ 0\ \text{m})}\left[0.060\ 0(3.00)\right] = \boxed{2.81\times10^{-3}\ \text{N/C}}$$

(b) The field is $\boxed{\text{tangent to the electric field line passing through at point } P_2 \text{ and counterclockwise}}$.

P31.41 A problem similar to this is discussed in Example 31.7.

(a) $\oint \vec{\mathbf{E}}\cdot d\vec{\ell} = \left|\dfrac{d\Phi_B}{dt}\right|$ where $\Phi_B = BA = \mu_0 nI\left(\pi r^2\right)$

$$2\pi rE = \mu_0 n\left(\pi r^2\right)\frac{dI}{dt}$$
$$2\pi rE = \mu_0 n\left(\pi r^2\right)\frac{d}{dt}(5.00\sin 100\pi t)$$
$$= \mu_0 n\left(\pi r^2\right)(5.00)(100\pi)\cos 100\pi t$$

Solving for the electric field gives

$$E = \frac{\mu_0 n\left(\pi r^2\right)(5.00)(100\pi)(\cos 100\pi t)}{2\pi r}$$
$$= 250\mu_0 n\pi r\cos 100\pi t$$

Substituting numerical values and suppressing units,

$$E = 250\left(4\pi\times10^{-7}\right)\left(1.00\times10^{3}\right)\pi(0.0100)\cos 100\pi t$$
$$= \left(9.87\times10^{-3}\right)\cos 100\pi t$$

$$\boxed{\begin{array}{l}E = 9.87\cos 100\pi t \text{ where } E \text{ is in} \\ \text{millivolts/meter and } t \text{ is in seconds.}\end{array}}$$

(b) If a viewer looks at the solenoid along its axis, and if the current is increasing in the counterclockwise direction, the magnetic flux is increasing toward the viewer; the electric field always opposes increasing magnetic flux; therefore, by the right-hand rule, the electric field lines are $\boxed{\text{clockwise}}$.

Section 31.5 Generators and Motors

P31.42 (a) Use Equation 31.11, where B is the horizontal component of the magnetic field because the coil rotates about a vertical axis:

$$\begin{aligned}\mathcal{E}_{\max} &= NB_{\text{horizontal}}A\omega \\ &= 100\left(2.00\times10^{-5}\text{ T}\right)\left(0.200\text{ m}\right)^2 \\ &\quad\times\left[\left(1500\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right] \\ &= 1.26\times10^{-2}\text{ V} = \boxed{12.6\text{ mV}}\end{aligned}$$

(b) Maximum emf occurs when the magnetic flux through the coil is changing the fastest. This occurs at the moment when the flux is zero, which is $\boxed{\text{when the plane of the coil is parallel to the magnetic field}}$.

P31.43 The emf induced in a rotating coil is directly proportional to the angular speed of the coil. Thus,

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{\omega_2}{\omega_1}$$

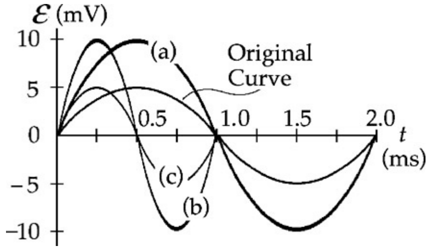
or

$$\mathcal{E}_2 = \left(\frac{\omega_2}{\omega_1}\right)\mathcal{E}_1 = \left(\frac{500\text{ rev/min}}{900\text{ rev/min}}\right)\left(24.0\text{ V}\right) = \boxed{13.3\text{ V}}$$

P31.44 The induced emf is proportional to the number of turns and the angular speed.

(a) Doubling the number of turns has this effect:

$$\boxed{\text{amplitude doubles and period is unchanged}}$$



ANS FIG. P31.44

(b) Doubling the angular velocity has this effect:

$$\boxed{\text{doubles the amplitude and cuts the period in half}}$$

(c) Doubling the angular velocity while reducing the number of turns to one half the original value has this effect:

$$\boxed{\text{amplitude unchanged and period is cut in half}}$$

P31.45 For the alternator,

$$\omega = (3\,000\text{ rev/min})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 314\text{ rad/s}$$

so

$$\begin{aligned}\mathcal{E} &= -N\frac{d\Phi_B}{dt} = -250\frac{d}{dt}\left[(2.50\times10^{-4})\cos(314t)\right]\\ &= +250(2.50\times10^{-4})(314)\sin(314t)\end{aligned}$$

(a) $\mathcal{E} = 19.6\sin(314t)$ where $\mathcal{E}$ is in volts and *t* is in seconds.

(b) $\boxed{\mathcal{E}_{\max} = 19.6\text{ V}}$

P31.46 Think of the semicircular conductor as enclosing half a coil of area $A = \frac{1}{2}\pi R^2$. There is no emf induced in the conductor until the magnetic flux through the area of the coil begins to change. The conductor is in the field region for only half a turn, so the flux changes over half a period $\frac{1}{2}T = \frac{1}{2}\left(\frac{2\pi}{\omega}\right) = \frac{\pi}{\omega}$. If we consider *t* = 0 to correspond to the time when the conductor is in the position shown in Figure P31.46 of the textbook, then there is no change in flux for a quarter of a turn, from *t* = 0 to $t = \pi/2\omega$, then the flux has a periodic behavior

$\Phi_B = AB\cos\omega t = \frac{1}{2}\pi R^2 B\cos\omega t$ for a half a turn, from $t = \pi/2\omega$ to $t = 3\pi/2\omega$, then there is no change in flux for the final quarter of a turn, from $t = 3\pi/2\omega$ to $t = 2\pi/\omega$, at the end of which the coil has returned to its starting position. While in the field region, the induced emf is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{1}{2}\pi R^2 B\frac{d}{dt}\cos\omega t = \frac{1}{2}\pi R^2\omega B\sin\omega t = \mathcal{E}_{\max}\sin\omega t$$

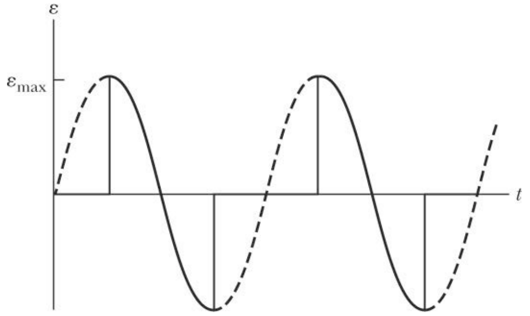
(a) The maximum emf is

$$\begin{aligned}\mathcal{E}_{\max} &= \frac{1}{2}\omega\pi R^2 B\\ &= \frac{1}{2}\left[\left(\frac{120\text{ rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{\text{rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]\pi(0.250\text{ m})^2(1.30\text{ T})\\ &= \boxed{1.60\text{ V}}\end{aligned}$$

(b) During the time period that the coil travels in the field region, the emf varies as $\mathcal{E}_{\max}\sin\omega t$ for half a period, from $+\mathcal{E}_{\max}$, at $t = \pi/2\omega$, to $-\mathcal{E}_{\max}$, at $t = 3\pi/2\omega$; therefore, the average emf is $\boxed{\text{zero}}$.

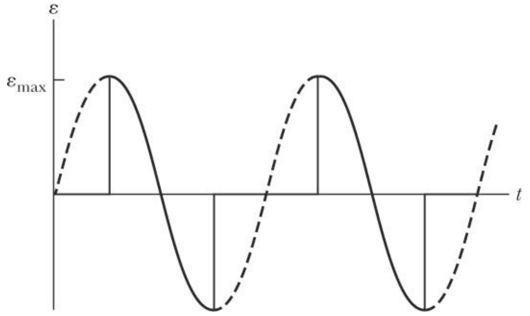
(c) The flux could also be written as $\Phi_B = \frac{1}{2}\pi R^2 B\cos\omega t$ so that it is a maximum at $t = 0$, but, in this case, the time period over which the flux changes would be from $t = 0$ to $t = 2\pi/\omega$, and the amplitude of the emf and its average would be the same as in the previous case; therefore, $\boxed{\text{no change in either answer}}$.

(d) The graph is



ANS. FIG. P31.46(d)

(e) If the time axis is chose so that the maximum emf occurs at the same time as it does in the figure of part (d) the graph is



ANS. FIG. P31.46(e)

P31.47 The magnetic field of the solenoid is given by

$$B = \mu_0 nI = \left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(200\ \text{m}^{-1}\right)(15.0\ \text{A})$$
$$= 3.77 \times 10^{-3}\ \text{T}$$

For the small coil, $\Phi_B = N\vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = NBA\cos\omega t = NB\left(\pi r^2\right)\cos\omega t$.

Thus, $\mathcal{E} = -\frac{d\Phi_B}{dt} = NB\pi r^2\omega\sin\omega t$

Substituting numerical values,

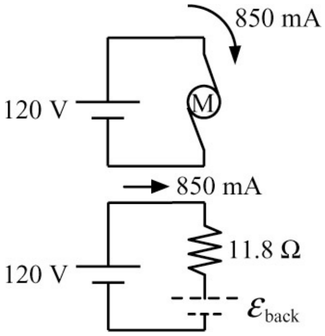
$$\mathcal{E} = (30.0)\left(3.77 \times 10^{-3}\ \text{T}\right)\pi\left(0.080\,0\ \text{m}\right)^2\left(4.00\pi\ \text{s}^{-1}\right)\sin\left(4.00\pi t\right)$$
$$= \boxed{(28.6\ \text{mV})\sin\left(4.00\pi t\right)}$$

P31.48 To analyze the actual circuit, we model it as the lower circuit diagram in ANS. FIG. P31.48.

(a) Kirchhoff's loop rule gives

$$+120\text{ V}-(0.850\text{ A})(11.8\ \Omega)-\mathcal{E}_{\text{back}}=0$$

$$\rightarrow\quad \mathcal{E}_{\text{back}}=\boxed{110\text{ V}}$$



ANS. FIG. P31.48

(b) The resistor is the device changing electrical work input into internal energy:

$$P=I^2R=(0.850\text{ A})^2(11.8\ \Omega)=\boxed{8.53\text{ W}}$$

(c) With no motion, the motor does not function as a generator, and $\mathcal{E}_{\text{back}}=0$. Then

$$120\text{ V}-I_c(11.8\ \Omega)=0\rightarrow I_c=10.2\text{ A}$$

$$P_c=I_c^2R=(10.2\text{ A})^2(11.8\ \Omega)=\boxed{1.22\text{ kW}}$$

P31.49 (a) The flux through the loop is

$$\begin{aligned}\Phi_B&=BA\cos\theta=BA\cos\omega t\\&=(0.800\text{ T})(0.010\,0\text{ m}^2)\cos 2\pi(60.0)t\\&=\boxed{(8.00\text{ mT}\cdot\text{m}^2)\cos(377t)}\end{aligned}$$

(b) $\mathcal{E}=-\dfrac{d\Phi_B}{dt}=\boxed{(3.02\text{ V})\sin(377t)}$

(c) $I=\dfrac{\mathcal{E}}{R}=\boxed{(3.02\text{ A})\sin(377t)}$

(d) $P=I^2R=\boxed{(9.10\text{ W})\sin^2(377t)}$

(e) $P=Fv=\tau\omega$ so $\tau=\dfrac{P}{\omega}=\boxed{(24.1\text{ mN}\cdot\text{m})\sin^2(377t)}$

Section 31.6 Eddy Currents

P31.50 The current in the magnet creates an  upward magnetic field, so the N and S poles on the solenoid core are shown correctly. On the rail in front of the brake, the upward flux of $\vec{\mathbf{B}}$ increases as the coil approaches, so a current is induced here to create a downward magnetic field. This is  clockwise current, so the S pole on the rail is shown correctly. On the rail behind the brake, the upward magnetic flux is decreasing. The induced current in the rail will produce upward magnetic field by being  counterclockwise as the picture correctly shows.

Additional Problems

***P31.51** (a) From Faraday's law of induction,

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \frac{d}{dt}(BA\cos\theta) = \frac{d}{dt}(BA) = A\frac{dB}{dt}$$

$$= \pi(0.060\ 0\ \text{m})^2\left(1.00\times10^4\ \text{T/s}\right)$$

$$= \boxed{113\ \text{V}}$$

(b) From Section 31.4, the electric field induced along the circumference of the circular area is given by

$$E = \frac{\mathcal{E}}{2\pi r} = \frac{113\ \text{V}}{2\pi(0.060\ 0\ \text{m})} = \boxed{300\ \text{V/m}}$$

***P31.52** Suppose we wrap twenty turns of wire into a flat compact circular coil of diameter 3 cm. Suppose we use a bar magnet to produce field 10^{-3} T through the coil in one direction along its axis. Suppose we then flip the magnet to reverse the flux in 10^{-1} s. The average induced emf is then

$$\bar{\mathcal{E}} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta[BA\cos\theta]}{\Delta t} = -NB\left(\pi r^2\right)\left(\frac{\cos 180° - \cos 0°}{\Delta t}\right)$$

$$\bar{\mathcal{E}} = -(20)\left(10^{-3}\ \text{T}\right)\pi\left(0.015\,0\ \text{m}\right)^2\left(\frac{-2}{10^{-1}\ \text{s}}\right)\boxed{\sim 10^{-4}\ \text{V}}$$

***P31.53** The magnitude of the average emf is given by

$$|\bar{\mathcal{E}}| = \mathrm{N}\frac{|\Delta\Phi_{\mathrm{B}}|}{\Delta\mathrm{t}} = \frac{\mathrm{NBA}(\Delta\cos\theta)}{\Delta\mathrm{t}}$$

$$= \frac{200(1.1\ \mathrm{T})\left(100\times10^{-4}\ \mathrm{m}^2\right)|\cos 180° - \cos 0°|}{0.10\ \mathrm{s}} = 44\ \mathrm{V}$$

The average current induced in the coil is therefore

$$\mathrm{I} = \frac{|\varepsilon|}{\mathrm{R}} = \frac{44\ \mathrm{V}}{5.0\ \Omega} = \boxed{8.8\ \mathrm{A}}$$

P31.54 (a) If the magnetic field were increasing, the flux would be increasing out of the page, so the induced current would tend to oppose the increase by generating a field into the page. The direction of such a current would be clockwise. This is the case here, so the field is $\boxed{\text{increasing}}$.

(b) The normal to the enclosed area can be taken to be parallel to the magnetic field, so the flux through the loop is

$$\Phi_{\mathrm{B}} = \mathrm{BA}\cos 0.00° = \mathrm{BA}$$

The rate of change of the flux is

$$\frac{\mathrm{d}\Phi_{\mathrm{B}}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\left(\mathrm{BA}\cos 0.00°\right) = \mathrm{A}\frac{\mathrm{dB}}{\mathrm{dt}}$$

and the induced emf is

$$|\mathcal{E}| = \left|-\frac{\mathrm{d}\Phi_{\mathrm{B}}}{\mathrm{dt}}\right| \quad \rightarrow \quad \mathrm{IR} = \mathrm{A}\frac{\mathrm{dB}}{\mathrm{dt}} = \pi \mathrm{r}^2\frac{\mathrm{dB}}{\mathrm{dt}}$$

Therefore,

$$\frac{\mathrm{dB}}{\mathrm{dt}} = \frac{\mathrm{IR}}{\pi \mathrm{r}^2} = \frac{\left(2.50\times10^{-3}\ \mathrm{A}\right)\left(0.500\ \Omega\right)}{\pi\left(0.080\ 0\ \mathrm{m}\right)^2}$$

$$= 0.062\ 2\ \mathrm{T/s}$$

$$= \boxed{62.2\ \mathrm{mT/s}}$$

P31.55 The emf through the hoop is given by

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_{\mathrm{B}}}{\mathrm{dt}} = -\mathrm{A}\frac{\mathrm{dB}}{\mathrm{dt}} = -0.160\frac{\mathrm{d}}{\mathrm{dt}}\left(0.350\mathrm{e}^{-\mathrm{t}/200}\right)$$

$$= \frac{(1.60)(0.350)}{200}\mathrm{e}^{-\mathrm{t}/200}$$

where $\mathcal{E}$ is in volts and t in seconds. For $t = 4.00$ s,

$$\mathcal{E} = \frac{\left(0.160\ \mathrm{m}^2\right)\left(0.350\ \mathrm{T}\right)}{2.00\ \mathrm{s}}\mathrm{e}^{-4.00/2.00} = \boxed{3.79\ \mathrm{mV}}$$

P31.56 The emf through the hoop is given by

$$\mathcal{E}=-\frac{d\Phi_B}{dt}=-A\frac{dB}{dt}=-A\frac{d}{dt}\left(B_{\text{max}}e^{-t/\tau}\right)=\boxed{\frac{AB_{\text{max}}}{\tau}e^{-t/\tau}}$$

P31.57 $\mathcal{E}=-N\dfrac{\Delta}{\Delta t}(BA\cos\theta)=-N\left(\pi r^2\right)\cos 0°\dfrac{\Delta B}{\Delta t}$

$$=-1\left(0.005\ 00\ \text{m}^2\right)(1)\left(\frac{1.50\ \text{T}-5.00\ \text{T}}{20.0\times10^{-3}\ \text{s}}\right)=0.875\ \text{V}$$

(a) $I=\dfrac{\mathcal{E}}{R}=\dfrac{0.875\ \text{V}}{0.020\,0\ \Omega}=\boxed{43.8\ \text{A}}$

(b) $P=\mathcal{E}I=(0.875\ \text{V})(43.8\ \text{A})=\boxed{38.3\ \text{W}}$

P31.58 (a) Motional emf produces a current $I=\dfrac{\mathcal{E}}{R}=\boxed{\dfrac{B\ell v}{R}}$.

(b) $\boxed{\text{Particle in equilibrium}}$

(c) The circuit encloses increasing flux of magnetic field into the page, so it tries to make its own field out of the page, by carrying counterclockwise current. The current flows upward in the bar, so the magnetic field produces a backward magnetic force $F_B=I\ell B$ (to the left) on the bar. This force increases until the bar has reached a speed when the backward force balances the applied force *F*:

$$F=F_B=I\ell B=\frac{\mathcal{E}}{R}\ell B=\frac{(B\ell v)}{R}\ell B=\frac{B^2\ell^2}{R}v$$

$$v=\frac{FR}{B^2\ell^2}=\frac{(0.600\ \text{N})(48.0\ \Omega)}{(0.400\ \text{T})^2(0.800\ \text{m})^2}=\boxed{281\ \text{m/s}}$$

(d) $I=\dfrac{\mathcal{E}}{R}=\dfrac{B\ell v}{R}=\dfrac{B\ell}{R}\dfrac{FR}{B^2\ell^2}=\dfrac{F}{B\ell}=\dfrac{0.600\ \text{N}}{(0.400\ \text{T})(0.800\ \text{m})}=\boxed{1.88\ \text{A}}$

(e) $P=I^2R=\left(\dfrac{F}{B\ell}\right)^2R=\left[\dfrac{0.600\ \text{N}}{(0.400\ \text{T})(0.800\ \text{m})}\right]^2(48.0\ \Omega)=\boxed{169\ \text{W}}$

(f) $P=Fv=F\dfrac{FR}{B^2\ell^2}=\dfrac{F^2R}{B^2\ell^2}=\dfrac{(0.600\ \text{N})^2(48.0\ \Omega)}{(0.400\ \text{T})^2(0.800\ \text{m})^2}=\boxed{169\ \text{W}}$

(g) $\boxed{\text{Yes.}}$

(h) $\boxed{\text{Increase}}$ because the speed is proportional to the resistance, as shown in part (c).

(i) $\boxed{\text{Yes.}}$

(j) $\boxed{\text{Larger}}$ because the speed is greater.

P31.59 $$\mathcal{E} = -N\frac{d}{dt}(BA\cos\theta) = -N\left(\pi r^2\right)\cos 0°\left(\frac{dB}{dt}\right)$$

$$\mathcal{E} = -(30.0)\left[\pi\left(2.70\times10^{-3}\right)^2\right](1) \times\frac{d}{dt}\left[50.0\times10^{-3} + \left(3.20\times10^{-3}\right)\sin(1\ 046\pi t)\right]$$

$$\mathcal{E} = -(30.0)\left[\pi\left(2.70\times10^{-3}\right)^2\right]\left[\left(3.20\times10^{-3}\right)(1\ 046\pi)\cos(1\ 046\pi t)\right]$$
$$= -\left(7.22\times10^{-3}\right)\cos(1\ 046\pi t)$$

$$\boxed{\mathcal{E} = -7.22\cos(1\ 046\pi t) \text{ where } \mathcal{E} \text{ is in millivolts and } t \text{ is in seconds.}}$$

P31.60 Model the loop as a particle under a net force. The two forces on the loop are the gravitational force in the downward direction and the magnetic force in the upward direction. The magnetic force arises from the current generated in the loop due to the motion of its lower edge through the magnetic field. As the loop falls, the motional emf $\mathcal{E} = Bwv$ induced in the bottom side of the loop produces a current $I = Bwv/R$ in the loop. From Newton's second law,

$$\sum F_y = ma_y \to F_B - F_g = Ma_y \to IwB - Mg = Ma_y$$
$$\to \left(\frac{Bwv}{R}\right)wB - Mg = Ma_y \to \frac{B^2w^2v}{MR} - g = a_y$$

The largest possible value of v, the terminal speed v_T, will occur when $a_y = 0$. Set $a_y = 0$ and solve for the terminal speed:

$$\frac{B^2w^2v_T}{MR} - g = 0 \quad \to \quad v_T = \frac{MgR}{B^2w^2}$$

Substituting numerical values,

$$v_T = \frac{(0.100\text{ kg})\left(9.80\text{ m/s}^2\right)(1.00\ \Omega)}{(1.00\text{ T})^2(0.500\text{ m})^2} = 3.92\text{ m/s}$$

This is the highest speed the loop can have while the upper edge is above the field, so it cannot possibly be moving at 4.00 m/s.

P31.61 For a counterclockwise trip around the left-hand loop, with $B = At$,

$$\frac{d}{dt}\left[At\left(2a^2\right)\cos 0°\right] - I_1(5R) - I_{PQ}R = 0$$

and for the right-hand loop,

$$\frac{d}{dt}\left[At a^2\right] + I_{PQ}R - I_2(3R) = 0$$

where $I_{PQ} = I_1 - I_2$ is the upward current in *QP*.

Thus, $2Aa^2 - 5R\left(I_{PQ} + I_2\right) - I_{PQ}R = 0$

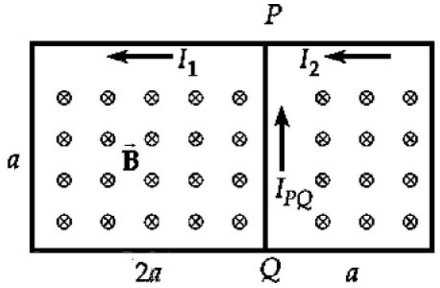
and $Aa^2 + I_{PQ}R = I_2(3R)$

$$2Aa^2 - 6RI_{PQ} - \frac{5}{3}\left(Aa^2 + I_{PQ}R\right) = 0$$

solving, $I_{PQ} = \dfrac{Aa^2}{23R}$ upward

and since $R = (0.100\ \Omega/\text{m})(0.650\ \text{m}) = 0.650\ 0\ \Omega$,

$$I_{PQ} = \frac{\left(1.00\times10^{-3}\ \text{T/s}\right)(0.650\ \text{m})^2}{23(0.065\ 0\ \Omega)} = \boxed{283\ \mu\text{A upward}}$$



ANS. FIG. P31.61

P31.62 (a) $I = \dfrac{dq}{dt} = \dfrac{\mathcal{E}}{R}$ where $\mathcal{E} = -N\dfrac{d\Phi_B}{dt}$ so $\displaystyle\int dq = \frac{N}{R}\int_{\Phi_1}^{\Phi_2} d\Phi_B$

and the charge passing any point in the circuit will be $|Q| = \dfrac{N}{R}\left(\Phi_2 - \Phi_1\right)$.

(b) $Q = \dfrac{N}{R}\left[BA\cos 0 - BA\cos\left(\dfrac{\pi}{2}\right)\right] = \dfrac{BAN}{R}$

so $B = \dfrac{RQ}{NA} = \dfrac{(200\ \Omega)\left(5.00\times10^{-4}\ \text{C}\right)}{(100)\left(40.0\times10^{-4}\ \text{m}^2\right)} = \boxed{0.250\ \text{T}}$

P31.63 The emf induced between the ends of the moving bar is

$$\mathcal{E} = B\ell v = (2.50\ \text{T})(0.350\ \text{m})(8.00\ \text{m/s}) = 7.00\ \text{V}$$

The left-hand loop contains decreasing flux away from you, so the induced current in it will be ☜ clockwise, to produce its own field directed away from you. Let I_1 represent the current flowing upward through the 2.00-Ω resistor. The right-hand loop will carry counterclockwise current. Let I_3 be the upward current in the 5.00-Ω resistor.

(a) Kirchhoff's loop rule then gives:

$$+7.00\text{ V} - I_1(2.00\ \Omega) = 0 \qquad \text{or} \qquad I_1 = \boxed{3.50\text{ A}}$$

$$\text{and}\quad +7.00\text{ V} - I_3(5.00\ \Omega) = 0 \qquad \text{or} \qquad I_3 = \boxed{1.40\text{ A}}$$

(b) The total power converted in the resistors of the circuit is

$$P = \mathcal{E}I_1 + \mathcal{E}I_3 = \mathcal{E}(I_1 + I_3) = (7.00\text{ V})(3.50\text{ A} + 1.40\text{ A})$$
$$= \boxed{34.3\text{ W}}$$

(c) *Method 1*: The current in the sliding conductor is downward with value I_2 = 3.50 A + 1.40 A = 4.90 A. The magnetic field exerts a force of $F_m = I\ell B = (4.90\text{ A})(0.350\text{ m})(2.50\text{ T}) = 4.29\text{ N}$ directed ☞ toward the right on this conductor. An outside agent must then exert a force of $\boxed{4.29\text{ N}}$ to the left to keep the bar moving.

Method 2: The agent moving the bar must supply the power according to $P = \vec{\mathbf{F}}\cdot\vec{\mathbf{v}} = Fv\cos 0°$. The force required is then:

$$F = \frac{P}{v} = \frac{34.3\text{ W}}{8.00\text{ m/s}} = \boxed{4.29\text{ N}}$$

P31.64 The enclosed flux is $\Phi_B = BA = B\pi r^2$. The particle moves according to

$$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}: \quad qvB\sin 90° = \frac{mv^2}{r} \quad \rightarrow \quad r = \frac{mv}{qB}$$

Thus, $$\Phi_B = \frac{B\pi m^2v^2}{q^2B^2}.$$

(a) $$v = \sqrt{\frac{\Phi_B q^2 B}{\pi m^2}} = \sqrt{\frac{(15\times10^{-6}\text{ T}\cdot\text{m}^2)(30\times10^{-9}\text{ C})^2(0.6\text{ T})}{\pi(2\times10^{-16}\text{ kg})^2}}$$
$$= \boxed{2.54\times10^5\text{ m/s}}$$

(b) Energy for the particle-electric field system is conserved in the firing process:

$$U_i = K_f: \quad q\Delta V = \frac{1}{2}mv^2$$

From which we obtain

$$\Delta V = \frac{mv^2}{2q} = \frac{(2\times10^{-16}\ \text{kg})(2.54\times10^5\ \text{m/s})^2}{2(30\times10^{-9}\ \text{C})} = \boxed{215\ \text{V}}$$

P31.65 The normal to the loop is horizontally north, at 35.0° to the magnetic field. We assume that $0.500\ \Omega$ is the total resistance around the circuit, including the ammeter.

$$Q = \int I\,dt = \int \frac{\mathcal{E}\,dt}{R} = \frac{1}{R}\int -\left(\frac{d\Phi_B}{dt}\right)dt = -\frac{1}{R}\int d\Phi_B$$

$$= -\frac{1}{R}\int d(BA\cos\theta) = -\frac{B\cos\theta}{R}\int_{A_1=a^2}^{A_2=0} dA$$

$$Q = -\left[\frac{B\cos\theta}{R}A\right]_{A_1=a^2}^{A_2=0} = \frac{B\cos\theta\, a^2}{R}$$

$$= \frac{(35.0\times10^{-6}\ \text{T})(\cos 35.0°)(0.200\ \text{m})^2}{0.500\ \Omega}$$

$$= \boxed{2.29\times10^{-6}\ \text{C}}$$

P31.66 (a) To find the induced current, we first compute the induced emf,

$$\mathcal{E} = B\ell v = (0.0800\ \text{T})(1.50\ \text{m})(3.00\ \text{m/s}) = 0.360\ \text{V}.$$

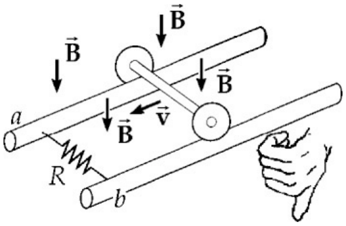
Then,

$$I = \frac{\mathcal{E}}{R} = \frac{0.360\ \text{V}}{0.400\ \Omega} = \boxed{0.900\ \text{A}}$$

(b) The applied force must balance the magnetic force

$$F = F_B = I\ell B$$

$$= (0.900\ \text{A})(1.50\ \text{m})(0.0800\ \text{T}) = \boxed{0.108\ \text{N}}$$



ANS. FIG. P31.66

(c) Since the magnetic flux $\vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$ between the axle and the resistor is in effect decreasing, the induced current is clockwise so that it produces a downward magnetic field to oppose the decrease in flux: thus, current flows through *R* from *b* to *a*. $\boxed{\text{Point b}}$ is at the higher potential.

(d) $\boxed{\text{No}}$. Magnetic flux will increase through a loop between the axle and the resistor to the left of *ab*. Here counterclockwise current will flow to produce an upward magnetic field to oppose the increase in flux. The current in *R* is still from *b* to *a*.

***P31.67** (a) From Equation 31.3, the emf induced in the loop is given by

$$\mathcal{E} = -N\frac{d}{dt}BA\cos\theta = -1\frac{d}{dt}\left(B\frac{\theta a^2}{2}\cos 0°\right)$$

$$= -\frac{Ba^2}{2}\frac{d\theta}{dt} = -\frac{1}{2}Ba^2\omega$$

Substituting numerical values,

$$\mathcal{E} = -\frac{1}{2}(0.500\ \text{T})(0.500\ \text{m})^2(2.00\ \text{rad/s})$$

$$= -0.125\ \text{V} = \boxed{0.125\ \text{V clockwise}}$$

The minus sign indicates that the induced emf produces clockwise current, to make its own magnetic field into the page.

(b) At this instant,

$$\theta = \omega t = (2.00\ \text{rad/s})(0.250\ \text{s}) = 0.500\ \text{rad}$$

The arc *PQ* has length

$$r\theta = (0.500\ \text{rad})(0.500\ \text{m}) = 0.250\ \text{m}$$

The length of the circuit is

$$0.500\ \text{m} + 0.500\ \text{m} + 0.250\ \text{m} = 1.25\ \text{m}$$

Its resistance is

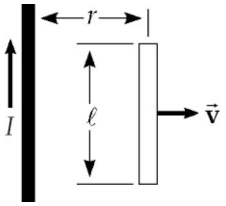
$$(1.25\ \text{m})(5.00\ \Omega/\text{m}) = 6.25\ \Omega$$

The current is then

$$I = \frac{\mathcal{E}}{R} = \frac{0.125\ \text{V}}{6.25\ \Omega} = \boxed{0.020\,0\ \text{A clockwise}}$$

P31.68 At a distance *r* from wire, $B = \frac{\mu_0 I}{2\pi r}$. Using $\mathcal{E} = B\ell v$, we find that

$$|\mathcal{E}| = \frac{\mu_0 v I \ell}{2\pi r}$$



ANS. FIG. P31.68

P31.69 (a) We use $\mathcal{E} = -N\frac{\Delta\Phi_B}{\Delta t}$, with $N = 1$.

Taking $a = 5.00 \times 10^{-3}$ m to be the radius of the washer, and $h =$ 0.500 m, the change in flux through the washer from the time it is released until it hits the tabletop is

$$\Delta\Phi_B = B_f A - B_i A = A(B_f - B_i) = \pi a^2\left(\frac{\mu_0 I}{2\pi(h+a)} - \frac{\mu_0 I}{2\pi a}\right)$$

$$= \frac{a^2\mu_0 I}{2}\left(\frac{1}{h+a} - \frac{1}{a}\right) = \frac{-\mu_0 a h I}{2(h+a)}$$

The time for the washer to drop a distance *h* (from rest) is:

$\Delta t = \sqrt{\frac{2h}{g}}$. Therefore,

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = \frac{\mu_0 a h I}{2(h+a)\Delta t} = \frac{\mu_0 a h I}{2(h+a)}\sqrt{\frac{g}{2h}} = \frac{\mu_0 a I}{2(h+a)}\sqrt{\frac{gh}{2}}$$

Substituting numerical values,

$$\mathcal{E} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(5.00\times10^{-3}\ \mathrm{m})(10.0\ \mathrm{A})}{2(0.500\ \mathrm{m} + 0.005\,00\ \mathrm{m})} \times\sqrt{\frac{(9.80\ \mathrm{m/s^2})(0.500\ \mathrm{m})}{2}}$$

$$= \boxed{97.4\ \mathrm{nV}}$$

(b) Since the magnetic flux going through the washer (into the plane of the page in the figure) is decreasing in time, a current will form in the washer so as to oppose that decrease. To oppose the decrease, the magnetic field from the induced current also must point into the plane of the page. Therefore, the current will flow in a $\boxed{\text{clockwise direction}}$.

P31.70 (a) We would need to know whether the field is increasing or decreasing.

(b) To find the resistance at maximum power, we note that

$$P = \mathcal{E}I = \frac{\mathcal{E}^2}{R} = \frac{\left(N\dfrac{dB}{dt}\pi r^2\cos 0°\right)^2}{R}$$

Solving for the resistance then gives

$$R = \frac{\left(N\dfrac{dB}{dt}\pi r^2\right)^2}{P} = \frac{\left[220(0.020\ \text{T/s})\pi(0.120\ \text{m})^2\right]^2}{160\ \text{W}} = \boxed{248\ \mu\Omega}$$

(c) $\boxed{\text{Higher resistance would reduce the power delivered.}}$

P31.71 Let θ represent the angle between the perpendicular to the coil and the magnetic field. Then $\theta = 0$ at $t = 0$ and $\theta = \omega t$ at all later times.

(a) The emf induced in the coil is given by

$$\mathcal{E} = -N\frac{d}{dt}(BA\cos\theta) = -NBA\frac{d}{dt}(\cos\omega t) = +NBA\,\omega\sin\omega t$$

The maximum value of $\sin\theta$ is 1, so the maximum voltage is

$$\mathcal{E}_{\text{max}} = NBA\omega = (60)(1.00\ \text{T})\left(0.020\ 0\ \text{m}^2\right)(30.0\ \text{rad/s})$$

$$= \boxed{36.0\ \text{V}}$$

(b) The rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(BA\cos\theta) = -BA\omega\sin\omega t$$

The minimum value of $\sin\theta$ is –1, so the maximum of $d\Phi_B/dt$ is

$$\left(\frac{d\Phi_B}{dt}\right)_{\text{max}} = +BA\omega = (1.00\ \text{T})(0.020\ 0\ \text{m}^2)(30.0\ \text{rad/s})$$

$$= \boxed{0.600\ \text{T}\cdot\text{m}^2/\text{s}}$$

(c) At $t = 0.050\ 0$ s,

$$\mathcal{E} = NBA\omega\sin\omega t = (36.0\ \text{V})\sin\left[(30.0\ \text{rad/s})(0.050\ 0\ \text{s})\right]$$
$$= (36.0\ \text{V})\sin(1.50\ \text{rad}) = (36.0\ \text{V})(\sin\ 85.9^\circ) = \boxed{35.9\ \text{V}}$$

(d) The emf is maximum when $\theta = 90°$, and $\vec{\tau} = \vec{\mu}\times\vec{\mathbf{B}}$, so

$$\tau_{\max} = \mu B\sin 90^\circ = NIAB = N\mathcal{E}_{\max}\frac{AB}{R}$$

and $\tau_{\max} = (60)(36.0\ \text{V})\dfrac{(0.020\ 0\ \text{m}^2)(1.00\ \text{T})}{10.0\ \Omega} = \boxed{4.32\ \text{N}\cdot\text{m}}$

P31.72 The emf induced in the loop is

$$\mathcal{E} = -\frac{d}{dt}(NBA) = -1\left(\frac{dB}{dt}\right)\pi a^2 = \pi a^2 K$$

(a) The charge on the fully-charged capacitor is

$$Q = C\mathcal{E} = \boxed{C\pi a^2 K}$$

(b) $\vec{\mathbf{B}}$ into the paper is decreasing; therefore, current will attempt to counteract this by producing a magnetic field into the page to oppose the decrease in flux. To do this, the current must be clockwise, so positive charge will go to the $\boxed{\text{upper plate}}$.

(c) $\boxed{\text{The changing magnetic field through the enclosed area of the loop induces a clockwise electric field within the loop, and this causes electric force to push on charges in the wire.}}$

P31.73 (a) The time interval required for the coil to move distance ℓ and exit the field is $\Delta t = \ell/v$, where v is the constant speed of the coil. Since the speed of the coil is constant, the flux through the area enclosed by the coil decreases at a constant rate. Thus, the instantaneous induced emf is the same as the average emf over the interval Δt, or

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{(0 - BA)}{t - 0} = N\frac{B\ell^2}{t} = \frac{NB\ell^2}{\ell/v} = \boxed{NB\ell v}$$

(b) The current induced in the coil is

$$I = \frac{\mathcal{E}}{R} = \boxed{\frac{NB\ell v}{R}}$$

(c) The power delivered to the coil is given by $P = I^2R$, or

$$P = \left(\frac{N^2B^2\ell^2v^2}{R^2}\right)R = \boxed{\frac{N^2B^2\ell^2v^2}{R}}$$

(d) The rate that the applied force does work must equal the power delivered to the coil, so $F_{\text{app}} \cdot v = P$ or

$$F_{\text{app}} = \frac{P}{v} = \frac{N^2B^2\ell^2v^2/R}{v} = \boxed{\frac{N^2B^2\ell^2v}{R}}$$

(e) As the coil is emerging from the field, the flux through the area it encloses is directed into the page and decreasing in magnitude. Thus, the *change* in the flux through the coil is directed out of the page. The induced current must then flow around the coil in such a direction as to produce flux into the page through the enclosed area, opposing the change that is occurring. This means that the current must flow $\boxed{\text{clockwise}}$ around the coil.

(f) As the coil is emerging from the field, the left side of the coil is carrying an induced current directed toward the top of the page through a magnetic field that is directed into the page. By the right-hand rule, this side of the coil will experience a magnetic force $\boxed{\text{directed to the left}}$, opposing the motion of the coil.

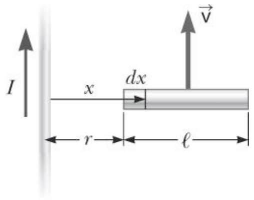
P31.74 The magnetic field at a distance x from wire is

$$B = \frac{\mu_0 I}{2\pi x}$$

The emf induced in an element in the bar of length dx is $|d\mathcal{E}| = Bv\,dx$. The total emf induced along the entire length of the bar is then

$$|\mathcal{E}| = \int_r^{r+\ell} Bv\,dx = \int_r^{r+\ell} \frac{\mu_0 I}{2\pi x} v\,dx = \frac{\mu_0 Iv}{2\pi}\int_r^{r+\ell}\frac{dx}{x} = \frac{\mu_0 Iv}{2\pi}\ln x\Big|_r^{r+\ell}$$

$$|\mathcal{E}| = \frac{\mu_0 Iv}{2\pi}\ln\left(\frac{r+\ell}{r}\right)$$



ANS. FIG. P31.74

P31.75 We are given

$$\Phi_B = \left(6.00t^3 - 18.0t^2\right)$$

Thus,

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -18.0t^2 + 36.0t$$

Maximum $\mathcal{E}$ occurs when $\frac{d\mathcal{E}}{dt} = -36.0t + 36.0 = 0$, which gives $t = 1.00$ s.

Therefore, the maximum current (at $t = 1.00$ s) is

$$I = \frac{\mathcal{E}}{R} = \frac{(-18.0 + 36.0)\ \text{V}}{3.00\ \Omega} = \boxed{6.00\ \text{A}}$$

P31.76 The magnetic field at a distance x from a long wire is $B = \frac{\mu_0 I}{2\pi x}$. We find an expression for the flux through the loop.

$$d\Phi_B = \frac{\mu_0 I}{2\pi x}(\ell dx)$$

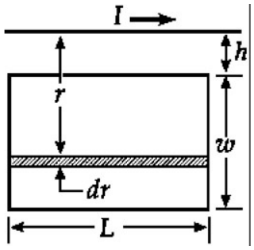
so $$\Phi_B = \frac{\mu_0 I \ell}{2\pi}\int_r^{r+w}\frac{dx}{x} = \frac{\mu_0 I \ell}{2\pi}\ln\left(1 + \frac{w}{r}\right)$$

Therefore,

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = \frac{\mu_0 I \ell v}{2\pi r}\frac{w}{(r+w)}$$

and $$I = \frac{\mathcal{E}}{R} = \boxed{\frac{\mu_0 I \ell v}{2\pi R r}\frac{w}{(r+w)}}$$

P31.77 The magnetic field produced by the current in the straight wire is perpendicular to the plane of the coil at all points within the coil. At a distance r from the wire, the magnitude of the field is $B = \frac{\mu_0 I}{2\pi r}$. Thus, the flux through an element of length L and width dr is



ANS. FIG. P31.77

$$d\Phi_B = BLdr = \frac{\mu_0 I L}{2\pi}\frac{dr}{r}$$

The total flux through the coil is

$$\Phi_B = \frac{\mu_0 I L}{2\pi}\int_h^{h+w}\frac{dr}{r} = \frac{\mu_0 I_{\max} L}{2\pi}\ln\left(\frac{h+w}{h}\right)\sin(\omega t + \phi)$$

Finally, the induced emf is

$$\begin{aligned}\mathcal{E} &= -N\frac{d\Phi_B}{dt}\\ &= -\frac{\mu_0 N I_{\max} L\omega}{2\pi}\ln\left(1+\frac{w}{h}\right)\cos(\omega t+\phi)\end{aligned}$$

$$\mathcal{E} = -\frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})(100)(50.0\ \mathrm{A})(0.200\ \mathrm{m})(200\pi\ \mathrm{rad/s})}{2\pi}\times\ln\left(1+\frac{0.050\ 0\ \mathrm{m}}{0.050\ 0\ \mathrm{m}}\right)\cos(\omega t+\phi)$$

$\mathcal{E} = \boxed{-87.1\cos(200\pi t+\phi)}$, where $\mathcal{E}$ is in millivolts and t is in seconds

The term $\sin(\omega t+\phi)$ in the expression for the current in the straight wire does not change appreciably when ωt changes by 0.10 rad or less. Thus, the current does not change appreciably during a time interval

$$\Delta t < \frac{0.10}{(200\pi\ \mathrm{s}^{-1})} = 1.6\times10^{-4}\ \mathrm{s}$$

We define a critical length,

$$c\Delta t = (3.00\times10^{8}\ \mathrm{m/s})(1.6\times10^{-4}\ \mathrm{s}) = 4.8\times10^{4}\ \mathrm{m}$$

equal to the distance to which field changes could be propagated during an interval of 1.6×10^{-4} s. This length is so much larger than any dimension of the coil or its distance from the wire that, although we consider the straight wire to be infinitely long, we can also safely ignore the field propagation effects in the vicinity of the coil. Moreover, the phase angle can be considered to be constant along the wire in the vicinity of the coil.

If the angular frequency ω were much larger, say, $200\pi\times10^{5}\ \mathrm{s}^{-1}$, the corresponding critical length would be only 48 cm. In this situation propagation effects would be important and the above expression for $\mathcal{E}$ would require modification. As a general rule we can consider field propagation effects for circuits of laboratory size to be negligible for frequencies, $f=\frac{\omega}{2\pi}$, that are less than about 10^6 Hz.

P31.78 (a) The induced emf is $\mathcal{E} = B\ell v$, where $B = \dfrac{\mu_0 I}{2\pi y}$, $\ell = 0.800$,

$v_f = v_i + gt = 9.80t$, and $y = y_f = y_i - \dfrac{1}{2}gt^2 = 0.800 - (4.90)t^2$ where I is in amperes, ℓ and y are in meters, v is in meters per second, and t in seconds.

Thus,

$$\mathcal{E} = \frac{(4\pi \times 10^{-7})(200)}{2\pi(0.800 - 4.90t^2)}(0.300)(9.80)t = \boxed{\frac{(1.18 \times 10^{-4})t}{0.800 - 4.90t^2}}$$

where $\mathcal{E}$ is in volts and t in seconds.

(b) The emf is $\boxed{\text{zero}}$ when $t = 0$.

(c) As $0.800 - 4.90t^2 \to 0$, $t \to 0.404$ s and the emf diverges to $\boxed{\text{infinity}}$.

(d) At $t = 0.300$ s,

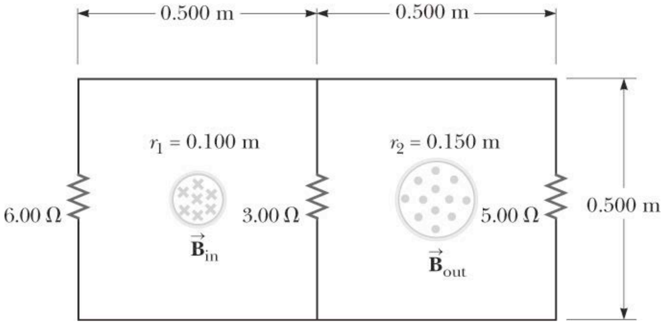
$$\mathcal{E} = \frac{(1.18 \times 10^{-4})(0.300)}{\left[0.800 - 4.90(0.300)^2\right]}\ \text{V} = \boxed{98.3\ \mu\text{V}}$$

Challenge Problems

P31.79 In the loop on the left, the induced emf is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi(0.100\ \text{m})^2(100\ \text{T/s}) = \pi\ \text{V}$$

and it attempts to produce a counterclockwise current in this loop.



ANS. FIG. P31.79

In the loop on the right, the induced emf is

$$|\mathcal{E}| = \frac{d\Phi_B}{dt} = \pi(0.150\text{ m})^2(100\text{ T/s}) = 2.25\pi\text{ V}$$

and it attempts to produce a clockwise current. Assume that I_1 flows down through the 6.00-Ω resistor, I_2 flows down through the 5.00-Ω resistor, and that I_3 flows up through the 3.00-Ω resistor.

From Kirchhoff's junction rule: $I_3 = I_1 + I_2$ **[1]**

Using the loop rule on the left loop: $6.00I_1 + 3.00I_3 = \pi$ **[2]**

Using the loop rule on the right loop: $5.00I_2 + 3.00I_3 = 2.25\pi$ **[3]**

Solving these three equations simultaneously,

$$I_3 = \frac{(\pi - 3I_3)}{6} + \frac{(2.25\pi - 3I_3)}{5}$$

which then gives

$$I_1 = \boxed{0.062\,3\text{ A}},\ I_2 = \boxed{0.860\text{ A}},\text{ and } I_3 = \boxed{0.923\text{ A}}$$

P31.80 (a) Consider an annulus of radius *r*, width *dr*, thickness *b*, and resistivity *ρ*. Around its circumference, a voltage is induced according to

$$\mathcal{E} = -N\frac{d}{dt}\vec{\mathbf{B}}\cdot\vec{\mathbf{A}} = -(1)\left[\frac{d}{dt}B_{\max}(\cos\omega t)\right]\pi r^2 = +B_{\max}\pi r^2\omega\sin\omega t$$

The resistance around the loop is $\frac{\rho\ell}{dA} = \frac{\rho(2\pi r)}{bdr}$. The eddy current in the ring is

$$dI = \frac{\mathcal{E}}{\text{resistance}} = \frac{B_{\max}\pi r^2\omega(\sin\omega t)}{\rho(2\pi r)/bdr} = \frac{B_{\max}rb\omega\sin\omega t}{2\rho}dr$$

The instantaneous power is

$$dP = \mathcal{E}\,dI = \frac{B_{\max}^2\pi r^3 b\omega^2\sin^2\omega t}{2\rho}dr$$

The time average of the function $\sin^2\omega t = \frac{1}{2} - \frac{1}{2}\cos 2\omega t$ is $\frac{1}{2} - 0 = \frac{1}{2}$, so the time-averaged power delivered to the annulus is

$$d\bar{P} = \frac{B_{\max}^2\pi r^3 b\omega^2}{4\rho}dr$$

The average power delivered to the disk is

$$P = \int dP = \int_0^R \frac{B_{\max}^2 \pi b\omega^2}{4\rho} r^3 dr$$

$$P = \frac{B_{\max}^2 \pi b\omega^2}{4\rho}\left(\frac{R^4}{4} - 0\right) = \boxed{\frac{\pi B_{\max}^2 R^4 b\omega^2}{16\rho}}$$

(b) When $B_{\max}$ doubles, $B_{\max}^2$ and P become $\boxed{4}$ times larger.

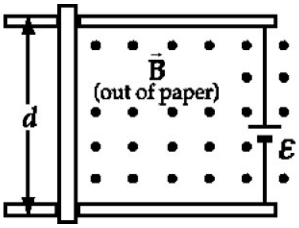
(c) When f doubles, $\omega = 2\pi f$ doubles, and ω^2 and P become $\boxed{4}$ times larger.

(d) When R doubles, R^4 and P become $2^4 = \boxed{16}$ times larger.

P31.81 The current in the rod is

$$I = \frac{\mathcal{E} + \mathcal{E}_{\text{induced}}}{R}$$

where $\mathcal{E}_{\text{induced}} = -Bdv$, because the induced emf opposes the emf of the battery. The force on the rod is related to the current and the velocity:



ANS. FIG. P31.81

$$F = m\frac{dv}{dt} = IBd$$

$$\frac{dv}{dt} = \frac{IBd}{m} = \frac{Bd}{mR}\left(\mathcal{E} + \mathcal{E}_{\text{induced}}\right) = \frac{Bd}{mR}\left(\mathcal{E} - Bvd\right)$$

To solve the differential equation, let $u = \mathcal{E} - Bvd \rightarrow \frac{du}{dt} = -Bd\frac{dv}{dt}$:

$$\frac{dv}{dt} = \frac{Bd}{mR}\left(\mathcal{E} - Bvd\right)$$

$$-\frac{1}{Bd}\frac{du}{dt} = \frac{Bd}{mR}u \quad \rightarrow \quad \int_{u_0}^{u}\frac{du}{u} = -\int_0^t \frac{(Bd)^2}{mR}dt$$

Integrating from $t = 0$ to $t = t$ gives $\ln\frac{u}{u_0} = -\frac{(Bd)^2}{mR}t$ or $\frac{u}{u_0} = e^{-B^2d^2t/mR}$.

Since $v = 0$ when $t = 0$, $u_0 = \mathcal{E}$; substituting $u = \mathcal{E} - Bvd$ gives

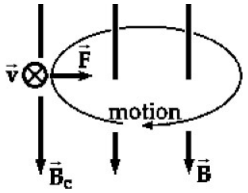
$$\mathcal{E} - Bvd = \mathcal{E}e^{-B^2d^2t/mR}$$

Therefore, $\boxed{v = \frac{\mathcal{E}}{Bd}\left(1 - e^{-B^2d^2t/mR}\right)}$.

P31.82 Suppose the magnetic field is vertically down. When an electron is moving away from you the force on it is in the direction given by

$q\vec{v}\times\vec{B}_c$ as – (away) × (down) = –  = – (left) = (right)

Therefore, the electrons circulate clockwise.



ANS. FIG. P31.82

(a) As the downward field increases, an emf is induced to produce some current that in turn produces an upward field to oppose the increasing downward field. This current is directed  counterclockwise, carried by negative electrons moving clockwise. Therefore the electric force on the electrons is clockwise and the original electron motion speeds up.

(b) At the circumference, we have

$$\sum F_c = ma_c \quad\rightarrow\quad |q|vB_c\sin 90° = \frac{mv^2}{r} \quad\rightarrow\quad mv = |q|rB_c$$

where B_c is the magnetic field at the circle's circumference.

The increasing magnetic field $\vec{B}_{av}$ in the area enclosed by the orbit produces a tangential electric field according to

$$\left|\oint\vec{E}\cdot d\vec{s}\right| = \left|-\frac{d}{dt}\vec{B}_{av}\cdot\vec{A}\right|$$

or

$$E(2\pi r) = \pi r^2\frac{dB_{av}}{dt} \quad\rightarrow\quad E = \frac{r}{2}\frac{dB_{av}}{dt}$$

Using this expression for E, we find the tangential force on the electron:

$$\sum F_t = ma_t \quad\rightarrow\quad |q|E = m\frac{dv}{dt}$$

$$|q|\frac{r}{2}\frac{dB_{av}}{dt} = m\frac{dv}{dt}$$

If the electron starts at rest and increases to final speed *v* as the magnetic field builds from zero to final value B_{av}, then integration of the last equation gives

$$|q|\frac{r}{2}\int_0^{B_{av}}\frac{dB_{av}}{dt}dt = m\int_0^{v}\frac{dv}{dt}dt \quad \rightarrow \quad |q|\frac{r}{2}B_{av} = mv$$

Thus, from the two expressions for *mv*, we have

$$|q|\frac{r}{2}B_{av} = mv = |q|rB_c \rightarrow B_{av} = 2B_c$$

P31.83 For the suspended mass, *M*:

$$\sum F = Mg - T = Ma$$

For the sliding bar, *m*:

$$\sum F = T - I\ell B = ma, \quad \text{where} \quad I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$$

Substituting the expression for current *I*, the first equation gives us

$$Mg - \frac{B^2\ell^2 v}{R} = (m + M)a \quad \rightarrow \quad a = \frac{dv}{dt} = \frac{Mg}{m + M} - \frac{B^2\ell^2 v}{R(M + m)}$$

The above equation can be written as

$$\int_0^v \frac{dv}{(\alpha - \beta v)} = \int_0^t dt \quad \text{where} \quad \alpha = \frac{Mg}{M + m} \quad \text{and} \quad \beta = \frac{B^2\ell^2}{R(M + m)}$$

Integrating,

$$\int_0^v \frac{dv}{(\alpha - \beta v)} = \int_0^t dt \quad \rightarrow \quad \frac{-1}{\beta}\ln(\alpha - \beta v)\Big|_0^v = t$$

Then,

$$\left[\ln(\alpha - \beta v) - \ln(\alpha)\right] = -\beta t$$

Solving for *v* gives

$$\ln\frac{(\alpha - \beta v)}{\alpha} = -\beta t \quad \rightarrow \quad 1 - \frac{\beta}{\alpha}v = e^{-\beta t}$$

$$v = \frac{\alpha}{\beta}\left(1 - e^{-\beta t}\right) = \boxed{\frac{MgR}{B^2\ell^2}\left[1 - e^{-B^2\ell^2 t/R(M+m)}\right]}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P31.2 (a) Each coil has a pulse of voltage tending to produce counterclockwise current as the projectile approaches, and then a pulse of clockwise voltage as the projectile recedes; (b) 625 m/s

P31.4 +9.82 mV

P31.6 2.26 mV

P31.8 160 A

P31.10 1.89×10^{-11} V

P31.12 (a) $\frac{\mu_0 n\pi r_2^2}{2R}\frac{\Delta I}{\Delta t}$; (b) $\frac{\mu_0^2 n\pi r_2^2}{4r_1 R}\frac{\Delta I}{\Delta t}$; (c) left

P31.14 $\mathcal{E} = -\left(1.42 \times 10^{-2}\right)\cos\left(120t\right)$, where *t* is in seconds and $\mathcal{E}$ is in V

P31.16 $\mathcal{E} = 68.2e^{-1.60t}$, where *t* is in seconds and $\mathcal{E}$ is in mV

P31.18 (a) See P31.18(a) for full explanation; (b) The emf induced in the coil is proportional to the line integral of the magnetic field around the circular axis of the toroid. Ampère's law says that this line integral depends only on the amount of current the coil encloses.

P31.20 (a) 0.013 3 A; (b) The current is counterclockwise in the lower loop and clockwise in the upper loop.

P31.22 (a) to the right; (b) out of the page; (c) to the right

P31.24 (a) 11.8 mV; (b) The wingtip on the pilot's left is positive; (c) no change; (d) No. If you try to connect the wings to a circuit containing the light bulb, you must run an extra insulated wire along the wing. In a uniform field the total emf generated in the one-turn coil is zero.

P31.26 1.00 m/s

P31.28 $\frac{Rmv}{B^2\ell^2}$

P31.30 The speed of the car is equivalent to about 640 km/h or 400 mi/h, much faster than the car could drive on the curvy road and much faster than any standard automobile could drive in general.

P31.32 $\frac{mgR\sin\theta}{B^2\ell^2\cos^2\theta}$

P31.34 (a) 0.729 m/s; (b) counterclockwise; (c) 0.650 mW; (d) Work is being done by the external force, which is transformed into internal energy in the resistor.

P31.36 (a) $\frac{N^2B^2w^2v}{R}$ to the left; (b) 0; (c) $\frac{N^2B^2w^2v}{R}$ to the left again

P31.38 (a) 6.00 μT; (b) Yes. The magnitude and direction of the Earth's field varies from one location to the other, so the induced voltage in the wire changes. Furthermore, the voltage will change if the tether cord or its velocity changes their orientation relative to the Earth's field; (c) Either the long dimension of the tether or the velocity vector could be parallel to the magnetic field at some instant.

P31.40 (a) 2.81×10^{-3} N/C; (b) tangent to the electric field line passing through at point P_2 and counterclockwise

P31.42 (a) 12.6 mV; (b) when the plane of the coil is parallel to the magnetic field

P31.44 (a) amplitude doubles and period is unchanged; (b) doubles the amplitude and cuts the period in half; (c) amplitude unchanged and period is cut in half

P31.46 (a) 1.60 V; (b) zero; (c) no change in either answer; (d) See ANS. FIG. P31.46(d); (e) See ANS. FIG. P31.46(e).

P31.48 (a) 110 V; (b) 8.53 W; (c) 1.22 kW

P31.50 See P31.50 for full explanation.

P31.52 $\sim 10^{-4}$ V

P31.54 (a) increasing; (b) 62.6 mT/s

P31.56 $\frac{AB_{\max}}{\tau}e^{-t/\tau}$

P31.58 (a) $\frac{B\ell v}{R}$; (b) particle in equilibrium; (c) 281 m/s; (d) 1.88 A; (e) 169 W; (f) 169 W; (g) yes; (h) increase; (i) yes; (j) larger

P31.60 3.92 m/s is the highest speed the loop can have while the upper edge is above the field, so it cannot possibly be moving at 4.00 m/s.

P31.62 (a) See P31.62(a) for full explanation; (b) 0.250 T

P31.64 (a) 2.54×10^{5} m/s; (b) 215 V

P31.66 (a) 0.900 A; (b) 0.108 N; (c) Point b; (d) no

P31.68 See P31.68 for full explanation.

P31.70 (a) We would need to know if the field is increasing or decreasing; (b) 248 $\mu\Omega$; (c) Higher resistance would reduce the power delivered.

P31.72	(a) $C\pi a^2K$; (b) upper plate; (c) The changing magnetic field through the enclosed area of the loop induces a clockwise electric field within the loop, and this causes electric force to push on charges in the wire
P31.74	See P31.74 for full explanation.
P31.76	$\frac{\mu_0 I\ell v}{2\pi Rr}\frac{w}{(r+w)}$
P31.78	(a) $\frac{(1.18\times10^{-4})t}{0.800-4.90t^2}$; (b) zero; (c) infinity; (d) 98.3 μV
P31.80	(a) $\frac{\pi B_{\max}^2R^4b\omega^2}{16\rho}$; (b) 4; (c) 4; (d) 16
P31.82	(a) See P31.82(a) for full description; (b) See P31.82(b) for full description.

32

Inductance

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ32.1 **(i)** Answer (a). The mutual inductance of two loops in free space—that is, ignoring the use of cores—is a maximum if the loops are coaxial. In this way, the maximum flux of the primary loop will pass through the secondary loop, generating the largest possible emf given the changing magnetic field due to the first.

(ii) Answer (c). The mutual inductance is a minimum if the magnetic field of the first coil lies in the plane of the second coil, producing no flux through the area the second coil encloses.

OQ32.2 Answer (c). The fine wire has considerable resistance, so a few seconds is many time constants. The final current depends on the resistance of the wire, which has not changed; the current is not affected by the inductance of the coil because the current is not changing.

OQ32.3 Answer (b). The inductance of a solenoid is proportional to the number of turns squared, so cutting the number of turns in half makes the inductance four times smaller. Doubling the current would by itself make the stored energy $(\frac{1}{2}\mathrm{Li}^2)$ four times larger, to just compensate.

OQ32.4 The ranking is $\Delta V_L > \Delta V_{1\,200\ \Omega} > 12.0\text{ V} > \Delta V_{12\ \Omega}$. Just before the switch is thrown, the voltage across the 12-Ω resistor is very nearly 12 V (we assume the resistance of the inductor is small). Just after the switch is thrown, the current is nearly the same, maintained by the inductor, but this current is diverted through the 1 200-Ω resistor; thus, the voltage across the 1 200-Ω resistor is much more than 12 V, about 1 200 V, because the same current in the 12-Ω resistor now passes through a resistor 100 times as large. By Kirchhoff's loop rule, the voltage across the coil is larger still.

OQ32.5 Answer (d). The inductance of a solenoid is proportional to the number of turns squared (N^2), to the cross-sectional area (A), and to the reciprocal of the length of its axis (L). Coil A has twice as many turns with the same length of wire, so its circumference must be half as large as that of coil B: therefore, its radius is half as large and its area one quarter as large. For coil A the inductance will be different by the factor $N^2A/L \sim [2^2(1/4)]/2 = 1/2$.

OQ32.6 Answer (a). The energy stored in the magnetic field of an inductor is proportional to the square of the current. Doubling I makes $U_B = \frac{1}{2}Li^2$ get four times larger.

OQ32.7 Answer (d). The emf across an inductor is zero whenever the current is constant (unchanging), large or small.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ32.1 (a) We can think of Henry's discovery of self-inductance as fundamentally new. Before a certain school vacation at the Albany Academy about 1830, one could visualize the universe as consisting of only one thing, matter. All the forms of energy then known (kinetic, gravitational, elastic, internal, electrical) belonged to chunks of matter. But the energy that temporarily maintains a current in a coil after the battery is removed is not energy that belongs to any bit of matter. This energy is vastly larger than the kinetic energy of the drifting electrons in the wires. This energy belongs to the magnetic field around the coil. Beginning in 1830, Nature has forced us to admit that the universe consists of matter and also of fields, massless and invisible, known only by their effects.

The idea of a field was not due to Henry, but rather to Faraday, to whom Henry personally demonstrated self-induction. Still the thesis stated in the question has an important germ of truth.

Henry precipitated a basic change if he did not cause it.

(b) A list today of what makes up the Universe might include quarks, electrons, muons, tauons, and neutrinos of matter; photons of electric and magnetic fields; W and Z particles; gluons; energy; charge; baryon number; three different lepton numbers; upness; downness; strangeness; charm; topness; and bottomness. Alternatively, the relativistic interconvertibility of mass and energy, and of electric and magnetic fields, can be used to make the list look shorter. Some might think of the conserved quantities energy, charge, . . . bottomness as properties of matter, rather than as things with their own existence.

CQ32.2 (a) The inductance of a coil is determined by (a) the geometry of the coil and (b) the "contents" of the coil. This is similar to the parameters that determine the capacitance of a capacitor and the resistance of a resistor. With an inductor, the most important factor in the geometry is the number of turns of wire, or turns per unit length. By the "contents" we refer to the material in which the inductor establishes a magnetic field, notably the magnetic properties of the core around which the wire is wrapped.

(b) No. The inductance of a coil is proportional to the flux through the coil per unit current, Φ/I, and the flux is proportional to the current I, so the inductance is independent of the current.

CQ32.3 When it is being opened. When the switch is initially standing open, there is no current in the circuit. Just after the switch is then closed, the inductor tends to maintain the zero-current condition, and there is very little chance of sparking. When the switch is standing closed, there is current in the circuit. When the switch is then opened, the current rapidly decreases. The induced emf is created in the inductor, and this emf tends to maintain the original current. Sparking occurs as the current bridges the air gap between the contacts of the switch.

CQ32.4 **(i)** (a) The bulb glows brightly right away, and then more and more faintly as the capacitor charges up. (b) The bulb gradually gets brighter and brighter, changing rapidly at first and then more and more slowly. (c) The bulb immediately becomes bright. (d) The bulb glows brightly right away, and then more and more faintly as the inductor starts carrying more and more current (the inductor eventually acts as a short).

(ii) (a) The bulb goes out immediately because current stops immediately (charge ceases to flow). (b) The bulb glows for a moment as a spark jumps across the switch. (c) The bulb stays lit for a while, gradually getting fainter and fainter as the capacitor discharges through the bulb. (d) The bulb suddenly glows brightly. Then its brightness decreases to zero, changing rapidly at first and then more and more slowly.

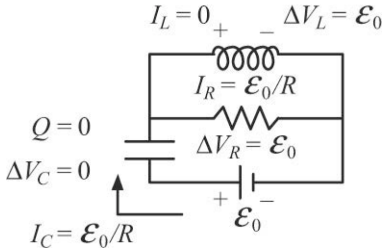
CQ32.5 (a) The coil has an inductance regardless of the nature of the current in the circuit. Inductance depends only on the coil geometry and its construction.

(b) Since the current is constant, the self-induced emf in the coil is zero, and the coil does not affect the steady-state current. (We assume the resistance of the coil is negligible.)

CQ32.6 (a) An object cannot exert a net force on itself. An object cannot create momentum out of nothing.

(b) A coil can induce an emf in itself. When it does so, the actual forces acting on charges in different parts of the loop add as vectors to zero. The term electromotive force does not refer to a force, but to a voltage.

CQ32.7 (a) The instant after the switch is closed, the capacitor acts as a closed switch, and the inductor acts to maintain zero current in itself. The situation is as shown in the circuit diagram of ANS. FIG. CQ32.7(a). The requested quantities are:

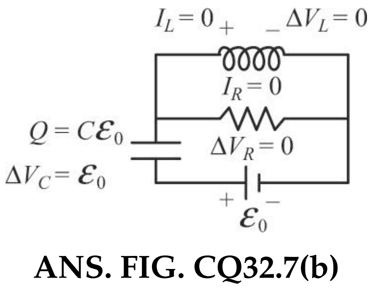


ANS. FIG. CQ32.7(a)

$$\boxed{I_L = 0,\ I_C = \frac{\mathcal{E}_0}{R},\ I_R = \frac{\mathcal{E}_0}{R}}$$

$$\boxed{\Delta V_L = \mathcal{E}_0,\ \Delta V_C = 0,\ \Delta V_R = \mathcal{E}_0}$$

(b) After the switch has been closed a long time, the capacitor acts as an open switch. The steady-state conditions shown in ANS. FIG. CQ32.7 (b) will exist. The currents and voltages are:



ANS. FIG. CQ32.7(b)

$$\boxed{I_L = 0,\ I_C = 0,\ I_R = 0}$$

$$\boxed{\Delta V_L = 0,\ \Delta V_C = \mathcal{E}_0,\ \Delta V_R = 0}$$

CQ32.8 When the capacitor is fully discharged, the current in the circuit is a maximum. The inductance of the coil is making the current continue to flow. At this time the magnetic field of the coil contains all the energy that was originally stored in the charged capacitor. The current has just finished discharging the capacitor and is proceeding to charge it up again with the opposite polarity.

CQ32.9 According to Equations 32.31 and 32.32, the oscillator is overdamped if $R > R_C = \sqrt{\frac{4L}{C}}$: it will not oscillate. If $R < R_C$, then the oscillator is underdamped and can go through several cycles of oscillation before the current falls below background noise.

CQ32.10 The energy stored in a capacitor is proportional to the square of the electric field, and the energy stored in an induction coil is proportional to the square of the magnetic field. The capacitor's energy is proportional to its capacitance, which depends on its geometry and the dielectric material inside. The coil's energy is proportional to its inductance, which depends on its geometry and the core material. The capacitor's energy is proportional to the charge it stores, the coil's energy is proportional to the current it holds.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 32.1 Self-Induction and Induction

***P32.1** The magnitude of the average induced emf for this coil is

$$|\bar{\mathcal{E}}| = L\frac{\Delta i}{\Delta t} = \left(3.00\times10^{-3}\text{ H}\right)\left(\frac{1.50\text{ A}-0.200\text{ A}}{0.200\text{ s}}\right) = 1.95\times10^{-2}\text{ V}$$

$$= \boxed{19.5\text{ mV}}$$

***P32.2** Treating the telephone cord as a solenoid, we have:

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(70.0)^2\pi\left(6.50\times10^{-3}\text{ m}\right)^2}{0.600\text{ m}}$$

$$= \boxed{1.36\ \mu\text{H}}$$

P32.3 The self-induced emf at any instant is

$$\mathcal{E}_L = -L\frac{di}{dt}$$

Its average value is

$$\mathcal{E}_{L,\text{ave}} = -L\left(\frac{I_f - I_i}{t}\right) = (-2.00\text{ H})\left(\frac{0 - 0.500\text{ A}}{1.00\times10^{-2}\text{ s}}\right)\left(\frac{\text{V}\cdot\text{s/A}}{1\text{ H}}\right)$$

$$= \boxed{+100\text{ V}}$$

P32.4 (a) The inductance of the solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(400)^2\left[\pi\left(2.50\times 10^{-2}\ \text{m}\right)^2\right]}{0.200\ \text{m}}$$

$$= 1.97\times 10^{-3}\ \text{H} = \boxed{1.97\ \text{mH}}$$

(b) From $|\mathcal{E}| = L(\Delta i/\Delta t)$,

$$\frac{\Delta I}{\Delta t} = \frac{|\mathcal{E}|}{L} = \frac{75.0\times 10^{-6}\ \text{V}}{1.97\times 10^{-3}\ \text{H}} = 38.0\times 10^{-3}\ \text{A/s} = \boxed{38.0\ \text{mA/s}}$$

P32.5 From $\left|\mathcal{E}\right| = L\left(\frac{\Delta i}{\Delta t}\right)$, we have

$$L = \frac{\mathcal{E}}{(\Delta i/\Delta t)} = \frac{24.0\times 10^{-3}\ \text{V}}{10.0\ \text{A/s}} = 2.40\times 10^{-3}\ \text{H}$$

From $L = \frac{N\Phi_B}{i}$, we have

$$\Phi_B = \frac{Li}{N} = \frac{\left(2.40\times 10^{-3}\ \text{H}\right)(4.00\ \text{A})}{500}$$

$$= \boxed{19.2\ \mu\text{T}\cdot\text{m}^2}$$

P32.6 (a) $B = \mu_0 ni = \left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{450}{0.120\ \text{m}}\right)(0.040\,0\ \text{A}) = \boxed{188\ \mu\text{T}}$

(b) $\Phi_B = BA = B\pi\left(\frac{15.0\times 10^{-3}\text{m}}{2}\right)^2 = \boxed{3.33\times 10^{-8}\ \text{T}\cdot\text{m}^2}$

(c) $L = \frac{N\Phi_B}{i} = \frac{450\Phi_B}{0.040\,0\ \text{A}} = \boxed{0.375\ \text{mH}}$

(d) $\boxed{\text{B and } \Phi_B \text{ are proportional to current; L is independent of current.}}$

P32.7 From $\left|\mathcal{E}\right| = L\left(\frac{\Delta i}{\Delta t}\right)$, we have

$$L = \frac{|\mathcal{E}|}{|\Delta i/\Delta t|} = \frac{|\mathcal{E}|(\Delta t)}{|\Delta i|} = \frac{\left(12.0\times 10^{-3}\ \text{V}\right)(0.500\ \text{s})}{|2.00\ \text{A} - 3.50\ \text{A}|}$$

$$= 4.00\times 10^{-3}\ \text{H} = \boxed{4.00\ \text{mH}}$$

P32.8 (a) In terms of its cross-sectional area (A), length (ℓ), and number of turns (N), the self inductance of a solenoid is given as $L = \mu_0 N^2 A/\ell$. Thus, for the given solenoid,

$$L = \frac{\mu_0 N^2 \left(\pi d^2/4\right)}{\ell}$$

$$= \frac{\left(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}\right)(580)^2\left[\pi\left(8.00\times 10^{-2}\ \mathrm{m}\right)^2/4\right]}{(0.36\ \mathrm{m})}$$

$$= 5.90\times 10^{-3}\ \mathrm{H} = \boxed{5.90\ \mathrm{mH}}$$

(b) $$|\mathcal{E}| = \left|-L\left(\frac{\Delta i}{\Delta t}\right)\right| = \left(5.90\times 10^{-3}\ \mathrm{H}\right)\left(+4.00\ \mathrm{A/s}\right)$$

$$= 23.6\times 10^{-3}\ \mathrm{V} = \boxed{23.6\ \mathrm{mV}}$$

P32.9 $|\mathcal{E}| = L\dfrac{di}{dt} = \left|\left(90.0\times 10^{-3}\right)\dfrac{d}{dt}\left(1.00t^2 - 6.00t\right)\right| = (90.0)\left|(2.00t - 6.00)\right|$, where $\mathcal{E}$ is in millivolts (mV) and t in seconds.

(a) At $t = 1.00$ s, $\mathcal{E} = \boxed{360\ \mathrm{mV}}$

(b) At $t = 4.00$ s, $\mathcal{E} = \boxed{180\ \mathrm{mV}}$

(c) $\mathcal{E} = (90.0)(2t - 6) = 0$ when $\boxed{t = 3.00\ \mathrm{s}}$

P32.10 The inductance is $L = \dfrac{\mu_0 N^2 A}{\ell}$ with $A = \pi r^2$. The induced emf as a function of time is $\mathcal{E}_L = -L\dfrac{di}{dt}$. By substitution we have

$$\mathcal{E}_L = -L\frac{di}{dt} = -\frac{\mu_0 N^2 \pi r^2}{\ell}\frac{di}{dt} \quad \text{and} \quad r = \left(\frac{-\mathcal{E}_L \ell}{\mu_0 N^2 \pi\, di/dt}\right)^{1/2}$$

Then $$r = \left(\frac{-(175\times 10^{-6}\ \mathrm{V})(0.160\ \mathrm{m})}{(4\pi\times 10^{-7}\ \mathrm{N/A^2})(420)^2\pi(-0.421\ \mathrm{A/s})}\right)^{1/2} = \boxed{9.77\ \mathrm{mm}}$$

P32.11 The emf is given by

$$\mathcal{E} = \mathcal{E}_0 e^{-kt} = -L\frac{di}{dt}$$

from which we obtain

$$di = -\frac{\mathcal{E}_0}{L}e^{-kt}dt$$

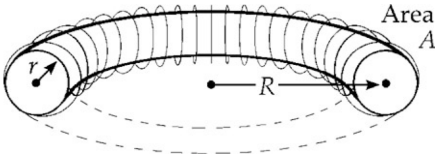
If we require $i \to 0$ as $t \to \infty$, the solution is $i = \dfrac{\mathcal{E}_0}{Lk}e^{-kt} = \dfrac{dq}{dt}$, so

$$Q = \int i\,dt = \int_0^\infty \frac{\mathcal{E}_0}{Lk}e^{-kt}\,dt = -\frac{\mathcal{E}_0}{Lk^2} \qquad \to \qquad \boxed{|Q| = \frac{\mathcal{E}_0}{Lk^2}}$$

P32.12 The inductance of a solenoid is $L = \dfrac{\mu_0 N^2 A}{\ell}$.

The long solenoid is bent into a circle of radius *R*, so its length $\ell \approx 2\pi R$; therefore, the inductance of the toroid is

$$L = \frac{\mu_0 N^2 A}{\ell} \approx \frac{\mu_0 N^2 (\pi r^2)}{2\pi R} = \frac{1}{2}\mu_0 N^2 \frac{r^2}{R}$$



ANS. FIG. P32.12

P32.13 Using the definition of self-inductance, $\mathcal{E} = -L\dfrac{di}{dt}$, we obtain

$$\begin{aligned}\mathcal{E} &= -L\frac{d}{dt}(I_i \sin\omega t) = -L\omega(I_i \cos\omega t)\\ &= -(10.0\times 10^{-3})[2\pi(60.0)](5.00)\cos\omega t\end{aligned}$$

$$\boxed{\mathcal{E} = -18.8\cos 120\pi t,\ \text{where } \mathcal{E} \text{ is in volts and } t \text{ is in seconds.}}$$

P32.14 The current change is linear, so $\mathcal{E} = -L\dfrac{di}{dt} = -L\dfrac{\Delta i}{\Delta t}$.

t = 0 to 4 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{-2.00\text{ mA}}{4.00\text{ ms}} = +2.00\text{ mV}$$

t = 4 to 8 ms:

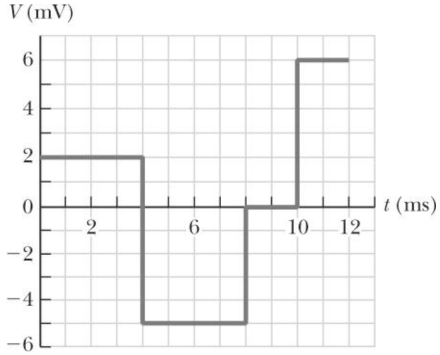
$$\mathcal{E} = -(4.00\text{ mH})\frac{+5.00\text{ mA}}{4.00\text{ ms}} = -5.00\text{ mV}$$

t = 8 to 10 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{0}{2.00\text{ ms}} = 0.00\text{ mV}$$

$t = 10$ to 12 ms:

$$\mathcal{E} = -(4.00\text{ mH})\frac{-3.00\text{ mA}}{2.00\text{ ms}} = +6.00\text{ mV}$$



ANS. FIG. P32.14

Section 32.2 *RL* Circuits

P32.15 (a) The inductance of a solenoid is

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\mu_0 N^2 \pi r^2}{\ell} = \frac{\mu_0 (510)^2 \pi (8.00\times10^{-3}\text{ m})^2}{0.140\text{ m}}$$
$$= 4.69\times10^{-4}\text{ H} = \boxed{0.469\text{ mH}}$$

(b) The time constant of the circuit is

$$\tau = \frac{L}{R} = \frac{4.69\times10^{-4}\text{ H}}{2.50\ \Omega} = 1.88\times10^{-4}\text{ s} = \boxed{0.188\text{ ms}}$$

P32.16 (a) At time *t*,

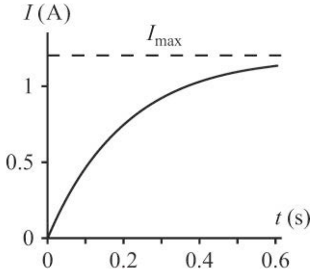
$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

where

$$\tau = \frac{L}{R} = \frac{2.00\text{ H}}{10.0\ \Omega} = 0.200\text{ s}$$

After a long time,

$$I_i = \frac{\mathcal{E}}{R}\left(1 - e^{-\infty}\right) = \frac{\mathcal{E}}{R}$$



ANS. FIG. P32.16

At $i(t) = 0.500 I_i$

$$(0.500)\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right)$$

so $\quad 0.500 = 1 - e^{-t/\tau}$

Isolating the constants on the right,

$$e^{-t/\tau} = 0.500$$
$$\ln\left(e^{-t/\tau}\right) = \ln(0.500)$$
$$t = \tau\left[-\ln(0.500)\right] = (0.200\text{ s})\left[-\ln(0.500)\right] = \boxed{0.139\text{ s}}$$

(b) Similarly, to reach 90% of I_i, $0.900 = 1 - e^{-t/\tau} \quad \rightarrow \quad e^{-t/\tau} = 0.100$

and $t = -\tau \ln(0.100)$

Thus,

$$t = -(0.200\text{ s})\ln(0.100) = \boxed{0.461\text{ s}}$$

P32.17 (a) Using $\tau = RC = \frac{L}{R}$, we get

$$R = \sqrt{\frac{L}{C}} = \sqrt{\frac{3.00\text{ H}}{3.00 \times 10^{-6}\text{ F}}} = 1.00 \times 10^{3}\ \Omega = \boxed{1.00\text{ k}\Omega}.$$

(b) The time constant is

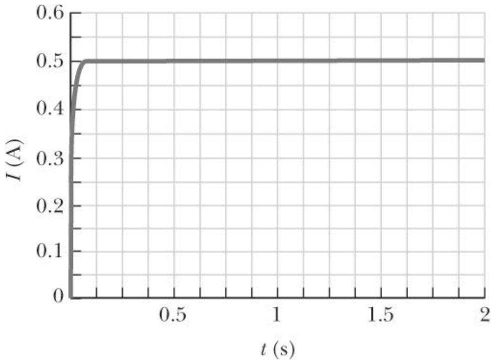
$$\tau = RC = \left(1.00 \times 10^{3}\ \Omega\right)\left(3.00 \times 10^{-6}\text{ F}\right)$$
$$= 3.00 \times 10^{-3}\text{ s} = \boxed{3.00\text{ ms}}$$

P32.18 The current builds exponentially according to:

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \frac{12.0\text{ V}}{24.0\ \Omega}\left(1 - e^{-t/\tau}\right)$$
$$= 0.500\left(1 - e^{-t/\tau}\right)$$

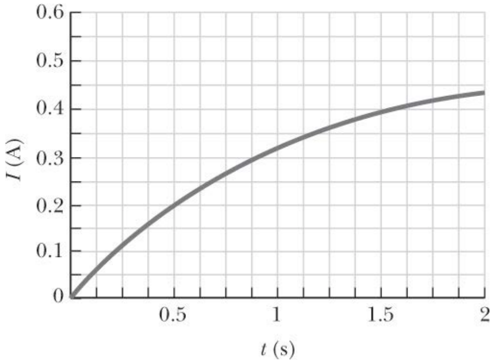
where current I is in amperes (A) and time t is in seconds (s).
The current increases from 0 to asymptotically approach 0.500 A. In case (a) the current jumps up essentially instantaneously. In case (b) it increases with a longer time constant, and in case (c) the increase is still slower.

(a) With "essentially zero" inductance, we take $\tau = \frac{L}{R} = 0.01$. ANS. FIG. P32.18(a) graphs $I(t)$ for this case.



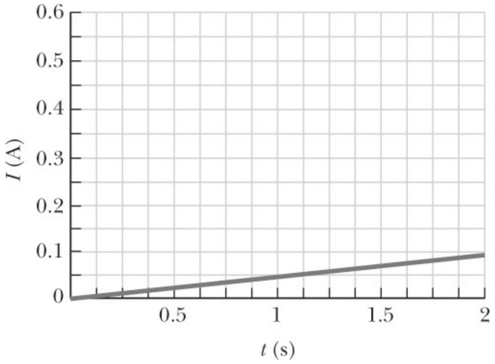
ANS. FIG. P32.18(a)

(b) We take $\tau = \frac{L}{R} = 1$. ANS. FIG. P32.18(b) graphs $I(t)$ for this case.



ANS. FIG. P32.18(b)

(c) We take $\tau = \frac{L}{R} = 10$. ANS. FIG. P32.18(c) graphs $I(t)$ for this case.



ANS. FIG. P32.18(c)

P32.19 (a) The two resistors are in parallel. Their resistance values 450 Ω and R are related to their equivalent resistance R_{eq} by

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R}+\frac{1}{450\,\Omega}$$

and the equivalent resistance is related to the time constant of the circuit by

$$\tau=\frac{L}{R_{\text{eq}}}\rightarrow\frac{1}{R_{\text{eq}}}=\frac{\tau}{L}$$

Thus,

$$\frac{1}{R_{\text{eq}}}=\frac{1}{R}+\frac{1}{450\ \Omega}=\frac{\tau}{L}$$

Solving for R,

$$\frac{1}{R}=\frac{\tau}{L}-\frac{1}{450\ \Omega}=\frac{15.0\times10^{-6}\ \text{s}}{5.00\times10^{-3}\ \text{H}}-\frac{1}{450\ \Omega}$$

which gives

$$R=1\ 290\ \Omega=\boxed{1.29\ \text{k}\Omega}$$

(b) The current will immediately begin to die from the value it had just before the switch was thrown to position *b*. Before the switch position was changed, the current was constant in time, so there was no emf induced in the inductor. The current was just

$$i=\frac{\Delta V}{R_{\text{eq}}}=\Delta V\frac{\tau}{L}=(24.0\ \text{V})\frac{15.0\times10^{-6}\ \text{s}}{5.00\times10^{-3}\ \text{H}}$$

$$=0.072\ 0\ \text{A}=\boxed{72.0\ \text{mA}}$$

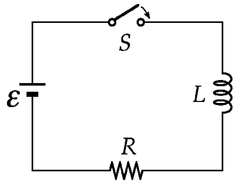
***P32.20** The current increases as a function of time as

$$i=I_i\left(1-e^{-t/\tau}\right)$$

Substituting,

$$0.980=1-e^{-3.00\times10^{-3}/\tau}$$

$$0.020\,0=e^{-3.00\times10^{-3}/\tau}$$



ANS. FIG. P32.20

Solving for the time constant gives

$$\tau=-\frac{3.00\times10^{-3}}{\ln(0.020\,0)}=7.67\times10^{-4}\ \text{s}$$

and since $\tau = \frac{L}{R}$,

$$L = \tau R = (7.67 \times 10^{-4}\ \text{s})(10.0\ \Omega) = \boxed{7.67\ \text{mH}}$$

***P32.21** For the increasing current $i = \frac{\mathcal{E}}{R}\left(1 - e^{-Lt/R}\right)$. The final value is $\frac{\mathcal{E}}{R}$, so the condition on Δt is

$$0.800\frac{\mathcal{E}}{R} = \frac{\mathcal{E}}{R}\left(1 - e^{-L\Delta t/R}\right)$$

$$e^{-L\Delta t/R} = 0.200$$

$$e^{+L\Delta t/R} = 5.00$$

$$\frac{L\Delta t}{R} = \ln 5.00$$

$$\Delta t = \frac{R\ln 5.00}{L}$$

At the moment when the battery is removed, the current in the coil is quite precisely $\frac{\mathcal{E}}{R}$. During the decrease, $i = I_i e^{-Lt/R} = \frac{\mathcal{E}}{R}e^{-Lt/R}$.

(a) at $t = \Delta t = \frac{R\ln 5.00}{L}$,

$$\frac{i}{I_i} = e^{-L\Delta t/R} = 0.200 = \boxed{20.0\%}$$

(b) at $t = 2\Delta t$,

$$\frac{i}{I_i} = e^{-L2\Delta t/R} = \left(e^{-L\Delta t/R}\right)^2 = (0.200)^2 = 0.040\ 0 = \boxed{4.00\%}$$

P32.22 Taking $\tau = \frac{L}{R}$, and $i = I_i e^{-t/\tau}$: $\frac{di}{dt} = I_i e^{-t/\tau}\left(-\frac{1}{\tau}\right)$

$iR + L\frac{di}{dt} = 0$ will be true if $I_i R\ e^{-t/\tau} + L\left(I_i e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right) = 0$

We have agreement because $\tau = \frac{L}{R}$.

P32.23 The current at this time is given by

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \frac{120\ \text{V}}{9.00\ \Omega}\left[1 - e^{-0.200/(7.00/9.00)}\right] = 3.02\ \text{A}$$

Then, $\Delta V_R = iR = (3.02)(9.00) = 27.2\ \text{V}$

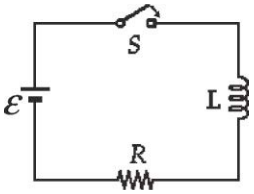
and $\Delta V_L = \mathcal{E} - \Delta V_R = 120 - 27.2 = \boxed{92.8\ \text{V}}$

P32.24 (a) $\tau = \frac{L}{R} = \frac{8.00\times 10^{-3}\ \text{H}}{4.00\ \Omega} = 2.00\times 10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$

(b) $i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) = \left(\frac{6.00\ \text{V}}{4.00\ \Omega}\right)\left(1 - e^{-0.250/2.00}\right) = \boxed{0.176\ \text{A}}$

(c) $I_i = \frac{\mathcal{E}}{R} = \frac{6.00\ \text{V}}{4.00\ \Omega} = \boxed{1.50\ \text{A}}$

(d) $0.800 = 1 - e^{-t/2.00\ \text{ms}} \rightarrow t = -(2.00\ \text{ms})\ln(0.200) = \boxed{3.22\ \text{ms}}$



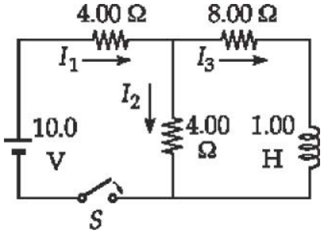
ANS. FIG. P32.24

P32.25 Name the currents as shown in ANS. FIG. P32.25. By Kirchhoff's laws:

$$i_1 = i_2 + i_3 \qquad \textbf{[1]}$$

$$+10.0\ \text{V} - 4.00i_1 - 4.00i_2 = 0 \qquad \textbf{[2]}$$

$$+10.0\ \text{V} - 4.00i_1 - 8.00i_3 - (1.00)\frac{di_3}{dt} = 0 \qquad \textbf{[3]}$$



ANS. FIG. P32.25

From [1] and [2],

$$+10.0 - 4.00i_1 - 4.00i_1 + 4.00i_3 = 0$$

$$i_1 = 0.500i_3 + 1.25\ \text{A}$$

Then [3] becomes $10.0\ \text{V} - 4.00(0.500i_3 + 1.25\ \text{A}) - 8.00i_3 - (1.00)\frac{di_3}{dt} = 0$

$$(1.00\ \text{H})\left(\frac{di_3}{dt}\right) + (10.0\ \Omega)i_3 = 5.00\ \text{V}$$

or $$5.00\ \text{V} - (10.0\ \Omega)i_3 - (1.00\ \text{H})\left(\frac{di_3}{dt}\right) = 0$$

which can be compared to the general form (Equation 32.6)

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

which has the solution (from Equation 32.7) $i = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$.

Thus, we have:

(a) $i_3 = \left(\frac{5.00\text{ V}}{10.0\ \Omega}\right)\left[1 - e^{-(10.0\ \Omega)t/1.00\text{ H}}\right] = \boxed{(0.500\text{ A})\left[1 - e^{-10t/\text{s}}\right]}$

(b) $i_1 = 1.25 + 0.500 I_3 = \boxed{1.50\text{ A} - (0.250\text{ A})e^{-10t/\text{s}}}$

P32.26 Refer to ANS. FIG. P32.25 above. Name the currents as shown. By Kirchhoff's laws:

$$i_1 = i_2 + i_3 \qquad \textbf{[1]}$$

$$\mathcal{E} - Ri_1 - Ri_2 = 0 \qquad \textbf{[2]}$$

$$\mathcal{E} - Ri_1 - 2Ri_3 - L\frac{di_3}{dt} = 0 \qquad \textbf{[3]}$$

From [1] and [2],

$$\mathcal{E} - Ri_1 - R(i_1 - i_3) = 0$$

$$\mathcal{E} - Ri_1 - Ri_1 + Ri_3 = 0$$

$$i_1 = \frac{1}{2}i_3 + \frac{\mathcal{E}}{2R}$$

Then [3] becomes

$$\mathcal{E} - R\left(\frac{1}{2}i_3 + \frac{\mathcal{E}}{2R}\right) - 2Ri_3 - L\frac{di_3}{dt} = 0$$

$$L\frac{di_3}{dt} + 2.5Ri_3 = \frac{\mathcal{E}}{2}$$

$$\frac{\mathcal{E}}{2} - 2.5Ri_3 - L\frac{di_3}{dt} = 0$$

which can be compared to the general form (Equation 32.6)

$$\mathcal{E} - iR - L\frac{di}{dt} = 0$$

which has the solution (from Equation 32.7)

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

Thus, we have:

(a) $$i_3 = \left(\frac{\mathcal{E}/2}{2.5R}\right)\left[1-e^{-2.5Rt/L}\right] = \boxed{\frac{\mathcal{E}}{5R}\left(1-e^{-5Rt/2L}\right)}$$

(b) $$i_1 = \frac{1}{2}i_3 + \frac{\mathcal{E}}{2R} = \frac{1}{2}\left[\frac{\mathcal{E}}{5R}\left(1-e^{-5Rt/2L}\right)\right] + \frac{\mathcal{E}}{2R}$$

$$i_1 = \frac{\mathcal{E}}{10R}\left(1-e^{-5Rt/2L}\right) + \frac{5\mathcal{E}}{10R} = \boxed{\frac{\mathcal{E}}{10R}\left(6-e^{-5Rt/2L}\right)}$$

P32.27 (a) When $i = 2.00$ A, the voltage across the resistor is

$$\Delta V_R = iR = (2.00\text{ A})(8.00\ \Omega) = 16.0\text{ V}$$

Kirchhoff's loop rule tells us that the sum of the changes in potential around the loop must be zero:

$$\mathcal{E} - \Delta V_R - \mathcal{E}_L = 36.0\text{ V} - 16.0\text{ V} - \mathcal{E}_L = 0$$

so $\mathcal{E}_L = 20.0$ V

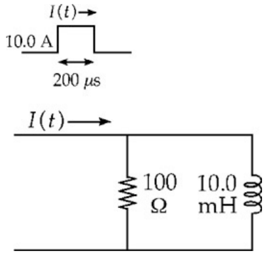
and $$\frac{\Delta V_R}{\mathcal{E}_L} = \frac{16.0\text{ V}}{20.0\text{ V}} = \boxed{0.800}$$

(b) Similarly, for $i = 4.50$ A, $\Delta V_R = iR = (4.50\text{ A})(8.00\ \Omega) = 36.0\text{ V}$ and

$$\mathcal{E} - \Delta V_R - \mathcal{E}_L = 36.0\text{ V} - 36.0\text{ V} - \mathcal{E}_L = 0$$

so $\mathcal{E}_L = \boxed{0}$

P32.28 For $t \le 0$, the current in the inductor is $\boxed{\text{zero}}$.



ANS. FIG. P32.28

For $0 \le t \le 200\ \mu s$, there will be current i_R in the resistor and I_L in the inductor so that $i = i_R + i_L = I_i = 10.0$ A. Assuming both currents are downward in ANS. FIG. P32.28, we apply Kirchhoff's loop rule going counterclockwise around the loop, and we find that

$$-i_R R + L\frac{di_L}{dt} = 0$$

Using $I_i = i_R + i_L \rightarrow i_R = I_i - i_L$, we have

$$-(I_i - i_L)R + L\frac{di_L}{dt} = 0$$

Then,

$$L\frac{dI_L}{dt} = \left(I_{\max} - I_L\right)R$$

$$\int_0^I \frac{di_L}{\left(I_i - I_L\right)} = \int_0^t \frac{R}{L}dt$$

$$-\ln\frac{\left(I_i - i_L\right)}{I_i} = \frac{R}{L}t$$

which gives

$$i_L = I_i\left(1 - e^{-Rt/L}\right)$$

We see that $t = 0$, $i_L = 0$ as we expect because of the back emf induced in the inductor. With the time constant

$$\tau = \frac{L}{R} = \frac{(10.0\text{ mH})}{(100\ \Omega)} = 1.00 \times 10^{-4}\text{ s}$$

we have

$$i_L = I_i\left(1 - e^{-t/\tau}\right) = \boxed{(10.0\text{ A})\left(1 - e^{-10\,000t/\text{s}}\right)} \qquad (0 \le t \le 200\ \mu\text{s})$$

At $t = 200\ \mu\text{s}$, $i = (10.00\text{ A})\left(1 - e^{-2.00}\right) = 8.65\text{ A}$; thereafter, the current decays. The loop rule gives the same result,

$$-i_R R + L\frac{di_L}{dt} = 0$$

but now $i_R + i_L = 0 \rightarrow i_R = -i_L$, so we have

$$i_L R + L\frac{di_L}{dt} = 0 \rightarrow L\frac{di_L}{dt} = -i_L R$$

$$\int_{I_i}^{I}\frac{di_L}{i_L} = -\int_{200\ \mu\text{s}}^{t}\frac{R}{L}dt$$

$$\ln\frac{i_L}{I_i} = -\frac{R}{L}(t - 200\ \mu\text{s}) \rightarrow i_L = I_i e^{-R(t-200\mu\text{s})/L}$$

For $t = 200\ \mu\text{s}$, $i_i = 8.65\text{ A}$, and for $t \ge 200\ \mu\text{s}$,

$$i = (8.65\text{ A})e^{-10\,000(t-200\ \mu\text{s})/\text{s}} = (8.65\text{ A})e^{-10\,000t/\text{s}+2.00}$$

$$= \left(8.65e^{2.00}\text{ A}\right)e^{-10\,000t/\text{s}} = \boxed{(63.9\text{ A})e^{-10\,000t/\text{s}}} \qquad (t \ge 200\ \mu\text{s})$$

P32.29 From Equation 32.7, $i = I_i\left(1 - e^{-t/\tau}\right)$. Therefore,

$$\frac{di}{dt} = -I_i\left(e^{-t/\tau}\right)\left(-\frac{1}{\tau}\right)$$

where

$$\tau = \frac{L}{R} = \frac{15.0\ \text{H}}{30.0\ \Omega} = 0.500\ \text{s}$$

Then, $\frac{di}{dt} = \frac{R}{L} I_i e^{-t/\tau}$ with $I_i = \frac{\mathcal{E}}{R}$

(a) At $t = 0$,

$$\frac{di}{dt} = \frac{R}{L} I_i e^0 = \frac{\mathcal{E}}{L} = \frac{100\ \text{V}}{15.0\ \text{H}} = \boxed{6.67\ \text{A/s}}$$

(b) At $t = 1.50$ s,

$$\frac{di}{dt} = \frac{\mathcal{E}}{L} e^{-t/\tau} = \left(6.67\ \text{A/s}\right) e^{-1.50/(0.500)} = \left(6.67\ \text{A/s}\right) e^{-3.00}$$
$$= \boxed{0.332\ \text{A/s}}$$

P32.30 (a) For a series connection, both inductors carry equal currents at every instant, so $\frac{di}{dt}$ is the same for both. The voltage across the pair is

$$L_{\text{eq}}\frac{di}{dt} = L_1\frac{di}{dt} + L_2\frac{di}{dt} \quad \rightarrow \quad L_{\text{eq}} = L_1 + L_2$$

(b) For a parallel connection, the voltage across each inductor is the same for both.

$$L_{\text{eq}}\frac{di}{dt} = L_1\frac{di_1}{dt} = L_2\frac{di_2}{dt} = \Delta V_L$$

where the currents are related by $i = i_1 + i_2$. Therefore,

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$
$$\frac{\Delta V_L}{L_{\text{eq}}} = \frac{\Delta V_L}{L_1} + \frac{\Delta V_L}{L_2} \quad \rightarrow \quad \frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}$$

(c) $L_{eq}\dfrac{di}{dt}+R_{eq}i=L_1\dfrac{di}{dt}+iR_1+L_2\dfrac{di}{dt}+iR_2$

Now i and $\dfrac{di}{dt}$ are independent quantities under our control, so functional equality requires both $L_{eq}=L_1+L_2$ and $R_{eq}=R_1+R_2$.

(d) Yes. The relations $\Delta V=L_{eq}\dfrac{di}{dt}+R_{eq}i=L_1\dfrac{di_1}{dt}+R_1i_1=L_2\dfrac{di_2}{dt}+R_2i_2$, where $i=i_1+i_2$ and $\dfrac{di}{dt}=\dfrac{di_1}{dt}+\dfrac{di_2}{dt}$, must always be true.

We may choose to keep the currents constant in time. Then, from $i=i_1+i_2$, we have

$$\frac{\Delta V_L}{R_{eq}}=\frac{\Delta V_L}{R_1}+\frac{\Delta V_L}{R_2}\quad\rightarrow\quad\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}$$

We may choose to make the current oscillate so that at a given moment it is zero. Then, from $\dfrac{di}{dt}=\dfrac{di_1}{dt}+\dfrac{di_2}{dt}$, as in part (b), we have $\dfrac{1}{L_{eq}}=\dfrac{1}{L_1}+\dfrac{1}{L_2}$.

P32.31 (a) The equation for current buildup is obtained by combining Equations 32.7 and 32.8:

$$i=\frac{\mathcal{E}\left(1-e^{-Rt/L}\right)}{R}$$

We proceed step-by-step to solve for t in terms of the other quantities, all of which are given:

$$iR/\mathcal{E}=1-e^{-Rt/L}$$

so $e^{-Rt/L}=1-iR/\mathcal{E}$

and $-Rt/L=\ln(1-iR/\mathcal{E})$

then,

$$t=-(L/R)\ln(1-iR/\mathcal{E})$$

$$t=-(0.140\text{ H}/4.90\ \Omega)\ln[1-(0.220\text{ A})(4.90\ \Omega)/6.00\text{ V}]$$

$$=-(0.028\,6\text{ s})\ln(0.820)$$

$$t=-(0.028\,6\text{ s})(-0.198)=\boxed{5.66\text{ ms}}$$

(b) We now make the general equation refer to a different instant. The current after ten seconds is

$$i = \left(\frac{6.00\text{ V}}{4.90\ \Omega}\right)\left(1 - e^{(-35.0\text{ s}^{-1})(10.0\text{ s})}\right) = (1.22\text{ A})(1 - e^{-350}) = \boxed{1.22\text{ A}}$$

(c) The equation for current decrease after the battery is removed is $i = \dfrac{\mathcal{E}}{R}e^{-Rt/L}$. We solve for *t*:

$$\frac{iR}{\mathcal{E}} = e^{-Rt/L} \quad \text{or} \quad \frac{\mathcal{E}}{iR} = e^{+Rt/L}$$

Then,

$$\ln(\mathcal{E}/iR) = Rt/L \quad \text{and} \quad t = (L/R)\ \ln(\mathcal{E}/iR)$$

Substituting,

$$\begin{aligned} t &= (0.140\text{ H}/4.90\ \Omega)\ \ln[6.00\text{ V}/(0.160\text{ A}\cdot 4.90\ \Omega)] \\ &= (0.028\ 6\text{ s})(\ln 7.65) = \boxed{58.1\text{ ms}} \end{aligned}$$

Section 32.3 Energy in a Magnetic Field

P32.32 The inductance of the solenoid is

$$L = N\frac{\Phi_B}{i} = 200\frac{3.70\times10^{-4}\text{ Wb}}{1.75\text{ A}} = 0.042\ 3\text{ H}$$

The energy stored is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(0.042\ 3\text{ H})(1.75\text{ A})^2 = 0.064\ 8\text{ J} = \boxed{64.8\text{ mJ}}$$

P32.33 For a solenoid of length ℓ, the inductance is $L = \dfrac{\mu_0 N^2 A}{\ell}$.

Thus, since $U_B = \dfrac{1}{2}Li^2 = \dfrac{\mu_0 N^2 A i^2}{2\ell}$, the stored energy is

$$\begin{aligned} U_B &= \frac{(4\pi\times10^{-7}\text{ N/A}^2)(68)^2\pi(6.00\times10^{-3}\text{ m})^2(0.770\text{ A})^2}{2\ (0.080\ 0)\text{ m}} \\ &= \boxed{2.44\times10^{-6}\text{ J}} \end{aligned}$$

P32.34 From Equation 32.7, $i = \dfrac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$:

(a) The maximum current, after a long time *t*, is $i = I_i = \dfrac{\mathcal{E}}{R} = 2.00\text{ A}$.

At that time, the inductor is fully energized and

$$P = i(\Delta V) = (2.00\text{ A})(10.0\text{ V}) = \boxed{20.0\text{ W}}$$

(b) $P_{\text{lost}} = i^2R = (2.00\text{ A})^2(5.00\ \Omega) = \boxed{20.0\text{ W}}$

(c) The inductor has no resistance: $P_{\text{inductor}} = i\left(\Delta V_{\text{drop}}\right) = \boxed{0}$

(d) $U_B = \dfrac{1}{2}Li^2 = \dfrac{1}{2}(10.0\text{ H})(2.00\text{ A})^2 = \boxed{20.0\text{ J}}$

P32.35 (a) The energy density stored by the electric field is

$$u_E = \epsilon_0\frac{E^2}{2} = \left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\frac{(100\ \text{V/m})^2}{2}\left(\frac{\text{J/C}}{\text{V}}\right)^2\left(\frac{\text{N}\cdot\text{m}}{\text{J}}\right)$$

$$= 4.43\times10^{-8}\ \frac{\text{J}}{\text{m}^3} = \boxed{44.3\ \text{nJ/m}^3}$$

(b) The energy density stored by the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(0.500\times10^{-4}\text{ T}\right)^2}{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)}\left(\frac{\text{N/A}\cdot\text{m}}{\text{T}}\right)$$

$$= 9.95\times10^{-4}\ \frac{\text{N}}{\text{m}^2}\left(\frac{\text{m}}{\text{m}}\right) = 9.95\times10^{-4}\ \frac{\text{N}\cdot\text{m}}{\text{m}^3} = \boxed{995\ \mu\text{J/m}^3}$$

P32.36 We compute the integral:

$$\int_0^\infty e^{-2Rt/L}\,dt = -\frac{L}{2R}\int_0^\infty e^{-2Rt/L}\left(\frac{-2Rdt}{L}\right) = -\frac{L}{2R}e^{-2Rt/L}\Big|_0^\infty$$

$$= -\frac{L}{2R}\left(e^{-\infty} - e^0\right) = \frac{L}{2R}(0-1) = \boxed{\frac{L}{2R}}$$

***P32.37** The current in the circuit at time *t* is $i = I_i\left(1 - e^{-t/\tau}\right)$, where $I_i = \dfrac{\mathcal{E}}{R}$, and the energy stored in the inductor is $U_B = \dfrac{1}{2}Li^2$.

(a) As $t \to \infty$, $I \to I_i = \dfrac{\mathcal{E}}{R} = \dfrac{24.0\text{ V}}{8.00\ \Omega} = 3.00\text{ A}$, and

$$U_B = \frac{1}{2}LI_i^2 = \frac{1}{2}(4.00\text{ H})(3.00\text{ A})^2 = \boxed{18.0\text{ J}}$$

(b) At $t = \tau$, $I = I_i\left(1 - e^{-1}\right) = (3.00\text{ A})(1 - 0.368) = 1.90\text{ A}$, and

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(4.00\text{ H})(1.90\text{ A})^2 = \boxed{7.19\text{ J}}$$

P32.38 (a) $P = i\Delta V = (3.00\text{ A})(22.0\text{ V}) = \boxed{66.0\text{ W}}$

(b) $P = i\Delta V_R = i^2R = (3.00\text{ A})^2(5.00\ \Omega) = \boxed{45.0\text{ W}}$

(c) METHOD 1: We treat the real inductor as an ideal inductor (with no resistance) in series with an ideal resistor (with no inductance). When the current is 3.00 A, Kirchhoff's loop rule reads

$$+22.0\text{ V} - (3.00\text{ A})(5.00\ \Omega) - \Delta V_L = 0$$
$$\Delta V_L = 7.00\text{ V}$$

The power being stored in the inductor is

$$i\Delta V_L = (3.00\text{ A})(7.00\text{ V}) = \boxed{21.0\text{ W}}$$

METHOD 2: We do not treat the real inductor as an ideal inductor in series with an ideal resistor.

We wish to find the rate at which energy is being delivered to the inductor. As discussed in Section 32.3, $U_B = \frac{1}{2}Li^2 \rightarrow \frac{dU_B}{dt} = Li\frac{di}{dt}$. We know *L* (0.040 0 H) and *i* (3.00 A); we need to evaluate the term $\frac{di}{dt}$. From Equations 32.7 and 32.8 (or Equation 32.9),

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \rightarrow \frac{di}{dt} = \frac{\mathcal{E}}{L}e^{-t/\tau}$$

because $\tau = \frac{L}{R}$. Also,

$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-t/\tau}\right) \rightarrow e^{-t/\tau} = 1 - \frac{iR}{\mathcal{E}}$$

Therefore,

$$\frac{dU_B}{dt} = Li\frac{di}{dt} = Li\left(\frac{\mathcal{E}}{L}e^{-t/\tau}\right) = i\mathcal{E}e^{-t/\tau} = i\mathcal{E}\left(1 - \frac{iR}{\mathcal{E}}\right) = i(\mathcal{E} - iR)$$

When *i* = 3.00 A,

$$\frac{dU_B}{dt} = i(\mathcal{E} - iR) = (3.00\text{ A})\left[22.0\text{ V} - (3.00\text{ A})(5.00\ \Omega)\right]$$
$$= \boxed{21.0\text{ W}}$$

(d) $\boxed{\text{The power supplied by the battery is equal to the sum of the power delivered to the internal resistance of the coil and the power stored in the magnetic field.}}$

(e) $\boxed{\text{Yes.}}$

(f) $\boxed{\text{Just after } t = 0\text{, the current is very small, so the power delivered to the internal resistance of the coil } (iR^2) \text{ is nearly zero, but the rate of the change of the current is large, so the power delivered to the magnetic field } (L\,di/dt) \text{ is large, and nearly all the battery power is being stored in the magnetic field. Long after the connection is made, the current is not changing, so no power is being stored in the magnetic field, and all the battery power is being delivered to the internal resistance of the coil.}}$

P32.39 (a) The magnetic energy density is given by

$$u_B = \frac{B^2}{2\mu_0} = \frac{(4.50\text{ T})^2}{2(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})} = 8.06\times 10^6\text{ J/m}^3 = \boxed{8.06\text{ MJ}}$$

(b) The magnetic energy stored in the field equals *u* times the volume of the solenoid (the volume in which *B* is non-zero).

$$U_B = u_B V = (8.06\times 10^6\text{ J/m}^3)\left[(0.260\text{ m})\pi(0.031\,0\text{ m})^2\right]$$
$$= \boxed{6.32\text{ kJ}}$$

Section 32.4 Mutual Inductance

P32.40 We use Equation 32.17, $|\mathcal{E}_2| = \left|-M\frac{di_1}{dt}\right|$, from which we obtain the mutual inductance:

$$M = \frac{|\mathcal{E}_2|}{|di_1/dt|} = \frac{0.096\,0\text{ V}}{1.20\text{ A/s}} = 0.080\,0\text{ H} = \boxed{80.0\text{ mH}}$$

P32.41 Let the changing current in coil 1 induce an emf in coil 2. Then,

$$\begin{aligned}\mathcal{E}_2 &= -M\frac{di_1}{dt} = -\left(100\times10^{-6}\right)\frac{d}{dt}\left[10.0\sin\left(1.00\times10^{3}t\right)\right]\\ &= -\left(100\times10^{-6}\right)(10.0)\left(1.00\times10^{3}\right)\cos\left(1.00\times10^{3}t\right)\\ &= -(1.00)\cos\left(1.00\times10^{3}t\right)\end{aligned}$$

Therefore, the peak emf is $\left(\mathcal{E}_2\right)_{\max} = \boxed{1.00\text{ V}}$.

P32.42 The current is given by $i = I_i e^{-\alpha t}\sin\omega t$, with $I_i = 5.00$, $\alpha = 0.025\,0$, and $\omega = 120\pi$. Then,

$$\begin{aligned}\frac{di}{dt} &= \frac{d}{dt}\left[I_i e^{-\alpha t}\sin\omega t\right]\\ &= I_i\left(-\alpha e^{-\alpha t}\right)\sin\omega t + I_i e^{-\alpha t}\left(\omega\cos\omega t\right)\\ &= I_i e^{-\alpha t}\left(-\alpha\sin\omega t + \omega\cos\omega t\right)\end{aligned}$$

where $\frac{di}{dt}$ is in amperes per second, I_i is in amperes, and t in seconds.

At $t = 0.800$ s,

$$\begin{aligned}\frac{di}{dt} &= (5.00)e^{-0.020\,0}\left\{-\left(0.025\,0\right)\sin\left[0.800(120\pi)\right]\right.\\ &\qquad\left.+120\pi\cos\left[0.800(120\pi)\right]\right\}\\ &= 1.85\times10^{3}\ \text{A/s}\end{aligned}$$

Thus, from $\mathcal{E}_2 = -M\frac{di_1}{dt}$,

$$M = \frac{-\mathcal{E}_2}{di_1/dt} = \frac{+3.20\text{ V}}{1.85\times10^{3}\ \text{A/s}} = \boxed{1.73\text{ mH}}$$

P32.43 (a) The mutual inductance of the coils is

$$M = \frac{N_B\Phi_{BA}}{i_A} = \frac{700\left(90.0\times10^{-6}\,\text{Wb}\right)}{3.50\text{ A}} = \boxed{18.0\text{ mH}}$$

(b) The inductance of coil A is

$$L_A = \frac{\Phi_A}{i_A} = \frac{400\left(300\times10^{-6}\,\text{Wb}\right)}{3.50\text{ A}} = \boxed{34.3\text{ mH}}$$

(c) The emf induced in the other coil is

$$\left|\varepsilon_B\right| = \left|-M\frac{di_A}{dt}\right| = (18.0\text{ mH})\left(0.500\ \text{A/s}\right) = \boxed{9.00\text{ mV}}$$

P32.44 (a) Solenoid S_1 creates a nearly uniform field everywhere inside it, given by $B_1 = \mu_0 N_1 i/\ell$. The flux through one turn of solenoid S_2 is

$$\mu_0 \pi R_2^2 N_1 i/\ell$$

The emf induced in solenoid S_2 is

$$-(\mu_0 \pi R_2^2 N_1 N_2/\ell)(di/dt)$$

The mutual inductance is

$$\boxed{M_{12} = \mu_0 \pi R_2^2 N_1 N_2/\ell}$$

(b) Solenoid S_2 creates a nearly uniform field everywhere inside it, given by $B_2 = \mu_0 N_2 i_2/\ell$ and nearly zero field outside. The flux through one turn of solenoid 1 is

$$\mu_0 \pi R_2^2 N_2 i_2/\ell$$

The emf induced in solenoid 1 is

$$-(\mu_0 \pi R_2^2 N_1 N_2/\ell)(di_2/dt)$$

The mutual inductance is

$$\boxed{M_{12} = \mu_0 \pi R_2{}^2 N_1 N_2/\ell.}$$

(c) $\boxed{\text{They are the same.}}$

P32.45 Assume the long wire carries current *I*. Then the magnitude of the magnetic field it generates at distance *x* from the wire is $B = \dfrac{\mu_0 I}{2\pi x}$, and this field passes perpendicularly through the plane of the loop. The flux through the loop is

$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int B(\ell dx) = \frac{\mu_0 I \ell}{2\pi} \int_{0.400\text{ mm}}^{1.70\text{ mm}} \frac{dx}{x}$$

$$= \frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{1.70}{0.400}\right)$$

The mutual inductance between the wire and the loop is then

$$M = \frac{N_2 \Phi_{12}}{I_1} = \frac{N_2 \mu_0 \cancel{I} \ell}{2\pi \cancel{I}} \ln\left(\frac{1.70}{0.400}\right)$$

$$= \frac{1(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A})(2.70 \times 10^{-3}\text{ m})}{2\pi} \ln\left(\frac{1.70}{0.400}\right)$$

$$M = 7.81 \times 10^{-10}\text{ H} = \boxed{781\text{ pH}}$$

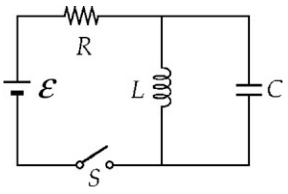
P32.46 (a) A current *i* in the large loop of radius *R* produces a magnetic field of magnitude $B = \frac{\mu_0 i}{2R}$ at its center. Because the radius of the small loop *r*<< *R*, we may treat the flux through the small loop as being approximately $\Phi_B = BA\cos 0.00° = \left(\frac{\mu_0 i}{2R}\right)A = \frac{\mu_0 \pi r^2 i}{2R}$. The mutual inductance of the loops is then

$$M = \frac{\Phi_B}{i} = \frac{\mu_0 \pi r^2}{2R}$$

(b) $$M = \frac{\mu_0 \pi r^2}{2R} = \frac{(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})\pi(0.020\ 0\ \mathrm{m})^2}{2(0.200\ \mathrm{m})} = 3.95\times10^{-9}\ \mathrm{H}$$

$$= \boxed{3.95\ \mathrm{nH}}$$

Section 32.5 Oscillations in an *LC* Circuit



ANS. FIG. P32.47

P32.47 When the switch has been closed for a long time, battery, resistor, and coil carry constant current $I_i = \frac{\mathcal{E}}{R}$. When the switch is opened, current in battery and resistor drops to zero, but the coil carries this same current for a moment as oscillations begin in the *LC* loop.

We interpret the problem to mean that the voltage amplitude of these oscillations is ΔV, in $\frac{1}{2}C(\Delta V)^2 = \frac{1}{2}LI_i^2$.

Then,

$$L = \frac{C(\Delta V)^2}{I_i^2} = \frac{C(\Delta V)^2 R^2}{\mathcal{E}^2} = \frac{(0.500\times10^{-6}\ \mathrm{F})(150\ \mathrm{V})^2(250\ \Omega)^2}{(50.0\ \mathrm{V})^2}$$

$$= 0.281\ \mathrm{H} = \boxed{281\ \mathrm{mH}}$$

P32.48 This radio is a radiotelephone on a ship, according to frequency assignments made by international treaties, laws, and decisions of the National Telecommunications and Information Administration.

The resonance frequency is $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Thus, $$C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{\left[2\pi(6.30\times10^6\ \mathrm{Hz})\right]^2(1.05\times10^{-6}\ \mathrm{H})} = \boxed{608\ \mathrm{pF}}$$

P32.49 At different times, $(U_C)_{\text{max}} = (U_L)_{\text{max}}$, so

$$\left[\frac{1}{2}C(\Delta V)^2\right]_{\text{max}} = \frac{1}{2}LI_i^2$$

Then,

$$I_i = \sqrt{\frac{C}{L}}(\Delta V)_{\text{max}} = \sqrt{\frac{1.00\times 10^{-6}\text{ F}}{10.0\times 10^{-3}\text{ H}}}(40.0\text{ V})$$

$$= 0.400\text{ A} = \boxed{400\text{ mA}}$$

P32.50 From the angular frequency of oscillation of the circuit, we have

$$\omega = \frac{1}{\sqrt{LC}} = 2\pi f$$

Solving for the inductance gives

$$L = \frac{1}{C(2\pi f)^2} = \frac{1}{(8.00\times 10^{-6}\text{ F})[2\pi(120\text{ Hz})]^2}$$

$$= \boxed{0.220\text{ H}}$$

P32.51 At different times, the maximum energy stored in the capacitor is equal to the maximum energy stored in the inductor.

$$\left[\frac{1}{2}C(\Delta V)^2\right]_{\text{max}} = \frac{1}{2}LI_i^2$$

so

$$(\Delta V_C)_{\text{max}} = \sqrt{\frac{L}{C}}I_i = \sqrt{\frac{20.0\times 10^{-3}\text{ H}}{0.500\times 10^{-6}\text{ F}}}(0.100\text{ A}) = \boxed{20.0\text{ V}}$$

P32.52 Find the energy stored in the circuit from Equation 32.27:

$$U = \frac{Q_{\text{max}}^2}{2C} = \frac{(200\times 10^{-6}\text{ C})^2}{2(50.0\times 10^{-6}\text{ F})} = 4.00\times 10^{-4}\text{ J} = 400\ \mu\text{J}$$

If the energy is split equally between the capacitor and inductor at some instant, the energy would be half this value, or 200 μJ. Therefore, there would be no time when each component stores 250 μJ.

P32.53 (a) The frequency of oscillation of the circuit is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.100\text{ H})(1.00\times 10^{-6}\text{ F})}} = \boxed{503\text{ Hz}}$$

(b) The maximum charge on the capacitor is

$$Q = C\mathcal{E} = \left(1.00\times10^{-6}\text{ F}\right)\left(12.0\text{ V}\right) = \boxed{12.0\ \mu\text{C}}$$

(c) To find the maximum current I_i, we equate

$$\frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}LI_i^2$$

Then solve for I_i to obtain

$$I_i = \mathcal{E}\sqrt{\frac{C}{L}} = 12.0\text{ V}\sqrt{\frac{1.00\times10^{-6}\text{ F}}{0.100\text{ H}}} = \boxed{37.9\text{ mA}}$$

(d) The total energy the circuit possesses at $t = 3.00$ s and at all times is

$$U = \frac{1}{2}C\mathcal{E}^2 = \frac{1}{2}\left(1.00\times10^{-6}\text{ F}\right)\left(12.0\text{ V}\right)^2 = \boxed{72.0\ \mu\text{J}}$$

P32.54 At $t = 0$ the capacitor charge is at its maximum value, so $\phi = 0$ in

$$Q = Q_{\max}\cos(\omega t + \phi) = Q_{\max}\cos\left(\frac{t}{\sqrt{LC}}\right)$$

Substituting the given information, the charge at 2 ms is

$$\begin{aligned} Q &= (105\times10^{-6}\text{ C})\cos\left(\frac{2.00\times10^{-3}\text{ s}}{\sqrt{(3.30\text{ H})(840\times10^{-12}\text{ F})}}\right) \\ &= (105\times10^{-6}\text{ C})\cos\left(38.0\text{ rad}\right) \\ &= 1.01\times10^{-4}\text{ C} \end{aligned}$$

(a) Then the energy in the capacitor is

$$U_C = \frac{Q^2}{2C} = \frac{\left(1.01\times10^{-4}\text{ C}\right)^2}{2\left(840\times10^{-12}\text{ F}\right)} = \boxed{6.03\text{ J}}$$

(c) The constant total energy is that originally of the capacitor:

$$U = \frac{Q_{\max}^2}{2C} = \frac{\left(1.05\times10^{-4}\text{ C}\right)^2}{2\left(840\times10^{-12}\text{ F}\right)} = \boxed{6.56\text{ J}}$$

(b) So the inductor's energy is the remaining

$$U_L = 6.56\text{ J} - 6.03\text{ J} = \boxed{0.529\text{ J}}$$

P32.55 (a) The angular frequency of oscillations is

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.082\,0\text{ H})(17.0\times10^{-6}\text{ F})}} = 847\text{ rad/s} = 2\pi f$$

so $f = \boxed{135\text{ Hz}}$

(b) The charge on the capacitor is

$$Q = Q_{\max}\cos\omega t = (180\ \mu\text{C})\cos[(847\text{ rad/s})(0.001\,00\text{ s})]$$
$$= \boxed{119\ \mu\text{C}}$$

(c) The current in the circuit is given by Equation 32.23:

$$i = \frac{dq}{dt} = -\omega Q_{\max}\sin\omega t$$
$$= -(847\text{ rad/s})(180\ \mu\text{C})\sin[(847\text{ Hz})(0.001\,00\text{ s})]$$
$$= \boxed{-114\text{ mA}}$$

Section 32.6 The *RLC* Circuit

P32.56 We choose to call positive current clockwise in Figure 32.15. It drains charge from the capacitor according to $i = -\frac{dq}{dt}$. A clockwise trip around the circuit then gives

$$+\frac{q}{C} - iR - L\frac{di}{dt} = 0$$

or $+\frac{q}{C} + \frac{dq}{dt}R + L\frac{d}{dt}\frac{dq}{dt} = 0,$ identical to Equation 32.28.

P32.57 (a) The frequency of damped oscillations is given by Equation 32.32:

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$
$$= \sqrt{\frac{1}{(2.20\times10^{-3}\text{H})(1.80\times10^{-6}\text{F})} - \left(\frac{7.60}{2(2.20\times10^{-3}\text{H})}\right)^2}$$
$$= 1.58\times10^4\ \text{ rad/s}$$

Therefore, $f_d = \frac{\omega_d}{2\pi} = \frac{1.58\times10^4\ \text{ rad/s}}{2\pi} = \boxed{2.51\text{ kHz}}$.

(b) Critical damping occurs when $\omega_d = 0$, or when

$$R_c = \sqrt{\frac{4L}{C}} = \sqrt{\frac{4\left(2.20\times 10^{-3}\text{ H}\right)}{1.80\times 10^{-6}\text{ F}}} = \boxed{69.9\ \Omega}$$

P32.58 (a) The angular frequency of undamped oscillations is

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.500\text{ H})\left(0.100\times 10^{-6}\text{ F}\right)}} = 4.47\times 10^3\text{ rad/s}$$

$$= \boxed{4.47\ \text{krad/s}}$$

(b) The frequency of the damped oscillations is

$$\omega_d = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

$$= \sqrt{\frac{1}{(0.500\text{ H})\left(0.100\times 10^{-6}\text{ F}\right)} - \left[\frac{1.00\times 10^3\ \Omega}{2(0.500\text{ H})}\right]^2}$$

$$= \boxed{4.36\ \text{krad/s}}$$

(c) $$\frac{\Delta\omega}{\omega_0}\times 100\% = \frac{\omega_d - \omega_0}{\omega_0}\times 100\% = \frac{4.36 - 4.47}{4.47}\times 100\% = \boxed{-2.53\%}$$

P32.59 (a) The charge on the capacitor is given by Equation 32.31:

$$q = Q_{\max} e^{-Rt/2L}\cos\omega_d t \qquad \text{so} \qquad I_i \propto e^{-Rt/2L}$$

When the amplitude of the oscillation falls to 50.0% of its initial value, we have

$$0.500 = e^{-Rt/2L} \qquad \text{and} \qquad \frac{Rt}{2L} = -\ln(0.500)$$

Then,

$$t = -\frac{2L}{R}\ln(0.500) = \boxed{0.693\left(\frac{2L}{R}\right)}$$

(b) The initial energy of the circuit is $U_0 \propto Q_{\max}^2$. When $U = 0.500U_0$,

$$q = \sqrt{0.500}Q_{\max} = 0.707Q_{\max}$$

Then,

$$t = -\frac{2L}{R}\ln(0.707) = \boxed{0.347\left(\frac{2L}{R}\right)} \quad \text{(half as long as part (a))}$$

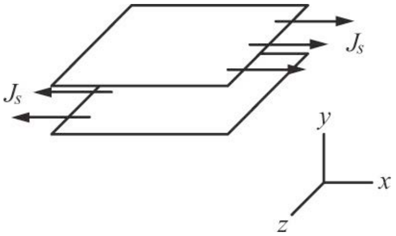
Additional Problems

P32.60 (a) Let Q represent the magnitude of the opposite charges on the plates of a parallel plate capacitor, the two plates having area A and separation d. The negative plate creates an electric field $\vec{\mathbf{E}} = \frac{Q}{2\epsilon_0 A}$ toward itself. It exerts on the positive plate force $\vec{\mathbf{F}} = \frac{Q^2}{2\epsilon_0 A}$ toward the negative plate. The total field between the plates is $\frac{Q}{\epsilon_0 A}$. The energy density is $u_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 \frac{Q^2}{\epsilon_0^2 A^2} = \frac{Q^2}{2\epsilon_0 A^2}$. Modeling this as a negative or inward pressure, we have for the force on one plate $F = PA = \frac{Q^2}{2\epsilon_0 A}$, in agreement with our first analysis.

(b) The lower of the two current sheets shown creates above it magnetic field $\vec{\mathbf{B}} = \frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right)$. Let ℓ and w represent the length and width of each sheet. The upper sheet carries current $J_s w$ and feels force

$$\vec{\mathbf{F}} = I\vec{\ell} \times \vec{\mathbf{B}} = J_s w\left[\ell\hat{\mathbf{i}} \times \left(-\frac{\mu_0 J_s}{2}\hat{\mathbf{k}}\right)\right] = \frac{\mu_0 w \ell J_s^2}{2}\hat{\mathbf{j}}$$

The force per area is $P = \frac{F}{\ell w} = \boxed{\frac{\mu_0 J_s^2}{2}}$.



ANS. FIG. P32.60(b)

(c) Between the two sheets, each sheet contributes the same field, so the total magnetic field is $\frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) + \frac{\mu_0 J_s}{2}\left(-\hat{\mathbf{k}}\right) = \mu_0 J_s \hat{\mathbf{k}}$, with magnitude $\boxed{B = \mu_0 J_s}$. Outside the space they enclose, the fields of the separate sheets are in opposite directions and add to $\boxed{\text{zero}}$.

(d) $u_B = \frac{1}{2\mu_0}B^2 = \frac{\mu_0^2 J_s^2}{2\mu_0} = \boxed{\frac{\mu_0 J_s^2}{2}}$

(e) $\boxed{\text{The energy density found in part (d) agrees with the magnetic pressure found in part (b).}}$

P32.61 (a) The voltage across the inductor is given by

$$\mathcal{E}_L = -L\frac{di}{dt} = -(1.00\text{ mH})\frac{d(20.0t)}{dt} = \boxed{-20.0\text{ mV}}$$

(b) The charge that flows into the capacitor is

$$q = \int_0^t I\,dt = \int_0^t (20.0t)\,dt = 10.0t^2$$

Going across the capacitor in the directon of the current, the potential drops from the positive to the negative side, so

$$\Delta V_C = \frac{-q}{C} = \frac{-10.0t^2}{1.00\times 10^{-6}\text{ F}} = \boxed{-10.0t^2}$$

where ΔV_C is in megavolts and *t* is in seconds.

(c) When $\frac{q^2}{2C} \geq \frac{1}{2}Li^2$, or

$$\frac{\left(-10.0t^2\right)^2}{2\left(1.00\times 10^{-6}\right)} \geq \frac{1}{2}\left(1.00\times 10^{-3}\right)(20.0t)^2,$$

then $100t^4 \geq \left(400\times 10^{-9}\right)t^2$

The earliest time this is true is at

$$t = \sqrt{4.00\times 10^{-9}}\ s = \boxed{63.2\ \mu\text{s}}$$

P32.62 (a) The voltage across the inductor is given by

$$\mathcal{E}_L = -L\frac{di}{dt} = -L\frac{d}{dt}(Kt) = \boxed{-LK}$$

(b) The current into the capacitor is $i = \frac{dq}{dt}$, so the charge that flows into the capacitor is

$$q = \int_0^t i\,dt = \int_0^t Kt\,dt = \frac{1}{2}Kt^2$$

Going across the capacitor in the direction of the current, the potential drops from the positive to the negative side, so

$$\Delta V_C = \frac{-q}{C} = \boxed{-\frac{Kt^2}{2C}}$$

(c) When $\frac{1}{2}C(\Delta V_C)^2 \geq \frac{1}{2}LI^2$,

$$\frac{1}{2}C\left(\frac{K^2t^4}{4C^2}\right) \geq \frac{1}{2}L\left(K^2t^2\right)$$

Thus, the energy in the capacitor begins to exceed the energy in the inductor after $t = \boxed{2\sqrt{LC}}$.

P32.63 The total energy equals the energy in the capacitor and inductor:

$$\frac{1}{2}\frac{Q^2}{C} = \frac{1}{2C}\left(\frac{Q}{2}\right)^2 + \frac{1}{2}Li^2$$

so $$i = \sqrt{\frac{3Q^2}{4CL}}$$

The flux through each turn of the coil is

$$\Phi_B = \frac{Li}{N} = \boxed{\frac{q}{2N}\sqrt{\frac{3L}{C}}}$$

where *N* is the number of turns.

P32.64 (a) The inductor has no resistance, therefore it has voltage across it. It behaves as a $\boxed{\text{short circuit}}$.

(b) The battery sees an equivalent resistance

$$4.00\ \Omega + \left(\frac{1}{4.00\ \Omega} + \frac{1}{8.00\ \Omega}\right)^{-1} = 6.67\ \Omega$$

The battery current is

$$\frac{10.0\ \text{V}}{6.67\ \Omega} = 1.50\ \text{A}$$

The voltage across the parallel combination of resistors is

$$10.0\ \text{V} - (1.50\ \text{A})(4.00\ \Omega) = 4.00\ \text{V}$$

The current in the 8-Ω resistor and the inductor is

$$\frac{4.00\ \text{V}}{8.00\ \Omega} = \boxed{500\ \text{mA}}$$

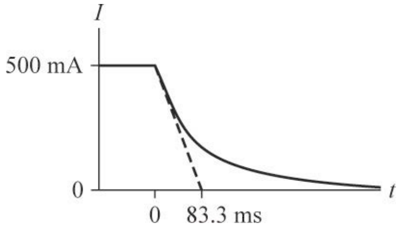
(c) The energy stored in the inductor for $t < 0$ is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(1.00\ \text{H})(0.500\ \text{A})^2 = \boxed{125\ \text{mJ}}$$

(d) $\boxed{\text{The energy becomes 125 mJ of additional internal energy in the 8-}\Omega\text{ resistor and the 4-}\Omega\text{ resistor in the middle branch.}}$

(e) See ANS. FIG. P32.64(e). The current decreases from 500 mA toward zero, showing exponential decay with a time constant

$$\tau = \frac{L}{R} = \frac{1.00\ \text{H}}{3(4.00\ \Omega)} = 0.083\ 3\ \text{s} = 8.33\ \text{ms}$$



ANS. FIG. P32.64(e)

P32.65 The voltages are related as

$$\mathcal{E} - iR - L\frac{di}{dt} = 0 \quad \rightarrow \quad \mathcal{E} = iR + L\frac{di}{dt} = 0$$

When the current is increasing:

$$9.00\ \text{V} = (2.00\ \text{A})R + L(0.500\ \text{A/s}) \qquad \textbf{[1]}$$

When the current is decreasing:

$$5.00\ \text{V} = (2.00\ \text{A})R + L(-0.500\ \text{A/s}) \qquad \textbf{[2]}$$

(a) Subtracting [2] from [1] gives

$$9.00\ \text{V} - 5.00\ \text{V} = L(1.00\ \text{A/s}) \rightarrow \boxed{L = 4.00\ \text{H}}$$

(b) Substituting the value for L in either equation gives

$$7.00\ \text{V} = (2.00\ \text{A})R \rightarrow \boxed{R = 3.50\ \Omega}$$

P32.66 (a) Initially, the current is zero because of the emf induced in the coil resists an increase in the current. $\boxed{\text{Just after the circuit is connected, the potential difference across the resistor is 0 and the emf across the coil is 24.0 V.}}$

(b) After several seconds, the current has reached a steady value and does not change. $\boxed{\text{After several seconds, the potential difference across the resistor is 24.0 V and that across the coil is 0.}}$

(c) The resistor voltage and inductor voltage always add to 24 V. The resistor voltage increases monotonically, so the two voltages are equal to each other, both being 12.0 V, just once. The time is given by

$$V = iR = R\mathcal{E}/R(1 - e^{-Rt/L})$$

Substituting,

$$12\text{ V} = 24\text{ V}(1\ -\ e^{-6\Omega t/0.005\text{ H}})$$

$$0.5 = e^{-1\,200\,t} \rightarrow 1\,200\,t = \ln\ 2 \quad \rightarrow \quad t = 0.578\text{ ms}$$

$$\boxed{\begin{array}{l}\text{The two voltages are equal to each other, both being 12.0 V,}\\ \text{just once, at 0.578 ms after the circuit is connected.}\end{array}}$$

(d) There is now no battery in the circuit, so the current decays to zero. The resistor and inductor are in parallel because they have common connections on each side. $\boxed{\text{As the current decays the potential difference across the resistor is always equal to the emf across the coil.}}$

P32.67 (a) At the center, $B \approx \dfrac{N\mu_0 i}{2R}$.

So the coil creates flux through itself

$$\Phi_B = BA\cos\theta = \frac{N\mu_0 i}{2R}\pi R^2 \cos 0° = \frac{1}{2}N\mu_0\pi iR$$

The inductance is

$$L = N\frac{\Phi_B}{i} \approx N\left(\frac{N\mu_0\pi IR}{2i}\right) \approx \boxed{\frac{1}{2}\mu_0\pi N^2 R}$$

(b) To find the inductance of the circuit, we compute its radius from

$$2\pi R = 3(0.300\text{ m}) \quad \rightarrow \quad R = 0.143\text{ m}$$

Then, from the expression found in part(a), the inductance is

$$L \approx \frac{1}{2}\mu_0\pi N^2 R = \frac{1}{2}\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\pi\left(1^2\right)(0.143\text{ m})$$

$$= 2.83 \times 10^{-7}\ \text{H}$$

$$\boxed{L \sim 10^{-7}\ \text{H}}$$

(c) The time constant is

$$\tau = \frac{L}{R} \approx \frac{2.83\times10^{-7}\ \text{H}}{270\ \Omega} = 1.05\times10^{-9}\ \text{s} \approx \boxed{10^{-9}\ \text{s}} \qquad \boxed{\frac{L}{R} \sim 1\ \text{ns}}$$

P32.68 We calculate the angular frequency of the circuit from Equation 32.32:

$$\omega_d = \left[\frac{1}{LC} - \left(\frac{R}{2L}\right)^2\right]^{1/2}$$

$$= \left[\frac{1}{(32.0\times10^{-3}\ \text{H})(500\times10^{-6}\ \text{F})} - \left(\frac{16.0\ \Omega}{2(32.0\times10^{-3}\ \text{H})}\right)^2\right]^{1/2}$$

$$= 0$$

The fact that the angular frequency at which the circuit oscillates is zero tells you that the circuit is critically damped. There will be no decaying oscillations. The critical resistance is given by

$$R_c = \sqrt{\frac{4L}{C}} = \sqrt{\frac{4(32.0\times10^{-3}\ \text{H})}{500\times10^{-6}\ \text{F}}} = 16.0\ \Omega$$

which is just the resistance that you are using for your experiment.

P32.69 The emf across the inductor is given by

$$\mathcal{E} = -L\frac{di}{dt} = -L\frac{\Delta i}{\Delta t} = -50\frac{\Delta i}{\Delta t}$$

where the rate of change of current $\frac{\Delta i}{\Delta t}$ is in amperes per second (A/s), and the induced emf $\mathcal{E}$ is in millivolts (mV).

Between $t = 0$ and $t = 1$ ms: $\frac{\Delta i}{\Delta t} = 2$ $\mathcal{E} = -100$ mV

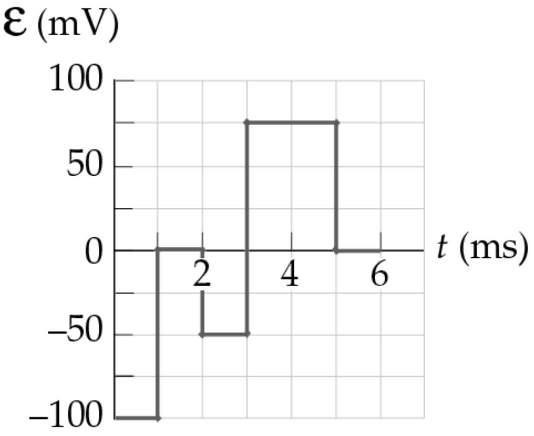
Between $t = 1$ ms and $t = 2$ ms: $\frac{\Delta i}{\Delta t} = 0$ $\mathcal{E} = 0$

Between $t = 2$ ms and $t = 3$ ms: $\frac{\Delta i}{\Delta t} = 1$ $\mathcal{E} = -50$ mV

Between $t = 3$ ms and $t = 5$ ms: $\frac{\Delta i}{\Delta t} = -\frac{3}{2}$ $\mathcal{E} = +75$ mV

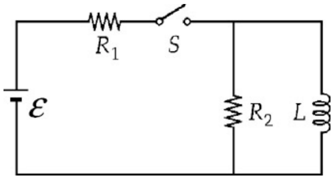
Between $t = 5$ ms and $t = 6$ ms: $\frac{\Delta i}{\Delta t} = 0$ $\mathcal{E} = 0$

The graph of $\mathcal{E}$ is shown in ANS. FIG. P32.69.



ANS. FIG. P32.69

P32.70 (a) $\boxed{i_1 = i_2 + i}$



ANS. FIG. P32.70

(b) For the left-hand loop,

$$\boxed{\mathcal{E} - i_1R_1 - i_2R_2 = 0}$$

(c) For the outside loop,

$$\boxed{\mathcal{E} - i_1R_1 - L\frac{di}{dt} = 0}$$

(d) Substitute the equation for i_1 from part (a) into the equation in part (b):

$$\mathcal{E} - (i_2 + i)R_1 - i_2R_2 = 0 \quad \rightarrow \quad i_2 = \frac{\mathcal{E} - iR_1}{R_1 + R_2}$$

Substitute the equation for i_1 from part (a) into the equation in part (c):

$$\mathcal{E} - (i_2 + i)R_1 - L\frac{di}{dt} = 0 \quad \rightarrow \quad i_2 = \frac{\mathcal{E} - L\frac{di}{dt}}{R_1} - i$$

Equate the two expressions for i_2:

$$\frac{\mathcal{E} - iR_1}{R_1 + R_2} = \frac{\mathcal{E} - L\frac{di}{dt}}{R_1} - i$$

Expanding and solving,

$$\mathcal{E} - L\frac{di}{dt} = \left(\frac{\mathcal{E} - iR_1}{R_1 + R_2} + i\right)R_1$$

$$= \left[\frac{\mathcal{E} - iR_1 + i(R_1 + R_2)}{R_1 + R_2}\right]R_1 = \left(\frac{\mathcal{E} + iR_2}{R_1 + R_2}\right)R_1$$

$$L\frac{di}{dt} = \mathcal{E} - \left(\frac{\mathcal{E} + iR_2}{R_1 + R_2}\right)R_1$$

$$= \frac{\mathcal{E}(R_1 + R_2) - (\mathcal{E} + iR_2)R_1}{R_1 + R_2} = \frac{\mathcal{E}(R_2) - (iR_2)R_1}{R_1 + R_2}$$

And finally,

$$\mathcal{E}\frac{R_2}{R_1 + R_2} - i\frac{R_1R_2}{R_1 + R_2} - L\frac{di}{dt} = 0$$

Calling $\mathcal{E}' = \mathcal{E}\frac{R_2}{R_1 + R_2}$ and $R' = \frac{R_1R_2}{R_1 + R_2}$, the equation for *i* can be written

$$\boxed{\mathcal{E}' - iR' - L\frac{di}{dt} = 0}$$

(e) This is of the same form as the Equation 32.6 in the text for a simple *RL* circuit, so its solution is of the same form as Equation 32.7:

$$i = \frac{\mathcal{E}'}{R'}\left(1 - e^{-R't/L}\right)$$

where

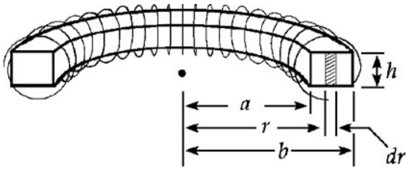
$$\frac{\mathcal{E}'}{R'} = \frac{\mathcal{E}R_2/(R_1 + R_2)}{R_1R_2/(R_1 + R_2)} = \frac{\mathcal{E}}{R_1}$$

Thus,

$$i = \frac{\mathcal{E}}{R_1}\left(1 - e^{-R't/L}\right) \quad \text{where} \quad R' = \frac{R_1R_2}{R_1 + R_2}$$

P32.71 See ANS. FIG. 32.71. The magnetic field inside the toroid is given by

$$B = \frac{\mu_0 Ni}{2\pi r}$$



ANS. FIG. P32.71

The magnetic flux through a cross-sectional area $A = h(b - a)$ is given by

$$\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 N i}{2\pi r} h\,dr = \frac{\mu_0 N i h}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 N i h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Thus, the inductance is

$$L = \frac{N\Phi_B}{i} = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Substituting numerical values, we obtain

$$L = \frac{\mu_0 (500)^2 (0.010\ 0\ \text{m})}{2\pi}\ln\left(\frac{12.0\ \text{cm}}{10.0\ \text{cm}}\right) = \boxed{91.2\ \mu\text{H}}$$

P32.72 See ANS. FIG. P23.71. $B = \dfrac{\mu_0 N i}{2\pi r}$ inside the toroid.

Calculate the flux through a cross-sectional area $A = h(b - a)$:

$$\Phi_B = \int B\,dA = \int_a^b \frac{\mu_0 N i}{2\pi r} h\,dr = \frac{\mu_0 N i h}{2\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 N i h}{2\pi}\ln\left(\frac{b}{a}\right)$$

Thus, the inductance is

$$L = \frac{N\Phi_B}{i} = \boxed{\frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right)}$$

P32.73 (a) $U_B = \dfrac{1}{2}Li^2 = \dfrac{1}{2}(50.0\ \text{H})(50.0\times10^3\ \text{A})^2 = \boxed{6.25\times10^{10}\ \text{J}}$

(b) Two adjacent turns are parallel wires carrying current in the same direction. Since the loops have such large radius, a one-meter section can be regarded as straight.

Then one wire creates a field of $B = \dfrac{\mu_0 i}{2\pi r}$.

This causes a force on the next wire of $F = i\ell B\sin\theta$,

giving a force per unit length $\dfrac{F}{\ell} = i\dfrac{\mu_0 i}{2\pi r}\sin 90° = \dfrac{\mu_0 i^2}{2\pi r}$.

Evaluating,

$$\frac{F}{\ell} = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\frac{\left(50.0\times10^3\ \text{A}\right)^2}{2\pi(0.250\ \text{m})} = \boxed{2\ 000\ \text{N/m}}$$

P32.74 For an *RL* circuit, $i = I_i e^{-(R/L)t}$ and

$$\frac{i}{I_i} = 1 - 10^{-9} = e^{-(R/L)t} \cong 1 - \frac{R}{L}t$$

$$\frac{R}{L}t = 10^{-9}$$

so

$$R_{\max} = \frac{L(10^{-9})}{t} = \frac{(3.14\times10^{-8}\ \text{H})(10^{-9})}{(2.50\ \text{yr})}\left(\frac{1\ \text{yr}}{3.16\times10^{7}\text{s}}\right)$$

$$= \boxed{3.97\times10^{-25}\ \Omega}$$

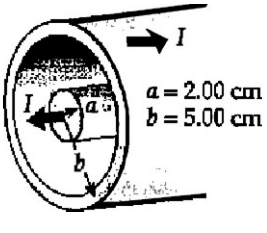
(If the ring were of purest copper, of diameter 1 cm, and cross-sectional area 1 mm^2, its resistance would be at least 10^{-6} Ω.)

P32.75 Find the current in the cylinder.

$$P = i\Delta V \quad \rightarrow \quad i = \frac{P}{\Delta V} = \frac{1.00\times10^{9}\ \text{W}}{200\times10^{3}\ \text{V}} = 5.00\times10^{3}\ \text{A}$$

From Ampère's law,

$$B(2\pi r) = \mu_0 i_{\text{enclosed}} \qquad \text{or} \qquad B = \frac{\mu_0 i_{\text{enclosed}}}{2\pi r}$$



ANS. FIG. P32.75

(a) At $r = a = 0.020\ 0$ m, $i_{\text{enclosed}} = 5.00\times10^{3}\ \text{A}$ and

$$B = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5.00\times10^{3}\ \text{A})}{2\pi(0.020\ 0\ \text{m})} = 0.050\ 0\ \text{T} = \boxed{50.0\ \text{mT}}$$

(b) At $r = b = 0.050\ 0$ m, $i_{\text{enclosed}} = i = 5.00\times10^{3}\ \text{A}$ and

$$B = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5.00\times10^{3}\ \text{A})}{2\pi(0.050\ 0\ \text{m})} = 0.020\ 0\ \text{T} = \boxed{20.0\ \text{mT}}$$

(c) The energy density is $u_B = \frac{B^2}{2\mu_0}$:

$$U_B = \int u\,dV = \int_a^b \frac{[B(r)]^2(2\pi r\ell dr)}{2\mu_0} = \frac{\mu_0 i^2\ell}{4\pi}\int_a^b \frac{dr}{r} = \frac{\mu_0 i^2 \ell}{4\pi}\ln\left(\frac{b}{a}\right)$$

$$U_B = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5.00\times10^3\ \text{A})^2(1\,000\times10^3\ \text{m})}{4\pi}\times\ln\left(\frac{5.00\ \text{cm}}{2.00\ \text{cm}}\right)$$

$$= 2.29\times10^6\ \text{J} = \boxed{2.29\ \text{MJ}}$$

(d) The magnetic field created by the inner conductor exerts a force of repulsion on the current in the outer sheath. The strength of this field, from part (b), is 20.0 mT. Consider a small rectangular section of the outer cylinder of length ℓ and width w.

It carries a current of $(5.00\times10^3\ \text{A})\left(\frac{w}{2\pi(0.050\,0\ \text{m})}\right)$

and experiences an outward force

$$F = i\ell B\sin\theta = \frac{(5.00\times10^3\ \text{A})w}{2\pi(0.050\,0\ \text{m})}\ell(20.0\times10^{-3}\ \text{T})\sin 90.0°$$

The pressure on it is

$$P = \frac{F}{A} = \frac{F}{w\ell} = \frac{(5.00\times10^3\ \text{A})(20.0\times10^{-3}\ \text{T})\cancel{w\ell}}{2\pi(0.050\,0\ \text{m})\cancel{w\ell}} = \boxed{318\ \text{Pa}}$$

P32.76 (a) The magnetic field inside the solenoid is given by $B = \frac{\mu_0 N i}{\ell}$:

$$B = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1\,400)(2.00\ \text{A})}{1.20\ \text{m}}$$

$$= 2.93\times10^{-3}\ \text{T} = \boxed{2.93\ \text{mT}}$$

ANS. FIG. P32.76

(b) The energy density of the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{\left(2.93\times10^{-3}\ \text{T}\right)^2}{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)} = \left(3.42\ \text{J/m}^3\right)\left(\frac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)$$
$$= 3.42\ \text{N/m}^2 = \boxed{3.42\ \text{Pa}}$$

(c) The supercurrents must be clockwise to produce a downward magnetic field to cancel the upward field of the current in the windings.

(d) The field of the windings is upward and radially outward around the top of the solenoid. It exerts a force radially inward and upward on each bit of the clockwise supercurrent. The total force on the supercurrents in the bar is upward. You can think of it as a force of repulsion between the solenoid with its north end pointing up, and the core, with its north end pointing down.

(e) $F = PA = (3.42\ \text{Pa})\left[\pi\left(1.10\times10^{-2}\ \text{m}\right)^2\right] = 1.30\times10^{-3}\ \text{N} = \boxed{1.30\ \text{mN}}$

Note that we have not proved that energy density is pressure. In fact, it is not in some cases. Chapter 21 proved that the pressure is two-thirds of the translational energy density in an ideal gas.

P32.77 From Ampère's law, the magnetic field at distance $r \le R$ is found as:

$$B(2\pi r) = \mu_0 j\left(\pi r^2\right) = \mu_0\left(\frac{i}{\pi R^2}\right)\left(\pi r^2\right),, \quad \text{or} \quad B = \frac{\mu_0 i r}{2\pi R^2}$$

The magnetic energy per unit length within the wire is then

$$\frac{U_B}{\ell} = \int_0^R \frac{B^2}{2\mu_0}\left(2\pi r\,dr\right) = \frac{\mu_0 i^2}{4\pi R^4}\int_0^R r^3\,dr = \frac{\mu_0 i^2}{4\pi R^4}\left(\frac{R^4}{4}\right) = \boxed{\frac{\mu_0 i^2}{16\pi}}$$

This is independent of the radius of the wire.

Challenge Problems

P32.78 (a) It has a magnetic field, and it stores energy, so $L = \frac{2U_B}{i^2}$ is non-zero.

(b) Every field line goes through the rectangle between the conductors.

(c) When the wires carry current *i*, magnetic flux passes through the rectangle bordered by the wires (surface to surface of the wires):

$$L = \frac{\Phi}{i} = \frac{1}{i}\int_{y=a}^{w-a} B\,dA$$

where *y* is measured from the center of the lower wire, *dA* is a rectangular area element of length *x* and width *dy*, and *B* is the magnitude of the net magnetic field generated by the upper and lower wires that passes through *dA*. The inductance is

$$L = \frac{1}{i}\int_{a}^{w-a}\left[\frac{\mu_0 i}{2\pi y} + \frac{\mu_0 i}{2\pi(w-y)}\right]x\,dy$$

We can simplify this calculation by noting that by the symmetry of the arrangement, each conductor contributes equally to the field that passes through the area between them. Thus, the total inductance of both wires is twice the inductance of one wire. The inductance due to the lower wire is

$$L_{\text{lower}} = \frac{1}{i}\int_{a}^{w-a}\frac{\mu_0 i}{2\pi y}x\,dy = \frac{\mu_0 x}{2\pi}\ln y\Big|_{a}^{w-a} = \frac{\mu_0 x}{2\pi}\left[\ln(w-a) - \ln a\right]$$
$$= \frac{\mu_0 x}{2\pi}\ln\left(\frac{w-a}{a}\right)$$

The inductance due to both wires is twice this: $L = \dfrac{\mu_0 x}{\pi}\ln\left(\dfrac{w-a}{a}\right)$.

P32.79 The total magnetic energy is the volume integral of the energy density,

$$u_B = \frac{B^2}{2\mu_0}$$

Because *B* changes with position, u_B is not constant. For $B = B_0\left(\dfrac{R}{r}\right)^2$,

$$u_B = \left(\frac{B_0^2}{2\mu_0}\right)\left(\frac{R}{r}\right)^4$$

Next, we set up an expression for the magnetic energy in a spherical shell of radius *r* and thickness *dr*. Such a shell has a volume $4\pi r^2 dr$, so the energy stored in it is

$$dU_B = u_B\left(4\pi r^2 dr\right) = \left(\frac{2\pi B_0^2 R^4}{\mu_0}\right)\frac{dr}{r^2}$$

We integrate this expression for $r = R$ to $r = \infty$ to obtain the total magnetic energy outside the sphere. This gives

$$U_B = \boxed{\frac{2\pi B_0^2 R^3}{\mu_0}}$$

Substituting numerical values,

$$U_B = \frac{2\pi B_0^2 R^3}{\mu_0} = \frac{2\pi\left(5.00\times10^{-5}\ \text{T}\right)^2\left(6.00\times10^{6}\ \text{m}\right)^3}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)}$$

$$= \boxed{2.70\times10^{18}\ \text{J}}$$

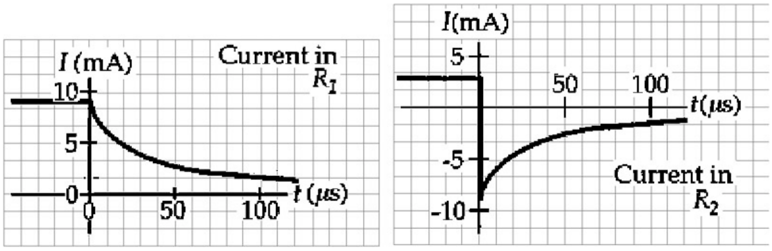
P32.80 (a) While steady-state conditions exist, a 9.00 mA flows clockwise around the right loop of the circuit. Immediately after the switch is opened, a 9.00 mA current will flow around the outer loop of the circuit. Applying Kirchhoff's loop rule going clockwise around this loop gives:

$$+\mathcal{E} - \left[(2.00 + 6.00)\times10^3\ \Omega\right]\left(9.00\times10^{-3}\ \text{A}\right) = 0$$

$$\mathcal{E} = \boxed{72.0\ \text{V}}$$

(b) Starting at point *a*, the potential rises across the inductor then falls across resistors R_2 and R_1. The positive answer in part (a) means that $\boxed{\text{point b}}$ is the higher potential.

(c) The currents in R_1 and R_2 are shown in ANS. FIG. P32.80(c).below. After $t = 0$, the current in R_1 decreases from an initial value of 9.00 mA according to $i = I_i e^{-Rt/L}$. Taking the original current direction as positive in each resistor, the current decreases from +9.00 mA (to the right) to zero in R_1. In R_2 the current jumps from +3.00 mA (downward) to –9.00 mA (upward) and then decreases in magnitude to zero. The time constant of each decay is 0.4 H/8 000 Ω = 50 μs. Thus we draw each current dropping to $1/e$ = 36.8% of its original value = 3.3 μA at the 50 μs instant.



ANS. FIG. P32.80(c)

(d) After the switch is opened, the current around the outer loop decays as

$$i = I_i e^{-Rt/L}$$

with $I_i = 9.00$ mA, $R = 8.00$ kΩ, and $L = 0.400$ H.

Thus, when the current has reached a value $i = 2.00$ mA, the elapsed time is:

$$t = \left(\frac{L}{R}\right)\ln\left(\frac{I_i}{i}\right) = \left(\frac{0.400\text{ H}}{8.00\times 10^3\ \Omega}\right)\ln\left(\frac{9.00\text{ mA}}{2.00\text{ mA}}\right)$$

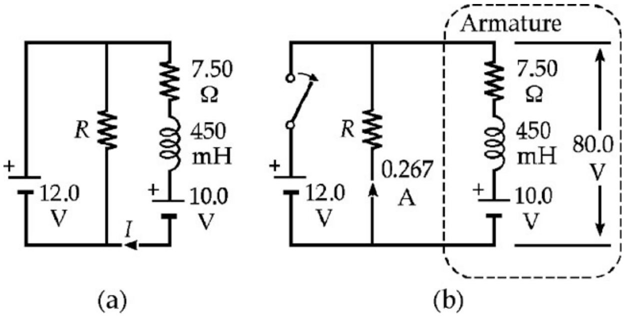
$$= 7.52\times 10^{-5}\text{ s} = \boxed{75.2\ \mu\text{s}}$$

P32.81 When the switch is closed, as shown in ANS. FIG. P32.81(a), the current in the inductor is *i*:

$$12.0 - 7.50i - 10.0 = 0 \rightarrow i = 0.267\text{ A}$$

When the switch is opened, as shown in ANS. FIG. P32.81(b), the initial current in the inductor remains at 0.267 A. Across resistor *R* and the armature, $iR = \Delta V$ and

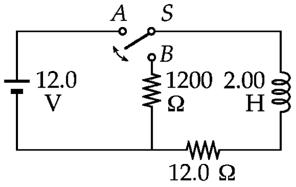
$$(0.267\text{ A})R \le 80.0\text{ V} \quad \rightarrow \quad \boxed{R \le 300\ \Omega}$$



ANS. FIG. P32.81

***P32.82** (a) After a long time,

$$i = \frac{\mathcal{E}}{R} = \frac{12.0\text{ V}}{12.0\ \Omega} = \boxed{1.00\text{ A}}$$



ANS. FIG. P32.82

(b) With the switch thrown to position *b*, the emf is no longer part of the circuit. The initial current is 1.00 A:

$$\Delta V_{12} = (1.00\text{ A})(12.00\ \Omega) = \boxed{12.0\text{ V}}$$

$$\Delta V_{1\,200} = (1.00\text{ A})(1\,200\ \Omega) = \boxed{1.20\text{ kV}}$$

$$\Delta V_L = \Delta V_R = 1\,200\text{ V} + 12.0\text{ V} = \boxed{1.21\text{ kV}}$$

(c) With the switch thrown to position b, $\mathcal{E} = 0$, so

$$\Delta V_L = \Delta V_R = iR_{\text{eff}} = i(1\ 200\ \Omega + 12.0\ \Omega) = i(1.21\ \text{k}\Omega)$$

Then, when the voltage across the inductor has reached 12.0 V,

$$i = \frac{\Delta V_L}{R_{\text{eff}}} = \frac{12.0\ \text{V}}{1.21\ \text{k}\Omega} = 9.90 \times 10^{-3}\ \text{A}$$

The current in the inductor decays as $i = I_i e^{-Rt/L}$. Solving for the time t,

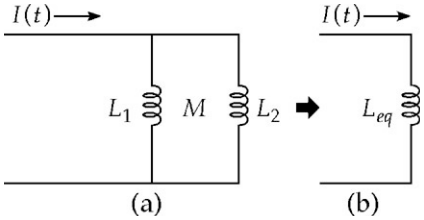
$$t = \frac{L}{R}\ln\left(\frac{I_i}{i}\right) = \left(\frac{2.00\ \text{H}}{1.21\ \text{k}\Omega}\right)\ln\left(\frac{1.00\ \text{A}}{9.90 \times 10^{-3}\ \text{A}}\right) = \boxed{7.62 \times 10^{-3}\ \text{s}}$$

P32.83 With $i = i_1 + i_2$, the voltage across the pair is:

$$\Delta V = -L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = -L_{eq}\frac{di}{dt} \qquad \textbf{[1]}$$

and

$$\Delta V = -L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = -L_{eq}\frac{di}{dt} \qquad \textbf{[2]}$$



ANS. FIG. P32.83

So, from [1], we have

$$-\frac{di_1}{dt} = \frac{\Delta V}{L_1} + \frac{M}{L_1}\frac{di_2}{dt}$$

which, when substituted into [2], gives

$$-L_2\frac{di_2}{dt} + M\left(\frac{(\Delta V)}{L_1} + \frac{M}{L_1}\frac{di_2}{dt}\right) = \Delta V$$

$$\left(-L_1L_2 + M^2\right)\frac{di_2}{dt} = \Delta V\left(L_1 - M\right) \qquad \textbf{[3]}$$

From [2], $-\frac{di_2}{dt} = \frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{di_1}{dt}$, which, when substituted into [1], gives

$$-L_1\frac{di_1}{dt} + M\left(\frac{\Delta V}{L_2} + \frac{M}{L_2}\frac{di_1}{dt}\right) = \Delta V$$

$$\left(-L_1L_2 + M^2\right)\frac{di_1}{dt} = \Delta V\left(L_2 - M\right) \qquad \textbf{[4]}$$

Adding [3] to [4], we have

$$\left(-L_1L_2 + M^2\right)\frac{di}{dt} = \Delta V\left(L_1 + L_2 - 2M\right)$$

So, $$L_{eq} = -\frac{\Delta V}{di/dt} = \boxed{\frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P32.2	1.36 μH
P32.4	(a) 1.97 mH; (b) 38.0 mA/s
P32.6	(a) 188 μT; (b) $3.33 \times 10^{-8}\ \text{T} \cdot \text{m}^2$; (c) 0.375 mH; (d) *B* and Φ_B are proportional to current: *L* is independent of current.
P32.8	(a) 5.90 mH; (b) 23.6 mV
P32.10	9.77 mm
P32.12	See P32.12 for full explanation.
P32.14	See ANS. FIG. P32.14.
P32.16	(a) 0.139 s; (b) 0.461 s
P32.18	See ANS. FIGs. P32.18(a), (b), and (c).
P32.20	7.67 mH
P32.22	See P32.22 for full explanation.
P32.24	(a) 2.00 ms; (b) 0.176 A; (c) 1.50 A; (d) 3.22 ms
P32.26	(a) $\dfrac{\mathcal{E}}{5R}\left(1 - e^{-5Rt/2L}\right)$; (b) $\dfrac{\mathcal{E}}{10R}\left(6 - e^{-5Rt/2L}\right)$
P32.28	For $t \le 0$, the current in the inductor is zero; for $0 \le t \le 200\ \mu$s, $i_L = (10.0\ \text{A})\left(1 - e^{-10\,000t/s}\right)$; for $t \ge 200\ \mu$s,, $(63.9\ \text{A})e^{-10\,000t/s}$
P32.30	(a) See P32.30(a) for full explanation; (b) See P32.30(b) for full explanation; (c) See P32.30(c) for full explanation; (d) Yes. See P32.30(d) for full explanation.
P32.32	64.8 mJ
P32.34	(a) 20.0 W; (b) 20.0 W; (c) 0; (d) 20.0 J
P32.36	$\dfrac{L}{2R}$
P32.38	(a) 66.0 W; (b) 45.0 W; (c) 21.0 W; (d) The power supplied by the battery is equal to the sum of the power delivered to the internal resistance of the coil and the power stored in the magnetic field; (e) Yes; (f) Just after $t = 0$, the current is very small, so the power delivered to the internal resistance of the coil (iR^2) is nearly zero, but the rate of the change of the current is large, so the power delivered to the magnetic field (*Ldi/dt*) is large, and nearly all the battery power is being stored in the magnetic field. Long after the connection is made, the current is not changing, so no power is being stored in the

magnetic field, and all the battery power is being delivered to the internal resistance of the coil.

P32.40 80.0 mH

P32.42 1.73 mH

P32.44 (a) $M_{12} = \mu_0 \pi R_2^{\ 2} N_1 N_2 / \ell$; (b) $M_{12} = \mu_0 \pi R_2^{\ 2} N_1 N_2 / \ell$; (c) They are the same.

P32.46 (a) See P32.46(a) for full explanation; (b) 3.95 nH

P32.48 608 pF

P32.50 0.220 H

P32.52 If the energy is split equally between the capacitor and inductor at some instant, the energy would be half this value, or 200 μJ. Therefore, there would be no time when each component stores 250 μJ.

P32.54 (a) 6.03 J; (b) 0.529 J; (c) 6.56 J

P32.56 See P32.56 for full explanation.

P32.58 (a) 4.47 krad/s; (b) 4.36 krad/s; (c) –2.53%

P32.60 (a) See P32.60(a) for full explanation; (b) $\frac{\mu_0 J_s^2}{2}$; (c) $B = \mu_0 J_s$, zero; (d) $\frac{\mu_0 J_s^2}{2}$; (e) The energy density found in part (d) agrees with the magnetic pressure found in part (b).

P32.62 (a) –*LK*; (b) $-\frac{Kt^2}{2C}$; (c) $2\sqrt{LC}$

P32.64 (a) short circuit; (b) 500 mA; (c) 125 mJ; (d) The energy becomes 125 mJ of additional internal energy in the 8-Ω resistor and the 4-Ω resistor in the middle branch; (e) See ANS FIG P32.64(e).

P32.66 (a) Just after the circuit is connected, the potential difference across the register is 0, and the emf across the coil is 24.0 V; (b) After several seconds, the potential difference across the resistor is 24.0 V and that across the coil is 0; (c) The two voltages are equal to each other, both being 12.0 V, just once, at 0.578 ms after the circuit is connected; (d) As the current decays, the potential difference across the resistor is always equal to the emf across the coil.

P32.68 See P32.68 for full explanation.

P32.70 (a) $i_1 = i_2 + i$; (b) $\mathcal{E} - i_1R_1 - i_2R_2 = 0$; (c) $\mathcal{E} - i_1R_1 - L\dfrac{di}{dt} = 0$;

(d) $\mathcal{E}' - iR' - L\dfrac{di}{dt} = 0$; (e) See P32.70(e) for full explanation.

P32.72 $\dfrac{\mu_0 N^2 h}{2\pi}\ln\left(\dfrac{b}{a}\right)$

P32.74 $3.97 \times 10^{-25}\ \Omega$

P32.76 (a) 2.93 mT; (b) 3.42 Pa; (c) The supercurrents must be clockwise to produce a downward magnetic field to cancel the upward field of the current in the windings; (d) The field of the windings is upward and radially outward around the top of the solenoid. It exerts a force radially inward and upward on each bit of the clockwise supercurrent. The total force on the supercurrents in the bar is upward; (e) 1.30 mN

P32.78 (a) It has a magnetic field, and it stores energy, so $L = \dfrac{2U_B}{i^2}$ is non-zero;

(b) Every field line goes through the rectangle between the conductors; (c) See P32.78(c) for full explanation.

P32.80 (a) 72.0 V; (b) point b; (c) See ANS. FIG. P32.80(c); (d) 75.2 μs

P32.82 (a) 1.00 A; (b) $\Delta V_{12} = 12.0\text{ V}$, $\Delta V_{1\,200} = 1.20\text{ kV}$, $\Delta V_L = 1.21\text{ kV}$

33

Alternating-Current Circuits

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ33.1 Answer (c). When a power source, AC or DC, is first connected to a *RL* combination, the presence of the inductor impedes the buildup of a current in the circuit. The value of the current starts at zero and increases as the back emf induced across the inductor decreases in magnitude.

OQ33.2 **(i)** Answer (e). Inductive reactance, $X_L = \omega L$, doubles when the frequency doubles, so the rms current is halved $\left(I_{\text{rms}} = \Delta V_{\text{rms}}/X_L\right)$.

(ii) Answer (b). Capacitive reactance, $X_C = 1/\omega C$, is cut in half when frequency doubles, so the rms current doubles $\left(I_{\text{rms}} = \Delta V_{\text{rms}}/X_C\right)$.

(iii) Answer (d). The resistance remains unchanged $\left(I_{\text{rms}} = \Delta V_{\text{rms}}/R\right)$.

OQ33.3 Answer (a). The voltage across the capacitor is proportional to the stored charge. This charge, and hence the voltage ΔV_C, is a maximum when the current has zero value and is in the process of reversing direction after having been flowing in one direction for a half cycle. Thus, the voltage across the capacitor lags behind the current by 90°. The capacitive reactance, $X_C = 1/\omega C$, decreases as frequency increases, causing the impedance to decrease and the current to increase.

OQ33.4 **(i)** Answer (d). $\Delta V_{\text{avg}} = \dfrac{\Delta V_{\text{max}}}{2}$.

(ii) Answer (c). The average of the squared voltage is $\left([\Delta V]^2\right)_{\text{avg}} = \dfrac{(\Delta V_{\text{max}})^2}{2}$. Then its square root is $\Delta V_{\text{rms}} = \dfrac{\Delta V_{\text{max}}}{\sqrt{2}}$.

OQ33.5 Answer (d). If the voltage is a maximum when the current is zero, the voltage is either leading or lagging the current by 90° (or a quarter cycle) in phase. Thus, the element could be *either* an inductor or a capacitor. It could not be a resistor since the voltage across a resistor is always in phase with the current. If the current and voltage were out of phase by 180°, one would be a maximum in one direction when the other was a maximum value in the opposite direction.

OQ33.6 **(i)** Answer (e). The voltage varies between +170 V and –170 V.

(ii) Answer (c). The average of a sine waveform is zero.

(iii) Answer (b). $\Delta V_{\text{rms}} = \Delta V_{\text{max}}/\sqrt{2} = 170\ \text{V}/\sqrt{2} = 120\ \text{V}$.

OQ33.7 **(i)** Answer (a). We have:

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

$$= \left\{(20.0\ \Omega)^2 + \left[2\pi(500\ \text{Hz})(120\times 10^{-3}\ \text{H}) - \frac{1}{2\pi(500\ \text{Hz})(0.750\times 10^{-6}\ \text{F})}\right]^2\right\}^{1/2}$$

$$Z = 51.5\ \Omega$$

and $I_{\text{rms}} = \Delta V_{\text{rms}}/Z = (120\ \text{V})/(51.5\ \Omega) = 2.33\ \text{A}$.

(ii) Answer (b). At the resonance frequency, $X_L = X_C$ and the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2} = R$. Thus, the rms current is

$$I_{\text{rms}} = \Delta V_{\text{rms}}/Z = (120\ \text{V})/(20.0\ \Omega) = 6.00\ \text{A}$$

OQ33.8 Answer (e) is *false*. In an *RLC* circuit, the instantaneous voltages Δv_R, Δv_L, and Δv_C (across the resistance, inductance, and capacitance respectively) are not in phase with each other. The instantaneous voltage Δv_R is in phase with the current, Δv_L leads the current by 90°, while Δv_C lags behind the current by 90°. The instantaneous values of these three voltages do add algebraically to give the instantaneous voltages across the *RLC* combination, but the rms voltages across these components do not add algebraically. The rms voltages across the three components must be added as vectors (or phasors) to obtain the correct rms voltage across the *RLC* combination.

OQ33.9 Answer (c). At resonance the inductive reactance and capacitive reactance cancel out.

OQ33.10 Answer (c). At resonance the inductive reactance and capacitive reactance add to zero: $\phi = \tan^{-1}[(X_L - X_C)/R] = 0$.

OQ33.11 The ranking is (a) > (d) > (b) > (c) > (e). At the resonance frequency $f_0 = 1\ 000$ Hz both X_L and X_C are equal: call their mutual value X. A high-Q value means the resonance has a small width, so X_L and X_C are also much larger than R at f_0. Inductive reactance X_L is proportional to frequency, and capacitive reactance X_C is inversely proportional to frequency. In terms of X, the choices have the values: (a) $f = f_0/2$, so $X_C = 2X$. (b) $f = 3f_0/2$, so $X_C = 2X/3$. (c) $f = f_0/2$, so $X_L = X/2$. (d) $f = 3f_0/2$, so $X_L = 3X/2$. (e) R is independent of frequency, and R is less than X. Thus, we have (a) $2X$ > (d) $3X/2$ > (b) $2X/3$ > (c) $X/2$ > (e) less than X.

OQ33.12 Answer (e). The battery produces a constant current in the primary coil, which generates a constant flux through the secondary coil. With no change in flux through the secondary coil, there is no induced voltage across the secondary coil.

OQ33.13 Answer (c). AC ammeters and voltmeters read rms values. With an oscilloscope you can read a maximum voltage, or test whether the average is zero.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ33.1 (a) The Q factor determines the selectivity of the radio receiver. For example, a receiver with a very low Q factor will respond to a wide range of frequencies and might pick up several adjacent radio stations at the same time. To discriminate between 102.5 MHz and 102.7 MHz requires a high-Q circuit.

(b) Typically, lowering the resistance in the circuit is the way to get a higher quality resonance.

CQ33.2 (a) The second letter in each word stands for the circuit element.

For an inductor *L*, the emf $\mathcal{E}$ leads the current *I*—thus ELI. For a capacitor *C*, the current leads the voltage across the device. In a circuit in which the capacitive reactance is larger than the inductive reactance, the current leads the source emf—thus ICE.

(b) CIVIL – in a capacitor C the current (I) leads voltage (represented by V), voltage leads current in an inductor L.

CQ33.3 The voltages are not added in a scalar form, but in a vector form, as shown in the phasor diagrams throughout the chapter. Kirchhoff's loop rule is true at any instant, but the voltages across different circuit elements are not simultaneously at their maximum values. Do not forget that an inductor can induce an emf in itself and that the voltage across it is 90° *ahead* of the current in the circuit in phase.

CQ33.4 (a) In an *RLC* series circuit, the phase angle depends on the source frequency. At very low frequency, the capacitor dominates the impedance and the phase angle is near –90°. At very high frequencies, the inductor dominates the impedance and the phase angle is near –90°.

(b) When the inductive reactance equals the capacitive reactance, the frequency is the resonance frequency; the phase angle is zero.

CQ33.5 In 1881, an assassin shot President James Garfield. The bullet was lost in his body. Alexander Graham Bell invented the metal detector in an effort to save the President's life. The coil is preserved in the Smithsonian Institution. The detector was thrown off by metal springs in Garfield's mattress, a new invention itself. Surgeons went hunting for the bullet in the wrong place. Garfield died.

CQ33.6 (a) The person is doing work at a rate of $P = Fv\cos\theta$.

(b) Compare the previous equation to $P = \Delta V_{\text{rms}} I_{\text{rms}} \cos\phi$. One can consider the emf as the "force" that moves the charges through the circuit, and the current as the "speed" of the moving charges. The cos θ factor measures the effectiveness of the cause in producing the effect. Theta is an angle in real space for the vacuum cleaner and phi is the analogous angle of phase difference between the emf and the current in the circuit.

CQ33.7 The circuit can be considered an *RLC* series circuit.

(a) Yes. The circuit is in resonance because the inductive reactance and capcitive reactance are equal, so the total impedance $Z = R$.

(b) Total power output by the emf $P_{\text{emf}} = I^2R_{\text{total}}$, where $R_{\text{total}} = 10\ \Omega$ (source resistance) + $10\ \Omega$ (load resistance) = $20\ \Omega$. Power delivered to the load $P_{\text{load}} = I^2R_L$, where $R_L = 10\ \Omega$. Fraction of average power delivered to the load to average power delivered by the source of emf:

$$\frac{P_{\text{load}}}{P_{\text{emf}}} = \frac{I^2R_{\text{load}}}{I^2R_{\text{total}}} = \frac{R_{\text{L}}}{R_{\text{total}}} = \frac{R_{\text{L}}}{R_{\text{source}} + R_{\text{L}}} = \frac{10\ \Omega}{20\ \Omega} = 0.5$$

(c) The resistance of the load could be increased to make a greater *fraction* of the emf's power go to the load. Then the emf would put out a lot less power and less power would reach the load.

CQ33.8 No. A voltage is only induced in the secondary coil if the flux through the core changes in time. No changing current, no changing flux, no induced voltage.

CQ33.9 (a) The capacitive reactance is proportional to the inverse of the frequency. At higher and higher frequencies, the capacitive reactance approaches zero, making a capacitor behave like a wire.

(b) As the frequency goes to zero, the capacitive reactance approaches infinity—the resistance of an open circuit.

CQ33.10 The ratio of turns indicates the ratio of voltages: $N_1/N_2 = \Delta V_1/\Delta V_2$, where $\Delta V_2 = 120$ V; therefore, $\Delta V_1 = 12$ kV. In its intended use, the transformer takes in energy by electric transmission at 12 kV and puts out nearly the same energy by electric transmission at 120 V. With the small generator putting energy into the secondary side of the transformer at 120 V, the primary side has 12 kV induced across it. It is deadly dangerous for the repairman.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 33.1 AC Sources

Section 33.2 Resistors in an AC Circuit

P33.1 (a) The rms voltage across the resistor is given by

$$\Delta V_{R,\text{rms}} = I_{\text{rms}}R = (8.00\ \text{A})(12.0\ \Omega) = \boxed{96.0\ \text{V}}$$

(b) From Equation 33.5,

$$\Delta V_{R,\max} = \sqrt{2}\left(\Delta V_{R,\text{rms}}\right) = \sqrt{2}(96.0\ \text{V}) = \boxed{136\ \text{V}}$$

(c) From Equation 33.4,

$$I_{\max} = \sqrt{2}I_{\text{rms}} = \sqrt{2}(8.00\text{ A}) = \boxed{11.3\text{ A}}$$

(d) $P_{\text{avg}} = I_{\text{rms}}^2 R = (8.00\text{ A})^2(12.0\ \Omega) = \boxed{768\text{ W}}$

P33.2 The rms voltage is

$$\Delta V_{\text{rms}} = \frac{170\text{ V}}{\sqrt{2}} = 120\text{ V}$$

(a) $P = \dfrac{(\Delta V_{\text{rms}})^2}{R} \quad \rightarrow \quad R = \dfrac{(120\text{ V})^2}{75.0\text{ W}} = \boxed{193\ \Omega}$

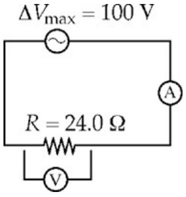
(b) $R = \dfrac{(120\text{ V})^2}{100\text{ W}} = \boxed{144\ \Omega}$

P33.3 Each meter reads the rms value.

(a) $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{R} = \dfrac{\left(\Delta V_{\max}/\sqrt{2}\right)}{R} = \dfrac{\left(100\text{ V}/\sqrt{2}\right)}{24.0\ \Omega} = \boxed{2.95\text{ A}}$

(b) The voltage across the resistor is the same as that across the power supply:

$$\Delta V_{\text{rms}} = \frac{\Delta V_{\max}}{\sqrt{2}} = \frac{100\text{ V}}{\sqrt{2}} = \boxed{70.7\text{ V}}$$



ANS. FIG. P33.3

P33.4 (a) We compute the peak voltage from the rms voltage:

$$\Delta V_{R,\max} = \sqrt{2}\left(\Delta V_{R,\text{rms}}\right) = \sqrt{2}(120\text{ V}) = \boxed{170\text{ V}}$$

(b) From the definition of power,

$$P_{\text{avg}} = I_{\text{rms}}^2 R = \frac{\Delta V_{\text{rms}}^2}{R}$$

Solving for the resistance,

$$R = \frac{\Delta V_{\text{rms}}^2}{P_{\text{avg}}} = \frac{(120\text{ V})^2}{60.0\text{ W}} = \boxed{2.40 \times 10^2\ \Omega}$$

(c) $\boxed{\text{Because } P_{\text{avg}} = \dfrac{(\Delta V_{\text{rms}})^2}{R} \rightarrow R = \dfrac{(\Delta V_{\text{rms}})^2}{P_{\text{avg}}}, \text{ a 100-W bulb has less resistance than a 60.0-W bulb.}}$

P33.5 The current as a function of time is $i = \dfrac{\Delta v}{R} = \left(\dfrac{\Delta V_{\text{max}}}{R}\right)\sin\omega t$. Given the value of *t*, we want to identify a point with

$$0.600\frac{\Delta V_{\text{max}}}{R} = \frac{\Delta V_{\text{max}}}{R}\sin(\omega t)$$

or $\omega t = \sin^{-1} 0.600$

To find the lowest frequency we choose the smallest angle satisfying this relation:

$$(0.007\ 00\text{ s})\omega = \sin^{-1}(0.600) = 0.644\text{ rad}$$

Thus, $\omega = 91.9\text{ rad/s} = 2\pi f$ so $\boxed{f = 14.6\text{ Hz}}$

P33.6 (a) From Equation 33.3, $\Delta v_R = \Delta V_{\text{max}}\sin\omega t$. To find the angular frequency, we write

$$\Delta v_R = 0.250(\Delta V_{\text{max}})$$

so $\sin\omega t = 0.250$

or $\omega t = \sin^{-1}(0.250)$

The smallest angle for which this is true is $\omega t = 0.253\text{ rad}$. Thus, if $t = 0.010\ 0\text{ s}$,

$$\omega = \frac{0.253\text{ rad}}{0.010\ 0\text{ s}} = \boxed{25.3\text{ rad/s}}$$

(b) The second time when $\Delta v_R = 0.250(\Delta V_{\text{max}})$, $\omega t = \sin^{-1}(0.250)$ again. For this occurrence, $\omega t = \pi - 0.253\text{ rad} = 2.89\text{ rad}$ (to understand why this is true, recall the identity $\sin(\pi - \theta) = \sin\theta$ from trigonometry). Thus,

$$t = \frac{2.89\text{ rad}}{25.3\text{ rad/s}} = \boxed{0.114\text{ s}}$$

P33.7 We are given $\Delta V_{\max} = 15.0\text{ V}$ and $R_{\text{total}} = 8.20\ \Omega + 10.4\ \Omega = 18.6\ \Omega$. The maximum current in the circuit is

$$I_{\max} = \frac{\Delta V_{\max}}{R_{\text{total}}} = \frac{15.0\text{ V}}{18.6\ \Omega} = 0.806\text{ A} = \sqrt{2}I_{\text{rms}}$$

And the power delivered to the speakers is

$$P_{\text{speaker}} = I_{\text{rms}}^2 R_{\text{speaker}} = \left(\frac{0.806\text{ A}}{\sqrt{2}}\right)^2 (10.4\ \Omega) = \boxed{3.38\text{ W}}$$

P33.8 All lamps are connected in parallel with the voltage source, so $\Delta V_{\text{rms}} = 120\text{ V}$ for each lamp. Also, the current is $I_{\text{rms}} = P_{\text{avg}}/\Delta V_{\text{rms}}$ and the resistance is $R = \Delta V_{\text{rms}}/I_{\text{rms}}$.

(a) For the 150-W bulbs,

$$I_{\text{rms}} = \frac{150\text{ W}}{120\text{ V}} = 1.25\text{ A}$$

For the 100-W bulb,

$$I_{\text{rms}} = \frac{100\text{ W}}{120\text{ V}} = 0.833\text{ A}$$

$\boxed{\text{The rms current in each 150-W bulb is 1.25 A. The rms current in the 100-W bulb is 0.833 A.}}$

(b) The resistance in bulbs 1 and 2 is

$$R_1 = R_2 = \frac{120\text{ V}}{1.25\text{ A}} = \boxed{96.0\ \Omega}$$

and the resistance in bulb 3 is

$$R_3 = \frac{120\text{ V}}{0.833\text{ A}} = \boxed{144\ \Omega}$$

(c) The bulbs are in parallel, so

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{96.0\ \Omega} + \frac{1}{96.0\ \Omega} + \frac{1}{144\ \Omega}$$

$$R_{\text{eq}} = \boxed{36.0\ \Omega}$$

Section 33.3 Inductors in an AC Circuit

P33.9 Inductive reactance is proportional to frequency.

At 50.0 Hz,

$$\frac{X'_L}{X_L} = \frac{f'}{f}$$

$$X'_L = X_L \frac{f'}{f} = \frac{50.0\,\text{Hz}}{60.0\,\text{Hz}}(54.0\ \Omega) = 45.0\ \Omega$$

The maximum current is

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} = \frac{\sqrt{2}\left(\Delta V_{\text{rms}}\right)}{X_L} = \frac{\sqrt{2}(100\ \text{V})}{45.0\ \Omega} = \boxed{3.14\ \text{A}}$$

P33.10 (a) $X_L = \omega L = \dfrac{\Delta V_{\max}}{I_{\max}}$

$$L = \frac{\Delta V_{\max}}{\omega I_{\max}} = \frac{100\ \text{V}}{2\pi(50.0\ \text{Hz})(7.50\ \text{A})} = \boxed{0.0424\ \text{H}}$$

(b) From $I_{\max} = \dfrac{\Delta V_{\max}}{X_L} = \dfrac{\Delta V_{\max}}{\omega L}$, we see that is current inversely proportional to angular frequency:

$$\frac{I_{\max}}{I'_{\max}} = \frac{\omega'}{\omega}$$

$$\omega' = \omega \frac{I_{\max}}{I'_{\max}} = \left[2\pi(50.0\ \text{Hz})\right]\frac{7.50\ \text{A}}{2.50\ \text{A}} = \boxed{942\ \text{rad/s}}$$

P33.11 The inductive reactance is

$$X_L = \omega L = (65.0\ \pi\,\text{s}^{-1})(70.0 \times 10^{-3}\ \text{V}\cdot\text{s/A}) = 14.3\ \Omega$$

The amplitude of the current is

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} = \frac{80.0\ \text{V}}{14.3\ \Omega} = 5.60\ \text{A}$$

Equation 33.7 lets us evaluate the current:

$$i = -I_{\max}\cos\omega t = -(5.60\ \text{A})\cos\left[\left(65.0\pi\ \text{s}^{-1}\right)(0.015\ 5\ \text{s})\right]$$

$$= -(5.60\ \text{A})\cos(3.17\ \text{rad}) = \boxed{+5.60\ \text{A}}$$

P33.12 The relationship between current, inductance, and maximum voltage is

$$I_{\text{rms}} = \frac{\Delta V_{L,\text{rms}}}{X_L} = \frac{\left(\Delta V_{L,\max}/\sqrt{2}\right)}{\omega L} = \frac{\Delta V_{L,\max}}{\sqrt{2}(2\pi)fL}$$

In order to restrict the current to $I_{\text{rms}} < 2.00 \times 10^{-3}$ A, we require

$$\frac{\Delta V_{L,\max}}{\sqrt{2}(2\pi)fL} < 2.00 \times 10^{-3}\text{ A}$$

Solving for the inductance then gives

$$L > \frac{\Delta V_{L,\max}}{\sqrt{2}(2\pi)f\left(2.00 \times 10^{-3}\text{ A}\right)}$$

$$= \frac{4.00\text{ V}}{\sqrt{2}(2\pi)(300.0\text{ Hz})\left(2.00 \times 10^{-3}\text{ A}\right)}$$

or $L > \boxed{0.750\text{ H}}$

P33.13 (a) $X_L = 2\pi f L = 2\pi(80.0\text{ Hz})\left(25.0 \times 10^{-3}\text{ H}\right) = \boxed{12.6\ \Omega}$

(b) $I_{\text{rms}} = \dfrac{\Delta V_{L,\text{rms}}}{X_L} = \dfrac{78.0\text{ V}}{X_L} = \boxed{6.21\text{ A}}$

(c) $I_{\max} = \sqrt{2}\, I_{\text{rms}} = \sqrt{2}(6.21\text{ A}) = \boxed{8.78\text{ A}}$

P33.14 In the inductor, because $U_B = \frac{1}{2}Li_L^2 = 0$ when $t = 0$, $i_L = I_{\max}\sin(\omega t)$. Then,

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{\Delta V_{\text{rms}}}{\omega L} = \frac{120\text{ V}}{\left[2\pi(60.0)\text{ s}^{-1}\right](0.020\,0\text{ H})} = 15.9\text{ A}$$

and $I_{\max} = \sqrt{2} I_{\text{rms}} = \sqrt{2}(15.9\text{ A}) = 22.5\text{ A}$

the current in the inductor is

$$i_L = I_{\max}\sin\omega t = (22.5\text{ A})\sin\left[2\pi(60.0)\text{ s}^{-1}\cdot\left(\frac{1}{180}\text{ s}\right)\right]$$
$$= (22.5\text{ A})\sin 120° = 19.5\text{ A}$$

and the energy stored is

$$U_B = \frac{1}{2}Li_L^2 = \frac{1}{2}(0.020\,0\text{ H})(19.5\text{ A})^2 = \boxed{3.80\text{ J}}$$

P33.15 The flux Φ_B through each turn of the inductor is related to the inductance by

$$L = \frac{N\Phi_B}{i}$$

Then, for an N-turn inductor,

$$N\Phi_{B,\,\max} = LI_{\max} = \frac{X_L}{\omega}\frac{\left(\Delta V_{L,\,\max}\right)}{X_L} = \frac{\left(\Delta V_{L,\,\max}\right)}{\omega}$$

$$N\Phi_{B,\,\max} = \frac{\sqrt{2}\left(\Delta V_{L,\,\mathrm{rms}}\right)}{2\pi f} = \frac{120\ \mathrm{V}}{\sqrt{2}\pi\left(60.0\ \mathrm{s}^{-1}\right)} = \boxed{0.450\ \mathrm{Wb}}$$

P33.16 We are given: $\Delta v = 120\sin 30.0\pi t$ where Δv is in volts and t in seconds, and $L = 0.500$ H.

(a) By inspection, $\omega = 30\pi\ \mathrm{rad/s}$, so

$$f = \frac{\omega}{2\pi} = \frac{30\pi\ \mathrm{rad/s}}{2\pi} = \boxed{15.0\ \mathrm{Hz}}.$$

(b) Also by inspection, $\Delta V_{L,\max} = 120\ \mathrm{V}$, so that

$$\Delta V_{L,\mathrm{rms}} = \frac{\Delta V_{L,\max}}{\sqrt{2}} = \frac{120\ \mathrm{V}}{\sqrt{2}} = \boxed{84.9\ \mathrm{V}}$$

(c) $X_L = 2\pi fL = \omega L = \left(30\pi\ \mathrm{rad/s}\right)\left(0.500\ \mathrm{H}\right) = \boxed{47.1\ \Omega}$

(d) $I_{\mathrm{rms}} = \dfrac{\Delta V_{L,\mathrm{rms}}}{X_L} = \dfrac{84.9\ \mathrm{V}}{47.1\ \Omega} = \boxed{1.80\ \mathrm{A}}$

(e) $I_{\max} = \sqrt{2}\,I_{\mathrm{rms}} = \sqrt{2}\left(1.80\ \mathrm{A}\right) = \boxed{2.55\ \mathrm{A}}$

Section 33.4 Capacitors in an AC Circuit

P33.17 Current leads voltage by 90° in a capacitor, and because charge is proportional to voltage, current leads charge by 90°. If $\Delta v_C = \Delta V_{\max}\sin\omega t$, then $q = C\left(\Delta V_{\max}\right)\sin\omega t$ so that the stored energy is

$U_C = \dfrac{q^2}{2C} = 0$ when $t = 0$. Therefore, the current is given by

$$i_C = I_{\max}\sin\left(\omega t + 90°\right) = \frac{\Delta V_{\max}}{X_C}\sin\left(\omega t + 90°\right)$$

The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(60.0/\text{s})(1.00 \times 10^{-3}\ \text{C/V})} = 2.65\ \Omega$$

and the current at $t = \frac{1}{180}$ s is

$$\begin{aligned} i_C &= \frac{\Delta V_{\max}}{X_C}\sin(\omega t + \phi) \\ &= \frac{\sqrt{2}(120\ \text{V})}{2.65\ \Omega}\sin\left[2\pi(60.0\ \text{s}^{-1})\cdot\left(\frac{1}{180}\ \text{s}\right) + \frac{\pi}{2}\right] \\ &= -32.0\ \text{A} \end{aligned}$$

The magnitude of the current is $\boxed{32.0\ \text{A}}$.

P33.18 (a) $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(60.0\ \text{Hz})(12.0 \times 10^{-6}\ \text{F})} = \boxed{221\ \Omega}$

(b) $I_{\text{rms}} = \frac{\Delta V_{C,\text{rms}}}{X_C} = \frac{36.0\ \text{V}}{221\ \Omega} = \boxed{0.163\ \text{A}}$

(c) $I_{\max} = \sqrt{2}\, I_{\text{rms}} = \boxed{0.230\ \text{A}}$

(d) $\boxed{\text{No.}}$ Current leads voltage, and thus charge, by 90° in a capacitor. The current reaches its maximum value one-quarter cycle before the voltage reaches its maximum value. From the definition of capacitance, the capacitor reaches its maximum charge when the voltage across it is also a maximum. Consequently, the maximum charge and the maximum current do not occur at the same time.

P33.19 (a) We require $X_C = \frac{1}{2\pi f C} < 175\ \Omega$, or

$$\frac{1}{2\pi f(22.0 \times 10^{-6}\ \text{F})} < 175\ \Omega$$

Solving,

$$\frac{1}{2\pi(22.0 \times 10^{-6}\ \text{F})(175\ \Omega)} < f$$

or $\boxed{f > 41.3\ \text{Hz}}$

(b) As a function of capacitance C, $X_C(C) \propto \frac{1}{C}$, so

$$X_C(C = 44.0\ \mu\text{F}) = \frac{1}{2} X_C(C = 22.0\ \mu\text{F})$$

or new $X_C = \frac{1}{2}$ old X_C.

Therefore, $\boxed{X_C < 87.5\ \Omega}$

P33.20 (a) By inspection, $\Delta V_{C,\max} = 98.0$ V, so

$$\Delta V_{C,\text{rms}} = \frac{\Delta V_{C,\max}}{\sqrt{2}} = \frac{98.0\ \text{V}}{\sqrt{2}} = \boxed{69.3\ \text{V}}.$$

(b) Also by inspection, $\omega = 80\pi$ rad/s, so

$$f = \frac{\omega}{2\pi} = \frac{80\pi\ \text{rad/s}}{2\pi\ \text{rad}} = \boxed{40.0\ \text{Hz}}$$

(c) We can find the capacitive reactance from

$$X_C = \frac{\Delta V_{C,\max}}{I_{\max}} = \frac{98.0\ \text{V}}{0.500\ \text{A}} = 196\ \Omega$$

and since

$$X_C = \frac{1}{2\pi fC} = \frac{1}{\omega C}$$

solving for the capacitance gives

$$C = \frac{1}{\omega X_C} = \frac{1}{(80\pi\ \text{rad/s})(196\ \Omega)} = 2.03 \times 10^{-5}\ \text{F} = \boxed{20.3\ \mu\text{F}}$$

P33.21 We combine the steps in the equation

$$I_{\max} = \frac{\Delta V_{\max}}{X_C} = \Delta V_{\max}\omega C = \Delta V_{\max}(2\pi fC)$$

Then,

$$I_{\max} = (48.0\ \text{V})(2\pi)(90.0\ \text{Hz})(3.70 \times 10^{-6}\ \text{F}) = \boxed{100\ \text{mA}}$$

P33.22 The maximum charge is given by

$$Q_{\max} = C(\Delta V_{\max}) = C\left[\sqrt{2}(\Delta V_{\text{rms}})\right] = \boxed{\sqrt{2}C(\Delta V_{\text{rms}})}$$

P33.23 The maximum current in the capacitor is given by

$$I_{\max} = \sqrt{2}I_{\text{rms}} = \frac{\sqrt{2}\left(\Delta V_{\text{rms}}\right)}{X_C} = \sqrt{2}\left(\Delta V_{\text{rms}}\right)2\pi f C$$

(a) For the North American electrical outlet,

$$I_{\max} = \sqrt{2}\left(120\text{ V}\right)2\pi\left(60.0/\text{s}\right)\left(2.20\times10^{-6}\text{ C/V}\right) = \boxed{141\text{ mA}}$$

(b) For the European electrical outlet,

$$I_{\max} = \sqrt{2}\left(240\text{ V}\right)2\pi\left(50.0/\text{s}\right)\left(2.20\times10^{-6}\text{ F}\right) = \boxed{235\text{ mA}}$$

Section 33.5 The *RLC* Series Circuit

P33.24 We first determine the reactances of the circuit. The capacitive reactance is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi(50.0)\left(65.0\times10^{-6}\text{ F}\right)} = 49.0\ \Omega$$

the inductive reactance is,

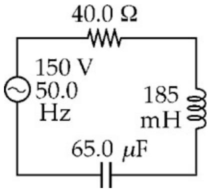
$$X_L = \omega L = 2\pi(50.0)\left(185\times10^{-3}\text{ H}\right) = 58.1\ \Omega$$

and the impedance Z of the circuit is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(40.0\ \Omega)^2 + (58.1\ \Omega - 49.0\ \Omega)^2}$$
$$= 41.0\ \Omega$$

The current in the circuit is then

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{150\text{ V}}{41.0\ \Omega} = 3.66\text{ A}$$



ANS. FIG. P33.24

(a) The maximum voltage between points *a* and *b* is the potential drop across the resistor:

$$\Delta V_R = I_{\max}R = (3.66\text{ A})(40.0\ \Omega) = \boxed{146\text{ V}}$$

(b) The maximum voltage between points *b* and *c* is the potential drop across the coil:

$$\Delta V_L = I_{\max} X_L = (3.66\ \text{A})(58.1\ \Omega) = 212.5\ \text{V} = \boxed{212\ \text{V}}$$

(c) The maximum voltage between points *c* and *d* is the potential drop across the capacitor:

$$\Delta V_C = I_{\max} X_C = (3.66\ \text{A})(49.0\ \Omega) = 179.1\ \text{V} = \boxed{179\ \text{V}}$$

(d) The potential drop between points *b* and *d* is

$$\Delta V_L - \Delta V_C = 212.5\ \text{V} - 179.1\ \text{V} = \boxed{33.4\ \text{V}}$$

P33.25 The resistance of the circuit is $R = 300\ \Omega$. The inductive reactance of the circuit is

$$X_L = \omega L = 2\pi\left(\frac{500}{\pi}\ \text{s}^{-1}\right)(0.200\ \text{H}) = 200\ \Omega$$

The capacitive reactance of the circuit is

$$X_C = \frac{1}{\omega C} = \left[2\pi\left(\frac{500}{\pi}\ \text{s}^{-1}\right)(11.0\times 10^{-6}\ \text{F})\right]^{-1} = 90.9\ \Omega$$

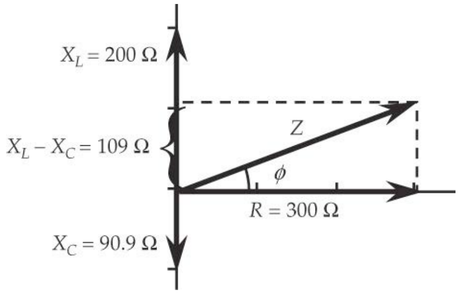
The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300\ \Omega)^2 + (200\ \Omega - 90.0\ \Omega)^2} = 319\ \Omega$$

and

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{200\ \Omega - 90.9\ \Omega}{300\ \Omega}\right) = 20.0°$$

The phasor diagram is shown in ANS. FIG. P33.25.



ANS. FIG. P33.25

P33.26 (a) From the time dependence of the voltage, we recognize that $\omega = 100\ \text{s}^{-1}$. The resistance of the circuit is $R = 68.0\ \Omega$, the inductive reactance of the circuit is

$$X_L = \omega L = (100\ \text{s}^{-1})(0.160\ \text{H}) = 16.0\ \Omega$$

The capacitive reactance of the circuit is

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(100\ \text{s}^{-1}\right)\left(99.0\times 10^{-6}\ \text{F}\right)} = 101\ \Omega$$

Therefore, the impedance of the circuit is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$
$$= \sqrt{\left(68.0\ \Omega\right)^2 + \left(16.0\ \Omega - 101\ \Omega\right)^2} = \boxed{109\ \Omega}$$

(b) The maximum current in the circuit is

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{40.0\ \text{V}}{109\ \Omega} = \boxed{0.367\ \text{A}}$$

(c) $\boxed{\omega = 100\ \text{rad/s}}$

(d) We find ϕ from

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{16.0\ \Omega - 101\ \Omega}{68.0\ \Omega}\right)$$
$$= -0.896\ \text{rad} = \boxed{-51.3^\circ}$$

P33.27 (a) The inductive reactance of the circuit is

$$X_L = \omega L = 2\pi\left(50.0\ \text{s}^{-1}\right)\left(150\times 10^{-3}\ \text{H}\right) = \boxed{47.1\ \Omega}$$

(b) The capacitive reactance of the circuit is

$$X_C = \frac{1}{\omega C} = \left[2\pi\left(50.0\ \text{s}^{-1}\right)\left(5.00\times 10^{-6}\ \text{F}\right)\right]^{-1} = \boxed{637\ \Omega}$$

(c) The impedance of the circuit is

$$Z = \frac{\Delta V_{\text{max}}}{I_{\text{max}}} = \frac{240\ \text{V}}{0.100\ \text{A}} = 2.40\times 10^3\ \Omega = \boxed{2.40\ \text{k}\Omega}$$

(d) From the definition of impedance,

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

Solving for the resistance gives

$$R = \sqrt{Z^2 - \left(X_L - X_C\right)^2}$$
$$= \sqrt{\left(2.40\times 10^3\ \Omega\right)^2 - \left(47.1\ \Omega - 637\ \Omega\right)^2}$$
$$= 2.33\times 10^3\ \Omega = \boxed{2.33\ \text{k}\Omega}$$

(e) $$\phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right] = \tan^{-1}\left(\frac{47.1\ \Omega - 637\ \Omega}{2.33\times 10^3\ \Omega}\right) = \boxed{-14.2^\circ}$$

P33.28 From the definitions of inductive and capacitive reactance, $X_L = \omega L$ and $X_C = \dfrac{1}{\omega C}$. Setting these equal and solving for the angular frequency gives

$$\omega L = \frac{1}{\omega C}$$

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(57.0\times10^{-6}\right)\left(57.0\times10^{-6}\right)}} = 1.75\times10^{4}\ \text{rad/s}$$

Then,

$$f = \frac{\omega}{2\pi} = \boxed{2.79\ \text{kHz}}$$

P33.29 The reactance of the inductor is

$$X_L = 2\pi f L = 2\pi\left(60.0\ \text{s}^{-1}\right)\left(0.460\ \text{H}\right) = 173\ \Omega$$

The reactance of the capacitor is

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(60.0\ \text{s}^{-1}\right)\left(21.0\times10^{-6}\ \text{F}\right)} = 126\ \Omega$$

(a) $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{173\ \Omega - 126\ \Omega}{150\ \Omega}\right) = \boxed{17.4^\circ}$

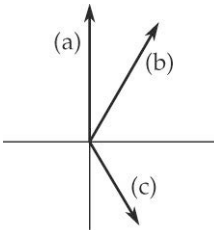
(b) Since $X_L > X_C$, ϕ is positive, so $\boxed{\text{voltage leads the current}}$. This means that the power-supply or total voltage goes through each maximum, zero-crossing, and minimum earlier in time than the current does.

P33.30 The Phasors for the three cases are shown in ANS. FIG. P33.30.

(a) 25.0 sin ωt at $\omega t = 90.0^\circ$

(b) 30.0 sin ωt at $\omega t = 60.0^\circ$

(c) 18.0 sin ωt at $\omega t = 300^\circ$



ANS. FIG. P33.30

P33.31 (a) We first find the impedances of the inductor and the capacitor:

$$X_L = \omega L = 2\pi(50.0)(400\times10^{-3}) = 126\ \Omega$$

and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi(50.0)(4.43\times10^{-6})} = 719\ \Omega$

We then compute the impedance of the circuit:

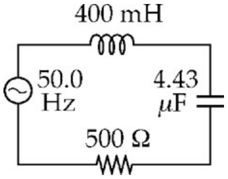
$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{500^2 + (126-719)^2} = 776\ \Omega$$

Then, from the equation for a series RLC circuit,

$$\Delta V_{\max} = I_{\max} Z = (250\times10^{-3})(776) = \boxed{194\ \text{V}}$$

(b) $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{126-719}{500}\right) = \boxed{-49.9°}$

Thus, the $\boxed{\text{current leads the voltage}}$.



ANS. FIG. P33.31

P33.32 (a) The capacitive reactance of the circuit is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\ \text{Hz})(30.0\times10^{-6}\ \text{F})} = \boxed{88.4\ \Omega}$$

(b) The impedance of the circuit is

$$Z = \sqrt{R^2 + (0 - X_C)^2} = \sqrt{R^2 + X_C^2} = \sqrt{(60.0\ \Omega)^2 + (88.4\ \Omega)^2}$$
$$= \boxed{107\ \Omega}$$

(c) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{1.20\times10^2\ \text{V}}{107\ \Omega} = \boxed{1.12\ \text{A}}$

(d) The phase angle in this *RC* circuit is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 88.4\ \Omega}{60.0\ \Omega}\right) = -55.8°$$

Since $\phi < 0$, $\boxed{\text{the voltage lags behind the current by 55.8°}}$.

(e) Adding an inductor will change the impedance, and hence the current in the circuit. The current could be larger or smaller, depending on the inductance added. The largest current would result when the inductive reactance equals the capacitive reactance, the impedance has its minimum value, equal to 60.0 Ω, and the current in the circuit is

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{\Delta V_{\max}}{R} = \frac{1.20\times 10^2 \text{ V}}{60.0\ \Omega} = 2.00 \text{ A}$$

P33.33 Let X_C represent the initial capacitive reactance. Moving the plates to half their original separation doubles the capacitance ($C = \frac{\epsilon_0 A}{d}$) and cuts $X_C = \frac{1}{\omega C}$ in half.

For the current to double, the total impedance must be cut in half, or $Z_i = 2Z_f$. Then,

$$\sqrt{R^2 + \left(X_L - X_C\right)^2} = 2\sqrt{R^2 + \left(X_L - \frac{X_C}{2}\right)^2}$$

$$R^2 + \left(R - X_C\right)^2 = 4\left[R^2 + \left(R - \frac{X_C}{2}\right)^2\right]$$

$$2R^2 - 2RX_C + X_C^2 = 8R^2 - 4RX_C + X_C^2$$

$$\boxed{X_C = 3R}$$

Section 33.6 Power in an AC Circuit

P33.34 The power factor for a series *RLC* circuit is given by

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

The circuit in this problem has no capacitance, so the power factor becomes

$$\cos\phi = \frac{R}{\sqrt{R^2 + X_L{}^2}}$$

In order for the power factor to be equal to 1.00, we would have to have $X_L = 0$, which would require either L or f to be zero. Because this is not the case, the situation is impossible.

P33.35 From the definition of impedance, $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$, we have

$$\left(X_L - X_C\right) = \sqrt{Z^2 - R^2}$$

Substituting numerical values,

$$\left(X_L - X_C\right) = \sqrt{(75.0\ \Omega)^2 - (45.0\ \Omega)^2} = 60.0\ \Omega$$

The phase angle of the circuit is then

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{60.0\ \Omega}{45.0\ \Omega}\right) = 53.1°$$

The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{210\ \text{V}}{75.0\ \Omega} = 2.80\ \text{A}$$

Therefore, the power delivered to the circuit is

$$P = \left(\Delta V_{\text{rms}}\right) I_{\text{rms}} \cos\phi = (210\ \text{V})(2.80\ \text{A})\cos(53.1°) = \boxed{353\ \text{W}}$$

P33.36 The rms voltage of the power supply is

$$\Delta V_{\text{rms}} = \frac{100\ \text{V}}{\sqrt{2}} = 70.7\ \text{V}$$

In order to calculate the impedance, we first need the capacitive and inductive reactances:

$$X_C = \frac{1}{\omega C} = \frac{1}{\left(1\,000\ \text{s}^{-1}\right)\left(5.00 \times 10^{-6}\ \text{F}\right)} = 200\ \Omega$$

$$X_L = \omega L = (1\,000\ \text{s}^{-1})(0.500\ \text{H}) = 500\ \Omega$$

Next,

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$= \sqrt{(400\ \Omega)^2 + (500\ \Omega - 200\ \Omega)^2} = 500\ \Omega$$

The rms current is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{70.7\ \text{V}}{500\ \Omega} = 0.141\ \text{A}$$

The average power is $P_{\text{avg}} = I_{\text{rms}}^2 R = (0.141\ \text{A})^2(400\ \Omega) = \boxed{8.00\ \text{W}}$

P33.37 The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{160\ \text{V}}{80.0\ \Omega} = 2.00\ \text{A}$$

and the average power delivered to the circuit is

$$P_{\text{avg}} = I_{\text{rms}}^2 R = (2.00\ \text{A})^2 (22.0\ \Omega) = \boxed{88.0\ \text{W}}$$

P33.38 Given $v = \Delta V_{\text{max}} \sin(\omega t) = (90.0\ \text{V})\sin(350t)$, observe that $\Delta V_{\text{max}} = 90.0\ \text{V}$ and $\omega = 350\ \text{rad/s}$. Also, the net reactance is

$$X_L - X_C = 2\pi fL - \frac{1}{2\pi fC} = \omega L - \frac{1}{\omega C}$$

(a) To find the impedance, we first compute

$$\begin{aligned} X_L - X_C &= \omega L - \frac{1}{\omega C} \\ &= (350\ \text{rad/s})(0.200\ \text{H}) - \frac{1}{(350\ \text{rad/s})(25.0 \times 10^{-6}\ \text{F})} \\ &= -44.3\ \Omega \end{aligned}$$

so the impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0\ \Omega)^2 + (-44.3\ \Omega)^2} = \boxed{66.8\ \Omega}$$

(b) The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{max}}/\sqrt{2}}{Z} = \frac{90.0\ \text{V}}{\sqrt{2}(66.8\ \Omega)} = \boxed{0.953\ \text{A}}$$

(c) The phase difference between the applied voltage and the current is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{-44.3\ \Omega}{50.0\ \Omega}\right) = -41.5°$$

so the average power delivered to the circuit is

$$\begin{aligned} P_{\text{avg}} &= I_{\text{rms}} \Delta V_{\text{rms}} \cos\phi = I_{\text{rms}}\left(\frac{\Delta V_{\text{max}}}{\sqrt{2}}\right)\cos\phi \\ &= (0.953\ \text{A})\left(\frac{90.0\ \text{V}}{\sqrt{2}}\right)\cos(-41.5°) = \boxed{45.4\ \text{W}} \end{aligned}$$

P33.39 The power is given by

$$P_{\text{avg}} = I_{\text{rms}} \Delta V_{\text{rms}} \cos\phi = I_{\text{rms}}^2 R$$

(a) Then,

$$P = I_{\text{rms}}\left(\Delta V_{\text{rms}}\right)\cos\phi = (9.00\text{ A})(180\text{ V})\cos(-37.0°)$$
$$= 1.29\times10^{3}\text{ W}$$

Then, from $P = I_{\text{rms}}^2 R$,

$$R = \frac{P}{I_{\text{rms}}^2} = \frac{1.29\times10^{3}\text{ W}}{(9.00\text{ A})^2} = \boxed{16.0\ \Omega}$$

(b) From the definition of phase angle, $\tan\phi = \dfrac{X_L - X_C}{R}$,

$$X_L - X_C = R\tan\phi = (16.0\ \Omega)\tan(-37.0°) = \boxed{-12.0\ \Omega}$$

P33.40 For this circuit, $R = 20.0\ \Omega$, the capacitive reactance is $X_C = 0$, and the inductive reactance is

$$X_L = \omega L = 2\pi\left(60.0\text{ s}^{-1}\right)(0.025\,0\text{ H}) = 9.42\ \Omega$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(20.0\ \Omega)^2 + (9.42\ \Omega)^2} = 22.1\ \Omega$$

(a) The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{120\text{ V}}{22.1\ \Omega} = \boxed{5.43\text{ A}}$$

(b) The phase angle of the circuit is

$$\phi = \tan^{-1}\left(\frac{9.42}{20.0}\right) = 25.2°$$

so the power factor is

$$\cos\phi = \boxed{0.905}$$

(c) For the power factor to equal 1, we require $\phi = 0$, and this can only occur if $X_L = X_C$, or

$$9.42\ \Omega = \frac{1}{2\pi\left(60.0\text{ s}^{-1}\right)C} \rightarrow C = \boxed{281\ \mu\text{F}}$$

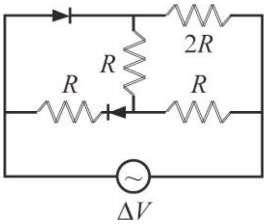
(d) For the power to equal that before the capacitor was installed, or $P_b = P_d$, we require

$$\left(\Delta V_{\text{rms}}\right)_b\left(I_{\text{rms}}\right)_b\cos\phi_b = \frac{\left(\Delta V_{\text{rms}}\right)_d^2}{R}$$

Solving for the rms voltage gives

$$\begin{aligned}\left(\Delta V_{\text{rms}}\right)_d &= \sqrt{R\left(\Delta V_{\text{rms}}\right)_b\left(I_{\text{rms}}\right)_b\cos\phi_b}\\ &= \sqrt{(20.0\ \Omega)(120\ \text{V})(5.43\ \text{A})(0.905)}\\ &= \boxed{109\ \text{V}}\end{aligned}$$

P33.41 One-half of the time, the left side of the generator is positive, the top diode conducts, and the bottom diode switches off. The power supply sees resistance



ANS. FIG. P33.41

$$\left[\frac{1}{2R}+\frac{1}{2R}\right]^{-1}=R$$

and the power is $\dfrac{\left(\Delta V_{\text{rms}}\right)^2}{R}$.

The other half of the time the right side of the generator is positive, the upper diode is an open circuit, and the lower diode has zero resistance. The equivalent resistance is then

$$R_{\text{eq}}=R+\left[\frac{1}{3R}+\frac{1}{R}\right]^{-1}=\frac{7R}{4}$$

and $$P=\frac{\left(\Delta V_{\text{rms}}\right)^2}{R_{\text{eq}}}=\frac{4\left(\Delta V_{\text{rms}}\right)^2}{7R}$$

The overall time average power is:

$$P_{\text{avg}}=\frac{\left[\left(\Delta V_{\text{rms}}\right)^2/R\right]+\left[4\left(\Delta V_{\text{rms}}\right)^2/7R\right]}{2}=\boxed{\frac{11\left(\Delta V_{\text{rms}}\right)^2}{14R}}$$

Section 33.7 Resonance in a Series *RLC* Circuit

P33.42 We are given $L = 0.020\,0$ H, $C = 100\times10^{-9}$ F, $R = 20.0\ \Omega$, and $\Delta V_{\max} = 100$ V.

(a) The resonant frequency for a series *RLC* circuit is

$$f=\frac{\omega_0}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{1}{LC}}=\boxed{3.56\ \text{kHz}}$$

(b) At resonance,

$$I_{\max} = \frac{\Delta V_{\max}}{R} = \boxed{5.00\text{ A}}$$

(c) From Equation 33.38,

$$Q = \frac{\omega_0 L}{R} = \boxed{22.4}$$

(d) At resonance, the amplitude of the voltage across the inductor is

$$\Delta V_{L,\max} = X_L I_{\max} = \omega_0 L I_{\max} = \boxed{2.24\text{ kV}}$$

P33.43 The circuit is to be in resonance when

$$\omega_0 L = \frac{1}{\omega_0 C}$$

Solving for the capacitance gives

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(99.7\text{ MHz})^2(1.40\ \mu\text{V}\cdot\text{s/A})}$$
$$= \boxed{1.82\text{ pF}}$$

P33.44 (a) The resonance frequency of a *RLC* circuit is $f_0 = 1/2\pi\sqrt{LC}$. Thus, the inductance is

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2\left(9.00\times10^{9}\text{ Hz}\right)^2\left(2.00\times10^{-12}\text{ F}\right)}$$
$$= 1.56\times10^{-10}\text{ H} = \boxed{156\text{ pH}}$$

(b) At resonance,

$$X_L = X_C = \frac{1}{2\pi f_0 C} = \frac{1}{2\pi\left(9.00\times10^{9}\text{ Hz}\right)\left(2.00\times10^{-12}\text{ F}\right)}$$
$$= \boxed{8.84\ \Omega}$$

P33.45 The resonance frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$. Thus, if $\omega = 2\omega_0$, then

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}}$$

and $$X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + 2.25\left(\frac{L}{C}\right)}$$

so the rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25\left(L/C\right)}}$$

The power delivered to the circuit is

$$P = \left(I_{\text{rms}}\right)^2 R = \left(\frac{\Delta V_{\text{rms}}}{Z}\right)^2 R = \frac{\left(\Delta V_{\text{rms}}\right)^2}{Z^2} R = \frac{\left(\Delta V_{\text{rms}}\right)^2 R}{R^2 + 2.25\left(L/C\right)}$$

and the energy delivered in one period is $E = P\Delta t$:

$$E = P\Delta t = \frac{\left(\Delta V_{\text{rms}}\right)^2 R}{R^2 + 2.25\left(L/C\right)}\left(\frac{2\pi}{\omega}\right) = \frac{\left(\Delta V_{\text{rms}}\right)^2 RC}{R^2 C + 2.25L}\left(\pi\sqrt{LC}\right)$$

$$= \frac{4\pi\left(\Delta V_{\text{rms}}\right)^2 RC\sqrt{LC}}{4R^2 C + 9L}$$

Substituting numerical values,

$$E = \frac{4\pi\left(50.0\text{ V}\right)^2 (10.0\ \Omega)\left(100\times10^{-6}\text{ F}\right)\left[\left(10.0\times10^{-3}\text{ H}\right)\left(100\times10^{-6}\text{ F}\right)\right]^{1/2}}{4\left(10.0\ \Omega\right)^2\left(100\times10^{-6}\text{ F}\right) + 9\left(10.0\times10^{-3}\text{ H}\right)}$$

$$= \boxed{242\text{ mJ}}$$

P33.46 The resonance frequency is $\omega_0 = \frac{1}{\sqrt{LC}}$. Thus, if $\omega = 2\omega_0$, then

$$X_L = \omega L = \left(\frac{2}{\sqrt{LC}}\right)L = 2\sqrt{\frac{L}{C}}$$

and $$X_C = \frac{1}{\omega C} = \frac{\sqrt{LC}}{2C} = \frac{1}{2}\sqrt{\frac{L}{C}}$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + 2.25\left(\frac{L}{C}\right)}$$

so the rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{\Delta V_{\text{rms}}}{\sqrt{R^2 + 2.25\left(L/C\right)}}$$

The power delivered to the circuit is

$$P=\left(I_{\text{rms}}\right)^2 R=\left(\frac{\Delta V_{\text{rms}}}{Z}\right)^2 R=\frac{\left(\Delta V_{\text{rms}}\right)^2}{Z^2}R=\frac{\left(\Delta V_{\text{rms}}\right)^2 R}{R^2+2.25\left(L/C\right)}$$

and the energy delivered in one period is $E=P\Delta t$:

$$E=P\Delta t=\frac{\left(\Delta V_{\text{rms}}\right)^2 R}{R^2+2.25\left(L/C\right)}\left(\frac{2\pi}{\omega}\right)=\frac{\left(\Delta V_{\text{rms}}\right)^2 RC}{R^2C+2.25L}\left(\pi\sqrt{LC}\right)$$

$$=\boxed{\frac{4\pi\left(\Delta V_{\text{rms}}\right)^2 RC\sqrt{LC}}{4R^2C+9L}}$$

P33.47 (a) To find the capacitance, we note that $f=\dfrac{1}{2\pi\sqrt{LC}}$. Solving for the capacitance *C* gives

$$C=\frac{1}{4\pi^2 f^2 L}$$

$$=\frac{1}{4\pi^2\left(1.00\times10^{10}\ \text{Hz}\right)^2\left(400\times10^{-12}\ \text{H}\right)}$$

$$=6.33\times10^{-13}\ \text{F}=\boxed{0.633\ \text{pF}}$$

(b) From Equation 26.15 for the capacitance of parallel plates with a dielectric, we have

$$C=\frac{\kappa\in_0 A}{d}=\frac{\kappa\in_0 \ell^2}{d}$$

Solving for the edge length gives

$$\ell=\left(\frac{Cd}{\kappa\in_0}\right)^{1/2}=\left[\frac{\left(6.33\times10^{-13}\ \text{F}\right)\left(1.00\times10^{-3}\ \text{m}\right)}{(1)\left(8.85\times10^{-12}\ \text{F}\right)}\right]^{1/2}$$

$$=8.46\times10^{-3}\ \text{m}=\boxed{8.46\ \text{mm}}$$

(c) The inductive reactance of the circuit at resonance, equal to the capacitive reactance, is

$$X_L=2\pi fL=2\pi\left(1.00\times10^{10}\ \text{Hz}\right)\left(400\times10^{-12}\ \text{H}\right)=\boxed{25.1\ \Omega}$$

Section 33.8 The Transformer and Power Transmission

P33.48 (a) The output voltage is found from Equation 33.41, $\Delta v_2 = \dfrac{N_2}{N_1}\Delta v_1$. Therefore,

$$\Delta v_2 = \frac{1}{13}(120\text{ V}) = \boxed{9.23\text{ V}}$$

(b) Assuming an ideal transformer, $P_2 = P_1$. Therefore,

$$\Delta V_2 I_2 = \Delta V_1 I_1 = (120\text{ V})(0.0200\text{ A}) = \boxed{2.40\text{ W}}$$

P33.49 The rms primary voltage is

$$\Delta V_{1,\text{rms}} = \frac{170\text{ V}}{\sqrt{2}} = 120\text{ V}$$

The rms voltage across the bigger coil is

$$\Delta V_{2,\text{rms}} = \left(\frac{N_2}{N_1}\right)\Delta V_{1,\text{rms}} = \left(\frac{2\,000}{350}\right)(120\text{ V}) = \boxed{687\text{ V}}$$

P33.50 (a) The total resistance of the transmission line is

$$R = \left(4.50\times10^{-4}\ \Omega/\text{m}\right)\left(6.44\times10^{5}\text{ m}\right) = 290\ \Omega$$

and the rms current in the line is

$$I_\text{rms} = \frac{P}{\Delta V_\text{rms}} = \frac{5.00\times10^{6}\text{ W}}{5.00\times10^{5}\text{ V}} = 10.0\text{ A}$$

The power loss during transmission is

$$P_\text{loss} = I_\text{rms}^2 R = (10.0\text{ A})^2(290\ \Omega) = \boxed{29.0\text{ kW}}$$

(b) The fraction of input power lost is

$$\frac{P_\text{loss}}{P} = \frac{2.90\times10^{4}\text{ W}}{5.00\times10^{6}\text{ W}} = \boxed{5.80\times10^{-3}}$$

(c) $\boxed{\text{It is impossible to transmit so much power at such low voltage.}}$ Maximum power transfer occurs when load resistance equals the line resistance of 290 Ω, and is $\dfrac{\left(4.50\times10^{3}\text{ V}\right)^2}{2\cdot2(290\ \Omega)} = 17.5\text{ kW}$, far below the required 5 000 kW.

P33.51 We find the voltage in the primary coil:

$$\frac{\Delta V_1}{\Delta V_2} = \frac{N_1}{N_2} \rightarrow \Delta V_1 = \Delta V_2 \frac{N_1}{N_2} = (25.0\text{ V})(2.50) = 62.5\text{ V}$$

The (average) power delivered to the load resistance is

$$P = I_2 \Delta V_2 = \frac{(\Delta V_2)^2}{R_2} = \frac{(25.0\text{ V})^2}{50.0\ \Omega} = 12.5\text{ W}$$

which is equal to the power delivered to the primary coil; thus, the (rms) current on the primary side is

$$I_1 = \frac{P}{\Delta V_1} = \frac{12.5\text{ W}}{62.5\text{ V}} = 0.200\text{ A}$$

On the primary side of the transformer, the voltages across the resistor and transformer (inductor) are 90° out of phase. Therefore,

$$(\Delta V_{\text{rms}})^2 = (\Delta V_{L,\text{rms}})^2 + (\Delta V_{R,\text{rms}})^2$$

$$(80.0\text{ V})^2 = (62.5\text{ V})^2 + (\Delta V_{R,\text{rms}})^2$$

$$\Delta V_{R,\text{rms}} = 49.9\text{ V}$$

and

$$\Delta V_{R,\text{rms}} = 49.9\text{ V} = I_{\text{rms}} R$$

$$R = \frac{\Delta V_{R,\text{rms}}}{I_{\text{rms}}} = \frac{49.9\text{ V}}{0.200\text{ A}} = \boxed{250\ \Omega}$$

P33.52 The capacitive reactance of this "circuit" is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\text{ Hz})(20.0\times 10^{-12}\text{ F})} = 1.33\times 10^8\ \Omega$$

and the impedance is

$$Z = \sqrt{(50.0\times 10^3\ \Omega)^2 + (1.33\times 10^8\ \Omega)^2} \approx 1.33\times 10^8\ \Omega$$

The rms current is then

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{5\,000\text{ V}}{1.33\times 10^8\ \Omega} = 3.77\times 10^{-5}\text{ A}$$

and the rms voltage across the person's body is

$$(\Delta V_{\text{rms}})_{\text{body}} = I_{\text{rms}} R_{\text{body}} = (3.77\times 10^{-5}\text{ A})(50.0\times 10^3\ \Omega)$$

$$= \boxed{1.88\text{ V}}$$

Section 33.9 Rectifiers and Filters

P33.53 For this *RC* high-pass filter, the voltage gain ratio is

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{I_{\max}R}{I_{\max}Z} = \frac{R}{\sqrt{R^2 + X_C^2}}$$

With a capacitance of 613 μF and a frequency of 600 Hz, the capacitive reactance is

$$X_C = \frac{1}{2\pi(600\text{ Hz})(6.13\times10^{-4}\text{ F})} = 0.433\ \Omega$$

and $$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{0.500\ \Omega}{\sqrt{(0.500\ \Omega)^2 + (0.433\ \Omega)^2}} = \boxed{0.756}$$

P33.54 (a) The amplitude of the input voltage is

$$\Delta V_{\text{in}} = I_{\max}Z = I_{\max}\sqrt{R^2 + X_C^2} = I_{\max}\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

The amplitude of the output voltage is $\Delta V_{\text{out}} = I_{\max}R$. The gain ratio is

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{I_{\max}R}{I_{\max}\sqrt{R^2 + (1/\omega C)^2}} = \boxed{\frac{R}{\sqrt{R^2 + (1/\omega C)^2}}}$$

(b) As $\omega \to 0$, $\dfrac{1}{\omega C} \to \infty$, and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \boxed{0}$.

(c) As $\omega \to \infty$, $\dfrac{1}{\omega C} \to 0$, and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \dfrac{R}{R} = \boxed{1}$.

P33.55 (a) The input power is 8 W, and the useful output power is given by

$$I\Delta V = (0.3\text{ A})(9\text{ V}) = 2.7\text{ W}$$

The efficiency is then

$$\text{efficiency} = \frac{\text{useful output}}{\text{total input}} = \frac{2.7\text{ W}}{8\text{ W}} = 0.34 \to \boxed{34\%}$$

(b) Total input power = Total output power

$$8\text{ W} = 2.7\text{ W} + \text{wasted power}$$

$$\text{wasted power} = \boxed{5.3\text{ W}}$$

(c) $E = P\Delta t = \left[6(8\text{ W})\right]\left[(31\text{ d})\left(\dfrac{86\,400\text{ s}}{1\text{ d}}\right)\right] = 1.29\times 10^{8}\text{ J}\left(\dfrac{\$0.110}{3.6\times 10^{6}\text{ J}}\right)$

$= \boxed{\$3.9}$

P33.56 (a) The amplitude of the input voltage is

$$\Delta V_{\text{in}} = IZ = I_{\max}\sqrt{R^2 + X_C^2} = I_{\max}\sqrt{R^2 + (1/\omega C)^2}$$

The amplitude of the output voltage is

$$\Delta V_{\text{out}} = I_{\max}X_C = \frac{I_{\max}}{\omega C}.$$

The gain ratio is therefore

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} = \frac{I_{\max}/\omega C}{I_{\max}\sqrt{R^2 + (1/\omega C)^2}} = \boxed{\frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}}$$

(b) As $\omega \to 0$, $\dfrac{1}{\omega C} \to \infty$ and *R* becomes negligible in comparison.

Then,

$$\frac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \frac{1/\omega C}{1/\omega C} = \boxed{1}$$

(c) As $\omega \to \infty$, $\dfrac{1}{\omega C} \to 0$ and $\dfrac{\Delta V_{\text{out}}}{\Delta V_{\text{in}}} \to \boxed{0}$.

(d) We start with

$$\frac{1}{2} = \frac{1/\omega C}{\sqrt{R^2 + (1/\omega C)^2}}$$

Solving,

$$R^2 + \left(\frac{1}{\omega C}\right)^2 = \frac{4}{\omega^2 C^2}$$

or $R^2\omega^2 C^2 = 3.$

Then,

$$\omega = 2\pi f = \frac{\sqrt{3}}{RC} \quad\to\quad f = \boxed{\frac{\sqrt{3}}{2\pi RC}}$$

Additional Problems

P33.57 (a) We determine the number of turns from

$$\left(\Delta V_{2,\text{ rms}}\right)=\frac{N_2}{N_1}\left(\Delta V_{1,\text{ rms}}\right)$$

solving,

$$N_2=\frac{(80)(2\,200\text{ V})}{110\text{ V}}=\boxed{1\,600\text{ windings}}$$

(b) Assuming ideal conditions,

$$I_{1,\text{ rms}}\left(\Delta V_{1,\text{ rms}}\right)=I_{2,\text{ rms}}\left(\Delta V_{2,\text{ rms}}\right)$$

Solving for the rms current in the primary then gives

$$I_{1,\text{ rms}}=\frac{(1.50\text{ A})(2\,200\text{ V})}{110\text{ V}}=\boxed{30.0\text{ A}}$$

(c) For 95.0% efficiency,

$$0.950I_{1,\text{ rms}}\left(\Delta V_{1,\text{ rms}}\right)=I_{2,\text{ rms}}\left(\Delta V_{2,\text{ rms}}\right)$$

and the rms current in the primary is

$$I_{1,\text{ rms}}=\frac{(1.20\text{ A})(2\,200\text{ V})}{(0.950)(110\text{ V})}=\boxed{25.3\text{ A}}$$

P33.58 From Equation 33.35, the resonance frequency for this circuit is

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi\sqrt{LC}}$$

$$=\frac{1}{2\pi\sqrt{\left(2.80\times10^{-6}\text{ H}\right)\left(0.910\times10^{-12}\text{ F}\right)}}=99.7\text{ MHz}$$

This frequency is not in the range of North American AM frequencies, which can be found from Internet research to be 520 kHz – 1 610 kHz. The frequency above is appropriate for an North American FM radio station.

P33.59 (a) The maximum voltage is given by

$$\Delta V_{\max}=\sqrt{\left(\Delta V_R\right)^2+\left(\Delta V_L-\Delta V_C\right)^2}$$

$$=\sqrt{(20.0\text{ V})^2+(25.0\text{ V}-15.0\text{ V})^2}$$

$$=\boxed{22.4\text{ V}}$$

(b) $$\phi = \tan^{-1}\left(\frac{\Delta V_L - \Delta V_C}{\Delta V_R}\right) = \tan^{-1}\left(\frac{25.0\text{ V} - 15.0\text{ V}}{20.0\text{ V}}\right) = \boxed{26.6^\circ}$$

(c) The current amplitude $I_{\max}$ determines the voltage amplitude in each component of the circuit. We know the resistance R, so

$$\Delta V_R = I_{\max}R \quad\rightarrow\quad I_{\max} = \frac{\Delta V_R}{R} = \frac{20.0\text{ V}}{75.0\ \Omega} = \boxed{0.267\text{ A}}$$

(d) For the entire circuit,

$$\Delta V_{\max} = I_{\max}Z \quad\rightarrow\quad Z = \frac{\Delta V_{\max}}{I_{\max}} = \boxed{83.9\ \Omega}$$

(e) For the capacitor,

$$\Delta V_C = I_{\max}X_C \quad\rightarrow\quad X_C = \frac{\Delta V_C}{I_{\max}} = \frac{15.0\text{ V}}{I_{\max}} = 56.3\ \Omega = \frac{1}{2\pi fC}$$

therefore,

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60.0\text{ Hz})X_C} = 4.72\times10^{-5}\text{ F} = \boxed{47.2\ \mu\text{F}}$$

(f) For the inductor,

$$\Delta V_L = I_{\max}X_L = I_{\max}(2\pi fL)$$

therefore,

$$L = \frac{\Delta V_L}{2\pi f I_{\max}} = \frac{25.0\text{ V}}{2\pi(60.0\text{ Hz})I_{\max}} = \boxed{0.249\text{ H}}$$

(g) The average power delivered to the circuit is

$$P_{\text{avg}} = I_{\text{rms}}^2 R = \left(\frac{I_{\max}}{\sqrt{2}}\right)^2 R = \boxed{2.67\text{ W}}$$

P33.60 We identify that $R = 200\ \Omega$, $L = 663$ mH, $C = 26.5\ \mu$F, $\omega = 377$ rad/s, and $\Delta V_{\max} = 50.0$ V. So,

$$\omega L = 250\ \Omega,\ \text{and}\ 1/\omega C = 100\ \Omega$$

The impedance is

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{(200\ \Omega)^2 + (250\ \Omega - 100\ \Omega)^2}$$
$$= 250\ \Omega$$

(a) The amplitude of the current is

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{50.0\text{ V}}{250\ \Omega} = \boxed{0.200\text{ A}}$$

The phase angle of the voltage relative to the current is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \boxed{36.8^\circ}$$

with $\Delta v \nearrow$ leading $i \rightarrow$

(b) $\Delta V_{R,\,\max} = I_{\max}R = \boxed{40.0\text{ V}}$ at $\boxed{\phi = 0^\circ}$

(c) $\Delta V_{C,\,\max} = I_{\max}X_C = \boxed{20.0\text{ V}}$ at $\boxed{\phi = -90.0^\circ}$ (I leads ΔV)

(d) $\Delta V_{L,\,\max} = I_{\max}X_L = \boxed{50.0\text{ V}}$ at $\boxed{\phi = +90.0^\circ}$ (ΔV leads I)

P33.61 Consider a two-wire transmission line: each wire has resistance R, the total power transmitted is P, and the current in the wires is $I_{\text{rms}} = \dfrac{P}{\Delta V_{\text{rms}}}$. The power loss is 1.00% of the transmitted power P. Therefore,

$$P_{\text{loss}} = I_{\text{rms}}^2 R_{\text{line}} = \frac{P}{100}$$

$$P_{\text{loss}} = \left(\frac{P}{\Delta V_{\text{rms}}}\right)^2 (2R) = \frac{P}{100}$$

Solving for the resistance gives

$$R = \frac{(\Delta V_{\text{rms}})^2}{200P}$$

The resistance of one wire is

$$R = \frac{\rho_{\text{Cu}}\ell}{A} = \frac{(\Delta V_{\text{rms}})^2}{200P}$$

Solving for the area gives

$$A = \frac{\pi d^2}{4} = \frac{200\rho_{\text{Cu}}P\ell}{(\Delta V_{\text{rms}})^2}$$

and the diameter is

$$d = \sqrt{\frac{800\rho_{\text{Cu}}P\ell}{\pi(\Delta V_{\text{rms}})^2}}$$

$$= \sqrt{\frac{800(1.7\times10^{-8}\ \Omega\cdot\text{m})\,(20\ 000\text{ W})(18\ 000\text{ m})}{\pi\left(1.50\times10^3\text{ V}\right)^2}}$$

$$= 0.026\text{ m} = \boxed{2.6\text{ cm}}$$

P33.62 Consider a two-wire transmission line: each wire has resistance R, the total power transmitted is P, and the current in the wires is $I_{\text{rms}} = \dfrac{P}{\Delta V_{\text{rms}}}$. The fractional power loss is f of the transmitted power P. Therefore,

$$P_{\text{loss}} = I_{\text{rms}}^2 R_{\text{line}} = fP$$

$$P_{\text{loss}} = \left(\frac{P}{\Delta V_{\text{rms}}}\right)^2 (2R) = fP \quad \to \quad R = \frac{f\left(\Delta V_{\text{rms}}\right)^2}{2P}$$

The resistance of one wire is

$$R = \frac{\rho_{\text{Cu}}\ell}{A} = \frac{\left(\Delta V_{\text{rms}}\right)^2}{200P}$$

Solving for the area gives

$$A = \frac{\pi d^2}{4} = \frac{200\rho_{\text{Cu}}P\ell}{\left(\Delta V_{\text{rms}}\right)^2}$$

and the diameter is

$$\boxed{d = \sqrt{\frac{8\rho_{\text{Cu}}\,\ell P}{\pi f\left(\Delta V_{\text{rms}}\right)^2}}}$$

P33.63 (a) The impedance is given by

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$$

From which we obtain

$$X_C = X_L \pm \sqrt{Z^2 - R^2}$$

$$X_C = X_L + \sqrt{Z^2 - R^2}$$
$$= 700\ \Omega + \sqrt{(760\ \Omega)^2 - (400\ \Omega)^2} = 1\ 346\ \Omega = 1.35\ \text{k}\Omega$$

or

$$X_C = X_L - \sqrt{Z^2 - R^2}$$
$$= 700\ \Omega - \sqrt{(760\ \Omega)^2 - (400\ \Omega)^2} = 53.8\ \Omega$$

$\boxed{X_C \text{ could be } 53.8\ \Omega \text{ or it could be } 1.35\ \text{k}\Omega.}$

(b) The power delivered to the circuit is given by

$$P = (I_{\text{rms}})^2 R = \frac{(\Delta V_{\text{rms}})^2}{Z^2} R$$

If the power is decreased as the frequency is raised, then the impedance is increased, so the inductive reactance is greater than the capacitive reactance, and the circuit must be above resonance:

$$X_L > X_C \quad \to \quad \omega L > 1/\omega C \quad \to \quad \omega > 1/\sqrt{LC} \quad \to \quad \omega > \omega_0$$

Therefore, the inductive reactance 700 Ω and the $\boxed{\text{capacitive reactance is } 53.8\ \Omega.}$

(c) Now,

$$X_C = X_L \pm \sqrt{Z^2 - R^2} = 700 \pm \sqrt{(760)^2 - (200)^2} = 700 \pm 733$$

Here $X_C = 700 - 733 = -33\ \Omega$ is impossible, but $X_C = 700 + 733 = 1433 = 1.43\ \text{k}\Omega$ is possible.

$\boxed{X_C \text{ must be } 1.43\ \text{k}\Omega.}$

P33.64 The equation for $\Delta v(t)$ during the first period (using $y = mx + b$) is:

$$\Delta v = \frac{2(\Delta V_{\max})t}{T} - \Delta V_{\max} = \Delta V_{\max}\left[\frac{2t}{T} - 1\right]$$

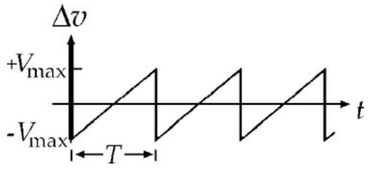
Therefore,

$$\left[(\Delta v)^2\right]_{\text{avg}} = \frac{1}{T}\int_0^T [\Delta v(t)]^2\,dt = \frac{(\Delta V_{\max})^2}{T}\int_0^T \left[\frac{2}{T}t - 1\right]^2 dt$$

$$\left[(\Delta v)^2\right]_{\text{avg}} = \frac{(\Delta V_{\max})^2}{T}\left(\frac{T}{2}\right)\frac{[2t/T - 1]^3}{3}\Bigg|_{t=0}^{t=T}$$

$$= \frac{(\Delta V_{\max})^2}{6}\left[(+1)^3 - (-1)^3\right] = \frac{(\Delta V_{\max})^2}{3}$$

Then, $$\Delta V_{\text{rms}} = \sqrt{\left[(\Delta v)^2\right]_{\text{avg}}} = \sqrt{\frac{(\Delta V_{\max})^2}{3}} = \boxed{\frac{\Delta V_{\max}}{\sqrt{3}}}$$



ANS. FIG. P33.64

P33.65 The turns ratio is the factor of change in voltage:

$$\frac{N_1}{N_2}=\frac{\Delta V_1}{\Delta V_2}$$

with $Z_1=\frac{\Delta V_1}{I_1}$ and $Z_2=\frac{\Delta V_2}{I_2}$

we have $\frac{N_1}{N_2}=\frac{Z_1 I_1}{Z_2 I_2}$

(a) Since $\frac{I_1}{I_2}=\frac{N_2}{N_1}$, we find

$$\frac{N_1}{N_2}=\frac{Z_1 N_2}{Z_2 N_1}\quad\text{so}\quad\frac{N_1^2}{N_2^2}=\frac{Z_1}{Z_2}\quad\text{and}\quad\frac{N_1}{N_2}=\boxed{\sqrt{\frac{Z_1}{Z_2}}}$$

(b) $\frac{N_1}{N_2}=\sqrt{\frac{8\,000\ \Omega}{8.00\ \Omega}}=\boxed{31.6}$

P33.66 (a) The angular frequency is $\omega=2\pi(60.0\text{ Hz})=377/\text{s}^{-1}$.

When *S* is open, *R*, *L*, and *C* are in series with the source, with impedance

$$Z=\frac{\Delta V_{\text{rms}}}{I_{\text{rms}}}$$

squaring both sides and substituting the definition of impedance gives

$$R^2+(X_L-X_C)^2=\left(\frac{\Delta V_{\text{rms}}}{I_{\text{rms}}}\right)^2=\left(\frac{20.0\text{ V}}{0.183\text{ A}}\right)^2=1.194\times10^4\ \Omega^2\quad\textbf{[1]}$$

When *S* is in position *a*, a parallel combination of two *R*'s presents equivalent resistance $\frac{R}{2}$, in series with *L* and *C*. The square of the impedance is then

$$\left(\frac{R}{2}\right)^2+(X_L-X_C)^2=\left(\frac{20.0\text{ V}}{0.298\text{ A}}\right)^2=4.504\times10^3\ \Omega^2\quad\textbf{[2]}$$

When *S* is in position *b*, the current bypasses the inductor. *R* and *C* are in series with the source, and the square of the impedance is

$$R^2+X_C^2=\left(\frac{20.0\text{ V}}{0.137\text{ A}}\right)^2=2.131\times10^4\ \Omega^2\quad\textbf{[3]}$$

Subtract equation [2] from equation [1]:

$$\frac{3}{4}R^2 = 7.440\times 10^3\ \Omega^2 \quad\rightarrow\quad \boxed{R = 99.6\ \Omega}$$

Only the positive root is physical, thus there is only one value for *R*.

(b) We have shown than only one resistance value is possible. Now equation [3] gives

$$X_C = \left[2.131\times 10^4\ \Omega^2 - (99.6\ \Omega)^2\right]^{1/2} = 106.7\ \Omega = \frac{1}{\omega C}$$

Only the positive root is physical, thus there is only one value for *C*.

$$C = (\omega X_C)^{-1} = \left[(377\ \text{s}^{-1})106.7\ \Omega\right]^{-1}$$
$$= 2.49\times 10^{-5}\ \text{F} = \boxed{24.9\ \mu\text{F}}$$

(c) Now equation [1] gives

$$X_L - X_C = \pm\left[1.194\times 10^4\ \Omega^2 - (99.6\ \Omega)^2\right]^{1/2} = \pm 44.99\ \Omega$$

The equation shows us that there are two possible values for *L*.

$$X_L = X_C - 44.99\ \Omega = 106.7\ \Omega - 44.99\ \Omega = 61.74\ \Omega = \omega L$$

or

$$X_L = X_C + 44.99\ \Omega = 106.7\ \Omega + 44.99\ \Omega = 151.7\ \Omega = \omega L$$

then

$$L = \frac{X_L}{\omega} = 0.164\ \text{H or } 0.402\ \text{H} = \boxed{164\ \text{mH or } 402\ \text{mH}}$$

(d) From the calculations above, we see that $\boxed{\text{only one value for } R \text{ and only one value for } C \text{ are possible. Two values for } L \text{ are possible.}}$

P33.67 (a) We can use $\sin A + \sin B = 2\sin\left(\frac{A}{2}+\frac{B}{2}\right)\cos\left(\frac{A}{2}-\frac{B}{2}\right)$ to find the sum of the two sine functions to be

$$E_1 + E_2 = (24.0\ \text{cm})\sin(4.50t + 35.0°)\cos 35.0°$$
$$E_1 + E_2 = (19.7\ \text{cm})\sin(4.50t + 35.0°)$$

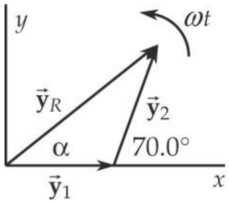
Thus, the total wave has amplitude $\boxed{19.7\ \text{cm}}$ and has a constant phase difference of $\boxed{35.0°}$ from the first wave.

(b) Refer to ANS. FIG. P33.67(b). In units of cm, the resultant phasor is

$$\vec{\mathbf{y}}_R = \vec{\mathbf{y}}_1 + \vec{\mathbf{y}}_2 = \left(12.0\hat{\mathbf{i}}\right) + \left(12.0\cos(70.0°)\hat{\mathbf{i}} + 12.0\sin(70.0)\hat{\mathbf{j}}\right)$$

$$= 16.1\hat{\mathbf{i}} + 11.3\hat{\mathbf{j}}$$

$$\vec{\mathbf{y}}_R = \sqrt{(16.1)^2 + (11.3)^2} \text{ at } \tan^{-1}\left(\frac{11.3}{16.1}\right) = \boxed{19.7 \text{ cm at } 35.0°}$$



ANS. FIG. P33.67(b)

(c) $\boxed{\text{The answers are identical.}}$

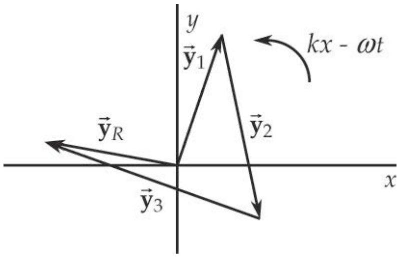
(d) Refer to ANS. FIG. P33.67(d). Adding the three waves yields

$$\vec{\mathbf{y}}_R = 12.0\cos(70.0°)\hat{\mathbf{i}} + 12.0\sin(70.0°)\hat{\mathbf{j}}$$
$$+ 15.5\cos(-80.0°)\hat{\mathbf{i}} + 15.5\sin(-80.0°)\hat{\mathbf{j}}$$
$$+ 17.0\cos(160°)\hat{\mathbf{i}} + 17.0\sin(160°)\hat{\mathbf{j}}$$

$$\vec{\mathbf{y}}_R = -9.18\hat{\mathbf{i}} + 1.83\hat{\mathbf{j}} = \boxed{9.36 \text{ cm at } 169°}$$

The wave function of the total wave is

$$y_R = (9.36 \text{ cm})\sin(15x - 4.5t + 169°)$$



ANS. FIG. P33.67(d)

P33.68 (a) $\boxed{\text{Higher. At the resonance frequency, } X_L = X_C\text{. As the frequency increases, } X_L \text{ goes up and } X_C \text{ goes down.}}$

(b) $\boxed{\text{It is possible. We have three independent equations in the three unknowns } L, C\text{, and the certain } f.}$

(c) The equations are $\omega_0^2 = \dfrac{1}{LC} = 2\ 000\ \text{s}^{-1}$, $X_C = \dfrac{1}{\omega C} = 8.00\ \Omega$, and $X_L = \omega L = 12.0\ \Omega$. From the inductive reactance,

$$X_L = \omega L \quad \rightarrow \omega = \frac{X_L}{L}$$

then from the capacitive reactance,

$$X_C \omega = \frac{1}{\omega C} \frac{X_L}{L}$$

solving for the angular frequency gives

$$\omega^2 = \frac{X_L}{X_C} \frac{1}{LC} = \frac{X_L}{X_C} \omega_0^2 = \left(\frac{12.0\ \Omega}{8.00\ \Omega} \right) \left(2\ 000\ \text{s}^{-1} \right)^2$$

from which we obtain

$$\omega = 2\ 450\ \text{s}^{-1}$$

Then,

$$L = \frac{X_L}{\omega} = \frac{12.0\ \Omega}{2\ 450\ \text{s}^{-1}} = 4.90 \times 10^{-3}\ \text{H} = \boxed{4.90\ \text{mH}}$$

$$C = \frac{1}{\omega X_C} = \frac{1}{\left(2\ 450\ \text{s}^{-1}\right)\left(8.00\ \Omega\right)} = 5.10 \times 10^{-5}\ \text{F} = \boxed{51.0\ \mu\text{F}}$$

P33.69 (a) The lowest-frequency standing-wave pattern is N-A-N. The distance between the clamps we represent as $d = d_{\text{NN}} = \dfrac{\lambda}{2}$. The speed of transverse waves on the string is $v = f\lambda = \sqrt{\dfrac{T}{\mu}} = f2d$.

The magnetic force on the wire oscillates at 60 Hz, so the wire will oscillate in resonance at 60 Hz. From the speed of transverse waves,

$$v = f\lambda = \sqrt{\frac{T}{\mu}} = f2d$$

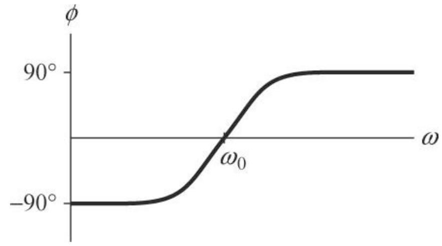
we obtain the period as

$$T = 4\mu f^2 d^2 = 4\left(19.0 \times 10^{-3}\ \text{kg/m}\right)\left(60.0\ \text{Hz}\right)^2 d^2$$
$$= \left(274\ \text{kg/m} \cdot \text{s}^2\right) d^2$$

$$\boxed{\text{Tension } T \text{ and separation } d \text{ must be related by } T = 274d^2 \text{ where } T \text{ is in newtons and } d \text{ is in meters.}}$$

(b) $\boxed{\text{One possibility is } T = 10.9\text{ N and } d = 0.200\text{ m.}}$ Any values of T and d related according to this expression will work. We did not need to use the value of the current and magnetic field.

P33.70 (a) See the graph in ANS. FIG. P33.70(a).



ANS. FIG. P33.70(a)

(b) $\phi = \tan^{-1}\left(\dfrac{\omega L - 1/\omega C}{R}\right)$ changes from –90° for $\omega = 0$ to 0 at the resonance frequency to +90° as ω goes to infinity.

The slope of the graph is $d\phi/d\omega$:

$$\frac{d\phi}{d\omega} = \frac{1}{1+\left(\dfrac{\omega L - 1/\omega C}{R}\right)^2}\frac{1}{R}\left(L - \frac{1}{C}(-1)\frac{1}{\omega^2}\right)$$

$$= \boxed{\frac{R}{R^2 + (\omega L - 1/\omega C)^2}\left(L + \frac{1}{\omega^2 C}\right)}$$

(c) At resonance we have $\omega_0^2 = 1/LC$; substituting, we find the slope at the resonance point is

$$\left.\frac{d\phi}{d\omega}\right|_{\omega_0} = \frac{1}{R+0^2}\left(L + \frac{LC}{C}\right) = \frac{2L}{R} = \frac{2Q}{\omega_0}$$

where $Q = \omega_0 L/R$.

P33.71 (a) When ωL is very large, Z is large for the bottom branch, so it carries negligible current. Also, $\dfrac{1}{\omega C}$ will be negligible compared to R for the top branch, so

$$I = \frac{V}{Z} \approx \frac{V}{R} = \frac{45.0\text{ V}}{200\ \Omega} = \boxed{0.225\text{ A}}$$

flows in the power supply and the top branch.

(b) Now $\frac{1}{\omega C}$ is very large in the top branch and ωL is very small compared to R in the bottom branch; the generator and bottom branch carry

$$I = \frac{V}{Z} \approx \frac{V}{R} = \frac{45.0\text{ V}}{100\ \Omega} = \boxed{0.450\text{ A}}$$

P33.72 (a) With both switches closed, the current goes only through the generator and resistor.

$$\boxed{i = \frac{\Delta V_{\max}}{R}\cos\omega t}$$

(b) $$\boxed{P = \frac{1}{2}\frac{(\Delta V_{\max})^2}{R}}$$

(c) $$\boxed{i = \frac{\Delta V_{\max}}{\sqrt{R^2+\omega^2L^2}}\cos\left[\omega t + \tan^{-1}\left(\frac{\omega L}{R}\right)\right]}$$

(d) For $0 = \phi = \tan^{-1}\left(\frac{\omega_0 L - (1/\omega_0 C)}{R}\right)$,

We require $\omega_0 L = \frac{1}{\omega_0 C}$, so $\boxed{C = \frac{1}{\omega_0^2 L}}$

(e) The frequency is the resonance frequency: $Z = \boxed{R}$

For parts (f) and (g), the circuit is at resonance, so $Z = R$ and $X_C = X_L = \omega_0 L$.

(f) To find the maximum energy stored in the capacitor, we start with

$$U = \frac{1}{2}C(\Delta V_C)^2 = \frac{1}{2}C(IX_C)^2$$

When $I = I_{\max}$,

$$U_{\max} = \frac{1}{2}CI_{\max}^2X_C^2 = \frac{1}{2}C\left(\frac{\Delta V_{\max}}{R}\right)^2(\omega_0 L)^2 = \boxed{\frac{(\Delta V_{\max})^2 L}{2R^2}}$$

(g) $$U_{\max} = \frac{1}{2}LI_{\max}^2 = \boxed{\frac{1}{2}L\frac{(\Delta V_{\max})^2}{R^2}}$$

(h) Now $\omega = 2\omega_0 = \dfrac{2}{\sqrt{LC}}$, so

$$\phi = \tan^{-1}\left(\frac{\omega L-(1/\omega C)}{R}\right) = \tan^{-1}\left(\frac{2\sqrt{L/C}-(1/2)\sqrt{L/C}}{R}\right)$$

$$= \boxed{\tan^{-1}\left(\frac{3}{2R}\sqrt{\frac{L}{C}}\right)}$$

(i) Now $\omega L = \dfrac{1}{2}\dfrac{1}{\omega C}$, so $\omega = \boxed{\dfrac{1}{\sqrt{2LC}}} = \dfrac{\omega_0}{\sqrt{2}}$

P33.73 (a) The inductive reactance of the circuit is

$$X_L = 2\pi fL = 2\pi(50.0\text{ Hz})(0.250\text{ H}) = \boxed{78.5\ \Omega}$$

(b) The capacitive reactance of the circuit is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(50.0\text{ Hz})(2.00\times10^{-6}\text{ F})}$$

$$= 1.59\times10^3\ \Omega = \boxed{1.59\text{ k}\Omega}$$

(c) The impedance of the circuit is

$$Z = \sqrt{R^2+(X_L-X_C)^2} = \sqrt{(150\ \Omega)^2+(78.5\ \Omega-1590\ \Omega)^2}$$

$$= 1.52\times10^3\ \Omega = \boxed{1.52\text{ k}\Omega}$$

(d) The maximum current is

$$I_{\max} = \frac{\Delta V_{\max}}{Z} = \frac{2.10\times10^2\text{ V}}{1.52\times10^3\ \Omega} = 0.138\text{ A} = \boxed{138\text{ mA}}$$

(e) $\phi = \tan^{-1}\left(\dfrac{X_L-X_C}{R}\right) = \tan^{-1}\left(\dfrac{78.5\ \Omega-1590\ \Omega}{150\ \Omega}\right) = \boxed{-84.3°}$

(f) $\cos\phi = \cos\left[\tan^{-1}\left(\dfrac{78.5\ \Omega-1\,590\ \Omega}{150\ \Omega}\right)\right] = \boxed{0.098\,7}$

(g) The power input into the circuit is

$$P = I_{\text{rms}}\Delta V_{\text{rms}}\cos\phi = \frac{(\Delta V_{\text{rms}})^2}{Z}\cos\phi = \frac{\left(\Delta V_{\max}/\sqrt{2}\right)^2}{Z}\cos\phi$$

$$= \frac{(\Delta V_{\max})^2}{2Z}\cos\phi$$

$$P = \frac{(210\text{ V})^2}{2(1.52\times10^3\ \Omega)}(0.098\,7) = \boxed{1.43\text{ W}}$$

P33.74 (a) We are given $X_L = X_C = 1\,884\ \Omega$ when $f = 2\,000$ Hz. The impedance is then

$$L = \frac{X_L}{2\pi f} = \frac{1\,884\ \Omega}{4\,000\pi\ \text{rad/s}} = 0.150\ \text{H}$$

and the capacitance is

$$C = \frac{1}{(2\pi f)X_C} = \frac{1}{(4\,000\pi\ \text{rad/s})(1\,884\ \Omega)}$$
$$= 42.2\ \text{nF}$$

therefore,

$$X_L = 2\pi f(0.150\ \text{H})$$

$$X_C = \frac{1}{(2\pi f)(4.22\times10^{-8}\ \text{F})}$$

and

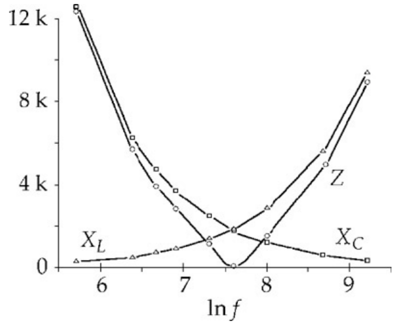
$$Z = \sqrt{(40.0\ \Omega)^2 + (X_L - X_C)^2}$$

TABLE P33.74 lists the inductive reactance, the capacitive reactance, and the impedance for the frequencies listed in the problem statement.

f (Hz)	X_L (Ω)	X_C (Ω)	Z (Ω)
300	283	12 600	12 300
600	565	6 280	5 720
800	754	4 710	3 960
1 000	942	3 770	2 830
1 500	1 410	2 510	1 100
2 000	1 880	1 880	40
3 000	2 830	1 260	1 570
4 000	3 770	942	2 830
6 000	5 650	628	5 020
10 000	9 420	377	9 040

TABLE P33.74

(b) ANS. FIG. P33.74(b) shows a graph of X_L, X_C, and Z as a function of the frequency f.



ANS. FIG. P33.74(b)

P33.75 The resonance frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.050\ 0\ \text{H})(5.00 \times 10^{-6}\ \text{F})}} = 318\ \text{Hz}$$

The operating frequency is $f = f_0/2 = 159$ Hz. We can calculate the impedance at this frequency. The inductive reactance is

$$X_L = 2\pi f L = 2\pi(159\ \text{Hz})(0.050\ 0\ \text{H}) = 50.0\ \Omega$$

The capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(159\ \text{Hz})(5.00 \times 10^{-6}\ \text{F})} = 200\ \Omega$$

The impedance is

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{8.00^2 + (50.0 - 200)^2}\ \Omega = 150\ \Omega$$

So the current is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{400\ V}{150\ \Omega} = 2.66\ \text{A}$$

The power delivered by the source is the power delivered to the resistor:

$$P = (2.66\ \text{A})^2(8.00\,\Omega) = \boxed{56.7\ \text{W}}$$

P33.76 (a) At resonance,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\left(1.00 \times 10^{-3}\ \text{H}\right)\left(1.00 \times 10^{-9}\ \text{F}\right)}} = 1.00 \times 10^{6}\ \text{rad/s}$$

At that point,

$$Z = R = 1.00\ \Omega \quad \text{and} \quad I = \frac{1.00\ \text{V}}{1.00\ \Omega} = 1.00\ \text{A}$$

The power is

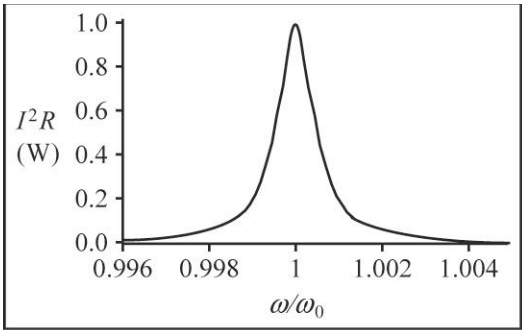
$$P = I^2R = (1.00\ \text{A})^2(1.00\ \Omega) = 1.00\ \text{W}$$

We compute the power at some other angular frequencies, listed in TABLE P33.76.

$\frac{\omega}{\omega_0}$	$\omega L\ (\Omega)$	$\frac{1}{\omega C}\ (\Omega)$	$Z\ (\Omega)$	$P = I_{\text{rms}}^2 R\ (W)$
0.9990	999.0	1001.0	2.24	0.19984
0.9991	999.1	1000.9	2.06	0.23569
0.9993	999.3	1000.7	1.72	0.33768
0.9995	999.5	1000.5	1.41	0.49987
0.9997	999.7	1000.3	1.17	0.73524
0.9999	999.9	1000.1	1.02	0.96153
1.0000	1000	1000.0	1.00	1.00000
1.0001	1000.1	999.9	1.02	0.96154
1.0003	1000.3	999.7	1.17	0.73535
1.0005	1000.5	999.5	1.41	0.50012
1.0007	1000.7	999.3	1.72	0.33799
1.0009	1000.9	999.1	2.06	0.23601
1.0010	1001	999.0	2.24	0.20016

TABLE P33.76

ANS. FIG. P33.76 shows a graph of the results tabulated above.



ANS. FIG. P33.76

(b) The angular frequencies giving half the maximum power are

$$0.999\,5 \times 10^6\ \text{rad/s} \quad \text{and} \quad 1.000\,5 \times 10^6\ \text{rad/s}$$

so the full width at half the maximum is

$$\Delta\omega = (1.000\,5 - 0.999\,5) \times 10^6 \text{ rad/s}$$

$$\Delta\omega = 1.00 \times 10^3 \text{ rad/s}$$

Since $\Delta\omega = 2\pi\,\Delta f$, $\Delta f = 159$ Hz

and for comparison,

$$\frac{R}{2\pi L} = \frac{1.00\ \Omega}{2\pi\left(1.00 \times 10^{-3}\,\text{H}\right)} = 159\text{ Hz}$$

The two quantities agree.

Challenge Problems

P33.77 We start with

$$\frac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}} = \frac{IR}{IZ} = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

$$= \frac{8.00\ \Omega}{\sqrt{(8.00\ \Omega)^2 + \left[2\pi fL - 1/2\pi f C\right]^2}}$$

Then, at 200 Hz,

$$\left(\frac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}}\right)^2 = \left(\frac{R}{Z}\right)^2 = \frac{1}{4} = \frac{(8.00\ \Omega)^2}{(8.00\ \Omega)^2 + \left[400\pi L - 1/400\pi C\right]^2}$$

and at 4 000 Hz,

$$\left(\frac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}}\right)^2 = \left(\frac{R}{Z}\right)^2 = \frac{1}{4} = \frac{(8.00\ \Omega)^2}{(8.00\ \Omega)^2 + \left[8000\pi L - 1/8000\pi C\right]^2}$$

At the low frequency, $X_L - X_C < 0$. This reduces to

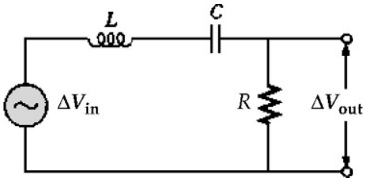
$$400\pi L - \frac{1}{400\pi C} = -13.9\ \Omega \qquad \textbf{[1]}$$

For the high frequency half-voltage point,

$$8\,000\pi L - \frac{1}{8000\pi C} = +13.9\ \Omega \qquad \textbf{[2]}$$

Solving equations [1] and [2] simultaneously gives

(a) $L = \boxed{580\ \mu\text{H}}$



ANS. FIG. P33.77(a)

(b) $C = \boxed{54.6\ \mu\text{F}}$

(c) When $X_L = X_C$, $\dfrac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}} = \left(\dfrac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}}\right)_{\text{max}} = \boxed{1.00}$

(d) $X_L = X_C$ requires

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(5.80\times10^{-4}\ \text{H}\right)\left(5.46\times10^{-5}\ \text{F}\right)}} = \boxed{894\ \text{Hz}}$$

(e) $\boxed{\text{At 200 Hz}}$, $\dfrac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}} = \dfrac{R}{Z} = \dfrac{1}{2}$ and $X_C > X_L$, so the phasor diagram is as shown in ANS. FIG. P33.77(e).

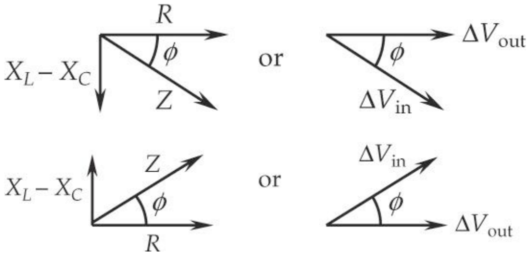
$$\boxed{\phi =} -\cos^{-1}\left(\frac{R}{Z}\right) = -\cos^{-1}\left(\frac{1}{2}\right) = \boxed{-60.0^\circ}$$

so $\boxed{\left(\Delta v_{\text{out}}\ \text{leads}\ \Delta v_{\text{in}}\right)}$.

$\boxed{\text{At } f_0}$, $X_L = X_C$ so

$$\boxed{\phi = 0\ (\Delta v_{\text{out}} \text{ is in phase with } \Delta v_{\text{in}})}\,.$$

$\boxed{\text{At 4 000 Hz}}$, $\dfrac{\Delta v_{\text{out}}}{\Delta v_{\text{in}}} = \dfrac{R}{Z} = \dfrac{1}{2}$ and $X_L - X_C > 0$.



ANS. FIG. P33.77(e)

Thus, $\boxed{\phi =} \cos^{-1}\left(\dfrac{1}{2}\right) = \boxed{+60.0^\circ}$

or $\boxed{\Delta v_{\text{out}}\ \text{lags}\ \Delta v_{\text{in}}}$.

(f) At 200 Hz and at 4 kHz,

$$P = \frac{\left(\Delta v_{\text{out, rms}}\right)^2}{R} = \frac{\left[(1/2)\Delta v_{\text{in, rms}}\right]^2}{R} = \frac{(1/4)\left(\Delta v_{\text{in, max}}/\sqrt{2}\right)^2}{R}$$

$$= \frac{(1/2)(10.0\text{ V})^2}{4(8.00\ \Omega)} = \boxed{1.56\text{ W}}$$

At f_0,

$$P = \frac{\left(\Delta v_{\text{out, rms}}\right)^2}{R} = \frac{\left(\Delta v_{\text{in, rms}}\right)^2}{R} = \frac{\left(\Delta v_{\text{in, max}}/\sqrt{2}\right)^2}{R} = \frac{(1/2)(10.0\text{ V})^2}{(8.00\ \Omega)}$$

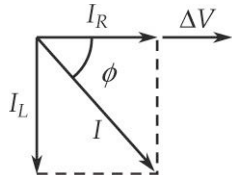
$$= \boxed{6.25\text{ W}}$$

(g) We take

$$Q = \frac{\omega_0 L}{R} = \frac{2\pi f_0 L}{R} = \frac{2\pi(894\text{ Hz})\left(5.80\times10^{-4}\text{ H}\right)}{8.00\ \Omega} = \boxed{0.408}$$

P33.78 (a) $I_{R,\text{ rms}} = \dfrac{\Delta V_{\text{rms}}}{R} = \dfrac{100\text{ V}}{80.0\ \Omega} = \boxed{1.25\text{ A}}$

(b) The total current will $\boxed{\text{lag}}$ the applied voltage as seen in the phasor diagram shown in ANS. FIG. P33.78.



ANS. FIG. P33.78

$$I_{L,\text{ rms}} = \frac{\Delta V_{\text{rms}}}{X_L}$$

$$= \frac{100\text{ V}}{2\pi\left(60.0\text{ s}^{-1}\right)(0.200\text{ H})} = 1.33\text{ A}$$

Thus, the phase angle is:

$$\phi = \tan^{-1}\left(\frac{I_{L,\text{ rms}}}{I_{R,\text{ rms}}}\right) = \tan^{-1}\left(\frac{1.33\text{ A}}{1.25\text{ A}}\right) = \boxed{46.7°}$$

P33.79 We are given $L = 2.00$ H, $C = 10.0 \times 10^{-6}$ F, $R = 10.0\ \Omega$, and $\Delta v = (100\sin\omega t)$.

(a) The resonance frequency ω_0 produces the maximum current and thus the maximum power delivery to the resistor.

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2.00\text{ H})\left(10.0\times10^{-6}\text{ F}\right)}} = \boxed{224\text{ s}^{-1}}$$

(b) At the resonance frequency ω_0, the impedance $Z = R$, and

$$\begin{aligned} P &= I_{\text{rms}}^2 R \\ &= \left(\frac{\Delta V_{\text{rms}}}{Z}\right)^2 R = \frac{(\Delta V_{\text{rms}})^2}{Z^2} R = \frac{(\Delta V_{\text{rms}})^2}{R^2} R = \frac{\left(\Delta V_{\max}/\sqrt{2}\right)^2}{R} \\ &= \frac{(100\text{ V})^2}{2(10.0\ \Omega)} = \boxed{500\text{ W}} \end{aligned}$$

(c) Now,

$$P = \frac{1}{2} P_{\max} \quad \rightarrow \quad \frac{(\Delta V_{\text{rms}})^2}{Z^2} R = \frac{1}{2} \frac{(\Delta V_{\text{rms}})^2}{R}$$

So, $Z^2 = 2R^2$, or

$$R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = 2R^2$$

which is a fourth order equation in ω. But this can be simplified to two equations:

$$\omega L - \frac{1}{\omega C} = \pm R \quad \rightarrow \quad \omega^2 LC \mp \omega CR - 1 = 0$$

The angular frequency ω must be positive, so we solve for the positive roots. (In the following, we suppress all units.)

For $\omega^2 LC - \omega CR - 1 = 0$,

$$\begin{aligned} \omega &= \frac{-(-CR) + \sqrt{(-CR)^2 - 4LC(-1)}}{2LC} \\ &= \frac{R + \sqrt{R^2 + 4L/C}}{2L} \\ &= \frac{10.0 + \sqrt{(10.0)^2 + 4(2.00)/(10.0 \times 10^{-6})}}{2(2.00)} = \boxed{226\text{ s}^{-1}} \end{aligned}$$

For $\omega^2 LC + \omega CR - 1 = 0$,

$$\begin{aligned} \omega &= \frac{-(CR) + \sqrt{(-CR)^2 - 4LC(-1)}}{2LC} \\ &= \frac{-R + \sqrt{R^2 + 4L/C}}{2L} \\ &= \frac{-10.0 + \sqrt{(10.0)^2 + 4(2.00)/(10.0 \times 10^{-6})}}{2(2.00)} = \boxed{221\text{ s}^{-1}} \end{aligned}$$

P33.80 The currents in the three components of the circuit are

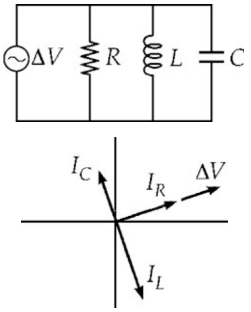
$$I_R = \frac{\Delta V_{\text{rms}}}{R},\ I_L = \frac{\Delta V_{\text{rms}}}{X_L} = \frac{\Delta V_{\text{rms}}}{\omega L},\ \text{and } I_C = \frac{\Delta V_{\text{rms}}}{X_C} = \frac{\Delta V_{\text{rms}}}{(\omega C)^{-1}}$$

(a) Then,

$$I_{\text{rms}} = \left[I_R^2 + (I_C - I_L)^2\right]^{1/2} = \boxed{\Delta V_{\text{rms}}\left[\left(\frac{1}{R^2}\right) + \left(\omega C - \frac{1}{\omega L}\right)^2\right]^{1/2}}$$

(b) $\tan\phi = \dfrac{I_C - I_L}{I_R} = \Delta V_{\text{rms}}\left[\dfrac{1}{X_C} - \dfrac{1}{X_L}\right]\left(\dfrac{1}{\Delta V_{\text{rms}}/R}\right)$

$$\boxed{\tan\phi = R\left(\frac{1}{X_C} - \frac{1}{X_L}\right)}$$



ANS. FIG. P33.80

P33.81 We have $P = I_{\text{rms}}^2 R = \left(\dfrac{\Delta V_{\text{rms}}}{Z}\right)^2 R = \dfrac{(\Delta V_{\text{rms}})^2}{Z^2}R$, and

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Therefore,

$$Z^2 = \frac{(\Delta V_{\text{rms}})^2 R}{P} \quad \rightarrow \quad R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{(\Delta V_{\text{rms}})^2 R}{P}$$

$$\left(\omega L - \frac{1}{\omega C}\right)^2 = \frac{(\Delta V_{\text{rms}})^2 R}{P} - R^2$$

which is a fourth order equation in ω. But this can be simplified to two equations:

$$\omega L - \frac{1}{\omega C} = \pm A \quad \rightarrow \quad \omega^2 LC \mp \omega CA - 1 = 0$$

where $A = \sqrt{\frac{(\Delta V_{\text{rms}})^2 R}{P} - R^2}$.

We will solve for ω when $\Delta V_{\text{rms}} = 100$ V and $P = 250$ W. From Figure P33.24, we have $R = 40.0\ \Omega$, $L = 185$ mH $= 0.185$ H, and $C = 65.0\ \mu$F $= 65.0 \times 10^{-6}$ F.

The quantity A is

$$\begin{aligned} A &= \sqrt{\frac{(\Delta V_{\text{rms}})^2 R}{P} - R^2} = \sqrt{\frac{(120\text{ V})^2(40.0\ \Omega)}{250\text{ W}} - (40.0\ \Omega)^2} \\ &= \sqrt{704}\ \Omega \end{aligned}$$

The angular frequency ω must be positive, so we solve for the positive roots. (In the following, we suppress all units.)

For $\omega^2 LC - \omega CA - 1 = 0$,

$$\begin{aligned} \omega &= \frac{-(-CA) + \sqrt{(-CA)^2 - 4LC(-1)}}{2LC} \\ &= \frac{A + \sqrt{A^2 + 4L/C}}{2L} \\ &= \frac{\sqrt{704} + \sqrt{704 + 4(0.185)/(65.0 \times 10^{-6})}}{2(0.185)} \\ &= 226\text{ s}^{-1} = 2\pi f \quad \rightarrow \quad f = \boxed{58.7\text{ Hz}} \end{aligned}$$

For $\omega^2 LC + \omega CA - 1 = 0$,

$$\begin{aligned} \omega &= \frac{-(CA) + \sqrt{(-CA)^2 - 4LC(-1)}}{2LC} \\ &= \frac{-A + \sqrt{A^2 + 4L/C}}{2L} \\ &= \frac{-\sqrt{704} + \sqrt{704 + 4(0.185)/(65.0 \times 10^{-6})}}{2(0.185)} \\ &= 225\text{ s}^{-1} = 2\pi f \quad \rightarrow \quad f = \boxed{35.9\text{ Hz}} \end{aligned}$$

There are two answers because $\boxed{\text{the circuit can be either above or below resonance.}}$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P33.2 (a) 193 Ω; (b) 144 Ω

P33.4 (a) 170 V; (b) $2.40 \times 10^2\ \Omega$;
(c) Because $P_{\text{avg}} = \dfrac{(\Delta V_{\text{rms}})^2}{R} \quad \rightarrow \quad R = \dfrac{(\Delta V_{\text{rms}})^2}{P_{\text{avg}}}$, a 100-W bulb has less resistance than a 60.0-W bulb.

P33.6 (a) 25.3 rad/s; (b) 0.114 s

P33.8 (a) The rms current in each 150-W bulb is 1.25 A. The rms current in the 100-W bulb is 0.833 A; (b) $R_1 = 96.0\ \Omega$, $R_2 = 96.0\ \Omega$, and $R_3 = 144\ \Omega$; (c) 36.0 Ω

P33.10 (a) 0.0424 H; (b) 942 rad/s

P33.12 0.750 H

P33.14 3.80 J

P33.16 (a) 15.0 Hz; (b) 84.9 V; (c) 47.1 Ω; (d) 1.80 A; (e) 2.55 A

P33.18 (a) 221 Ω; (b) 0.163 A; (c) 0.230 A; (d) no

P33.20 (a) 69.3 V; (b) 40.0 Hz; (c) 20.3 μF

P33.22 $\sqrt{2}C(\Delta V_{\text{rms}})$

P33.24 (a) 146 V ; (b) 212 V; (c) 179 V; (d) 33.4 V

P33.26 (a) 109 Ω; (b) $I_{\text{max}} = 0.367$ A; (c) $\omega = 100$ rad/s;
(d) $\phi = -0.896 \text{ rad} = -51.3°$

P33.28 2.79 kHz

P33.30 See ANS. FIG P33.30.

P33.32 (a) 88.4 Ω; (b) 107 Ω; (c) 1.12 A; (d) the voltage lags behind the current by 55.8°; (e) Adding an inductor will change the impedence, and hence the current in the circuit. The current could be larger or smaller, depending on the inductance added. The largest current would result when the inductive reactance equals the capacitive reactance, the impedance has its minimum value, equal to 60.0 Ω, and the current in the circuit is

$$I_{\text{max}} = \frac{\Delta V_{\text{max}}}{Z} = \frac{\Delta V_{\text{max}}}{R} = \frac{1.20 \times 10^2 \text{ V}}{60.0\ \Omega} = 2.00 \text{ A}$$

P33.34 In order for the power factor to be equal to 1.00, we would have to have $X_L = 0$, which would require either L or f to be zero. Because this is not the case, the situation is impossible.

P33.36	8.00 W
P33.38	(a) 66.8 Ω; (b) 0.953 A; (c) 45.4 W
P33.40	(a) 5.43 A; (b) 0.905; (c) 281 μF; (d) 109 V
P33.42	(a) 3.56 kHz; (b) 5.00 A; (c) 22.4; (d) 2.24 kV
P33.44	(a) 156 pH; (b) 8.84 Ω
P33.46	$\frac{4\pi RC\sqrt{LC}\left(\Delta V_{\text{rms}}\right)^2}{4R^2C+9L}$
P33.48	(a) 9.23 V; (b) 2.40 W
P33.50	(a) 29.0 kW; (b) 5.80×10^{-3}; (c) It is impossible to transmit so much power at such low voltage.
P33.52	1.88 V
P33.54	(a) $\frac{R}{\sqrt{R^2+\left(1/\omega C\right)^2}}$; (b) 0; (c) 1
P33.56	(a) $\frac{1/\omega C}{\sqrt{R^2+\left(1/\omega C\right)^2}}$; (b) 1; (c) 0; (d) $\frac{\sqrt{3}}{2\pi RC}$
P33.58	The resonance frequency for this circuit is not in the North American AM frequency range.
P33.60	(a) 0.200 A, 36.8°; (b) 40.0 V at $\phi = 0°$; (c) 20.0 V at $\phi = -90.0°$; (d) 50.0 V at $\phi = +90.0°$
P33.62	$\sqrt{\frac{8\rho_{\text{Cu}}\ell P}{\pi f\left(\Delta V_{\text{rms}}\right)^2}}$
P33.64	See P33.64 for full explanation.
P33.66	(a) R = 99.6 Ω; (b) 24.9 μF; (c) 164 mH or 402 mH; (d) Only one value for R and only one value for C are possible. Two values for L are possible.
P33.68	(a) Higher. At the resonance frequency, $X_L = X_C$. As the frequency increases, X_L goes up and X_C goes down; (b) It is possible. We have three independent equations in the three unknowns L, C, and the certain f; (c) L = 4.90 mH and C = 51.0 μF
P33.70	(a) See ANS. FIG. P33.70(a); (b) $\frac{R}{R^2+\left(\omega L-1/\omega C\right)^2}\left(L+\frac{1}{\omega^2 C}\right)$; (c) See P33.70(c) for full explanation.

P33.72 (a) $i = \dfrac{\Delta V_{\max}}{R}\cos\omega t$; (b) $P = \dfrac{1}{2}\dfrac{(\Delta V_{\max})^2}{R}$;

(c) $i = \dfrac{\Delta V_{\max}}{\sqrt{R^2+\omega^2 L^2}}\cos\left[\omega t + \tan^{-1}\left(\dfrac{\omega L}{R}\right)\right]$; (d) $C = \dfrac{1}{\omega_0^2 L}$; (e) R;

(f) $\dfrac{(\Delta V_{\max})^2 L}{2R^2}$; (g) $\dfrac{1}{2}L\dfrac{(\Delta V_{\max})^2}{R^2}$; (h) $\tan^{-1}\left(\dfrac{3}{2R}\sqrt{\dfrac{L}{C}}\right)$; (i) $\dfrac{1}{\sqrt{2LC}}$

P33.74 (a) See Table P33.74; (b) See ANS. FIG. P33.74(b).

P33.76 (a) See ANS. FIG. P33.76; (b) See P33.76 for full explanation.

P33.78 (a) 1.25 A; (b) lag, 46.7°

P33.80 (a) $\Delta V_{\text{rms}}\left[\left(\dfrac{1}{R^2}\right)+\left(\omega C - \dfrac{1}{\omega L}\right)^2\right]^{1/2}$; (b) $\tan\phi = R\left(\dfrac{1}{X_C} - \dfrac{1}{X_L}\right)$

34

Electromagnetic Waves

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ34.1 **(i)** Answer (c). Both the light intensity and the gravitational force follow inverse-square laws.

(ii) Answer (a). The smaller grain presents less face area and feels a smaller force due to light pressure.

OQ34.2 **(i)** Answer (c). **(ii)** Answer (c). **(iii)** Answer (c). **(iv)** Answer (b). **(v)** Answer (b). The same amount of energy passes through concentric spheres of increasing area as the wave travels outward from its source, so the amplitude and the intensity, which is proportional to the square of the amplitude, decrease.

OQ34.3 Answer (b). Frequency, wavelength, and the speed of light are related:

$$f\lambda = c \quad \rightarrow \quad \lambda = \frac{c}{f} = \frac{3.00\times 10^{8}\ \text{m/s}}{2.45\times 10^{9}\ \text{Hz}} = 0.122\ \text{m} = 12.2\ \text{cm}$$

OQ34.4 **(i)** Answer (a). According to $f = 1/2\pi\sqrt{LC}$, to make f half as large, the capacitance should be made four times larger.

(ii) Answer (b). According to $f\lambda = c$, if frequency is halved, wavelength is doubled.

OQ24.5 Answer (e). Accelerating charge, changing electric field, or changing magnetic field can be the source of a radiated electromagnetic wave.

OQ34.6 Answers (c) and (d). The relationship between frequency, wavelength, and the speed of a wave is $f\lambda = v$. In a vacuum, all electromagnetic waves travel at the same speed: $v = c$. Electromagnetic waves, consisting of oscillating electric and magnetic fields, are transverse waves.

OQ34.7 **(i)** through **(v)** have the same answer (c). The same amount of energy passes through equal areas parallel to the yz plane as the wave travels in the $+x$ direction, so the amplitude and the intensity, which is proportional to the square of the amplitude, do not change.

OQ34.8 **(i)** Answer (b). Electric and magnetic fields both carry the same energy, so their amplitudes are proportional to each other.

(ii) Answer (a). The intensity is proportional to the square of the amplitude.

OQ34.9 Answer (d). The peak values of the electric and magnetic field components of an electromagnetic wave are related by $E_{\max}/B_{\max} = c$, where c is the speed of light in vacuum. Thus,

$$E_{\max} = cB_{\max} = \left(3.00\times10^{8}\ \mathrm{m/s}\right)\left(1.50\times10^{-7}\ \mathrm{T}\right) = 45.0\ \mathrm{N/C}$$

OQ34.10 **(i)** The ranking is c > b > d > e > a. Gamma rays have the shortest wavelength.

(ii) The ranking is a > e > d > b > c. According to $f\lambda = c$, as wavelength decreases, frequency increases.

(iii) The ranking is a = b = c = d = e. All electromagnetic waves travel at the speed of light c in vacuum, which is assumed here.

OQ34.11 Answer (d). An electromagnetic wave travels in the direction of the Poynting vector: $\vec{\mathbf{S}} = \vec{\mathbf{E}}\times\vec{\mathbf{B}}/\mu_0$.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ34.1 The entire room and its contents have a soft glow. Incandescent light bulbs shine brightly in the infrared, but fluorescent lights do not. The top of a computer monitor glows brighter than the screen, which glows faintly. Windowpanes appear dark if they are cool, and a patch of wall where sunlight falls glows brighter than where the sunlight does not fall. Heating resistors or warm air outlets shine, and the air near to them has a faint glow, but cold air outlets are dark, and the nearby air has no glow.

CQ34.2 Electromagnetic waves carry momentum. Recalling what we learned in Chapter 9, the impulse imparted by a particle that bounces elastically off a wall is twice that imparted by an object that sticks to a wall. Similarly, the impulse, and hence the pressure exerted by a wave reflecting from a surface, must be twice that exerted by a wave that is absorbed.

CQ34.3 No. Radio waves travel at a finite speed, the speed of light. Radio waves can travel around the curved surface of the Earth, bouncing between the ground and the ionosphere, which has an altitude that is small when compared to the radius of the Earth. The distance across the lower forty-eight states is approximately 5 000 km, requiring a transit time of $\dfrac{5\times10^{6}\text{ m}}{3\times10^{8}\text{ m/s}}\sim10^{-2}\text{ s}$.

CQ34.4

Sound	*Light*
1) Sound is a longitudinal wave.	1) Light is a transverse wave.
2) Sound requires a material medium.	2) Light does not require a material medium.
3) Sound in air moves at hundreds of meters per second.	3) Light in air moves at hundreds of millions of meters per second.
4) The speed of sound through a medium, depending the material of the medium, can be faster or slower than that in air.	4) The speed of light through materials is less than in vacuum.

5) Sound propagates by a chain reaction of density and pressure disturbances recreating each other.	5) Light propagates by a chain reaction of electric and magnetic fields recreating each other.
6) Audible sound has frequencies over a range of three decades (ten octaves) from 20 Hz to 20 kHz.	6) Visible light has frequencies over a range of less than one octave, from 430 to 700 THz (THz = Terahertz = 10^{12} Hz).
7) Audible sound has wavelengths of ordinary size (1.7 cm to 17 m).	7) Visible light has wavelengths of very small size (400 nm to 750 nm).

CQ34.5 The changing magnetic field of the solenoid induces eddy currents in the conducting core. This is accompanied by I^2R conversion by heating of electrically-transmitted energy into internal energy in the conductor.

CQ34.6 (a) The electric and magnetic fields of the light wave oscillate in time at each point in space, like sports fans in a grandstand when the crowd does "the wave."

(b) The wave transports energy.

CQ34.7 An infrared photograph records the infrared light *reflected*, but also *emitted* by a person's face. When a person blushes or exercises or becomes excited, warmer areas glow brighter in the infrared. A person's nostrils and the openings of the ear canals are bright; brighter still are just the pupils of the eyes.

CQ34.8 No, they do not. Specifically, Gauss's law in magnetism prohibits magnetic monopoles. If magnetic monopoles existed, then the magnetic field lines would not have to be closed loops, but could begin or terminate on a magnetic monopole, as they can in Gauss's law in electrostatics.

CQ34.9 Different stations have transmitting antennas at different locations. For best reception align your rabbit ears perpendicular to the straight-line path from your TV to the transmitting antenna. The transmitted signals are also polarized. The polarization direction of the wave can be changed by reflection from surfaces—including the atmosphere—and through Kerr rotation—a change in polarization axis when passing through an organic substance. In your home, the plane of polarization is determined by your surroundings, so antennas need to be adjusted to align with the polarization of the wave.

CQ34.10 Consider a typical metal rod antenna for a car radio. Charges in the rod respond to the electric field portion of the carrier wave. Variations in the amplitude of the incoming radio wave cause the electrons in the rod to vibrate with amplitudes emulating those of the carrier wave. Likewise, for frequency modulation, the variations of the frequency of the carrier wave cause constant-amplitude vibrations of the electrons in the rod but at frequencies that imitate those of the carrier.

CQ34.11 The Poynting vector $\vec{\mathbf{S}}$ describes the energy flow associated with an electromagnetic wave. The direction of $\vec{\mathbf{S}}$ is along the direction of propagation and the magnitude of $\vec{\mathbf{S}}$ is the rate at which electromagnetic energy crosses a unit surface area perpendicular to the direction of $\vec{\mathbf{S}}$.

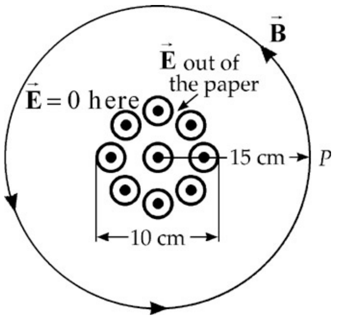
CQ24.12 The frequency of EM waves in a microwave oven, typically 2.45 GHz, is chosen to be in a band of frequencies absorbed by water molecules. The plastic and the glass contain no water molecules. Plastic and glass have very different absorption frequencies from water, so they may not absorb any significant microwave energy and remain cool to the touch.

CQ34.13 Maxwell included a term in Ampère's law to account for the contributions to the magnetic field by changing electric fields, by treating those changing electric fields as "displacement currents."

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 34.1 Displacement Current and the Generalized Form of Ampère's Law

P34.1 (a) We use the right-hand rule for both real and displacement currents. Thus, the direction of $\vec{\mathbf{B}}$ is *counterclockwise*, and the direction of $\vec{\mathbf{B}}$ at P is $\boxed{\text{upwards.}}$



ANS. FIG. P34.1

(b) We use the extended form of Ampère's law, Equation 34.7. Since no moving charges are present, $I = 0$ and we have

$$\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$

In order to evaluate the integral, we make use of the symmetry of the situation. Symmetry requires that no particular direction from the center can be any different from any other direction. Therefore, there must be *circular symmetry* about the central axis. We know the magnetic field lines are circles about the axis. Therefore, as we travel around such a magnetic field circle, the magnetic field remains constant in magnitude. Setting aside until later the determination of the *direction* of $\vec{\mathbf{B}}$, we integrate $\oint \vec{\mathbf{B}} \cdot d\vec{\ell}$ around the circle at $R = 0.150$ m to obtain $2\pi RB$.

Differentiating the expression $\Phi_E = AE$, we have

$$\frac{d\Phi_E}{dt} = \left(\frac{\pi d^2}{4}\right)\frac{dE}{dt}$$

Thus, $\oint \vec{\mathbf{B}} \cdot d\vec{\ell} = 2\pi RB = \mu_0 \epsilon_0 \left(\frac{\pi d^2}{4}\right)\frac{dE}{dt} \quad \rightarrow \quad B = \frac{\mu_0 \epsilon_0 d^2}{8R}\frac{dE}{dt}$

Substituting numerical values,

$$B = \frac{(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A})(8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2})(0.100\ \mathrm{m})^2}{8(0.150\ \mathrm{m})} \times (20.0\ \mathrm{V/m\cdot s})$$

$$= \boxed{1.85\times 10^{-18}\ \mathrm{T}}$$

P34.2 For the capacitor,

$$\frac{d\Phi_E}{dt} = \frac{d}{dt}(EA) = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0}$$

(a) $$\frac{dE}{dt} = \frac{I}{\epsilon_0 A} = \frac{0.200\ \mathrm{A}}{(8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2})\left[\pi(10.0\times 10^{-2}\ \mathrm{m})\right]}$$

$$= \boxed{7.19\times 10^{11}\ \mathrm{V/m\cdot s}}$$

(b) $\oint B\cdot ds = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt}: \qquad 2\pi rB = \cancel{\epsilon_0}\mu_0 \frac{d}{dt}\left[\frac{Q}{\cancel{\epsilon_0}A}\cdot \pi r^2\right]$

$$B = \frac{\mu_0 Ir}{2A} = \frac{\mu_0(0.200\ \mathrm{A})(5.00\times 10^{-2}\ \mathrm{m})}{2\left[\pi(10.0\times 10^{-2}\ \mathrm{m})^2\right]} = \boxed{2.00\times 10^{-7}\ \mathrm{T}}$$

P34.3 The electric field in the space between the plates is $E = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q}{\epsilon_0 A}$.

The flux of this field is $\Phi_E = \vec{\text{E}} \cdot \vec{\text{A}} = \left(\dfrac{Q}{\epsilon_0 A}\right) A \cos 0^\circ = \dfrac{Q}{\epsilon_0}$.

(a) The rate of change of flux is

$$\frac{d\Phi_E}{dt} = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0} = \frac{0.100\text{ A}}{8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2}$$
$$= \boxed{11.3\times 10^{9}\text{ V}\cdot\text{m/s}}$$

(b) The displacement current is defined as

$$I_d = \epsilon_0 \frac{d\Phi_E}{dt}$$
$$= (8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1.13\times 10^{10}\text{ N}\cdot\text{m}^2/\text{C}\cdot\text{s})$$
$$= \boxed{0.100\text{ A}}$$

Section 34.2 Maxwell's Equations and Hertz's Discoveries

P34.4 $\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}}$ so

$\vec{\mathbf{a}} = \dfrac{-e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right]$ where

$$\vec{\mathbf{v}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 10.0 & 0 & 0 \\ 0 & 0 & 0.400 \end{vmatrix} = -(4.00\text{ T}\cdot\text{m/s})\hat{\mathbf{j}}$$

Then

$$\vec{\mathbf{a}} = \left(\frac{-1.60\times 10^{-19}\text{ C}}{9.11\times 10^{-31}\text{ kg}}\right)$$
$$\times\left[(2.50\text{ V/m})\hat{\mathbf{i}} + (5.00\text{ V/m})\hat{\mathbf{j}} - (4.00\text{ T}\cdot\text{m/s})\hat{\mathbf{j}}\right]$$
$$= \left(-1.76\times 10^{11}\right)\left[2.50\hat{\mathbf{i}} + 1.00\hat{\mathbf{j}}\right]\text{ m/s}^2$$
$$\vec{\mathbf{a}} = \boxed{\left(-4.39\hat{\mathbf{i}} - 1.76\hat{\mathbf{j}}\right)\times 10^{11}\text{ m/s}^2}$$

P34.5 The net force on the proton is the Lorentz force, as described by

$$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}}\times\vec{\mathbf{B}} \quad \text{so that} \quad \vec{\mathbf{a}} = \frac{e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right]$$

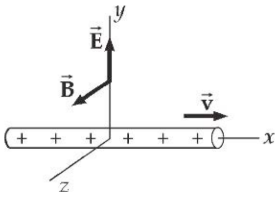
Taking the cross product of $\vec{\mathbf{v}}$ and $\vec{\mathbf{B}}$,

$$\vec{\mathbf{v}}\times\vec{\mathbf{B}} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ 200 & 0 & 0 \\ 0.200 & 0.300 & 0.400 \end{vmatrix} = -200(0.400)\hat{\mathbf{j}} + 200(0.300)\hat{\mathbf{k}}$$

Then, $$\vec{\mathbf{a}} = \frac{e}{m}\left[\vec{\mathbf{E}} + \vec{\mathbf{v}}\times\vec{\mathbf{B}}\right] = \left(\frac{1.60\times10^{-19}}{1.67\times10^{-27}}\right)\left[50.0\,\hat{\mathbf{j}} - 80.0\,\hat{\mathbf{j}} + 60.0\,\hat{\mathbf{k}}\right]\text{ m/s}^2$$

$$= \boxed{\left(-2.87\times10^{9}\hat{\mathbf{j}} + 5.75\times10^{9}\hat{\mathbf{k}}\right)\text{ m/s}^2}$$

P34.6 (a) The very long rod creates the same electric field that it would if stationary. We apply Gauss's law to a cylinder, concentric with the rod, of radius r = 20.0 cm and length ℓ:



ANS. FIG. P34.6

$$\oint\vec{\mathbf{E}}\cdot d\vec{\mathbf{A}} = \frac{q_{\text{inside}}}{\epsilon_0}$$

$$E\left(2\pi r\ell\right)\cos 0° = \frac{\lambda\ell}{\epsilon_0}$$

$$\vec{\mathbf{E}} = \frac{\lambda}{2\pi\epsilon_0 r} \text{ radially outward}$$

$$= \frac{35.0\times10^{-9}\text{ C/m}}{2\pi\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)(0.200\text{ m})}\hat{\mathbf{j}}$$

$$= \boxed{3.15\times10^{3}\hat{\mathbf{j}}\text{ N/C}}$$

(b) The charge in motion constitutes a current of

$$(35.0\times10^{-9}\text{ C/m})\times(15.0\times10^{6}\text{ m/s}) = 0.525\text{ A}$$

This current creates a magnetic field.

$$\vec{\mathbf{B}} = \frac{\mu_0 I}{2\pi r}$$

$$= \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(0.525\text{ A})}{2\pi(0.200\text{ m})}\hat{\mathbf{k}} = \boxed{5.25\hat{\mathbf{k}}\times10^{-7}\text{ T}}$$

(c) The Lorentz force on the electron is $\vec{\mathbf{F}} = q\vec{\mathbf{E}} + q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$.

$$\begin{aligned}\vec{\mathbf{F}} &= \left(-1.60\times10^{-19}\text{ C}\right)\left(3.15\times10^{3}\,\hat{\mathbf{j}}\text{ N/C}\right)\\ &\quad+\left(-1.60\times10^{-19}\text{ C}\right)\left(240\times10^{6}\,\hat{\mathbf{i}}\text{ m/s}\right)\\ &\quad\times\left(5.25\times10^{-7}\,\hat{\mathbf{k}}\text{ T}\right)\end{aligned}$$

$$\begin{aligned}\vec{\mathbf{F}} &= 5.04\times10^{-16}\left(-\hat{\mathbf{j}}\right)\text{ N} + 2.02\times10^{-17}\left(+\hat{\mathbf{j}}\right)\text{ N}\\ &= \boxed{4.83\left(-\hat{\mathbf{j}}\right)\times10^{-16}\text{ N}}\end{aligned}$$

Section 34.3 Plane Electromagnetic Waves

***P34.7** (a) From Equation 34.20,

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{1\ 150\times10^{3}\text{ s}^{-1}} = 261\text{ m}$$

so

$$\frac{180\text{ m}}{261\text{ m}} = \boxed{0.690\text{ wavelengths}}$$

(b) From Equation 34.20,

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{98.1\times10^{6}\text{ s}^{-1}} = 3.06\text{ m}$$

so

$$\frac{180\text{ m}}{3.06\text{ m}} = \boxed{58.9\text{ wavelengths}}$$

***P34.8** From Equation 34.20,

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{27.33\times10^{6}\text{ Hz}} = \boxed{11.0\text{ m}}$$

P34.9 (a) Since the light from this star travels at 3.00×10^{8} m/s, the last bit of light will hit the Earth in

$$t = \frac{d}{c} = \frac{6.44\times10^{18}\text{ m}}{2.998\times10^{8}\text{ m/s}} = 2.15\times10^{10}\text{ s} = \boxed{681\text{ years}}$$

(b) From Table C.4 (in Appendix C of the textbook), the average Earth-Sun distance is $d = 1.496 \times 10^{11}$ m, giving the transit time as

$$t = \frac{d}{c} = \left(\frac{1.496 \times 10^{11}\ \text{m}}{2.998 \times 10^{8}\ \text{m/s}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{8.32\ \text{min}}$$

(c) Also from Table C.4, the average Earth-Moon distance is $d = 3.84 \times 10^{8}$ m, giving the time for the round trip as

$$t = \frac{2d}{c} = \frac{2\left(3.84 \times 10^{8}\ \text{m}\right)}{2.998 \times 10^{8}\ \text{m/s}} = \boxed{2.56\ \text{s}}$$

P34.10 From $f\lambda = c,$ we have

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^{8}\ \text{m/s}}{632.8 \times 10^{-9}\ \text{m}} = \boxed{4.738 \times 10^{14}\ \text{Hz}}$$

P34.11 In the fundamental mode, there is a single loop in the standing wave between the plates. Therefore, the distance between the plates is equal to half a wavelength.

$$\lambda = 2L = 2(2.00\ \text{m}) = 4.00\ \text{m}$$

Thus, $$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \text{m/s}}{4.00\ \text{m}} = 7.50 \times 10^{7}\ \text{Hz} = \boxed{75.0\ \text{MHz}}.$$

P34.12 $\frac{E}{B} = c$ or $\frac{220\ \text{V/m}}{B} = 3.00 \times 10^{8}\ \text{m/s},$ so

$$B = 7.33 \times 10^{-7}\ \text{T} = \boxed{733\ \text{nT}}$$

P34.13 From Equation 34.17,

$$v = \frac{1}{\sqrt{\kappa\mu_0 \epsilon_0}} = \frac{1}{\sqrt{1.78}}c = 0.750c = \boxed{2.25 \times 10^{8}\ \text{m/s}}$$

***P34.14** Time to reach object

$$= \frac{1}{2}(\text{total time of flight}) = \frac{1}{2}\left(4.00 \times 10^{-4}\ \text{s}\right) = 2.00 \times 10^{-4}\ \text{s}$$

Thus,

$$d = vt = \left(3.00 \times 10^{8}\ \text{m/s}\right)\left(2.00 \times 10^{-4}\ \text{s}\right) = 6.00 \times 10^{4}\ \text{m} = \boxed{60.0\ \text{km}}$$

P34.15 (a) $c = f\lambda$ gives the frequency as

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\ \text{m/s}}{50.0\ \text{m}} = \boxed{6.00 \times 10^{6}\ \text{Hz}}$$

(b) $c = E/B$ gives the magnetic field amplitude as

$$B = \frac{E}{c} = \frac{22.0\text{ V/m}}{3.00\times10^{8}\text{ m/s}} = 7.33\times10^{-8}\text{ T} = 73.3\text{ nT}$$

$\vec{\mathbf{B}}$ must be directed along **negative** ***z*** **direction** when $\vec{\mathbf{E}}$ is in the negative *y* direction, so that $\vec{\mathbf{S}} = \vec{\mathbf{E}}\times\vec{\mathbf{B}}/\mu_0$ will propagate in the direction $\left(-\hat{\mathbf{j}}\right)\times\left(-\hat{\mathbf{k}}\right) = +\hat{\mathbf{i}}$. So,

$$\vec{\mathbf{B}}_{\text{max}} = \boxed{-73.3\hat{\mathbf{k}}\text{ nT}}$$

(c) $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{50.0\text{m}} = 0.126\text{ m}^{-1}$

and $\omega = 2\pi f = 2\pi\left(6.00\times10^{6}\text{ s}^{-1}\right) = 3.77\times10^{7}\text{ rad/s}$.

Then,

$$\vec{\mathbf{B}} = \vec{\mathbf{B}}_{\text{max}}\cos\left(kx-\omega t\right) = \boxed{-73.3\hat{\mathbf{k}}\cos\left(0.126x - 3.77\times10^{7}t\right)\text{ nT}}$$

P34.16 $E = E_{\text{max}}\cos\left(kx-\omega t\right)$

$$\frac{\partial E}{\partial x} = -E_{\text{max}}\sin\left(kx-\omega t\right)(k) \rightarrow \frac{\partial^2 E}{\partial x^2} = -E_{\text{max}}\cos\left(kx-\omega t\right)\left(k^2\right)$$
$$\frac{\partial E}{\partial t} = -E_{\text{max}}\sin\left(kx-\omega t\right)(-\omega) \rightarrow \frac{\partial^2 E}{\partial t^2} = -E_{\text{max}}\cos\left(kx-\omega t\right)(-\omega)^2$$

We must show: $\dfrac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\dfrac{\partial^2 E}{\partial t^2}$

That is, $-\left(k^2\right)E_{\text{max}}\cos\left(kx-\omega t\right) = -\mu_0\epsilon_0(-\omega)^2 E_{\text{max}}\cos\left(kx-\omega t\right)$.

But this is true, because $\dfrac{k^2}{\omega^2} = \left(\dfrac{1}{f\lambda}\right)^2 = \dfrac{1}{c^2} = \mu_0\epsilon_0$.

The proof for the wave of the magnetic field follows precisely the same steps.

P34.17 Since the separation of the burn marks is $d_{\text{A to A}} = 6\text{ cm} \pm 5\% = \dfrac{\lambda}{2}$, then $\lambda = 12\text{ cm }\pm5\%$ and

$$v = \lambda f = \left(0.12\text{ m }\pm5\%\right)\left(2.45\times10^{9}\text{ s}^{-1}\right)$$
$$= \boxed{2.9\times10^{8}\text{ m/s }\pm5\%}$$

P34.18 The amplitudes of the electric and magnetic fields are in the correct ratio so that $E_{\max}/B_{\max} = c$. The ratio of ω to k, however, must also equal the speed of light:

$$\frac{\omega}{k} = \frac{3.00 \times 10^{15}\ \mathrm{s^{-1}}}{9.00 \times 10^{6}\ \mathrm{m^{-1}}} = 3.33 \times 10^{8}\ \mathrm{m/s}$$

This value is higher than the speed of light in a vacuum, so the wave as described is impossible.

P34.19 The wave is of the form $E_y = E_{\max}\sin(kx - \omega t)$.

(a) 100 V/m is the amplitude of the electric field, so the amplitude of the magnetic field is

$$B_{\max} = \frac{E_{\max}}{c} = \frac{100\ \mathrm{V/m}}{3.00\times 10^{8}\ \mathrm{m/s}} = 3.33\times 10^{-7}\ \mathrm{T} = \boxed{0.333\ \mu\mathrm{T}}$$

(b) We compare the given wave function with $y = A\sin(kx - \omega t)$ to see that the wave number is $k = 1.00 \times 10^{7}\ \mathrm{m^{-1}}$. With $k = 2\pi/\lambda$, we then have the wavelength as

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{1.00\times 10^{7}\ \mathrm{m^{-1}}} = \boxed{0.628\ \mu\mathrm{m}}$$

(c) The frequency is

$$f = \frac{c}{\lambda} = \frac{3.00\times 10^{8}\ \mathrm{m/s}}{6.28\times 10^{-7}\ \mathrm{m}} = \boxed{4.77\times 10^{14}\ \mathrm{Hz}}$$

Section 34.4 Energy Carried by Electromagnetic Waves

P34.20 From Equation 17.7, we recall that the intensity I a distance r from a point or spherical source is inversely proportional to the square of the distance: $I = P/4\pi r^2$. At the Earth, $r_1 = 1.496 \times 10^{11}$ m, the intensity is $I_1 = I_E$, then at distance r_2, the intensity $I_2 = 3I_E$. Then,

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

and

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = \left(1.496\times 10^{11}\ \mathrm{m}\right)\sqrt{\frac{1}{3}} = \boxed{8.64\times 10^{10}\ \mathrm{m}}$$

P34.21 In time interval Δt, sunlight travels distance $\Delta x = c\Delta t$. The intensity of the sunlight passing into a volume $\Delta V = A\Delta x$ in time Δt is

$$S = I = \frac{U}{A\Delta t} = \frac{U}{A\,\Delta x/c} = \frac{Uc}{V} = uc$$

$$\frac{\text{Energy}}{\text{Unit Volume}} = u = \frac{I}{c} = \frac{1\,000\text{ W/m}^2}{3.00\times10^8\text{ m/s}} = \boxed{3.33\ \mu\text{J/m}^3}$$

P34.22 (a) $$\frac{P}{\text{area}} = \frac{\text{energy}}{\Delta t\cdot\text{area}} = \frac{600\times10^3\text{ Wh}}{(30\text{ d})(13.0\text{ m})(9.50\text{ m})}\left(\frac{1\text{ d}}{24\text{ h}}\right) = \boxed{6.75\text{ W/m}^2}$$

(b) The car uses gasoline at the rate of $(55\text{ mi/h})\left(\frac{\text{gal}}{25\text{ mi}}\right)$. Its rate of energy conversion is

$$P = 44.0\times10^6\text{ J/kg}\left(\frac{2.54\text{ kg}}{1\text{ gal}}\right)(55\text{ mi/h})\left(\frac{\text{gal}}{25\text{ mi}}\right)\left(\frac{1\text{ h}}{3\,600\text{ s}}\right)$$
$$= 6.83\times10^4\text{ W}$$

Its power-per-footprint-area is

$$\frac{P}{\text{area}} = \frac{6.83\times10^4\text{ W}}{(2.10\text{ m})(4.90\text{ m})} = \boxed{6.64\times10^3\text{ W/m}^2}$$

(c) A powerful automobile that is running on sunlight would have to carry on its roof a solar panel huge compared with the size of the car.

(d) Agriculture and forestry for food and fuels, space heating of large and small buildings, water heating, and heating for drying and many other processes are current and potential applications of solar energy.

P34.23 Power output = (power input)(efficiency).

Thus, $$\text{Power input} = \frac{\text{Power out}}{\text{eff}} = \frac{1.00\times10^6\text{ W}}{0.300} = 3.33\times10^6\text{ W}$$

and $$A = \frac{P}{I} = \frac{3.33\times10^6\text{ W}}{1.00\times10^3\text{ W/m}^2} = \boxed{3.33\times10^3\text{ m}^2}$$

P34.24 (a) $$\vec{\mathbf{E}}\cdot\vec{\mathbf{B}} = \left(80.0\hat{\mathbf{i}} + 32.0\hat{\mathbf{j}} - 64.0\hat{\mathbf{k}}\right)(\text{N/C})\cdot\left(0.200\hat{\mathbf{i}} + 0.080\,0\hat{\mathbf{j}} + 0.290\hat{\mathbf{k}}\right)\ \mu\text{T}$$

$$\vec{\mathbf{E}}\cdot\vec{\mathbf{B}} = (16.0 + 2.56 - 18.56)\ \mu\text{T}\cdot\text{N/C}^2 = \boxed{0}$$

(b) $$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$$

$$= \left(\frac{1}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}}\right)\left[\left(80.0\hat{\mathbf{i}} + 32.0\hat{\mathbf{j}} - 64.0\hat{\mathbf{k}}\right)\ \text{N/C}\right]$$

$$\times\left[\left(0.200\hat{\mathbf{i}} + 0.080\ 0\hat{\mathbf{j}} + 0.290\hat{\mathbf{k}}\right)\ \mu\text{T}\right]$$

$$\vec{\mathbf{S}} = \frac{\left(6.40\hat{\mathbf{k}} - 23.2\hat{\mathbf{j}} - 6.40\hat{\mathbf{k}} + 9.28\hat{\mathbf{i}} - 12.8\hat{\mathbf{j}} + 5.12\hat{\mathbf{i}}\right)\times10^{-6}\ \text{W/m}^2}{4\pi\times10^{-7}}$$

$$\vec{\mathbf{S}} = \boxed{\left(11.5\hat{\mathbf{i}} - 28.6\hat{\mathbf{j}}\right)\ \text{W/m}^2}$$

$$= 30.9\ \text{W/m}^2\ \text{at}\ -68.1°\ \text{from the}\ +x\ \text{axis}$$

P34.25 (a) $$I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{\left(3.00\times10^6\ \text{V/m}\right)^2}{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times10^8\ \text{m/s}\right)}$$

$$I = \boxed{1.19\times10^{10}\ \text{W/m}^2}$$

(b) $$P = IA = I\pi r^2 = \left(1.19\times10^{10}\ \text{W/m}^2\right)\pi\left(\frac{5.00\times10^{-3}\ \text{m}}{2}\right)^2$$

$$= \boxed{2.34\times10^5\ \text{W}}$$

P34.26 The energy put into the water in each container by electromagnetic radiation can be written as $\Delta E = eP\Delta t = eIA\Delta t$, where e is the percentage absorption efficiency. This energy has the same effect as heat in raising the temperature of the water:

$$eIA\Delta t = mc\Delta T = \rho Vc\Delta T$$

$$\Delta T = \frac{eI\ell^2\Delta t}{\rho\ell^3 c} = \frac{eI\Delta t}{\rho\ell c}$$

where ℓ is the edge dimension of the container and c the specific heat of water. For the small container,

$$\Delta T = \frac{0.700\left(25.0\times10^3\ \text{W/m}^2\right)(480\ \text{s})}{\left(10^3\ \text{kg/m}^3\right)(0.060\ 0\ \text{m})\left(4\,186\ \text{J/kg}\cdot°\text{C}\right)} = \boxed{33.4°\text{C}}$$

For the larger,

$$\Delta T = \frac{0.910\left(25.0\times10^3\ \text{W/m}^2\right)(480\ \text{s})}{\left(10^3\ \text{kg/m}^3\right)(0.120\ \text{m})\left(4\,186\ \text{J/°C}\right)} = \boxed{21.7°\text{C}}$$

P34.27 (a) $B_{\max} = \dfrac{E_{\max}}{c}$: $B_{\max} = \dfrac{7.00 \times 10^5 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.33 \text{ mT}}$

(b) $I = \dfrac{E_{\max}^2}{2\mu_0 c}$:

$$I = \frac{\left(7.00 \times 10^5 \text{ V/m}\right)^2}{2\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)\left(3.00 \times 10^8 \text{ m/s}\right)} = 6.50 \times 10^8 \text{ W/m}^2$$
$$= \boxed{650 \text{ MW/m}^2}$$

(c) $I = \dfrac{P}{A}$: $P = IA = \left(6.50 \times 10^8 \text{ W/m}^2\right)\left[\dfrac{\pi}{4}\left(1.00 \times 10^{-3} \text{ m}\right)^2\right] = \boxed{511 \text{ W}}$

P34.28 (a) We assume that the starlight moves through space without any of it being absorbed. The radial distance is

$$20 \text{ ly} = 20c\left(1 \text{ yr}\right) = 20\left(3.00 \times 10^8 \text{ m/s}\right)\left(3.16 \times 10^7 \text{ s}\right)$$
$$= 1.89 \times 10^{17} \text{ m}$$
$$I = \frac{P}{4\pi r^2} = \frac{4.00 \times 10^{28} \text{ W}}{4\pi\left(1.89 \times 10^{17} \text{ m}\right)^2} = 8.88 \times 10^{-8} \text{ W/m}^2$$
$$= \boxed{88.8 \text{ nW/m}^2}$$

(b) The Earth presents the projected target area of a flat circle:

$$P = IA = \left(8.88 \times 10^{-8} \text{ W/m}^2\right)\left[\pi\left(6.37 \times 10^6 \text{ m}\right)^2\right]$$
$$= 1.13 \times 10^7 \text{ W} = \boxed{11.3 \text{ MW}}$$

P34.29 The Poynting vector is

$$S_{\text{avg}} = \frac{Power}{A} = \frac{Power}{4\pi r^2}.$$

In meters,

$$r = \left(5.00 \text{ mi}\right)\left(1\,609 \text{ m/mi}\right) = 8.04 \times 10^3 \text{ m}$$

and the intensity of the wave is

$$S = \frac{250 \times 10^3 \text{ W}}{4\pi(8\,045 \text{ m})^2} = \boxed{307\ \mu\text{W/m}^2}$$

P34.30 (a) The intensity of the broadcast waves is

$$I = \frac{B_{\max}^2 c}{2\mu_0} = \frac{P}{4\pi r^2}$$

solving,

$$B_{\max} = \sqrt{\left(\frac{P}{4\pi r^2}\right)\left(\frac{2\mu_0}{c}\right)} = \sqrt{\left(\frac{P}{2\pi r^2}\right)\left(\frac{\mu_0}{c}\right)}$$

$$= \sqrt{\frac{\left(10.0\times 10^{3}\ \text{W}\right)\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2\pi\left(5.00\times 10^{3}\ \text{m}\right)^2\left(3.00\times 10^{8}\ \text{m/s}\right)}} = \boxed{5.16\times 10^{-10}\ \text{T}}$$

(b) Since the magnetic field of the Earth is approximately 5 × 10^{-5} T, the Earth's field is some 100 000 times stronger.

P34.31 The average Poynting flux is

$$S_{\text{avg}} = \frac{P_{\text{avg}}}{4\pi r^2} = \frac{E_{\max}^2}{2\mu_0 c}$$

solving,

$$E_{\max} = \sqrt{2\mu_0 c S_{\text{avg}}} = \sqrt{\mu_0 c \frac{P_{\text{avg}}}{2\pi r^2}}$$

$$= \sqrt{\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times 10^{8}\ \text{m/s}\right)\frac{4.00\times 10^{3}\ \text{W}}{2\pi\left[4.00\left(1\,609\ \text{m}\right)\right]^2}}$$

$$= 0.076\,1\ \text{V/m}$$

The maximum emf (amplitude) induced in a length *L* of wire is

$$\Delta V_{\max} = E_{\max} L = \left(76.1\ \text{mV/m}\right)\left(0.650\ \text{m}\right) = \boxed{49.5\ \text{mV}}$$

***P34.32** Power $= SA = \dfrac{E_{\max}^2}{2\mu_0 c}\left(4\pi r^2\right)$

Solving for *r*,

$$r = \sqrt{\frac{P\mu_0 c}{2\pi E_{\max}^2}} = \sqrt{\frac{(100\ \text{W})\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times 10^{8}\ \text{m/s}\right)}{2\pi(15.0\ \text{V/m})^2}}$$

$$= \boxed{5.16\ \text{m}}$$

***P34.33** (a) $P = I^2 R = 150\ \text{W}$

$$A = 2\pi r L = 2\pi\left(0.900\times 10^{-3}\ \text{m}\right)\left(0.080\,0\ \text{m}\right) = 4.52\times 10^{-4}\ \text{m}^2$$

$$S = \frac{P}{A} = \frac{150\ \text{W}}{4.52\times 10^{-4}\ \text{m}^2} = \boxed{332\ \text{kW/m}^2}\ \text{(points radially inward)}$$

(b) $$B = \frac{\mu_0 I}{2\pi r} = \frac{(1.00\ \text{A})\left(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2\pi\left(0.900\times 10^{-3}\ \text{m}\right)} = \boxed{222\ \mu\text{T}}$$

$$E = \frac{\Delta V}{\Delta x} = \frac{IR}{L} = \frac{150\ \text{V}}{0.080\,0\ \text{m}} = \boxed{1.88\ \text{kV/m}}$$

Note that these values yield $S = \dfrac{EB}{\mu_0} = 332\ \text{kW/m}^2$, in agreement with the result from part (a).

P34.34 (a) $E_{\text{rms}} = cB_{\text{rms}} = \left(3.00\times10^{8}\ \text{m/s}\right)\left(1.80\times10^{-6}\ \text{T}\right) = \boxed{540\ \text{V/m}}$

(b) From Equation 34.25,

$$u_{\text{avg}} = \frac{\left(B_{\text{max}}\right)^2}{2\mu_0} = \frac{\left(B_{\text{rms}}\right)^2}{\mu_0} = \frac{\left(1.80\times10^{-6}\ \text{T}\right)^2}{4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}} = \boxed{2.58\ \mu\text{J/m}^3}$$

(c) $S_{\text{avg}} = cu_{\text{avg}} = \left(3.00\times10^{8}\ \text{m/s}\right)\left(2.58\times10^{-6}\ \text{J/m}^3\right) = \boxed{773\ \text{W/m}^2}$

Section 34.5 Momentum and Radiation Pressure

P34.35 The intensity of the beam is $I = \dfrac{P_{\text{power}}}{\pi r^2}$, where $r = 1.00 \times 10^{-3}$ m. By Equation 34.29, the radiation pressure on the mirror is

$$P = \frac{2S}{c} = \frac{2I}{c} = \frac{2P_{\text{power}}}{\pi r^2 c}$$

$$= \frac{2\left(25.0\times10^{-3}\ \text{W}\right)}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^2\left(3.00\times10^{8}\ \text{m/s}\right)} = \boxed{5.31\times10^{-5}\ \text{N/m}^2}$$

P34.36 For complete absorption, from equation 34.27,

$$P = \frac{S}{c} = \frac{25.0\ \text{W/m}^2}{3.00\times10^{8}\ \text{m/s}} = 8.33\times10^{-8}\ \text{N/m}^2 = \boxed{83.3\ \text{nPa}}$$

P34.37 (a) $I = \dfrac{P}{\pi r^2} = \dfrac{E_{\text{max}}^2}{2\mu_0 c}$, and $r = 1.00 \times 10^{-3}$ m:

$$E_{\text{max}} = \sqrt{\frac{2\mu_0 cP}{\pi r^2}}$$

$$= \sqrt{\frac{2\left[4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right]\left(3.00\times10^{8}\ \text{m/s}\right)\left(15.0\times10^{-3}\ \text{W}\right)}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^2}}$$

$$= 1.90\times10^{8}\ \text{J} = \boxed{1.90\ \text{kN/C}}$$

(b) The beam carries power *P*. The amount of energy Δ*E* in the length of a beam of length ℓ is the amount of power that passes a point in time interval $\Delta t = \ell/c$:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta E}{\ell/c}$$

or $$\Delta E = \frac{P\ell}{c} = \frac{15.0\times10^{-3}\ \text{W}}{3.00\times10^{8}\ \text{m/s}}(1.00\ \text{m}) = \boxed{50.0\ \text{pJ}}.$$

(c) From Equation 34.27 and our result in part (b), the momentum and energy carried a light beam are related by

$$p = \frac{T_{\text{ER}}}{c} = \frac{\Delta E}{c} = \frac{50.0\times10^{-12}\ \text{J}}{3.00\times10^{8}\ \text{m/s}} = \boxed{1.67\times10^{-19}\ \text{kg}\cdot\text{m/s}}$$

P34.38 (a) $$I = \frac{P}{\pi r^2} = \frac{E_{\max}^2}{2\mu_0 c} \quad\rightarrow\quad E_{\max} = \boxed{\sqrt{\frac{2\mu_0 cP}{\pi r^2}}}$$

(b) The beam carries power *P*. The amount of energy Δ*E* in the length of a beam of length ℓ is the amount of power that passes a point in time interval $\Delta t = \ell/c$:

$$P = \frac{\Delta E}{\Delta t} = \frac{\Delta E}{\ell/c} \quad\rightarrow\quad \Delta E = \boxed{\frac{P\ell}{c}}$$

(c) From Equation 34.27 and our result in part (b), the momentum and energy carried a light beam are related by

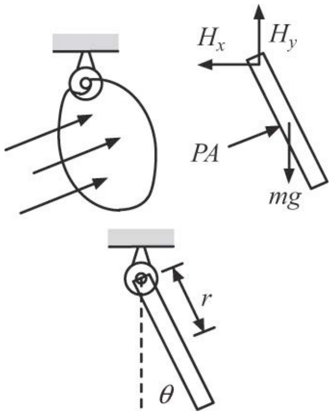
$$p = \frac{T_{\text{ER}}}{c} = \frac{\Delta E}{c} = \boxed{\frac{P\ell}{c^2}}$$

P34.39 The radiation pressure on the disk is

$$P = \frac{S}{c} = \frac{I}{c} = \frac{F}{A} = \frac{F}{\pi r^2}.$$

Thus, $$F = \frac{\pi r^2 I}{c}$$

Because the force acts uniformly over the surface of the disk, we may consider it to be acting at the center of the disk when calculating its torque. Take torques about the hinge:



ANS. FIG. P34.39

$\sum \tau = 0$:

$$H_x(0) + H_y(0) - mgr\sin\theta + \frac{\pi r^2 Ir}{c} = 0$$

Solving for the angle gives

$$\begin{aligned}\theta &= \sin^{-1}\left(\frac{\pi r^2 I}{mgc}\right)\\ &= \sin^{-1}\left[\frac{\pi(0.400\text{ m})^2(10.0\times10^6\text{ W/ m}^2)}{(0.024\,0\text{ kg})(9.80\text{ m/ s}^2)(3.00\times10^8\text{ m/s})}\right]\\ &= \sin^{-1}0.071\,2 = \boxed{4.09°}\end{aligned}$$

P34.40 (a) The light pressure on the absorbing Earth is $P = \frac{S}{c} = \frac{I}{c}$.

The force is

$$\begin{aligned}F = PA = \frac{I}{c}(\pi R^2) &= \frac{(1\,370\text{ W/m}^2)\pi(6.37\times10^6\text{ m})^2}{3.00\times10^8\text{ m/s}}\\ &= \boxed{5.82\times10^8\text{ N}}\end{aligned}$$

away from the Sun.

(b) The attractive gravitational force exerted on Earth by the Sun is

$$\begin{aligned}F_g &= \frac{GM_SM_M}{r_M^2}\\ &= \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.991\times10^{30}\text{ kg})(5.98\times10^{24}\text{ kg})}{(1.496\times10^{11}\text{ m})^2}\\ &= 3.55\times10^{22}\text{ N}\end{aligned}$$

which is $\boxed{6.10\times10^{13}\text{ times stronger}}$ compared to the repulsive force in part (a).

P34.41 (a) The magnitude of the momentum transferred to the assumed totally reflecting surface in time interval Δt is (from Equation 34.29)

$$\Delta p = \frac{2T_{ER}}{c} = \frac{2SA\Delta t}{c}$$

Then the momentum transfer is

$$\Delta\vec{\mathbf{p}} = \frac{2\vec{\mathbf{S}}A\Delta t}{c} = \frac{2(6.00\,\hat{\mathbf{i}}\text{ W/m}^2)(40.0\times10^{-4}\text{ m}^2)(1.00\text{ s})}{3.00\times10^8\text{ m/s}}$$

$$\Delta\vec{\mathbf{p}} = \boxed{1.60\times10^{-10}\,\hat{\mathbf{i}}\text{ kg}\cdot\text{m/s each second}}$$

(b) The force is

$$\vec{\mathbf{F}} = PA\hat{\mathbf{i}} = \frac{2SA}{c}\hat{\mathbf{i}} = \frac{2\left(6.00\ \mathrm{W/m^2}\right)\left(40.0\times10^{-4}\ \mathrm{m^2}\right)\left(1.00\ \mathrm{s}\right)}{3.00\times10^{8}\ \mathrm{m/s}}$$

$$= \boxed{1.60\times10^{-10}\ \hat{\mathbf{i}}\ \mathrm{N}}$$

(c) The answers are the same. Force is the time rate of momentum transfer.

P34.42 (a) If P_S is the total power radiated by the Sun, and r_E and r_M are the radii of the orbits of the planets Earth and Mars, then the intensities of the solar radiation at these planets are:

$$I_E = \frac{P_S}{4\pi r_E^2} \quad \text{and} \quad I_M = \frac{P_S}{4\pi r_M^2}$$

Thus,

$$I_M = I_E\left(\frac{r_E}{r_M}\right)^2 = \left(1\,370\ \mathrm{W/m^2}\right)\left(\frac{1.496\times10^{11}\ \mathrm{m}}{2.28\times10^{11}\ \mathrm{m}}\right)^2$$

$$= \boxed{590\ \mathrm{W/m^2}}$$

(b) Mars intercepts the power falling on its circular face:

$$P_M = I_M\left(\pi R_M^2\right) = \left(590\ \mathrm{W/m^2}\right)\left[\pi\left(3.37\times10^{6}\ \mathrm{m}\right)^2\right]$$

$$= \boxed{2.10\times10^{16}\ \mathrm{W}}$$

(c) If Mars behaves as a perfect absorber, it feels pressure

$$P = \frac{S_M}{c} = \frac{I_M}{c},$$

so the light-pressure force is

$$F_L = PA = \frac{I_M}{c}\left(\pi R_M^2\right) = \frac{P_M}{c} = \frac{2.10\times10^{16}\ \mathrm{W}}{3.00\times10^{8}\ \mathrm{m/s}} = \boxed{7.01\times10^{7}\ \mathrm{N}}$$

(d) Using our results from above, we have $F_L = I_M\dfrac{\pi R_M^2}{c}$ and $I_M = I_E\dfrac{r_E^2}{r_M^2}$, so the light-pressure force on Mars is $F_L = I_E\dfrac{r_E^2}{r_M^2}\dfrac{\pi R_M^2}{c}$. The attractive gravitational force exerted on Mars by the Sun is $F_g = \dfrac{GM_SM_M}{r_M^2}$. Their ratio is

$$\frac{F_g}{F_L} = \frac{GM_SM_M}{\cancel{r_M^2}}\cdot\frac{1}{I_E}\frac{\cancel{r_M^2}}{r_E^2}\frac{c}{\pi R_M^2} = \left(\frac{cGM_S}{\pi I_E r_E^2}\right)\frac{M_M}{R_M^2}$$

Suppressing units,

$$\frac{F_g}{F_L}=\left[\frac{\left(3.00\times10^{8}\right)\left(6.67\times10^{-11}\right)\left(1.991\times10^{30}\right)}{\pi\left(1\,370\right)\left(1.496\times10^{11}\right)^{2}}\right]\left(\frac{M_M}{R_M^2}\right)$$

$$\frac{F_g}{F_L}=\left(414\text{ m}^2/\text{kg}\right)\frac{M_M}{R_M^2}=\left(414\text{ m}^2/\text{kg}\right)\frac{\left(6.42\times10^{23}\text{ kg}\right)}{\left(3.37\times10^{6}\text{ m}\right)^{2}}$$

$$=2.34\times10^{13}$$

The attractive gravitational force exerted on Mars by the Sun is $\boxed{\sim10^{13}\text{ times stronger}}$ than the repulsive light-pressure force of part (c).

(e) The expression for the ratio of the gravitational force to the light-pressure force for Earth is similar to that used in part (d) for Mars (replace *M* with *E*):

$$\frac{F_g}{F_L}=\left(414\text{ m}^2/\text{kg}\right)\frac{M_E}{R_E^2}=\left(414\text{ m}^2/\text{kg}\right)\frac{\left(5.98\times10^{24}\text{ kg}\right)}{\left(6.37\times10^{6}\text{ m}\right)^{2}}$$

$$=6.10\times10^{13}$$

$\boxed{\text{The values are similar for both planets because both the forces follow inverse-square laws. The force ratios are not identical for the two planets because of their different radii and masses.}}$

P34.43 (a) The radiation pressure is

$$P=\frac{2S}{c}=\frac{2I}{c}$$

The force on area *A* is

$$F=PA=\frac{2\left(1\,370\text{ W/m}^2\right)}{3.00\times10^{8}\text{ m/s}}\left(6.00\times10^{5}\text{ m}^2\right)=\boxed{5.48\text{ N}}$$

(b) The acceleration is:

$$a=\frac{F}{m}=\frac{5.48\text{ N}}{6\,000\text{ kg}}=9.13\times10^{-4}\text{ m/s}^2$$

$$=\boxed{913\ \mu\text{m/s}^2\text{ away from the Sun}}$$

(c) It will arrive at time *t*, where $d=\frac{1}{2}at^2$ or,

$$t=\sqrt{\frac{2d}{a}}=\sqrt{\frac{2\left(3.84\times10^{8}\text{ m}\right)}{\left(9.13\times10^{-4}\text{ m/s}^2\right)}}=9.17\times10^{5}\text{ s}=\boxed{10.6\text{ days}}$$

Section 34.6 Production of Electromagnetic Waves by an Antenna

P34.44 (a) The wavelength of an ELF wave of frequency 75.0 Hz is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{75.0\ \text{Hz}} = 4.00\times10^{6}\ \text{m}$$

The length of a quarter-wavelength antenna would be

$$L = 1.00\times10^{6}\ \text{m} = \boxed{1.00\times10^{3}\ \text{km}}$$

or $$L = (1\,000\ \text{km})\left(\frac{0.621\ \text{mi}}{1.00\ \text{km}}\right) = \boxed{621\ \text{mi}}$$

(b) $\boxed{\text{While the project may be theoretically possible, it is not very practical.}}$

P34.45 (a) $$h = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3.00\times10^{8}\ \text{m/s}}{4(560\times10^{3}\ \text{Hz})} = \boxed{134\ \text{m}}$$

(b) $$h = \frac{\lambda}{4} = \frac{c}{4f} = \frac{3.00\times10^{8}\ \text{m/s}}{4(1600\times10^{3}\ \text{Hz})} = \boxed{46.9\ \text{m}}$$

P34.46 (a) The magnetic field $\vec{\mathbf{B}} = \frac{1}{2}\mu_0 J_{\max}\cos(kx-\omega t)\hat{\mathbf{k}}$ applies for $x > 0$, since it describes a wave moving in the $\hat{\mathbf{i}}$ direction. The electric field direction must satisfy $\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}}$ as $\hat{\mathbf{i}} = \hat{\mathbf{j}}\times\hat{\mathbf{k}}$ so the direction of the electric field is $\hat{\mathbf{j}}$ when the cosine is positive. For its magnitude we have $E = cB$, so altogether we have

$$\boxed{\vec{\mathbf{E}} = \frac{1}{2}\mu_0 c J_{\max}\cos(kx-\omega t)\hat{\mathbf{j}}}.$$

(b) $$\vec{\mathbf{S}} = \frac{1}{\mu_0}\vec{\mathbf{E}}\times\vec{\mathbf{B}} = \frac{1}{\mu_0}\frac{1}{4}\mu_0^2 c J_{\max}^2\cos^2(kx-\omega t)\hat{\mathbf{i}}$$

$$\boxed{\vec{\mathbf{S}} = \frac{1}{4}\mu_0 c J_{\max}^2\cos^2(kx-\omega t)\hat{\mathbf{i}}}$$

(c) The intensity is the magnitude of the Poynting vector averaged over one or more cycles. The average of the cosine-squared function is $\frac{1}{2}$, so $\boxed{I = \frac{1}{8}\mu_0 c J_{\max}^2}$.

(d) $$J_{\max} = \sqrt{\frac{8I}{\mu_0 c}} = \sqrt{\frac{8\left(570\text{ W/m}^2\right)}{4\pi\times10^{-7}\left(\text{Tm/A}\right)3\times10^8\text{ m/s}}} = \boxed{3.48\text{ A/m}}$$

***P34.47** For the proton, Newton's second law gives

$$\sum F = ma: \qquad qvB\sin 90.0° = \frac{mv^2}{R}.$$

The period and frequency of the proton's circular motion are therefore:

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB} = \frac{2\pi\left(1.67\times10^{-27}\text{ kg}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.350\text{ T}\right)} = 1.87\times10^{-7}\text{ s}$$

and $f = 5.34\times10^6\text{ Hz}$.

The charge will radiate at this same frequency, with

$$\lambda = \frac{c}{f} = \frac{3.00\times10^8\text{ m/s}}{5.34\times10^6\text{ Hz}} = \boxed{56.2\text{ m}}$$

P34.48 For the proton, $\sum F = ma$ yields

$$qvB\sin 90.0° = \frac{mv^2}{R} \rightarrow v = \frac{qBR}{m}$$

The period of the proton's circular motion is therefore:

$$T = \frac{2\pi R}{v} = \frac{2\pi m}{qB}$$

The frequency of the proton's motion is $f = \dfrac{1}{T}$

The charge will radiate electromagnetic waves at this frequency, with

$$\lambda = \frac{c}{f} = cT = \boxed{\frac{2\pi mc}{qB}}$$

P34.49 Refer to ANS. FIG. P34.49. For any wavelength:

(a) Constructive interference occurs when $d\cos\theta = n\lambda$ for some integer *n*.

$$\cos\theta = n\frac{\lambda}{d} = n\left(\frac{\lambda}{\lambda/2}\right) = 2n \qquad n = 0, \pm1, \pm2, \ldots$$

$\therefore$ strong signal @ $\theta = \cos^{-1}0 = 90°, 270°$, or

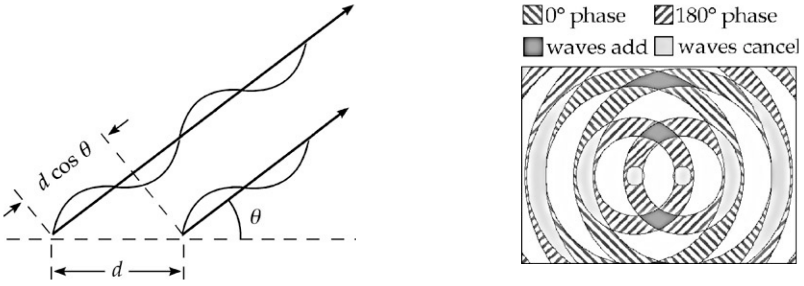
$\boxed{\text{along the perpendicular bisector of the line segment joining the antennas.}}$

(b) Destructive interference occurs when

$$d\cos\theta = \left(\frac{2n+1}{2}\right)\lambda: \qquad \cos\theta = 2n+1$$

$\therefore$ weak signal @ $\theta = \cos^{-1}(\pm 1) = 0°,\ 180°$, or

$\boxed{\text{along the extensions of the line segment joining the antennas.}}$



ANS. FIG. P34.49

Section 34.7 The Spectrum of Electromagnetic Waves

P34.50 (a) $f = \frac{c}{\lambda} = \frac{3\times 10^8\ \text{m/s}}{1.7\ \text{m}}\ \boxed{\sim 10^8\ \text{Hz}}\ \boxed{\text{radio wave}}$

(b) 1 000 pages, 500 sheets, is about 3 cm thick so one sheet is about 6×10^{-5} m thick.

$$f = \frac{3.00\times 10^8\ \text{m/s}}{6\times 10^{-5}\ \text{m}}\ \boxed{\sim 10^{13}\ \text{Hz}}\ \boxed{\text{infrared}}$$

P34.51 (a) $f\lambda = c$ gives $\left(5.00\times 10^{19}\ \text{Hz}\right)\lambda = 3.00\times 10^8\ \text{m/s}$:

$$\boxed{\lambda = 6.00\times 10^{-12}\ \text{m} = 6.00\ \text{pm}}$$

(b) $f\lambda = c$ gives $\left(4.00\times 10^9\ \text{Hz}\right)\lambda = 3.00\times 10^8\ \text{m/s}$:

$$\boxed{\lambda = 0.075\ 0\ \text{m} = 7.50\ \text{cm}}$$

P34.52 The time interval for the radio signal to travel 100 km is:

$$\Delta t_r = \frac{100\times 10^3\ \text{m}}{3.00\times 10^8\ \text{m/s}} = 3.33\times 10^{-4}\ \text{s}$$

The sound wave travels 3.00 m across the room in:

$$\Delta t_s = \frac{3.00\ \text{m}}{343\ \text{m/s}} = 8.75\times 10^{-3}\ \text{s}$$

Therefore, $\boxed{\text{listeners 100 km away}}$ will receive the news before the people in the newsroom by a total time difference of

$$\Delta t = 8.75\times10^{-3}\ \text{s} - 3.33\times10^{-4}\ \text{s} = 8.41\times10^{-3}\ \text{s}$$

P34.53 From $f\lambda = c$,

Channel 3: f = 60.0 MHz to 66.0 MHz.

(a) Channel 4: f = 66.0 MHz to 72.0 MHz, $\lambda = \boxed{4.17\text{ m to }4.55\text{ m}}$.

72.0–76.0 MHz is reserved for non-TV purposes.

Channel 5: f = 76.0 MHz to 82.0 MHz.

(b) Channel 6: f = 82.0 MHz to 88.0 MHz, $\lambda = \boxed{3.41\text{ m to }3.66\text{ m}}$.

88.0–174 MHz is reserved for non-TV purposes.

Channel 7: f = 174 MHz to 180 MHz.

(c) Channel 8: f = 180 MHz to 186 MHz, $\lambda = \boxed{1.61\text{ m to }1.67\text{ m}}$.

Additional Problems

***P34.54** From the electromagnetic spectrum chart and accompanying text discussion, the following identifications are made:

Frequency, f	Wavelength, $\lambda = \dfrac{c}{f}$	Classification
2 Hz = 2×10^{0} Hz	150 Mm	Radio
2 KHz = 2×10^{3} Hz	150 km	Radio
2 MHz = 2×10^{6} Hz	150 m	Radio
2 GHz = 2×10^{9} Hz	15 cm	Microwave
2 THz = 2×10^{12} Hz	150 μm	Infrared
2 PHz = 2×10^{15} Hz	150 nm	Ultraviolet
2 EHz = 2×10^{18} Hz	150 pm	X-ray
2 ZHz = 2×10^{21} Hz	150 fm	Gamma ray
2 YHz = 2×10^{24} Hz	150 am	Gamma ray

Wavelength, λ	Frequency, $f = \frac{c}{\lambda}$	Classification
2 km = 2×10^3 m	1.5×10^5 Hz	Radio
2 m = 2×10^0 m	1.5×10^8 Hz	Radio
2 mm = 2×10^{-3} m	1.5×10^{11} Hz	Microwave
2 μm = 2×10^{-6} m	1.5×10^{14} Hz	Infrared
2 nm = 2×10^{-9} m	1.5×10^{17} Hz	Ultraviolet/X-ray
2 pm = 2×10^{-12} m	1.5×10^{20} Hz	X-ray/Gamma ray
2 fm = 2×10^{-15} m	1.5×10^{23} Hz	Gamma ray
2 am = 2×10^{-18} m	1.5×10^{26} Hz	Gamma ray

P34.55 (a) From $P = SA$, we have

$$P = \left(1\,370\ \text{W/m}^2\right)\left[4\pi\left(1.496\times10^{11}\ \text{m}\right)^2\right] = \boxed{3.85\times10^{26}\ \text{W}}$$

(b) $S = \dfrac{E_{\text{max}}^2}{2\mu_0 c}$ so

$$E_{\text{max}} = \sqrt{2\mu_0 cS}$$

$$= \sqrt{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times10^8\ \text{m/s}\right)\left(1\,370\ \text{W/m}^2\right)}$$

$$= \boxed{1.02\ \text{kV/m}}$$

(c) $S = \dfrac{cB_{\text{max}}^2}{2\mu_0}$ so

$$B_{\text{max}} = \sqrt{\frac{2\mu_0 S}{c}} = \sqrt{\frac{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1\,370\ \text{W/m}^2\right)}{3.00\times10^8\ \text{m/s}}}$$

$$= \boxed{3.39\ \mu\text{T}}$$

P34.56 We use the relationship between energy density and electric field magnitude that we studied previously for a static field. The energy density can be written as

$$u_E = \tfrac{1}{2}\epsilon_0 E_{\text{max}}^2$$

so $$E_{\max} = \sqrt{\frac{2u_E}{\epsilon_0}} = \sqrt{\frac{2\left(4.00 \times 10^{-14}\ \text{N}\cdot\text{m}^2\right)}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}} = \boxed{95.1\ \text{mV/m}}.$$

P34.57 The wavelength is found from

$$f\lambda = c \rightarrow \lambda = \frac{c}{f} = \frac{3.00\times10^8\ \text{m/s}}{5.45\times10^{14}\ \text{Hz}} = \boxed{5.50\times10^{-7}\ \text{m}}$$

P34.58 The angular frequency of the wave is

$$\omega = 2\pi f = 2\pi\left(3.00\times10^9\ \text{s}^{-1}\right) = 1.88\times10^{10}\ \text{s}^{-1}$$

and the wave number is

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = 2\pi\left(\frac{3.00\times10^9\ \text{s}^{-1}}{3.00\times10^8\ \text{m/s}}\right) = 20.0\pi\ \text{m}^{-1} = 62.8\ \text{m}^{-1}$$

Also,

$$B_{\max} = \frac{E}{c} = \frac{300\ \text{V/m}}{3.00\times10^8\ \text{m/s}} = 1.00\ \mu\text{T}$$

Then,

$$\boxed{E = 300\cos\left(62.8x - 1.88\times10^{10}t\right)}$$

$$\boxed{B = 1.00\cos\left(62.8x - 1.88\times10^{10}t\right)}$$

where E is in volts per meter (V/m), B is in microtesla (μT), x is in meters, and t is in seconds.

***P34.59** (a) The power incident on the mirror is:

$$P_I = IA = \left(1\ 370\ \text{W/m}^2\right)\left[\pi(100\ \text{m})^2\right] = 4.30\times10^7\ \text{W}.$$

The power reflected through the atmosphere is

$$P_R = 0.746\left(4.30\times10^7\ \text{W}\right) = \boxed{3.21\times10^7\ \text{W}}$$

(b) $$S = \frac{P_R}{A} = \frac{3.21\times10^7\ \text{W}}{\pi\left(4.00\times10^3\ \text{m}\right)^2} = \boxed{0.639\ \text{W/m}^2}$$

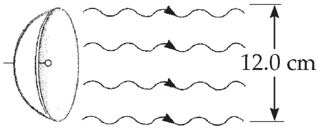
(c) Noon sunshine in St. Petersburg produces this power-per-area on a horizontal surface:

$$\frac{P_N}{A} = 0.746\left(1\ 370\ \text{W/m}^2\right)\sin 7.00° = 125\ \text{W/m}^2$$

The radiation intensity received from the mirror is

$$\left(\frac{0.639\ \text{W/m}^2}{125\ \text{W/m}^2}\right)100\% = \boxed{0.513\%}$$ of that from the noon Sun in January.

***P34.60** (a) $\lambda = \frac{c}{f} = \frac{3.00\times 10^8\ \text{m/s}}{20.0\times 10^9\ \text{s}^{-1}} = \boxed{1.50\ \text{cm}}$



ANS. FIG. P34.60

(b) $U = P(\Delta t) = (25.0\times 10^3\ \text{J/s})(1.00\times 10^{-9}\ \text{s})$

$= 25.0\times 10^{-6}\ \text{J} = \boxed{25.0\ \mu\text{J}}$

(c) $u_{\text{avg}} = \frac{U}{V} = \frac{U}{(\pi r^2)\ell}$

$$= \frac{U}{(\pi r^2)c(\Delta t)} = \frac{25.0\times 10^{-6}\ \text{J}}{\pi(0.060\ 0\ \text{m})^2(3.00\times 10^8\ \text{m/s})(1.00\times 10^{-9}\ \text{s})}$$

$u_{\text{avg}} = 7.37\times 10^{-3}\ \text{J/m}^3 = \boxed{7.37\ \text{mJ/m}^3}$

(d) $E_{\max} = \sqrt{\frac{2u_{\text{av}}}{\epsilon_0}} = \sqrt{\frac{2(7.37\times 10^{-3}\ \text{J/m}^3)}{8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2}} = 4.08\times 10^4\ \text{V/m}$

$= \boxed{40.8\ \text{kV/m}}$

$B_{\max} = \frac{E_{\max}}{c} = \frac{4.08\times 10^4\ \text{V/m}}{3.00\times 10^8\ \text{m/s}} = 1.36\times 10^{-4}\ \text{T} = \boxed{136\ \mu\text{T}}$

(e) $F = PA = \left(\frac{S}{c}\right)A = u_{\text{av}}A = (7.37\times 10^{-3}\ \text{J/m}^3)\pi(0.060\ 0\ \text{m})^2$

$= 8.33\times 10^{-5}\ \text{N} = \boxed{83.3\ \mu\text{N}}$

P34.61 Suppose you cover a 1.7 m × 0.3 m section of beach blanket. Suppose the elevation angle of the Sun is 60°. Then the effective target area you fill in the Sun's light is

$$A = (1.7\ \text{m})(0.3\ \text{m})\cos 30° = 0.4\ \text{m}^2$$

Now $I = \frac{P}{A} = \frac{\Delta E}{A\Delta t}$, so

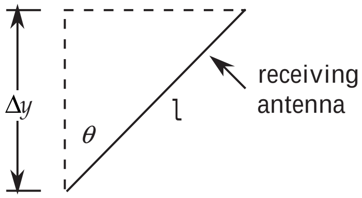
$$\Delta E = IA\Delta t = (0.5)\left[(0.6)(1\,370\ \text{W/m}^2)\right](0.4\ \text{m}^2)(3\,600\ \text{s})$$

$$\boxed{\sim 10^6\ \text{J}}$$

P34.62 $P = \frac{(\Delta V)^2}{R}$ or $P \propto (\Delta V)^2$

$\Delta V = (-)E_y\cdot\Delta y = E_y\cdot\ell\cos\theta$

$\Delta V \propto \cos\theta$ so $P \propto \cos^2\theta$



ANS. FIG. P34.62

(a) $\theta = 15.0°$: $P = P_{\max}\cos^2(15.0°) = 0.933P_{\max} = \boxed{93.3\%}$

(b) $\theta = 45.0°$: $P = P_{\max}\cos^2(45.0°) = 0.500P_{\max} = \boxed{50.0\%}$

(c) $\theta = 90.0°$: $P = P_{\max}\cos^2(90.0°) = \boxed{0}$

P34.63 The gravitational force exerted by the Sun on the particle is given by

$$F_{\text{grav}} = \frac{GM_S m}{R^2} = \left(\frac{GM_S}{R^2}\right)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]$$

where M_S = mass of Sun, r = radius of particle, and R = distance from Sun to particle. The force exerted by solar radiation on the particle is given by $F_{\text{rad}} = PA$, and since the particle absorbs all the radiation, by Equation 34.28, we have

$$F_{\text{rad}} = PA = \frac{S}{c}\pi r^2$$

When the particle is in equilibrium, the gravitational force toward the Sun is balanced by the force of radiation away from the Sun, $F_{\text{rad}} = F_{\text{grav}}$, so

$$\frac{S}{c}\pi r^2 = \left(\frac{GM_S}{R^2}\right)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]$$

Solving for r, the radius of the particle, then gives

$$r = \frac{3SR^2}{4cGM_S\rho}$$

Suppressing units,

$$r = \frac{3(214)\left(3.75\times10^{11}\right)^2}{4\left(3.00\times10^{8}\right)\left(6.67\times10^{-11}\right)\left(1.991\times10^{30}\right)(1\ 500)}$$
$$= 3.78\times10^{-7}\ \text{m} = \boxed{378\ \text{nm}}$$

P34.64 The gravitational force exerted by the Sun on the particle is given by

$$F_{\text{grav}} = \frac{GM_S m}{R^2} = \left(\frac{GM_S}{R^2}\right)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]$$

where M_S = mass of Sun, r = radius of particle, and R = distance from Sun to particle. The force exerted by solar radiation on the particle is given by $F_{\text{rad}} = PA$, and since the particle absorbs all the radiation, by Equation 34.28, we have

$$F_{\text{rad}} = PA = \frac{S}{c}\pi r^2$$

When the particle is in equilibrium, the gravitational force toward the Sun is balanced by the force of radiation away from the Sun, $F_{\text{rad}} = F_{\text{grav}}$, so

$$\frac{S}{c}\pi r^2 = \left(\frac{GM_S}{R^2}\right)\left[\rho\left(\frac{4}{3}\pi r^3\right)\right]$$

Solving for *r*, the radius of the particle, then gives

$$r = \boxed{\frac{3SR^2}{4cGM_S\rho}}$$

P34.65 (a) The magnetic-field amplitude is

$$B_{\max} = \frac{E_{\max}}{c} = \frac{0.200\times10^{-6}\text{ V/m}}{3.00\times10^{8}\text{ m/s}} = \boxed{6.67\times10^{-16}\text{ T}}$$

(b) The intensity is the Poynting vector averaged over one or more cycles, given by

$$S_{\text{avg}} = \frac{E_{\max}^2}{2\mu_0 c} = \frac{\left(0.200\times10^{-6}\text{ V/m}\right)^2}{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(3.00\times10^{8}\text{ m/s}\right)}$$
$$= \boxed{5.31\times10^{-17}\text{ W/m}^2}$$

(c) The power tells how fast the antenna receives energy. It is

$$P = S_{\text{avg}}A = S_{\text{avg}}\pi\left(\frac{d}{2}\right)^2 = \left(5.31\times10^{-17}\text{ W/m}^2\right)\pi\left(\frac{20.0\text{ m}}{2}\right)^2$$
$$= \boxed{1.67\times10^{-14}\text{ W}}$$

(d) The force tells how fast the antenna receives momentum. It is

$$F = PA = \left(\frac{S_{\text{avg}}}{c}\right)A = \left(\frac{5.31\times10^{-17}\text{ W/m}^2}{3.00\times10^{8}\text{ m/s}}\right)\pi\left(\frac{20.0\text{ m}}{2}\right)^2$$
$$= \boxed{5.56\times10^{-23}\text{ N}}$$

(approximately the weight of 3 000 hydrogen atoms!)

P34.66 Of the intensity $S = 1\,370\text{ W/m}^2$, the 38.0% that is reflected exerts a pressure

$$P_1 = \frac{2S_r}{c} = \frac{2(0.380)S}{c}$$

The absorbed light exerts pressure

$$P_2 = \frac{S_a}{c} = \frac{0.620S}{c}$$

Altogether the pressure at the subsolar point on Earth is

(a) $P_{\text{total}} = P_1 + P_2 = \dfrac{1.38S}{c} = \dfrac{1.38\left(1370\ \text{W/m}^2\right)}{3.00 \times 10^8\ \text{m/s}} = \boxed{6.30 \times 10^{-6}\ \text{Pa}}$

(b) Compared to normal atmospheric pressure,

$$\frac{P_a}{P_{\text{total}}} = \frac{1.01 \times 10^5\ \text{N/m}^2}{6.30 \times 10^{-6}\ \text{N/m}^2}$$

$$= \boxed{1.60 \times 10^{10}\ \text{times smaller than atmospheric pressure}}$$

P34.67 The mirror intercepts power

$$P = I_1 A_1 = \left(1.00 \times 10^3\ \text{W/m}^2\right)\left[\pi\left(0.500\ \text{m}\right)^2\right] = 785\ \text{W}.$$

(a) In the image, $I_2 = \dfrac{P}{A_2}$, so

$$I_2 = \frac{785\ \text{W}}{\pi\left(0.020\,0\ \text{m}\right)^2} = \boxed{625\ \text{kW/m}^2}$$

(b) $I_2 = \dfrac{E_{\max}^2}{2\mu_0 c}$, so

$$E_{\max} = \sqrt{2\mu_0 c I_2}$$

$$= \sqrt{2\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00 \times 10^8\ \text{m/s}\right)\left(6.25 \times 10^5\ \text{W/m}^2\right)}$$

$$= \boxed{21.7\ \text{kN/C}}$$

(c) $B_{\max} = \dfrac{E_{\max}}{c} = \boxed{72.4\ \mu\text{T}}$

(d) We obtain the time interval from

$$0.400\left(P\Delta t\right) = mc\Delta T$$

solving,

$$\Delta t = \frac{mc\Delta T}{0.400P} = \frac{\left(1.00\ \text{kg}\right)\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(100^\circ\text{C} - 20.0^\circ\text{C}\right)}{0.400\left(785\ \text{W}\right)}$$

$$= 1.07 \times 10^3\ \text{s} = \boxed{17.8\ \text{min}}$$

P34.68 (a) In $\vec{\mathbf{E}} = \dfrac{q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}} = \dfrac{\Phi}{4\pi r^2}\hat{\mathbf{r}} = \dfrac{487\ \text{N}\cdot\text{m}^2/\text{C}}{4\pi r^2}\hat{\mathbf{r}}$,

$$\boxed{\vec{\mathbf{E}} = \frac{38.8}{r^2}\hat{\mathbf{r}}\ \text{where}\ \vec{\mathbf{E}}\ \text{is in volts per meter and}\ r\ \text{is in meters.}}$$

(b) The radiated intensity is

$$I = \frac{P}{4\pi r^2} = \frac{E_{\text{max}}^2}{2\mu_0 c}$$

solving,

$$E_{\text{max}} = \sqrt{\frac{2\mu_0 cP}{4\pi r^2}} = \frac{1}{r}\sqrt{\frac{\mu_0 cP}{2\pi}}$$

$$= \frac{1}{r}\sqrt{\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times10^{8}\ \text{m/s}\right)\left(25.0\ \text{W}\right)}{2\pi}}$$

$$\boxed{E_{\text{max}} = \frac{38.7}{r}\ \text{where } E \text{ is in volts per meter and } r \text{ is in meters.}}$$

(c) For $E_{\text{max}} = \frac{38.7}{r} = 3.00\times10^{6} \rightarrow r = 1.29\times10^{-5} = 12.9\times10^{-6}$, so *r* is $\boxed{\text{12.9 }\mu\text{m, but the expression in part (b) does not apply if this point is inside the source.}}$

(d) From part (c), we see that $\boxed{\text{in the radiated wave, the field amplitude is inversely proportional to distance. As the distance doubles, the amplitude is cut in half.}}$

(e) $\boxed{\text{In the static case, the field is inversely proportional to the square of the distance. As the distance doubles, the field is reduced by a factor of 4.}}$

P34.69 (a) At steady state, $P_{\text{in}} = P_{\text{out}}$ and the power radiated out is $P_{\text{out}} = e\sigma AT^4$. Thus,

$$T = \left[\frac{P_{\text{out}}}{e\sigma A}\right]^{1/4} = \left[\frac{900\ \text{W/m}^2}{0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4}$$

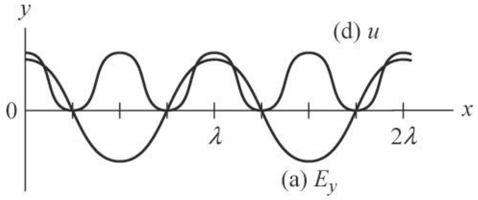
$$= \boxed{388\ \text{K}} = 115°\text{C}$$

(b) The box of horizontal area *A* presents projected area *A* sin 50.0° perpendicular to the sunlight. Then by the same reasoning,

$$0.900\left(1\,000\ \text{W/m}^2\right)A\sin 50.0°$$
$$= 0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)AT^4$$

or $$T = \left[\frac{\left(900\ \text{W/m}^2\right)\sin 50.0°}{0.700\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4} = \boxed{363\ \text{K}} = 90.0°\text{C}$$

P34.70 (a) See ANS. FIG. P34.70



ANS. FIG. P34.70

(b) $u_E = \frac{1}{2}\epsilon_0 E^2 = \boxed{\frac{1}{2}\epsilon_0 E_{\text{max}}^2 \cos^2(kx)}$

(c) $u_B = \frac{1}{2\mu_0}B^2 = \boxed{\frac{1}{2\mu_0}B_{\text{max}}^2 \cos^2(kx)}$

(d) Note that

$$u_B = \frac{1}{2\mu_0}\frac{E_{\text{max}}^2}{c^2}\cos^2(kx) = \frac{1}{2\mu_0}\frac{E_{\text{max}}^2}{\left(1/\mu_0\epsilon_0\right)}\cos^2(kx)$$
$$= \frac{1}{2}\epsilon_0 E_{\text{max}}^2\cos^2(kx) = u_E$$

Therefore, $u = u_E + u_B = \boxed{\epsilon_0 E_{\text{max}}^2 \cos^2(kx)}$.

(e) $E_\lambda = \int_0^\lambda uA\,dx$

$$E_\lambda = \int_0^\lambda \epsilon_0 E_{\text{max}}^2\cos^2(kx)A\,dx = \int_0^\lambda \epsilon_0 E_{\text{max}}^2 A\left[\frac{1}{2}+\frac{1}{2}\cos(2kx)\right]A\,dx$$
$$= \frac{1}{2}\epsilon_0 E_{\text{max}}^2 A\,x\Big|_0^\lambda + \frac{\epsilon_0 E_{\text{max}}^2}{4k}A\sin(2kx)\Big|_0^\lambda$$
$$= \frac{1}{2}\epsilon_0 E_{\text{max}}^2 A\lambda + \frac{\epsilon_0 E_{\text{max}}^2}{4k}A\left[\sin(4\pi)-\sin(0)\right]$$
$$= \boxed{\frac{1}{2}\epsilon_0 E_{\text{max}}^2 \lambda A}$$

(f) $P = \frac{E_\lambda}{T} = \frac{1}{2}\frac{\epsilon_0 E_{\text{max}}^2\lambda A}{\left(1/f\right)} = \frac{1}{2}\epsilon_0 E_{\text{max}}^2\left(\lambda f\right)A = \boxed{\frac{1}{2}\epsilon_0 cE_{\text{max}}^2 A}$

(g) $I = \frac{P}{A} = \frac{\frac{1}{2}\epsilon_0 cE_{\text{max}}^2 A}{A} = \boxed{\frac{1}{2}\epsilon_0 cE_{\text{max}}^2}$

(h) From part (g), we have

$$\frac{1}{2}\epsilon_0 cE_{\max}^2 = \frac{\mu_0}{\mu_0}\frac{\epsilon_0 cE_{\max}^2}{2} = (\mu_0 \epsilon_0)\frac{cE_{\max}^2}{2\mu_0} = \frac{1}{c^2}\frac{cE_{\max}^2}{2\mu_0} = \frac{E_{\max}^2}{2\mu_0 c}$$

$$\boxed{\text{The result in part (g) agrees with } I = \frac{E_{\max}^2}{2\mu_0 c} \text{ in Equation 34.24.}}$$

P34.71 The bead is black, so we assume it absorbs all light that strikes it. The bead presents an effective face of area $A = \pi r^2$ to the light. Since we assume the bead to be perfectly absorbing, the light pressure, from Equation 34.28, is

$$P = \frac{S_{\text{av}}}{c} = \frac{I}{c} = \frac{F_\ell}{A}$$

so the light force is $F_\ell = \frac{I}{c}A$.

(a) The light force balances the weight, $F_\ell = F_g$, so

$$\frac{I}{c}\pi r^2 = mg$$

solving,

$$I = \frac{mgc}{\pi r^2} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)gc}{\pi r^2} = \frac{4}{3}\rho cgr$$

$$= \frac{4}{3}\left(\frac{0.200\times 10^{-3}\text{ kg}}{10^{-6}\text{ m}^3}\right)(3.00\times 10^8\text{ m/s})(9.80\text{ m/s}^2)$$

$$\times(0.500\times 10^{-3}\text{ m})$$

$$I = \boxed{3.92\times 10^8\text{ W/m}^2}$$

(b) The minimum power required is

$$P = IA = (3.92\times 10^8\text{ W/m}^2)\pi(0.500\times 10^{-3}\text{ m})^2 = \boxed{308\text{ W}}$$

P34.72 The bead is black, so we assume it absorbs all light that strikes it. The bead presents an effective face of area $A = \pi r^2$ to the light. Since we assume the bead to be perfectly absorbing, the light pressure, from Equation 34.28, is

$$P = \frac{S_{\text{av}}}{c} = \frac{I}{c} = \frac{F_\ell}{A}$$

so the light force is $F_\ell = \frac{I}{c}A$.

(a) The light force balances the weight, $F_\ell = F_g$, so

$$\frac{I}{c}\pi r^2 = mg$$

solving,

$$I = \frac{mgc}{\pi r^2} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)gc}{\pi r^2} = \boxed{\frac{4}{3}\rho cgr}$$

(b) The minimum power required is

$$P = IA = \left(\frac{4}{3}\rho cgr\right)\left(\pi r^2\right) = \boxed{\frac{4}{3}\pi\rho cgr^3}$$

P34.73 (a) A hemisphere is half a sphere:

$$m = \rho V = \rho\left[\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)\right] = 5.50 + 4(0.800)\text{ kg} = 8.70\text{ kg}$$

$$r = \left(\frac{6m}{\rho 4\pi}\right)^{1/3} = \left(\frac{6(8.7\text{ kg})}{(990\text{ kg/m}^3)4\pi}\right)^{1/3} = \boxed{0.161\text{ m}}$$

(b) $A = \frac{1}{2}4\pi r^2 = 2\pi(0.161\text{ m})^2 = \boxed{0.163\text{ m}^2}$

(c) $P = e\sigma AT^4$ and $T = 31.0 + 273.0 = 304$ K:

$$P = 0.970\left(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)\left(0.163\text{ m}^2\right)(304\text{ K})^4$$
$$= \boxed{76.8\text{ W}}$$

(d) $I = \frac{P}{A} = e\sigma T^4$

$$I = 0.970\left(5.67\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(304\text{ K})^4$$
$$= \boxed{470\text{ W/m}^2}$$

(e) $I = \frac{E_{\max}^2}{2\mu_0 c}$

$$E_{\max} = (2\mu_0 cI)^{1/2}$$
$$= \left[2\left(4\pi\times 10^{-7}\text{ Tm/A}\right)\left(3.00\times 10^8\text{ m/s}\right)\left(470\text{ W/m}^2\right)\right]^{1/2}$$
$$= \boxed{595\text{ N/C}}$$

(f) $E_{\max} = cB_{\max} \rightarrow B_{\max} = \dfrac{595\ \text{N/C}}{3\times 10^8\ \text{m/s}} = \boxed{1.98\ \mu\text{T}}$

(g) Each kitten has radius $r_k = \left(\dfrac{6m}{\rho 4\pi}\right)^{1/3} = \left[\dfrac{6(0.800)}{990\times 4\pi}\right]^{1/3} = 0.072\ 8\ \text{m}$ and radiating area $2\pi(0.072\ 8\ \text{m})^2 = 0.033\ 3\ \text{m}^2$. The mother cat has area $2\pi\left[\dfrac{6(5.50)}{990\times 4\pi}\right]^{2/3} = 0.120\ \text{m}^2$. The total glowing area is $0.120\ \text{m}^2 + 4(0.033\ 3\ \text{m}^2) = 0.254\ \text{m}^2$ and has power output $P = IA = (470\ \text{W/m}^2)(0.254\ \text{m}^2) = \boxed{119\ \text{W}}$.

P34.74 (a) On the right side of the equation,

$$\frac{\text{C}^2(\text{m/s}^2)^2}{(\text{C}^2/\text{N}\cdot\text{m}^2)(\text{m/s})^3} = \frac{\text{N}\cdot\text{m}^2\cdot\text{C}^2\cdot\text{m}^2\cdot\text{s}^3}{\text{C}^2\cdot\text{s}^4\cdot\text{m}^3} = \frac{\text{N}\cdot\text{m}}{\text{s}} = \frac{\text{J}}{\text{s}} = \text{W}$$

(b) $F = ma = qE$, or

$$a = \frac{qE}{m} = \frac{(1.60\times 10^{-19}\ \text{C})(100\ \text{N/C})}{9.11\times 10^{-31}\ \text{kg}} = \boxed{1.76\times 10^{13}\ \text{m/s}^2}$$

(c) The radiated power is then:

$$P = \frac{q^2a^2}{6\pi \in_0 c^3} = \frac{(1.60\times 10^{-19}\ \text{C})^2(1.76\times 10^{13}\ \text{m/s}^2)^2}{6\pi(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(3.00\times 10^8\ \text{m/s})^3}$$

$$= \boxed{1.75\times 10^{-27}\ \text{W}}$$

(d) $F = ma_c = m\left(\dfrac{v^2}{r}\right) = qvB$,

so $v = \dfrac{qBr}{m}$

The proton accelerates at

$$a = \frac{v^2}{r} = \frac{q^2B^2r}{m^2} = \frac{(1.60\times 10^{-19}\ \text{C})^2(0.350\ \text{T})^2(0.500\ \text{m})}{(1.67\times 10^{-27}\ \text{kg})^2}$$

$$= 5.62\times 10^{14}\ \text{m/s}^2$$

The proton then radiates

$$P = \frac{q^2a^2}{6\pi \in_0 c^3} = \frac{\left(1.60\times 10^{-19}\ \text{C}\right)^2\left(5.62\times 10^{14}\ \text{m/s}^2\right)^2}{6\pi\left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(3.00\times 10^{8}\ \text{m/s}\right)^3}$$

$$= \boxed{1.80\times 10^{-24}\ \text{W}}$$

P34.75 We take R to be the planet's distance from its star, and r to be the radius of the planet.

(a) The effective area of the planet over which it absorbs light is its projection onto a plane perpendicular to the light from its sun. The projected area of a planet of radius r is πr^2, so $\boxed{\text{the planet absorbs light over area } \pi r^2}$.

(b) $\boxed{\text{The planet radiates over its entire surface area, } 4\pi r^2.}$

(c) At steady-state, $P_{\text{in}} = P_{\text{out}}$:

$$eI_{\text{in}}\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4$$

$$e\left(\frac{6.00\times 10^{23}\ \text{W}}{4\pi R^2}\right)\left(\pi r^2\right) = e\sigma\left(4\pi r^2\right)T^4\text{, so that}$$

$$6.00\times 10^{23}\ \text{W} = 16\pi\sigma R^2T^4$$

$$R = \sqrt{\frac{6.00\times 10^{23}\ \text{W}}{16\pi\sigma T^4}}$$

$$= \sqrt{\frac{6.00\times 10^{23}\ \text{W}}{16\pi\left(5.67\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(310\ \text{K}\right)^4}} = \boxed{4.77\times 10^{9}\ \text{m}}$$

Challenge Problems

P34.76 We are given $f = 90.0$ MHz and $E_{\text{max}} = 200\ \text{mV/m} = 2.00\times 10^{-3}\ \text{V/m}$

(a) The wavelength of the wave is $\lambda = \dfrac{c}{f} = \boxed{3.33\ \text{m}}$

(b) Its period is $T = \dfrac{1}{f} = 1.11\times 10^{-8}\ \text{s} = \boxed{11.1\ \text{ns}}$

(c) We obtain the maximum value of the magnetic field from

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = 6.67\times 10^{-12}\ \text{T} = \boxed{6.67\ \text{pT}}$$

(d) $\boxed{\vec{\mathbf{E}} = \left(2.00\times10^{-3}\right)\cos 2\pi\left(\dfrac{x}{3.33} - 90.0\times10^{6}t\right)\hat{\mathbf{j}}}$

$\boxed{\vec{\mathbf{B}} = \left(6.67\times10^{-12}\right)\cos 2\pi\left(\dfrac{x}{3.33} - 90.0\times10^{6}t\right)\hat{\mathbf{k}}}$

where $\vec{\mathbf{E}}$ is in V/m, $\vec{\mathbf{B}}$ in tesla, *x* in meters, and *t* in seconds.

(e) $I = \dfrac{E_{\text{max}}^2}{2\mu_0 c} = \dfrac{\left(2.00\times10^{-3}\text{ V/m}\right)^2}{2\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(3.00\times10^{8}\text{ m/s}\right)}$

$= \boxed{5.31\times10^{-9}\text{ W/m}^2}$

(f) From Equation 34.26, $I = cu_{\text{avg}}$ so $u_{\text{avg}} = \dfrac{I}{c} = \boxed{1.77\times10^{-17}\text{ J/m}^3}$

(g) From Equation 34.30, the pressure is

$$P = \frac{2I}{c} = \frac{(2)\left(5.31\times10^{-9}\text{ W/m}^2\right)}{3.00\times10^{8}\text{ m/s}} = \boxed{3.54\times10^{-17}\text{ Pa}}$$

P34.77 (a) The magnetic field has amplitude

$$B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{175\text{ V/m}}{3.00\times10^{8}\text{ m/s}} = 5.83\times10^{-7}\text{ T} = \boxed{583\text{ nT}}$$

(b) The wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.015\,0\text{ m}} = \boxed{419\text{ m}^{-1}}$$

(c) The angular frequency is

$$\omega = kc = \left(419\text{ m}^{-1}\right)\left(3.00\times10^{8}\text{ m/s}\right) = 1.26\times10^{11}\text{ s}^{-1}$$

(d) $\vec{\mathbf{S}} \propto \vec{\mathbf{E}}\times\vec{\mathbf{B}}$, $\vec{\mathbf{S}}$ is in the *x* direction, and $\vec{\mathbf{E}}$ vibrates in the *y* direction (*xy* plane), so $\vec{\mathbf{B}}$ must vibrate in the *z* direction, thus $\boxed{\vec{\mathbf{B}}\text{ vibrates in the } xz \text{ plane}}$.

(e) The magnitude of the average Poynting vector is the wave intensity

$$S_{\text{avg}} = \frac{E_{\text{max}}B_{\text{max}}}{2\mu_0} = \frac{(175\text{ V/m})\left(5.83\times10^{-7}\text{ T}\right)}{2\left(4\pi\times10^{-7}\text{ N/A}^2\right)} = 40.6\text{ W/m}^2$$

The Poynting vector itself points in the direction of energy transport:

$$\vec{\mathbf{S}}_{\text{avg}} = \boxed{40.6\hat{\mathbf{i}}\text{ W/m}^2}$$

(f) For perfect reflection, the pressure is

$$P_r = \frac{2S}{c} = \frac{2\left(40.6\ \text{W/m}^2\right)}{3.00\times 10^8\ \text{m/s}} = 2.71\times 10^{-7}\ \text{N/m}^2 = \boxed{271\ \text{nPa}}$$

(g) From Newton's second law,

$$a = \frac{\sum F}{m} = \frac{PA}{m} = \frac{\left(2.71\times 10^{-7}\ \text{N/m}^2\right)\left(0.750\ \text{m}^2\right)}{0.500\ \text{kg}}$$

$$= 4.07\times 10^{-7}\ \text{m/s}^2$$

$$\vec{\mathbf{a}} = \boxed{407\hat{\mathbf{i}}\ \text{nm/s}^2}$$

P34.78 We can approximate the magnetic field as uniform over the area of the loop while it oscillates in time as $B = B_{\max}\cos\omega t$. The induced voltage is

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(BA\cos\theta) = -A\frac{d}{dt}\left(B_{\max}\cos\omega t\cos\theta\right)$$

or $$\mathcal{E} = AB_{\max}\,\omega(\sin\omega t\cos\theta)$$

(a) Since the angular frequency is $\omega = 2\pi f$, and the area of the loop is πr^2, the amplitude of this emf is

$$\boxed{\mathcal{E}_{\max} = 2\pi^2 r^2 f B_{\max}\cos\theta}$$

where θ is the angle between the magnetic field and the normal to the loop.

(b) If $\vec{\mathbf{E}}$ is vertical, $\vec{\mathbf{B}}$ is horizontal, so $\boxed{\text{the plane of the loop should be vertical}}$ and $\boxed{\text{the plane should contain the line of sight of the transmitter}}$.

P34.79 (a) From the particle under a net force model, the acceleration of the astronaut is

$$a = \frac{F}{m} = \frac{1}{m}\frac{dp}{dt}$$

where dp/dt is the rate of change of momentum of the astronaut. From the momentum version of the isolated system model, the rate of change of momentum of the astronaut is equal in magnitude to that of the radiation from the flashlight. The momentum of the radiation leaving the flashlight can be evaluated from Equation 34.27, assuming the same equation for complete absorption applies to complete emission. Therefore, the acceleration of the astronaut can be written as

$$a = \frac{1}{m}\frac{d}{dt}\left(\frac{T_{\text{ER}}}{c}\right) = \frac{1}{mc}\frac{dT_{\text{ER}}}{dt} = \frac{P}{mc}$$

where P is the power of the radiation leaving the flashlight. Because all three variables on the right side of this equation are constant, the acceleration of the astronaut is constant and we can use the particle under constant acceleration model. The position of the astronaut is given by,

$$x = x_i + v_i t + \frac{1}{2}at^2 = 0 + 0 + \frac{1}{2}\left(\frac{P}{mc}\right)t^2$$

where we have defined the initial position of the astronaut as $x = 0$ and recognized that the astronaut begins from rest. Solve for the time at which the astronaut is at a position x:

$$t = \sqrt{\frac{2mcx}{P}}$$

Substituting numerical values,

$$t = \sqrt{\frac{2(110\text{ kg})(3.00\times10^8\text{ m/s})(10.0\text{ m})}{100\text{ W}}} = 8.12\times10^4\text{ s}$$

$$= \boxed{22.6\text{ h}}$$

(b) There are no external forces on the astronaut–flashlight system, so the system is isolated for momentum. Apply the conservation of momentum principle along an axis parallel to the direction of travel of the astronaut and the flashlight:

$$\Delta p = 0 \quad \rightarrow \quad p_i = p_f \quad \rightarrow \quad 0 = (m - m_f)v - m_f(v_{\text{rel}} - v)$$

Solve for the speed of the astronaut:

$$v = \left(\frac{m_f}{m}\right)v_{\text{rel}}$$

Because this speed is constant, we can use the particle under constant velocity model to find the time interval required for the astronaut to arrive back at her spacecraft:

$$\Delta t = \frac{\Delta x}{v} = \frac{m}{m_f}\left(\frac{\Delta x}{v_{\text{rel}}}\right)$$

Substituting numerical values,

$$\Delta t = \left(\frac{110\text{ kg}}{3.00\text{ kg}}\right)\frac{10.0\text{ m}}{12.0\text{ m/s}} = \boxed{30.6\text{ s}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P34.2	(a) $7.19\times10^{11}\ \text{V/m}\cdot\text{s}$; (b) 2.00×10^{-7} T
P34.4	$\left(-4.39\hat{\mathbf{i}}-1.76\hat{\mathbf{j}}\right)\times10^{11}\ \text{m/s}^2$
P34.6	(a) $3.15\times10^{3}\hat{\mathbf{j}}\ \text{N/C}$; (b) $5.25\hat{\mathbf{k}}\times10^{-7}\ \text{T}$; (c) $4.83\left(-\hat{\mathbf{j}}\right)\times10^{-16}\ \text{N}$
P34.8	11.0 m
P34.10	4.738×10^{14} Hz
P34.12	733 nT
P34.44	60.0 km
P34.16	See P34.16 for full explanation.
P34.18	The ratio of ω to k is higher than the speed of light in a vacuum, so the wave as described is impossible.
P34.20	8.64×10^{10} m
P34.22	(a) 6.75 W/m^2; (b) 6.64×10^3 W/m^2; (c) A powerful automobile running on sunlight would have to carry on its roof a solar panel that is huge compared to the size of the car; (d) Agriculture and forestry for food and fuels, space heating of large and small buildings, water heating, and heating for drying and many other processes are current and potential applications of solar energy.
P34.24	(a) 0; (b) $\left(11.5\hat{\mathbf{i}}-28.6\hat{\mathbf{j}}\right)\ \text{W/m}^2$
P34.26	For the small container, 33.4° and for the larger container, 21.7°
P34.28	(a) 88.8 nW/m^2; (b) 11.3 MW
P34.30	(a) 5.16×10^{-10} T; (b) Since the magnetic field of the Earth is approximately 5×10^{-10} T, the Earth's field is some 100 000 times stronger.
P34.32	5.16 m
P34.34	(a) 540 V/m; (b) 2.58 μJ/m^3; (c) 773 W/m^2
P34. 36	83.3 nPa
P34.38	(a) $\sqrt{\dfrac{2\mu_0 cP}{\pi r^2}}$; (b) $\dfrac{P\ell}{c}$; (c) $\dfrac{P\ell}{c^2}$
P34.40	(a) 5.82×10^{8} N; (b) 6.10×10^{13} times stronger

P34.42 (a) 590 W/m^2; (b) 2.10×10^{16} W; (c) 7.01×10^7 N; (d) ~10^{13} times stronger; (e) The values are similar for both planets because both the forces follow inverse-square laws. The force ratios are not identical for the two planets because of their different radii and masses.

P34.44 (a) 1.00×10^3 km or 621 mi; (b) While the project may be theoretically possible, it is not very practical.

P34.46 (a) $\frac{1}{2}\mu_0 c J_{\max}\cos(kx-\omega t)\hat{\mathbf{j}}$; (b) $\frac{1}{4}\mu_0 c J_{\max}^2\cos^2(kx-\omega t)\hat{\mathbf{i}}$; (c) $\frac{1}{8}\mu_0 c J_{\max}^2$; (d) 3.48 A/m

P34.48 $\dfrac{2\pi mc}{qB}$

P34.50 (a) ~10^8 Hz radio wave; (b) ~10^{13} Hz infrared

P34.52 Listeners 100 km away will receive the news before the people in the newsroom.

P34.54 See table in P34.54 for full description.

P34.56 95.1 mV/m

P34.58 $E = 300\cos(62.8x - 1.88 \times 10^{10} t)$ and $B = 1.00\cos(62.8x - 1.88 \times 10^{10} t)$

P34.60 (a) 1.50 cm; (b) 25.0 μJ; (c) 7.37 mJ/m^3; (d) $E_{\max}$ = 40.8 kV/m, $B_{\max}$ = 136 μT; (e) 83.3 μN

P34.62 (a) 93.3%; (b) 50.0%; (c) 0

P34.64 $\dfrac{3SR^2}{4cGM_S\rho}$

P34.66 (a) 6.30×10^{-6} Pa; (b) 1.60×10^{10} times smaller than atmospheric pressure

P34.68 (a) $\vec{\mathbf{E}} = \dfrac{38.8}{r^2}\hat{\mathbf{r}}$, where $\vec{\mathbf{E}}$ is in volts per meter and r is in meters;

(b) $E_{\max} = \dfrac{38.7}{r}$ where E is in volts per meter and r is in meters;

(c) 12.9 μm, but the expression in part (b) does not apply if this point is inside the source; (d) From part (c), we see that in the radiated wave, the field amplitude is inversely proportional to distance. As the distance doubles, the amplitude is cut in half; (e) In the static case, the field is inversely proportional to the square of distance. As the distance doubles, the field is reduced by a factor of 4.

P34.70 (a) See ANS. FIG. P34.70; (b) $\frac{1}{2}\epsilon_0 E_{\max}^2 \cos^2(kx)$; (c) $\frac{1}{2\mu_0}B_{\max}^2\cos^2(kx)$; (d) $\epsilon_0 E_{\max}^2\cos^2(kx)$; (e) $\frac{1}{2}\epsilon_0 E_{\max}^2\lambda A$; (f) $\frac{1}{2}\epsilon_0 cE_{\max}^2 A$; (g) $\frac{1}{2}\epsilon_0 cE_{\max}^2$; (h) The result in part (g) agrees with $I = \frac{E_{\max}^2}{2\mu_0 c}$ in Equation 34.24.

P34.72 (a) $\frac{4}{3}\rho cgr$; (b) $\frac{4}{3}\pi\rho cgr^3$

P34.74 (a) See P34.74(a) for full proof; (b) 1.76×10^{13} m/s^2; (c) 1.75×10^{-27} W; (d) 1.80×10^{-24} W

P34.76 (a) 3.33 m; (b) 11.1 ns; (c) 6.67 pT; (d) $\vec{\mathbf{E}} = \left(2.00\times10^{-3}\right)\cos 2\pi\left(\frac{x}{3.33} - 90.0\times10^6 t\right)\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = \left(6.67\times10^{-12}\right)\cos 2\pi\left(\frac{x}{3.33} - 90.0\times10^6 t\right)\hat{\mathbf{k}}$; (e) 5.31×10^{-9} W/m^2; (f) 1.77×10^{-17} J/m^2; (g) 3.54×10^{-17} Pa

P34.78 (a) $\mathcal{E}_{\max} = 2\pi^2 r^2 f B_{\max}\cos\theta$; (b) The plane of the loop should be vertical and the plane should contain the line of sight of the transmitter.

35

The Nature of Light and the Principles of Ray Optics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ35.1 The ranking is answer e, c, b, a, d. We consider the quantity λ/d : the smaller it is, the better the ray approximation works. The quantity λ/d is about (a) 0.34 m/1 m $\approx$ 0.3, (b) 0.7 μm/2 mm $\approx$ 0.000 3, (c) 0.4 μm/2 mm $\approx$ 0.000 2, (d) 300 m/1 m $\approx$ 300, (e) 1 nm/1 mm $\approx$ 0.000 001.

OQ35.2 Answer (c). As light travels from one medium to another, both the wavelength of the light and the index of refraction of the medium will change, but the product λn is constant: $\lambda_2 n_2 = \lambda_{\text{air}} n_{\text{air}}$. In going from air into a second medium of index n, according to Equation 25.6, $n = \lambda/\lambda_n$ = 495 nm/434 nm = 1.14.

OQ35.3 Answer (b). In going from carbon disulfide ($n_1 = 1.63$) to crown glass ($n_2 = 1.52$), the critical angle for total internal reflection is

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.52}{1.63}\right) = 68.8°$$

OQ35.4 Answers (a), (b), and (c) are all correct statements. The frequency of a wave does not change when it travels from one medium to another: $f_1 = f_2 \rightarrow n_1\lambda_1 = n_2\lambda_2$; also, Snell's law of refraction states $n_1 \sin\theta_1 = n_2 \sin\theta_2$. By their definitions, $n = c/v = c/f\lambda$ and $\sin\theta = 1/\csc\theta$. Thus, Snell's law can take these alternate forms:

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2} \rightarrow \frac{v_1}{\sin\theta_1} = \frac{v_2}{\sin\theta_2} \rightarrow \frac{\csc\theta_1}{n_1} = \frac{\csc\theta_2}{n_2} \rightarrow \frac{\lambda_1}{\sin\theta_1} = \frac{\lambda_2}{\sin\theta_2}$$

Snell originally stated his law in terms of cosecants.

OQ35.5 Answer (e). The index of refraction of glass is greater than that of air, which means the speed of light in glass is slower than in air ($n = c/v$). The frequency does not change, but because the speed decreases, the wavelength also decreases.

OQ35.6 Answer (b). When light is in water, the relationships between the values of its speed and wavelength to the values of the same quantities in air are $n_{\text{water}} = \dfrac{c}{v_{\text{water}}} \rightarrow v_{\text{water}} = \dfrac{c}{n_{\text{water}}} = \dfrac{3}{4}c$, and

$$n_{\text{water}}\lambda_{\text{water}} = n_{\text{air}}\lambda_{\text{air}} \rightarrow \lambda_{\text{water}} = \left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)\lambda_{\text{air}} \approx \frac{3}{4}\lambda_{\text{air}}.$$

OQ35.7 Answer (c). Water has a greater index of refraction than air. In passing from one of these media into the other, light will be refracted (deviated in direction) unless the angle of incidence is zero (in which case, the angle of refraction is also zero). Because the angle of refraction can be zero only if the angle of incidence is zero, ray *B* cannot be correct. In refraction, the incident ray and the refracted ray are never on the same side of the line normal to the surface at the point of contact, so ray *A* cannot be correct. Also in refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$; thus, if $n_2 > n_1$, then $\theta_2 < \theta_1$: the refracted ray makes a smaller angle with the normal in the medium having the higher index of refraction. Therefore, rays *D* and *E* cannot be correct, leaving only ray *C* as a likely path.

OQ35.8 Answer (c). The time interval is 10^4 m/(3×10^8 m/s) = 33 μs.

OQ35.9 Answer (c). For any medium, other than vacuum, the index of refraction for red light is slightly lower (closer to 1) than that for blue light. This means that when light goes from vacuum (or air) into glass, the red light deviates from its original direction less than does the blue light. Also, as the light reemerges from the glass into vacuum (or air), the red light again deviates less than the blue light. If the two surfaces of the glass are parallel to each other, the red and blue rays will emerge traveling parallel to each other, but displaced laterally from one another. The sketch that best illustrates this process is *C*.

OQ35.10 For a wave to experience total internal reflection, it must be traveling in the medium in which it moves slower, in which it has a greater index of refraction.

(i) Answer (a). Water has a greater index of refraction than air.

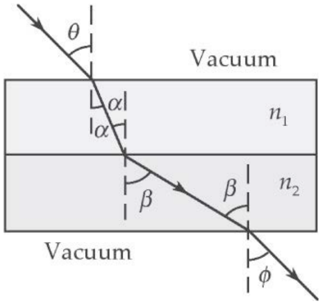
(ii) Answer (c). The sound travels slower in air than in water.

OQ35.11 Answer (c). Consider the sketch in ANS. FIG. OQ35.11 and apply Snell's law to the refraction at each of the three surfaces. Because the surfaces are parallel, the resulting equations are

$(1.00)\sin\theta = n_1 \sin\alpha$ (Top surface)

$n_1 \sin\alpha = n_2 \sin\beta$ (Middle surface)

$n_2 \sin\beta = (1.00)\sin\phi$ (Bottom surface)



ANS. FIG. OQ35.11

These equations allow us to equate the left side of the first equation with the right side of the last equation:

$$(1.00)\sin\theta = (1.00)\sin\phi \rightarrow \phi = \theta$$

OQ35.12 Color A travels slower in the glass of the prism. Light with the greater change in speed will have the greater deviation in direction.

OQ35.13 Answer (c). We want a big difference between indices of refraction to have total internal reflection under the widest range of conditions.

OQ35.14 Answer (a). In a dispersive medium, the index of refraction is largest for the shortest wavelength. Thus, the violet light will be refracted (or bent) the most as it passes through a surface of the crown glass.

OQ35.15 Answer (b). For a wave to experience total internal reflection, it must be traveling in the medium in which it moves slower, in which it has a greater index of refraction. A light ray, in attempting to go from a medium with index of refraction n_1 into a second medium with index of refraction n_2, will undergo total internal reflection if $n_2 < n_1$ and if the ray strikes the surface at an angle of incidence greater than or equal to the critical angle.

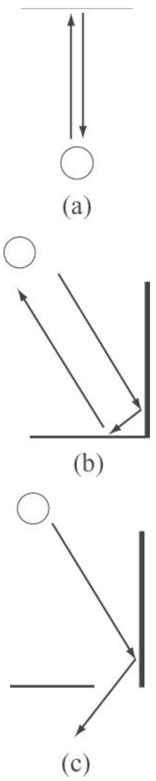
ANSWERS TO CONCEPTUAL QUESTIONS

CQ35.1 The water level in a clear glass is observable because light is refracted as it passes from air to water to air. The index of liquid helium is very close to that of air, so very little refraction occurs as light travels from air to helium to the air.

CQ35.2 At the altitude of the plane the surface of the Earth need not block off the lower half of the rainbow. Thus, the full circle can be seen. You can see such a rainbow by climbing on a stepladder above a garden sprinkler in the middle of a sunny day. Set the sprinkler for fine mist. Do not let the slippery children fall from the ladder.

CQ35.3 (a) We assume that you and the child are always standing close together. For a flat wall to make an echo of a sound that you make, you must be standing along a normal to the wall. You must be on the order of 100 m away, to make the transit time sufficiently long that you can hear the echo separately from the original sound. Your sound must be loud enough so that you can hear it even at this considerable range. In ANS. FIG. CQ35.3(a), the circle represents an area in which you can be standing. The arrows represent rays of sound.

(b) Now suppose two vertical perpendicular walls form an inside corner that you can see. Some of the sound you radiate horizontally will be headed generally toward the corner. It will reflect from both walls with high efficiency to reverse in direction and come back to you, as shown in ANS. FIG. CQ35.3(b). You can stand anywhere reasonably far away to hear a retroreflected echo of sound you produce.



ANS FIG. CQ35.3

(c) If the two walls are not perpendicular, the inside corner will not produce retroreflection. You will generally hear no echo of your shout or clap.

(d) If two perpendicular walls have a reasonably narrow gap between them at the corner, you can still hear a clear echo. It is not the corner line itself that retroreflects the sound, but the perpendicular walls on both sides of the corner. [ANS. FIG. CQ35.3(b) applies also in this case.]

(e) At some angles, sound will reflect from the first wall but not the second; rather, it will pass into the breezeway, as shown in ANS. FIG. CQ35.3(c), so there will be no echo.

CQ35.4 The stealth fighter is designed so that adjacent panels are not joined at right angles, to prevent any retroreflection of radar signals. This means that radar signals directed at the fighter will not be channeled back toward the detector by reflection. Just as with sound, radar signals can be treated as *diverging* rays, so that any ray that is by chance reflected back to the detector will be too weak in intensity to distinguish from background noise.

CQ35.5 "Immediately around the dark shadow of my head, I see a halo brighter than the rest of the dewy grass." It is called the *heiligenschein*. Cellini believed that it was a miraculous sign of divine favor pertaining to him alone. Apparently none of the people to whom he showed it told him that they could see halos around their own shadows but not around Cellini's. Thoreau knew that each person had his own halo. He did not draw any ray diagrams but assumed that it was entirely natural. Between Cellini's time and Thoreau's, the Enlightenment and Newton's explanation of the rainbow had happened. Today the effect is easy to see whenever your shadow falls on a retroreflecting traffic sign, license plate, or road stripe. When a bicyclist's shadow falls on a paint stripe marking the edge of the road, her halo races along with her.

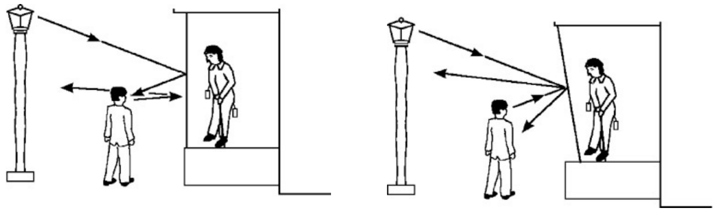
CQ35.6 An echo is an example of the reflection of sound. Hearing the noise of a distant highway on a cold morning, when you cannot hear it after the ground warms up, is an example of acoustical refraction. You can use a rubber inner tube (or balloon of the same shape) inflated with helium as an acoustical lens to concentrate sound in the way a lens can focus light: the speed of sound is greater in helium, so wavefronts passing through the helium speed ahead of wavefronts passing through the air in the doughnut hole of the tube, so that the overall shape of the wavefronts changes from plane to concave, resulting in a focusing of the wave. At your next party, see if you can experimentally find the approximate focal point!

CQ35.7 Highly silvered mirrors reflect about 98% of the incident light. With a 2-mirror periscope, that results in approximately a 4% decrease in intensity of light as the light passes through the periscope. This may not seem like much, but in low-light conditions, that lost light may mean the difference between being able to distinguish an enemy armada or an iceberg from the sky beyond. Using prisms results in total internal reflection, meaning that 100% of the incident light is reflected through the periscope. That is the "total" in total internal reflection.

CQ35.8 Diamond has higher index of refraction than glass and consequently a smaller critical angle for total internal reflection. A brilliant-cut diamond is shaped to admit light from above, reflect it totally at the converging facets on the underside of the jewel, and let the light escape only at the top. Glass will have less light internally reflected.

CQ35.9 If a laser beam enters a sugar solution with a concentration gradient (density and index of refraction increasing with depth), then the laser beam will be progressively bent downward (toward the normal) as it passes into regions of greater index of refraction.

CQ35.10 With a vertical shop window, streetlights and his own reflection can impede the window shopper's clear view of the display. The tilted shop window can put these reflections out of the way. Windows of airport control towers are also tilted like this, as are automobile windshields.



ANS. FIG. CQ35.10

CQ35.11 (a) Light from the lamps along the edges of the sheet enters the plastic, and then the front and back faces of the plastic totally internally reflect it, wherever the plastic has an interface with air. If the refractive index of the grease is intermediate between 1.55 and 1.00, some of this light can leave the plastic into the grease and leave the grease into the air. The surface of the grease is rough, so the grease can send out light in all directions. The customer sees the grease shining against a black background.

(b) The spotlight method of producing the same effect is much less efficient. With it, the blackboard absorbs much of the light from the spotlight.

(c) The refractive index of the grease must be less than 1.55. Perhaps the best choice would be $\sqrt{1.55 \times 1.00} = 1.24$.

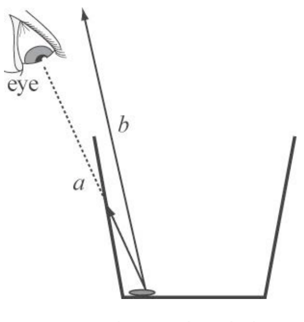
CQ35.12 A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction because they have different densities at different temperatures. When the sun makes a blacktop road hot, an apparent wet spot is bright due to refraction of light from the bright sky. The light, originally headed a little below the horizontal, always bends up as it first enters and then leaves sequentially hotter, lower-density, lower-index layers of air closer to the road surface.

CQ35.13 Light rays coming from parts of the pencil under water are bent away from the normal as they emerge into the air above. The rays enter the eye (or camera) at angles closer to the horizontal, thus the parts of the pencil under water appear closer to the surface than they actually are, so the pencil appears bent. See CQ35.16 for an illustration of a related effect.

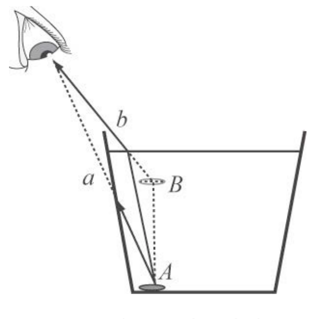
CQ35.14 No. The speed of light v in any medium except vacuum is less than the speed of light c in vacuum. By definition, the index of refraction $n = c/v$, thus the index of any material medium is always greater than 1. A material with an index less than 1 is impossible.

CQ35.15 Light travels through a vacuum at a speed of 300 000 km per second. Thus, an image we see from a distant star or galaxy must have been generated some time ago. For example, the star Altair is 16 light-years away; if we look at an image of Altair today, we know only what was happening 16 years ago. This may not initially seem significant, but astronomers who look at other galaxies can gain an idea of what galaxies looked like when they were significantly younger. Thus, it actually makes sense to speak of "looking backward in time."

CQ35.16 With no water in the cup, light rays from the coin do not reach the eye because they are blocked by the side of the cup. With water in the cup, light rays are bent away from the normal as they leave the water so that some reach the eye.



ANS. FIG. CQ35.16(a)



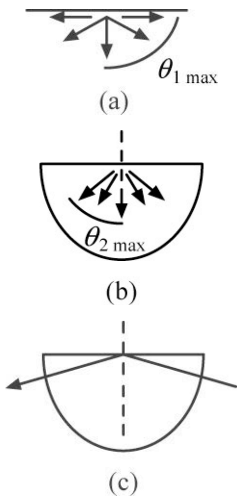
ANS. FIG. CQ35.16(b)

In ANS. FIG. CQ35.16(a), ray a is blocked by the side of the cup so it cannot enter the eye, and ray b misses the eye. In ANS. FIG. CQ35.16(b), ray a is still blocked by the side of the cup, but ray b refracts at the water's surface so that it reaches the eye. Ray b seems to come from position B, directly above the coin at position A.

CQ35.17 (a) Scattered light rays leave the center of the photograph, shown in ANS. FIG. CQ35.17(a), in all horizontal directions between $\theta_1 = 0°$ and 90° from the normal. When the light rays immediately enter the water they are gathered into a fan, shown in ANS. FIG. CQ35.17(b), between 0° and $\theta_{2\max}$ given by

$$\begin{aligned} n_1 \sin\theta_1 &= n_2 \sin\theta_2 \\ 1.00 \sin 90 &= 1.333 \sin\theta_{2\ \max} \\ \theta_{2\ \max} &= 48.6° \end{aligned}$$

The light rays leave the cylinder without deviation because they travel along the normal everywhere they strike the surface of the glass, so the viewer only receives light from the center of the photograph when he has turned by an angle less than 48.6°.



ANS. FIG. CQ35.17

(b) When the paperweight is turned farther, light at the back surface undergoes total internal reflection, shown in ANS. FIG. CQ35.17(c). The viewer sees things outside the globe on the far side.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 35.1 The Nature of Light

Section 35.2 Measurements of the Speed of Light

***P35.1** We find the energy of the photons from Equation 35.1, $E = hf$.

(a) $E = hf = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(5.00\times10^{17}\ \text{Hz}\right)\left(\dfrac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$

$= \boxed{2.07\times10^{3}\ \text{eV}} = 2.07\ \text{keV}$

(b) $E = hf = \dfrac{hc}{\lambda}$

$= \dfrac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{3.00\times10^{2}\ \text{nm}}\left(\dfrac{1\ \text{nm}}{10^{-9}\ \text{m}}\right)\left(\dfrac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$

$= \boxed{4.14\ \text{eV}}$

P35.2 (a) The Moon's radius is 1.74×10^{6} m and the Earth's radius is 6.37×10^{6} m. The total distance traveled by the light is:

$$\begin{aligned} d &= 2\left(3.84\times10^{8}\ \text{m} - 1.74\times10^{6}\ \text{m} - 6.37\times10^{6}\ \text{m}\right) \\ &= 7.52\times10^{8}\ \text{m} \end{aligned}$$

This takes 2.51 s, so

$$v = \frac{7.52 \times 10^8 \text{ m}}{2.51 \text{ s}} = \boxed{3.00 \times 10^8 \text{ m/s}}$$

(b) The sizes of the objects need to be taken into account. Otherwise the answer would be too large by 2%.

P35.3 The experiment is most convincing if the wheel turns fast enough to pass outgoing light through one notch and returning light through the next. This requires $\Delta t = \frac{2\ell}{c}$, or

$$\theta = \omega \Delta t = \omega\left(\frac{2\ell}{c}\right)$$

so
$$\omega = \frac{c\theta}{2\ell} = \frac{(2.998 \times 10^8 \text{ m/s})[2\pi/(720)]}{2(11.45 \times 10^3 \text{ m})} = \boxed{114 \text{ rad/s}}$$

The returning light would be blocked by a tooth at one-half the angular speed, giving another data point.

P35.4 The difference is due to the extra time light takes to cross Earth's orbit. From $\Delta x = c\Delta t$, we have

$$c = \frac{\Delta x}{\Delta t} = \frac{2(1.50 \times 10^8 \text{ km})(1000 \text{ m/km})}{(22.0 \text{ min})(60.0 \text{ s/min})} = \boxed{2.27 \times 10^8 \text{ m/s}}$$

Section 35.3 The Ray Approximation in Ray Optics

Section 35.4 Analysis Model: Wave Under Reflection

Section 35.5 Analysis Model: Wave Under Refraction

P35.5 (a) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{6.328 \times 10^{-7} \text{ m}} = \boxed{4.74 \times 10^{14} \text{ Hz}}$

(b) $\lambda_{\text{glass}} = \frac{\lambda_{\text{air}}}{n} = \frac{632.8 \text{ nm}}{1.50} = \boxed{422 \text{ nm}}$

(c) $v_{\text{glass}} = \frac{c_{\text{air}}}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = \boxed{2.00 \times 10^8 \text{ m/s}}$

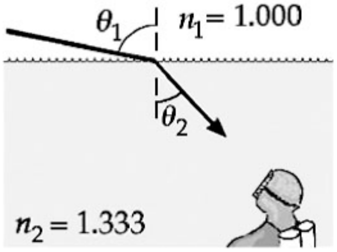
P35.6 Refracted light enters the diver's eyes. The angle of refraction θ_2 is 45.0°. From Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

Solving,

$$\theta_1 = \sin^{-1}(1.333\sin 45.0°)$$

$$= 70.5° \text{ from the vertical} \rightarrow \boxed{19.5° \text{ above the horizon}}$$



ANS. FIG. P35.6

P35.7 We find the angle of incidence from Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Solving,

$$1.333\sin\theta_1 = 1.52\sin 19.6° \quad \rightarrow \quad \theta_1 = 22.5°$$

The angle of reflection of the beam in water is then also $\boxed{22.5°}$.

P35.8 (a) The dashed lines are parallel, and alternate interior angles are equal between parallel lines, so the angle of refraction law at the air-oil interface is 20.0°. Applying Snell's law,

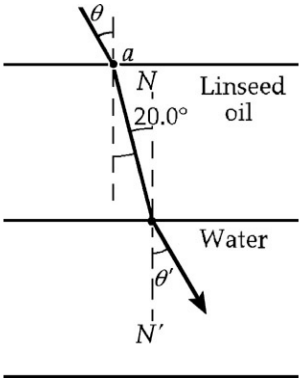
$$n_{\text{air}}\sin\theta = n_{\text{oil}}\sin\alpha$$
$$1.00\sin\theta = 1.48\sin 20.0°$$

yields $\boxed{\theta = 30.4°}$.

(b) The angle of incidence $\alpha = 20.0°$. Applying Snell's law at the oil-water interface,

$$n_{\text{water}}\sin\theta' = n_{\text{oil}}\sin\alpha$$
$$1.33\sin\theta' = 1.48\sin 20.0°$$

yields $\boxed{\theta' = 22.3°}$.



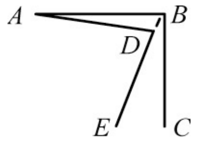
ANS. FIG. P35.8

P35.9 (a) flint glass: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.66} = \boxed{1.81\times10^8 \text{ m/s}}$

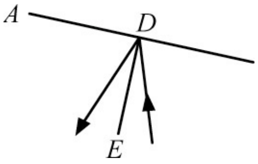
(b) water: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{1.333} = \boxed{2.25\times10^8 \text{ m/s}}$

(c) cubic zirconia: $v = \dfrac{c}{n} = \dfrac{3.00\times10^8 \text{ m/s}}{2.20} = \boxed{1.36\times10^8 \text{ m/s}}$

P35.10 (a) Let AB be the originally horizontal ceiling, BC its originally vertical normal, AD the new ceiling, and DE its normal. Then angle $BAD = \phi$. By definition DE is perpendicular to AD and BC is perpendicular to AB. Then the angle between DE extended and BC is ϕ because angles are equal when their sides are perpendicular, right side to right side and left side to left side.



ANS. FIG. P35.10(a)



ANS. FIG. P35.10(b)

(b) Now $CBE = \phi$ is the angle of incidence of the vertical light beam. Its angle of reflection is also ϕ. The angle between the vertical incident beam and the reflected beam is 2ϕ.

(c) $\tan 2\phi = \dfrac{1.40\text{ cm}}{720\text{ cm}} = 0.001\,94$ $\boxed{\phi = 0.055\,7°}$

P35.11 From Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. Thus, when $\theta_1 = 45.0°$ and the first medium is air ($n_1 = 1.00$), we have $\sin\theta_2 = (1.00)\sin 45.0°/n_2$.

(a) For quartz, $n_2 = 1.458$:

$$\theta_2 = \sin^{-1}\left(\frac{(1.00)\sin 45.0°}{1.458}\right) = \boxed{29.0°}$$

(b) For carbon disulfide, $n_2 = 1.628$:

$$\theta_2 = \sin^{-1}\left(\frac{(1.00)\sin 45.0°}{1.628}\right) = \boxed{25.7°}$$

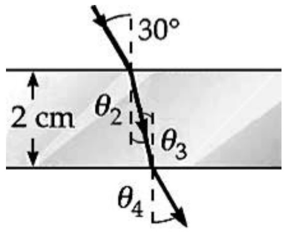
(c) For water, $n_2 = 1.333$:

$$\theta_2 = \sin^{-1}\left(\frac{(1.00)\sin 45.0°}{1.333}\right) = \boxed{32.0°}$$

P35.12 At entry, the wave under refraction model, expressed as $n_1 \sin\theta_1 = n_2 \sin\theta_2$, gives

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.000 \sin 30.0°}{1.50}\right) = \boxed{19.5°}$$

To do ray optics, you must remember some geometry. The surfaces of entry and exit are parallel so their normals are parallel. Then angle θ_2 of refraction at entry and the angle θ_3 of incidence at exit are alternate interior angles formed by the ray as a transversal cutting



ANS. FIG. P35.12

parallel lines. Therefore, $\theta_3 = \theta_2 = \boxed{19.5°}$.

At the exit point, $n_2 \sin\theta_3 = n_1 \sin\theta_4$ gives

$$\theta_4 = \sin^{-1}\left(\frac{n_2 \sin\theta_3}{n_1}\right) = \sin^{-1}\left(\frac{1.50 \sin 19.5°}{1.000}\right) = \boxed{30.0°}$$

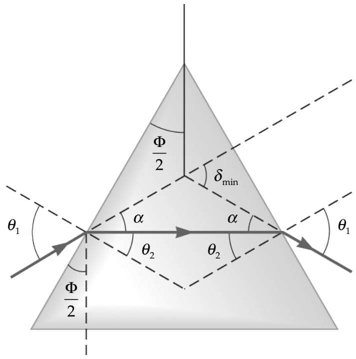
Because θ_1 and θ_4 are equal, the departing ray in air is parallel to the original ray.

P35.13 Taking Φ to be the apex angle and $\delta_{\min}$ to be the angle of minimum deviation (See ANS. FIG. P35.13), from Equation 35.9, the index of refraction of the prism material is

$$n = \frac{\sin\left[\left(\Phi + \delta_{\min}\right)/2\right]}{\sin\left(\Phi/2\right)}$$

Solving for $\delta_{\min}$,

$$\begin{aligned}\delta_{\min} &= 2\sin^{-1}\left(n \sin\frac{\Phi}{2}\right) - \Phi \\ &= 2\sin^{-1}\left[(2.20)\sin(25.0°)\right] - 50.0° \\ &= \boxed{86.8°}\end{aligned}$$

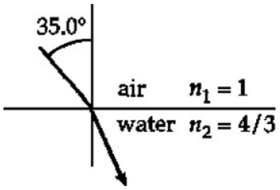


ANS. FIG. P35.13

P35.14 (a) The law of refraction $n_1 \sin\theta_1 = n_2 \sin\theta_2$ can be put into the more general form

$$\frac{c}{v_1}\sin\theta_1 = \frac{c}{v_2}\sin\theta_2$$

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$



ANS. FIG. P35.14

This is equivalent to Equation 35.3. This form applies to all kinds of waves that move through space.

In air at 20°C, the speed of sound is 343 m/s. From Table 17.1, the speed of sound in water at 25.0°C is 1493 m/s. The angle of incidence is 13.0°:

$$\frac{\sin 13.0°}{343 \text{ m/s}} = \frac{\sin\theta_2}{1\,493 \text{ m/s}}$$

$$\theta_2 = \boxed{78.3°}$$

(b) The wave keeps constant frequency in all media:

$$f = \frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\lambda_2 = \frac{v_2\lambda_1}{v_1} = \frac{1\ 493\ \text{m/s}(0.589\ \text{m})}{343\ \text{m/s}} = \boxed{2.56\ \text{m}}$$

(c) Using Snell's law,

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$
$$1.333 \sin\theta_2 = 1.000\ 293 \sin 13.0°$$
$$\theta_2 = \boxed{9.72°}$$

(d) $\lambda_2 = \frac{v_2\lambda_1}{v_1} = \frac{n_1\lambda_1}{n_2} = \frac{1.000\ 293(589\ \text{nm})}{1.333} = \boxed{442\ \text{nm}}$

(e) The light wave slows down as it moves from air to water, but the sound wave speeds up by a large factor. The light wave bends toward the normal and its wavelength shortens, but the sound wave bends away from the normal and its wavelength increases.

***P35.15** From the wave under refraction model, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, we solve for the index of refraction n_2 in the substance:

$$n_2 = \frac{1.333 \sin 37.0°}{\sin 25.0°} = 1.90$$

Then, from the definition of index of refraction,

$$n_2 = 1.90 = \frac{c}{v}: \qquad v = \frac{c}{1.90} = 1.58 \times 10^8\ \text{m/s} = \boxed{158\ \text{Mm/s}}$$

***P35.16** (a) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1.00 \sin 30.0° = n \sin 19.24°$

$n = \boxed{1.52}$

(c) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{6.328 \times 10^{-7}\ \text{m}} = \boxed{4.74 \times 10^{14}\ \text{Hz}}$ in air and in syrup.

(d) $v = \frac{c}{n} = \frac{3.00 \times 10^8\ \text{m/s}}{1.52} = 1.98 \times 10^8\ \text{m/s} = \boxed{198\ \text{Mm/s}}$

(b) $\lambda = \frac{v}{f} = \frac{1.98 \times 10^8\ \text{m/s}}{4.74 \times 10^{14}\ \text{s}^{-1}} = \boxed{417\ \text{nm}}$

***P35.17** (a) The angle of incidence at the first surface is $\theta_{1i} = \boxed{30.0°}$, and the angle of refraction is

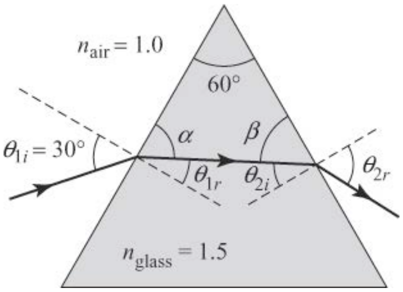
$$\theta_{1r} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_{1i}}{n_{\text{glass}}}\right) = \sin^{-1}\left(\frac{1.0\sin 30°}{1.5}\right) = \boxed{19°}$$

Also, $\alpha = 90° - \theta_{1r} = 71°$ and $\beta = 180° - 60° - \alpha = 49°$.

Therefore, the angle of incidence at the second surface is $\theta_{2i} = 90° - \beta = \boxed{41°}$. The angle of refraction at this surface is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{\text{glass}}\sin\theta_{2i}}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{1.5\sin 41°}{1.0}\right) = \boxed{77°}$$

ANS. FIG. P35.17 traces the path of the ray of light.

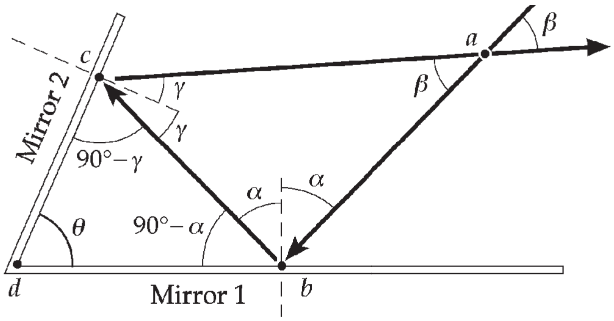


ANS. FIG. P35.17

(b) The angle of reflection at each surface equals the angle of incidence at that surface. Thus,

$$(\theta_1)_{\text{reflection}} = \theta_{1i} = \boxed{30°}, \text{ and } (\theta_1)_{\text{reflection}} = \theta_{2i} = \boxed{41°}$$

***P35.18** ANS. FIG. P35.18 shows the path of the light ray. α and γ are angles of incidence at mirrors 1 and 2.



ANS. FIG. P35.18

For triangle *abca*,

$$2\alpha + 2\gamma + \beta = 180°$$

or $\beta = 180° - 2(\alpha + \gamma)$. **[1]**

Now for triangle *bcdb*,

$$(90.0° - \alpha) + (90.0° - \gamma) + \theta = 180°$$

or $\theta = \alpha + \gamma$. **[2]**

Substituting equation [2] into equation [1] gives $\boxed{\beta = 180° - 2\theta}$.

Note: From equation [2], $\gamma = \theta - \alpha$. Thus, the ray will follow a path like that shown only if $\alpha < \theta$. For $\alpha > \theta$, γ is negative and multiple reflections from each mirror will occur before the incident and reflected rays intersect.

***P35.19** Consider glass with an index of refraction of 1.50, which is 3.00 mm thick. The speed of light in the glass is

$$\frac{3.00 \times 10^8 \text{ m/s}}{1.50} = 2.00 \times 10^8 \text{ m/s}$$

The extra travel time is

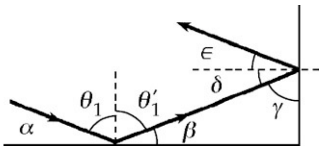
$$\frac{3.00 \times 10^{-3} \text{ m}}{2.00 \times 10^8 \text{ m/s}} - \frac{3.00 \times 10^{-3} \text{ m}}{3.00 \times 10^8 \text{ m/s}} \boxed{\sim 10^{-11} \text{ s}}$$

For light of wavelength 600 nm in vacuum and wavelength $\frac{600 \text{ nm}}{1.50} = 400 \text{ nm}$ in glass, the extra optical path, in wavelengths, is

$$\frac{3 \times 10^{-3} \text{ m}}{4 \times 10^{-7} \text{ m}} - \frac{3 \times 10^{-3} \text{ m}}{6 \times 10^{-7} \text{ m}} \boxed{\sim 10^3 \text{ wavelengths}}$$

P35.20 (a) **Method One:**

The incident ray makes angle $\alpha = 90° - \theta_1$ with the first mirror. In ANS. FIG. P35.20, the law of reflection implies that $\theta_1 = \theta_1'$



ANS. FIG. P35.20

Then,

$$\beta = 90° - \theta_1' = 90 - \theta_1 = \alpha.$$

In the triangle made by the mirrors and the ray passing between them,

$$\beta + 90° + \gamma = 180°$$
$$\gamma = 90° - \beta$$

Further, $\delta = 90° - \gamma = \beta = \alpha$

and $\epsilon = \delta = \alpha$.

Thus the final ray makes the same angle with the first mirror as did the incident ray. Its direction is opposite to the incident ray.

Method Two:

The vector velocity of the incident light has a component v_y perpendicular to the first mirror and a component v_x perpendicular to the second. The v_y component is reversed upon the first reflection, which leaves v_x unchanged. The second reflection reverses v_x and leaves v_y unchanged. The doubly reflected ray then has velocity opposite to the incident ray.

(b) The incident ray has velocity $v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$. If all of these components are non-zero, the light will reflect from each mirror because each component carries the light into the mirror that is perpendicular to that component: for example, the x component of velocity carries the light into the mirror in the yz plane. Each reflection reverses one component and leaves the other two unchanged. After all the reflections, the light has velocity $-v_x\hat{\mathbf{i}} - v_y\hat{\mathbf{j}} - v_z\hat{\mathbf{k}}$, opposite to the incident ray.

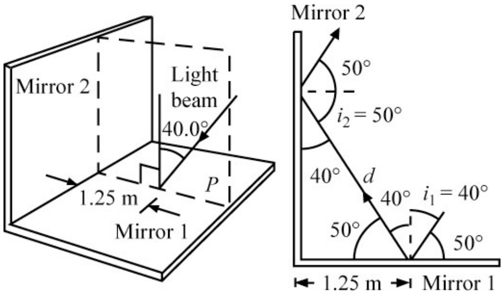
P35.21 (a) From geometry,

$$1.25\text{ m} = d\sin 40.0°$$

so $d = \boxed{1.94\text{ m}}$.

(b) $\boxed{50.0°\text{ above the horizontal}}$

or parallel to the incident ray.



ANS. FIG. P35.21

P35.22 (a) At entry, $n_1\sin\theta_1 = n_2\sin\theta_2$,

or $1.00\sin 30.0° = 1.50\sin\theta_2$,

which gives $\theta_2 = 19.5°$.

The distance h the light travels in the medium is given by

$$\cos\theta_2 = \frac{2.00\text{ cm}}{h}$$

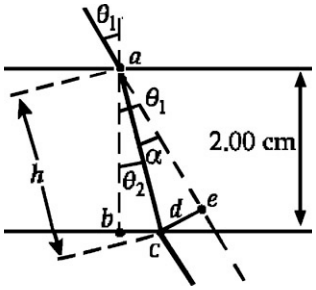
or $$h = \frac{2.00\text{ cm}}{\cos 19.5°} = 2.12\text{ cm}.$$

The angle of deviation upon entry is

$$\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$$

The offset distance comes from $\sin\alpha = \frac{d}{h}$:

$$d = (2.12\text{ cm})\sin 10.5° = \boxed{0.387\text{ cm}}$$



ANS. FIG. P35.22

(b) The speed of light in the material is

$$\text{v} = \frac{c}{n} = \frac{3.00\times 10^8\text{ m/s}}{1.50} = 2.00\times 10^8\text{ m/s}$$

The distance h traveled by the light is h = 2.12 cm. The time interval is

$$\Delta t = \frac{h}{\text{v}} = \frac{2.12\times 10^{-2}\text{ m}}{2.00\times 10^8\text{ m/s}} = 1.06\times 10^{-10}\text{ s} = \boxed{106\text{ ps}}$$

P35.23 From Table 35.1, the index of refraction of ice is 1.309. The pulses are in step with each other until one enters the ice, then that pulse slows down. The difference in the times of arrival of the pulses is

$$\Delta t = \frac{L}{\text{v}_{\text{ice}}} - \frac{L}{\text{v}_{\text{air}}} = \frac{L}{c/n_{\text{ice}}} - \frac{L}{c/n_{\text{air}}} = (n_{\text{ice}} - n_{\text{air}})\frac{L}{c}$$

$$\Delta t = (1.309 - 1.000)\frac{6.20\text{ m}}{3.00\times 10^8\text{ m/s}} = 6.39\times 10^{-9}\text{ s} = \boxed{6.39\text{ ns}}$$

P35.24 Refraction proceeds according to

$$(1.00)\sin\theta_1 = (1.66)\sin\theta_2 \qquad \textbf{[1]}$$

(a) For the normal component of velocity to be constant,

$$v_1\cos\theta_1 = v_2\cos\theta_2 \quad\text{or}\quad (c)\cos\theta_1 = \left(\frac{c}{1.66}\right)\cos\theta_2 \qquad \textbf{[2]}$$

We multiply equations [1] and [2], obtaining:

$$\sin\theta_1\cos\theta_1 = \sin\theta_2\cos\theta_2 \quad\text{or}\quad \sin 2\theta_1 = \sin 2\theta_2$$

We do not consider the case $\theta_1 = 0$. The physical solution is

$$2\theta_1 = 180° - 2\theta_2 \quad \text{or} \quad \theta_2 = 90.0° - \theta_1$$

Then equation [1] becomes:

$$\sin\theta_1 = 1.66\cos\theta_1$$
$$\tan\theta_1 = 1.66$$
$$\theta_1 = 58.9°$$

$\boxed{\text{Yes, if the angle of incidence is 58.9°.}}$

(b) $\boxed{\text{No. Both the reduction in speed and the bending toward the normal reduce the component of velocity parallel to the interface. This component cannot remain constant for a nonzero angle of incidence.}}$

P35.25 (a) As measured from the diagram, the incidence angle is 60°, and the refraction angle is 35°. From Snell's law, $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{n_1}{n_2} = \dfrac{v_2}{v_1}$, then $\dfrac{\sin 35°}{\sin 60°} = \dfrac{v_2}{c}$ and the speed of light in the block is $\boxed{2.0\times 10^8 \text{ m/s}}$.

(b) The frequency of the light does not change upon refraction. Knowing the wavelength in a vacuum, we can use the speed of light in a vacuum to determine the frequency: $c = f\lambda$, thus $3.00\times 10^8 = f\left(632.8\times 10^{-9}\right)$, so the frequency is $\boxed{4.74\times 10^{14} \text{ Hz}}$.

(c) To find the wavelength of light in the block, we use the same wave speed relation, $v = f\lambda$, so $2.0\times 10^8 = \left(4.74\times 10^{14}\right)\lambda$, so $\lambda_{\text{glass}} = 4.20\times 10^{-7} = \boxed{420 \text{ nm}}$.

P35.26 From Snell's law, the angle of refraction θ inside the liver is

$$\sin\theta = \left(\frac{n_{\text{medium}}}{n_{\text{liver}}}\right)\sin 50.0°$$

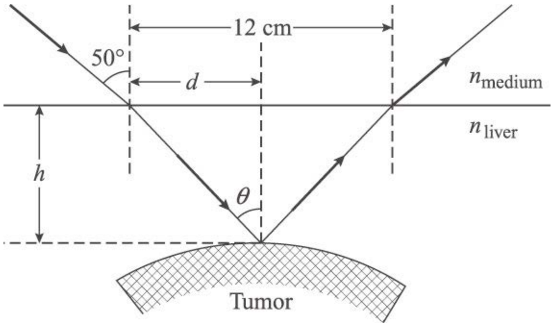
But $\quad \dfrac{n_{\text{medium}}}{n_{\text{liver}}} = \dfrac{c/v_{\text{medium}}}{c/v_{\text{liver}}} = \dfrac{v_{\text{liver}}}{v_{\text{medium}}} = 0.900,$

so $\quad \theta = \sin^{-1}\left[(0.900)\sin 50.0°\right] = 43.6°.$

From the law of reflection,

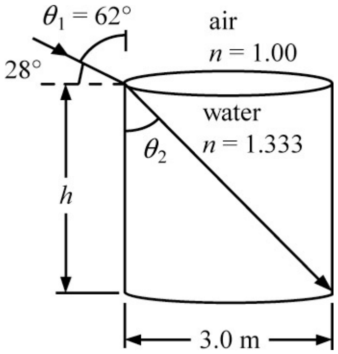
$$d = \frac{12.0 \text{ cm}}{2} = 6.00 \text{ cm}$$

and $$h = \frac{d}{\tan\theta} = \frac{6.00\text{ cm}}{\tan 43.6°} = \boxed{6.30\text{ cm}}.$$



ANS. FIG. P35.26

P35.27 The refracted sunlight does not illuminate any part of the bottom when it strikes its far inside edge:



ANS. FIG. P35.27

$$\sin\theta_1 = n_w \sin\theta_2$$

$$\sin\theta_2 = \frac{1}{1.333}\sin\theta_1$$

$$= \frac{1}{1.333}\sin(90.0° - 28.0°) = 0.662$$

$$\theta_2 = \sin^{-1}(0.662) = 41.5°$$

$$h = \frac{d}{\tan\theta_2} = \frac{3.00\text{ m}}{\tan 41.5°} = \boxed{3.39\text{ m}}$$

P35.28 Note for use in every part (refer to ANS. FIG. P35.28): from apex angle Φ,

$$\Phi + (90.0° - \theta_2) + (90.0° - \theta_3) = 180°$$

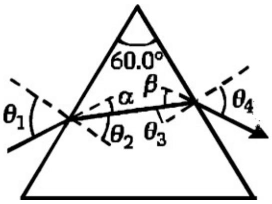
so $$\theta_3 = \Phi - \theta_2$$

At the first surface the deviation is

$$\alpha = \theta_1 - \theta_2$$

At exit, the deviation is

$$\beta = \theta_4 - \theta_3$$



ANS. FIG. P35.28

The total deviation is therefore

$$\delta = \alpha + \beta = \theta_1 + \theta_4 - \theta_2 - \theta_3 = \theta_1 + \theta_4 - \Phi$$

(a) At entry,

$$n_1\sin\theta_1 = n_2\sin\theta_2 \quad \text{or} \quad \theta_2 = \sin^{-1}\left(\frac{\sin 48.6°}{1.50}\right) = 30.0°$$

Thus, $\theta_3 = 60.0° - 30.0° = 30.0°$

At exit,

$$1.50\sin 30.0° = 1.00\sin\theta_4$$

or $\theta_4 = \sin^{-1}\left[1.50\sin(30.0°)\right] = 48.6°$

so the path through the prism is symmetric when $\theta_1 = 48.6°$.

(b) $\delta = 48.6° + 48.6° - 60.0° = \boxed{37.2°}$

(c) At entry,

$$\sin\theta_2 = \frac{\sin 45.6°}{1.50} \Rightarrow \theta_2 = 28.4°$$

$$\theta_3 = 60.0° - 28.4° = 31.6°$$

At exit,

$$\sin\theta_4 = 1.50\sin(31.6°) \Rightarrow \theta_4 = 51.7°$$

$$\delta = 45.6° + 51.7° - 60.0° = \boxed{37.3°}$$

(d) At entry,

$$\sin\theta_2 = \frac{\sin 51.6°}{1.50} \Rightarrow \theta_2 = 31.5°$$

$$\theta_3 = 60.0° - 31.5° = 28.5°$$

At exit,

$$\sin\theta_4 = 1.50\sin(28.5°) \Rightarrow \theta_4 = 45.7°$$

$$\delta = 51.6° + 45.7° - 60.0° = \boxed{37.3°}$$

P35.29 The index of refraction at 700 nm is $n(700 \text{ nm}) = 1.458$.

(a) $(1.00)\sin 75.0° = 1.458\sin\theta_2$; $\theta_2 = \boxed{41.5°}$

(b) Refer to ANS. FIG. P35.29. Let

$$\theta_3 + \beta = 90.0° \text{ and } \theta_2 + \alpha = 90.0°$$

then,

$$\alpha + \beta + 60.0° = 180°$$

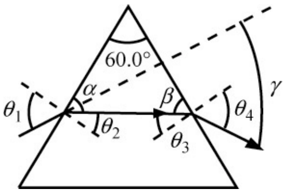
So

$$\alpha + \beta + 60.0° = 180°$$
$$(90.0° - \theta_2) + (90.0° - \theta_3) + 60.0° = 180°$$
$$60.0° - \theta_2 - \theta_3 = 0 \Rightarrow 60.0° - 41.5° = \theta_3 = \boxed{18.5°}$$

(c) $1.458 \sin 18.5° = 1.00 \sin \theta_4 \quad \rightarrow \quad \theta_4 = \boxed{27.6°}$

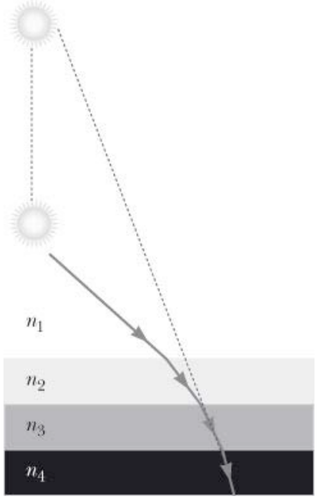
(d) $\gamma = (\theta_1 - \theta_2) + (\theta_4 - \theta_3)$

$$\gamma = (75.0° - 41.5°) + (27.6° - 18.5°) = \boxed{42.6°}$$



ANS. FIG. P35.29

P35.30 The index of refraction of the atmosphere decreases with increasing altitude because of the decrease in density of the atmosphere with increasing altitude. As indicated in the ray diagram, the sun located at S below the horizon appears to be located at S′.



ANS. FIG. P35.30

P35.31 For sheets 1 and 2 as described,

$$n_1 \sin 26.5° = n_2 \sin 31.7°$$
$$0.849 n_1 = n_2$$

For the trial with sheets 3 and 2,

$$n_3 \sin 26.5° = n_2 \sin 36.7°$$
$$0.747 n_3 = n_2$$

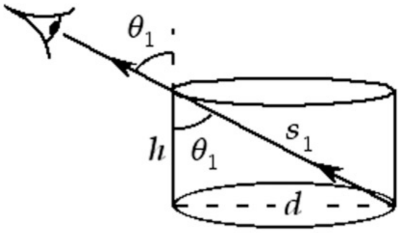
Equate the two expressions for n_2:

$$0.747 n_3 = 0.849 n_1$$
$$n_3 = 1.14 n_1$$

For the third trial,

$$n_1 \sin 26.5° = n_3 \sin\theta_3 = 1.14 n_1 \sin\theta_3$$
$$\theta_3 = \boxed{23.1°}$$

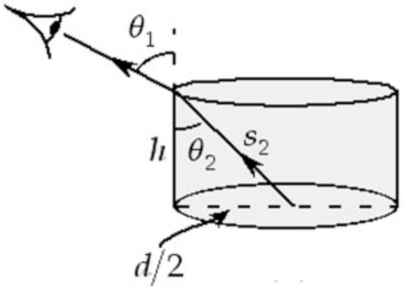
P35.32 (a) Before the container is filled, the ray's path is as shown in ANS. FIG. P35.32(a). From this figure, observe that



ANS. FIG. P35.32(a)

$$\sin\theta_1 = \frac{d}{s_1} = \frac{d}{\sqrt{h^2 + d^2}}$$
$$= \frac{1}{\sqrt{(h/d)^2 + 1}}$$

After the container is filled, the ray's path is shown in ANS. FIG. P35.32(b). From this figure, we find that



ANS. FIG. P35.32(b)

$$\sin\theta_2 = \frac{d/2}{s_2} = \frac{d/2}{\sqrt{h^2 + (d/2)^2}}$$
$$= \frac{1}{\sqrt{4(h/d)^2 + 1}}$$

From Snell's law, we have

$$1.00 \sin\theta_1 = n \sin\theta_2$$
$$\frac{1.00}{\sqrt{(h/d)^2 + 1}} = \frac{n}{\sqrt{4(h/d)^2 + 1}}$$
$$4(h/d)^2 + 1 = n^2 (h/d)^2 + n^2$$
$$(h/d)^2 (4 - n^2) = n^2 - 1 \rightarrow \boxed{\frac{h}{d} = \sqrt{\frac{n^2 - 1}{4 - n^2}}}$$

(b) For water, $n = 1.333$.

$$\frac{h}{d} = \sqrt{\frac{n^2 - 1}{4 - n^2}}$$

$$\frac{h}{8.00\text{ cm}} = \sqrt{\frac{(1.333)^2 - 1}{4 - (1.333)^2}} = \boxed{4.73\text{ cm}}$$

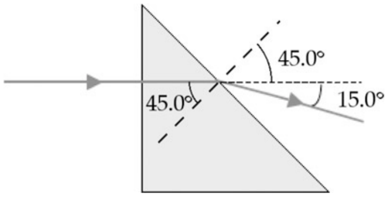
(c) $\boxed{\text{For } n = 1,\ h = 0.\ \text{For } n = 2,\ h = \infty.\ \text{For } n > 2,\ h \text{ has no real solution.}}$

P35.33 Since the light ray strikes the first surface at normal incidence, it passes into the prism without deviation. Thus, the angle of incidence at the second surface (hypotenuse of the triangular prism) is $\theta_1 = 45.0°$ as shown in the sketch at the right. The angle of refraction is

$$\theta_2 = 45.0° + 15.0° = 60.0°$$

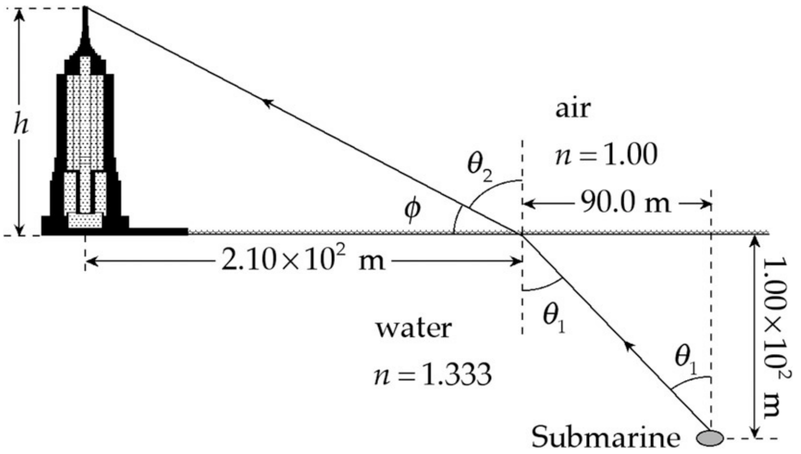
and Snell's law gives the index of refraction of the prism material as

$$n_1 = \frac{n_2 \sin\theta_2}{\sin\theta_1} = \frac{(1.00)\sin(60.0°)}{\sin(45.0°)} = \boxed{1.22}$$



ANS. FIG. P35.33

P35.34 (a) A sketch illustrating the situation and the two triangles needed in the solution is given in ANS. FIG. P35.34.



ANS. FIG. P35.34

(b) From the triangle under water, the angle of incidence θ_1 at the water surface is

$$\tan\theta_1 = \frac{90.0\text{ m}}{100\text{ m}} \quad \rightarrow \theta_1 = \boxed{42.0°}$$

(c) Snell's law gives the angle of refraction as

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{water}}\sin\theta_1}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{(1.333)\sin 42.0°}{1.00}\right) = \boxed{63.1°}$$

(d) The refracted beam makes angle $\phi = 90.0° - \theta_2 = \boxed{26.9°}$ with the horizontal.

(e) In the triangle above the water,

$$h = (210\text{ m})\tan\phi = (210\text{ m})\tan 26.9° = \boxed{107\text{ m}}$$

P35.35 The reflected ray and refracted ray are perpendicular to each other, and the angle of reflection θ_1 and the angle of refraction θ_2 are related by

$$\theta_1 + 90.0° + \theta_2 = 180.0° \rightarrow \theta_2 = 90.0° - \theta_1$$

Then, from Snell's law,

$$\sin\theta_1 = \frac{n_g\sin\theta_2}{n_{\text{air}}}$$

$$= n_g\sin(90° - \theta_1) = n_g\cos\theta_1$$

Thus, $\dfrac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1 = n_g$ or $\boxed{\theta_1 = \tan^{-1}(n_g)}$

Section 35.6 Huygens's Principle

Section 35.7 Dispersion

P35.36 Using Snell's law gives

(a) $$\theta_{\text{red}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{red}}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 83.0°}{1.331}\right) = \boxed{48.2°}$$

(b) $$\theta_{\text{blue}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{blue}}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 83.0°}{1.340}\right) = \boxed{47.8°}$$

P35.37 Using Snell's law gives

$$\theta_{\text{red}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{red}}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 50.00°}{1.455}\right)$$

and $\theta_{\text{violet}} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_i}{n_{\text{violet}}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 50.00°}{1.468}\right)$

Thus, the dispersion is $\theta_{\text{red}} - \theta_{\text{violet}} = \boxed{0.314°}$

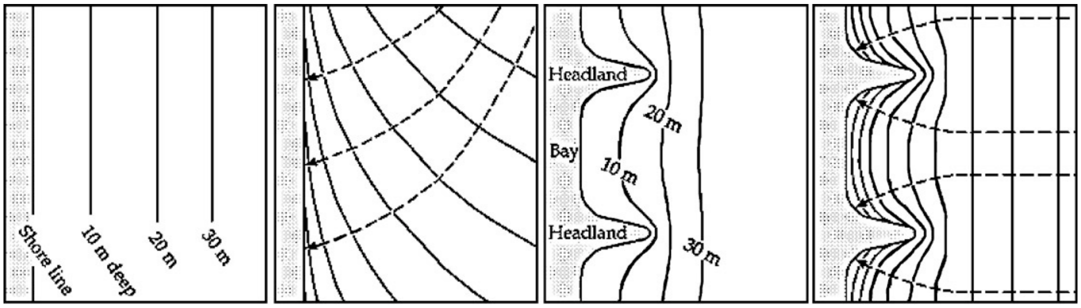
P35.38 Recall that if a wave slows down as it passes from one medium into another, its rays tend to bend toward the normal, unless it has normal incidence. Example: the case when light passes from air into water.

(a) For the diagrams of contour lines and wave fronts and rays, see ANS. FIG. P35.38(a) below.

(b) As the waves move to shallower water, the wave fronts slow down, and those closer to shore slow down more. The rays tend to bend toward the normal of the contour lines; or equivalently, the wave fronts bend to become more nearly parallel to the contour lines. See ANS. FIG. P35.38(b) below.

(c) For the diagrams of contour lines and wave fronts and rays, see ANS. FIG. P35.38(c) below.

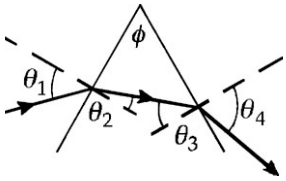
(d) We suppose that the headlands are steep underwater, as they are above water. The rays are everywhere perpendicular to the wave fronts of the incoming refracting waves. As shown, because the rays tend to bend toward the normal of the contour lines, the rays bend toward the headlands and deliver more energy per length at the headlands. See ANS. FIG. P35.38(d) below.



(a) Contour lines (b) Wave fronts and rays (c) Contour lines (d) Wave fronts and rays

ANS. FIG. P35.38

P35.39 For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$.



ANS. FIG. P35.39

Using ANS. FIG. P35.39,

$$(\theta_2)_{\text{violet}} = \sin^{-1}\left(\frac{\sin 50.0°}{1.66}\right) = 27.48°$$

$$(\theta_2)_{\text{red}} = \sin^{-1}\left(\frac{\sin 50.0°}{1.62}\right) = 28.22°$$

For the outgoing ray,

$$(90.0° - \theta_2) + (90.0° - \theta_3) + 60.0° = 180.0°$$
$$\theta_3 = 60.0° - \theta_2$$

and

$$\sin\theta_4 = n\sin\theta_3: \quad (\theta_4)_{\text{violet}} = \sin^{-1}[1.66\sin 32.52°] = 63.17°$$
$$(\theta_4)_{\text{red}} = \sin^{-1}[1.62\sin 31.78°] = 58.56°$$

The angular dispersion is the difference

$$\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}} = 63.17° - 58.56° = \boxed{4.61°}$$

P35.40 For the incoming ray, $\sin\theta_2 = \dfrac{\sin\theta_1}{n}$. Using ANS. FIG. P35.40,

$$(\theta_2)_{\text{violet}} = \sin^{-1}\left(\frac{\sin\theta}{n_V}\right)$$

$$(\theta_2)_{\text{red}} = \sin^{-1}\left(\frac{\sin\theta}{n_R}\right)$$

For the outgoing ray,

$$(90.0° - \theta_2) + (90.0° - \theta_3) + \Phi = 180.0°$$
$$\theta_3 = \Phi - \theta_2$$

and

$$\sin\theta_4 = n\sin\theta_3: \quad (\theta_4)_{\text{violet}} = \sin^{-1}\left\{n_V \sin\left[\Phi - \sin^{-1}\left(\frac{\sin\theta}{n_V}\right)\right]\right\}$$

$$(\theta_4)_{\text{red}} = \sin^{-1}\left\{n_R \sin\left[\Phi - \sin^{-1}\left(\frac{\sin\theta}{n_R}\right)\right]\right\}$$

The angular dispersion is the difference

$$\Delta\theta_4 = (\theta_4)_{\text{violet}} - (\theta_4)_{\text{red}}$$

$$= \boxed{\sin^{-1}\left\{n_V \sin\left[\Phi - \sin^{-1}\left(\frac{\sin\theta}{n_V}\right)\right]\right\} - \sin^{-1}\left\{n_R \sin\left[\Phi - \sin^{-1}\left(\frac{\sin\theta}{n_R}\right)\right]\right\}}$$

Section 35.8 Total Internal Reflection

P35.41 From Equation 35.10,

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.33}{1.50} \rightarrow \theta_c = \boxed{62.5°}$$

P35.42 From Equation 35.10, $\sin\theta_c = \frac{n_2}{n_1}$, where $n_2 = 1.000\ 293$. Values for n_1 come from Table 35.1,

(a) $\theta_c = \sin^{-1}\left(\frac{1.000\ 293}{2.20}\right) = \boxed{27.0°}$

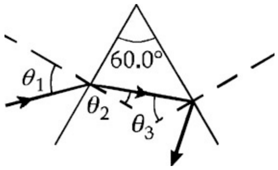
(b) $\theta_c = \sin^{-1}\left(\frac{1.000\ 293}{1.66}\right) = \boxed{37.1°}$

(c) $\theta_c = \sin^{-1}\left(\frac{1.000\ 293}{1.309}\right) = \boxed{49.8°}$

P35.43 The prism is in air, so at the first refraction,

$$1.00\sin\theta_1 = n\sin\theta_2$$

The angle of incidence θ_3 must be less than the critical angle at the second surface to emerge from the other side.



ANS. FIG. P35.43

$$\theta_3 < \theta_c$$

$$\theta_3 < \sin^{-1}\theta_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\left(\frac{1.00}{1.50}\right)$$

$$\theta_3 < 41.8°$$

The angles θ_2 and θ_3 are related by

$$\begin{aligned}(90.0° - \theta_2) + (90.0° - \theta_3) + 60.0° &= 180.0° \\ \theta_2 &= 60.0° - \theta_3\end{aligned}$$

Thus, to avoid total internal reflection at the second surface (i.e., have $\theta_3 < 41.8°$), it is necessary that $\theta_2 > 18.2°$.

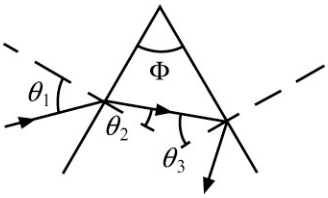
Since $\sin\theta_1 = n\sin\theta_2$, this becomes

$$\sin\theta_1 > 1.50\sin 18.2° = 0.468$$

or $\theta_1 > \boxed{27.9°}$.

P35.44 The prism is in air, so at the first refraction,

$$1.00\sin\theta_1 = n\sin\theta_2$$



ANS. FIG. P35.44

The angle of incidence θ_3 must be less than the critical angle at the second surface to emerge from the other side.

$$\theta_3 < \theta_c$$

$$\theta_3 < \sin^{-1}\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.00}{n}\right)$$

The angles θ_2 and θ_3 are related:

$$(90.0° - \theta_2) + (90.0° - \theta_3) + \Phi = 180°$$

which gives $\theta_2 = \Phi - \theta_3$.

Thus, to have $\theta_3 < \sin^{-1}\left(\frac{1.00}{n}\right)$ and avoid total internal reflection at the second surface, it is necessary that $\theta_2 > \Phi - \sin^{-1}\left(\frac{1.00}{n}\right)$.

Since $\sin\theta_1 = n\sin\theta_2$, this requirement becomes

$$\sin\theta_1 > n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]$$

or $$\theta_1 > \boxed{\sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]\right)}.$$

Through the application of trigonometric identities,

$$\theta_1 > \boxed{\sin^{-1}\left(\sqrt{n^2 - 1}\sin\Phi - \cos\Phi\right)}$$

P35.45 At the upper surface,

$$\sin\theta_c = \frac{n_{\text{air}}}{n_{\text{pipe}}} = \frac{1.00}{1.36} = 0.735 \quad \rightarrow \quad \theta_c = 47.3°$$

Geometry shows that the angle of refraction at the end is

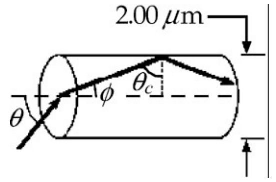
$$\phi = 90.0° - \theta_c = 90.0° - 47.3° = 42.7°$$

Then, by Snell's law at the end,

$$1.00\sin\theta = 1.36\sin 42.7°$$

gives $\boxed{\theta = 67.2°}$.

The 2-μm diameter is unnecessary information.



ANS. FIG. P35.45

P35.46 (a) Using the index of refraction values listed in Table 35.1, we find

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.000}{2.419} \quad \rightarrow \quad \theta_c = \boxed{24.42°}$$

(b) $\boxed{\text{Because the angle of incidence (35.0°) is greater than the critical angle, the light is totally reflected at } P.}$

(c) $\sin\theta_c = \dfrac{n_2}{n_1} = \dfrac{1.333}{2.419} \quad \rightarrow \quad \theta_c = \boxed{33.44°}$

(d) The angle of incidence is 35.0°. $\boxed{\text{Yes. In this case, the angle of incidence is just larger than the critical angle, so the light ray again undergoes total internal reflection at } P.}$

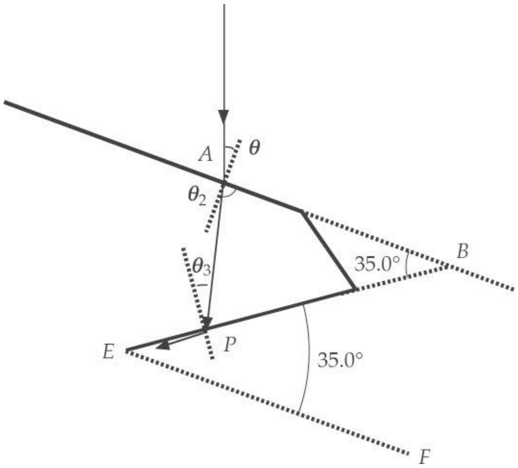
(e) The angle of incidence must be reduced below the critical angle for light to exit the diamond, so the diamond should be rotated $\boxed{\text{clockwise}}$.

(f) Rotating the diamond by angle θ clockwise changes the angle of incidence θ_1 at point *A* from 0.00° to θ, causing the angle of refraction θ_2 inside the diamond to change from 0.00°:

$$n_1\sin\theta_1 = n_2\sin\theta_2$$

$$1.333\sin\theta_1 = 2.419\sin\theta_2$$

Refer to ANS. FIG. P35.46. What is the angle of incidence at *P*? Extending a line from points *A* and *P* parallel to the surfaces of the diamond until they meet at point *B*, we form a triangle *ABP*.



ANS. FIG. P35.46

The angle at vertex *B* is 35.0° because the extended line *AB* is parallel to the line *EF* extended from the base of the diamond. From the sum of the interior angles of *ABP*, we find the incident angle θ_3 at point *P*:

$$\left(90.0^\circ - \theta_2\right) + \left(90.0^\circ - \theta_3\right) + 35.0^\circ = 180$$

$$\theta_3 = 35.0^\circ - \theta_2$$

At *P*, we require that the angle of incidence θ_3 results in an angle of refraction of 90.0°:

$$2.419 \sin\theta_3 = 1.333 \sin 90.0^\circ$$

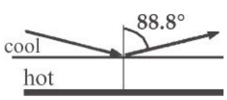
$$2.419 \sin\left(35.0^\circ - \theta_2\right) = 1.333$$

$$35.0^\circ - \theta_2 = \sin^{-1}\frac{1.333}{2.419}$$

solving gives $\theta_2 = 1.561^\circ$. Then, from above,

$$1.333 \sin\theta_1 = 2.419 \sin\theta_2 \;\rightarrow\; \theta = \boxed{2.83^\circ}$$

P35.47 The line of sight is 1.20° below the horizontal, so the angle of reflection of the light reaching the truck driver's eyes is 90.0° – 1.20° = 88.8°.



ANS. FIG. P35.47

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$n_2 = n_1 \sin 88.8^\circ = \left(1.000\,293\right)\sin 88.8^\circ = \boxed{1.000\,07}$$

Note: Mirages are caused by a continuous variation in index of refraction of the air rather than by total internal reflection. In this problem, the intent is to recognize that the result of the variation in index of refraction is equivalent to the result of a total internal reflection occurring at a single layer of hot air just above the surface of the roadway. This problem MODELS the phenomenon as a total internal reflection.

P35.48 (a) $\dfrac{\sin\theta_2}{\sin\theta_1} = \dfrac{v_2}{v_1}$ and $\theta_2 = 90.0°$ at the critical angle.

$$\frac{\sin 90.0°}{\sin\theta_c} = \frac{1\ 850\ \text{m/s}}{343\ \text{m/s}} \quad \text{so} \quad \theta_c = \sin^{-1}(0.185) = \boxed{10.7°}.$$

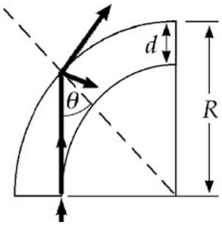
(b) Sound can be totally reflected if it is traveling in the medium where it travels slower: $\boxed{\text{air}}$.

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected,}}$ so the wall is a good mirror.

P35.49 (a) If any ray escapes it will be a ray along the inner edge, because it has the smallest angle of incidence. Its angle of incidence is described by $\sin\theta = \dfrac{R-d}{R}$ and by $n\sin\theta > 1\sin 90°$. Then

$$\frac{n(R-d)}{R} > 1 \quad \rightarrow \quad nR - nd > R$$

$$\rightarrow \quad nR - R > nd \quad \rightarrow \quad R > \boxed{\frac{nd}{n-1}}$$



ANS. FIG. P35.49

(b) As $d \rightarrow 0$, $\boxed{R_{\min} \rightarrow 0}$. $\boxed{\text{Yes: for very small } d\text{, the light strikes the interface at very large angles of incidence.}}$

(c) As n increases, $\boxed{R_{\min} \text{ decreases. Yes: as } n \text{ increases, the critical angle becomes smaller.}}$

(d) As n decreases toward 1, $R_{\min}$ increases. $\boxed{R_{\min} \rightarrow \infty\text{. Yes: as } n \rightarrow 1\text{, the critical angle becomes close to 90° and any bend will allow the light to escape.}}$

(e) $R_{\min} = \dfrac{1.40\left(100\times10^{-6}\text{ m}\right)}{0.40} = 350\times10^{-6}\text{ m} = \boxed{350\ \mu\text{m}}$

P35.50 (a) In the gasoline gauge, skylight from above travels down the plastic. The rays close to the vertical are totally reflected from the sides of the slab and from both facets at the lower end of the plastic, where it is not immersed in gasoline. This light returns up inside the plastic and makes it look bright. Where the plastic is immersed in gasoline, with index of refraction of about 1.50, total internal reflection should not happen. The light passes out of the lower end of the plastic with little reflected, making this part of the gauge look dark.

(b) To ensure total internal reflection at the plastic-air interface, the critical angle must be less than the angle of incidence, about 45.0°. This places a lower limit on the index of refraction of the plastic:

$$\theta_c \le 45.0^\circ$$

$$\sin\theta_c \le \sin 45.0^\circ$$

$$\frac{1}{n} \le \sin 45.0^\circ \quad\to\quad \boxed{n \ge 1.41}$$

To prevent total internal reflection at the plastic-gasoline interface, the critical angle must be greater than the angle of incidence. This places an upper limit on the index of refraction of the plastic:

$$\theta_c \ge 45.0^\circ$$

$$\sin\theta_c \ge \sin 45.0^\circ$$

$$\frac{1.50}{n} \ge \sin 45.0^\circ \quad\to\quad \boxed{n \le 2.12}$$

Additional Problems

***P35.51** Using Snell's law, the index of refraction of the liquid is found to be

$$n_{\text{liquid}} = \frac{n_{\text{air}}\sin\theta_i}{\sin\theta_r}$$

Thus, the critical angle for light going from this liquid into air is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{liquid}}}\right) = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{air}}\sin\theta_i/\sin\theta_r}\right)$$

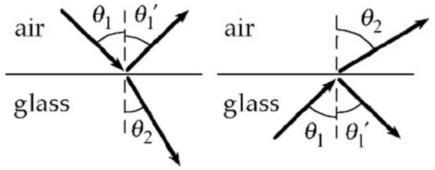
$$= \sin^{-1}\left(\frac{\sin\theta_r}{\sin\theta_i}\right) = \sin^{-1}\left(\frac{\sin 22.0^\circ}{\sin 30.0^\circ}\right) = \boxed{48.5^\circ}$$

P35.52 (a) $\theta_1' = \theta_1 = \boxed{30.0°}$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$1.00 \sin 30.0° = 1.55 \sin\theta_2$$

$$\theta_2 = \boxed{18.8°}$$



ANS. FIG. P35.52

(b) $\theta_1' = \theta_1 = \boxed{30.0°}$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{1.55 \sin 30.0°}{1}\right) = \boxed{50.8°}$$

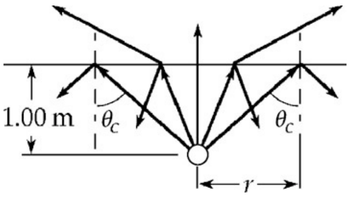
(c), (d) The other entries are computed similarly, and are shown in Table P35.52 below.

(c) air into glass, angles in degrees			(d) glass into air, angles in degrees		
incidence	reflection	refraction	incidence	reflection	refraction
0	0	0	0	0	0
10.0	10.0	6.43	10.0	10.0	15.6
20.0	20.0	12.7	20.0	20.0	32.0
30.0	30.0	18.8	30.0	30.0	50.8
40.0	40.0	24.5	40.0	40.0	85.1
50.0	50.0	29.6	50.0	50.0	none*
60.0	60.0	34.0	60.0	60.0	none*
70.0	70.0	37.3	70.0	70.0	none*
80.0	80.0	39.4	80.0	80.0	none*
90.0	90.0	40.2	90.0	90.0	none*

*total internal reflection

TABLE P35.52

P35.53 The critical angle is found by imagining the refracted ray just grazing the surface ($\theta_2 = 90°$). The index of refraction of water is $n_1 = 1.333$, and $n_2 = 1.00$ for air, so $n_1 \sin\theta_c = n_2 \sin 90°$ gives $\theta_c = \sin^{-1}(1/1.333) = \sin^{-1}(0.750) = 48.6°$.



ANS. FIG. P35.53

The radius then satisfies

$$\tan\theta_c = \frac{r}{1.00\ \text{m}}$$

So the diameter is

$$d = 2\left[(1.00\ \text{m})\tan\theta_c\right]$$

$$d = (2.00\ \text{m})\tan 48.6° = \boxed{2.27\ \text{m}}$$

P35.54 If the light ray to the eyes of the scuba diver makes an angle of 38.0° with the horizontal, it makes an angle of 52.0° with the normal to the water surface. This is larger than the critical angle of 48.8° found in Example 35.6, however. Therefore, no light from above the water will approach the scuba diver's eyes from this direction. The light approaching from this direction will be that originating underwater and reflected downward from the surface. The Sun will be seen somewhere within a circle whose edge is $90.0° - 48.8° = 41.2°$ above the horizontal.

P35.55 From the textbook Figure P35.55, we have $w = 2b + a$, so

$$b = \frac{w-a}{2} = \frac{700\ \mu\text{m} - 1\ \mu\text{m}}{2} = 349.5\ \mu\text{m}$$

$$\tan\theta_2 = \frac{b}{t} = \frac{349.5\ \mu\text{m}}{1\,200\ \mu\text{m}} = 0.291 \quad \rightarrow \quad \theta_2 = 16.2°$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

For refraction at entry,

$$\theta_1 = \sin^{-1}\frac{n_2 \sin\theta_2}{n_1} = \sin^{-1}\left(\frac{1.55 \sin 16.2°}{1.00}\right) = \sin^{-1} 0.433 = \boxed{25.7°}$$

P35.56 The incident light reaches the left-hand mirror at distance

$$(1.00\ \text{m})\tan 5.00° = 0.087\,5\ \text{m}$$

above its bottom edge. The reflected light first reaches the right-hand mirror at height

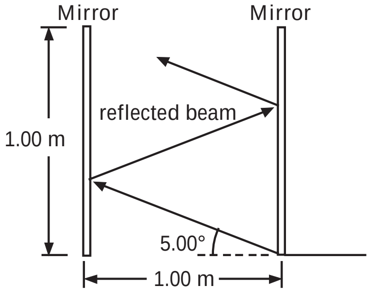
$$2(0.087\,5\text{ m}) = 0.175\text{ m}$$

It bounces between the mirrors with this distance between points of contact with either. Since

$$\frac{1.00\text{ m}}{0.175\text{ m}} = 5.72$$

the light reflects

$\boxed{\text{five times from the right-hand mirror and six times from the left}}$.



ANS. FIG. P35.56

***P35.57** (a) The fraction reflected is

$$\frac{S_1'}{S_1} = \left[\frac{n_2 - n_1}{n_2 + n_1}\right]^2 = \left[\frac{1.52 - 1.00}{1.52 + 1.00}\right]^2 = \boxed{0.042\,6}$$

(b) If medium 1 is glass and medium 2 is air,

$$\frac{S_1'}{S_1} = \left[\frac{n_2 - n_1}{n_2 + n_1}\right]^2 = \left[\frac{1.00 - 1.52}{1.00 + 1.52}\right]^2 = 0.042\,6$$

There is $\boxed{\text{no difference}}$.

***P35.58** (a) With $n_1 = 1$ and $n_2 = n$, the reflected fractional intensity is

$$\frac{S_1'}{S_1} = \left(\frac{n-1}{n+1}\right)^2$$

The remaining intensity must be transmitted:

$$\frac{S_2}{S_1} = 1 - \left(\frac{n-1}{n+1}\right)^2 = \frac{(n+1)^2 - (n-1)^2}{(n+1)^2} = \frac{n^2 + 2n + 1 - n^2 + 2n - 1}{(n+1)^2}$$

$$= \boxed{\frac{4n}{(n+1)^2}}$$

(b) At entry, $\frac{S_2}{S_1} = \frac{4n}{(n+1)^2} = \frac{4(2.419)}{(2.419+1)^2} = 0.828.$

At exit, $\frac{S_3}{S_2} = 0.828.$

Overall, $\frac{S_3}{S_1} = \left(\frac{S_3}{S_2}\right)\left(\frac{S_2}{S_1}\right) = (0.828)^2 = 0.685$

or $\boxed{68.5\%}$.

P35.59 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = 1.00\,293$ be its value at Earth's surface. Then,

$$n(x) = 1.000\ 00 + \left(\frac{1.002\ 93 - 1.000\ 00}{h}\right)x$$

$$= 1.000\ 00 + \left(\frac{0.002\ 93}{h}\right)x$$

(a) The total time interval required to traverse the atmosphere is

$$\Delta t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx: \quad \Delta t = \frac{1}{c}\int_0^h \left[1.000\ 00 + \left(\frac{0.002\ 93}{h}\right)x\right]dx$$

$$\Delta t = \frac{h}{c} + \frac{0.002\ 93}{ch}\left(\frac{h^2}{2}\right)$$

$$= \frac{h}{c}\left(\frac{2.002\ 93}{2}\right) = \frac{100\times 10^3\ \text{m}}{3.00\times 10^8\ \text{m/s}}\left(\frac{2.002\ 93}{2}\right)$$

$$= 3.33\times 10^{-4}\ \text{s} = \boxed{334\ \mu\text{s}}$$

(b) The travel time in the absence of an atmosphere would be $\frac{h}{c}$.

Thus, the time in the presence of an atmosphere is

$$\frac{h/c\left(\frac{2.002\ 93}{2}\right) - h/c}{h/c} = \left(\frac{0.002\ 93}{2}\right)\times 100\% = \boxed{0.147\%}$$

P35.60 Let $n(x)$ be the index of refraction at distance x below the top of the atmosphere and $n(x = h) = n$ be its value at the planet surface.

Then, $n(x) = 1.00 + \left(\frac{n - 1.00}{h}\right)x$

(a) The total time interval required to traverse the atmosphere is

$$\Delta t = \int_0^h \frac{dx}{v} = \int_0^h \frac{n(x)}{c}dx : \Delta t = \frac{1}{c}\int_0^h \left[1.00 + \left(\frac{n - 1.00}{h}\right)x\right]dx$$

$$\Delta t = \frac{h}{c} + \frac{(n - 1.00)}{ch}\left(\frac{h^2}{2}\right) = \boxed{\frac{h}{c}\left(\frac{n + 1.00}{2}\right)}$$

(b) The travel time in the absence of an atmosphere would be $\frac{h}{c}$.

Thus, the time in the presence of an atmosphere is

$$\boxed{\left(\frac{n + 1.00}{2}\right)\text{ times larger}}$$

P35.61 Let the air and glass be medium 1 and 2, respectively. By Snell's law,

$$n_2 \sin\theta_2 = n_1 \sin\theta_1$$

or $$1.56\sin\theta_2 = \sin\theta_1$$

But the conditions of the problem are such that $\theta_1 = 2\theta_2$, so $1.56\sin\theta_2 = \sin 2\theta_2$ We now use the double-angle trig identity suggested:

$$1.56\sin\theta_2 = 2\sin\theta_2\cos\theta_2$$

or $$\cos\theta_2 = \frac{1.56}{2} = 0.780$$

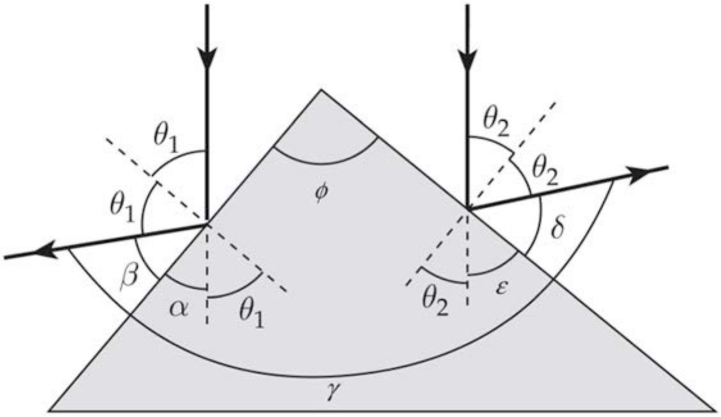
Thus, $\theta_2 = 38.7°$ and $\theta_1 = 2\theta_2 = \boxed{77.5°}$.

P35.62 In ANS. FIG. P35.62, observe on the left side of the prism that $\beta = 90° - \theta_1$ and $\alpha = 90° - \theta_1$. Thus, $\beta = \alpha$. Similarly, on the right side of the prism, $\delta = 90° - \theta_2$ and $\varepsilon = 90° - \theta_2$, giving $\delta = \varepsilon$. The incident rays are initially parallel, so observe that the angle between the reflected rays is $\gamma = (\alpha + \beta) + (\varepsilon + \delta)$, so $\gamma = 2(\alpha + \varepsilon)$. Finally, observe that the left side of the prism is sloped at angle α from the vertical, and the right side is sloped at angle ε. The angle ϕ is related to the other angles by

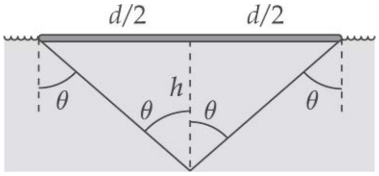
$$\phi + (90° - \alpha) + (90° - \varepsilon) = 180° \rightarrow \phi = \alpha + \varepsilon$$

Thus, we obtain the result

$$\gamma = 2(\alpha + \varepsilon) = 2\phi \rightarrow \phi = \frac{1}{2}\gamma$$



ANS. FIG. P35.62



ANS. FIG. P35.63

P35.63 Light from the diamond reflects totally at the water's surface at incident angles greater than the critical angle θ_c. The circular raft must cover the area of the surface through which light from the diamond could emerge. Thus, it must form the base of an inverted cone (with apex at the diamond) whose half angle is at least the critical angle.

$$\theta \geq \theta_c$$

$$\tan\theta \geq \tan\theta_c$$

$$\frac{d/2}{h} \geq \tan\theta_c \quad \rightarrow \quad h \leq \frac{d}{2\tan\theta_c}$$

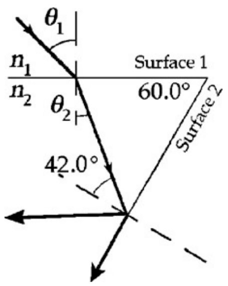
The critical angle at the water-air boundary is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{1.000}{1.333}\right) = 48.61°$$

Thus, the maximum depth of the water is

$$h_{\max} = \frac{d}{2\tan\theta_c} = \frac{4.54\text{ m}}{2\tan 48.61°} = \boxed{2.00\text{ m}}$$

***P35.64** Consider an insulated box with the imagined one-way mirror forming one face, installed so that 90% of the electromagnetic radiation incident from the outside is transmitted to the inside and only a lower percentage of the electromagnetic waves from the inside make it through to the outside. Suppose the interior and exterior of the box are originally at the same temperature. Objects within and without are radiating and absorbing electromagnetic waves. They would all maintain constant temperature if the box had an open window. With the glass letting more energy in than out, the interior of the box will rise in temperature. But this is impossible, according to Clausius's statement of the second law. This reduction to a contradiction proves that it is impossible for the one-way mirror to exist.



ANS. FIG. P35.65

P35.65 Define n_1 to be the index of refraction of the surrounding medium and n_2 to be that for the prism material. We can use the critical angle of 42.0° to find the ratio $\frac{n_2}{n_1}$:

$$n_2 \sin 42.0° = n_1 \sin 90.0°$$

So, $\dfrac{n_2}{n_1} = \dfrac{1}{\sin 42.0°} = 1.49$

Call the angle of refraction θ_2 at the surface 1. The ray inside the prism forms a triangle with surfaces 1 and 2, so the sum of the interior angles of this triangle must be 180°.

Thus, $(90.0° - \theta_2) + 60.0° + (90.0° - 42.0°) = 180°$

Therefore, $\theta_2 = 18.0°$.

Applying Snell's law at surface 1, $n_1 \sin\theta_1 = n_2 \sin 18.0°$:

$$\sin\theta_1 = \left(\frac{n_2}{n_1}\right)\sin\theta_2 = 1.49 \sin 18.0°$$

gives $\boxed{\theta_1 = 27.5°}$

P35.66 The number *N* of reflections the beam makes before exiting at the other end is equal to the length of the slab divided by the component of the displacement of the beam for each reflection:

$$N = \frac{L}{(t / \tan\theta_2)} = \frac{L \tan\theta_2}{t}$$

where θ_2 is the refracted angle as the beam enters the material. Substitute for this refracted angle in terms of the incident angle by using Snell's law:

$$N = \frac{L}{t} \tan\left[\sin^{-1}\left(\frac{n_1 \sin\theta_1}{n_2}\right)\right]$$

Substitute numerical values:

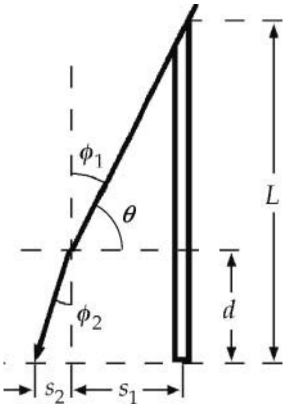
$$N = \frac{0.420 \text{ m}}{0.003\ 10 \text{ m}} \tan\left[\sin^{-1}\left(\frac{(1)\sin 50.0°}{1.48}\right)\right]$$
$$= 81.96 \rightarrow 81 \text{ reflections}$$

Therefore, the beam will exit after making 81 reflections, so it does not make 85 reflections.

P35.67 A light beam passing the top of the pole makes an angle θ of 40.0° with the horizontal, so its angle of incidence at the water is $\phi_1 = 90.0° - \theta$. It enters the water's surface at distance from the pole

$$s_1 = \frac{L - d}{\tan\theta}$$

and has an angle of refraction ϕ_2 from $1.00 \sin\phi_1 = n \sin\phi_2$.



ANS. FIG. P35.67

The beam reaches the bottom after traveling the horizontal distance

$$s_2 = d\tan\phi_2$$

The whole shadow length is

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left[\sin^{-1}\left(\frac{\sin\phi_1}{n}\right)\right]$$

Because $\sin\phi_1 = \sin(90.0° - \theta) = \cos\theta$, we find that

$$s_1 + s_2 = \frac{L-d}{\tan\theta} + d\tan\left[\sin^{-1}\left(\frac{\cos\theta}{n}\right)\right]$$

$$= \frac{2.00\text{ m}}{\tan 40.0°} + (2.00\text{ m})\tan\left[\sin^{-1}\left(\frac{\cos 40.0°}{1.33}\right)\right] = \boxed{3.79\text{ m}}$$

P35.68 From Table 35.1, the index of refraction of polystyrene is 1.49.

(a) For polystyrene *surrounded by air*, total internal reflection requires

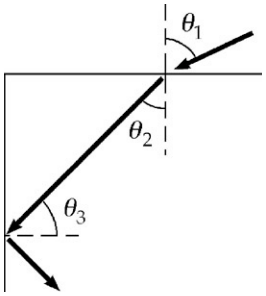
$$\theta_3 \geq \theta_c = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

Then from geometry, $\theta_2 = 90.0° - \theta_3 \leq 47.8°$.

From Snell's law,

$$\sin\theta_1 = 1.49\sin\theta_2 \leq 1.49\sin 47.8°$$
$$\sin\theta_1 \leq 1.10$$

Any angle θ_1 satisfies this equation.



ANS. FIG. P35.68

$$\boxed{\text{Total internal reflection occurs for all values of } \theta, \text{ or the maximum angle is } 90°.}$$

(b) For polystyrene *surrounded by water*, $\theta_3 = \sin^{-1}\left(\frac{1.33}{1.49}\right) = 63.2°$

and $\theta_2 = 26.8°$.

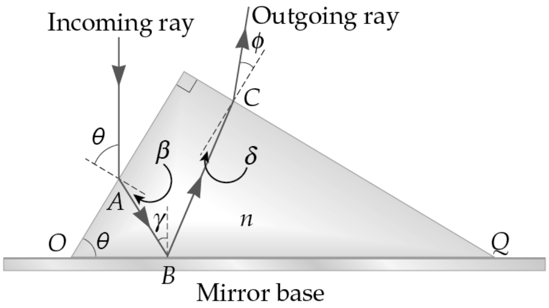
From Snell's law, $\theta_1 = \boxed{30.3°}$.

(c) From Table 35.1, the index of carbon disulfide is 1.628 > 1.49. $\boxed{\text{Total internal reflection never occurs as the light moves from lower-index polystyrene into higher-index carbon disulfide.}}$

P35.69 From ANS. FIG. P35.69, observe that the angle of incidence at *A* is the same as the prism angle at point *O*. Given that $\theta = 60.0°$, application of By Snell's law at point *A*:

$$1.50 \sin\beta = (1.00)\sin 60.0°$$

$$\sin\beta = \frac{\sin 60.0°}{1.50}$$



ANS. FIG. P35.69

From triangle *AOB*, we calculate the angle of incidence and reflection, γ, at point *B*:

$$\theta + (90.0° - \beta) + (90.0° - \gamma) = 180° \quad \text{or} \quad \gamma = \theta - \beta$$

Now, we find the angle of incidence at point *C* using triangle *BCQ*:

$$(90.0° - \gamma) + (90.0° - \delta) + (90.0° - \theta) = 180°$$

or

$$\delta = 90.0° - (\theta + \gamma) = 90.0° - (\theta + \theta - \beta) = 90.0° - 2\theta + \beta$$

Finally, application of Snell's law at point *C* gives

$$(1.00)\sin\phi = (1.50)\sin\delta$$

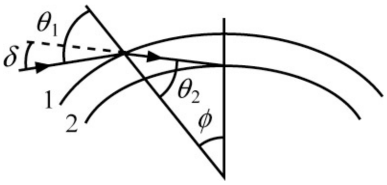
or

$$\phi = \sin^{-1}\left[1.50\sin(90.0° - 2\theta + \beta)\right]$$

$$= \sin^{-1}\left\{1.50\sin\left[90.0° - 2(60.0°) + \sin^{-1}\left(\frac{\sin 60.0°}{1.50}\right)\right]\right\}$$

$$= \boxed{7.91°}$$

P35.70 (a) The optical day is longer. Incoming sunlight is refracted downward at the top of the atmosphere, so an observer can see the rising Sun when it is still geometrically below the horizon. Light from the setting Sun reaches her after the Sun is below the horizon geometrically.



ANS. FIG. P35.70

(b) ANS. FIG. P35.70 illustrates optical sunrise. At the center of the Earth,

$$\cos\phi = \frac{6.37\times10^6\text{ m}}{6.37\times10^6\text{ m} + 8\,614}$$

$$\phi = 2.98°$$

$$\theta_2 = 90 - 2.98° = 87.0°$$

At the top of the atmosphere

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$1\sin\theta_1 = 1.000\,293 \sin 87.0°$$

$$\theta_1 = 87.4°$$

Deviation upon entry is

$$\delta = |\theta_1 - \theta_2|$$

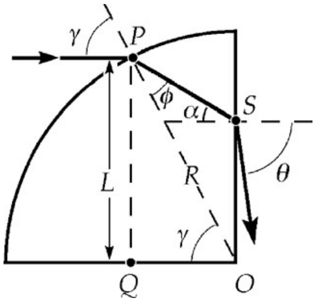
$$\delta = 87.364° - 87.022° = 0.342°$$

Sunrise of the optical day is before geometric sunrise by

$0.342°\left(\dfrac{86\,400\text{ s}}{360°}\right) = 82.2\text{ s}.$ Optical sunset occurs later too, so the optical day is longer by $\boxed{164\text{ s}}$.

P35.71 Observe in ANS. FIG. P35.71 that the angle of incidence at point *P* is γ, and using triangle *OPQ*:

$$\sin\gamma = \frac{L}{R}$$



ANS. FIG. P35.71

Also, $\cos\gamma = \sqrt{1-\sin^2\gamma} = \dfrac{\sqrt{R^2 - L^2}}{R}$

Apply Snell's law at point *P*:

$$1.00\sin\gamma = n\sin\phi$$

Thus, $\sin\phi = \dfrac{\sin\gamma}{n} = \dfrac{L}{nR}$

and $\cos\phi = \sqrt{1-\sin^2\phi} = \dfrac{\sqrt{n^2R^2 - L^2}}{nR}.$

From triangle *OPS*, $\phi + (\alpha + 90.0°) + (90.0° - \gamma) = 180°$, or the angle of incidence at point *S* is $\alpha = \gamma - \phi$. Then, applying Snell's law at point *S* gives $1.00\sin\theta = n\sin\alpha = n\sin(\gamma-\phi)$

or
$$\begin{aligned}\sin\theta &= n\sin(\gamma-\phi)\\ &= n\left[\sin\gamma\cos\phi - \cos\gamma\sin\phi\right]\\ &= n\left[\left(\frac{L}{R}\right)\frac{\sqrt{n^2R^2-L^2}}{nR} - \frac{\sqrt{R^2-L^2}}{R}\left(\frac{L}{nR}\right)\right]\\ &= \frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)\end{aligned}$$

thus, $\theta = \boxed{\sin^{-1}\left[\frac{L}{R^2}\left(\sqrt{n^2R^2-L^2} - \sqrt{R^2-L^2}\right)\right]}$;

or, using from above $\sin\gamma = \dfrac{L}{R} \rightarrow \gamma = \sin^{-1}\dfrac{L}{R}$ and $\phi = \sin^{-1}\dfrac{L}{nR}$,

$$\sin\theta = n\sin(\gamma-\phi) = n\sin\left(\sin^{-1}\frac{L}{R} - \sin^{-1}\frac{L}{nR}\right)$$

$$\theta = \boxed{\sin^{-1}\left[n\sin\left(\sin^{-1}\frac{L}{R} - \sin^{-1}\frac{L}{nR}\right)\right]}$$

P35.72 $\delta = \theta_1 - \theta_2 = 10.0°$ and $n_1\sin\theta_1 = n_2\sin\theta_2$ with $n_1 = 1,\ n_2 = \dfrac{4}{3}.$

Thus, $\theta_1 = \sin^{-1}(n_2\sin\theta_2) = \sin^{-1}\left[n_2\sin(\theta_1 - 10.0°)\right].$

(You can use a calculator to home in on an approximate solution to this equation, testing different values of θ_1 until you find that $\theta_1 = \boxed{36.5°}$. Alternatively, you can solve for θ_1 exactly, as shown below.)

We are given that $\sin\theta_1 = \dfrac{4}{3}\sin(\theta_1 - 10.0°).$

This is the sine of a difference, so

$$\frac{3}{4}\sin\theta_1 = \sin\theta_1\cos 10.0° - \cos\theta_1\sin 10.0°$$

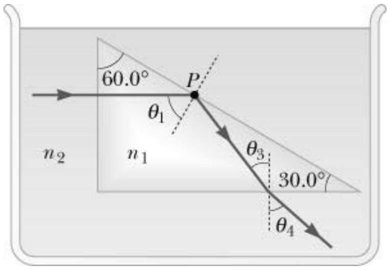
Rearranging, $\sin 10.0° \cos\theta_1 = \left(\cos 10.0° - \frac{3}{4}\right)\sin\theta_1$,

$$\frac{\sin 10.0°}{\cos 10.0° - 0.750} = \tan\theta_1$$

and $\theta_1 = \tan^{-1}(0.740) = \boxed{36.5°}$.

P35.73 (a) From the geometry shown in ANS. FIG. P35.73, observe that $\theta_1 = 60.0°$. Also, from the law of reflection, $\theta_2 = \theta_1 = 60.0°$.

Therefore, $\alpha = 90.0° - \theta_2 = 30.0°$, and $\theta_3 + 90.0° = 180 - \alpha - 30.0°$ or $\theta_3 = 30.0°$.



ANS. FIG. P35.73

Then, since the prism is immersed in water ($n_2 = 1.333$), Snell's law gives

$$\theta_4 = \sin^{-1}\left(\frac{n_{\text{glass}}\sin\theta_3}{n_2}\right) = \sin^{-1}\left(\frac{(1.66)\sin 30.0°}{1.333}\right) = \boxed{38.5°}$$

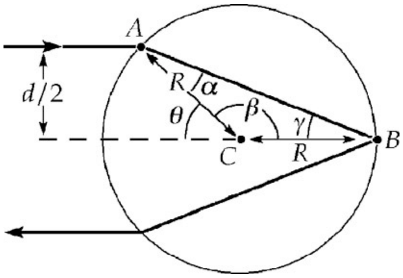
(b) For refraction to occur at point *P*, it is necessary that $\theta_c > \theta_1$. Thus,

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_{\text{glass}}}\right) > \theta_1, \text{ which gives}$$

$$n_2 > n_{\text{glass}}\sin\theta_1 = (1.66)\sin 60.0° = \boxed{1.44}$$

P35.74 As shown in ANS. FIG. P35.74, the angle of incidence at point *A* is:

$$\theta = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°$$



ANS. FIG. P35.74

If the emerging ray is to be parallel to the incident ray, the path must be symmetric about the centerline *CB* of the cylinder. In the isosceles triangle *ABC*,

$$\gamma = \alpha \quad \text{and} \quad \beta = 180° - \theta$$

Therefore, $\alpha + \beta + \gamma = 180°$

becomes $2\alpha + 180° - \theta = 180°$

or $\alpha = \frac{\theta}{2} = 15.0°.$

Then, applying Snell's law at point *A*,

$$n\sin\alpha = 1.00\sin\theta$$

$$n = \frac{\sin\theta}{\sin\alpha} = \frac{\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

P35.75 Applying Snell's law at points A, B, and C gives

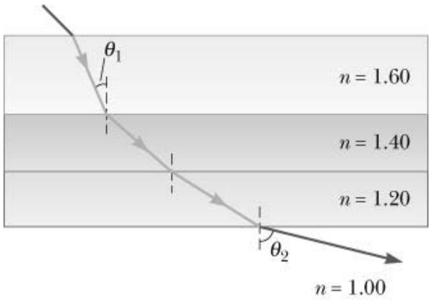
$$1.40\sin\alpha = 1.60\sin\theta_1 \qquad [1]$$

$$1.20\sin\beta = 1.40\sin\alpha \qquad [2]$$

and $$1.00\sin\theta_2 = 1.20\sin\beta \qquad [3]$$

Combining equations [1], [2], and [3] yields

$$\sin\theta_2 = 1.60\sin\theta_1 \qquad [4]$$



ANS. FIG. P35.75

Note that equation [4] is exactly what Snell's law would yield if the second and third layers of this "sandwich" were ignored. This will always be true if the surfaces of all the layers are parallel to each other.

(a) If $\theta_1 = 30.0°$, then equation [4] gives

$$\theta_2 = \sin^{-1}\left(1.60\sin 30.0°\right) = \boxed{53.1°}$$

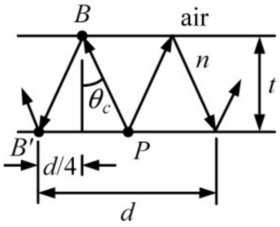
(b) At the critical angle of incidence on the lowest surface, $\theta_2 = 90.0°$. Then, equation [4] gives

$$\theta_1 = \sin^{-1}\left(\frac{\sin\theta_2}{1.60}\right) = \sin^{-1}\left(\frac{\sin 90.0°}{1.60}\right) = 38.7°$$

Total internal reflection will occur for $\boxed{\theta_1 \geq 38.7°}$.

P35.76 (a) At the boundary of the air and glass, the critical angle is given by

$$\sin\theta_c = \frac{1}{n}$$



ANS. FIG. P35.76

Consider the critical ray *PBB′*:

$$\tan\theta_c = \frac{d/4}{t} \quad \text{or} \quad \frac{\sin\theta_c}{\cos\theta_c} = \frac{d}{4t}$$

Squaring the last equation gives:

$$\frac{\sin^2\theta_c}{\cos^2\theta_c} = \frac{\sin^2\theta_c}{1-\sin^2\theta_c} = \left(\frac{d}{4t}\right)^2$$

Since $\sin\theta_c = \frac{1}{n}$, this becomes $\frac{1}{n^2-1} = \left(\frac{d}{4t}\right)^2$ or

$$\boxed{n = \sqrt{1+\left(\frac{4t}{d}\right)^2}}$$

(b) Solving for *d*,

$$d = \frac{4t}{\sqrt{n^2-1}}$$

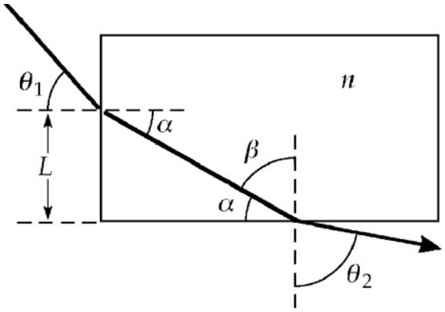
Thus, if *n* = 1.52 and *t* = 0.600 cm, $d = \frac{4(0.600\text{ cm})}{\sqrt{(1.52)^2-1}} = \boxed{2.10\text{ cm}}$

(c) Since violet light has a larger index of refraction, it will lead to a smaller critical angle and the inner edge of the white halo will be tinged with $\boxed{\text{violet}}$ light.

P35.77 (a) Given that $\theta_1 = 45.0°$ and $\theta_2 = 76.0°$,

Snell's law at the first surface gives

$$n \sin\alpha = 1.00 \sin 45.0° \qquad \textbf{[1]}$$



ANS. FIG. P35.77

Observe that the angle of incidence at the second surface is

$$\beta = 90.0° - \alpha$$

Thus, Snell's law at the second surface yields

$$n \sin\beta = n \sin(90.0° - \alpha) = 1.00 \sin 76.0°$$

or $n \cos\alpha = \sin 76.0°$. **[2]**

Dividing equation [1] by equation [2], we obtain

$$\tan\alpha = \frac{\sin 45.0°}{\sin 76.0°} = 0.729$$

or $\alpha = 36.1°$.

Then, from equation [1],

$$n = \frac{\sin 45.0°}{\sin\alpha} = \frac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$$

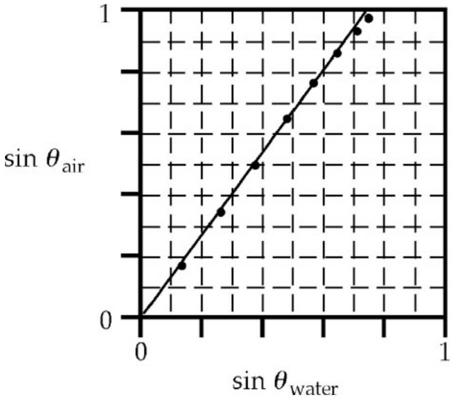
(b) From the sketch, observe that the distance the light travels in the plastic is $d = \dfrac{L}{\sin\alpha}$. Also, the speed of light in the plastic is $v = \dfrac{c}{n}$, so the time required to travel through the plastic is

$$\Delta t = \frac{d}{v} = \frac{nL}{c \sin\alpha} = \frac{1.20(0.500 \text{ m})}{(3.00 \times 10^8 \text{ m/s}) \sin 36.1°}$$

$$= 3.40 \times 10^{-9} \text{ s} = \boxed{3.40 \text{ ns}}$$

P35.78 (a) See graph in ANS. FIG. P35.78.

$\sin\theta_1$	$\sin\theta_2$	$\frac{\sin\theta_1}{\sin\theta_2}$
0.174	0.131	1.330 4
0.342	0.261	1.312 9
0.500	0.379	1.317 7
0.643	0.480	1.338 5
0.766	0.576	1.328 9
0.866	0.647	1.339 0
0.940	0.711	1.322 0
0.985	0.740	1.331 5



ANS. FIG. P35.78

(b) The straightness of the graph line demonstrates Snell's proportionality of the sine of the angle of refraction to the sine of the angle of incidence.

(c) The slope of the line is $\bar{n} = 1.327\ 6 \pm 0.01$

The equation $\sin\theta_1 = n\sin\theta_2$ shows that this slope is the index of refraction, $n = \boxed{1.328 \pm 0.8\%}$

P35.79 (a) We see the Sun moving from east to west across the sky. Its angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{86\ 400 \text{ s}} = 7.27\times10^{-5} \text{ rad/s}$$

The direction of sunlight crossing the cell from the window changes at this rate, moving on the opposite wall at speed

$$v = r\omega = (2.37 \text{ m})(7.27\times10^{-5} \text{ rad/s})$$
$$= 1.72\times10^{-4} \text{ m/s} = \boxed{0.172 \text{ mm/s}}$$

(b) The mirror folds into the cell the motion that would occur in a room twice as wide:

$$v = r\omega = 2(0.174 \text{ mm/s}) = \boxed{0.345 \text{ mm/s}}$$

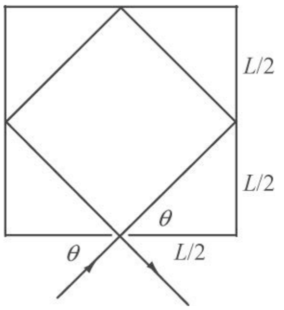
(c), (d) As the Sun moves southward and upward at 50.0°, we may regard the corner of the window as fixed, and both patches of light move $\boxed{\text{northward and downward at } 50.0°}$.

P35.80 Because the enclosure is square and the beam enters at bottom center, and because a light beam travels the same path regardless of its direction on the path, we expect the beam pattern to be symmetric about a vertical line passing through the opening. Therefore, the beam enters the opening at the same angle it exits, the beam strikes each side mirror at the same height, and the beam forms a zigzag pattern that intersects itself at a point (or points) above the center opening; thus, the beam must reflect off the top mirror at its center. Also, because of the law of reflection, the path of the beam is symmetric about a horizontal line passing through the points where the beam reflects off a side mirror.

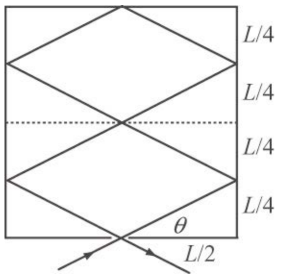
(a) Call the length of each side of the square L. If the beam is to strike each mirror once, the beam must strike each side mirror at its center, at height $L/2$ after traveling a horizontal distance $L/2$. Therefore,

$$\tan\theta = \frac{L/2}{L/2} = 1 \;\rightarrow\; \theta = 45.0^\circ$$

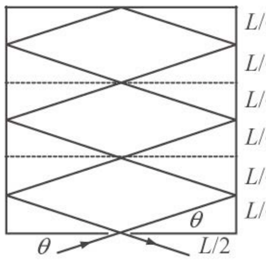
The beam will exit the enclosure if it enters at angle $\boxed{45.0^\circ}$, as shown in ANS. FIG. P35.80(a).



ANS. FIG. P35.80(a)



ANS. FIG. P35.80(b)



ANS. FIG. P35.80(c)

(b) Because the path of the beam is symmetric about a horizontal lines passing through the points where the beam reflects off a side mirror, we can divide the square enclosure into vertically stacked rectangular areas, each a mirror image of the one below. In each, the ray passes upward through the bottom center of the rectangle and exits at its top center until it reflects off the top mirror, then the ray passes back downward through each center until it exits the enclosure. The pattern of the ray's path is repeated in each rectangle. If the enclosure is divided into n rectangles, the height of each rectangle is L/n, and the beam strikes a side mirror at height $L/2n$ within each rectangle. Therefore, the angle of entry at the opening is

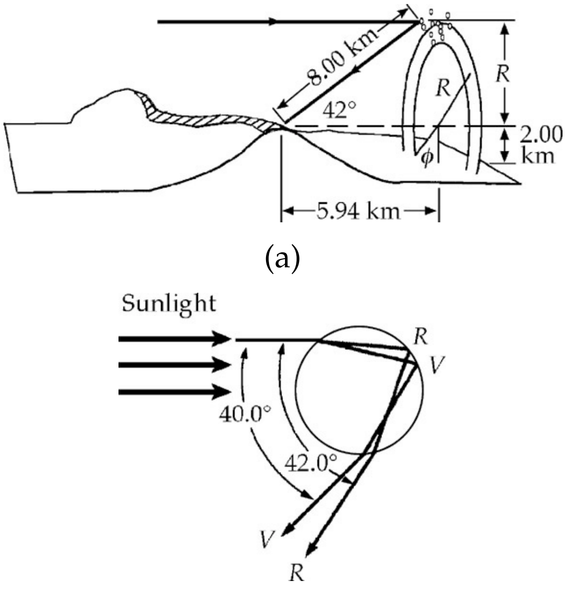
$$\tan\theta = \frac{L/2n}{L/2} = \frac{1}{n}$$

The cases for $n = 2$ and 3 are shown in ANS. FIG. 35.80(b) and (c) above.

$$\boxed{\text{Yes. The ray will exit if it enters at an angle } \theta \text{ that satisfies the condition } \tan\theta = \frac{1}{n}, \text{ where } n = 1,\ 2,\ 3,\ \ldots}$$

Challenge Problems

P35.81 Horizontal light rays from the setting Sun pass above the hiker. The light rays are twice refracted and once reflected, as in ANS. FIG. P35.81(b). The most intense light reaching the hiker, that which represents the visible rainbow, is located between angles of 40° and 42° from the hiker's shadow.



ANS. FIG. P35.81

The hiker sees a greater percentage of the violet inner edge, so we consider the red outer edge. The radius R of the circle of droplets is

$$R = (8.00\text{ km})\sin 42.0° = 5.35\text{ km}$$

Then the angle ϕ, between the vertical and the radius where the bow touches the ground, is given by

$$\cos\phi = \frac{2.00\text{ km}}{R} = \frac{2.00\text{ km}}{5.35\text{ km}} = 0.374$$

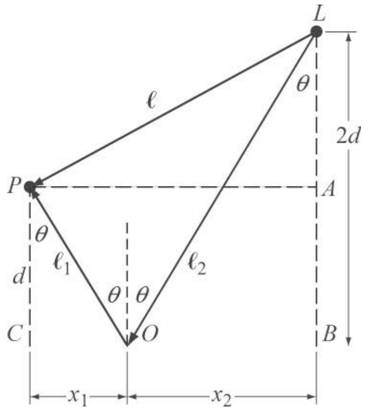
or $\phi = 68.1°$.

The angle filled by the visible bow is

$$360° - (2 \times 68.1°) = 224°$$

so the visible bow is $\frac{224°}{360°} = \boxed{62.2\% \text{ of a circle}}$.

P35.82 The geometry of the situation is shown in ANS. FIG. P35.82, where *P* is the person and *L* is the lightbulb.



ANS. FIG. P35.82

We have used the law of reflection to claim that the angles on either side of the dashed line at *O* are equal. From triangle *OPC*, we see that

$$\cos\theta = \frac{d}{\ell_1} \quad \text{and} \quad \sin\theta = \frac{x_1}{\ell_1}$$

which can be rearranged to give

$$\ell_1 = \frac{d}{\cos\theta} \quad \text{and} \quad x_1 = \ell_1 \sin\theta \qquad [1]$$

Similarly, from triangle *OLB*,

$$\cos\theta = \frac{2d}{\ell_2} \quad \text{and} \quad \sin\theta = \frac{x_2}{\ell_2}$$

which can be rearranged to give

$$\ell_2 = \frac{2d}{\cos\theta} \quad \text{and} \quad x_2 = \ell_2 \sin\theta \qquad [2]$$

Let $n = 3.10$ from the problem statement. The condition given in the problem is expressed as

$$\ell_1 + \ell_2 = n\ell \qquad [3]$$

Substitute for ℓ_1 and ℓ_2 from equations [1] and [2]:

$$\frac{d}{\cos\theta} + \frac{2d}{\cos\theta} = n\ell \rightarrow \frac{3d}{\cos\theta} = n\ell \qquad [4]$$

From triangle *APL*, apply the Pythagorean theorem:

$$\ell^2 = d^2 + (x_1 + x_2)^2$$

Substitute for x_1 and x_2 from equations [1] and [2]:

$$\ell^2 = d^2 + (\ell_1 \sin\theta + \ell_2 \sin\theta)^2 = d^2 + (\ell_1 + \ell_2)^2 \sin^2\theta$$

Substitute from equation [3]:

$$\ell^2 = d^2 + n^2\ell^2\sin^2\theta \rightarrow \ell^2\left(1 - n^2\sin^2\theta\right) = d^2 \qquad \textbf{[5]}$$

Eliminate ℓ between equations [4] and [5]:

$$\left(\frac{3d}{n\cos\theta}\right)^2\left(1 - n^2\sin^2\theta\right) = d^2 \rightarrow 9 - 9n^2\sin^2\theta = n^2\cos^2\theta$$

Simplify this expression:

$$9 = 9n^2\sin^2\theta + n^2\cos^2\theta = 8n^2\sin^2\theta + n^2\sin^2\theta + n^2\cos^2\theta$$

$$= 8n^2\sin^2\theta + n^2 \rightarrow \sin\theta = \sqrt{\frac{9 - n^2}{8n^2}}$$

If we now substitute $n = 3.10$, we see that there is no real solution for $\sin\theta$. Therefore, it is impossible for the distances to be in this relationship. The largest value that n can have is 3.00, which leads to an incident angle of 0°.

In fact, we could have solved this problem more elegantly (and quickly!) by realizing that the largest ratio of distances would be obtained by bringing the person and the lightbulb as close together as possible given the condition on their distances from the mirror. This would be done by aligning them both above O in the figure so that the light strikes the mirror at normal incidence. Then, the person and lightbulb are separated by a distance d, and the light travels a distance $3d$. This gives a maximum ratio of 3.00 and we see that a ratio of 3.10 is impossible.

P35.83 (a) Calling the angle between the dashed line in Figure P35.83 and the reflected laser beam θ, we see that

$$\tan\theta = \frac{x}{L/2} = \frac{2x}{L} \rightarrow x = \frac{1}{2}L\tan\theta$$

Differentiate with respect to time to find the speed of the laser spot on the wall:

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(\frac{1}{2}L\tan\theta\right) = \frac{1}{2}L\sec^2\theta\frac{d\theta}{dt} \qquad \textbf{[1]}$$

From Figure P35.83, we see that

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{4x^2 + L^2}}{L} \qquad \textbf{[2]}$$

Because the incident ray is stationary, as the mirror turns through angle ϕ, its normal rotates through angle ϕ, so the angle of incidence increases by ϕ as does the angle of reflection. Therefore, the reflected ray rotates through 2ϕ. As a consequence, the angular speed of the reflected ray is twice that of the mirror:

$$\omega_{\text{reflected ray}} = \frac{d\theta}{dt} = 2\omega \qquad \textbf{[3]}$$

Substitute equations [2] and [3] into equation [1]:

$$v = \frac{1}{2}L\left(\frac{4x^2 + L^2}{L^2}\right)2\omega = \boxed{\left(\frac{4x^2 + L^2}{L}\right)\omega}$$

(b) The variable in this expression is x, so we can minimize the speed by setting $x = \boxed{0}$.

(c) Let $x = 0$ in the expression for v:

$$v = \left(\frac{4(0)^2 + L^2}{L}\right)\omega = \boxed{L\omega}$$

(d) The maximum speed occurs when the reflected laser beam arrives at a corner of the room, where $x = L/2$:

$$v = \left(\frac{4(L/2)^2 + L^2}{L}\right)\omega = \boxed{2L\omega}$$

(e) Between the minimum and maximum speed, the reflected laser beam rotates through $\pi/4$ radians, so the mirror rotates through $\pi/8$ radians. Therefore,

$$\Delta t = \frac{\Delta\theta}{\omega} = \boxed{\frac{\pi}{8\omega}}$$

P35.84 (a) In the textbook Figure P35.84, we have $r_1 = \sqrt{a^2 + x^2}$ and $r_2 = \sqrt{b^2 + (d - x)^2}$. The speeds in the two media are $v_1 = c/n_1$ and $v_2 = c/n_2$ so the travel time for the light from P to Q is indeed

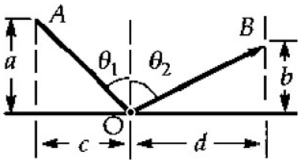
$$\Delta t = \frac{r_1}{v_1} + \frac{r_2}{v_2} = \frac{n_1\sqrt{a^2 + x^2}}{c} + \frac{n_2\sqrt{b^2 + (d - x)^2}}{c}$$

(b) Now $\dfrac{d(\Delta t)}{dx} = \dfrac{n_1}{2c}\dfrac{2x}{\sqrt{a^2 + x^2}} + \dfrac{n_2}{2c}\dfrac{2(d - x)(-1)}{\sqrt{b^2 + (d - x)^2}} = 0$ is the requirement for minimal travel time, which simplifies to

$$\frac{n_1 x}{\sqrt{a^2 + x^2}} = \frac{n_2(d - x)}{\sqrt{b^2 + (d - x)^2}}$$

(c) Now $\sin\theta_1 = \dfrac{x}{\sqrt{a^2+x^2}}$ and $\sin\theta_2 = \dfrac{d-x}{\sqrt{b^2+(d-x)^2}}$, so we have $n_1\sin\theta_1 = n_2\sin\theta_2$.

P35.85 In ANS. FIG. P35.85, a ray travels along path *AM* from point *A* to the mirror, reflects and travels along path *MB* from the mirror to point *B*. Point *A* is a vertical distance *a* above the mirror, and point *B* is a vertical distance *b* above the mirror. Points *A* and *B* are a horizontal distance *d* apart. The ray strikes the mirror at point *M* which is a horizontal distance *x* from point *A*. The angle of incidence is θ_1 and the angle of reflection is θ_2.



ANS. FIG. P35.85

We have $AM = \sqrt{a^2+x^2}$ and $MB = \sqrt{b^2+(d-x)^2}$. The travel time for the light from *A* to *B* is

$$\Delta t = \frac{AM}{c} + \frac{MB}{c} = \frac{\sqrt{a^2+x^2}}{c} + \frac{\sqrt{b^2+(d-x)^2}}{c}$$

We require a minimal travel time, so

$$\frac{d(\Delta t)}{dx} = \frac{1}{2c}\frac{2x}{\sqrt{a^2+x^2}} + \frac{1}{2c}\frac{2(d-x)(-1)}{\sqrt{b^2+(d-x)^2}} = 0$$

which simplifies to

$$\frac{x}{\sqrt{a^2+x^2}} = \frac{(d-x)}{\sqrt{b^2+(d-x)^2}}$$

This expression is equivalent to

$$\sin\theta_1 = \sin\theta_2 \;\rightarrow\; \theta_1 = \theta_2$$

P35.86 (a) Assume the viewer is far away to the right. In ANS. FIG. P35.86(a), a ray directed toward the viewer comes tangentially from the edge of the glowing sphere and emerges from the atmosphere at angle θ_2. The apparent radius of the glowing sphere is R_3 as shown. For the figure, we see that

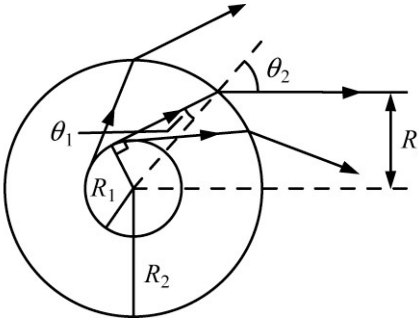
$$\sin\theta_1 = \frac{R_1}{R_2} \quad \text{and} \quad \sin\theta_2 = \frac{R_3}{R_2}$$

Then,

$$n\sin\theta_1 = 1.00\sin\theta_2$$

and

$$n\frac{R_1}{R_2} = \frac{R_3}{R_2} \quad \rightarrow \quad \boxed{R_3 = nR_1}$$



ANS. FIG. P35.86(a)

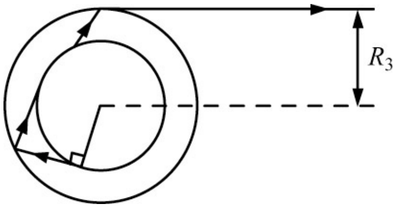
(b) If a ray is to come tangentially from the edge of the glowing sphere and emerge from the atmosphere, the incident angle θ_1 must be less than the critical angle, $\theta_1 < \theta_c$. Then,

$$\sin\theta_1 < \sin\theta_c = \frac{1}{n}$$

and

$$\frac{R_1}{R_2} < \frac{1}{n} \quad \rightarrow \quad nR_1 < R_2 \quad \rightarrow \quad R_2 > nR_1$$

This is not so for the case we consider here.



ANS. FIG. P35.86(b)

Thus, the ray considered in part (a) undergoes total internal reflection. In this case a ray traveling toward the viewer must emerge tangentially from the atmosphere, as shown in ANS. FIG. P35.86(b), so the apparent radius of the glowing sphere is the same as the radius of the atmosphere: $\boxed{R_3 = R_2}$.

***P35.87** Define $T = \dfrac{4n}{(n+1)^2}$ as the transmission coefficient for one encounter with an interface. For diamond and air, it is 0.828, as in Problem P35.58. As shown in ANS. FIG. P35.87, the total amount transmitted is

$$\begin{aligned}&T^2 + T^2(1-T)^2 + T^2(1-T)^4\\&+T^2(1-T)^6 + \ldots + T^2(1-T)^{2n} + \ldots\end{aligned}$$

We have $1-T = 1-0.828 = 0.172,$ so the total transmission is

$$(0.828)^2\left[1+(0.172)^2+(0.172)^4+(0.172)^6+\ldots\right]$$

To sum this series, define

$$F = 1+(0.172)^2+(0.172)^4+(0.172)^6+\ldots$$

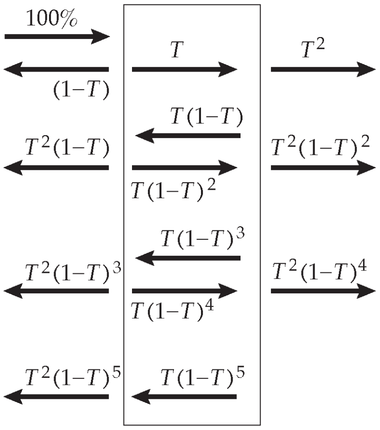
Note that $(0.172)^2 F = (0.172)^2+(0.172)^4+(0.172)^6+\ldots,$ and

$$1+(0.172)^2 F = 1+(0.172)^2+(0.172)^4+(0.172)^6+\ldots = F$$

Then,

$$1 = F-(0.172)^2 F \quad \text{or} \quad F = \frac{1}{1-(0.172)^2}.$$

The overall transmission is then $\dfrac{(0.828)^2}{1-(0.172)^2} = 0.706$ or $\boxed{70.6\%}$.



ANS. FIG. P35.87

ANSWERS TO EVEN-NUMBERED PROBLEMS

P35.2	(a) 3.00×10^8 m/s; (b) The sizes of the objects need to be taken into account. Otherwise the answer would be too large by 2%.
P35.4	2.27×10^8 m/s
P35.6	19.5° above the horizon
P35.8	(a) $\theta = 30.4°$; (b) $\theta' = 22.3°$
P35.10	(a) See P35.10(a) for full explanation; (b) Now $CBE = \phi$ is the angle of incidence of the vertical light beam. Its angle of reflection is also ϕ. The angle between the vertical incident beam and the reflected beam is 2ϕ; (c) $\phi = 0.055\,7°$
P35.12	$\theta_2 = 19.5°$; $\theta_3 = 19.5°$; $\theta_4 = 30.0°$
P35.14	(a) 78.3°; (b) 2.56 m; (c) 9.72°; (d) 442 nm; (e) The light wave slows down as it moves from air to water, but the sound wave speeds up by a larger factor. The light wave bends toward the normal and its wavelength shortens, but the sound wave bends away from the normal and its wavelength increases.
P35.16	(a) 1.52; (b) 417 nm; (c) 4.74×10^{14} Hz; (d) 198 Mm/s
P35.18	$\beta = 180° - 2\theta$
P35.20	(a) See P35.20(a) for full explanation; (b) See P35.20(b) for full explanation.
P35.22	(a) 0.387 cm; (b) 106 ps
P35.24	(a) Yes, if the angle of incidence is 58.9°; (b) No. Both the reduction in speed and the bending toward the normal reduce the component of velocity parallel to the interface. This component cannot remain constant for a nonzero angle of incidence.
P35.26	6.30 cm
P35.28	(a) See P35.28(a) for full explanation; (b) 37.2°; (c) 37.3°; (d) 37.3°
P35.30	The index of refraction of the atmosphere decreases with increasing altitude because of the decrease in density of the atmosphere with increasing altitude. As indicated in the ray diagram, the Sun located at S below the horizon appears to be located at S′.
P35.32	(a) $\dfrac{h}{d} = \sqrt{\dfrac{n^2-1}{4-n^2}}$; (b) 4.73 cm; (c) For $n = 1$, $h = 0$. For $n = 2$, $h = \infty$. For $n > 2$, h has no real solution.
P35.34	(a) See ANS. FIG. P35.34; (b) 42.0°; (c) 63.1°; (d) 26.9°; (e) 107 m

P35.36 (a) 48.2°; (b) 47.8°

P35.38 (a) See ANS. FIG. P35.38(a); (b) As the waves move to shallower water, the wave fronts slow down, and those closer to shore slow down more. The rays tend to bend toward the normal of the contour lines; or equivalently, the wave fronts bend to become more nearly parallel to the contour lines; (c) See ANS. FIG. P35.38(c); (d) We suppose that the headlands are steep underwater, as they are above water. The rays are everywhere perpendicular to the wave fronts of the incoming refracting waves. As shown, because the rays tend to bend toward the normal of the contour lines, the rays bend toward the headlands and deliver more energy per length at the headlands.

P35.40 $\sin^{-1}\left\{n_V \sin\left[\Phi - \sin^{-1}\left(\frac{\sin\theta}{n_V}\right)\right]\right\} - \sin^{-1}\left\{n_R \sin\left[\Phi - \sin^{-1}\left(\frac{\sin\theta}{n_R}\right)\right]\right\}$

P35.42 (a) 27.0°; (b) 37.1°; (c) 49.8°

P35.44 $\theta_1 > \sin^{-1}\left(n\sin\left[\Phi - \sin^{-1}\left(\frac{1.00}{n}\right)\right]\right)$; $\theta_1 > \sin^{-1}\left(\sqrt{n^2 - 1}\sin\Phi - \cos\Phi\right)$

P35.46 (a) 24.42°; (b) Because the angle of incidence (35.0°) is greater than the critical angle, the light is totally reflected at *P*; (c) 33.44°; (d) Yes. In this case, the angle of incidence is just larger than the critical angle, so the light ray again undergoes total internal reflection at *P*; (e) clockwise; (f) 2.83°

P35.48 (a) 10.7°; (b) air; (c) Sound in air falling on the wall from directions is 100% reflected.

P35.50 (a) See P35.50(a) for full explanation; (b) $n \geq 1.41$ and $n \leq 2.12$

P35.52 (a) angle of incidence: 30.0°, angle of refraction: 18.8°; (b) angle of incidence: 30.0°, angle of refraction: 50.8°; (c) and (d) See TABLE P35.52.

P35.54 No light from above the water will approach the scuba diver's eyes from 48.8° found in Example 35.6.

P35.56 Five times from the right-hand mirror and six times from the left.

P35.58 (a) $\frac{4n}{(n+1)^2}$; (b) 68.5%

P35.60 (a) $\frac{h}{c}\left(\frac{n+1.00}{2}\right)$; (b) $\left(\frac{n+1.00}{2}\right)$ times larger

P35.62 See P35.62 for full explanation.

P35.64 Consider an insulated box with the imagined one-way mirror forming one face, installed so that 90% of the electromagnetic radiation incident from the outside is transmitted to the inside and only a lower percentage of the electromagnetic waves from the inside make it through to the outside. Suppose the interior and exterior of the box are originally at the same temperature. Objects within and without are radiating and absorbing electromagnetic waves. They would all maintain constant temperature if the box had an open window. With the glass letting more energy in than out, the interior of the box will rise in temperature. But this is impossible, according to Clausius's statement of the second law. This reduction to a contradiction proves that it is impossible for the one-way mirror to exist.

P35.66 The beam will exit after making 81 reflections, so it does not make 85 reflections.

P35.68 (a) Total internal reflection occurs for all values of θ, or the maximum angle is 90°; (b) 30.3°; (c) Total internal reflection never occurs as the light moves from lower-index polystyrene to higher-index carbon disulfide.

P35.70 (a) The optical day is longer; (b) 164 s

P35.72 36.5°

P35.74 1.93

P35.76 (a) $n=\sqrt{1+\left(\dfrac{4t}{d}\right)^2}$; (b) 2.10 cm; (c) violet

P35.78 (a) See ANS. FIG. P35.78; (b) The straightness of the graph line demonstrates Snell's proportionality of the sine of the angle of refraction to the sine of the angle of incidence; (c) $1.328 \pm 0.8\%$

P35.80 (a) 45.0°; (b) Yes. The ray will exit if it enters at an angle θ that satisfies the condition $\tan\theta = \dfrac{1}{n}$, where $n = 1, 2, 3, \ldots$

P35.82 The person and lightbulb are separated by a distance d, and the light travels at a distance $3d$. This gives a maximum ratio of 3.00, and we see that a ratio of 3.10 is impossible.

P35.84 (a–c) See P35.84 for full explanations.

P35.86 (a) $R_3 = nR_1$; (b) $R_3 = R_2$

36

Image formation

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ36.1 Answer (b). A change in the medium in contact with the outer surface will result in a change in refraction at the outer surface if the surface is curved. Refraction should be limited to the inner surface because the medium inside (air) does not change. The outer surface should be flat so that it will not produce a fuzzy or distorted image for the diver when the mask is used either in air or in water.

OQ36.2 **(i)** Answer (c). The image is an upright and virtual at first then inverted and real. A concave (converging) mirror can produce real and virtual images depending on the object distance.

(ii) Answer (c). When the object passes through the focal point, the image switches from virtual to real.

OQ36.3 Answer (b). A converging lens forms real, inverted images of real objects located outside the focal point.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}: \qquad \frac{1}{50.0\,\text{cm}}+\frac{1}{q}=\frac{1}{15.0\,\text{cm}} \quad \rightarrow \quad q=21.4\,\text{cm}$$

$$M=\frac{-q}{p}=\frac{-21.4\text{ cm}}{50.0\text{ cm}}=-0.429$$

The positive image distance confirms that the image is real, and the negative magnification confirms that the image is inverted. Also, $M = -0.429$ tells us the image is smaller than the object.

OQ36.4 **(i)** Answer (e). A converging lens forms real, inverted images of real objects located farther than the focal length ($p > f$), and virtual, upright images of real objects located closer than the focal length ($p < f$).

(ii) Answers (a) and (c). A diverging lens forms a virtual, upright, and diminished image of any real object located any distance from the lens.

OQ36.5 Answer (d). The entire image is visible, but only at half the intensity. Each point on the object is a source of rays that travel in all directions. Thus, light from all parts of the object goes through all unblocked parts of the lens and forms an image. If you block part of the lens, you are blocking some of the rays, but the remaining ones still come from all parts of the object.

OQ36.6 Answer (d). The image is upright, so the magnification is positive:

$$M=\frac{-q}{p}: \qquad +1.50=\frac{-q}{30.0\text{ cm}} \quad \rightarrow \quad q=-45.0\text{ cm}$$

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}: \qquad \frac{1}{30.0\text{ cm}}+\frac{1}{-45.0\text{ cm}}=\frac{1}{f} \quad \rightarrow \quad f=90.0\text{ cm}$$

OQ36.7 Answer (b). For lens 1, the object distance $p_1 = 50.0$ cm:

$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f_1}: \qquad \frac{1}{50.0\text{ cm}}+\frac{1}{q_1}=\frac{1}{15.0\text{ cm}} \quad \rightarrow \quad q_1=21.4\text{ cm}$$

The image distance is positive, so the image is real and forms 21.4 cm to the right of lens 1.

The image of lens 1 is the object of lens 2. For lens 2, the object distance $p_2 = 35.0\text{ cm} - 21.4\text{ cm} = 13.6\text{ cm}$:

$$\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f_2}: \qquad \frac{1}{13.6\text{ cm}}+\frac{1}{q_2}=\frac{1}{10.0\text{ cm}} \quad \rightarrow \quad q_2=38.0\text{ cm}$$

The image distance is positive, so the image is real and forms 38.0 cm to the right of lens 2.

From Equation 36.18, the overall magnification is

$$M = M_1M_2 = \left(\frac{-q_1}{p_1}\right)\left(\frac{-q_2}{p_2}\right) = \left(\frac{-21.4\,\text{cm}}{50.0\,\text{cm}}\right)\left(\frac{-38.0\,\text{cm}}{13.6\,\text{cm}}\right) = 1.20$$

OQ36.8 Answer (c). The amount of light focused on the film by a camera is proportional to the area of the aperture through which the light enters the camera. Since the area of a circular opening varies as the square of the diameter of the opening, the light reaching the film is proportional to the square of the diameter of the aperture. Thus, increasing this diameter by a factor of 3 increases the amount of light by a factor of 9.

OQ36.9 Answer (b). The angle of refraction for the light coming from the fish to the person is 60°. The angle of incidence is smaller, so the fish is deeper than it appears. [Refer to CQ35.16.]

OQ36.10 The ranking is c > e > a > d > b. In case (c) the object distance is effectively infinite. In (e) the object distance is very large compared to the focal length, but not infinite. In (a) the object distance is a little larger than the focal length. In (d) the object distance is equal to the focal length. In (b) the object distance is less than the focal length.

OQ36.11 Answer (d). We can answer this question conceptually by noting that if the lens were surrounded by water, parallel light rays passing into and out of the lens would experience smaller changes in the index of refraction, so they would bend less, and so would focus farther from the lens.

We can answer this question quantitatively if we consider the derivation of the lens makers' equation (Equation 36.15) for the general case of the lens being surrounded by a medium of index n_0. We would conclude that Equation 36.15 takes the general form

$$\frac{1}{f} = \left(\frac{n}{n_0} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

So, for a lens of crown glass ($n = 1.52$, from Table 35.1) surrounded by air, $n_0 = 1$, we have

$$\frac{1}{f} = (1.52 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{15.0\text{ cm}}$$

but for a lens surrounded by water, $n_0 = 1.333$, and we have

$$\frac{1}{f} = \left(\frac{1.52}{1.333} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$= \frac{\left(\frac{1.52}{1.333} - 1\right)}{(1.52-1)}\left[(1.52-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\right] = \frac{\left(\frac{1.52}{1.333} - 1\right)}{(1.52-1)}\frac{1}{15.0\text{ cm}}$$

$$f = 55.6\text{ cm}$$

OQ36.12 Answer (e). At the smallest distance the object and image distances are equal, $p = q$:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \quad \frac{1}{p} + \frac{1}{p} = \frac{1}{f}$$

$$\frac{2}{p} = \frac{1}{f} \quad \rightarrow \quad p = 2f = q$$

OQ36.13 **(i)** Answers (a) and (c). The image of a real object formed by a plane mirror is always an upright and virtual image, which is the same size as the object and located as far behind the mirror as the object is in front of the mirror.

(ii) Answer (e). A concave (converging) mirror forms real, inverted images of real objects located outside the focal point ($p > f$), and virtual, upright images of real objects located inside the focal point ($p < f$) of the mirror.

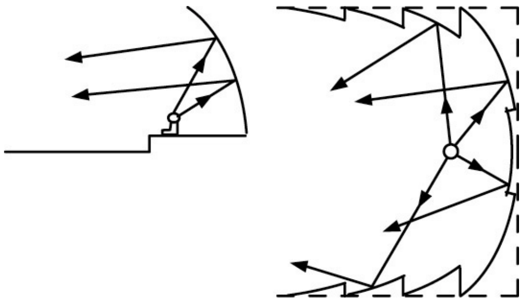
(iii) Answer (a) and (c). With a real object in front of a convex (diverging) mirror, the image is always virtual, upright, and diminished in size, and located between the mirror and the focal point.

OQ36.14 Answer (b). The image is upright, and corresponding parts of the object and image are the same distance from the mirror.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ36.1 (a) Yes.

(b) You have likely seen a Fresnel mirror for sound. The diagram represents first a side view of a band shell. It is a concave mirror for sound, designed to channel sound into a beam toward the audience in front of the band shell. Sections of its surface can be kept at the right orientations as they are pushed around inside a rectangular box to form an auditorium with good diffusion of sound from stage to audience, with a floor plan suggested by the second part of the diagram.



ANS. FIG. CQ36.1

CQ36.2 (a) The focal point is defined as the location of the image formed by rays originally parallel to the axis. An object at a large but finite distance will radiate rays nearly but not exactly parallel. Infinite object distance describes the definite limiting case in which these rays become parallel.

(b) To measure the focal length of a converging lens, set it up to form an image of the farthest object you can see outside a window. The image distance will be equal to the focal length within one percent or better if the object distance is a hundred times larger or more.

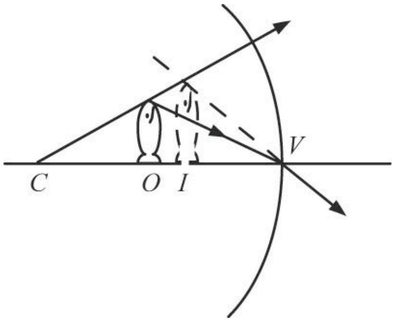
CQ36.3 Because when you look at the ƎƆИA⅃UᙠMA in your rear view mirror, the apparent left-right inversion clearly displays the name of the AMBULANCE behind you. Do not jam on your brakes when a MIAMI city bus is right behind you.

CQ36.4 Chromatic aberration arises because a material medium’s refractive index can be wavelength dependent. A mirror changes the direction of light by reflection, not refraction. Light of all wavelengths follows the same path according to the law of reflection, so no chromatic aberration happens.

CQ36.5 (a) Yes. If the converging lens is immersed in a liquid with an index of refraction significantly greater than that of the lens itself, it will make light from a distant source diverge.

(b) No. This is not the case with a converging (concave) mirror, as the law of reflection has nothing to do with the indices of refraction.

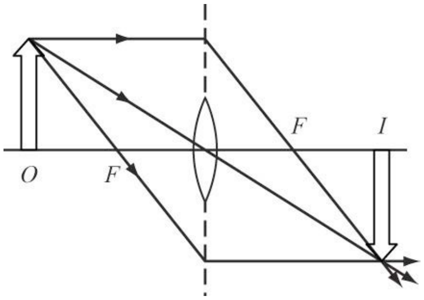
CQ36.6 As in the diagram, let the center of curvature *C* of the fishbowl and the bottom of the fish define the optical axis, intersecting the fishbowl at vertex *V*. A ray from the top of the fish that reaches the bowl surface along a radial line through *C* has angle of incidence zero and angle of refraction zero. This ray exits from the bowl unchanged in direction. A ray from the top of the fish to *V* is refracted to bend away from the normal. Its extension back inside the fishbowl determines the location of the image and the characteristics of the image. The image is upright, virtual, and enlarged.



ANS. FIG. CQ36.6

CQ36.7 (a) An infinite number. In general, an infinite number of rays leave each point of any object and travel in all directions. Note that the three principal rays that we use for imaging are just a subset of the infinite number of rays.

(b) All three principal rays can be drawn in a ray diagram, provided that we extend the plane of the lens as shown in Figure CQ36.7.



ANS. FIG. CQ36.7

CQ36.8 With the meniscus design, when you direct your gaze near the outer circumference of the lens you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Thus, the lens minimally distorts the direction to the object you are looking at. If you wear glasses, turn them around and look through them the wrong way to maximize this distortion.

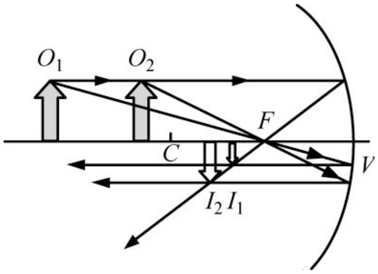
CQ36.9 Note that an object at infinity has an image at the focal point of a converging lens, and an object at the focal point of a converging lens has its image at infinity, so we may conclude that the farther an object is from a lens, the closer the image is to the focal point of the lens. Therefore, we expect the image of the farther tree to form closer to the lens, so we conclude that the screen should be moved toward the lens.

We can verify our conclusion using the lens equation:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad\rightarrow\quad \frac{1}{q}=\frac{1}{f}-\frac{1}{p}=\frac{p-f}{fp}=\frac{1-f/p}{f} \quad\rightarrow\quad q=\frac{f}{1-f/p}$$

For $p = x$, $q = \dfrac{f}{1-f/x}$, and for $p' = 2x$, $q' = \dfrac{f}{1-f/2x} < q$, so our conclusion is correct.

CQ36.10 In the diagram, only two of the three principal rays have been used to locate images to reduce the amount of visual clutter. The upright shaded arrows are the objects, and the correspondingly numbered inverted arrows are the images. As you can see, object 2 is closer to the focal point than object 1, and image 2 is farther to the left than image 1.



ANS. FIG. CQ36.10

CQ36.11 The eyeglasses on the left are diverging lenses that correct for nearsightedness. If you look carefully at the edge of the person's face through the lens, you will see that everything viewed through these glasses is reduced in size. The eyeglasses on the right are converging lenses, which correct for farsightedness. These lenses make everything that is viewed through them look larger.

CQ36.12 The eyeglass wearer's eye is at an object distance from the lens that is quite small—the eye is on the order of 10^{-2} meter from the lens. The focal length of an eyeglass lens is several decimeters, positive or negative. Therefore the image distance will be similar in magnitude to the object distance. The onlooker sees a sharp image of the eye behind the lens. Look closely at Figure CQ36.11a and notice that the wearer's eyes seem not only to be smaller, but also positioned a bit behind the plane of his face—namely, behind where they would be if he were not wearing glasses. Similarly, in Figure CQ36.11b, his eyes seem to be magnified and in front of the plane of his face. We as observers take the light information coming from the object through the lens and perceive or photograph the image as if it were an object.

CQ36.13 Absolutely. Only absorbed light, not transmitted light, contributes internal energy to a transparent object. A clear lens can stay ice-cold and solid as megajoules of light energy pass through it.

CQ36.14 Make the mirror an efficient reflector (shiny). Make it reflect to the image even rays far from the axis, by giving it a parabolic shape. Most important, make it large in diameter to intercept a lot of solar power. And you get higher temperature if the image is smaller, as you get with shorter focal length; and if the furnace enclosure is an efficient absorber (black).

CQ36.15 The artist's statements are accurate, perceptive, and eloquent. The image you see is "almost one's whole surroundings," including things behind you and things farther in front of you than the globe is, but nothing eclipsed by the opaque globe or by your head. For example, we cannot see Escher's index and middle fingers or their reflections in the globe.

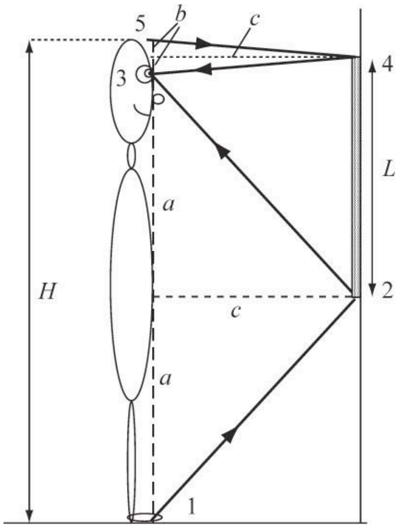
The point halfway between your eyes is indeed the focus in a figurative sense, but it is not an optical focus. The principal axis will always lie in a line that runs through the center of the sphere and the bridge of your nose (between your eyes). Outside the globe, you are at the center of your observable universe. If you close one eye, the center of the looking-glass world may hop over to the location of the image of your open eye (depending on which eye is dominant).

CQ36.16 Both words are inverted, but the word OXIDE looks the same when inverted.

CQ36.17 Yes, the mirror equation and the magnification equation apply to plane mirrors. A curved mirror is made flat by increasing its radius of curvature without bound, so that its focal length goes to infinity. From $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=0$ we have $\frac{1}{p}=-\frac{1}{q}$; therefore, $p=-q$. The virtual image is as far behind the mirror as the object is in front. The magnification is $M=-\frac{q}{p}=\frac{p}{p}=1$. The image is right side up and actual size.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 36.1 Images Formed by Flat Mirrors

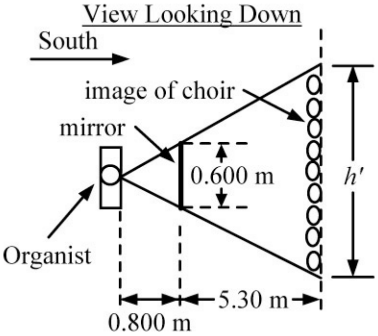


ANS. FIG. P36.1

P36.1 ANS. FIG. P36.1 shows the path of rays reflected by a mirror of minimum height: rays from the person's feet and top of his head travel along the respective paths 123 and 543 to his eyes. The rays reflect at the bottom and top of the mirror. Because of the law of reflection, the paths can be considered to form the hypotenuses of two pairs of right triangles with common base c: two large similar right triangles with height a, and two small similar right triangles with height b.

Rays from his feet enter his eyes a vertical distance $2a$ from the ground. The rays from the top of his head enter his eyes a distance $2b$ from the top of his head. His full height is $H = 2a + 2b$. The mirror has height $L = a + b$. We see then that

$$L = a + b = \frac{H}{2} = \frac{178\text{ cm}}{2} = \boxed{89\text{ cm}}$$



ANS. FIG. P36.2

P36.2 The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror. The image of the choir is 0.800 m + 5.30 m = 6.10 m from the organist. Using similar triangles:

$$\frac{h'}{0.600\text{ m}} = \frac{6.10\text{ m}}{0.800\text{ m}}$$

or $$h' = (0.600\text{ m})\left(\frac{6.10\text{ m}}{0.800\text{ m}}\right) = \boxed{4.58\text{ m}}$$

P36.3 (a) $\boxed{\text{Younger}}$. Light takes a finite time to travel from an object to the mirror and then to the eye.

(b) I stand about 40 cm from my bathroom mirror. I scatter light, which travels to the mirror and back to me in time interval

$$\Delta t = \frac{2d}{c} = \frac{0.8\text{ m}}{3\times10^{8}\text{ m/s}} \sim \boxed{10^{-9}\text{ s}},$$ showing me a view of myself as I was then.

P36.4 The mirrors are 6.00 m apart.

(1) The first image in the left mirror is 2.00 m behind the mirror, or 2.00 m + 2.00 m = $\boxed{4.00\text{ m}}$ from the position of the person.

(2) The first image in the right mirror is located 4.00 m behind the right mirror, but this location is 4.00 m + 6.00 m = 10.0 ft from the left mirror. Thus, the second image in the left mirror is 10.00 m behind the mirror, or 10.00 m + 2.00 m = $\boxed{12.00\text{ m}}$ from the person.

(3) The first image in the left mirror forms an image in the right mirror. This first image is 2.00 m + 6.00 m = 8.00 m from the right mirror, and, thus, an image 8.00 m behind the right mirror is formed. This image in the right mirror also forms an image in the left mirror. The distance from this image in the right mirror to the left mirror is 8.00 m + 6.00 m = 14.00 m. The third image in the left mirror is, thus, 14.00 m behind the mirror, or 14.00 m + 2.00 m = $\boxed{16.00\text{ m}}$ from the person.

P36.5 For a plane mirror, $q = -p$. Recall from common experience that the position of an image does not shift as a viewer rotates. Thus, to a viewer looking toward a mirror that is turned by 45°, the image distance still follows this rule.

(a) The upper mirror M_1 produces a virtual, actual-sized image I_1 according to

$$M_1 = -\frac{q_1}{p_1} = +1$$

As shown in ANS. FIG. P36.5, this image is a distance p_1 above the upper mirror. It is the object for mirror M_2, at object distance

$$p_2 = p_1 + h$$

The lower mirror produces a virtual, actual-sized, right-side-up image according to

$$q_2 = -p_2 = -(p_1 + h)$$

with

$$M_2 = -\frac{q_2}{p_2} = +1 \quad \text{and} \quad M_{\text{overall}} = M_1 M_2 = 1.$$

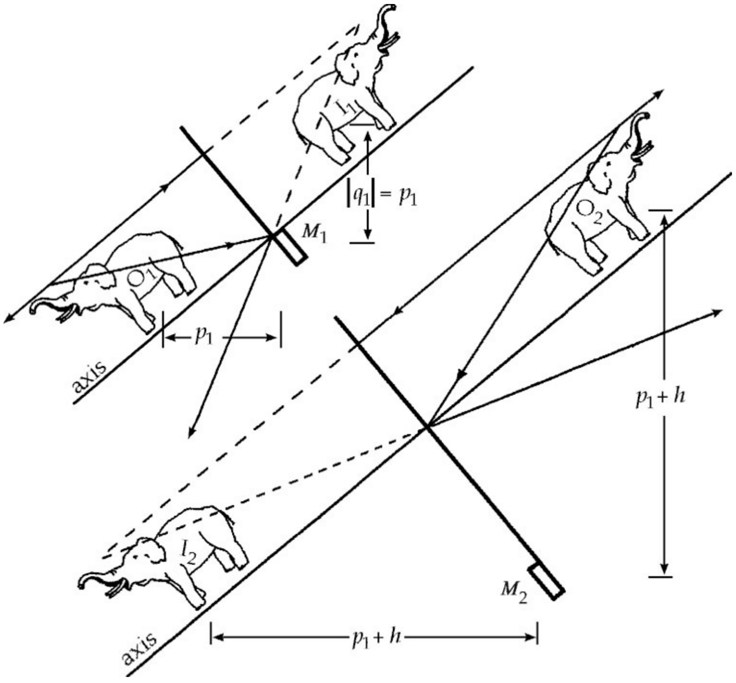
Thus the final image is at distance $\boxed{p_1 + h\text{, behind the lower mirror}}$.

(b) It is $\boxed{\text{virtual}}$.

(c) $\boxed{\text{Upright}}$

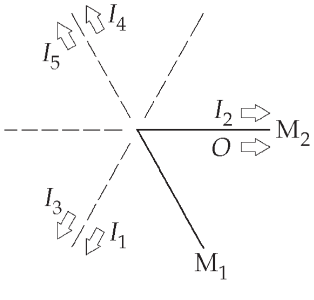
(d) With magnification $\boxed{+1.00}$.

(e) $\boxed{\text{No}}$. Left and right are not reversed. In a top view of the periscope, parallel rays from the right and left sides of the object stay parallel and on the right and left. The first mirror switches left and right, but the second mirror switches them again; so, overall left and right are not reversed.



ANS. FIG. P36.5

***P36.6** A graphical construction, shown in ANS. FIG. P36.6, produces 5 images, with images I_1 and I_2 directly into the mirrors from the object *O*, and (O, I_3, I_4) and (I_2, I_1, I_5) forming the vertices of equilateral triangles.



ANS. FIG. P36.6

P36.7 We assume that she looks only at images in the nearest mirror. The mirrors are 3.00 m apart.

(a) With her palm located 1.00 m in front of the nearest mirror, that she sees its image $\boxed{\text{1.00 m behind the nearest mirror}}$.

(b) The nearest mirror shows $\boxed{\text{the palm}}$ of her hand.

(c) Her hand is 2.00 m from the farthest mirror, so its image forms 2.00 m behind the farthest mirror, but this image is 2.00 m + 3.00 m = 5.00 m from the nearest mirror, so the image she sees is $\boxed{\text{5.00 m behind the nearest mirror}}$.

(d) The image is that of $\boxed{\text{the back of her hand}}$ reflected in the farthest mirror.

(e) The farthest mirror forms an image of the first image of part (a), which is 1.00 m + 3.00 m = 4.00 m from the farthest mirror; this image is then 4.00 m behind the farthest mirror, so it is 4.00 m + 3.00 m = 7.00 m in front of the nearest mirror, so the image she sees is $\boxed{\text{7.00 m behind the nearest mirror}}$.

(f) This is the image of $\boxed{\text{the palm}}$ reflected back from the nearest to the farthest and back to the nearest mirror.

(g) Since all images are located behind the mirror, and all images result from light reflected in a mirror, $\boxed{\text{all are virtual images}}$.

Section 36.2 Images Formed by Spherical Mirrors

P36.8 (a) A concave mirror is a converging mirror, so the focal length $f = +20.0$ cm. Then, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$ gives

$$\frac{1}{50.0\text{ cm}} + \frac{1}{q} = \frac{1}{20.0\text{ cm}} \quad \rightarrow \quad q = +33.3\text{ cm}$$

Since $q > 0$, the image is located $\boxed{\text{33.3 cm in front of the mirror}}$.

(b) $$M = -\frac{q}{p} = -\frac{(33.3\text{ cm})}{50.0\text{ cm}} = \boxed{-0.666}$$

(c) The image distance is positive, so the image is $\boxed{\text{real}}$.

(d) The magnification is negative, so the image is $\boxed{\text{inverted}}$.

P36.9 We apply the mirror equation using the sign conventions listed in the textbook chapter.

(i) The mirror equation gives

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{40.0\text{ cm}} \quad \rightarrow \quad q = 13.3\text{ cm}$$

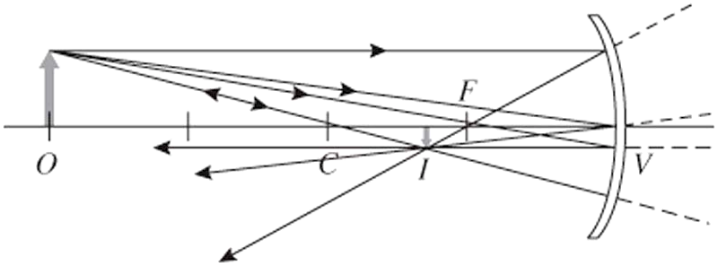
and $$M = \frac{q}{p} = -\frac{13.3\text{ cm}}{40.0\text{ cm}} = -0.333$$

(a) The image is $\boxed{\text{13.3 cm}}$ in front of the mirror.

(b) The image distance is positive, so the image is $\boxed{\text{real}}$.

(c) The magnification is negative, so the image is $\boxed{\text{inverted}}$.

(d) From above, $M = \boxed{-0.333}$. The value of *M* indicates that the image is inverted and one-third the height of the object.



ANS. FIG. P36.9(i)

The ray diagram traced in ANS. FIG. P36.9(i) shows this identification more clearly, and that the image is inverted.

(ii) Again, from the mirror equation,

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{10.0\text{ cm}} - \frac{1}{20.0\text{ cm}} \quad \rightarrow \quad q = 20.0\text{ cm}$$

and

$$M = \frac{q}{p} = -\frac{20.0\text{ cm}}{20.0\text{ cm}} = -1.00$$

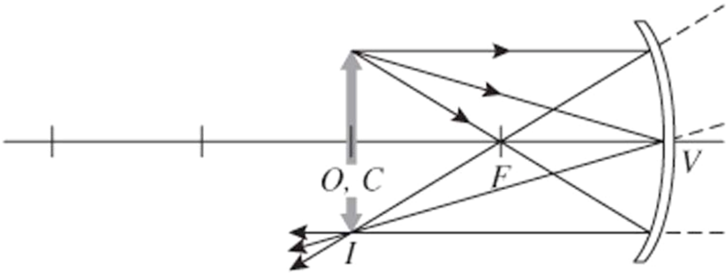
The ray diagram for this case is shown in ANS. FIG. P36.9(ii).

(a) The image is $\boxed{20.0\text{ cm}}$ in front of the mirror.

(b) The image distance is positive, so the image is $\boxed{\text{real}}$.

(c) The magnification is negative, so the image is $\boxed{\text{inverted}}$.

(d) From above, $M = \boxed{-1.00}$. The value of *M* indicates that the image is inverted and the same height as the object in this special case.



ANS. FIG. P36.9(ii)

(iii) (a) The object is now at the focal point of the mirror. Following the same steps gives

$$q = \frac{1}{2/R - 1/p} = \frac{1}{2/(20.0\text{ cm}) - 1/(10.0\text{ cm})} = \frac{1}{0} = \infty$$

We can say that no image is formed, or that the image is at an infinite distance. The ray diagram for this case is shown in ANS. FIG. P36.9(iii).

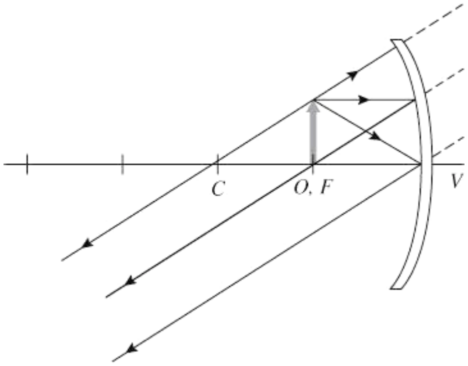
(b) In this special case the reflected rays do not intersect. We cannot classify the image as real or virtual as $\boxed{\text{no image is formed}}$

(c) We cannot classify the image as upright or inverted as $\boxed{\text{no image is formed}}$.

A screen placed at a large distance in front of the mirror can intercept the reflected light energy, showing the appearance of an upside-down real image, but it is not sharp for any finite distance. You can look into the mirror to view the image as a right side up virtual image, with your eye focused on infinity.

(d) The magnification is $M = -\frac{q}{p} = -\frac{\infty}{20.0\text{ cm}} = \infty$.

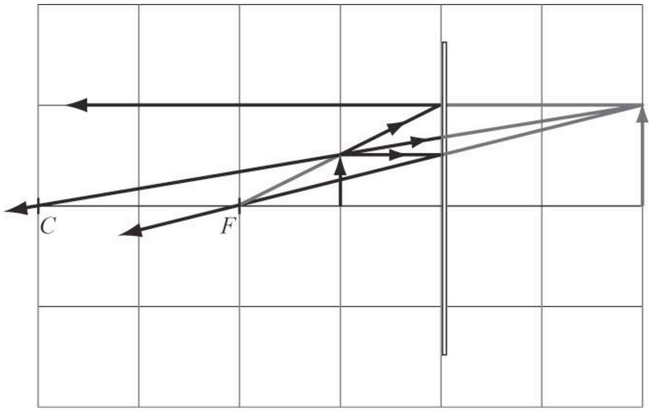
In this special case, if we say $\boxed{\text{no image is formed}}$ at a finite distance, it has no finite magnification. If we say the image is at infinity, then its height and its magnification are also infinite. There is no physical difference between $+\infty$ and $-\infty$.



ANS. FIG. P36.9(iii)

P36.10 (a) To approximate paraxial rays, the rays should be drawn so that they reflect at the vertical plane that passes through the vertex of the mirror, rather than at the mirror's surface, as done in the textbook. For this reason, the concave surface of the mirror appears flat in ANS. FIG. P36.10.

(b) $\boxed{q = -40.0\text{ cm, so the image is behind the mirror}}$.



ANS. FIG. P36.10

(c) $\boxed{M = +2.00\text{, so the image is enlarged and upright}}$.

(d) The mirror is concave (converging), so $f = +40.0$ cm.

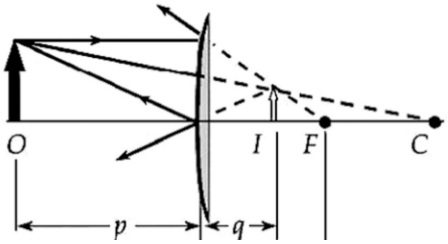
$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{40.0\text{ cm}} - \frac{1}{20.0\text{ m}} \quad \rightarrow \quad q = -40.0\text{ cm}$$

and $M = \dfrac{-q}{p} = \dfrac{-(-40.0\text{ cm})}{20.0\text{ cm}} = +2.00$

P36.11 The convex mirror is described by

$$f = \frac{R}{2} = \frac{-40.0\text{ cm}}{2} = -20.0\text{ cm}$$

ANS. FIG. P36.11 shows the ray diagram for this situation.



ANS. FIG. P36.11

(a) Then $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f}$ gives

$$q = \frac{1}{1/f - 1/p} = \frac{1}{1/(-20.0\text{ cm}) - 1/(30.0\text{ cm})} = \boxed{-12.0\text{ cm}}$$

The magnification factor is

$$M = -\frac{q}{p} = -\left(\frac{-12.0\text{ cm}}{30.0\text{ cm}}\right) = \boxed{+0.400}$$

The image is behind the mirror, upright, virtual, and diminished.

(b) Following the same steps,

$$q = \frac{1}{1/f - 1/p} = \frac{1}{1/(-20.0\text{ cm}) - 1/(60.0\text{ cm})} = \boxed{-15.0\text{ cm}}$$

and $M = \frac{-q}{p} = -\left(\frac{-15.0\text{ cm}}{60.0\text{ cm}}\right) = \boxed{+0.250}$.

(c) Since $M > 0$, the images are $\boxed{\text{both upright}}$.

P36.12 (a) The mirror is convex (diverging), so

$$f = -\frac{R}{2} = -\frac{0.550\text{ m}}{2} = -0.275\text{ m}$$

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{-0.275\text{ m}} - \frac{1}{10.0\text{ m}}$$

gives $q = -0.267\text{ m} = \boxed{-26.7\text{ cm}}$.

The image distance is negative; thus, the image is virtual. The image is 26.7 cm behind the mirror.

(b) $M = \frac{-q}{p} = -\frac{-0.267}{10.0\text{ m}} = +0.026\,7$

The magnification is positive, so the image is $\boxed{\text{upright}}$.

(c) From above, $M = \boxed{0.026\,7}$.

P36.13 (a) The mirror is convex (diverging), so $f = -10.0$ cm.

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{-10.0\text{ cm}} - \frac{1}{30.0\text{ m}} \quad \rightarrow \quad q = \boxed{-7.50\text{ cm}}$$

The image distance is negative; thus, the image is virtual. The image is 7.50 cm behind the mirror.

(b) From $M = \frac{-q}{p} = -\frac{-7.50}{30.0\text{ cm}} = +0.250$, we see that the magnification is positive, so the image is $\boxed{\text{upright}}$.

(c) $M = \frac{h'}{h} \quad \rightarrow \quad h' = Mh = +0.250(2.00\text{ cm}) = \boxed{0.500\text{ cm}}$

P36.14 (a) Since the object is in front of the mirror, $p > 0$, and $p = 1.00$ cm. With the image behind the mirror, the image is virtual, so $q < 0$, and $q = -10.0$ cm. The mirror equation gives for the radius of curvature

$$\frac{1}{f} = \frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.00\text{ cm}} + \frac{1}{-10.0\text{ cm}} \quad \rightarrow \quad f = \frac{R}{2} = 1.11\text{ cm}$$

$$R = \boxed{+2.22\text{ cm}}$$

A positive radius means the mirror is converging, so it is a concave mirror.

(b) The magnification is $M = -\frac{q}{p} = -\frac{(-10.0\text{ cm})}{1.00\text{ cm}} = \boxed{+10.0}$.

P36.15 The niche acts as a cylindrical mirror that reflects sound. This is a mirror with a vertical axis and a radius $R = 2.50$ m: its focal length $f = \frac{R}{2} = 1.25\text{ m}$. To the extent that we can treat sound as being composed of "rays of sound," we can find the point of focus of sound waves by using the same method we use for rays of light.

In a vertical plane the sound disperses as usual, but that radiated in a horizontal plane is concentrated in a sound image at distance q from the back of the niche, where

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} \quad \rightarrow \quad \frac{1}{2.00\text{ m}} + \frac{1}{q} = \frac{1}{1.25\text{ m}}$$

$$q = \boxed{3.33\text{ m from the deepest point in the niche}}.$$

P36.16 A convex mirror *diverges* light rays incident upon it, so the mirror in this problem cannot focus the Sun's rays to a point.

P36.17 From the definition of magnification, $M = -\frac{q}{p}$, which gives

$$q = -Mp = -0.013\,0(30\text{ cm}) = -0.390\text{ cm}$$

Then, from the mirror-lens equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{2}{R}$$

$$\frac{1}{30.0\text{ cm}} + \frac{1}{-0.390\text{ cm}} = \frac{2}{R}$$

$$R = -0.790\text{ cm}$$

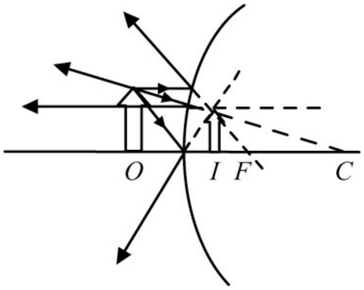
The cornea is convex, with radius of curvature $\boxed{0.790\text{ cm}}$.

P36.18 The ball is a convex mirror with a diameter of 8.50 cm:

$$R = -4.25\text{ cm} \quad \text{and} \quad f = \frac{R}{2} = -2.125\text{ cm}$$

(a) We have

$$M = \frac{3}{4} = -\frac{q}{p} \quad \rightarrow \quad q = -\frac{3}{4}p$$



ANS. FIG. P36.18

By the mirror equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-(3/4)p} = \frac{1}{-2.125\text{ cm}}$$

or $$\frac{3}{3p} - \frac{4}{3p} = \frac{1}{-2.125\text{ cm}} = \frac{-1}{3p} \quad \rightarrow \quad p = +0.708\text{ m}$$

The object is $\boxed{\text{0.708 m in front of the sphere}}$.

(b) From ANS. FIG. P36.18, the image is $\boxed{\text{upright}}$, virtual, and diminished.

P36.19 (a) The image is inverted and 4.00 times larger, so the magnification is

$$M = -4.00 = -\frac{q}{p} \quad \rightarrow \quad q = 4.00p$$

Thus the image is farther from the mirror than the object.

The object and images distances are related by

$$q - p = 0.600\text{ m} = 4.00p - p = 3.00p \quad \rightarrow \quad p = 0.200\text{ m},$$

and $q = 0.800$ m.

By the mirror equation,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.200\text{ m}} + \frac{1}{0.800\text{ m}} \quad \rightarrow \quad f = \boxed{0.160\text{ m}}$$

(b) A convex (diverging) mirror forms an upright, virtual image, so the magnification is

$$M = +0.500 = -\frac{q}{p} \quad \rightarrow \quad q = -0.500p$$

The image is virtual, so it is behind the mirror, and the image distance is negative. The object and images distances are related by

$$|q| + p = 0.600\text{ m} = -q + p = -(-0.500p) + p = 1.50p$$

$$p = 0.400\text{ m} \quad \to \quad q = -0.200\text{ m}$$

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f} = \frac{1}{0.400\text{ m}} + \frac{1}{-0.200\text{ m}} \quad \to \quad f = \boxed{-0.400\text{ m}}$$

P36.20 (a) The image is inverted, and a > 1 times larger, so the magnification is

$$M = -a = -\frac{q}{p} \quad \to \quad q = ap$$

Thus the image is farther from the mirror than the object.

The object and image distances are related by

$$q - p = d = ap - p = (a-1)p \quad \to \quad p = \frac{d}{a-1}, \qquad q = \frac{ad}{a-1}$$

By the mirror equation,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{a-1}{d} + \frac{a-1}{ad} = \frac{a(a-1)+(a-1)}{ad} = \frac{a^2-1}{ad}$$

$$f = \boxed{\frac{ad}{a^2-1}}$$

(b) The image is upright, and a < 1, so the magnification is:

$$M = a = -\frac{q}{p} \quad \to \quad q = -ap$$

The image is virtual, so it is behind the mirror, and the image distance is negative. The object and image distances are related by

$$|q| + p = d = -q + p = -(-ap) + p = (a+1)p$$

$$p = \frac{d}{1+a} \qquad \text{and} \qquad q = -ap = \frac{-ad}{1+a}$$

By the mirror equation,

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1+a}{d} + \frac{1+a}{-ad} = \frac{a(1+a)-(1+a)}{ad} = \frac{a^2-1}{ad}$$

$$f = \boxed{\frac{ad}{a^2-1}}$$

P36.21 From the magnification equation,

$$M = \frac{h'}{h} = \frac{+4.00\text{ cm}}{10.0\text{ cm}} = +0.400 = -\frac{q}{p}$$

which gives $q = -0.400p$, so the image must be virtual.

(a) It is a (diverging) $\boxed{\text{convex}}$ mirror that produces a diminished, upright virtual image.

(b) We must have

$$p + |q| = 42.0 \text{ cm} = p - q$$

$$p = 42.0 \text{ cm} + q$$

$$p = 42.0 \text{ cm} - 0.400p$$

$$p = \frac{42.0 \text{ cm}}{1.40} = 30.0 \text{ cm}$$

The mirror is $\boxed{\text{at the 30.0-cm mark}}$.

(c) $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{1}{30 \text{ cm}} + \dfrac{1}{-0.400(30 \text{ cm})} = \dfrac{1}{f} = -0.050\,0/\text{cm}$

$\boxed{f = -20.0 \text{ cm}}$

The ray diagram looks like Figure 36.13(c) in the text.

P36.22 (a) Since the mirror is concave, $R > 0$, giving $f = \dfrac{R}{2} = +12.0 \text{ cm}$. The magnification is positive because the image is upright:

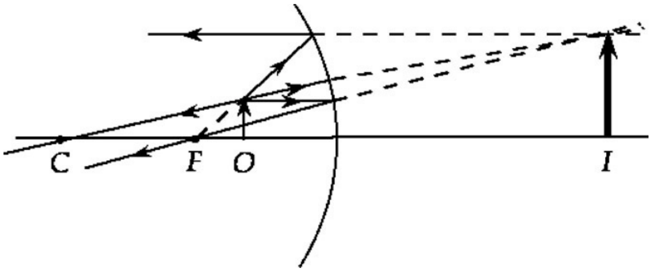
$$M = -\frac{q}{p} = +3 \quad \rightarrow \quad q = -3p$$

The mirror equation is then

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{p} - \frac{1}{3p} = \frac{2}{3p} = \frac{1}{12.0 \text{ cm}} \quad \rightarrow \quad p = \boxed{8.00 \text{ cm}}$$

(b) ANS. FIG. P36.22(b) shows the principal ray diagram for this situation.



ANS. FIG. P36.22(b)

(c) The image distance is negative, so the image is $\boxed{\text{virtual}}$. The rays of light do not actually come from the position of the image.

P36.23 Assume that the object distance is the same in both cases (i.e., her face is the same distance from the hubcap regardless of which way it is turned). Also realize that the near image (q = –10.0 cm) occurs when using the convex side of the hubcap. Applying the mirror equation to both cases gives:

concave side: $R = |R|$, $q = -30.0$ cm

$$\frac{1}{p} - \frac{1}{30.0} = \frac{2}{|R|} \quad \text{or} \quad \frac{2}{|R|} = \frac{30.0\text{ cm} - p}{(30.0\text{ cm})p} \qquad \textbf{[1]}$$

convex side: $R = -|R|$, $q = -10.0$ cm

$$\frac{1}{p} - \frac{1}{10.0} = -\frac{2}{|R|} \quad \text{or} \quad \frac{2}{|R|} = \frac{p - 10.0\text{ cm}}{(10.0\text{ cm})p} \qquad \textbf{[2]}$$

(a) Equating equations [1] and [2] gives:

$$\frac{30.0\text{ cm} - p}{3.00} = p - 10.0\text{ cm} \quad \text{or} \quad p = 15.0\text{ cm}$$

Thus, her face is $\boxed{15.0\text{ cm}}$ from the hubcap.

(b) Using the above result (p = 15.0 cm) in equation [1] gives:

$$\frac{2}{|R|} = \frac{30.0\text{ cm} - 15.0\text{ cm}}{(30.0\text{ cm})(15.0\text{ cm})} \quad \text{or} \quad \frac{2}{|R|} = \frac{1}{30.0\text{ cm}}$$

and $|R| = 60.0$ cm.

The radius of the hubcap is $\boxed{60.0\text{ cm}}$.

P36.24 (a) We assume the object is real; thus the object distance p is positive. The mirror is convex, so it is a diverging mirror, and we have $f = -|f| = -8.00$ cm. The image is virtual, so $q = -|q|$. Since we also know that $|q| = p/3$, the mirror equation gives

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{p} - \frac{3}{p} = \frac{1}{f} \quad \text{or} \quad -\frac{2}{p} = \frac{1}{-8.00\text{ cm}}$$

so $p = +16.0$ cm

This means that the object is $\boxed{\text{16.0 cm from the mirror}}$.

(b) The magnification is $M = -q/p = +|q|/p = +1/3 = \boxed{+0.333}$.

(c) Thus, the image is $\boxed{\text{upright}}$ and one-third the size of the object.

P36.25 (a) The image forms on a screen, so it is real and in front of the mirror, so $q = p + 5.00$ m, because p is positive. The magnification is

$$M = -\frac{q}{p} = -5.00 \quad \text{or} \quad q = 5.00p$$

Therefore,

$$p + 5.00\text{ m} = 5.00p \quad \rightarrow \quad p = 1.25\text{ m}$$

and $q = p + 5.00\text{ m} = 6.25\text{ m}$. From

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{1.25\text{ m}}+\frac{1}{6.25\text{ m}} \quad \rightarrow \quad f = +1.04\text{ m}$$

The focal length is positive, so the mirror is a converging mirror: $\boxed{\text{concave}}$.

(b) $f = +1.04\text{ m} = \dfrac{R}{2} \quad \rightarrow \quad R = \boxed{2.08\text{ m}}$

(c) From part (a), $p = 1.25$ m; the mirror should be $\boxed{\text{1.25 m from the object}}$.

P36.26 (a) The image starts from a point whose height above the mirror vertex is given by

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$$

$$\frac{1}{3.00\text{ m}}+\frac{1}{q}=\frac{1}{0.500\text{ m}} \quad \rightarrow \quad q = 0.600\text{ m}$$

As the ball falls, p decreases and q increases. Ball and image pass when $q_1 = p_1$. When this is true,

$$\frac{1}{p_1}+\frac{1}{p_1}=\frac{1}{0.500\text{ m}}=\frac{2}{p_1} \quad \rightarrow \quad p_1 = 1.00\text{ m},$$

which is at the focal point.

As the ball passes the focal point, the image switches from infinitely far above the mirror to infinitely far below the mirror. As the ball approaches the mirror from above, the virtual image approaches the mirror from below, reaching it together when $p_2 = q_2 = 0$.

(b) The falling ball passes its real image when it has fallen

$$\Delta y = 3.00\text{ m} - 1.00\text{ m} = 2.00\text{ m} = \frac{1}{2}gt^2$$

which gives $t = \sqrt{\dfrac{2(2.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{0.639\text{ s}}$.

The ball reaches its virtual image when it reaches the surface of the mirror, which is when it has traversed

$$\Delta y = 3.00\text{ m} - 0 = 3.00\text{ m} = \frac{1}{2}gt^2$$

which gives $t = \sqrt{\dfrac{2(3.00\text{ m})}{9.80\text{ m/s}^2}} = \boxed{0.782\text{ s}}$.

P36.27 (a) The flat mirror produces an image according to $q = -p = -24.0$ cm. The image is behind the mirror, with the distance from your eyes given by

$$1.55\text{ m} + 24.0\text{ m} = \boxed{25.6\text{ m}}$$

(b) The image is the same size as the object, so

$$\theta = \frac{h}{d} = \frac{1.50\text{ m}}{25.6\text{ m}} = \boxed{0.058\ 7\text{ rad}}$$

(c) $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{2}{R}$

$$\frac{1}{24\text{ m}} + \frac{1}{q} = \frac{2}{(-2\text{ m})} \quad \rightarrow \quad q = -0.960\text{ m}$$

This image is behind the mirror, distant from your eyes by

$$1.55\text{ m} + 0.960\text{ m} = \boxed{2.51\text{ m}}$$

(d) The image size is given by $M = \dfrac{h'}{h} = -\dfrac{q}{p}$:

$$h' = -h\frac{q}{p} = -1.50\text{ m}\left(\frac{-0.960\text{ m}}{24\text{ m}}\right) = 0.060\ 0\text{ m}$$

So its angular size at your eye is $\theta' = \dfrac{h'}{d} = \dfrac{0.06\text{ m}}{2.51\text{ m}} = \boxed{0.023\ 9\text{ rad}}$.

(e) Your brain assumes that the car is 1.50 m high and calculates its distance as

$$d' = \frac{h}{\theta'} = \frac{1.50\text{ m}}{0.023\ 9} = \boxed{62.8\text{ m}}$$

***P36.28** The focal length of the mirror may be found from the given object and image distances as

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$$

Solving for the focal length *f* gives

$$f=\frac{pq}{p+q}=\frac{(152\text{ cm})(18.0\text{ cm})}{152\text{ cm}+18.0\text{ cm}}=+16.1\text{ cm}$$

For an upright image twice the size of the object, the magnification is

$$M=-\frac{q}{p}=+2.00$$

which gives $q=-2.00p$.

Then, using the mirror equation again, $\frac{1}{f}=\frac{1}{p}+\frac{1}{q}$ becomes

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{p}-\frac{1}{2.00p}=\frac{2-1}{2.00p}=\frac{1}{f}$$

or $$p=\frac{f}{2.00}=\frac{16.1\text{ cm}}{2.00}=\boxed{8.05\text{ cm}}$$

Section 36.3 Images Formed by Refraction

P36.29 The image forms within the rod.

$$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}\quad\rightarrow\quad\frac{1.00}{p}+\frac{1.50}{q}=\frac{1.50-1.00}{6.00\text{ cm}}=\frac{1}{12.0\text{ cm}}$$

(a) $$\frac{1.00}{20.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}\quad\rightarrow\quad q=\boxed{45.0\text{ cm}}$$

(b) $$\frac{1.00}{10.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}\quad\rightarrow\quad q=\boxed{-90.0\text{ cm}}$$

(c) $$\frac{1.00}{3.0\text{ cm}}+\frac{1.50}{q}=\frac{1}{12.0\text{ cm}}\quad\rightarrow\quad q=\boxed{-6.00\text{ cm}}$$

P36.30 $$\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}=0\quad\text{and}\quad R\rightarrow\infty$$

$$q=-\frac{n_2}{n_1}p=-\frac{1}{1.309}(50.0\text{ cm})=-38.2\text{ cm}$$

Thus, the virtual image of the dust speck is $\boxed{\text{38.2 cm below the top surface}}$ of the ice.

P36.31 For a plane refracting (water) surface $(R \to \infty)$

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \quad \text{becomes} \quad q = -\frac{n_2}{n_1}p$$

(a) When the pool is full, $p = 2.00$ m and

$$q = -\left(\frac{1.00}{1.333}\right)(2.00\text{ m}) = -1.50\text{ m}$$

or the pool appears to be $\boxed{1.50\text{ m}}$ deep.

(b) If the pool is half filled, then $p = 1.00$ m and $q = -0.750$ m. Thus, the bottom of the pool appears to be 0.750 m below the water surface or $\boxed{1.75\text{ m}}$ below ground level.

P36.32 Since the center of curvature of the surface is on the side the light comes from, $R < 0$ giving $R = -4.00$ cm. For the line, $p = 4.00$ cm; then,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

becomes

$$\frac{1.00}{q} = \frac{1.00 - 1.50}{-4.00\text{ cm}} - \frac{1.50}{4.00\text{ cm}}$$

or $\qquad q = -4.00$ cm

Thus, the magnification $M = \frac{h'}{h} = -\left(\frac{n_1}{n_2}\right)\frac{q}{p}$ gives

$$h' = -\left(\frac{n_1 q}{n_2 p}\right)h = -\frac{1.50(-4.00\text{ cm})}{1.00(4.00\text{ cm})}(2.50\text{ mm}) = \boxed{3.75\text{ mm}}$$

P36.33 The water's surface has no curvature. When $R \to \infty$, the equation $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$, which describes image formation at a single refracting surface, becomes $q = -p\left(\frac{n_2}{n_1}\right)$. We use this to locate the final images of the two surfaces of the glass plate. First, find the image the glass forms of the *bottom* of the plate. [From Table 35.1, for flint glass, $n_1 = 1.66$.]

$$q_{B1} = -\left(\frac{1.33}{1.66}\right)(8.00\text{ cm}) = -6.41\text{ cm}$$

This virtual image is 6.41 cm below the top surface of the glass, or 18.41 cm below the water surface. Next, use this image as an object and locate the image the water forms of the bottom of the plate.

$$q_{B2} = -\left(\frac{1.00}{1.33}\right)(18.41\text{ cm}) = -13.84\text{ cm}$$

$$= 13.84\text{ cm below the water surface}$$

Now find image the water forms of the *top* surface of the glass.

$$q_3 = -\left(\frac{1}{1.33}\right)(12.0\text{ cm}) = -9.02\text{ cm}$$

$$= 9.02\text{ cm below the water surface}$$

Therefore, the apparent thickness of the glass is

$$\Delta t = 13.84\text{ cm} - 9.02\text{ cm} = \boxed{4.82\text{ cm}}$$

P36.34 Refer to Figure P36.34 in the textbook. In the right triangle lying between *O* and the center of the curved surface, $\tan\theta_1 = h/p$. In the right triangle lying between *I* and the center of the surface, tan $\theta_2 = -h'/q$. We need the negative sign because the image height is counted as negative while the angle is not. We substitute into the given

$$n_1 \tan\theta_1 = n_2 \tan\theta_2$$

to obtain

$$n_1 h/p = -n_2 h'/q$$

Then the magnification, defined by $M = h'/h$, is given by

$$M = h'/h = -n_1 q/n_2 p$$

P36.35 From Equation 36.8 for image formation by a single refracting surface,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

We solve for q to find

$$q = \frac{n_2 R p}{p(n_2 - n_1) - n_1 R}.$$

In this case, $n_1 = 1.50$, $n_2 = 1.00$, $p = 10.0$ cm, and $R = -15.0$ cm.

So the image location is

$$q = \frac{(1.00)(-15.0\text{ cm})(10.0\text{ cm})}{(10.0\text{ cm})(1.00 - 1.50) - (1.50)(-15.0\text{ cm})} = -8.57\text{ cm}$$

$\boxed{\text{apparent depth is 8.57 cm}}$

P36.36 The center of curvature is on the object side, so the radius of curvature is negative: $R = -|R| = -225$ cm.

(a) (i) $p = 5.00$ cm:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

$$\frac{1.333}{5.00\text{ cm}} + \frac{1.000}{q} = \frac{1.000 - 1.333}{-225\text{ cm}} \quad \rightarrow \quad q = -3.77\text{ cm}$$

The image is virtual and $\boxed{\text{3.77 cm from the front wall, in the water}}$.

(ii) $p = 25.0$ cm:

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$$

$$\frac{1.333}{25.0\text{ cm}} + \frac{1.000}{q} = \frac{1.000 - 1.333}{-225\text{ cm}} \quad \rightarrow \quad q = -19.3\text{ cm}$$

The image is virtual and $\boxed{\text{19.3 cm from the front wall, in the water}}$.

(b) From Problem 34, the magnification is $M = -\frac{n_1 q}{n_2 p}$.

(i) $M = -\frac{n_1 q}{n_2 p} = -\frac{1.333(-3.77\text{ cm})}{1.00(5.00\text{ cm})} = \boxed{+1.01}$

(ii) $M = -\frac{n_1 q}{n_2 p} = -\frac{1.333(-19.2\text{ cm})}{1.000(25.0\text{ cm})} = \boxed{+1.03}$

(c) $\boxed{\text{The plastic has uniform thickness, so the surfaces of entry and exit for any particular ray are very nearly parallel. The ray is slightly displaced, but it would not be changed in direction by going through the plastic wall with air on both sides. Only the difference between the air and water is responsible for the refraction of the light.}}$

(d) $\boxed{\text{Yes}}$

(e) If $p = |R|$, from $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} = \frac{n_1 - n_2}{|R|}$ we have

$$\frac{n_1}{|R|} + \frac{n_2}{q} = \frac{n_1 - n_2}{|R|} \quad \rightarrow \quad \frac{n_2}{q} = \frac{-n_2}{|R|}$$

then $q = -|R|$.

If $p > |R|$ (but also $p < 4.00|R|$, if the image is to be virtual—see **NOTE** below), then

$$p > |R| \quad \rightarrow \quad \frac{1}{|R|} > \frac{1}{p} \quad \rightarrow \quad \frac{1}{|R|} - \frac{1}{p} > 0$$

and

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_1 - n_2}{|R|}$$

$$\frac{n_2}{q} = \frac{n_1}{|R|} - \frac{n_2}{|R|} - \frac{n_1}{p}$$

$$\frac{1}{q} = -\frac{1}{|R|} + \frac{n_1}{n_2}\left(\frac{1}{|R|} - \frac{1}{p}\right)$$

$$\frac{1}{q} = -\frac{1}{|R|} + (1.333)\left(\frac{1}{|R|} - \frac{1}{p}\right)$$

$$\frac{1}{|q|} = \frac{1}{|R|} - (1.333)\left(\frac{1}{|R|} - \frac{1}{p}\right) < \frac{1}{|R|} \quad \rightarrow \quad |q| > |R|$$

[Assuming that $p < 4.00|R|$.] For example, if $p = 2|R|$,

$$\frac{1}{q} = -\frac{1}{|R|} + (1.333)\left(\frac{1}{|R|} - \frac{1}{2|R|}\right) = \frac{1}{|R|}\left(-1 + \frac{1.333}{2}\right) = \frac{-0.3335}{|R|}$$

$$q = -3.00|R|$$

$$M = -\frac{n_1 q}{n_2 p} = -\frac{1.333(-3.00|R|)}{1.000(2|R|)} = +2.00$$

Summarizing our results:

$$\boxed{\begin{array}{l}\text{If } p = |R|\text{, then } q = -p = -|R|\text{; if } p > |R|\text{, then } |q| > |R|\text{. For example, if}\\ p = 2|R|\text{, then } q = -3.00|R| \text{ and } M = +2.00.\end{array}}$$

NOTE: In the equation $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} = \frac{n_1 - n_2}{|R|}$, the term $\frac{n_1 - n_2}{|R|}$ is positive because $n_1 > n_2$. If the image is to be virtual, then q must be negative, and so the term $(n_1 - n_2)/|R|$ must be less than n_1/p:

$$\frac{n_1 - n_2}{|R|} < \frac{n_1}{p} \quad \rightarrow \quad p < \frac{n_1}{n_1 - n_2}|R| = \frac{1.333}{1.333 - 1.000}|R| = 4.00|R|$$

P36.37 For a plane surface ($R = \infty$), $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $q = -\frac{n_2 p}{n_1}$.

Thus, the magnitudes of the rate of change in the image and object positions are related by

$$\left|\frac{dq}{dt}\right| = \frac{n_2}{n_1}\left|\frac{dp}{dt}\right|$$

If the fish swims toward the wall with a speed of 2.00 cm/s, the speed of the image is given by

$$v_{\text{image}} = \left|\frac{dq}{dt}\right| = \frac{1.00}{1.33}(2.00\text{ cm/s}) = \boxed{1.50\text{ cm/s}}$$

Section 36.4 Images Formed by Thin Lenses

***P36.38** (a) From $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25.0\text{ cm}} - \frac{1}{26.0\text{ cm}}$, we obtain

$\boxed{q = 650\text{ cm}}$.

The image is $\boxed{\text{real, inverted, and enlarged}}$.

(b) From $\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{1}{25.0\text{ cm}} - \frac{1}{24.0\text{ cm}}$, we obtain

$\boxed{q = -600\text{ cm}}$.

The image is $\boxed{\text{virtual, upright, and enlarged}}$.

***P36.39** (a) From the mirror-and-lens equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \qquad \frac{1}{32.0\text{ cm}} + \frac{1}{8.00\text{ cm}} = \frac{1}{f}$$

so $\boxed{f = 6.40\text{ cm}}$.

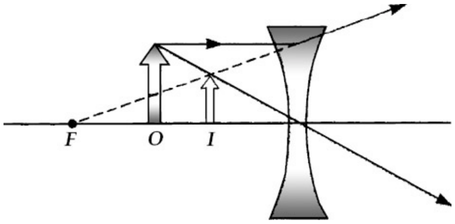
(b) $M = -\frac{q}{p} = -\frac{8.00\text{ cm}}{32.0\text{ cm}} = \boxed{-0.250}$

(c) Since $f > 0$, the lens is $\boxed{\text{converging}}$.

P36.40 (a) From the mirror-and-lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$:

$$\frac{1}{20.0\text{ cm}}+\frac{1}{q}=\frac{1}{(-32.0\text{ cm})}$$

$$\rightarrow \quad q=-\left(\frac{1}{20.0}+\frac{1}{32.0}\right)^{-1}=-12.3\text{ cm}$$



ANS. FIG. P36.40

Refer to ANS. FIG. P36.40. The image distance is negative, hence the image is virtual; thus, it forms $\boxed{\text{12.3 cm to the left of the lens}}$.

(b) $M=-\frac{q}{p}=-\frac{(-12.3\text{ cm})}{20.0\text{ cm}}=\boxed{0.615}$

(c) See the ray diagram shown in ANS. FIG. P36.40.

P36.41 The image is inverted:

$$M=\frac{h'}{h}=\frac{-1.80\text{ m}}{0.024\ 0\text{ m}}=-75.0 \ \rightarrow\ q=75.0p$$

The distance from slide to screen $d = p + q = 3.00$ m:

$$d=p+q=p+75.0p=76.0p$$

$$p=\frac{d}{76.0}=\frac{3.00\text{ m}}{76.0}=0.039\ 5\text{ m}$$

$$p = 39.5\text{ mm}$$

and $q = 75.0p = 2.96$ m.

(a) $\frac{1}{f}=\frac{1}{p}+\frac{1}{q}=\frac{1}{0.039\ 5\text{ m}}+\frac{1}{2.96\text{ m}} \ \rightarrow\ f=0.039\ 0\text{ m}=\boxed{39.0\text{ mm}}$

(b) From above, $p=\boxed{39.5\text{ mm}}$.

P36.42 (a) We are told that $p = 5f$. From the thin lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, we have

$$\frac{1}{5.00f}+\frac{1}{q}=\frac{1}{f} \quad \rightarrow \quad \frac{1}{q}=\frac{1}{f}-\frac{1}{5.00f}=\frac{4.00}{5.00f}$$

or $q = \dfrac{5.00}{4.00} f = +1.25 f$

The image distance is positive, hence the image is real.

$\boxed{\text{The image is in back of the lens at a distance of } 1.25f \text{ from the lens.}}$

(b) $M = -\dfrac{q}{p} = -\dfrac{1.25f}{5.00f} = \boxed{-0.250}$

(c) From part (a), the image distance is positive, hence the image is $\boxed{\text{real}}$.

P36.43 Let R_1 = outer radius and R_2 = inner radius:

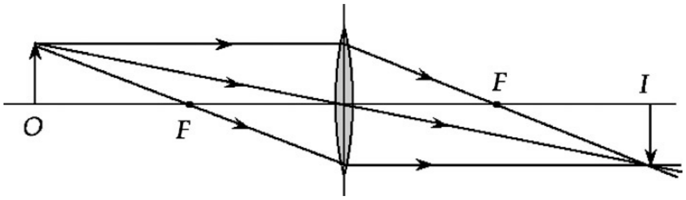
$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = (1.50-1)\left[\frac{1}{2.00\text{ m}} - \frac{1}{2.50\text{ cm}}\right]$$
$$= 0.050\ 0\text{ cm}^{-1}$$

so $f = \boxed{20.0\text{ cm}}$.

P36.44 Your scale drawings should look similar to those given below:

(i) See diagram in ANS. FIG. P36.44(i).

(a) A carefully drawn-to-scale version of ANS FIG. P36.44(i) should yield an inverted image $\boxed{\text{20.0 cm in back of the lens}}$ and the same size as the object.



ANS. FIG. P36.44(i)

(b) The image forms behind the lens, so the image is $\boxed{\text{real}}$.

(c) The figure shows that the image is $\boxed{\text{inverted}}$.

(d) The height of the image is the same as the height of the object, so $\boxed{M = -1.00}$.

(e) $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f}$: $\quad \dfrac{1}{20.0\text{ cm}} + \dfrac{1}{q} = \dfrac{1}{10.0\text{ cm}} \rightarrow q = +20.0\text{ cm}$

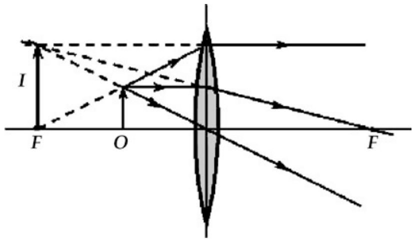
A positive image distance means that the image is real.

The magnification is $M = -\dfrac{q}{p} = -\dfrac{+20.0\text{ cm}}{20.0\text{ cm}} = -1.00$

A negative magnification means that the image is inverted.

$\boxed{\text{Algebraic answers agree, and we can express values to three significant figures: } q = 20.0 \text{ cm}, M = -1.00.}$

(ii) See diagram in ANS. FIG. P36.44(ii).



ANS. FIG. P36.44(ii)

(a) A carefully drawn-to-scale version of ANS FIG. P36.44(ii) should yield an upright, virtual image located $\boxed{\text{10 cm in front of the lens}}$ and twice the size of the object.

(b) The image forms in front of the lens, so the image is $\boxed{\text{virtual}}$.

(c) The figure shows that the image is $\boxed{\text{upright}}$.

(d) The height of the image is twice that of the object, so $\boxed{M = +2.00}$.

(e) $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}: \quad \frac{1}{5.00 \text{ cm}}+\frac{1}{q}=\frac{1}{10.0 \text{ cm}} \quad \to \quad q = -10.0 \text{ cm}$

A negative image distance means that the image is virtual.

The magnification is $M = -\frac{q}{p} = -\frac{(-10.0 \text{ cm})}{5.00 \text{ cm}} = +2.00$

A positive magnification means that the image is upright.

$\boxed{\text{Algebraic answers agree, and we can express values to three significant figures: } q = -10.0 \text{ cm}, M = +2.00.}$

(f) $\boxed{\text{Small variations from the correct directions of rays can lead to significant errors in the intersection point of the rays. These variations may lead to the three principal rays not intersecting at a single point.}}$

P36.45 In parts (a) and (b), the images are real, so the image distances are positive.

(a) $q = +20.0$ cm:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}: \quad \frac{1}{p}+\frac{1}{20.0 \text{ cm}}=\frac{1}{10.0 \text{ cm}} \quad \to \quad p = +20.0 \text{ cm}$$

The object distance is positive, so the object is real.

The object is $\boxed{\text{20.0 cm from the lens on the front side}}$.

(b) $q = +50.0$ cm:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}: \quad \frac{1}{p}+\frac{1}{50.0\text{ cm}}=\frac{1}{10.0\text{ cm}}$$

$$p = +12.5\text{ cm}$$

The object distance is positive, so the object is real.

The object is $\boxed{\text{12.5 cm from the lens on the front side}}$.

(c and d) Now, the images in parts (a) and (b) are virtual, so the image distances are negative.

(c) $q = -20.0$ cm:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}: \quad \frac{1}{p}+\frac{1}{-20.0\text{ cm}}=\frac{1}{10.0\text{ cm}}$$

$$p = +6.67\text{ cm}$$

The object distance is positive, so the object is real.

The object is $\boxed{\text{6.67 cm from the lens on the front side}}$.

(d) $q = -50.0$ cm:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}: \quad \frac{1}{p}+\frac{1}{-50.0\text{ cm}}=\frac{1}{10.0\text{ cm}}$$

$$p = +8.33\text{ cm}$$

The object distance is positive, so the object is real.

The object is $\boxed{\text{8.33 cm from the lens on the front side}}$.

P36.46 Use the thin lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$. The magnification is $M=-\frac{q}{p}$.

(i) $p = 40.0$ cm: $\quad \frac{1}{40.0\text{ cm}}+\frac{1}{q}=\frac{1}{-20.0\text{ cm}} \quad \rightarrow \quad q = -13.3\text{ cm}$

$$M=-\frac{q}{p}=-\frac{(-13.3\text{ cm})}{40.0\text{ cm}}=+0.333$$

(a) The image forms $\boxed{\text{13.3 cm in front of the lens}}$.

(b) The object distance is negative, so the image is $\boxed{\text{virtual}}$.

(c) The magnification is positive, so the image is $\boxed{\text{upright}}$.

(d) From above, $M = \boxed{+0.333}$

(ii) $p = 20.0$ cm: $\dfrac{1}{20.0\text{ cm}} + \dfrac{1}{q} = \dfrac{1}{-20.0\text{ cm}} \quad \rightarrow \quad q = -10.0\text{ cm}$

$$M = -\frac{q}{p} = -\frac{(-10.0\text{ cm})}{20.0\text{ cm}} = +0.500$$

(a) The image forms $\boxed{\text{10.0 cm in front of the lens}}$.

(b) The object distance is negative, so the image is $\boxed{\text{virtual}}$.

(c) The magnification is positive, so the image is $\boxed{\text{upright}}$.

(d) From above, $M = \boxed{+0.500}$

(iii) $p = 10.0$ cm: $\dfrac{1}{10.0\text{ cm}} + \dfrac{1}{q} = \dfrac{1}{-20.0\text{ cm}} \quad \rightarrow \quad q = -6.67\text{ cm}$

$$M = -\frac{q}{p} = -\frac{(-6.67\text{ cm})}{10.0\text{ cm}} = +0.667$$

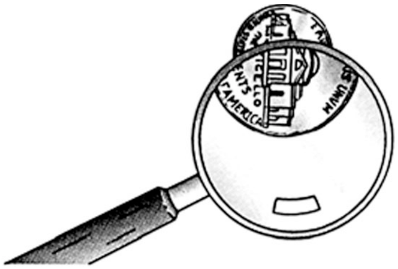
(a) The image forms $\boxed{\text{6.67 cm in front of the lens}}$.

(b) The object distance is negative, so the image is $\boxed{\text{virtual}}$.

(c) The magnification is positive, so the image is $\boxed{\text{upright}}$.

(d) From above, $M = \boxed{+0.667}$

P36.47 We are looking at an enlarged, upright, virtual image. Therefore, $M = +2$ and not -2. Looking through the lens, you see the image beyond the lens. Therefore, the image is virtual, with $q = -2.84$ cm.

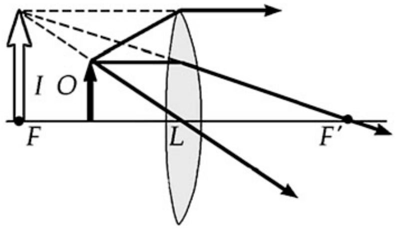


ANS. FIG. P36.47(a)

Now, $M = \dfrac{h'}{h} = 2 = -\dfrac{q}{p}$

so $p = -\dfrac{q}{2} = 1.42$ cm

A check is that p is positive, as it must be for a real object.



ANS. FIG. P36.47(b)

Thus,

$$f = \left(\frac{1}{p} + \frac{1}{q}\right)^{-1}$$

$$= \left[\frac{1}{1.42\text{ cm}} + \frac{1}{(-2.84\text{ cm})}\right]^{-1} = \boxed{2.84\text{ cm}}$$

P36.48 From the thin lens equation, since the focal length of the lens is constant,

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}:\; p^{-1}+q^{-1}=\text{constant}$$

Differentiating both sides with respect to ρ then gives

$$-1p^{-2}-1q^{-2}\frac{dq}{dp}=0$$

$$\frac{dq}{dp}=-\frac{q^2}{p^2}\quad\rightarrow\quad\boxed{dq=-\frac{q^2}{p^2}dp}$$

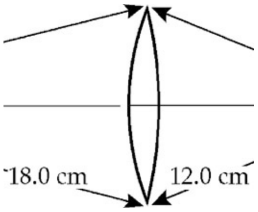
P36.49 We apply the lens maker's equation. The centers of curvature of the lens surfaces are on opposite sides, so the second surface has a negative radius

(a) $$\frac{1}{f}=(n-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=(0.440)\left[\frac{1}{12.0\text{ cm}}-\frac{1}{(-18.0\text{ cm})}\right]$$

$$f=\boxed{16.4\text{ cm}}$$

(b) $$\frac{1}{f}=(0.440)\left[\frac{1}{18.0\text{ cm}}-\frac{1}{(-12.0\text{ cm})}\right]$$

$$f=\boxed{16.4\text{ cm}}$$



ANS. FIG. P36.49

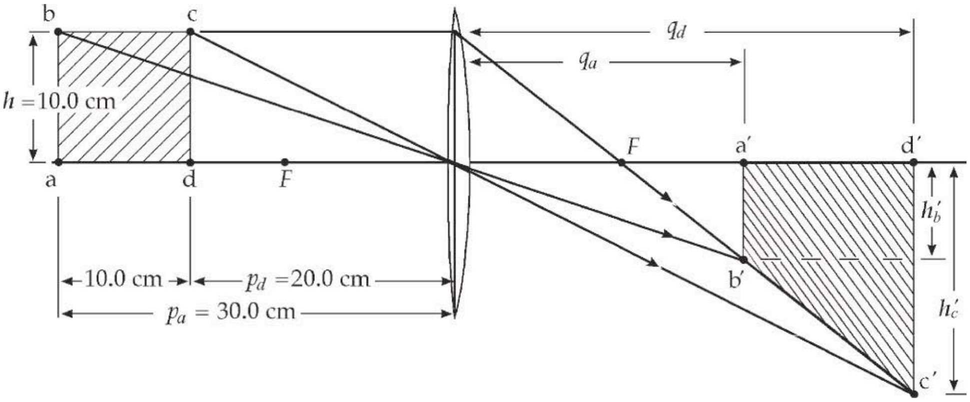
P36.50 (a) $$\frac{1}{p_a}+\frac{1}{q_a}=\frac{1}{f}\text{ becomes }\frac{1}{30.0\text{ cm}}+\frac{1}{q_a}=\frac{1}{14.0\text{ cm}}\;\rightarrow\;\boxed{q_a=26.3\text{ cm}}$$

$$\frac{1}{p_d}+\frac{1}{q_d}=\frac{1}{f}\text{ becomes }\frac{1}{20.0\text{ cm}}+\frac{1}{q_d}=\frac{1}{14.0\text{ cm}}\;\rightarrow\;\boxed{q_d=46.7\text{ cm}}$$

$$h_b'=hM_a=h\left(\frac{-q_a}{p_a}\right)=(10.0\text{ cm})\left(\frac{-26.3\text{ cm}}{30.0\text{ cm}}\right)=\boxed{-8.75\text{ cm}}$$

$$h_c'=hM_d=h\left(\frac{-q_d}{p_d}\right)=(10.0\text{ cm})\left(\frac{-46.7\text{ cm}}{20.0\text{ cm}}\right)=\boxed{-23.3\text{ cm}}$$

(b) See ANS. FIG. P36.50(b).



ANS. FIG. P36.50(b)

The square is imaged as a trapezoid.

(c) The equation follows from $h'/h = -q/p$ and $1/p + 1/q = 1/f$.

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \text{becomes} \quad \frac{1}{p}+\frac{1}{q}=\frac{1}{14\text{ cm}} \quad \text{or} \quad \frac{1}{p}=\frac{1}{14\text{ cm}}-\frac{1}{q}.$$

$$|h'| = |hM| = \left|h\left(\frac{-q}{p}\right)\right| = (10.0\text{ cm})q\left(\frac{1}{14\text{ cm}}-\frac{1}{q}\right)$$

(d) $\boxed{\text{The integral stated adds up the areas of ribbons covering the whole image, each with vertical dimension } |h'| \text{ and horizontal width } dq.}$

(e) We have

$$\int_{q_a}^{q_d} |h'|dq = (10.0\text{ cm})\left(\frac{q^2}{28.0\text{ cm}}-q\right)\Bigg|_{26.3\text{ cm}}^{46.7\text{ cm}}$$

$$= (10.0\text{ cm})\left[\frac{(46.7\text{ cm})^2-(26.3\text{ cm})^2}{28.0\text{ cm}} - 46.7\text{ cm} + 26.3\text{ cm}\right]$$

$$= \boxed{328\text{ cm}^2}$$

P36.51 In $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ or $p^{-1}+q^{-1}=$ constant, we differentiate with respect to time:

$$-1(p^{-2})\frac{dp}{dt}-1(q^{-2})\frac{dq}{dt}=0$$

$$\frac{dq}{dt}=\frac{-q^2}{p^2}\frac{dp}{dt}$$

We must find the momentary image location q:

$$\frac{1}{20.0\text{ m}}+\frac{1}{q}=\frac{1}{0.300\text{ m}}$$

$$q = 0.305\text{ m}$$

Now $$\frac{dq}{dt}=-\frac{(0.305\text{ m})^2}{(20.0\text{ m})^2}(5.00\text{ m/s})=-0.001\ 16\text{ m/s}=1.16\text{ mm/s}.$$

(a) The speed is $\boxed{1.16\text{ mm/s}}$.

(b) Increasing q is away from the lens, negative q is $\boxed{\text{toward the lens}}$. The motion of the image is towards the lens because dq/dt is negative.

P36.52 Let the object distance be p. Then the image distance is $d-p$. Set up the lens equation:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}\rightarrow\frac{1}{p}+\frac{1}{d-p}=\frac{1}{f}$$

Rearrange the equation to generate the following quadratic equation:

$$p^2-dp+df=0$$

Solve with the quadratic formula:

$$p=\frac{d\pm\sqrt{d^2-4df}}{2} \qquad \textbf{[1]}$$

Substitute numerical values:

$$p=\frac{2.00\text{ m}\pm\sqrt{(2.00\text{ m})^2-4(2.00\text{ m})(0.600\text{ m})}}{2}$$

$$=\frac{2.00\text{ m}\pm\sqrt{-0.800\text{ m}^2}}{2}$$

This expression has no real solutions. Therefore, we cannot find even one position between the object and the screen at which an image is formed on the screen. From equation [1], we see that a real value of p will result only if $d^2>4df$, or $d>4f$, in which case the plus/minus sign in equation [1] will give us two real values for p.

***P36.53** From the thin lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, we obtain

$$q_1=\frac{p_1f_1}{p_1-f_1}=\frac{(4.00\text{ cm})(8.00\text{ cm})}{4.00\text{ cm}-8.00\text{ cm}}=-8.00\text{ cm}$$

The magnification by the first lens is

$$M_1 = -\frac{q_1}{p_1} = -\frac{(-8.00\text{ cm})}{4.00\text{ cm}} = +2.00$$

The virtual image formed by the first lens is the object for the second lens, so

$$p_2 = 6.00\text{ cm} + |q_1| = 6.00\text{ cm} + 8.00\text{ cm} = +14.00\text{ cm}$$

and the thin lens equation gives

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(14.0\text{ cm})(-16.0\text{ cm})}{14.0\text{ cm} - (-16.0\text{ cm})} = -7.47\text{ cm}$$

The magnification by the second lens is

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-7.47\text{ cm})}{14.0\text{ cm}} = +0.533$$

so the overall magnification is

$$M = M_1 M_2 = (+2.00)(+0.533) = +1.07$$

The position of the final image is $\boxed{\text{7.47 cm in front of the second lens}}$, and its height is

$$h' = Mh = M_1 M_2 = (+1.07)(1.00\text{ cm}) = \boxed{1.07\text{ cm}}$$

Since $M > 0$, the final image is $\boxed{\text{upright}}$, and since $q_2 < 0$, this image is $\boxed{\text{virtual}}$.

Section 36.5 Lens Abberations

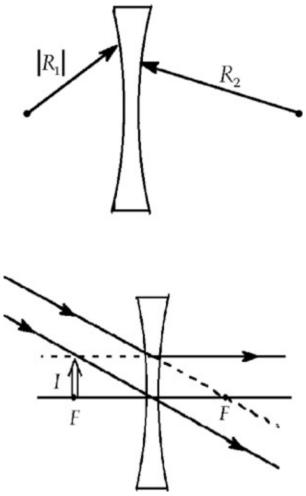
P36.54 Rays from a very distant object are effectively parallel, and the lens is diverging; therefore, the image is virtual and forms at the focal point.

(a) The focal length of the lens is given by

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.53 - 1.00)\left(\frac{1}{-32.5\text{ cm}} - \frac{1}{42.5\text{ cm}}\right)$$
$$f = -34.7\text{ cm}$$

Note that R_1 is negative because the center of curvature of the first surface is on the virtual image side.

The violet image forms at $\boxed{-34.7\text{ cm}}$



ANS FIG. P36.54

(b) For red light,

$$\frac{1}{f} = (1.51 - 1.00)\left(\frac{1}{-32.5\text{ cm}} - \frac{1}{42.5\text{ cm}}\right)$$
$$f = -36.1\text{ cm}$$

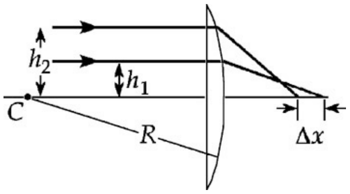
The red image forms at $\boxed{-36.1\text{ cm}}$.

P36.55 Ray h_1 is undeviated at the plane surface and strikes the second surface at angle of incidence given by

$$\theta_1 = \sin^{-1}\left(\frac{h_1}{R}\right) = \sin^{-1}\left(\frac{0.500\text{ cm}}{20.0\text{ cm}}\right) = 1.43°$$

Then,

$$1.00\sin\theta_2 = 1.60\sin\theta_1 = (1.60)\left(\frac{0.500}{20.0\text{ cm}}\right)$$
$$\theta_2 = 2.29°$$



ANS. FIG. P36.55

The angle this emerging ray makes with the horizontal is $\theta_2 - \theta_1 = 0.860°$.

The ray crosses the axis at a point farther out by f_1 (the focal length):

$$f_1 = \frac{h_1}{\tan(\theta_2 - \theta_1)} = \frac{0.500 \text{ cm}}{\tan(0.860°)} = 33.3 \text{ cm}$$

Because of the curved surface of the lens, the point of exit for this ray is horizontally slightly to the left of the lens vertex (where the principal axis intersects the curved surface of the lens), by the distance

$$R(1 - \cos\theta_1) = 20.0 \text{ cm}\left[1 - \cos(1.43°)\right] = 0.006\,25 \text{ cm}$$

Therefore, ray h_1 crosses the axis at this distance from the vertex:

$$x_1 = f_1 - R(1 - \cos\theta_1) = 33.3 \text{ cm} - 0.006\,25 \text{ cm} = 33.3 \text{ cm}$$

Now we repeat the above calculation for ray h_2:

$$\theta = \sin^{-1}\left(\frac{12.0 \text{ cm}}{20.0 \text{ cm}}\right) = 36.9°$$

Then,

$$1.00 \sin\theta_2 = 1.60 \sin\theta_1 = (1.60)\left(\frac{12.00}{20.0}\right) \rightarrow \theta_2 = 73.7°$$

$$f_2 = \frac{h_2}{\tan(\theta_1 - \theta_2)} = \frac{12.0 \text{ cm}}{\tan 36.8°} = 16.0 \text{ cm}$$

$$\begin{aligned} x_2 &= f_2 - R(1 - \cos\theta_2) \\ &= (16.0 \text{ cm}) - 20.0 \text{ cm}\left[1 - \cos(36.9°)\right] = 12.0 \text{ cm} \end{aligned}$$

Now $$\Delta x = x_1 - x_2 = 33.3 \text{ cm} - 12.0 \text{ cm} = \boxed{21.3 \text{ cm}}$$

Section 36.6 The Camera

P36.56 The same light intensity is received from the subject, and the same light energy on the film is required:

$$IA_1\Delta t_1 = IA_2\Delta t_2$$

$$\frac{\pi d_1^2}{4}\Delta t_1 = \frac{\pi d_2^2}{4}\Delta t_2$$

Substituting f-stops and shutter speeds,

$$\left(\frac{f}{4}\right)^2\left(\frac{1}{15} \text{ s}\right) = d_2^2\left(\frac{1}{125} \text{ s}\right)$$

solving,

$$d_2 = \sqrt{\frac{125}{15}}\frac{f}{4} = \frac{f}{1.39} = \boxed{\frac{f}{1.4}}$$

We can verify this by noting that changing the shutter speed from $\frac{1}{15\text{ s}}$ to $\frac{1}{125\text{ s}}$ is approximately a factor of 8 decrease in the exposure time, and requires a three *f*-stop increase (each increasing the area by a factor of 2), from *f*-4 down to *f*-2.8, *f*-2.0, and *f*-1.4.

***P36.57** To properly focus the image of a distant object, the lens must be at a distance equal to the focal length from the film ($q_1 = 65.0$ mm). For the closer object:

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f}$$

becomes $\frac{1}{2\ 000\text{ mm}} + \frac{1}{q_2} = \frac{1}{65.0\text{ mm}}$,

and $q_2 = (65.0\text{ mm})\left(\frac{2\ 000}{2\ 000 - 65.0}\right)$.

The lens must be moved $\boxed{\text{away from the film}}$ by a distance

$$D = q_2 - q_1 = (65.0\text{ mm})\left(\frac{2\ 000}{2\ 000 - 65.0}\right) - 65.0\text{ mm} = \boxed{2.18\text{ mm}}$$

Section 36.7 The Eye

P36.58 The lens should take parallel light rays from a very distant object ($p = \infty$) and make them diverge from a virtual image at the woman's far point, which is 25.0 cm beyond the lens, at $q = -25.0$ cm.

(a) $P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{\infty} - \frac{1}{0.250\text{ m}} = \boxed{-4.00\text{ diopters}}$

(b) The power is negative: a $\boxed{\text{diverging lens}}$.

***P36.59** The corrective lens must form an upright, virtual image at the near point of the eye (i.e., $q = -60.0$ cm in this case) for objects located 25.0 cm in front of the eye ($p = +25.0$ cm). From the thin-lens equation,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

the required focal length of the corrective lens is

$$f = \frac{pq}{p+q} = \frac{(25.0\text{ cm})(-60.0\text{ cm})}{25.0\text{ cm} - 60.0\text{ cm}} = \boxed{+42.9\text{ cm}}$$

and the power (in diopters) of this lens will be

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{+0.429\text{ m}} = \boxed{+2.33\text{ diopters}}$$

***P36.60** (a) $f = \frac{1}{P} = \frac{1}{-4.00\text{ diopters}} = -0.250\text{ m} = \boxed{-25.0\text{ cm}}$

(b) The corrective lens forms virtual images of very distant objects ($p \to \infty$) at $q = f = -25.0$ cm. Thus, the person must be very $\boxed{\text{nearsighted}}$, unable to see objects clearly when they are more than 25.0 cm + 2.00 cm = 27.0 cm from the eye.

(c) If contact lenses are to be worn, the far point of the eye will be 27.0 cm in front of the lens, so the needed focal length will be $f = q = -27.0$ cm, and the power is

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{-0.270\text{ m}} = \boxed{-3.70\text{ diopters}}$$

P36.61 For starlight going through a nearsighted person's glasses,

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

$$\frac{1}{\infty} + \frac{1}{(-0.800\text{ m})} = \frac{1}{f} = -1.25\text{ diopters}$$

For a nearby object (the image is virtual),

$$\frac{1}{p} + \frac{1}{(-0.180\text{ m})} = -1.25\text{ m}^{-1}$$

so $p = \boxed{23.2\text{ cm}}$.

***P36.62** (a) When the child clearly sees objects at her far point $(p_{\text{max}} = 125\text{ cm})$, the lens-cornea combination has assumed a focal length suitable for forming the image on the retina ($q = 2.00$ cm). The thin-lens equation gives the optical power under these conditions as

$$P_{\text{far}} = \frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.25\text{ m}} + \frac{1}{0.020\ 0\text{ m}}$$
$$= +50.8\text{ diopters}$$

When the eye is focused ($q = 2.00$ cm) on objects at her near point $\left(p_{\text{min}} = 10.0\text{ cm}\right)$, the optical power of the lens-cornea combination is

$$P_{\text{near}} = \frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.100\text{ m}} + \frac{1}{0.020\ 0\text{ m}}$$
$$= +60.0\text{ diopters}$$

Therefore, the range of the power of the lens-cornea combination is $\boxed{+50.8\text{ diopters } \leq P \leq 60.0\text{ diopters}}$.

(b) If the child is to see very distant objects ($p \to \infty$) clearly, her eyeglass lens must form an erect, virtual image at the far point of her eye ($q = -125$ cm). The optical power of the required lens is

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-0.125\text{ m}} = \boxed{-0.800\text{ diopters}}$$

Since the power, and hence the focal length, of this lens is negative, it is $\boxed{\text{diverging}}$.

***P36.63** (a) The upper portion of the lens should form an upright, virtual image of very distant objects $\left(p \approx \infty\right)$ at the far point of the eye ($q = -1.50$ m). The thin-lens equation then gives $f = q = -1.50$ m, so the needed power is

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{-1.50\text{ m}} = \boxed{-0.667\text{ diopters}}$$

(b) The lower part of the lens should form an upright, virtual image at the near point of the eye ($q = -30.0$ cm) when the object distance is $p = 25.0$ cm. From the thin-lens equation,

$$f = \frac{pq}{p+q} = \frac{(25.0\text{ cm})(-30.0\text{ cm})}{25.0\text{ cm} - 30.0\text{ cm}} = +1.50 \times 10^2\text{ cm} = \boxed{+1.50\text{ m}}$$

Therefore, the power is $P = \frac{1}{f} = \frac{1}{+1.50\text{ m}} = \boxed{+0.667\text{ diopters}}$.

***P36.64** $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ so $\frac{1.00}{\infty} + \frac{1.40}{21.0\text{ mm}} = \frac{1.40 - 1.00}{6.00\text{ mm}}$

and $0.066\ 7 = 0.066\ 7$.

They agree. $\boxed{\text{The image is inverted, real, and diminished.}}$

***P36.65** (a) $\boxed{\text{Yes, a single lens can correct the patient's vision.}}$ The patient needs corrective action in both the near vision (to allow clear viewing of objects between 45.0 cm and the normal near point of 25.0 cm) and the distant vision (to allow clear viewing of objects more than 85.0 cm away). A single lens solution is for the patient to wear a bifocal or progressive lens. Alternately, the patient must purchase two pairs of glasses, one for reading, and one for distant vision.

(b) To correct the near vision, the lens must form an upright, virtual image at the patient's near point (q = –45.0 cm) when a real object is at the normal near point (p = +25.0 cm). The thin-lens equation gives the needed focal length as

$$f=\frac{pq}{p+q}=\frac{(25.0\text{ cm})(-45.0\text{ cm})}{25.0\text{ cm}-45.0\text{ cm}}=\boxed{+56.3\text{ cm}}$$

so the required power in diopters is

$$P=\frac{1}{f_{\text{in meters}}}=\frac{1}{+0.563\text{ m}}=\boxed{+1.78\text{ diopters}}$$

(c) To correct the distant vision, the lens must form an upright, virtual image at the patient's far point (q = –85.0 cm) for the most distant objects ($p\to\infty$). The thin-lens equation gives the needed focal length as $f = q$ = –85.0 cm, so the needed power is

$$P=\frac{1}{f_{\text{in meters}}}=\frac{1}{-0.850\text{ m}}=\boxed{-1.18\text{ diopters}}$$

Section 36.8 The Simple Magnifyer

P36.66 (a) Angular magnification is a maximum when the image is at the near point of the eye: q = 25.0 cm. From the thin lens equation:

$$\frac{1}{p}+\frac{1}{(-25.0\text{ cm})}=\frac{1}{5.00\text{ cm}}\quad\text{or}\quad p=\boxed{4.17\text{ cm}}$$

(b) From Equation 36.24,

$$M=-\frac{q}{p}=1+\frac{25.0\text{ cm}}{f}=1+\frac{25.0\text{ cm}}{5.00\text{ cm}}=\boxed{6.00}$$

Section 36.9 The Compound Microscope

P36.67 Using Equation 36.26,

$$M \approx -\left(\frac{L}{f_o}\right)\left(\frac{25.0\text{ cm}}{f_e}\right) = -\left(\frac{23.0\text{ cm}}{0.400\text{ cm}}\right)\left(\frac{25.0\text{ cm}}{2.50\text{ cm}}\right) = \boxed{-575}$$

Section 36.10 The Telescope

P36.68 $f_o = 20.0$ m, $f_e = 0.025\,0$ m

(a) From Equation 36.27, The angular magnification produced by this telescope is

$$m = -\frac{f_o}{f_e} = \boxed{-800}$$

(b) Since $m < 0$, the image is $\boxed{\text{inverted}}$.

P36.69 Let I_0 represent the intensity of the light from the nebula and θ_0 its angular diameter. With the first telescope, the image diameter h' on the film is given by

$$\theta_o = -\frac{h'}{f_o} \text{ as } h' = -\theta_o(2\,000\text{ mm})$$

The light power captured by the telescope aperture is

$$P_1 = I_0 A_1 = I_0\left[\frac{\pi(200\text{ mm})^2}{4}\right]$$

and the light energy focused on the film during the exposure is

$$E_1 = P_1\Delta t_1 = I_0\left[\frac{\pi(200\text{ mm})^2}{4}\right](1.50\text{ min})$$

Likewise, the light power captured by the aperture of the second telescope is

$$P_2 = I_0 A_2 = I_0\left[\frac{\pi(60.0\text{ mm})^2}{4}\right]$$

and the light energy is

$$E_2 = I_0\left[\frac{\pi(60.0\text{ mm})^2}{4}\right]\Delta t_2$$

Therefore, to have the same light energy per unit area, it is necessary that

$$\frac{I_0\left[\pi(60.0\text{ mm})^2/4\right]\Delta t_2}{\pi\left[\theta_o(900\text{ mm})^2/4\right]}=\frac{I_0\left[\pi(200\text{ mm})^2/4\right](1.50\text{ min})}{\pi\left[\theta_o(2\,000\text{ mm})^2/4\right]}$$

The required exposure time with the second telescope is

$$\Delta t_2=\frac{(200\text{ mm})^2(900\text{ mm})^2}{(60.0\text{ mm})^2(2\,000\text{ mm})^2}(1.50\text{ min})=\boxed{3.38\text{ min}}$$

P36.70 (a) The mirror-and-lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, gives

$$q=\frac{1}{1/f-1/p}=\frac{1}{(p-f)/fp}=\frac{fp}{p-f}$$

Then,

$$M=\frac{h'}{h}=-\frac{q}{p}=-\frac{f}{p-f}$$

gives $h'=\frac{fh}{f-p}$

(b) For $p >> f$, $f-p\approx -p$. Then, $h'=\boxed{-\frac{hf}{p}}$

(c) Suppose the telescope observes the space station at the zenith:

$$h'=-\frac{hf}{p}=-\frac{(108.6\text{ m})(4.00\text{ m})}{407\times10^3\text{ m}}=\boxed{-1.07\text{ mm}}$$

Additional Problems

P36.71 (a) For the lens in air,

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\frac{1}{79.0\text{ cm}}=(1.55-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

For the same lens in water,

$$\frac{1}{f'}=\left(\frac{n_2}{n_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Substituting,

$$\frac{1}{f'}=\left(\frac{1.55}{1.333}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

By division,

$$\frac{1/79.0\text{ cm}}{1/f'}=\frac{0.55}{\left(\frac{1.55}{1.333}-1\right)}=\frac{f'}{79.0\text{ cm}}\rightarrow f'=\boxed{267\text{ cm}}$$

(b) The path of a reflected ray does not depend on the refractive index of the medium which the reflecting surface bounds. Therefore the focal length of a mirror does not change when it is put into a different medium: $f'=\frac{R}{2}=f=\boxed{79.0\text{ cm}}$.

P36.72 The real image formed by the concave mirror serves as a real object for the convex mirror with $p = 50$ cm and $q = -10$ cm. Therefore,

$$\frac{1}{f}=\frac{1}{p}+\frac{1}{q}\quad\rightarrow\quad\frac{1}{f}=\frac{1}{50.0\text{ cm}}+\frac{1}{(-10.0\text{ cm})}$$

gives $f = -12.5$ cm and $R=2f=\boxed{-25.0\text{ cm}}$.

P36.73 Only a diverging lens gives an upright, diminished image. Therefore, the image is virtual and between the object and the lens (the image is closer to the lens), and $q < 0$. We have

$$d=p-|q|=p+q,\quad\text{and}\quad M=-\frac{q}{p},$$

so $q=-Mp$ and $d=p-Mp$.

Therefore, $p=\frac{d}{1-M}$:

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{1}{p}+\frac{1}{-Mp}=\frac{-M+1}{-Mp}=\frac{(1-M)^2}{-Md}$$

$$f=\frac{-Md}{(1-M)^2}=\frac{-(0.500)(20.0\text{ cm})}{(1-0.500)^2}=\boxed{-40.0\text{ cm}}$$

P36.74 For a single lens, an object and its image cannot be on opposite sides of the lens if the image is upright. The object and image must be on the same side of the lens; thus the image is virtual, and $q < 0$. Because the image is upright, $M > 0$.

If the image is between the object and the lens (the image is closer to the lens), we have

$$d = p - |q| = p + q, \quad \text{so} \quad q = d - p:$$

$$M = -\frac{q}{p} \quad \text{so} \quad q = -Mp = d - p \;\rightarrow\; p = \frac{d}{1-M}$$

Substituting into the thin lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, gives

$$\frac{1}{p}+\frac{1}{(-Mp)}=\frac{1}{f}$$

Solving,

$$\frac{M}{Mp}+\frac{1}{(-Mp)}=\frac{1}{f}=\frac{M-1}{Mp}=\frac{M-1}{M}\left(\frac{1-M}{d}\right)=-\frac{(1-M)^2}{Md}$$

$$\rightarrow \boxed{f=\frac{-Md}{(1-M)^2}}$$

Since M is positive, the lens is diverging.

If the object is between the image and the lens (the object is closer to the lens), the lens is converging. We have

$$d = |q| - p = -q - p \quad \rightarrow \quad q = -d - p$$

Substituting into the thin lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, gives

$$\frac{1}{p}+\frac{1}{(-Mp)}=\frac{1}{f}$$

Solving,

$$\frac{M}{Mp}+\frac{1}{(-Mp)}=\frac{1}{f}=\frac{M-1}{Mp}=\frac{M-1}{M}\left(\frac{M-1}{d}\right)=\frac{(M-1)^2}{Md}$$

$$\rightarrow \boxed{f=\frac{Md}{(M-1)^2}}$$

Since M is positive, the lens is converging.

***P36.75** The lens for the left eye forms an upright, virtual image at q_L = –50.0 cm when the object distance is p_L = 25.0 cm, so the thin lens equation $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, gives its focal length as

$$f_L=\frac{p_Lq_L}{p_L+q_L}=\frac{(25.0\text{ cm})(-50.0\text{ cm})}{25.0\text{ cm}-50.0\text{ cm}}=50.0\text{ cm}$$

Similarly for the other lens, q_R = –100 cm when p_R = 25.0 cm, and f_R = 33.3 cm.

(a) Using the lens for the left eye as the objective,

$$m=\frac{f_o}{f_c}=\frac{50.0\text{ cm}}{33.3\text{ cm}}=\boxed{1.50}$$

(b) Using the lens for the right eye as the eyepiece and, for maximum magnification, requiring that the final image be formed at the normal near point (q_e =–25.0 cm) gives the object distance for the eyepiece as

$$p_e=\frac{q_ef_e}{q_e-f_e}=\frac{(-25.0\text{ cm})(33.3\text{ cm})}{-25.0\text{ cm}-33.3\text{ cm}}=+14.3\text{ cm}$$

The maximum magnification by the eyepiece is then

$$m_e=1+\frac{25.0\text{ cm}}{f_e}=1+\frac{25.0\text{ cm}}{33.3\text{ cm}}=+1.75$$

and the image distance for the objective is

$$q_1=L-p_e=10.0\text{ cm}-14.3\text{ cm}=-4.28\text{ cm}$$

The thin lens equation then gives the object distance for the objective as

$$p_1=\frac{q_1f_1}{q_1-f_1}=\frac{(-4.28\text{ cm})(50.0\text{ cm})}{-4.28\text{ cm}-50.0\text{ cm}}=+3.95\text{ cm}$$

The magnification by the objective is then

$$M_1=-\frac{q_1}{p_1}=-\frac{(-4.28\text{ cm})}{3.95\text{ cm}}=+1.08$$

and the overall magnification is

$$m=M_1m_e=(+1.08)(+1.75)=\boxed{1.90}$$

***P36.76** The image will be inverted. With $h = 6.00$ cm, we require $h' = -1.00$ mm.

(a) $M = \dfrac{h'}{h} = -\dfrac{q}{p}$ gives

$$q = -p\frac{h'}{h} = -(50.0\text{ mm})\left(\frac{-1.00\text{ mm}}{60.0\text{ mm}}\right) = \boxed{0.833\text{ mm}}$$

(b) From $\dfrac{1}{p} + \dfrac{1}{q} = \dfrac{1}{f} = \dfrac{1}{50.0\text{ mm}} + \dfrac{1}{0.833\text{ mm}}$, we obtain

$$f = \boxed{0.820\text{ mm}}$$

P36.77 (a) Start with the first pass through the lens.

$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{80.0\text{ cm}} - \frac{1}{100\text{ cm}}$$

gives $q_1 = +400$ cm or 400 cm to right of the lens.

The object of the mirror is 400 cm – 100 cm = 300 cm to the right of the mirror, so the object is virtual. Therefore, for the mirror, $p_2 = -300$ cm:

$$\frac{1}{q_2} = \frac{1}{f_2} - \frac{1}{p_2} = \frac{1}{(-50.0\text{ cm})} - \frac{1}{(-300\text{ cm})}$$

gives $q_2 = -60.0$ cm or 60.0 cm to the right of the mirror.

The image formed by the mirror is 100 cm + 60 cm = 160 cm to the right of the lens. Therefore, for the second pass through the lens, $p_3 = 160$ cm:

$$\frac{1}{q_3} = \frac{1}{f_1} - \frac{1}{p_3} = \frac{1}{80.0\text{ cm}} - \frac{1}{160\text{ cm}}$$

or $q_3 = \boxed{160\text{ cm to the left of lens}}$.

(b) $M_1 = -\dfrac{q_1}{p_1} = -\dfrac{400\text{ cm}}{100\text{ cm}} = -4.00 \qquad M_2 = -\dfrac{q_2}{p_2} = -\dfrac{(-60.0\text{ cm})}{(-300\text{ cm})} = -\dfrac{1}{5}$

$M_3 = -\dfrac{q_3}{p_3} = -\dfrac{160\text{ cm}}{160\text{ cm}} = -1 \qquad M = M_1M_2M_3 = \boxed{-0.800}$

(c) Since $M < 0$ the final image is $\boxed{\text{inverted}}$.

P36.78 (a) We start with the final image and work backward. From Figure P36.78, the final image is virtual (to left of lens 2) and $x = 30.0$ cm, so

$$q_2 = -(50.0\text{ cm} - 30.0\text{ cm}) = -20.0\text{ cm}$$

The thin lens equation then gives

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}:\ \frac{1}{p_2} + \frac{1}{-20.0\text{ cm}} = \frac{1}{20.0\text{ cm}} \rightarrow p_2 = +10.0\text{ cm}$$

The image formed by the first lens serves as the object for the second lens and is located 10.0 cm in front of the second lens.

Thus, $q_1 = 50.0\text{ cm} - 10.0\text{ cm} = 40.0$ cm and the thin lens equation gives

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}:\ \frac{1}{p_1} + \frac{1}{40.0\text{ cm}} = \frac{1}{10.0\text{ cm}} \rightarrow p_1 = +13.3\text{ cm}$$

The original object should be located $\boxed{\text{13.3 cm in front of the first lens}}$.

(b) The overall magnification is

$$M = M_1M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{40.0\text{ cm}}{13.3\text{ cm}}\right)\left(-\frac{(-20.0\text{ cm})}{10.0\text{ cm}}\right)$$
$$= \boxed{-6.00}$$

(c) Since $M < 0$, the final image is $\boxed{\text{inverted}}$.

(d) Since $q_2 < 0$, it is $\boxed{\text{virtual}}$.

P36.79 (a) With light going through the piece of glass from left to right, the radius of the first surface is positive and that of the second surface is negative according to the sign convention of Table 36.2. Thus, $R_1 = +2.00$ cm and $R_2 = -4.00$ cm. Applying $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ to the first surface gives

$$\frac{1.00}{1.00\text{ cm}} + \frac{1.50}{q_1} = \frac{1.50 - 1.00}{+2.00\text{ cm}}$$

which yields $q_1 = -2.00$ cm. The first surface forms a virtual image 2.00 cm to the left of that surface and 16.0 cm to the left of the second surface.

The image formed by the first surface is the object for the second surface, so $p_2 = +16.0\text{ cm}$ and $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ gives

$$\frac{1.50}{16.0\text{ cm}}+\frac{1.00}{q_2}=\frac{1.00-1.50}{-4.00\text{ cm}} \quad \text{or} \quad q_2 = +32.0\text{ cm}$$

$\boxed{\text{32.0 cm to the right of the second surface}}$

(b) The final image distance is positive, so the image is $\boxed{\text{real}}$.

P36.80 (a) When the meterstick coordinate of the object is 0, its object distance is $p_i = 32$ cm. When the meterstick coordinate of the object is x, its object distance is $p = 32\text{ cm} - x$. The image distance from the lens is given by the thin lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$ (in the following, all variables are in units of cm, and units are suppressed). Substituting,

$$\frac{1}{32.0-x}+\frac{1}{q}=\frac{1}{26.0}$$

Solving for q then gives

$$\frac{1}{q}=\frac{1}{26.0}-\frac{1}{(32.0-x)}=\frac{(32.0-x)-26.0}{26.0(32.0-x)}=\frac{6.0-x}{26.0(32.0-x)}$$

$$q=\frac{832-26.0x}{6.0-x}$$

The image distance q is measured from the position of the lens. The image coordinate on the meterstick is

$$x' = 32.0+q = 32.0+\frac{832-26.0x}{6.0-x}=\frac{32.0(6.0-x)+832-26.0x}{6.0-x}$$

$$\boxed{x' = \frac{1024-58.0x}{6.0-x} \text{ where } x \text{ and } x' \text{ are in centimeters.}}$$

(b) The image starts at the position $x_i' = 171$ cm and moves in the positive x direction, faster and faster, and as the object approaches the position $x = 6$ cm (the focal point of the lens), the image goes out to infinity. At the instant the object is at $x = 6$ cm, the rays from the top of the object are parallel as they leave the lens: their intersection point can be described as at $x' = \infty$ to the right or equally well at $x' = -\infty$ on the left. From $x' = -\infty$ the image continues moving to the right, now slowing down. It reaches, for example, –280 cm when the object is at 8 cm, and –55 cm when the object is finally at 12 cm.

object position (cm)	image position (cm)
x	x'
0	170.7
1	193.2
2	227.0
3	283.3
4	396.0
5	734.0
6	infinity
7	–618.0
8	–280.0
9	–167.3
10	–111.0
11	–77.2
12	–54.7

(c) $\boxed{\text{The image moves to infinity and beyond—meaning it moves forward to infinity (on the right), jumps back to minus infinity (on the left), and then proceeds forward again.}}$

(d) $\boxed{\text{The image usually travels to the right, except when it jumps from plus infinity (right) to minus infinity (left).}}$

P36.81 (a) $$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f_1}=\frac{2}{R} \rightarrow \frac{1}{q_1}=\frac{1}{f_1}-\frac{1}{p_1}=\frac{1}{10.0\text{ cm}}-\frac{1}{12.5\text{ cm}}$$

so $q_1 = 50.0$ cm (a real image, to left of mirror).This serves as an object for the lens (a virtual object, to left of lens) with object distance $p_2 = 25.0\text{ cm} - 50.0\text{ cm} = -25.0\text{ cm}$, so

$$\frac{1}{q_2}=\frac{1}{f_2}-\frac{1}{p_2}=\frac{1}{(-16.7\text{ cm})}-\frac{1}{(-25.0\text{ cm})}$$

so $q_2 = -50.3\text{ cm}$ (a virtual image),

meaning 50.3 cm to the right of the lens. Thus, the final image is located $\boxed{\text{25.3 cm to right of mirror}}$.

(b) The final image distance is negative (–50.3 cm), so the image is $\boxed{\text{virtual}}$.

Calculate the overall magnification $M = M_1M_2$:

$$M_1 = -\frac{q_1}{p_1} = -\frac{50.0\text{ cm}}{12.5\text{ cm}} = -4.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-50.3\text{ cm})}{(-25.0\text{ cm})} = -2.01$$

Then $M = M_1M_2 = 8.05$.

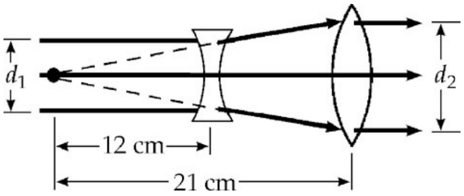
(c) The magnification is positive, so the image is $\boxed{\text{upright}}$.

(d) From above, $M = M_1M_2 = \boxed{8.05}$.

P36.82 (a) Have the beam pass through the diverging lens first, then the converging lens. The rays of light entering the diverging lens are parallel, so they behave as though they come from an object at infinity ($p = \infty$):

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \qquad \frac{1}{\infty}+\frac{1}{q}=\frac{1}{-12.0\text{ cm}}$$

or $q = -12.0\text{ cm}$.



ANS. FIG. P36.82

Use this image as a real object for the converging lens, placing it at the focal point on the object side of the lens, at $p = 21.0\text{ cm}$. Then

$$\frac{1}{p}+\frac{1}{q}=\frac{1}{f} \quad \rightarrow \quad \frac{1}{21\text{ cm}}+\frac{1}{q}=\frac{1}{21\text{ cm}}$$

or $q = \infty$.

The exiting rays will be parallel. The lenses must be 21.0 cm – 12.0 cm = 9.00 cm apart.

(b) Refer to ANS. FIG. P36.82. By similar triangles,

$$\frac{d_2}{d_1} = \frac{21.0 \text{ cm}}{12.0 \text{ cm}} = \boxed{1.75 \text{ times}}$$

P36.83 (a) $I = \frac{P}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi\left(1.60 \times 10^{-2} \text{ m}\right)^2} = \boxed{1.40 \text{ kW/m}^2}$

(b) $I = \frac{P}{4\pi r^2} = \frac{4.50 \text{ W}}{4\pi(7.20 \text{ m})^2} = \boxed{6.91 \text{ mW/m}^2}$

(c) $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}: \frac{1}{7.20 \text{ m}} + \frac{1}{q} = \frac{1}{0.350 \text{ m}}$

$\rightarrow \quad q = 0.368 \text{ m}$

$$M = \frac{h'}{3.20 \text{ cm}} = -\frac{q}{p} = -\frac{0.368 \text{ m}}{7.20 \text{ m}}$$

$\rightarrow \quad h' = \boxed{0.164 \text{ cm}}$

(d) The lens intercepts power given by

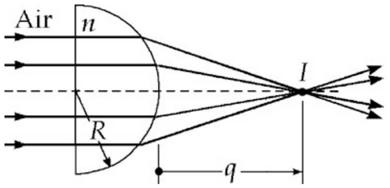
$$P = IA = \left(6.91 \times 10^{-3} \text{ W/m}^2\right)\left[\frac{\pi}{4}(0.150 \text{ m})^2\right]$$

and puts it all onto the image where

$$I = \frac{P}{A} = \frac{\left(6.91 \times 10^{-3} \text{ W/m}^2\right)\left[\pi(15.0 \text{ cm})^2/4\right]}{\pi(0.164 \text{ cm})^2/4}$$

$$I = \boxed{58.1 \text{ W/m}^2}$$

P36.84 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the second surface, for which $R = -6.00$ cm. The incident rays are parallel, so $p = \infty$.



ANS. FIG. P36.84

Then, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

becomes $0 + \frac{1}{q} = \frac{1.00 - 1.56}{-6.00 \text{ cm}}$

and $\boxed{q = 10.7 \text{ cm}}$.

P36.85 Use the lens makers' equation, Equation 36.15, and the conventions of Table 36.2. The first lens has focal length described by

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_{1,1}} - \frac{1}{R_{1,2}}\right) = (n_1 - 1)\left(\frac{1}{\infty} - \frac{1}{R}\right) = \frac{1 - n_1}{R}$$

For the second lens

$$\frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{R_{2,1}} - \frac{1}{R_{2,2}}\right) = (n_2 - 1)\left(\frac{1}{+R} - \frac{1}{-R}\right) = +\frac{2(n_2 - 1)}{R}$$

Let an object be placed at any distance p_1 large compared to the thickness of the doublet. The first lens forms an image according to

$$\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f_1}$$

$$\frac{1}{q_1} = \frac{1 - n_1}{R} - \frac{1}{p_1}$$

This virtual ($q_1 < 0$) image (to the left of lens 1) is a real object for the second lens at distance $p_2 = -q_1$. For the second lens

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$$

$$\frac{1}{q_2} = \frac{2n_2 - 2}{R} - \frac{1}{p_2} = \frac{2n_2 - 2}{R} + \frac{1}{q_1} = \frac{2n_2 - 2}{R} + \frac{1 - n_1}{R} - \frac{1}{p_1}$$

$$= \frac{2n_2 - n_1 - 1}{R} - \frac{1}{p_1}$$

Then $\frac{1}{p_1} + \frac{1}{q_2} = \frac{2n_2 - n_1 - 1}{R}$ so the doublet behaves like a single lens with $\frac{1}{f} = \frac{2n_2 - n_1 - 1}{R}$.

P36.86 Find the image position for light traveling to the left through the lens:

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f_L} \rightarrow q = \frac{pf_L}{p - f_L} = \frac{(0.300\text{ m})(0.200\text{ m})}{0.300\text{ m} - 0.200\text{ m}} = 0.600\text{ m}$$

Therefore, this image forms 0.600 m to the left of the lens. Find the image formed by light traveling to the right toward the mirror from an object distance of 1.30 m – 0.300 m = 1.00 m:

$$\frac{1}{p_M} + \frac{1}{q_M} = \frac{1}{f_M}$$

Solving and substituting numerical values gives

$$q_M = \frac{p_M f_M}{p_M - f_M} = \frac{(1.00\text{ m})(0.500\text{ m})}{1.00\text{ m} - 0.500\text{ m}} = 1.00\text{ m}$$

This image forms at the position of the original object. Therefore, as light continues to the left through the lens, it will form an image at a position 0.600 m to the left of the lens. As a result, *both* images form at the *same* position and there are not two locations at which the student can hold a screen to see images formed by this system.

P36.87 For the first lens, the thin lens equation gives

$$q_1 = \frac{f_1 p_1}{p_1 - f_1} = \frac{(-6.00\text{ cm})(12.0\text{ cm})}{12.0\text{ cm} - (-6.00\text{ cm})} = -4.00\text{ cm}$$

The first lens forms an image 4.00 cm to its left. The rays between the lenses diverge from this image, so the second lens receives diverging light. It sees a real object at distance

$$p_2 = d - (-4.00\text{ cm}) = d + 4.00\text{ cm}$$

For the second lens, when we require that $q_2 \to \infty$, the mirror-lens equation becomes $p_2 = f_2 = 12.0$ cm.

Since the object for the converging lens must be 12.0 cm to its left, and since this object is the image for the diverging lens, which is 4.00 cm to **its** left, the two lenses must be separated by 8.00 cm.

Mathematically,

$$d + 4.00\text{ cm} = f_2 = 12.0\text{ cm} \;\to\; d = \boxed{8.00\text{ cm}}$$

P36.88 For the first lens, the thin lens equation gives

$$q_1 = \frac{f_1 p}{p - f_1}$$

We require that $q_2 \to \infty$ for the second lens; the thin lens equation gives $p_2 = f_2$, where, in this case,

$$p_2 = d - q_1 = d - \frac{f_1 p}{p - f_1}$$

Therefore, from $p_2 = f_2$,

$$d - \frac{f_1 p}{p - f_1} = f_2$$

$$d = \frac{f_1 p}{p - f_1} + f_2 = \frac{f_1 p + f_2(p - f_1)}{p - f_1} = \boxed{\frac{p(f_1 + f_2) - f_1 f_2}{p - f_1}}$$

P36.89 The inverted image is formed by light that leaves the object and goes directly through the lens, never having reflected from the mirror. For the formation of this inverted image, we have

$$M = -\frac{q_1}{p_1} = -1.50 \quad \text{giving} \quad q_1 = +1.50 p_1$$

The thin lens equation then gives (with p and q in centimeters)

$$\frac{1}{p_1} + \frac{1}{1.50 p_1} = \frac{1}{10.0}$$

$$\frac{1.50}{1.50 p_1} + \frac{1}{1.50 p_1} = \frac{1}{10.0}$$

$$\frac{2.50}{1.50 p_1} = \frac{1}{10.0}$$

giving $p_1 = 10.0\left(\frac{2.50}{1.50}\right) = \boxed{16.7 \text{ cm}}$.

The upright image is formed by light that passes through the lens after reflecting from the mirror. The object for the lens in this upright image formation is the image formed by the mirror. In order for the lens to form the upright image at the same location as the inverted image, the image formed by the mirror must be located at the position of the original object (so the object distances, and hence image distances, are the same for both the inverted and upright images formed by the lens). Therefore, the object distance and the image distance for the mirror are equal, and their common value is

$$q_{\text{mirror}} = p_{\text{mirror}} = 40.0 - p_1 = 40.0 - 16.7 = +23.3$$

The mirror equation, $\frac{1}{p_{\text{mirror}}} + \frac{1}{q_{\text{mirror}}} = \frac{1}{f_{\text{mirror}}}$, then gives

$$\frac{1}{f_{\text{mirror}}} = \frac{1}{23.3 \text{ cm}} + \frac{1}{23.3 \text{ cm}} = \frac{2}{23.3 \text{ cm}}$$

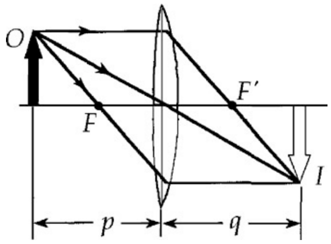
or $f_{\text{mirror}} = +\frac{23.3 \text{ cm}}{2} = \boxed{+11.7 \text{ cm}}$.

P36.90 (a) In the first situation, $\frac{1}{p_1} + \frac{1}{q_1} = \frac{1}{f}$, and

$$p_1 + q_1 = 1.50 \rightarrow q_1 = 1.50 - p_1$$

where f, p, and q are in meters.

Substituting, we have $\boxed{\frac{1}{f} = \frac{1}{p_1} + \frac{1}{1.50 - p_1}}$.



ANS. FIG. P36.90

(b) In the second situation, $\frac{1}{p_2}+\frac{1}{q_2}=\frac{1}{f}$,

$$p_2 = p_1 + 0.900\text{ m} \text{ and } q_2 = q_1 - 0.900\text{ m} = 0.600\text{ m} - p_1,$$

where f, p, and q are in meters.

Substituting, we have $\boxed{\frac{1}{f}=\frac{1}{p_1+0.900}+\frac{1}{0.600-p_1}}$.

(c) Both lens equation are equal:

$$\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f}=\frac{1}{p_2}+\frac{1}{q_2}$$

$$\frac{1}{p_1}+\frac{1}{1.50-p_1}=\frac{1}{p_1+0.900}+\frac{1}{0.600-p_1}$$

$$\frac{1.50-p_1+p_1}{p_1(1.50-p_1)}=\frac{0.600-p_1+p_1+0.900}{(p_1+0.900)(0.600-p_1)}$$

$$\frac{1.50}{p_1(1.50-p_1)}=\frac{1.50}{(p_1+0.900)(0.600-p_1)}$$

Simplified, this becomes

$$p_1(1.50-p_1)=(p_1+0.900)(0.600-p_1)$$

$$1.50p_1-p_1^2=(0.600-0.900)p_1+(0.900)(0.600)-p_1^2$$

$$1.80p_1=0.540$$

$$p_1=\boxed{0.300\text{ m}}$$

(d) From part (a), $\frac{1}{f}=\frac{1}{p_1}+\frac{1}{1.50-p_1}$:

$$\frac{1}{f}=\frac{1}{0.300}+\frac{1}{1.50-0.300}$$

$$f=\boxed{0.240\text{ m}}$$

P36.91 (a) For the mirror, $f = \frac{R}{2} = +1.50$ m. In addition, because the distance to the Sun is so much larger than any other distances, we can take $p = \infty$.

The mirror equation, $\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$, then gives $q = f = \boxed{1.50 \text{ m in front of the mirror}}$.

(b) Now, in $M = -\frac{q}{p} = \frac{h'}{h}$,

the magnification is nearly zero, but we can be more precise: $\frac{h}{p} = 0.533°$ is the angular diameter of the object. Thus,

$$h' = -\frac{h}{p}q = -\left[(0.533°)\left(\frac{\pi \text{ rad}}{180°}\right)\right](1.50 \text{ m}) = -0.014\,0 \text{ m}$$
$$= -1.40 \text{ cm}$$

and the image diameter is $\boxed{1.40 \text{ cm}}$.

P36.92 (a) For lens one, as shown in the top panel in ANS. FIG. P36.92,

$$\frac{1}{40.0 \text{ cm}} + \frac{1}{q_1} = \frac{1}{30.0 \text{ cm}}$$
$$q_1 = 120 \text{ cm}$$

This real image is the object of the second lens: $I_1 = O_2$; it is *behind* the lens, as shown in the middle panel in ANS. FIG. P36.92, so it is a virtual object for the second lens. That is, the object distance is

$$p_2 = 110 \text{ cm} - 120 \text{ cm} = -10.0 \text{ cm}$$
$$\frac{1}{-10.0 \text{ cm}} + \frac{1}{q_2} = \frac{1}{-20.0 \text{ cm}}:$$
$$q_2 = \boxed{20.0 \text{ cm}}$$

(b) From part (a),

$$M_1 = -\frac{q_1}{p_1} = -\frac{120 \text{ cm}}{40.0 \text{ cm}} = -3.00$$

$$M_2 = -\frac{q_2}{p_2} = -\frac{20.0 \text{ cm}}{(-10.0 \text{ cm})} = +2.00$$

$$M_{\text{overall}} = M_1 M_2 = \boxed{-6.00}$$

(c) $M_{\text{overall}} < 0$, so final image is $\boxed{\text{inverted}}$.

(d) If lens two is a converging lens (bottom panel in ANS. FIG. P36.92):

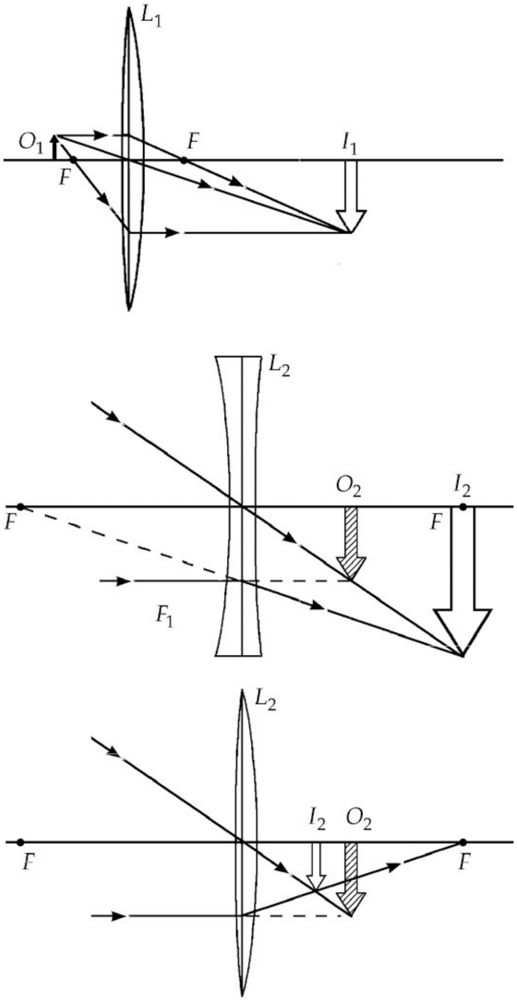
$$\frac{1}{-10.0\text{ cm}} + \frac{1}{q_2} = \frac{1}{20.0\text{ cm}}$$

$$q_2 = \boxed{6.67\text{ cm}}$$

$$M_2 = -\frac{6.67\text{ cm}}{(-10.0\text{ cm})} = +0.667$$

$$M_{\text{overall}} = M_1M_2 = \boxed{-2.00}$$

Again, $M_{\text{overall}} < 0$ and the final image is $\boxed{\text{inverted}}$.



ANS. FIG. P36.92

Challenge Problems

P36.93 (a) For the light the mirror intercepts, the power is given by

$$P = I_0 A = I_0 \pi R_a^2$$

Substituting,

$$350 \text{ W} = \left(1\,000 \text{ W/m}^2\right)\pi R_a^2$$

and $R_a = \boxed{0.334 \text{ m or larger}}$.

(b) In $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}=\frac{2}{R}$ we have $p \to \infty$, so $q = \frac{R}{2}$ and

$$M = \frac{h'}{h} = -\frac{q}{p},$$

so $$h' = -q\left(\frac{h}{p}\right) = -\left(\frac{R}{2}\right)\left[0.533°\left(\frac{\pi \text{ rad}}{180°}\right)\right] = -\left(\frac{R}{2}\right)(9.30 \text{ m rad})$$

where $\frac{h}{p}$ is the angle the Sun subtends.

The intensity at the image is then

$$I = \frac{P}{\pi h'^2/4} = \frac{4I_0\pi R_a^2}{\pi h'^2} = \frac{4I_0 R_a^2}{h'^2}$$

$$I = \frac{4I_0 R_a^2}{(R/2)^2\left(9.30\times 10^{-3} \text{ rad}\right)^2}$$

$$120\times 10^3 \text{ W/m}^2 = \frac{16\left(1\,000 \text{ W/m}^2\right)R_a^2}{R^2\left(9.30\times 10^{-3} \text{ rad}\right)^2}$$

$$\frac{R_a^2}{R^2} = 6.49\times 10^{-4}$$

So, $\boxed{\frac{R_a}{R} = 0.025\,5 \text{ or larger}}$.

P36.94 (a) From the thin lens equation,

$$\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1} = \frac{1}{5 \text{ cm}} - \frac{1}{7.5 \text{ cm}} \rightarrow q_1 = 15 \text{ cm}$$

and, from the definition of magnification,

$$M_1 = -\frac{q_1}{p_1} = -\frac{15 \text{ cm}}{7.5 \text{ cm}} = -2$$

Then, for a combination of two lenses,

$$M = M_1M_2: \; 1 = (-2)M_2$$

or

$$M_2 = -\frac{1}{2} = -\frac{q_2}{p_2} \rightarrow p_2 = 2q_2$$

From the thin lens equation for the second lens,

$$\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}: \; \frac{1}{2q_2} + \frac{1}{q_2} = \frac{1}{10\text{ cm}} \rightarrow q_2 = 15\text{ cm}, p_2 = 30\text{ cm}$$

So the distance between the object and the screen is

$$p_1 + q_1 + p_2 + q_2 = 7.5\text{ cm} + 15\text{ cm} + 30\text{ cm} + 15\text{ cm} = \boxed{67.5\text{ cm}}$$

(b) In the following, if no units are shown, assume all distances (*p*, *q*, and *f*) are in units of cm.

For lens 1, we have $\frac{1}{p_1'} + \frac{1}{q_1'} = \frac{1}{f_1} = \frac{1}{5}$. Solve for q_1' in terms of p_1':

$$q_1' = \frac{5p_1'}{p_1' - 5} \qquad \textbf{[1]}$$

Now we have $M_1' = -\frac{q_1'}{p_1'} = -\frac{5}{p_1' - 5}$, using [1]. From $M' = M_1'M_2' = 3$, we have

$$M_2' = \frac{M'}{M_1'} = -\frac{3}{5}(p_1' - 5) = -\frac{q_2'}{p_2'}$$

$$q_2' = \frac{3}{5}p_2'(p_1' - 5) \qquad \textbf{[2]}$$

Substitute [2] into the lens equation for lens 2, $\frac{1}{p_2'} + \frac{1}{q_2'} = \frac{1}{f_2} = \frac{1}{10\text{ cm}}$, and obtain p_2' in terms of p_1':

$$p_2' = \frac{10(3p_1' - 10)}{3(p_1' - 5)} \qquad \textbf{[3]}$$

Substitute [3] into [2], to obtain q_2' in terms of p_1':

$$q_2' = 2(3p_1' - 10) \qquad \textbf{[4]}$$

We know that the distance from object to the screen is a constant:

$$p_1' + q_1' + p_2' + q_2' = \text{ a constant} \qquad \textbf{[5]}$$

Using [1], [3], and [4], and the value obtained in part (a), [5] becomes

$$p_1' + \frac{5p_1'}{p_1' - 5} + \frac{10\left(3p_1' - 10\right)}{3\left(p' - 5\right)} + 2\left(3p_1' - 10\right) = 67.5 \qquad \textbf{[6]}$$

Multiplying equation [6] by $3\left(p_1' - 5\right)$, we have

$$\left[3\left(p_1' - 5\right)\right]p_1' + 15p_1' + 10\left(3p_1' - 10\right)$$

$$+ 2\left(3p_1' - 10\right)\left[3\left(p_1' - 5\right)\right] = 67.5\left[3\left(p_1' - 5\right)\right]$$

$$3p_1'^2 \cancel{-15p_1' + 15p_1'} + 30p_1'$$

$$- 100 + 6\left(3p_1'^2 - 25p_1' + 50\right) = 202.5p_1' - 1012.5$$

$$3p_1'^2 + 30p_1' - 100 + 18p_1'^2 - 150p_1' + 300 - 202.5p_1' + 1012.5 = 0$$

This reduces to the quadratic equation

$$21p_1'^2 - 322.5p_1' + 1\,212.5 = 0$$

which has solutions $p_1' = 8.784 \text{ cm}$ and 6.573 cm.

Case 1: $p_1' = 8.784 \text{ cm}$

$$\therefore\; p_1' - p_1 = 8.784 \text{ cm} - 7.50 \text{ cm} = 1.28 \text{ cm}$$

From [4]: $q_2' = 32.7 \text{ cm}$

$$\therefore\; q_2' - q_2 = 32.7 \text{ cm} - 15.0 \text{ cm} = 17.7 \text{ cm}$$

Case 2: $p_1' = 6.573 \text{ cm}$

$$\therefore\; p_1' - p_1 = 6.573 \text{ cm} - 7.50 \text{ cm} = -0.927 \text{ cm}$$

From [4]: $q_2' = 19.44 \text{ cm}$

$$\therefore\; q_2' = q_2 = 19.44 \text{ cm} - 15.0 \text{ cm} = 4.44 \text{ cm}$$

From these results it is concluded that:

The lenses can be displaced in two ways. The first lens can be moved 1.28 cm farther from the object and the second lens 17.7 cm toward the object. Alternatively, the first lens can be moved 0.927 cm toward the object and the second lens 4.44 cm toward the object.

P36.95 (a) The lens makers' equation, $\frac{1}{f}=(n-1)\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$, becomes:

$$\frac{1}{5.00\text{ cm}}=(n-1)\left[\frac{1}{9.00\text{ cm}}-\frac{1}{(-11.0\text{ cm})}\right]$$

giving $n=\boxed{1.99}$.

(b) As the light passes through the lens for the first time, the thin lens equation, $\frac{1}{p_1}+\frac{1}{q_1}=\frac{1}{f}$, becomes:

$$\frac{1}{8.00\text{ cm}}+\frac{1}{q_1}=\frac{1}{5.00\text{ cm}}$$

giving $q_1 = 13.3$ cm, and $M_1=-\frac{q_1}{p_1}=-\frac{13.3\text{ cm}}{8.00\text{ cm}}=-1.67$.

This image becomes the object for the concave mirror with:

$$p_M = 20.0\text{ cm}-q_1 = 20.0\text{ cm}-13.3\text{ cm}=6.67\text{ cm}$$

and $f=\frac{R}{2}=+4.00$ cm.

The mirror equation becomes: $\frac{1}{6.67\text{ cm}}+\frac{1}{q_M}=\frac{1}{4.00\text{ cm}}$,

giving $q_M = 10.0$ cm,

and $M_2=-\frac{q_M}{p_M}=-\frac{10.0\text{ cm}}{6.67\text{ cm}}=-1.50$.

The image formed by the mirror serves as a real object for the lens on the second pass of the light through the lens, with

$$p_3 = 20.0\text{ cm}-q_M=+10.0\text{ cm}$$

The thin lens equation yields: $\frac{1}{10.0\text{ cm}}+\frac{1}{q_3}=\frac{1}{5.00\text{ cm}}$,

or $q_3 = 10.0$ cm

and $M_3=-\frac{q_3}{p_3}=-\frac{10.0\text{ cm}}{10.0\text{ cm}}=-1.00$.

The final image is a real image located $\boxed{\text{10.0 cm to the left of the lens}}$.

(c) From above, we find the overall magnification:

$$M_{\text{total}}=M_1M_2M_3=\boxed{-2.50}$$

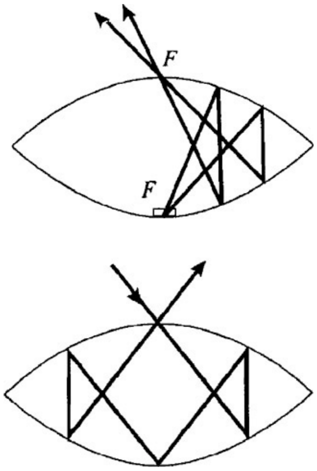
(d) The overall magnification is negative, so the final image is $\boxed{\text{inverted}}$.

P36.96 (a) The object is located at the focal point of the upper mirror. Thus, the upper mirror creates an image at infinity (i.e., parallel rays leave this mirror). For the upper mirror, the object is real, and the mirror equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, gives

$$\frac{1}{7.50\text{ cm}}+\frac{1}{q_1}=\frac{1}{7.50\text{ cm}}$$

$$\rightarrow\ q_1\approx\infty\text{ (very large)}$$

The lower mirror focuses these parallel rays at its focal point, located at the hole in the upper mirror. For the lower mirror, the object is virtual (behind the mirror), $p_2\approx-\infty$:



ANS. FIG. P36.96

$$\frac{1}{-\infty}+\frac{1}{q_2}=\frac{1}{7.50\text{ cm}}\ \rightarrow\ q_2=7.50\text{ cm}$$

The overall magnification is

$$M=m_1m_2=\left(\frac{-q_1}{p_1}\right)\left(\frac{-q_2}{p_2}\right)=\left(\frac{\infty}{7.50\text{ cm}}\right)\left(\frac{7.50\text{ cm}}{-\infty}\right)=-1$$

Thus, the $\boxed{\text{image is real, inverted, and actual size}}$.

(b) Light travels the same path regardless of direction, so light shined on the image is directed to the actual object inside, and the light then reflects and is directed back to the outside. Light directed into the hole in the upper mirror reflects as shown in the lower figure, to behave as if it were reflecting from the image.

P36.97 First, we solve for the image formed by light traveling to the left through the lens. The object distance is $p_L=p$, so

$$\frac{1}{p_L}+\frac{1}{q_L}=\frac{1}{f_L}\ \rightarrow\ \frac{1}{q_L}=\frac{1}{f_L}-\frac{1}{p}$$

Next, we solve for the image formed by light traveling to the right and reflecting off the mirror. The object distance is $p_M=d-p$, so

$$\frac{1}{p_M}+\frac{1}{q_M}=\frac{1}{f_M}\ \rightarrow\ \frac{1}{q_M}=\frac{1}{f_M}-\frac{1}{p_M}=\frac{p_M-f_M}{f_Mp_M}$$

$$q_M=\frac{f_Mp_M}{p_M-f_M}=\frac{f_M(d-p)}{d-p-f_M}$$

If q_M is positive (real image), the image formed by the mirror will be to its left, and if q_M is negative (virtual image), the image formed by the mirror will be to its right; for either case, the image formed by the mirror acts as an object for the lens at a distance p_L':

$$p_L' = d - q_M = d - \frac{f_M(d-p)}{(d-p)-f_M} = \frac{d(d-p-f_M)-f_M(d-p)}{d-p-f_M}$$

We solve for the position of the final image q'_L:

$$\frac{1}{q_L'} = \frac{1}{f_L} - \frac{1}{p_L'} = \frac{1}{f_L} - \frac{d-p-f_M}{d(d-p-f_M)-f_M(d-p)}$$

For the two images formed by the lens to be at the same place,

$$\frac{1}{q_L} = \frac{1}{q_L'} \rightarrow \frac{1}{f_L} - \frac{1}{p_L} = \frac{1}{f_L} - \frac{1}{p_L'} \rightarrow p_L' = p_L$$

Therefore,

$$\frac{d(d-p-f_M)-f_M(d-p)}{d-p-f_M} = p$$

$$d(d-p-f_M)-f_M(d-p) = p(d-p-f_M)$$

$$d^2 - pd - f_M d - f_M d + f_M p = pd - p^2 - f_M p$$

$$d^2 - 2(p+f_M)d + (2f_M p + p^2) = 0$$

Solving for d then gives

$$d = \frac{2(p+f_M) \pm \sqrt{4(p+f_M)^2 - 4(1)(2f_M p + p^2)}}{2(1)}$$

$$d = \frac{2(p+f_M) \pm \sqrt{4p^2 + 8f_M p + 4f_M^2 - 8f_M p - 4p^2}}{2}$$

$$d = \frac{2(p+f_M) \pm \sqrt{4f_M^2}}{2} = (p+f_M) \pm f_M$$

Therefore, $\boxed{d = p \text{ and } d = p + 2f_M}$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P36.2	4.58 m
P36.4	(1) 4.00 m; (2) 12.00 m; (3) 16.00 m
P36.6	See ANS. FIG. P36.6 for the locations of the five images.
P36.8	(a) 33.3 cm in front of the mirror; (b) –0.666; (c) real; (d) inverted
P36.10	(a) See ANS FIG P36.10; (b) $q = -40.0$ cm, so the image is behind the mirror; (c) $M = +2.00$, so the image is enlarged and upright; (d) See P36.10(d) for full explanation.
P36.12	(a) –26.7 cm; (b) upright; (c) 0.026 7
P36.14	(a) +2.22 cm; (b) +10.0
P36.16	A convex mirror *diverges* light rays incident upon it, so the mirror in this problem cannot focus the Sun's rays to a point.
P36.18	(a) 0.708 m in front of the sphere; (b) upright
P36.20	(a) $\frac{ad}{a^2-1}$; (b) $\frac{ad}{a^2-1}$
P36.22	(a) 8.00 cm; (b) See ANS. FIG. P36.22(b); (c) virtual
P36.24	(a) 16.0 cm from the mirror; (b) +0.333; (c) upright
P36.26	(a) See P36.26(a) for full explanation; (b) real image at 0.639 s and virtual image at 0.782 s
P36.28	8.05 cm
P36.30	38.2 cm below the top surface
P36.32	3.75 mm
P36.34	See P36.34 for full explanation.
P36.36	(a) (i) 3.77 cm from the front of the wall, in the water, (ii) 19.3 cm from the front wall, in the water; (b) (i) +1.01, (ii) +1.03; (c) The plastic has uniform thickness, so the surfaces of entry and exit for any particular ray are very nearly parallel. The ray is slightly displaced, but it would not be changed in direction by going through the plastic wall with air on both sides. Only the difference between the air and water is responsible for the refraction of the light; (d) yes; (e) If $p = \lvert R \rvert$, then $q = -p = -\lvert R \rvert$; if $p > \lvert R \rvert$, then $\lvert q \rvert > \lvert R \rvert$. For example, if $p = 2\lvert R \rvert$, then $q = -3.00\lvert R \rvert$ and $M = +2.00$.
P36.38	(a) 650 cm, real, inverted, enlarged; (b) –600 cm, virtual, upright, enlarged

P36.40 (a) 12.3 cm to the left of the lens; (b) 0.615; (c) See ANS. FIG. P36.40.

P36.42 (a) The image is in back of the lens at a distance of $1.25f$ from the lens; (b) –0.250; (c) real

P36.44 (i) See ANS. FIG P36.44(i): (a) 20.0 cm in back of the lens, (b) real, (c) inverted, (d) $M = -1.00$, (e) Algebraic answers agree, and we can express values to three significant figures: $q = 20.0$ cm, $M = -1.00$; (ii) See ANS. FIG. P36.44(ii): (a) 10 cm front of the lens, (b) virtual, (c) upright, (d) $M = +2.00$, (e) Algebraic answers agree, and we can express values to three significant figures: $q = -10.0$ cm, $M = +2.00$, (f) Small variations from the correct directions of rays can lead to significant errors in the intersection point of the rays. These variations may lead to the three principal rays not intersecting at a single point.

P36.46 (i): (a) 13.3 cm in front of the lens, (b) virtual, (c) upright, (d) +0.333; (ii): (a) 10.0 cm in front of the lens, (b) virtual, (c) upright, (d) +0.500; (iii): (a) 6.67 cm in front of the lens, (b) virtual, (c) upright, (d) +0.667

P36.48 $dq = -\frac{q^2}{p^2}dp$

P36.50 (a) $q_a = 26.3$ cm, $q_d = 46.7$ cm, –8.75 cm, –23.3 cm; (b) See ANS. FIG. P36.50(b); (c) See P36.50(c) for full explanation; (d) The integral stated adds up the areas of ribbons covering the whole image, each with vertical dimension $|h'|$ and horizontal width dq; (e) 328 cm^2.

P36.52 See P36.52 for full explanation.

P36.54 (a) –34.7 cm; (b) –36.1 cm

P36.56 $f/1.4$

P36.58 (a) –4.00 diopters; (b) diverging lens

P36.60 (a) –25.0 cm; (b) nearsighted; (c) –3.70 diopters

P36.62 (a) $+50.8 \text{ diopters} \le P \le 60.0 \text{ diopters}$; (b) –0.800 diopters, diverging

P36.64 The image is inverted, real, and diminished.

P36.66 (a) 4.17 cm; (b) 6.00

P36.68 (a) –800; (b) inverted

P36.70 (a) See P36.70(a) for full explanation; (b) $-\frac{hf}{p}$; (c) –1.07 mm

P36.72 –25.0 cm

P36.74 $f = \frac{-Md}{(1-M)^2}$ when the lens is diverging; $f = \frac{Md}{(M-1)^2}$ when the lens is converging

P36.76 (a) 0.833 mm; (b) 0.820 mm

P36.78 (a) 13.3 cm in front of the first lens; (b) –6.00; (c) inverted; (d) virtual

P36.80 (a) $x' = \dfrac{1024 - 58.0x}{6.0 - x}$ where x and x' is are in centimeters; (b) See P36.80(b) for full explanation; (c) The image moves to infinity and beyond—meaning it moves forward to infinity (on the right), jumps back to minus infinity (on the left), and then proceeds forward again; (d) The image usually travels to the right, except when it jumps from plus infinity (right) to minus infinity (left).

P36.82 (a) See P36.82(a) for full explanation; (b) 1.75 times

P36.84 $q = 10.7$ cm

P36.86 See P36.86 for full explanation

P36.88 $\dfrac{p\left(f_1 + f_2\right) - f_1 f_2}{p - f_1}$

P36.90 (a) $\dfrac{1}{f} = \dfrac{1}{p_1} + \dfrac{1}{1.50 - p_1}$; (b) $\dfrac{1}{f} = \dfrac{1}{p_1 + 0.900} + \dfrac{1}{0.600 - p_1}$; (c) 0.300 m; (d) 0.240 m

P36.92 (a) 20.0 cm; (b) –6.00; (c) inverted; (d) $q_2 = 6.67$ cm and $M_{\text{overall}} = -2.00$, inverted

P36.94 (a) 67.5 cm; (b) The lenses can be displaced in two ways. The first lens can be moved 1.28 cm farther from the object and the second lens 17.7 cm toward the object. Alternatively, the first lens can be moved 0.927 cm toward the object and the second lens 4.44 cm toward the object.

P36.96 (a) The image is real, inverted, and actual size; (b) Light travels the same path regardless of direction, so light shined on the image is directed to the actual object inside, and the light then reflects and is directed back to the outside. Light directed into the hole in the upper mirror reflects as shown in the lower figure, to behave as if it were reflecting from the image.

37

Wave Optics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ37.1 **(i)** Answer (a). If the mirrors do not move the character of the interference stays the same.

(ii) Answer (c). The light waves destructively interfere so they are initially out of phase by 180°. Moving the mirror by $\lambda/2$ changes the path difference by $2(\lambda/2) = \lambda$, so the waves go in phase then back out of phase to their original phase relation.

OQ37.2 **(i)** The ranking is b > a > c = d. The angles in the interference pattern are small, so we can make a good approximation of their values: $d\sin\theta = m\lambda \rightarrow \theta \approx m\lambda/d$. Thus for $m = 1$, $\theta \approx \lambda/d$, which we estimate in each case: (a) $0.450\ \mu\text{m}/400\ \mu\text{m} \approx 1.1 \times 10^{-3}$ rad (b) $0.7\ \mu\text{m}/400\ \mu\text{m} \approx 1.8 \times 10^{-3}$ rad (c) and (d) $0.7\ \mu\text{m}/800\ \mu\text{m} \approx 0.9 \times 10^{-3}$ rad.

(ii) The ranking is b = d > a > c. Now we consider the distance

$$y = L\tan\theta \approx L\sin\theta = L\left(m\lambda/d\right) \quad \rightarrow \quad y \approx mL\lambda/d$$

Thus for $m = 1$, $y \approx L\lambda/d$, which we estimate in each case:

(a) (4 m) (0.45 μm/400 μm) ≈ 4.5 mm; (b) (4 m)(0.7 μm/400 μm) ≈ 7 mm; (c) (4 m)(0.7 μm/800 μm) ≈ 3.5 mm;
(d) (8 m)(0.7 μm/800 μm) ≈ 7 mm.

OQ37.3 Answer (c). Underwater, the wavelength of the light decreases according to $\lambda_{\text{water}} = \lambda_{\text{air}}/n_{\text{water}}$. Since the angles between positions of light and dark bands, being small, are approximately proportional to λ, the underwater fringe separations decrease.

OQ37.4 **(i)** Answer (c). The distance between nodes is half a wavelength.

(ii) Answer (d). The reflected light travels through the same path twice because it reflects, so moving the mirror one-quarter wavelength, 125 nm, results in a path change of one-half wavelength, 250 nm, which results in destructive interference.

(iii) Answer (e). The wavelength of the light in the film is 500 nm/2 = 250 nm. If the film is made 62.5 nm thicker (one-quarter wavelength in the film), the light reflecting inside the film has a path length 125 nm greater. This is half a wavelength, which reverses constructive into destructive interference.

OQ37.5 Answer (d). There are 180° phase changes occurring in the reflections at both the air-oil boundary and the oil-water boundary; thus the relative phase change from reflection is zero. The condition for constructive interference in the reflected light is

$$2t = m\frac{\lambda}{n} \quad \to \quad t = m\frac{\lambda}{2n}$$

where m is any integer. The minimum non-zero thickness of the oil which will strongly reflect 530-nm light is $m = 1$:

$$t = m\frac{\lambda}{2n} = (1)\frac{530\text{ nm}}{2(1.25)} = 212\text{ nm}$$

O37.6 Answer (a). For the second-order bright fringe,

$$d\sin\theta = 2\lambda$$

$$\sin\theta = 2\left(\frac{500\times10^{-9}\text{ m}}{2.00\times10^{-5}\text{ m}}\right)$$

$$\theta = 0.050\ 0\text{ rad}$$

OQ37.7 **(i)** Answer (b). If the oil film is brightest where it is thinnest, then $n_{\text{air}} < n_{\text{oil}} < n_{\text{flint glass}}$. With this condition, light reflecting from both the top and the bottom surface of the oil film will undergo 180° phase changes. Then these two beams will be in phase with each other where the film is very thin. This is the condition for constructive interference as the thickness of the oil film decreases toward zero. If the oil film is dark where it is thinnest, then $n_{\text{air}} < n_{\text{oil}} > n_{\text{crown glass}}$. In this case, reflecting light undergoes a 180° phase change upon reflection from the top surface but no 180° phase change upon reflection from the bottom surface of the oil. The two reflected beams are 180° out of phase and interfere destructively as the oil film thickness goes to zero.

(ii) Yes. It should have a lower refractive index than both kinds of glass.

(iii) Yes. It should have a higher refractive index than both kinds of glass.

(iv) No. Its refractive index cannot be both greater than 1.66 and less than 1.52.

OQ37.8 Answer (b). With two fine slits separated by a distance d slightly less than λ, the equation $d\sin\theta = 0$ has the usual solution $\theta = 0$, but $d\sin\theta = \lambda$ has no solution: there is no first-order maximum. However, $d\sin\theta = \frac{1}{2}\lambda$ has a solution: first-order minima flank the central maximum on each side.

OQ37.9 **(i)** Answer (a). The angular position of the mth-order bright fringe in a double-slit interference pattern is given by $d\sin\theta_m = m\lambda$. The distance y_m of the mth-order bright fringe from the center of the pattern is given by $y_m = L\tan\theta_m$, where L is the distance to the screen. The spacing between successive bright fringes is

$$\begin{aligned}\Delta y = y_{m+1} - y_m &= L\left(\tan\theta_{m+1} - \tan\theta_m\right)\\ &\approx L\left(\sin\theta_{m+1} - \sin\theta_m\right)\\ &= L\frac{\left[(m+1) - m\right]}{d}\lambda = \frac{L}{d}\lambda\end{aligned}$$

because the angles are small, and for small angles (in radians) $\sin\theta \simeq \tan\theta$. As L increases, the spacing Δy increases.

(ii) Answer (b). From our result above, we see that as d increases, the spacing Δy decreases.

OQ37.10 Answer (b). If the thickness of the oil film were smaller than half of the wavelengths of visible light, no colors would appear. If the thickness of the oil film were much larger, the colors would overlap to mix to white or gray.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ37.1 A camera lens will have more than one element, to correct (at least) for chromatic aberration. It will have several surfaces, each of which would reflect some fraction of the incident light. To maximize light throughout, the surfaces need antireflective coatings. The coating thickness is chosen to produce destructive interference for reflected light of a particular wavelength.

CQ37.2 Due to gravity, the soap film tends to sag in its holder, being quite thin at the top and becoming thicker as one moves toward the bottom of the holding ring. Because light reflecting from the front surface of the film experiences a 180° phase change, and light reflecting from the back surface of the film does not (see Figure 37.10 in the textbook), the film must be a minimum of a half wavelength thick before it can produce constructive interference in the reflected light. Thus, the light must be striking the film at some distance from the top of the ring before the thickness is sufficient to produce constructive interference for any wavelength in the visible portion of the spectrum.

CQ37.3 The light from the flashlights consists of many different wavelengths (that's why it's white) with random time differences between the light waves. There is no *coherence* between the two sources. The light from the two flashlights does not maintain a constant phase relationship over time. These three equivalent statements mean no possibility of an interference pattern.

CQ37.4 Typically, a thin air film forms between the lens and the glass plate. Light reflecting from the upper surface of the air film (lower surface of the lens) can interfere with light reflecting from the lower surface of the air film (upper surface of the flat glass plate). The light reflecting from the *lower* surface of the air film undergoes a 180° phase change on reflection while the light reflecting from the *upper* surface of the air film does not. (a) Where there is negligible distance between the surfaces, at the center of the pattern you will see a dark spot because of the destructive interference associated with the 180° phase shift. (b) Colored rings surround the dark spot. If the lens is a perfect sphere and the plate is perfectly flat, the rings are perfect circles. On the fine scale of the wavelength of visible light, distorted rings reveal bumps and hollows that cause variation in the air film between the glass surfaces.

CQ37.5 The waves interfere destructively at some places and interfere constructively at others. The total energy is not lost, it is just rearranged. The energy that does not go into the dark fringes is shifted into the bright fringes.

CQ37.6 Every color produces its own double-slit interference pattern, so if white light is used, the central maximum is white and the first-order maxima are full spectra running from violet to red. Each higher-order maximum is in principle a full spectrum, but it can partially overlap with the next order maximum, so the pattern for a specific color is hard to distinguish. Using monochromatic light eliminates this problem.

CQ37.7 (a) Two waves interfere constructively if their path difference is zero, or an integral multiple of the wavelength, according to $\delta = m\lambda$, with $m = 0, 1, 2, 3, \ldots$.

(b) Two waves interfere destructively if their path difference is a half wavelength, or an odd multiple of $\frac{\lambda}{2}$, described by

$$\delta = \left(m + \frac{1}{2}\right)\lambda, \text{ with } m = 0, 1, 2, 3, \ldots.$$

CQ37.8 Each liquid forms a film which causes interference of light reflected off the top and bottom surfaces of the film. Since the liquids would have an index greater than that of air, light reflected off the top surface of each film would undergo a 180° phase change. When the films become sufficiently thin, the type of interference that occurs, constructive or destructive, depends on whether the reflected wave does or does not undergo a 180° phase change. If the index of one liquid is less than that of water, light reflected off the bottom surface of the film (off the water surface) will be shifted by 180°, so the overall interference will be constructive, and the film will appear bright. If the index of the other liquid is greater than that of water, light reflected off the bottom surface of the film will not be shifted, so the overall interference will be destructive, and the film will appear dark.

CQ37.9 Yes. A single beam of laser light going into the slits divides up into several fuzzy-edged beams diverging from the point halfway between the slits.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 27.1 Young's Double-Slit Experiment

Section 27.2 Analysis Model: Waves in Interference

***P37.1** The angular locations of the bright fringes (or maxima) is given by Equation 37.2:

$$d\sin\theta = m\lambda$$

Solving for m and substituting 30.0° gives

$$m = \frac{d\sin\theta}{\lambda} = \frac{\left(3.20\times10^{-4}\ \text{m}\right)\sin 30.0^\circ}{500\times10^{-9}\ \text{m}} = 320$$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead at $\theta = 0$. There are therefore $\boxed{\text{641 maxima}}$.

P37.2 The location of the dark fringe of order m (measured from the position of the central maximum) is given by

$$(y_{\text{dark}})_m = \left(m+\frac{1}{2}\right)\left(\frac{L\lambda}{d}\right)$$

where $m = 0, \pm1, \pm2, \ldots$ Thus, the spacing between the first and second dark fringes will be

$$\begin{aligned}\Delta y &= \left(y_{\text{dark}}\right)_{m=1} - \left(y_{\text{dark}}\right)_{m=0} \\ &= \left(1+\frac{1}{2}\right)\left(\frac{L\lambda}{d}\right) - \left(0+\frac{1}{2}\right)\left(\frac{L\lambda}{d}\right) = \frac{L\lambda}{d}\end{aligned}$$

or $$\Delta y = \frac{\left(5.30\times10^{-7}\ \text{m}\right)\left(2.00\ \text{m}\right)}{0.300\times10^{-3}\ \text{m}} = 3.53\times10^{-3}\ \text{m} = \boxed{3.53\ \text{mm}}$$

P37.3 The location of the bright fringe of order m (measured from the position of the central maximum) is

$$d\sin\theta = m\lambda \qquad m = 0, \pm1, \pm2, \ldots.$$

For first bright fringe to the side, $m = 1$. Thus, the wavelength of the laser light must be

$$\begin{aligned}\lambda &= d\sin\theta = (0.200\times10^{-3}\text{m})\sin 0.181^\circ \\ &= 6.32\times10^{-7}\ \text{m} = \boxed{632\ \text{nm}}\end{aligned}$$

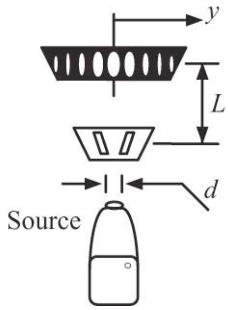
P37.4 The location of the bright fringes for small angles is given by Equation 37.7:

$$y_{\text{bright}} = \frac{\lambda L}{d}m$$

For $m = 1$,

$$\lambda = \frac{y_{\text{bright}}}{L} = \frac{(3.40\times10^{-3}\ \text{m})(0.500\times10^{-3}\ \text{m})}{3.30\ \text{m}} = \boxed{515\ \text{nm}}$$

P37.5 In the equation $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$, the first minimum is described by $m = 0$ and the tenth by $m = 9$:



ANS. FIG. P37.5

$$\sin\theta = \frac{\lambda}{d}\left(9+\frac{1}{2}\right) = 9.5\frac{\lambda}{d}$$

Also, $\tan\theta = \frac{y}{L}$. But, for small θ, $\sin\theta \approx \tan\theta$.

Thus, $d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y}$:

$$d = \frac{9.5(5\,890\times10^{-10}\ \text{m})(2.00\ \text{m})}{7.26\times10^{-3}\ \text{m}} = 1.54\times10^{-3}\ \text{m} = \boxed{1.54\ \text{mm}}$$

P37.6 We use Equation 37.2, $d\sin\theta_{\text{bright}} = m\lambda$, to find the angle for the $m = 1$ fringe:

$$\sin\theta_{\text{bright}} = \frac{m\lambda}{d} = \frac{(1)(1.00\times10^{-2}\ \text{m})}{8.00\times10^{-3}\ \text{m}} = 1.25$$

The sine of the angle is greater than 1, which is impossible. Therefore, there is no $m = 1$ fringe on the screen whose position can be measured. In fact, there is no interference pattern at all, just a bright area of microwaves directly behind the double slit.

P37.7 We do not use the small-angle approximation $\sin\theta \approx \tan\theta$ here because the angle is greater than 10°. For the first bright fringe, $m = 1$, and we have

$$d\sin\theta = m\lambda = \lambda$$

and $$d = \frac{\lambda}{\sin\theta} = \frac{620\times10^{-9}\ \text{m}}{\sin 15.0°} = 2.40\times10^{-6}\ \text{m} = \boxed{240\ \mu\text{m}}$$

P37.8 (a) For a bright fringe of order m, the path difference is $\delta = m\lambda$, where $m = 0, 1, 2, \ldots$ At the location of the third order bright fringe,

$$\delta = m\lambda = 3(589\times10^{-9}\ \text{m}) = 1.77\times10^{-6}\ \text{m} = \boxed{1.77\ \mu\text{m}}$$

(b) For a dark fringe, the path difference is $\delta = \left(m + \frac{1}{2}\right)\lambda$, where $m = 0, 1, 2, \ldots$ At the third dark fringe, $m = 2$ and

$$\delta = \left(2 + \frac{1}{2}\right)\lambda = \frac{5}{2}(589 \text{ nm}) = 1.47 \times 10^3 \text{ nm} = \boxed{1.47\ \mu\text{m}}$$

P37.9 (a) For the bright fringe,

$$y_{\text{bright}} = \frac{m\lambda L}{d}, \text{ where } m = 1$$

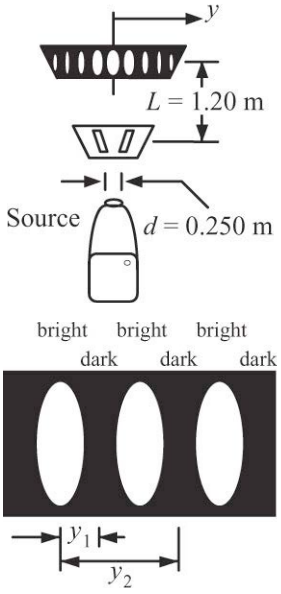
$$y = \frac{(546.1 \times 10^{-9} \text{ m})(1.20 \text{ m})}{0.250 \times 10^{-3} \text{ m}} = 2.62 \times 10^{-3} \text{ m}$$

$$= \boxed{2.62 \text{ mm}}$$

(b) If you have trouble remembering whether the equation with $m\lambda$ or the equation with $\left(m + \frac{1}{2}\right)\lambda$ applies to a particular situation, you can remember that a zero-order bright band is in the center, and dark bands are halfway between bright bands. Thus, the made-up equation $d \sin\theta = (count)\lambda$ describes them all, with *count* = 0, 1, 2, … for bright bands, and with *count* = 0.5, 1.5, 2.5, … for dark bands.

Then, for the dark bands,

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(m + \frac{1}{2}\right); m = 0, 1, 2, 3, \ldots$$



ANS. FIG. P37.9

$$\Delta y = y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1 + \frac{1}{2}\right) - \left(0 + \frac{1}{2}\right)\right] = \frac{\lambda L}{d}$$

$$= \frac{(546.1 \times 10^{-9} \text{ m})(1.20 \text{ m})}{0.250 \times 10^{-3} \text{ m}}$$

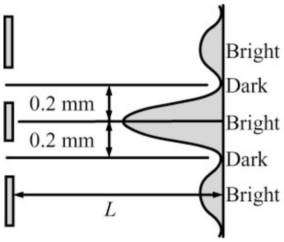
$$\Delta y = \boxed{2.62 \text{ mm}}$$

P37.10 Taking $m = 0$ and $y = 0.200$ mm in Equations 37.3 and 37.4 gives

$$L \approx \frac{2dy}{\lambda} = \frac{2(0.400 \times 10^{-3} \text{ m})(0.200 \times 10^{-3} \text{ m})}{442 \times 10^{-9} \text{ m}} = 0.362 \text{ m}$$

$$L \approx \boxed{36.2 \text{ cm}}$$

Geometric optics or a particle theory of light would incorrectly predict bright regions opposite the slits and darkness in between. But, as this example shows, interference can produce just the opposite.



ANS. FIG. P37.10

***P37.11** $\lambda = \dfrac{340\ \text{m/s}}{2\ 000\ \text{Hz}} = 0.170\ \text{m}$

The maxima are located at $d\sin\theta = m\lambda$:

$m = 0$ gives $\theta = 0°$

$m = 1$ gives $\theta = \sin^{-1}\left(\dfrac{\lambda}{d}\right) = \sin^{-1}\left(\dfrac{0.170\ \text{m}}{0.350\ \text{m}}\right) = 29.1°$

$m = 2$ gives $\theta = \sin^{-1}\left(\dfrac{2\lambda}{d}\right) = \sin^{-1}\left[\dfrac{2(0.170\ \text{m})}{0.350\ \text{m}}\right] = 76.3°$

$m = 3$ has no solution, since $\sin\theta > 1$.

The minima are located at $d\sin\theta = \left(m + \dfrac{1}{2}\right)\lambda$:

$m = 0$ gives $\theta = \sin^{-1}\left(\dfrac{\lambda}{2d}\right) = \sin^{-1}\left[\dfrac{0.170\ \text{m}}{2(0.350\ \text{m})}\right] = 14.1°$

$m = 1$ gives $\theta = \sin^{-1}\left(\dfrac{3\lambda}{2d}\right) = \sin^{-1}\left[\dfrac{3(0.170\ \text{m})}{2(0.350\ \text{m})}\right] = 46.8°$

$m = 2$ has no solution, since $\sin\theta > 1$.

$\boxed{\text{We have maxima at } 0°, 29.1°, \text{ and } 76.3°\text{; minima at } 14.1° \text{ and } 46.8°}$.

P37.12 The wavelength $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{2\ 000\ \text{s}^{-1}} = 0.171\ 5\ \text{m}$ is on the same order of size as the slit separation $d = 0.300$ m, so we may treat this as a double-slit diffraction problem.

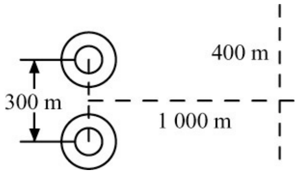
(a) $d\sin\theta = m\lambda$ so $(0.300\ \text{m})\sin\theta = 1(0.171\ 5\ \text{m})$ and $\theta = \boxed{34.9°}$.

(b) $d\sin\theta = m\lambda$ so $d\sin 34.9° = 1(0.030\ 0\ \text{m})$ and $d = \boxed{5.25\ \text{cm}}$.

(c) $\left(1.00\times10^{-6}\text{ m}\right)\sin34.9° = (1)\lambda$ so $\lambda = 572$ nm.

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{5.72\times10^{-7}\text{ m}} = \boxed{5.24\times10^{14}\text{ Hz}}$$

P37.13 Note, with the conditions given, the small-angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. We treat the interference as a Fraunhofer pattern.



ANS. FIG. P37.13

(a) At the $m = 2$ maximum,

$$\tan\theta = \frac{400\text{ m}}{1\,000\text{ m}} = 0.400 \rightarrow \theta = 21.8°$$

So $\lambda = \dfrac{d\sin\theta}{m} = \dfrac{(300\text{ m})\sin21.8°}{2} = \boxed{55.7\text{ m}}$.

(b) The next minimum encountered is the $m = 2$ minimum, and at that point,

$$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

which becomes $d\sin\theta = \dfrac{5}{2}\lambda$,

or $\sin\theta = \dfrac{5}{2}\dfrac{\lambda}{d} = \dfrac{5}{2}\left(\dfrac{55.7\text{ m}}{300\text{ m}}\right) = 0.464 \rightarrow \theta = 27.7°$,

so $y = (1\,000\text{ m})\tan27.7° = 524$ m.

Therefore, the car must travel an additional

$$524\text{ m} - 400\text{ m} = \boxed{124\text{ m}}$$

If we considered Fresnel interference, we would more precisely find

(a) $\lambda = \dfrac{1}{2}\left(\sqrt{550^2 + 1\,000^2}\text{ m} - \sqrt{250^2 + 1\,000^2}\text{ m}\right) = 55.2$ m and

(b) 123 m.

P37.14 Location of A = central maximum, location of B = first minimum.

So, $\Delta y = \left[y_{\text{min}} - y_{\text{max}}\right] = \dfrac{\lambda L}{d}\left(0 + \dfrac{1}{2}\right) - 0 = \dfrac{1}{2}\dfrac{\lambda L}{d} = 20.0$ m.

Thus, $d = \dfrac{\lambda L}{2(20.0\text{ m})} = \dfrac{(3.00\text{ m})(150\text{ m})}{40.0\text{ m}} = \boxed{11.3\text{ m}}$.

P37.15 The angle θ of the 50th-order fringe is given by

$$d\sin\theta = m\lambda \;\rightarrow\; \theta = \sin^{-1}\left(\frac{50\lambda}{d}\right)$$

The distance x from the slit to the screen and the distance y of the mth-order fringe from the center of the central maximum are related by $\tan\theta = \frac{y}{x}$. As the student approaches the screen at speed v, the distances x and y decrease but their ratio stays the same. Therefore,

$$\tan\theta = \frac{y}{x} \;\rightarrow\; y = x\tan\theta$$

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta = -v\tan\theta$$

where dy/dt is negative because the distance y shrinks. The *speed* of the fringe is

$$v_{50\text{th-order}} = \left|\frac{dy}{dt}\right| = v\tan\theta = v\tan\left[\sin^{-1}\left(\frac{m\lambda}{d}\right)\right]$$

Thus, the speed of the 50th-order fringe is

$$v_{50\text{th-order}} = (3.00\text{ m/s})\tan\left\{\sin^{-1}\left[\frac{50(632.8\times10^{-9}\text{ m})}{0.300\times10^{-3}\text{ m}}\right]\right\}$$

$$= \boxed{0.318\text{ m/s}}$$

P37.16 The angle θ of the mth-order fringe is given by

$$d\sin\theta = m\lambda \;\rightarrow\; \theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

The distance x from the slit to the screen and the distance y of the mth-order fringe from the center of the central maximum are related by $\tan\theta = \frac{y}{x}$. As the student approaches the screen at speed v, the distances x and y decrease but their ratio stays the same. Therefore,

$$\tan\theta = \frac{y}{x} \;\rightarrow\; y = x\tan\theta$$

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta = -v\tan\theta$$

where dy/dt is negative because the distance y shrinks. Thus, the speed of the mth-order fringe is

$$v_{m\text{th-order}} = \left|\frac{dy}{dt}\right| = v\tan\theta = \boxed{v\tan\left[\sin^{-1}\left(\frac{m\lambda}{d}\right)\right]}$$

P37.17 As shown in the figure to the right, the height of the radio telescope dish is $h = d_2 \sin\theta$, and the path difference in the waves reaching the telescope is

$$\delta = d_2 - d_1 = d_2(1 - \sin\alpha)$$

where

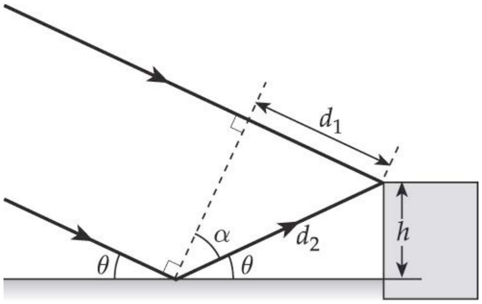
$$\theta + \alpha + \theta = 90° \quad \rightarrow \quad \alpha = 90° - 2\theta$$

If the first minimum $(\delta = \lambda/2)$ occurs when $\theta = 25.0°$, then

$$\alpha = 90° - 2(25.0°) = 40.0°\text{ , and}$$

$$d_2 = \frac{\delta}{1 - \sin\alpha} = \frac{(250\text{ m})/2}{1 - \sin 40.0°} = 350\text{ m}$$

Thus, the height $h = d_2 \sin\theta = 350\text{ m}\sin 25.0° = \boxed{148\text{ m}}$



ANS. FIG. P37.17

P37.18 For a double-slit system, the path difference of the two wave fronts arriving at a screen is $\delta = d\sin\theta$ and the phase difference is

$$\phi = \frac{2\pi}{\lambda}\delta = \frac{2\pi}{\lambda}d\sin\theta \approx \frac{2\pi}{\lambda}d\left(\frac{y}{L}\right)$$

(a) For $\theta = 0.500°$,

$$\phi = \frac{2\pi}{\lambda}d\sin\theta$$

$$\phi = \frac{2\pi}{(500\times10^{-9}\text{ m})}(0.120\times10^{-3}\text{ m})\sin(0.500°) = \boxed{13.2\text{ rad}}$$

(b) $$\phi \approx \frac{2\pi}{\lambda}d\left(\frac{y}{L}\right) = \frac{2\pi}{(500\times10^{-9}\text{ m})}(0.120\times10^{-3}\text{ m})\left(\frac{5.00\times10^{-3}\text{ m}}{1.20\text{ m}}\right)$$

$$= \boxed{6.28\text{ rad}}$$

(c) If $\phi = 0.333 \text{ rad} = \dfrac{2\pi d \sin\theta}{\lambda}$, then

$$\theta = \sin^{-1}\left(\frac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\frac{\left(500\times10^{-9}\text{ m}\right)\left(0.333\text{ rad}\right)}{2\pi\left(0.120\times10^{-3}\text{ m}\right)}\right]$$

$$\theta = \boxed{1.27\times10^{-2\,\circ}}$$

(d) If $d\sin\theta = \dfrac{\lambda}{4}$, then

$$\theta = \sin^{-1}\left(\frac{\lambda}{4d}\right) = \sin^{-1}\left[\frac{500\times10^{-9}\text{ m}}{4\left(0.120\times10^{-3}\text{ m}\right)}\right]$$

$$\theta = \boxed{5.97\times10^{-2\,\circ}}$$

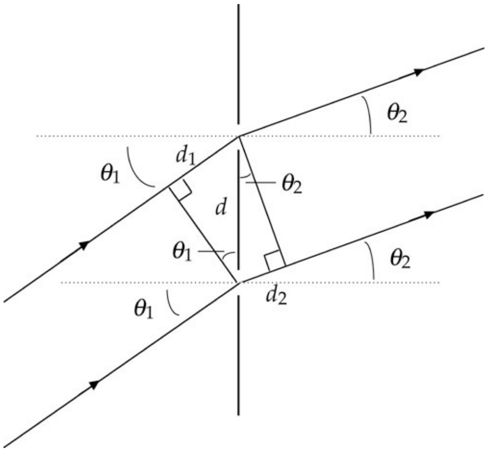
P37.19 From the diagram, the path difference between rays 1 and 2 is

$$\delta = d_1 - d_2 = d\sin\theta_1 - d\sin\theta_2$$

For constructive interference, this path difference must be equal to an integral number of wavelengths:

$$d\sin\theta_1 - d\sin\theta_2 = m\lambda$$

$$\sin\theta_1 - \sin\theta_2 = \frac{m\lambda}{d} \quad\rightarrow\quad \theta_2 = \sin^{-1}\left(\sin\theta_1 - \frac{m\lambda}{d}\right)$$



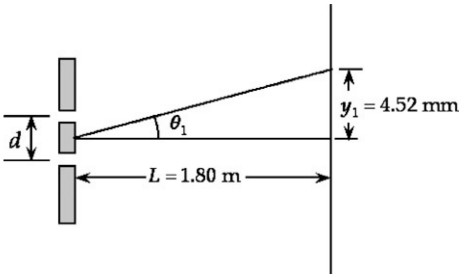
ANS. FIG. P37.19

P37.20 (a) $y = 50y_{\text{bright}} = 50\left(4.52\times10^{-3}\text{ m}\right) = 0.226\text{ m} = \boxed{22.6\text{ cm}}$

(b) $\tan\theta_1 = \dfrac{\left(y_{\text{bright}}\right)_{m=1}}{L} = \dfrac{4.52\times10^{-3}\text{ m}}{1.80\text{ m}} = \boxed{2.51\times10^{-3}}$

(c) From (b), $\theta_1 = \tan^{-1}\left(\dfrac{4.52\times10^{-3}\text{ m}}{1.80\text{ m}}\right) = 0.144°$

$\rightarrow \quad \sin\theta_1 = 2.51\times10^{-3}$



ANS. FIG. P37.20

The sine and the tangent are very nearly the same, but only because the angle is small. From $d\sin\theta_{\text{bright}} = m\lambda$, for $m = 1$:

$$\lambda = \frac{d\sin\theta_1}{1} = \frac{(2.40\times10^{-4}\text{ m})\sin(0.144°)}{1} = \boxed{6.03\times10^{-7}\text{ m}}$$

(d) From $\delta = d\sin\theta = m\lambda$ for the order m bright fringe,

$$\theta_{50} = \sin^{-1}\left(\frac{50\lambda}{d}\right) = \sin^{-1}(50\sin\theta_1) = \sin^{-1}[50\sin(0.144°)]$$
$$= \boxed{7.21°}$$

(e) $y_5 = L\tan\theta_5 = (1.80\text{ m})\tan(7.21°) = 2.26\times10^{-2}\text{ m} = \boxed{2.28\text{ cm}}$

(f) The two answers are close but do not agree exactly. The fringes are not laid out linearly on the screen as assumed in part (a), and this nonlinearity is evident for relatively large angles such as 7.21°.

P37.21 (a) The path difference $\delta = d\sin\theta$, and when $L >> y$:

$$\delta = \frac{yd}{L} = \frac{(1.80\times10^{-2}\text{ m})(1.50\times10^{-4}\text{ m})}{1.40\text{ m}}$$
$$= 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}$$

(b) $\dfrac{\delta}{\lambda} = \dfrac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00$, or $\boxed{\delta = 3.00\lambda}$

(c) Point P will be a $\boxed{\text{maximum}}$ because the path difference is an integer multiple of the wavelength.

P37.22 Observe that the pilot must not only home in on the airport, but must be headed in the right direction when she arrives at the end of the runway.

(a) $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^{8}\ \text{m/s}}{30.0 \times 10^{6}\ \text{s}^{-1}} = \boxed{10.0\ \text{m}}$

(b) The first side maximum is at an angle given by $d \sin\theta = (1)\lambda$.

$$(40.0\ \text{m})\sin\theta = 10.0\ \text{m} \qquad \theta = 14.5°$$

The 2.00 km is the length of the hypotenuse of a triangle with angle θ:

$$y = L\sin\theta = (2\ 000\ \text{m})\sin 14.5° = \boxed{500\ \text{m}}$$

(c) The intent is to inform the pilot which signal corresponds to the central maximum. The signal of 10-m wavelength in parts (a) and (b) would show maxima at 0°, 14.5°, 30.0°, 48.6°, and 90°. A signal of wavelength, say, 11.23 m, would show maxima at 0°, 16.3°, 34.2°, and 57.3°. The only value in common is 0°. A strong signal for both frequencies would indicate that the airplane was traveling along the central maximum, thus, straight on the runway. If λ_1 and λ_2 were related by a ratio of small integers in $\dfrac{\lambda_1}{\lambda_2} = \dfrac{n_1}{n_2}$, equivalent to $\dfrac{f_2}{f_1} = \dfrac{n_1}{n_2}$, then the equations $d\sin\theta = n_2\lambda_1$ and $d\sin\theta = n_1\lambda_2$ would both be satisfied for the same nonzero angle. The pilot could approach on an inappropriate bearing, and run off the runway immediately after touchdown.

Section 37.3 Intensity Distribution of the Double-Slit Interference Pattern

P37.23 We use Equation 37.14,

$$I = I_{\max}\cos^2\left(\frac{\pi y d}{\lambda L}\right)$$

Solving and substituting then gives

$$\frac{I}{I_{\max}} = \cos^2\left[\frac{\pi(6.00 \times 10^{-3}\ \text{m})(1.80 \times 10^{-4}\ \text{m})}{(656.3 \times 10^{-9}\ \text{m})(0.800\ \text{m})}\right] = \boxed{0.968}$$

P37.24 We use trigonometric identities to write

$$E_1 + E_2 = 6.00\sin(100\pi t) + 8.00\sin(100\pi t + \pi/2)$$

$$= 6.00\sin(100\pi\ t) + \left[8.00\sin(100\pi t)\cos(\pi/2) + 8.00\cos(100\pi t)\sin(\pi/2)\right]$$

$$E_1 + E_2 = 6.00\sin(100\pi\ t) + 8.00\cos(100\pi\ t)$$

and

$$E_R\sin(100\pi t + \phi) = E_R\sin(100\pi t)\cos\phi + E_R\cos(100\pi t)\sin\phi$$

The equation $E_1 + E_2 = E_R\sin(100\pi t + \phi)$ is satisfied if we require

$$600 = E_R^-\cos\phi \quad \text{and} \quad 8.00 = E_R\sin\phi$$

or $$(6.00)^2 + (8.00)^2 = E_R{}^2(\cos^2\phi + \sin^2\phi) \rightarrow \boxed{E_R = 10.0}$$

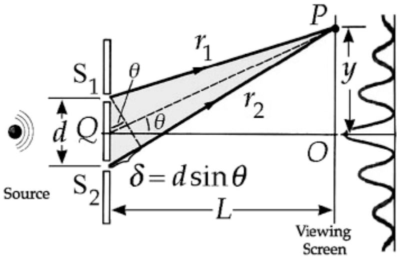
and $$\tan\phi = \sin\phi/\cos\phi = 8.00/6.00 = 1.33 \rightarrow \boxed{\phi = 53.1°}$$

P37.25 We will use Equation 37.14 for intensity in a double-slit interference pattern, which is

$$I = I_{\max}\cos^2\left[\frac{\pi d\sin\theta}{\lambda}\right]$$

For small θ, from ANS. FIG. P37.25,

$$\sin\theta \approx \frac{y}{L}$$



ANS. FIG. P37.25

Substituting and solving gives

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\sqrt{\frac{I}{I_{\max}}}$$

Next, with $I = 0.750 I_{\max}$, we can substitute a value for each variable:

$$y = \frac{(6.00 \times 10^{-7}\ \text{m})(1.20\ \text{m})}{\pi(2.50 \times 10^{-3}\ \text{m})}\cos^{-1}\sqrt{0.750} = \boxed{48.0\ \mu\text{m}}$$

P37.26 (a) The resultant amplitude is

$$E_r = E_0\sin\omega t + E_0\sin(\omega t + \phi) + E_0\sin(\omega t + 2\phi)$$

where $\phi = \dfrac{2\pi}{\lambda}d\sin\theta$.

Expanding,

$$E_r = E_0\left(\sin\omega t + \sin\omega t\cos\phi + \cos\omega t\sin\phi + \sin\omega t\cos 2\phi + \cos\omega t\sin 2\phi\right)$$

$$E_r = E_0\left(\sin\omega t\right)\left(1 + \cos\phi + 2\cos^2\phi - 1\right) + E_0\left(\cos\omega t\right)\left(\sin\phi + 2\sin\phi\cos\phi\right)$$

$$E_r = E_0\left(1 + 2\cos\phi\right)\left(\sin\omega t\cos\phi + \cos\omega t\sin\phi\right)$$

$$= E_0\left(1 + 2\cos\phi\right)\sin\left(\omega t + \phi\right)$$

Then the intensity is

$$I \propto E_r^2 = E_0^2\left(1 + 2\cos\phi\right)^2\left(\frac{1}{2}\right)$$

where we have substituted the time average of $\sin^2\left(\omega t + \phi\right)$, which is $\frac{1}{2}$. The maximum intensity occurs at $\phi = 0$:

$$I_{\max} \propto E_0^2\left(1 + 2\cos 0\right)^2\left(\frac{1}{2}\right) = \frac{9}{2}E_0^2$$

Therefore, the ratio of intensity to maximum intensity is

$$\frac{I}{I_{\max}} = \frac{E_0^2\left(1 + 2\cos\phi\right)^2\left(\frac{1}{2}\right)}{\frac{9}{2}E_0^2} = \frac{\left(1 + 2\cos\phi\right)^2}{9}$$

$$I = \frac{I_{\max}}{9}\left(1 + 2\cos\phi\right)^2$$

$$\boxed{I = \frac{I_{\max}}{9}\left[1 + 2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]^2}$$

(b) Look at the $N = 3$ graph in the textbook Figure 37.7. The intensity is zero at two places between the relative maxima, attained where $\cos\phi = -\frac{1}{2}$. The relative secondary maximum in the middle occurs at $\cos\phi = -1.00$, where $I = \frac{I_{\max}}{9}[1 - 2]^2 = \frac{I_{\max}}{9}$.

(c) The larger local maximum happens where $\cos\phi = +1.00$, giving $I = \frac{I_{\max}}{9}[1 + 2]^2 = I_{\max}$. The ratio of intensities at primary versus secondary maxima is $\boxed{9:1}$.

P37.27 (a) From Equation 37.14,

$$I = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right)$$

with $\phi = \dfrac{2\pi}{\lambda}d\sin\theta$. This gives

$$\frac{I}{I_{\max}} = \cos^2\left(\frac{\phi}{2}\right)$$

Therefore,

$$\phi = 2\cos^{-1}\sqrt{\frac{I}{I_{\max}}} = 2\cos^{-1}\sqrt{0.640} = \boxed{1.29\text{ rad}}$$

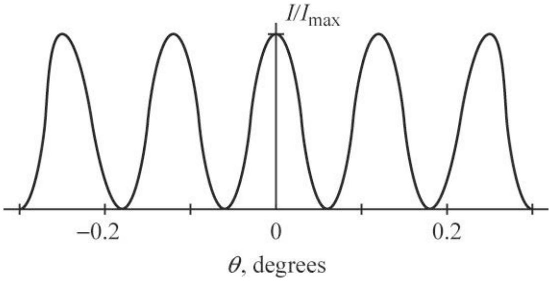
P37.28 In $I_{\text{avg}} = I_{\max}\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$ for angles between −0.3° and +0.3° we may take $\sin\theta = \theta$ (in radians) to find

$$I = I_{\max}\cos^2\left[\frac{\pi(250\ \mu\text{m})\ \theta}{0.546\ \mu\text{m}}\right]$$

This equation is correct assuming θ is in radians; but we can then equally well substitute in values for θ in degrees and interpret the argument of the cosine function as a number of degrees. We get the same answers for θ negative and for θ positive. We evaluate

θ degrees	−0.30	−0.25	−0.20	−0.15	−0.10	−0.05	0.0
$I/I_{\max}$	0.101	1.00	0.092	0.659	0.652	0.096	1.00
θ degrees	0.05	0.10	0.15	0.20	0.25	0.30	
$I/I_{\max}$	0.096	0.652	0.659	0.092	1.00	0.101	

TABLE P37.28



ANS. FIG. P37.28

The cosine-squared function has maximum values of 1 at $\theta = 0$, at $\theta = 0.125°$, and at $\theta = 0.250°$. It has minimum values of zero halfway between the maximum values. The graph then has the appearance shown.

P37.29 (a) From Equation 37.9,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\cdot\frac{y}{\sqrt{y^2 + D^2}}$$

$$\phi \approx \frac{2\pi yd}{\lambda D} = \frac{2\pi\left(0.850\times10^{-3}\text{ m}\right)\left(2.50\times10^{-3}\text{ m}\right)}{\left(600\times10^{-9}\text{ m}\right)\left(2.80\text{ m}\right)} = \boxed{7.95\text{ rad}}$$

(b) $$\frac{I}{I_{\max}} = \frac{\cos^2\left[\left(\pi d/\lambda\right)\sin\theta\right]}{\cos^2\left[\left(\pi d/\lambda\right)\sin\theta_{\max}\right]} = \frac{\cos^2\left(\phi/2\right)}{\cos^2 m\pi}$$

$$\frac{I}{I_{\max}} = \cos^2\frac{\phi}{2} = \cos^2\left(\frac{7.95\text{ rad}}{2}\right) = \boxed{0.453}$$

Section 37.4 Change of Phase Due to Reflection

Section 37.5 Interference in Thin Films

P37.30 (a) With phase reversal in the reflection at the outer surface of the soap film and no reversal on reflection from the inner surface, the condition for constructive interference in the light reflected from the soap bubble is

$$2t = \left(m + \frac{1}{2}\right)\lambda_n = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad\rightarrow\quad 2nt = \left(m + \frac{1}{2}\right)\lambda$$

$$\lambda = \frac{2nt}{\left(m + \frac{1}{2}\right)}$$

where $m = 0, 1, 2, \ldots$. For the lowest order reflection ($m = 0$), and the wavelength is

$$\lambda = \frac{2nt}{\left(0 + 1/2\right)} = \frac{2\left(1.33\right)\left(120\text{ nm}\right)}{1/2} = \boxed{638\text{ nm}}$$

(b) $\boxed{\text{A thicker film would require a higher order of reflection, so use a larger value of } m.}$

(c) From (a) above, for a given wavelength, the thickness would be

$$t=\left(m+\frac{1}{2}\right)\frac{\lambda}{2n}=\left(m+\frac{1}{2}\right)\frac{638\text{ nm}}{2(1.33)}$$

The next greater thickness of soap film that can strongly reflect 638 nm light corresponds to $m = 1$, giving

$$t=\left(m+\frac{1}{2}\right)\frac{\lambda}{2n}=\left(1+\frac{1}{2}\right)\frac{638\text{ nm}}{2(1.33)}=\boxed{360\text{ nm}}$$

and the third such thickness (corresponding to $m = 2$) is

$$t=\left(m+\frac{1}{2}\right)\frac{\lambda}{2n}=\left(2+\frac{1}{2}\right)\frac{638\text{ nm}}{2(1.33)}=\boxed{600\text{ nm}}$$

P37.31 The layers are air, oil, and water. Because $1 < 1.25 < 1.33$, light reflected both from the top and from the bottom surface of the oil suffers phase reversal. For constructive interference we require

$$2t=\frac{m\lambda_{\text{cons}}}{n}$$

and for destructive interference,

$$2t=\frac{\left[m+(1/2)\right]\lambda_{\text{des}}}{n}$$

Then, $\dfrac{\lambda_{\text{cons}}}{\lambda_{\text{dest}}}=1+\dfrac{1}{2m}=\dfrac{640\text{ nm}}{512\text{ nm}}=1.25$ and $m = 2$

Therefore, $t=\dfrac{2(640\text{ nm})}{2(1.25)}=\boxed{512\text{ nm}}$.

P37.32 There are a total of two phase reversals caused by reflection, one at the top and one at the bottom surface of the coating.

$$2nt=\left(m+\frac{1}{2}\right)\lambda \quad \text{so} \quad t=\left(m+\frac{1}{2}\right)\frac{\lambda}{2n}$$

The minimum thickness of the film is therefore

$$t=\left(\frac{1}{2}\right)\frac{(500\text{ nm})}{2(1.30)}=\boxed{96.2\text{ nm}}$$

P37.33 Treating the anti-reflectance coating like a camera-lens coating (two phase reversals caused by reflection, one at the top and one at the bottom surface of the coating),

$$2t=\left(m+\frac{1}{2}\right)\frac{\lambda}{n} \rightarrow 2nt=\left(m+\frac{1}{2}\right)\lambda$$

(destructive interference)

Let $m = 0$. Then,

$$t = \frac{\lambda}{4n} = \frac{3.00\text{ cm}}{4(1.50)} = \boxed{0.500\text{ cm}}$$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar to 1.50 cm. Then the coating would exhibit maximum reflection!

P37.34 (a) The film thickness is $t = 1.00 \times 10^{-5}$ cm $= 1.00 \times 10^{-7}$ m $= 100$ nm. Since the light undergoes a 180° phase change at each surface of the film, the condition for *constructive* interference is

$$2t = m\frac{\lambda}{n}, \quad \text{or} \quad \lambda = \frac{2nt}{m} = \frac{2(1.38)(100\text{ nm})}{m} = \frac{276\text{ nm}}{m}$$

Therefore, the wavelengths intensified in the reflected light are, for $m = 1$, 2, and 3:

$$\lambda = \boxed{276\text{ nm},\ 138\text{ nm},\ 92.0\text{ nm}}$$

(b) $\boxed{\text{No visible wavelengths are intensified.}}$ Because $m \geq 1$, all reflection maxima are in the ultraviolet and beyond.

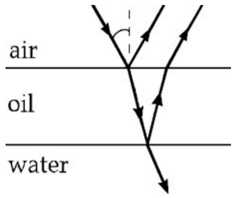
P37.35 If the path length difference $\Delta = \lambda$, the transmitted light will be bright. Since $\Delta = 2d = \lambda$,

$$d_{\min} = \frac{\lambda}{2} = \frac{580\text{ nm}}{2} = \boxed{290\text{ nm}}$$

P37.36 (a) The light reflected from the top of the oil film undergoes phase reversal. Since 1.45 > 1.33, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \rightarrow 2nt = \left(m + \frac{1}{2}\right)\lambda$$

or $$\lambda_m = \frac{2nt}{m + 1/2} = \frac{2(1.45)(280\text{ nm})}{m + 1/2} = \frac{812\text{ nm}}{m + 1/2}.$$



ANS. FIG. P37.36

Substituting for m gives:

$m = 0,\ \lambda_0 = 1\,620$ nm (infrared)

$m = 1,\ \lambda_1 = 541$ nm (green)

$m = 2,\ \lambda_2 = 325$ nm (ultraviolet)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in reflected light is $\boxed{\text{green}}$.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2t = m\frac{\lambda}{n}$$

or $$\lambda_m = \frac{2nt}{m} = \frac{812\text{ nm}}{m}.$$

Substituting for m gives:

$m = 1,\ \lambda_1 = 812$ nm (near infrared)

$m = 2,\ \lambda_2 = 406$ nm (violet)

$m = 3,\ \lambda_3 = 271$ nm (ultraviolet)

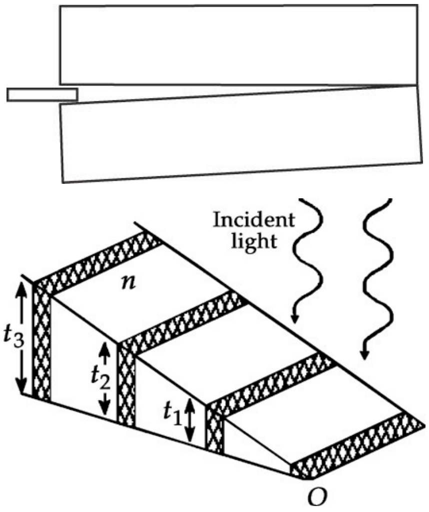
Of these, the only wavelength visible to the human eye (and hence the dominant wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is $\boxed{\text{violet}}$.

P37.37 For destructive interference in the air,

$$2t = m\lambda$$

The first dark fringe occurs at the end where the plates meet, where destructive interference occurs because of the phase reversal caused by light reflecting from the top of the lower glass slide. For 30 dark fringes, including the one where the plates meet, $m = 29$ and

$$t = \frac{n\lambda}{2} = \frac{29(600\text{ nm})}{2}$$
$$= 8.70\times10^{-6}\text{ m} = 8.70\ \mu\text{m}$$



ANS. FIG. P37.37

The *diameter* of the wire is the same as the thickness:

$$d = t = \boxed{8.70\ \mu\text{m}}$$

P37.38 Light waves are partially reflected and transmitted by the partially aluminized glass surfaces on the front and back surfaces of the filter. For maximum transmission, we want destructive interference between the waves reflected from the front and back surfaces of the film: the result of this interference is that most light of the H_α line is transmitted through the filter.

(a) If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface of the filter (glass-filter interface) suffers no phase reversal and light reflected from the back surface of the filter (filter-glass interface) does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film: $2t = \frac{\lambda}{n}$.

$$t = \frac{\lambda}{2n} = \frac{656.3\text{ nm}}{2(1.378)} = \boxed{238\text{ nm}}$$

(b) The filter will undergo thermal expansion. As t increases in $2nt = \lambda$, so does $\boxed{\lambda\text{ increase}}$.

(c) Destructive interference for reflected light happens when $2t = \frac{2\lambda}{n}$:

$$\lambda = nt = 1.378(238\text{ nm}) = \boxed{328\text{ nm}}\quad(\text{near ultraviolet})$$

P37.39 Reflection off the lower glass plate causes a phase reversal. The condition for bright fringes is

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \qquad m = 0,\ 1,\ 2,\ 3,\ \ldots.$$

From ANS. FIG. P37.39, observe that

$$t = R(1 - \cos\theta) \approx R\left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{R}{2}\left(\frac{r}{R}\right)^2 = \frac{r^2}{2R}$$

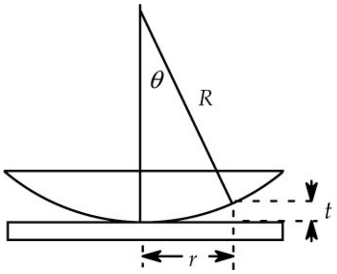
The condition for a bright fringe becomes

$$\frac{r^2}{R} = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$$

Thus, for fixed m and λ, nr^2 = constant.

Therefore,

$$n_{\text{liquid}} r_{fi}^2 = n_{\text{air}} r^2 \quad \text{and} \quad n_{\text{liquid}} = (1.00)\frac{(1.50\text{ cm})^2}{(1.31\text{ cm})^2} = \boxed{1.31}$$



ANS. FIG. P37.39

P37.40 (a) The missing wavelength in reflected light is caused by destructive interference. The index of the coating (1.38) is greater than that of air (1.00), and the index of the glass (1.52) is greater than that of the coating; therefore, light waves reflected off the front and back surfaces of the coating undergo phase reversals. For destructive interference,

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \qquad m = 0, 1, 2, 3, \ldots \quad \text{and} \quad n = 1.38$$

For the minimum thickness, $m = 0$:

$$2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n} \quad \to \quad t = \frac{\lambda}{4n} = \frac{540\text{ nm}}{4(1.38)} = \boxed{97.8\text{ nm}}$$

(b) Yes. Destructive interference occurs when $2nt = (m + \frac{1}{2})\lambda$ (Eq. 37.17), where m is an integer. (There is a phase change at both faces of the film in Figure P37.40.) Hence, for m = 1, 2, ... we obtain thicknesses of 293 nm, 489 nm, ...

P37.41 For total darkness, we want destructive interference for reflected light for both 400 nm and 600 nm. With phase reversal at just one reflecting surface (the bottom glass plate), the condition for destructive interference is

$$2n_{\text{air}}t = m\lambda \qquad m = 0, 1, 2, \ldots .$$

The least common multiple of these two wavelengths is 1 200 nm, so we get no reflected light at $2(1.00)t = 3(400\text{ nm}) = 2(600\text{ nm}) = 1\,200$ nm, so $t = 600$ nm at this second dark fringe.

By similar triangles, $\dfrac{600\text{ nm}}{x} = \dfrac{0.050\,0\text{ mm}}{10.0\text{ cm}}$

or the distance from the contact point is

$$x = \left(600\times10^{-9}\text{ m}\right)\left(\frac{0.100\text{ m}}{5.00\times10^{-5}\text{ m}}\right) = \boxed{1.20\text{ mm}}$$

Section 37.6 The Michelson Interferometer

P37.42 When the mirror on one arm is displaced by $\Delta\ell$, the path difference changes by $2\Delta\ell$. A shift resulting in the reversal between dark and bright fringes requires a path length change of one-half wavelength. Therefore, $2\Delta\ell = \frac{m\lambda}{2}$, where in this case, $m = 250$.

$$\Delta\ell = m\frac{\lambda}{4} = \frac{(250)\left(6.328\times10^{-7}\text{ m}\right)}{4} = \boxed{39.6\ \mu\text{m}}$$

***P37.43** Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \frac{2L}{\lambda}$. It changes to $m_2 = \frac{2L}{\lambda/n_{\text{gas}}} = \frac{2n_{\text{gas}}L}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \frac{2L}{\lambda}\left(n_{\text{gas}} - 1\right)$, or the index of refraction of the gas is

$$n_{\text{gas}} = 1 + \frac{N\lambda}{2L} = 1 + \frac{(160)\left(600\times10^{-9}\text{ m}\right)}{2\left(5.00\times10^{-2}\text{ m}\right)} = \boxed{1.001}$$

P37.44 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \frac{2L}{\lambda}$. It changes to $m_2 = \frac{2L}{\lambda/n} = \frac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \frac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = \boxed{1 + \frac{N\lambda}{2L}}$$

Additional Problems

P37.45 The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{60.0\times10^{6}\ \text{s}^{-1}} = 5.00\ \text{m}$$

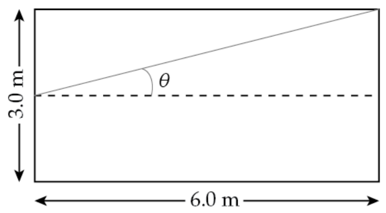
Along the line *AB* the two traveling waves going in opposite directions add to give a standing wave. The two transmitters are exactly 2.00 wavelengths apart and the signal from *B*, when it arrives at *A*, will always be in phase with transmitter *B*. Since *B* is 180° out of phase with *A*, the two signals always interfere destructively at the position of *A* to form a node.

The first antinode (point of constructive interference) is located at distance

$$\frac{\lambda}{4} = \frac{5.00\ \text{m}}{4} = \boxed{1.25\ \text{m}}\ \text{from the node at } A$$

***P37.46** From ANS. FIG. P37.46, we note that the angle between the center line of the speakers and the corners of the room is

$$\theta = \tan^{-1}\left(\frac{1.5\ \text{m}}{6.0\ \text{m}}\right) = 14.0°$$



ANS. FIG. P37.46

In order for no other maxima to be heard, the $m = 1$ maximum must be more than 14.0° away from the central maximum. From Equation 37.2, the condition for constructive interference is

$$d\sin\theta_{\text{bright}} = m\lambda$$

or

$$\lambda = \frac{d\sin\theta_{\text{bright}}}{m} = \frac{v}{f}$$

where $v = 343$ m/s is the speed of sound. Solving for f and substituting $m = 1$ and $\theta = 14.0°$ then gives

$$f = \frac{v}{\lambda} = \frac{mv}{d\sin\theta_{\text{bright}}} = \frac{(1)(343\ \text{m/s})}{(1.0\ \text{m})\sin 14.0°} = \boxed{1.4\times10^{2}\ \text{Hz}}$$

P37.47 The same source will radiate light into the sugar solution with wavelength $\lambda_n = \dfrac{\lambda}{n}$. In other words, the condition for bright fringes becomes

$$d\sin\theta = m\lambda_n \rightarrow d\sin\theta = m\frac{\lambda}{n}$$

Also, for small angles, as is the case here

$$\sin\theta \approx \tan\theta = \frac{y}{L}$$

The first side bright fringe ($m = 1$) is separated from the central bright fringe by distance y described by

$$d\sin\theta = m\frac{\lambda}{n} \quad\rightarrow\quad d\left(\frac{y}{L}\right) = \frac{\lambda}{n}$$

solving for y gives

$$y = \frac{\lambda L}{nd} = \frac{\left(560\times10^{-9}\ \text{m}\right)\left(1.20\ \text{m}\right)}{\left(1.38\right)\left(30.0\times10^{-6}\ \text{m}\right)} = 1.62\times10^{-2}\ \text{m} = \boxed{1.62\ \text{cm}}$$

P37.48 (a) Where fringes of the two colors coincide we have

$$d\sin\theta = m\lambda = m'\lambda', \quad \text{requiring} \quad \frac{\lambda}{\lambda'} = \frac{m'}{m}$$

(b) $\lambda = 430$ nm, $\lambda' = 510$ nm

$$\therefore \frac{m'}{m} = \frac{430\ \text{nm}}{510\ \text{nm}} = \frac{43}{51}$$

which cannot be reduced any further. Then $m = 51$, $m' = 43$. Then,

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{(51)\left(430\times10^{-9}\ \text{m}\right)}{0.025\times10^{-3}\ \text{m}}\right] = 61.3^\circ$$

and

$$y_m = L\tan\theta_m = \left(1.5\ \text{m}\right)\tan 61.3^\circ = \boxed{2.74\ \text{m}}$$

P37.49 (a) Refer to ANS. FIG. P37.49. By similar triangles, the distance x between consecutive *like* interference fringes (bright-to-bright, or dark-to-dark) is to the change in thickness Δt of the air gap as the entire length of a plate ℓ (14.0 cm) is to the diameter d of the fiber (equal to the thickness of the air gap at the open end of the gap):

$$\frac{x}{\Delta t} = \frac{\ell}{d}$$

where, say, between consecutive destructive interference fringes

$$2t = \left(m + \frac{1}{2}\right)\lambda \quad\rightarrow\quad \Delta t = \frac{\lambda}{2}$$

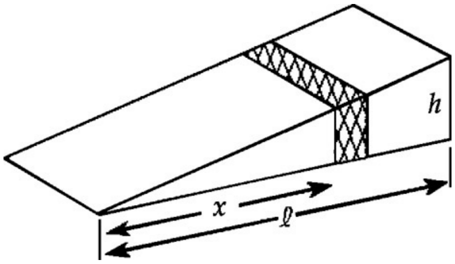
Combing the two relations gives

$$\frac{x}{\lambda/2}=\frac{\ell}{d}$$

and solving for the diameter d of the fiber then gives

$$d=\frac{\ell\lambda}{2x}=\frac{\left(14.0\times10^{-2}\text{ m}\right)\left(650\times10^{-9}\text{ m}\right)}{2\left(0.580\times10^{-3}\text{ m}\right)}$$

$$=7.84\times10^{-5}\text{ m}=\boxed{78.4\ \mu\text{m}}$$



ANS. FIG. P37.49

P37.50 Assume the distance between gaps is 2 cm.

(a) Two adjacent directions of constructive interference for 600-nm light are described by $d\sin\theta=m\lambda$, with $\theta_0=0$. Then,

$$d\sin\theta=m\lambda$$

$$\left(2\times10^{-2}\text{ m}\right)\sin\theta_1=1\left(600\times10^{-9}\text{ m}\right)$$

Thus, $\theta_1=2\times10^{-3}{}^\circ$,

and $\theta_1-\theta_0\boxed{\sim10^{-3}{}^\circ}$.

(b) We choose $\theta_1=20°$. Then,

$$\left(2\times10^{-2}\text{ m}\right)\sin20°=(1)\lambda$$

Which gives $\lambda=7$ mm. The frequency is then

$$f=\frac{c}{\lambda}=\frac{3\times10^{8}\text{ m/s}}{7\times10^{-3}\text{ m}}\boxed{\sim10^{11}\text{ Hz}}$$

(c) Millimeter waves are $\boxed{\text{microwaves}}$.

P37.51 Constructive interference occurs where the phases of the waves differ by integral multiples m of 2π:

$$\left(\frac{2\pi x_1}{650}-924\pi t+\frac{\pi}{6}\right)-\left(\frac{2\pi x_2}{650}-924\pi t+\frac{\pi}{8}\right)=2\pi m$$

which becomes

$$\frac{2\pi(x_1 - x_2)}{650} + \left(\frac{\pi}{6} - \frac{\pi}{8}\right) = 2\pi m$$

$$\frac{(x_1 - x_2)}{650} + \frac{1}{12} - \frac{1}{16} = m$$

$\boxed{x_1 - x_2 = \left(m - \tfrac{1}{48}\right)650, \text{ where } x_1 \text{ and } x_2 \text{ are in nanometers and } m = 0, 1, -1, 2, -2, 3, -3, \ldots}$

P37.52 A bright line for the green light requires

$$d\sin\theta \approx d\tan\theta = m_1\lambda_1$$

$$d\frac{y}{L} = m_1\lambda_1$$

Similarly, a blue interference maximum requires

$$d\frac{y}{L} = m_2\lambda_2$$

for integers m_1 and m_2. Thus,

$$m_1(540\text{ nm}) = m_2(450\text{ nm})$$

$$\frac{m_2}{m_1} = \frac{540\text{ nm}}{450\text{ nm}} = \frac{6}{5}$$

and smallest integers satisfying the equation are $m_1 = 5$ and $m_2 = 6$.

Then for both,

$$d\frac{y}{L} = 2\ 700\text{ nm}$$

which gives

$$y = (2\ 700\text{ nm})\frac{L}{d} = (2.7\ \mu\text{m})\left(\frac{1.4\text{ m}}{150\ \mu\text{m}}\right) = \boxed{2.52\text{ cm}}$$

P37.53 If the center point on the screen is to be a dark spot rather than bright, passage through the plastic must delay the light by one-half wavelength. Calling the thickness of the plastic t, $\frac{t}{\lambda} + \frac{1}{2} = \frac{t}{\lambda/n} = \frac{nt}{\lambda}$ or

$t = \boxed{\frac{\lambda}{2(n-1)}}$ where n is the index of refraction for the plastic.

P37.54 There is no phase shift upon reflection from the upper surface (glass to air) of the film, but there will be a shift of $\frac{\lambda}{2}$ due to the reflection at the lower surface of the film (air to metal).

The total phase difference in the two reflected beams is then

$$\delta = 2nt + \frac{\lambda}{2}$$

For constructive interference, $\delta = m\lambda$, or

$$2(1.00)t + \frac{\lambda}{2} = m\lambda$$

Thus, the film thickness for the *m*th order bright fringe is

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = m\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

and the thickness for the $m - 1$ bright fringe is:

$$t_{m-1} = (m-1)\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}$$

Therefore, the change in thickness required to go from one bright fringe to the next is

$$\Delta t = t_m - t_{m-1} = \frac{\lambda}{2}$$

To go through 200 bright fringes, the change in thickness of the air film must be

$$200\left(\frac{\lambda}{2}\right) = 100\lambda$$

Thus, the increase in the length of the rod is

$$\Delta L = 100\lambda = 100\left(5.00 \times 10^{-7}\text{ m}\right) = 5.00 \times 10^{-5}\text{ m}$$

From $\Delta L = L_i \alpha\, \Delta T$

we have: $\alpha = \dfrac{\Delta L}{L_i \Delta T} = \dfrac{5.00 \times 10^{-5}\text{ m}}{(0.100\text{ m})(25.0°\text{C})} = \boxed{20.0 \times 10^{-6}{}°\text{C}^{-1}}$

P37.55 Since 1 < 1.25 < 1.34, light reflected from top and bottom surfaces of the oil undergoes phase reversal. The path difference is then $2t$, thus

$$2t = m\lambda_n = \frac{m\lambda}{n}$$

for maximum reflection, with $m = 1$ for the given first-order condition and $n = 1.25$. So

$$t = \frac{m\lambda}{2n} = \frac{1(500\text{ nm})}{2(1.25)} = 200\text{ nm}$$

The volume we assume to be constant:

$$1.00\text{ m}^3 = (200\text{ nm})A$$

The area is then

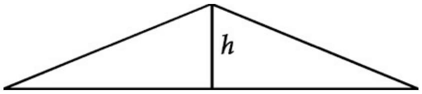
$$A = \frac{1.00\text{ m}^3}{(200\times10^{-9}\text{ m})} = 5.00\times10^6\text{ m}^2 = \boxed{5.00\text{ km}^2}$$

P37.56 The interfering waves travel either along the hypotenuses or the bases of the right triangles. The total length of the two bases is 15.0 km. The condition for destructive interference for minimum height h is

$$2\sqrt{(15.0\times10^3\text{ m})^2 + h^2} - 2(15.0\times10^3\text{ m}) = \lambda/2 = 175\text{ m}$$

$$2\sqrt{(15.0\times10^3\text{ m})^2 + h^2} = 30.175\times10^3\text{ m}$$

$$h = 1.62\times10^3\text{ m} = \boxed{1.62\text{ km}}$$



ANS. FIG. P37.56

P37.57 We may treat this problem as a double slit experiment where the second slit is the mirror image of the source, 1.00 cm below the mirror plane; however, we must remember that the light undergoes a $\frac{\pi}{2}$ phase shift at the mirror, so light and dark fringes are interchanged in the interference pattern. Thus, for destructive interference, the path length must differ by $m\lambda$. For dark for the first dark fringe (modifying Equation 37.7), we have

$$y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1(5.00\times10^{-7}\text{ m})(100\text{ m})}{(2.00\times10^{-2}\text{ m})} = \boxed{2.50\text{ mm}}$$

P37.58 From Equation 37.14, for wavelength $\lambda_1 = 600\text{ nm}$,

$$\frac{I}{I_{\text{max}}} = \cos^2\left(\frac{\pi yd}{\lambda_1 L}\right) = 0.810$$

$$\frac{\pi yd}{L} = \lambda_1\cos^{-1}\left(\frac{I_1}{I_{\text{max}}}\right)^{1/2} = (600\text{ nm})\cos^{-1}(0.810)^{1/2} = 271\text{ nm}$$

For the same y, d, and L, let λ_2 be the wavelength for which

$$\frac{I_2}{I_{2,\max}} = 0.640$$

Then,

$$\lambda_2 = \frac{\pi yd/L}{\cos^{-1}\left(I_2/I_{2,\max}\right)^{1/2}} = \frac{271\text{ nm}}{\cos^{-1}(0.640)^{1/2}} = \boxed{421\text{ nm}}$$

Note that in this problem, $\cos^{-1}\left(\dfrac{I}{I_{\max}}\right)^{1/2}$ must be expressed in radians.

P37.59 As with any air gap between glass plates, light reflecting off the lower plate undergoes a phase reversal. Thus, for the mth-dark fringe after the first fringe ($m = 0$), with the gap filled with air:

$$2nt = m\lambda$$

where $n = 1.00$ and $m = 1, 2, \ldots, 84$. So, at the widest edge of the wedge,

$$t = \frac{84\lambda}{2} = 42\lambda$$

When submerged in water,

$$2nt = m\lambda$$

$$m = \frac{2nt}{\lambda} = \frac{2(1.33)\left[(42)\lambda\right]}{\lambda} = 111.7 = 111$$

So, counting the first fringe ($m = 0$), the total number of fringes is $m + 1 = \boxed{112\text{ dark fringes}}$

P37.60 Refer to Figure P37.60. Call t the thickness of the sheet. With the sheet in place, the central maximum corresponds to zero phase difference. Thus, the added distance δ traveled by the light from the lower slit introduces a phase difference equal to that introduced by the plastic film sheet. Call the original length of the path from the upper slit to the screen D; then, the original number of wavelengths along distance D are

$$N_0 = \frac{D}{\lambda_a}$$

where λ_a is the wavelength in air. With the plastic sheet in the path, the number of wavelengths changes to

$$N = \frac{D-t}{\lambda_a} + \frac{t}{\lambda_p} = \frac{D-t}{\lambda_a} + \frac{t}{\lambda_a/n} = \frac{D-t+nt}{\lambda_a} = \frac{D+(n-1)t}{\lambda_a}$$

where λ_a is the wavelength in plastic. The phase difference introduced by the plastic sheet is

$$\delta\phi = 2\pi\left(N - N_0\right) = 2\pi\left[\frac{D + (n-1)t}{\lambda_a} - \frac{D}{\lambda_a}\right] = 2\pi\frac{(n-1)t}{\lambda_a}$$

The corresponding difference in **path length** δ is

$$\delta = \delta\phi\left(\frac{\lambda_a}{2\pi}\right) = \left[2\pi\frac{(n-1)t}{\lambda_a}\right]\left(\frac{\lambda_a}{2\pi}\right) = t(n-1)$$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle θ may be expressed as

$$\sin\theta = \frac{\delta}{d} = \frac{(n-1)t}{d} \quad \rightarrow \quad \theta = \sin^{-1}\left[\frac{(n-1)t}{d}\right]$$

The height y of the central maximum is given by

$$\frac{y'}{L} = \tan\theta$$

from which we obtain

$$y = \boxed{L\tan\left\{\sin^{-1}\left[\frac{(n-1)t}{d}\right]\right\} = \frac{(n-1)Lt}{\sqrt{d^2 - (n-1)^2 t^2}}}$$

P37.61 From Figure P37.61, observe that the distance that the ray travels from the top of the transmitter to the ground is

$$x = \sqrt{h^2 + \left(\frac{d}{2}\right)^2}$$

$$= \sqrt{(35.0\text{ m})^2 + \left(\frac{50.0\text{ m}}{2}\right)^2} = \sqrt{1850\text{ m}^2} = 43.0\text{ m}$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves (transmitter-to-ground-to-receiver and transmitter-to-receiver) is

$$\delta = 2x + \frac{\lambda}{2} - d$$

For constructive interference,

$$2x + \frac{\lambda}{2} - d = m\lambda \;\rightarrow\; \lambda = \frac{2x - d}{\left(m - \frac{1}{2}\right)}$$

and for destructive interference

$$2x+\frac{\lambda}{2}-d=\left(m+\frac{1}{2}\right)\lambda \;\rightarrow\; \lambda=\frac{2x-d}{m}$$

(a) The longest wavelength that interferes constructively is, for $m = 1$,

$$\lambda=\frac{2x-d}{\left(1-\frac{1}{2}\right)}=14x-2d=4\sqrt{1850\text{ m}^2}-2(50.0\text{ m})=\boxed{72.0\text{ m}}$$

(b) The longest wavelength that interferes destructively is, for $m = 1$,

$$\lambda=\frac{2x-d}{1}=2\sqrt{1850\text{ m}^2}-50.0\text{ m}=\boxed{36.0\text{ m}}$$

P37.62 From Figure P37.57, observe that the distance that the ray travels from the top of the transmitter to the ground is

$$x=\sqrt{h^2+\left(\frac{d}{2}\right)^2}=\frac{\sqrt{4h^2+d^2}}{2}$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves (transmitter-to-ground-to-receiver and transmitter-to-receiver) is

$$\delta=2x+\frac{\lambda}{2}-d$$

For constructive interference,

$$2x+\frac{\lambda}{2}-d=m\lambda \;\rightarrow\; \lambda=\frac{2x-d}{\left(m-\frac{1}{2}\right)}$$

and for destructive interference

$$2x+\frac{\lambda}{2}-d=\left(m+\frac{1}{2}\right)\lambda \;\rightarrow\; \lambda=\frac{2x-d}{m}$$

(a) The longest wavelength that interferes constructively is, for $m = 1$,

$$\lambda=\frac{2x-d}{\left(1-\frac{1}{2}\right)}=4x-2d=\frac{4\sqrt{4h^2+d^2}}{2}-2d=\boxed{2\sqrt{4h^2+d^2}-2d}$$

(b) The longest wavelength that interferes destructively is, for $m = 1$,

$$\lambda=\frac{2x-d}{1}=\boxed{\sqrt{4h^2+d^2}-d}$$

P37.63 (a) There is a phase reversal by reflection at the flat plate. Constructive interference in the reflected light requires $2t = \left(m + \frac{1}{2}\right)\lambda$.

The first bright ring has $m = 0$ and the 55th has $m = 54$, so at the edge of the lens

$$t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2} = (54.5)\frac{650 \times 10^{-9}\text{ m}}{2} = 17.7\ \mu\text{m}$$

Now from the geometry in textbook Figure P37.59, we can find the distance t from the curved surface down to the flat plate by considering distances measured from the center of curvature:

$$\sqrt{R^2 - r^2} = R - t \quad \text{or} \quad R^2 - r^2 = R^2 - 2Rt + t^2$$

Solving for R gives

$$R = \frac{r^2 + t^2}{2t} = \frac{\left(5.00 \times 10^{-2}\text{ m}\right)^2 + \left(1.77 \times 10^{-5}\text{ m}\right)^2}{2\left(1.77 \times 10^{-5}\text{ m}\right)} = \boxed{70.6\text{ m}}$$

(b) $\frac{1}{f} = (n-1)\left(\frac{1}{R_2} - \frac{1}{R_2}\right) = 0.520\left(\frac{1}{\infty} - \frac{1}{-70.6\text{ m}}\right)$ so $f = \boxed{136\text{ m}}$

P37.64 Reflection off the top surface of the wedge produced a phase reversal, but light reflecting off the bottom surface produces no phase change. Thus, a first *dark* fringe occurs at the thin end of the wedge. For bright fringes in the thin film, the thickness is given by Equation 37.17:

$$t = \frac{\left(m + \frac{1}{2}\right)\lambda}{2n}$$

The first fringe corresponds to $m = 0$, the second to $m = 1$, etc.; so the Nth fringe corresponds to $N = m + 1$.

To find how many fringes are present, we solve for m by setting $t = h$:

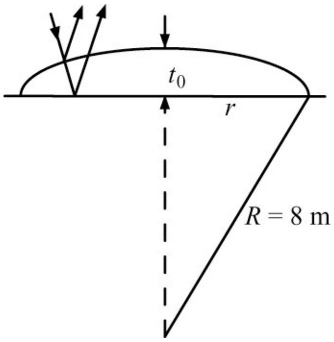
$$m + \frac{1}{2} = \frac{2nt}{\lambda} = \frac{2nh}{\lambda} = \frac{2(1.50)\left(1.00 \times 10^{-3}\text{ m}\right)}{\left(632.8 \times 10^{-9}\text{ m}\right)} = 4\ 740$$

$$\therefore \quad m = 4\ 740$$

So, the number of fringes is $N = m + 1 = 4\ 741$. This number is less than 5000.

P37.65 Light reflecting from the upper interface of the air layer suffers no phase change, while light reflecting from the lower interface is reversed 180°. Then there is indeed a dark fringe at the outer circumference of the lens, and a dark fringe wherever the air thickness t satisfies

$$2t = m\lambda, \quad m = 0, 1, 2, \ldots.$$



ANS. FIG. P37.65

(a) At the central dark spot, $m = 50$ and

$$t_0 = \frac{50\lambda}{2}$$
$$= 25\left(589\times10^{-9}\text{ m}\right) = 1.47\times10^{-5}\text{ m} = \boxed{14.7\ \mu\text{m}}$$

(b) In the right triangle,

$$R^2 = r^2 + \left(R - t_0\right)^2$$
$$(8.00\text{ m})^2 = r^2 + \left(8.00\text{ m} - 1.47\times10^{-5}\text{ m}\right)^2$$
$$\cancel{(8.00\text{ m})^2} = r^2 + \cancel{(8.00\text{ m})^2}$$
$$-2(8.00\text{ m})\left(1.47\times10^{-5}\text{ m}\right) + 2.16\times10^{-10}\text{ m}^2$$
$$r^2 = 2(8.00\text{ m})\left(1.47\times10^{-5}\text{ m}\right) - 2.16\times10^{-10}\text{ m}^2$$

The last term is negligible. Then,

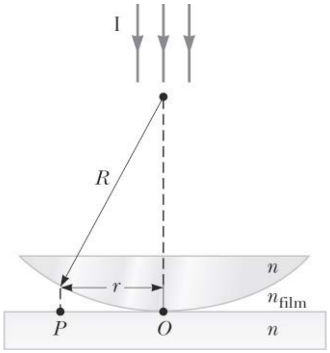
$$r = \sqrt{2(8\text{ m})\left(1.47\times10^{-5}\text{ m}\right)} = 1.53\times10^{-2}\text{ m} = \boxed{1.53\text{ cm}}$$

(c) $$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{\infty} - \frac{1}{8.00\text{ m}}\right)$$

$$\boxed{f = -16.0\text{ m}}$$

P37.66 The shift between the waves reflecting from the top and bottom surfaces of the film at the point where the film has thickness t is $\delta = 2tn_{\text{film}} + \frac{\lambda}{2}$, with the factor of $\frac{\lambda}{2}$ being due to a phase reversal at *one* of the surfaces.

For the dark rings (destructive interference), the total shift should be $\delta = \left(m + \frac{1}{2}\right)\lambda$ with $m = 0, 1, 2, 3, \ldots$. This requires that $t = \frac{m\lambda}{2n_{\text{film}}}$. To find t in terms of r and R,



ANS. FIG. P37.66

$$R^2 = r^2 + (R-t)^2 \rightarrow r^2 = 2Rt + t^2$$

Since t is much smaller than R, $t^2 << 2Rt$, therefore

$$r^2 \approx 2Rt = 2R\left(\frac{m\lambda}{2n_{\text{film}}}\right)$$

Thus, $\boxed{r \approx \sqrt{\frac{m\lambda R}{n_{\text{film}}}}}$ where m is an integer.

P37.67 Refer to the solution of P37.57. We may treat this as a double-slit interference problem, where $d = 2h$, but with maxima and minima interchanged because of phase reversal caused by the reflection off the mirror:

$$d\sin\theta = 2h\sin\theta = \left(m + \frac{1}{2}\right)\lambda \quad \text{bright fringe}$$

and $\sin\theta \approx \tan\theta = \frac{y}{L}$ for small angles; hence,

$$2h\sin\theta = \left(m + \frac{1}{2}\right)\lambda$$

$$2h\left(\frac{y}{L}\right) = \left(m + \frac{1}{2}\right)\lambda$$

The spacing between consecutive fringes corresponding to m and $m + 1$ is

$$2h\left(\frac{\Delta y}{L}\right) = \lambda$$

so

$$h = \frac{L\lambda}{2\Delta y} = \frac{(2.00\text{ m})(606\times 10^{-9}\text{ m})}{2(1.20\times 10^{-3}\text{ m})}$$

$$= 5.05\times 10^{-4}\text{ m} = \boxed{0.505\text{ mm}}$$

P37.68 (a) For a linear function taking the value $n = 1.90$ at $y = 0$ and $n = 1.33$ at $y = 20.0$ cm, we write

$$n(y) = 1.90 + (1.33 - 1.90)y/(20.0\text{ cm})$$

or $\boxed{n(y) = 1.90 - 0.0285\,y/\text{cm}}$

(b) The optical path length is

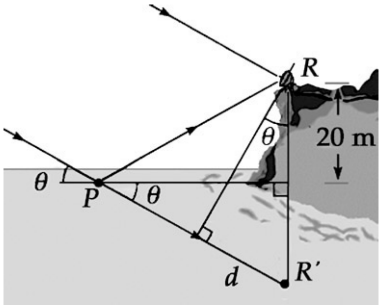
$$\int_0^{20.0\text{ cm}} n(y)\,dy = \int_0^{20.0\text{ cm}} [1.90 - 0.0285\,y/\text{cm}]\,dy$$

$$= 1.90y - \frac{0.0285y^2}{2}\Bigg|_0^{20.0\text{ cm}}$$

$$= 38.0\text{ cm} - 5.7\text{ cm} = \boxed{32.3\text{ cm}}$$

(c) A wavefront slows down as it travels deeper into the mixture to regions of greater index of refraction. The lower part of the wavefront travels more slowly than the upper part; the result is that the wavefront bends, becoming more horizontal. The path is similar to that of a beam crossing the boundary between a medium of lesser to a medium of greater index of refraction, as, for example, from air into water: the beam tends to bend toward the normal. The difference is that the change in direction is gradual rather than sudden. $\boxed{\text{The beam will continuously curve downward.}}$

P37.69 One radio wave reaches the receiver R directly from the distant source at an angle θ above the horizontal. The other wave undergoes phase reversal as it reflects from the water at P. The distance from P to R is the same as from P to R', where R' is the mirror image of the telescope. Therefore, the path difference is d.



ANS. FIG. P37.69

Constructive interference first occurs for a path difference of

$$d = \frac{\lambda}{2} \qquad \textbf{[1]}$$

The angles θ in the figure are equal because they each form part of a right triangle with a shared angle at *R′*.

So the path difference is

$$d = 2(20.0\text{ m})\sin\theta = (40.0\text{ m})\sin\theta$$

The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times 10^{8}\text{ m/s}}{60.0\times 10^{6}\text{ Hz}} = 5.00\text{ m}$$

Substituting for *d* and λ in equation [1],

$$(40.0\text{ m})\sin\theta = \frac{5.00\text{ m}}{2}$$

Solving for the angle θ,

$$\theta = \sin^{-1}\left(\frac{5.00\text{ m}}{80.0\text{ m}}\right) = \boxed{3.58°}$$

P37.70 One phase reversal occurs by reflection off the front of the soap film.

(a) Bright bands are observed when $2nt = \left(m + \frac{1}{2}\right)\lambda$.

Hence, the first bright band (*m* = 0) corresponds to $nt = \frac{\lambda}{4}$.

By similar triangles, the distance *x* from the top where a fringe occurs is proportional to the thickness *t* of the film:

$$\frac{x_1}{x_2} = \frac{t_1}{t_2}$$

Thus, we have

$$x_2 = x_1\left(\frac{t_2}{t_1}\right) = x_1\left(\frac{\lambda_2}{\lambda_1}\right) = (3.00\text{ cm})\left(\frac{680\text{ nm}}{420\text{ nm}}\right) = \boxed{4.86\text{ cm}}$$

(b) $$t_1 = \frac{\lambda_1}{4n} = \frac{420\text{ nm}}{4(1.33)} = \boxed{78.9\text{ nm}}$$

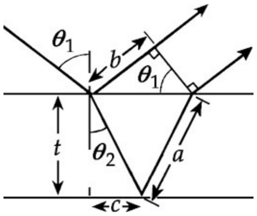
$$t_2 = \frac{\lambda_2}{4n} = \frac{680\text{ nm}}{4(1.33)} = \boxed{128\text{ nm}}$$

(c) $\theta \approx \tan\theta = \frac{t_1}{x_1} = \frac{78.9\text{ nm}}{3.00\text{ cm}} = \boxed{2.63\times10^{-6}\text{ rad}}$

Challenge Problems

P37.71 Refer to ANS. FIG. P37.71 for the geometry of the situation. At the air-film interface, Snell's law gives

$$1.00\sin 30.0° = 1.38\sin\theta_2 \quad\rightarrow\quad \theta_2 = 21.2°$$



ANS. FIG. P37.71

Call *t* the unknown thickness of the film. Then,

$$\cos 21.2° = \frac{t}{a} \quad\rightarrow\quad a = \frac{t}{\cos 21.2°}$$

$$\tan 21.2° = \frac{c}{t} \quad\rightarrow\quad c = t\tan 21.2°$$

$$\sin\theta_1 = \frac{b}{2c} \quad\rightarrow\quad b = 2t(\tan 21.2°)(\sin 30.0°)$$

The net shift for the second ray, including the phase reversal on reflection of the first, is

$$2an - b - \frac{\lambda}{2}$$

where the factor *n* accounts for the shorter wavelength in the film. For constructive interference, we require

$$2an - b - \frac{\lambda}{2} = m\lambda$$

The minimum thickness will occur when $m = 0$ and will be given by

$$2an - b - \frac{\lambda}{2} = 0$$

Then,

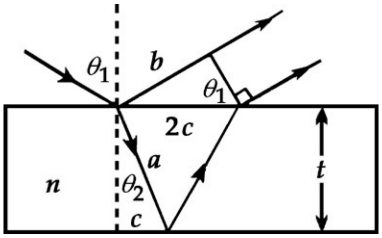
$$\frac{\lambda}{2} = 2an - b = 2\frac{nt}{\cos 21.2°} - 2t(\tan 21.2°)(\sin 30.0°)$$

and $$\frac{590\text{ nm}}{2}=\left[\frac{2(1.38)}{\cos 21.2°}-2(\tan 21.2°)(\sin 30.0°)\right]t=2.57t$$

which gives $t=\boxed{115\text{ nm}}$.

P37.72 The shift between the two reflected waves is $\delta = 2na - b - \frac{\lambda}{2}$, where a and b are as shown in the ray diagram in ANS. FIG. P37.72, n is the index of refraction, and the term $\frac{\lambda}{2}$ is due to phase reversal at the top surface. For constructive interference, $\delta = m\lambda$, where m has integer values. This condition becomes

$$2na-b=\left(m+\frac{1}{2}\right)\lambda \qquad \textbf{[1]}$$



ANS. FIG. P37.72

From the figure's geometry,

$$a=\frac{t}{\cos\theta_2}$$

$$c=a\sin\theta_2=\frac{t\sin\theta_2}{\cos\theta_2}$$

$$b=2c\sin\theta_1=\frac{2t\sin\theta_2}{\cos\theta_2}\sin\theta_1$$

Also, from Snell's law, $\sin\theta_1 = n\sin\theta_2$.

Thus, $$b=\frac{2nt\sin^2\theta_2}{\cos\theta_2}.$$

With these results, the condition for constructive interference given in equation [1] becomes:

$$2n\left(\frac{t}{\cos\theta_2}\right)-\frac{2nt\sin^2\theta_2}{\cos\theta_2}=\left(m+\frac{1}{2}\right)\lambda$$

$$\frac{2nt}{\cos\theta_2}\left(1-\sin^2\theta_2\right)=\left(m+\frac{1}{2}\right)\lambda$$

$$2nt\frac{\left(1-\sin^2\theta_2\right)}{\sqrt{1-\sin^2\theta_2}}=\left(m+\frac{1}{2}\right)\lambda$$

or $$2nt\sqrt{1-\sin^2\theta_2}=\left(m+\frac{1}{2}\right)\lambda$$

Using $\sin\theta_1 = n\sin\theta_2 \rightarrow \sin\theta_2 = \sin\theta_1/n$, we have finally

$$\boxed{2nt\sqrt{1-\frac{\sin^2\theta_1}{n^2}} = \left(m+\tfrac{1}{2}\right)\lambda, \text{ where } m = 0, 1, 2, \ldots}$$

P37.73 (a) Minimum: $2nt = m\lambda_2$ for $m = 0, 1, 2, \ldots$

Maximum: $2nt = \left(m'+\frac{1}{2}\right)\lambda_1$ for $m' = 0, 1, 2, \ldots$

Note that m and m' are distinct integer values, and must be consecutive because no intensity minima are observed between λ_1 and λ_2.

Also, $\lambda_1 > \lambda_2 \rightarrow \left(m'+\frac{1}{2}\right) < m$, so $m' = m - 1$.

Thus, we have

$$2nt = m\lambda_2 = \left(m'+\frac{1}{2}\right)\lambda_1 = \left[(m-1)+\frac{1}{2}\right]\lambda_1$$

$$m\lambda_2 = \left(m-\frac{1}{2}\right)\lambda_1$$

$$2m\lambda_2 = 2m\lambda_1 - \lambda_1$$

so $\boxed{m = \frac{\lambda_1}{2\left(\lambda_1-\lambda_2\right)}}$.

(b) $m = \dfrac{500\,\text{nm}}{2\left(500\,\text{nm}-370\,\text{nm}\right)} = 1.92 \rightarrow 2$ (wavelengths measured to ±5 nm)

Minimum: $2nt = m\lambda_2$

$2(1.40)t = 2(370\text{ nm})$ $t = 264\text{ nm}$

Maximum: $2nt = \left(m'+\frac{1}{2}\right)\lambda = \left(m-1+\frac{1}{2}\right)\lambda = 1.5\lambda$

$2(1.40)t = 1.5(500\text{ nm}) \rightarrow t = 268\text{ nm}$

Film thickness = $\boxed{266\text{ nm}}$

P37.74 The amplitude of the light from slit 1 is three times that from slit 2; therefore, the magnitude of the light arriving at the screen at some point *P* is

$$\begin{aligned} E_P &= E_1 + E_2 = 3E_0\sin(\omega t) + E_0\sin(\omega t+\phi) \\ &= E_0\left[3\sin\omega t + \sin(\omega t+\phi)\right] \\ \frac{E_P}{E_0} &= 3\sin(\omega t) + \sin(\omega t+\phi) \\ &= 3\sin(\omega t) + \left[\sin(\omega t)\cos(\phi) + \cos(\omega t)\sin(\phi)\right] \\ &= \sin(\omega t)\left[3+\cos(\phi)\right] + \cos(\omega t)\sin(\phi) \end{aligned}$$

The square of this expression is

$$\begin{aligned} \left(\frac{E_P}{E_0}\right)^2 = \sin^2(\omega t)\left[3+\cos(\phi)\right]^2 \\ &+ 2\sin(\omega t)\cos(\omega t)\left[3+\cos(\phi)\right]\sin(\phi) \\ &+ \cos^2(\omega t)\sin^2(\phi) \end{aligned}$$

$$\begin{aligned} \left(\frac{E_P}{E_0}\right)^2 = \sin^2(\omega t)\left[3+\cos(\phi)\right]^2 + \sin(2\omega t)\left[3+\cos(\phi)\right]\sin(\phi) \\ &+ \cos^2(\omega t)\sin^2(\phi) \end{aligned}$$

and the time average of this expression is

$$\begin{aligned} \overline{\left(\frac{E_P}{E_0}\right)^2} &= \frac{1}{2}\left[3+\cos(\phi)\right]^2 + \frac{1}{2}\sin^2(\phi) \\ &= \frac{1}{2}\left[9+6\cos(\phi)+\cos^2(\phi)+\sin^2(\phi)\right] = \frac{1}{2}\left[10+6\cos(\phi)\right] \end{aligned}$$

because the time average of $\sin^2(\omega t)$ and $\cos^2(\omega t)$ is $\frac{1}{2}$, and the time average of $\sin(2\omega t)$ is zero. Using the identity

$$\cos(\phi) = \cos\left(\frac{\phi}{2}+\frac{\phi}{2}\right) = 2\cos^2\left(\frac{\phi}{2}\right) - 1$$

we have

$$\begin{aligned} \overline{\left(\frac{E_P}{E_0}\right)^2} &= \frac{1}{2}\left[10+6\cos(\phi)\right] = \frac{1}{2}\left[10+6\left(2\cos^2\left(\frac{\phi}{2}\right)-1\right)\right] \\ &= \frac{1}{2}\left[4+12\cos^2\left(\frac{\phi}{2}\right)\right] = 2\left[1+3\cos^2\left(\frac{\phi}{2}\right)\right] \end{aligned}$$

Intensity is proportional to the time average of the square of the amplitude, so

$$I \propto \overline{E_P^2} = 2E_0^2\left[1+3\cos^2\left(\frac{\phi}{2}\right)\right]$$

At the central maximum, $\phi = 0$, so the maximum intensity is

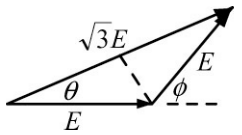
$$I_{\max} \propto 2E_0^2\left[1+3\cos^2(0)\right] = 2E_0^2(4) = 8E_0^2$$

Thus, we have

$$\frac{I}{I_{\max}} = \frac{2E_0^2\left[1+3\cos^2\left(\frac{\phi}{2}\right)\right]}{8E_0^2} = \frac{1}{4}\left[1+3\cos^2\left(\frac{\phi}{2}\right)\right]$$

$$I = \frac{I_{\max}}{4}\left[1+3\cos^2\left(\frac{\phi}{2}\right)\right]$$

P37.75 Represent the light radiated from each slit to point *P* as a phasor. The two have very nearly equal amplitudes *E*. Since intensity is proportional to amplitude squared, we are told they add to amplitude $\sqrt{3}E$. As shown in the figure, the triangle representing the sum of phasors may be divided into two right triangles whose common side that bisects the line of length $\sqrt{3}E$. From either triangle, we see that



ANS. FIG. P37.75

$$\cos\theta = \frac{\sqrt{3}\,E/2}{E} \quad \rightarrow \quad \theta = 30°$$

Next, the obtuse angle between the two phasors is 180 – 30 – 30 = 120°, and so $\phi = 180 - 120° = 60°$.

The phase difference between the two phasors is caused by the path difference from S to the slits, $\delta = \overline{SS_2} - \overline{SS_1}$, according to $\frac{\delta}{\lambda} = \frac{\phi}{360°}$, $\delta = \lambda\frac{60°}{360°} = \frac{\lambda}{6}$. Then

$$\delta = \sqrt{L^2+d^2} - L = \frac{\lambda}{6}$$

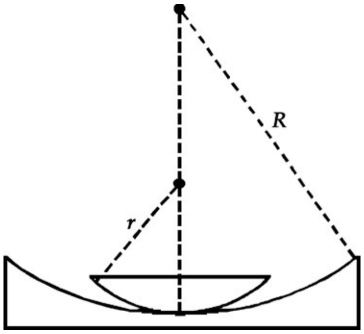
$$L^2 + d^2 = L^2 + \frac{2L\lambda}{6} + \frac{\lambda^2}{36} \quad \rightarrow \quad d^2 = \frac{2L\lambda}{6} + \frac{\lambda^2}{36}$$

The last term is negligible, so

$$d=\left(\frac{2L\lambda}{6}\right)^{1/2}=\sqrt{\frac{2(1.2\text{ m})(620\times10^{-9}\text{ m})}{6}}=\boxed{0.498\text{ mm}}$$

P37.76 For bright rings the gap *t* between surfaces is given by $2t=\left(m+\frac{1}{2}\right)\lambda$.

The first bright ring has *m* = 0 and the hundredth has *m* = 99.



ANS. FIG. P37.76

So, $t=\frac{1}{2}(99.5)(500\times10^{-9}\text{ m})=24.9\ \mu\text{m}$

Call r_b the ring radius. From the geometry shown in ANS. FIG. P37.76,

$$t=\left(r-\sqrt{r^2-r_b^2}\right)-\left(R-\sqrt{R^2-r_b^2}\right)$$

$$=r-r\sqrt{1-\left(\frac{r_b}{r}\right)^2}-R+R\sqrt{1-\left(\frac{r_b}{R}\right)^2}$$

Since $r_b \ll r$, we can expand in binomial series:

$$t=r-r\left(1-\frac{1}{2}\frac{r_b^2}{r^2}\right)-R+R\left(1-\frac{1}{2}\frac{r_b^2}{R^2}\right)=\frac{1}{2}\frac{r_b^2}{r}-\frac{1}{2}\frac{r_b^2}{R}$$

$$r_b=\left[\frac{2t}{1/r-1/R}\right]^{1/2}=\left[\frac{2(24.9\times10^{-6}\text{ m})}{1/4.00\text{ m}-1/12.0\text{ m}}\right]^{1/2}=\boxed{1.73\text{ cm}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P37.2	3.53 mm
P37.4	515 nm
P37.6	The sine of the angle for $m = 1$ fringe is greater than 1, which is impossible.
P37.8	(a) 1.77 μm; (b) 1.47 μm
P37.10	36.2 cm
P37.12	(a) 34.9°; (b) 5.25 cm; (c) 5.24×10^{14} Hz
P37.14	11.3 m
P37.16	$v\tan\left[\sin^{-1}\left(\frac{m\lambda}{d}\right)\right]$
P37.18	(a) 13.2 rad; (b) 6.28 rad; (c) 1.27×10^{-2} deg; (d) 5.97×10^{-2} deg
P37.20	(a) 22.6 cm; (b) 2.51×10^{-3}; (c) 6.03×10^{-7} m; (d) 7.21°; (e) 2.28 cm; (f) The two answers are close but do not agree exactly. The fringes are not laid out linearly on the screen as assumed in part (a), and this nonlinearity is evident for relatively large angles such as 7.21°.
P37.22	(a) 10 m; (b) 500 m; (c) See P37.22(c) for full explanation.
P37.24	$E_R = 10.0$ and $\phi = 53.1°$
P37.26	(a) $I = \frac{I_{\max}}{9}\left[1+2\cos\left(\frac{2\pi d\sin\theta}{\lambda}\right)\right]^2$; (b) See P37.24(b) for full explanation; (c) 9:1
P37.28	See ANS. FIG. P37.28.
P37.30	(a) 638 nm; (b) A thicker film would require a higher order of reflection, so use a larger value of m; (c) 360 nm, 600 nm
P37.32	96.2 nm
P37.34	(a) 276 nm, 138 nm, 92.0 nm; (b) No visible wavelengths are intensified.
P37.36	(a) green; (b) violet
P37.38	(a) 238 nm; (b) λ increase; (c) 328 nm
P37.40	(a) 97.8 nm; (b) Yes. Destructive interference occurs when $2nt = (m+\frac{1}{2})\lambda$ (Eq. 37.17), where m is an integer. (There is a phase change at both faces of the film in Figure P37.40.) Hence, for $m = 1, 2,\ldots$ we obtain thicknesses of 293 nm, 489 nm, . . .

P37.42	39.6 μm
P37.44	$1+\dfrac{N\lambda}{2L}$
P37.46	1.4×10^2 Hz
P37.48	(a) See P37.48(a) for full explanation; (b) 2.74 m
P37.50	(a) ~10^{-3} degree; (b) ~10^{11} Hz; (c) microwaves
P37.52	2.52 cm
P37.54	20.0×10^{-6} °C^{-1}
P37.56	1.62 km
P37.58	421 nm
P37.60	$y = L\tan\left\{\sin^{-1}\left[\dfrac{(n-1)t}{d}\right]\right\} = \dfrac{(n-1)Lt}{\sqrt{d^2-(n-1)^2t^2}}$
P37.62	(a) $2\sqrt{4h^2+d^2}-2d$; (b) $\sqrt{4h^2+d^2}-d$
P37.64	The number of fringes is $N = m + 1 = 474\ 1$. This number is less than 5 000.
P37.66	$r \approx \sqrt{\dfrac{m\lambda R}{n_{\text{film}}}}$
P37.68	(a) $n(y) = 1.90 - 0.0285\,y/\text{cm}$; (b) 32.3 cm; (c) The beam will continuously curve downward.
P37.70	(a) 4.86 cm; (b) 78.9 nm, 128 nm; (c) 2.63×10^{-6} rad
P37.72	$2nt\sqrt{1-\dfrac{\sin^2\theta_1}{n^2}} = \left(m+\tfrac{1}{2}\right)\lambda,$ where $m = 0, 1, 2, \ldots$
P37.74	See P37.74 for full explanation.
P37.76	1.73 cm

38

Diffraction Patterns and Polarization

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ38.1 Answer (a). Glare, as usually encountered when driving or boating, is horizontally polarized. Reflected light is polarized in the same plane as the reflecting surface. As unpolarized light hits a shiny horizontal surface, the atoms on the surface absorb and then reemit the light energy as a reflection. We can model the surface as containing conduction electrons free to vibrate easily along the surface, but not to move easily out of surface. The light emitted from a vibrating electron is partially or completely polarized along the plane of vibration, thus horizontally.

OQ38.2 Answer (c). The polarization state of a light beam that is reflected by a metallic surface is not changed; therefore, a beam of light that is not polarized before it is reflected is not polarized after it is reflected by a metallic surface.

OQ38.3 Answer (b). The wavelength will be much smaller than with visible light, so there will be no noticeable diffraction pattern.

OQ38.4 Answer (b). In a single slit diffraction pattern, dark fringes occur where $\sin\theta_{\text{dark}} = m\lambda/a \approx \tan\theta_{\text{dark}} = y_{\text{dark}}/L,$ and *m* is any non-zero integer.

Thus, the width of the slit, *a*, in the described situation, must be

$$a = \frac{m\lambda L}{\left(y_{\text{dark}}\right)_1} = \frac{(1)\lambda L}{\left(y_{\text{dark}}\right)_1} = \frac{\left(5.00\times 10^{-7}\text{ m}\right)(1.00\text{ m})}{5.00\times 10^{-3}\text{ m}}$$
$$= 1.00\times 10^{-4}\text{ m} = 0.100\text{ mm}$$

OQ38.5 Answer (d). The central maximum lies between the first-order minima defined by the relation $\sin\theta_{\text{dark}} = m\lambda/a = \lambda/a.$ Because the angle is small, $\sin\theta_{\text{dark}} \approx \tan\theta_{\text{dark}} = y_{\text{dark}}/L,$ so the width of the central maximum is proportional to $L\lambda/a$. Thus, the central maximum becomes twice as wide if the slit width *a* becomes half as wide.

OQ38.6 The ranking is (e) > (c) >(a) > (b) > (d). The central maximum lies between the first-order minima defined by the relation $\sin\theta_{\text{dark}} = m\lambda/a = \lambda/a$. Because the angle is small, $\sin\theta_{\text{dark}} \approx \tan\theta_{\text{dark}} = y_{\text{dark}}/L$, so the width of the central maximum is proportional to $L\lambda/a$. We consider the value of $L\lambda/a$: (a) $L\lambda_0/a$, (b) $f = c/\lambda_0$, so for $f' = 3/2\, f$, $\lambda' = 2/3\,\lambda_0$, and the width is $L\left(2/3\,\lambda_0\right)/a = 2/3\left(L\lambda_0/a\right)$, (c) $L\left(1.5\lambda_0\right)/a = 32\left(L\lambda_0/a\right)$, (d) $L\lambda_0/(2a) = 1/2\left(L\lambda_0/a\right)$, (e) $(2L)\lambda_0/a = 2\left(L\lambda_0/a\right)$.

OQ38.7 Answer (b). From Malus' law, the intensity of the light transmitted through a polarizer (analyzer) having its transmission axis oriented at angle 45° to the plane of polarization of the incident polarized light is $I = I_{\text{max}}\cos^2 45° = I_{\text{max}}/2$. Therefore, the intensity passing through the second polarizer having its transmission axis oriented at angle $\theta = 90° - 45° = 45°$ is $I = (I_{\text{max}}/2)\cos^2 45° = I_{\text{max}}/4$.

OQ38.8 Answer (e). Diffraction of light as it passes through, or reflects from, the objective element of a telescope can cause the images of two sources having a small angular separation to overlap and fail to be seen as separate images. According to Equation 38.6, $\theta_{\text{min}} = 1.22\,\lambda/D$, the minimum angular separation θ_{min} two sources must have in order to be seen as separate sources is inversely proportional to the diameter *D* of the objective element. Thus, using a large-diameter objective element in a telescope increases its resolution.

OQ38.9 Answer (e). The bright colored patterns are the result of interference between light reflected from the upper surface of the oil and light reflected from the lower surface of the oil film.

OQ38.10 Answer (b). No diffraction effects are observed because the separation distance between adjacent ribs is so much greater than the wavelength of x-rays. Diffraction does not limit the resolution of an x-ray image. Diffraction might sometimes limit the resolution of a sonogram.

OQ38.11 Answer (a). The grooves in a diffraction grating are not electrically conducting. Sending light through a diffraction grating is not like sending a vibration on a rope through a picket fence: there is no moving substance that could collide with the groove of the grating, so the grating could not prevent the wave from passing though it.

OQ38.12 Answer (c). The ability to resolve light sources depends on diffraction, not on intensity.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ38.1 The crystal cannot produce diffracted beams of visible light. The wavelengths of visible light are some hundreds of nanometers. There is no angle whose sine is greater than 1. Bragg's law, $2d\sin\theta = m\lambda$, cannot be satisfied for a wavelength much larger than the distance between atomic planes in the crystal.

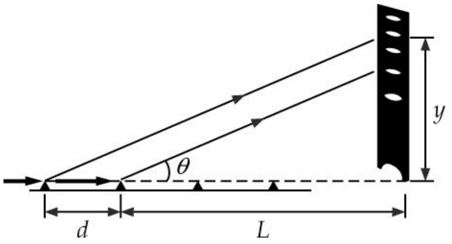
CQ38.2 The wavelength of visible light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around an obstacle the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

CQ38.3 Since the obsidian is opaque, a standard method of measuring incidence and refraction angles and using Snell's Law is ineffective. Reflect unpolarized light from the horizontal surface of the obsidian through a vertically polarized filter. Change the angle of incidence until you observe that none of the reflected light is transmitted through the filter. This means that the reflected light is completely horizontally polarized, and that the incidence and reflection angles are the polarization angle. According to Equation 38.10, the tangent of the polarization angle is the index of refraction of the obsidian.

CQ38.4 (a) Light from the sky is partially polarized.

(b) Light from the blue sky that is polarized at 90° to the polarization axis of the glasses will be blocked, making the sky look darker as compared to the clouds.

CQ38.5 Consider incident light nearly parallel to the horizontal ruler. Suppose it scatters from bumps at distance d apart to produce a diffraction pattern on a vertical wall a distance L away. At a point of height y, where $\theta = \frac{y}{L}$ gives the scattering angle θ, the character of the interference is determined by the shift δ between beams scattered by adjacent bumps, where $\delta = d\cos\theta \approx d\left(1+\frac{\theta^2}{2}\right)$. Bright spots appear for $\delta = m\lambda$, where m = 1, 2, 3, ….



ANS. FIG. CQ38.5

For small θ, these equations combine and reduce to $m\lambda = d\left(1+\frac{y_m^2}{2L^2}\right)$. Measurement of the heights y_m of bright spots allows calculation of the wavelength of the light. [Note that if a maximum occurs at $\theta = \frac{y}{L} \approx 0$, then scattered light from a bump constructively interferes with scattered light from the next bump in front, which constructively interferes with scattered light from the next bump…; thus $\lambda = d$.]

CQ38.6 First think about the glass without a coin and about one particular point P on the screen. We can divide up the area of the glass into ring-shaped zones centered on the line joining P and the light source, with successive zones contributing alternately in-phase and out-of-phase with the light that takes the straight-line path to P. These Fresnel zones have nearly equal areas. An outer zone contributes only slightly less to the total wave disturbance at P than does the central circular zone. Now insert the coin. If P is in line with its center, the coin will block off the light from some particular number of zones. The first unblocked zone around its circumference will send light to P with significant amplitude. Zones farther out will predominantly interfere destructively with each other, and the Arago spot is bright. Slightly off the axis there is nearly complete destructive interference, so most of the geometrical shadow is dark. A bug on the screen crawling out past the edge of the geometrical shadow would in effect see the central few zones coming out of eclipse. As the light from them interferes alternately constructively and destructively, the bug moves through bright and dark fringes on the screen. The diffraction pattern is shown in Figure 38.3 in the text.

CQ38.7 The skin on the tip of a finger has a series of closely spaced ridges and swirls on it. When the finger touches a smooth surface, the oils from the skin will be deposited on the surface in the pattern of the closely spaced ridges. The clear spaces between the lines of deposited oil can serve as the slits in a crude diffraction grating and produce a colored spectrum of the light passing through or reflecting from the glass surface.

CQ38.8 (a) The diffraction pattern of a hair is the same as the diffraction pattern produced by a single slit of the same width.

(b) The central maximum is flanked by minima. Measure the width 2y of the central maximum between the minima bracketing it. Because the angle is small, you can use

$$\begin{aligned}\sin\theta_{\text{dark}} &\approx \tan\theta_{\text{dark}}\\ m\lambda/a &\approx y/L\end{aligned}$$

to find the width a of the hair.

CQ38.9 The condition for constructive interference is that the three radio signals arrive at the city in phase. We know the speed of the waves (it is the speed of light c), the angular bearing θ of the city east of north from the broadcast site, and the distance d between adjacent towers. The wave from the westernmost tower must travel an extra distance $2d\sin\theta$ to reach the city, compared to the signal from the eastern tower. For each cycle of the carrier wave, the western antenna would transmit first, the center antenna after a time delay $\dfrac{d\sin\theta}{c}$, and the eastern antenna after an additional equal time delay.

CQ38.10 The correct orientation is vertical. If the horizontal width of the opening is equal to or less than the wavelength of the sound, then the equation $a\sin\theta = (1)\lambda$ has the solution $\theta = 90°$, or has no solution. The central diffraction maximum covers the whole seaward side. If the vertical height of the opening is large compared to the wavelength, then the angle in $a\sin\theta = (1)\lambda$ will be small, and the central diffraction maximum will form a thin horizontal sheet.

Featured in the motion picture *M*A*S*H* (20th Century Fox, Aspen Productions, 1970) is a loudspeaker mounted on an exterior wall of an Army barracks. It has an approximately rectangular aperture, and it is installed incorrectly. The longer side is horizontal, to maximize sound spreading in a vertical plane and to minimize sound radiated in different horizontal directions.

CQ38.11 Audible sound has wavelengths on the order of meters or centimeters, while visible light has a wavelength on the order of half a micrometer. In this world of breadbox-sized objects, $\frac{\lambda}{a}$ is large for sound, and sound diffracts around walls with doorways. But $\frac{\lambda}{a}$ is a tiny fraction for visible light passing ordinary-size objects or apertures, so light changes its direction by only very small angles when it diffracts.

Another way of phrasing the answer: We can see by a small angle around a small obstacle or around the edge of a small opening. The side fringes in Figure 38.1 and the Arago spot in the center of Figure 38.3 show this diffraction. We cannot always hear around corners. Out-of-doors, away from reflecting surfaces, have someone a few meters distant face away from you and whisper. The high-frequency, short-wavelength, information-carrying components of the sound do not diffract around his head enough for you to understand his words.

Suppose an opera singer loses the tempo and cannot immediately get it from the orchestra conductor. Then the prompter may make rhythmic kissing noises with her lips and teeth. Try it—you will sound like a birdwatcher trying to lure out a curious bird. This sound is clear on the stage but does not diffract around the prompter's box enough for the audience to hear it.

CQ38.12 Consider vocal sound moving at 340 m/s and of frequency 3 000 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{340\text{ m/s}}{3\,000\text{ Hz}} = 0.113\text{ m}$$

If your mouth, for horizontal dispersion, behaves similarly to a slit 6.00 cm wide, then $a\sin\theta = m\lambda$ predicts no diffraction minima. You are a nearly isotropic source of this sound. It spreads out from you nearly equally in all directions. On the other hand, if you use a megaphone with width 60.0 cm at its wide end, then $a\sin\theta = m\lambda$ predicts the first diffraction minimum at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{0.113\text{ m}}{0.600\text{ m}}\right) = 10.9^\circ$$

This suggests that the sound is radiated mostly toward the front into a diverging beam of angular diameter only about 20°. With less sound energy wasted in other directions, more is available for your intended auditors. We could check that a distant observer to the side or behind you receives less sound when a megaphone is used.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 38.2 Diffraction Patterns from Narrow Slits

***P38.1** (a) According to Equation 38.1, dark bands (minima) occur where

$$\sin\theta = m\frac{\lambda}{a}$$

For the first minimum, $m = 1$, and the distance from the center of the central maximum is

$$y_1 = L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right)$$

Thus, the needed distance to the screen is

$$L = y_1\left(\frac{a}{\lambda}\right) = \left(0.85\times10^{-3}\text{ m}\right)\left(\frac{0.75\times10^{-3}\text{ m}}{587.5\times10^{-9}\text{ m}}\right) = \boxed{1.1\text{ m}}$$

(b) The width of the central maximum is

$$2y_1 = 2\left(0.85\text{ mm}\right) = \boxed{1.7\text{ mm}}$$

P38.2 From Equation 38.1, with $m = 1$,

$$\sin\theta = \frac{\lambda}{a} = \frac{6.328\times10^{-7}\text{ m}}{3.00\times10^{-4}\text{ m}} = 2.11\times10^{-3}$$

Then,

$$\frac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta \approx \theta \text{ (for small } \theta) \quad \rightarrow \quad y = 2.11\text{ mm}$$

and $2y = \boxed{4.22\text{ mm}}$

P38.3 If the speed of sound is 343 m/s,

$$\lambda = \frac{v}{f} = \frac{343\text{ m/s}}{650\text{ s}^{-1}} = 0.528\text{ m}$$

Diffraction minima occur at angles described by $a\sin\theta = m\lambda$.

$$\left(1.10\text{ m}\right)\sin\theta_1 = 1\left(0.528\text{ m}\right) \quad \theta_1 = \pm28.7°$$

$$\left(1.10\text{ m}\right)\sin\theta_2 = 2\left(0.528\text{ m}\right) \quad \theta_2 = \pm73.6°$$

$$\left(1.10\text{ m}\right)\sin\theta_3 = 3\left(0.523\text{ m}\right) \quad \theta_3 \text{ nonexistent}$$

(a) There are $\boxed{\text{four}}$ minima.

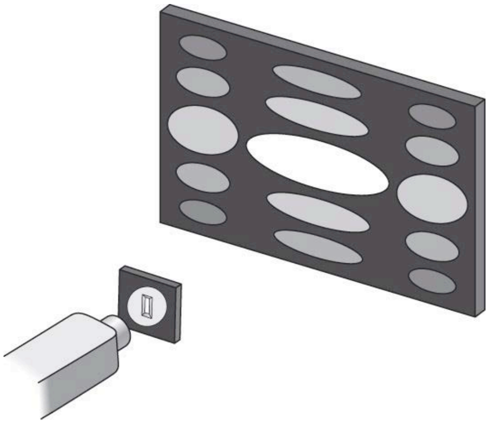
(b) $\boxed{\theta = \pm28.7°, \pm73.6°}$

P38.4 (a) Refer to ANS. FIG. P38.4. The rectangular patch on the wall is wider than it is tall. The aperture will be taller than it is wide. For horizontal spreading we have

$$\tan\theta_{\text{width}} = \frac{y_{\text{width}}}{L} = \frac{0.110\text{ m}/2}{4.5\text{ m}} = 0.012\,2$$

$$a_{\text{width}}\sin\theta_{\text{width}} = 1\lambda$$

$$a_{\text{width}} = \frac{632.8\times10^{-9}\text{ m}}{0.012\,2} = 5.18\times10^{-5}\text{ m} = \boxed{51.8\ \mu\text{m}}$$



ANS. FIG. P38.4

(b) For vertical spreading, similarly

$$\tan\theta_{\text{height}} = \frac{0.006\text{ m}/2}{4.5\text{ m}} = 0.000\,667$$

$$a_{\text{height}} = \frac{1\lambda}{\sin\theta_h} = \frac{632.8\times10^{-9}\text{ m}}{0.000\,667} = 9.49\times10^{-4}\text{ m} = \boxed{949\ \mu\text{m}}$$

(c) The longer dimension in the central bright patch is $\boxed{\text{horizontal}}$.

(d) The longer dimension of the aperture is $\boxed{\text{vertical}}$.

(e) $\boxed{\text{A smaller distance between aperture edges causes a wider diffraction angle. The longer dimension of each rectangle is 18.3 times larger than the smaller dimension.}}$

P38.5 For destructive interference, from Equation 38.1,

$$\sin\theta = m\frac{\lambda}{a} = \frac{\lambda}{a} = \frac{5.00\text{ cm}}{36.0\text{ cm}} = 0.139$$

and $\theta = 7.98°$. Then,

$$\frac{y}{L} = \tan\theta$$

gives $y = L\tan\theta = (6.50\text{ m})\tan 7.98° = 0.912\text{ m}$

$$= \boxed{91.2\text{ cm}}$$

P38.6 In a single slit diffraction pattern, with the slit having width a, the dark fringe of order m occurs at angle θ_m, where $\sin\theta_m = m(\lambda/a)$ and $m = \pm1,\ \pm2,\ \pm3, \ldots$. The location, on a screen located distance L from the slit, of the dark fringe of order m (measured from $y = 0$ at the center of the central maximum) is

$$(y_{\text{dark}})_m = L\tan\theta_m \approx L\sin\theta_m = m\lambda\left(\frac{L}{a}\right)$$

(a) The central maximum extends from the $m = +1$ dark fringe on one side to the $m = -1$ dark fringe on the other side, so the width of this central maximum is

$$\text{Central max. width} = (y_{\text{dark}})_{m=1} - (y_{\text{dark}})_{m=-1}$$
$$= (1)\left(\frac{\lambda L}{a}\right) - (-1)\left(\frac{\lambda L}{a}\right) = \frac{2\lambda L}{a}$$

Therefore,

$$L = \frac{a(\text{Central max. width})}{2\lambda}$$
$$= \frac{(0.200\times10^{-3}\text{ m})(8.10\times10^{-3}\text{ m})}{2(5.40\times10^{-7}\text{ m})} = \boxed{1.50\text{ m}}$$

(b) The first order bright fringe extends from the $m = 1$ dark fringe to the $m = 2$ dark fringe, or

$$(\Delta y_{\text{bright}})_1 = (y_{\text{dark}})_{m=2} - (y_{\text{dark}})_{m=1} = 2\left(\frac{\lambda L}{a}\right) - 1\left(\frac{\lambda L}{a}\right) = \frac{\lambda L}{a}$$
$$= \frac{(5.40\times10^{-7}\text{ m})(1.50\text{ m})}{0.200\times10^{-3}\text{ m}}$$
$$= 4.05\times10^{-3}\text{ m} = \boxed{4.05\text{ mm}}$$

Note that the width of the first order bright fringe is exactly one half the width of the central maximum.

P38.7 In the equation for single-slit diffraction minima at small angles,

$$\frac{y}{L} \approx \sin\theta_{\text{dark}} = \frac{m\lambda}{a}$$

we take differences between the first and third dark fringes, to see that

$$\frac{\Delta y}{L} = \frac{\Delta m\lambda}{a} \quad \text{with} \quad \Delta y = 3.00\times10^{-3}\text{ m} \quad \text{and} \quad \Delta m = 3 - 1 = 2$$

The width of the slit is then

$$a = \frac{\lambda L \Delta m}{\Delta y} = \frac{\left(690\times10^{-9}\text{ m}\right)(0.500\text{ m})(2)}{\left(3.00\times10^{-3}\text{ m}\right)} = \boxed{2.30\times10^{-4}\text{ m}}$$

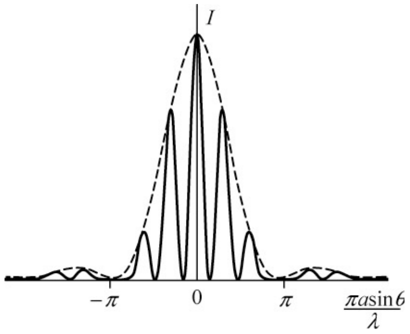
P38.8 Use the small-angle approximation: $\frac{y}{L} \approx \sin\theta = \frac{m\lambda}{a}$.

Then, $\frac{\Delta y}{L} = \left|\frac{m_2\lambda}{a} - \frac{m_1\lambda}{a}\right| = \left|m_2 - m_1\right|\frac{\lambda}{a} \rightarrow \boxed{a = \frac{\lambda L\left|m_2 - m_1\right|}{\Delta y}}$.

P38.9 The diffraction envelope shows a broad central maximum flanked by zeros at $a\sin\theta = 1\lambda$ and $a\sin\theta = 2\lambda$. That is, the zeros are at $(\pi a \sin\theta)/\lambda = \pi,\ -\pi,\ 2\pi,\ -2\pi, \ldots$ Noting that the distance between slits is $d = 9\ \mu\text{m} = 3a$, we say that within the diffraction envelope the interference pattern shows closely spaced maxima at $d\sin\theta = m\lambda$, giving $(\pi 3a\sin\theta)/\lambda = m\pi$ or

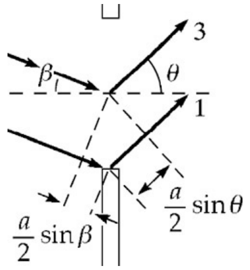
$$(\pi a\sin\theta)/\lambda = 0,\ \pi/3,\ -\pi/3,\ 2\pi/3,\ -2\pi/3$$

The third-order interference maxima are missing because they fall at the same directions as diffraction minima, but the fourth order can be visible at $(\pi a\sin\theta)/\lambda = 4\pi/3$ and $-4\pi/3$ as diagrammed.



ANS. FIG. P38.9

P38.10 Equation 38.1 states that $\sin\theta = \frac{m\lambda}{a}$, where $m = \pm1,\ \pm2,\ \pm3,\ \ldots$. The requirement for $m = 1$ is from an analysis of the extra path distance traveled by ray 1 compared to ray 3 in the textbook Figure 38.5. This extra distance must be equal to $\frac{\lambda}{2}$ for destructive interference. When the source rays approach the slit at an angle β, there is a distance added to the path difference (of ray 1 compared to



ANS. FIG. P38.10

ray 3) of $\frac{a}{2}\sin\beta$. Then, for destructive interference,

$$\frac{a}{2}\sin\beta + \frac{a}{2}\sin\theta = \frac{\lambda}{2} \quad \text{so} \quad \sin\theta = \frac{\lambda}{a} - \sin\beta$$

Dividing the slit into 4 parts leads to the second order minimum:

$$\frac{a}{4}\sin\beta + \frac{a}{4}\sin\theta = \frac{\lambda}{2} \quad \text{so} \quad \sin\theta = \frac{2\lambda}{a} - \sin\beta$$

Dividing the slit into 6 parts gives the third order minimum:

$$\sin\theta = \frac{3\lambda}{a} - \sin\beta$$

Generalizing, we obtain the condition for the mth order minimum:

$$\sin\theta = \frac{m\lambda}{a} - \sin\beta \qquad m = \pm 1,\ \pm 2,\ \pm 3,\ \ldots$$

P38.11 First we find where we are. The angle to the side is small so

$$\sin\theta \approx \tan\theta = \frac{y}{L} = \frac{4.10\times 10^{-3}\text{ m}}{1.20\text{ m}} = 3.417\times 10^{-3}$$

The parameter controlling the intensity is

$$\frac{\pi a\sin\theta}{\lambda} = \frac{\pi\left(4.00\times 10^{-4}\text{ m}\right)\left(3.417\times 10^{-3}\right)}{546.1\times 10^{-9}\text{ m}} = 7.862\text{ rad}$$

This is between 2π and 3π, so the point analyzed is off in the second side fringe. The fractional intensity is

$$\frac{I}{I_{\max}} = \left[\frac{\sin\left(\pi a\sin\theta/\lambda\right)}{\pi a\sin\theta/\lambda}\right]^2 = \left[\frac{\sin(7.862\text{ rad})}{7.862\text{ rad}}\right]^2 = \boxed{1.62\times 10^{-2}}$$

P38.12 (a) Double-slit interference maxima are at angles given by $d\sin\theta = m\lambda$.

For $m = 0$, $\theta_0 = \boxed{0°}$

For $m = 1$, $(2.80\ \mu\text{m})\sin\theta = 1(0.501\,5\ \mu\text{m})$:

$\theta_1 = \sin^{-1}(0.179) = \boxed{\pm 10.3°}$

Similarly, for $m = 2, 3, 4$, and 5,

$\theta_2 = \boxed{\pm 21.0°}$, $\theta_3 = \boxed{\pm 32.5°}$, $\theta_4 = \boxed{\pm 45.8°}$, and $\theta_5 = \boxed{\pm 63.6°}$

For $m > 5$, there are no maxima.

(b) Thus, there are $5 + 5 + 1 = \boxed{11}$ directions for interference maxima.

(c) We check for missing orders by looking for single-slit diffraction minima, at $a\sin\theta = m\lambda$.

For $m = 1$, $(0.700\ \mu\text{m})\sin\theta = 1(0.501\,5\ \mu\text{m})$ and $\theta_1 = \boxed{\pm 45.8°}$

Thus, there is no bright fringe at this angle.

(d) From our answer to (c), $\boxed{\text{two}}$.

(e) $\boxed{\text{two}}$

(f) Two are missing because slit-slit minimum occur where a double-slit maximum would be: $\boxed{\text{nine}}$

(g) $$I = I_{\max}\left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi\sin\theta/\lambda}\right]^2$$

At $\theta = 63.6°$,

$$\frac{\pi a\sin\theta}{\lambda} = \frac{\pi(0.700\ \mu\text{m})\sin 63.6°}{(0.501\,5\ \mu\text{m})} = 3.93\ \text{rad} = 225°$$

and $I = \boxed{0.032\,4 I_{\max}}$

P38.13 With the screen locations of the dark fringe of order m at

$$(y_{\text{dark}})_m = L\tan\theta_m \approx L\sin\theta_m = m(\lambda L/a) \quad \text{for } m = \pm 1, \pm 2, \pm 3, \ldots$$

the width of the central maximum is

$$\Delta y_{\substack{\text{central}\\\text{maximum}}} = (y_{\text{dark}})_{m=+1} - (y_{\text{dark}})_{m=-1} = 2(\lambda L/a)$$

so

$$\lambda = \frac{a\left(\Delta y_{\substack{\text{central}\\\text{maximum}}}\right)}{2L} = \frac{(0.600\times 10^{-3}\ \text{m})(2.00\times 10^{-3}\ \text{m})}{2(1.30\ \text{m})}$$
$$= 4.62\times 10^{-7}\ \text{m} = \boxed{462\ \text{nm}}$$

Section 38.3 Resolution of Single-Slit and Circular Apertures

P38.14 We assume Rayleigh's criterion applies to the cat's eye with pupil narrowed. For a single slit (not a round aperture), for small angles

$$\theta \approx \sin\theta = \frac{\lambda}{a} = \frac{500\times 10^{-9}\ \text{m}}{0.500\times 10^{-3}\ \text{m}} = \boxed{1.00\times 10^{-3}\ \text{rad}}$$

P38.15 Using Rayleigh's criterion,

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 0.100°\left(\frac{\pi \text{ rad}}{180°}\right) = 1.75\times10^{-3}\text{ rad}$$

and $$D = 1.22\left(\frac{\lambda}{\theta_{\min}}\right) = 1.22\left(\frac{3.00\times10^{-3}\text{ m}}{\theta_{\min}}\right) = \boxed{2.10\text{ m}}$$

P38.16 Using Rayleigh's criterion, $\theta_{\min} = 1.22\dfrac{\lambda}{D} = \dfrac{y}{L}$. Therefore,

$$L = \frac{yD}{1.22\lambda} = \frac{(2.80\times10^{-2}\text{ m})(0.600\times10^{-3}\text{ m})}{1.22(550\times10^{-9}\text{ m})} = \boxed{25.0\text{ m}}$$

P38.17 Using Rayleigh's criterion, $\theta_{\min} = 1.22\dfrac{\lambda}{D} = \dfrac{y}{L}$. Therefore,

$$y = 1.22\left(\frac{\lambda}{D}\right)L = 1.22\left(\frac{500\times10^{-9}\text{ m}}{58.0\times10^{-2}\text{ m}}\right)(270\times10^{3}\text{ m})$$
$$= \boxed{0.284\text{ m}}$$

P38.18 (a) The limiting angle for the resolution of the microscope is

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{589\times10^{-9}\text{ m}}{9.00\times10^{-3}\text{ m}}\right) = 7.98\times10^{-5}\text{ rad}$$
$$= \boxed{79.8\ \mu\text{rad}}$$

(b) For a smaller angle of diffraction we choose the smallest visible wavelength, $\boxed{\text{violet}}$ at 400 nm, to obtain

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{400\times10^{-9}\text{ m}}{9.00\times10^{-3}\text{ m}}\right) = 5.42\times10^{-5}\text{ rad}$$
$$= \boxed{54.2\ \mu\text{rad}}$$

(c) The wavelength in water is shortened to its vacuum value divided by the index of refraction. $\boxed{\text{The resolving power is improved, with the minimum resolvable angle becoming}}$

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{589\times10^{-9}\text{ m}/1.33}{9.00\times10^{-3}\text{ m}}\right) = 6.00\times10^{-5}\text{ rad}$$
$$= \boxed{60.0\ \mu\text{rad}}$$

Better than water for many purposes is oil immersion.

P38.19 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. We are given

$$L = 250\times10^3\ \text{m},\ \lambda = 5.00\times10^{-7}\ \text{m, and } d = 5.00\times10^{-3}\ \text{m}$$

The smallest object the astronauts can resolve is given by Rayleigh's criterion, $\theta_{\text{min}} = 1.22\dfrac{\lambda}{D} = \dfrac{y}{L}$. Therefore,

$$y = 1.22\frac{\lambda}{D}L = 1.22\left(\frac{5.00\times10^{-7}\ \text{m}}{5.00\times10^{-3}\ \text{m}}\right)\left(250\times10^3\ \text{m}\right) = \boxed{30.5\ \text{m}}$$

P38.20 Undergoing diffraction from a circular opening, the beam spreads into a cone of half-angle

$$\theta_{\text{min}} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{632.8\times10^{-9}\ \text{m}}{0.005\ 00\ \text{m}}\right) = 1.54\times10^{-4}\ \text{rad}$$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\text{min}}\left(1.00\times10^4\ \text{m}\right) = 1.544\ \text{m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = \boxed{3.09\ \text{m}}$.

***P38.21** The limit of resolution in air is

$$\theta_{\text{min}}\big|_{\text{air}} = 1.22\frac{\lambda}{D} = 0.60\ \mu\text{rad}$$

In oil, the limiting angle of resolution will be

$$\theta_{\text{min}}\big|_{\text{oil}} = 1.22\frac{\lambda_{\text{oil}}}{D} = 1.22\frac{\left(\lambda/n_{\text{oil}}\right)}{D} = \frac{1}{n_{\text{oil}}}\left(1.22\frac{\lambda}{D}\right)$$

or $$\theta_{\text{min}}\big|_{\text{oil}} = \frac{\theta_{\text{min}}\big|_{\text{air}}}{n_{\text{oil}}} = \frac{0.60\ \mu\text{rad}}{1.5} = \boxed{0.40\ \mu\text{rad}}$$

P38.22 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. We take $\theta_{\text{min}} = \dfrac{d}{L} = 1.22\dfrac{\lambda}{D}$, where θ_{min} is the smallest angular separation of two objects for which they are resolved by an aperture of diameter D, d is the separation of the two objects, and L is the maximum distance of the aperture from the two objects at which they can be resolved.

(a) Two objects can be resolved if their angular separation is greater than $\theta_{\min}$. Thus, $\theta_{\min}$ should be as small as possible. Therefore, light with the smaller of the two given wavelengths is easier to resolve, i.e., $\boxed{\text{blue}}$.

(b) $$L = \frac{Dd}{1.22\lambda} = \frac{\left(5.20\times10^{-3}\text{ m}\right)\left(2.80\times10^{-2}\text{ m}\right)}{1.22\lambda} = \frac{1.193\times10^{-4}\text{ m}^2}{\lambda}$$

Thus for λ = 640 nm, L = 186 m, and for λ = 440 nm, L = 271 m. The viewer with the assumed diffraction-limited vision could resolve adjacent tubes of blue in the range $\boxed{\text{186 m to 271 m}}$, but cannot resolve adjacent tubes of red in this range.

P38.23 When the pupil is open wide, it appears that the resolving power of human vision is limited by the coarseness of light sensors on the retina. But we use Rayleigh's criterion as a handy indicator of how good our vision might be. According to this criterion, two dots separated center-to-center by 2.00 mm would overlap when

$$\theta_{\min} = \frac{d}{L} = 1.22\frac{\lambda}{D}$$

Thus, $$L = \frac{dD}{1.22\lambda} = \frac{\left(2.00\times10^{-3}\text{ m}\right)\left(5.00\times10^{-3}\text{ m}\right)}{1.22\left(500\times10^{-9}\text{ m}\right)} = \boxed{16.4\text{ m}}.$$

P38.24 We are given D = 2.10 m and L = 9 000 m. The wavelength of the Coast Guard radar is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{15.0\times10^{9}\text{ Hz}} = 0.020\,0\text{ m}$$

From Rayleigh's criterion, $\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{d}{L}$. Therefore,

$$d = 1.22\left[\frac{(0.020\ 0\text{ m})(9\ 000\text{ m})}{2.10\text{ m}}\right] = \boxed{105\text{ m}}$$

Section 38.4 The Diffraction Grating

P38.25 The first order maximum occurs at 20.5°, so $\sin\theta = \sin 20.5° = 0.350$, and, from Equation 38.7,

$$d = \frac{\lambda}{\sin\theta} = \frac{632.8\text{ nm}}{0.350} = 1.81\times10^{3}\text{ nm}$$

Therefore, the line spacing $= \boxed{1.81\ \mu\text{m}}$

P38.26 The ruling engine that cut the diffraction grating (or the aluminum plate from which the gelatin or plastic was cast) sliced each centimeter into two thousand divisions. So the grating spacing is

$$d = \frac{1.00 \times 10^{-2}\ \text{m}}{2\ 000} = 5.00 \times 10^{-6}\ \text{m}$$

The light is deflected according to $d \sin\theta = m\lambda$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{1\left(640 \times 10^{-9}\ \text{m}\right)}{5.00 \times 10^{-6}\ \text{m}}\right] = \boxed{7.35°}$$

P38.27 The sound has wavelength $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{37.2 \times 10^{3}/\text{s}} = 9.22 \times 10^{-3}\ \text{m}$. Each diffracted beam is described by $d\ \sin\theta = m\lambda, \quad m = 0, 1, 2, \ldots$

The zero-order beam is at $m = 0$, $\theta = 0$. The beams in the first order of interference are to the left and right at

$$\theta = \sin^{-1}\left(\frac{1\lambda}{d}\right) = \sin^{-1}\left(\frac{9.22 \times 10^{-3}\ \text{m}}{1.30 \times 10^{-2}\ \text{m}}\right) = \sin^{-1} 0.709 = 45.2°$$

For a second-order beam we would need

$$\theta = \sin^{-1}\left(\frac{2\lambda}{d}\right) = \sin^{-1}\left[2(0.709)\right] = \sin^{-1}(1.42)$$

No angle, smaller or larger than 90°, has a sine greater than 1. Then a diffracted beam does not exist for the second order or any higher order. The whole answer is then:

(a) There are $\boxed{\text{three}}$ beams.

(b) The beams are at $\boxed{0°, +45.2°, -45.2°}$.

P38.28 (a) $d = \dfrac{10^{-2}\ \text{m}}{3\ 660} = 2.732 \times 10^{-6}\ \text{m} = 2\ 732\ \text{nm}$

$\lambda = \dfrac{d\sin\theta}{m}$, and $m = 1$: At $\theta = 10.1°$, $\lambda = \boxed{479\ \text{nm}}$

At $\theta = 13.7°$, $\lambda = \boxed{647\ \text{nm}}$.

At $\theta = 14.8°$, $\lambda = \boxed{698\ \text{nm}}$.

(b) $d = \dfrac{\lambda}{\sin\theta_1}$ and $2\lambda = d\sin\theta_2$ so $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\lambda}{\lambda/\sin\theta_1} = 2\sin\theta_1$.

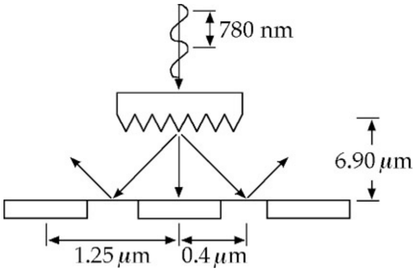
Therefore, if $\theta_1 = 10.1°$ then $\sin\theta_2 = 2\sin(10.1°)$ gives $\theta_2 = \boxed{20.5°}$.

Similarly, for $\theta_1 = 13.7°$, $\theta_2 = \boxed{28.3°}$ and for $\theta_1 = 14.8°$, $\theta_2 = \boxed{30.7°}$.

P38.29 For a side maximum, $\tan\theta = \dfrac{y}{L} = \dfrac{0.400\ \mu\text{m}}{6.90\ \mu\text{m}}$, which gives $\theta = 3.32°$.

Then, from $d\sin\theta = m\lambda$,

$$d = \frac{(1)(780\times10^{-9}\ \text{m})}{\sin 3.32°} = 13.5\ \mu\text{m}$$



ANS. FIG. P38.29

The number of grooves per millimeter

$$= \frac{1\times10^{-3}\ \text{m}}{13.5\times10^{-6}\ \text{m}} = \boxed{74.2}$$

P38.30 The grating spacing is

$$d = \frac{1.00\times10^{-3}\ \text{m}}{250} = 4.00\times10^{-6}\ \text{m} = 4\ 000\ \text{nm}$$

Solving for m in Equation 38.7 gives

$$d\sin\theta = m\lambda \quad \rightarrow \quad m = \frac{d\sin\theta}{\lambda}$$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$$m_{\text{max}} = \frac{d\sin\theta_{\text{max}}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{700\ \text{nm}} = 5.71$$

or $\boxed{\text{5 orders is the maximum}}$.

(b) The highest order in which the violet end of the spectrum can be seen is:

$$m_{\text{max}} = \frac{d\sin\theta_{\text{max}}}{\lambda} = \frac{(4\ 000\ \text{nm})\sin 90.0°}{400\ \text{nm}} = 10.0$$

or $\boxed{\text{10 orders in the short-wavelength region}}$.

P38.31 The grating spacing is

$$d = \frac{1.00\times10^{-2}\ \text{m}}{4\,200} = 2.38\times10^{-6}\ \text{m} = 2\,380\ \text{nm}$$

Solving for the angle θ from Equation 38.7, $d\sin\theta = m\lambda$, gives

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right)$$

Then, $$y = L\tan\theta = L\tan\left[\sin^{-1}\left(\frac{m\lambda}{d}\right)\right]$$

Thus, $$\Delta y = L\left\{\tan\left[\sin^{-1}\left(\frac{m\lambda_2}{d}\right)\right] - \tan\left[\sin^{-1}\left(\frac{m\lambda_1}{d}\right)\right]\right\}$$

For $m = 1$,

$$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{589.6}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{589}{2\,380}\right)\right]\right\}$$
$$= 0.554\ \text{mm}$$

For $m = 2$,

$$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{2(589.6)}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{2(589)}{2\,380}\right)\right]\right\}$$
$$= 1.54\ \text{mm}$$

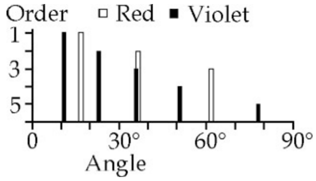
For $m = 3$,

$$\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\frac{3(589.6)}{2\,380}\right)\right] - \tan\left[\sin^{-1}\left(\frac{3(589)}{2\,380}\right)\right]\right\}$$
$$= 5.04\ \text{mm}$$

Thus, the observed order must be $\boxed{m = 2}$.

P38.32 The grating spacing is

$$d = \frac{1.00\times10^{-2}\ \text{m}}{4\,500} = 2.22\times10^{-6}\ \text{m}$$



ANS. FIG. P38.32

In the 1st-order spectrum, diffraction angles are given by

$$\sin\theta = \frac{\lambda}{d}: \qquad \sin\theta_1 = \frac{656\times10^{-9}\ \text{m}}{2.22\times10^{-6}\ \text{m}} = 0.295$$

so that for red $\theta_1 = 17.17°$

and for blue $$\sin\theta_2 = \frac{434\times10^{-9}\ \text{m}}{2.22\times10^{-6}\ \text{m}} = 0.195,$$

so that $\theta_2 = 11.26°$.

The angular separation is in first-order,

$$\Delta\theta = 17.17° - 11.26° = \boxed{5.91°}$$

In the second-order spectrum,

$$\Delta\theta = \sin^{-1}\left(\frac{2\lambda_1}{d}\right) - \sin^{-1}\left(\frac{2\lambda_2}{d}\right) = \boxed{13.2°}$$

Again, in the third order,

$$\Delta\theta = \sin^{-1}\left(\frac{3\lambda_1}{d}\right) - \sin^{-1}\left(\frac{3\lambda_2}{d}\right) = \boxed{26.5°}$$

Since the red does not appear in the fourth-order spectrum, the answer is complete.

P38.33 The principal maxima are defined by $d\sin\theta = m\lambda$, where $m = 0, 1, 2, \ldots$ For $m = 1$, $\lambda = d\sin\theta$.

Here, θ is the angle between the central ($m = 0$) and the first order ($m = 1$) maxima. The value of θ can be determined from the information given about the distance between maxima and the grating-to-screen distance:

$$\tan\theta = \frac{0.488 \text{ m}}{1.72 \text{ m}} = 0.284$$

so $\theta = 15.8°$ and $\sin\theta = 0.273$.

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$$d = \frac{1}{5\,310 \text{ cm}^{-1}} = 1.88 \times 10^{-4} \text{ cm} = 1.88 \times 10^{3} \text{ nm}$$

The wavelength is

$$\lambda = d\sin\theta = \left(1.88 \times 10^{3} \text{ nm}\right)(0.273) = \boxed{514 \text{ nm}}$$

P38.34 From Equation 38.7, $\sin\theta = \dfrac{m\lambda}{d}$

Therefore, taking the ends of the visible spectrum to be $\lambda_v = 400$ nm and $\lambda_r = 750$ nm, the ends of the different order spectra are defined by:

End of second order: $\sin\theta_{2r} = \dfrac{2\lambda_r}{d} = \dfrac{1\,500 \text{ nm}}{d}$

Start of third order: $\sin\theta_{3v} = \dfrac{3\lambda_v}{d} = \dfrac{1\,200 \text{ nm}}{d}$

Thus, it is seen that $\boxed{\theta_{2r} > \theta_{3v} \text{ and these orders must overlap}}$ regardless of the value of the grating spacing *d*.

P38.35 (a) We use the grating equation $d\sin\theta = m\lambda$:

$$d = \frac{m\lambda}{\sin\theta} = \frac{3\left(5.00\times10^{-7}\text{ m}\right)}{\sin 32.0°} = 2.83\times10^{-6}\text{ m}$$

Thus the grating gauge is

$$\frac{1}{d} = 3.53\times10^{5}\text{ grooves/m} = \boxed{3.53\times10^{3}\text{ grooves/cm}}$$

(b) For any interference maximum for this light going through this grating,

$$\sin\theta = m\left(\frac{\lambda}{d}\right) = \frac{m\left(5.00\times10^{-7}\text{ m}\right)}{2.83\times10^{-6}\text{ m}} = m(0.177)$$

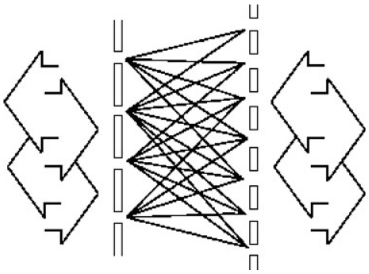
For $\sin\theta \le 1$, we require that $m(0.177) \le 1$ or $m \le 5.66$. Because m must be an integer, its maximum value is really 5. Therefore, the total number of maxima is $2m+1 = \boxed{11}$.

P38.36 (a) The several narrow parallel slits make a diffraction grating. The zeroth- and first-order maxima are separated according to

$$d\sin\theta = (1)\lambda \qquad \sin\theta = \frac{\lambda}{d} = \frac{632.8\times10^{-9}\text{ m}}{1.2\times10^{-3}\text{ m}}$$

$$\theta = \sin^{-1}(0.000\,527) = 0.000\,527\text{ rad}$$

$$y = L\tan\theta = (1.40\text{ m})(0.000\,527) = \boxed{0.738\text{ mm}}$$



ANS. FIG. P38.36

(b) Many equally spaced transparent lines appear on the film. It is itself a diffraction grating. When the same light is sent through the film, it produces interference maxima separated according to

$$d\sin\theta = (1)\lambda \quad \rightarrow \quad \sin\theta = \frac{\lambda}{d} = \frac{632.8\times10^{-9}\text{ m}}{0.738\times10^{-3}\text{ m}} = 0.000\,857$$

$$y = L\tan\theta = (1.40\text{ m})(0.000\,857) = 1.20\text{ mm}$$

An image of the original set of slits appears on the screen. If the screen is removed, light diverges from the real images with the same wave fronts reconstructed as the original slits produced. Reasoning from the mathematics of Fourier transforms, Gabor showed that light diverging from any object, not just a set of slits, could be used. In the picture, the slits or maxima on the left are separated by 1.20 mm. The slits or maxima on the right are separated by 0.738 mm. The length difference between any pair of lines is an integer number of wavelengths. Light can be sent through equally well toward the right or toward the left.

P38.37 Fifteen bright spots means that the central maximum and seven orders of side maxima appear.

(a) If the seventh order is at less than 90°, the eighth order might be nearly ready to appear according to

$$d\sin\theta = m\lambda$$

$$d(1) = 8\left(654\times10^{-9}\text{ m}\right) \rightarrow d = 5.23\ \mu\text{m}$$

$$d = 5.23\times10^{-6}\text{ m} = \boxed{5.23\ \mu\text{m}}$$

(b) If the seventh order is just at 90°,

$$d\sin\theta = m\lambda$$

$$d(1) = 7\left(654\times10^{-9}\text{ m}\right)$$

$$d = 4.58\times10^{-6}\text{ m} = \boxed{4.58\ \mu\text{m}}$$

Section 38.5 Diffraction of X-Rays by Crystals

P38.38 The atomic planes in this crystal are shown in Figure 38.22 of the text. The diffraction they produce is described by Bragg's law,

$$2d\sin\theta = m\lambda: \qquad \sin\theta = \frac{m\lambda}{2d} = \frac{1\left(0.140\times10^{-9}\text{ m}\right)}{2\left(0.281\times10^{-9}\text{ m}\right)} = 0.249$$

and $\boxed{\theta = 14.4^\circ}$.

P38.39 The grazing angle is measured from the surface, as shown in Figure 38.23. Then, from $2d\sin\theta = m\lambda$,

$$\lambda = \frac{2d\sin\theta}{m}$$

$$= \frac{2\left(0.353\times10^{-9}\text{ m}\right)\sin 7.60^\circ}{1} = 9.34\times10^{-11}\text{ m} = \boxed{0.093\,4\text{ nm}}$$

P38.40 From $2d\sin\theta = m\lambda$,

$$\sin\theta = \frac{m\lambda}{2d} = \frac{2(0.166\text{ nm})}{2(0.314\text{ nm})} = 0.529$$

and $\boxed{\theta = 31.9°}$

P38.41 (a) By Bragg's law, $2d\sin\theta = m\lambda$, and $m = 2$:

$$\lambda = 2d\sin\theta = 2(0.250\text{ nm})\sin 12.6° = \boxed{0.109\text{ nm}}$$

(b) We obtain the number of orders from

$$\frac{m\lambda}{2d} = \sin\theta \le 1 \rightarrow m \le \frac{2d}{\lambda} = \frac{2(0.250\text{ nm})}{0.109\text{ nm}} = 4.59$$

The order-number must be an integer, so the largest value m can have is 4: $\boxed{\text{four}}$ orders can be observed.

Section 38.6 Polarization of Light Waves

P38.42 In Equation 38.10, $\tan\theta_p = n_2/n_1$, the index of refraction n_2 of the solid material must be larger than that of air ($n_1 = 1.00$). Therefore, we must have $\tan\theta_p > 1$. For this to be true, we must have $\theta_p > 45°$, so $\theta_p = 41.0°$ is not possible.

P38.43 We define the initial angle, at which all the light is transmitted, to be $\theta = 0$. Turning the disk to another angle will then reduce the transmitted light by an intensity factor as described by $I = I_{\max}\cos^2\theta$.

Then, $$\theta = \cos^{-1}\sqrt{\frac{I}{I_{\max}}}$$

(a) For $I = I_{\max}/3.00$,

$$\theta = \cos^{-1}\left(\frac{I}{I_{\max}}\right)^{1/2} = \cos^{-1}\frac{1}{\sqrt{3.00}} = \cos^{-1}0.577 = \boxed{54.7°}$$

(b) Now $\theta = \cos^{-1}\left(\frac{I}{I_{\max}}\right)^{1/2} = \cos^{-1}\frac{1}{\sqrt{5.00}} = \cos^{-1}0.447 = \boxed{63.4°}$

(c) The largest factor of intensity reduction requires the largest crossing angle,

$$\theta = \cos^{-1}\left(\frac{I}{I_{\max}}\right)^{1/2} = \cos^{-1}\frac{1}{\sqrt{10.0}} = \cos^{-1}0.316 = \boxed{71.6°}$$

P38.44 By Brewster's law, for light in air ($n = 1.00$) reflecting off a surface of index n,

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{n}{1.00} = n$$

$$n = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$$

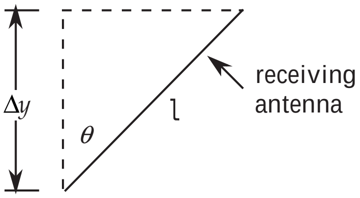
P38.45 The intensity of unpolarized light passing through the first polarizing filter is reduced by 1/2. The second transmits $\cos^2 30.0° = \frac{3}{4}$.

$$\frac{I}{I_{\text{max}}} = \frac{1}{2}\times\frac{3}{4} = \frac{3}{8} = \boxed{0.375}$$

***P38.46** $P = \frac{(\Delta V)^2}{R}$ or $P \propto (\Delta V)^2$

$$\Delta V = (-)E_y\cdot\Delta y = E_y\cdot\ell\cos\theta$$

$$\Delta V \propto \cos\theta \quad \text{so} \quad P \propto \cos^2\theta$$



ANS. FIG. P38.46

(a) $\theta = 15.0°$:

$$P = P_{\text{max}}\cos^2(15.0°) = 0.933P_{\text{max}} = \boxed{93.3\%}$$

(b) $\theta = 45.0°$: $P = P_{\text{max}}\cos^2(45.0°) = 0.500P_{\text{max}} = \boxed{50.0\%}$

(c) $\theta = 90.0°$: $P = P_{\text{max}}\cos^2(90.0°) = \boxed{0.00\%}$

P38.47 Let the first sheet have its axis at angle θ to the original plane of polarization, and let each further sheet have its axis turned by the same angle.

The first sheet passes intensity $I_{\text{max}}\cos^2\theta$,

The second sheet passes $\left(I_{\text{max}}\cos^2\theta\right)\cos^2\theta = I_{\text{max}}\cos^4\theta$,

and the nth sheet lets through $I_{\text{max}}\cos^{2n}\theta \geq 0.90I_{\text{max}}$, where $\theta = \frac{45°}{n}$.

Try different integers to find n; for example,

$$\cos^{2\times5}\left(\frac{45°}{5}\right) = 0.885, \quad \cos^{2\times6}\left(\frac{45°}{6}\right) = 0.902, \quad \cos^{2\times7}\left(\frac{45°}{7}\right) = 0.915$$

(a) So $n = \boxed{6}$.

(b) $\theta = \frac{45°}{6} = \boxed{7.50°}$

P38.48 (a) Let I_0 represent the intensity of unpolarized light incident on the first polarizer. The intensity of unpolarized light passing through a polarizing filter is reduced by 1/2, so the first filter lets through 1/2 of the incident intensity. Of the light reaching them, the second filter passes $\cos^2 45° = 1/2$ and the third filter also $\cos^2 45° = 1/2$. The transmitted intensity is then

$$I_0\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = 0.125I_0$$

The reduction in intensity is by a factor of $1.00 - 0.125 = \boxed{0.875}$ of the incident intensity.

(b) By the same logic as in part (a) we have transmitted

$$I_0\left(\frac{1}{2}\right)\left(\cos^2 30.0°\right)\left(\cos^2 30.0°\right)\left(\cos^2 30.0°\right) = \left(\frac{I_0}{2}\right)\left(\cos^2 30.0°\right)^3$$
$$= 0.211I_0$$

Then the fraction absorbed is $1.00 - 0.211 = \boxed{0.789}$.

(c) Yet again we compute transmission

$$I_0\left(\frac{1}{2}\right)\left(\cos^2 15.0°\right)^6 = 0.330I_0$$

And the fraction absorbed is $1.00 - 0.330 = \boxed{0.670}$.

(d) We can get more and more of the incident light through the stack of ideal filters, approaching 50%, by reducing the angle between the transmission axes of each one and the next.

P38.49 For the polarizing angle,

$$\frac{n_{\text{sapphire}}}{n_{\text{air}}} = \tan\theta_p \quad \text{and} \quad \theta_p = \tan^{-1}\left(\frac{n_{\text{sapphire}}}{1.00}\right)$$

For the critical angle for total internal reflection,

$$n_{\text{sapphire}}\sin\theta_c = n_{\text{air}}\sin 90^\circ = 1.00 \quad \text{so} \quad n_{\text{sapphire}} = \frac{1}{\sin\theta_c}$$

Therefore,

$$\theta_p = \tan^{-1}\left(\frac{1}{\sin\theta_c}\right) = \tan^{-1}\left(\frac{1}{\sin 34.4°}\right) = \boxed{60.5°}$$

P38.50 For the polarizing angle,

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{n}{1} = n$$

and $$\sin\theta_c = \frac{n_2}{n_1} = \frac{1}{n}$$

Thus, $$\tan\theta_p = \frac{1}{\sin\theta_c}:$$

$$\boxed{\theta_p = \tan^{-1}\left(\frac{1}{\sin\theta_c}\right) \quad \text{or} \quad \theta_p = \tan^{-1}\left(\csc\theta_c\right) \quad \text{or} \quad \theta_p = \cot^{-1}\left(\sin\theta_c\right)}$$

***P38.51** From Malus's law, the intensity of the light transmitted by the first polarizer is $I_1 = I_i\cos^2\theta_1$. The plane of polarization of this light is parallel to the axis of the first plate and is incident on the second plate. Malus's law gives the intensity transmitted by the second plate as

$$I_2 = I_1\cos^2\left(\theta_2 - \theta_1\right) = I_i\cos^2\theta_1\cos^2\left(\theta_2 - \theta_1\right)$$

This light is polarized parallel to the axis of the second plate and is incident upon the third plate. A final application of Malus's law gives the transmitted intensity as

$$I_f = I_2\cos^2\left(\theta_3 - \theta_2\right) = I_i\cos^2\theta_1\cos^2\left(\theta_2 - \theta_1\right)\cos^2\left(\theta_3 - \theta_2\right)$$

With $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$, this result yields

$$I_f = (10.0\text{ units})\left(\cos^2 20.0°\right)\left(\cos^2 20.0°\right)\left(\cos^2 20.0°\right) = \boxed{6.89\text{ units}}$$

***P38.52** Half of the unpolarized light passes through the first sheet. The light that passes through the first sheet is polarized at 45° relative to the second sheet, and the light that passes through the second sheet is polarized at 45° relative to the third sheet. The fraction of transmitted light is given by two successive applications of Malus's law:

$$\frac{I}{I_{\max}} = \frac{1}{2}\left(\cos^2 45.0°\right)\left(\cos^2 45.0°\right) = \boxed{\frac{1}{8}}$$

Additional Problems

P38.53 (a) We assume the first side maximum is at $a\sin\theta = 1.5\lambda$. (Its location is determined more precisely in Problem 71.) Then the required fractional intensity is

$$\frac{I}{I_{\max}} = \left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 = \left[\frac{\sin(1.5\pi)}{1.5\pi}\right]^2 = \frac{1}{2.25\pi^2} = \boxed{0.045\ 0}$$

(b) Proceeding as in part (a), we assume $a\sin\theta = 2.5\lambda$:

$$\frac{I}{I_{\max}} = \left[\frac{\sin(\pi a\sin\theta/\lambda)}{\pi a\sin\theta/\lambda}\right]^2 = \left[\frac{\sin(2.5\pi)}{2.5\pi}\right]^2 = \frac{1}{6.25\pi^2} = \boxed{0.016\ 2}$$

P38.54 (a) $\boxed{\text{One slit, as the central maximum is twice as wide as the other maxima.}}$ A two-slit pattern has evenly spaced fringes (within a one-slit diffraction envelope).

(b) For precision, we measure from the second minimum on one side of the center to the second minimum on the other side:

$$2y = (11.7 - 6.3)\text{ cm} = 5.4\text{ cm} \rightarrow y = 2.7\text{ cm}$$

$$\tan\theta = \frac{y}{L} = \frac{0.027\text{ m}}{2.60\text{ m}} \approx \sin\theta$$

$$a\sin\theta = m\lambda$$

$$a = \frac{m\lambda}{\sin\theta} = \frac{2\left(632.8\times10^{-9}\text{ m}\right)}{\left(\dfrac{0.027\text{ m}}{2.60\text{ m}}\right)} = 1.22\times10^{-4}\text{ m}$$

$$= \boxed{0.122\text{ mm wide}}$$

P38.55 Figure 38.23 of the text shows the situation. This is Bragg diffraction for water waves.

$$2d\sin\theta = m\lambda \quad \text{or} \quad \lambda = \frac{2d\sin\theta}{m}$$

$m = 1$: $$\lambda_1 = \frac{2(2.80\text{ m})\sin 80.0°}{1} = \boxed{5.51\text{ m}}$$

$m = 2$: $$\lambda_2 = \frac{2(2.80\text{ m})\sin 80.0°}{2} = \boxed{2.76\text{ m}}$$

$m = 3$: $$\lambda_3 = \frac{2(2.80\text{ m})\sin 80.0°}{3} = \boxed{1.84\text{ m}}$$

P38.56 For dark fringes in an interference pattern formed by a single slit, $a\sin\theta = m\lambda$. By the small-angle approximation, $\sin\theta \approx \tan\theta = \dfrac{y}{L}$.

Substituting, we have $a\dfrac{y}{L} = 2\lambda$ and

$$\lambda = \frac{ya}{2L} = \frac{\left(1.40\times10^{-3}\text{ m}\right)\left(0.800\times10^{-3}\text{ m}\right)}{2\left(85.0\times10^{-2}\text{ m}\right)}$$

$$= 6.59\times10^{-7}\text{ m} = \boxed{659\text{ nm}}$$

P38.57 The first minimum is at $a\sin\theta = (1)\lambda$.

This has no solution if $\dfrac{\lambda}{a} > 1$,

or if $a < \lambda = \boxed{632.8 \text{ nm}}$.

P38.58 (a) With light in effect moving through vacuum, Rayleigh's criterion limits the resolution according to

$$\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{d}{L}$$

The diameter of the aperture is then

$$D = \frac{1.22\lambda L}{d} = \frac{1.22\left(885\times10^{-9}\text{ m}\right)\left(12\ 000\text{ m}\right)}{2.30\text{ m}}$$
$$= 0.005\ 63\text{ m} = \boxed{5.63\text{ mm}}$$

(b) $\boxed{\text{The assumption is unreasonable. Over a horizontal path of 12 km in air, density variations associated with convection (“heat waves,” or what an astronomer calls “seeing”) would make the motorcycles completely unresolvable with any optical device.}}$

P38.59 (a) We first determine the wavelength of 1.40-GHz radio waves from $\lambda = \dfrac{v}{f}$:

$$\lambda = \frac{3.00\times10^{8}\text{ m/s}}{1.40\times10^{9}\text{ s}^{-1}} = 0.214\text{ m}$$

Applying Rayleigh's criterion, $\theta_{\min} = 1.22\dfrac{\lambda}{D}$, we obtain

$$\theta_{\min} = 1.22\left(\frac{0.214\text{ m}}{3.60\times10^{4}\text{ m}}\right) = \boxed{7.26\ \mu\text{rad}}$$

$$\theta_{\min} = \left(7.26\ \mu\text{rad}\right)\left(\frac{180\times60\times60\text{ s}}{\pi}\right) = \boxed{1.50\text{ arc seconds}}$$

(b) To determine the separation between the clouds, we use $\theta_{\min} = \dfrac{d}{L}$:

$$d = \theta_{\min}L = \left(7.26\times10^{-6}\text{ rad}\right)\left(26\,000\text{ ly}\right) = \boxed{0.189\text{ ly}}$$

(c) It is not true for humans, but we assume the hawk's visual acuity is limited only by Rayleigh's criterion, $\theta_{\min} = 1.22\dfrac{\lambda}{D}$. Substituting numerical values,

$$\theta_{\min} = 1.22\left(\frac{500\times10^{-9}\text{ m}}{12.0\times10^{-3}\text{ m}}\right) = 50.8\ \mu\text{rad} = \boxed{10.5\text{ seconds of arc}}$$

(d) Following the same procedure as in part (b), we have

$$d = \theta_{\min}L = \left(50.8\times10^{-6}\text{ rad}\right)(30.0\text{ m}) = 1.52\times10^{-3}\text{ m} = \boxed{1.52\text{ mm}}$$

***P38.60** Differentiating Equation 38.7, $d\sin\theta = m\lambda$, gives

$$d(\cos\theta)d\theta = md\lambda$$

or $$d\sqrt{1-\sin^2\theta}\Delta\theta \approx m\Delta\lambda.$$

Plugging in for $\sin\theta$,

$$d\sqrt{1-\frac{m^2\lambda^2}{d^2}}\Delta\theta \approx m\Delta\lambda$$

so $$\Delta\theta \approx \frac{\Delta\lambda}{\sqrt{\left(d^2/m^2\right)-\lambda^2}}.$$

***P38.61** The grid spacing is

$$d = \frac{10^{-3}\text{ m}}{400} = 2.50\times10^{-6}\text{ m}$$

(a) From Equation 38.7, $d\sin\theta = m\lambda$:

$$\theta_a = \sin^{-1}\left[\frac{2\left(541\times10^{-9}\text{ m}\right)}{2.50\times10^{-6}\text{ m}}\right] = \boxed{25.6°}$$

(b) In water,

$$\lambda = \frac{541\times10^{-9}\text{ m}}{1.333} = 4.06\times10^{-7}\text{ m}$$

and $$\theta_b = \sin^{-1}\left[\frac{2\left(4.06\times10^{-7}\text{ m}\right)}{2.50\times10^{-6}\text{ m}}\right] = \boxed{18.9°}$$

(c) $d\sin\theta_a = 2\lambda$ and $d\sin\theta_b = \dfrac{2\lambda}{n} \rightarrow dn\sin\theta_b = 2\lambda$

Each equals 2λ: therefore $n\sin\theta_b = (1)\sin\theta_a$.

P38.62 We check to see if the $m = 15$ interference maximum is visible.

We find the sine of the angle for the $m = m_{\text{double}}$ two-slit interference maximum:

$$m_{\text{double}}\lambda = d\sin\theta_{\text{bright}} \rightarrow \sin\theta_{\text{bright}} = \frac{m_{\text{double}}\lambda}{d} \qquad [1]$$

Then find the sine of the angle for the $m = m_{\text{single}}$ single-slit interference minimum:

$$\sin\theta_{\text{dark}} = \frac{m_{\text{single}}\lambda}{a} \qquad [2]$$

Divide equation [2] by equation [1]:

$$\frac{\sin\theta_{\text{dark}}}{\sin\theta_{\text{bright}}} = \frac{m_{\text{single}}\,\lambda/a}{m_{\text{double}}\,\lambda/d} = \frac{m_{\text{single}}}{m_{\text{double}}}\frac{d}{a}$$

Now let the angle of the single-slit minimum be equal to that of the double-slit maximum:

$$1 = \frac{m_{\text{single}}}{m_{\text{double}}}\frac{d}{a} = \frac{m_{\text{single}}}{m_{\text{double}}}\frac{30.0\ \mu m}{2.00\ \mu m} = 15\frac{m_{\text{single}}}{m_{\text{double}}}$$

which gives $m_{\text{double}} = 15m_{\text{single}}$.

Therefore, the $m_{\text{single}} = 1$ minimum aligns with the $m_{\text{double}} = 15$ maximum so that the $m_{\text{double}} = 15$ maximum has zero intensity and could not startle the co-worker.

P38.63 With a grazing angle of 36.0° (measured from the surface), the angle of incidence is 54.0°, which equals the polarizing angle:

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{n}{1.00} = n = \tan 54.0° = 1.38$$

In the liquid,

$$\lambda_n = \frac{\lambda}{n} = \frac{750\text{ nm}}{1.38} = \boxed{545\text{ nm}}$$

P38.64 (a) Bragg's law applies to the space lattice of melanin rods. Consider the planes $d = 0.25\ \mu$m apart. For light at near-normal incidence, strong reflection happens for the wavelength given by $2d\sin\theta = m\lambda$. The longest wavelength reflected strongly corresponds to $m = 1$:

$$2\left(0.25\times10^{-6}\text{ m}\right)\sin 90° = \lambda = 500\text{ nm}$$

This is the blue-green color.

(b) For light incident at grazing angle 60°, $2d\sin\theta = m\lambda$ gives $2\left(0.25\times10^{-6}\text{ m}\right)\sin 60° = \lambda = 433$ nm. This is violet.

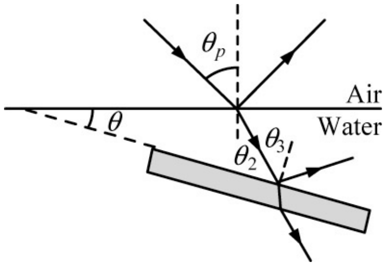
(c) Your two eyes receive light reflected from the feather at different angles, so they receive light incident at different angles and containing different colors reinforced by constructive interference.

(d) The longest wavelength that can be reflected with extra strength by these melanin rods is the one we computed first, 500 nm blue-green.

(e) If the melanin rods were farther apart (say 0.32 μm) they could reflect red with constructive interference.

P38.65 In ANS. FIG. P38.65, light strikes the liquid at the polarizing angle θ_p, enters the liquid at angle θ_2, and then strikes the slab at the angle θ_3, which is equal to the polarizing angle θ'_p. The angle between the water surface and the surface of the slab, θ, is related to the other angles by (from the triangle)

$$\theta + \left(90° + \theta_2\right) + \left(90° - \theta_3\right) = 180° \quad \rightarrow \quad \theta = \theta_3 - \theta_2$$



ANS. FIG. P38.65

For the air-to-water interface,

$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} \quad \rightarrow \quad \theta_p = 53.1°$$

and $$(1.00)\sin\theta_p = (1.33)\sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9°$$

For the water-to-slab interface,

$$\tan\theta_3 = \tan\theta_p = \frac{n_{\text{slab}}}{n_{\text{water}}} = \frac{n}{1.33} = \frac{1.62}{1.33}$$

$$\theta_3 = 50.6°$$

The angle between surfaces is $\theta = \theta_3 - \theta_2 = \boxed{13.7°}$.

P38.66 Refer to ANS. FIG. P38.65 above. Light strikes the liquid at the polarizing angle θ_p, enters the liquid at angle θ_2, and then strikes the slab at the angle θ_3, which is equal to the polarizing angle θ_p'. The angle between the water surface and the surface of the slab, θ, is related to the other angles by (from the triangle)

$$\theta + (90° + \theta_2) + (90° - \theta_3) = 180° \quad \rightarrow \quad \theta = \theta_3 - \theta_2$$

Also,

$$\theta_p + 90° + \theta_2 = 180°$$
$$\theta_p = 90° - \theta_2$$

For the air-to-liquid interface,

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{n_{\text{liquid}}}{n_{\text{air}}} = \frac{n_\ell}{1} = \frac{\sin\theta_p}{\cos\theta_p} = \frac{\sin(90° - \theta_2)}{\cos(90° - \theta_2)}$$
$$= \frac{\cos\theta_2}{\sin\theta_2} = \frac{1}{\tan\theta_2}$$

So, $$\tan\theta_2 = \frac{1}{n_\ell} \quad \rightarrow \quad \theta_2 = \tan^{-1}\left(\frac{1}{n_\ell}\right)$$

For the water-to-slab interface,

$$\tan\theta_3 = \tan\theta_p' = \frac{n_{\text{slab}}}{n_{\text{liquid}}} = \frac{n}{n_\ell} \quad \rightarrow \quad \theta_3 = \tan^{-1}\left(\frac{n}{n_\ell}\right)$$

Therefore,

$$\theta = \theta_3 - \theta_2 \quad \rightarrow \quad \boxed{\theta = \tan^{-1}\left(\frac{n}{n_\ell}\right) - \tan^{-1}\left(\frac{1}{n_\ell}\right)}$$

P38.67 For the limiting angle of resolution between lines we assume

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{(550 \times 10^{-9}\ \text{m})}{(5.00 \times 10^{-3}\ \text{m})} = 1.34 \times 10^{-4}\ \text{rad}$$

Assuming a picture screen with vertical dimension ℓ, the minimum viewing distance for no visible lines is found from $\theta_{\min} = \frac{\ell/485}{L}$. The desired ratio is then

$$\frac{L}{\ell} = \frac{1}{485\theta_{\min}} = \frac{1}{485(1.34 \times 10^{-4}\ \text{rad})} = \boxed{15.4}$$

When the pupil of a human eye is wide open, its actual resolving power is significantly poorer than Rayleigh's criterion suggests.

P38.68 (a) We require

$$\theta_{\min} = 1.22\frac{\lambda}{D} = \frac{\text{radius of diffraction disk}}{L} = \frac{D/2}{L}.$$

Then, $\boxed{D^2 = 2.44\lambda L}$.

(b) $D = \sqrt{2.44\left(500\times 10^{-9}\text{ m}\right)\left(0.150\text{ m}\right)} = 4.28\times 10^{-4}\text{ m} = \boxed{428\ \mu\text{m}}$

P38.69 (a) Constructive interference of light of wavelength λ on the screen is described by $d\sin\theta = m\lambda$ and, because $\tan\theta = \dfrac{y}{L}$, we may write $\sin\theta = \dfrac{y}{\sqrt{L^2+y^2}}$. Therefore,

$$(d)y\left(L^2+y^2\right)^{-1/2} = m\lambda$$

Differentiating with respect to *y* gives

$$(d)\left(L^2+y^2\right)^{-1/2} + (d)y\left(-\frac{1}{2}\right)\left(L^2+y^2\right)^{-3/2}(0+2y) = m\frac{d\lambda}{dy}$$

$$\frac{(d)}{\left(L^2+y^2\right)^{1/2}} - \frac{(d)y^2}{\left(L^2+y^2\right)^{3/2}} = m\frac{d\lambda}{dy} = \frac{(d)\left(L^2+y^2\right)-(d)y^2}{\left(L^2+y^2\right)^{3/2}}$$

$$\rightarrow\ \frac{d\lambda}{dy} = \frac{(d)L^2}{m\left(L^2+y^2\right)^{3/2}}$$

(b) Here $d\sin\theta = m\lambda$ gives, for *m* = 1,

$$\frac{10^{-2}\text{ m}}{8\ 000}\sin\theta = 1\left(550\times 10^{-9}\text{ m}\right)$$

or $$\theta = \sin^{-1}\left(\frac{550\times 10^{-9}\text{ m}}{1.25\times 10^{-6}\text{ m}}\right) = 26.1^\circ$$

Then,

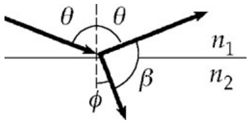
$$y = L\tan\theta = (2.40\text{ m})\tan 26.1^\circ = 1.18\text{ m}$$

So we have

$$\frac{d\lambda}{dy} = \frac{(d)L^2}{m\left(L^2+y^2\right)^{3/2}} = \frac{\left(1.25\times 10^{-6}\text{ m}\right)(2.40\text{ m})^2}{(1)\left[(2.4\text{ m})^2+(1.18\text{ m})^2\right]^{3/2}}$$

$$= 3.77\times 10^{-7}\frac{\text{m}}{\text{m}} = 3.77\times 10^{-7}\frac{10^9\text{nm}}{10^2\text{cm}} = \boxed{3.77\ \text{nm/cm}}$$

P38.70 (a) Applying Snell's law gives $n_2 \sin\phi = n_1 \sin\theta$. From the sketch in ANS. FIG. P38.70(a), we also see that:



ANS. FIG. P38.70(a)

$$\theta + \phi + \beta = \pi\,, \quad \text{or} \quad \phi = \pi - (\theta + \beta)$$

Using the given identity,

$$\sin\phi = \sin\pi\cos(\theta+\beta) - \cos\pi\sin(\theta+\beta)$$

which reduces to,

$$\sin\phi = \sin(\theta+\beta)$$

Applying the identity again,

$$\sin\phi = \sin\theta\cos\beta + \cos\theta\sin\beta$$

Snell's law then becomes,

$$n_2(\sin\theta\cos\beta + \cos\theta\sin\beta) = n_1\sin\theta$$

or (after dividing by $\cos\theta$):

$$n_2(\tan\theta\cos\beta + \sin\beta) = n_1\tan\theta$$

Solving for $\tan\theta$ gives:

$$\boxed{\tan\theta = \frac{n_2\sin\beta}{n_1 - n_2\cos\beta}}$$

(b) If $\beta = 90.0°$, the above result becomes:

$$\tan\theta = \frac{n_2\sin 90°}{n_1 - n_2\cos 90°} = \frac{n_2}{n_1}\text{, which is Brewster's law}$$

P38.71 From $I = I_{\max}\left(\dfrac{\sin\phi}{\phi}\right)^2$ we find

$$\frac{dI}{d\phi} = I_{\max}2\left(\frac{\sin\phi}{\phi}\right)\left(\frac{\phi\cos\phi - [\sin\phi]1}{\phi^2}\right)$$

and require that it be zero. The possibility $\sin\phi = 0$ locates all of the minima and the central maximum, according to

$$\phi = 0,\ \pi,\ 2\pi,\ \ldots; \qquad \phi = \frac{\pi a\sin\theta}{\lambda} = 0,\ \pi,\ 2\pi,\ \ldots;$$

$$a\sin\theta = 0,\ \lambda,\ 2\lambda,\ \ldots$$

The side maxima are found from

$$\phi\cos\phi - \sin\phi = 0 \quad \text{or} \quad \tan\phi = \phi$$

This has solutions

$$\phi = 4.493\,4,\ \phi = 7.7253, \text{ and others.}$$

(a) $\boxed{\phi = 4.49 \text{ compared to the prediction from the approximation of } 1.5\pi = 4.71.}$

(b) $\boxed{\phi = 7.73 \text{ compared to the prediction from the approximation of } 2.5\pi = 7.85.}$

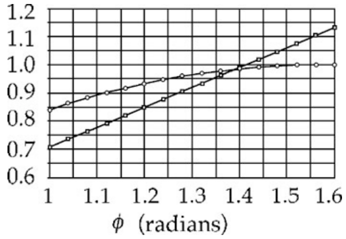
P38.72 (a) From Equation 38.2, $\dfrac{I}{I_{\text{max}}} = \left[\dfrac{\sin(\phi)}{\phi}\right]^2$ where we define $\phi \equiv \dfrac{\pi a \sin\theta}{\lambda}$.

Therefore, when $\dfrac{I}{I_{\text{max}}} = \dfrac{1}{2}$ we must have

$$\frac{\sin\phi}{\phi} = \frac{1}{\sqrt{2}}, \quad \text{or} \quad \boxed{\sin\phi = \frac{\phi}{\sqrt{2}}}$$

(b) Let $y_1 = \sin\phi$ and $y_2 = \dfrac{\phi}{\sqrt{2}}$.

A plot of y_1 and y_2 in the range $1.00 \le \phi \le \dfrac{\pi}{2}$ is shown in ANS. FIG. P38.72(b).



ANS. FIG. P38.72(b)

The solution to the transcendental equation is found to be $\boxed{\phi = 1.39 \text{ rad}}$.

(c) $\dfrac{\pi a \sin\theta}{\lambda} = \phi$ gives $\sin\theta = \left(\dfrac{\phi}{\pi}\right)\dfrac{\lambda}{a}$. If $\dfrac{\lambda}{a}$ is small, then $\theta \approx \left(\dfrac{\phi}{\pi}\right)\dfrac{\lambda}{a}$.

This gives the half-width, measured away from the maximum at $\theta = 0$. The pattern is symmetric, so the full width is given by

$$\Delta\theta = \left(\frac{\phi}{\pi}\right)\frac{\lambda}{a} - \left(-\left(\frac{\phi}{\pi}\right)\frac{\lambda}{a}\right) = 2\left(\frac{\phi}{\pi}\right)\frac{\lambda}{a} = 2\left(\frac{1.39\text{ rad}}{\pi}\right)\frac{\lambda}{a}$$
$$= \boxed{\frac{0.885\lambda}{a}}$$

(d)

ϕ	$\sqrt{2}\sin\phi$	
1	1.19	bigger than ϕ
2	1.29	smaller than ϕ
1.5	1.41	smaller
1.4	1.394	
1.39	1.391	bigger
1.395	1.392	
1.392	1.391 7	smaller
1.391 5	1.391 54	bigger
1.391 52	1.391 55	bigger
1.391 6	1.391 568	smaller
1.391 58	1.391 563	
1.391 57	1.391 561	
1.391 56	1.391 558	
1.391 559	1.391 557 8	
1.391 558	1.391 557 5	
1.391 557	1.391 557 3	
1.391 557 4	1.391 557 4	

We get the answer as 1.391 557 4 to seven digits after 17 steps. Clever guessing, like using the value of $\sqrt{2}\sin\phi$ as the next guess for ϕ, could reduce this to around 13 steps.

P38.73 (a) The angles of bright beams diffracted from the grating are given by $d\sin\theta = m\lambda$. The angular dispersion is defined as the derivative $\dfrac{d\theta}{d\lambda}$:

$$d\cos\theta\frac{d\theta}{d\lambda} = m \quad \rightarrow \quad \frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta}$$

(b) For the average wavelength

$$\frac{579.065\text{ nm} + 576.959\text{ nm}}{2} = 578.012\text{ nm}$$

$d\sin\theta = m\lambda$ gives

$$\frac{0.020\,0\text{ m}}{8\,000}\sin\theta = 2\left(578.012\times 10^{-9}\text{ m}\right)$$

and $\theta = \sin^{-1}\dfrac{2\times 578\times 10^{-9}\text{ m}}{2.5\times 10^{-6}\text{ m}} = 27.5°$

The separation angle between the lines is, for

$$\Delta\lambda = 576.959\text{ nm} - 579.065\text{ nm} = 2.106\text{ nm}$$

and

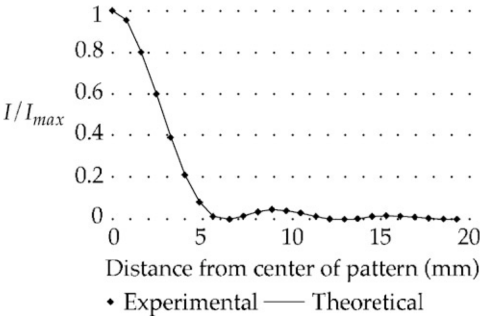
$$\Delta\theta = \frac{d\theta}{d\lambda}\Delta\lambda = \frac{m}{d\cos\theta}\Delta\lambda$$

$$= \frac{2}{2.5\times 10^{-6}\text{ m}\cos 27.5°}\left(2.106\times 10^{-9}\text{ m}\right)$$

$$= 0.001\,90 = 0.001\,90\text{ rad} = 0.001\,90\text{ rad}\left(\frac{180°}{\pi\text{ rad}}\right)$$

$$= \boxed{0.109°}$$

P38.74 (a) See ANS. FIG. P38.74.



ANS. FIG. P38.74

(b) The first minimum in the single-slit diffraction pattern occurs at

$$\sin\theta = \frac{\lambda}{a} \approx \frac{y_{\min}}{L}$$

Thus, the slit width is given by

$$a = \frac{\lambda L}{y_{\min}}$$

For a minimum located at $y_{\min} = 6.36\text{ mm} \pm 0.08\text{ mm}$, the width is

$$a = \frac{(632.8\times 10^{-9}\text{ m})(1.00\text{ m})}{6.36\times 10^{-3}\text{ m}} = \boxed{99.5\ \mu\text{m} \pm 1\%}$$

Challenge Problems

P38.75 (a) The *E* and *O* rays, in phase at the surface of the plate, will have a phase difference

$$\theta = \left(\frac{2\pi}{\lambda}\right)\delta$$

after traveling distance *d* through the plate. Here δ is the difference in the *optical path* lengths of these rays. The optical path length between two points is the product of the actual path length *d* and the index of refraction. Therefore,

$$\delta = |dn_O - dn_E|$$

The absolute value is used since $\frac{n_O}{n_E}$ may be more or less than unity. Therefore,

$$\theta = \left(\frac{2\pi}{\lambda}\right)|dn_O - dn_E| = \boxed{\left(\frac{2\pi}{\lambda}\right)d|n_O - n_E|}$$

(b) $$d = \frac{\lambda\theta}{2\pi|n_O - n_E|} = \frac{(550\times 10^{-9}\text{ m})(\pi/2)}{2\pi|1.544 - 1.553|} = 1.53\times 10^{-5}\text{ m} = \boxed{15.3\ \mu\text{m}}$$

P38.76 (a) The concave mirror of the spy satellite is probably about 2 m in diameter, and is surely not more than 5 m in diameter. That is the size of the largest piece of glass successfully cast to a precise shape, for the mirror of the Hale telescope on Mount Palomar. If the spy satellite had a larger mirror, its manufacture could not be kept secret, and it would be visible from the ground. Outer space is probably closer than your state capitol, but the satellite is surely above 200-km altitude, for reasonably low air friction. We find the distance between barely resolvable objects at a distance of 200 km, seen in yellow light through a 5-m aperture:

$$\frac{y}{L} = \theta_{\text{min}} = 1.22\frac{\lambda}{D}$$

$$y = \left(2\times10^{7}\text{ m}\right)(1.22)\left(\frac{6\times10^{-7}\text{ m}}{5\text{ m}}\right) = 3\text{ cm}$$

Thus the snooping spy satellite cannot see the difference between III and II or IV on a license plate. $\boxed{\text{A resolution of about 3 cm would make it difficult to read a license plate.}}$

(b) $\boxed{\text{No}}$. The resolution is too large. It cannot count coins spilled on a sidewalk, much less read the dates on them.

Considering atmospheric image distortion caused by variations in air density and temperature, the distance between barely resolvable objects is more like, assuming a limiting angle of one second of arc,

$$\left(2\times10^{7}\text{ m}\right)(1\text{ s})\left(\frac{1°}{3\,600\text{ s}}\right)\left(\frac{\pi\text{ rad}}{180°}\right) = 97\text{ cm} \approx 1\text{ m}$$

P38.77 (a) From Equation 38.1, $\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right)$. In this case $m = 1$ and

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{7.50\times10^{9}\text{ Hz}} = 4.00\times10^{-2}\text{ m}$$

Thus,

$$\theta = \sin^{-1}\left(\frac{4.00\times10^{-2}\text{ m}}{6.00\times10^{-2}\text{ m}}\right) = \boxed{41.8°}$$

(b) From Equation 38.2,

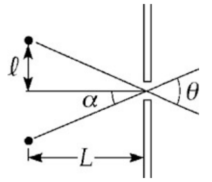
$$\frac{I}{I_{\text{max}}} = \left[\frac{\sin(\phi)}{\phi}\right]^{2} \quad\text{where}\quad \phi = \frac{\pi a\sin\theta}{\lambda}$$

When $\theta = 15.0°$,

$$\phi = \frac{\pi(0.060\,0\text{ m})\sin 15.0°}{0.040\,0\text{ m}} = 1.22\text{ rad}$$

and $\frac{I}{I_{\text{max}}} = \left[\frac{\sin(1.22\text{ rad})}{1.22\text{ rad}}\right]^{2} = \boxed{0.592}$

(c) $\sin\theta = \dfrac{\lambda}{a}$, so $\theta = 41.8°$:



ANS. FIG. P38.77(c)

This is the minimum angle subtended by the two sources at the slit. Refer to ANS. FIG. P38.77(c). Let α be the half angle between the sources, each a distance $\ell = 0.100$ m from the center line and a distance L from the slit plane. Then,

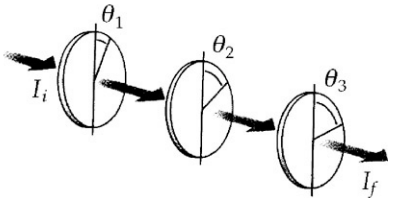
$$L = \ell\cot\alpha = (0.100\text{ m})\cot\left(\frac{41.8°}{2}\right) = \boxed{0.262\text{ m}}$$

P38.78 For incident unpolarized light of intensity I_{max}, the average value of the cosine-squared function is one-half, so the intensity after transmission by the first disk is $I = \frac{1}{2}I_{\text{max}}$.

After transmitting 2nd disk: $I = \frac{1}{2}I_{\text{max}}\cos^2\theta$

After transmitting 3rd disk: $I = \frac{1}{2}I_{\text{max}}\cos^2\theta\cos^2(90° - \theta)$

where the angle between the first and second disk is $\theta = \omega t$.



ANS. FIG. P38.78

Using trigonometric identities $\cos^2\theta = \frac{1}{2}(1 + \cos 2\theta)$

and $\cos^2(90° - \theta) = \sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$,

we have $I = \frac{1}{2}I_{\text{max}}\left[\frac{(1 + \cos 2\theta)}{2}\right]\left[\frac{(1 - \cos 2\theta)}{2}\right]$

$$I = \frac{1}{8}I_{\text{max}}(1 - \cos^2 2\theta) = \frac{1}{8}I_{\text{max}}\left(\frac{1}{2}\right)(1 - \cos 4\theta)$$

Since $\theta = \omega t$, the intensity of the emerging beam is given by

$$\boxed{I = \frac{1}{16}I_{\text{max}}(1 - \cos 4\omega t)}$$

P38.79 The energy in the central maximum we can estimate in Figure P38.79 as proportional to

$$(\text{width})(\text{height}) = (2\pi)I_{\max}$$

As in Problem P38.71, the maximum height of the first side maximum is approximately

$$I = I_{\max}\left[\frac{\sin(\phi)}{\phi}\right]^2 = I_{\max}\left[\frac{\sin(3\pi/2)}{3\pi/2}\right]^2 = \frac{4I_{\max}}{9\pi^2}$$

Then the energy in one side maximum is proportional to $\pi\left(\frac{4I_{\max}}{9\pi^2}\right)$, and that in both of the first side maxima together is proportional to $2\pi\left(\frac{4I_{\max}}{9\pi^2}\right)$.

Similarly and more precisely, and always with the same proportionality constant, the energy in both of the second side maxima is proportional to $2\pi\left(\frac{4I_{\max}}{25\pi^2}\right)$.

The energy in all of the side maxima together is proportional to

$$2\pi\left(\frac{4I_{\max}}{\pi^2}\right)\left(\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\frac{1}{9^2}+\cdots\right)$$

$$= 2\pi\left(\frac{4I_{\max}}{\pi^2}\right)\left(\frac{\pi^2}{8}-1\right) = I_{\max}\left(\pi-\frac{8}{\pi}\right) = 0.595I_{\max}$$

The ratio of the energy in the central maximum to the total energy is then

$$\frac{(2\pi)I_{\max}}{(2\pi)I_{\max}+0.595I_{\max}} = \frac{1}{1+0.595/(2\pi)} = 0.913 = 91.3\%$$

Our calculation is only a rough estimate, because the shape of the central maximum in particular is not just a vertically-stretched cycle of a cosine curve. It is slimmer than that.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P38.2	4.22 mm
P38.4	(a) 51.8 μm; (b) 949 μm; (c) horizontal; (d) vertical; (e) A smaller distance between aperture edges causes a wider diffraction angle. The longer dimension of each rectangle is 18.3 times larger than the smaller dimension.
P38.6	(a) 1.50 m; (b) 4.05 mm
P38.8	$a = \dfrac{\lambda L\lvert m_2 - m_1\rvert}{\Delta y}$
P38.10	See P38.10 for full explanation.
P38.12	(a) $\theta_0 = 0°$, $\theta_1 = \pm 10.3°$, $\theta_2 = \pm 21.0$, $\theta_3 = \pm 32.5°$, $\theta_4 = \pm 45.8°$, $\theta_5 = \pm 63.6°$; (b) 11, (c) $\theta_1 = \pm 45.8°$, (d) two, (e) two, (f) nine, (g) 0.032 4 I_{max}
P38.14	1.00×10^{-3} rad
P38.16	25.0 m
P38.18	(a) 79.8 μrad; (b) violet, 54.2 μrad; (c) The resolving power is improved, with the minimum resolvable angle becoming 60.0 μrad.
P38.20	3.09 m
P38.22	(a) Blue; (b) 186 m to 271 m
P38.24	105 m
P38.26	7.35°
P38.28	(a) 479 nm, 647 nm, 698 nm; (b) 20.5°, 28.3°, 30.7°
P38.30	(a) 5 orders is the maximum; (b) 10 orders in the short-wavelength region
P38.32	5.91°, 13.2°, 26.5°
P38.34	$\theta_{2r} > \theta_{3v}$ and these orders must overlap.
P38.36	(a) 0.738 mm; (b) See P38.36(b) for full explanation.
P38.48	$\theta = 14.4°$
P38.40	$\theta = 31.9°$
P38.42	See P38.42 for full explanation.
P38.44	1.11
P38.46	(a) 93.3%; (b) 50.0%; (c) 0.00%

P38.48 (a) 0.875; (b) 0.789; (c) 0.670; (d) We can get more and more of the incident light through the stack of ideal filters, approaching 50%, by reducing the angle between the transmission axes of each one and the next.

P38.50 $\theta_p = \tan^{-1}\left(\frac{1}{\sin\theta_c}\right)$ or $\theta_p = \tan^{-1}\left(\csc\theta_c\right)$ or $\theta_p = \cot^{-1}\left(\sin\theta_c\right)$

P38.52 1/8

P38.54 (a) One slit, as the central maximum is twice as wide as the other maxima; (b) 0.122 mm wide

P38.56 659 nm

P38.58 (a) 5.63 mm; (b) The assumption is unreasonable. Over a horizontal path of 12 km in air, density variation associated with convection would make the motorcycles completely unresolvable with any optical device.

P38.60 See P38.60 for full explanation.

P38.62 See P38.62 for full explanation.

P38.64 (a–e) See P38.64 for full explanations.

P38.66 $\theta = \tan^{-1}\left(\frac{n}{n_\ell}\right) - \tan^{-1}\left(\frac{1}{n_\ell}\right)$

P38.68 (a) $D^2 = 2.44\lambda L$; (b) 428 μm

P38.70 (a) $\tan\theta = \frac{n_2\sin\beta}{n_1 - n_2\cos\beta}$; (b) See P38.70(b) for full explanation.

P38.72 (a) $\sin\phi = \frac{\phi}{\sqrt{2}}$; (b) $\phi = 1.39$ rad; (c) $\frac{0.885\lambda}{a}$; (d) 17 steps (13 with clever guessing)

P38.74 (a) See ANS FIG P38.74; (b) 99.5 μm ± 1%

P38.76 (a) A resolution of about 3 cm would make it difficult to read a license plate; (b) No

P38.78 $\frac{1}{16}I_{\max}\left(1-\cos 4\omega t\right)$

39

Relativity

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ39.1 **(i)** Answer (a). **(ii)** Answer (c). **(iii)** Answer (d). There is no upper limit on the momentum or energy of an electron. As the speed of the electron approaches c, the factor γ tends to infinity, so both the kinetic energy, $K = (\gamma - 1)mc^2$, and momentum, $p = \gamma mv$, tend to infinity.

OQ39.2 Answer (d). The relativistic time dilation effect is symmetric between the observers.

OQ39.3 Answers (b) and (c). According to the second postulate of special relativity (the constancy of the speed of light), both observers will measure the light speed to be c.

OQ39.4 Answer (c). An oblate spheroid. The dimension in the direction of motion would be contracted but the dimension perpendicular to the motion would be unaltered.

OQ39.5 Answer (e). The astronaut is moving with constant velocity and is therefore in an inertial reference frame. According to the principle of relativity, all the laws of physics are the same in her reference frame as in any other inertial reference frame. Thus, she should experience no effects due to her motion through space.

OQ39.6 Answer (b). The dimension parallel to the direction of motion is reduced by the factor γ and the other dimensions are unchanged.

OQ39.7 **(i)** Answer (c). The Earth observer measures the clock in orbit to run slower.

(ii) Answer (b). They are not synchronized. They both tick at the same rate after return, but a time difference has developed between the two clocks.

OQ39.8 Answer (a) > (c) > (b). The relativistic momentum of a particle is $p = \sqrt{E^2 - E_R^2}/c$, where E is the total energy of the particle, and $E_R = mc^2$ is its rest energy ($E_R = 0$ for the photon). In this problem, each of the particles has the same total energy E. Thus, the particle with the smallest rest energy (photon < electron < proton) has the greatest momentum.

OQ39.9 Answers (d) and (e). The textbook refers to the postulate summarized in choice (d) as the principle of relativity, and to the postulate in choice (e) as the constancy of the speed of light.

OQ39.10 Answer (b). By the postulate of the constancy of the speed of light, light from any source travels in vacuum at speed c.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ39.1 The star and the planet orbit about their common center of mass, thus the star moves in an elliptical path. Just like the light from a star in a binary star system, the spectrum of light from the star would undergo a cyclic series of Doppler shifts depending on the star's speed and direction of motion relative to the observer. The repetition rate of the Doppler shift pattern is the period of the orbit. Information about the orbit size can be calculated from the size of the Doppler shifts.

CQ39.2 Suppose a railroad train is moving past you. One way to measure its length is this: You mark the tracks at the cowcatcher forming the front of the moving engine at 9:00:00 AM, while your assistant marks the tracks at the back of the caboose at the same time. Then you find the distance between the marks on the tracks with a tape measure.

You and your assistant must make the marks simultaneously in your frame of reference, for otherwise the motion of the train would make its length different from the distance between marks.

CQ39.3 (a) Yours does. From your frame of reference, the clocks on the train run slow, so the symphony takes a longer time interval to play on the train.

(b) The observer's on the train does. From the train's frame of reference, your clocks run slow, so the symphony takes a longer time interval to play for you.

(c) Each observer measures his symphony as finishing first.

CQ39.4 Get a *Mr. Tompkins* book by George Gamow for a wonderful fictional exploration of this question. Because of time dilation, your trip to work would be short, so your coffee would not have time to become cold, and you could leave home later. Driving home in a hurry, you push on the gas pedal not to increase your speed by very much, but rather to make the blocks get shorter. Big Doppler shifts in wave frequencies make red lights look green as you approach them, alter greatly the frequencies of car horns, and make it very difficult to tune a radio to a station. High-speed transportation is very expensive because a small change in speed requires a large change in kinetic energy, resulting in huge fuel use. Crashes would be disastrous because a speeding car has a great amount of kinetic energy, so a collision would generate great damage. There is a five-day delay in transmission when you watch the Olympics in Australia on live television. It takes ninety-five years for sunlight to reach Earth.

CQ39.5 Acceleration is indicated by a curved line. This can be seen in the middle of Speedo's world-line in Figure 39.11, where he turns around and begins his trip home.

CQ39.6 (a) Any physical theory must agree with experimental measurements within some domain. Newtonian mechanics agrees with experiment for objects moving slowly compared to the speed of light. Relativistic mechanics agrees with experiment for objects moving at relativistic speeds.

(b) It is well established that Newtonian mechanics applies to objects moving at speeds a lot less than light, but Newtonian mechanics fails at relativistic speeds. If relativistic mechanics is to be the better theory, it must apply to all physically possible speeds. Relativistic mechanics at nonrelativistic speeds must reduce to Newtonian mechanics, and it does.

CQ39.7 No. The principle of relativity implies that nothing can travel faster than the speed of light in a *vacuum*, which is 300 Mm/s. The electron would emit light in a conical shock wave of Cerenkov radiation.

CQ39.8 According to $\vec{\mathbf{p}} = \gamma m\vec{\mathbf{u}}$, doubling the speed u will make the momentum of an object increase by the factor $2\left[\dfrac{c^2 - u^2}{c^2 - 4u^2}\right]^{1/2}$.

CQ39.9 As the object approaches the speed of light, its kinetic energy grows without limit. It would take an infinite investment of work to accelerate the object to the speed of light.

CQ39.10 A microwave pulse is reflected from a moving object. The waves that are reflected back are Doppler shifted in frequency according to the speed of the target. The receiver in the radar gun detects the reflected wave and compares its frequency to that of the emitted pulse. Using the frequency shift, the speed can be calculated to high precision. Be forewarned: this technique works if you are either traveling toward or away from your local law enforcement agent!

CQ39.11 Running "at a speed near that of light" means some other observer measures you to be running near the speed of light. To you, you are at rest in your own inertial frame. You would see the same thing that you see when looking at a mirror when at rest. The theory of relativity tells us that all experiments will give the same results in all inertial frames of reference.

CQ39.12 **(i)** Solving for the image location q in terms of the object location p and the focal length f gives

$$q = \frac{pf}{p - f}$$

We note that when $p = f$, the image is formed at infinity. Let us, for example, take an object initially a distance $p_i = 2f$ from the mirror. Its speed, in approaching f in a finite amount of time is

$$v = \frac{p - f}{\Delta t} = \frac{2f - f}{\Delta t} = \frac{f}{\Delta t}$$

At the same time, the location of the image moves from $q_i = (2f)f/(2f - f) = 2f$ to $q_f = \infty$, i.e., covering an infinite distance in a finite amount of time. The speed of the image thus exceeds the speed of light c.

(ii) For simplicity, we assume that the distant screen is curved with a radius of curvature R. The linear speed of the spot on the screen is then given by $v = \omega R$, where ω is the angular speed of rotation of the laser pointer. With sufficiently large ω and R, the speed of the spot moving on the screen can exceed c.

(iii) Neither of these examples violates the principle of relativity. In the first case, the image transtions from being real to being virtual when $p = f$. In the second case, we have the intersection of a light beam with a screen. A point of tranition or intersection is not made of matter so it has no mass, and hence no energy. A bug momentarily at the intersection point could squeak or reflect light. A second bug would have to wait for sound or light to travel across the distance between the first bug and himself, to get the message; neither of these actions would result in communication reaching the second bug sooner than the intersection point reaches him.

CQ39.13 Special relativity describes the relationship between physical quantities and laws in inertial reference frames: that is, reference frames that are not accelerating. General relativity describes the relationship between physical quantities and laws in all reference frames.

CQ39.14 Because of gravitational time dilation, the downstairs clock runs more slowly because it is closer to the Earth and hence in a stronger gravitational field than the upstairs clock.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 39.1 The Principle of Galilean Relativity

P39.1 By Equation 4.20, $\vec{\mathbf{u}}_{PA} = \vec{\mathbf{u}}_{PB} + \vec{\mathbf{v}}_{BA}$, with motion in one dimension,

$$\mathrm{u}_{baseball,\,\text{ground}} = \mathrm{u}_{baseball,\,\text{truck}} + \vec{\mathbf{v}}_{\text{truck, ground}}$$
$$\mathrm{u}_{baseball,\,\text{ground}} = -20.0\text{ m/s} + 10.0\text{ m/s} = -10.0\text{ m/s}$$

In other words, $\boxed{\text{10.0 m/s toward the left in Figure P39.1}}$.

P39.2 In the laboratory frame of reference, Newton's second law is valid: $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$. Laboratory observer 1 watches some object accelerate under applied forces. Call the instantaneous velocity of the object $\vec{\mathbf{v}}_1 = \vec{\mathbf{v}}_{O1}$ (the velocity of object O relative to observer 1 in laboratory frame) and its acceleration $\dfrac{d\vec{\mathbf{v}}_1}{dt} = \vec{\mathbf{a}}_1$. A second observer has instantaneous velocity $\vec{\mathbf{v}}_{21}$ relative to the first. In general, the velocity of the object in the frame of the second observer is

$$\vec{\mathbf{v}}_2 = \vec{\mathbf{v}}_{O2} = \vec{\mathbf{v}}_{O1} + \vec{\mathbf{v}}_{12} = \vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{21}$$

(a) If the relative instantaneous velocity $\vec{\mathbf{v}}_{21}$ of the second observer is *constant*, the second observer measures the acceleration

$$\vec{\mathbf{a}}_2 = \frac{d\vec{\mathbf{v}}_2}{dt} = \frac{d\vec{\mathbf{v}}_1}{dt} = \vec{\mathbf{a}}_1$$

This is the same as that measured by the first observer. In this nonrelativistic case, they measure the same forces and masses as well. Thus, the second observer also confirms that $\vec{\mathbf{F}} = m\vec{\mathbf{a}}$.

(b) If the second observer's frame is accelerating, then the instantaneous relative velocity $\vec{\mathbf{v}}_{21}$ is *not constant*. The second observer measures an acceleration of

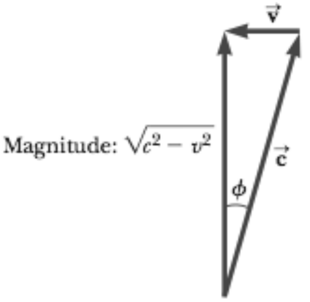
$$\vec{\mathbf{a}}_2 = \frac{d\vec{\mathbf{v}}_2}{dt} = \frac{d\left(\vec{\mathbf{v}}_1 - \vec{\mathbf{v}}_{21}\right)}{dt} = \vec{\mathbf{a}}_1 - \frac{d\left(\vec{\mathbf{v}}_{21}\right)}{dt} = \vec{\mathbf{a}}_1 - \vec{\mathbf{a}}',$$

where $\frac{d\left(\vec{\mathbf{v}}_{21}\right)}{dt} = \vec{\mathbf{a}}'$

The observer in the accelerating frame measures the acceleration of the mass as being $\vec{\mathbf{a}}_2 = \vec{\mathbf{a}}_1 - \vec{\mathbf{a}}'$. If Newton's second law held for the accelerating frame, that observer would expect to find valid the relation $\vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}_2$, or $\vec{\mathbf{F}}_1 = m\vec{\mathbf{a}}_2$ (since $\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_2$ and the mass is unchanged in each). But, instead, the accelerating frame observer finds that $\vec{\mathbf{F}}_2 = m\vec{\mathbf{a}}_2 - m\vec{\mathbf{a}}'$, which is *not* Newton's second law.

P39.3 From the triangle in ANS. FIG. P39.3,

$$\phi = \sin^{-1}\left(\frac{v}{c}\right) = \sin^{-1}\left(\frac{29.8\times10^{3}\text{ m/s}}{2.998\times10^{8}\text{ m/s}}\right)$$
$$= \boxed{5.70\times10^{-3}\text{ degrees} = 9.94\times10^{-5}\text{ rad}}$$



ANS. FIG. P39.3

P39.4 In the rest frame,

$$p_i = m_1 v_{1i} + m_2 v_{2i} = (2\,000\text{ kg})(20.0\text{ m/s}) + (1\,500\text{ kg})(0\text{ m/s})$$
$$= 4.00\times 10^4\text{ kg}\cdot\text{m/s}$$
$$p_f = (m_1 + m_2)v_f = (2\,000\text{ kg} + 1\,500\text{ kg})v_f$$

Since $p_i = p_f$,

$$v_f = \frac{p_i}{m_1 + m_2} = \frac{4.00\times 10^4\text{ kg}\cdot\text{m/s}}{2\,000\text{ kg} + 1\,500\text{ kg}} = 11.429\text{ m/s}$$

In the moving frame, these velocities are all reduced by +10.0 m/s.

$$v'_{1i} = v_{1i} - v' = 20.0\text{ m/s} - (+10.0\text{ m/s}) = 10.0\text{ m/s}$$
$$v'_{2i} = v_{2i} - v' = 0\text{ m/s} - (+10.0\text{ m/s}) = -10.0\text{ m/s}$$
$$v'_f = 11.429\text{ m/s} - (+10.0\text{ m/s}) = 1.429\text{ m/s}$$

Our initial momentum is then

$$p'_i = m_1 v'_{1i} + m_2 v'_{2i}$$
$$= (2\,000\text{ kg})(10.0\text{ m/s}) + (1\,500\text{ kg})(-10.0\text{ m/s})$$
$$= 5\,000\text{ kg}\cdot\text{m/s}$$

and our final momentum has the same value:

$$p'_f = (2\,000\text{ kg} + 1\,500\text{ kg})v'_f = (3\,500\text{ kg})(1.429\text{ m/s})$$
$$= 5\,000\text{ kg}\cdot\text{m/s}$$

Section 39.2 The Michelson-Morley Experiment

Section 39.3 Einstein's Principle of Relativity

Section 39.4 Consequences of the Special Theory of Relativity

P39.5 In the rest frame of the spacecraft, the Earth-star gap travels past it at speed u. The distance from Earth to the star is a proper length in the Earth's frame:

$$L = \frac{L_P}{\gamma} = L_P\sqrt{1-\left(\frac{u}{c}\right)^2}$$

Solving for the speed of the spacecraft gives,

$$u = c\sqrt{1-\left(\frac{L}{L_P}\right)^2} = c\sqrt{1-\left(\frac{2.00\text{ ly}}{5.00\text{ ly}}\right)^2} = \boxed{0.917c}$$

P39.6 (a) The length of the meter stick measured by the observer moving at speed $v = 0.900\ c$ relative to the meter stick is

$$L = L_p/\gamma = L_p\sqrt{1-(v/c)^2} = (1.00\text{ m})\sqrt{1-(0.900)^2} = \boxed{0.436\text{ m}}$$

(b) If the observer moves relative to Earth in the direction opposite the motion of the meter stick relative to Earth, the velocity of the observer relative to the meter stick is greater than that in part (a). The measured length of the meter stick will be $\boxed{\text{less than } 0.436\text{ m}}$ under these conditions, but so small it is unobservable.

P39.7 A clock running at one-half the rate of a clock at rest takes twice the time to register the same time interval: $\Delta t = 2\Delta t_p$.

$$\Delta t = \frac{\Delta t_p}{\left[1-(v/c)^2\right]^{1/2}} \qquad \text{so} \qquad v = c\left[1-\left(\frac{\Delta t_p}{\Delta t}\right)^2\right]^{1/2}$$

For $\Delta t = 2\Delta t_p$,

$$v = c\left[1-\left(\frac{\Delta t_p}{2\Delta t_p}\right)^2\right]^{1/2} = c\left[1-\frac{1}{4}\right]^{1/2} = \boxed{0.866c}$$

P39.8 For $\dfrac{v}{c} = 0.990$, $\gamma = 7.09$.

(a) The muon's lifetime as measured in the Earth's rest frame is

$$\Delta t = \frac{L_P}{v} = \frac{4.60\text{ km}}{0.990c} = \left[\frac{4.60\times10^3\text{ m}}{0.990\left(3.00\times10^8\text{ m/s}\right)}\right]$$
$$= 1.55\times10^{-5}\text{ s} = 15.5\ \mu\text{s}$$

and the lifetime measured in the muon's rest frame is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1}{7.09}(15.5\ \mu\text{s}) = \boxed{2.18\ \mu\text{s}}$$

(b) In the muon's frame, the Earth is approaching the muon at speed $v = 0.990c$. During the time interval the muon exists, the Earth travels the distance

$$d = v\Delta t_p = v\frac{\Delta t}{\gamma} = v\frac{L_P}{\gamma v} = \frac{L_P}{\gamma}$$
$$= \left(4.60\times10^3\text{ m}\right)\sqrt{1-(0.990)^2} = \boxed{649\text{ m}}$$

P39.9 From Equation 39.9 for length contraction,

$$L = L_p\sqrt{1-\frac{v^2}{c^2}}$$

we solve for the speed v of the meterstick:

$$v = c\sqrt{1-\left(\frac{L}{L_p}\right)^2}$$

Taking $L = \frac{L_p}{2}$ where, $L_p = 1.00$ m, gives

$$v = c\sqrt{1-\left(\frac{L_p/2}{L_p}\right)^2} = c\sqrt{1-\frac{1}{4}} = \boxed{0.866c}$$

P39.10 (a) The time interval between pulses as measured by the astronaut is a proper time:

$$\Delta t_p = \left(\frac{1\text{ min}}{75.0\text{ beats}}\right)$$

The time interval between pulses as measured by the Earth observer is then:

$$\Delta t = \gamma\Delta t_p = \frac{1}{\sqrt{1-(0.500)^2}}\left(\frac{1\text{ min}}{75.0\text{ beats}}\right) = 1.54\times10^{-2}\text{ min/beat}$$

Thus, the Earth observer records a pulse rate of

$$\frac{1}{\Delta t} = \frac{1}{\gamma\Delta t_p} = \sqrt{1-(0.500)^2}\left(\frac{75.0\text{ beats}}{1\text{ min}}\right) = \boxed{65.0\text{ beats/min}}$$

(b) From part (a), the pulse rate is

$$\frac{1}{\Delta t} = \frac{1}{\gamma\Delta t_p} = \sqrt{1-(0.990)^2}\left(\frac{75.0\text{ beats}}{1\text{ min}}\right) = \boxed{10.5\text{ beats/min}}$$

That is, the life span of the astronaut (reckoned by the duration of the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

P39.11 For the light as observed, $\lambda = 650$ nm and $\lambda' = 520$ nm. From Equation 39.10,

$$f' = \frac{c}{\lambda'} = \sqrt{\frac{1+v/c}{1-v/c}}f = \sqrt{\frac{1+v/c}{1-v/c}}\frac{c}{\lambda}$$

Solving for the velocity,

$$\sqrt{\frac{1+v/c}{1-v/c}}=\frac{\lambda}{\lambda'} \quad\rightarrow\quad 1+\frac{v}{c}=\left(\frac{\lambda}{\lambda'}\right)^2\left(1-\frac{v}{c}\right)$$

Then,

$$\frac{v}{c}\left[1+\left(\frac{\lambda}{\lambda'}\right)^2\right]=\left(\frac{\lambda}{\lambda'}\right)^2-1$$

$$\frac{v}{c}=\frac{\left(\frac{\lambda}{\lambda'}\right)^2-1}{1+\left(\frac{\lambda}{\lambda'}\right)^2}=\frac{\left(\frac{650\text{ nm}}{520\text{ nm}}\right)^2-1}{1+\left(\frac{650\text{ nm}}{520\text{ nm}}\right)^2}=0.220$$

or $v=\boxed{0.220c}=6.59\times10^7\text{ m/s}$

P39.12 The spacecraft are identical, so they have the same proper length; thus, your measurements and the astronaut's measurements are reciprocal.

(a) You measure the proper length of your spacecraft to be

$$\boxed{L_p=20.0\text{ m}}$$

(b) You measure the length *L* of the astronaut's spacecraft to be

$$\boxed{L=19.0\text{ m}}$$

(c) From the astronaut's measurement of the length *L* of your spacecraft,

$$L=\frac{L_p}{\gamma}=L_p\sqrt{1-\left(\frac{u}{c}\right)^2}$$

we solve for the speed of the astronaut's spacecraft relative to yours:

$$\frac{u}{c}=\sqrt{1-\left(\frac{L}{L_p}\right)^2}=\sqrt{1-\left(\frac{19.0\text{ m}}{20.0\text{ m}}\right)^2}=0.312$$

or $u=\boxed{0.312c}$

P39.13 The astronaut's measured time interval is a proper time in her reference frame. Therefore, according to an observer on Earth,

$$\Delta t=\gamma\,\Delta t_p=\frac{\Delta t_p}{\sqrt{1-(v/c)^2}}=\frac{3.00\text{ s}}{\sqrt{1-(0.800)^2}}=\boxed{5.00\text{ s}}$$

P39.14 From the definition of γ,

$$\gamma = \frac{1}{\sqrt{1-\left(v^2/c^2\right)}} = 1.010\ 0$$

we solve for the speed:

$$v = c\sqrt{1-\left(\frac{1}{\gamma}\right)^2} = c\sqrt{1-\left(\frac{1}{1.010\ 0}\right)^2} = \boxed{0.140c}$$

P39.15 The observer measures the proper length of the tunnel, 50.0 m, but measures the train contracted to length

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} = 100\text{ m}\sqrt{1-\left(0.950\right)^2} = 31.2\text{ m}$$

shorter than the tunnel by 50.0 – 31.2 = 18.8 m.

$\boxed{\text{The trackside observer measures the length to be 31.2 m, so the supertrain is measured to fit in the tunnel, with 18.8 m to spare.}}$

***P39.16** (a) The lifetime of the pi meson measured by an observer on Earth is given by

$$\Delta t = \gamma\Delta t_p = \frac{\Delta t_p}{\sqrt{1-\left(v/c\right)^2}} = \frac{2.6\times 10^{-8}\text{ s}}{\sqrt{1-\left(0.98\right)^2}} = \boxed{1.3\times 10^{-7}\text{ s}}$$

(b) The distance travelled before the meson decays is

$$d = v\Delta t = 0.98\left(3.0\times 10^8\text{ m/s}\right)\left(1.3\times 10^{-7}\text{ s}\right) = \boxed{38\text{ m}}$$

(c) In the absence of time dilation, the meson would travel a distance

$$d = v\Delta t = 0.98\left(3.0\times 10^8\text{ m/s}\right)\left(2.6\times 10^{-8}\text{ s}\right) = \boxed{7.6\text{ m}}$$

***P39.17** (a) The 0.800*c* and the 20.0 ly are measured in the Earth frame, so in this frame,

$$\Delta t = \frac{x}{v} = \frac{20.0\text{ ly}}{0.800c} = \left(\frac{20.0\text{ ly}}{0.800c}\right)\left(\frac{1\ c}{1\text{ ly/yr}}\right) = \boxed{25.0\text{ yr}}$$

(b) We see a clock on the meteoroid moving, so we do not measure proper time; that clock measures proper time.

$$\Delta t = \gamma\Delta t_p:$$

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{25.0\text{ yr}}{1/\sqrt{1-v^2/c^2}} = 25.0\text{ yr}\sqrt{1-0.800^2}$$

$$= 25.0\text{ yr}\left(0.600\right) = \boxed{15.0\text{ yr}}$$

(c) Method one: We measure the 20.0 ly on a stick stationary in our frame, so it is proper length. The tourist measures it to be contracted to

$$L = \frac{L_p}{\gamma} = \frac{20.0\text{ ly}}{1/\sqrt{1-0.800^2}} = \frac{20.0\text{ ly}}{1.67} = \boxed{12.0\text{ ly}}$$

Method two: The tourist sees the Earth approaching at 0.800*c*:

$$(0.800\text{ ly/yr})(15.0\text{ yr}) = \boxed{12.0\text{ ly}}$$

***P39.18** The relativistic density is

$$\frac{E_R}{c^2V} = \frac{\gamma mc^2}{c^2V} = \frac{\gamma m}{V} = \frac{m}{\left(L_p\right)^3\left[1-(u/c)^2\right]}$$

$$= \frac{8.00\text{ g}}{(1.00\text{ cm})^3\left[1-(0.900)^2\right]} = \boxed{42.1\text{ g/cm}^3}$$

P39.19 The spaceship is measured by the Earth observer to be length-contracted to

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{or} \qquad L^2 = L_p^2\left(1-\frac{v^2}{c^2}\right)$$

Also, the contracted length is related to the time required to pass overhead by

$$L = v\Delta t \qquad \text{or} \qquad L^2 = v^2(\Delta t)^2 = \frac{v^2}{c^2}(c\Delta t)^2$$

Equating these two expressions gives $L_p^2 - L_p^2\frac{v^2}{c^2} = (c\Delta t)^2\frac{v^2}{c^2}$.

or $$\left[L_p^2 + (c\Delta t)^2\right]\frac{v^2}{c^2} = L_p^2$$

Using the given values $L_p = 300$ m and $\Delta t = 0.750\times10^{-6}$ s, this becomes

$$\left(1.41\times10^5\text{ m}^2\right)\frac{v^2}{c^2} = 9.00\times10^4\text{ m}^2$$

giving $v = \boxed{0.800c}$

P39.20 The spaceship is measured by Earth observers to be of length L, where

$$L = L_p\sqrt{1-\frac{v^2}{c^2}}$$

and $L = v\Delta t$

$$v\Delta t = L_p\sqrt{1-\frac{v^2}{c^2}} \qquad \text{and} \qquad v^2\Delta t^2 = L_p^2\left(1-\frac{v^2}{c^2}\right)$$

Solving for *v*,

$$v^2\left(\Delta t^2 + \frac{L_p^2}{c^2}\right) = L_p^2$$

giving $\boxed{v = \dfrac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}}$

P39.21 (a) When the source moves away from an observer, the observed frequency is

$$f' = f\left(\frac{c+v}{c-v}\right)^{1/2} = f\left(\frac{c-v_S}{c+v_S}\right)^{1/2}$$

where $v = v_{\text{source}} = -v_S$ because the source is moving away from the observer.

When $v_s \ll c$, the binomial expansion gives

$$\left(\frac{c-v_S}{c+v_S}\right)^{1/2} = \left[1-\left(\frac{v_S}{c}\right)\right]^{1/2}\left[1+\left(\frac{v_S}{c}\right)\right]^{-1/2}$$

$$\approx\left(1-\frac{v_S}{2c}\right)\left(1-\frac{v_S}{2c}\right)\approx\left(1-\frac{v_S}{c}\right)$$

So, $f' \approx f\left(1-\dfrac{v_S}{c}\right)$

The observed wavelength is found from $c = \lambda' f' = \lambda f$:

$$\lambda' = \frac{\lambda f}{f'} \approx \frac{\lambda f}{f\left(1-v_S/c\right)} = \frac{\lambda}{1-v_S/c}$$

$$\Delta\lambda = \lambda' - \lambda = \lambda\left(\frac{1}{1-v_S/c}-1\right) = \lambda\left(\frac{v_S/c}{1-v_S/c}\right)$$

Since $1-\dfrac{v_S}{c}\approx 1$, $\boxed{\dfrac{\Delta\lambda}{\lambda}\approx\dfrac{v_S}{c}}$

(b) We use the equation from part (a) with the given values:

$$v_S = c\left(\frac{\Delta\lambda}{\lambda}\right) = c\left(\frac{20.0\text{ nm}}{397\text{ nm}}\right) = \boxed{0.050\,4c}$$

P39.22 We find Cooper's speed from Newton's second law:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

Solving,

$$v = \left[\frac{GM}{(R+h)}\right]^{1/2} = \left[\frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\ \mathrm{kg})}{(6.37\times10^6\ \mathrm{m} + 0.160\times10^6\ \mathrm{m})}\right]^{1/2}$$

$$= 7.82\times10^3 = 7.82\ \mathrm{km/s}$$

Then the time period of one orbit is

$$T = \frac{2\pi(R+h)}{v} = \frac{2\pi(6.53\times10^6\ \mathrm{m})}{7.82\times10^3\ \mathrm{m/s}} = 5.25\times10^3\ \mathrm{s}$$

(a) The time difference for 22 orbits is

$$\Delta t - \Delta t_p = (\gamma - 1)\Delta t_p = \left[\left(1 - \frac{v^2}{c^2}\right)^{-1/2} - 1\right](22T)$$

$$\Delta t - \Delta t_p \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2} - 1\right)(22T) = \frac{1}{2}\left(\frac{7.82\times10^3\ \mathrm{m/s}}{3.00\times10^8\ \mathrm{m/s}}\right)^2$$

$$\times 22(5.25\times10^3\ \mathrm{s}) = \boxed{39.2\ \mu\mathrm{s}}$$

(b) For each one orbit Cooper aged less by

$$\Delta t - \Delta t_p = \frac{39.2\ \mu\mathrm{s}}{22} = 1.78\ \mu\mathrm{s}$$

The press report is $\boxed{\text{accurate to one digit}}$.

P39.23 (a) The mirror is approaching the source. Let f_m be the frequency as seen by the mirror. Thus,

$$f_m = f\sqrt{\frac{c+v}{c-v}}$$

After reflection, the mirror acts as a source, approaching the receiver. If f' is the frequency of the reflected wave,

$$f' = f_m\sqrt{\frac{c+v}{c-v}}$$

Combining gives

$$\boxed{f' = \frac{c+v}{c-v}f}$$

(b) Using the above result, the beat frequency is

$$f_{\text{beat}} = f' - f = f' = \frac{c+v}{c-v}f - f = f\left(\frac{c+v}{c-v} - 1\right)$$

$$f_{\text{beat}} = f\left(\frac{c+v-(c-v)}{c-v}\right) = f\left(\frac{2v}{c-v}\right) \approx f\frac{2v}{c} = \frac{2v}{c/f}$$

$$f_{\text{beat}} = \frac{2v}{\lambda}$$

(c) The wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{10.0\times10^{9}\ \text{Hz}} = 0.030\ 0\ \text{m}$$

The beat frequency is therefore,

$$f_{\text{beat}} = \frac{2v}{\lambda} = \frac{(2)(30.0\ \text{m/s})}{(0.030\,0\ \text{m})} = 2\ 000\ \text{Hz} = \boxed{2.00\ \text{kHz}}$$

(d) From part (b), $v = \dfrac{f_{\text{beat}}\lambda}{2}$, so

$$\Delta v = \frac{\Delta f_{\text{beat}}\lambda}{2} = \frac{(5.0\ \text{Hz})(0.030\,0\ \text{m})}{2}$$

$$= \boxed{0.075\,0\ \text{m/s} \approx 0.17\ \text{mi/h}}$$

P39.24 (a) In the Earth frame, Speedo's trip lasts for a time

$$\Delta t = \frac{\Delta x}{v} = \frac{20.0\ \text{ly}}{0.950\text{c}} = 21.05\ \text{yr}$$

Speedo's age advances only by the proper time interval

$$\Delta t_p = \frac{\Delta t}{\gamma} = 21.05\ \text{yr}\sqrt{1-(0.950)^2} = 6.574\ \text{yr}$$

during his trip. Similarly for Goslo,

$$\Delta t_p = \frac{\Delta x}{v}\sqrt{1-\frac{v^2}{c^2}} = \frac{20.0\ \text{ly}}{0.750\ \text{ly/yr}}\sqrt{1-(0.750)^2} = 17.64\ \text{yr}$$

While Speedo has landed on Planet X and is waiting for his brother, he ages by

$$\frac{20.0\ \text{ly}}{0.750\ \text{ly/yr}} - \frac{20.0\ \text{ly}}{0.950\ \text{ly/yr}} = 5.614\ \text{yr}$$

From their departure to when the twins meet, Speedo has aged (6.574 yr + 5.614 yr) = 12.19 yr, and Goslo has aged 17.64 years, for an age difference of

$$17.64\text{ yr} - \left(6.574\text{ yr} + 5.614\text{ yr}\right) = \boxed{5.45\text{ yr}}$$

(b) $\boxed{\text{Goslo}}$ is older.

P39.25 This problem is slightly more difficult than most, for the simple reason that your calculator probably cannot hold enough decimal places to yield an accurate answer. However, we can bypass the difficulty by noting the approximation

$$\sqrt{1-\frac{v^2}{c^2}} \approx 1-\frac{v^2}{2c^2}$$

Squaring both sides shows that when v/c is small, these two terms are equivalent.

We evaluate $\dfrac{v}{c} = \left(\dfrac{1\,000\times 10^3\text{ m/h}}{3.00\times 10^8\text{ m/s}}\right)\left(\dfrac{1\text{ h}}{3\,600\text{ s}}\right) = 9.26\times 10^{-7}$

From Equation 39.7, the dilated time interval is

$$\Delta t = \gamma\Delta t_p = \frac{\Delta t_p}{\sqrt{1-\dfrac{v^2}{c^2}}}$$

Rearranging, our approximation yields

$$\Delta t_p = \left(\sqrt{1-\frac{v^2}{c^2}}\right)\Delta t \approx \left(1-\frac{v^2}{2c^2}\right)\Delta t$$

and $$\Delta t - \Delta t_p = \frac{v^2}{2c^2}\Delta t$$

Substituting,

$$\Delta t - \Delta t_p = \frac{\left(9.26\times 10^{-7}\right)^2}{2}(3\,600\text{ s})$$

Thus, the time lag of the moving clock is

$$\Delta t - \Delta t_p = 1.54\times 10^{-9}\text{ s} = \boxed{1.54\text{ ns}}$$

P39.26 The orbital speed of the Earth is as described by Newton's second law:

$$\sum F = ma: \quad \frac{Gm_S m_E}{r^2} = \frac{m_E v^2}{r}$$

Solving for the speed,

$$v = \sqrt{\frac{Gm_S}{r}} = \sqrt{\frac{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times 10^{30}\ \text{kg}\right)}{1.496\times 10^{11}\ \text{m}}}$$
$$= 2.98\times 10^4\ \text{m/s}$$

The maximum frequency received by the extraterrestrials is

$$f'_{\max} = f\sqrt{\frac{1+v/c}{1-v/c}}$$
$$= \left(57.0\times 10^6\ \text{Hz}\right)\sqrt{\frac{1+\left(2.98\times 10^4\ \text{m/s}\right)/\left(3.00\times 10^8\ \text{m/s}\right)}{1-\left(2.98\times 10^4\ \text{m/s}\right)/\left(3.00\times 10^8\ \text{m/s}\right)}}$$
$$= 57.005\,66\times 10^6\ \text{Hz}$$

The minimum frequency received is

$$f'_{\min} = f\sqrt{\frac{1+v/c}{1-v/c}}$$
$$= \left(57.0\times 10^6\ \text{Hz}\right)\sqrt{\frac{1-\left(2.98\times 10^4\ \text{m/s}\right)/\left(3.00\times 10^8\ \text{m/s}\right)}{1+\left(2.98\times 10^4\ \text{m/s}\right)/\left(3.00\times 10^8\ \text{m/s}\right)}}$$
$$= 56.994\,34\times 10^6\ \text{Hz}$$

The difference, which allows them figure out the speed of our planet, is

$$\left(57.005\,66 - 56.994\,34\right)\times 10^6\ \text{Hz} = \boxed{1.13\times 10^4\ \text{Hz}}$$

Section 39.5 The Lorentz Transformation Equations

P39.27 (a) From the Lorentz transformation, the separations between the blue-light and red-light events are described by

$$\Delta x' = \gamma\left(\Delta x - v\Delta t\right):\ 0 = \gamma\left[2.00\ \text{m} - v\left(8.00\times 10^{-9}\ \text{s}\right)\right]$$

$$v = \frac{2.00\ \text{m}}{8.00\times 10^{-9}\ \text{s}} = \boxed{2.50\times 10^8\ \text{m/s}}$$

so

$$\gamma = \frac{1}{\sqrt{1-\left(2.50\times10^{8}\text{ m/s}\right)^{2}/\left(3.00\times10^{8}\text{ m/s}\right)^{2}}} = 1.81$$

(b) Again from the Lorentz transformation, $x' = \gamma(x - vt)$:

$$x' = 1.81\left[3.00\text{ m} - \left(2.50\times10^{8}\text{ m/s}\right)\left(1.00\times10^{-9}\text{ s}\right)\right] = \boxed{4.98\text{ m}}$$

(c) $t' = \gamma\left(t - \frac{v}{c^2}x\right)$:

$$t' = 1.81\left[1.00\times10^{-9}\text{ s} - \frac{\left(2.50\times10^{8}\text{ m/s}\right)}{\left(3.00\times10^{8}\text{ m/s}\right)^{2}}\left(3.00\text{ m}\right)\right]$$

$$t' = \boxed{-1.33\times10^{-8}\text{ s}}$$

P39.28 Let Shannon be fixed in reference from S and see the two light-emission events with coordinates $x_1 = 0$, $t_1 = 0$, $x_2 = 0$, $t_2 = 3.00\ \mu$s. Let Kimmie be fixed in reference frame S′ and give the events coordinate $x_1' = 0$, $t_1' = 0$, $t_2' = 9.00\ \mu$s.

(a) Then we have

$$t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right)$$

$$9.00\ \mu\text{s} = \frac{1}{\sqrt{1 - v^2/c^2}}\left(3.00\ \mu\text{s} - 0\right)$$

$$\sqrt{1 - \frac{v^2}{c^2}} = \frac{1}{3}$$

$$\boxed{v = 0.943c}$$

(b) The coordinate separation of the events is

$$\Delta x' = x_2' - x_1' = \gamma\left[\left(x_2 - x_1\right) - v\left(t_2 - t_1\right)\right]$$

$$= 3\left[0 - (0.943c)\left(3.00\times10^{-6}\text{ s}\right)\right]\left(\frac{3.00\times10^{8}\text{ m/s}}{c}\right)$$

$$= -2.55\times10^{3}\text{ m}$$

$$|\Delta x'| = \boxed{2.55\times10^{3}\text{ m}}$$

The later pulse is to the left of the origin.

P39.29 The rod's length perpendicular to the motion is the same in both the proper frame of the rod and in the frame in which the rod is moving–our frame:

$$\ell_y = \ell \sin\theta = \ell_{Py}$$

where ℓ_{Py} is the y component of the proper length.

We are given: $\ell = 2.00\text{ m}$, and $\theta = 30.0°$, both measured in our reference frame. Also,

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-0.995^2}} \approx 10.0$$

As observed in our frame,

$$\ell_x = \ell\cos\theta = (2.00\text{ m})\cos 30.0° = 1.73\text{ m}$$

and $$\ell_y = \ell\sin\theta = (2.00\text{ m})\sin 30.0° = 1.00\text{ m}$$

ℓ_{Px} is a proper length, related to ℓ_x by $\ell_x = \dfrac{\ell_{Px}}{\gamma}$.

Therefore, $$\ell_{Px} = 10.0\ell_x = 17.3\text{ m}$$

and $$\ell_{Py} = \ell_y = 1.00\text{ m}$$

(a) $$\ell_P = \sqrt{(\ell_{Px})^2 + (\ell_{Py})^2} = \sqrt{\left(\frac{\ell_x}{\gamma}\right)^2 + (\ell_y)^2} = \boxed{17.4\text{ m}}$$

(b) In the proper frame,

$$\theta_2 = \tan^{-1}\left(\frac{\ell_{Py}}{\ell_{Px}}\right) = \tan^{-1}\left(\frac{\ell_y}{\gamma\ell_x}\right) = \tan^{-1}\left(\frac{\tan 30.0°}{\gamma}\right) = \boxed{3.30°}$$

***P39.30** (a) $L_0^2 = L_{0x}^2 + L_{0y}^2$ and $L^2 = L_x^2 + L_y^2$.

Since the motion is in the x direction, the length of the rod in the y direction does not change: $L_y = L_{0y} = L_0\sin\theta_0$ and

$$L_x = L_{0x}\sqrt{1-\left(\frac{v}{c}\right)^2} = (L_0\cos\theta_0)\sqrt{1-\left(\frac{v}{c}\right)^2}$$

Thus,

$$L^2 = L_0^2\cos^2\theta_0\left[1-\left(\frac{v}{c}\right)^2\right] + L_0^2\sin^2\theta_0 = L_0^2\left[1-\left(\frac{v}{c}\right)^2\cos^2\theta_0\right]$$

or $$\boxed{L = L_0\left[1-\left(\frac{v}{c}\right)^2\cos^2\theta_0\right]^{1/2}}.$$

(b) $$\tan\theta = \frac{L_y}{L_x} = \frac{L_{0y}}{L_{0x}\sqrt{1-(v/c)^2}} = \boxed{\gamma\tan\theta_0}$$

P39.31 We use the Lorentz transformation equations 39.11. In frame S, we may take $t = 0$ for both events, so the coordinates of event A are (x = 50.0 m, $y = 0$, $z = 0$, $t = 0$), and the coordinates of event B are (x = 150 m, $y = 0$, $z = 0$, $t = 0$). The time coordinates of event A in frame S′ are

$$\begin{aligned} t'_A &= \gamma\left(t_A - \frac{v}{c^2}x_A\right) \\ &= \frac{1}{\sqrt{1-(0.800)^2}}\left(0 - \frac{0.800c}{c^2}(150\text{ m})\right) \\ &= 1.667\left(-\frac{120\text{ m}}{3.00\times10^8\text{ m/s}}\right) \\ &= -6.67\times10^{-7}\text{ s} \end{aligned}$$

The time coordinates of event B in frame S′ are

$$\begin{aligned} t'_B &= \gamma\left(t_B - \frac{v}{c^2}x_B\right) \\ &= \frac{1}{\sqrt{1-(0.800)^2}}\left(0 - \frac{0.800c}{c^2}(50.0\text{ m})\right) \\ &= 1.667\left(-\frac{40.0\text{ m}}{3.00\times10^8\text{ m/s}}\right) \\ &= -2.22\times10^{-7}\text{ s} \end{aligned}$$

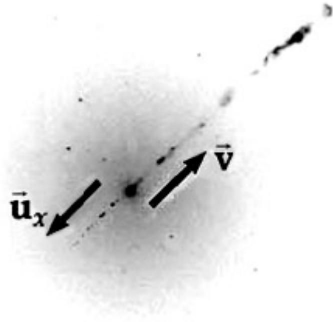
We see that event B occurred earlier. The time elapsed between the events was

$$\Delta t' = t'_A - t'_B = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right) = -\gamma\frac{v}{c^2}\Delta x$$

$$= -1.667\left(\frac{80.0\text{ m}}{3.00\times10^8\text{ m/s}}\right) = -4.44\times10^{-7}\text{ s} = \boxed{444\text{ ns}}$$

Section 39.6 The Lorentz Velocity Transformation Equations

P39.32 Take the galaxy as the unmoving frame. Arbitrarily define the jet moving upward to be the object, and the jet moving downward to be the "moving" frame:



ANS. FIG. P39.32

u'_x = velocity of other jet in frame of jet

u_x = velocity of other jet in frame of galaxy center = $0.750c$

v = speed of galaxy center in frame of jet = $-0.750c$

From Equation 39.16, the speed of the upward-moving jet as measured from the downward-moving jet is

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.750c - (-0.750c)}{1 - (0.750c)(-0.750c)/c^2} = \frac{1.50c}{1 + 0.750^2}$$
$$= \boxed{0.960c}$$

P39.33 The question is equivalent to asking for the speed of the patrol craft in the frame of the enemy craft.

u'_x = velocity of patrol craft in frame of enemy craft

u_x = velocity of patrol craft in frame of Earth

v = speed of Earth in frame of enemy craft

From Equation 39.16,

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.900c - 0.800c}{1 - (0.900)(0.800)} = \boxed{0.357c}$$

***P39.34** Let frame S be the Earth frame of reference. Then $v = -0.700c$.

The components of the velocity of the first spacecraft are

$$u_x = (0.600c)\cos 50.0° = 0.386c$$

and $u_y = (0.600c)\sin 50.0° = 0.460c.$

As measured from the S' frame of the second spacecraft,

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} = \frac{0.386c - (-0.700c)}{1 - \left[(0.386c)(-0.700c)/c^2\right]}$$
$$= \frac{1.086c}{1.27} = 0.855c$$

and

$$u'_y = \frac{u_y}{\gamma\left(1 - u_x v/c^2\right)} = \frac{0.460c\sqrt{1-(0.700)^2}}{1-(0.386)(-0.700)}$$
$$= \frac{0.460c(0.714)}{1.27} = 0.258c$$

The magnitude of $\vec{\mathbf{u}}'$ is $\sqrt{(0.855c)^2 + (0.285c)^2} = \boxed{0.893c}$

and its direction is at $\tan^{-1}\left(\dfrac{0.258c}{0.855c}\right) = \boxed{16.8° \text{ above the } x' \text{ axis}}$.

***P39.35** Taking to the right as positive, it is given that the velocity of the rocket relative to observer *A* is $v_{RA} = +0.92c$. If observer *B* observes the rocket to have a velocity $v_{RB} = -0.95c$, the velocity of observer *B* relative to the rocket is $v_{BR} = +0.95c$. The relativistic velocity addition relation then gives the velocity of *B* relative to the stationary observer *A* as

$$v_{BA} = \frac{v_{BR} + v_{RA}}{1 + \dfrac{v_{BR}v_{RA}}{c^2}} = \frac{+0.95c + 0.92c}{1 + \dfrac{(0.95c)(0.92c)}{c^2}} = +0.998c$$

or $\boxed{0.998c \text{ toward the right}}$

Section 39.7 Relativistic Linear Momentum

P39.36 (a) $p = \gamma m u$; for an electron moving at 0.010 0*c*,

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}} = \frac{1}{\sqrt{1-(0.010\,0)^2}} = 1.000\,05 \approx 1.00$$

Thus, $p = 1.00\left(9.11\times10^{-31}\text{ kg}\right)(0.010\,0)\left(3.00\times10^{8}\text{ m/s}\right)$

$$p = \boxed{2.73\times10^{-24}\text{ kg}\cdot\text{m/s}}$$

(b) Following the same steps as used in part (a), we find at 0.500*c*, $\gamma = 1.15$ and

$$p = \boxed{1.58\times10^{-22}\text{ kg}\cdot\text{m/s}}$$

(c) At 0.900*c*, $\gamma = 2.29$ and

$$p = \boxed{5.64\times10^{-22}\text{ kg}\cdot\text{m/s}}$$

P39.37 (a) The momentum condition

$$p = \gamma m u = 3mu \quad \rightarrow \quad \gamma = 3$$

From the definition of γ,

$$\gamma = \frac{1}{\sqrt{1-(u/c)^2}} \quad \rightarrow \quad u = c\sqrt{1-\frac{1}{\gamma^2}}$$

$$u = c\sqrt{1-\frac{1}{3^2}} = c\sqrt{\frac{8}{9}} = \boxed{\frac{2\sqrt{2}}{3}c = 0.943c = 2.83\times10^8 \text{ m/s}}$$

(b) From part (a), we see the mass of the particle drops out. $\boxed{\text{The result would be the same}}$.

***P39.38** From the definition of relativistic linear momentum,

$$p = \frac{mu}{\sqrt{1-(u/c)^2}}$$

we obtain

$$1-\frac{u^2}{c^2} = \frac{m^2u^2}{p^2}$$

which gives:

$$1 = u^2\left(\frac{m^2}{p^2}+\frac{1}{c^2}\right)$$

or $c^2 = u^2\left(\dfrac{m^2c^2}{p^2}+1\right)$ and $\boxed{u = \dfrac{c}{\sqrt{(m^2c^2/p^2)+1}}}$.

***P39.39** (a) Classically,

$$p = mv = m(0.990c) = (1.67\times10^{-27}\text{ kg})(0.990)(3.00\times10^8\text{ m/s})$$

$$= \boxed{4.96\times10^{-19}\text{ kg}\cdot\text{m/s}}$$

(b) By relativistic calculations,

$$p = \frac{mu}{\sqrt{1-(u/c)^2}} = \frac{m(0.990c)}{\sqrt{1-(0.990)^2}}$$

$$= \frac{(1.67\times10^{-27}\text{ kg})(0.990)(3.00\times10^8\text{ m/s})}{\sqrt{1-(0.990)^2}}$$

$$= \boxed{3.52\times10^{-18}\text{ kg}\cdot\text{m/s}}$$

(c) $\boxed{\text{No}}$, neglecting relativistic effects at such speeds would introduce an approximate 86% error in the result.

P39.40 We can express the proportion relating the speeding fine to the excess momentum as $\frac{F}{\$80.0} = \frac{\left(p_u - p_{90\text{ km/h}}\right)}{\left(p_{190\text{ km/h}} - p_{90\text{ km/h}}\right)}$, where F is the fine, $p_u = \frac{mu}{\sqrt{1-(u/c)^2}}$ is the magnitude of the vehicle's momentum at speed u, and $c = 1.08 \times 10^9$ km/h. After substitution of the expression for momentum, the proportion becomes

$$\frac{F}{\$80.0} = \frac{\left[\frac{mu}{\sqrt{1-(u/c)^2}} - \frac{m(90.0\text{ km/h})}{\sqrt{1-(90.0\text{ km/h}/c)^2}}\right]}{\left[\frac{m(190.0\text{ km/h})}{\sqrt{1-(190.0\text{ km/h}/c)^2}} - \frac{m(90.0\text{ km/h})}{\sqrt{1-(90.0\text{ km/h}/c)^2}}\right]}$$

$$\approx \frac{\frac{u}{\sqrt{1-(u/c)^2}} - (90.0\text{ km/h})}{100.0\text{ km/h}}$$

(a) For $u = 1\,090$ km/h,

$$\frac{F}{\$80.0} \approx \frac{\frac{(1\,090\text{ km/h})}{\sqrt{1-(1\,090\text{ km/h}/1.08\times10^9\text{ km/h})^2}} - (90.0\text{ km/h})}{100.0\text{ km/h}}$$

$$\approx \frac{(1\,090\text{ km/h})-(90.0\text{ km/h})}{100.0\text{ km/h}} = \frac{1\,000\text{ km/h}}{100.0\text{ km/h}} = 10$$

$$F = \boxed{\$800}$$

(b) For $u = 1\,000\,000\,090$ km/h,

$$\frac{F}{\$80.0} \approx \left(\frac{1}{100\text{ km/h}}\right)$$

$$\left[\frac{(1\,000\,000\,090\text{ km/h})}{\sqrt{1-(1\,000\,000\,090\text{ km/h}/1.08\times10^9\text{ km/h})^2}} - (90.0\text{ km/h})\right]$$

$$\frac{F}{\$80.0} \approx \frac{(2.648)(1\,000\,000\,090\text{ km/h})-(90.0\text{ km/h})}{100.0\text{ km/h}}$$

$$F = \boxed{\$2.12\times10^9}$$

P39.41 The ratio of relativistic to classical momentum is

$$\frac{p-mu}{mu}=\frac{\gamma mu-mu}{mu}=\gamma-1$$

From the definition of γ,

$$\gamma-1=\frac{1}{\sqrt{1-(u/c)^2}}-1\approx 1+\frac{1}{2}\left(\frac{u}{c}\right)^2-1=\frac{1}{2}\left(\frac{u}{c}\right)^2$$

The ratio is then

$$\frac{p-mu}{mu}\approx\frac{1}{2}\left(\frac{90.0\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2=\boxed{4.51\times10^{-14}}$$

P39.42 Using the relativistic form, $p=\frac{mu}{\sqrt{1-(u/c)^2}}=\gamma mu,$ we find the difference Δp from the classical momentum, mu:

$$\Delta p=\gamma mu-mu=(\gamma-1)mu$$

(a) The difference is 1.00% when $(\gamma-1)mu=0.010\,0\gamma mu$:

$$\gamma=\frac{1}{0.990}=\frac{1}{\sqrt{1-(u/c)^2}}$$

thus,

$$1-\left(\frac{u}{c}\right)^2=(0.990)^2,\text{ and}\qquad u=\boxed{0.141c}$$

(b) The difference is 10.0% when $(\gamma-1)mu=0.100\gamma mu$:

$$\gamma=\frac{1}{0.900}=\frac{1}{\sqrt{1-(u/c)^2}}$$

thus, $1-\left(\frac{u}{c}\right)^2=(0.900)^2$ and $u=\boxed{0.436c}$

P39.43 Relativistic momentum of the system of fragments must be conserved. For total momentum to be zero after as it was before, we must have, with subscript 2 referring to the heavier fragment, and subscript 1 to the lighter, $p_2=p_1$,

or $$\gamma_2 m_2 u_2=\gamma_1 m_1 u_1=\frac{2.50\times10^{-28}\text{ kg}}{\sqrt{1-(0.893)^2}}\times(0.893c)$$

or $$\frac{\left(1.67\times10^{-27}\text{ kg}\right)u_2}{\sqrt{1-\left(u_2/c\right)^2}}=\left(4.960\times10^{-28}\text{ kg}\right)c$$

Proceeding to solve, we find

$$\left(\frac{1.67\times10^{-27}}{4.960\times10^{-28}}\frac{u_2}{c}\right)^2=1-\frac{u_2^2}{c^2}$$

$$12.3\frac{u_2^2}{c^2}=1\qquad\text{and}\qquad u_2=\boxed{0.285c}$$

Section 39.8 Relativistic Energy

***P39.44** We use the equation $\Delta E=\left(\gamma_1-\gamma_2\right)mc^2$. For an electron, $mc^2=0.511$ MeV.

(a) $$\Delta E=\left(\sqrt{\frac{1}{(1-0.810)}}-\sqrt{\frac{1}{(1-0.250)}}\right)mc^2=\boxed{0.582\text{ MeV}}$$

(b) $$\Delta E=\left(\sqrt{\frac{1}{1-(0.990)^2}}-\sqrt{\frac{1}{1-0.810}}\right)mc^2=\boxed{2.45\text{ MeV}}$$

***P39.45** (a) $K=E-E_R=5E_R$

$$E=6E_R=6\left(9.11\times10^{-31}\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)^2=4.92\times10^{-13}\text{ J}$$

$$=\boxed{3.07\text{ MeV}}$$

(b) $E=\gamma mc^2=\gamma E_R$

Thus, $\gamma=\dfrac{E}{E_R}=6=\dfrac{1}{\sqrt{1-u^2/c^2}}$ which yields $\boxed{u=0.986c}$

P39.46 (a) To find the speed of the protons with $E=\gamma mc^2=400mc^2$, we write

$$\gamma=\frac{1}{\sqrt{1-\left(u/c\right)^2}}\quad\rightarrow\quad u=c\sqrt{1-\frac{1}{\gamma^2}}$$

So, $$u=c\sqrt{1-\frac{1}{\left(400\right)^2}}=\boxed{0.999\,997c}$$

(b) From Example 39.9, for a proton, $mc^2 = 938$ MeV. Then

$$K = (\gamma - 1)mc^2 = 399(938\text{ MeV}) = \boxed{3.74\times 10^5\text{ MeV}}$$

P39.47 At $u = 0.950c$, it will be useful to know the gamma factor:

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = \frac{1}{\sqrt{1 - 0.950^2}} = 3.20$$

(a) The rest energy is

$$\begin{aligned} E_R &= mc^2 = (1.67\times 10^{-27}\text{ kg})(2.998\times 10^8\text{ m/s})^2 \\ &= 1.50\times 10^{-10}\text{ J} \\ &= 1.50\times 10^{-10}\text{ J}\left(\frac{1\text{ eV}}{1.60\times 10^{-19}\text{ J}}\right) = \boxed{938\text{ MeV}} \end{aligned}$$

(We use a value for c accurate to four digits so that we can be sure to get an answer accurate to three digits. Through the rest of the book we will use values for physical constants accurate to four digits or to three, whichever we like. We will still quote answers to three digits, and you can still think of the last digit as uncertain.)

(b) The total energy is

$$E = \gamma mc^2 = \gamma E_R = (3.20)(938\text{ MeV}) = \boxed{3.00\text{ GeV}}$$

(c) The kinetic energy is

$$K = E - E_R = 3.00\text{ GeV} - 938\text{ MeV} = \boxed{2.07\text{ GeV}}$$

P39.48 (a) Using the classical equation,

$$K = \frac{1}{2}mu^2 = \frac{1}{2}(78.0\text{ kg})(1.06\times 10^5\text{ m/s})^2 = \boxed{4.38\times 10^{11}\text{ J}}$$

(b) Using the relativistic equation, $K = \left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)mc^2$:

$$\begin{aligned} K &= \left[\frac{1}{\sqrt{1-\left(\frac{1.06\times 10^5}{2.998\times 10^8}\right)^2}} - 1\right](78.0\text{ kg})(3.00\times 10^8\text{ m/s})^2 \\ &= \boxed{4.38\times 10^{11}\text{J}} \end{aligned}$$

(c) When $\frac{u}{c} \ll 1$, the binomial series expansion gives

$$\left[1-\left(\frac{u}{c}\right)^2\right]^{-1/2} \approx 1+\frac{1}{2}\left(\frac{u}{c}\right)^2$$

Thus, $\left[1-\left(\frac{u}{c}\right)^2\right]^{-1/2} - 1 \approx \frac{1}{2}\left(\frac{u}{c}\right)^2$ and the relativistic expression for kinetic energy becomes $K \approx \frac{1}{2}\left(\frac{u}{c}\right)^2 mc^2 = \frac{1}{2}mu^2$. That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results. In this situation the two kinetic energy values are experimentally indistinguishable. The fastest-moving macroscopic objects launched by human beings move sufficiently slowly compared to light that relativistic corrections to their energy are negligible.

P39.49 The work–kinetic energy theorem is $W = \Delta K = K_f - K_i$, which for relativistic speeds (u comparable to c) is:

$$W = \left(\frac{1}{\sqrt{1-u_f^2/c^2}} - 1\right)mc^2 - \left(\frac{1}{\sqrt{1-u_i^2/c^2}} - 1\right)mc^2$$

or, simplified,

$$W = \left(1/\sqrt{1-u_f^2/c^2} - 1/\sqrt{1-u_i^2/c^2}\right)mc^2$$

From our specialized equation,

(a) $$W = \left(\frac{1}{\sqrt{1-0.750^2}} - \frac{1}{\sqrt{1-0.500^2}}\right) \times \left(1.67 \times 10^{-27}\text{ kg}\right)\left(3.00 \times 10^{8}\text{ m/s}\right)^2$$

$$W = (1.512 - 1.155)\left(1.50 \times 10^{-10}\text{ J}\right) = \boxed{5.37 \times 10^{-11}\text{ J}} = 336\text{ MeV}$$

(b) $$W = \left(\frac{1}{\sqrt{1-0.995^2}} - \frac{1}{\sqrt{1-0.500^2}}\right) \left(1.67 \times 10^{-27}\text{ kg}\right)\left(3.00 \times 10^{8}\text{ m/s}\right)^2$$

$$W = \left(10.01 - 1.155\right)\left(1.50 \times 10^{-10}\text{ J}\right) = \boxed{1.33 \times 10^{-9}\text{ J}} = 8.32\text{ GeV}$$

P39.50 The relativistic kinetic energy of an object of mass *m* and speed *u* is $K_r = \left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$. The classical equation is $K_c = \frac{1}{2}mu^2$. Their ratio is

$$\frac{K_r}{K_c} = \frac{\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2}{\frac{1}{2}mu^2} = \frac{2\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)}{u^2/c^2}$$

$$= 2\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right)\frac{1}{u^2/c^2}$$

$$\frac{K_r}{K_c} = 2\left(\frac{1}{\sqrt{1-(0.100)^2}} - 1\right)\frac{1}{(0.100)^2} = 1.007\ 56$$

For still smaller speeds the agreement will be still better.

P39.51 Given $E = 2mc^2$, where $mc^2 = 938$ MeV from Example 39.9. We use Equation 39.27:

$$E^2 = p^2c^2 + (mc^2)^2$$

$$(2mc^2)^2 = p^2c^2 + (mc^2)^2$$

$$4(mc^2)^2 = p^2c^2 + (mc^2)^2 \quad \rightarrow \quad p^2c^2 = 3(mc^2)^2$$

Solving for the momentum then gives

$$p = \sqrt{3}\frac{(mc^2)}{c} = \sqrt{3}\frac{(938\ \text{MeV})}{c} = \boxed{1.62 \times 10^3\ \text{MeV}/c}$$

P39.52 (a) $E = \gamma mc^2 = 20.0$ GeV with $mc^2 = 0.511$ MeV for electrons.

Thus, $\gamma = \frac{20.0 \times 10^9\ \text{eV}}{0.511 \times 10^6\ \text{eV}} = \boxed{3.91 \times 10^4}$.

(b) $\gamma = \frac{1}{\sqrt{1-(u/c)^2}} \quad \rightarrow \quad u = c\sqrt{1 - \frac{1}{\gamma^2}} = \boxed{0.999\ 999\ 999\ 7c}$

(c) $L = L_p\sqrt{1 - \left(\frac{u}{c}\right)^2} = \frac{L_p}{\gamma} = \frac{3.00 \times 10^3\ \text{m}}{3.91 \times 10^4} = 7.67 \times 10^{-2}\ \text{m} = \boxed{7.67\ \text{cm}}$

P39.53 (a) $E = 2.86 \times 10^5$ J leaves the system, so the final mass is $\boxed{\text{smaller}}$.

(b) The mass-energy relation says that $E = mc^2$. Therefore,

$$m = \frac{E}{c^2} = \frac{2.86 \times 10^5 \text{ J}}{\left(3.00 \times 10^8 \text{ m/s}\right)^2} = \boxed{3.18 \times 10^{-12} \text{ kg}}$$

(c) $\boxed{\text{It is too small a fraction of 9.00 g to be measured}}$.

P39.54 The loss of mass in the nuclear reactor is

$$\begin{aligned}\Delta m &= \frac{E}{c^2} = \frac{P\,\Delta t}{c^2} \\ &= \frac{0.800\left(1.00 \times 10^9 \text{ J/s}\right)\left(3.00 \text{ yr}\right)\left(3.16 \times 10^7 \text{ s/yr}\right)}{\left(3.00 \times 10^8 \text{ m/s}\right)^2} \\ &= \boxed{0.842 \text{ kg}}\end{aligned}$$

P39.55 The power output of the Sun is

$$P = \frac{dE}{dt} = \frac{d\left(mc^2\right)}{dt} = c^2\frac{dm}{dt} = 3.85 \times 10^{26} \text{ W}$$

Thus, $$\frac{dm}{dt} = \frac{3.85 \times 10^{26} \text{ J/s}}{\left(3.00 \times 10^8 \text{ m/s}\right)^2} = \boxed{4.28 \times 10^9 \text{ kg/s}}$$

P39.56 Total energy is conserved. The photon must have enough energy to be able to create an electron and a positron, both having the same rest mass:

$$E_\gamma \geq 2m_e c^2 = 1.02 \text{ MeV} \quad \rightarrow \quad E_\gamma \geq \boxed{1.02 \text{ MeV}}$$

P39.57 We use Equation 39.23 for relativistic kinetic energy.

(a) The change in kinetic energy of the spaceship is the minimum energy required to accelerate the spaceship. From Equation 39.23, relativistic kinetic energy is given by

$$K = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1 - u^2/c^2}} - 1\right)mc^2$$

The change in kinetic energy is then

$$\begin{aligned}\Delta K &= \left(\gamma_f - 1\right)mc^2 - \left(\gamma_i - 1\right)mc^2 = \left(\gamma_f - \gamma_i\right)mc^2\\ &= \left(\frac{1}{\sqrt{1-u_f^2/c^2}} - \frac{1}{\sqrt{1-u_i^2/c^2}}\right)mc^2\\ &= \left(\frac{1}{\sqrt{1-(0.700)^2}} - \frac{1}{\sqrt{1-0}}\right)mc^2\\ &= (1.40-1)\left(2.40\times10^6\ \text{kg}\right)\left(2.998\times10^8\ \text{m/s}\right)^2\\ &= \boxed{8.63\times10^{22}\ \text{J}}\end{aligned}$$

(b) We use Einstein's famous mass-energy relation, and equate the rest energy of the fuel to the change in kinetic energy of the spacecraft:

$$E = mc^2 = \Delta K$$

The required mass of fuel is then

$$m = \frac{\Delta K}{c^2} = \left(\frac{1}{\sqrt{1-(0.700)^2}} - 1\right)\left(2.40\times10^6\ \text{kg}\right) = \boxed{9.61\times10^5\ \text{kg}}$$

P39.58 We are told to start from $E = \gamma mc^2$ and $p = \gamma mu$. Squaring both equations gives

$$E^2 = \left(\gamma mc^2\right)^2 \qquad \text{and} \qquad p^2 = \left(\gamma mu\right)^2$$

We choose to multiply the second equation by c^2 and subtract it from the first:

$$E^2 - p^2c^2 = \left(\gamma mc^2\right)^2 - (\gamma mu)^2c^2$$

We factor to obtain

$$E^2 - p^2c^2 = \gamma^2\left[\left(mc^2\right)\left(mc^2\right) - \left(mc^2\right)\left(mu^2\right)\right]$$

Extracting the (mc^2) factors gives

$$E^2 - p^2c^2 = \gamma^2\left(mc^2\right)^2\left(1 - \frac{u^2}{c^2}\right)$$

We substitute the definition of γ:

$$E^2 - p^2c^2 = \left(1 - \frac{u^2}{c^2}\right)^{-1}\left(mc^2\right)^2\left(1 - \frac{u^2}{c^2}\right)$$

The γ^2 factors divide out, leaving

$$E^2 - p^2c^2 = \left(mc^2\right)^2$$

P39.59 From $K = (\gamma - 1)mc^2 = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2$, we have

$$\frac{K}{mc^2} + 1 = \frac{1}{\sqrt{1-u^2/c^2}} = \frac{K + mc^2}{mc^2}$$

$$1 - \frac{u^2}{c^2} = \frac{m^2c^4}{\left(K + mc^2\right)^2}$$

$$\frac{u^2}{c^2} = 1 - \frac{\left(mc^2\right)^2}{\left(K + mc^2\right)^2}$$

$$u = c\left[1 - \left(\frac{mc^2}{K + mc^2}\right)^2\right]^{1/2}$$

(a) Electron: $u = c\left[1 - \left(\dfrac{0.511}{2.511}\right)^2\right]^{1/2} = \boxed{0.979c}$

(b) Proton: $u = c\left[1 - \left(\dfrac{938}{940}\right)^2\right]^{1/2} = \boxed{0.065\,2c}$

(c) $\dfrac{u_{\text{electron}}}{u_{\text{proton}}} = \dfrac{0.979c}{0.065\,2c} = \boxed{15.0}$

In this case the electron is moving relativistically, but the classical expression $\frac{1}{2}mv^2$ is accurate to two digits for the proton.

(d) Electron: $u = c\left[1 - \left(\dfrac{0.511}{2\,000.511}\right)^2\right]^{1/2} = \boxed{0.999\,999\,97c}$

Proton: $u = c\left[1 - \left(\dfrac{938}{2\ 938}\right)^2\right]^{1/2} = \boxed{0.948c}$

Then,

$$\frac{u_{\text{electron}}}{u_{\text{proton}}} = \frac{c\left[1-\left(\dfrac{0.511}{2\,000.511}\right)^2\right]^{1/2}}{c\left[1-\left(\dfrac{938}{2\;938}\right)^2\right]^{1/2}} = \boxed{1.06}$$

As the kinetic energies of both particles become large, their speeds approach c. By contrast, classically the speed would become large without any finite limit.

P39.60 The kinetic energy of the car is given by

$$K = (\gamma - 1)mc^2 = \left(\left(1 - u^2/c^2\right)^{-1/2} - 1\right)mc^2$$

We use the series expansion from Appendix B.5:

$$K = mc^2\left[1 + \left(-\frac{1}{2}\right)(-u^2/c^2) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{2}(-u^2/c^2)^2 + \cdots - 1\right]$$

$$K = \frac{1}{2}mu^2 + \frac{3}{8}m\frac{u^4}{c^2} + \cdots$$

The actual kinetic energy, given by this relativistic equation, is $\boxed{\text{larger}}$ than the classical $\frac{1}{2}mu^2$.

The difference, for m = 1 000 kg and u = 25 m/s, is

$$\frac{3}{8}m\frac{u^4}{c^2} = \frac{3}{8}(1\,000\text{ kg})\frac{(25\text{ m/s})^4}{(3.00\times10^8\text{ m/s})^2} = 1.6\times10^{-9}\text{ J } \boxed{\sim 10^{-9}\text{ J}}$$

P39.61 We use, together, both the energy version and the momentum version of the isolated system model. By conservation of system energy,

$$m_\pi c^2 = \gamma m_\mu c^2 + |p_{\bar{v}}|c$$

By conservation of system momentum:

$$p_{\bar{v}} = -p_\mu = -\gamma m_\mu u$$

Substituting the second equation into the first,

$$m_\pi c^2 = \gamma m_\mu c^2 + \gamma m_\mu uc$$

Simplified, this equation then reads

$$m_\pi = m_\mu(\gamma + \gamma u/c)$$

Substituting the masses,

$$273m_e = (207m_e)(\gamma + \gamma u/c)$$

where the rest energy of an electron is

$$m_ec^2 = 0.511 \text{ MeV}$$

Numerically,

$$\frac{273m_e}{207m_e} = \frac{1 + u/c}{\sqrt{1 - (u/c)^2}} = \sqrt{\frac{1 + u/c}{1 - u/c}}$$

Solving for the muon speed,

$$\frac{u}{c} = \frac{273^2 - 207^2}{273^2 + 207^2} = 0.270$$

Therefore,

$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}} = 1.038\ 5$$

(a) and the muon's kinetic energy is

$$K_\mu = (0.038\ 5)(207 \times 0.511 \text{ MeV}) = \boxed{4.08 \text{ MeV}}$$

(b) The energy of the antineutrino is

$$K_{\bar{\nu}} = (273 \times 0.511 \text{ MeV}) - (207 \times 0.511 \text{ MeV} + 4.08 \text{ MeV})$$
$$= \boxed{29.6 \text{ MeV}}$$

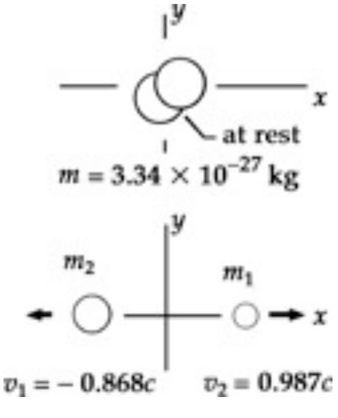
P39.62 (a) The initial system is $\boxed{\text{isolated}}$.

(b) $\boxed{\text{Isolated system: conservation of energy, and isolated system: conservation of momentum.}}$

(c) We must conserve both energy and relativistic momentum of the system of fragments. With subscript 1 referring to the 0.987*c* particle and subscript 2 to the 0.868*c* particle,

$$\gamma_1 = \frac{1}{\sqrt{1-(0.987)^2}} = \boxed{6.22} \text{ and}$$

$$\gamma_2 = \frac{1}{\sqrt{1-(0.868)^2}} = \boxed{2.01}$$



ANS. FIG. P39.62

(d) Conservation of energy gives $E_1 + E_2 = E_{\text{total}}$

which is

$$\gamma_1 m_1 c^2 + \gamma_2 m_2 c^2 = m_{\text{total}} c^2$$

or $\quad 6.22m_1 + 2.01m_2 = 3.34 \times 10^{-27}\ \text{kg}$

This reduces to: $\boxed{3.09m_1 + m_2 = 1.66 \times 10^{-27}\ \text{kg}}$ **[1]**

(e) Since the final momentum of the system must equal zero, $p_1 = p_2$ gives

$$\gamma_1 m_1 u_1 = \gamma_2 m_2 u_2$$

or $\quad (6.22)(0.987c)m_1 = (2.01)(0.868c)m_2$

which becomes $\boxed{m_2 = 3.52m_1}$ **[2]**

(f) Substituting [2] into [1] gives

$$3.09m_1 + 3.52m_1 = 1.66 \times 10^{-27}\ \text{kg}$$

thus, $m_1 = \boxed{2.51 \times 10^{-28}\ \text{kg}}$ and $m_2 = \boxed{8.84 \times 10^{-28}\ \text{kg}}$

P39.63 Let $m = 1.99 \times 10^{-26}$ kg, and $\vec{\mathbf{u}} = u\hat{\mathbf{i}} = 0.500c\hat{\mathbf{i}}$. An isolated system of two particles of mass m and $m' = m/3$ collide with the respective velocities $\vec{\mathbf{u}}$ and $-\vec{\mathbf{u}}$, resulting in a particle with mass M and velocity $\vec{\mathbf{v}}_f = v_f\hat{\mathbf{i}}$. By conservation of the x component of momentum (γmu):

$$\frac{mu}{\sqrt{1-u^2/c^2}} + \frac{m(-u)}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}}$$

$$\frac{2mu}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}} \qquad \textbf{[1]}$$

By conservation of total energy (γmc^2):

$$\frac{mc^2}{\sqrt{1-u^2/c^2}} + \frac{mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}}$$

$$\frac{4mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}} \qquad \textbf{[2]}$$

To start solving, we divide the momentum equation [1] by the energy equation [2], giving $v_f = \dfrac{2u}{4} = \dfrac{u}{2}$.

Then, substituting the value of the final speed back into the energy equation [2], we get

$$\frac{Mc^2}{\sqrt{1-u^2/4c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

$$\frac{2Mc^2}{\sqrt{4-u^2/c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

$$M = \frac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}} = \frac{2\left(1.99\times10^{-26}\text{ kg}\right)\sqrt{4-(0.500)^2}}{3\sqrt{1-(0.500)^2}}$$

$$M = \boxed{2.97\times10^{-26}\text{ kg}}$$

P39.64 (a) By conservation of the *x* component of momentum (γmu):

$$\frac{mu}{\sqrt{1-u^2/c^2}} + \frac{m(-u)}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}}$$

$$\frac{2mu}{3\sqrt{1-u^2/c^2}} = \frac{Mv_f}{\sqrt{1-v_f^2/c^2}} \qquad \textbf{[1]}$$

By conservation of total energy (γmc^2):

$$\frac{mc^2}{\sqrt{1-u^2/c^2}} + \frac{mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}}$$

$$\frac{4mc^2}{3\sqrt{1-u^2/c^2}} = \frac{Mc^2}{\sqrt{1-v_f^2/c^2}} \qquad \textbf{[2]}$$

To start solving, we divide the momentum equation [1] by the energy equation [2], giving $v_f = \frac{2u}{4} = \frac{u}{2}$. Then, substituting the value of the final speed back into the energy equation [2], we get

$$\frac{Mc^2}{\sqrt{1-u^2/4c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

$$\frac{2Mc^2}{\sqrt{4-u^2/c^2}} = \frac{4mc^2}{3\sqrt{1-u^2/c^2}}$$

$$\boxed{M = \frac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}}}$$

(b) As $u \to 0$, $M = \dfrac{2m\sqrt{4-u^2/c^2}}{3\sqrt{1-u^2/c^2}} \to \dfrac{2m\sqrt{4}}{3\sqrt{1}} = \boxed{\dfrac{4m}{3}}$

(c) $\boxed{\text{The answer to part (b) is in agreement with the classical result, which is the arithmetic sum of the masses of the two colliding particles.}}$

Section 39.9 The General Theory of Relativity

P39.65 (a) For the satellite, Newton's second law gives

$$\sum F = ma: \quad \frac{GM_E m}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$$

which gives

$$GM_E T^2 = 4\pi^2 r^3$$

Solving for the orbital radius,

$$r = \left(\frac{GM_E T^2}{4\pi^2}\right)^{1/3}$$

$$r = \left[\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(43\,080\ \text{s}\right)^2}{4\pi^2}\right]^{1/3}$$

$$= \boxed{2.66\times10^7\ \text{m}}$$

(b) $v = \dfrac{2\pi r}{T} = \dfrac{2\pi\left(2.66\times10^7\ \text{m}\right)}{43\,080\ \text{s}} = \boxed{3.87\times10^3\ \text{m/s}}$

(c) From the relationship of frequency and period:

$$f = \frac{1}{T} \quad \to \quad df = -\frac{dT}{T^2} = -f\left(\frac{dT}{T}\right) \quad \to \quad \frac{df}{f} = -\frac{dT}{T}$$

We see the fractional decrease in frequency is equal in magnitude to the fractional change in period.

The small fractional decrease in frequency received is equal in magnitude to the fractional increase in period of the moving oscillator due to time dilation:

$$\frac{df}{f} = -\frac{dT}{T} = -\frac{\gamma\Delta t_P - \Delta t_P}{\Delta t_P} = -(\gamma - 1)$$

$$\frac{df}{f}=-\left(\frac{1}{\sqrt{1-(v/c)^2}}-1\right)=1-\frac{1}{\sqrt{1-(v/c)^2}}$$

$$\approx 1-\left[1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right]=-\frac{1}{2}\left(\frac{v}{c}\right)^2$$

$$=-\frac{1}{2}\left[\left(\frac{3.87\times10^3\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2\right]=\boxed{-8.34\times10^{-11}}$$

(d) The orbit altitude is large compared to the radius of the Earth, so we must use

$$U_g=-\frac{GM_Em}{r}$$

The change in gravitational potential energy is

$$\Delta U_g=-\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)$$

$$\times\left(5.98\times10^{24}\text{ kg}\right)m\left[\frac{1}{2.66\times10^7\text{ m}}-\frac{1}{6.37\times10^6\text{ m}}\right]$$

$$=\left(4.76\times10^7\text{ J/kg}\right)m$$

Then

$$\frac{\Delta f}{f}=\frac{\Delta U_g}{mc^2}=\frac{\left(4.76\times10^7\text{ J/kg}\right)\cancel{m}}{\cancel{m}\left(3.00\times10^8\text{ m/s}\right)^2}=\boxed{+5.29\times10^{-10}}$$

(e) $-8.34\times10^{-11}+5.29\times10^{-10}=\boxed{+4.46\times10^{-10}}$

Additional Problems

P39.66 (a) When $K_e=K_p$, $m_ec^2\left(\gamma_e-1\right)=m_pc^2\left(\gamma_p-1\right)$

In this case, $m_ec^2=0.511\text{ MeV}$, $m_pc^2=938\text{ MeV}$

and $\gamma_e=\left[1-\left(0.750\right)^2\right]^{-1/2}=1.511\,9$

Substituting, $\gamma_p = 1 + \dfrac{m_e c^2(\gamma_e - 1)}{m_p c^2} = 1 + \dfrac{(0.511\text{ MeV})(1.511\,9 - 1)}{938\text{ MeV}}$

$= 1.000\,279$

But $\gamma_p = \dfrac{1}{\left[1 - (u_p/c)^2\right]^{1/2}}$

Therefore, $u_p = c\sqrt{1 - \gamma_p^{-2}} = \boxed{0.023\,6c}$

(b) When $p_e = p_p$, $\gamma_p m_p u_p = \gamma_e m_e u_e$ or $\gamma_p u_p = \dfrac{\gamma_e m_e u_e}{m_p}$

Thus, $\gamma_p u_p = \dfrac{(1.511\,9)(0.511\ \text{MeV}/c^2)(0.750c)}{938\ \text{MeV}/c^2} = 6.177\,2 \times 10^{-4} c$

and $\dfrac{u_p}{c} = 6.177\,2 \times 10^{-4}\sqrt{1 - \left(\dfrac{u_p}{c}\right)^2}$

which yields $u_p = \boxed{6.18 \times 10^{-4} c} = 185\text{ km/s}$

P39.67 The original rest energy of four protons is

$E_R = 4(938.78\text{ MeV}) = 3\,755.12\text{ MeV}$

The energy given off is

$|\Delta E| = (3\,755.12 - 3\,728.4)\text{ MeV} = 26.7\text{ MeV}$

The fractional energy released is

$$\frac{|\Delta E|}{E_R} = \frac{26.7\text{ MeV}}{3\,755\text{ MeV}} \times 100\% = \boxed{0.712\%}$$

P39.68 From the particle under constant speed model, find the travel time for Speedo from Goslo's reference frame:

$$\Delta t = \frac{d}{u} = \frac{2(50\text{ ly})}{0.85c}\left(\frac{c \cdot \text{yr}}{\text{ly}}\right) = 118\text{ yr}$$

Therefore, when Speedo arrives back on Earth, 118 years have passed and Goslo would have to be 158 years old. Furthermore, Speedo will be 102 years old. Perhaps future medical breakthroughs may extend the life expectancy to 158 years and beyond, but that is impossible at present.

***P39.69** (a) Consider the raindrops moving toward the station, at speed v. They receive radio waves with the Doppler-enhanced frequency $f' = f\sqrt{\frac{c+v}{c-v}}$, where $f = 2.85$ GHz. These raindrops reflect the waves at frequency f'. The waves are received by the station with another upward Doppler shift in frequency to

$$f'' = f'\sqrt{\frac{c+v}{c-v}} = f\sqrt{\frac{c+v}{c-v}}\sqrt{\frac{c+v}{c-v}}$$

$$2.85 \times 10^9 \text{ Hz} + 254 \text{ Hz} = 2.85 \times 10^9 \text{ Hz}\left(\frac{c+v}{c-v}\right)$$

$$1 + 8.91 \times 10^{-8} = \frac{c+v}{c-v}$$

$$c + 8.91 \times 10^{-8} c - v - 8.91 \times 10^{-8} v = c + v$$

$$8.91 \times 10^{-8} c = 2.000\,000\,089 v$$

$$v = \left(4.46 \times 10^{-8}\right)\left(3.00 \times 10^8 \text{ m/s}\right) = \boxed{13.4 \text{ m/s}}$$

The same calculation with 254 Hz replaced by –254 Hz applies to the receding raindrops and given the same velocity magnitude. Thus the velocities are 13.4 m/s toward the station and 13.4 m/s away from the station.

(b) Radio waves travel to the rain and back again in $180\ \mu\text{s}$, so the one-way distance is $\frac{1}{2}\left(3.00 \times 10^8 \text{ m/s}\right)\left(180 \times 10^{-6} \text{ s}\right) = 27\,000 \text{ m}$. The frequency shifts indicate the batch of raindrops are whirling around a common center separated by 1° of arc. The diameter of the vortex is

$$s = r\theta = \left(27\,000 \text{ m}\right)(1°)\left(\frac{\pi \text{ rad}}{180°}\right) = 471 \text{ m}$$

Its radius is therefore $\frac{1}{2}(471 \text{ m}) = 236 \text{ m}$ and the angular speed of the rain is $\omega = \frac{v}{r} = \frac{13.4 \text{ m/s}}{236 \text{ m}} = \boxed{0.056\,7 \text{ rad/s}}$. A Doppler weather radar computer performs a calculation like this to detect a "tornado vortex signature."

***P39.70** From energy conservation, we have

$$\frac{\left(1\,400 \text{ kg}\right)c^2}{\sqrt{1-0^2}} + \frac{\left(900 \text{ kg}\right)c^2}{\sqrt{1-0.850^2}} = \frac{Mc^2}{\sqrt{1-v^2/c^2}}$$

$$(3\,108\text{ kg})\sqrt{1-\frac{v^2}{c^2}} = M$$

From momentum conservation, we have

$$0+\frac{(900\text{ kg})(0.850c)}{\sqrt{1-0.850^2}} = \frac{Mv}{\sqrt{1-v^2/c^2}}$$

$$(1\,452\text{ kg})\sqrt{1-\frac{v^2}{c^2}} = \frac{Mv}{c}$$

(a) Dividing the momentum equation by the energy equation gives

$$\frac{v}{c}=\frac{1\,452}{3\,108}=0.467,\quad\text{or}\qquad \boxed{v=0.467c}$$

(b) Now by substitution, $(3\,108\text{ kg})\sqrt{1-0.467^2} = \boxed{M=2.75\times10^3\text{ kg}}$.

***P39.71** (a) Observers on Earth measure the distance to Andromeda to be

$$d=2.00\times10^6\text{ ly}=(2.00\times10^6\text{ ly})c$$

The time for the trip, in Earth's frame of reference, is

$$\Delta t=\gamma\Delta t_p=\frac{30.0\text{ yr}}{\sqrt{1-(v/c)^2}}$$

The required speed is then

$$v=\frac{d}{\Delta t}=\frac{(2.00\times10^6\text{ ly})c}{(30.0\text{ yr})/\sqrt{1-(v/c)^2}}$$

which gives, suppressing units,

$$(1.50\times10^{-5})(v/c)=\sqrt{1-(v/c)^2}$$

Squaring both sides of this equation and solving for *v/c* yields

$$\frac{v}{c}=\frac{1}{\sqrt{1+2.25\times10^{-10}}}$$

Then, the approximation $\frac{1}{\sqrt{1+x}}=1-\frac{x}{2}$ gives

$$\frac{v}{c}=1-\frac{2.25\times10^{-10}}{2}=\boxed{1-1.12\times10^{-10}}$$

(b) Let $\frac{v}{c}=\frac{1}{\sqrt{1+x}}$, where $x = 2.25\times10^{-10}$.. Then,

$$1-\left(\frac{v}{c}\right)^2=1-\frac{1}{1+x}=\frac{x}{1+x}$$

and

$$\gamma=\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}}=\sqrt{\frac{1+x}{x}}=\sqrt{1+\frac{1}{x}}$$

The kinetic energy of the spacecraft is given by

$$KE=(\gamma-1)mc^2=\left(\sqrt{1+\frac{1}{x}}-1\right)mc^2$$

Thus,

$$KE=\left(\sqrt{1+\frac{1}{2.25\times10^{-10}}}-1\right)\left(1.00\times10^6\text{ kg}\right)\left(3.00\times10^8\text{ m/s}\right)^2$$

$$=5.99991\times10^{27}\text{ J}=\boxed{6.00\times10^{27}\text{ J}}$$

(c) The cost of this energy is

$$\text{cost}=KE\times\text{rate}=\left[\left(6.00\times10^{27}\text{ J}\right)\left(\frac{1\text{ kWh}}{3.60\times10^6\text{ J}}\right)\right](\$0.13/\text{kWh})$$

$$=\boxed{\$2.17\times10^{20}}$$

***P39.72** In this case, the proper time is T_0 (the time measured by the students on a clock at rest relative to them). The dilated time measured by the professor is:

$$\Delta t=\gamma T_0$$

where $\Delta t=T+t$. Here *T* is the time she waits before sending a signal and *t* is the time required for the signal to reach the students. Thus, we have:

$$T+t=\gamma T_0 \qquad \textbf{[1]}$$

To determine the travel time *t*, realize that the distance the students will have moved beyond the professor before the signal reaches them is:

$$d=v(T+t)$$

The time required for the signal to travel this distance is:

$$t=\frac{d}{c}=\left(\frac{v}{c}\right)(T+t)$$

Solving for *t* gives:

$$t = \frac{(v/c)T}{1-(v/c)}$$

Substituting this into equation [1] yields:

$$T + \frac{(v/c)T}{1-(v/c)} = \gamma T_0$$

or $$\frac{T}{1-v/c} = \gamma T_0 .$$

Then

$$T = T_0 \frac{1-(v/c)}{\sqrt{1-(v^2/c^2)}} = T_0 \frac{1-(v/c)}{\sqrt{[1+(v/c)][1-(v/c)]}}$$

$$= \boxed{T_0 \sqrt{\frac{1-(v/c)}{1+(v/c)}}}$$

***P39.73** (a) The proper lifetime is measured in the ship's reference frame, and Earth-based observers measure a dilated lifetime of

$$\Delta t = \gamma \Delta t_p = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \frac{15.0 \text{ yr}}{\sqrt{1-(0.700)^2}} = \boxed{21.0 \text{ yr}}$$

(b) As measured by mission control, the distance to the ship is

$$d = v\Delta t = (0.700c)(21.0 \text{ yr}) = [(0.700)(1.00 \text{ ly/yr})](21.0 \text{ yr})$$
$$= \boxed{14.7 \text{ ly}}$$

(c) Looking out the rear window, the astronauts see Earth recede at a rate of $v = 0.700c$. The distance it has receded, as measured by the astronauts, when the batteries fail is

$$d = v(\Delta t_p) = (0.700c)(15.0 \text{ yr}) = [(0.700)(1.00 \text{ ly/yr})](15.0 \text{ yr})$$
$$= \boxed{10.5 \text{ ly}}$$

(d) Mission control gets signals for 21.0 yr while the battery is operating and then for 14.7 yr after the battery stops powering the transmitter, 14.7 ly away. The total time that signals are received is $21.0 \text{ yr} + 14.7 \text{ yr} = \boxed{35.7 \text{ yr}}$.

P39.74 (a) We let *H* represent K/mc^2. Then,

$$H + 1 = \frac{1}{\sqrt{1-u^2/c^2}}$$

so $1-u^2/c^2=\dfrac{1}{H^2+2H+1}$

Solving,

$$\frac{u^2}{c^2}=1-\frac{1}{H^2+2H+1}=\frac{H^2+2H}{H^2+2H+1}$$

and $\boxed{u=c\left(\dfrac{H^2+2H}{H^2+2H+1}\right)^{1/2}}$

(b) $\boxed{u \text{ goes to } 0 \text{ as } K \text{ goes to } 0.}$

(c) $\boxed{u \text{ approaches } c \text{ as } K \text{ increases without limit.}}$

(d) The acceleration is given by

$$a=\frac{du}{dt}=\frac{d}{dt}\left[c\left(\frac{H^2+2H}{H^2+2H+1}\right)^{1/2}\right]$$

$$a=c\frac{1}{2}\left(\frac{H^2+2H}{H^2+2H+1}\right)^{-1/2}$$

$$\times\left(\frac{\left[H^2+2H+1\right][2H+2]-\left[H^2+2H\right][2H+2]}{\left[H^2+2H+1\right]^2}\right)$$

$$\times\frac{d(K/mc^2)}{dt}$$

$$a=c\left(\frac{H^2+2H+1}{H^2+2H}\right)^{1/2}\left(\frac{H+1}{[H+1]^4}\right)\frac{P}{mc^2}$$

$$=\boxed{\frac{P}{mcH^{1/2}(H+2)^{1/2}(H+1)^2}}$$

where $P=\dfrac{dK}{dt}$.

(e) When *H* is small ($H\ll 1$), we have approximately

$$a=\frac{P}{mcH^{1/2}(2)^{1/2}(1)^2}=\frac{P}{mcH^{1/2}2^{1/2}}=\frac{P}{mc\left(\dfrac{K}{mc^2}\right)^{1/2}2^{1/2}}$$

$$=\frac{P}{(2mK)^{1/2}}$$

in agreement with the nonrelativistic case.

(f) When *H* is large the acceleration approaches

$$a = \frac{P}{mcH^{1/2}(H+2)^{1/2}(H+1)^2} \to \frac{P}{mcH^{1/2}(H)^{1/2}(H)^2} = \frac{P}{mcH^3}$$

$$= \frac{P}{mc\left(\dfrac{K}{mc^2}\right)^3} = \frac{m^2c^5P}{K^3}$$

(g) $\boxed{\text{As energy is steadily imparted to the particle, the particle's acceleration decreases. It decreases steeply, proportionally to } 1/K^3 \text{ at high energy. In this way the particle's speed cannot reach or surpass a certain upper limit, which is the speed of light in vacuum.}}$

***P39.75** (a) From Problem 71,

$$\frac{v}{c} = \frac{1}{\sqrt{1+2.25\times10^{-10}}}$$

and

$$\gamma = \frac{1}{\sqrt{1-\left(\dfrac{1}{\sqrt{1+2.25\times10^{-10}}}\right)^2}} = \frac{1}{\sqrt{1-\left(\dfrac{1}{1+2.25\times10^{-10}}\right)}}$$

$$= \boxed{6.67\times10^4}$$

(b) The astronaut's speed, from Problem 71, is

$$v = \frac{1}{\sqrt{1+2.25\times10^{-10}}}c$$

The time difference between the astronaut's trip and that of the beam of light is then

$$\Delta t = \frac{d}{v} - \frac{d}{c} = d\left(\frac{1}{v} - \frac{1}{c}\right) = \frac{d}{c}\left(\frac{c}{v} - 1\right) = \frac{d}{c}\left(\sqrt{1+x} - 1\right) \approx \frac{d}{c}\left(1 + \frac{x}{2} - 1\right)$$

$$= \frac{d}{c}\left(\frac{x}{2}\right)$$

Where $x = 2.25\times10^{-10}$. Substituting numerical values,

$$\Delta t = \frac{(2.00\times10^6\ \text{ly})(9.46\times10^{15}\ \text{m/ly})}{3.00\times10^8\ \text{m/s}}\left(\frac{2.25\times10^{-10}}{2}\right)$$

$$= 7\ 095\ \text{s} = \boxed{1.96\ \text{h}}$$

P39.76 The energy of the first fragment is given by

$$E_1^2 = p_1^2c^2 + \left(m_1c^2\right)^2 = \left(1.75\text{ MeV}\right)^2 + \left(1.00\text{ MeV}\right)^2$$

$$E_1 = 2.02\text{ MeV}$$

For the second,

$$E_2^2 = \left(2.00\text{ MeV}\right)^2 + \left(1.50\text{ MeV}\right)^2$$

$$E_2 = 2.50\text{ MeV}$$

(a) Energy is conserved, so the unstable object had $E = E_1 + E_2 = 4.52\text{ MeV}$. Each component of momentum is conserved, so for the original object

$$p^2 = p_x^2 + p_y^2 = \left(\frac{1.75\text{ MeV}}{c}\right)^2 + \left(\frac{2.00\text{ MeV}}{c}\right)^2$$

Then, using Equation 39.27, we find the mass of the original object:

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

$$\left(4.52\text{ MeV}\right)^2 = \left[\left(1.75\text{ MeV}\right)^2 + \left(2.00\text{ MeV}\right)^2\right] + \left(mc^2\right)^2$$

$$\boxed{m = \frac{3.65\text{ MeV}}{c^2}}$$

(b) Now $E = \gamma mc^2$ gives

$$4.52\text{ MeV} = \frac{1}{\sqrt{1 - u^2/c^2}}3.65\text{ MeV}$$

$$1 - \frac{u^2}{c^2} = 0.654 \quad \text{which gives} \quad \boxed{u = 0.589c}$$

P39.77 The relativistic kinetic energy of such a proton is

$$K = \left(\gamma - 1\right)mc^2 = 10^{13}\text{ MeV}$$

Its rest energy is

$$mc^2 = \left(1.67 \times 10^{-27}\text{ kg}\right)\left(2.998 \times 10^8\text{ m/s}\right)^2 \times \left(\frac{1\text{ eV}}{1.60 \times 10^{-19}\text{ kg}\cdot\text{m}^2/\text{s}^2}\right) = 938\text{ MeV}$$

So $\quad 10^{13}\text{ MeV} = (\gamma - 1)(938\text{ MeV})$

and therefore, $\gamma = 1.07 \times 10^{10}$. The proton's speed in the galaxy's reference frame can be found from

$$\gamma = 1/\sqrt{1 - u^2/c^2} \qquad \text{so} \qquad 1 - u^2/c^2 = 8.80 \times 10^{-21}$$

and $$u = c\sqrt{1 - 8.80 \times 10^{-21}} \approx \left(1 - 4.40 \times 10^{-21}\right)c \approx 3.00 \times 10^8 \text{ m/s}$$

The proton's speed is nearly as large as the speed of light. In the galaxy frame, the traversal time is

$$\Delta t = x/u = 10^5 \text{ light years}/c = 10^5 \text{ years}$$

(a) This is dilated from the proper time measured in the proton's frame. The proper time interval is found from $\Delta t = \gamma \Delta t_p$:

$$\Delta t_p = \Delta t/\gamma = 10^5 \text{ yr}/1.07 \times 10^{10} = 9.38 \times 10^{-6} \text{ years} = 296 \text{ s}$$

$$\Delta t \ \boxed{\sim 10^2 \text{ s or } 10^3 \text{ s}}$$

(b) The proton sees the galaxy moving by at a speed nearly equal to c, passing in 296 s:

$$\begin{aligned}\Delta L_{\text{proton frame}} &= u\Delta t_p = \left(3.00 \times 10^8 \text{ m/s}\right)(296 \text{ s}) \\ &= 8.88 \times 10^7 \text{ km} \sim 10^8 \text{ km}\end{aligned}$$

$$\begin{aligned}\Delta L_{\text{proton frame}} &= \left(8.88 \times 10^{10} \text{ m}\right)\left(\frac{1 \text{ ly}}{9.46 \times 10^{15} \text{ m}}\right) \\ &= 9.39 \times 10^{-6} \text{ ly} \ \boxed{\sim 10^{-5} \text{ ly}}\end{aligned}$$

P39.78 Look at the situation from the instructors' viewpoint since they are at rest relative to the clock, and hence measure the proper time. The Earth moves with velocity $v = -0.280c$ relative to the instructors while the students move with a velocity $u' = -0.600c$ relative to Earth. Using the velocity addition equation, the velocity of the students relative to the instructors (and hence the clock) is:

$$u = \frac{v + u'}{1 + vu'/c^2} = \frac{(-0.280c) - (0.600c)}{1 + (-0.280c)(-0.600c)/c^2} = -0.753c$$

(students relative to clock)

(a) With a proper time interval of $\Delta t_p = 50.0$ min, the time interval measured by the students is:

$$\Delta t = \gamma \, \Delta t_p \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1 - (0.753c)^2/c^2}} = 1.52$$

Thus, the students measure the exam to last

$$T = 1.52(50.0\text{ min}) = \boxed{76.0\text{ minutes}}$$

(b) The duration of the exam as measured by observers on Earth is:

$$\Delta t = \gamma\,\Delta t_p \quad \text{with} \quad \gamma = \frac{1}{\sqrt{1-(0.280c)^2/c^2}} \quad \text{so}$$

$$T = 1.04(50.0\text{ min}) = \boxed{52.1\text{ minutes}}$$

P39.79 (a) The speed of light in water is $c/1.33$, so the electron's speed is $1.10c/1.333$. Then

$$\gamma = \frac{1}{\sqrt{1-(1.10/1.333)^2}} = 1.770$$

and the total energy is

$$E = \gamma mc^2 = 1.770(0.511\text{ MeV}) = \boxed{0.905\text{ MeV}}$$

(b) The electron's kinetic energy is

$$K = E - mc^2 = 0.905\text{ MeV} - 0.511\text{ MeV} = \boxed{0.394\text{ MeV}}$$

(c) The electron's momentum is found from

$$pc = \sqrt{E^2-(mc^2)^2} = \sqrt{\gamma^2-1}\;\; mc^2$$
$$= \sqrt{\gamma^2-1}(0.511\text{ MeV}) = 0.747\text{ MeV}$$

and

$$p = \boxed{0.747\frac{\text{MeV}}{c}} = \frac{0.747\times10^6\,(1.602\times10^{-19}\text{ J})}{3.00\times10^8\text{ m/s}}$$
$$= \boxed{3.99\times10^{-22}\text{ kg}\cdot\text{m/s}}$$

(d) From Figure 17.11, the angle between the particle (source of waves) and the shock wave is

$$\sin\theta = v/v_S$$

where v is the wave speed, which is the speed of light in water, and v_S is the source speed. Then

$$\sin\theta = v/v_S = 1/1.10 \quad \rightarrow \quad \theta = \boxed{65.4°}$$

P39.80 (a) From Equation 39.18, the speed of light in the laboratory frame is

$$u = \frac{v + \frac{c}{n}}{1 + \frac{v\left(c/n\right)}{c^2}} = \frac{c\ (1 + nv/c)}{n\ (1 + v/nc)}$$

(b) When v is much less than c we have

$$u = \frac{c}{n}\left(1 + \frac{nv}{c}\right)\left(1 + \frac{v}{nc}\right)^{-1} \approx \frac{c}{n}\left(1 + \frac{nv}{c}\right)\left(1 - \frac{v}{nc}\right)$$

$$\approx \frac{c}{n}\left(1 + \frac{nv}{c} - \frac{v}{nc}\right) = \frac{c}{n} + v - \frac{v}{n^2}$$

(c) If light travels at speed c/n in the water, and the water travels at speed v, then the Galilean velocity transformation Equation 4.20 would indeed give $c/n + v$ for the speed of light in the moving water. The third term $-v/n^2$ does represent a relativistic effect that was observed decades before the Michelson-Morley experiment. It is a piece of twentieth-century physics that dropped into the nineteenth century. We could say that light is intrinsically relativistic.

(d) To take the limit as v approaches c we must go back to $u = \frac{c\ (1 + nv/c)}{n\ (1 + v/nc)}$. As $v \to c$,

$$u \to \frac{c\ (1 + nc/c)}{n\ (1 + c/nc)} = \frac{c(1+n)}{n+1} = \boxed{c}$$

P39.81 (a) Assuming the Sun-mass system is isolated, the energy (work) required to remove a mass m from the Sun's surface to infinity is equal to the change in potential energy of the system. If the work equals the rest energy mc^2, then

$$W = \Delta E = \Delta K + \Delta U = 0 + \left(U_f - U_i\right)$$

$$mc^2 = 0 - \left(-\frac{GM_s m}{R_g}\right)$$

$$mc^2 = \frac{GM_s m}{R_g} \quad \rightarrow \quad R_g = \frac{GM_s}{c^2}$$

(b) $$R_g = \frac{GM_s}{c^2} = \frac{\left(6.67 \times 10^{-11}\ \text{N.m}^2/\text{kg}^2\right)\left(1.99 \times 10^{30}\ \text{kg}\right)}{\left(3.00 \times 10^8\ \text{m/s}\right)^2}$$

$$R_g = 1.47 \times 10^3\ \text{m} = \boxed{1.47\ \text{km}}$$

P39.82 We find the speed of the electrons after accelerating through a potential difference ΔV from Equation 39.23:

$$K = e\Delta V = (\gamma - 1)mc^2 = \left(\frac{1}{\sqrt{1-(u/c)^2}} - 1\right)mc^2$$

then

$$\frac{1}{\sqrt{1-(u/c)^2}} = \frac{e\Delta V}{mc^2} + 1 = \frac{e\Delta V + mc^2}{mc^2}$$

or

$$1-(u/c)^2 = \left(\frac{mc^2}{e\Delta V + mc^2}\right)^2$$

Solving,

$$\frac{u}{c} = \sqrt{1-\left(\frac{\mathrm{m}}{\mathrm{e\Delta V/c^2 + m}}\right)^2}$$

Substituting numerical values and suppressing units,

$$\frac{u}{c} = \sqrt{1-\left[\frac{(9.11\times 10^{-31}\text{ kg})}{\dfrac{(1.60\times 10^{-19}\text{ C})(8.40\times 10^{4}\text{ V})}{(3.00\times 10^{8}\text{ m/s})^2}+9.11\times 10^{-31}\text{ kg}}\right]^2}$$

$$u = 0.512c$$

Because this speed is more than half the speed of light, there is no way to double its speed, regardless of the increased accelerating voltage. If the accelerating voltage is quadrupled to 336 kV, the speed of the electrons rises to $u = 0.798c$.

P39.83 (a) Take the spaceship as the primed frame, moving toward the right at $v = +0.600c$. Then $u'_x = +0.800c$, and

$$u_x = \frac{u'_x + v}{1+(u'_x v)/c^2} = \frac{0.800c + 0.600c}{1+(0.800)(0.600)} = \boxed{0.946c}$$

(b) $L = \dfrac{L_p}{\gamma}$: $\quad L = (0.200\text{ ly})\sqrt{1-(0.600)^2} = \boxed{0.160\text{ ly}}$

(c) The aliens observe the 0.160-ly interval decreasing because the probe reduces it from one end at $0.800c$ and the Earth reduces it at the other end at $0.600c$.

Thus, $$\text{time} = \frac{0.160 \text{ ly}}{0.800c + 0.600c} = \boxed{0.114 \text{ yr}}$$

(d) In Earth's reference frame, the kinetic energy of the landing craft is

$$K = \left(\frac{1}{\sqrt{1 - u^2/c^2}} - 1 \right) mc^2$$

$$K = \left(\frac{1}{\sqrt{1 - (0.946)^2}} - 1 \right) \left(4.00 \times 10^5 \text{ kg}\right)\left(3.00 \times 10^8 \text{ m/s}\right)^2$$

$$= \boxed{7.50 \times 10^{22} \text{ J}}$$

P39.84 (a) Take m = 1.00 kg. The classical kinetic energy is

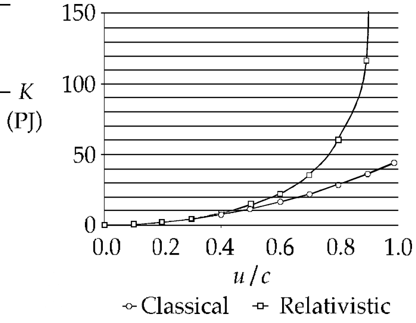
$$K_c = \frac{1}{2} mu^2 = \frac{1}{2} mc^2 \left(\frac{u}{c} \right)^2 = \left(4.50 \times 10^{16} \text{ J}\right)\left(\frac{u}{c} \right)^2$$

and the actual kinetic energy is

$$K_r = \left(\frac{1}{\sqrt{1 - (u/c)^2}} - 1 \right) mc^2 = \left(9.00 \times 10^{16} \text{ J}\right)\left(\frac{1}{\sqrt{1 - (u/c)^2}} - 1 \right)$$

Using these expressions, we generate the graph in ANS. GRAPH P39.84.

$\frac{u}{c}$	K_c (J)	K_r (J)
0.000	0.000	0.000
0.100	0.045×10^{16}	$0.045\,3 \times 10^{16}$
0.200	0.180×10^{16}	0.186×10^{16}
0.300	0.405×10^{16}	0.435×10^{16}
0.400	0.720×10^{16}	0.820×10^{16}
0.500	1.13×10^{16}	1.39×10^{16}
0.600	1.62×10^{16}	2.25×10^{16}
0.700	2.21×10^{16}	3.60×10^{16}
0.800	2.88×10^{16}	6.00×10^{16}
0.900	3.65×10^{16}	11.6×10^{16}
0.990	4.41×10^{16}	54.8×10^{16}



ANS. GRAPH P39.84

(b) $K_c = 0.990K_r$, when $\frac{1}{2}\left(\frac{u}{c}\right)^2 = 0.990\left[\frac{1}{\sqrt{1-(u/c)^2}} - 1\right]$, yielding

$$u = \boxed{0.115c}$$

(c) Similarly, $K_c = 0.950K_r$ when $u = \boxed{0.257c}$.

(d) $K_c = 0.500K_r$ when $u = \boxed{0.786c}$.

P39.85 Both observers measure the speed of light to be c.

(a) Call the total travel time Δt_S. An observer at rest relative to the mirror sees the light travel a distance $d_1 = d$ from the spacecraft to the mirror, but a distance $d_2 = d - v\Delta t_S$ from the mirror back to the spacecraft because the spacecraft has traveled the distance $v\Delta t_S$ forward. Therefore, the total distance traveled by the light is

$$D = d_1 + d_2$$

$$d + (d - v\Delta t_S) = c\Delta t_S$$

$$\Delta t_S = \frac{2d}{c+v} = \frac{2d}{c + 0.650c} = \frac{2(5.66\times10^{10}\text{ m})}{1.650(3.00\times10^8\text{ m/s})} = \boxed{229\text{ s}}$$

(b) The observer in the spacecraft measures a length-contracted initial distance to the mirror of

$$L = d\sqrt{1-\frac{v^2}{c^2}}$$

and the mirror moving toward the ship at speed v. Consider the motion of the light toward the mirror in time interval Δt_1: light travels toward the mirror at speed c while the mirror travels toward the spacecraft at speed v; together, they travel the distance L:

$$c\Delta t_1 + v\Delta t_1 = L$$

$$\Delta t_1 = \frac{L}{c+v}$$

When light strikes the mirror, it is a distance $L' = L - v\Delta t_1$ from the spacecraft. The light must travel back through this same distance to return to the spacecraft:

$$c\Delta t_2 = L - v\Delta t_1 \quad \rightarrow \quad \Delta t_2 = \frac{L}{c} - \frac{v}{c}\Delta t_1$$

The total travel time is

$$\Delta t_1 + \Delta t_2 = \frac{L}{c+v} + \frac{L}{c} - \frac{v}{c}\Delta t_1 = \frac{L}{c+v} + \frac{L}{c} - \frac{v}{c}\left(\frac{L}{c+v}\right)$$

$$= \frac{Lc + L(c+v) - Lv}{c(c+v)} = \frac{2L\cancel{c}}{\cancel{c}(c+v)} = \frac{2}{(c+v)} d\sqrt{1-\frac{v^2}{c^2}}$$

$$= \frac{2}{(c+v)} d\frac{\sqrt{c^2-v^2}}{c}$$

$$\Delta t_1 + \Delta t_2 = \frac{2d}{c}\sqrt{\frac{c-v}{c+v}} = \frac{2d}{c}\sqrt{\frac{c-0.650c}{c+0.650c}}$$

$$= \frac{2\left(5.66\times 10^{10}\text{ m}\right)}{\left(3.00\times 10^{8}\text{ m/s}\right)}\sqrt{\frac{0.350}{1.650}}$$

$$= \boxed{174\text{ s}}$$

P39.86 Both observers measure the speed of light to be c.

(a) Call the total travel time Δt_S. An observer at rest relative to the mirror sees the light travel a distance $d_1 = d$ from the spacecraft to the mirror, but a distance $d_2 = d - v\Delta t_S$ from the mirror back to the spacecraft because the spacecraft has traveled the distance $v\Delta t_S$ forward. Therefore, the total distance traveled by the light is

$$D = d_1 + d_2$$

$$d + \left(d - v\Delta t_S\right) = c\Delta t_S$$

$$\Delta t_S = \boxed{\frac{2d}{c+v}}$$

(b) The observer in the spacecraft measures a length-contracted initial distance to the mirror of

$$L = d\sqrt{1-\frac{v^2}{c^2}}$$

and the mirror moving toward the ship at speed v. Consider the motion of the light toward the mirror in time interval Δt_1: light travels toward the mirror at speed c while the mirror travels toward the spacecraft at speed v; together, they travel the distance L:

$$c\Delta t_1 + v\Delta t_1 = L$$

$$\Delta t_1 = \frac{L}{c+v}$$

When light strikes the mirror, it is a distance $L' = L - v\Delta t_1$ from the spacecraft. The light must travel back through this same distance to return to the spacecraft:

$$c\Delta t_2 = L - v\Delta t_1 \quad \rightarrow \quad \Delta t_2 = \frac{L}{c} - \frac{v}{c}\Delta t_1$$

The total travel time is

$$\begin{aligned}
\Delta t_1 + \Delta t_2 &= \frac{L}{c+v} + \frac{L}{c} - \frac{v}{c}\Delta t_1 \\
&= \frac{L}{c+v} + \frac{L}{c} - \frac{v}{c}\left(\frac{L}{c+v}\right) \\
&= \frac{Lc + L(c+v) - Lv}{c(c+v)} = \frac{2L\cancel{c}}{\cancel{c}(c+v)} = \frac{2}{(c+v)}d\sqrt{1-\frac{v^2}{c^2}} \\
&= \frac{2}{(c+v)}d\frac{\sqrt{c^2-v^2}}{c} \\
\Delta t_1 + \Delta t_2 &= \boxed{\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}
\end{aligned}$$

P39.87 Since the total momentum is zero before decay, it is necessary that after the decay

$$p_{\text{nucleus}} = p_{\text{photon}} = \frac{E_\gamma}{c} = \frac{14.0\text{ keV}}{c}$$

Also, for the recoiling nucleus, $E^2 = p^2c^2 + \left(mc^2\right)^2$ with

$$Mc^2 = 8.60\times 10^{-9}\text{ J} = 5.38\times 10^{10}\text{ eV} = 5.38\times 10^{7}\text{ keV}$$

Thus, $\left(Mc^2 + K\right)^2 = \left(14.0\text{ keV}\right)^2 + \left(Mc^2\right)^2$

or

$$\left(1+\frac{K}{Mc^2}\right)^2 = \left(\frac{14.0\text{ keV}}{Mc^2}\right)^2 + 1$$

Because the term $\left(\frac{14.0\text{ keV}}{Mc^2}\right)^2 \ll 1$, evaluating $\left(\frac{14.0\text{ keV}}{Mc^2}\right)^2 + 1$ on a calculator gives 1.

We need to expand $\left(\dfrac{14.0\text{ keV}}{Mc^2}\right)^2 + 1$ using the Binomial Theorem:

$$1+\frac{K}{Mc^2}=\sqrt{1+\left(\frac{14.0\text{ keV}}{Mc^2}\right)^2}\approx 1+\frac{1}{2}\left(\frac{14.0\text{ keV}}{Mc^2}\right)^2$$

$$K\approx\frac{(14.0\text{ keV})^2}{2Mc^2}=\frac{\left(14.0\times10^3\text{ eV}\right)^2}{2\left(53.8\times10^9\text{ eV}\right)}=\boxed{1.82\times10^{-3}\text{ eV}}$$

Challenge Problems

P39.88 (a) At any speed, the momentum of the particle is given by

$$p=\gamma mu=\frac{mu}{\sqrt{1-(u/c)^2}}$$

With Newton's law expressed as $F=qE=\dfrac{dp}{dt}$, we have

$$qE=\frac{d}{dt}\left[mu\left(1-\frac{u^2}{c^2}\right)^{-1/2}\right]$$

$$qE=m\left(1-\frac{u^2}{c^2}\right)^{-1/2}\frac{du}{dt}+\frac{1}{2}mu\left(1-\frac{u^2}{c^2}\right)^{-3/2}\left(\frac{2u}{c^2}\right)\frac{du}{dt}$$

so $$\frac{qE}{m}=\frac{du}{dt}\left[\frac{1-u^2/c^2+u^2/c^2}{\left(1-u^2/c^2\right)^{3/2}}\right]$$

and $$\boxed{a=\frac{du}{dt}=\frac{qE}{m}\left(1-\frac{u^2}{c^2}\right)^{3/2}}$$

(b) For u small compared to c, the relativistic expression reduces to the classical $a=\dfrac{qE}{m}$. As u approaches c, the acceleration approaches zero, so that the object can never reach the speed of light.

(c) We can use the result of (a) to find the velocity u at time t:

$$a = \frac{du}{dt} = \frac{qE}{m}\left(1 - \frac{u^2}{c^2}\right)^{3/2} \quad \rightarrow \quad \int_0^u \frac{du}{\left(1 - u^2/c^2\right)^{3/2}} = \int_0^t \frac{qE}{m}dt$$

$$\frac{u}{\left(1 - u^2/c^2\right)^{1/2}} = \frac{qEt}{m}$$

$$u^2 = \left(\frac{qEt}{m}\right)^2\left(1 - \frac{u^2}{c^2}\right)$$

$$\boxed{u = \frac{qEct}{\sqrt{m^2c^2 + q^2E^2t^2}}}$$

Now, we can use this result to find position x at time t:

$$\frac{dx}{dt} = u = \frac{qEct}{\sqrt{m^2c^2 + q^2E^2t^2}}$$

$$x = \int_0^t u dt = qEc\int_0^t \frac{tdt}{\sqrt{m^2c^2 + q^2E^2t^2}} = \frac{c}{qE}\sqrt{m^2c^2 + q^2E^2t^2}\Bigg|_0^t$$

$$\boxed{x = \frac{c}{qE}\left(\sqrt{m^2c^2 + q^2E^2t^2} - mc\right)}$$

P39.89 (a) Take the two colliding protons as the system

$$E_1 = K + mc^2 \qquad E_2 = mc^2$$

$$E_1^2 = p_1^2c^2 + m^2c^4 \qquad p_2 = 0$$

In the final state,

$$E_f = K_f + Mc^2 = p_f^2c^2 + M^2c^4$$

By energy conservation, $E_1 + E_2 = E_f$, so

$$E_1^2 + 2E_1E_2 + E_2^2 = E_f^2$$

$$p_1^2c^2 + m^2c^4 + 2\left(K + mc^2\right)mc^2 + m^2c^4$$
$$= p_f^2c^2 + M^2c^4$$

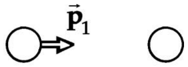
By conservation of momentum, $p_1 = p_f$, so

$$\cancel{p_1^2c^2} + m^2c^4 + 2\left(K + mc^2\right)mc^2 + m^2c^4$$
$$= \cancel{p_f^2c^2} + M^2c^4$$

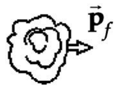
and we have then

$$M^2c^4 = 2Kmc^2 + 4m^2c^4 = \frac{4Km^2c^4}{2mc^2} + 4m^2c^4$$

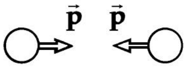
$$Mc^2 = 2mc^2\sqrt{1+\frac{K}{2mc^2}}$$



initial



final



initial (beams)

final (beams)

ANS. FIG. P39.89

(b) By contrast, for colliding beams we have, in the original state,

$$E_1 = K + mc^2 \qquad E_2 = K + mc^2$$

In the final state,

$$E_f = Mc^2$$

$$E_1 + E_2 = E_f:$$

$$K + mc^2 + K + mc^2 = Mc^2$$

$$Mc^2 = 2mc^2\left(1+\frac{K}{2mc^2}\right)$$

P39.90 We choose to write down the answer to part (b) first.

(b) Consider a hermit who lives on an asteroid halfway between the Sun and Tau Ceti, stationary with respect to both. Just as our spaceship is passing him, he also sees the blast waves from both explosions. Judging both stars to be stationary, this observer concludes that $\boxed{\text{the two stars blew up simultaneously}}$.

(a) We in the spaceship moving past the hermit do not calculate the explosions to be simultaneous. We measure the distance we have traveled from the Sun as

$$L = L_p\sqrt{1-\left(\frac{v}{c}\right)^2} = (6.00\text{ ly})\sqrt{1-(0.800)^2} = 3.60\text{ ly}$$

We see the Sun flying away from us at 0.800*c* while the light from the Sun approaches at 1.00*c*. Thus, the gap between the Sun and its blast wave has opened at 1.80*c*, and the time we calculate to have elapsed since the Sun exploded is

$$\frac{3.60\text{ ly}}{1.80c} = 2.00\text{ yr}$$

We see Tau Ceti as moving toward us at 0.800*c*, while its light approaches at 1.00*c*, only 0.200*c* faster. We measure the gap between that star and its blast wave as 3.60 ly and growing at 0.200*c*. We calculate that it must have been opening for

$$\frac{3.60\text{ ly}}{0.200c} = 18.0\text{ yr}$$

and conclude that $\boxed{\text{Tau Ceti exploded 16.0 years before the Sun}}$.

P39.91 (a) Since Dina is in the same reference frame, S′, as Owen, she measures the ball to have the same speed Owen observes, namely

$$|u_x'| = \boxed{0.800c}$$

(b) Within the frame S′, the ball travels 1.80×10^{12} m at a speed of 0.800*c*, so

$$\Delta t' = \frac{L_p}{|u_x'|} = \frac{1.80\times10^{12}\text{ m}}{0.800(3.00\times10^8\text{ m/s})} = \boxed{7.50\times10^3\text{ s}}$$

(c) In the S frame, the distance between Dina and Owen is a proper length; therefore,

$$L = L_p\sqrt{1-\frac{v^2}{c^2}} = (1.80\times10^{12}\text{ m})\sqrt{1-\frac{(0.600c)^2}{c^2}} = \boxed{1.44\times10^{12}\text{ m}}$$

Since *v* = 0.600*c* and $u_x' = -0.800c$, the velocity Ed measures for the ball is

$$u_x = \frac{u_x'+v}{1+u_x'v/c^2} = \frac{(-0.800c)+(0.600c)}{1+(-0.800)(0.600)} = \boxed{-0.385c}$$

(d) Ed measures the ball and Dina to be initially separated by 1.44×10^{12} m. Dina's motion at 0.600*c* and the ball's motion at 0.385*c* cover this distance from both ends. The gap closes at the rate 0.600*c* + 0.385*c* = 0.985*c*, so the ball and catcher meet after a time

$$\Delta t = \frac{1.44 \times 10^{12}\ \text{m}}{0.985\left(3.00 \times 10^{8}\ \text{m/s}\right)} = \boxed{4.88 \times 10^{3}\ \text{s}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P39.2	(a–b) See P39.2 for full explanation.
P39.4	See P39.4 for full explanation.
P39.6	(a) 0.436 m; (b) less than 0.436 m
P39.8	(a) $2.18\ \mu\text{s}$; (b) 649 m
P39.10	65.0 beats/min; (b) 10.5 beats/min
P39.12	(a) $L_p = 20.0$ m; (b) $L = 19.0$ m; (c) $0.312c$
P39.14	$0.140c$
P39.16	(a) 1.3×10^{-7} s; (b) 38 m; (c) 7.6 m
P39.18	42.1 g/cm^3
P39.20	$v = \dfrac{cL_p}{\sqrt{c^2\Delta t^2 + L_p^2}}$
P39.22	(a) $39.2\ \mu\text{s}$; (b) accurate to one digit
P39.24	(a) 5.45 yr; (b) Goslo
P39.26	1.13×10^4 Hz
P39.28	(a) $v = 0.943c$; (b) 2.55×10^3 m
P39.30	(a) $L = L_0\left[1-\left(\dfrac{v}{c}\right)^2\cos^2\theta_0\right]^{1/2}$; (b) $\gamma\tan\theta_0$
P39.32	$0.960c$
P39.34	0.893c, 16.8° above the x' axis
P39.36	(a) $2.73\times10^{-24}\ \text{kg}\cdot\text{m/s}$; (b) $1.58\times10^{-22}\ \text{kg}\cdot\text{m/s}$; (c) $5.64\times10^{-22}\ \text{kg}\cdot\text{m/s}$
P39.38	$u = \dfrac{c}{\sqrt{\left(m^2c^2/p^2\right)+1}}$
P39.40	(a) \$800; (b) \$$2.12 \times 10^9$
P39.42	(a) $0.141c$; (b) $0.436c$
P39.44	(a) 0.582 MeV; (b) 2.45 MeV
P39.46	(a) $0.999997c$; (b) 3.74×10^5 MeV
P39.48	(a) 4.38×10^{11} J; (b) 4.38×10^{11}; (c) See P39.48(c) for full explanation.

P39.50 See P39.50 for full explanation.

P39.52 (a) 3.91×10^4; (b) $0.9999999997c$; (c) 7.67 cm

P39.54 0.842 kg

P39.56 1.20 MeV

P39.58 See P39.58 for full explanation.

P39.60 larger; $\sim 10^{-9}$ J

P39.62 (a) isolated; (b) isolated system: conservation of energy and isolated system: conservation of momentum; (c) 6.22 and 2.01; (d) $3.09m_1 + m_2 = 1.66 \times 10^{-27}$ kg; (e) $m_2 = 3.52m_1$; (f) $m_1 = 2.51 \times 10^{-28}$ kg and $m_2 = 8.84 \times 10^{-28}$ kg

P39.64 (a) $M = \dfrac{2m\sqrt{4 - u^2/c^2}}{3\sqrt{1 - u^2/c^2}}$; (b) $\dfrac{4m}{3}$; (c) The answer to part (b) is in agreement with the classical result, which is the arithmetic sum of the masses of the two colliding particles.

P39.66 (a) $0.023\,6c$; (b) $6.18 \times 10^{-4}c$

P39.68 When Speedo arrives back on Earth, 118 years have passed, and Goslo would be 158 years old. That is impossible at the present time.

P39.70 (a) $0.467c$; (b) 2.75×10^3 kg

P39.72 See P39.72 for full explanation.

P39.74 (a) $u = c\left(\dfrac{H^2 + 2H}{H^2 + 2H + 1}\right)^{1/2}$; (b) u goes to 0 as K goes to 0; (c) u approaches c as K increases without limit; (d) $\dfrac{P}{mcH^{1/2}(H+2)^{1/2}(H+1)^2}$; (e) See P39.74(e) for full explanation; (f) See P39.74(f) for full explanation; (g) As energy is steadily imparted to particle, the particle's acceleration decreases. It decreases steeply, proportionally to $1/K^3$ at high energy. In this way the particle's speed cannot reach or surpass a certain upper limit, which is the speed of light in vacuum.

P39.76 (a) $m = \dfrac{3.65\text{ MeV}}{c^2}$; (b) $v = 0.589c$

P39.78 (a) 76.0 minutes; (b) 52.1 minutes

P39.80 (a–c) See P39.80 for full explanation; (d) c

P39.82 Because the speed of the electrons after accelerating through a potential difference ΔV is more than half the speed of light, there is no way to double its speed, regardless of the increased accelerating voltage.

P39.84 (a) See ANS. GRAPH P39.84; (b) 0.115c; (c) 0.257c; (d) 0.786c

P39.86 (a) $\dfrac{2d}{c+v}$; (b) $\dfrac{2d}{c}\sqrt{\dfrac{c-v}{c+v}}$

P39.88 (a) $a = \dfrac{du}{dt} = \dfrac{qE}{m}\left(1-\dfrac{u^2}{c^2}\right)^{3/2}$; (b) For u small compared to c, the relativistic expression reduces to the classical $a = \dfrac{qE}{m}$. As u approaches c, the acceleration approaches zero, so that the object can never reach the speed of light; (c) $u = \dfrac{qEct}{\sqrt{m^2c^2 + q^2E^2t^2}}$ and $x = \dfrac{c}{qE}\left(\sqrt{m^2c^2 + q^2E^2t^2} - mc\right)$

P39.90 (a) Tau Ceti exploded 16.0 years before the Sun; (b) The two stars blew up simultaneously.

40

Introduction to Quantum Physics

CHAPTER OUTLINE

* An asterisk indicates a question or problem item new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ40.1 The ranking is d > a = e > b > c. The wavelength is described by $\lambda = h/p$ in all cases. For photons, the momentum is given by $p = E/c$, so (a) is the same as (e), and (d) has a wavelength ten times larger. For the particles with mass, $pc = (E^2 - m^2c^4)^{1/2} = ([K + mc^2]^2 - m^2c^4)^{1/2} = (K^2 + 2Kmc^2)^{1/2}$. Thus a particle with larger mass has more momentum for the same kinetic energy, and a shorter wavelength.

OQ40.2 Answer (a). The x-ray photon transfers some of its energy to the electron. Thus, its frequency must decrease.

OQ40.3 Answer (b). In Compton scattering, a photon of energy $E = hf = hc/\lambda$ is scattered from an electron at rest. The scattering sets the electron into motion: the electron gains kinetic energy, so the photon loses energy. Because the photon has less energy, its frequency is smaller than E/h and its wavelength is larger than hc/E.

OQ40.4 **(i)** Answer (d). Because $P = IV$, the power input to the filament has increased by $8 \times 2 = 16$ times. The filament radiates this greater power according to Stefan's law, so its absolute temperature is higher by the fourth root of 16: it is two times higher.

(ii) Answer (d). By Wien's displacement law, the wavelength emitted with the highest intensity is inversely proportional the temperature: the temperature is twice as large, so the wavelength is half as large.

OQ40.5 Answer (a) and (c). One form of Heisenberg's uncertainty relation is $\Delta x\, \Delta p_x \geq \hbar/2\pi$, which says that one cannot determine both the position and momentum of a particle with arbitrary accuracy. Another form of this relation is $\Delta E\, \Delta t \geq \hbar/2\pi$, which sets a limit on how accurately the energy can be determined in a finite time interval.

OQ40.6 Answer: (a). The stopping potential is 1.00 V, so the maximum kinetic energy is 1.00 eV. From Equation 40.9,

$$K_{\max} = hf - \phi = hc/\lambda - \phi$$

$$\lambda = \frac{hc}{\phi + K_{\max}} = \frac{1240\ \text{eV}\cdot\text{nm}}{(2.50\ \text{eV} + 1.00\ \text{eV})} = 354\ \text{nm}$$

OQ40.7 Answer (c). UV light has the highest frequency of the three, and hence each photon delivers more energy to a skin cell. This explains why you can become sunburned on a cloudy day: clouds block visible light and infrared, but not much ultraviolet. You usually do not become sunburned through window glass, even though you can see the visible light from the Sun coming through the window, because the glass absorbs much of the ultraviolet and reemits it as infrared.

OQ40.8 Answer (d). Electron diffraction by crystals, first detected by the Davisson-Germer experiment in 1927, confirmed de Broglie's hypothesis and, of the listed choices, most clearly demonstrates the wave nature of electrons.

OQ40.9 Answer (c). We obtain the momentum of the electron from

$$K = \frac{1}{2}mu^2 = \frac{p^2}{2m} = e\Delta V \qquad \rightarrow \qquad p = \sqrt{2me\Delta V}$$

The de Broglie wavelength is then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2me\Delta V}}$$

$$= \frac{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times 10^{-31}\ \text{kg}\right)\left(1.60\times 10^{-19}\ \text{C}\right)\left(50.0\ \text{V}\right)}}$$

$$= 1.74\times 10^{-10}\ \text{m} = 0.174\ \text{nm}$$

OQ40.10 The ranking is: electron, proton, helium nucleus. The comparative masses of the particles of interest are $m_p \approx 1\,840 m_e$ and $m_{He} \approx 4m_p$. Assuming each particle is classical, its wavelength is inversely proportional to its mass: $\lambda = h/p = h/mv$.

OQ40.11 **(i)** (a) and (c). Electrons and protons possess mass, therefore they have rest energy $E_R = mc^2$. Photons do not have rest energy—they are never at rest.

(ii) (a) and (c). The electron and the proton have charges $-e$ and $+e$, respectively; the photon has no charge.

(iii) (a), (b), and (c). The electron and proton carry energy $E = \sqrt{p^2c^2 + (mc^2)^2} = K + mc^2$; the photon carries energy $E = hf$.

(iv) (a), (b), and (c). The electron and proton carry momentum $p = \gamma mu$, the photon carries momentum $p = E/c$, where E is its energy.

(v) Answer (b). Because it is light.

(vi) (a), (b), and (c). Each has the same de Broglie wavelength $\lambda = h/p$.

OQ40.12 Answer (a). If we set $K = \frac{1}{2}mu^2 = \frac{p^2}{2m} = e\Delta V$, which is the same for both particles, then we see that the momentum is $p = \sqrt{2me\Delta V}$, so the electron has the smaller momentum and therefore the longer wavelength $\left(\lambda = \frac{h}{p} = \frac{h}{\sqrt{2me\Delta V}}\right)$.

OQ40.13 Answer (b). Diffraction, polarization, interference, and refraction are all processes associated with waves. However, to understand the photoelectric effect, we must think of the energy transmitted as light coming in discrete packets, or quanta, called photons. Thus, the photoelectric effect most clearly demonstrates the particle nature of light.

OQ40.14 Answer (c). For the same uncertainty in speed, the particle with the smaller mass has the smaller uncertainty in momentum, $\Delta p_x = m\Delta v_x$, thus greater uncertainty in its position: $\Delta x \geq \frac{\hbar}{2\pi\Delta p_x} = \frac{\hbar}{2\pi m\Delta v_x}$. The mass of the electron is smaller than that of the proton, thus its minimum possible uncertainty in position is greater than that of the proton.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ40.1 In general, a turn of wire receives energy by two energy transfer mechanisms: (1) electrical transmission and (2) absorption of electromagnetic radiation from neighboring turns. Each turn of wire emits radiation similar to blackbody radiation. For most turns, the electromagnetic radiation absorbed comes from two neighbors. The turns on the end, however, have only one neighbor so they receive less energy input by electromagnetic radiation than the others. As a result, they operate at a lower temperature and do not glow as brightly.

CQ40.2 The Compton effect describes the *scattering* of photons from electrons, while the photoelectric effect predicts the ejection of electrons due to the *absorption* of photons by a material.

CQ40.3 Any object of macroscopic size—including a grain of dust—has an undetectably small wavelength, so any diffraction effects it might exhibit are very small, effectively undetectable. Recall historically how the diffraction of sound waves was at one time well known, but the diffraction of light was not.

CQ40.4 No. The second metal may have a larger work function than the first, in which case the incident photons may not have enough energy to eject photoelectrons.

CQ40.5 The stopping potential measures the kinetic energy of the most energetic photoelectrons. Each of them has gotten its energy from a single photon. According to Planck's $E = hf$, the photon energy depends on the frequency of the light. The intensity controls only the number of photons reaching a unit area in a unit time.

CQ40.6 Wave theory predicts that the photoelectric effect should occur at any frequency, provided the light intensity is high enough, or provided that the light shines on the surface for a sufficient time interval so that enough energy is delivered to the surface to eject electrons. However, as seen in the photoelectric experiments, the light must have a sufficiently high frequency for the effect to occur, and that electrons are either ejected almost immediately (less than 10^9 seconds after the surface is illuminated) or not at all, regardless of the intensity.

CQ40.7 Ultraviolet light has shorter wavelength and higher photon energy than any wavelength of visible light.

CQ40.8 Our eyes are not able to detect all frequencies of electromagnetic waves. For example, all objects that are above 0 K in temperature emit electromagnetic radiation in the infrared region. This describes *everything* in a dark room. We are only able to see objects that emit or reflect electromagnetic radiation in the visible portion of the spectrum.

CQ40.9 An electron has both classical-wave and classical-particle characteristics. In single- and double-slit diffraction and interference experiments, electrons behave like classical waves. An electron has mass and charge. It carries kinetic energy and momentum in parcels of definite size, as classical particles do. At the same time it has a particular wavelength and frequency. Since an electron displays characteristics of both classical waves and classical particles, it is neither a classical wave nor a classical particle. It is customary to call it a *quantum particle*, but another invented term, such as "wavicle," could serve equally well.

CQ40.10 A photon can interact with the photographic film at only one point. A few photons would only give a few dots of exposure, apparently randomly scattered.

CQ40.11 The wavelength of violet light is on the order of $\frac{1}{2}\ \mu\text{m}$, while the de Broglie wavelength of an electron can be 4 orders of magnitude smaller. The resolution is better (recall Rayleigh's criterion) because the diffraction effects are smaller.

CQ40.12 Light has both classical-wave and classical-particle characteristics. In single- and double-slit experiments light behaves like a wave. In the photoelectric effect light behaves like a particle. Light may be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time light may be characterized as a stream of photons, each carrying a discrete energy, hf. Since light displays *both* wave and particle characteristics, perhaps it would be fair to call light a "wavicle." It is customary to call a photon a *quantum particle*, different from a classical particle.

CQ40.13 Comparing Equation 40.9 with the slope-intercept form of the equation for a straight line, $y = mx + b$, we see

(a) that the slope in Figure 40.11 in the text is Planck's constant h and

(b) that the y intercept is $-\phi$, the negative of the work function.

(c) If a different metal were used, the slope would remain the same but the work function would be different. Thus, data for different metals appear as parallel lines on the graph.

CQ40.14 The discovery of electron diffraction by Davisson and Germer was a fundamental advance in our understanding of the motion of material particles. Newton's laws fail to properly describe the motion of an object with small mass. It moves as a wave, not as a classical particle. Proceeding from this recognition, the development of quantum mechanics made possible describing the motion of electrons in atoms; understanding molecular structure and the behavior of matter at the atomic scale, including electronics, photonics, and engineered materials; accounting for the motion of nucleons in nuclei; and studying elementary particles.

CQ40.15 The spacing between repeating structures on the surface of the feathers or scales is on the order of 1/2 the wavelength of light. An optical microscope would not have the resolution to see such fine detail, while an electron microscope can. The electrons can have much shorter wavelength.

CQ40.16 The *intensity* of electron waves in some small region of space determines the *probability* that an electron will be found in that region.

CQ40.17 The first flaw is that the Rayleigh–Jeans law predicts that the intensity of short wavelength radiation emitted by a black body approaches infinity as the wavelength decreases. This is known as the *ultraviolet catastrophe*. The second flaw is the prediction of much more power output from a black body than is shown experimentally. The intensity of radiation from the black body is given by the area under the red $I(\lambda, T)$ *vs.* λ curve in Figure 40.5 in the text, not by the area under the blue curve.

Planck's Law dealt with both of these issues and brought the theory into agreement with the experimental data by adding an exponential term to the denominator that depends on $1/\lambda$. This keeps both the predicted intensity from approaching infinity as the wavelength decreases and the area under the curve finite.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 40.1 Blackbody Radiation and Planck's Hypothesis

P40.1 The absolute temperature of the heating element is

$$T = 150°\text{C} + 273 = 423\ \text{K}$$

The peak wavelength is, from Equation 40.2,

$$\lambda_{\max} T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$$

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{T} = \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{423\ \text{K}} = 6.85 \times 10^{-6}\ \text{m}$$

or $\boxed{6.85\ \mu\text{m, which is in the infrared region of the spectrum.}}$

P40.2 (a) From Equation 40.2,

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{2900\ \text{K}} = \boxed{999\ \text{nm}}$$

(b) The wavelength emitted at the greatest intensity is in the infrared (greater than 700 nm), and according to the graph in Active Figure 40.3, much more energy is radiated at wavelengths longer than $\lambda_{\max}$ than at shorter wavelengths.

P40.3 (a) For lightning,

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{T} \sim \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{10^4\ \text{K}} \boxed{\sim 10^{-7}\ \text{m}}$$

For the explosion,

$$\lambda_{\max} \sim \frac{2.898 \times 10^{-3}\ \text{m} \cdot \text{K}}{10^7\ \text{K}} \boxed{\sim 10^{-10}\ \text{m}}$$

(b) $\boxed{\text{Lightning: ultraviolet; explosion: x-ray and gamma ray}}$

P40.4 (a) The peak radiation occurs at approximately 560 nm wavelength. From Wien's displacement law,

$$T = \frac{0.289\ 8 \times 10^{-2}\ \text{m} \cdot \text{K}}{\lambda_{\max}} = \frac{0.289\ 8 \times 10^{-2}\ \text{m} \cdot \text{K}}{560 \times 10^{-9}\ \text{m}} \approx \boxed{5\ 200\ \text{K}}$$

(b) Clearly, a firefly is not at this temperature, so $\boxed{\text{this is not blackbody radiation}}$.

P40.5 The energy of a single 500-nm photon is:

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{500\times10^{-9}\ \text{m}}$$
$$= 3.98\times10^{-19}\ \text{J}$$

The energy entering the eye each second

$$E = P\Delta t = IA\Delta t$$
$$= \left(4.00\times10^{-11}\ \text{W/m}^2\right)\left[\frac{\pi}{4}\left(8.50\times10^{-3}\ \text{m}\right)^2\right]\left(1.00\ \text{s}\right)$$
$$= 2.27\times10^{-15}\ \text{J}$$

The number of photons required to yield this energy is

$$n = \frac{E}{E_\gamma} = \frac{2.27\times10^{-15}\ \text{J}}{3.98\times10^{-19}\ \text{J/photon}} = \boxed{5.71\times10^{3}\ \text{photons}}$$

P40.6 (i) Planck's equation is $E = hf$. The photon energies are:

(a) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(620\times10^{12}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$$
$$= \boxed{2.57\ \text{eV}}$$

(b) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.10\times10^{9}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$$
$$= \boxed{1.28\times10^{-5}\ \text{eV}}$$

(c) $$E = hf = \left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(46.0\times10^{6}\ \text{s}^{-1}\right)\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)$$
$$= \boxed{1.91\times10^{-7}\ \text{eV}}$$

(ii) Wavelengths:

(a) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{620\times10^{12}\ \text{Hz}} = 4.84\times10^{-7}\ \text{m} = \boxed{484\ \text{nm}}$$

(b) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{3.10\times10^{9}\ \text{Hz}} = 9.68\times10^{-2}\ \text{m} = \boxed{9.68\ \text{cm}}$$

(c) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{46.0\times10^{6}\ \text{Hz}} = \boxed{6.52\ \text{m}}$$

(iii) Part of spectrum:

(a) $\boxed{\text{visible light (blue)}}$

(b) $\boxed{\text{radio wave}}$

(c) $\boxed{\text{radio wave}}$

P40.7 From Wien's displacement law,

(a) $$T = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{970 \times 10^{-9}\ \text{m}} \approx \boxed{2.99 \times 10^{3}\ \text{K}}$$

(b) $$T = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{145 \times 10^{-9}\ \text{m}} \approx \boxed{2.00 \times 10^{4}\ \text{K}}$$

P40.8 Each photon has an energy

$$E = hf = \left(6.626 \times 10^{-34}\right)\left(99.7 \times 10^{6}\right) = 6.61 \times 10^{-26}\ \text{J}$$

This implies that there are

$$\frac{150 \times 10^{3}\ \text{J/s}}{6.61 \times 10^{-26}\ \text{J/photon}} = \boxed{2.27 \times 10^{30}\ \text{photons/s}}$$

P40.9 From Equation 40.2, Wien's displacement law,

$$T = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{560 \times 10^{-9}\ \text{m}} = \boxed{5.18 \times 10^{3}\ \text{K}}$$

P40.10 (a) From Stefan's law (Equation 40.1), $P = eA\sigma T^4$. If the sun emits as a black body, $e = 1$.

$$T = \left(\frac{P}{eA\sigma}\right)^{1/4}$$

$$= \left[\frac{3.85 \times 10^{26}\ \text{W}}{1\left[4\pi\left(6.96 \times 10^{8}\ \text{m}\right)^2\right]\left(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)}\right]^{1/4}$$

$$= \boxed{5.78 \times 10^{3}\ \text{K}}$$

(b) $$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{5.78 \times 10^{3}\ \text{K}}$$

$$= 5.01 \times 10^{-7}\ \text{m} = \boxed{501\ \text{nm}}$$

P40.11 Planck's radiation law, Equation 40.6, gives the intensity-per-wavelength (W/m^2-wavelength). Because the range of the wavelengths is small, we treat the wavelength as the average $\bar{\lambda} = (\lambda_1 + \lambda_2)/2$. Taking E to be the average photon energy and n to be the number of photons emitted each second, we multiply by area and wavelength range to have energy-per-time leaving the hole:

$$\begin{aligned} P &= I(\bar{\lambda}, T)\bar{\lambda}A \\ &= \frac{2\pi hc^2}{\left[(\lambda_1 + \lambda_2)/2\right]^5 \left(e^{2hc/\left[(\lambda_1+\lambda_2)k_BT\right]} - 1\right)}(\lambda_2 - \lambda_1)\pi(d/2)^2 \\ &= En = nhf \end{aligned}$$

where $\bar{E} \approx h\bar{f} \approx \dfrac{hc}{\bar{\lambda}} = \dfrac{2hc}{\lambda_1 + \lambda_2}$

Solving for n,

$$n = \frac{P}{E} = \frac{8\pi^2 cd^2(\lambda_2 - \lambda_1)}{(\lambda_1 + \lambda_2)^4\left(e^{2hc/\left[(\lambda_1+\lambda_2)k_BT\right]} - 1\right)}$$

Substituting numerical values and suppressing units,

$$n = \frac{8\pi^2(3.00\times10^8\ \text{m/s})(0.050\ 0\times10^{-3}\ \text{m})^2(1.00\times10^{-9}\ \text{m})}{(1\ 001\times10^{-9}\ \text{m})^4\left(e^{\frac{2(6.626\times10^{-34})(3.00\times10^8)}{(1\ 001\times10^{-9})(1.38\times10^{-23})(7.50\times10^3)}} - 1\right)}$$

$$n = \frac{5.90\times10^{16}/\text{s}}{(e^{3.84} - 1)} = \boxed{1.30\times10^{15}/\text{s}}$$

P40.12 (a) From Stefan's law,

$$\begin{aligned} P &= eA\sigma T^4 \\ &= 1(20.0\times10^{-4}\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(5\ 000\ \text{K})^4 \\ &= \boxed{7.09\times10^4\ \text{W}} \end{aligned}$$

(b) From Wien's displacement law,

$$\lambda_{\text{max}}T = \lambda_{\text{max}}(5\ 000\ \text{K}) = 2.898\times10^{-3}\ \text{m}\cdot\text{K} \Rightarrow \lambda_{\text{max}} = \boxed{580\ \text{nm}}$$

(c) We compute:

$$\frac{hc}{k_B T} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1.38\times10^{-23}\ \text{J/K}\right)\left(5\,000\ \text{K}\right)} = 2.88\times10^{-6}\ \text{m}$$

The power per wavelength interval is

$$P(\lambda) = AI(\lambda) = \frac{2\pi hc^2 A}{\lambda^5\left[\exp\left(hc/\lambda k_B T\right)-1\right]},$$

and

$$\begin{aligned}2\pi hc^2 A &= 2\pi\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)^2\left(20.0\times10^{-4}\ \text{m}^2\right)\\ &= 7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}\end{aligned}$$

$$\begin{aligned}P(580\ \text{nm}) &= \frac{7.50\times10^{-19}\ \text{J}\cdot\text{m}^4/\text{s}}{\left(580\times10^{-9}\ \text{m}\right)^5\left[\exp\left(2.88\ \mu\text{m}/0.580\ \mu\text{m}\right)-1\right]}\\ &= \frac{1.15\times10^{13}\ \text{J/m}\cdot\text{s}}{e^{4.973}-1}\\ &= \boxed{7.99\times10^{10}\ \text{W/m}}\end{aligned}$$

(d)–(i) The other values are computed similarly:

	λ	$\dfrac{hc}{\lambda k_B T}$	$e^{hc/\lambda k_B T}-1$	$\dfrac{2\pi hc^2 A}{\lambda^5}$	$\mathcal{P}(\lambda)$, W/m
(d)	1.00 nm	2882.6	7.96×10^{1251}	7.50×10^{26}	9.42×10^{-1226}
(e)	5.00 nm	576.5	2.40×10^{250}	2.40×10^{23}	1.00×10^{-227}
(f)	400 nm	7.21	1347	7.32×10^{13}	5.44×10^{10}
(g)	700 nm	4.12	60.4	4.46×10^{12}	7.38×10^{10}
(h)	1.00 mm	0.00288	0.00289	7.50×10^{-4}	0.260
(i)	10.0 cm	2.88×10^{-5}	2.88×10^{-5}	7.50×10^{-14}	2.60×10^{-9}

(j) We approximate the area under the $P(\lambda)$ versus λ curve, between 400 nm and 700 nm, as the product of the average power per wavelength times the range of wavelength:

$$\begin{aligned}P &= P\left(\bar{\lambda}\right)\Delta\lambda\\ &= \frac{\left[(5.44+7.38)\times10^{10}\ \text{W/m}\right]}{2}\left[(700-400)\times10^{-9}\ \text{m}\right]\\ &= 1.92\times10^{4}\ \text{W}\ \boxed{\approx 19\ \text{kW}}\end{aligned}$$

P40.13 (a) The mass of the sphere is

$$m = \rho V = \rho\left(\frac{4}{3}\pi r^3\right) = \left(7.86\times10^3\ \text{kg/m}^3\right)\left[\frac{4}{3}\pi\left(0.020\ 0\ \text{m}\right)^3\right]$$

$$= \boxed{0.263\ \text{kg}}$$

(b) From Stefan's law,

$$P = \sigma AeT^4 = \sigma\left(4\pi r^2\right)eT^4$$

$$P = \left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left[4\pi\left(0.020\ 0\ \text{m}\right)^2\right]\left(0.860\right)\left(293\ \text{K}\right)^4$$

$$= \boxed{1.81\ \text{W}}$$

(c) It emits but does not absorb radiation, so its temperature must drop according to

$$Q = mc\Delta T = mc\left(T_f - T_i\right) \qquad \rightarrow \qquad \frac{dQ}{dt} = mc\frac{dT_f}{dt}$$

$$\frac{dT_f}{dt} = \frac{dQ/dt}{mc} = \frac{-P}{mc}$$

$$= \frac{-1.81\ \text{J/s}}{\left(0.263\ \text{kg}\right)\left(448\ \text{J/kg}\cdot\text{C°}\right)}$$

$$= \boxed{-0.015\ 3\ \text{°C/s} = -0.919\ \text{°C/min}}$$

(d) $\lambda_{\max}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$

$$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{293\ \text{K}} = 9.89\times10^{-6}\ \text{m} = \boxed{9.89\ \mu\text{m}}\ \text{(infrared)}$$

(e) $$E = hf = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{9.89\times10^{-6}\ \text{m}} = \boxed{2.01\times10^{-20}\ \text{J}}$$

(f) The energy output each second is carried by photons according to

$$P = \left(\frac{N}{\Delta t}\right)E$$

$$\frac{N}{\Delta t} = \frac{P}{E} = \frac{1.81\ \text{J/s}}{2.01\times10^{-20}\ \text{J/photon}} = \boxed{8.98\times10^{19}\ \text{photon/s}}$$

Matter is coupled to radiation quite strongly, in terms of photon numbers.

P40.14 Planck's radiation law is

$$I(\lambda,\ T)=\frac{2\pi hc^2}{\lambda^5\left(e^{hc/\lambda k_BT}-1\right)}.$$

For long wavelengths, the exponent $hc/\lambda k_{\mathrm{B}}T$ is small. Using the series expansion

$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$$

Planck's law reduces to

$$I(\lambda,\ T)=\frac{2\pi hc^2}{\lambda^5\left[\left(1+hc/\lambda k_{\mathrm{B}}T+\cdots\right)-1\right]}\approx\frac{2\pi hc^2}{\lambda^5\left(hc/\lambda k_{\mathrm{B}}T\right)}=\frac{2\pi ck_{\mathrm{B}}T}{\lambda^4}$$

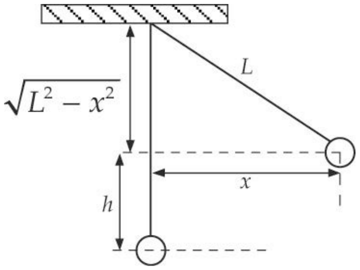
which is the Rayleigh–Jeans law, for very long wavelengths.

P40.15 From the figure, at maximum horizontal displacement *x*, the bob is at height $h=L-\sqrt{L^2-x^2}$. Then the pendulum's total energy is

$$E=mgh=mg\left(L-\sqrt{L^2-x^2}\right)$$

$$E=(1.00\text{ kg})\left(9.80\text{ m/s}^2\right)\left(1.00\text{ m}-\sqrt{(1.00\text{ m})^2-(0.030\ 0\text{ m})^2}\right)$$

$$=4.41\times10^{-3}\text{ J}$$



ANS. FIG. P40.15

The frequency of oscillation is

$$f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{g}{L}}=\frac{1}{2\pi}\sqrt{\frac{9.80\text{ m/s}^2}{1.00\text{ m}}}=0.498\text{ Hz}$$

The energy is quantized:

$$E=nhf$$

Therefore,

$$n=\frac{E}{hf}=\frac{4.41\times10^{-3}\text{ J}}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(0.498\text{ s}^{-1}\right)}$$

$$=\boxed{1.34\times10^{31}}$$

***P40.16** (a) The physical length of the pulse is

$$\ell = vt = \left(3.00\times10^{8}\text{ m/s}\right)\left(14.0\times10^{-12}\text{ s}\right) = \boxed{4.20\text{ mm}}$$

(b) We find the number of photons from

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{694.3\times10^{-9}\text{ m}} = 2.86\times10^{-19}\text{ J}$$

Then,

$$N = \frac{3.00\text{ J}}{2.86\times10^{-19}\text{ J}} = \boxed{1.05\times10^{19}\text{ photons}}$$

(c) The volume of the beam is

$$V = (4.20\text{ mm})\left[\pi(3.00\text{ mm})^{2}\right] = 119\text{ mm}^{3}$$

The number of photons per cubic millimeter is

$$n = \frac{1.05\times10^{19}\text{ photons}}{119\text{ mm}^{3}} = \boxed{8.82\times10^{16}\text{ mm}^{-3}}$$

Section 40.2 The Photoelectric Effect

***P40.17** (a) The cutoff wavelength is given by Equation 40.12:

$$\lambda_c = \frac{hc}{\phi} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(2.998\times10^{8}\text{ m/s}\right)}{(4.20\text{ eV})\left(1.602\times10^{-19}\text{ J/eV}\right)} = \boxed{295\text{ nm}}$$

which corresponds to a frequency of

$$f_c = \frac{c}{\lambda_c} = \frac{2.998\times10^{8}\text{ m/s}}{295\times10^{-9}\text{ m}} = \boxed{1.02\times10^{15}\text{ Hz}}$$

(b) We find the stopping potential from $\frac{hc}{\lambda} = \phi + e\Delta V_S$:

$$\frac{\left(6.626\times10^{-34}\right)\left(2.998\times10^{8}\right)}{180\times10^{-9}} = (4.20\text{ eV})\left(1.602\times10^{-19}\text{ J/eV}\right) + \left(1.602\times10^{-19}\right)\Delta V_S$$

Therefore, $\boxed{\Delta V_S = 2.69\text{ V}}$.

P40.18 (a) At the cutoff wavelength, the energy of the photons is equal to the work function ($K_{\text{max}} = 0$):

$$\frac{hc}{\lambda} = \phi \qquad \rightarrow \qquad \lambda = \frac{hc}{\phi} = \frac{1\,240\text{ nm}\cdot\text{eV}}{4.31\text{ eV}} = \boxed{288\text{ nm}}$$

(b) This is the cutoff frequency:

$$f = \frac{c}{\lambda} = \frac{3.00\times 10^{8}\text{ m/s}}{288\times 10^{-9}\text{ m}} = \boxed{1.04\times 10^{15}\text{ Hz}}$$

(c) The maximum kinetic energy is the difference between the energy of the photons and the work function:

$$K_{\text{max}} = E - \phi = 5.50\text{ eV} - 4.31\text{ eV} = \boxed{1.19\text{ eV}}$$

P40.19 (a) Einstein's photoelectric effect equation is $K_{\text{max}} = hf - \phi$ and the energy required to raise an electron through a 1-V potential is 1 eV, so that

$$K_{\text{max}} = e\Delta V_s = 0.376\text{ eV}$$

The energy of a photon from the mercury lamp is:

$$hf = \frac{hc}{\lambda} = \frac{(6.626\times 10^{-34}\text{ J}\cdot\text{s})(2.998\times 10^{8}\text{ m/s})}{546.1\times 10^{-9}\text{ m}}\left(\frac{1\text{ eV}}{1.602\times 10^{-19}\text{ J}}\right)$$

$$= \frac{1\,240\text{ eV}\cdot\text{nm}}{546.1\text{ nm}} = 2.27\text{ eV}$$

Therefore, the work function for this metal is:

$$\phi = hf - K_{\text{max}} = 2.27\text{ eV} - 0.376\text{ eV} = \boxed{1.89\text{ eV}}$$

(b) For the yellow light, $\lambda = 587.5$ nm and the photon energy is

$$hf = \frac{hc}{\lambda} = \frac{1\,240\text{ eV}\cdot\text{nm}}{587.5\text{ nm}} = 2.11\text{ eV}$$

Therefore the maximum energy that can be given to an ejected electron is

$$K_{\text{max}} = hf - \phi = 2.11\text{ eV} - 1.89\text{ eV} = 0.216\text{ eV}$$

so the stopping voltage is

$$\Delta V_s = \boxed{0.216\text{ V}}$$

P40.20 (a) The energy of a photon with a wavelength of 400 nm is

$$E = \frac{hc}{\lambda} = \frac{\left(6.63\times 10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times 10^{8}\text{ m/s}\right)}{400\times 10^{-9}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times 10^{-19}\text{ J}}\right)$$

$$= 3.11\text{ eV}$$

$$\boxed{\text{The energy of a photon with wavelength 400 nm is calculated to be 3.11 eV. Now compare this energy with the given work functions. Of these metals, only lithium shows the photoelectric effect because its work function is less than the energy of the photon.}}$$

(b) For lithium,

$$\begin{aligned} K_{\max} &= E - \phi \\ &= \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{400\times10^{-9}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) \\ &\qquad - 2.30\text{ eV} \\ &= \boxed{0.808\text{ eV}} \end{aligned}$$

P40.21 The maximum kinetic energy of the electrons is

$$\begin{aligned} K_{\max} &= \frac{1}{2}mu_{\max}^2 = \frac{1}{2}(9.11\times10^{-31}\text{kg})(4.60\times10^{5}\text{m/s})^2 \\ &= 9.64\times10^{-20}\text{ J} = 0.602\text{ eV} \end{aligned}$$

(a) The work function is

$$\phi = E - K_{\max} = \frac{1\,240\text{ eV}\cdot\text{nm}}{625\text{ nm}} - 0.602\text{ eV} = \boxed{1.38\text{ eV}}$$

(b) At the cutoff frequency, the energy of the photons equals the work function:

$$E = hf = \phi \qquad \to \qquad f = \frac{\phi}{h} = \frac{1.38\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)$$

$$= \boxed{3.34\times10^{14}\text{ Hz}}$$

P40.22 (a) The energy needed is $E = 1.00\text{ eV} = 1.60\times10^{-19}$ J.

The energy absorbed in time interval Δt is

$$E = P\Delta t = IA\Delta t$$

So,

$$\Delta t = \frac{E}{IA} = \frac{1.60\times10^{-19}\text{ J}}{(500\text{ J/s}\cdot\text{m}^2)\left[\pi(2.82\times10^{-15}\text{ m})^2\right]} = 1.28\times10^{7}\text{ s}$$

$$= \boxed{148\text{ days}}$$

(b) $\boxed{\text{The result for part (a) does not agree at all with the experimental observations.}}$

P40.23 Ultraviolet photons will be absorbed to knock electrons out of the sphere with maximum kinetic energy $K_{\max} = hf - \phi$. As the sphere loses charge, it becomes more positive relative to $V = 0$ at $r = \infty$. Eventually, the sphere will accumulate enough charge $+Q$ that the potential difference between the sphere's surface and infinity reaches the stopping potential of the photoelectrons, at which point no more electrons can escape.

$$K_{\max} = \frac{hc}{\lambda} - \phi = e\Delta V_s \quad \rightarrow \quad \Delta V_s = \frac{1}{e}\left(\frac{hc}{\lambda} - \phi\right)$$

and $$\frac{k_e Q}{r} = \Delta V_s = \frac{1}{e}\left(\frac{hc}{\lambda} - \phi\right).$$

Therefore,

$$Q = \frac{r}{ek_e}\left(\frac{hc}{\lambda} - \phi\right)$$

Solving for Q gives

$$Q = \frac{5.00 \times 10^{-2}\ \text{m}}{\left(1.602 \times 10^{-19}\ \text{C}\right)\left(8.99 \times 10^{9}\ \text{N} \cdot \text{m}^2/\text{C}^2\right)}$$
$$\times \left\{\left[\frac{\left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(2.998 \times 10^{8}\ \text{m/s}\right)}{200 \times 10^{-9}\ \text{m}} - 4.70\ \text{eV}\left(\frac{1.602 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)\right]\right\}$$
$$= \boxed{8.34 \times 10^{-12}\ \text{C}}$$

P40.24 (a) The energy of photons is

$$E = \frac{hc}{\lambda} = \frac{1\,240\ \text{nm} \cdot \text{eV}}{150\ \text{nm}} = \boxed{8.27\ \text{eV}}$$

(b) The photon energy is larger than the work function.

(c) $KE_{\max} = E - \phi = 8.27\ \text{eV} - 6.35\ \text{eV} = \boxed{1.92\ \text{eV}}$

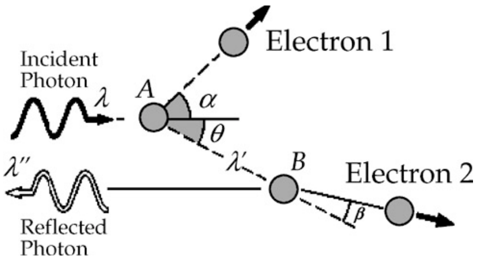
(d) $K_{\max} = e\Delta V_s \quad \rightarrow \quad \Delta V_s = \dfrac{K_{\max}}{e} = \dfrac{1.92\ \text{eV}}{e} = \boxed{1.92\ \text{V}}$

Section 40.3 The Compton Effect

P40.25 From the Compton shift equation, the wavelength shift of the scattered x-rays is

$$\begin{aligned}\Delta\lambda &= \frac{h}{m_e c}(1-\cos\theta)\\ &= \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})}(1-\cos 55.0°)\\ &= 1.03\times10^{-12}\ \text{m}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = \boxed{1.03\times10^{-3}\ \text{nm}}\end{aligned}$$

P40.26 We note that $\lambda'' - \lambda = (\lambda'' - \lambda') + (\lambda' - \lambda)$.



ANS. FIG. P40.26

At *A*, the scattering angle is θ, and

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta)$$

At *B*, the scattering angle is $180° - \theta$, and

$$\lambda'' - \lambda' = \frac{h}{m_e c}\left[1-\cos(180°-\theta)\right] = \frac{h}{m_e c}\left[1+\cos\theta\right]$$

Therefore,

$$\begin{aligned}\lambda'' - \lambda &= (\lambda'' - \lambda') + (\lambda' - \lambda)\\ &= \frac{h}{m_e c}(1+\cos\theta) + \frac{h}{m_e c}(1-\cos\theta) = \frac{2h}{m_e c}\\ &= \frac{2(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})}\\ &= \boxed{4.85\times10^{-12}\ \text{m}}\end{aligned}$$

***P40.27** This is Compton scattering through 180°:

$$E_0 = \frac{hc}{\lambda_0} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(0.110\times10^{-9}\ \text{m})(1.60\times10^{-19}\ \text{J/eV})} = 11.3\ \text{keV}$$

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = (2.43\times10^{-12}\ \text{m})(1-\cos180°)$$

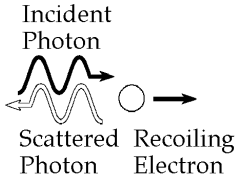
$$= 4.85\times10^{-12}\ \text{m}$$

$$\lambda' = \lambda_0 + \Delta\lambda = 0.115\ \text{nm},\ \text{so}\ E' = \frac{hc}{\lambda'} = 10.8\ \text{keV}.$$

By conservation of momentum for the photon-electron system,

$$\frac{h}{\lambda_0}\hat{\mathbf{i}} = \frac{h}{\lambda'}(-\hat{\mathbf{i}}) + p_e\hat{\mathbf{i}}$$

and $$p_e = h\left(\frac{1}{\lambda_0} + \frac{1}{\lambda'}\right),$$



ANS. FIG. P40.27

$$p_e = (6.626\times10^{-34}\ \text{J}\cdot\text{s})\left(\frac{(3.00\times10^{8}\ \text{m/s})/c}{1.60\times10^{-19}\ \text{J/eV}}\right) \times\left(\frac{1}{0.110\times10^{-9}\ \text{m}} + \frac{1}{0.115\times10^{-9}\ \text{m}}\right)$$

$$= \boxed{\frac{22.1\ \text{keV}}{c}}$$

By conservation of system energy, 11.3 keV = 10.8 keV + K_e, so that

$\boxed{K_e = 478\ \text{eV}}$.

Check: $$E^2 = p^2c^2 + m_e^2c^4 \ \text{or}\ (m_ec^2 + K_e)^2 = (pc)^2 + (m_ec^2)^2$$

$$(511\ \text{keV} + 0.478\ \text{keV})^2 = (22.1\ \text{keV})^2 + (511\ \text{keV})^2$$

$$2.62\times10^{5}\ \text{keV}^2 = 2.62\times10^{5}\ \text{keV}^2$$

P40.28 (a) and (b) From $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta)$ we calculate the wavelength of the scattered photon. For example, at $\theta = 30°$ we have

$$\lambda' + \Delta\lambda = 120\times10^{-12}\ \text{m} + \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(2.998\times10^{8}\ \text{m/s})}(1-\cos30.0°)$$

$$= 120.3\times10^{-12}\ \text{m}$$

The electron carries off the energy the photon loses:

$$\begin{aligned} K_e &= \frac{hc}{\lambda_0} - \frac{hc}{\lambda'} \\ &= \frac{\left(6.626 \times 10^{-34}\ \mathrm{J \cdot s}\right)\left(2.998 \times 10^{8}\ \mathrm{m/s}\right)}{\left(1.602 \times 10^{-19}\ \mathrm{J/eV}\right)} \\ &\qquad \times \left(\frac{1}{120 \times 10^{-12}\ \mathrm{m}} - \frac{1}{120.3 \times 10^{-12}\ \mathrm{m}} \right) \\ &= 27.9\ \mathrm{eV} \end{aligned}$$

The other entries are computed similarly.

θ, degrees	0	30	60	90	120	150	180
λ', pm	120.0	120.3	121.2	122.4	123.6	124.5	124.8
K_e, eV	0	27.9	104	205	305	376	402

(c) 180°. We could answer like this: The photon imparts the greatest momentum to the originally stationary electron in a head-on collision. Here the photon recoils straight back and the electron has maximum kinetic energy.

P40.29 With $K_e = E'$ and $K_e = E_0 - E'$, we have $E' = E_0 - E' \quad \rightarrow \quad E' = \frac{E_0}{2}$.

We also have $\lambda' = \frac{hc}{E'}$; therefore, $\lambda' = \frac{hc}{E_0/2} = 2\frac{hc}{E_0} = 2\lambda_0$.

By the Compton equation,

$$\lambda' = \lambda_0 + \lambda_C\left(1 - \cos\theta\right) \quad \rightarrow \quad 2\lambda_0 = \lambda_0 + \lambda_C\left(1 - \cos\theta\right)$$

Therefore,

$$1 - \cos\theta = \frac{\lambda_0}{\lambda_C} = \frac{0.001\,60}{0.002\,43} \quad \rightarrow \quad \theta = \boxed{70.0^\circ}$$

P40.30 (a) To compute the Compton shift, we first determine the electron's kinetic energy:

$$\begin{aligned} K &= \frac{1}{2}m_e u^2 = \frac{1}{2}\left(9.11 \times 10^{-31}\ \mathrm{kg}\right)\left(1.40 \times 10^{6}\ \mathrm{m/s}\right)^2 \\ &= 8.93 \times 10^{-19}\ \mathrm{J} = 5.58\ \mathrm{eV} \end{aligned}$$

Then,

$$E_0 = \frac{hc}{\lambda_0} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{0.800\ \text{nm}} = 1\ 550\ \text{eV}$$

$$E' = E_0 - K \quad \text{and} \quad \lambda' = \frac{hc}{E'} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{1\ 550\ \text{eV} - 5.58\ \text{eV}} = 0.803\ \text{nm}$$

and the Compton shift is

$$\Delta\lambda = \lambda' - \lambda_0 = 0.002\ 89\ \text{nm} = \boxed{2.89\ \text{pm}}$$

(b) $\Delta\lambda = \lambda_C\left(1 - \cos\theta\right)$

$$\cos\theta = 1 - \frac{\Delta\lambda}{\lambda_C} = 1 - \frac{0.002\ 89\ \text{nm}}{0.002\ 43\ \text{nm}} = -0.189 \quad \rightarrow \quad \boxed{\theta = 101°}$$

P40.31 The photon has momentum $p_0 = E_0/c = h/\lambda_0$ before scattering and momentum $p' = h/\lambda'$ after scattering. The electron momentum after scattering is p_e.

(a) Conservation of momentum in the x direction gives

$$p_0 = p'\cos\theta + p_e\cos\theta$$

or $$\frac{h}{\lambda_0} = \left(\frac{h}{\lambda'} + p_e\right)\cos\theta. \qquad \textbf{[1]}$$

Conservation of momentum in the y direction gives

$$0 = p'\sin\theta - p_e\sin\theta$$

which (neglecting the trivial solution $\theta = 0$) gives

$$p_e = p' = \frac{h}{\lambda'} \qquad \textbf{[2]}$$

Substituting [2] into [1] gives

$$\frac{h}{\lambda_0} = \frac{2h}{\lambda'}\cos\theta$$

or $$\lambda' = 2\lambda_0\cos\theta. \qquad \textbf{[3]}$$

Substitute [3] into the Compton equation:

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\left(2\lambda_0\cos\theta\right) - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

Solving,

$$\left(2\lambda_0 + \frac{h}{m_e c}\right)\cos\theta = \lambda_0 + \frac{h}{m_e c}$$

$$\left(2\frac{hc}{E_0} + \frac{h}{m_e c}\right)\cos\theta = \frac{hc}{E_0} + \frac{h}{m_e c}$$

$$\frac{1}{m_e c^2 E_0}\left(2m_e c^2 + E_0\right)\cos\theta = \frac{1}{m_e c^2 E_0}\left(m_e c^2 + E_0\right)$$

$$\cos\theta = \frac{m_e c^2 + E_0}{2m_e c^2 + E_0} = \frac{0.511\text{ MeV} + 0.880\text{ MeV}}{2(0.511\text{ MeV}) + 0.880\text{ MeV}} = 0.731$$

$$\rightarrow \quad \theta = \boxed{43.0°}$$

(b) Using equation [3]:

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0(2\cos\theta)} = \frac{E_0}{2\cos\theta} = \frac{0.880\text{ MeV}}{2\cos 43.0°} = \boxed{0.602\text{ MeV}}$$

Then,

$$p' = \frac{E'}{c} = \frac{0.602\text{ MeV}}{c} = \boxed{3.21\times10^{-22}\text{ kg}\cdot\text{m/s}}$$

(c) From energy conservation:

$$K_e = E_0 - E' = 0.880\text{ MeV} - 0.602\text{ MeV} = \boxed{0.278\text{ MeV}}$$

From equation [2],

$$p_e = p' = \frac{0.602\text{ MeV}}{c}\left(\frac{c}{3.00\times10^{-22}\text{ m/s}}\right)\left(\frac{1.60\times10^{-23}\text{ J}}{1\text{ MeV}}\right)$$

$$= \boxed{3.21\times10^{-22}\text{ kg}\cdot\text{m/s}}$$

P40.32 The photon has momentum $p_0 = E_0/c = h/\lambda_0$ before scattering and momentum $p' = h/\lambda'$ after scattering. The electron momentum after scattering is p_e.

(a) Conservation of momentum in the *x* direction gives

$$p_0 = p'\cos\theta + p_e\cos\theta$$

or $$\frac{h}{\lambda_0} = \left(\frac{h}{\lambda'} + p_e\right)\cos\theta. \qquad \textbf{[1]}$$

Conservation of momentum in the *y* direction gives

$$0 = p'\sin\theta - p_e\sin\theta$$

which (neglecting the trivial solution $\theta = 0$) gives

$$p_e = p' = \frac{h}{\lambda'} \tag{2}$$

Substituting [2] into [1] gives

$$\frac{h}{\lambda_0} = \frac{2h}{\lambda'}\cos\theta$$

or $\lambda' = 2\lambda_0 \cos\theta.$ **[3]**

Substitute [3] into the Compton equation:

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$$

$$(2\lambda_0 \cos\theta) - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$$

$$\left(2\lambda_0 + \frac{h}{m_e c}\right)\cos\theta = \lambda_0 + \frac{h}{m_e c}$$

$$\left(2\frac{hc}{E_0} + \frac{h}{m_e c}\right)\cos\theta = \frac{hc}{E_0} + \frac{h}{m_e c}$$

$$\frac{1}{m_e c^2 E_0}\left(2m_e c^2 + E_0\right)\cos\theta = \frac{1}{m_e c^2 E_0}\left(m_e c^2 + E_0\right)$$

$$\cos\theta = \frac{m_e c^2 + E_0}{2m_e c^2 + E_0} \quad \rightarrow \quad \boxed{\theta = \cos^{-1}\left(\frac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)}$$

(b) Using equation [3]:

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0(2\cos\theta)} = \frac{E_0}{2\cos\theta} = \boxed{\frac{E_0\left(2m_e c^2 + E_0\right)}{2\left(m_e c^2 + E_0\right)}}$$

Then, $p' = \dfrac{E'}{c} = \boxed{\dfrac{E_0\left(2m_e c^2 + E_0\right)}{2c\left(m_e c^2 + E_0\right)}}.$

(c) From energy conservation:

$$K_e = E_0 - E' = E_0 - \frac{E_0\left(2m_e c^2 + E_0\right)}{2\left(m_e c^2 + E_0\right)}$$

$$= \frac{2E_0\left(m_e c^2 + E_0\right) - E_0\left(2m_e c^2 + E_0\right)}{2\left(m_e c^2 + E_0\right)}$$

$$K_e = \frac{\left(2E_0 m_e c^2 + 2E_0^2\right) - \left(2E_0 m_e c^2 + E_0^2\right)}{2\left(m_e c^2 + E_0\right)}$$

$$= \frac{2E_0 m_e c^2 + 2E_0^2 - 2E_0 m_e c^2 - E_0^2}{2\left(m_e c^2 + E_0\right)} = \boxed{\frac{E_0^2}{2\left(m_e c^2 + E_0\right)}}$$

From equation [2],

$$p_e = p' = \boxed{\frac{E_0\left(2m_e c^2 + E_0\right)}{2c\left(m_e c^2 + E_0\right)}}$$

P40.33 (a) From $\Delta\lambda = \frac{h}{m_e c}\left(1 - \cos\theta\right)$:

$$\Delta\lambda = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(2.998 \times 10^{8}\ \text{m/s}\right)}\left(1 - \cos 37.0°\right)$$

$$= 4.89 \times 10^{-13}\ \text{m} = \boxed{4.89 \times 10^{-4}\ \text{nm}}$$

(b) $\lambda_0 = \frac{hc}{E} = \frac{1\,240\ \text{eV}\cdot\text{nm}}{300 \times 10^{3}\ \text{eV}} = 4.13 \times 10^{-3}\ \text{nm}$

and

$$\lambda' = \lambda_0 + \Delta\lambda = 4.62 \times 10^{-12}\ \text{m} = 4.62 \times 10^{-3}\ \text{nm}$$

$$E' = \frac{hc}{\lambda'} = \frac{1\,240\ \text{eV}\cdot\text{nm}}{4.62 \times 10^{-3}\ \text{nm}} = 2.68 \times 10^{5}\ \text{eV} = \boxed{268\ \text{keV}}$$

(c) $K_e = E_0 - E' = \boxed{31.7\ \text{keV}}$

P40.34 (a) It is, because Compton's equation and the conservation of vector momentum give three independent equations in the unknowns λ', λ_0, and u.

(b) Assuming the photon is incident along the *x* direction, the equations are

$$\lambda' - \lambda_0 = \frac{h}{m_e c}\left(1 - \cos 90.0°\right) \rightarrow \lambda' = \lambda_0 + \frac{h}{m_e c} \qquad \textbf{[1]}$$

and

$$\Delta p_x = 0 \rightarrow \frac{h}{\lambda_0} = \gamma m_e u \cos 20.0°$$

$$\Delta p_y = 0 \rightarrow \frac{h}{\lambda'} = \gamma m_e u \sin 20.0°$$

Dividing the latter two equations gives

$$\frac{\lambda_0}{\lambda'} = \tan 20.0° \qquad \textbf{[2]}$$

Substituting equation [2] into equation [1] gives

$$\lambda' = \lambda' \tan 20.0° + \frac{h}{m_e c}$$

$$\lambda' = \frac{h}{m_e c\left(1-\tan 20.0°\right)} = \frac{hc}{m_e c^2\left(1-\tan 20.0°\right)}$$

$$= \frac{1240 \text{ eV}\cdot\text{nm}}{\left(0.511\times 10^6 \text{ eV}\right)\left(1-\tan 20.0°\right)}$$

$$= 3.82\times 10^{-3} \text{ nm} = 3.82\times 10^{-12} \text{ m} = \boxed{3.82 \text{ pm}}$$

P40.35 We treat the electron non-relativistically because

$$\frac{u}{c} = \frac{2.18\times 10^6 \text{ m/s}}{3.00\times 10^3 \text{ m/s}} = 0.007\ 27 < 0.01$$

The electron's final kinetic energy is

$$K_f = \frac{1}{2} m_e u^2.$$

This is the energy lost by the photon:

$$\Delta E = hf_0 - hf' = \frac{hc}{\lambda_0} - \frac{hc}{\lambda'} = K_f \qquad \textbf{[1]}$$

From the Compton equation, we have

$$\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c}\left(1-\cos\theta\right) \qquad \textbf{[2]}$$

$$\lambda' = \lambda_0 + \frac{h}{m_e c}\left(1-\cos\theta\right) \qquad \textbf{[3]}$$

Substitute [2] and [3] into [1]:

$$K_f = \frac{hc}{\lambda_0} - \frac{hc}{\lambda'} = \frac{\left(\lambda_0 - \lambda'\right)hc}{\lambda_0\lambda'} = \frac{h}{m_e c}\left(1-\cos\theta\right)\frac{hc}{\lambda_0\lambda'}$$

$$K_f = \frac{h^2 c\left(1-\cos\theta\right)}{m_e c\lambda_0\left[\lambda_0 + \dfrac{h}{m_e c}\left(1-\cos\theta\right)\right]}$$

Solving,

$$m_e c\lambda_0\left[\lambda_0+\frac{h}{m_e c}(1-\cos\theta)\right]=\frac{h^2c}{K_f}(1-\cos\theta)$$

$$m_e c\lambda_0^2+h(1-\cos\theta)\lambda_0-\frac{h^2c}{K_f}(1-\cos\theta)=0$$

(a) Solve for λ_0:

$$\lambda_0=\frac{h(1-\cos\theta)\pm\sqrt{\left[h(1-\cos\theta)\right]^2-4(m_e c)\left[-\frac{h^2c}{K_f}(1-\cos\theta)\right]}}{2m_e c}$$

$$\lambda_0=\frac{h(1-\cos\theta)\pm\sqrt{\left[h(1-\cos\theta)\right]^2+\left[\frac{4h^2m_e c^2}{\frac{1}{2}m_e u^2}(1-\cos\theta)\right]}}{2m_e c}$$

$$\lambda_0=\frac{h(1-\cos\theta)\pm\sqrt{\left[h(1-\cos\theta)\right]^2+\left[\frac{8h^2c^2}{u^2}(1-\cos\theta)\right]}}{2m_e c}$$

$$=\frac{h(1-\cos\theta)}{2m_e c}\left\{1\pm\sqrt{1+\left[\frac{8c^2}{u^2(1-\cos\theta)}\right]}\right\}$$

Only the positive answer is physical:

$$\lambda_0=\frac{h(1-\cos\theta)}{2m_e c}\left\{1+\sqrt{1+\left[\frac{8c^2}{u^2(1-\cos\theta)}\right]}\right\}$$

$$=\frac{\left(6.63\times10^{-34}\ \mathrm{J\cdot s}\right)(1-\cos17.4^\circ)}{2\left(9.11\times10^{-31}\ \mathrm{kg}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}$$

$$\times\left\{1+\sqrt{1+\left[\frac{8\left(3.00\times10^{8}\ \mathrm{m/s}\right)^2}{\left(2.18\times10^{6}\ \mathrm{m/s}\right)^2(1-\cos17.4^\circ)}\right]}\right\}$$

$$=1.01\times10^{-10}\ \mathrm{m}=\boxed{0.101\ \mathrm{nm}}$$

(b) From [3],

$$\lambda'=\lambda_0+\frac{h}{m_e c}(1-\cos\theta)$$

Substituting,

$$\lambda' = \frac{h(1-\cos\theta)}{2m_e c}\left\{1+\sqrt{1+\left[\frac{8c^2}{u^2(1-\cos\theta)}\right]}\right\}+\frac{h}{m_e c}(1-\cos\theta)$$

$$= \frac{h}{m_e c}(1-\cos\theta)\left\{\frac{3}{2}+\frac{1}{2}\sqrt{1+\left[\frac{8c^2}{u^2(1-\cos\theta)}\right]}\right\}$$

$$= 1.0116\times 10^{-10}\text{ m}$$

The electron scattering angle is ϕ. By conservation of momentum in the transverse direction:

$$0 = \frac{h}{\lambda'}\sin\theta - m_e u\sin\phi \quad \rightarrow \quad \sin\phi = \frac{h}{\lambda' m_e u}\sin\theta$$

$$\phi = \sin^{-1}\left(\frac{h}{\lambda' m_e u}\sin\theta\right)$$

$$= \sin^{-1}\left(\frac{6.63\times 10^{-34}\text{ J}\cdot\text{s}}{\lambda'(9.11\times 10^{-31}\text{ kg})(2.18\times 10^{6}\text{ m/s})}\sin 17.4°\right)$$

$$= \boxed{80.7°}$$

P40.36 Maximum energy loss appears as maximum increase in wavelength, which occurs for scattering angle 180°. Then,

$$\Delta\lambda = \left(\frac{h}{mc}\right)(1-\cos 180°) = \frac{2h}{mc}$$

where m is the mass of the target particle. The fractional energy loss is

$$\frac{E_0 - E'}{E_0} = \frac{hc/\lambda_0 - hc/\lambda'}{hc/\lambda_0} = \frac{\lambda' - \lambda_0}{\lambda'} = \frac{\Delta\lambda}{\lambda_0 + \Delta\lambda} = \frac{2h/mc}{\lambda_0 + 2h/mc}$$

Further, $\lambda_0 = \frac{hc}{E_0}$, so $\frac{E_0 - E'}{E_0} = \frac{2h/mc}{hc/E_0 + 2h/mc} = \frac{2E_0}{mc^2 + 2E_0}$.

(a) For scattering from a free electron, $mc^2 = 0.511$ MeV, so

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511\text{ MeV})}{0.511\text{ MeV} + 2(0.511\text{ MeV})} = \boxed{0.667}$$

(b) For scattering from a free proton, $mc^2 = 938$ MeV, and

$$\frac{E_0 - E'}{E_0} = \frac{2(0.511\text{ MeV})}{938\text{ MeV} + 2(0.511\text{ MeV})} = \boxed{0.001\,09}$$

Section 40.4 The Nature of Electromagnetic Waves

P40.37 With photon energy $E = hf = 10.0$ eV, a photon would have

$$f = \frac{E}{h} = \frac{10.0\left(1.602\times10^{-19}\ \text{J}\right)}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 2.42\times10^{15}\ \text{Hz}$$

and

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{2.41\times10^{15}\ \text{Hz}} = 124\ \text{nm}$$

$\boxed{\text{To have photon energy 10 eV or greater, according to this definition, ionizing radiation is the ultraviolet light, x-rays, and } \gamma \text{ rays with wavelength shorter than 124 nm; that is, with frequency higher than } 2.42\times10^{15}\ \text{Hz.}}$

P40.38 The photon energy is

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{633\times10^{-9}\ \text{m}} = 3.14\times10^{-19}\ \text{J}$$

The power carried by the beam is

$$\left(2.00\times10^{18}\ \text{photons/s}\right)\left(3.14\times10^{-19}\ \text{J/photon}\right) = 0.628\ \text{W}$$

Its intensity is the average Poynting vector

$$I = S_{\text{avg}} = \frac{P}{\pi r^2} = \frac{0.628\ \text{W}}{\pi\left(\dfrac{1.75\times10^{-3}\ \text{m}}{2}\right)^2} = 2.61\times10^{5}\ \text{W/m}^2$$

(a) To find the electric field, we use

$$S_{\text{avg}} = \frac{1}{\mu_0}E_{\text{rms}}B_{\text{rms}} = \frac{E_{\text{max}}^2}{2\mu_0 c}$$

Solving,

$$\begin{aligned} E_{\text{max}} &= \left(2\mu_0 c S_{\text{avg}}\right)^{1/2} \\ &= \Big[2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\times10^{8}\ \text{m/s}\right) \\ &\qquad \times\left(2.61\times10^{5}\ \text{W/m}^2\right)\Big]^{1/2} \\ &= 1.40\times10^{4}\ \text{N/C} = \boxed{14.0\ \text{kV/m}} \end{aligned}$$

(b) $B_{\max} = \dfrac{E_{\max}}{c} = \dfrac{1.40 \times 10^{4}\ \text{N/C}}{3.00 \times 10^{8}\ \text{m/s}} = 4.68 \times 10^{-5}\ \text{T} = \boxed{46.8\ \mu\text{T}}$

(c) Each photon carries momentum $\dfrac{E}{c}$. The beam transports momentum at the rate $\dfrac{P}{c}$. It imparts momentum to a perfectly reflecting surface at the rate

$$\frac{2P}{c} = \text{force} = \frac{2(0.628\ \text{W})}{3.00 \times 10^{8}\ \text{m/s}} = 4.19 \times 10^{-9}\ \text{N} = \boxed{4.19\ \text{nN}}$$

(d) The block of ice absorbs energy $mL = P\Delta t$ melting

$$m = \frac{P\Delta t}{L} = \frac{(0.628\ \text{W})\left[1.50(3\,600\ \text{s})\right]}{3.33 \times 10^{5}\ \text{J/kg}} = 1.02 \times 10^{-2}\ \text{kg} = \boxed{10.2\ \text{g}}$$

Section 40.5 The Wave Properties of Particles

P40.39 (a) $p = \dfrac{h}{\lambda} = \dfrac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{4.00 \times 10^{-7}\ \text{m}} = \boxed{1.66 \times 10^{-27}\ \text{kg} \cdot \text{m/s}}$

(b) From $p = m_e u$,

$$u = \frac{p}{m_e} = \frac{1.66 \times 10^{-27}\ \text{kg} \cdot \text{m/s}}{9.11 \times 10^{-31}\ \text{kg}} = 1.82 \times 10^{3}\ \text{m/s} = \boxed{1.82\ \text{km/s}}$$

P40.40 (a) Electron: $\lambda = \dfrac{h}{p}$ and $K = \dfrac{1}{2} m_e u^2 = \dfrac{m_e^2 u^2}{2m_e} = \dfrac{p^2}{2m_e}$,

so $p = \sqrt{2m_e K}$

and $\lambda = \dfrac{h}{\sqrt{2m_e K}} = \dfrac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31}\ \text{kg})(3.00)(1.60 \times 10^{-19}\ \text{J})}}$

$= 7.09 \times 10^{-10}\ \text{m} = \boxed{0.709\ \text{nm}}$

(b) Photon: $\lambda = \dfrac{c}{f}$ and $E = hf$, so $f = \dfrac{E}{h}$,

and $\lambda = \dfrac{hc}{E} = \dfrac{1240\ \text{eV} \cdot \text{nm}}{3.00\ \text{eV}} = \boxed{413\ \text{nm}}$.

P40.41 Since the de Broglie wavelength is $\lambda = \dfrac{h}{p}$, the electron momentum is:

$$p = \frac{h}{\lambda} \approx \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{1.00 \times 10^{-11}\ \text{m}} = 6.626 \times 10^{-23}\ \text{kg}\cdot\text{m/s}$$

(a) For electrons, the relativistic answer is more precisely correct. Suppressing units,

$$K_e = \sqrt{p^2c^2 + \left(m_ec^2\right)^2} - m_ec^2 = \sqrt{(pc)^2 + \left(m_ec^2\right)^2} - m_ec^2$$

$$= \sqrt{\left[\left(6.626\times10^{-23}\right)\left(2.998\times10^{8}\right)\left(\frac{1\ \text{MeV}}{1.602\times10^{-13}\ \text{J}}\right)\right]^2 + (0.511)^2}$$

$$-0.511$$

$$= 0.014\ 8\ \text{MeV} = \boxed{14.8\ \text{keV}}$$

$$\boxed{\text{or, ignoring relativistic correction,}}$$

$$K_e = \frac{p^2}{2m_e} = \frac{\left(6.626\times10^{-23}\right)^2}{2\left(9.11\times10^{-31}\right)}\left(\frac{1\ \text{keV}}{1.602\times10^{-16}\ \text{J}}\right) = \boxed{15.1\ \text{keV}}$$

(b) For photons (suppressing units):

$$E_\gamma = pc = \left(6.626\times10^{-23}\right)\left(2.998\times10^{8}\right)\left(\frac{1\ \text{keV}}{1.602\times10^{-16}\ \text{J}}\right)$$

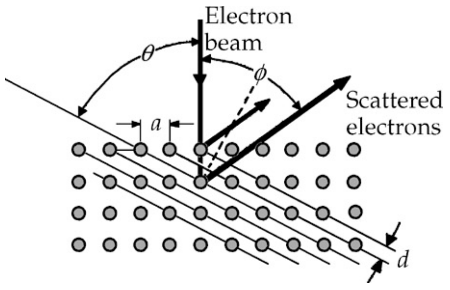
$$= \boxed{124\ \text{keV}}$$

P40.42 The de Broglie wavelength of the proton is

$$\lambda = \frac{h}{p} = \frac{h}{mu} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(1.00\times10^{6}\ \text{m/s}\right)}$$

$$= \boxed{3.97\times10^{-13}\ \text{m}}$$

P40.43 Refer to Figure P40.43 (or ANS. FIG. P40.43). For Bragg reflection, the angle θ is measured from the reflecting plane to the incident beam, as shown in Figure 38.23. Angle ϕ is measured from the incident beam to the reflected (scattered) beam. The law of reflection applies relative to the normal to the plane (the dashed line), so the angles of incidence and reflection are equal to $\phi/2$. The angle between the reflecting plane and



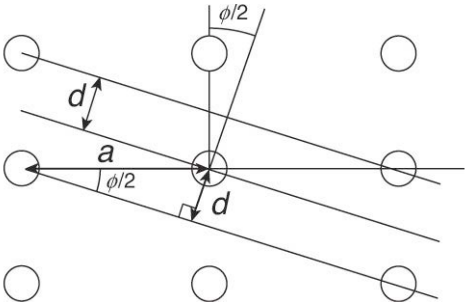
ANS. FIG. P40.43(a)

the normal is 90°, so

$$\theta + \frac{\phi}{2} = 90°$$

From the condition for Bragg reflection, we have

$$m\lambda = 2d\sin\theta = 2d\sin\left(90° - \frac{\phi}{2}\right)$$
$$= 2d\cos\left(\frac{\phi}{2}\right)$$



ANS. FIG. P40.43(b)

The vertical beam is incident along the normal to the horizontal lattice planes which contain atoms that are separated by distance a, and the reflecting lattice planes form the angle $\phi/2$ with the horizontal planes because the normal to the reflecting planes forms the angle $\phi/2$ with the vertical beam. Therefore, the spacing of the reflecting lattice planes is $d = a\sin\left(\frac{\phi}{2}\right)$.

Thus, for the first maximum, with $m = 1$,

$$\lambda = 2\left[a\sin\left(\frac{\phi}{2}\right)\right]\cos\left(\frac{\phi}{2}\right) = a\sin\phi$$

We know that

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_eK}}$$
$$= \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(54.0\times1.60\times10^{-19}\ \text{J}\right)}} = 1.67\times10^{-10}\ \text{m}$$

Therefore, the lattice spacing is

$$a = \frac{\lambda}{\sin\phi} = \frac{1.67\times10^{-10}\ \text{m}}{\sin 50.0°} = 2.18\times10^{-10} = \boxed{0.218\ \text{nm}}$$

P40.44 (a) $\lambda \sim 10^{-14}$ m or less, so $p = \frac{h}{\lambda} \sim \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{10^{-14}\ \text{m}} \approx 10^{-19}\ \text{kg}\cdot\text{m/s}$ or more. The energy of the electron is, suppressing units,

$$E = \sqrt{p^2c^2 + m_e^2c^4}$$
$$\sim \sqrt{\left(10^{-19}\right)^2\left(3\times10^8\right)^2 + \left(9\times10^{-31}\right)^2\left(3\times10^8\right)^4}$$

or $E \sim 10^{-11}\ \text{J} \sim 10^8\ \text{eV}$ or more so that

$$K = E - m_e c^2 \sim 10^8 \text{ eV} - \left(0.5 \times 10^6 \text{ eV}\right) \boxed{\sim 10^8 \text{ eV}} \text{ or more}$$

(b) If the nucleus contains ten protons, the electric potential energy of the electron-nucleus system would be

$$U_e = \frac{k_e q_1 q_2}{r} \sim \frac{\left(9 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left[10\left(1.60 \times 10^{-19} \text{ C}\right)\right](-e)}{0.5 \times 10^{-14} \text{ m}}$$

$$\boxed{\sim -10^6 \text{ eV}}$$

(c) With its $K + U_e \sim 10^8 \text{ eV} >> 0$, $\boxed{\text{the electron could not be confined to the nucleus}}$.

P40.45 (a) From $E = \gamma mc^2$,

$$\gamma = \frac{E}{mc^2} = \frac{20\ 000 \text{ MeV}}{0.511 \text{ MeV}} = \boxed{3.91 \times 10^4}$$

(b) We find the momentum of the particle from

$$pc = \left[E^2 - \left(mc^2\right)^2\right]^{1/2} = \left[\left(20\ 000 \text{ MeV}\right)^2 - \left(0.511 \text{ MeV}\right)^2\right]$$

$$= 20.0 \text{ GeV}$$

Then,

$$p = \boxed{20.0 \text{ GeV}/c = 1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}}$$

(c) The electron's wavelength is

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{1.07 \times 10^{-17} \text{ kg} \cdot \text{m/s}} = \boxed{6.21 \times 10^{-17} \text{ m}}$$

(d) $\boxed{\text{The wavelength is two orders of magnitude smaller than the size of the nucleus.}}$

P40.46 Given the assumption in the problem statement, for significant diffraction to occur, we must have

$$w \le 10\lambda = 10\left(\frac{h}{p}\right) = 10\left(\frac{h}{mu}\right)$$

where u is the speed of the student as he passes through the doorway. The variable we do not know here is the speed u, so let's solve for it:

$$u \le 10\left(\frac{h}{mw}\right)$$

This expression will give the upper limit to the speed of the student.

Substitute numerical values:

$$u \le 10\left[\frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{(80\ \text{kg})(0.75\ \text{m})}\right] = 1.1 \times 10^{-34}\ \text{m/s}$$

This is an extremely low velocity. It is impossible for the student to walk this slowly. At this speed, if the thickness of the wall in which the door is built is 15 cm, the time interval required for the student to pass through the door is 1.4×10^{33} s, which is 10^{15} times the age of the Universe.

P40.47 (a) For the electron,

$$K = (\gamma - 1)m_e c^2 \qquad \text{and} \qquad \lambda = \frac{h}{p} = \frac{h}{\gamma m_e u}$$

For the photon,

$$E_{\text{ph}} = K \qquad \text{and} \qquad \lambda_{\text{ph}} = \frac{c}{f} = \frac{ch}{E} = \frac{ch}{K} = \frac{ch}{(\gamma - 1)m_e c^2}$$

Then the ratio is

$$\frac{\lambda_{\text{ph}}}{\lambda} = \frac{ch}{(\gamma-1)m_e c^2}\frac{\gamma m_e u}{h} = \boxed{\frac{\gamma}{\gamma - 1}\frac{u}{c},\ \text{where } \gamma = \frac{1}{\sqrt{1 - u^2/c^2}}}$$

(b) For $u = 0.900c$,

$$\frac{\lambda_{\text{ph}}}{\lambda} = \frac{1}{\sqrt{1-(0.900)^2}}\frac{(0.900)}{\left[\left(1/\sqrt{1-(0.900)^2}\right) - 1\right]} = \boxed{1.60}$$

(c) The ratio for a particular particle speed does not depend on the particle mass: There would be $\boxed{\text{no change.}}$

(d) For $u = 0.001\,00c$,

$$\frac{\lambda_{\text{ph}}}{\lambda} = \frac{1}{\sqrt{1-(0.001\,00)^2}}\frac{(0.001\,00)}{\left[\left(1/\sqrt{1-(0.001\,00)^2}\right) - 1\right]} = \boxed{2.00 \times 10^3}$$

(e) As $\dfrac{u}{c} \to 1$, $\gamma \to \infty$ and $\gamma - 1$ becomes nearly equal to γ. Then,

$$\frac{\lambda_\gamma}{\lambda_m} \to \frac{\gamma}{\gamma}(1) = \boxed{1}$$

(f) As $\dfrac{u}{c} \to 0$, $\left(1 - \dfrac{u^2}{c^2}\right)^{-1/2} - 1 \approx 1 - \left(-\dfrac{1}{2}\right)\dfrac{u^2}{c^2} - 1 = \dfrac{1}{2}\dfrac{u^2}{c^2}$ and

$$\frac{\lambda_\gamma}{\lambda_m} \to 1\frac{u/c}{(1/2)(u^2/c^2)} = \frac{2c}{u} \to \boxed{\infty}$$

P40.48 (a) $E^2 = p^2c^2 + m^2c^4$ with $E = hf$,

$$p = \frac{h}{\lambda} \quad \text{and} \quad mc = \frac{h}{\lambda_C}$$

Substituting, we find that

$$h^2 f^2 = \frac{h^2c^2}{\lambda^2} + \frac{h^2c^2}{\lambda_C^2} \quad \text{and} \quad \left(\frac{f}{c}\right)^2 = \frac{1}{\lambda^2} + \frac{1}{\lambda_C^2}$$

(b) No. For a photon, $\frac{f}{c} = \frac{1}{\lambda}$. The third term $\frac{1}{\lambda_C}$ in the equation above for particles with mass shows that $\boxed{\text{they will always have a different frequency from photons of the same wavelength}}$.

Section 40.6 A New Model: The Quantum Particle

P40.49 (a) The particle is freely moving, so we attribute no potential energy to it. Its energy is

$$E = K = \frac{1}{2}mu^2 = hf = \left(\frac{h}{2\pi}\right)(2\pi f) = \hbar\omega$$

For its momentum we have

$$p = mu = \frac{h}{\lambda} = \left(\frac{h}{2\pi}\right)\left(\frac{2\pi}{\lambda}\right) = \hbar k$$

Thus,

$$\omega = \frac{K}{\hbar} \quad \text{and} \quad k = \frac{p}{\hbar}$$

Then the phase speed is

$$v_{\text{phase}} = f\lambda = \left(\frac{mu^2}{2h}\right)\left(\frac{h}{mu}\right) = \boxed{\frac{u}{2}}$$

(b) We see that the phase speed is only one-half of the experimentally measurable speed u at which the quantum particle transports mass, energy, and momentum. In the textbook's Active Figure 28.17, individual wave crests would move forward more slowly than their envelope moves forward, so individual crests would appear to move backward relative to the packet containing them.

P40.50 As a bonus, we begin by proving that the phase speed $v_p = \dfrac{\omega}{k}$ is not the speed of the particle.

$$v_p = \frac{\omega}{k} = \frac{\sqrt{p^2c^2 + m^2c^4}\hbar}{\hbar\gamma mu} = \frac{\sqrt{\gamma^2m^2u^2c^2 + m^2c^4}}{\sqrt{\gamma^2m^2u^2}}$$

$$= c\sqrt{1 + \frac{c^2}{\gamma^2u^2}} = c\sqrt{1 + \frac{c^2}{u^2}\left(1 - \frac{u^2}{c^2}\right)} = c\sqrt{1 + \frac{c^2}{u^2} - 1} = \frac{c^2}{u}$$

In fact, the phase speed is larger than the speed of light! A point of constant phase in the wave function carries no mass, no energy, and no information.

Now for the group speed:

$$v_g = \frac{d\omega}{dk} = \frac{d(\hbar\omega)}{d(\hbar k)} = \frac{dE}{dp} = \frac{d}{dp}\sqrt{m^2c^4 + p^2c^2}$$

$$= \frac{1}{2}\left(m^2c^4 + p^2c^2\right)^{-1/2}\left(0 + 2pc^2\right) = \sqrt{\frac{p^2c^4}{p^2c^2 + m^2c^4}}$$

$$= c\sqrt{\frac{\gamma^2m^2u^2}{\gamma^2m^2u^2 + m^2c^2}}$$

$$= c\sqrt{\frac{u^2/\left(1 - u^2/c^2\right)}{u^2/\left(1 - u^2/c^2\right) + c^2}} = c\sqrt{\frac{u^2/\left(1 - u^2/c^2\right)}{\left(u^2 + c^2 - u^2\right)/\left(1 - u^2/c^2\right)}} = u$$

It is this speed at which mass, energy, and momentum are transported.

Section 40.7 The Double-Slit Experiment Revisited

P40.51 (a) $\lambda = \dfrac{h}{mu} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(0.400\ \text{m/s}\right)} = 9.92\times10^{-7}\ \text{m} = \boxed{992\ \text{nm}}$

(b) For destructive interference in a multiple-slit experiment, $d\sin\theta = \left(m + \dfrac{1}{2}\right)\lambda$, with $m = 0$ for the first minimum. Also, $\dfrac{y}{L} = \tan\theta \approx \sin\theta = \left(\dfrac{1}{2}\right)\dfrac{\lambda}{d}$, so

$$y = L\tan\theta = \frac{\lambda L}{2d} = \frac{\left(9.92\times10^{-7}\ \text{m}\right)\left(10.0\ \text{m}\right)}{2\left(1.00\times10^{-3}\ \text{m}\right)} = \boxed{4.96\ \text{mm}}$$

(c) $\boxed{\text{No; there is no way to identify the slit through which the neutron passed. Even if one neutron at a time is incident on the pair of slits, an interference pattern still develops on the detector array. Therefore, each neutron in effect passes through both slits.}}$

P40.52 We find the speed of each electron from energy conservation in the firing process:

$$E = 0 = K_f + U_f = \frac{1}{2}mu^2 - \mathrm{eV}$$

$$u = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(45.0\ \mathrm{V}\right)}{9.11\times10^{-31}\ \mathrm{kg}}} = 3.98\times10^{6}\ \mathrm{m/s}$$

The time of flight is

$$\Delta t = \frac{\Delta x}{u} = \frac{0.280\ \mathrm{m}}{3.98\times10^{6}\ \mathrm{m/s}} = 7.04\times10^{-8}\ \mathrm{s}$$

The current when electrons are 28 cm apart is

$$I = \frac{q}{t} = \frac{e}{\Delta t} = \frac{1.60\times10^{-19}\ \mathrm{C}}{7.04\times10^{-8}\ \mathrm{s}} = \boxed{2.27\times10^{-12}\ \mathrm{A}}$$

P40.53 Consider the first bright band away from the center:

$$d\sin\theta = m\lambda$$

$$\left(0.060\,0\times10^{-6}\ \mathrm{m}\right)\sin\left[\tan^{-1}\left(\frac{0.400\times10^{-3}\ \mathrm{m}}{20.0\times10^{-2}\ \mathrm{m}}\right)\right] = (1)\lambda = 1.20\times10^{-10}\ \mathrm{m}$$

And since $\lambda = \frac{h}{p} = \frac{h}{m_e u}$, so $m_e u = \frac{h}{\lambda}$,

and

$$K = \frac{1}{2}m_e u^2 = \frac{m_e^2 u^2}{2m_e} = \frac{h^2}{2m_e\lambda^2} = e\Delta V \quad \rightarrow \quad \Delta V = \frac{h^2}{2em_e\lambda^2}$$

Therefore,

$$\Delta V = \frac{\left(6.626\times10^{-34}\ \mathrm{J\cdot s}\right)^2}{2\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(9.11\times10^{-31}\ \mathrm{kg}\right)\left(1.20\times10^{-10}\ \mathrm{m}\right)^2}$$

$$= \boxed{105\ \mathrm{V}}$$

Section 40.8 The Uncertainty Principle

P40.54 (a) The uncertainty principle states $\Delta p \Delta x = m\Delta u \Delta x \geq \frac{\hbar}{2}$, so

$$\Delta u \geq \frac{h}{4\pi m \Delta x} = \frac{2\pi \text{ J}\cdot\text{s}}{4\pi(2.00 \text{ kg})(1.00 \text{ m})} = \boxed{0.250 \text{ m/s}}$$

(b) The duck might move by (0.250 m/s)(5.00 s) = 1.25 m. With an original position uncertainty of 1.00 m, we can think of Δx growing to $1.00 \text{ m} + 1.25 \text{ m} = \boxed{2.25 \text{ m}}$.

P40.55 The uncertainty principle states $\Delta x \Delta p_x \geq \frac{\hbar}{2}$, where $\Delta p_x = m\Delta u$ and $\hbar = h/2\pi$.

Both the electron and bullet have a velocity uncertainty

$$\Delta u = (0.000\ 100)(500 \text{ m/s}) = 0.050\ 0 \text{ m/s}$$

For the electron, the minimum uncertainty in position is

$$\Delta x = \frac{h}{4\pi m \Delta u} = \frac{6.626\times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(9.11\times 10^{-31} \text{ kg})(0.050\ 0 \text{ m/s})} = \boxed{1.16 \text{ mm}}$$

For the bullet,

$$\Delta x = \frac{h}{4\pi m \Delta u} = \frac{6.626\times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(0.020\ 0 \text{ kg})(0.050\ 0 \text{ m/s})} = \boxed{5.28\times 10^{-32} \text{ m}}$$

P40.56 The momentum of the block is $p = mv$, and if the mass is known precisely, the uncertainty in the momentum is $\Delta p = m\Delta v$. From the uncertainty principle, $\Delta x \, \Delta p_x \geq \hbar/2$, so if there is an uncertainty of $\Delta x = 0.150 \text{ cm} = 1.50\times 10^{-3} \text{ m}$ in the position of the particle, the minimum uncertainty in its speed is

$$(\Delta v_x)_{\min} = \frac{(\Delta p_x)_{\min}}{m} = \frac{h}{4\pi m (\Delta x)_{\max}}$$

$$= \frac{6.63\times 10^{-34} \text{ J}\cdot\text{s}}{4\pi(0.500 \text{ kg})(1.50\times 10^{-3} \text{ m})} = \boxed{7.03\times 10^{-32} \text{ m/s}}$$

P40.57 The maximum time one can use in measuring the energy of the particle is equal to the lifetime of the particle, or $\Delta t_{\text{max}} \approx 2\ \mu\text{s}$. One form of the uncertainty principle is $\Delta E\,\Delta t \geq \hbar/2$. Thus, the minimum uncertainty one can have in the measurement of a muon's energy is

$$\Delta E_{\text{min}} = \frac{h}{4\pi\,\Delta t_{\text{max}}} = \frac{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(2\times 10^{-6}\ \text{s}\right)} = \boxed{3\times 10^{-29}\ \text{J} \approx 2\times 10^{-10}\ \text{eV}}$$

P40.58 Assume the rifle is firing horizontally and let the distance between the rifle and the target be *L*. The uncertainty in the vertical position of the particle as it leaves the end of the rifle is $\Delta y = 2.00$ mm. The uncertainty principle will allow us to approximate the uncertainty in the vertical momentum of the particles (ignoring gravitational acceleration):

$$\Delta y \Delta p_y \geq \frac{\hbar}{2} \quad \rightarrow \quad \Delta p_y \geq \frac{\hbar}{2\Delta y}$$

The time interval for the particle to reach the screen is, from the particle under constant velocity model,

$$\Delta t = \frac{L}{v_x}$$

During this time interval, again from the particle under constant velocity model, the particle moves in the vertical direction by a distance (again ignoring gravitational effects)

$$\Delta y_t = v_y \Delta t = v_y \frac{L}{v_x} = p_y \frac{L}{p_x}$$

where Δy_t is the vertical distance though which the particle moves when it arrives at the target and p_y is the vertical momentum of the particle. Because the particles begin with zero vertical momentum, let's assume that the vertical momentum of the particles is on the order of the uncertainty in the vertical momentum. Then,

$$\Delta y_t \approx \frac{\hbar}{2\Delta y}\ \frac{L}{p_x}$$

What we don't know in this expression is the distance *L*, so let's solve for it:

$$L \approx \frac{2p_x \Delta y \Delta y_t}{\hbar}$$

Substitute numerical values:

$$L \approx \frac{2(0.001\ 00\ \text{kg})(100\ \text{m/s})(0.002\ 00\ \text{m})(0.010\ 0\ \text{m})}{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}$$
$$\approx 4 \times 10^{28}\ \text{m}$$

According to Table 1.1, this distance is two orders of magnitude larger than the distance from the Earth to the most remote known quasar. In conclusion, then, for rifles fired at targets at reasonable distances away, a spread of 1.00 cm *due to the uncertainty principle* would be impossible.

P40.59 With $\Delta x = 1\times10^{-14}\ \text{m}$, the uncertainty principle requires

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\left(1 \times 10^{-14}\ \text{m}\right)} = 5.3\times10^{-21}\ \text{kg}\cdot\text{m/s}$$

The average momentum of the particle bound in a stationary nucleus is zero. The uncertainty in momentum measures the standard deviation of the momentum, so we take $p \approx 5.3\times10^{-21}\ \text{kg}\cdot\text{m/s}$.

For an electron, the non-relativistic approximation $p = m_e u$ would predict $u \approx 6\times10^{9}\ \text{m/s}$, which is impossible because u cannot be greater than c. Thus, a better solution would be to use

$$E = \left[\left(m_e c^2\right)^2 + \left(pc\right)^2\right]^{1/2} \approx 9.9\ \text{MeV} = \gamma m_e c^2$$

to find the speed (with $m_e c^2 = 0.511$ MeV):

$$\gamma \approx 19.4 = \frac{1}{\sqrt{1-u^2/c^2}} \quad \text{so} \quad u \approx 0.998\ 67c$$

For a proton,

$$u = \frac{p}{m} = \frac{5.3\times10^{-21}\ \text{kg}\cdot\text{m/s}}{1.67\times10^{-27}\ \text{kg}} = 3.2\times10^{6}\ \text{m/s} = 0.011c$$

about one-hundredth the speed of light.

Additional Problems

P40.60 From each wavelength we find the corresponding frequency using the relation $\lambda f = c$, where c is the speed of light:

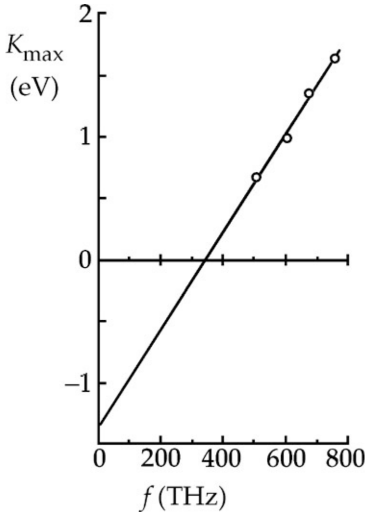
For $\lambda_1 = 588 \times 10^{-9}\ \text{m}$ $\quad f_1 = \frac{c}{\lambda_1} = 5.10 \times 10^{14}\ \text{Hz}$

For $\lambda_2 = 505 \times 10^{-9}$ m $\quad f_2 = 5.94 \times 10^{14}$ Hz

$\lambda_3 = 445 \times 10^{-9}$ m $\quad f_3 = 6.74 \times 10^{14}$ Hz

$\lambda_4 = 399 \times 10^{-9}$ m $\quad f_4 = 7.52 \times 10^{14}$ Hz

(a) We plot each point on an energy versus frequency graph, as shown in ANS. FIG. P40.60. We extend a straight line through the set of 4 points, as far as the negative *y* intercept.



ANS. FIG. P40.60

(b) Our basic equation is $K_{\text{max}} = hf - \phi$. Therefore, an experimental value for Planck's constant is the slope of the *K*-*f* graph, which can be found from a least-squares fit or from reading the graph as:

$$h_{\text{exp}} = \frac{\text{Rise}}{\text{Run}} = \frac{1.25 \text{ eV} - 0.25 \text{ eV}}{6.5 \times 10^{14} \text{ Hz} - 4.0 \times 10^{14} \text{ Hz}}$$

$$= 4.0 \times 10^{-15} \text{ eV} \cdot \text{s} = \boxed{6.4 \times 10^{-34} \text{ J} \cdot \text{s}}$$

From the scatter of the data points on the graph, we estimate the uncertainty of the slope to be about 3%. Thus we choose to show two significant figures in writing the experimental value of Planck's constant.

(c) Again from the linear equation $K_{\text{max}} = hf - \phi$, the work function for the metal surface is the negative of the *y*-intercept of the graph, so

$$\phi_{\text{exp}} = -(-1.4 \text{ eV}) = \boxed{1.4 \text{ eV}}$$

Based on the range of slopes that appear to fit the data, the estimated uncertainty of the work function is 5%.

***P40.61** From the circular path the electrons follow in the magnetic field, the magnetic force is centripetal,

$$F = ma: \qquad evB = \frac{m_e v^2}{R} \quad \rightarrow \quad m_e v = eBR$$

so the maximum kinetic energy is seen to be:

$$\begin{aligned} K_{\max} &= \frac{1}{2}m_e v^2 = \frac{(m_e v)^2}{2m_e} = \frac{e^2B^2R^2}{2m_e} \\ &= \frac{(1.602\times10^{-19}\text{ C})^2(2.00\times10^{-5}\text{ T})^2(0.200\text{ m})^2}{2(9.11\times10^{-31}\text{ kg})} \\ &= 2.25\times10^{-19}\text{ J} = 1.40\text{ eV} \end{aligned}$$

From the photoelectric equation,

$$K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$$

Thus, the work function is

$$\phi = \frac{hc}{\lambda} - K_{\max} = \frac{1\,240\text{ eV}\cdot\text{nm}}{450\text{ nm}} - 1.40\text{ eV} = \boxed{1.36\text{ eV}}$$

P40.62 From the circular path the electrons follow in the magnetic field, the magnetic force is centripetal,

$$F = ma: \qquad evB = \frac{m_e v^2}{R} \quad \rightarrow \quad m_e v = eBR$$

so maximum kinetic energy is seen to be:

$$K_{\max} = \frac{1}{2}m_e v^2 = \frac{(m_e v)^2}{2m_e} = \frac{e^2B^2R^2}{2m_e}$$

From the photoelectric equation,

$$K_{\max} = hf - \phi = \frac{hc}{\lambda} - \phi$$

Thus, the work function is

$$\phi = \frac{hc}{\lambda} - K_{\max} = \boxed{\frac{hc}{\lambda} - \frac{e^2B^2R^2}{2m_e}}$$

P40.63 The condition on electric power delivered to the filament is

$$P = I\,\Delta V = \frac{(\Delta V)^2}{R} = \frac{(\Delta V)^2 A}{\rho \ell} = \frac{(\Delta V)^2 \pi r^2}{\rho \ell}, \quad \text{so} \quad \ell = \frac{(\Delta V)^2 \pi r^2}{\rho P}.$$

Here $P = 75.0\text{ W}$, $\rho = 7.13\times10^{-7}\,\Omega\cdot\text{m}$, and $\Delta V = 120\text{ V}$. As the filament radiates in steady state, it must emit all of this power through its lateral surface area $P = \sigma eAT^4 = \sigma e2\pi r\ell T^4$.

(a) We combine the conditions by substitution:

$$P = \sigma e2\pi r\left[\frac{(\Delta V)^2\pi r^2}{\rho P}\right]T^4$$

$$r^3 = \frac{\rho P^2}{2\sigma e(\Delta V)^2\pi^2T^4}$$

$$= \frac{(7.13\times10^{-7}\,\Omega\cdot\text{m})(75.0\text{ W})^2}{2(5.67\times10^{-8}\text{ W/m}^2\text{K}^4)(0.450)(120\text{ V})^2\pi^2(2\ 900\text{ K})^4}$$

$$r = \boxed{1.98\times10^{-5}\text{ m}}$$

$$r = \left(\frac{P\rho\ell}{\pi(\Delta V)^2}\right)^{1/2} = \left[\frac{(75.0\text{ W})\ (7.13\times10^{-7}\,\Omega\cdot\text{m})\ (0.333\text{ m})}{\pi(120\text{ V})^2}\right]^{1/2}$$

$$= \boxed{1.98\times10^{-5}\text{ m}}$$

(b) $$\ell = \frac{(\Delta V)^2\pi r^2}{\rho P} = \frac{(120\text{ V})^2\pi r^2}{(7.13\times10^{-7}\,\Omega\cdot\text{m})(75.0\text{ W})} = \boxed{0.333\text{ m}}$$

P40.64 We first isolate the terms involving ϕ in Equations 40.13 and 40.14,

$$\gamma m_e u\cos\phi = \frac{h}{\lambda_0} - \frac{h}{\lambda'}\cos\theta$$

$$\gamma m_e u\sin\phi = \frac{h}{\lambda'}\sin\theta$$

We then square and add to eliminate ϕ:

$$(\gamma m_e u\cos\phi)^2 + (\gamma m_e u\sin\phi)^2 = \left(\frac{h}{\lambda_0} - \frac{h}{\lambda'}\cos\theta\right)^2 + \left(\frac{h}{\lambda'}\sin\theta\right)^2$$

$$\gamma^2 m_e^2 u^2 = h^2\left[\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

$$\frac{u^2/c^2}{(1-u^2/c^2)} = \frac{h^2}{m_e^2c^2}\left[\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

Defining $b = \frac{h^2}{m_e^2c^2}\left[\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}\right]$, the above equation becomes

$$\frac{u^2/c^2}{\left(1-u^2/c^2\right)} = b \qquad \to \qquad u^2/c^2 = b\left(1-u^2/c^2\right)$$

$$u^2/c^2 = \frac{b}{(1+b)}$$

Substitute into Equation 40.12 for the cutoff wavelength,

$$1+\left(\frac{h}{m_ec}\right)\left[\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right] = \gamma = \left(1-\frac{b}{1+b}\right)^{-1/2} = \sqrt{1+b}$$

Squaring each side then gives

$$1+\frac{2h}{m_ec}\left[\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right] + \frac{h^2}{m_e^2c^2}\left[\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right]^2$$

$$= 1+\left(\frac{h^2}{m_e^2c^2}\right)\left[\frac{1}{\lambda_0^2} + \frac{1}{\lambda'^2} - \frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

Eliminating terms,

$$\cancel{c^2} + \frac{2h}{m_ec}\left[\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right] + \frac{h^2}{m_e^2c^2}\left[\bcancel{\frac{1}{\lambda_0^2}} - \frac{2}{\lambda_0\lambda'} + \xcancel{\frac{1}{\lambda'^2}}\right]$$

$$= \cancel{c^2} + \left(\frac{h^2}{m_e^2c^2}\right)\left[\cancel{\frac{1}{\lambda_0^2}} + \xcancel{\frac{1}{\lambda'^2}} - \frac{2\cos\theta}{\lambda_0\lambda'}\right]$$

$$\frac{\bcancel{2}h}{m_ec}\left[\frac{1}{\lambda_0} - \frac{1}{\lambda'}\right] - \left(\frac{h^2}{m_e^2c^2}\right)\left(\frac{\bcancel{2}}{\lambda_0\lambda'}\right) = -\left(\frac{h^2}{m_e^2c^2}\right)\left(\frac{\bcancel{2}\cos\theta}{\lambda_0\lambda'}\right)$$

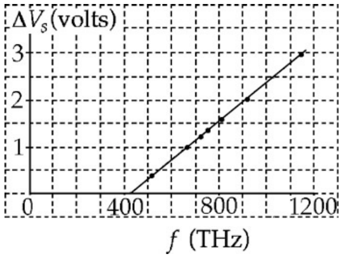
$$m_ec\left[\frac{\lambda'-\lambda_0}{\cancel{\lambda_0\lambda'}}\right] - h\left(\frac{1}{\cancel{\lambda_0\lambda'}}\right) = -h\left(\frac{\cos\theta}{\cancel{\lambda_0\lambda'}}\right)$$

$$m_ec\left(\lambda'-\lambda_0\right) - h = -h\cos\theta$$

Rearranging this gives Equation 40.11,

$$\lambda' - \lambda_0 = \left(\frac{h}{m_ec}\right)(1-\cos\theta)$$

P40.65 We use $\Delta V_S = \left(\frac{h}{e}\right)f - \frac{\phi}{e}$.



ANS. FIG. P40.65

From two points on the graph in ANS. FIG. P40.65,

$$0 = \left(\frac{h}{e}\right)\left(4.1\times 10^{14}\text{ Hz}\right) - \frac{\phi}{e}$$

and

$$3.3\text{ V} = \left(\frac{h}{e}\right)\left(12\times 10^{14}\text{ Hz}\right) - \frac{\phi}{e}$$

Combining these two expressions we find:

(a) $\phi = \boxed{1.7\text{ eV}}$

(b) $\frac{h}{e} = \boxed{4.2\times 10^{-15}\text{ V}\cdot\text{s}}$

(c) At the cutoff wavelength, $\frac{hc}{\lambda_c} = \phi = \left(\frac{h}{e}\right)\frac{ec}{\lambda_c}$, or

$$\lambda_c = \left(4.2\times 10^{-15}\text{ V}\cdot\text{s}\right)\left(1.60\times 10^{-19}\text{ C}\right) \times \frac{\left(3.00\times 10^{8}\text{ m/s}\right)}{\left(1.7\text{ eV}\right)\left(1.60\times 10^{-19}\text{ J/eV}\right)}$$

$$= \boxed{7.3\times 10^{2}\text{ nm}}$$

P40.66 Equation 40.11 states $\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda' - \lambda_0$ for the scattered photon. The initial energy of a photon is $E_0 = hc/\lambda_0$. Its energy after scattering is

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda_0 + \Delta\lambda} = hc\left[\lambda_0 + \frac{h}{m_e c}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2\lambda_0}(1-\cos\theta)\right]^{-1}$$

$$E' = \frac{hc}{\lambda_0}\left[1 + \frac{hc}{m_e c^2\lambda_0}(1-\cos\theta)\right]^{-1} = E_0\left[1 + \frac{E_0}{m_e c^2}(1-\cos\theta)\right]^{-1}$$

P40.67 (a) We use energy conservation in the daredevil-Earth system to find the speed of the daredevil just before he makes a splash:

$$mgy_i = \frac{1}{2}mu_f^2$$

gives

$$u_f = \sqrt{2gy_i} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(50.0\text{ m}\right)} = 31.3\text{ m/s}$$

The de Broglie wavelength is then

$$\lambda = \frac{h}{mu} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\left(75.0\text{ kg}\right)\left(31.3\text{ m/s}\right)} = \boxed{2.82\times10^{-37}\text{ m}}$$

This is too small to be observable.

(b) Equation 40.26 gives us the energy-lifetime version of the uncertainty principle:

$$\Delta E\Delta t \geq \frac{\hbar}{2}$$

substituting numerical values,

$$\Delta E \geq \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(5.00\times10^{-3}\text{ s}\right)} = \boxed{1.05\times10^{-32}\text{ J}}$$

(c) We find the percent error from

$$\frac{\Delta E}{E} = \frac{1.05\times10^{-32}\text{ J}}{\left(75.0\text{ kg}\right)\left(9.80\text{ m/s}^2\right)\left(50.0\text{ m}\right)} = \boxed{2.87\times10^{-35}\%}$$

P40.68 The definition of the Compton wavelength is $\lambda_C = h/m_ec$. The de Broglie wavelength is $\lambda = h/p$. We take the ratio of the Compton wavelength to the de Broglie wavelength, and square it:

$$\left(\frac{\lambda_C}{\lambda}\right)^2 = \frac{p^2}{\left(m_ec\right)^2}$$

From Equation 39.27, the momentum for a slowly-moving or rapidly-moving object is described by

$$p^2 = \frac{E^2 - m_e{}^2c^4}{c^2}$$

Substituting and simplifying,

$$\left(\frac{\lambda_C}{\lambda}\right)^2 = \frac{\left(E^2 - m_e^2c^4\right)}{\left(m_ec^2\right)^2} = \left(\frac{E}{m_ec^2}\right)^2 - 1$$

and $$\frac{\lambda_C}{\lambda} = \sqrt{\left(\frac{E}{m_e c^2}\right)^2 - 1}$$

P40.69 (a) We find the energy of one photon:

$$\begin{aligned} hf &= K_{\max} + \phi \\ &= \frac{1}{2}\left(9.11\times10^{-31}\text{ kg}\right)\left(420\times10^{3}\text{ m/s}\right)^2 \\ &\qquad + \left(3.44\text{ eV}\right)\left(\frac{1.6\times10^{-19}\text{ J}}{1\text{ eV}}\right) = 6.31\times10^{-19}\text{ J} \end{aligned}$$

The number intensity of photon bombardment is

$$\begin{aligned} \frac{I}{hf} &= \frac{550\text{ J/s}\cdot\text{m}^2}{6.31\times10^{-19}\text{ J/photon}}\left(\frac{1\text{ m}^2}{10^4\text{ cm}^2}\right)\left(\frac{1\text{ electron emitted}}{1\text{ photon absorbed}}\right) \\ &= \boxed{8.72\times10^{16}\ \frac{\text{electrons}}{\text{s}\cdot\text{cm}^2}} \end{aligned}$$

(b) The density of the current the imagined electrons comprise is

$$\begin{aligned} J &= \left(8.72\times10^{16}\ \frac{\text{electrons}}{\text{s}\cdot\text{cm}^2}\right)\left(1.60\times10^{-19}\ \frac{\text{C}}{\text{electron}}\right) \\ &= 0.014\ 0\ \frac{\text{C}}{\text{s}\cdot\text{cm}^2} = \boxed{14.0\text{ mA/cm}^2} \end{aligned}$$

(c) Many photons are likely reflected or give their energy to the metal as internal energy, so the actual current is probably a small fraction of 14.0 mA.

P40.70 From the uncertainty principle,

$$\Delta E\Delta t \ge \frac{\hbar}{2} \qquad \rightarrow \qquad \Delta\left(mc^2\right)\Delta t = \frac{\hbar}{2}$$

Therefore,

$$\begin{aligned} \frac{\Delta m}{m} &= \frac{h}{4\pi c^2(\Delta t)m} = \frac{h}{4\pi(\Delta t)E_R} \\ &= \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi\left(8.70\times10^{-17}\text{ s}\right)\left(135\text{ MeV}\right)}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) \\ &= \boxed{2.81\times10^{-8}} \end{aligned}$$

P40.71 (a) To find the de Broglie wavelength of the neutron, we first determine its momentum,

$$\begin{aligned} p = mu &= \sqrt{2mE} \\ &= \sqrt{2\left(1.67\times10^{-27}\text{ kg}\right)\left(0.040\,0\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} \\ &= 4.62\times10^{-24}\text{ kg}\cdot\text{m/s} \end{aligned}$$

Then,

$$\lambda = \frac{h}{mu} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4.62\times10^{-24}\text{ kg}\cdot\text{m/s}} = 1.43\times10^{-10}\text{ m} = \boxed{0.143\text{ nm}}$$

(b) $\boxed{\text{This is of the same order of magnitude as the spacing between atoms in a crystal.}}$

(c) $\boxed{\text{Because the wavelength is about the same as the spacing, diffraction effects should occur.}}$

A diffraction pattern with maxima and minima at the same angles can be produced with x-rays, with neutrons, and with electrons of much higher kinetic energy, by using incident quantum particles with the same wavelength.

Challenge Problems

***P40.72** (a) At the top of the ladder, the woman holds a pellet inside a small region Δx_i. Thus, the uncertainty principle requires her to release it with typical horizontal momentum $\Delta p_x = m\Delta v_x = \dfrac{\hbar}{2\Delta x_i}$. It falls to the floor in a travel time given by $H = 0 + \dfrac{1}{2}gt^2$ as $t = \sqrt{\dfrac{2H}{g}}$, so the total width of the impact points is

$$\Delta x_f = \Delta x_i + \left(\Delta v_x\right)t = \Delta x_i + \left(\frac{\hbar}{2m\Delta x_i}\right)\sqrt{\frac{2H}{g}} = \Delta x_i + \frac{A}{\Delta x_i}$$

where $A = \dfrac{\hbar}{2m}\sqrt{\dfrac{2H}{g}}$.

To minimize Δx_f, we require $\dfrac{d\left(\Delta x_f\right)}{d\left(\Delta x_i\right)} = 0$ or $1 - \dfrac{A}{\Delta x_i^2} = 0$,

so $\Delta x_i = \sqrt{A}$.

The minimum width of the impact points is

$$\left(\Delta x_f\right)_{\min} = \left(\Delta x_i + \frac{A}{\Delta x_i}\right)\Bigg|_{\Delta x_i=\sqrt{A}} = 2\sqrt{A} = \boxed{\sqrt{\frac{2\hbar}{m}}\left(\frac{2H}{g}\right)^{1/4}}$$

(b) $$\left(\Delta x_f\right)_{\min} = \left[\frac{2\left(1.054\ 6\times 10^{-34}\ \text{J}\cdot\text{s}\right)}{5.00\times 10^{-4}\ \text{kg}}\right]^{1/2}\left[\frac{2(2.00\ \text{m})}{9.80\ \text{m/s}^2}\right]^{1/4}$$

$$= \boxed{5.19\times 10^{-16}\ \text{m}}$$

P40.73 (a) $\boxed{\text{The Doppler shift increases the apparent frequency of the incident light.}}$

(b) If $v = 0.280c$,

$$f' = f\sqrt{\frac{1+v/c}{1-v/c}} = \left(7.00\times 10^{14}\ \text{Hz}\right)\sqrt{\frac{1.28}{0.720}} = 9.33\times 10^{14}\ \text{Hz}$$

Therefore,

$$\phi = hf'$$

$$= \left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(9.33\times 10^{14}\ \text{Hz}\right)\left(\frac{1\ \text{eV}}{1.602\times 10^{-19}\ \text{J}}\right)$$

$$= \boxed{3.86\ \text{eV}}$$

(c) At $v = 0.900c$,

$$f' = f\sqrt{\frac{1+v/c}{1-v/c}} = \left(7.00\times 10^{14}\ \text{Hz}\right)\sqrt{\frac{1.900}{0.100}} = 3.05\times 10^{15}\ \text{Hz}$$

and

$$K_{\max} = hf' - \phi$$

$$= \left[\left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.05\times 10^{15}\ \text{Hz}\right)\left(\frac{1.00\ \text{eV}}{1.602\times 10^{-19}\ \text{J}}\right)\right] - 3.86\ \text{eV}$$

$$= \boxed{8.76\ \text{eV}}$$

P40.74 We show that if all of the energy of a photon is transmitted to an electron, momentum will not be conserved. In general, a photon of energy $E_0 = hc/\lambda_0$ scatters off an electron at rest, resulting in the photon having energy $E' = hc/\lambda'$ and the electron having kinetic energy K_e. Energy conservation requires $E_0 = E' + K_e$, or

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'} + m_e c^2(\gamma - 1)$$

If the photon is absorbed, then $E' = hc/\lambda' = 0$, and the above equation becomes

$$\frac{hc}{\lambda_0} = m_e c^2 (\gamma - 1) \qquad \textbf{[1]}$$

Because the photon is absorbed, momentum conservation requires the momentum of the electron be in the same direction as the momentum of the original photon:

$$p_0 = \frac{E}{c} = \frac{h}{\lambda_0} = \gamma m_e u \qquad \textbf{[2]}$$

From [1], we find that

$$\gamma = \frac{h}{\lambda_0 m_e c} + 1 \qquad \textbf{[3]}$$

and $$u = c\sqrt{1 - \left(\frac{\lambda_0 m_e c}{h + \lambda_0 m_e c}\right)^2} \qquad \textbf{[4]}$$

Substituting [3] and [4] into [2] reveals the inconsistency:

$$\begin{aligned}\frac{h}{\lambda_0} &= \left(1 + \frac{h}{\lambda_0 m_e c}\right) m_e c \sqrt{1 - \left(\frac{\lambda_0 m_e c}{h + \lambda_0 m_e c}\right)^2} \\ &= \frac{\lambda_0 m_e c + h}{\lambda_0}\sqrt{\frac{h(h + 2\lambda_0 m_e c)}{(h + \lambda_0 m_e c)^2}} = \frac{h}{\lambda_0}\sqrt{\frac{h + 2\lambda_0 m_e c}{h}}\end{aligned}$$

This is impossible, so all of the energy of a photon cannot be transmitted to an electron.

P40.75 (a) Starting with Planck's law,

$$I(\lambda, T) = \frac{2\pi hc^2}{\lambda^5 \left[e^{hc/\lambda k_B T} - 1\right]}$$

the total power radiated per unit area

$$\int_0^\infty I(\lambda, T)\, d\lambda = \int_0^\infty \frac{2\pi hc^2}{\lambda^5 \left[e^{hc/\lambda k_B T} - 1\right]} d\lambda$$

Change variables by letting $x = \dfrac{hc}{\lambda k_B T}$, so $dx = -\dfrac{hc}{k_B T \lambda^2} d\lambda$.

Note that as λ varies from $0 \to \infty$, x varies from $\infty \to 0$.

Then,

$$\int_0^\infty I(\lambda,\ T)d\lambda = -\frac{2\pi k_B^4 T^4}{h^3c^2}\int_\infty^0 \frac{x^3}{(e^x-1)}dx = \frac{2\pi k_B^4T^4}{h^3c^2}\left(\frac{\pi^4}{15}\right)$$

Therefore,

$$\boxed{\int_0^\infty I(\lambda,\ T)d\lambda = \left(\frac{2\pi^5k_B^4}{15h^3c^2}\right)T^4 = \sigma T^4}$$

(b) From part (a),

$$\sigma = \frac{2\pi^5k_B^4}{15h^3c^2} = \frac{2\pi^5\left(1.38\times10^{-23}\ \text{J/K}\right)^4}{15\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3\left(3.00\times10^8\ \text{m/s}\right)^2}$$

$$\sigma = \boxed{5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4}$$

P40.76 (a) Planck's law states

$$I(\lambda,\ T) = \frac{2\pi hc^2}{\lambda^5\left[e^{hc/\lambda k_BT}-1\right]} = 2\pi hc^2\lambda^{-5}\left[e^{hc/\lambda k_BT}-1\right]^{-1}.$$

To find the wavelength at which this distribution has a maximum, compute

$$\frac{dI}{d\lambda} = 2\pi hc^2\left\{-5\lambda^{-6}\left[e^{hc/\lambda k_BT}-1\right]^{-1} - \lambda^{-5}\left[e^{hc/\lambda k_BT}-1\right]^{-2}e^{hc/\lambda k_BT}\left(-\frac{hc}{\lambda^2k_BT}\right)\right\} = 0$$

$$\frac{dI}{d\lambda} = \frac{2\pi hc^2}{\lambda^6\left[e^{hc/\lambda k_BT}-1\right]}\left\{-5+\frac{hc}{\lambda k_BT}\frac{e^{hc/\lambda k_BT}}{\left[e^{hc/\lambda k_BT}-1\right]}\right\} = 0$$

Letting $x = \dfrac{hc}{\lambda k_BT}$, the condition for a maximum becomes $\dfrac{xe^x}{e^x-1} = 5$. We zero in on the solution to this transcendental equation by iterations as shown in the table on the following page.

x	$xe^x/(e^x-1)$
4.000 00	4.074 629 4
4.500 00	4.550 552 1
5.000 00	5.033 918 3
4.900 00	4.936 762 0
4.950 00	4.985 313 0
4.975 00	5.009 609 0
4.963 00	4.997 945 2
4.969 00	5.003 776 7
4.966 00	5.000 860 9

x	$xe^x/(e^x-1)$
4.964 50	4.999 403 0
4.965 50	5.000 374 9
4.965 00	4.999 889 0
4.965 25	5.000 132 0
4.965 13	5.000 015 3
4.965 07	4.999 957 0
4.965 10	4.999 986 2
4.965 115	5.000 000 8

The solution is found to be

$$x = \frac{hc}{\lambda_{\text{max}} k_B T} = 4.965\,115 \qquad \text{and} \qquad \lambda_{\text{max}} T = \frac{hc}{4.965\,115 k_B}$$

(b) Thus,

$$\lambda_{\text{max}} T = \frac{(6.626\,075 \times 10^{-34}\ \text{J} \cdot \text{s})(2.997\,925 \times 10^{8}\ \text{m/s})}{4.965\,115(1.380\,658 \times 10^{-23}\ \text{J/K})}$$

$$= \boxed{2.897\,755 \times 10^{-3}\ \text{m} \cdot \text{K}}$$

This result agrees with Wien's experimental value of $\lambda_{\text{max}} T = 2.898 \times 10^{-3}\ \text{m} \cdot \text{K}$ for this constant.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P40.2 (a) 999 nm; (b) The wavelength emitted at the greatest intensity is in the infrared (greater than 700 nm), and according to the graph in Active Figure 40.3, much more energy is radiated at wavelengths longer than $\lambda_{\max}$ than at shorter wavelengths.

P40.4 (a) 5 200 K; (b) This is not blackbody radiation.

P40.6 i: (a) 2.57 eV, (b) 1.28×10^{-5} eV, (c) 1.91×10^{-7} eV; ii: (a) 484 nm, (b) 9.68 cm, (c) 6.52 m; iii: (a) visible light (blue), (b) radio wave, (c) radio wave

P40.8 2.27×10^{30} photon/s

P40.10 (a) 5.78×10^{3} K; (b) 501 nm

P40.12 (a) 7.09×10^{4} W; (b) 580 nm; (c) 7.99×10^{10} W/m; (d–i) See table in P40.12; (j) ≈ 19 kW

P40.14 See P40.14 for full explanation.

P40.16 (a) 4.20 mm; (b) 1.05×10^{19} photons; (c) 8.82×10^{16} mm^{-3}

P40.18 (a) 288 nm; (b) 1.04×10^{15} Hz; (c) 1.19 eV

P40.20 (a) The energy of a photon with wavelength 400 nm is calculated to be 3.11 eV. Now compare this energy with the given work functions. Of these metals, only lithium shows the photoelectric effect because its work function is less than the energy of the photon; (b) 0.808 eV

P40.22 (a) 148 days; (b) The result for part (a) does not agree at all with the experimental observations.

P40.24 (a) 8.27 eV; (b) The photon energy is larger than the work function; (c) 1.92 eV; (d) 1.92 V

P40.26 4.85×10^{-12} m

P40.28 (a and b) See P40.28 for full answer; (c) 180°. We could answer like this: The photon imparts the greatest momentum to the originally stationary electron in a head-on collision. Here the photon recoils straight back, and the electron has maximum kinetic energy.

P40.30 (a) 2.89 pm; (b) $\theta = 101°$

P40.32 (a) $\theta = \cos^{-1}\left(\dfrac{m_e c^2 + E_0}{2m_e c^2 + E_0}\right)$; (b) $E' = \dfrac{E_0\left(2m_e c^2 + E_0\right)}{2\left(m_e c^2 + E_0\right)}$, $p' = \dfrac{E_0\left(2m_e c^2 + E_0\right)}{2c\left(m_e c^2 + E_0\right)}$;

(c) $K_e = \dfrac{E_0^2}{2\left(m_e c^2 + E_0\right)}$, $p_e = \dfrac{E_0\left(2m_e c^2 + E_0\right)}{2c\left(m_e c^2 + E_0\right)}$

P40.34	(a) It is because Compton's equation and the conservation of vector momentum give three independent equations in the unknowns λ', λ_0, and *u*; (b) 3.82 pm
P40.36	(a) 0.667; (b) 0.001 09
P40.38	(a) 14.0 kV/m; (b) $46.8\ \mu\text{T}$; (c) 4.19 nN; (d) 10.2 g
P40.40	(a) 0.709 nm; (b) 413 nm
P40.42	3.97×10^{-13} m
P40.44	(a) $\sim 10^{8}$ eV; (b) $\sim -10^{6}$ eV; (c) The electron could not be confined to the nucleus.
P40.46	The speed with which the student passes through the door is an extremely low velocity. It is impossible for the student to walk this slowly. At this speed, if the thickness of the wall in which the door is built is 15 cm, the time interval required for the student to pass through the door is 1.4×10^{33} s, which is 10^{15} times the age of the Universe.
P40.48	(a) See P40.48(a) for full explanation; (b) They will always have a different frequency from photons of the same wavelength.
P40.50	See P40.50 for the full explanation.
P40.52	2.27×10^{-12} A
P40.54	(a) 0.250 m/s; (b) 2.25 m
P40.56	7.03×10^{-32} m/s
P40.58	For the rifles fired at targets at reasonable distances away, a spread of 1.00 cm *due to the uncertainty principle* would be impossible.
P40.60	(a) See graph in ANS. FIG. P40.60 (b) $6.4 \times 10^{-34}\ \text{J} \cdot \text{s}$; (c) 1.4 eV
P40.62	$\dfrac{hc}{\lambda} - \dfrac{e^2B^2R^2}{2m_e}$
P40.64	See P40.64 for full explanation.
P40.66	See P40.66 for full explanation.
P40.68	See P40.68 for full explanation.
P40.70	2.81×10^{-8}
P40.72	(a) See P40.72(a) for full explanation; (b) 5.19×10^{-16} m
P40.74	See P40.74 for full explanation.
P40.76	(a) See P40.76 for full explanation; (b) $2.897\,755 \times 10^{-3}\ \text{m} \cdot \text{K}$

41

Quantum Mechanics

CHAPTER OUTLINE

41.1 The Wave Function

41.2 Analysis Model: Quantum Particle Under Boundary Conditions

41.3 The Schrödinger Equation

41.4 A Particle in a Well of Finite Height

41.5 Tunneling Through a Potential Energy Barrier

41.6 Applications of Tunneling

41.7 The Simple Harmonic Oscillator

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ41.1 Answer (b). Fewer particles are reflected as the height of the potential barrier decreases and approaches the energy of the particles. By Equations 41.22 and 41.23, the transmission coefficient $T \approx e^{-2CL}$, where $C = \sqrt{2m(U-E)}/\hbar$, increases as $U - E$ decreases, so the reflection coefficient $R = 1 - T \approx 1 - e^{-2CL}$ decreases as $U - E$ decreases.

OQ41.2 The ranking is answer (b) > (a) > (c) > (e) > (d). From Equation 41.14, consider the quantity

$$E = \left(\frac{h^2}{8mL^2}\right)n^2:$$

(a) $\left[\dfrac{h^2}{8m_1(3\text{ nm})^2}\right](1)^2 = \dfrac{1}{9}\left(\dfrac{h^2}{8m_1}\text{ nm}^{-1}\right)$

(b) $\left[\dfrac{h^2}{8m_1(3\text{ nm})^2}\right](2)^2 = \dfrac{4}{9}\left(\dfrac{h^2}{8m_1}\text{ nm}^{-1}\right)$

(c) $\left[\dfrac{h^2}{8(2m_1)(3\text{ nm})^2}\right](1)^2 = \dfrac{1}{18}\left(\dfrac{h^2}{8m_1}\text{ nm}^{-1}\right)$

(d) $\left[\dfrac{(0)^2}{8m_1(3\text{ nm})^2}\right](1)^2 = 0$

(e) $\left[\dfrac{h^2}{8m_1(6\text{ nm})^2}\right](1)^2 = \dfrac{1}{36}\left(\dfrac{h^2}{8m_1}\text{ nm}^{-1}\right)$

OQ41.3 (a) True. Examples: An electron has mass and charge, but it can also display interference effects.

(b) False. An electron has rest energy $E_R = m_e c^2$.

(c) True. A moving electron possesses kinetic energy.

(d) True. $p = m_e u$.

(e) True.

OQ41.4 (a) True. Examples: A photon behaves as a particle in the photoelectric effect and as a wave in double-slit interference.

(b) True. A photon cannot have rest energy (mass) because it is never at rest: it travels at the speed of light.

(c) True. $E = hf$.

(d) True. $p = E/c$.

(e) True.

OQ41.5 Answer (d). The probability of finding the particle is at the antinodes (places of greatest amplitude) of the standing wave.

OQ41.6 Compare the ground state wave functions in Figures 41.4 and 41.7 in the text. In the square well with infinitely high walls, the particle's simplest wave function has strict nodes separated by the length L of the well. The particle's wavelength is $2L$, its momentum $h/2L$, and its energy $p^2/2m = h^2/8mL^2$. In the well with walls of only finite height, the wave function has nonzero amplitude at the walls, and it extends outside the walls.

(i) Answer (a). The ground state wave function extends somewhat outside the walls of the finite well, so the particle's wavelength is longer.

(ii) Answer (b). The particle's momentum in its ground state is smaller because $p = h/\lambda$ and the wave function has a larger wavelength.

(iii) Answer (b). The particle has less energy because is has smaller momentum.

OQ41.7 Answer (e). From the relation between the square of the wave function and the probability *P* of finding the particle in the interval Δx = (7 nm – 4 nm) = 3 nm, we have

$$|\psi|^2 \Delta x = P \qquad \rightarrow \qquad \psi = \sqrt{\frac{P}{\Delta x}} = \sqrt{\frac{0.48}{3\text{ nm}}} = 0.40\text{ nm}^{-1}$$

OQ41.8 Answer (a). Because of the exponential tailing of the wave function within the barrier, the tunneling current is more sensitive to the width of the barrier than to its height. Notice that the exponent term *CL* in the transmission coefficient $T \approx e^{-2CL}$, where $C = \sqrt{2m(U-E)}/\hbar$, decreases more if *L* decreases than if *U* decreases by the same percentage.

OQ41.9 Answer (c). Other points see a wider potential-energy barrier and carry much less tunneling current.

OQ41.10 Answer (d). The probability of finding the particle is greatest at the place of greatest amplitude of the wave function. The next most likely place is point *b*, after that, points *a* and *e* appear to be equally probable. The particle would never be found at point *c*.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ41.1 Consider the Heisenberg uncertainty principle. It implies that electrons initially moving at the same speed and accelerated by an electric field through the same distance *need not* all have the same measured speed after being accelerated. Perhaps the philosopher could have said "it is necessary for the very existence of science that the same conditions always produce the same results within the uncertainty of the measurements."

CQ41.2 Consider a particle bound to a restricted region of space. If its minimum energy were zero, then the particle could have zero momentum and zero uncertainty in its momentum. At the same time, the uncertainty in its position would not be infinite, but equal to the width of the region. In such a case, the uncertainty product $\Delta x \Delta p_x$ would be zero, violating the uncertainty principle. This contradiction proves that the minimum energy of the particle is not zero.

CQ41.3 The motion of the quantum particle does not consist of moving through successive points. The particle has no definite position. It can sometimes be found on one side of a node and sometimes on the other side, but never at the node itself. There is no contradiction here, for the quantum particle is moving as a wave. It is not a classical particle. In particular, the particle does not speed up to infinite speed to cross the node.

CQ41.4 (a) $\psi(x)$ becomes infinite as $x \to \infty$.

(b) $\psi(x)$ is discontinuous and becomes infinite at $x = \pi/2, 3\pi/2, \ldots$

CQ41.5 A particle's wave function represents its state, containing all the information there is about its location and motion. The squared absolute value of its wave function tells where we would classically think of the particle as spending most its time. $|\Psi|^2$ is the probability distribution function for the position of the particle.

CQ41.6 In quantum mechanics, particles are treated as wave functions, not classical particles. In classical mechanics, the kinetic energy is never negative. That implies that $E \geq U$. Treating the particle as a wave, the Schrödinger equation predicts that there is a nonzero probability that a particle can tunnel through a barrier—a region in which $E < U$.

CQ41.7 Both (d) and (e) are not physically significant. Wave function (d) is not acceptable because ψ is not single-valued. Wave function (e) is not acceptable because ψ is discontinuous (as is its slope).

CQ41.8 Newton's 1st and 2nd laws are used to determine the motion of a particle of large mass. The Schrödinger equation is not used to determine the motion of a particle of small mass; rather, it is used to determine the state of the wave function of a particle of small mass. In particular, the states of atomic electrons are confined-wave states whose wave functions are solutions to the Schrödinger equation. Anything that we can know about a particle comes from its wave function.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 41.1 The Wave Function

P41.1 (a) The wave function,

$$\psi(x) = Ae^{i\left(5\times10^{10}x\right)} = A\cos\left(5\times10^{10}x\right) + iA\sin\left(5\times10^{10}x\right)$$

will go through one full cycle between $x_1 = 0$ and $(5.00 \times 10^{10})x_2 = 2\pi$. The wavelength is then

$$\lambda = x_2 - x_1 = \frac{2\pi}{5.00\times10^{10}\ \text{m}^{-1}} = \boxed{1.26\times10^{-10}\ \text{m}}$$

To say the same thing, we can inspect $Ae^{i\left(5\times10^{10}x\right)}$ to see that the wave number is $k = 5.00 \times 10^{10}\ \text{m}^{-1} = 2\pi/\lambda$.

(b) Since $\lambda = h/p$, the momentum is

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{1.26\times10^{-10}\ \text{m}} = \boxed{5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

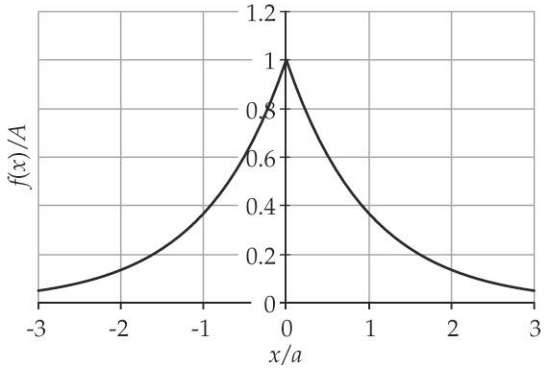
(c) The electron's kinetic energy is

$$K = \frac{1}{2}mu^2 = \frac{p^2}{2m}$$

$$= \frac{\left(5.27\times10^{-24}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)}\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) = \boxed{95.3\ \text{eV}}$$

[We use u to represent the speed of a particle with mass in chapters 39, 40, and 41.]

P41.2 (a) See ANS. FIG. P41.2 for a graph of $\frac{f(x)}{A} = e^{-|x|/a}$ for the range $-3 < \frac{x}{a} < 3$.



ANS. FIG. P41.2

(b) Normalization requires

$$\int_{\text{all space}} |\psi|^2\,dx = 1:$$

$$\int_{-\infty}^{\infty} A^2 e^{-2|x|/a}dx = 2\int_0^{\infty} A^2 e^{-2|x|/a}dx = 1$$

$$-aA^2e^{-2|x|/a}\Big|_0^{\infty} = aA^2 = 1 \quad \rightarrow \quad A = \boxed{\frac{1}{\sqrt{a}}}$$

(c) $P = \int_{-a}^{a} \frac{e^{-2|x|/a}}{a}dx = 2\int_0^a \frac{e^{-2|x|/a}}{a}dx = -e^{-2x/a}\Big|_0^a = -e^{-2}+1 = \boxed{0.865}$

P41.3 (a) Normalization requires

$$\int_{\text{all space}} |\psi|^2\,dx = 1:$$

$$\int_0^{1.00} A^2x^2dx = 1$$

$$\frac{A^2x^3}{3}\Bigg|_0^{1.00} = \frac{A^{23}}{3} = 1 \qquad \rightarrow \qquad \boxed{A = \sqrt{3}}$$

(b) $P = \int_{0.300}^{0.400} 3x^2\,dx = x^3\Big|_{0.300}^{0.400} = (0.400)^2 - (0.300)^2 = \boxed{0.037\ 0}$

(c) The expectation value is

$$\langle x \rangle = \int_{\text{all space}} \psi^* x\psi\,dx = \int_0^{1.00} 3x^3\,dx = \frac{3x^4}{4}\Bigg|_0^{1.00} = \boxed{0.750}$$

P41.4 The probability is given by

$$P = \int_{-a}^{a} |\psi(x)|^2 = \int_{-a}^{a} \frac{a}{\pi\left(x^2+a^2\right)}dx = \left(\frac{a}{\pi}\right)\left(\frac{1}{a}\right)\tan^{-1}\left(\frac{x}{a}\right)\Bigg|_{-a}^{a}$$

$$P = \frac{1}{\pi}\left[\tan^{-1}1 - \tan^{-1}(-1)\right] = \frac{1}{\pi}\left[\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right] = \boxed{\frac{1}{2}}$$

Section 41.2 Analysis Model: Quantum Particle Under Boundary Conditions

P41.5 (a) The energy of a quantum particle confined to a line segment is

$$E_n = \frac{h^2 n^2}{8mL^2}$$

Here we have for the ground state

$$E_1 = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2 (1)^2}{8\left(1.67 \times 10^{-27}\text{kg}\right)\left(2.00 \times 10^{-14}\text{m}\right)^2}$$

$$= 8.22 \times 10^{-14}\text{J} = \boxed{0.513\ \text{MeV}}$$

and for the first and second excited states, which are states 2 and 3,

$$E_2 = 4E_1 = \boxed{2.05\ \text{MeV}} \quad \text{and} \quad E_3 = 9E_1 = \boxed{4.62\ \text{MeV}}$$

(b) $\boxed{\text{They do; the MeV is the natural unit for energy radiated by an atomic nucleus.}}$

Stated differently: Scattering experiments show that an atomic nucleus is a three-dimensional object always less than 15 fm in diameter. This one-dimensional box 20 fm long is a good model in energy terms.

P41.6 From Equation 41.14, the allowed energy levels of a particle in a box is

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2, \qquad n = 1, 2, 3, \ldots$$

(a) For $L = 1.00$ nm,

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2$$

$$= \left(\frac{1\ \text{eV}}{1.60\times 10^{-19}\ \text{J}}\right)\left[\frac{\left(6.626\times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(9.11\times 10^{-31}\ \text{kg}\right)\left(1.00\times 10^{-9}\ \text{m}\right)^2}\right]n^2$$

$$= 0.377n^2 = 6\ \text{eV}$$

$$\boxed{n \approx 4}$$

(b) For $n = 4$, $E_n = 0.377(4)^2 = \boxed{6.03\ \text{eV}}$

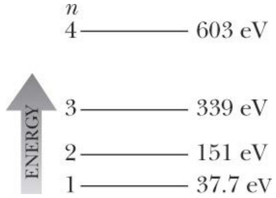
P41.7 (a) From Equation 41.14, the allowed energy levels of an electron in a box is

$$E_n = \left(\frac{h^2}{8m_e L^2}\right)n^2 \qquad n = 1, 2, 3, \ldots$$

Substituting numerical values,

$$E_n = \left[\frac{\left(6.626 \times 10^{-34}\ \mathrm{J \cdot s}\right)^2}{8\left(9.11 \times 10^{-31}\ \mathrm{kg}\right)\left(0.100 \times 10^{-9}\ \mathrm{m}\right)^2} \right] n^2$$

$$= \left(6.02 \times 10^{-18}\ \mathrm{J}\right) n^2 = (37.7\ \mathrm{eV}) n^2$$



ANS. FIG. P41.7

(b) When the electron falls from higher level n_i to lower level n_f, it emits energy

$$\Delta E_n = \left(\frac{h^2}{8 m_e L^2} \right)\left(n_i^2 - n_f^2\right) = (37.7\ \mathrm{eV})\left(n_i^2 - n_f^2\right)$$

by emitting a photon of wavelength

$$\lambda = \frac{hc}{\Delta E_n} = \frac{8 m_e c L^2}{h\left(n_i^2 - n_f^2\right)}$$

$$= \frac{8\left(9.109 \times 10^{-31}\ \mathrm{kg}\right)\left(2.998 \times 10^{8}\ \mathrm{m/s}\right)\left(0.100 \times 10^{-9}\ \mathrm{m}\right)^2}{\left(6.626 \times 10^{-34}\ \mathrm{J \cdot s}\right)\left(n_i^2 - n_f^2\right)} \times \left(\frac{1\ \mathrm{nm}}{10^{-9}\ \mathrm{m}} \right)$$

$$= \frac{33.0\ \mathrm{nm}}{\left(n_i^2 - n_f^2\right)}$$

For example, for the transition $4 \rightarrow 3$, the wavelength is

$$\lambda = \frac{33.0\ \mathrm{nm}}{(4)^2 - (3)^2} = 4.71\ \mathrm{nm}$$

The wavelengths produced by all possible transitions are:

Transition	$4 \rightarrow 3$	$4 \rightarrow 2$	$4 \rightarrow 1$	$3 \rightarrow 2$	$3 \rightarrow 1$	$2 \rightarrow 1$
λ (nm)	4.71	2.75	2.20	6.59	4.12	11.0

P41.8 The energy of the photon is

$$E = \frac{hc}{\lambda} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{6.06\ \text{mm}}\left(\frac{1\ \text{mm}}{10^6\,\text{nm}}\right) = 2.05\times10^{-4}\text{eV}$$

The allowed energies of the proton in the box are

$$\begin{aligned}E_n &= \left(\frac{h^2}{8mL^2}\right)n^2\\ &= \left[\frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{8\left(1.673\times10^{-27}\ \text{kg}\right)\left(1.00\times10^{-9}\ \text{m}\right)^2}\right]\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right)n^2\\ &=\left(2.05\times10^{-4}\ \text{eV}\right)n^2\end{aligned}$$

The smallest possible energy for a transition between states is from $n = 1$ to $n = 2$, which has energy

$$\Delta E_n = \left(2.05\times10^{-4}\,\text{eV}\right)\left(2^2-1^2\right) = 6.14\times10^{-4}\,\text{eV}$$

The photon does not have enough energy to cause this transition. The photon energy would be sufficient to cause a transition from $n = 0$ to $n = 1$, but the $n = 0$ state does not exist for the particle in a box.

P41.9 From Equation 41.14,

$$\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_eL^2}\right)\left[2^2-1^2\right] = \frac{3h^2}{8m_eL^2}$$

Solving for the length of the box then gives

$$\begin{aligned}L &= \sqrt{\frac{3h\lambda}{8m_ec}}\\ &= \sqrt{\frac{3\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(694.3\times10^{-9}\ \text{m}\right)}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}}\\ &= 7.95\times10^{-10}\ \text{m} = \boxed{0.795\ \text{nm}}\end{aligned}$$

P41.10 From Equation 41.14,

$$\Delta E = \frac{hc}{\lambda} = \left(\frac{h^2}{8m_eL^2}\right)\left[2^2-1^2\right] = \frac{3h^2}{8m_eL^2}$$

Solving for the length of the box then gives

$$L = \boxed{\sqrt{\frac{3h\lambda}{8m_ec}}}$$

P41.11 From Equation 41.14, the allowed energy levels of a particle in a box is

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2 = n^2 E_1 \qquad n = 1, 2, 3, \ldots$$

For a proton (m = 1.673 × 10^{-27} kg) in a 10.0-fm wide box:

$$\begin{aligned} E_1 &= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(1.673\times10^{-27}\ \text{kg})(10.0\times10^{-15}\ \text{m})^2} \\ &= 3.28\times10^{-13}\ \text{J}\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) = 2.05\times10^{6}\ \text{eV} = 2.05\ \text{MeV} \end{aligned}$$

(a) The energy of the emitted photon is

$$E = \Delta E_n = E_2 - E_1 = (2)^2 E_1 - E_1 = 3E_1 = \boxed{6.14\ \text{MeV}}$$

(b) The wavelength of the photon is

$$\begin{aligned} \lambda &= \frac{hc}{E} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{6.14\times10^{6}\ \text{eV}} \\ &= 2.02\times10^{-4}\ \text{nm} = 2.02\times10^{-13}\ \text{m} = 202\times10^{-15}\ \text{m} = \boxed{202\ \text{fm}} \end{aligned}$$

(c) This is a $\boxed{\text{gamma ray}}$, according to the electromagnetic spectrum chart in Chapter 34.

P41.12 The ground state energy of a particle (mass m) in a 1-dimensional box of width L is $E_1 = \dfrac{h^2}{8mL^2}$.

(a) For a proton (m = 1.67 × 10^{-27} kg) in a 0.200-nm wide box:

$$\begin{aligned} E_1 &= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(1.67\times10^{-27}\ \text{kg})(2.00\times10^{-10}\ \text{m})^2} \\ &= 8.22\times10^{-22}\ \text{J} = \boxed{5.13\times10^{-3}\ \text{eV}} \end{aligned}$$

(b) For an electron (m = 9.11 × 10^{-31} kg) in the same size box:

$$\begin{aligned} E_1 &= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{8(9.11\times10^{-31}\ \text{kg})(2.00\times10^{-10}\ \text{m})^2} \\ &= 1.51\times10^{-18}\ \text{J} = \boxed{9.41\ \text{eV}} \end{aligned}$$

(c) $\boxed{\text{The electron has a much higher energy because it is much less massive.}}$

P41.13 $E_1 = 2.00 \text{ eV} = 3.20 \times 10^{-19}$ J. For the ground state,

$$E_1 = \frac{h^2}{8m_eL^2}$$

(a) The length of the region is

$$L = \frac{h}{\sqrt{8m_eE_1}} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{8\left(9.11\times10^{-31}\text{ kg}\right)\left(3.20\times10^{-19}\text{ J}\right)}}$$

$$= 4.34\times10^{-10}\text{ m} = \boxed{0.434\text{ nm}}$$

(b) For the excited states, $E_n = \left(\frac{h^2}{8m_eL^2}\right)n^2 = n^2E_1$. For the first excited state, $\Delta E = E_2 - E_1 = 4E_1 - E_1 = 3E_1 = \boxed{6.00\text{ eV}}$

P41.14 (a) The classical kinetic energy of the particle is

$$K = \frac{1}{2}mv^2 = \frac{1}{2}\left(4.00\times10^{-3}\text{ kg}\right)\left(1.00\times10^{-3}\text{ m/s}\right)^2$$

$$= \boxed{2.00\times10^{-9}\text{ J}}$$

(b) The length *L* can be found from

$$E = \left(\frac{h^2}{8mL^2}\right)n^2$$

Solving,

$$L = n\sqrt{\frac{h^2}{8mE}} = 2\sqrt{\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{8\left(4.00\times10^{-3}\text{ kg}\right)\left(2.00\times10^{-9}\text{ J}\right)}}$$

$$= \boxed{1.66\times10^{-28}\text{ m}}$$

(c) $\boxed{\text{No. The length of the box would have to be much smaller than the size of a nucleus } (\sim 10^{-14}\text{ m}) \text{ to confine the particle.}}$

***P41.15** (a) The energies of the confined electron are $E_n = \frac{h^2}{8m_eL^2}n^2$. Its energy gain in the quantum jump from state 1 to state 4 is $\frac{h^2}{8m_eL^2}\left(4^2 - 1^2\right)$, and this is the photon energy $\frac{h^2 15}{8m_eL^2} = hf = \frac{hc}{\lambda}$.

Then $8m_ecL^2 = 15h\lambda$ and $\boxed{L = \left(\frac{15h\lambda}{8m_ec}\right)^{1/2}}$.

(b) Let λ' represent the wavelength of the photon emitted:

$$\frac{hc}{\lambda'} = \frac{h^2}{8m_eL^2}4^2 - \frac{h^2}{8m_eL^2}2^2 = \frac{12h^2}{8m_eL^2}$$

Then $\frac{hc}{\lambda}\frac{\lambda'}{hc} = \frac{h^2 15\left(8m_eL^2\right)}{8m_eL^2 12h^2} = \frac{5}{4}$ and $\boxed{\lambda' = 1.25\lambda}$.

P41.16 (a) From $\Delta x\Delta p \geq \frac{\hbar}{2}$, with $\Delta x = L$, $\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{\hbar}{2L}$, so the uncertainty in momentum must be at least $\Delta p \approx \boxed{\frac{\hbar}{2L}}$.

(b) Its energy is all kinetic, so

$$E = \frac{p^2}{2m} = \frac{(\Delta p)^2}{2m} \approx \boxed{\frac{\hbar^2}{8mL^2}} = \frac{h^2}{(4\pi)^2 8mL^2}$$

(c) Compare the result of part (b) to the result $h^2/8mL^2$ for the wave function as a standing wave. $\boxed{\text{This estimate is too low by } 4\pi^2 \approx 40 \text{ times, but it correctly displays the pattern of dependence of the energy on the mass and on the length of the well.}}$

P41.17 (a) $\int_{-\infty}^{\infty} |\psi|^2\,dx = 1$ becomes

$$A^2 \int_{-L/4}^{L/4} \cos^2\left(\frac{2\pi x}{L}\right)dx = A^2 \int_{-L/4}^{L/4} \frac{1 + \cos\left[2\left(2\pi x/L\right)\right]}{2}dx = 1$$

$$\frac{A^2}{2}\left[x + \frac{L}{4\pi}\cos\left(\frac{4\pi x/L}{L}\right)\right]\Bigg|_{-L/4}^{L/4} = 1$$

$$\frac{A^2}{2}\left(\frac{L}{2}\right) = 1 \rightarrow \boxed{A = \frac{2}{\sqrt{L}}}$$

(b) The probability of finding the particle between 0 and $\frac{L}{8}$ is

$$\int_0^{L/8} |\psi|^2\,dx = A^2 \int_0^{L/8} \cos^2\left(\frac{2\pi x}{L}\right)dx = \frac{A^2}{2}\left[x + \frac{L}{4\pi}\cos\left(\frac{4\pi x}{L}\right)\right]\Bigg|_0^{L/8}$$

$$\frac{1}{2}\left(\frac{4}{L}\right)\left[\frac{L}{8} + \frac{L}{4\pi}\sin\left(\frac{\pi}{2}\right)\right] = \frac{1}{4} + \frac{1}{2\pi} = \boxed{0.409}$$

P41.18 Normalization requires $\int\limits_{\text{all space}} |\psi|^2 dx = 1$:

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right)dx = \int_0^L A^2 \frac{1-\cos\left[2(\pi x/L)\right]}{2}dx = 1$$

$$= \frac{A^2}{2}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^L = 1$$

$$= \frac{A^2}{2}\left[L - \frac{L}{2\pi}\sin 2\pi\right]_0^L = \frac{A^2 L}{2} = 1$$

$$A = \sqrt{\frac{2}{L}}$$

P41.19 (a) The expectation value is $\langle x \rangle = \int_0^L \psi^* x\psi\, dx$:

$$\langle x \rangle = \int_0^L x\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_0^L x\left\{\frac{1-\cos\left[2(\pi x/L)\right]}{2}\right\}dx$$

$$= \frac{1}{L}\int_0^L x\left(1-\cos\frac{4\pi x}{L}\right)dx$$

From integral tables, we find that

$$\langle x \rangle = \frac{1}{L}\frac{x^2}{2}\bigg|_0^L - \frac{1}{L}\frac{L^2}{16\pi^2}\left[\frac{4\pi x}{L}\sin\frac{4\pi x}{L} + \cos\frac{4\pi x}{L}\right]_0^L = \boxed{\frac{L}{2}}$$

(b) The probability of finding the particle in the range $0.490L \le x \le 0.510L$ is

$$P = \frac{2}{L}\int_{0.490L}^{0.510L} \sin^2\left(\frac{2\pi x}{L}\right)dx = \frac{2}{L}\int_{0.490L}^{0.510L} \frac{1-\cos\left[2(2\pi x/L)\right]}{2}dx$$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{4\pi x}{L}\right)\right]_{0.490L}^{0.510L}$$

$$= 0.020 - \frac{1}{4\pi}\left(\sin 2.04\pi - \sin 1.96\pi\right) = \boxed{5.26\times 10^{-5}}$$

(c) The probability of finding the particle in the range $0.240L \le x \le 0.260L$ is

$$P = \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{4\pi x}{L}\right)\right]_{0.240L}^{0.260L} = \boxed{3.99\times 10^{-2}}$$

(d) $\boxed{\text{In the } n = 2 \text{ graph in the text's Figure 41.4(b), it is more probable to find the particle either near } x = L/4 \text{ or } x = 3L/4 \text{ than at the center, where the probability density is zero. Nevertheless, the symmetry of the distribution means that the average position is } x = L/2.}$

P41.20 (a) The most probable positions of the particle are $\boxed{x = L/4,\ L/2, \text{ and } 3L/4}$.

(b) $\boxed{\text{We look for } \sin(3\pi x/L) \text{ taking on its extreme values 1 and } -1 \text{ so that the squared wave function is as large as it can be. The result can also be found by studying Figure 41.4b.}}$ The most probable locations are at the antinodes of the standing wave pattern $n = 3$, which has three antinodes that are equally spaced, one at the center, and two a distance $L/4$ from either end.

P41.21 (a) The probability of finding the electron between $x = 0$ and $x = 0.100$ nm $= L/3$ is

$$\int_0^{L/3} |\psi_1|^2\,dx = \frac{2}{L}\int_0^{L/3} \sin^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^{L/3} \frac{1-\cos\left[2\left(\pi x/L\right)\right]}{2}\,dx$$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]_0^{L/3}$$

$$= \frac{1}{3} - \frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right) = \frac{1}{3} - \frac{0.866}{2\pi} = \boxed{0.196}$$

(b) Classically, the particle moves back and forth steadily, spending equal time intervals in each third of the line. $\boxed{\text{The classical probability is 0.333, which is significantly larger.}}$

(c) The probability is

$$\int_0^{L/3} |\psi_{99}|^2\,dx = \frac{2}{L}\int_0^{L/3} \sin^2\left(\frac{99\pi x}{L}\right)dx = \frac{1}{L}\int_0^{L/3}\left[1-\cos\left(\frac{198\pi x}{L}\right)\right]dx$$

$$= \frac{1}{L}\left[x - \frac{L}{198\pi}\sin\left(\frac{198\pi x}{L}\right)\right]_0^{L/3}$$

$$= \frac{1}{3} - \frac{1}{198\pi}\sin(66\pi) = \frac{1}{3} - 0 = 0.333$$

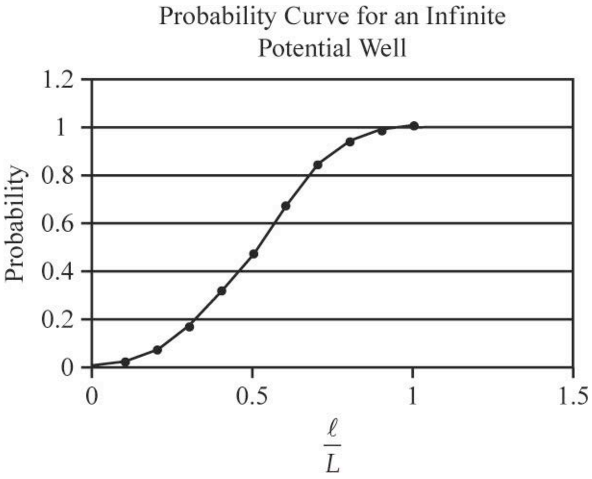
The probability is $\boxed{0.333 \text{ for both classical and quantum models}}$.

P41.22 (a) From Equation 41.13, $\psi_1(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{\pi x}{L}\right)$. The probability of finding the particle between $x = 0$ and $x = \ell$ is

$$\int_0^\ell |\psi_1|^2\,dx = \frac{2}{L}\int_0^\ell \sin^2\left(\frac{\pi x}{L}\right)dx = \frac{2}{L}\int_0^\ell \frac{1-\cos\left[2(\pi x/L)\right]}{2}dx$$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]\Bigg|_0^\ell = \boxed{\frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi \ell}{L}\right)}$$

(b) The probability function is sketched in ANS. FIG. P41.22(b).



ANS. FIG. P41.22(b)

(c) $\boxed{\text{The wave function is zero for } x < 0 \text{ and for } x > L.}$

The wave function is zero for $x < 0$ and for $x > L$. The probability at $\ell = 0$ must be zero because the particle is never found at $x < 0$ or exactly at $x = 0$. The probability at $\ell = L$ must be 1 for normalization: the particle is always found somewhere in the range $0 < x < L$.

(d) The probability of finding the particle between $x = 0$ and $x = \ell$ is $\frac{2}{3}$, and between $x = \ell$ and $x = L$ is $\frac{1}{3}$.

Thus, $$\int_0^\ell |\psi_1|^2\,dx = \frac{2}{3}$$

$$\therefore \frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi\ell}{L}\right) = \frac{2}{3}$$

or, defining $u = \frac{\ell}{L}$, $$u - \frac{1}{2\pi}\sin 2\pi u = \frac{2}{3}$$

This equation for u can be solved by homing in on the solution with a calculator, the result being $u = \frac{\ell}{L} = 0.585$, or $\ell = \boxed{0.585L}$ to three digits.

P41.23 (a) The probability is

$$P = \int_0^{L/3} |\psi_1|^2\,dx = \frac{2}{L}\int_0^{L/3} \sin^2\left(\frac{\pi x}{L}\right)dx$$

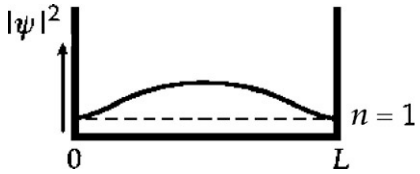
$$= \frac{2}{L}\int_0^{L/3} \frac{1-\cos\left[2(\pi x/L)\right]}{2}dx$$

$$= \frac{1}{L}\left[x - \frac{L}{2\pi}\sin\left(\frac{2\pi x}{L}\right)\right]\Bigg|_0^{L/3} = \frac{1}{L}\left[\frac{L}{3} - \frac{L}{2\pi}\sin\left(\frac{2\pi}{3}\right)\right]$$

$$= \frac{1}{3} - \frac{1}{2\pi}\sin\left(\frac{2\pi}{3}\right)$$

$$= \left(\frac{1}{3} - \frac{\sqrt{3}}{4\pi}\right) = \boxed{0.196}$$

(b) The probability density is symmetric about $x = \frac{L}{2}$. Thus, the probability of finding the particle between $x = \frac{2L}{3}$ and $x = L$ is the same, 0.196. Therefore, the probability of finding it in the range $\frac{L}{3} \le x \le \frac{2L}{3}$ is $P = 1.00 - 2(0.196) = \boxed{0.609}$.



ANS. FIG. P41.23(b)

Section 41.3 The Schrödinger Equation

P41.24 From $\psi = Ae^{i(kx - \omega t)}$ **[1]**

we evaluate

$$\frac{d\psi}{dx} = ikAe^{i(kx - \omega t)}$$

and $$\frac{d^2\psi}{dx^2} = -k^2Ae^{i(kx - \omega t)} \qquad \textbf{[2]}$$

We substitute equations [1] and [2] into the Schrödinger equation, so that Equation 41.15,

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$$

becomes the test equation

$$\left(-\frac{\hbar^2}{2m}\right)\left(-k^2Ae^{i(kx - \omega t)}\right) + 0 = EAe^{i(kx - \omega t)} \qquad \textbf{[3]}$$

The wave function $\psi = Ae^{i(kx - \omega t)}$ is a solution to the Schrödinger equation if equation [3] is true. Both sides depend on A, x, and t in the same way, so we can cancel several factors, and determine that we have a solution if

$$\frac{\hbar^2k^2}{2m} = E$$

But this is true for a nonrelativistic particle with mass in a region where the potential energy is zero, since

$$\frac{\hbar^2k^2}{2m} = \frac{1}{2m}\left(\frac{h}{2\pi}\right)^2\left(\frac{2\pi}{\lambda}\right)^2 = \underbrace{\frac{(h/\lambda)^2}{2m} = \frac{p^2}{2m}}_{\text{using de Broglie's equation}}$$

$$= \frac{m^2u^2}{2m} = \tfrac{1}{2}mu^2 = \underbrace{K = K + U}_{\text{recall } U=0} = E$$

where K is the kinetic energy. Therefore, the given wave function does satisfy Equation 41.15.

P41.25 (a) Given the function

$$\psi(x) = A\cos kx + B\sin kx$$

Its derivative with respect to x is

$$\frac{\partial\psi}{\partial x} = -kA\sin kx + kB\cos kx$$

And its second derivative is

$$\frac{\partial^2\psi}{\partial x^2} = -k^2 A\cos kx - k^2 B\sin kx$$
$$= -k^2\left(A\cos kx + B\sin kx\right) = -k^2\psi$$

The Schrödinger equation is satisfied if

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi, \qquad \text{where} \quad U = 0:$$

$$-\frac{\hbar^2}{2m}\left(-k^2\psi\right) = E\psi \quad \rightarrow \quad \frac{\hbar^2 k^2}{2m}\psi = E\psi$$

This is true as an identity (functional equality) for all x if $E = \frac{\hbar^2 k^2}{2m}$, which is true because $E = K + U = K + 0 = K$, and

$$\frac{\hbar^2 k^2}{2m} = \frac{1}{2m}\left(\frac{h}{2\pi}\right)^2\left(\frac{2\pi}{\lambda}\right)^2 = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2 = \frac{p^2}{2m} = K$$

(b) From part (a), $E = \boxed{\frac{\hbar^2 k^2}{2m}}$.

P41.26 (a) These are standing wave patterns with nodes at the ends and n antinodes.

For $n = 1$, the wave function is

$$\psi_1(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{\pi x}{L}\right)$$

and the probability density is

$$P_1(x) = \left|\psi_1(x)\right|^2 = \frac{2}{L}\cos^2\left(\frac{\pi x}{L}\right)$$

For $n = 2$, the wave function is

$$\psi_2(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$$

and the probability density is

$$P_2(x) = \left|\psi_2(x)\right|^2 = \frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)$$

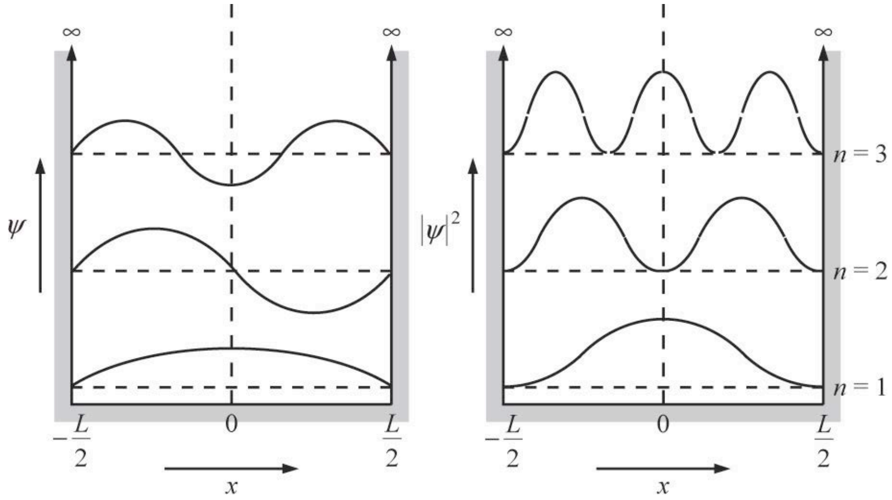
For $n = 3$, the wave function is

$$\psi_3(x) = \sqrt{\frac{2}{L}}\cos\left(\frac{3\pi x}{L}\right)$$

and the probability density is

$$P_3(x) = |\psi_3(x)|^2 = \frac{2}{L}\cos^2\left(\frac{3\pi x}{L}\right)$$

(b) The wave functions and probability densities are shown in ANS. FIG. P41.26(b).



ANS. FIG. P41.26(b)

P41.27 (a) Setting the total energy *E* equal to zero and rearranging the Schrödinger equation to isolate the potential energy function gives

$$\left(\frac{\hbar^2}{2m}\right)\frac{d^2\psi}{dx^2} + U(x)\psi = 0$$

$$U(x) = \left(\frac{\hbar^2}{2m}\right)\frac{1}{\psi}\frac{d^2\psi}{dx^2}$$

If $\psi(x) = Axe^{-x^2/L^2}$

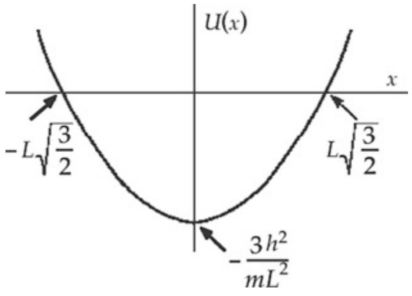
Then,

$$\frac{d^2\psi}{dx^2} = \left(4Ax^3 - 6AxL^2\right)\frac{e^{-x^2/L^2}}{L^4}$$

or $\frac{d^2\psi}{dx^2} = \frac{\left(4x^2 - 6L^2\right)}{L^4}\psi(x)$

and $\boxed{U(x) = \frac{\hbar^2}{2mL^2}\left(\frac{4x^2}{L^2} - 6\right)}$

(b) U(x) is sketched in ANS. FIG. P41.27(b).



ANS. FIG. P41.27(b)

P41.28 (a) $\psi(x)=A\left(1-\frac{x^2}{L^2}\right) \rightarrow \frac{d\psi}{dx}=-\frac{2Ax}{L^2} \rightarrow \frac{d^2\psi}{dx^2}=-\frac{2A}{L^2}$

Schrödinger's equation:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+U\psi=E\psi$$

becomes

$$-\frac{\hbar^2}{2m}\left(-\frac{2A}{L^2}\right)+\frac{\left(-\hbar^2x^2\right)}{mL^2\left(L^2-x^2\right)}A\left(1-\frac{x^2}{L^2}\right)=EA\left(1-\frac{x^2}{L^2}\right)$$

$$-\frac{\hbar^2}{\cancel{2}m}\left(-\frac{\cancel{2}}{L^2}\right)+\frac{\left(-\hbar^2x^2\right)\cancel{\left(L^2-x^2\right)}}{mL^4\cancel{\left(L^2-x^2\right)}}=E\left(1-\frac{x^2}{L^2}\right)$$

$$\frac{\hbar^2}{mL^2}+\frac{\left(-\hbar^2x^2\right)}{mL^4}=E\left(1-\frac{x^2}{L^2}\right)$$

$$\frac{\hbar^2}{mL^2}\left(1-\frac{x^2}{L^4}\right)=E\left(1-\frac{x^2}{L^2}\right)$$

This will be true for all *x* if $\boxed{E=\frac{\hbar^2}{L^2m}}$.

(b) Note that the wave function $\psi(x)$ is an even function; therefore, we may write the normalization condition as

$$\int_{-L}^{L}|\psi|^2\,dx=1=\int_{-L}^{L}A^2\left(1-\frac{x^2}{L^2}\right)^2dx=2\int_0^L A^2\left(1-\frac{x^2}{L^2}\right)^2dx$$

$$=2A^2\int_0^L\left(1-\frac{2x^2}{L^2}+\frac{x^4}{L^4}\right)dx$$

Solving,

$$1 = 2A^2\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^4}\right]_0^L = 2A^2\left[L - \frac{2}{3}L + \frac{L}{5}\right]$$

$$= A^2\left(\frac{16L}{15}\right) \quad \rightarrow \quad \boxed{A = \sqrt{\frac{15}{16L}}}$$

(c) As in part (b), because the wave function is an even function, the probability is

$$P = \int_{-L/3}^{L/3} \psi^2\,dx = \int_0^{L/3} \psi^2\,dx = 2\frac{15}{16L}\int_0^{L/3}\left(1 - \frac{2x^2}{L^2} + \frac{x^4}{L^4}\right)dx$$

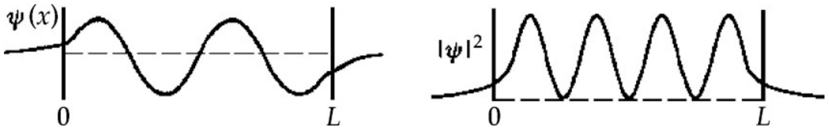
$$= \frac{15}{8L}\left[x - \frac{2x^3}{3L^2} + \frac{x^5}{5L^5}\right]_0^{L/3}$$

$$= \frac{15}{8L}\left[\frac{L}{3} - \frac{2L}{81} + \frac{L}{1215}\right] = \frac{47}{81} = \boxed{0.580}$$

Section 41.4 A Particle in a Well of Finite Height

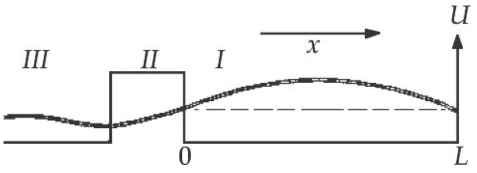
P41.29 (a) For $n = 4$, the wave function has two maxima and two minima (four extrema), as shown in the left-hand panel of ANS. FIG. P41.29.

(b) For $n = 4$, the probability function has four maxima. as shown in the right-hand panel of ANS. FIG. P41.29.



ANS. FIG. P41.29

P41.30 (a) See ANS. FIG. P41.30(a).



ANS. FIG. P41.30(a)

(b) The wavelength inside the box is 2*L*. The wave function penetrates the wall, but the wavelength of the transmitted wave traveling to the left is the same, $\boxed{2L}$, because *U* = 0 on both sides of the wall, so the energy and momentum and, therefore, the wavelength, are the same.

Section 41.5 Tunneling Through a Potential Energy Barrier

P41.31 The decay constant for the wave function inside the barrier is:

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}$$

$$= \frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left(10.0\text{ eV}-5.00\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}}{6.626\times10^{-34}\text{ J}\cdot\text{s}/2\pi}$$

$$= 1.14\times10^{10}\text{ m}^{-1}$$

(a) The approximate probability of transmission is

$$T \approx e^{-2CL} = e^{-2\left(1.14\times10^{10}\text{ m}^{-1}\right)\left(2.00\times10^{-10}\text{ m}\right)} = \boxed{0.010\ 3}$$

or a 1% chance of transmission.

(b) $R = 1 - T = \boxed{0.990}$, a 99% chance of reflection.

P41.32 (a) $T = e^{-2CL}$, where

$$C = \frac{\sqrt{2m(U-E)}}{\hbar}$$

$$= \frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)(5.00-4.50)\left(1.60\times10^{-19}\text{ J}\right)}}{1.055\times10^{-34}\text{ J}\cdot\text{s}}$$

$$= 3.62\times10^{9}\text{ m}^{-1}$$

and $T = e^{-2CL} = \exp\left[-2\left(3.62\times10^{9}\text{ m}^{-1}\right)\left(950\times10^{-12}\text{ m}\right)\right]$

$$= \exp(-6.88) = \boxed{1.03\times10^{-3}}$$

(b) We require $e^{-2CL} = 10^{-6}$. Taking logarithms,

$$-2CL = \ln 10^{-6} = -6\ln 10$$

$$L = \frac{3\ln 10}{C} = \frac{3\ln 10}{3.62\times10^{9}\text{ m}^{-1}} = 1.91\times10^{-9}\text{ m} = \boxed{1.91\text{ nm}}$$

P41.33 The original tunneling probability is $T = e^{-2CL}$, where

$$\begin{aligned} C &= \frac{\sqrt{2m(U-E)}}{\hbar} \\ &= \frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)(20.0-12.0)\left(1.60\times10^{-19}\text{ J}\right)}}{6.626\times10^{-34}\text{ J}\cdot\text{s}/2\pi} \\ &= 1.448\ 1\times10^{10}\text{ m}^{-1} \end{aligned}$$

The photon energy is $hf = \frac{hc}{\lambda} = \frac{1\,240\text{ eV}\cdot\text{nm}}{546\text{ nm}} = 2.27\text{ eV}$, to make the electron's new kinetic energy 12.0 + 2.27 = 14.27 eV and its decay coefficient inside the barrier

$$\begin{aligned} C' &= \frac{\sqrt{2m(U-E)}}{\hbar} \\ &= \frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)(20.0-14.27)\left(1.60\times10^{-19}\text{ J}\right)}}{6.626\times10^{-34}\text{ J}\cdot\text{s}/2\pi} \\ &= 1.225\ 5\times10^{10}\text{ m}^{-1} \end{aligned}$$

Now the factor of increase in transmission probability is

$$\frac{e^{-2C'L}}{e^{-2CL}} = e^{2L(C-C')} = e^{2\left(1.00\times10^{-9}\text{ m}\right)\left(0.223\times10^{10}\text{ m}^{-1}\right)} = e^{4.45} = \boxed{85.9}$$

Section 41.6 Applications of Tunneling

P41.34 With the wave function proportional to e^{-CL}, the transmission coefficient and the tunneling current are proportional to $|\psi|^2$, to e^{-2CL}. Then,

$$\frac{I(0.500\text{ nm})}{I(0.515\text{ nm})} = \frac{e^{-2(10.0/\text{nm})(0.500\text{ nm})}}{e^{-2(10.0/\text{nm})(0.515\text{ nm})}} = e^{20.0(0.015)} = \boxed{1.35}$$

P41.35 With transmission coefficient e^{-2CL}, the fractional change in transmission is

$$\begin{aligned} \frac{e^{-2(10.0/\text{nm})L} - e^{-2(10.0/\text{nm})(L+0.002\,00\text{ nm})}}{e^{-2(10.0/\text{nm})L}} &= 1 - e^{-20.0(0.002\,00)} \\ &= 0.039\ 2 = \boxed{3.92\%} \end{aligned}$$

Section 41.7 The Simple Harmonic Oscillator

P41.36 (a) The wave function is given by $\psi = Axe^{-bx^2}$, so

$$\frac{d\psi}{dx} = Ae^{-bx^2} - 2bx^2Ae^{-bx^2}$$

and

$$\frac{d^2\psi}{dx^2} = \left[-2bxAe^{-bx^2} - 4bxAe^{-bx^2}\right] + 4b^2x^3e^{-bx^2} = -6b\psi + 4b^2x^2\psi$$

Substitute into Equation 41.24:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

$$-\frac{\hbar^2}{2m}\left[-6b\psi + 4b^2x^2\psi\right] + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

$$\frac{3b\hbar^2}{m}\psi - \frac{2b^2\hbar^2}{m}x^2\psi = -\frac{1}{2}m\omega^2x^2\psi + E\psi$$

For this to be true as an identity, the coefficients of like terms must be the same for all values of x. So we must have both

$$\frac{2b^2\hbar^2}{m} = \frac{1}{2}m\omega^2 \quad \rightarrow \quad b^2 = \frac{m^2\omega^2}{4\hbar^2} \quad \text{and} \quad \frac{3b\hbar^2}{m} = E$$

(b) Therefore, $\boxed{b = \frac{m\omega}{2\hbar}}$ and $E = \frac{3b\hbar^2}{m} = \boxed{\frac{3}{2}\hbar\omega}$

(c) The energy levels are $E_n = \left(n + \frac{1}{2}\right)\hbar\omega = \frac{3}{2}\hbar\omega$, so $n = 1$, which corresponds to the $\boxed{\text{first excited state}}$.

P41.37 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator. From $E = \hbar\omega$, we have

$$\frac{hc}{\lambda} = \hbar\sqrt{\frac{k}{m}} = \frac{h}{2\pi}\sqrt{\frac{k}{m}}$$

or

$$\lambda = 2\pi c\sqrt{\frac{m}{k}} = 2\pi\left(3.00\times10^8 \text{ m/s}\right)\left(\frac{9.11\times10^{-31} \text{ kg}}{8.99 \text{ N/m}}\right)^{1/2}$$

$$= \boxed{600 \text{ nm}}$$

P41.38 The longest wavelength corresponds to minimum photon energy, which must be equal to the spacing between energy levels of the oscillator, which is (from Equation 41.28)

$$E = \hbar\omega$$

$$\frac{hc}{\lambda} = \hbar\sqrt{\frac{k}{m}} = \frac{h}{2\pi}\sqrt{\frac{k}{m}}$$

$$\lambda = \boxed{2\pi c\sqrt{\frac{m}{k}}}$$

P41.39 (a) With $\psi = Be^{-(m\omega/2\hbar)x^2}$, the normalization condition $\int\limits_{\text{all } x} |\psi|^2\,dx = 1$ becomes

$$1 = \int_{-\infty}^{\infty} B^2 e^{-2(m\omega/2\hbar)x^2}\,dx = 2B^2\int_0^{\infty} e^{-(m\omega/\hbar)x^2}\,dx$$

$$= 2B^2\frac{1}{2}\sqrt{\frac{\pi}{m\omega/\hbar}} = B^2\sqrt{\frac{\pi\hbar}{m\omega}}$$

where Table B.6 in Appendix B was used to evaluate the integral.

Thus, $\boxed{B = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}}$.

(b) For small δ, the probability of finding the particle in the range $-\frac{\delta}{2} < x < \frac{\delta}{2}$ is

$$\int_{-\delta/2}^{\delta/2} |\psi|^2\,dx \approx \delta|\psi(0)|^2 = \delta B^2 e^{-0} = \boxed{\delta\left(\frac{m\omega}{\pi\hbar}\right)^{1/2}}$$

P41.40 (a) For the center of mass to be fixed, $m_1u_1 + m_2u_2 = 0$. Then

$$u = |u_1| + |u_2| = |u_1| + \frac{m_1}{m_2}|u_1| = \frac{m_2 + m_1}{m_2}|u_1|$$

and

$$|u_1| = \frac{m_2 u}{m_1 + m_2}$$

Also,

$$u = \frac{m_2}{m_1}|u_2| + |u_2| = \left(\frac{m_2 + m_1}{m_1}\right)|u_2| \quad\rightarrow\quad |u_2| = \frac{m_1 u}{m_1 + m_2}$$

Substitute for $|u_1|$ and $|u_2|$:

$$\frac{1}{2}m_1u_1^2+\frac{1}{2}m_2u_2^2+\frac{1}{2}kx^2=\frac{1}{2}\frac{m_1m_2^2u^2}{\left(m_1+m_2\right)^2}+\frac{1}{2}\frac{m_2m_1^2u^2}{\left(m_1+m_2\right)^2}+\frac{1}{2}kx^2$$

$$=\frac{1}{2}\frac{m_1m_2\left(m_1+m_2\right)}{\left(m_1+m_2\right)^2}u^2+\frac{1}{2}kx^2$$

$$=\frac{1}{2}\mu u^2+\frac{1}{2}kx^2$$

(b) Because the total energy is constant

$$\frac{d}{dx}\left(\frac{1}{2}\mu u^2+\frac{1}{2}kx^2\right)=0$$

$$0=\frac{1}{2}\mu 2u\frac{du}{dx}+\frac{1}{2}k2x=\mu\frac{dx}{dt}\frac{du}{dx}+kx=\mu\frac{du}{dt}+kx=\mu a+kx$$

$$\mu a=-kx$$

$$a=-\frac{kx}{\mu}$$

This is the condition for simple harmonic motion; the acceleration of the equivalent particle is a negative constant times the displacement from equilibrium.

(c) By identification with $a=-\omega^2x$,

$$\omega=\sqrt{\frac{k}{\mu}}=2\pi f \text{ and } \boxed{f=\frac{1}{2\pi}\sqrt{\frac{k}{\mu}}}$$

P41.41 (a) With $\langle x\rangle=0$ and $\langle p_x\rangle=0$, the average value of x^2 is $(\Delta x)^2$ and the average value of p_x^2 is $\left(\Delta p_x\right)^2$. We know $\Delta x\geq\frac{\hbar}{2\Delta p_x}$.

The average of the energy is constant:

$$\langle E\rangle=\left\langle\frac{p_x^2}{2m}\right\rangle+\left\langle\frac{k}{2}x^2\right\rangle=\frac{\langle p_x^2\rangle}{2m}+\frac{k}{2}\langle x^2\rangle$$

$$E=\frac{\left(\Delta p_x\right)^2}{2m}+\frac{k}{2}(\Delta x)^2\geq\frac{\left(\Delta p_x\right)^2}{2m}+\frac{k}{2}\left(\frac{\hbar}{2\Delta p_x}\right)^2$$

$$E\geq\frac{\left(\Delta p_x\right)^2}{2m}+\frac{k\hbar^2}{8\left(\Delta p_x\right)^2}$$

We rewrite the last equation as $E\geq\frac{p_x^2}{2m}+\frac{k\hbar^2}{8p_x^2}$

(b) To minimize *E* as a function of $(\Delta p_x)^2$, we require

$$\frac{d}{d\left[(\Delta p_x)^2\right]}\left[\frac{(\Delta p_x)^2}{2m}+\frac{k\hbar^2}{8(\Delta p_x)^2}\right]=0$$

$$\frac{1}{2m}+\frac{k\hbar^2}{8}(-1)\frac{1}{(\Delta p_x)^4}=0$$

Then

$$\frac{k\hbar^2}{8(\Delta p_x)^4}=\frac{1}{2m}\;\rightarrow\;(\Delta p_x)^2=\left(\frac{2mk\hbar^2}{8}\right)^{1/2}=\frac{\hbar\sqrt{mk}}{2}$$

and

$$E\geq\frac{(\Delta p_x)^2}{2m}+\frac{k\hbar^2}{8(\Delta p_x)^2}=\frac{\hbar\sqrt{mk}}{2(2m)}+\frac{k\hbar^2 2}{8\hbar\sqrt{mk}}$$

$$=\frac{\hbar}{4}\sqrt{\frac{k}{m}}+\frac{\hbar}{4}\sqrt{\frac{k}{m}}=\frac{\hbar}{2}\sqrt{\frac{k}{m}}$$

Therefore, $E_{\min}=\frac{\hbar}{2}\sqrt{\frac{k}{m}}=\boxed{\frac{\hbar\omega}{2}}$

P41.42 Equation 41.26 is $\psi=Be^{-(m\omega/2\hbar)x^2}$, so

$$\frac{d\psi}{dx}=-\left(\frac{m\omega}{\hbar}\right)x\psi\quad\text{and}\quad\frac{d^2\psi}{dx^2}=\left(\frac{m\omega}{\hbar}\right)^2x^2\psi+\left(-\frac{m\omega}{\hbar}\right)\psi$$

Substitute into Equation 41.24:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}+\frac{1}{2}m\omega^2x^2\psi=E\psi$$

$$-\frac{\hbar^2}{2m}\left[\left(\frac{m\omega}{\hbar}\right)^2x^2\psi+\left(-\frac{m\omega}{\hbar}\right)\psi\right]+\frac{1}{2}m\omega^2x^2\psi=E\psi$$

$$\cancel{-\frac{1}{2}m\omega^2x^2\psi}+\left(\frac{\hbar\omega}{2}\right)\psi+\cancel{\frac{1}{2}m\omega^2x^2\psi}=E\psi$$

$$\left(\frac{\hbar\omega}{2}\right)\psi=E\psi$$

which is satisfied provided that $E=\frac{\hbar\omega}{2}$.

Additional Problems

P41.43 (a) The particle's wavelength is

$$\lambda = \frac{2L}{n} = \frac{2L}{1} = \boxed{2.00 \times 10^{-10}\ \text{m}}$$

(b) Its momentum is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{2.00 \times 10^{-10}\ \text{m}} = \boxed{3.31 \times 10^{-24}\ \text{kg} \cdot \text{m/s}}$$

(c) And its energy is

$$E = \frac{p^2}{2m} = \frac{\left(3.31 \times 10^{-24}\ \text{kg} \cdot \text{m/s}\right)^2}{2\left(2.00 \times 10^{-28}\ \text{kg}\right)} = \boxed{0.171\ \text{eV}}$$

P41.44 (a) From Equation 41.4 for $\psi(x) = Ae^{ikx}$, the first and second derivatives are

$$\frac{d}{dx}\left(Ae^{ikx}\right) = ikAe^{ikx} \quad \text{and} \quad \frac{d^2\psi}{dx^2} = -k^2 Ae^{ikx}$$

Then

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -\frac{\hbar^2}{2m}\left(-k^2 Ae^{ikx}\right) = \frac{\hbar^2 k^2}{2m}\left(Ae^{ikx}\right)$$

$$= \frac{1}{2m}\left(\frac{h}{\cancel{2\pi}}\right)^2\left(\frac{\cancel{2\pi}}{\lambda}\right)^2\left(Ae^{ikx}\right)$$

$$= \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2\left(Ae^{ikx}\right) = \frac{p^2}{2m}\psi = K\psi$$

(b) For $\psi(x) = A\sin\left(\frac{2\pi x}{\lambda}\right) = A\sin kx$,

$$\frac{d}{dx}(A\sin kx) = Ak\cos kx \qquad \text{and} \qquad \frac{d^2\psi}{dx^2} = -Ak^2\sin kx.$$

Then, similarly to the proof in part (a),

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -\frac{\hbar^2}{2m}\left(-Ak^2\sin kx\right) = \frac{\hbar^2 k^2}{2m}\left(Ak^2\sin kx\right) = \frac{p^2}{2m}\psi$$
$$= K\psi$$

P41.45 From Equation 41.13, $\psi_2(x)=\sqrt{\frac{2}{L}}\sin\left(\frac{2\pi x}{L}\right)$.

The probability of finding the particle between $x = 0$ and $x = L/4$ is

$$\int_0^{L/4}|\psi|^2\,dx=\int_0^{L/4}\frac{2}{L}\sin^2\left(\frac{2\pi x}{L}\right)dx=\frac{\cancel{2}}{L}\int_0^{L/4}\frac{1-\cos\left[2\left(2\pi x/L\right)\right]}{\cancel{2}}dx$$

$$\frac{1}{L}\left[x-\frac{L}{4\pi}\sin\left(\frac{4\pi x}{L}\right)\right]\Bigg|_0^{L/4}=\frac{1}{L}\left[\frac{L}{4}-\frac{L}{4\pi}\sin(\pi)\right]=\frac{1}{4}=\boxed{0.250}$$

P41.46 If we had $n = 0$ for a quantum particle in a box, its momentum would be zero. The uncertainty in its momentum would be zero. The uncertainty in its position would not be infinite, but just equal to the width of the box. Then the uncertainty product would be zero, to violate the uncertainty principle. The contradiction shows that the quantum number cannot be zero. In its ground state the particle has some nonzero zero-point energy.

P41.47 $T=e^{-2CL}$, where $C=\frac{\sqrt{2m(U-E)}}{\hbar}$ and where m is in kilograms, and U and E are in joules.

(a) We compute

$$C=\frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left[\left(0.010\ 0\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)\right]}}{1.055\times10^{-34}\text{ J}\cdot\text{s}}$$
$$=5.12\times10^{8}\text{ m}^{-1}$$

Then,

$$2CL=2\left(5.12\times10^{8}\text{ m}^{-1}\right)\left(0.100\times10^{-9}\text{ m}\right)=0.102$$

and $T=e^{-0.102}=\boxed{0.903}$

(b) We compute

$$C=\frac{\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left[\left(1.00\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)\right]}}{1.055\times10^{-34}\text{ J}\cdot\text{s}}$$
$$=5.12\times10^{9}\text{ m}^{-1}$$

Then,

$$2CL=2\left(5.12\times10^{9}\text{ m}^{-1}\right)\left(0.100\times10^{-9}\text{ m}\right)=1.02$$

and $T=e^{-1.02}=\boxed{0.359}$

(c) We compute

$$C=\frac{\sqrt{2\left(6.65\times10^{-27}\ \text{kg}\right)\left[\left(1.00\times10^{6}\ \text{eV}\right)\left(1.60\times10^{-19}\ \text{J/eV}\right)\right]}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}$$

$$=4.37\times10^{14}\ \text{m}^{-1}$$

Then,

$$2CL=2\left(4.37\times10^{14}\ \text{m}^{-1}\right)\left(1.00\times10^{-15}\ \text{m}\right)=0.875$$

and $T=e^{-0.875}=\boxed{0.417}$

(d) We compute

$$2CL=2\frac{\sqrt{2\left(8.00\ \text{kg}\right)\left(1.00\ \text{J}\right)}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}}\left(0.020\ 0\ \text{m}\right)=1.52\times10^{33}$$

Then,

$$T=e^{-1.52\times10^{33}}=e^{(\ln 10)(-1.52\times10^{33}/\ln 10)}=\boxed{10^{-6.59\times10^{32}}}$$

P41.48 From Equation 41.14, the energy levels of an electron in an infinitely deep potential well are proportional to n^2. If the energy of the ground state, $n=1$, is $E_1=0.300$ eV, the energy levels of the states $n=2$, 3, and 4 are

$$E_2=2^2\left(0.300\ \text{eV}\right)=1.20\ \text{eV}$$

$$E_3=3^2\left(0.300\ \text{eV}\right)=2.70\ \text{eV}$$

$$E_4=4^2\left(0.300\ \text{eV}\right)=4.80\ \text{eV}$$

(a) For the transition from the $n=3$ level to the $n=1$ level, the electron loses energy

$$E=\frac{hc}{\lambda}=E_3-E_1=2.70\ \text{eV}-0.300\ \text{eV}=2.40\ \text{eV}$$

$$\lambda=\frac{hc}{\Delta E}=\frac{1240\ \text{eV}\cdot\text{nm}}{2.40\ \text{eV}}=517\ \text{nm}$$

(b) For the transition from level 2 to level 1,

$$E=1.20\ \text{eV}-0.300\ \text{eV}=0.900\ \text{eV}$$

and

$$\lambda=\frac{hc}{\Delta E}=\frac{1240\ \text{eV}\cdot\text{nm}}{0.900\ \text{eV}}=1\ 380\ \text{nm}=\boxed{1.38\ \mu\text{m}}$$

This photon, with wavelength greater than 700 nm, is in the $\boxed{\text{infrared}}$ region.

In like manner, we find

for 3 to 2: $\Delta E = 1.50$ eV, and $\lambda = \boxed{827\text{ nm, infrared}}$

for 4 to 1: $\Delta E = 4.50$ eV, and $\lambda = \boxed{275\text{ nm, ultraviolet}}$

for 4 to 2: $\Delta E = 3.60$ eV, and $\lambda = \boxed{344\text{ nm, near ultraviolet}}$

for 4 to 3: $\Delta E = 2.10$ eV, and $\lambda = \boxed{590\text{ nm, yellow-orange visible}}$

P41.49 (a) From $E = hf$, the frequency is

$$f = \frac{E}{h} = \frac{(1.80\text{ eV})}{(6.626\times10^{-34}\text{ J}\cdot\text{s})}\left(\frac{1.602\times10^{-19}\text{ J}}{1.00\text{ eV}}\right)$$

$$= 4.35\times10^{14}\text{ Hz} = 435\times10^{12}\text{ Hz} = \boxed{435\text{ THz}}$$

(b) The wavelength of the emitted photon is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{4.35\times10^{14}\text{ Hz}} = 6.89\times10^{-7}\text{ m} = \boxed{689\text{ nm}}$$

(c) We use $\Delta E\Delta t \geq \frac{\hbar}{2}$, so

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{h}{4\pi(\Delta t)} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{4\pi(2.00\times10^{-6}\text{ s})}$$

$$\Delta E \geq 2.64\times10^{-29}\text{ J} = 1.65\times10^{-10}\text{ eV} = 165\times10^{-12}\text{ eV}$$

$$= 165\text{ peV}$$

The uncertainty is $\boxed{165\text{ peV or more}}$.

P41.50 Suppose the marble has mass 20 g. Suppose the wall of the box is 12 cm high and 2 mm thick. While it is inside the wall,

$$U = mgy = (0.02\text{ kg})(9.8\text{ m/s}^2)(0.12\text{ m}) = 0.023\,5\text{ J}$$

and

$$E = K = \frac{1}{2}mu^2 = \frac{1}{2}(0.02\text{ kg})(0.8\text{ m/s})^2 = 0.006\,4\text{ J}$$

Then,

$$C = \frac{\sqrt{2m(U-E)}}{\hbar} = \frac{\sqrt{2(0.02\text{ kg})(0.017\,1\text{ J})}}{1.055\times10^{-34}\text{ J}\cdot\text{s}} = 2.5\times10^{32}\text{ m}^{-1}$$

and the transmission coefficient is

$$e^{-2CL} = e^{-2\left(2.5\times10^{32}\right)\left(2\times10^{-3}\right)} = e^{-10\times10^{29}} = e^{-2.30\left(4.3\times10^{29}\right)}$$

$$= 10^{-4.3\times10^{29}} = \boxed{\sim 10^{-10^{30}}}$$

P41.51 (a) From Equation 41.14, the allowed energy levels are

$$E_n = \left(\frac{h^2}{8mL^2}\right)n^2 \qquad n = 1, 2, 3, \ldots$$

The energy of the absorbed photon is

$$E = \Delta E_n = E_3 - E_1 = \left(\frac{h^2}{8m_eL^2}\right)(3)^2 - \left(\frac{h^2}{8m_eL^2}\right)(1)^2 = 8\left(\frac{h^2}{8m_eL^2}\right)$$

We determine the length of the box from

$$\frac{hc}{\lambda} = \frac{h^2}{m_eL^2} \qquad \rightarrow \qquad \boxed{L = \left(\frac{h\lambda}{m_ec}\right)^{1/2}}$$

(b) The energy lost during the $n = 3$ to $n = 2$ transition is

$$E' = E_3 - E_2 = \left(\frac{h^2}{8m_eL^2}\right)(3)^2 - \left(\frac{h^2}{8m_eL^2}\right)(2)^2 = 5\left(\frac{h^2}{8m_eL^2}\right)$$

The wavelength of the emitted photon is then

$$\frac{hc}{\lambda'} = \frac{5h^2}{8m_eL^2} = \frac{5h^2}{8\cancel{m_e}}\left(\frac{\cancel{m_e}c}{h\lambda}\right) \quad \rightarrow \quad \boxed{\lambda' = \frac{8}{5}\lambda}$$

P41.52 $\langle x^2\rangle = \int_{-\infty}^{\infty} x^2|\psi|^2\,dx$

For a one-dimensional box of width L, from Equation 41.18,

$$\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right)$$

$$\langle x^2\rangle = \frac{2}{L}\int_0^L x^2\sin^2\left(\frac{n\pi x}{L}\right)dx$$

With the substitution

$$y = \frac{n\pi x}{L} \;\rightarrow\; dy = \frac{n\pi}{L}dx$$

$$x = \frac{L}{n\pi}y \;\rightarrow\; dx = \frac{L}{n\pi}dy$$

the integral becomes (from integral tables)

$$\begin{aligned}\langle x^2\rangle &= \frac{2}{L}\left(\frac{L}{n\pi}\right)^3\int_0^{n\pi} x^2\sin^2 y\,dy\\
&= \frac{2L^2}{(n\pi)^3}\left[\frac{y^3}{6}-\left(\frac{y^2}{4}-\frac{1}{8}\right)\sin 2y-\frac{y}{4}\cos 2y\right]\Bigg|_0^{n\pi}\\
&= \frac{2L^2}{(n\pi)^3}\left[\frac{(n\pi)^3}{6}-\frac{n\pi}{4}\cos 2(n\pi)\right]\\
&= \frac{2L^2}{(n\pi)^3}\left[\frac{(n\pi)^3}{6}-\frac{n\pi}{4}\right]=\frac{L^2}{3}-\frac{L^2}{2n^2\pi^2}\end{aligned}$$

P41.53 (a) The requirements that $\frac{n\lambda}{2}=L$ and $p=\frac{h}{\lambda}=\frac{nh}{2L}$ are still valid. From the relativistic energy of the particle,

$$E=\sqrt{(pc)^2+(mc^2)^2}\Rightarrow E_n=\sqrt{\left(\frac{nhc}{2L}\right)^2+(mc^2)^2}$$

its kinetic energy is therefore

$$K_n=E_n-mc^2=\boxed{\sqrt{\left(\frac{nhc}{2L}\right)^2+(mc^2)^2}-mc^2}$$

(b) Taking $L = 1.00\times10^{-12}$ m, $m = 9.11\times10^{-31}$ kg, and $n = 1$, we find

$$\begin{aligned}K_n &= \sqrt{\left(\frac{nhc}{2L}\right)^2+(mc^2)^2}-mc^2\\
&= \left\{\left[\frac{(1)(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{2(1.00\times10^{-12}\text{ m})}\right]^2\right.\\
&\quad\left.+\left[(9.11\times10^{-31}\text{ kg})\left(2.998\times10^8\ \frac{\text{m}}{\text{s}}\right)^2\right]^2\right\}^{1/2}\\
&\qquad -(9.11\times10^{-31}\text{ kg})\left(2.998\times10^8\ \frac{\text{m}}{\text{s}}\right)^2\\
&= \boxed{4.68\times10^{-14}\text{ J}}\end{aligned}$$

(c) The particle's nonrelativistic energy is

$$E_1=\frac{h^2}{8mL^2}=\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{8(9.11\times10^{-31}\text{ kg})(1.00\times10^{-12}\text{ m})^2}=6.02\times10^{-14}\text{ J}$$

Comparing this to K_1, we see that this value is too large by $\boxed{28.6\%}$.

P41.54 Looking at Figure 41.7, we see that wavelengths for a particle in a finite well are longer than those for a particle in an infinite well. Therefore, the energies of the allowed states should be lower for a finite well than for an infinite well. As a result, the photons from the source have too much energy to be absorbed or, equivalently, the photons have a frequency that is too high. In order to lower their apparent frequency using the Doppler shift, the source would have to move *away* from the particle in the finite square well, not *toward* it.

P41.55 (a) For a particle with wave function

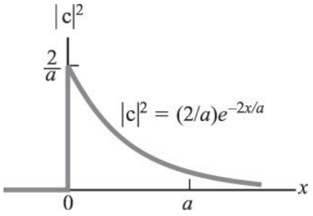
$$\psi(x) = \begin{cases} \sqrt{\dfrac{2}{a}}e^{-x/a} & \text{for } x > 0 \\ 0 & \text{for } x < 0 \end{cases}$$

The probability densities are

$$|\psi(x)|^2 = 0 \qquad \text{for } x < 0$$

and $$|\psi^2(x)| = \frac{2}{a}e^{-2x/a} \qquad \text{for } x > 0.$$

ANS. FIG. P41.55. shows a sketch of the probability density for this particle.



ANS. FIG. P41.55

(b) The probability is obtained from

$$\text{Prob}(x < 0) = \int_{-\infty}^{0} |\psi(x)|^2\,dx = \int_{-\infty}^{0} (0)\,dx = \boxed{0}$$

(c) For the wave function to be normalized, we require

$$\int_{-\infty}^{\infty} |\psi(x)|^2\,dx = \int_{-\infty}^{0} |\psi|^2\,dx + \int_{0}^{\infty} |\psi|^2\,dx = 1$$

Performing the integration gives

$$\int_{-\infty}^{0} 0dx + \int_{0}^{\infty}\left(\frac{2}{a}\right)e^{-2x/a}dx = 0 - e^{-2x/a}\Big|_0^{\infty} = -\left(e^{-\infty} - 1\right) = 1$$

(d) The probability is obtained from

$$\text{Prob}(0 < x < a) = \int_0^a |\psi|^2\, dx = \int_0^a \left(\frac{2}{a}\right) e^{-2x/a}\, dx = -e^{-2x/a}\Big|_0^a$$

$$= 1 - e^{-2} = \boxed{0.865}$$

P41.56 (a) Taking $L_x = L_y = L$, we see that the expression for E becomes

$$\boxed{E = \frac{h^2}{8m_e L^2}\left(n_x^2 + n_y^2\right)}$$

The general form of the wave function is

$$\psi \sim \sin\left(\frac{n_x \pi x}{L}\right)\sin\left(\frac{n_y \pi y}{L}\right)$$

For a normalizable wave function, neither n_x nor n_y can be zero, otherwise $\psi = 0$.

(b) The ground state corresponds to $\boxed{n_x = n_y = 1}$.

(c) The energy of the ground state is

$$E_{1,\,1} = \frac{h^2}{8m_e L^2}\left(1^2 + 1^2\right) = \boxed{\frac{h^2}{4m_e L^2}}$$

(d) For the first excited state, $\boxed{n_x = 1 \text{ and } n_y = 2, \text{ or } n_x = 2 \text{ and } n_y = 1}$.

(e) For the second excited state, $\boxed{n_x = 2 \text{ and } n_y = 2}$.

(f) The second excited state, corresponding to $n_x = 2$, $n_y = 2$, has an energy of

$$E_{2,\,2} = \frac{h^2}{8m_e L^2}\left(2^2 + 2^2\right) = \boxed{\frac{h^2}{m_e L^2}}$$

(g) The energy difference between the ground state and the second excited state is

$$\Delta E = E_{2,\,2} - E_{1,\,1} = \frac{h^2}{m_e L^2} - \frac{h^2}{4m_e L^2} = \boxed{\frac{3h^2}{4m_e L^2}}$$

(h) $\Delta E = \dfrac{3h^2}{4m_e L^2} = \dfrac{hc}{\lambda} \quad \rightarrow \quad \lambda = \boxed{\dfrac{4m_e c L^2}{3h}}$

P41.57 (a) The expectation value is

$$\langle x\rangle_0 = \int_{-\infty}^{\infty} x\left(\frac{a}{\pi}\right)^{1/2} e^{-ax^2}dx = \boxed{0}$$

since the integrand is an odd function of x.

(b) The expectation value is

$$\langle x\rangle_1 = \int_{-\infty}^{\infty} x\left(\frac{4a^3}{\pi}\right)^{1/2} x^2e^{-ax^2}dx = \boxed{0}$$

since the integrand is an odd function of x.

(c) The expectation value is

$$\langle x\rangle_{01} = \int_{-\infty}^{\infty} x\frac{1}{2}\left(\psi_0+\psi_1\right)^2 dx = \frac{1}{2}\langle x\rangle_0 + \frac{1}{2}\langle x\rangle_1 + \int_{-\infty}^{\infty} x\psi_0(x)\psi_1(x)dx$$

The first two terms are zero, from (a) and (b). Thus,

$$\langle x\rangle_{01} = \int_{-\infty}^{\infty} x\left(\frac{a}{\pi}\right)^{1/4} e^{-ax^2/2}\left(\frac{4a^3}{\pi}\right)^{1/4} xe^{-ax^2/2}dx = 2\left(\frac{2a^2}{\pi}\right)^{1/2}\int_0^{\infty} x^2e^{-ax^2}dx$$

$$= 2\left(\frac{2a^2}{\pi}\right)^{1/2}\frac{1}{4}\left(\frac{\pi}{a^3}\right)^{1/2}$$

$$= \boxed{\frac{1}{\sqrt{2a}}}$$

Where we have used Table B.6 in the Appendix to evaluate the integral.

P41.58 With one slit open,

$$P_1 = |\psi_1|^2 \quad \text{or} \quad P_2 = |\psi_2|^2$$

With both slits open,

$$P = |\psi_1 + \psi_2|^2$$

At a maximum, the wave functions are in phase

$$P_{\text{max}} = \left(|\psi_1| + |\psi_2|\right)^2$$

At a minimum, the wave functions are out of phase,

$$P_{\text{min}} = \left(|\psi_1| - |\psi_2|\right)^2$$

Now,

$$\frac{P_1}{P_2} = \frac{|\psi_1|^2}{|\psi_2|^2} = 25.0\,,$$

so $$\frac{|\psi_1|}{|\psi_2|} = 5.00$$

and $$\frac{P_{\max}}{P_{\min}} = \frac{\left(|\psi_1| + |\psi_2|\right)^2}{\left(|\psi_1| - |\psi_2|\right)^2} = \frac{\left(5.00|\psi_2| + |\psi_2|\right)^2}{\left(5.00|\psi_2| - |\psi_2|\right)^2} = \frac{(6.00)^2}{(4.00)^2} = \frac{36.0}{16.0} = \boxed{2.25}$$

Challenge Problems

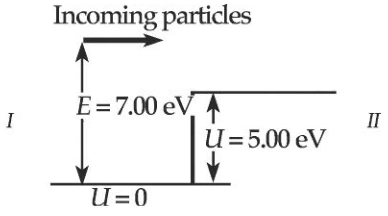
P41.59 (a) The claim is that Schrödinger's equation

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E - U)\psi$$

has the solutions

$$\psi_1 = Ae^{ik_1 x} + Be^{-ik_1 x} \quad [\text{region } I]$$

$$\psi_2 = Ce^{ik_2 x} \quad [\text{region } II]$$



ANS. FIG. P41.59(a)

Check that the solution for region *I* satisfies Schrödinger's equation:

$$\frac{\partial^2 \psi_1}{\partial x^2} = -\frac{2m}{\hbar^2} E\psi_1$$

$$\frac{\partial^2}{\partial x^2}\left(Ae^{ik_1 x}\right) + \frac{\partial^2}{\partial x^2}\left(Be^{-ik_1 x}\right) = -\frac{2m}{\hbar^2} E\left(Ae^{ik_1 x} + Be^{-ik_1 x}\right)$$

$$-k_1^2\left(Ae^{ik_1 x}\right) - k_1^2\left(Be^{-ik_1 x}\right) = -\frac{2m}{\hbar^2} E\left(Ae^{ik_1 x} + Be^{-ik_1 x}\right)$$

$$-k_1^2\left(Ae^{ik_1 x} + Be^{-ik_1 x}\right) = -\frac{2m}{\hbar^2} E\left(Ae^{ik_1 x} + Be^{-ik_1 x}\right)$$

The last line is true if $k_1^2 = \frac{2m}{\hbar^2}E$, which it is because

$$E = \frac{p^2}{2m} = \frac{\left(\hbar k_1\right)^2}{2m} \rightarrow k_1 = \frac{\sqrt{2mE}}{\hbar}$$

Therefore, the equation is satisfied in region *I*.

Check that the solution for region *II* satisfies Schrödinger's equation:

$$\frac{\partial^2 \psi_2}{\partial x^2} = -\frac{2m}{\hbar^2}(E-U)\psi_2$$

$$\frac{\partial^2}{\partial x^2}\left(Ce^{ik_2x}\right) = -\frac{2m}{\hbar^2}(E-U)\left(Ce^{ik_2x}\right)$$

$$-k_2^2\left(Ce^{ik_2x}\right) = -\frac{2m}{\hbar^2}(E-U)\left(Ce^{ik_2x}\right)$$

The last line is true if $k_2^2 = \frac{2m}{\hbar^2}(E-U)$, which it is because

$$E = \frac{p^2}{2m} + U = \frac{(\hbar k_2)^2}{2m} \rightarrow k_2 = \frac{\sqrt{2m(E-U)}}{\hbar}$$

Therefore, the equation is satisfied in region *II*. We apply boundary conditions. Matching functions and derivatives at $x = 0$, we find that

$(\psi_1)_0 = (\psi_2)_0$ gives $A + B = C$,

and $\left(\frac{d\psi_1}{dx}\right)_0 = \left(\frac{d\psi_2}{dx}\right)_0$ gives $k_1(A-B) = k_2C$.

Then $B = \frac{1-k_2/k_1}{1+k_2/k_1}A$ and $C = \frac{2}{1+k_2/k_1}A$.

Incident wave Ae^{ikx} reflects Be^{-ikx}, with probability

$$R = \frac{B^2}{A^2} = \frac{(1-k_2/k_1)^2}{(1+k_2/k_1)^2} = \boxed{\frac{(k_1-k_2)^2}{(k_1+k_2)^2}}$$

(b) With $E = 7.00$ eV and $U = 5.00$ eV:

$$\frac{k_2}{k_1} = \sqrt{\frac{E-U}{E}} = \sqrt{\frac{2.00\text{ eV}}{7.00\text{ eV}}} = 0.535$$

The reflection probability is $R = \frac{(1-0.535)^2}{(1+0.535)^2} = \boxed{0.092\ 0}$.

(c) The probability of transmission is $T = 1 - R = \boxed{0.908}$.

P41.60 (a) The potential energy of the system is given by

$$U = \frac{e^2}{4\pi \in_0 d}\left[\left(-1+\frac{1}{2}-\frac{1}{3}\right)+\left(-1+\frac{1}{2}\right)+(-1)\right] = \frac{(-7/3)e^2}{4\pi \in_0 d}$$

$$= \boxed{-\frac{7k_e e^2}{3d}}$$

(b) There are two electrons, each with minimum energy E_1. From Equation 41.14, the total energy is

$$K = 2E_1 = \frac{2h^2}{8m_e(3d)^2} = \boxed{\frac{h^2}{36m_e d^2}}$$

(c) The total energy of the system is

$$E = K + U = \frac{h^2}{36m_e d^2} - \frac{7k_e e^2}{3d}$$

For a minimum, we require $\frac{dE}{d(d)} = 0$. Differentiating,

$$\frac{dE}{d(d)} = 0$$

$$\frac{d}{d(d)}\left(\frac{h^2}{36m_e d^2} - \frac{7k_e e^2}{3d}\right) = 0$$

$$(-2)\frac{h^2}{36m_e d^3} - (-1)\frac{7k_e e^2}{3d^2} = 0$$

$$\frac{h^2}{18m_e d^3} = \frac{7k_e e^2}{3d^2}$$

$$d = \frac{3h^2}{7(18m_e)k_e e} = \frac{h^2}{42m_e k_e e^2}$$

Substituting numerical values,

$$d = \frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})^2}{(42)(9.11\times10^{-31}\ \mathrm{kg})(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\ \mathrm{C})^2}$$
$$= 4.99\times10^{-11}\ \mathrm{m} = \boxed{49.9\ \mathrm{pm}}$$

(d) The lithium spacing is d and the number of atoms N in volume V is related by $Nd^3 = V$, and the density is $\frac{Nm}{V}$, where m is the mass of one atom. We have:

$$\text{density} = \frac{Nm}{V} = \frac{Nm}{Nd^3} = \frac{m}{d^3}$$

From which we obtain

$$d = \left(\frac{m}{\text{density}}\right)^{1/3} = \left[\frac{\dfrac{6.94\text{ g}}{\text{mol}}\left(\dfrac{1\text{ mol}}{6.022\times 10^{23}\text{ atoms}}\right)}{0.530\ \dfrac{\text{g}}{\text{cm}^3}}\right]^{1/3}$$

$$= 2.79\times 10^{-8}\text{ cm} = 2.79\times 10^{-10}\text{ mm} = 279\text{ pm}$$

$\boxed{\text{The lithium interatomic spacing of 280 pm is 5.59 times larger. Therefore, it is of the same order of magnitude as the interatomic spacing } 2d \text{ here.}}$

P41.61 The wave functions and probability densities are the same as those shown in Active Figure 41.4 of the textbook. From Equation 41.13, the wave functions are

$$\psi_n = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi x}{L}\right) \text{ where } n = 1, 2, 3\ldots$$

(a) For $n = 1$,

$$P_1 = \int_{0.150\text{ nm}}^{0.350\text{ nm}} |\psi_1|^2\,dx = \left(\frac{2}{1.00\text{ nm}}\right)\int_{0.150}^{0.350}\sin^2\left(\frac{\pi x}{1.00\text{ nm}}\right)dx$$

$$= (2.00/\text{nm})\left[\frac{x}{2} - \frac{1.00\text{ nm}}{4\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$$

In the above result we used

$$\int \sin^2(ax)\,dx = \left(\frac{x}{2}\right) - \left(\frac{1}{4a}\right)\sin(2ax)$$

Therefore,

$$P_1 = (1.00/\text{nm})\left[x - \frac{1.00\text{ nm}}{2\pi}\sin\left(\frac{2\pi x}{1.00\text{ nm}}\right)\right]_{0.150\text{ nm}}^{0.350\text{ nm}}$$

$$P_1 = (1.00/\text{nm})\Big\{0.350\text{ nm} - 0.150\text{ nm}$$

$$- \frac{1.00\text{ nm}}{2\pi}\left[\sin(0.700\pi) - \sin(0.300\pi)\right]\Big\}$$

$$= \boxed{0.200}$$

(b) $$P_2 = \frac{2}{1.00}\int_{0.150}^{0.350} \sin^2\left(\frac{2\pi x}{1.00}\right)dx = 2.00\left[\frac{x}{2} - \frac{1.00}{8\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$$

$$P_2 = 1.00\left[x - \frac{1.00}{4\pi}\sin\left(\frac{4\pi x}{1.00}\right)\right]_{0.150}^{0.350}$$

$$= 1.00\left\{(0.350 - 0.150) - \frac{1.00}{4\pi}\left[\sin(1.40\pi) - \sin(0.600\pi)\right]\right\}$$

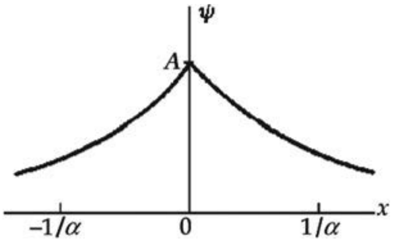
$$= \boxed{0.351}$$

Using $E_n = \dfrac{n^2h^2}{8mL^2}$, we find that

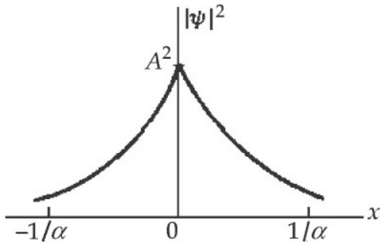
(c) $E_1 = \boxed{0.377 \text{ eV}}$ and

(d) $E_2 = \boxed{1.51 \text{ eV}}$

P41.62 (a) and (b) The Wave functions are shown in ANS. FIG. P41.62(a) and ANS. FIG. P41.62(b).



ANS. FIG. P41.62(a)



ANS. FIG. P41.62(b)

(c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$. The function can be normalized. It describes a particle bound near $x = 0$.

(d) Since ψ is symmetric,

$$\int_{-\infty}^{\infty} |\psi|^2\,dx = 2\int_{0}^{\infty} |\psi|^2\,dx = 1$$

or $$2A^2\int_0^{\infty} e^{-2\alpha x}\,dx = \left(\frac{2A^2}{-2\alpha}\right)\left(e^{-\infty} - e^0\right) = 1.$$

This gives $\boxed{A = \sqrt{\alpha}}$.

(e) The probability of finding the particle between $-1/2\alpha$ and $+1/2\alpha$ is

$$P_{(-1/2\alpha)\to(1/2\alpha)} = 2\left(\sqrt{a}\right)^2 \int_{x=0}^{1/2\alpha} e^{-2\alpha x}\,dx = \left(\frac{2\alpha}{-2\alpha}\right)\left(e^{-2\alpha/2\alpha} - 1\right)$$

$$= \left(1 - e^{-1}\right) = \boxed{0.632}$$

P41.63 (a) Recall from Section 41.7 that the potential energy of a harmonic oscillator is $\frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$. We can find the energy of the oscillator *E* by substituting the wave function into the Schrödinger equation.

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi \quad \to \quad \frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

From $\psi = Bxe^{-(m\omega/2\hbar)x^2}$, we have

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} + Bx\left(-\frac{m\omega}{2\hbar}\right)2xe^{-(m\omega/2\hbar)x^2}$$

$$= Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2e^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = Bx\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)2xe^{-(m\omega/2\hbar)x^2}$$

$$- B\left(\frac{m\omega}{\hbar}\right)x^2\left(-\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2}$$

$$\frac{d^2\psi}{dx^2} = -3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2x^3e^{-(m\omega/2\hbar)x^2}$$

Substituting the above into the Schrödinger equation, we have

$$\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2}m\omega^2x^2\psi = E\psi$$

$$\frac{-\hbar^2}{2m}\left[-3B\left(\frac{m\omega}{\hbar}\right)xe^{-(m\omega/2\hbar)x^2} + B\left(\frac{m\omega}{\hbar}\right)^2x^3e^{-(m\omega/2\hbar)x^2}\right]$$

$$+\frac{1}{2}m\omega^2x^2\left[Bxe^{-(m\omega/2\hbar)x^2}\right]$$

$$= E\left[Bxe^{-(m\omega/2\hbar)x^2}\right]$$

$$\left(\frac{3\hbar\omega}{2}\right)\left[Bxe^{-(m\omega/2\hbar)x^2}\right] + \cancel{\left(-\frac{1}{2}m\omega^2x^2\right)\left[Bxe^{-(m\omega/2\hbar)x^2}\right]}$$

$$+ \cancel{\left(\frac{1}{2}m\omega^2x^2\right)\left[Bxe^{-(m\omega/2\hbar)x^2}\right]}$$

$$= E\left[Bxe^{-(m\omega/2\hbar)x^2}\right]$$

$$\left(\frac{3\hbar\omega}{2}\right)\left(Bxe^{-(m\omega/2\hbar)x^2}\right) = E\left(Bxe^{-(m\omega/2\hbar)x^2}\right)$$

The last line is true if $\boxed{E = \frac{3\hbar\omega}{2}}$.

(b) We never find the particle at $\boxed{x = 0}$ because $\psi = 0$ there.

(c) ψ is maximized if

$$\frac{d\psi}{dx} = Be^{-(m\omega/2\hbar)x^2} - B\left(\frac{m\omega}{\hbar}\right)x^2 e^{-(m\omega/2\hbar)x^2} = 0$$

$$1 - \left(\frac{m\omega}{\hbar}\right)x^2 = 0$$

which is true at $\boxed{x = \pm\sqrt{\frac{\hbar}{m\omega}}}$.

(d) We require $\int_{-\infty}^{\infty} |\psi|^2\, dx = 1$:

$$1 = \int_{-\infty}^{\infty} B^2 x^2 e^{-(m\omega/\hbar)x^2}\, dx = 2B^2 \int x^2 e^{-(m\omega/\hbar)x^2}\, dx$$

$$= 2B^2 \frac{1}{4}\sqrt{\frac{\pi}{(m\omega/\hbar)^3}} = B^2 \frac{\pi^{1/2}}{2}\left(\frac{\hbar}{m\omega}\right)^{3/2}$$

Then,

$$B = \frac{2^{1/2}}{\pi^{1/4}}\left(\frac{m\omega}{\hbar}\right)^{3/4} = \boxed{\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/4}}$$

(e) At $x = 2(\hbar/m\omega)^{1/2}$, the potential energy is

$$\frac{1}{2}m\omega^2 x^2 = \frac{1}{2}m\omega^2\left(\frac{4\hbar}{m\omega}\right) = 2\hbar\omega$$

This is larger than the total energy $\frac{3\hbar\omega}{2}$, so there is $\boxed{\text{zero}}$ classical probability of finding the particle here.

(f) The actual probability is given by

$$P = |\psi|^2\, dx = \left(Bxe^{-(m\omega/2\hbar)x^2}\right)^2 \delta$$

$$P = \delta B^2 x^2 e^{-(m\omega/\hbar)x^2} = \delta\left(\frac{4m^3\omega^3}{\pi\hbar^3}\right)^{1/2}\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)x^2}$$

$$= \delta\frac{2}{\pi^{1/2}}\left(\frac{m^{3/2}\omega^{3/2}}{\hbar^{3/2}}\right)\left(\frac{4\hbar}{m\omega}\right)e^{-(m\omega/\hbar)4(\hbar/m\omega)} = \boxed{8\delta\left(\frac{m\omega}{\hbar\pi}\right)^{1/2} e^{-4}}$$

P41.64 (a) To find the normalization constant, we note that $\int_0^L |\psi|^2\, dx = 1$, or

$$A^2\int_0^L\left[\sin^2\left(\frac{\pi x}{L}\right)+16\sin^2\left(\frac{2\pi x}{L}\right)+8\sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\right]dx=1$$

Noting that

$$\int_0^L \sin^2\left(\frac{\pi x}{L}\right)dx = \int_0^L \frac{1-\cos\left[2(\pi x/L)\right]}{2}dx$$

$$= \left[\frac{x}{2}-\frac{L}{\pi}\frac{\sin(2\pi x/L)}{2}\right]_0^L = \frac{L}{2}$$

the integral becomes

$$\int_0^L |\psi|^2\, dx = A^2\left[\left(\frac{L}{2}\right)+16\left(\frac{L}{2}\right)+8\int_0^L \sin\left(\frac{\pi x}{L}\right)\sin\left(\frac{2\pi x}{L}\right)dx\right]$$

$$1 = A^2\left\{\left(\frac{L}{2}\right)+16\left(\frac{L}{2}\right)\right.$$

$$\left.+8\int_0^L \sin\left(\frac{\pi x}{L}\right)\left[2\sin\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)\right]dx\right\}$$

$$1 = A^2\left[\frac{17L}{2}+16\int_0^L \sin^2\left(\frac{\pi x}{L}\right)\cos\left(\frac{\pi x}{L}\right)dx\right]$$

$$1 = A^2\left[\frac{17L}{2}+\frac{16L}{3\pi}\sin^3\left(\frac{\pi x}{L}\right)\Bigg|_{x=0}^{x=L}\right] = A^2\left(\frac{17L}{2}\right)$$

$$\rightarrow \boxed{A=\sqrt{\frac{2}{17L}}}$$

(b) To determine the relationship between *A* and *B*, we note that $\int_{-a}^{a} |\psi|^2\, dx = 1$. Therefore,

$$\int_{-a}^{a}\left[|A|^2\cos^2\left(\frac{\pi x}{2a}\right)+|B|^2\sin^2\left(\frac{\pi x}{a}\right)\right.$$

$$\left.+2|A||B|\cos\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{a}\right)\right]dx=1$$

Noting that

$$\int_{-a}^{a} \sin^2\left(\frac{\pi x}{2a}\right)dx = \int_{-a}^{a} \frac{1-\cos\left[2\left(\pi x/2a\right)\right]}{2}dx$$

$$= \left[\frac{x}{2} + \frac{2L}{\pi}\frac{\sin\left(\pi x/a\right)}{2}\right]\Bigg|_{-a}^{a} = a$$

and

$$\int_{-a}^{a} \cos^2\left(\frac{\pi x}{2a}\right)dx = \int_{-a}^{a} \frac{1+\cos\left[2\left(\pi x/2a\right)\right]}{2}dx$$

$$= \left[\frac{x}{2} + \frac{2L}{\pi}\frac{\sin\left(\pi x/a\right)}{2}\right]\Bigg|_{-a}^{a} = a$$

the integral becomes

$$|A|^2 a + |B|^2 a + \int_{-a}^{a}\left[2|A||B|\cos\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{a}\right)\right]dx = 1$$

The third term is:

$$2|A||B|\int_{-a}^{a} \cos\left(\frac{\pi x}{2a}\right)\left[2\sin\left(\frac{\pi x}{2a}\right)\cos\left(\frac{\pi x}{2a}\right)\right]dx$$

$$= 4|A||B|\int_{-a}^{a} \cos^2\left(\frac{\pi x}{2a}\right)\sin\left(\frac{\pi x}{2a}\right)dx$$

$$= \frac{8a|A||B|}{3\pi}\cos^3\left(\frac{\pi x}{2a}\right)\Bigg|_{-a}^{a} = 0$$

so the whole integral is

$$a\left(|A|^2 + |B|^2\right) = 1\text{, giving } \boxed{|A|^2 + |B|^2 = \frac{1}{a}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P41.2 (a) See ANS. FIG. P41.2; (b) $\frac{1}{\sqrt{a}}$; (c) 0.865

P41.4 $\frac{1}{2}$

P41.6 (a) $n \approx 4$; (b) 6.03 eV

P41.8 The photon does not have the smallest possible energy to cause the transition between states $n = 1$ to $n = 2$.

P41.10 $\sqrt{\frac{3h\lambda}{8m_e c}}$

P41.12 (a) 5.13×10^{-3} eV; (b) 9.41 eV; (c) The electron has a much higher energy because it is much less massive.

P41.14 (a) 2.00×10^{-9} J; (b) 1.66×10^{-28} m; (c) No. The length of the box would have to be much smaller than the size of a nucleus ($\sim 10^{-14}$ m) to confine the particle.

P41.16 (a) $\frac{\hbar}{2L}$; (b) $\hbar^2/8mL^2$; (c) This estimate is too low by $4\pi^2 \approx 40$ times, but it correctly displays the pattern of dependence of the energy on the mass and on the length of the well.

P41.18 See P41.18 for full explanation.

P41.20 (a) $x = L/4$, $L/2$, and $3\,L/4$; (b) We look for $\sin(3\pi x/L)$ taking on its extreme values 1 and –1 so that the squared wave function is as large as it can be. The result can also be found by studying Figure 41.4b.

P41.22 (a) $\frac{\ell}{L} - \frac{1}{2\pi}\sin\left(\frac{2\pi\ell}{L}\right)$; (b) See ANS FIG P41.22(b); (c) The wave function is zero for $x < 0$ and for $x > L$. The probability at $\ell = 0$ must be zero because the particle is never found at $x < 0$ or exactly at $x = 0$. The probability at $\ell = L$ must be 1 for normalization: the particle is always found somewhere in the range $0 < x < L$; (d) $0.585L$

P41.24 See P41.24 for complete solution.

P41.26 (a) $n = 1$: $\psi_1(x) = \sqrt{\dfrac{2}{L}}\cos\left(\dfrac{\pi x}{L}\right)$; $P_1(x) = |\psi_1(x)|^2 = \dfrac{2}{L}\cos^2\left(\dfrac{\pi x}{L}\right)$,

$n = 2$: $\psi_2(x) = \sqrt{\dfrac{2}{L}}\sin\left(\dfrac{2\pi x}{L}\right)$; $P_2(x) = |\psi_2(x)|^2 = \dfrac{2}{L}\sin^2\left(\dfrac{2\pi x}{L}\right)$,

$n = 3$: $\psi_3(x) = \sqrt{\dfrac{2}{L}}\cos\left(\dfrac{3\pi x}{L}\right)$; $P_3(x) = |\psi_3(x)|^2 = \dfrac{2}{L}\cos^2\left(\dfrac{3\pi x}{L}\right)$;

(b) See ANS FIG. P41.26(b).

P41.28 (a) $\dfrac{\hbar^2}{L^2 m}$; (b) $\sqrt{\dfrac{15}{16L}}$; (c) 0.580

P41.30 (a) See ANS. FIG. P41.30(a); (b) $2L$

P41.32 (a) 1.03×10^{-3}; (b) 1.91 nm

P41.34 1.35

P41.36 (a) See P41.36(a) for full explanation; (b) $b = \dfrac{m\omega}{2\hbar}$ and $\dfrac{3}{2}\hbar\omega$; (c) first excited state

P41.38 $2\pi c\sqrt{\dfrac{m}{k}}$

P41.40 (a) See P41.40(a) for full explanation; (b) See P41.40(b) for full explanation; (c) $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{\mu}}$

P41.42 See P41.42 for full explanation.

P41.44 (a–b) See P41.44(a) and (b) for full explanations.

P41.46 See P41.46 for full explanation.

P41.48 (a) See P41.48(a) for full proof; (b) For 2 to 1, $\lambda = 1.38\ \mu$m, infrared; For 3 to 2, $\lambda = 827$ nm, infrared; For 4 to 1, $\lambda = 275$ nm, ultraviolet; For 4 to 2, $\lambda = 344$ nm, near ultraviolet; For 4 to 3, $\lambda = 590$ nm, yellow-orange visible.

P41.50 $\sim 10^{-10^{30}}$

P41.52 See P41.52 for full explanation.

P41.54 Looking at Figure 41.7, we see that wavelengths for a particle in a finite well are longer than those for a particle in an infinite well. Therefore, the energies of the allowed states should be lower for a finite well than for an infinite well. As a result, the photons from the source have too much energy to be absorbed or, equivalently, the photons have a frequency that is too high. In order to lower their apparent frequency using the Doppler shift, the source would have to move *away* from the particle in the finite square well, not *toward* it.

P41.56 (a) $E = \frac{h^2}{8m_eL^2}\left(n_x^2 + n_y^2\right)$; (b) $n_x = n_y = 1$; (c) $\frac{h^2}{4m_eL^2}$; (d) $n_x = 1$ and $n_y = 2$, or $n_x = 2$ and $n_y = 1$; (e) $n_x = 2$ and $n_y = 2$; (f) $\frac{h^2}{m_eL^2}$; (g) $\frac{3h^2}{4m_eL^2}$; (h) $\frac{4m_ecL^2}{3h}$

P41.58 2.25

P41.60 (a) $-\frac{7k_ee^2}{3d}$; (b) $\frac{h^2}{36m_ed^2}$; (c) 49.9 pm; (d) The lithium interatomic spacing of 280 pm is 5.59 times larger. Therefore, it is of the same order of magnitude as the interatomic spacing $2d$ here.

P41.62 (a) See ANS. FIG. P41.62(a); (b) See ANS. FIG. P41.62(b); (c) ψ is continuous and $\psi \to 0$ as $x \to \pm\infty$. The function can be normalized. It describes a particle bound near $x = 0$; (d) $A = \sqrt{\alpha}$; (e) 0.632

P41.64 (a) $A = \sqrt{\frac{2}{17L}}$; (b) $|A|^2 + |B|^2 = \frac{1}{a}$

42

Atomic Physics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ42.1 **(i)** Answer (e). **(ii)** Answer (c). The M means that the electron falls into the M shell, for which $n = 3$. The β means the electron comes from two shells above M: the O shell, for which $n = 5$. M_α would refer to $4 \to 3$ and M_β refers to $5 \to 3$.

OQ42.2 Answer (c). All states associated with $\ell = 2$ are referred to as *d* states. Thus, all 10 possible quantum states having $n = 3$, $\ell = 2$ are called 3*d* states.

OQ42.3 Answer (d). Wavelengths of the hydrogen spectrum are given by $1/\lambda = R_H\left(1/n_f^2 - 1/n_i^2\right)$, where R_H is the Rydberg constant. For the transition $n_i = 5$ to $n_f = 3$, we have

$$\frac{1}{\lambda} = \left(1.097\times10^7\ \text{m}^{-1}\right)\left(\frac{1}{3^2} - \frac{1}{5^2}\right) = 7.80\times10^5\ \text{m}^{-1}$$

$$\lambda = 1.28\times10^{-6}\ \text{m}$$

OQ42.4 Answer (e). With a principal quantum number of $n = 3$, there are 3 possible values of the orbital quantum number, $\ell = 0,\ 1,\ 2.$ There are a total of $2(2\ell+1)$ possible quantum states for each value of ℓ, $2\ell+1$ possible values of the orbital magnetic quantum number m_ℓ, and 2 possible spin orientations $(m_s = \pm\frac{1}{2})$ for each value of m_ℓ. Thus, the number of states are

3*s* states $(n = 3, \ell = 0)$: $2[2(0) + 1] = 2$

3*p* states $(n = 3, \ell = 1)$: $2[2(1) + 1] = 6$

3*d* states $(n = 3, \ell = 2)$: $2[2(2) + 1] = 10$

The grand total of $n = 3$ states is $2 + 6 + 10 = 18$.

OQ42.5 Answer (c). It is an experimental fact the charge on the electron is quantized. The Bohr model does not introduce this as a new assumption.

OQ42.6 **(i)** Answer (b). **(ii)** Answer (e). From the discussion of Equations 42.8 and 42.9, $K = \frac{k_e e^2}{2r}$ and $U_e = -\frac{k_e e^2}{r}$. If

$$-E = K + U_e = +\frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = -\frac{k_e e^2}{2r}, \text{ then } E = \frac{k_e e^2}{2r}.$$

Therefore, $K = E$ and $U_e = -2E$.

OQ42.7 Answer (e). The structure of the periodic table is the result of the Pauli exclusion principle, which states that no two electrons in an atom can ever have the same set of values for the set of quantum numbers n, ℓ, m_ℓ, and m_s.

OQ42.8 (a) Yes, provided that the energy of the photon is *precisely* enough to put the electron into one of the allowed energy states. Strangely—more precisely non-classically—enough, if the energy of the photon is not sufficient to put the electron into a particular excited energy level, the photon will not interact with the atom at all!

(b) Yes, a photon of any energy greater than 13.6 eV will ionize the atom. Any energy above 13.6 eV will go into kinetic energy of the newly liberated electron.

OQ42.9 Answers (b) and (e). Choice (b) is not possible because the Pauli exclusion principle limits the number of electrons in any *p* subshell to a maximum of 6. Choice (e) is impossible because the selection rules of quantum mechanics limit the maximum value of ℓ to $n - 1$. Thus, a 2*d* state ($n = 2$, $\ell = 2$) cannot exist.

OQ42.10 Answer (e). Since the electron is in some bound quantum state of the atom, the atom is not ionized and choice (a) is false. The fact that the electron is in a *d* state means that its orbital quantum number is $\ell = 2$, so choice (b) is false. Also, since the maximum value of ℓ is $n - 1$, choice (c) is false. Finally, the ground state of hydrogen is a 1*s* state, so choice (d) is false. Choice (e) is true because the magnitude of the orbital angular momentum is $L = \sqrt{\ell(\ell+1)}\hbar = \sqrt{2(2+1)}\hbar = \sqrt{6}\hbar$.

OQ42.11 **(i)** In order of energy change, the ranking is a > d > c > b.

(ii) In order of decreasing photon wavelength, the ranking is c = d > b > a.

We calculate the energy of the photon according to $\Delta E = (13.6\text{ eV})\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$, where $\Delta E > 0$ means the photon is absorbed and $\Delta E < 0$ means the photon is emitted. We calculate the wavelength according to $\lambda = \frac{hc}{\Delta E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{\Delta E}$.

(a) $n_i = 2$ and $n_f = 5$, $\Delta E = 2.86$ eV (absorption) $\lambda = 434$ nm

(b) $n_i = 5$ and $n_f = 3$, $\Delta E = -0.967$ eV (emission) $\lambda = 1\,280$ nm

(c) $n_i = 7$ and $n_f = 4$, $\Delta E = -0.572$ eV (emission) $\lambda = 2\,170$ nm

(d) $n_i = 4$ and $n_f = 7$, $\Delta E = 0.572$ eV (absorption) $\lambda = 2\,170$ nm

OQ42.12 Answer (c). The photon carries energy, thus an electron must lose energy.

OQ42.13 (a) Yes. As $n \to \infty$, $E_n = -13.6\text{ eV}/n^2 \to 0$, and the electron remains in a bound state.

(b) No. To produce a spectral line, the electron must make a transition from a higher energy bound state to a lower energy bound state. The greatest frequency is that of the Lyman series limit, caused by the transition from $n = \infty$ to $n = 1$.

(c) Yes. Photons with large wavelengths, corresponding to low photon energies, can be produced by transitions between adjacent states with *n* large.

OQ42.14 **(i)** Answer (d). The spin quantum number $m_s = \pm 1/2$.

(ii) Answers (c) and (d). The orbital magnetic quantum number m_ℓ has the range $-\ell, -\ell+1, \ldots, -1, 0, 1, \ldots, \ell-1, \ell,$ and spin quantum number $m_s = \pm 1/2$.

(iii) Answers (b) and (c). The orbital quantum number has values $\ell = 0, 1, 2, \ldots, n-1$, and, as stated above, m_ℓ can be zero.

OQ42.15 Answer (a). The bombarding electron can give up all or part of its kinetic energy to the atom. The energy required to raise the atom from its ground state to its first excited state is

$$\Delta E = E_2 - E_1 = -\frac{13.6 \text{ eV}}{2^2} - \left(-\frac{13.6 \text{ eV}}{1^2}\right) = 10.2 \text{ eV}$$

The bombarding electron can give up this energy to the atom and carry off the remaining 0.3 eV.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ42.1 Stimulated emission coerces atoms to emit photons along a specific axis and in phase rather than in the random directions and phases of spontaneously emitted photons. The photons that are emitted through stimulation can be made to accumulate over time. The fraction allowed to escape constitutes the intense, collimated, and coherent laser beam. If this process relied solely on spontaneous emission, the emitted photons would not exit the laser tube or crystal in the same direction. Neither would they be coherent with one another.

CQ42.2 In a neutral helium atom, one electron can be modeled as moving in an electric field created by the nucleus and the other electron. According to Gauss's law, if the electron is above the ground state it moves in the electric field of a net charge of +2*e* – 1*e* = +1*e*. We say the nuclear charge is *screened* by the inner electron. The electron in a He^+ ion moves in the field of the unscreened nuclear charge of 2 protons. Then the potential energy function for the electron is about double that of one electron in the neutral atom.

CQ42.3 Fundamentally, three quantum numbers describe an orbital wave function because we live in three-dimensional space. They arise mathematically from boundary conditions on the wave function, expressed as a product of a function of r, a function of θ, and a function of ϕ.

CQ42.4 Bohr's theory pictures the electron as moving in a flat circle like a classical particle described by $\sum F = ma$. Schrödinger's theory pictures the electron as a cloud of probability amplitude in the three-dimensional space around the hydrogen nucleus, with its motion described by a wave equation. In the Bohr model, angular momentum can take the values $L = n\hbar$, $n = 1, 2, 3, \ldots$, so the ground-state angular momentum is $1\hbar$; in the Schrödinger model, angular momentum can take the values $L = \sqrt{\ell(\ell+1)}\hbar$, $\ell = 0, 1, \ldots, n-1$, so the ground-state angular momentum $(n = 1 \rightarrow \ell = 0)$ is zero. Both models predict that the electron's energy is limited to discrete energy levels, given by $-13.6\text{ eV}/n^2$, with $n = 1, 2, 3, \ldots$.

CQ42.5 Practically speaking, no. Ions have a net charge and the magnetic force $q\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$ would deflect the beam, making it difficult to separate the atoms with different orientations of magnetic moments.

CQ42.6 The deflecting force on an atom with a magnetic moment is proportional to the *gradient* of the magnetic field. Thus, atoms with oppositely directed magnetic moments would be deflected in *opposite* directions in an inhomogeneous magnetic field.

CQ42.7 If the exclusion principle were not valid, the elements and their chemical behavior would be grossly different because every electron would end up in the lowest energy level of the atom. All matter would be nearly alike in its chemistry and composition, since the shell structures of all elements would be identical. Most materials would have a much higher density. The spectra of atoms and molecules would be very simple, and there would be very little color in the world.

CQ42.8 Bohr modeled the electron as moving in a perfect circle, with zero uncertainty in its radial coordinate. Then its radial velocity is always zero with zero uncertainty. Bohr's theory violates the uncertainty principle by making the uncertainty product $\Delta r \Delta p_r$ be zero, less than the minimum allowable $\hbar/2$.

CQ42.9 The three elements have similar electronic configurations. Each has filled inner shells plus one electron in an outer *s* orbital. Their single outer electrons largely determine their chemical interactions with other atoms.

CQ42.10 Each of the electrons must have at least one quantum number different from the quantum numbers of each of the other electrons. They can differ (in m_s) by being spin-up or spin-down. They can also differ (in ℓ) in angular momentum. Those electrons with $\ell = 1$ can differ (in m_ℓ) in orientation of angular momentum. For $n = 2$, $\ell = 0$ or 1. If $\ell = 0$, $m_\ell = 0$, and $m_s = \pm 1/2$, for a total of two different states. For $\ell = 1$, $m_\ell = -1, 0, +1$, and $m_s = \pm 1/2$, for a total of six different states.

CQ42.11 If an electron moved like a hockey puck, it could have any arbitrary frequency of revolution around an atomic nucleus. If it behaved like a charge in a radio antenna, it would radiate light with frequency equal to its own frequency of oscillation. Thus, the electron in hydrogen atoms would emit a continuous spectrum, electromagnetic waves of all frequencies smeared together.

CQ42.12 No. Laser light is collimated. The energy generally travels in the same direction. The intensity of a laser beam stays remarkably constant, independent of the distance it has traveled.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 42.1 Atomic Spectra of Gases

P42.1 (a) The wavelengths in the Lyman series of hydrogen are given by

$$\frac{1}{\lambda} = R_H\left(1 - \frac{1}{n^2}\right)$$

where $n = 2, 3, 4, \ldots$, and the Rydberg constant is $R_H = 1.097\,373\,2 \times 10^7\ \text{m}^{-1}$. This can also be written as

$$\lambda = \left(\frac{1}{R_H}\right)\left(\frac{n^2}{n^2 - 1}\right)$$

therefore, the first three wavelengths in this series are

$$\lambda_1 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left(\frac{2^2}{2^2 - 1}\right) = 1.215 \times 10^{-7}\ \text{m}$$

$$= \boxed{121.5\ \text{nm}}$$

$$\lambda_2 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left(\frac{3^2}{3^2 - 1}\right) = 1.025 \times 10^{-7}\ \text{m}$$

$$= \boxed{102.5\ \text{nm}}$$

$$\lambda_3 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left(\frac{4^2}{4^2 - 1}\right) = 9.720 \times 10^{-8}\ \text{m}$$

$$= \boxed{97.20\ \text{nm}}$$

(b) These wavelengths are all in the $\boxed{\text{ultraviolet}}$ of the spectrum.

P42.2 (a) The wavelengths in the Paschen series of hydrogen are given by

$$\frac{1}{\lambda} = R_H\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$$

where $n = 4, 5, 6, \ldots$, and the Rydberg constant is $R_H = 1.097\ 373\ 2 \times 10^7\ \text{m}^{-1}$. This can also be written as

$$\lambda = \left(\frac{1}{R_H}\right)\left(\frac{9n^2}{n^2 - 9}\right)$$

therefore, the first three wavelengths in this series are

$$\lambda_1 = \frac{1}{1.097\ 373\ 2 \times 10^7\ \text{m}^{-1}}\left[\frac{9(4)^2}{4^2 - 9}\right] = 1.875 \times 10^{-6}\ \text{m}$$
$$= \boxed{1\ 875\ \text{nm}}$$

$$\lambda_2 = \frac{1}{1.097\ 373\ 2 \times 10^7\ \text{m}^{-1}}\left[\frac{9(5)^2}{5^2 - 9}\right] = 1.281 \times 10^{-6}\ \text{m}$$
$$= \boxed{1\ 281\ \text{nm}}$$

$$\lambda_1 = \frac{1}{1.097\ 373\ 2 \times 10^7\ \text{m}^{-1}}\left[\frac{9(6)^2}{6^2 - 9}\right] = 1.094 \times 10^{-6}\ \text{m}$$
$$= \boxed{1\ 094\ \text{nm}}$$

(b) These wavelengths are all in the $\boxed{\text{infrared}}$ region of the spectrum.

P42.3 (a) The fifth excited state must lie above the second excited state by the photon energy

$$E_{52} = hf = \frac{hc}{\lambda} = \frac{\left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(3.00 \times 10^8\ \text{m/s}\right)}{520 \times 10^{-9}\ \text{m}}$$
$$= 3.82 \times 10^{-19}\ \text{J}$$

The sixth excited state exceeds the second in energy by

$$E_{62} = \frac{\left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(3.00 \times 10^8\ \text{m/s}\right)}{410 \times 10^{-9}\ \text{m}} = 4.85 \times 10^{-19}\ \text{J}$$

Then the sixth excited state is above the fifth by

$$\left(4.85 - 3.82\right) \times 10^{-19}\ \text{J} = 1.03 \times 10^{-19}\ \text{J}$$

In the 6 to 5 transition the atom emits a photon with the infrared wavelength

$$\lambda = \frac{hc}{E_{65}} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{1.03\times10^{-19}\ \text{J}}$$

$$= 1.94\times10^{-6}\ \text{m} = \boxed{1.94\ \mu\text{m}}$$

P42.4 (a) Denote the energy level *n* of the atom by E_n. For the transition $m \to 1$, the energy of the emitted photon and its wavelength λ_{m1} are related thus:

$$\Delta E_{m1} = E_m - E_1 = \frac{hc}{\lambda_{m1}}$$

For the transition $n \to 1$, the energy of the emitted photon and its wavelength λ_{n1} are related similarly:

$$\Delta E_{n1} = E_n - E_1 = \frac{hc}{\lambda_{n1}}$$

Therefore, for the transition $m \to n$, the energy of the emitted photon and its wavelength λ_{mn} (where *m* is the higher state, so $\lambda_{n1} > \lambda_{m1}$) can be related as

$$\Delta E_{mn} = E_m - E_n = \frac{hc}{\lambda_{mn}}$$

$$\Delta E_{mn} = (E_m - E_1) - (E_n - E_1) = \frac{hc}{\lambda_{mn}}$$

$$= \frac{hc}{\lambda_{m1}} - \frac{hc}{\lambda_{n1}} = \frac{hc}{\lambda_{mn}} \qquad \to \qquad \frac{1}{\lambda_{mn}} = \frac{1}{\lambda_{m1}} - \frac{1}{\lambda_{n1}}$$

This result may be written as $\boxed{\lambda_{mn} = \left|\dfrac{1}{1/\lambda_{m1} - 1/\lambda_{n1}}\right|}$.

(b) Multiply the result of part (a) by 2π and apply the definition $k_{ij} = 2\pi/\lambda_{ij}$:

$$2\pi\left(\frac{1}{\lambda_{mn}} = \left|\frac{1}{\lambda_{m1}} - \frac{1}{\lambda_{n1}}\right|\right) \qquad \to \qquad \boxed{k_{mn} = |k_{m1} - k_{n1}|}$$

P42.5 Our equation is $\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$ where $R_H = 1.097\ 373\ 2\times10^7\ \text{m}^{-1}$.

With our notation we have identified what Rydberg did not know, that the integers are the principal quantum numbers of the original and final atomic states in the photon emission process. For the Lyman series, we have $n_f = 1$, and $n_i = 2, 3, 4, \ldots$ We solve for the quantum number of the original state

$$\frac{1}{n_i^2} = \frac{1}{n_f^2} - \frac{1}{R_H\lambda} \quad \rightarrow \quad n_i = \left(\frac{1}{n_f^2} - \frac{1}{R_H\lambda}\right)^{-1/2}$$

(a) and substitute the given values.

$$n_i = \left(\frac{1}{1^2} - \frac{1}{94.96\times10^{-9}\,\text{m}\times1.097\times10^7\,\text{m}^{-1}}\right)^{-1/2} = \boxed{5}$$

The electron makes a transition from energy level 5 to the ground state to emit light in this spectral line.

(b) and (c) By Figure 42.8, spectral lines in the Balmer and Paschen series all have much longer wavelengths, since much smaller energy losses put the atom into energy levels 2 or 3. The expressions

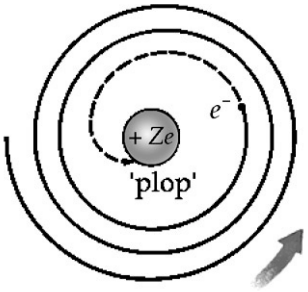
$$n_i = \left(\frac{1}{2^2} - \frac{1}{94.96\times10^{-9}\,\text{m}\times1.097\times10^7\,\text{m}^{-1}}\right)^{-1/2}$$

and $$n_i = \left(\frac{1}{3^2} - \frac{1}{94.96\times10^{-9}\,\text{m}\times1.097\times10^7\,\text{m}^{-1}}\right)^{-1/2}$$

are imaginary quantities, not real positive integers. The Lyman-delta wavelength given cannot be part of the Balmer or the Paschen series.

Section 42.2 Early Models of the Atom

P42.6 According to a classical model, the electron moving as a particle in uniform circular motion about the proton in the hydrogen atom experiences a force $k_e e^2/r^2$; and from Newton's second law, $F = ma$, its acceleration is $k_e e^2/m_e r^2$.



ANS. FIG. P42.6

(a) Using the fact that the Coulomb constant is $k_e = \dfrac{1}{4\pi \in_0}$, the centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{F}{m_e} = \frac{e^2}{4\pi\in_0 r^2 m_e} \qquad \rightarrow \qquad m_e v^2 = \frac{e^2}{4\pi\in_0 r}$$

The total energy is

$$E = K + U = \frac{m_e v^2}{2} - \frac{e^2}{4\pi\in_0 r} = -\frac{e^2}{8\pi\in_0 r}$$

Substitute the expressions for *E* and *a* into the relation for $\dfrac{dE}{dt}$:

$$\frac{dE}{dt} = \frac{-1}{6\pi\in_0}\frac{e^2 a^2}{c^3}$$

$$\frac{e^2}{8\pi\in_0 r^2}\frac{dr}{dt} = \frac{-e^2}{6\pi\in_0 c^3}\left(\frac{e^2}{4\pi\in_0 r^2 m_e}\right)^2$$

Therefore, $\dfrac{dr}{dt} = -\dfrac{e^4}{12\pi^2\in_0^2 m_e^2 c^3}\left(\dfrac{1}{r^2}\right)$.

(b) From the result of part (a), we have

$$T = \int_0^T dt = -\int_{2.00\times10^{-10}\text{ m}}^{0} \frac{12\pi^2\in_0^2 r^2 m_e^2 c^3}{e^4} dr$$

$$= \int_0^{2.00\times10^{-10}\text{ m}} \frac{12\pi^2\in_0^2 r^2 m_e^2 c^3}{e^4} dr$$

$$= \frac{12\pi^2\in_0^2 m_e^2 c^3}{e^4}\frac{r^3}{3}\bigg|_0^{2.00\times10^{-10}}$$

$$= \frac{12\pi^2\left(8.85\times10^{-12}\text{ C}\right)^2\left(9.11\times10^{-31}\text{ kg}\right)^2\left(3.00\times10^8\text{ m/s}\right)^3}{\left(1.60\times10^{-19}\text{ C}\right)^4}$$

$$\times\frac{\left(2.00\times10^{-10}\text{ m}\right)^3}{3}$$

$$= 8.46\times10^{-10}\text{ s} = \boxed{0.846\text{ ns}}$$

Since atoms last much longer than 0.8 ns, the classical laws (fortunately!) do not hold for systems of atomic size.

P42.7 (a) The point of closest approach is found when

$$E = K_i + U_i = K_f + U_f$$

$$K_i + 0 = 0 + \frac{k_e q_\alpha q_{Au}}{r_{min}}$$

$$\rightarrow \quad r_{min} = \frac{k_e(2e)(79e)}{K_i}$$

$$r_{min} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(158)(1.602\times10^{-19}\text{ C})^2}{(4.00\times10^6\text{ eV})(1.602\times10^{-19}\text{ J/eV})}$$

$$= \boxed{5.69\times10^{-14}\text{ m}}$$

(b) The maximum force exerted on the alpha particle is

$$F_{max} = \frac{k_e q_\alpha q_{Au}}{r_{min}^2} = \frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(158)(1.602\times10^{-19}\text{ C})^2}{(5.69\times10^{-14}\text{ m})^2}$$

$$= \boxed{11.3\text{ N}}$$

away from the nucleus.

Section 42.3 Bohr's Model of the Hydrogen Atom

***P42.8** From the equation just above Equation 42.9 in the text, $\frac{1}{2}m_e v^2 = \frac{k_e e^2}{2r}$, we have

$$v^2 = \frac{k_e e^2}{m_e r}$$

and using

$$r_n = \frac{n^2\hbar^2}{m_e k_e e^2}$$

we obtain

$$v_n^2 = \frac{k_e e^2}{m_e(n^2\hbar^2/m_e k_e e^2)}$$

or

$$v_n = \frac{k_e e^2}{n\hbar}$$

***P42.9** We use $E_n = \frac{-13.6\text{ eV}}{n^2}$. To ionize the atom when the electron is in the *n*th level, it is necessary to add an amount of energy given by

$$E = -E_n = \frac{13.6\text{ eV}}{n^2}$$

(a) Thus, in the ground state where *n* = 1, we have $\boxed{E = 13.6\text{ eV}}$.

(b) In the *n* = 3 level, $E = \frac{13.6\text{ eV}}{3^2} = \boxed{1.51\text{ eV}}$.

P42.10 The allowed energy levels of the hydrogen atom are given by

$$E_n = -\frac{13.6\text{ eV}}{n^2} \quad \text{where } n = 1, 2, 3\ldots$$

A transition in which a lower state n_i absorbs a photon of energy ΔE results in a higher state n_f, and energy is conserved:

$$E_i + \Delta E = E_f$$

or

$$\Delta E = E_f - E_i = -\frac{13.6\text{ eV}}{n_f^2} - \left(-\frac{13.6\text{ eV}}{n_i^2}\right) = 13.6\text{ eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

(a) For the transition $n_i = 2$ to $n_f = 5$,

$$\Delta E = 13.6\text{ eV}\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = \boxed{2.86\text{ eV}}$$

(b) For the transition $n_i = 4$ to $n_f = 6$,

$$\Delta E = 13.6\text{ eV}\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = \boxed{0.472\text{ eV}}$$

P42.11 The allowed energy levels of the hydrogen atom are given by

$$E_n = -\frac{13.6\text{ eV}}{n^2} \quad \text{where } n = 1, 2, 3\ldots$$

In a transition for higher state n_i to lower state n_f, a photon of energy ΔE is emitted, and energy is conserved:

$$E_i + \Delta E = E_f$$

or

$$\Delta E = E_f - E_i = -\frac{13.6\text{ eV}}{n_f^2} - \left(-\frac{13.6\text{ eV}}{n_i^2}\right) = 13.6\text{ eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

(a) For the transition $n_i = 5$ to $n_f = 3$,

$$\Delta E = 13.6\text{ eV}\left(\frac{1}{3^2} - \frac{1}{5^2}\right) = \boxed{0.967\text{ eV}}$$

(b) To find the wavelength of the emitted photon, we use Equation 42.5:

$$\Delta E = 0.967\text{ eV} = hf = \frac{hc}{\lambda}$$

Solving,

$$\lambda = \frac{hc}{\Delta E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{0.967\text{ eV}} = 1\,282\text{ nm} = \boxed{1.28\ \mu\text{m}}$$

(c) The frequency of the emitted photon is

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{1\,282\times10^{-9}\text{ m}} = \boxed{2.34\times10^{14}\text{ Hz}}$$

P42.12 (a) The longest wavelength implies lowest frequency and smallest energy. The electron makes a transition from $n = 3$ to $n = 2$:

$$\Delta E = -\frac{13.6\text{ eV}}{3^2} + \frac{13.6\text{ eV}}{2^2} = \boxed{1.89\text{ eV}}$$

(b) The photon's wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{(1.89\text{ eV})} = \boxed{656\text{ nm}}$$

This is the red Balmer-alpha line, which gives its characteristic color to the chromosphere of the Sun and to photographs of the Orion nebula.

(c) The shortest wavelength implies highest frequency and greatest energy. The electron makes a transition from $n = \infty$ to $n = 2$:

$$\Delta E = -\frac{13.6\text{ eV}}{\infty} + \frac{13.6\text{ eV}}{2^2} = \boxed{3.40\text{ eV}}$$

(d) The photon's wavelength is

$$\lambda = \frac{c}{f} = \frac{hc}{\Delta E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{3.40\text{ eV}} = \boxed{365\text{ nm}}$$

(e) This is the Balmer series limit, $\boxed{365\text{ nm}}$, in the near ultraviolet.

P42.13 (a) From the equation just above Equation 42.9 in the text, $v_1 = \sqrt{\dfrac{k_e e^2}{m_e r_1}}$

where, from Equation 42.10,

$$r_1 = (1)^2 a_0 = 0.005\,29 \text{ nm} = 5.29 \times 10^{-11} \text{ m}$$

Substituting numerical values,

$$v_1 = \sqrt{\frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{\left(9.11 \times 10^{-31} \text{ kg}\right)\left(5.29 \times 10^{-11} \text{ m}\right)}}$$

$$= \boxed{2.19 \times 10^6 \text{ m/s}}$$

(b) The kinetic energy of the electron is

$$K_1 = \frac{1}{2} m_e v_1^2 = \frac{1}{2}\left(9.11 \times 10^{-31} \text{ kg}\right)\left(2.19 \times 10^6 \text{ m/s}\right)^2$$

$$= 2.18 \times 10^{-18} \text{ J} = \boxed{13.6 \text{ eV}}$$

(c) The electric potential energy of the atom is

$$U_1 = -\frac{k_e e^2}{r_1} = -\frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19} \text{ C}\right)^2}{5.29 \times 10^{-11} \text{ m}}$$

$$= -4.35 \times 10^{-18} \text{ J} = \boxed{-27.2 \text{ eV}}$$

***P42.14** Each atom gives up its kinetic energy in emitting a photon, so

$$\frac{1}{2} m v^2 = \frac{hc}{\lambda} = \frac{\left(6.626 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{\left(1.216 \times 10^{-7} \text{ m}\right)}$$

$$= 1.63 \times 10^{-18} \text{ J}$$

Their speed before the collision is

$$v = \sqrt{\frac{2(1.63 \times 10^{-18} \text{ J})}{1.67 \times 10^{-27} \text{ kg}}} = \boxed{4.42 \times 10^4 \text{ m/s}}$$

***P42.15** (a) The speed of the moon in its orbit is

$$v = \frac{2\pi r}{T} = \frac{2\pi\left(3.84 \times 10^8 \text{ m}\right)}{2.36 \times 10^6 \text{ s}} = 1.02 \times 10^3 \text{ m/s}$$

so,

$$L = mvr = \left(7.36 \times 10^{22} \text{ kg}\right)\left(1.02 \times 10^3 \text{ m/s}\right)\left(3.84 \times 10^8 \text{ m}\right)$$

$$= \boxed{2.89 \times 10^{34} \text{ kg} \cdot \text{m}^2/\text{s}}$$

(b) We have $L = n\hbar,$

or $$n = \frac{L}{\hbar} = \frac{2.89\times10^{34}\ \text{kg}\cdot\text{m}^2/\text{s}}{1.055\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{2.74\times10^{68}}.$$

(c) We have $$n\hbar = L = mvr = m\left(\frac{GM_e}{r}\right)^{1/2} r,$$

so $$r = \frac{\hbar^2}{m^2GM_e}n^2 = Rn^2 \text{ and } \frac{\Delta r}{r} = \frac{(n+1)^2R - n^2R}{n^2R} = \frac{2n+1}{n^2},$$

which is approximately equal to $\frac{2}{n} = \boxed{7.30\times10^{-69}}$.

P42.16 (a) The collection of excited atoms must make these six transitions to get back to state one: $4\to1$, $4\to2$, and $4\to3$; $3\to1$ and $3\to2$; $2\to1$. Thus, the absorbed photon changes the atomic state from 1 to 4:

$$E_1 + hf = E_4 \quad \to \quad hf = E_4 - E_1, \quad \text{where} \quad E_n = -\frac{13.6\text{ eV}}{n^2}$$

The incoming photons have energy

$$hf = \Delta E = E_f - E_i = -0.850\text{ eV} - (-13.6\text{ eV}) = 12.75\text{ eV} = \frac{hc}{\lambda}$$

and wavelength

$$\lambda = \frac{hc}{\Delta E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{12.75\text{ eV}} = \boxed{97.3\text{ nm}}$$

(b) The longest of the six wavelengths corresponds to the lowest photon energy, emitted in the transition $4\to3$. By energy conservation, $E_4 = hf + E_3$ and

$$hf = E_4 - E_3 = 13.6\text{ eV}\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 0.661\text{ eV} = \frac{hc}{\lambda}$$

which gives

$$\lambda = \frac{hc}{E_4 - E_3} = \frac{1\,240\text{ eV}\cdot\text{nm}}{0.661\text{ eV}} = 1\,876\text{ nm} = \boxed{1.88\ \mu\text{m}}$$

(c) The wavelength is in the $\boxed{\text{infrared}}$ region of the spectrum.

(d) The wavelength is part of the $\boxed{\text{Paschen}}$ series, since the lower state has $n = 3$.

(e) The shortest wavelength emitted is from the transition $4\to1$, and it is the same as the wavelength absorbed: $\boxed{97.3\text{ nm}}$.

(f) The wavelength is in the $\boxed{\text{ultraviolet}}$ region of the spectrum.

(g) The wavelength is part of the $\boxed{\text{Lyman}}$ series, since the lower state has *n* = 1.

P42.17 (a) From Equation 42.12,

$$r_n = n^2 a_0 = n^2(0.052\,9\text{ nm})$$

and $r_3 = (3)^2(0.052\,9\text{ nm}) = \boxed{0.476\text{ nm}}$

(b) Using Equation 42.8, we calculate the momentum of the electron:

$$m_e v_2 = m_e\sqrt{\frac{k_e e^2}{m_e r_2}} = \sqrt{\frac{m_e k_e e^2}{r_2}}$$

$$= \sqrt{\frac{(9.11\times10^{-31}\text{ kg})(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\text{ C})^2}{0.476\times10^{-9}\text{ m}}}$$

$$= 6.64\times10^{-25}\text{ kg}\cdot\text{m/s}$$

The de Broglie wavelength for the electron is

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{6.64\times10^{-25}\text{ kg}\cdot\text{m/s}} = 9.97\times10^{-10}\text{ m} = \boxed{0.997\text{ nm}}$$

P42.18 We note, during our calculations, that the nominal velocity of the electron is less than 1% of the speed of light; therefore, we do not need to use relativistic equations.

(a) By Bohr's theory and Equation 42.12,

$$r_n = n^2 a_0$$

$$r_2 = (2)^2(0.052\,9\text{ nm}) = \boxed{0.212\text{ nm}}$$

(b) Since $m_e vr = n\hbar$,

$$p = m_e v = \frac{n\hbar}{r} = \frac{2(1.054\,6\times10^{-34}\text{ J}\cdot\text{s})}{2.12\times10^{-10}\text{ m}}$$

$$= \boxed{9.97\times10^{-25}\text{ kg}\cdot\text{m/s}}$$

(c) $\vec{\mathbf{L}} = \vec{\mathbf{r}}\times\vec{\mathbf{p}}$ becomes

$$L_2 = m_e v_2 r_2 = (9.97\times10^{-25}\text{ kg}\cdot\text{m/s})(0.212\times10^{-9}\text{ m})$$

$$= \boxed{2.11\times10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}}$$

(d) Next, the speed is

$$v = \frac{p}{m_e} = \frac{9.97\times10^{-25}\text{ kg}\cdot\text{m/s}}{9.11\times10^{-31}\text{ kg}} = 1.09\times10^{6}\text{ m/s}$$

So the kinetic energy is $K = \frac{1}{2}m_e v^2$:

$$K = \frac{\left(9.11\times 10^{-31}\text{ kg}\right)\left(1.09\times 10^{6}\text{m/s}\right)^2}{2}$$

$$= \frac{5.45\times 10^{-19}\text{ J}}{1.602\times 10^{-19}\text{ J/eV}} = \boxed{3.40\text{ eV}}$$

(e) From Chapter 25, the electric potential energy is $U = k_e\frac{q_1q_2}{r}$:

$$U = -\frac{k_e e^2}{r} = -\frac{\left(8.99\times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602\times 10^{-19}\text{ C}\right)^2}{2.12\times 10^{-10}\text{ m}}$$

$$= -1.09\times 10^{-18}\text{ J} = \boxed{-6.80\text{ eV}}$$

(f) Thus the total energy is

$$E = K + U = -5.45\times 10^{-19}\text{ J} = \boxed{-3.40\text{ J}}$$

P42.19 (a) The photon has energy 2.28 eV, and $\frac{13.6\text{ eV}}{2^2} = 3.40\text{ eV}$ is required to ionize a hydrogen atom from state *n* = 2. So while the photon cannot ionize a hydrogen atom pre-excited to *n* = 2, it can ionize a hydrogen atom in the $n = \boxed{3}$ state, with energy $-\frac{13.6\text{ eV}}{3^2} = -1.51\text{ eV}$.

(b) The electron thus freed can have kinetic energy

$$K_e = 2.28\text{ eV} - 1.51\text{ eV} = 0.769\text{ eV} = \frac{1}{2}m_e v^2$$

Therefore,

$$v = \sqrt{\frac{2\left(0.769\text{ eV}\right)\left(1.60\times 10^{-19}\text{ J/eV}\right)}{9.11\times 10^{-31}\text{ kg}}} = 5.20\times 10^{5}\text{ m/s}$$

$$= \boxed{520\text{ km/s}}$$

P42.20 (a) From the Bohr theory, we find the speed of the electron:

$$L = m_e vr = n\hbar \quad \rightarrow \quad v = \frac{n\hbar}{m_e r}$$

The period of its orbital motion is $T = \frac{2\pi r}{v} = \frac{2\pi r m_e r}{n\hbar}$.

Substituting the orbital radius $r = \dfrac{n^2\hbar^2}{m_e k_e e^2}$, we find

$$T = \frac{2\pi m_e n^4\hbar^4}{n\hbar m_e^2 k_e^2 e^4} = \frac{2\pi\hbar^3}{m_e k_e^2 e^4} n^3$$

Thus we have the periods determined in terms of the ground-state period

$$t_0 = \frac{2\pi\hbar^3}{m_e k_e^2 e^4}$$

$$= \frac{2\pi(1.054\ 6\times10^{-34}\ \text{J}\cdot\text{s})^3}{\left(9.11\times10^{-31}\ \text{kg}\right)(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)^2(1.602\times10^{-19}\ \text{C})^4}$$

$$= 1.52\times10^{-16}\ \text{s} = 152\times10^{-18}\ \text{s} = \boxed{152\ \text{as}}$$

(b) In the *n* = 2 state, the period is

$$T = t_0 n^3 = t_0(2)^3 = 8t_0 = 1.22\times10^{-15}\ \text{s}$$

The number of orbits completed in the excited state is

$$N = \frac{10\times10^{-6}\ \text{s}}{T} = \frac{10\times10^{-6}\ \text{s}}{1.22\times10^{-15}\ \text{s}} = \boxed{8.23\times10^{9}\ \text{revolutions}}$$

(c) Its lifetime in electron years is comparable to the lifetime of the Sun in Earth years, so we can think of it as a long time.

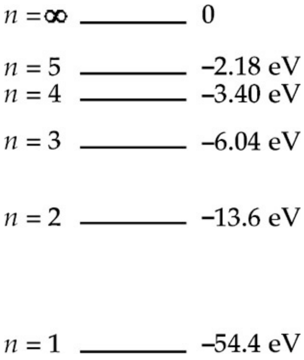
P42.21 (a) The energy levels of a hydrogen-like ion whose charge number is *Z* are given by

$$E_n = (-13.6\ \text{eV})\frac{Z^2}{n^2}$$

Thus for helium (*Z* = 2), the energy levels are

$$\boxed{E_n = -\frac{54.4\ \text{eV}}{n^2} \quad n = 1,\ 2,\ 3,\ \ldots}$$

The enegy level diagram for helium is shown in ANS. FIG. P42.21.



ANS. FIG. P42.21

(b) For He$^+$, *Z* = 2, so we see that the ionization energy (the energy required to take the electron from the *n* = 1 to the *n* = ∞ state) is

$$E = E_\infty - E_1 = 0 - \frac{(-13.6\ \text{eV})(2)^2}{(1)^2} = \boxed{54.4\ \text{eV}}$$

Section 42.4 The Quantum Model of the Hydrogen Atom

P42.22 The reduced mass of positronium is **less** than hydrogen, so the photon energy will be **less** for positronium than for hydrogen. This means that the wavelength of the emitted photon will be **longer** than 656.3 nm. On the other hand, helium has about the same reduced mass but more charge than hydrogen, so its transition energy will be **larger**, corresponding to a wavelength **shorter** than 656.3 nm.

All the factors in the given equation are constant for this problem except for the reduced mass and the nuclear charge. Therefore, the wavelength corresponding to the energy difference for the transition can be found simply from the ratio of mass and charge variables.

For hydrogen, $\mu = \dfrac{m_p m_e}{m_p + m_e} \approx m_e$. The photon energy is $\Delta E = E_3 - E_2$.

Its wavelength is $\lambda = 656.3$ nm, where $\lambda = \dfrac{c}{f} = \dfrac{hc}{\Delta E}$.

(a) For positronium, $\mu = \dfrac{m_e m_e}{m_e + m_e} = \dfrac{m_e}{2}$,

so the energy of each level is one half as large as in hydrogen. The photon energy is inversely proportional to its wavelength, so for positronium,

$$\lambda_{32} = 2(656.3 \text{ nm}) = \boxed{1.31\ \mu\text{m}} \quad \text{(in the infrared region)}$$

(b) For He$^+$, $\mu \approx m_e$, $q_1 = e$, and $q_2 = 2e$, so the transition energy is $2^2 = 4$ times larger than hydrogen. Then,

$$\lambda_{32} = \left(\frac{656}{4}\right) \text{nm} = \boxed{164 \text{ nm}} \quad \text{(in the ultraviolet region)}$$

P42.23 (a) For this problem, refer to the equation from Problem 22, with $q_1 = q_2 = e$. For a particular transition from n_i to n_f,

$$\Delta E_H = -\frac{\mu_H k_e^2 e^4}{2\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{hc}{\lambda_H}$$

and $$\Delta E_D = -\frac{\mu_D k_e^2 e^4}{2\hbar^2}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \frac{hc}{\lambda_D},$$

where $\mu_H = \dfrac{m_e m_p}{m_e + m_p}$ and $\mu_D = \dfrac{m_e m_D}{m_e + m_D}$.

By division, $\frac{\Delta E_H}{\Delta E_D} = \frac{\mu_H}{\mu_D} = \frac{\lambda_D}{\lambda_H}$ or $\lambda_D = \left(\frac{\mu_H}{\mu_D}\right)\lambda_H$. Then,

$$\boxed{\lambda_H - \lambda_D = \left(1 - \frac{\mu_H}{\mu_D}\right)\lambda_H}$$

(b) $$\frac{\mu_H}{\mu_D} = \left(\frac{m_e m_p}{m_e + m_p}\right)\left(\frac{m_e + m_D}{m_e m_D}\right)$$

$$= \frac{(1.007\ 276\ \text{u})(0.000\ 549\ \text{u} + 2.013\ 553\ \text{u})}{(0.000\ 549\ \text{u} + 1.007\ 276\ \text{u})(2.013\ 553\ \text{u})}$$

$$= 0.999\ 728$$

$$\lambda_H - \lambda_D = (1 - 0.999\ 728)(656.3\ \text{nm}) = \boxed{0.179\ \text{nm}}$$

P42.24 (a) The uncertainty principle is represented by $\Delta x \Delta p \geq \frac{\hbar}{2}$.

Thus, if $\Delta x = r$, $\boxed{\Delta p \geq \frac{\hbar}{2r}}$.

(b) The minimum uncertainty would be attained only if the wave function had a particular (gaussian) waveform. We assume that the momentum uncertainty is just twice as large as its minimum possible value: $\Delta p = \frac{\hbar}{r}$. Then the kinetic energy is

$$K = \frac{p^2}{2m_e} \approx \frac{(\Delta p)^2}{2m_e} = \boxed{\frac{\hbar^2}{2m_e r^2}}$$

(c) The electric potential energy is $U = -\frac{k_e e^2}{r}$ so the total energy is

$$E = K + U \approx \boxed{\frac{\hbar^2}{2m_e r^2} - \frac{k_e e^2}{r}}.$$

(d) To minimize *E* as a function of *r*, we require

$$\frac{dE}{dr} = -\frac{\hbar^2}{m_e r^3} + \frac{k_e e^2}{r^2} = 0 \quad \rightarrow \quad r = \boxed{\frac{\hbar^2}{m_e k_e e^2} = a_0} \text{ (the Bohr radius)}$$

(e) Then the energy is

$$E = \frac{\hbar^2}{2m_e}\left(\frac{m_e k_e e^2}{\hbar^2}\right)^2 - k_e e^2\left(\frac{m_e k_e e^2}{\hbar^2}\right) = -\frac{m_e k_e^2 e^4}{2\hbar^2}$$

Substituting numerical values,

$$E = -\frac{m_e k_e^2 e^4}{2\hbar^2} = -\frac{m_e k_e^2 e^4}{2\hbar^2}$$

$$= -\frac{\left(9.109\times10^{-31}\text{ kg}\right)\left(8.988\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)^2\left(1.602\times10^{-19}\text{ C}\right)^4}{2\left(6.626\times10^{-34}\text{ J}\cdot\text{s}/2\pi\right)^2}$$

$$= -2.179\times10^{-18}\text{ J}\left(\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right)$$

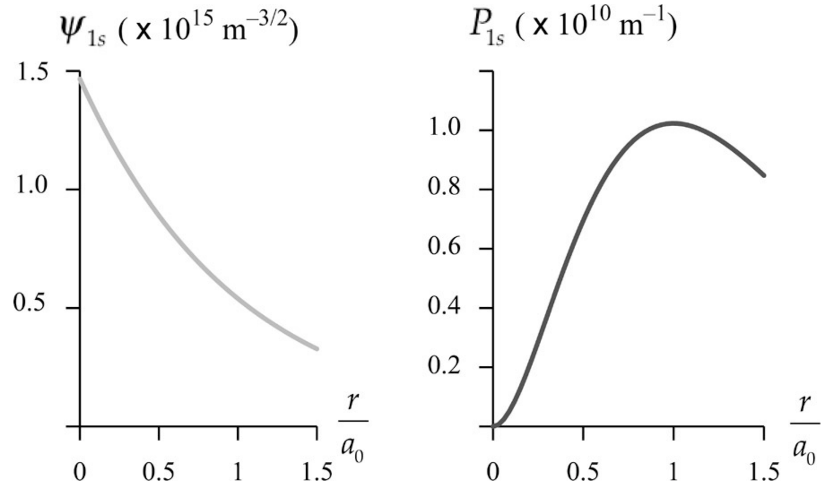
$$= \boxed{-13.6\text{ eV}}$$

(f) With our particular choice for the momentum uncertainty as double its minimum possible value, $\boxed{\text{we find our results are in agreement with the Bohr theory.}}$

Section 42.5 The Wave Functions for Hydrogen

P42.25 $\psi_{1s}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$ is the ground state hydrogen wave function.

$P_{1s}(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0}$ is the ground state radial probability distribution function. The plots are shown in ANS. FIG. P42.25.



ANS. FIG. P42.25

P42.26 (a) We first find the first and second derivatives of the wave function:

$$\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0} \quad \rightarrow \quad \frac{2}{r}\frac{d\psi}{dr} = \frac{2}{r}\left(\frac{-1}{\sqrt{\pi a_0^5}} e^{-r/a_0}\right) = -\frac{2}{ra_0}\psi$$

and $\dfrac{d^2\psi}{dr^2} = \dfrac{1}{\sqrt{\pi a_0^7}} e^{-r/a_0} = \dfrac{1}{a_0^2}\psi$

Substitution into the Schrödinger equation to test the validity of the solution yields

$$-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2} - \frac{2}{ra_0}\right)\psi - \frac{e^2}{4\pi \epsilon_0 r}\psi = E\psi$$

But, from Equation 42.11, $a_0 = \dfrac{\hbar^2}{m_e k_e e^2} = \dfrac{\hbar^2 (4\pi \epsilon_0)}{m_e e^2}$, thus

$$-\frac{\hbar^2}{2m_e}\left(\frac{1}{a_0^2} - \frac{2}{ra_0}\right)\psi - \frac{e^2}{4\pi \epsilon_0 r}\psi = E\psi$$

$$-\frac{\hbar^2}{2m_e}\frac{1}{a_0^2}\psi + \left[\frac{\hbar^2}{m_e r}\frac{1}{a_0}\psi - \frac{e^2}{4\pi \epsilon_0 r}\psi\right] = E\psi$$

$$-\frac{\hbar^2}{2m_e}\frac{1}{a_0^2}\psi + \left[\frac{\cancel{\hbar^2}}{\cancel{m_e} r}\frac{\cancel{m_e} e^2}{4\pi \epsilon_0 \cancel{\hbar^2}} - \frac{e^2}{4\pi \epsilon_0 r}\right]\psi = E\psi$$

$$-\frac{\hbar^2}{2m_e}\frac{1}{a_0^2}\psi = E\psi$$

The Schrödinger equation is satisfied if $E = -\dfrac{\hbar^2}{2m_e}\dfrac{1}{a_0^2}$.

(b) Substituting $a_0 = \dfrac{\hbar^2}{m_e k_e e^2}$ for one factor of a_0, we find that

$$E = -\frac{\hbar^2}{2m_e}\frac{1}{a_0^2} = -\frac{\cancel{\hbar^2}}{2\cancel{m_e}}\frac{1}{a_0}\frac{\cancel{m_e} k_e e^2}{\cancel{\hbar^2}} = \boxed{E = -\frac{k_e e^2}{2a_0}}$$

P42.27 The wave function given is

$$\psi = \frac{1}{\sqrt{3}}\frac{1}{(2a_0)^{3/2}}\frac{r}{a_0}e^{-r/2a_0}$$

so, by Equation 42.24,

$$P_r = 4\pi r^2 |\psi|^2 = 4\pi r^2 \frac{r^2}{24a_0^5} e^{-r/a_0}$$

Setting $\dfrac{dP}{dr} = \dfrac{4\pi}{24a_0^5}\left[4r^3 e^{-r/a_0} + r^4\left(-\dfrac{1}{a_0}\right)e^{-r/a_0}\right] = 0$, we obtain

$$\left[4r^3 + -\frac{r^4}{a_0}\right] = 0$$

Solving for *r*, this is a maximum at $\boxed{r = 4a_0}$.

P42.28 (a) $\int |\psi|^2 dV = 4\pi \int_0^\infty |\psi|^2 r^2 dr = 4\pi\left(\dfrac{1}{\pi a_0^3}\right)\int_0^\infty r^2 e^{-2r/a_0} dr$. Using integral tables,

$$\int |\psi|^2 dV = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_0^\infty = \left(-\frac{2}{a_0^2}\right)\left(-\frac{a_0^2}{2}\right) = \boxed{1}$$

so the wave function as given is normalized.

(b) $P_{a_0/2\to 3a_0/2} = 4\pi \int_{a_0/2}^{3a_0/2} |\psi|^2 r^2 dr = 4\pi\left(\dfrac{1}{\pi a_0^3}\right)\int_{a_0/2}^{3a_0/2} r^2 e^{-2r/a_0} dr$. Again, using integral tables,

$$P_{a_0/2\to 3a_0/2} = -\frac{2}{a_0^2}\left[e^{-2r/a_0}\left(r^2 + a_0 r + \frac{a_0^2}{2}\right)\right]_{a_0/2}^{3a_0/2}$$

$$= -\frac{2}{a_0^2}\left[e^{-3}\left(\frac{17a_0^2}{4}\right) - e^{-1}\left(\frac{5a_0^2}{4}\right)\right] = \boxed{0.497}$$

P42.29 The hydrogen ground-state radial probability density is, from Equation 42.25,

$$P_{1s}(r) = 4\pi r^2 |\psi_{1s}|^2 = \frac{4r^2}{a_0^3}\exp\left(-\frac{2r}{a_0}\right)$$

The number of observations at $2a_0$ is, by proportion,

$$N = (1\,000)\frac{P_{1s}(2a_0)}{P_{1s}(a_0/2)} = (1\,000)\frac{(2a_0)^2}{(a_0/2)^2}\frac{e^{-4a_0/a_0}}{e^{-a_0/a_0}} = (1\,000)(16)e^{-3}$$

$$= \boxed{797 \text{ times}}$$

Section 42.6 Physical Interpretation of the Quantum Numbers

P42.30 (a) In the 3*d* subshell, $n = 3$ and $\ell = 2$, we have

n	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+1	+1	0	0	–1	–1	–2	–2
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(a total of 10 states.)

(b) In the 3*p* subshell, $n = 3$ and $\ell = 1$, we have

n	3	3	3	3	3	3
ℓ	1	1	1	1	1	1
m_ℓ	+1	+1	0	0	–1	–1
m_s	+1/2	–1/2	+1/2	–1/2	+1/2	–1/2

(a total of 6 states.)

***P42.31** From Equation 42.27, $L = \sqrt{\ell(\ell+1)}\hbar$ (suppressing units):

$$4.714 \times 10^{-34} = \sqrt{\ell(\ell+1)}\left(\frac{6.626 \times 10^{-34}}{2\pi}\right)$$

$$\ell(\ell+1) = \frac{\left(4.714 \times 10^{-34}\right)^2 (2\pi)^2}{\left(6.626 \times 10^{-34}\right)^2} = 1.998 \times 10^1 \approx 20 = 4(4+1)$$

so $\boxed{\ell = 4}$.

P42.32 (a) For a 3*d* state, $n = 3$ and $\ell = 2$. Therefore,

$$L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar} = 2.58 \times 10^{-34}\ \text{J}\cdot\text{s}$$

(b) m_ℓ can have the values –2, –1, 0, 1, and 2,

so $\boxed{L_z \text{ can have the values } -2\hbar,\ -\hbar,\ 0,\ \hbar \text{ and } 2\hbar}$.

(c) Using the relation $\cos\theta = \frac{L_z}{L}$, we find the possible values of θ:

$$\boxed{145°,\ 114°,\ 90.0°,\ 65.9°,\ \text{and } 35.3°}$$

P42.33 From Equation 42.27, $L = \sqrt{\ell(\ell+1)}\hbar$.

(a) For the *d* state, $\ell = 2$, and $L = \boxed{\sqrt{6}\hbar = 2.58\times 10^{-34}\ \text{J}\cdot\text{s}}$.

(b) For the *f* state, $\ell = 3$, and $L = \sqrt{12}\hbar = \boxed{2\sqrt{3}\hbar = 3.65\times 10^{-34}\ \text{J}\cdot\text{s}}$.

P42.34 (a) For *n* = 1, we have $\ell = 0$, $m_\ell = 0$, $m_s = \pm\frac{1}{2}$.

n	ℓ	m_ℓ	m_s
1	0	0	–1/2
1	0	0	+1/2

This yields $2n^2 = 2(1)^2 = \boxed{2}$ sets.

(b) For *n* = 2, we have

n	ℓ	m_ℓ	m_s
2	0	0	±1/2
2	1	–1	±1/2
2	1	0	±1/2
2	1	+1	±1/2

This yields $2n^2 = 2(2)^2 = \boxed{8}$ sets.

Note that the number is twice the number of m_ℓ values. Also, for each ℓ there are $(2\ell+1)$ different m_ℓ values. Finally, l can take on values ranging from 0 to *n* – 1.

So the general expression is $\text{number} = \sum_{\ell=0}^{n-1} 2(2\ell+1)$.

The series is an arithmetic progression like 2 + 6 + 10 + 14.

The sum is $\sum_{0}^{n-1} 4\ell + \sum_{0}^{n-1} 2 = 4\left[\frac{n^2 - n}{2}\right] + 2n = 2n^2$.

(c) *n* = 3: 2(1) + 2(3) + 2(5) = 2 + 6 + 10 = 18 or $2n^2 = 2(3)^2 = \boxed{18}$

(d) *n* = 4: 2(1) + 2(3) + 2(5) + 2(7) = 32 or $2n^2 = 2(4)^2 = \boxed{32}$

(e) *n* = 5: 32 + 2(9) = 32 + 18 = 50 or $2n^2 = 2(5)^2 = \boxed{50}$

P42.35 In the N shell, *n* = 4. For *n* = 4, ℓ can take on values of 0, 1, 2, and 3. For each value of ℓ, m_ℓ can be $-\ell$ to ℓ in integral steps. Thus, the maximum value for m_ℓ is 3. Since $L_z = m_\ell \hbar$, the maximum value for L_z is $L_z = \boxed{3\hbar}$.

P42.36 (a) Modeling it as a solid sphere, the density of a proton is,

$$\rho = \frac{m}{V} = \frac{1.67 \times 10^{-27}\ \text{kg}}{(4/3)\pi\left(1.00 \times 10^{-15}\ \text{m}\right)^3} = \boxed{3.99 \times 10^{17}\ \text{kg/m}^3}$$

(b) The radius of an electron modelled as a solid sphere is,

$$r = \left(\frac{3m}{4\pi\rho}\right)^{1/3} = \left[\frac{3\left(9.11 \times 10^{-31}\ \text{kg}\right)}{4\pi\left(3.99 \times 10^{17}\ \text{kg/m}^3\right)}\right]^{1/3}$$
$$= 8.17 \times 10^{-17}\ \text{m} = 81.7 \times 10^{-18}\ \text{m} = \boxed{8.17\ \text{am}}$$

(c) The moment of inertia of the spinning electron is

$$I = \frac{2}{5}mr^2 = \frac{2}{5}\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(8.17 \times 10^{-17}\ \text{m}\right)^2$$
$$= 2.43 \times 10^{-63}\ \text{kg}\cdot\text{m}^2$$

$$L_z = I\omega = \frac{\hbar}{2} = \frac{Iv}{r}$$

Therefore,

$$v = \frac{\hbar r}{2I} = \frac{\left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(8.17 \times 10^{-17}\ \text{m}\right)}{2\pi\left(2 \times 2.43 \times 10^{-63}\ \text{kg}\cdot\text{m}^2\right)}$$
$$= 1.77 \times 10^{12}\ \text{m/s} = \boxed{1.77\ \text{Tm/s}}$$

(d) It is $5.91 \times 10^3 c$, which is huge compared with the speed of light and impossible.

P42.37 The 5th excited state has $n = 6$, energy $E_6 = \dfrac{-13.6\text{ eV}}{(6)^2} = -0.378\text{ eV}$.

The atom loses this much energy:

$$E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(1\,090\times10^{-9}\text{ m}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} = 1.14\text{ eV}$$

to end up with energy $-0.378\text{ eV} - 1.14\text{ eV} = -1.52\text{ eV}$

which is the energy in state 3: $-\dfrac{13.6\text{ eV}}{3^3} = -1.51\text{ eV}$

While $n = 3$, ℓ can be as large as 2, giving angular momentum

$$L = \sqrt{\ell(\ell+1)}\hbar = \boxed{\sqrt{6}\hbar = 2.58\times10^{-34}\text{ J}\cdot\text{s}}$$

P42.38 The energy of the photon is

$$E_{ph} = \frac{1\,240\text{ eV}\cdot\text{nm}}{88.0\text{ eV}} = 14.1\text{ eV}$$

The maximum energy of the ejected photoelectron from the aluminum surface is

$$K_{max} = E_{ph} - \phi = 14.1\text{ eV} - 4.08\text{ eV} = 10.0\text{ eV}$$

where the work function ϕ for aluminum is found from Table 40.1. This electron energy is not enough to excite the hydrogen atom from its ground state to even the first exited state.

P42.39 The 3*d* subshell has $n = 3$ and $\ell = 2$. Also, we have $s = 1$. Altogether we can have $n = 3$, $\ell = 2$, $m_\ell = -2, -1, 0, 1, 2$, $s = 1$, and $m_s = -1, 0, 1$, leading to the following table:

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	–2	–2	–2	–1	–1	–1	0	0	0	1	1	1	2	2	2
s	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
m_s	–1	0	1	–1	0	1	–1	0	1	–1	0	1	–1	0	1

Section 42.7 The Exclusion Principle and the Periodic Table

P42.40 (a) $\boxed{\text{The 4s subshell}}$, for potassium and calcium, before the 3*d* subshell starts to fill for scandium through zinc.

(b) $\boxed{\begin{array}{l}\text{We would expect [Ar]}3d^4 4s^2\text{ to have lower energy, but}\\ \text{[Ar]}3d^5 4s^1\text{ has more unpaired spins and lower energy}\\ \text{according to Hund's rule.}\end{array}}$

(c) It is the ground-state configuration of $\boxed{\text{chromium}}$.

P42.41 (a) $\boxed{1s^2 2s^2 2p^4}$

(b) For the 1*s* electrons,

$$n = 1,\ \ell = 0,\ m_\ell = 0,\ m_s = +\frac{1}{2} \text{ and } -\frac{1}{2}$$

For the two 2*s* electrons,

$$n = 2,\ \ell = 0,\ m_\ell = 0,\ m_s = +\frac{1}{2} \text{ and } -\frac{1}{2}$$

For the four 2*p* electrons,

$$n = 2,\ \ell = 1,\ m_\ell = -1,\ 0,\ 1, \text{ and } m_s = +\frac{1}{2} \text{ and } -\frac{1}{2}$$

P42.42 Electronic configuration: sodium to argon

Orbitals 1*s*, 2*s*, and 2*p* are filled (and not shown).

	3*s*		3*p*		4*s*	
Na^{11}	⇑					$[1s^2 2s^2 2p^6]3s^1$
Mg^{12}	⇑⇓					$[1s^2 2s^2 2p^6]3s^2$
Al^{13}	⇑⇓	⇑				$[1s^2 2s^2 2p^6]3s^2 3p^1$
Si^{14}	⇑⇓	⇑	⇑			$[1s^2 2s^2 2p^6]3s^2 3p^2$
P^{15}	⇑⇓	⇑	⇑	⇑		$[1s^2 2s^2 2p^6]3s^2 3p^3$
S^{16}	⇑⇓	⇑⇓	⇑	⇑		$[1s^2 2s^2 2p^6]3s^2 3p^4$
Cl^{17}	⇑⇓	⇑⇓	⇑⇓	⇑		$[1s^2 2s^2 2p^6]3s^2 3p^5$
Ar^{18}	⇑⇓	⇑⇓	⇑⇓	⇑⇓		$[1s^2 2s^2 2p^6]3s^2 3p^6$
K^{19}	⇑⇓	⇑⇓	⇑⇓	⇑⇓	⇑	$[1s^2 2s^2 2p^6 3s^2 3p^6]4s^1$

P42.43 In the table of electronic configurations in the text, or on a periodic table, we look for the element whose last electron is in a 3*p* state and which has three electrons outside a closed shell. Its electron configuration then ends in $3s^2\,3p^1$. The element is $\boxed{\text{aluminum}}$.

P42.44 (a) Note that the possible values for ℓ range from zero to $n - 1$.

$n + \ell$	1	2	3	4	5	6	7
subshell	1*s*	2*s*	2*p*, 3*s*	3*p*, 4*s*	3*d*, 4*p*, 5*s*	4*d*, 5*p*, 6*s*	4*f*, 5*d*, 6*p*, 7*s*

The order is $\boxed{1s,\ 2s,\ 2p,\ 3s,\ 3p,\ 4s,\ 3d,\ 4p,\ 5s,\ 4d,\ 5p,\ 6s,\ 4f,\ 5d,\ 6p,\ 7s}$.

P42.45 (a) For electron one and also for electron two, $n = 3$ and $\ell = 1$; possible values are $m_\ell = 1, 0, -1$ and $m_s = 1/2, -1/2$. The exclusion principle requires that the electrons cannot have identical sets of quantum numbers. The possible states are listed here in columns giving the other quantum numbers:

electron	m_ℓ	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0
one	m_s	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
electron	m_ℓ	1	0	0	–1	–1	1	0	0	–1	–1	1	1	0	–1	–1
two	m_s	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$

electron	m_ℓ	0	0	0	0	0	–1	–1	–1	–1	–1	–1	–1	–1	–1	–1
one	m_s	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
electron	m_ℓ	1	1	0	–1	–1	1	1	0	0	–1	1	1	0	0	–1
two	m_s	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$

There are $6 \times 5 = \boxed{30}$ allowed states, since electron one can have any of three possible values for m_ℓ for both spin up and spin down, amounting to six states, and the second electron can have any of the other five states.

(b) Were it not for the exclusion principle, there would be $6 \times 6 = \boxed{36}$ possible states, six for each electron independently.

P42.46 Listing subshells in the order of filling, we have for element 110,

$$1s^2 2s^2 2p^6 3s^2 3p^6 4s^2 3d^{10} 4p^6 5s^2 4d^{10} 5p^6 6s^2 4f^{14} 5d^{10} 6p^6 7s^2 5f^{14} 6d^8$$

In order of increasing principal quantum number, this is

$$\boxed{1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2}$$

P42.47 In the ground state of sodium, the outermost electron is in an *s* state. This state is spherically symmetric, so it generates no magnetic field by orbital motion, and has the same energy no matter whether the electron is spin-up or spin-down. The energies of the states $3p\uparrow$ and $3p\downarrow$ above 3*s* are $hf_1 = \frac{hc}{\lambda}$ and $hf_2 = \frac{hc}{\lambda_2}$.

The energy difference is

$$2\mu_B B = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

so

$$\begin{aligned} B &= \frac{hc}{2\mu_B}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \\ &= \frac{(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{2(9.27 \times 10^{-24}\ \text{J/T})} \\ &\qquad \times \left(\frac{1}{588.995 \times 10^{-9}\ \text{m}} - \frac{1}{589.592 \times 10^{-9}\ \text{m}}\right) \\ &= \boxed{18.4\ \text{T}} \end{aligned}$$

Section 42.8 More on Atomic Spectra: Visible and X-Ray

***P42.48** Some electrons can give all their kinetic energy $K_e = e\Delta V$ to the creation of a single photon of x-radiation, with

$$hf = \frac{hc}{\lambda} = e\Delta V$$

$$\lambda = \frac{hc}{e\Delta V} = \frac{(6.626\ 1\times 10^{-34}\ \text{J}\cdot\text{s})(2.997\ 9\times 10^{8}\ \text{m/s})}{(1.602\ 2\times 10^{-19}\ \text{C})\Delta V}$$

$$= \boxed{\frac{1\ 240\ \text{nm}\cdot\text{V}}{\Delta V}}$$

P42.49 A photon of maximum energy or minimum wavelength is produced when the electron gives up all of its kinetic energy in a single collision within the target. Thus,

$$E_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = KE = e\Delta V$$

For a minimum wavelength of $\lambda_{\text{min}} = 70.0\ \text{pm} = 70.0\times 10^{-12}\ \text{m}$, the required accelerating voltage is

$$\Delta V = \frac{hc}{e\lambda_{\text{min}}} = \frac{(6.626\times 10^{-34}\ \text{J}\cdot\text{s})(3.00\times 10^{8}\ \text{m/s})}{(1.60\times 10^{-19}\ \text{C})(70.0\times 10^{-12}\ \text{m})}$$

$$= 1.77\times 10^{4}\ \text{V} = \boxed{17.7\ \text{kV}}$$

P42.50 The shortest wavelength is produced when the electron gives up all of its kinetic energy as a photon in a single collision within the target. For an accelerating voltage of 40.0 keV, the kinetic energy of the electrons is

$$KE = e\Delta V = e(40.0\ \text{kV}) = 40.0\ \text{keV}$$

For the shortest wavelength produced,

$$E_{\text{max}} = \frac{hc}{\lambda_{\text{min}}} = KE = e\Delta V$$

and $$\lambda_{\text{min}} = \frac{hc}{e\Delta V} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{(40.0\times 10^{3}\ \text{eV})} = \boxed{0.031\ 0\ \text{nm}}$$

P42.51 (a) For bismuth, *Z* = 83. Following Example 42.5, the electron in the M shell (*n* = 3) is shielded from the nuclear charge by one electron in the L shell (*n* = 1) and eight electrons in the K shell (*n* = 2). Its energy is

$$E_M \approx -(Z-9)^2\frac{13.6\ \text{eV}}{(3)^2} = -13.6\ \text{eV}\frac{(74)^2}{(3)^2}$$

The electrons in the L shell ($n = 2$) are shielded from the nuclear charge by one electron in the K shell, so (from page 1324)

$$E_L \approx -(Z-1)^2 \frac{13.6 \text{ eV}}{(2)^2} = -13.6 \text{ eV} \frac{(82)^2}{(2)^2}$$

When the electron drops from the M to the L shell of the atom, it emits a photon of energy

$$E_{\text{photon}} = E_M - E_L \approx 13.6 \text{ eV}\left[-\frac{(74)^2}{(3)^2} + \frac{(82)^2}{(2)^2}\right]$$

$$= 1.46 \times 10^4 \text{ eV} \approx \boxed{15 \text{ keV}}$$

(b) The wavelength of the emitted x-ray is given by

$$\lambda = \frac{1.240 \text{ keV}\cdot\text{nm}}{E} = \frac{1.240 \text{ keV}\cdot\text{nm}}{15 \text{ keV}}$$

$$\approx 0.083 \text{ nm} = \boxed{8.3 \times 10^{-11} \text{ m}}$$

P42.52 (a) For the 3*p* state, $E_n = \frac{-13.6 \text{ eV } Z_{\text{eff}}^2}{n^2}$ becomes

$$-3.0 \text{ eV} = \frac{-13.6 \text{ eV } Z_{\text{eff}}^2}{3^2} \quad \text{so} \quad Z_{\text{eff}} = \boxed{1.4}$$

For the 3*d* state

$$-1.5 \text{ eV} = \frac{-13.6 \text{ eV } Z_{\text{eff}}^2}{3^2} \quad \text{so} \quad Z_{\text{eff}} = \boxed{1.0}$$

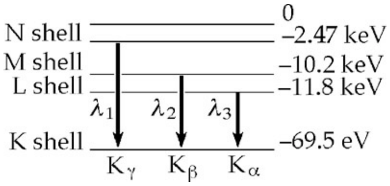
(b) When the outermost electron in sodium is promoted from the 3*s* state into a 3*p* state, its wave function still overlaps somewhat with the ten electrons below it. It therefore sees the $+11e$ nuclear charge not fully screened, and on the average moves in an electric field like that created by a particle with charge $+11e - 9.6e = 1.4e$. When this valence electron is lifted farther to a 3*d* state, it is essentially entirely outside the cloud of ten electrons below it, and moves in the field of a net charge $+11e - 10e = 1e$.

P42.53 (a) Recall $\ell \le n-1$. For $n = 3$, $\ell = 0, 1, 2$. $\boxed{\text{If } \ell = 2\text{, then } m_\ell = 2, 1, 0, -1, -2\text{; if } \ell = 1\text{, then } m_\ell = 1, 0, -1\text{; if } \ell = 0\text{, then } m_\ell = 0}$.

(b) The He^+ ion is a one-electron atom, so all states are have the same energy, determined by the principal quantum number *n*:

$$E_3 = -\frac{Z^2 E_0}{n^2} = -\frac{2^2(13.606 \text{ eV})}{3^2} = \boxed{-6.05 \text{ eV}}$$

***P42.54** The K series includes transitions from higher levels down to the K shell ($n = 1$). Transitions from higher n produce photons of higher energy. The ionization energy for the K shell is 69.5 keV, so the energy of the K shell is –69.5 keV.



ANS. FIG. P42.54

The photon energies are

$$E = \frac{hc}{\lambda} = \frac{1\,240\text{ eV}\cdot\text{nm}}{\lambda} = \frac{1.240\text{ keV}\cdot\text{nm}}{\lambda}$$

λ (nm)	photon energy (keV)	transition	energy of level (keV)	level
$\lambda_1 = 0.018\,5$	67.03	$n = 4 \to 1$	$-69.5 + 67.03 = -2.47$	N
$\lambda_2 = 0.020\,9$	59.3	$n = 3 \to 1$	$-69.5 + 59.3 = -10.2$	M
$\lambda_3 = 0.021\,5$	57.7	$n = 2 \to 1$	$-69.5 + 57.7 = -11.8$	L

The ionization energy for the K shell is 69.5 keV, so the ionization energies for the other shells are:

$$\boxed{\text{L shell} = 11.8\text{ keV}} \quad \boxed{\text{M shell} = 10.2\text{ keV}} \quad \boxed{\text{N shell} = 2.47\text{ keV}}$$

P42.55 Following the reasoning of Example 42.5, when the electron is in the K shell ($n = 1$), from Equation 42.37, its energy is

$$E_K \approx -Z^2(13.6\text{ eV})$$

When the electron was in the L shell ($n = 2$), the nuclear charge is shielded by one electron in the K shell, so (from page 1324)

$$E_L \approx -(Z-1)^2\frac{13.6\text{ eV}}{2^2}$$

When the electron drops from the L to the K shell of the atom, it emits a photon of energy (for $Z = 42$)

$$E_{\text{photon}} = E_L - E_K \approx (13.6\text{ eV})\left[-\frac{(41)^2}{4} + (42)^2\right] = 1.83\times10^4\text{ eV}$$

with wavelength

$$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{1\,240\text{ eV}\cdot\text{nm}}{1.83\times 10^{4}\text{ eV}} = 6.79\times 10^{-2}\text{ nm} = \boxed{0.068\text{ nm}}$$

P42.56 (a) All of the kinetic energy of an electron after its acceleration through a potential difference ΔV goes into producing a single photon:

$$E = \frac{hc}{\lambda} = e\Delta V \quad \rightarrow \quad \Delta V = \frac{hc}{e\lambda} = \frac{1240\ \cancel{e}\text{V}\cdot\text{nm}}{\cancel{e}\lambda} = \boxed{\frac{1240\text{ V}\cdot\text{nm}}{\lambda}}$$

(b) The potential difference is inversely proportional to the wavelength.

(c) Yes. It predicts a minimum wavelength of 33.5 pm when the accelerating voltage is 37 keV, in agreement with the minimum wavelength in the figure.

(d) Yes, but it might be unlikely for a very high energy electron to stop in a single interaction to produce a high-energy gamma ray, and it might be difficult to observe the very low intensity radio waves produced as bremsstrahlung by low-energy electrons.

(e) The potential difference goes to infinity as the wavelength goes to zero.

(f) The potential difference goes to zero as the wavelength goes to infinity.

P42.57 The concepts for this problem are discussed in Example 42.5. An electron makes a transition from the M to the K shell. From Equation 42.37, when the electron is in the K shell, its energy is

$$E_K \approx -Z^2(13.6\text{ eV})$$

When the electron was in the M shell, because nine electrons shield the nuclear charge—one in the L shell (n = 1) and eight in the K shell (n = 2)—its energy is

$$E_M \approx -(Z-9)^2\frac{13.6\text{ eV}}{(3)^2}$$

Thus, as the electron drops from the M to the K shell, it emits a photon of energy

$$\begin{aligned} E_{\text{photon}} = E_M - E_K &\approx (13.6\text{ eV})\left[-\frac{(Z-9)^2}{9} + Z^2\right] \\ &= (13.6\text{ eV})\left[-\left(\frac{Z^2 - 18Z + 81}{9}\right) + Z^2\right] \\ &= (13.6\text{ eV})\left(\frac{8}{9}Z^2 + 2Z - 9\right) = \frac{hc}{\lambda} \end{aligned}$$

Therefore, we have the relation

$$(13.6\text{ eV})\left(\frac{8}{9}Z^2 + 2Z - 9\right) = \frac{hc}{\lambda}$$

$$\frac{8}{9}Z^2 + 2Z - 9 = \frac{hc}{(13.6\text{ eV})\lambda} = \frac{(1\,240\text{ eV}\cdot\text{nm})}{(13.6\text{ eV})(0.101\text{ nm})} = 902.7$$

$$\frac{8}{9}Z^2 + 2Z - 911.7 = 0$$

Solving for *Z* gives

$$\begin{aligned} Z &= \frac{-(2) \pm \sqrt{(2)^2 - 4(8/9)(-911.7)}}{2(8/9)} = \frac{-1 \pm \sqrt{1 + (8/9)(911.7)}}{(8/9)} \\ &= \frac{-1 \pm 28.5}{(8/9)} \end{aligned}$$

The positive solution is physical:

$$Z = \frac{-1 + 28.5}{(8/9)} = 30.9$$

The nearest whole number for *Z* is 31, which corresponds to the element $\boxed{\text{gallium}}$.

Section 42.9 Spontaneous and Stimulated Transitions

Section 42.10 Lasers

P42.58 The electron in the E_3* state drops to the E_2 state, emitting a photon of energy *hf*. The process conserves energy:

$$E_3^* = E_2 + hf \quad \rightarrow \qquad hf = E_3^* - E_2$$

The photon's energy is

$$hf = E_3^* - E_2 = (20.66 - 18.70)\text{ eV} = 1.96\text{ eV} = \frac{hc}{\lambda}$$

and its wavelength is

$$\lambda = \frac{hc}{E} = \frac{1\,240\text{ eV}\cdot\text{nm}}{1.96\text{ eV}} = \boxed{633\text{ nm}}$$

P42.59 (a) We use Equation 42.5, $E = hf = 0.117$ eV, and solve for *f*:

$$f = \frac{E}{h} = \frac{0.117\text{ eV}}{6.626\times10^{-34}\text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-19}\text{ J}}{\text{eV}}\right)$$

$$= 2.83\times10^{13}\text{ s}^{-1} = 28.3\times10^{12}\text{ s}^{-1} = \boxed{28.3\text{ THz}}$$

(b) The wavelength of the laser is

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{2.83\times10^{13}\text{ s}^{-1}} = \boxed{10.6\ \mu\text{m}}$$

(c) This is in the $\boxed{\text{infrared}}$ portion of the electromagnetic spectrum.

P42.60 (a) We find the energy from

$$\Delta E = E_4^* - E_2 = \frac{hc}{\lambda}$$

Then,

$$\begin{aligned} E_2 &= E_4^* - \frac{hc}{\lambda} \\ &= 20.66\text{ eV} \\ &\quad - \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{543\times10^{-9}\text{ m}}\left(\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right) \\ &= \boxed{18.37\text{ eV}} \end{aligned}$$

(b) The light in the cavity is incident perpendicularly on the mirrors. Some of the light reflects off the front surface of the silicon dioxide layer while some enters the layer and then reflects off the titanium oxide layer on its other side. The index of refraction of titanium oxide (1.9–2.6) is greater the index of refraction of silicon dioxide (1.458), so there is automatically a 180° shift of the ray reflecting off the silicon dioxide. To minimize reflection at a vacuum wavelength of 632.8 nm, the net phase difference between reflected rays should be 180°, so the extra distance traveled by the ray passing into the silicon dioxide should be one whole wavelength:

$$2t = \frac{\lambda}{n}$$

$$t = \frac{\lambda}{2n} = \frac{632.8\text{ nm}}{2(1.458)} = \boxed{217\text{ nm}}$$

(c) For the green light to experience constructive interference, the net phase difference should be 360°, including contributions of 180° by reflection and 180° by extra distance traveled:

$$2t = \frac{\lambda}{2n}$$

$$t = \frac{\lambda}{4n} = \frac{543\text{ nm}}{4(1.458)} = \boxed{93.1\text{ nm}}$$

P42.61 The energy in each pulse is

$$E = P\Delta t = (1.00\times10^{6}\text{ W})(1.00\times10^{-8}\text{ s}) = 1.00\times10^{-2}\text{ J}$$

The energy of each photon is

$$E_\gamma = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{694.3\times10^{-9}\text{ m}}$$

$$= 2.86\times10^{-19}\text{ J}$$

So the number of photons in the pulse is

$$N = \frac{E}{E_\gamma} = \frac{1.00\times10^{-2}\text{ J}}{2.86\times10^{-19}\text{ J/photon}} = \boxed{3.49\times10^{16}\text{ photons}}$$

P42.62 (a) The equilibrium ratio is

$$\frac{N_4{}^*}{N_3} = \frac{N_g e^{-E_3/k_BT}}{N_g e^{-E_2/k_BT}} = e^{-(E_3-E_2)/k_BT} = e^{-\Delta E/k_BT}$$

where the temperature *T* = 27.0 °C + 273.15 = 300.2 K, and the energy difference (from Figure P42.60) is

$$\Delta E = E_4{}^* - E_3 = 20.66\text{ eV} - 18.70\text{ eV} = 1.96\text{ eV}$$

Substituting numerical values,

$$\frac{N_4^*}{N_3} = e^{-\Delta E/k_BT} = e^{-(1.96\text{ eV})\left(1.602\times10^{-19}\text{ J/eV}\right)/\left(1.381\times10^{-23}\text{ J/K}\right)(300.2\text{ K})}$$

$$= \boxed{1.26\times10^{-33}}$$

(b) Now, we require $\frac{N_4^*}{N_3} = e^{-\Delta E/k_BT} = 1.02$

where $\Delta E = E_4^* - E_3 = 1.96\text{ eV}$

Thus,

$$\ln(1.02) = -\frac{(1.96\text{ eV})\left(1.602\times10^{-19}\text{ J/eV}\right)}{\left(1.381\times10^{-23}\text{ J/K}\right)T}$$

$$T = \boxed{-1.15\times10^{6}\text{ K}}$$

(c) $\boxed{\text{The population inversion requires the temperature be negative.}}$ Because $\Delta E = E_4^* - E_3 > 0$, and in any real equilibrium state *T* > 0, the ratio $N_4^*/N_3 = e^{-\Delta E/k_BT} < 1$. Thus, a population inversion cannot happen in thermal equilibrium.

P42.63 (a) The energy of the pulse is spread over the area of a circle of radius $R = 15.0\ \mu$m:

$$I = \frac{P}{A} = \frac{\Delta E/\Delta t}{\pi R^2} = \frac{3.00\times10^{-3}\text{ J}/1.00\times10^{-9}\text{ s}}{\left[\pi\left(15.0\times10^{-6}\text{ m}\right)^2\right]}$$

$$= \boxed{4.24\times10^{15}\text{ W/m}^2}$$

(b) The absorbed energy falls within the area of a circle of radius $r = 0.300$ nm:

$$\Delta E = I\Delta tA = \left(\frac{\Delta E/\Delta t}{\pi R^2}\right)\Delta t\pi r^2 = \Delta E\left(\frac{r^2}{R^2}\right)$$

$$= \left(3.00\times10^{-3}\text{ J}\right)\left[\frac{\left(0.300\times10^{-9}\text{ m}\right)^2}{\left(15.0\times10^{-6}\text{ m}\right)^2}\right] = \boxed{1.20\times10^{-12}\text{ J}}$$

***P42.64** (a) The distance between nodes is $\frac{\lambda}{2}$, so we require solutions to

$$35.124\ 103 \text{ cm} = \frac{N}{2}\lambda,$$ where *N* is an integer and λ is in the required range. The midpoint of the range is 632.809 10 nm, giving

$$N_{\text{trial}} = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{632.809\ 1 \times 10^{9}\ \text{m}} = 1\ 110\ 101.07$$

So we try *N* = 1 110 101, 1 110 102, 1 110 100, 1 110 103, and so on:

$$\lambda_1 = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{1\ 110\ 101} = \boxed{632.809\ 14\ \text{nm}}$$

$$\lambda_2 = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{1\ 110\ 102} = \boxed{632.808\ 57\ \text{nm}}$$

$$\lambda_3 = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{1\ 110\ 100} = \boxed{632.809\ 71\ \text{nm}}$$

$$\lambda_{\text{trial}} = \frac{2\left(35.124\ 103 \times 10^{-2}\ \text{m}\right)}{1\ 110\ 103} = 632.808\ 00\ \text{nm}$$

outside the range. Thus the laser light has just $\boxed{\text{three}}$ wavelength components.

(b) The rms speed is obtained from $\frac{1}{2}m_0v^2 = \frac{3}{2}kT$. We use the periodic table for the mass of a neon atom. Then,

$$v = \sqrt{\frac{3kT}{m_0}} = \sqrt{\frac{3\left(1.38 \times 10^{-23}\ \text{J/K}\right)(393\ \text{K})}{20.18\ \text{u}}\left(\frac{1\ \text{u}}{1.66 \times 10^{-27}\ \text{kg}}\right)}$$
$$= \boxed{697\ \text{m/s}}$$

(c) For a neon atom moving toward one mirror at the rms speed as it emits, the Doppler shift is described by

$$f' = f\sqrt{\frac{c+v}{c-v}} = \frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{c+v}{c-v}}$$

$$\lambda' = \lambda\sqrt{\frac{c-v}{c+v}} = (632.809\ 1\ \text{nm})\sqrt{\frac{3 \times 10^{8} - 697}{3 \times 10^{8} + 697}} = 632.807\ 63\ \text{nm}$$

This is outside the given range. Many atoms are moving faster than the rms speed, so we should expect still more Doppler broadening of the resonance amplification peak.

Additional Problems

P42.65 To ionize the atom, it is necessary that $n_f \to \infty$. The required energy is then

$$\Delta E = E_f - E_i = -13.6\text{ eV}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = -13.6\text{ eV}\left(\frac{1}{\infty} - \frac{1}{n_i^2}\right)$$
$$= \frac{13.6\text{ eV}}{n_i^2}$$

(a) If $n_i = 1$, the required energy is

$$\Delta E = \frac{13.6\text{ eV}}{1^2} = \boxed{13.6\text{ eV}}$$

(b) If $n_i = 3$,

$$\Delta E = \frac{13.6\text{ eV}}{3^2} = \boxed{1.51\text{ eV}}$$

P42.66 The sliver atoms ($Z = 108$) move as particles initially traveling in the *x* direction at speed *u* with constant acceleration in the *z* direction:

$$\Delta z = \frac{1}{2}a(\Delta t)^2 = \frac{1}{2}\left(\frac{F_z}{m_0}\right)(\Delta t)^2 = \frac{\mu_z\left(dB_z/dz\right)}{2m_0}\left(\frac{\Delta x}{u}\right)^2$$

where $\mu_z = \dfrac{e\hbar}{2m_e}$, so

$$\Delta z = \frac{e\hbar}{4m_e m_0}\left(\frac{\Delta x}{u}\right)^2 \frac{dB_z}{dz}$$

Therefore,

$$\frac{dB_z}{dz} = \frac{4m_0 m_e \Delta z v^2}{e\hbar\Delta x^2}$$
$$= \frac{4\left[(108)\left(1.66\times10^{-27}\text{ kg}\right)\right]\left(9.11\times10^{-31}\text{ kg}\right)\left(10^{-3}\text{ m}\right)(100\text{ m/s})^2}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.05\times10^{-34}\text{ J}\cdot\text{s}\right)(1.00\text{ m})^2}$$
$$\frac{dB_z}{dz} = \boxed{0.389\text{ T/m}}$$

P42.67 The wave function for the 2*s* state is given by Equation 42.26:

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left[2 - \frac{r}{a_0}\right]e^{-r/2a_0}$$

(a) Taking $r = a_0 = 0.529\times10^{-10}$ m, we find

$$\psi_{2s}(a_0) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{0.529\times10^{-10}\ \text{m}}\right)^{3/2}[2-1]e^{-1/2}$$
$$= \boxed{1.57\times10^{14}\ \text{m}^{-3/2}}$$

(b) $\left|\psi_{2s}(a_0)\right|^2 = \left(1.57\times10^{14}\ \text{m}^{-3/2}\right)^2 = \boxed{2.47\times10^{28}\ \text{m}^{-3}}$

(c) Using Equation 42.24 and the result of part (b) gives

$$P_{2s}(a_0) = 4\pi a_0^2\left|\psi_{2s}(a_0)\right|^2 = \boxed{8.69\times10^{8}\ \text{m}^{-1}}$$

P42.68 (a) The energy difference between these two states is equal to the energy that is absorbed. Thus,

$$\Delta E = E_2 - E_1 = \frac{(-13.6\ \text{eV})}{4} - \frac{(-13.6\ \text{eV})}{1} = 10.2\ \text{eV}$$
$$= \boxed{1.63\times10^{-18}\ \text{J}}$$

(b) $E = \frac{3}{2}k_BT$ or $T = \frac{2E}{3k_B} = \frac{2\left(1.63\times10^{-18}\ \text{J}\right)}{3\left(1.38\times10^{-23}\ \text{J/K}\right)} = \boxed{7.88\times10^{4}\ \text{K}}$

P42.69 To evaluate the energy difference, we imagine the *z* component of the electron’s magnetic moment as continuously variable. In turning it from alignment with the field to the opposite direction, the field does work according to Equation 10.22,

$$W = \int dW = \int_0^{180^\circ}\tau\, d\theta = \int_0^{\pi}\mu B\sin\theta\, d\theta = -\mu B\cos\theta\Big|_0^{\pi} = 2\mu B$$

To make the electron flip, the photon must carry energy $\Delta E = 2\mu_B B = hf$. Therefore,

$$f = \frac{2\mu_B B}{h} = \frac{2\left(9.27\times10^{-24}\ \text{J/T}\right)(0.350\ \text{T})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}}$$
$$= 9.79\times10^{9}\ \text{Hz} = \boxed{9.79\ \text{GHz}}$$

P42.70 We suppose that the electron that makes the transition is shielded from the electric field of the full nuclear charge by the one K-shell electron originally below it. With $Z = 24$, its original energy is

$$E = -(Z-1)^2(13.6\ \text{eV})\left(\frac{1}{2^2}\right) = -1.80\ \text{keV}$$

Its final energy is $E = -Z^2(13.6\ \text{eV})\left(\frac{1}{1^2}\right) = -7.83\ \text{keV}$.

The magnitude of the electron's energy loss is

$$7.83\text{ keV} - 1.80\text{ keV} = 6.04\text{ keV}$$

Then, instead of coming out as an x-ray photon, this +6.04 keV can be transferred to the single 4*s* electron. Suppose that it is shielded by the 22 electrons in the K, L, and M shells. To break the outermost electron out of the atom, producing a Cr^{2+} ion, requires an energy investment of

$$E_{\text{ionize}} = \frac{(Z-22)^2(13.6\text{ eV})}{4^2} = \frac{2^2(13.6\text{ eV})}{16} = 3.40\text{ eV}$$

Then the remaining energy that can appear as kinetic energy is

$$K = |\Delta E| - E_{\text{ionize}} = 6\ 035\text{ eV} - 3.4\text{ eV} = \boxed{6.03\text{ keV}}$$

Because of conservation of momentum for the ion-electron system and the tiny mass of the electron compared to that of the Cr^{2+} ion, almost all of this kinetic energy will belong to the electron.

***P42.71** From Figure 42.20, a typical ionization energy is 8 eV. For internal energy to ionize most of the atoms we require

$$\frac{3}{2}k_BT = 8\text{ eV}:$$

$$T = \frac{2\times 8\left(1.60\times 10^{-19}\text{ J}\right)}{3\left(1.38\times 10^{-23}\text{ J/K}\right)}\boxed{\sim\text{ between }10^4\text{ K and }10^5\text{ K}}$$

***P40.72** From Equation 42.26,

$$\psi_{2s} = \frac{1}{4}(2\pi)^{-1/2}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0} = A\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$$

Differentiating gives

$$\frac{d\psi}{dr} = Ae^{-r/2a_0}\left(-\frac{2}{a_0}+\frac{r}{2a_0^2}\right)$$

Differentiating a second time gives,

$$\frac{d^2\psi}{dr^2} = \left(\frac{Ae^{-r/2a_0}}{a_0^2}\right)\left(\frac{3}{2}-\frac{r}{4a_0}\right)$$

Substituting into Schrödinger's equation and dividing by $Ae^{-r/2a_0}$, we will have a solution if

$$-\frac{5}{4}\frac{\hbar^2}{m_ea_0^2}+\frac{k_ee^2}{a_0}+\frac{\hbar^2 r}{8m_ea_0^3}+\frac{2\hbar^2}{m_ea_0r}-\frac{2k_ee^2}{r} = 2E-\frac{Er}{a_0}$$

Now with $a_0 = \dfrac{\hbar^2}{m_e e^2 k_e}$, this reduces to

$$-\frac{m_e e^4 k_e^2}{8\hbar^2}\left(2 - \frac{r}{a_0}\right) = E\left(2 - \frac{r}{a_0}\right)$$

This is true, so ψ_{2s} is a solution to the Schrödinger equation, provided $E = \dfrac{1}{4}E_1 = -3.40\text{ eV}$.

***P42.73** The expectation value of $1/r$ is found from

$$\left\langle \frac{1}{r} \right\rangle = \int_0^\infty \frac{4r^2}{a_0^3} e^{-2r/a_0} \frac{1}{r} dr = \frac{4}{a_0^3}\int_0^\infty r e^{-(2/a_0)r}\, dr = \frac{4}{a_0^3}\frac{1}{\left(\frac{2}{a_0}\right)^2} = \boxed{\frac{1}{a_0}}$$

We compare this to $\dfrac{1}{\langle r \rangle} = \dfrac{1}{\frac{3a_0}{2}} = \dfrac{2}{3a_0}$, and find that the average reciprocal value is $\boxed{\text{NOT}}$ the reciprocal of the average value.

P42.74 The fact that there are five values of the *z* component of orbital angular momentum tells us that there are five values of m_ℓ, which, in turn, tells us that $\ell = 2$. From Equation 42.28, we can find the maximum value of m_ℓ:

$$L_z = m_\ell \hbar \quad \to \quad m_\ell = \frac{L_z}{\hbar} = \frac{3.16 \times 10^{-34}}{1.055 \times 10^{-34}} = 3$$

In order to have a maximum value of m_ℓ equal to 3, we need to have $\ell = 3$, which is inconsistent with the first result.

P42.75 (a) The size of the quantum jump in the electron's energy is

$$\Delta E = \frac{e\hbar B}{m_e} = \frac{\left(1.60 \times 10^{-19}\text{ C}\right)\left(6.626 \times 10^{-34}\text{ J}\cdot\text{s}\right)\left(5.26\text{ T}\right)}{2\pi\left(9.11 \times 10^{-31}\text{ kg}\right)}$$

$$= 9.75 \times 10^{-23}\text{ J}\left(\frac{1\text{ eV}}{1.60 \times 10^{-19}\text{ J}}\right) = 6.09 \times 10^{-4}\text{ eV} = \boxed{609\ \mu\text{eV}}$$

(b) The energy available from the walls of the container is

$$k_B T = \left(1.38 \times 10^{-23}\text{ J/K}\right)\left(80 \times 10^{-3}\text{ K}\right) = 1.10 \times 10^{-24}\text{ J} = \boxed{6.9\ \mu\text{eV}}$$

(c) The photon's frequency is

$$f = \frac{\Delta E}{h} = \frac{9.75 \times 10^{-23}\text{ J}}{6.626 \times 10^{-34}\text{ J}\cdot\text{s}} = 1.47 \times 10^{11}\text{ Hz} = 147 \times 10^9\text{ Hz}$$

$$= \boxed{147\text{ GHz}}$$

(d) The photon's wavelength is

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^{8}\ \text{m/s}}{1.47 \times 10^{11}\ \text{Hz}} = 2.04 \times 10^{-3}\ \text{m} = \boxed{2.04\ \text{mm}}$$

P42.76 (a) Using the same procedure that was used in the Bohr model of the hydrogen atom, we apply Newton's second law to the Earth. We simply replace the Coulomb force by the gravitational force exerted by the Sun on the Earth and find

$$G\frac{M_S M_E}{r^2} = M_E \frac{v^2}{r} \qquad \textbf{[1]}$$

where *v* is the orbital speed of the Earth. Next, we apply the postulate that angular momentum of the Earth is quantized in multiples of $\hbar$:

$$M_E v r = n\hbar \qquad (n = 1,\ 2,\ 3,\ \ldots)$$

Solving for *v* gives

$$v = \frac{n\hbar}{M_E r} \qquad \textbf{[2]}$$

Substituting [2] into [1], we find

$$r = \frac{n^2\hbar^2}{GM_S M_E^2} \qquad \textbf{[3]}$$

(b) Solving equation [3] for *n* gives

$$n = \sqrt{GM_S r}\,\frac{M_E}{\hbar} \qquad \textbf{[4]}$$

Taking $M_S = 1.99 \times 10^{30}$ kg, $M_E = 5.98 \times 10^{24}$ kg, $r = 1.496 \times 10^{11}$ m, $G = 6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2$, and $\hbar = 1.055 \times 10^{-34}\ \text{J} \cdot \text{s}$, we find

$$n = \boxed{2.53 \times 10^{74}}$$

(c) We can use equation [3] to determine the radii for the orbits corresponding to the quantum numbers *n* and *n* + 1:

$$r_n = \frac{n^2\hbar^2}{GM_S M_E^2} \quad \text{and} \quad r_{n+1} = \frac{(n+1)^2\hbar^2}{GM_S M_E^2}$$

Hence, the separation between these two orbits is

$$\Delta r = \frac{\hbar^2}{GM_S M_E^2}\left[(n+1)^2 - n^2\right] = \frac{\hbar^2}{GM_S M_E^2}(2n+1)$$

Since *n* is very large, we can neglect the number 1 in the parentheses and express the separation as

$$\begin{aligned}\Delta r &\approx \frac{\hbar^2}{GM_S M_E^2}(2n)\\ &= \frac{\left(1.054\,6\times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{\left(6.67\times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times 10^{30}\ \text{kg}\right)\left(5.98\times 10^{24}\ \text{kg}\right)^2}\\ &\qquad\times\left[2\left(2.53\times 10^{74}\right)\right]\\ &= \boxed{1.18\times 10^{-63}\ \text{m}}\end{aligned}$$

(d) This number is *much smaller* than the radius of an atomic nucleus (~ 10^{-15} m), so the distance between quantized orbits of the Earth is too small to observe.

P42.77 The average squared separation distance is

$$\begin{aligned}\left\langle r^2\right\rangle &= \int_{\text{all space}}\psi_{1s}^{*} r^2 \psi_{1s}\,dV\\ &= \int_{r=0}^{\infty}\left(\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}\right) r^2 \left(\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}\right)4\pi r^2\,dr\\ &= \frac{4}{a_0^3}\int_0^{\infty} r^4 e^{-2r/a_0}\,dr\end{aligned}$$

We use $\int_0^{\infty} x^n e^{-ax}dx = \dfrac{n!}{a^{n+1}}$ from Table B.6:

$$\left\langle r^2\right\rangle = \frac{4}{a_0^3}\,\frac{4!}{\left(2/a_0\right)^5} = \frac{96a_0^2}{32} = 3a_0^2$$

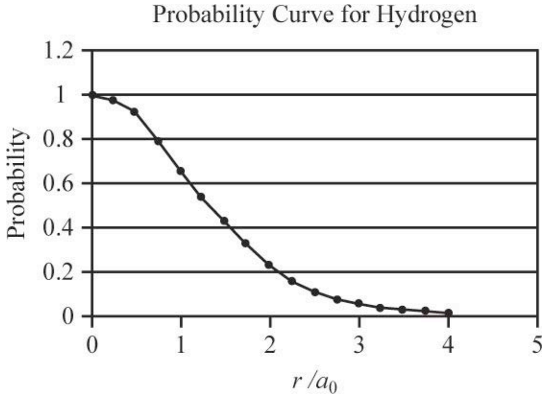
The root-mean-square uncertainty in *r* is

$$\begin{aligned}\Delta r &= \sqrt{\left\langle r^2\right\rangle - \left\langle r\right\rangle^2} = \left[3a_0^2 - \left(\frac{3a_0}{2}\right)^2\right]^{1/2}\\ &= \left(3a_0^2 - \frac{9a_0^2}{4}\right)^{1/2} = \left(\frac{3}{4}\right)^{1/2} a_0 = \boxed{0.866a_0}\end{aligned}$$

P42.78 (a) From Equations 42.22 – 42.25,

$$\begin{aligned}P &= \int_r^{\infty} P_{1s}(r')\,dr' = \frac{4}{a_0^3}\int_r^{\infty} r'^2 e^{-2r'/a_0}\,dr'\\ &= \left[-\left(\frac{2r'^2}{a_0^2} + \frac{2r'}{a_0} + 1\right)e^{-2r'/a_0}\right]_r^{\infty} = \boxed{\left(\frac{2r^2}{a_0^2} + \frac{2r}{a_0} + 1\right)e^{-2r/a_0}}\end{aligned}$$

(b) The graph is shown in ANS. FIG. P42.78.



ANS. FIG. P42.78

(c) The probability of finding the electron inside or outside the sphere of radius *r* is $\frac{1}{2}$:

$$\left(\frac{2r^2}{a_0^2}+\frac{2r}{a_0}+1\right)e^{-2r/a_0}=\frac{1}{2}\quad\text{or}\quad z^2+2z+2=e^z$$

where $z=\frac{2r}{a_0}$.

One can home in on a solution to this transcendental equation for *r* on a calculator, the result being $r=\boxed{1.34a_0}$ to three digits.

P42.79 (a) The energy emitted by the atom is

$$\Delta E=E_4-E_2=-13.6\text{ eV}\left(\frac{1}{4^2}-\frac{1}{2^2}\right)=2.55\text{ eV}$$

The wavelength of the photon produced is then

$$\lambda=\frac{hc}{E_\gamma}=\frac{hc}{\Delta E}=\frac{1\,240\text{ eV}\cdot\text{nm}}{2.55\text{ nm}}=\boxed{486\text{ nm}}$$

(b) Since momentum must be conserved, the photon and the atom go in opposite directions with equal magnitude momenta. Thus,

$p=m_{\text{atom}}v=\frac{h}{\lambda}$, or

$$v=\frac{h}{m_{\text{atom}}\lambda}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\left(1.67\times10^{-27}\text{ kg}\right)\left(4.86\times10^{-7}\text{ m}\right)}$$
$$=\boxed{0.816\text{ m/s}}$$

P42.80 (a) The energy of the ground state is

$$E_1 = -\frac{hc}{\lambda_{\text{series limit}}} = -\frac{1\ 240\ \text{eV}\cdot\text{nm}}{152.0\ \text{nm}} = \boxed{-8.16\ \text{eV}}$$

From the wavelength of the Lyman α line:

$$E_2 - E_1 = \frac{hc}{\lambda} = \frac{1\ 240\ \text{nm}\cdot\text{eV}}{202.6\ \text{nm}} = 6.12\ \text{eV}$$

$$E_2 = E_1 + 6.12\ \text{eV} = \boxed{-2.04\ \text{eV}}$$

The wavelength of the Lyman β line gives

$$E_3 - E_1 = \frac{1\ 240\ \text{nm}\cdot\text{eV}}{170.9\ \text{nm}} = 7.26\ \text{eV}$$

so $E_3 = \boxed{-0.902\ \text{eV}}$.

Next, using the Lyman γ line gives

$$E_4 - E_1 = \frac{1\ 240\ \text{nm}\cdot\text{eV}}{162.1\ \text{nm}} = 7.65\ \text{eV}$$

and $E_4 = \boxed{-0.508\ \text{eV}}$.

From the Lyman δ line,

$$E_5 - E_1 = \frac{1\ 240\ \text{nm}\cdot\text{eV}}{158.3\ \text{nm}} = 7.83\ \text{eV}$$

so $E_5 = \boxed{-0.325\ \text{eV}}$.

(b) For the Balmer series,

$$\frac{hc}{\lambda} = E_i - E_2, \text{ or } \lambda = \frac{1\ 240\ \text{nm}\cdot\text{eV}}{E_i - E_2}$$

For the α line, $E_i = E_3$ and so

$$\lambda_a = \frac{1\ 240\ \text{nm}\cdot\text{eV}}{(-0.902\ \text{eV}) - (-2.04\ \text{eV})} = \boxed{1\ 090\ \text{nm}}$$

Similarly, the wavelengths of the β line, γ line, and the short wavelength limit are found to be: $\boxed{811\ \text{nm, } 724\ \text{nm, and } 609\ \text{nm}}$.

(c) Using Equation 42.2, $\frac{1}{\lambda} = R_H\left(1 - \frac{1}{n^2}\right)$, the Lyman series for hydrogen contains the lines: α ($n = 2$) = 122 nm, β ($n = 3$) = 103 nm, γ($n = 4$) = 97.2 nm, δ ($n = 5$) = 94.9 nm, the short wavelength limit ($n \to \infty$) = 91.1 nm.

Computing 60.0% of the wavelengths of the spectral lines shown on the energy-level diagram gives:

$$0.600(202.6\text{ nm}) = \boxed{122\text{ nm}}$$

$$0.600(170.9\text{ nm}) = \boxed{103\text{ nm}}$$

$$0.600(162.1\text{ nm}) = \boxed{97.3\text{ nm}}$$

$$0.600(158.3\text{ nm}) = \boxed{95.0\text{ nm}}$$

$$0.600(152.0\text{ nm}) = \boxed{91.2\text{ nm}}$$

These are seen to be the wavelengths of the α, β, γ, and δ lines as well as the short wavelength limit for the Lyman series in Hydrogen.

(d) The observed wavelengths could be the result of Doppler shift when the source moves away from the Earth. The required speed of the source is found from

$$\frac{f'}{f} = \frac{\lambda}{\lambda'} = \sqrt{\frac{c-v}{c+v}} = 0.600 \quad \text{yielding} \quad v = 0.471c.$$

$$\boxed{\begin{array}{l}\text{The spectrum could be that of hydrogen, Doppler-shifted}\\ \text{by motion away from us at speed } 0.471c.\end{array}}$$

P42.81 We use Equation 42.26:

$$\psi_{2s}(r) = \frac{1}{4\sqrt{2\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left[2-\frac{r}{a_0}\right]e^{-r/2a_0}$$

(a) By Equation 42.24,

$$P(r) = 4\pi r^2|\psi|^2 = \boxed{\frac{r^2}{8a_0^3}\left(2-\frac{r}{a_0}\right)^2 e^{-r/a_0}}$$

(b) The derivative of the radial probability is

$$\frac{dP(r)}{dr} = \frac{1}{8a_0^3}\left[2r\left(2-\frac{r}{a_0}\right)^2 - 2r^2\left(\frac{1}{a_0}\right)\left(2-\frac{r}{a_0}\right) - r^2\left(2-\frac{r}{a_0}\right)^2\left(\frac{1}{a_0}\right)\right]e^{-r/a_0}$$

Simplifying the expression,

$$\frac{dP(r)}{dr} = \frac{1}{8a_0^3}\left(2 - \frac{r}{a_0}\right)e^{-r/a_0}\left[4r - \frac{2r^2}{a_0} - \frac{2r^2}{a_0} - \left(\frac{2r^2}{a_0} - \frac{r^3}{a_0^2}\right)\right]$$

$$= \frac{r}{8a_0^5}\left(2 - \frac{r}{a_0}\right)e^{-r/a_0}\left[4a_0^2 - 6ra_0 + r^2\right]$$

$$= \boxed{\frac{r}{8a_0^5}\left(2 - \frac{r}{a_0}\right)e^{-r/a_0}\left[r^2 - 6ra_0 + 4a_0^2\right]}$$

(c) Its extremes are given by

$$\frac{dP}{dr} = \frac{r}{8a_0^5}\left(2 - \frac{r}{a_0}\right)e^{-r/a_0}\left[r^2 - 6ra_0 + 4a_0^2\right] = 0$$

The roots of $\frac{dP}{dr} = 0$ at $\boxed{r = 0,\ r = 2a_0,\ \text{and}\ r = \infty}$ are minima with $P(r) = 0$ (as shown in Figure 42.12).

(d) We require $r^2 - 6ra_0 + 4a_0^2 = 0$. The solutions are

$$r = \frac{-(-6a_0) \pm \sqrt{(-6a_0)^2 - 4(1)(4a_0^2)}}{2} = \frac{6a_0 \pm \sqrt{20a_0^2}}{2} = \boxed{\left(3 \pm \sqrt{5}\right)a_0}$$

(e) We substitute the last two roots into $P(r)$ to determine the most probable value:

When $r = \left(3 - \sqrt{5}\right)a_0 = 0.764\,a_0$,

$$P(r) = \frac{(0.764\,a_0)^2}{8a_0^3}\left(2 - \frac{0.764\,a_0}{a_0}\right)^2 e^{-0.764}$$

$$= \frac{(0.764)^2}{8a_0}(2 - 0.764)^2 e^{-0.764} = \frac{0.051\,9}{a_0}$$

When $r = \left(3 + \sqrt{5}\right)a_0 = 5.236a_0$,

$$P(r) = \frac{(5.236\,a_0)^2}{8a_0^3}\left(2 - \frac{5.236\,a_0}{a_0}\right)^2 e^{-5.236}$$

$$= \frac{(5.236)^2}{8a_0}(2 - 5.236)^2 e^{-5.236} = \frac{0.191}{a_0}$$

Therefore, the most probable value of *r* is $\boxed{r = \left(3 + \sqrt{5}\right)a_0 \rightarrow P = 0.191/a_0}$.

P42.82 (a) One molecule's share of volume is,

$$\text{Al: } V = \left(\frac{27.0\text{ g}}{\text{mol}}\right)\left(\frac{1\text{ mol}}{6.02\times10^{23}\text{ molecules}}\right)\left(\frac{1.00\times10^{-6}\text{ m}^3}{2.70\text{ g}}\right)$$

$$= 1.66\times10^{-29}\text{ m}^3$$

$$D \approx \sqrt[3]{V} = \boxed{2.55\times10^{-10}\text{ m}\sim10^{-1}\text{ nm}}$$

$$\text{U: } V = \left(\frac{238\text{ g}}{\text{mol}}\right)\left(\frac{1\text{ mol}}{6.02\times10^{23}\text{ molecules}}\right)\left(\frac{1.00\times10^{-6}\text{ m}^3}{18.9\text{ g}}\right)$$

$$= 2.09\times10^{-29}\text{ m}^3$$

$$D \approx \sqrt[3]{V} = \boxed{2.76\times10^{-10}\text{ m}\sim10^{-1}\text{ nm}}$$

(b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge $+Ze-(Z-1)e=+e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is a_0/Z.

P42.83 (a) The length of the pulse is

$$\Delta L = c\Delta t = \left(3.00\times10^{8}\text{ m/s}\right)\left(14.0\times10^{-12}\text{ s}\right) = \boxed{4.20\text{ mm}}$$

(b) The energy of each photon is

$$E = \frac{hc}{\lambda} = 2.86\times10^{-19}\text{ J}$$

so the number of photons in the pulse is

$$N = \frac{3.00\text{ J}}{2.86\times10^{-19}\text{ J/photon}} = \boxed{1.05\times10^{19}\text{ photons}}$$

(c) The volume of the pulse is

$$V = \Delta L\pi r^2 = (4.20\text{ mm})\left[\pi(3.00\text{ mm})^2\right] = 119\text{ mm}^3$$

resulting in a photon density of

$$n = \frac{1.05\times10^{19}\text{ photons}}{119\text{ mm}^3} = \boxed{8.82\times10^{16}\text{ mm}^{-3}}$$

P42.84 (a) The length of the pulse is $\Delta L = \boxed{c\Delta t}$.

(b) The energy of each photon is $E = \frac{hc}{\lambda}$, so

$$N = \frac{T_{ER}}{E} = \boxed{\frac{\lambda T_{ER}}{hc}}$$

(c) The volume of the pulse is

$$V = \Delta L \pi \frac{d^2}{4} = c\Delta t \pi \frac{d^2}{4}$$

resulting in a photon density of

$$n = \frac{N}{V} = \frac{\lambda T_{ER}}{hc\left(c\Delta t \pi d^2/4\right)} = \boxed{\frac{4\lambda T_{ER}}{\pi hc^2 d^2 \Delta t}}$$

P42.85 The fermions are described by the exclusion principle. Two of them, one spin-up and one spin-down, will be in the ground energy level, in a standing wave pattern with one antinode:

$$d_{NN} = \frac{1}{2}\lambda = L \rightarrow \lambda = 2L = \frac{h}{p},$$

and $$p = \frac{h}{2L} \rightarrow K = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{h^2}{8mL^2}.$$

The third must be in the next higher level, in a standing wave pattern with two antinodes:

$$2d_{NN} = 2\left(\frac{\lambda}{2}\right) = L \rightarrow \lambda = L,$$

and $$p = \frac{h}{L} \rightarrow K = \frac{p^2}{2m} = \frac{h^2}{2mL^2}.$$

The total energy is then

$$\frac{h^2}{8mL^2} + \frac{h^2}{8mL^2} + \frac{h^2}{2mL^2} = \boxed{\frac{3h^2}{4mL^2}}$$

P42.86 An ionization energy of 4.10 eV means the ground state energy is –4.10 eV. The photon energies tell us the separation of the energy levels:

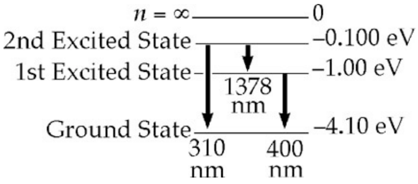
$$E = \frac{hc}{\lambda} = \frac{1\,240\text{ eV}\cdot\text{nm}}{\lambda} = \Delta E$$

Then, $\lambda_1 = 310$ nm, so $\Delta E_1 = 4.00$ eV

$$\lambda_2 = 400 \text{ nm}, \qquad \Delta E_2 = 3.10 \text{ eV}$$

$$\lambda_3 = 1\,378 \text{ nm}, \qquad \Delta E_3 = 0.900 \text{ eV}$$

The energy level diagram having the fewest levels and consistent with these energies is shown in ANS. FIG. P42.86.



ANS. FIG. 42.86

P42.87 The general radial probability distribution function is

$$P(r) = 4\pi r^2 |\psi|^2$$

With $\psi_{1s} = \left(\pi a_0^3\right)^{-1/2} e^{-r/a_0}$, it is $P(r) = 4r^2 a_0^{-3} e^{-2r/a_0}$.

The required probability is then

$$P = \int_{2.50a_0}^{\infty} P(r)dr = \int_{2.50a_0}^{\infty} \frac{4r^2}{a_0^3} e^{-2r/a_0}\, dr$$

Let $z = 2r/a_0$ and $dz = 2dr/a_0$. Then we want $P = \frac{1}{2}\int_{5.00}^{\infty} z^2 e^{-z}\, dz$.

Performing this integration by parts,

$$P = -\frac{1}{2}\left(z^2 + 2z + 2\right)e^{-z}\Big|_{5.00}^{\infty}$$

$$P = -\frac{1}{2}(0) + \frac{1}{2}(25.0 + 10.0 + 2.00)e^{-5.00} = \left(\frac{37}{2}\right)(0.006\ 74) = \boxed{0.125}$$

P42.88 From Equations 42.22 – 42.25,

$$P = \int_{\beta a_0}^{\infty} P_{1s}(r)dr = \int_{\beta a_0}^{\infty} \frac{4r^2}{a_0^3} e^{-2r/a_0}\, dr = \frac{1}{2}\int_{2\beta}^{\infty} z^2 e^{-z}\, dz, \quad \text{where} \quad z \equiv \frac{2r}{a_0}$$

$$P = -\frac{1}{2}\left(z^2 + 2z + 2\right)e^{-z}\Big|_{2\beta}^{\infty} = -\frac{1}{2}[0] + \frac{1}{2}\left[(2\beta)^2 + 4\beta + 2\right]e^{-2\beta}$$

$$= \boxed{e^{-2\beta}\left(2\beta^2 + 2\beta + 1\right)}$$

Challenge Problems

P42.89 (a) Let r represent the distance between the electron and the positron. The two move in a circle of radius $r/2$ around their center of mass with opposite velocities. The total angular momentum of the electron-positron system is quantized according to

$$L_n = \frac{mvr}{2} + \frac{mvr}{2} = n\hbar \qquad \text{where} \quad n = 1, 2, 3, \ldots$$

For each particle, $\sum F = ma$ expands to

$$\frac{k_e e^2}{r^2} = \frac{mv^2}{r/2}$$

We can eliminate $v = \dfrac{n\hbar}{mr}$ to find

$$\frac{k_e e^2}{r} = \frac{2mn^2\hbar^2}{m^2 r^2}$$

So the separation distances are

$$r = \frac{2n^2\hbar^2}{mk_e e^2} = 2a_0 n^2$$

Comparing this result to Equations 42.10 and 42.11, we see the allowed separation distances are two times the allowed radii of the Bohr hydrogen atom. Therefore,

$\boxed{r_n = 0.106n^2\text{, where } r_n \text{ is in nanometers and } n = 1, 2, 3,\ldots}$

(b) The orbital radii are $\dfrac{r}{2} = a_0 n^2$, the same as for the electron in hydrogen. The energy can be calculated from

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 - \frac{k_e e^2}{r}$$

Since $mv^2 = \dfrac{k_e e^2}{2r}$,

$$E = \frac{k_e e^2}{2r} - \frac{k_e e^2}{r} = -\frac{k_e e^2}{2r} = \frac{-k_e e^2}{4a_0 n^2}$$

Comparing this result to Equations 42.13 and 42.14, we see the allowed energies are half those of the Bohr hydrogen atom. Therefore,

$\boxed{E_n = -\dfrac{6.80}{n^2}\text{, where } E_n \text{ is in electron volts and } n = 1, 2, 3,\ldots}$

P42.90 (a) Suppose the atoms move in the +*x* direction. The absorption of a photon by an atom is a completely inelastic collision, described by

$$mv_i\hat{\mathbf{i}}+\frac{h}{\lambda}\left(-\hat{\mathbf{i}}\right)=mv_f\hat{\mathbf{i}} \quad \text{so} \quad v_f-v_i=-\frac{h}{m\lambda}$$

This happens promptly every time an atom has fallen back into the ground state, so it happens every $10^{-8}\text{ s}=\Delta t$. Then,

$$a=\frac{v_f-v_i}{\Delta t}=-\frac{h}{m\lambda\Delta t}\sim-\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\left(10^{-25}\text{ kg}\right)\left(500\times10^{-9}\text{ m}\right)\left(10^{-8}\text{ s}\right)}$$

$$\sim\boxed{-10^6\text{ m/s}^2}$$

(b) With constant average acceleration,

$$v_f^2=v_i^2+2a\Delta x \quad 0\sim\left(10^3\text{ m/s}\right)^2+2\left(-10^6\text{ m/s}^2\right)\Delta x$$

so $$\Delta x\sim\frac{\left(10^3\text{ m/s}\right)^2}{10^6\text{ m/s}^2}\boxed{\sim1\text{ m}}.$$

P42.91 (a) From Equation 42.13, the allowed energies are $E_n=\frac{k_ee^2}{2a_0}\left(\frac{1}{n^2}\right)$, where, from Equation 42.11, the Bohr radius is

$$a_0=\frac{\hbar^2}{m_ek_ee^2}=\frac{\left(h/2\pi\right)^2}{m_ek_ee^2}=\frac{h^2}{4\pi^2m_ek_ee^2}$$

Combining these gives

$$E_n=\frac{k_ee^2}{2}\frac{4\pi^2m_ek_ee^2}{h^2}\left(\frac{1}{n^2}\right)=\frac{2\pi^2m_ek_e^2e^4}{h^2}\left(\frac{1}{n^2}\right)$$

For a transition from state *n* to state *n* – 1,

$$hf=\Delta E=\left(\frac{2\pi^2m_ek_e^2e^4}{h^2}\right)\left(\frac{1}{(n-1)^2}-\frac{1}{n^2}\right)$$

$$hf=\Delta E=\left(\frac{2\pi^2m_ek_e^2e^4}{h^2}\right)\frac{n^2-\left(n^2-2n+1\right)}{n^2(n-1)^2}$$

which gives

$$f=\left(\frac{2\pi^2m_ek_e^2e^4}{h^3}\right)\frac{2n-1}{n^2(n-1)^2}$$

(b) As $n \to \infty$, we find the quantum result:

$$f \to \frac{2\pi^2 m_e k_e^2 e^4}{h^3}\frac{2}{n^3} = \frac{4\pi^2 m_e k_e^2 e^4}{h^3 n^3}$$

The classical frequency is $f = \frac{v}{2\pi r}$, where classically, from Equation 42.8, $v^2 = \frac{k_e e^2}{m_e r}$. By substituting, the relation for the classical frequency becomes

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi r}\sqrt{\frac{k_e e^2}{m_e r}} = \sqrt{\frac{k_e e^2}{4\pi^2 m_e r^3}}$$

From Equation 42.10, the radius $r = r_n = \frac{n^2\hbar^2}{m_e k_e e^2} = \frac{n^2 h^2}{4\pi^2 m_e k_e e^2}$; substituting this yields

$$f = \sqrt{\frac{k_e e^2}{4\pi^2 m_e r^3}} = \sqrt{\frac{k_e e^2}{4\pi^2 m_e}\left(\frac{4\pi^2 m_e k_e e^2}{n^2 h^2}\right)^3}$$

$$= \sqrt{\frac{\left(4\pi^2\right)^2 m_e^2 k_e^4 e^8}{n^6 h^6}} = \frac{4\pi^2 m_e k_e^2 e^4}{h^3 n^3}$$

$\boxed{\text{The classical frequency is } 4\pi^2 m_e k_e^2 e^4 / h^3 n^3.}$ We see that the Bohr result for large *n* reduces to the classical result.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P42.2 (a) 1 875 nm, 1 282 nm, 1 094 nm; (b) infrared

P42.4 (a) $\lambda_{mn} = \left|\dfrac{1}{1/\lambda_{m1} - 1/\lambda_{n1}}\right|$; (b) $k_{mn} = \left|k_{m1} - k_{n1}\right|$

P42.6 (a) See P42.6(a) for full explanation; (b) 0.846 ns

P42.8 See P42.8 for full explanation.

P42.10 (a) 2.86 eV; (b) 0.472 eV

P42.12 (a) 1.89 eV; (b) 656 nm; (c) 3.40 eV; (d) 365 nm; (e) 365 nm

P42.14 4.42×10^4 m/s

P42.16 (a) 97.3 nm; (b) 1.88 μm; (c) infrared; (d) Paschen; (e) 97.3 nm; (f) ultraviolet; (g) Lyman

P42.18 (a) 0.212 nm; (b) 9.97×10^{-25} kg · m/s; (c) 2.11×10^{-34} kg · m^2/s; (d) 3.40 eV; (e) –6.80 eV; (f) –3.40 eV

P42.20 (a) 152 as; (b) 8.23×10^9 revolutions; (c) Its lifetime in electron years is comparable to the lifetime of the Sun in Earth years, so we can think of it as a long time.

P42.22 (a) 1.31 μm; (b) 164 nm

P42.24 (a) $\dfrac{\hbar}{2r}$; (b) $\dfrac{\hbar^2}{2m_e r^2}$; (c) $\dfrac{\hbar^2}{2m_e r^2} - \dfrac{k_e e^2}{r}$; (d) $\dfrac{\hbar^2}{m_e k_e e^2} = a_0$; (e) –13.6 eV; (f) We find our results are in agreement with the Bohr theory.

P42.26 (a) See P42.26(a) for full explanation; (b) $E = -\dfrac{k_e e^2}{2a_0}$

P42.28 (a) 1; (b) 0.497

P42.30 (a) See P42.30(a) for a list of all sets; (b) See P42.30(b) for a list of all sets.

P42.32 (a) $\sqrt{6}\hbar$; (b) L_z can have the values $-2\hbar,\ -\hbar,\ 0,\ \hbar$ and $2\hbar$; (c) 145°, 114°, 90.0°, 65.9°, and 35.3°

P42.34 (a) 2; (b) 8; (c) 18; (d) 32; (e) 50

P42.36 (a) 3.99×10^{17} kg/m^3; (b) 8.17 am; (c) 1.77 Tm/s; (d) It is $5.91 \times 10^3 c$, which is huge compared with the speed of light and impossible.

P42.38 The electron energy is not enough to excite the hydrogen atom from its ground state to even the first excited state.

P42.40 (a) the 4*s* subshell; (b) We would expect $[\mathrm{Ar}]3d^44s^2$ to have lower energy, but $[\mathrm{Ar}]3d^54s^1$ has more unpaired spins and lower energy according to Hund's rule; (c) chromium

P42.42 See P42.42 for the complete table.

P42.44 1*s*, 2*s*, 2*p*, 3*s*, 3*p*, 4*s*, 3*d*, 4*p*, 5*s*, 4*d*, 5*p*, 6*s*, 4*f*, 5*d*, 6*p*, 7*s*

P42.46 $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^6 4d^{10} 4f^{14} 5s^2 5p^6 5d^{10} 5f^{14} 6s^2 6p^6 6d^8 7s^2$

P42.48 See P42.48 for full explanation.

P42.50 0.310 nm

P42.52 (a) For the 3*p* state, 1.4 and for the 3*d* state, 1.0; (b) See P42.52(b) for full explanation.

P42.54 L shell = 11.8 keV, M shell = 10.2 keV, N shell = 2.47 keV

P42.56 (a) $\dfrac{1240\ \mathrm{V \cdot nm}}{\lambda}$; (b) The potential difference is inversely proportional to the wavelength; (c) Yes. It predicts a minimum wavelength of 33.5 pm when the accelerating voltage is 37 keV, in agreement with the minimum wavelength in the figure; (d) Yes, but it might be unlikely for a very high energy electron to stop in a single interaction to produce a high-energy gamma ray, and it might be difficult to observe the very low intensity radio waves produced as bremsstrahlung by low-energy electrons; (e) The potential difference goes to infinity as the wavelength goes to zero; (f) The potential difference goes to zero as the wavelength goes to infinity.

P42.58 633 nm

P42.60 (a) 18.37 eV; (b) 217 nm; (c) 93.1 nm

P42.62 (a) 1.26×10^{-33}; (b) -1.15×10^6 K; (c) A population inversion cannot happen in thermal equilibrium.

P42.64 (a) $\lambda_1 = 632.809\ 14$ nm, $\lambda_2 = 632.808\ 57$ nm, $\lambda_3 = 632.809\ 71$ nm, three; (b) 697 m/s (c) See P42.64(c) for full description.

P42.66 0.389 T/m

P42.68 (a) 1.63×10^{-18} J; (b) 7.88×10^4 K

P42.70 5.39 keV

P42.72 See P42.72 for full explanation.

P42.74 In order to have a maximum value of m_ℓ equal to 3, we need to have $\ell = 3$, which is inconsistent with the first result.

P42.76 (a) See P42.76(a) for full explanation; (b) 2.53×10^{74}; (c) 1.18×10^{-63} m; (d) This number is *much smaller* than the radius of an atomic nucleus (~10^{-15} m), so the distance between quantized orbits of the Earth is too small to observe.

P42.78 (a) $\left(\frac{2r^2}{a_0^2} + \frac{2r}{a_0} + 1\right)e^{-2r/a_0}$; (b) See ANS. FIG. P42.78; (c) 1.34 a_0

P42.80 (a) –8.16 eV, –2.04 eV, –0.902 eV, –0.508 eV, –0.325 eV; (b) 1 090 nm, 811 nm, 724 nm, and 609 nm; (c) 122 nm, 103 nm, 97.3 nm, 95.0 nm, 91.2 nm; (d) The spectrum could be that of hydrogen, Doppler-shifted by motion away from us at speed 0.471*c*.

P42.82 (a) Al: 2.55×10^{-10} m ~ 10^{-1} nm and U: 2.76×10^{-10} m ~ 10^{-1} nm; (b) The outermost electron in any atom sees the nuclear charge screened by all the electrons below it. If we can visualize a single outermost electron, it moves in the electric field of net charge $+Ze - (Z - 1)e = +e$, the charge of a single proton, as felt by the electron in hydrogen. So the Bohr radius sets the scale for the outside diameter of every atom. An innermost electron, on the other hand, sees the nuclear charge unscreened, and the scale size of its (K-shell) orbit is a_0/Z.

P42.84 (a) $c\Delta t$; (b) $\frac{\lambda T_{ER}}{hc}$; (c) $\frac{4\lambda T_{ER}}{\pi h c^2 d^2 \Delta t}$

P42.86 See ANS. FIG P42.86 for the energy-level diagram.

P42.88 $e^{-2\beta}\left(2\beta^2 + 2\beta + 1\right)$

P42.90 (a) -10^6 m/s^2; (b) ~1 m

43

Molecules and Solids

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ43.1 (a) False. An infinite current would produce an infinite magnetic field that would penetrate the surface of the superconductor and destroy the superconducting properties. (b) False. There is no physical requirement that a superconductor carry a current. (c) True. (d) True. (e) True. Collisions do not occur between Cooper pairs and the lattice ions.

OQ43.2 Answer (b). At higher temperature, molecules are typically in higher rotational energy levels before as well as after infrared absorption.

OQ43.3 **(i)** Answer (c). Think of aluminum foil.

(ii) Answer (a). An example is NaCl, table salt.

(iii) Answer (b). Examples are elemental silicon and carborundum (silicon carbide).

OQ43.4 **(i)** Answer (b). The density of states is proportional to the energy to the one-half power.

(ii) Answer (a). Most states well above the Fermi energy are unoccupied.

OQ43.5 Answer (b). First consider electric conduction in a metal. The number of conduction electrons is essentially fixed. They conduct electricity by having drift motion in an applied electric field superposed on their random thermal motion. At higher temperature, the ion cores vibrate more and scatter more efficiently the conduction electrons flying among them. The mean time between collisions is reduced. The electrons have time to develop only a lower drift speed. The electric current is reduced, so we see the resistivity increasing with temperature.

Now consider an intrinsic semiconductor. At absolute zero its valence band is full and its conduction band is empty. It is an insulator, with very high resistivity. As the temperature increases, more electrons are promoted to the conduction band, leaving holes in the valence band. Then both electrons and holes move in response to an applied electric field. Thus we see the resistivity decreasing as temperature goes up.

OQ43.6 **(i)** and **(ii)** Answer (a) for both. Either kind of doping contributes more mobile charge carriers, either holes or electrons.

OQ43.7 The ranking is then b > d > c > a. If you start with a solid sample and raise its temperature, it will typically melt first, then start emitting lots of far infrared light, then emit light with a spectrum peaking in the near infrared, and later have its molecules dissociate into atoms. Rotation of a diatomic molecule involves less energy than vibration. Absorption and emission of microwave photons, of frequency ~10^{11} Hz, accompany excitation and de-excitation of rotational motion, while infrared photons, of frequency ~10^{13} Hz, accompany changes in the vibration state of typical simple molecules.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ43.1 A material can absorb a photon of energy greater than the energy gap, as an electron jumps into a higher energy state; therefore, silicon can absorb visible light, thus appearing opaque. If the photon does not have enough energy to raise the energy of the electron by the energy gap, then the photon will not be absorbed; therefore, diamond cannot absorb visible light, thus appearing transparent.

CQ43.2 Rotational, vibrational, and electronic (as discussed in Chapter 42) are the three major forms of excitation. Rotational energy for a diatomic molecule is on the order of $\frac{\hbar^2}{2I}$, where I is the moment of inertia of the molecule. A typical value for a small molecule is on the order of 1 meV = 10^{-3} eV. Vibrational energy is on the order of hf, where f is the vibration frequency of the molecule. A typical value is on the order of 0.1 eV. Electronic energy depends on the state of an electron in the molecule and is on the order of a few eV. The rotational energy can be zero, but neither the vibrational nor the electronic energy can be zero.

CQ43.3 From the rotational spectrum of a molecule, one can easily calculate the moment of inertia of the molecule using Equation 43.7 in the text. Note that with this method, only the spacing between adjacent energy levels needs to be measured. From the moment of inertia, the size of the molecule can be calculated, provided that the structure of the molecule is known.

CQ43.4 Along with arsenic (As), any other element in group V, such as phosphorus (P), antimony (Sb), and bismuth (Bi), would make good donor atoms. Each has 5 valence electrons. Any element in group III would make good acceptor atoms, such as boron (B), aluminum (Al), gallium (Ga), and indium (In). They all have only 3 valence electrons.

CQ43.5 The energy of the photon is given to the electron. The energy of a photon of visible light is sufficient to promote the electron from the lower-energy valence band to the higher-energy conduction band. This results in the additional electron in the conduction band and an additional hole—the energy state that the electron used to occupy—in the valence band.

CQ43.6 (a) In a metal, there is no energy gap between the valence and conduction bands, or the conduction band is partly full even at absolute zero in temperature. Thus an applied electric field is able to inject a tiny bit of energy into an electron to promote it to a state in which it is moving through the metal as part of an electric current. In an insulator, there is a large energy gap between a full valence band and an empty conduction band. An applied electric field is unable to give electrons in the valence band enough energy to jump across the gap into the higher energy conduction band. In a semiconductor, the energy gap between valence and conduction bands is smaller than in an insulator.

(b) At absolute zero the valence band is full and the conduction band is empty, but at room temperature thermal energy has promoted some electrons across the gap. Then there are some mobile holes in the valence band as well as some mobile electrons in the conduction band.

CQ43.7 (a) The two assumptions in the free-electron theory are that the conduction electrons are not bound to any particular atom, and that the nuclei of the atoms are fixed in a lattice structure. In this model, it is the “soup” of free electrons that are conducted through metals.

(b) The energy band model is more comprehensive than the free-electron theory. The energy band model includes an account of the more tightly bound electrons as well as the conduction electrons. It can be developed into a theory of the structure of the crystal and its mechanical and thermal properties.

CQ43.8 A molecule containing two atoms of D = ^{2}H, deuterium, has twice the mass of a molecule containing two atoms of ordinary hydrogen ^{1}H; therefore the deuterium molecule has twice the reduced mass of the hydrogen molecule. The atoms have the same electronic structure, so the molecules have the same interatomic spacing, and the same spring constant. Therefore, each vibrational energy level for D_2 is $1/\sqrt{2}$ times that of H_2. The moment of inertia of deuterium is twice as large and the rotational energies one-half as large as for the ordinary hydrogen molecule.

CQ43.9 Ionic bonds are ones between oppositely charged ions. One atom essentially steals an electron from another; for example, in table salt, NaCl, the chlorine atom takes the outer 3*s* electron from the sodium atom, resulting in two ions Cl^- and Na^+. A simple model of an ionic bond is the electrostatic attraction of a negatively charged latex balloon to a positively charged Mylar balloon.

Covalent bonds are ones in which atoms share electrons. Classically, two children playing a short-range game of catch with a ball models a covalent bond. On a quantum scale, the two atoms are sharing a wave function, so perhaps a better model would be two children using a single hula hoop.

Van der Waals bonds are weak electrostatic forces: the electric dipole-dipole force is analogous to the attraction between the opposite poles of two bar magnets, the dipole-induced dipole force is similar to a bar magnet attracting an iron nail or paper clip, and the dispersion force is analogous to an alternating-current electromagnet attracting a paper clip.

A hydrogen atom in a molecule is not ionized, but its electron can spend more time elsewhere than it does in the hydrogen atom. The hydrogen atom can be a location of net positive charge, and can weakly attract a zone of negative charge in another molecule.

CQ43.10 The atoms of crystalline substances form a regular array of ions in a lattice structure, and the atoms are close enough together to allow energy bands to form. The atoms of amorphous solids do not form a regular array, but they are close enough to produce energy bands. The atoms of gases do not form regular arrays and are too far apart to form energy bands.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 43.1 Molecular Bonds

P43.1 At the boiling or condensation temperature,

$$E = \frac{3}{2}k_B T \approx 10^{-3}\ \text{eV} = 10^{-3}\left(1.6\times10^{-19}\ \text{J}\right)$$

Solving for the temperature *T* gives,

$$T = \frac{E}{k_B} \approx \frac{2\left(1.6\times10^{-22}\ \text{J}\right)}{3\left(1.38\times10^{-23}\ \text{J/K}\right)}\ \boxed{\sim 10\ \text{K}}$$

P43.2 (a) The electrostatic force is

$$\begin{aligned} F &= \frac{q^2}{4\pi\epsilon_0 r^2} \\ &= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(5.00\times10^{-10}\ \text{m}\right)^2} \\ &= 9.21\times10^{-10}\ \text{N} = 921\times10^{-12}\ \text{N} \end{aligned}$$

or $\boxed{\text{921 pN toward the other ion.}}$

(b) The potential energy of the ion pair is

$$\begin{aligned} U &= \frac{-q^2}{4\pi\epsilon_0 r} \\ &= -\left[\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{5.00\times10^{-10}\ \text{m}}\right]\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) \\ &= \boxed{-2.88\ \text{eV}} \end{aligned}$$

P43.3 We are told that

$$\mathrm{K} + \mathrm{Cl} + 0.70\text{ eV} \rightarrow \mathrm{K}^{+} + \mathrm{Cl}^{-}$$

and $$\mathrm{Cl} + e^{-} \rightarrow \mathrm{Cl}^{-} + 3.6\text{ eV}$$

or $$\mathrm{Cl}^{-} \rightarrow \mathrm{Cl} + e^{-} - 3.6\text{ eV}$$

By substitution,

$$\mathrm{K} + \mathrm{Cl} + 0.7\text{ eV} \rightarrow \mathrm{K}^{+} + \mathrm{Cl} + e^{-} - 3.6\text{ eV}$$

$$\mathrm{K} + 4.3\text{ eV} \rightarrow \mathrm{K}^{+} + e^{-}$$

or the ionization energy of potassium is $\boxed{4.3\text{ eV}}$.

P43.4 (a) Because the ionization energy of K is 4.34 eV, we have the relation

$$\mathrm{K} + 4.34\text{ eV} \rightarrow \mathrm{K}^{+} + e^{-} \qquad \textbf{[1]}$$

and because the electron affinity of I is 3.06 eV, we have the relation

$$\mathrm{I} + e^{-} \rightarrow \mathrm{I}^{-} + 3.06\text{ eV}$$
$$\mathrm{I} - 3.06\text{ eV} \rightarrow \mathrm{I}^{-} - e^{-} \qquad \textbf{[2]}$$

Adding equations [1] and [2] gives

$$(\mathrm{K} + 4.34\text{ eV}) + (\mathrm{I} - 3.06\text{ eV}) \rightarrow (\mathrm{K}^{+} + e^{-}) + (\mathrm{I}^{-} - e)$$

$$\mathrm{K} + \mathrm{I} + (4.34\text{ eV} - 3.06\text{ eV}) \rightarrow \mathrm{K}^{+} + \mathrm{I}^{-}$$

$$\mathrm{K} + \mathrm{I} + 1.28\text{ eV} \rightarrow \mathrm{K}^{+} + \mathrm{I}^{-}$$

Therefore, the activation energy is $\boxed{E_a = 1.28\text{ eV}}$.

(b) We differentiate the given function:

$$\frac{dU}{dr} = \frac{4\in}{\sigma}\left[-12\left(\frac{\sigma}{r}\right)^{13} + 6\left(\frac{\sigma}{r}\right)^{7}\right]$$

Setting the expression above equal to 0, at $r = r_0$ we have

$$\frac{dU}{dr} = 0 \quad \rightarrow \quad \left(\frac{\sigma}{r_0}\right)^{13} = \frac{1}{2}\left(\frac{\sigma}{r_0}\right)^{7}$$

which gives

$$\left(\frac{\sigma}{r_0}\right)^{6} = 2^{-1} \quad \rightarrow \quad \sigma = 2^{-1/6} r_0 = 2^{-1/6}(0.305)\text{ nm}$$

or $\boxed{\sigma = 0.272\text{ nm}}$

Then also

$$U(r_0) = 4\epsilon\left[\left(\frac{2^{-1/6}r_0}{r_0}\right)^{12} - \left(\frac{2^{-1/6}r_0}{r_0}\right)^{6}\right] + E_a$$

$$= 4\epsilon\left[\frac{1}{4} - \frac{1}{2}\right] + E_a = -\epsilon + E_a$$

solving for ϵ gives

$$\epsilon = E_a - U(r_0) = 1.28\text{ eV} + 3.37\text{ eV}$$

$$= \boxed{4.65\text{ eV}}$$

(c) The force of attraction between the atoms is

$$F(r) = -\frac{dU}{dr} = \frac{4\epsilon}{\sigma}\left[12\left(\frac{\sigma}{r}\right)^{13} - 6\left(\frac{\sigma}{r}\right)^{7}\right]$$

To find the maximum force we calculate

$$\frac{dF}{dr} = \frac{4\epsilon}{\sigma^2}\left[-156\left(\frac{\sigma}{r}\right)^{14} + 42\left(\frac{\sigma}{r}\right)^{8}\right] = 0$$

$$\frac{\sigma}{r_{\text{break}}} = \left(\frac{42}{156}\right)^{1/6}$$

So at $r = r_{\text{break}}$, the force is a maximum:

$$F_{\max} = \frac{4(4.65\text{ eV})}{0.272\text{ nm}}\left[12\left(\frac{42}{156}\right)^{13/6} - 6\left(\frac{42}{156}\right)^{7/6}\right]$$

$$= \frac{-41.0\text{ eV}}{\text{nm}}\left(\frac{1.60\times10^{-19}\text{ N}\cdot\text{m}}{1\text{ eV}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right) = -6.55\text{ nN}$$

Therefore the applied force required to break the molecule is $\boxed{+6.55\text{ nN}}$ away from the center.

(d) To calculate the force constant, we expand *U*(*r*) as suggested in the problem statement:

$$U(r_0 + s) = 4\epsilon\left[\left(\frac{\sigma}{r_0 + s}\right)^{12} - \left(\frac{\sigma}{r_0 + s}\right)^{6}\right] + E_a$$

$$= 4\epsilon\left[\left(\frac{2^{-1/6}r_0}{r_0 + s}\right)^{12} - \left(\frac{2^{-1/6}}{r_0 + s}\right)^{6}\right] + E_a$$

Expanding,

$$U\left(r_0+s\right)=4\in\left[\frac{1}{4}\left(1+\frac{s}{r_0}\right)^{-12}-\frac{1}{2}\left(1+\frac{s}{r_0}\right)^{-6}\right]+E_a$$

$$=4\in\left[\frac{1}{4}\left(1-12\frac{s}{r_0}+78\frac{s^2}{r_0^2}-\cdots\right)\right.$$

$$\left.-\frac{1}{2}\left(1-6\frac{s}{r_0}+21\frac{s^2}{r_0^2}-\cdots\right)\right]+E_a$$

$$=\ \in-12\in\frac{s}{r_0}+78\in\frac{s^2}{r_0^2}-2\in+12\in\frac{s}{r_0}-42\in\frac{s^2}{r_0^2}+E_a+\cdots$$

$$=-\in+E_a+0\left(\frac{s}{r_0}\right)+36\in\frac{s^2}{r_0^2}+\cdots$$

or $\quad U\left(r_0+s\right)\approx U\left(r_0\right)+\frac{1}{2}ks^2$

where $k=\dfrac{72\in}{r_0^2}=\dfrac{72(4.65\text{ eV})}{(0.305\text{ nm})^2}=3\,599\text{ eV/nm}^2=\boxed{576\text{ N/m}}$

P43.5 (a) The minimum energy of the molecule at $r = r_0$ is found from

$$\frac{dU}{dr}=-12Ar_0^{-13}+6Br_0^{-7}=0$$

yielding

$$r_0=\left[\frac{2A}{B}\right]^{1/6}$$

$$=\left[\frac{2\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)}{1.488\times10^{-60}\text{ eV}\cdot\text{m}^6}\right]^{1/6}=7.42\times10^{-11}\text{ m}=\boxed{74.2\text{ pm}}$$

(b) The energy required to break up the molecule would separate the atoms from $r = r_0$ to $r = \infty$:

$$E=U\big|_{r=\infty}-U\big|_{r=r_0}=0-\left[\frac{A}{4A^2/B^2}-\frac{B}{2A/B}\right]=-\left[\frac{1}{4}-\frac{1}{2}\right]\frac{B^2}{A}=\frac{B^2}{4A}$$

$$E=\frac{\left(1.488\times10^{-60}\text{ eV}\cdot\text{m}^6\right)^2}{4\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)}=\boxed{4.46\text{ eV}}$$

This is also the equal to the binding energy, the amount of energy given up by the two atoms as they come together to form a molecule.

P43.6 (a) The minimum energy of the molecule at $r = r_0$ is found from

$$\frac{dU}{dr} = -12Ar_0^{-13} + 6Br_0^{-7} = 0$$

yielding

$$r_0 = \left[\frac{2A}{B}\right]^{1/6}$$

(b) The energy required to break up the molecule would separate the atoms from $r = r_0$ to $r = \infty$:

$$E = U\big|_{r=\infty} - U\big|_{r=r_0} = 0 - \left[\frac{A}{4A^2/B^2} - \frac{B}{2A/B}\right] = -\left[\frac{1}{4} - \frac{1}{2}\right]\frac{B^2}{A} = \boxed{\frac{B^2}{4A}}$$

Section 43.2 Energy States and Spectra of Molecules

P43.7 (a) Recall from Chapter 42 that the energy of the photon is given by

$$hf = \Delta E = \frac{\hbar^2}{2I}\left[2(2+1)\right] - \frac{\hbar^2}{2I}\left[1(1+1)\right] = \frac{\hbar^2}{2I}(4)$$

Then,

$$I = \frac{4(h/2\pi)^2}{2hf} = \frac{h}{2\pi^2 f} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi^2\left(2.30\times10^{11}\text{ Hz}\right)}$$

$$= \boxed{1.46\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

(b) The results are the same, suggesting that the bond length of the molecule does not change measurably between the two transitions.

P43.8 From Equations 43.4 and 43.3, the reduced mass and moment of inertia of CsI are

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(132.9\text{ u})(126.9\text{ u})}{132.9\text{ u} + 126.9\text{ u}}\left(\frac{1.66\times10^{-27}\text{ kg}}{\text{u}}\right)$$

$$= 1.08\times10^{-25}\text{ kg}$$

and

$$I = \mu r^2 = \left(1.08\times10^{-25}\text{ kg}\right)\left(0.127\times10^{-9}\text{ m}\right)^2$$

$$= 1.74\times10^{-45}\text{ kg}\cdot\text{m}^2$$

The allowed rotational energies (from Equation 43.6) are

$$E_{rot} = \frac{\hbar^2}{2I}J(J+1) = J(J+1)\left[\frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}/2\pi\right)^2}{2\left(1.74\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}\right]$$

$$= J(J+1)\left(3.20\times10^{-24}\text{ J}\right)\cdot\left(\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right)$$

$$= J(J+1)\left(2.00\times10^{-5}\text{ eV}\right)$$

(a) $J = 2$ gives

$$E_{rot} = 2(3)\frac{\hbar^2}{2I} = 1.20\times10^{-4}\text{ eV} = \boxed{0.120\text{ meV}}$$

(b) The photon that can cause the transition $J = 1 \rightarrow 2$ has energy

$$hf = \Delta E_{rot} = 2(2+1)\left(\frac{\hbar^2}{2I}\right) - 1(1+1)\left(\frac{\hbar^2}{2I}\right) = 4\left(\frac{\hbar^2}{2I}\right)$$

$$= 4\left(3.20\times10^{-24}\text{ J}\right) = 1.28\times10^{-23}\text{ J} = 7.99\times10^{-2}\text{ eV}$$

The frequency of the photon is

$$f = \frac{\Delta E_{rot}}{h} = \frac{1.28\times10^{-23}\text{ J}}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = 1.93\times10^{10}\text{ s}^{-1} = \boxed{19.3\text{ GHz}}$$

***P43.9** For the HCl molecule in the $J = 2$ rotational energy level, we are given the distance between nuclei, $r_0 = 0.127\,5$ nm. From Equation 43.6, the allowed rotational energies are

$$E_{rot} = \frac{\hbar^2}{2I}J(J+1)$$

Taking $J = 2$, we have $E_{rot} = 6\frac{\hbar^2}{2I} = \frac{3\hbar^2}{I} = \frac{1}{2}I\omega^2$,

or $$\omega = \sqrt{\frac{6\hbar^2}{I^2}} = \sqrt{6}\frac{\hbar}{I}$$

The moment of inertia of the molecule is given by Equation 43.3:

$$I = \mu r_0^2 = \left(\frac{m_1 m_2}{m_1 + m_2}\right)r_0^2$$

Substituting numerical values,

$$\begin{aligned} I &= \left[\frac{(1.008\ \text{u})(35.45\ \text{u})}{1.008\ \text{u} + 35.45\ \text{u}}\right] r_0^2 \\ &= (0.980\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)\left(1.275\times10^{-10}\ \text{m}\right)^2 \\ &= 2.64\times10^{-47}\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

Therefore,

$$\omega = \sqrt{6}\,\frac{\hbar}{I} = \frac{\sqrt{6}\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)}{2\pi(2.64\times10^{-47}\ \text{kg}\cdot\text{m}^2)} = \boxed{9.77\times10^{12}\ \text{rad/s}}$$

P43.10 (a) From Equation 43.10, the energy separation between the ground and first excited state is

$$\Delta E_{\text{vib}} = \frac{h}{2\pi}\sqrt{\frac{k}{\mu}} = hf \qquad \text{so} \qquad k = 4\pi^2 f^2 \mu$$

and the reduced mass is

$$\mu = \frac{k}{4\pi^2 f^2} = \frac{1530\ \text{N/m}}{4\pi^2(56.3\times10^{12}\ \text{s}^{-1})^2} = \boxed{1.22\times10^{-26}\,\text{kg}}$$

(b) From Equation 43.4, the reduced mass is

$$\begin{aligned} \mu &= \frac{m_1 m_2}{m_1 + m_2} = \frac{(14.007\ \text{u})(15.999\ \text{u})}{14.007\ \text{u} + 15.999\ \text{u}}\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) \\ &= \boxed{1.24\times10^{-26}\ \text{kg}} \end{aligned}$$

(c) $\boxed{\text{They agree because the small apparent difference can be attributed to uncertainty in the data.}}$

P43.11 (a) With *r* representing the distance of each atom from the center of mass, the moment of inertia is

$$\begin{aligned} mr^2 + mr^2 &= 2mr^2 \\ &= 2(1.008\ \text{u})\left(\frac{1.66\times10^{-27}\ \text{kg}}{\text{u}}\right)\left(\frac{0.750\times10^{-10}\ \text{m}}{2}\right)^2 \\ &= 4.71\times10^{-48}\ \text{kg}\cdot\text{m}^2 \end{aligned}$$

The allowed rotational energies (from Equation 43.6) are

$$E_{\text{rot}} = \frac{\hbar^2}{2I} J(J+1)$$

The $J = 0$ state has energy $E_{\text{rot}} = 0$, and the $J = 1$ state has energy

$$
\begin{aligned}
E_{\text{rot}} &= \frac{(h/2\pi)^2}{2I}(1)(2) = \frac{(h/2\pi)^2}{I} \\
&= \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}/2\pi\right)^2}{\left(4.71\times10^{-48}\ \text{kg}\cdot\text{m}^2\right)}\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) \\
&= 1.48\times10^{-2}\ \text{eV} = \boxed{0.014\ 8\ \text{eV}}
\end{aligned}
$$

(b) The energy of the photon that raises the molecule from 0 to 0.014 8 eV is 0.014 8 eV. The photon's wavelength is

$$
\begin{aligned}
\lambda &= \frac{h}{E} = \frac{1240\ \text{eV}\cdot\text{nm}}{0.0148\ \text{eV}} = 8.41\times10^4\ \text{nm} \\
&= 84.1\times10^3\ \text{nm} = \boxed{84.1\ \mu\text{m}}
\end{aligned}
$$

***P43.12** From Equation 43.10, the energy separation between the ground and first excited state is

$$\Delta E_{\text{vib}} = \hbar\sqrt{\frac{k}{\mu}} = \hbar\sqrt{\frac{k(m_1 + m_2)}{m_1m_2}}$$

Substituting numerical values,

$$
\begin{aligned}
\Delta E_{\text{vib}} &= \left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)\sqrt{\frac{(480\ \text{N/m})(35+1)}{(35)(1)\left(1.66\times10^{-27}\ \text{kg}\right)}} \\
&= 5.75\times10^{-20}\ \text{J} = 0.359\ \text{eV}
\end{aligned}
$$

To excite a transition with this energy difference, the wavelength of incident photons must be

$$\lambda = \frac{hc}{\Delta E_{\text{vib}}} = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{0.359\ \text{eV}} = 3.45\times10^3\ \text{nm}$$

The incident photons have a wavelength longer than this, which means they have less energy than 0.359 eV. Therefore, these photons cannot excite the molecule to the first excited state.

P43.13 The mass to consider is the molecule's reduced mass. Iodine has atomic mass 126.90 u and a hydrogen atom is 1.007 9 u, so the reduced mass of HI is

$$\mu = \frac{m_1m_2}{m_1+m_2} = \frac{(126.90\ \text{u})(1.007\ 9\ \text{u})}{126.90\ \text{u} + 1.007\ 9\ \text{u}} = 0.999\ 96\ \text{u}$$

Now for the energy of the ground state we have

$$E=\frac{1}{2}kA^2=\left(0+\frac{1}{2}\right)hf=\frac{1}{2}\frac{h}{2\pi}\sqrt{\frac{k}{\mu}}$$

So the amplitude is

$$A=\sqrt{\frac{h}{2\pi k}\sqrt{\frac{k}{\mu}}}=\left(\frac{h}{2\pi}\right)^{1/2}\left(\frac{1}{k\mu}\right)^{1/4}$$

(a) For HI we have

$$A=\left(\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi}\right)^{1/2}\times\left(\frac{1}{(320\text{ N/m})(0.999\ 96)(1.66\times10^{-27}\text{ kg})}\right)^{1/4}$$

$$=\boxed{1.20\times10^{-11}\text{ m}=12.0\text{ pm}}$$

(b) Flourine has an atomic mass of 18.998 4 u, so, for HF,

$$\mu=\frac{m_1m_2}{m_1+m_2}=\frac{(18.998\ 4\text{ u})(1.007\ 9\text{ u})}{18.998\ 4\text{ u}+1.007\ 9\text{ u}}=0.957\ 12\text{ u}$$

and

$$A=\left(\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi}\right)^{1/2}\times\left(\frac{1}{(970\text{ N/m})(0.957\ 12)(1.66\times10^{-27}\text{ kg})}\right)^{1/4}$$

$$=\boxed{9.22\times10^{-12}\text{ m}=9.22\text{ pm}}$$

P43.14 The energy of a rotational transition is $\Delta E=\left(\frac{\hbar^2}{I}\right)J$, where J is the rotational quantum number of the higher energy state (see Equation 43.7). We do not know J from the data. However,

$$\Delta E=\frac{hc}{\lambda}=\frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{\lambda}\left(\frac{1\text{ eV}}{1.602\times10^{-19}\text{ J}}\right)$$

For each observed wavelength,

λ (mm)	*E* (eV)
0.120 4	0.010 30
0.096 4	0.012 86
0.080 4	0.015 42
0.069 0	0.017 97
0.060 4	0.020 53

The $\Delta E'$s consistently increase by 0.002 56 eV.

$$E_1 = \frac{\hbar^2}{I} = 0.002\,56\text{ eV}$$

and

$$\begin{aligned} I &= \frac{\hbar^2}{E_1} = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{\left(0.002\,56\text{ eV}\right)}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) \\ &= \boxed{2.72\times10^{-47}\text{ kg}\cdot\text{m}^2} \end{aligned}$$

For the HCl molecule, the internuclear radius is

$$r = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{2.72\times10^{-47}}{1.62\times10^{-27}}}\text{ m} = 0.130\text{ nm}$$

P43.15 (a) The reduced mass of NaCl is

$$\begin{aligned} \mu &= \frac{m_{\text{Na}}m_{\text{Cl}}}{m_{\text{Na}}+m_{\text{Cl}}} = \frac{(22.99\text{ u})(35.45\text{ u})}{22.99\text{ u}+35.45\text{ u}}\left(\frac{1.66\times10^{-27}\text{ kg}}{\text{u}}\right) \\ &= \boxed{2.32\times10^{-26}\text{ kg}} \end{aligned}$$

(b) Its moment of inertia is

$$\begin{aligned} I &= \mu r^2 = \left(2.32\times10^{-26}\text{ kg}\right)\left(0.280\times10^{-9}\text{ m}\right)^2 \\ &= \boxed{1.82\times10^{-45}\text{ kg}\cdot\text{m}^2} \end{aligned}$$

(c) The wavelength of the emitted photon is found from:

$$\frac{hc}{\lambda}=\Delta E=\frac{\hbar^2}{2I}2(2+1)-\frac{\hbar^2}{2I}1(1+1)=\frac{3\hbar^2}{I}-\frac{\hbar^2}{I}=\frac{2\hbar^2}{I}=\frac{2h^2}{4\pi^2 I}$$

then,

$$\lambda=\frac{c4\pi^2 I}{2h}=\frac{\left(3.00\times10^{8}\text{ m/s}\right)4\pi^2\left(1.82\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}{2\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)}$$

$$=\boxed{1.62\text{ cm}}$$

P43.16 Masses m_1 and m_2 have the respective distances r_1 and r_2 from the center of mass. Then,

$$m_1r_1=m_2r_2 \quad\text{and}\quad r_1+r_2=r$$

So, $$r_1=\frac{m_2r_2}{m_1}$$

and thus, $$\frac{m_2r_2}{m_1}+r_2=r \quad\rightarrow\quad r_2=\frac{m_1r}{m_1+m_2}$$

Also, $$r_2=\frac{m_1r_1}{m_2}$$

thus, $$r_1+\frac{m_1r_1}{m_2}=r \quad\rightarrow\quad r_1=\frac{m_2r}{m_1+m_2}$$

The moment of inertia of the molecule is then

$$I=m_1r_1^2+m_2r_2^2=m_1\frac{m_2^2r^2}{\left(m_1+m_2\right)^2}+m_2\frac{m_1^2r^2}{\left(m_1+m_2\right)^2}$$

$$=\frac{m_1m_2\left(m_2+m_1\right)r^2}{\left(m_1+m_2\right)^2}=\frac{m_1m_2r^2}{m_1+m_2}=\boxed{\mu r^2}$$

P43.17 (a) The reduced mass of the O_2 is

$$\mu=\frac{m_Om_O}{m_O+m_O}=\frac{(16.00\text{ u})(16.00\text{ u})}{(16.00\text{ u})+(16.00\text{ u})}=8\text{ u}$$

$$=8\left(1.66\times10^{-27}\text{ kg}\right)=1.33\times10^{-26}\text{ kg}$$

The moment of inertia is then

$$I=\mu r^2=\left(1.33\times10^{-26}\text{ kg}\right)\left(1.20\times10^{-10}\text{ m}\right)^2$$

$$=1.91\times10^{-46}\text{ kg}\cdot\text{m}^2$$

The rotational energies are

$$E_{\text{rot}} = \frac{\hbar^2}{2I}J(J+1) = \frac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}/2\pi\right)^2}{2\left(1.91\times10^{-46}\ \text{kg}\cdot\text{m}^2\right)}J(J+1)$$

Thus,

$$E_{\text{rot}} = \left(2.91\times10^{-23}\ \text{J}\right)J(J+1)$$

and for $J = 0, 1, 2,$

$$E_{\text{rot}} = \boxed{0,\ 3.63\times10^{-4}\ \text{eV},\ 1.09\times10^{-3}\ \text{eV}}$$

(b) The vibrational energies are given by

$$\begin{aligned}E_{\text{vib}} &= \left(v+\frac{1}{2}\right)\hbar\sqrt{\frac{k}{\mu}}\\ &= \left(v+\frac{1}{2}\right)\left(\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi}\right)\sqrt{\frac{1\,177\ \text{N/m}}{8\left(1.66\times10^{-27}\ \text{kg}\right)}}\\ &= \left(v+\frac{1}{2}\right)\left(3.14\times10^{-20}\ \text{J}\right)\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right)\\ &= \left(v+\frac{1}{2}\right)(0.196\ \text{eV})\end{aligned}$$

For $v = 0, 1, 2$, $\boxed{E_{\text{vib}} = 0.098\,0\ \text{eV},\ 0.294\ \text{eV},\ 0.490\ \text{eV}}$.

P43.18 (a) In benzene, the dashed lines form equilateral triangles, so the carbon atoms are each 0.110 nm from the axis and each hydrogen atom is (0.110 + 0.100 nm) = 0.210 nm from the axis. Thus, the moment of inertia is given by

$$\begin{aligned}I = \sum mr^2 &= 6\left(1.99\times10^{-26}\ \text{kg}\right)\left(0.110\times10^{-9}\ \text{m}\right)^2\\ &\quad+6\left(1.67\times10^{-27}\ \text{kg}\right)\left(0.210\times10^{-9}\ \text{m}\right)^2\\ &= \boxed{1.89\times10^{-45}\ \text{kg}\cdot\text{m}^2}\end{aligned}$$

(b) The allowed rotational energies are then

$$\begin{aligned}E_{\text{rot}} &= \frac{\hbar^2}{2I}J(J+1) = \frac{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\left(1.89\times10^{-45}\ \text{kg}\cdot\text{m}^2\right)}J(J+1)\\ &= \left(2.95\times10^{-24}\ \text{J}\right)J(J+1) = \left(18.4\times10^{-6}\ \text{eV}\right)J(J+1)\end{aligned}$$

$$\boxed{\begin{array}{l}E_{\text{rot}} = 18.4J(J+1),\ \text{where } E_{\text{rot}} \text{ is in microelectron volts and}\\ J = 0, 1, 2, 3, \ldots\end{array}}$$

The first five of these allowed energies are:

$$E_{\text{rot}} = 0,\ 36.9\ \mu\text{eV},\ 111\ \mu\text{eV},\ 221\ \mu\text{eV, and } 369\ \mu\text{eV}$$

P43.19 We carry extra digits through the solution because part (c) involves the subtraction of two close numbers. The longest wavelength corresponds to the smallest energy difference between the rotational energy levels. It is between $J = 0$ and $J = 1$, namely

$$\Delta E_{\min} = \frac{\hbar^2}{I}$$

The wavelength is then

$$\lambda = \frac{hc}{\Delta E_{\min}} = \frac{hc}{\hbar^2/I} = \frac{4\pi^2 Ic}{h}$$

If $\mu = \dfrac{m_H m_{Cl}}{m_H + m_{Cl}}$ is the reduced mass, then

$$I = \mu r^2 = \mu\left(0.127\ 46 \times 10^{-9}\ \text{m}\right)^2$$

and therefore,

$$\lambda = \frac{4\pi^2\left[\mu\left(0.127\ 46 \times 10^{-9}\ \text{m}\right)^2\right]\left(2.997\ 925 \times 10^8\ \text{m/s}\right)}{6.626\ 075 \times 10^{-34}\ \text{J} \cdot \text{s}}$$

$$= \left(2.901\ 830 \times 10^{23}\ \text{m/kg}\right)\mu \qquad \textbf{[1]}$$

(a) For ^{35}Cl,

$$\mu_{35} = \frac{(1.007\ 825\text{u})(34.968\ 853\text{u})}{1.007\ 825\text{u} + 34.968\ 853\text{u}}\left(\frac{1.660\ 540 \times 10^{-27}\ \text{kg}}{\text{u}}\right)$$

$$= 1.626\ 653 \times 10^{-27}\ \text{kg}$$

From equation [1],

$$\lambda_{35} = \left(2.901\ 830 \times 10^{23}\ \text{m/kg}\right)\left(1.626\ 653 \times 10^{-27}\ \text{kg}\right)$$

$$= \boxed{472\ \mu\text{m}}$$

(b) For ^{37}Cl,

$$\mu_{37} = \frac{(1.007\ 825\text{u})(36.965\ 903\text{u})}{1.007\ 825\text{u} + 36.965\ 903\text{u}}\left(\frac{1.660\ 540 \times 10^{-27}\ \text{kg}}{\text{u}}\right)$$

$$= 1.629\ 118 \times 10^{-27}\ \text{kg}$$

From equation [1],

$$\lambda_{37} = \left(2.901\ 830 \times 10^{23}\ \text{m/kg}\right)\left(1.629\ 118 \times 10^{-27}\ \text{kg}\right) = \boxed{473\ \mu\text{m}}$$

(c) The separation in wavelength is

$$\lambda_{37} - \lambda_{35} = 472.742\,4\ \mu\text{m} - 472.027\,0\ \mu\text{m} = \boxed{0.715\ \mu\text{m}}$$

P43.20 We find an average spacing between peaks by counting 22 gaps (counting the central gap as two) between 7.96×10^{13} Hz and 9.24×10^{13} Hz:

$$\Delta f = \frac{(9.24 - 7.96)\times 10^{13}\ \text{Hz}}{22} = 0.058\,2 \times 10^{13}\ \text{Hz}$$

$$= 5.82 \times 10^{11}\ \text{Hz} = \frac{1}{h}\left(\frac{h^2}{4\pi^2 I}\right)$$

The moment of inertia is then

$$I = \frac{h}{4\pi^2 \Delta f} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{4\pi^2\left(5.82 \times 10^{11}\ \text{s}^{-1}\right)} = \boxed{2.88 \times 10^{-47}\ \text{kg}\cdot\text{m}^2}$$

P43.21 We carry extra digits through the solution because the given wavelengths are close together.

(a) The energy levels are given by

$$E_{vJ} = \left(v + \frac{1}{2}\right)hf + \frac{\hbar^2}{2I}J(J+1)$$

Therefore,

$$E_{00} = \frac{1}{2}hf,\ E_{11} = \frac{3}{2}hf + \frac{\hbar^2}{I},\ \text{and } E_{02} = \frac{1}{2}hf + \frac{3\hbar^2}{I}$$

Then,

$$\Delta E_1 = E_{11} - E_{00} = hf + \frac{\hbar^2}{I} = \frac{hc}{\lambda_1}$$

$$= \frac{\left(6.626\,075 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.997\,925 \times 10^{8}\ \text{m/s}\right)}{2.211\,2 \times 10^{-6}\ \text{m}}$$

$$\Delta E_1 = hf + \frac{\hbar^2}{I} = 8.983\,573 \times 10^{-20}\ \text{J} \qquad [1]$$

and

$$\Delta E_2 = E_{11} - E_{02} = hf - \frac{2\hbar^2}{I} = \frac{hc}{\lambda_2}$$

$$= \frac{\left(6.626\,075 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.997\,925 \times 10^{8}\ \text{m/s}\right)}{2.405\,4 \times 10^{-6}\ \text{m}}$$

$$\Delta E_2 = hf - \frac{2\hbar^2}{I} = 8.258\,284 \times 10^{-20}\ \text{J} \qquad [2]$$

Subtracting equation [2] from [1] gives,

$$\Delta E_1 - \Delta E_2 = \left(hf + \frac{\hbar^2}{I}\right) - \left(hf - \frac{2\hbar^2}{I}\right) = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

solving,

$$\frac{3\hbar^2}{I} = hc\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right) \quad \to \quad \frac{3h}{4\pi^2 I} = c\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

Then,

$$\begin{aligned}\frac{3h}{4\pi^2 I} &= c\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)\\ &= \left(2.997\,925\times 10^{8}\ \text{m/s}\right)\\ &\qquad \times\left(\frac{1}{2.211\,2\times 10^{-6}\ \text{m}} - \frac{1}{2.405\,4\times 10^{-6}\ \text{m}}\right)\\ &= 1.094\,6\times 10^{13}\ \text{s}^{-1}\end{aligned}$$

Solving for the moment of inertia then gives

$$I = \frac{3\left(6.626\,075\times 10^{-34}\ \text{J}\cdot\text{s}\right)}{4\pi^2\left(1.094\,6\times 10^{13}\ \text{s}^{-1}\right)} = \boxed{4.60\times 10^{-48}\ \text{kg}\cdot\text{m}^2}$$

(b) From equation [1]:

$$\begin{aligned}f_1 &= \frac{\Delta E_1}{h} - \frac{\hbar^2}{2\pi I}\\ &= \frac{8.983\,573\times 10^{-20}\ \text{J}}{6.626\,075\times 10^{-34}\ \text{J}\cdot\text{s}} - \frac{\left(6.626\,075\times 10^{-34}\ \text{J}\cdot\text{s}\right)}{2\pi\left(4.600\,060\times 10^{-48}\ \text{kg}\cdot\text{m}^2\right)}\\ &= \boxed{1.32\times 10^{14}\ \text{Hz}}\end{aligned}$$

(c) The moment of inertia of the molecule is given by $I = \mu r^2$, where μ is the reduced mass,

$$\mu = \frac{1}{2}m_H = \frac{1}{2}\left(1.007\,825\text{u}\right) = 8.367\,669\times 10^{-28}\ \text{kg}$$

The equilibrium separation distance is then,

$$r = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{4.600\,060\times 10^{-48}\ \text{kg}\cdot\text{m}^2}{8.367\,669\times 10^{-28}\ \text{kg}}} = \boxed{0.074\,1\ \text{nm}}$$

P43.22 The emission energies are the same as the absorption energies, but the final state must be below ($v = 1$, $J = 0$). The transition must satisfy $\Delta J = \pm 1$, so it must end with $J = 1$. To be lower in energy, the state must be ($v = 0$, $J = 1$). The emitted photon energy is therefore

$$\begin{aligned} hf_{\text{photon}} &= \left(E_{\text{vib}}\big|_{v=1} + E_{\text{rot}}\big|_{J=0}\right) - \left(E_{\text{vib}}\big|_{v=0} + E_{\text{rot}}\big|_{J=1}\right) \\ &= \left(E_{\text{vib}}\big|_{v=1} - E_{\text{vib}}\big|_{v=0}\right) - \left(E_{\text{rot}}\big|_{J=1} - E_{\text{rot}}\big|_{J=0}\right) \\ hf_{\text{photon}} &= hf_{\text{vib}} - hf_{\text{rot}} \end{aligned}$$

Thus, $$\begin{aligned} f_{\text{photon}} &= f_{\text{vib}} - f_{\text{rot}} = 6.42\times10^{13}\ \text{Hz} - 1.15\times10^{11}\ \text{Hz} \\ &= 6.41\times10^{13}\ \text{Hz} = \boxed{64.1\ \text{THz}} \end{aligned}$$

P43.23 The moment of inertia about the molecular axis is

$$I_y = \frac{2}{5}mr^2 + \frac{2}{5}mr^2 = \frac{4}{5}m\left(2.00\times10^{-15}\ \text{m}\right)^2$$

The moment of inertia about a perpendicular axis is

$$I_x = m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2 = \frac{m}{2}\left(2.00\times10^{-10}\ \text{m}\right)^2$$

The allowed rotational energies are $E_{\text{rot}} = \left(\dfrac{\hbar^2}{2I}\right)J(J+1)$, so the energy of the first excited state is $E_1 = \dfrac{\hbar^2}{I}$. The ratio is therefore

$$\begin{aligned} \frac{E_{1,\,y}}{E_{1,\,x}} &= \frac{\left(\hbar^2/I_y\right)}{\left(\hbar^2/I_x\right)} = \frac{I_x}{I_y} = \frac{(1/2)m\left(2.00\times10^{-10}\ \text{m}\right)^2}{(4/5)m\left(2.00\times10^{-15}\ \text{m}\right)^2} \\ &= \frac{5}{8}\left(10^5\right)^2 = \boxed{6.25\times10^9} \end{aligned}$$

Section 43.3 Bonding in Solids

P43.24 (a) Consider a cubical salt crystal of edge length 0.1 mm.

The number of atoms is $\left(\dfrac{10^{-4}\ \text{m}}{0.261\times10^{-9}\ \text{m}}\right)^3 = 6\times10^{16}\ \boxed{\sim 10^{17}}$.

(b) This number of salt crystals would have volume

$$\left(10^{-4}\ \text{m}\right)^3 6\times10^{16} = 6\times10^4\ \boxed{\sim 10^5\ \text{m}^3}$$

If it is cubic, it has edge length 40 m.

P43.25 The ionic cohesive energy is

$$U = -\frac{\alpha k_e e^2}{r_0}\left(1 - \frac{1}{m}\right)$$

$$= -(1.747\,6)(8.99\times10^9)\frac{(1.60\times10^{-19})^2}{(0.281\times10^{-9})}\left(1 - \frac{1}{8}\right)$$

$$= -1.25\times10^{-18}\ \text{J} = \boxed{-7.83\ \text{eV}}$$

P43.26 We assume the ions are all singly ionized. The total potential energy is obtained by summing over all interactions of our ion with others:

$$U = \sum_{i\neq j} k_e \frac{q_i q_j}{r_{ij}}$$

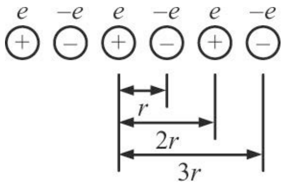
$$= -k_e\left[\frac{e^2}{r} + \frac{e^2}{r} - \frac{e^2}{2r} - \frac{e^2}{2r} + \frac{e^2}{3r} + \frac{e^2}{3r} - \frac{e^2}{4r} - \frac{e^2}{4r} + \cdots\right]$$

$$U = -2k_e\frac{e^2}{r}\left[1 - \tfrac{1}{2} + \tfrac{1}{3} - \tfrac{1}{4} + \cdots\right]$$

But from Appendix B.5,

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

Our series follows this pattern with $x = 1$, so the potential energy of one ion due to its interactions with all the others is



ANS. FIG. P43.26

$$U = (-2\ln 2)k_e\frac{e^2}{r} = \boxed{-k_e\alpha\frac{e^2}{r}\ \text{where } \alpha = 2\ln 2}$$

Section 43.4 Free-Electron Theory of Metals

Section 43.5 Band Theory of Solids

P43.27 Taking $E_F = 5.48$ eV for sodium at 800 K,

$$f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} \qquad \text{so} \qquad e^{(E-E_F)/k_BT} = \frac{1}{f(E)} - 1$$

Then, $$\frac{E - E_F}{k_BT} = \ln\left(\frac{1}{f(E)} - 1\right)$$

and $$E = E_F + k_B T \ln\left(\frac{1}{f(E)} - 1\right)$$

substituting numerical values,

$$E = 5.48\text{ eV} + \left(1.38\times10^{-23}\text{ J/K}\right)$$
$$\times\left(1.602\times10^{-19}\text{ eV/J}\right)(800\text{ K})\ln\left(\frac{1}{0.950}-1\right) = \boxed{5.28\text{ eV}}$$

P43.28 (a) $\boxed{\text{The Fermi energy is proportional to the spatial concentration of free electrons to the two-thirds power.}}$

(b) From Equation 43.25,

$$E_F = \frac{h^2}{2m}\left(\frac{3n_e}{8\pi}\right)^{2/3}$$
$$= \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(9.11\times10^{-31}\text{ kg}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}\left(\frac{3}{8\pi}\right)^{2/3} n_e^{2/3}$$

becomes

$$E_F = \left(3.65\times10^{-19}\right)n_e^{2/3}$$

where E_F is in electron volts and n_e in electrons per cubic meter.

(c) Copper has the greater concentration of free electrons by a factor of

$$\frac{n_e(\text{Cu})}{n_e(\text{K})} = \frac{8.46\times10^{-19}\text{ m}^{-3}}{1.40\times10^{-19}\text{ m}^{-3}} = \boxed{6.04}$$

(d) $\boxed{\text{Copper}}$ has the greater Fermi energy, 7.05 eV.

(e) The Fermi energy is larger by a factor of 7.05 eV/2.12 eV = $\boxed{0.333}$.

(f) This behavior agrees with the proportionality because $E_F \sim n_e^{2/3}$ and $6.04^{2/3} = 3.32$.

P43.29 The melting point of silver is 1 234 K. Its Fermi energy at 300 K is 5.48 eV. The approximate fraction of electrons excited is

$$\frac{k_B T}{E_F} = \frac{\left(1.38\times10^{-23}\text{ J/K}\right)\left(1\,234\text{ K}\right)}{\left(5.48\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)} \approx \boxed{2\%}$$

P43.30 (a) Setting the kinetic energy equal to the Fermi energy,

$$\frac{1}{2}mv^2 = 7.05\text{ eV}$$

we solve for the speed of the conduction electron as

$$v = \sqrt{\frac{2(7.05\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}{9.11\times10^{-31}\ \text{kg}}}$$

$$= 1.57\times10^{6}\ \text{m/s} = \boxed{1.57\ \text{Mm/s}}$$

(b) Compared to the drift velocity of 0.1 mm/s = 10^{-4} mm/s, the $\boxed{\text{speed is larger by ten orders of magnitude.}}$ The energy of an electron at room temperature is typically $k_BT = \frac{1}{40}$ eV.

P43.31 (a) From Equation 43.26,

$$E_{\text{avg}} = \frac{3}{5}E_F = 0.6(7.05\ \text{eV}) = \boxed{4.23\ \text{eV}}$$

(b) The average energy of a molecule in an ideal gas is $\frac{3}{2}k_BT$ so we have

$$T = \frac{2}{3}\frac{4.23\ \text{eV}}{1.38\times10^{-23}\ \text{J/K}}\frac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}} = \boxed{3.27\times10^{4}\ \text{K}}$$

P43.32 For edge *d* = 1.00 mm,

$$V = d^3 = \left(1.00\times10^{-3}\ \text{m}\right)^3 = 1.00\times10^{-9}\ \text{m}^3$$

The density of states is

$$g(E) = CE^{1/2} = \frac{8\sqrt{2}\pi m_e^{3/2}}{h^3}E^{1/2}$$

or

$$g(E) = \frac{8\sqrt{2}\pi\left(9.11\times10^{-31}\ \text{kg}\right)^{3/2}}{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3}\sqrt{(4.00\ \text{eV})\left(1.60\times10^{-19}\ \text{J/eV}\right)}$$

$$g(E) = 8.50\times10^{46}\ \text{m}^{-3}\cdot\text{J}^{-1} = 1.36\times10^{28}\ \text{m}^{-3}\cdot\text{eV}^{-1}$$

So, the total number of electrons is

$$N = [g(E)](\Delta E)V$$

$$= \left(1.36\times10^{28}\ \text{m}^{-3}\cdot\text{eV}^{-1}\right)(0.025\,0\ \text{eV})\left(1.00\times10^{-9}\ \text{m}^3\right)$$

$$= \boxed{3.40\times10^{17}\ \text{electrons}}$$

P43.33 For sodium, $M = 23.0\text{ g/mol}$ and $\rho = 0.971\text{ g/cm}^3$. Sodium contributes one electron per atom to the conduction band.

(a) The density of conduction electrons is

$$n_e = \frac{N_A\rho}{M} = \frac{(6.02\times10^{23}\ \text{electrons/mol})(0.971\text{ g/cm}^3)}{23.0\ \text{g/mol}}$$

$$n_e = 2.54\times10^{22}\ \text{electrons/cm}^3 = \boxed{2.54\times10^{28}\ \text{m}^{-3}}$$

(b) From Equation 43.25,

$$E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3n_e}{8\pi}\right)^{2/3}$$

$$= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})}\left[\frac{3(2.54\times10^{28}\ \text{m}^{-3})}{8\pi}\right]^{2/3}$$

$$= 5.05\times10^{-19}\ \text{J} = \boxed{3.15\text{ eV}}$$

P43.34 From Equation 43.24, the number density of free electrons is

$$n_e = \tfrac{2}{3}\frac{8\sqrt{2}\pi m_e^{3/2}}{h^3}E_F^{3/2}$$

$$= \tfrac{2}{3}\frac{8\sqrt{2}\pi(9.11\times10^{-31}\text{kg})^{3/2}}{(6.626\times10^{-34}\text{J}\cdot\text{s})^3}(5.48\text{eV})^{3/2}\left(\frac{1.602\times10^{-19}\text{J}}{1\text{ eV}}\right)^{3/2}$$

$$= 5.83\times10^{28}\text{m}^{-3}$$

Then, the number density of atoms in the metal is

$$n_{\text{atoms}} = \frac{nN_A}{V} = \frac{mN_A}{MV} = \frac{\rho N_A}{M}$$

$$= \frac{(4.90\times10^3\ \text{kg/m}^3)(6.02\times10^{23}\ \text{mol}^{-1})}{0.100\ \text{kg/mol}}$$

$$= 2.95\times10^{28}\ \text{m}^{-3}$$

Then the number of free electrons per atom is

$$\frac{n_e}{n_{\text{atoms}}} = \frac{5.83\times10^{28}\ \text{m}^{-3}}{2.95\times10^{28}\ \text{m}^{-3}} = 1.97$$

Therefore, there are approximately two free electrons per atom for this metal, not one.

P43.35 From Table 43.2, the Fermi energy for copper at 300 K is 7.05 eV. From Equation 43.19, the Fermi-Dirac distribution function, the occupation probability is

$$f(E)=\frac{1}{e^{(E-E_F)/k_BT}+1}=\frac{1}{e^{(0.99E_F-E_F)/k_BT}+1}$$

$$=\frac{1}{e^{\left[-0.01(7.05\text{ eV})(1.602\times10^{-19}\text{ J/eV})\right]/\left[(1.381\times10^{-23}\text{ J/K})(300\text{ K})\right]}+1}$$

$$=\frac{1}{e^{-2.72}+1}=\boxed{0.939}$$

P43.36 From Equation 43.19, the Fermi-Dirac distribution function, the occupation probability is

$$f(E)=\frac{1}{e^{(E-E_F)/k_BT}+1}=\frac{1}{e^{(\beta E_F-E_F)/k_BT}+1}=\boxed{\frac{1}{e^{(\beta-1)E_F/k_BT}+1}}$$

P43.37 Consider first the wave function in *x*. At $x = 0$ and $x = L$, $\psi = 0$. Therefore,

$\sin k_xL = 0$ and $k_xL = \pi, 2\pi, 3\pi, \ldots$

Similarly, $\sin k_yL = 0$ and $k_yL = \pi, 2\pi, 3\pi, \ldots$

and $\sin k_zL = 0$ and $k_zL = \pi, 2\pi, 3\pi, \ldots$

Then,

$$\psi = A\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)\sin\left(\frac{n_z\pi z}{L}\right)$$

From Schrödinger's Equation, $\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}=\frac{2m_e}{\hbar^2}(U-E)\psi$, we have inside the box, where $U = 0$,

$$\left(-\frac{n_x^2\pi^2}{L^2}-\frac{n_y^2\pi^2}{L^2}-\frac{n_z^2\pi^2}{L^2}\right)\psi=\frac{2m_e}{\hbar^2}(-E)\psi$$

Therefore,

$$\boxed{E=\frac{\hbar^2\pi^2}{2m_eL^2}\left(n_x^2+n_y^2+n_z^2\right)\qquad n_x,\ n_y,\ n_z=1,\ 2,\ 3,\ \ldots}$$

Outside the box we require $\psi = 0$. The minimum energy state inside the box is $n_x = n_y = n_z = 1$, with $E=\frac{3\hbar^2\pi^2}{2m_eL^2}$.

P43.38 The density of states at the energy *E* is $g(E) = CE^{1/2}$.

(a) Hence, the required ratio is

$$R_{\text{states}} = \frac{g(8.50\text{ eV})}{g(7.05\text{ eV})} = \frac{C(8.50)^{1/2}}{C(7.05)^{1/2}} = \boxed{1.10}$$

(b) From Equation 43.22, we see that the number of occupied states between energy *E* and energy *E* + *dE* is

$$N(E)dE = \frac{CE^{1/2}}{e^{(E-E_F)/k_BT}+1}dE$$

Hence, the required ratio is

$$R_{\text{occupied states}} = \frac{N(8.50\text{ eV})}{N(7.05\text{ eV})} = \sqrt{\frac{8.50}{7.05}}\left[\frac{e^{(7.05-7.05)/k_BT}+1}{e^{(8.50-7.05)/k_BT}+1}\right]$$

At *T* = 300 K, we compute

$$k_BT = \left(1.380\ 65\times10^{-23}\ \frac{\text{J}}{\text{K}}\right)(300.000\text{ K})\left(\frac{1\text{ eV}}{1.602\ 18\times10^{-19}\text{J}}\right)$$

$$= 0.025\ 852\ 0\text{ eV}$$

so $$R_{\text{occ.st}} = \left(\frac{8.50}{7.05}\right)^{1/2}\left(\frac{2}{e^{1.45/0.025\ 852\ 0}+1}\right) = \boxed{9.61\times10^{-25}}$$

With an exponent of 56.1, the derivative of the exponential function is so large that none of the digits in 9.61 is really significant. Different-looking answers would result from different choices of how precisely to represent the input data.

(c) The answer to part (b) is vastly smaller than the answer to (a). Very few states well above the Fermi energy are occupied at room temperature.

P43.39 We are to compute

$$E_{\text{avg}} = \frac{1}{n_e}\int_0^\infty EN(E)\ dE$$

where from Equation 43.22,

$$N(E) = \frac{CE^{1/2}}{e^{(E-E_F)/k_BT}+1} = Cf(E)E^{1/2}$$

with $$C = \frac{8\sqrt{2}\pi m_e^{3/2}}{h^3}$$

But at *T* = 0 the Fermi-Dirac distribution function is *f*(*E*) = 0 for $E > E_F$, and *f*(*E*) = 1 for $E < E_F$. So we can take $N(E) = CE^{1/2}$ just for energies up to the Fermi energy. The average we want is then

$$E_{avg} = \frac{1}{n_e}\int_0^{E_F} CE^{3/2}dE = \frac{2C}{5n_e}E_F^{5/2}$$

But from Equation 43.24, $\frac{C}{n_e} = \frac{3}{2}E_F^{-3/2}$, so

$$E_{avg} = \left(\frac{2}{5}\right)\left(\frac{3}{2}\right)\left(E_F^{-3/2}\right)E_F^{5/2} = \boxed{\frac{3}{5}E_F}$$

Section 43.6 Electrical Conduction in Metals, Insulators, and Semiconductors

P43.40 (a) If $\lambda \le 1.00\ \mu\text{m} = 1.00\times 10^3\ \text{nm}$, then photons of sunlight have energy

$$E \ge \frac{hc}{\lambda_{max}} = \frac{1240\ \text{eV}\cdot\text{nm}}{1.00\times 10^3\ \text{m}} = 1.24\ \text{eV}$$

$\boxed{\text{The gap should be less than or equal to 1.24 eV.}}$

(b) $\boxed{\text{Because silicon has an energy gap of 1.14 eV, it can absorb the energy of nearly all of the photons in sunlight and is an appropriate material for a solar energy collector.}}$

P43.41 (a) $E_g = 1.14$ eV for Si. The photon energy, given by $E = hf$, must be at least this energy. Then,

$$f = \frac{E}{h} = \frac{(1.14\ \text{eV})(1.602\times 10^{-19}\ \text{J/eV})}{6.626\times 10^{-34}\ \text{J}\cdot\text{s}}$$

$$= 2.76\times 10^{14}\ \text{Hz} = 276\times 10^{12}\ \text{Hz} = \boxed{276\ \text{THz}}$$

(b) From $c = \lambda f$,

$$\lambda = \frac{c}{f} = \frac{3.00\times 10^8\ \text{m/s}}{2.75\times 10^{14}\ \text{Hz}} = 1.09\times 10^{-6}\ \text{m} = \boxed{1.09\ \mu\text{m}}\ \text{(infrared)}$$

P43.42 (a) From Table 43.3, the energy gap for CdS is 2.42 eV, so photons of energy greater than 2.42 eV will be absorbed, corresponding to wavelengths shorter than

$$\lambda = \frac{hc}{E} = \frac{1240\ \text{eV}\cdot\text{nm}}{2.42\ \text{eV}} = 512\ \text{nm}$$

All the Balmer lines lie between the shortest (series limit) produced by the transition $n = \infty \to 2$, with energy

$$\Delta E = 13.6 \text{ eV}\left(\frac{1}{2^2}\right) = 3.40 \text{ eV}$$

is $$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV}\cdot\text{nm}}{3.40 \text{ eV}} = 365 \text{ nm}$$

The longest produced by the transition *n* = 3 → 2, with energy

$$\Delta E = 13.6 \text{ eV}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.89 \text{ eV}$$

is $$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV}\cdot\text{nm}}{1.89 \text{ eV}} = 656 \text{ nm}$$

All the hydrogen Balmer lines except for the red line at 656 nm will be absorbed.

(b) The red line at 656 nm will be transmitted.

P43.43 The energy-bang gap is

$$E_g = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{650 \text{ nm}} \approx \boxed{1.91 \text{ eV}}$$

P43.44 The wavelength 0.512 μm = 512 nm. The corresponding photon energy is just sufficient to promote an electron across the gap.

$$E_g = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{512 \text{ nm}} = \boxed{2.42 \text{ eV}}$$

P43.45 If the photon energy is 5.47 eV or higher, the diamond window will absorb the photons. Here,

$$(hf)_{\max} = \frac{hc}{\lambda_{\min}} = 5.47 \text{ eV}$$

which gives

$$\lambda_{\min} = \frac{hc}{5.47 \text{ eV}} = \frac{1240 \text{ eV}\cdot\text{nm}}{5.47 \text{ eV}} = \boxed{227 \text{ nm}}$$

P43.46 (a) In the Bohr model we replace k_e by k_e/κ and m_e by m^*. Then the radius of the first Bohr orbit, $a_0 = \frac{\hbar^2}{m_e k_e e^2}$ in hydrogen, changes to

$$a' = \frac{\hbar^2}{m^*(k_e/\kappa)e^2} = \frac{\hbar^2\kappa}{m^* k_e e^2} = \left(\frac{m_e}{m^*}\right)\kappa\frac{\hbar^2}{m_e k_e e^2} = \boxed{\left(\frac{m_e}{m^*}\right)\kappa\, a_0}$$

(b) Substituting numerical values,

$$a' = \left(\frac{m_e}{m^*}\right)\kappa\, a_0 = \left(\frac{m_e}{0.220 m_e}\right)(11.7)(0.052\,9 \text{ nm}) = \boxed{2.81 \text{ nm}}$$

(c) The energy levels for hydrogen are $E_n = -\frac{k_e e^2}{2a_0}\frac{1}{n^2}$. Making the replacements $k_e \to k_e/\kappa$ and $a_0 \to a'$, we have

$$E_n' = -\frac{(k_e/\kappa)e^2}{2\left[(m_e/m^*)\kappa a_0\right]}\frac{1}{n^2} = -\frac{k_e e^2}{2\kappa^2\left[(m_e/m^*)a_0\right]}\frac{1}{n^2}$$

$$= -\frac{1}{\kappa^2}\left(\frac{m^*}{m_e}\right)\left(\frac{k_e e^2}{2a_0}\frac{1}{n^2}\right) = \boxed{-\left(\frac{m^*}{m_e}\right)\frac{E_n}{\kappa^2}}$$

(d) For *n* = 1,

$$E_1' = -0.220\left(\frac{13.6 \text{ eV}}{11.7^2}\right) = \boxed{-0.0219 \text{ eV}}$$

Section 43.7 Semiconductor Devices

P43.47 Equation 43.27 is

$$I = I_0\left(e^{e\Delta V/k_BT} - 1\right)$$

Thus, $$e^{e(\Delta V)/k_BT} = 1 + \frac{I}{I_0}$$

and $$\Delta V = \frac{k_BT}{e}\ln\left(1 + \frac{I}{I_0}\right)$$

At *T* = 300 K,

$$\Delta V = \frac{(1.38\times 10^{-23} \text{ J/K})(300 \text{ K})}{1.60\times 10^{-19} \text{ C}}\ln\left(1 + \frac{I}{I_0}\right)$$

$$= (25.9 \text{ mV})\ln\left(1 + \frac{I}{I_0}\right)$$

(a) If $I = 9.00I_0$,

$$\Delta V = (25.9 \text{ mV})\ln(10.0) = \boxed{59.5 \text{ mV}}$$

(b) If $I = -0.900I_0$,

$$\Delta V = (25.9 \text{ mV})\ln(0.100) = \boxed{-59.5 \text{ mV}}$$

The basic idea behind a semiconductor device is that a large current or charge can be controlled by a small control voltage.

P43.48 (a) The current in the diode, and thus in all elements of the series circuit, is $I = I_0\left(e^{e\Delta V/k_BT} - 1\right)$. Appling Kirchhoff's loop rule in the direction of the current, going through the negative to the positive side of the battery, then through the diode, and then the resistor, we get

$$\mathcal{E} - \Delta V - IR = 0$$

$$\mathcal{E} - \Delta V - I_0R\left(e^{e\Delta V/k_BT} - 1\right) = 0$$

or $$\mathcal{E} - \Delta V = I_0R\left(e^{e\Delta V/k_BT} - 1\right)$$

(b) The graphs to be plotted are the voltage across the resistor,

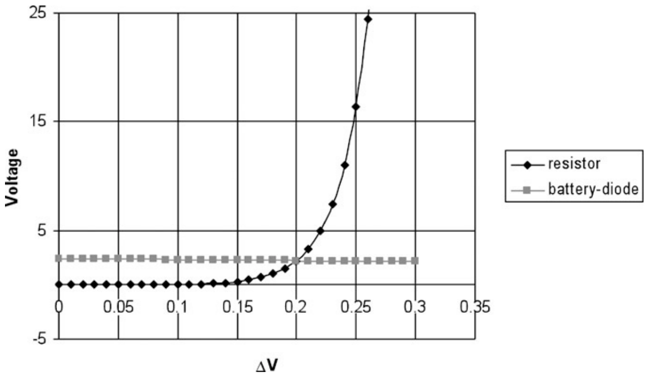
$$\Delta V_{\text{resistor}} = IR = \left(1.00\times10^{-6}\ \text{A}\right)(745\ \Omega)\left(e^{\Delta V/0.0250\ \text{V}} - 1\right)$$

$$= \left(7.45\times10^{-4}\ \text{A}\right)\left(e^{\Delta V/0.025\ \text{V}} - 1\right)$$

and the voltage across the battery and diode combined,

$$\Delta V_{BD} = \mathcal{E} - \Delta V = 2.42\ \text{V} - \Delta V$$

The graphs are plotted in ANS. FIG. P43.48 below.



ANS. FIG. P43.48

(c) The two graphs intersect at $\Delta V = 0.200$ V. The current is then

$$I = \left(1.00\times10^{-6}\ \text{A}\right)\left(e^{0.200\ \text{V}/0.0250\ \text{V}} - 1\right) = 2.98\times10^{-3}\ \text{A} = \boxed{2.98\ \text{mA}}$$

(d) The ohmic resistance of the diode is

$$R_{\text{ohmic}} = \frac{\Delta V}{I} = \frac{0.200\ \text{V}}{2.98\times10^{-3}\ \text{A}} = \boxed{67.1\ \Omega}$$

(e) The dynamic resistance of the diode is

$$R_{\text{dynamic}} = d(\Delta V)/dI = \left[dI/d(\Delta V)\right]^{-1}$$

where $I = I_0\left(e^{e\Delta V/k_BT} - 1\right)$. Then,

$$\frac{dI}{d(\Delta V)} = \frac{d}{d(\Delta V)}\left[I_0\left(e^{e\Delta V/k_BT} - 1\right)\right] = \frac{eI_0}{k_BT}e^{e\Delta V/k_BT}$$

Therefore,

$$R_{\text{dynamic}} = \frac{d(\Delta V)}{dI} = \left[\frac{dI}{d(\Delta V)}\right]^{-1} = \left[\frac{eI_0}{k_BT}e^{e\Delta V/k_BT}\right]^{-1}$$

$$= \left[\frac{1.00\times 10^{-6}\text{ A}}{0.0250\text{ V}}e^{0.200\text{ V}/0.0250\text{ V}}\right]^{-1} = \boxed{8.39\ \Omega}$$

P43.49 First, we evaluate I_0 in $I = I_0\left(e^{e\Delta V/k_BT} - 1\right)$, given that I = 200 mA when ΔV = 100 mV and T = 300 K.

$$\frac{e\Delta V}{k_BT} = \frac{(1.60\times 10^{-19}\text{ C})(0.100\text{ V})}{(1.38\times 10^{-23}\text{ J/K})(300\text{ K})} = 3.86$$

so $$I_0 = \frac{I}{e^{e(\Delta V)/k_BT} - 1} = \frac{200\text{ mA}}{e^{3.86} - 1} = 4.28\text{ mA}$$

If $V = -100$ mV, $\frac{e(\Delta V)}{k_BT} = -3.86$; and the current will be

$$I = I_0\left(e^{e\Delta V/k_BT} - 1\right) = (4.28\text{ mA})\left(e^{-3.86} - 1\right) = \boxed{-4.19\text{ mA}}$$

P43.50 From Equation 43.27, the current in the diode is a function of ΔV is

$$I(\Delta V) = I_0\left(e^{e\Delta V/k_BT} - 1\right)$$

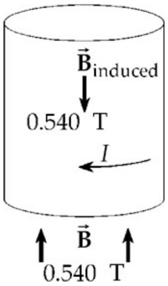
where k_BT = 0.025 0 eV. Therefore,

$$\frac{I(+\Delta V)}{I(-\Delta V)} = \frac{I_0\left(e^{e\Delta V/k_BT} - 1\right)}{I_0\left(e^{e(-\Delta V)/k_BT} - 1\right)} = \frac{e^{e\Delta V/k_BT} - 1}{e^{e(-\Delta V)/k_BT} - 1}$$

$$\frac{I(+1.00\text{ V})}{I(-1.00\text{ V})} = \frac{e^{1.00/0.0250} - 1}{e^{-1.00/0.0250} - 1} = \frac{e^{40} - 1}{e^{-40} - 1} = \boxed{-2.35\times 10^{17}}$$

Section 43.8 Superconductivity

P43.51 (a) See ANS. FIG. P43.51.



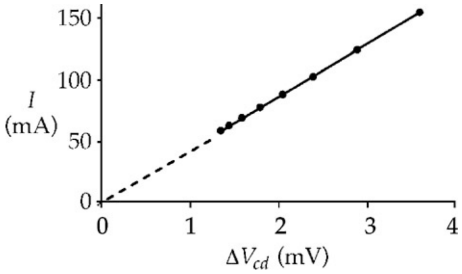
ANS. FIG. P43.51

(b) Treat the rod as a solenoid. For a surface current around the outside of the cylinder as shown, $B = \frac{N\mu_0 I}{\ell}$, or

$$NI = \frac{B\ell}{\mu_0} = \frac{(0.540\text{ T})(2.50\times10^{-2}\text{ m})}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = \boxed{10.7\text{ kA}}$$

P43.52 (a) In the definition of resistance, $\Delta V = IR$; if R is zero, then $\Delta V = 0$ for any value of the current.

(b) See ANS. FIG. P43.52. The graph is linear.



ANS. FIG. P43.52

(c) The graph shows a direct proportionality with resistance given by the reciprocal of the slope:

$$\text{slope} = \frac{\Delta I}{\Delta V} = \frac{1}{R} = \frac{(155-57.8)\text{ mA}}{(3.61-1.356)\text{ mV}} = 43.1\ \Omega^{-1}$$

so, $R = \boxed{0.023\,2\ \Omega}$

(d) The expulsion of magnetic flux, and therefore fewer current-carrying paths through the superconductor, could explain the decrease in current.

P43.53 By Faraday's law: $\frac{\Delta\Phi_B}{\Delta t} = L\frac{\Delta I}{\Delta t} = A\frac{\Delta B}{\Delta t}$, thus

$$\Delta I = \frac{A(\Delta B)}{L} = \frac{\left[\pi(0.010\,0\text{ m})^2\right](0.020\,0\text{ T})}{3.10\times10^{-8}\text{ H}} = 203\text{ A}$$

The current generated in the ring is $\boxed{\text{203 A to produce a magnetic field in the direction of the original field}}$ through the ring.

Additional Problems

43.54 For the N_2 molecule, $k = 2\,297$ N / m, $m = 2.32 \times 10^{-26}$ kg, and $r = 1.20 \times 10^{-10}$ m. The reduced mass is, from Equation 43.4,

$$\mu = \frac{mm}{m+m} = \frac{m}{2}$$

The frequency of vibration for the molecule is, from Equation 43.8,

$$\omega = \sqrt{\frac{k}{\mu}} = 4.45 \times 10^{14} \text{ rad/s}$$

and the moment of inertia is, from Equation 43.3,

$$\begin{aligned} I = \mu r^2 &= \left(1.16 \times 10^{-26} \text{ kg}\right)\left(1.20 \times 10^{-10} \text{ m}\right)^2 \\ &= 1.67 \times 10^{-46} \text{ kg} \cdot \text{m}^2 \end{aligned}$$

The allowed vibrational energies are, from Equation 43.9,

$$E_{vib} = \left(v + \frac{1}{2}\right)\hbar\omega, \text{ where } v = 1, 2, 3\ldots$$

The first excited vibrational state is above the vibrational ground state by the energy difference $\Delta E = \hbar\omega$. For the rotational state that is above the rotational ground state by the same energy difference, we require

$$\frac{\hbar^2}{2I} J(J+1) = \hbar\omega$$

or

$$\begin{aligned} J(J+1) = \frac{2I\omega}{\hbar} &= \frac{2\left(1.67 \times 10^{-46} \text{ kg} \cdot \text{m}^2\right)\left(4.45 \times 10^{14} \text{ rad/s}\right)}{1.055 \times 10^{-34} \text{ J} \cdot \text{s}} \\ &= 1\,410. \end{aligned}$$

Thus, by inspection, $\boxed{J = 37}$.

P43.55 From Equation 43.9, the allowed vibrational energies are

$$E_{vib} = \left(v + \frac{1}{2}\right)\hbar\omega, \text{ where } v = 1, 2, 3\ldots$$

For the vibrational energy level that is just below the dissociation energy, we require

$$E_{vib} = \left(v + \frac{1}{2}\right)\hbar\omega \le E_{max} = 4.48 \text{ eV}$$

or, $$v \le \frac{E_{\max}}{\hbar\omega} - \frac{1}{2} = \frac{E_{\max}}{\hbar\omega} - \frac{1}{2}$$

$$v \le \frac{(4.48\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{\left[6.626\times10^{-34}\text{ J}\cdot\text{s}/2\pi\right](8.28\times10^{14}\text{ rad/s})} - \frac{1}{2} = 7.7$$

Therefore, because *v* is an integer, $\boxed{v = 7}$.

P43.56 (a) The total potential energy $U_{\text{total}} = -\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}$, given by Equation 43.17, has its minimum value at the equilibrium spacing, $r = r_0$. At this point, $F = -\left.\frac{dU}{dr}\right|_{r=r_0} = 0$:

$$F = -\frac{d}{dr}\left(-\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}\right)\Bigg|_{r=r_0} = 0$$

$$= -\alpha\frac{k_e e^2}{r_0^2} + \frac{mB}{r_0^{m+1}} = 0$$

which gives

$$B = \alpha\frac{k_e e^2}{m} r_0^{m-1}$$

Substituting this value of *B* into *F*, we have

$$F = -\alpha\frac{k_e e^2}{r^2} + \frac{m}{r^{m+1}}\left(\alpha\frac{k_e e^2}{m} r_0^{m-1}\right) = -\alpha\frac{k_e e^2}{r^2}\left[1 - \left(\frac{r_0}{r}\right)^{m-1}\right]$$

(b) Let $r = r_0 + x$, so $r_0 = r - x$. Then assuming *x* is small we have,

$$F = -\alpha\frac{k_e e^2}{r^2}\left[1 - \left(\frac{r-x}{r}\right)^{m-1}\right] = -\alpha\frac{k_e e^2}{r^2}\left[1 - \left(1 - \frac{x}{r}\right)^{m-1}\right]$$

$$\approx -\alpha\frac{k_e e^2}{r^2}\left[1 - 1 + (m-1)\frac{x}{r}\right] \approx -\alpha\frac{k_e e^2}{r_0^3}(m-1)x$$

This is of the form of Hooke's law with spring constant

$$K = \frac{k_e\alpha e^2}{r_0^3}(m-1).$$

(c) Figure 38.22 (in Section 38.5 on electron diffraction) gives the distance from sodium ion to sodium ion as 0.562 737 nm. Therefore, the interatomic spacing in NaCl is

$$r_0 = (0.562\ 737\text{ nm})/2 = 0.281\ 369 \times 10^{-9}\text{ m}$$

Other problems in this chapter give the same information, or we could calculate it from the statement in Section 43.3 that the ionic cohesive energy for this crystal is –7.84 eV. Using Equation 43.17,

$$U_0 = -\alpha \frac{k_e e^2}{r_0}\left(1 - \frac{1}{m}\right) = -7.84 \text{ eV}$$

Solving for r_0,

$$\begin{aligned} r_0 &= -\alpha \frac{k_e e^2}{U_0}\left(1 - \frac{1}{m}\right) \\ &= -\alpha \frac{\left(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-19} \text{ C}\right)^2}{(-7.84 \text{ eV})\left(1.60\times 10^{-19} \text{ J/eV}\right)}\left(1 - \frac{1}{8}\right) \\ &= 2.81\times 10^{-10} \text{ m} \end{aligned}$$

The stiffness constant is then

$$\begin{aligned} K &= \alpha \frac{k_e e^2}{r_0^3}(m-1) \\ &= (1.747\,6)\frac{\left(8.99\times 10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)(1.60\times 10^{-19} \text{ C})^2(8-1)}{(2.81\times 10^{-10} \text{ m})^3} \\ &= 127 \text{ N/m} \end{aligned}$$

The vibration frequency of a sodium ion (m = 23.0 u) within the crystal is

$$\begin{aligned} f &= \frac{1}{2\pi}\sqrt{\frac{K}{m}} = \frac{1}{2\pi}\sqrt{\frac{127 \text{ N/m}}{23.0\left(1.66\times 10^{-27} \text{ kg}\right)}} \\ &= 9.18\times 10^{12} \text{ Hz} = \boxed{9.18 \text{ THz}} \end{aligned}$$

P43.57 Because the average energy required to break one van der Waals bond is 1.74×10^{-23} J, and because the bond is between two atoms of the same kind, the energy required to remove one helium atom from the bond is half the total:

$$1.74\times 10^{-23} \text{ J}/2 = 0.870\times 10^{-23} \text{ J}$$

Because each atom bonds with four other atoms, the energy required to remove one atom from all four bonds is

$$4\left(0.870\times 10^{-23} \text{ J}\right) = 3.48\times 10^{-23} \text{ J/atom}$$

The latent heat of fusion for helium (in joules per gram) is the total energy required to break the bonds of all the helium atoms in a mol, expressed as energy / unit mass:

$$L = \left(\frac{3.48\times 10^{-23} \text{ J}}{\text{atom}}\right)\left(\frac{6.02\times 10^{23} \text{ atoms}}{\text{mol}}\right)\left(\frac{1 \text{ mol}}{4.00 \text{ g}}\right) = \boxed{5.24 \text{ J/g}}$$

P43.58 We assume the potential well is that of a harmonic-oscillator. From Equation 43.9, the allowed energies of vibration of the molecule are

$$E_{\text{vib}} = \left(v + \frac{1}{2}\right)hf$$

To dissociate the atoms, enough energy must be supplied to raise their energy to the top of the potential well. The energy required to dissociate the atoms in the ground state ($v = 0$) is 4.48 eV; thus, the well depth is $\frac{1}{2}hf + 4.48\text{ eV}$. In the first excited vibrational state ($v = 1$), the dissociation energy is 3.96 eV; thus, the well depth is $\frac{3}{2}hf + 3.96\text{ eV}$. Then, the depth of the well is

$$\frac{1}{2}hf + 4.48\text{ eV} = \frac{3}{2}hf + 3.96\text{ eV}$$

from which we see that $hf = 0.52$ eV. Therefore, the depth of the well is

$$\frac{1}{2}hf + 4.48\text{ eV} = \frac{1}{2}(0.520\text{ eV}) + 4.48\text{ eV} = \boxed{4.74\text{ eV}}$$

P43.59 The total potential energy is given by Equation 43.17:

$$U_{\text{total}} = -\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}$$

The total potential energy has its minimum value U_0 at the equilibrium spacing, $r = r_0$. At this point, $\left.\frac{dU}{dr}\right|_{r=r_0} = 0$, or

$$\left.\frac{dU}{dr}\right|_{r=r_0} = \left.\frac{d}{dr}\left(-\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}\right)\right|_{r=r_0} = -\alpha\frac{k_e e^2}{r_0^2} + \frac{mB}{r_0^{m+1}} = 0$$

which gives

$$B = \alpha\frac{k_e e^2}{m}r_0^{m-1}$$

Substituting this value of B into U_{total}, we arrive at

$$U_0 = -\alpha\frac{k_e e^2}{r_0} + \alpha\frac{k_e e^2}{m}r_0^{m-1}\left(\frac{1}{r_0^m}\right) = -\alpha\frac{k_e e^2}{r_0}\left(1 - \frac{1}{m}\right)$$

P43.60 (a) The results of the spreadsheet are shown in two parts, in TABLE P43.60(a) and TABLE P43.60(b) for $f(E)=\dfrac{1}{e^{[(E/E_F-1)T_F]/T}+1}$. ANS. FIG. P43.60 shows the graphs of the tabulated values.

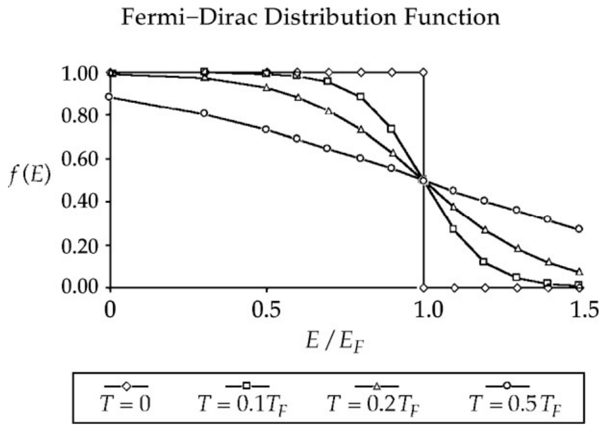
(b) The function is compared to the case $T = 0$. See the table and graphs below.

	$T = 0$		$T = 0.1T_F$	
$\dfrac{E}{E_F}$	$e^{[(E/E_F)-1](T_F/T)}$	$f(E)$	$e^{[(E/E_F)-1](T_F/T)}$	$f(E)$
0	$e^{-\infty}$	1.00	$e^{-10.0}$	1.000
0.500	$e^{-\infty}$	1.00	$e^{-5.00}$	0.993
0.600	$e^{-\infty}$	1.00	$e^{-4.00}$	0.982
0.700	$e^{-\infty}$	1.00	$e^{-3.00}$	0.953
0.800	$e^{-\infty}$	1.00	$e^{-2.00}$	0.881
0.900	$e^{-\infty}$	1.00	$e^{-1.00}$	0.731
1.00	e^{0}	0.500	e^{0}	0.500
1.10	$e^{+\infty}$	0.00	$e^{1.00}$	0.269
1.20	$e^{+\infty}$	0.00	$e^{2.00}$	0.119
1.30	$e^{+\infty}$	0.00	$e^{3.00}$	0.047 4
1.40	$e^{+\infty}$	0.00	$e^{4.00}$	0.018 0
1.50	$e^{+\infty}$	0.00	$e^{5.00}$	0.006 69

TABLE P43.60(a)

	$T = 0.2T_F$		$T = 0.5T_F$	
$\frac{E}{E_F}$	$e^{[(E/E_F)-1](T_F/T)}$	$f(E)$	$e^{[(E/E_F)-1](T_F/T)}$	$f(E)$
0	$e^{-5.00}$	0.993	$e^{-2.00}$	0.881
0.500	$e^{-2.50}$	0.924	$e^{-1.00}$	0.731
0.600	$e^{-2.00}$	0.881	$e^{-0.800}$	0.690
0.700	$e^{-1.50}$	0.818	$e^{-0.600}$	0.646
0.800	$e^{-1.00}$	0.731	$e^{-0.400}$	0.599
0.900	$e^{-0.500}$	0.622	$e^{-0.200}$	0.550
1.00	e^{0}	0.500	e^{0}	0.500
1.10	$e^{0.500}$	0.378	$e^{0.200}$	0.450
1.20	$e^{1.00}$	0.269	$e^{0.400}$	0.401
1.30	$e^{1.50}$	0.182	$e^{0.600}$	0.354
1.40	$e^{2.00}$	0.119	$e^{0.800}$	0.310
1.50	$e^{2.50}$	0.075 9	$e^{1.00}$	0.269

TABLE P43.60(b)



ANS. FIG. P43.60

P43.61 (a) For equilibrium, $\frac{dU}{dx} = 0$:

$$\frac{d}{dx}\left(Ax^{-3} - Bx^{-1}\right) = -3Ax^{-4} + Bx^{-2} = 0$$

$x \to \infty$ describes one equilibrium position, but the stable equilibrium position x_0 is at

$$-3Ax_0^{-4} + Bx_0^{-2} = 0$$

solving,

$$x_0^2 = \frac{3A}{B} \quad \to \quad x_0 = \sqrt{\frac{3A}{B}} = \sqrt{\frac{3\left(0.150 \text{ eV}\cdot\text{nm}^3\right)}{3.68 \text{ eV}\cdot\text{nm}}} = \boxed{0.350 \text{ nm}}$$

(b) The depth of the well is given by

$$U_0 = U\big|_{x=x_0} = \frac{A}{x_0^3} - \frac{B}{x_0} = \frac{AB^{3/2}}{3^{3/2}A^{3/2}} - \frac{BB^{1/2}}{3^{1/2}A^{1/2}}$$

$$U_0 = U\big|_{x=x_0} = -\frac{2B^{3/2}}{3^{3/2}A^{1/2}} = -\frac{2\left(3.68 \text{ eV}\cdot\text{nm}\right)^{3/2}}{3^{3/2}\left(0.150 \text{ eV}\cdot\text{nm}^3\right)^{1/2}} = \boxed{-7.02 \text{ eV}}$$

(c) The force on the particle is given by $F_x = -\frac{dU}{dx} = 3Ax^{-4} - Bx^{-2}$. To find the maximum force, we determine finite x_m such that $\left.\frac{dF}{dx}\right|_{x=x_m} = 0$.

$$\left.\frac{dF_x}{dx}\right|_{x=x_m} = \left[-12Ax^{-5} + 2Bx^{-3}\right]_{x=x_0} = 0$$

$$2Bx_m^{-3} = 12Ax_m^{-5}$$

$$x_m^2 = \frac{6A}{B} \quad \to \quad x_m = \sqrt{\frac{6A}{B}}$$

Then,

$$F_{\max} = 3A\left(\frac{B}{6A}\right)^2 - B\left(\frac{B}{6A}\right) = -\frac{B^2}{12A} = -\frac{\left(3.68 \text{ eV}\cdot\text{nm}\right)^2}{12\left(0.150 \text{ eV}\cdot\text{nm}^3\right)}$$

so $$F_{\max} = -7.52 \text{ eV/nm}\left(\frac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right)\left(\frac{1 \text{ nm}}{10^{-9} \text{ m}}\right) = -1.20\times10^{-9} \text{ N}$$

$$= -1.20 \text{ nN}$$

or, as a vector, $\boxed{-1.20\hat{\mathbf{i}} \text{ nN}}$.

P43.62 (a) For equilibrium, $\frac{dU}{dx} = 0$:

$$\frac{d}{dx}(Ax^{-3} - Bx^{-1}) = -3Ax^{-4} + Bx^{-2} = 0$$

$x \to \infty$ describes one equilibrium position, but the stable equilibrium position x_0 is at

$$-3Ax_0^{-4} + Bx_0^{-2} = 0$$

$$Bx_0^{-2} = 3Ax_0^{-4}$$

$$x_0^2 = \frac{3A}{B} \qquad \to \qquad \boxed{x_0 = \sqrt{\frac{3A}{B}}}$$

(b) The depth of the well is given by

$$U_0 = U\big|_{x=x_0} = \frac{A}{x_0^3} - \frac{B}{x_0} = \frac{AB^{3/2}}{3^{3/2}A^{3/2}} - \frac{BB^{1/2}}{3^{1/2}A^{1/2}} = \boxed{-2\sqrt{\frac{B^3}{27A}}}$$

(c) The force on the particle is given by $F_x = -\frac{dU}{dx} = 3Ax^{-4} - Bx^{-2}$.

To find the maximum force, we determine finite x_m such that

$$\left.\frac{dF_x}{dx}\right|_{x=x_m} = \left[-12Ax^{-5} + 2Bx^{-3}\right]_{x=x_0} = 0$$

$$2Bx_m^{-3} = 12Ax_m^{-5}$$

$$x_m^2 = \frac{6A}{B} \qquad \to \qquad x_m = \sqrt{\frac{6A}{B}}$$

then

$$F_{max} = 3A\left(\frac{B}{6A}\right)^2 - B\left(\frac{B}{6A}\right) = \boxed{-\frac{B^2}{12A}}$$

Challenge Problems

P43.63 (a) Refer to Example 43.2 for details. Since the interatomic potential is the same for both molecules, the spring constant is the same.

Then,

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$$

where

$$\mu_{12}=\frac{(12\text{ u})(16\text{ u})}{12\text{ u}+16\text{ u}}=6.86\text{ u}\quad\text{and}\quad\mu_{14}=\frac{(14\text{ u})(16\text{ u})}{14\text{ u}+16\text{ u}}=7.47\text{ u}$$

Therefore,

$$f_{14}=\frac{1}{2\pi}\sqrt{\frac{k}{\mu_{14}}}=\frac{1}{2\pi}\sqrt{\frac{k}{\mu_{12}}\left(\frac{\mu_{12}}{\mu_{14}}\right)}=f_{12}\sqrt{\frac{\mu_{12}}{\mu_{14}}}$$

$$=\left(6.42\times10^{13}\text{ Hz}\right)\sqrt{\frac{6.86\text{ u}}{7.47\text{ u}}}$$

$$=\boxed{6.15\times10^{13}\text{ Hz}}$$

(b) The equilibrium distance is the same for both molecules.

$$I_{14}=\mu_{14}r^2=\left(\frac{\mu_{14}}{\mu_{12}}\right)\mu_{12}r^2=\left(\frac{\mu_{14}}{\mu_{12}}\right)I_{12}$$

$$I_{14}=\left(\frac{7.47\text{ u}}{6.86\text{ u}}\right)\left(1.46\times10^{-46}\text{ kg}\cdot\text{m}^2\right)=\boxed{1.59\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

(c) The molecule can move to the (v = 1, J = 9) state or to the (v = 1, J = 11) state. The energy it can absorb is either

$$\Delta E=\frac{hc}{\lambda}=\left[\left(1+\frac{1}{2}\right)hf_{14}+11(11+1)\frac{\hbar^2}{2I_{14}}\right]$$

$$-\left[\left(0+\frac{1}{2}\right)hf_{14}+10(10+1)\frac{\hbar^2}{2I_{14}}\right]$$

$$\frac{hc}{\lambda}=hf_{14}+22\frac{\hbar^2}{2I_{14}}=hf_{14}+11\frac{h\hbar}{2\pi I_{14}}$$

$$\frac{c}{\lambda}=f_{14}+11\frac{\hbar}{2\pi I_{14}}$$

or

$$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\frac{1}{2}\right)hf_{14} + 9(9+1)\frac{\hbar^2}{2I_{14}}\right] - \left[\left(0-\frac{1}{2}\right)hf_{14} + 10(10+1)\frac{\hbar^2}{2I_{14}}\right]$$

$$\frac{hc}{\lambda} = hf_{14} - 20\frac{\hbar^2}{2I_{14}} = hf_{14} - 10\frac{h\hbar}{2\pi I_{14}}$$

$$\frac{c}{\lambda} = f_{14} - 10\frac{\hbar}{2\pi I_{14}}$$

The wavelengths it can absorb are then

$$\lambda = \frac{c}{f_{14} + 11\hbar/(2\pi I_{14})} \quad \text{or} \quad \lambda = \frac{c}{f_{14} - 10\hbar/(2\pi I_{14})}$$

These are,

$$\lambda = \frac{2.998\times 10^8 \text{ m/s}}{6.15\times 10^{13}\text{ Hz} + \dfrac{\left[11\left(1.055\times 10^{-34}\text{ J}\cdot\text{s}\right)\right]}{\left[2\pi\left(1.59\times 10^{-46}\text{ kg}\cdot\text{m}^2\right)\right]}} = \boxed{4.78\ \mu\text{m}}$$

or

$$\lambda = \frac{2.998\times 10^8 \text{ m/s}}{6.15\times 10^{13}\text{ Hz} - \dfrac{\left[10\left(1.055\times 10^{-34}\text{ J}\cdot\text{s}\right)\right]}{\left[2\pi\left(1.59\times 10^{-46}\text{ kg}\cdot\text{m}^2\right)\right]}} = \boxed{4.96\ \mu\text{m}}$$

P43.64 (a) At equilibrium separation $r = r_e$,

$$\left.\frac{dU}{dr}\right|_{r=r_e} = -2aB\left[e^{-a(r_e-r_0)} - 1\right]e^{-a(r_e-r_0)} = 0$$

We have neutral equilibrium as $r_e \to \infty$ and stable equilibrium at

$$e^{-a(r_e-r_0)} = 1 \qquad \to \qquad r_e = \boxed{r_0}$$

(b) At $r = r_0$, $U = 0$. As $r \to \infty$, $U \to B$. The depth of the well is $\boxed{B}$.

(c) We expand the potential in a Taylor series about the equilibrium point $r = r_0$:

$$U(r) \approx U(r_0) + \left.\frac{dU}{dr}\right|_{r=r_0}(r-r_0) + \frac{1}{2}\left.\frac{d^2U}{dr^2}\right|_{r=r_0}(r-r_0)^2$$

or,

$$U(r) \approx 0 + 0 + \frac{1}{2}(-2Ba)\left[-2ae^{-2a(r-r_0)} + ae^{-a(r-r_0)}\right]_{r=r_0}(r-r_0)^2$$
$$\approx Ba^2(r-r_0)^2$$

This is of the form

$$\frac{1}{2}kx^2 = \frac{1}{2}k(r-r_0)^2$$

for a simple harmonic oscillator with $k = 2Ba^2$.

Then, the molecule vibrates with frequency

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{a}{2\pi}\sqrt{\frac{2B}{\mu}} = \boxed{\frac{a}{\pi}\sqrt{\frac{B}{2\mu}}}$$

(d) The ground state energy is

$$\frac{1}{2}\hbar\omega = \frac{1}{2}hf = \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}$$

The energy at infinity is B. Therefore, to separate the nuclei to infinity requires energy

$$\boxed{B - \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P43.2	(a) 921 pN toward the other ion; (b) –2.88 eV
P43.4	(a) E_a = 1.28 eV; (b) σ = 0.272 nm, $\in$ = 4.65 eV; (c) +6.55 nN; (d) 576 N / m
P43.6	(a) $r_0 = \left[\frac{2A}{B}\right]^{1/6}$; (b) $\frac{B^2}{4A}$
P43.8	(a) 0.120 meV; (b) 19.3 GHz
P43.10	(a) 1.22×10^{-26} kg; (b) 1.24×10^{-26} kg; (c) They agree because the small apparent difference can be attributed to uncertainty in the data.
P43.12	The incident photons have a wavelength longer than this, which means they have less energy than 0.359 eV. Therefore, these photons cannot excite the molecule to the first excited state.
P43.14	$2.72 \times 10^{-47}\ \text{kg} \cdot \text{m}^2$
P43.16	μr^2
P43.18	(a) $1.89 \times 10^{-45}\ \text{kg} \cdot \text{m}^2$; (b) $E_{\text{rot}} = 18.4\ J(J + 1)$, where E_{rot} is in microelectron volts and J = 0, 1, 2, 3,….
P43.20	$2.88 \times 10^{-47}\ \text{kg} \cdot \text{m}^2$
P43.22	64.1 THz
P43.24	(a) ~10^{17}; (b) ~10^5 m^3
P43.26	$U = -k_e \alpha \frac{e^2}{r}$ where $\alpha = 2 \ln 2$
P43.28	(a) The Fermi energy is proportional to the spatial concentration of free electrons to the two-thirds power; (b) See P43.28(b) for full explanation; (c) 6.04; (d) Copper; (e) 0.333; (f) This behavior agrees with the proportionality because $E_F \sim n_e^{2/3}$ and $6.04^{2/3} = 3.32$.
P43.30	(a) 1.57 Mm / s; (b) The speed is larger by ten orders of magnitude.
P43.32	3.40×10^{17} electrons
P43.34	There are approximately two free electrons per atom for this metal, not one (see P43.34 for full explanation).
P43.36	$\frac{1}{e^{(\beta - 1)E_F / k_B T} + 1}$
P43.38	(a) 1.10; (b) 9.42×10^{-25}

P43.40 (a) The gap should be less than or equal to 1.24 eV; (b) Because silicon has an energy gap of 1.14 eV, it can absorb the energy of nearly all of the photons in sunlight and is an appropriate material for a solar energy collector.

P43.42 (a) All the hydrogen Balmer lines except for the red lien at 656 nm will be absorbed; (b) The red line at 656 nm will be transmitted.

P43.44 2.42 eV

P43.46 (a) $a' = \left(\frac{m_e}{m^*}\right)\kappa a_0$; (b) 2.81 nm; (c) $E'_n = -\left(\frac{m^*}{m_e}\right)\frac{E_n}{\kappa^2}$; (d) -0.0219 eV

P43.48 (a) See P43.48(a) for full explanation; (b) See ANS. FIG. P43.48; (c) 2.98 mA; (d) 67.1 Ω; (e) 8.39 Ω

P43.50 -2.35×10^{17}

P43.52 (a) In the definition of resistance $\Delta V = IR$, if R is zero then $\Delta V = 0$ for any value of current; (b) See ANS FIG P43.52; (c) 0.023 2 Ω; (d) Expulsion of magnetic flux, and therefore fewer current-carrying paths through the superconductor, could explain the decrease in current.

P43.54 $J = 37$

P43.56 (a) See P43.56(a) for full explanation; (b) See P43.56(b) for full explanation; (c) 9.18 THz

P43.58 4.74 eV

P43.60 (a–b) See P43.60 for full explanation.

P43.62 (a) $x_0 = \sqrt{\frac{3A}{B}}$; (b) $-2\sqrt{\frac{B^3}{27A}}$; (c) $-\frac{B^2}{12A}$

P43.64 (a) r_0; (b) B; (c) $\frac{a}{\pi}\sqrt{\frac{B}{2\mu}}$; (d) $B - \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}$

44

Nuclear Physics

CHAPTER OUTLINE

44.1	Some Properties of Nuclei
44.2	Nuclear Binding Energy
44.3	Nuclear Models
44.4	Radioactivity
44.5	The Decay Processes
44.6	Natural Radioactivity
44.7	Nuclear Reactions
44.8	Nuclear Magnetic Resonance and Magnetic Resonance Imaging

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ44.1 Answer (b). The frequency increases linearly with the magnetic field strength because the magnetic potential energy $-\vec{\boldsymbol{\mu}} \cdot \vec{\mathbf{B}}$ is proportional to the magnetic field strength.

OQ44.2 Answer (a). In the beta decay of ${}^{95}_{36}\text{Kr}$, the emitted particles are an electron, ${}^{0}_{-1}\text{e}$, and an antineutrino, $\bar{\nu}_e$. The emitted particles contain a total charge of $-e$ and zero nucleons. Thus, to conserve both charge and nucleon number, the daughter nucleus must be ${}^{95}_{37}\text{Rb}$, which contains $Z = 37$ protons and $A - Z = 95 - 37 = 58$ neutrons. (Recall that the electron and an antineutrino are produced by the decay on a neutron into a proton.)

OQ44.3 Answer (c). The emitted particle is not a nucleon because there is no change in nucleon number, and conservation of charge requires $15 = 16 + Z \to Z = -1$, so the emitted particle is an electron. From Equation 44.19, we see that ${}^{32}_{15}\mathrm{P}$ decays by means of beta decay: ${}^{32}_{15}\mathrm{P} \to {}^{32}_{16}\mathrm{S} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}_\mathrm{e}$.

OQ44.4 Answer (d). In a large sample, one half of the radioactive nuclei initially present remain in the sample after one half-life has elapsed. Hence, the fraction of the original number of radioactive nuclei remaining after n half-lives have elapsed is $(1/2)^n = 1/2^n$. In this case the number of half-lives that have elapsed is $\Delta t/T_{1/2} = 14\text{ d}/3.6\text{ d} \approx 4$. Therefore, the approximate fraction of the original sample that remains undecayed is $1/2^4 = 1/16$.

OQ44.5 **(i)** Answer (b). Since the samples are of the same radioactive isotope, their half-lives are the same.

(ii) Answer (b). When prepared, sample G has twice the activity (number of radioactive decays per second) of sample H. The activity of a sample experiences exponential decay also; therefore, after 5 half-lives, the activity of sample G is decreased by a factor of 2^5, and after 5 half-lives the activity of sample H is decreased by a factor of 2^5. So after 5 half-lives, the ratio of activities is still 2:1.

OQ44.6 Answer (b). A gamma ray photon carries no nucleon number and no charge, so there can be no change in these quantities.

OQ44.7 Answer (c). The nucleus ${}^{40}_{18}\mathrm{X}$ contains $A = 40$ total nucleons, of which $Z = 18$ are protons. The remaining $A - Z = 40 - 18 = 22$ are neutrons.

OQ44.8 Answer (b). Conservation of nucleon number requires $144 = 140 + A \to A = 4$, and conservation of charge requires $60 = 58 + Z \to Z = 2$. The particle is ${}^{4}_{2}\mathrm{X} = {}^{4}_{2}\mathrm{He}$.

OQ44.9 Answer (d). The Q value for the reaction ${}^{9}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \to {}^{12}_{6}\mathrm{C} + {}^{1}_{0}\mathrm{n}$ is (using masses from Table 44.2)

$$\begin{aligned} Q = (\Delta m)c^2 &= \left(m_{{}^{9}_{4}\mathrm{Be}} + m_{{}^{4}_{2}\mathrm{He}} - m_{{}^{12}_{6}\mathrm{C}} - m_\mathrm{n}\right)c^2 \\ &= \big[9.012\,182\text{ u} + 4.002\,603\text{ u} \\ &\qquad -12.000\,000\text{ u} - 1.008\,665\text{ u}\big] \times (931.5\text{ Mev/u}) \\ &= 5.70\text{ MeV} \end{aligned}$$

OQ44.10 **(i)** Answer (a). The liquid drop model gives a simpler account of a nuclear fission reaction, including the energy released and the probable fission product nuclei.

(ii) Answer (b). The shell model predicts magnetic moments by necessarily describing the spin and orbital angular momentum states of the nucleons.

(iii) Answer (b). Again, the shell model wins when it comes to predicting the spectrum of an excited nucleus, as it allows only quantized energy states, and thus only specific transitions.

OQ44.11 Answer (d). A free neutron can undergo beta decay into a proton plus an electron and an antineutrino because its mass is greater than the mass of a free proton. Energy conservation prevents a free proton from decaying into a neutron plus a positron and a neutrino. (A proton bound inside a nucleus can undergo beta decay into a neutron if the final mass of the nucleus is less than that of the original nucleus, as for example in the beta decay of sodium-22: $^{22}_{11}\text{Na} \to e^{+} + \nu + {}^{22}_{10}\text{Ne}.$)

OQ44.12 Answer (d). The reaction energy is the amount of energy released as a result of a nuclear reaction. Equation 44.29 in the text implies that the reaction energy is (initial mass – final mass) c^2. The Q-value is taken as positive for an exothermic reaction.

OQ44.13 Answer (c). To conserve nucleon number (mass number), it is necessary that $A + 4 = 234$, or $A = 230$. Conservation of charge (atomic number) demands that $Z + 2 = 90$, or $Z = 88$.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ44.1 The alpha particle and the daughter nucleus carry equal amounts of momentum in opposite directions. Since kinetic energy can be written as $\frac{p^2}{2m}$, the small-mass alpha particle has much more of the decay energy than the recoiling nucleus.

CQ44.2 The statement is false. Both patterns show monotonic decrease over time, but with very different shapes. For radioactive decay, maximum activity occurs at time zero. Cohorts of people now living will be dying most rapidly perhaps forty years from now. Everyone now living will be dead within less than two centuries, while the mathematical model of radioactive decay tails off exponentially forever. A radioactive nucleus never gets old. It has constant probability of decay however long it has existed.

CQ44.3 An alpha particle contains two protons and two neutrons. Because the nuclei of heavy hydrogen (D and T) contain only one proton, they cannot emit an alpha particle.

CQ44.4 In alpha decay, there are only two final particles, the alpha particle and the daughter nucleus. There are also two conservation principles, energy and momentum, that apply to the process. As a result, the alpha particle must be ejected with a discrete energy to satisfy both conservation principles. Beta decay, however, is a three-particle decay involving the beta particle, the neutrino (or antineutrino), and the daughter nucleus. As a result, the energy and momentum can be shared in a variety of ways among the three particles while still satisfying the two conservation principles. This explains why the beta particle can have a continuous range of energies.

CQ44.5 Carbon dating cannot generally be used to estimate the age of a rock, because the rock was not alive to receive carbon, and hence radioactive carbon-14, from the environment. Only the ages of objects that were once alive can be estimated with carbon dating.

CQ44.6 The larger rest energy of the neutron means that a free proton in space will not spontaneously decay into a neutron and a positron. When the proton is in the nucleus, however, you must consider the total rest energy of the nucleus. If it is energetically favorable for the nucleus to have one fewer proton and one more neutron, then the process of positron decay will occur to achieve this lower energy.

CQ44.7 I refers to nuclear spin quantum number.

(a) I_z may have $2I + 1 = 2\left(\frac{5}{2}\right) + 1 = 6$ values for $I = \frac{5}{2}$, namely $\frac{5}{2}$, $\frac{3}{2}$, $\frac{1}{2}$, $-\frac{1}{2}$, $-\frac{3}{2}$, and $-\frac{5}{2}$.

(b) For $I = 3$, there are $2I + 1 = 2(3) + 1 = 7$ possible values for I_z.

CQ44.8 Extra neutrons are required to overcome the increasing electrostatic repulsion of the protons. The neutrons participate in the net attractive effect of the nuclear force, but feel no Coulomb repulsion.

CQ44.9 Nuclei with more nucleons than bismuth-209 are unstable because the electrical repulsion forces among all of the protons is stronger than the nuclear attractive force between nucleons.

CQ44.10 The nuclear force favors the formation of neutron-proton pairs, so a stable nucleus cannot be too far away from having equal numbers of protons and neutrons. This effect sets the upper boundary of the zone of stability on the neutron-proton diagram. All of the protons repel one another electrically, so a stable nucleus cannot have too many protons. This effect sets the lower boundary of the zone of stability.

CQ44.11 Nucleus Y will be more unstable. The nucleus with the higher binding energy requires more energy to be disassembled into its constituent parts and has less available energy to release in a decay.

CQ44.12 After one half-life, one half the radioactive atoms have decayed. After the second half-life, one half of the remaining atoms have decayed. Therefore, $\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ of the original radioactive atoms have decayed after two half-lives.

CQ44.13 Long-lived progenitors at the top of each of the three natural radioactive series are the sources of our radium. As an example, thorium-232 with a half-life of 14 Gyr produces radium-228 and radium-224 at stages in its series of decays.

CQ44.14 Yes. The daughter nucleus can be left in its ground state or sometimes in one of a set of excited states. If the energy carried by the alpha particle is mysteriously low, the daughter nucleus can quickly emit the missing energy in a gamma ray.

CQ44.15 The alpha particle does not make contact with the nucleus because of electrostatic repulsion between the positively-charged nucleus and the $+2e$ alpha particle. To drive the alpha particle into the nucleus would require extremely high kinetic energy.

CQ44.16 The samples would have started with more carbon-14 than we first thought. We would increase our estimates of their ages.

CQ44.17 The photon and the neutrino are similar in that both particles have zero charge and little or no mass. (The photon has zero mass, but evidence suggests that neutrinos have a very small mass.) Both particles are capable of transferring both energy and momentum. They differ in that the photon has spin 1 and is involved in electromagnetic interactions, while the neutrino has spin $\frac{1}{2}$, interacts through the weak interaction, and is closely related to beta decay.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 44.1 Some Properties of Nuclei

P44.1 The average nuclear radii are $r = r_0A^{1/3}$, where $r_0 = 1.2 \times 10^{-15}$ m = 1.2 fm and A is the mass number.

(a) For ${}^{2}_{1}\mathrm{H}$, $r = (1.2\text{ fm})(2)^{1/3} = \boxed{1.5\text{ fm}}$

(b) For ${}^{60}_{27}\mathrm{Co}$, $r = (1.2\text{ fm})(60)^{1/3} = \boxed{4.7\text{ fm}}$

(c) For ${}^{197}_{79}\mathrm{Au}$, $r = (1.2\text{ fm})(197)^{1/3} = \boxed{7.0\text{ fm}}$

(d) For ${}^{239}_{94}\mathrm{Pu}$, $r = (1.2\text{ fm})(239)^{1/3} = \boxed{7.4\text{ fm}}$

P44.2 (a) Approximate nuclear radii are given by $r = r_0A^{1/3}$. Thus, if a nucleus of atomic number A has a radius approximately two-thirds that of ${}^{230}_{88}\mathrm{Ra}$, we should have

$$r = r_0A^{1/3} = \frac{2}{3}r_0(230)^{1/3}$$

or $$A = \frac{2^3}{3^3}(230) = \frac{8}{27}(230) \approx \boxed{68}$$

(b) $\boxed{\text{One possible nucleus is } {}^{68}_{30}\mathrm{Zn}}$.

(c) $\boxed{\text{Isotopes of other elements to the left and right of zinc in the periodic table (from manganese to bromine) may have the same mass number.}}$

P44.3 (a) The initial kinetic energy of the alpha particle must equal the electrostatic potential energy at the distance of closest approach.

$$K_i = U_f = \frac{k_eqQ}{r_{\min}}$$

$$r_{\min} = \frac{k_eqQ}{K_i} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(2)(79)(1.60\times10^{-19}\ \mathrm{C})^2}{(0.500\ \mathrm{MeV})(1.60\times10^{-13}\ \mathrm{J/MeV})}$$

$$= 4.55\times10^{-13}\ \mathrm{m} = 455\times10^{-15}\ \mathrm{m} = \boxed{455\ \mathrm{fm}}$$

(b) Following the same logic as in part (a),

$$K_i = \frac{1}{2}m_\alpha v_i^2 = \frac{k_eqQ}{r_{\min}}$$

Now, for $r_{\text{min}} = 300\text{ fm} = 300 \times 10^{-15}\text{ m}$, solving for the initial velocity gives

$$v_i = \sqrt{\frac{2k_e qQ}{m_\alpha r_{\text{min}}}}$$

$$= \sqrt{\frac{2\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)(2)(79)\left(1.602\times10^{-19}\text{ C}\right)^2}{\left(6.645\times10^{-27}\text{ kg}\right)\left(300\times10^{-15}\text{ m}\right)}}$$

$$v_i = \boxed{6.05\times10^{6}\text{ m/s}}$$

P44.4 An iron nucleus (in hemoglobin) has a few more neutrons than protons, but in a typical water molecule there are eight neutrons and ten protons. So protons and neutrons are nearly equally numerous in your body, each contributing mass (say) 35 kg:

(a) $35\text{ kg}\left(\dfrac{1\text{ nucleon}}{1.67\times10^{-27}\text{ kg}}\right)\boxed{\sim10^{28}\text{ protons}}$,

(b) and $\boxed{\sim10^{28}\text{ neutrons}}$.

(c) The electron number is precisely equal to the proton number, $\boxed{\sim10^{28}\text{ electrons}}$.

P44.5 (a) ${}^{65}_{29}\text{Cu}$ has an A number of 65, so the radius of its nucleus is

$$r = r_0A^{1/3} = (1.2\text{ fm})(65)^{1/3} = \boxed{4.8\text{ fm}}$$

(b) The volume of the nucleus, assumed to be spherical in shape, is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left[r_0^3A\right] = \frac{4}{3}\pi\left[\left(1.2\times10^{-15}\text{ m}\right)^3(65)\right]$$

$$= \boxed{4.7\times10^{-43}\text{ m}^3}$$

(c) The density of the nucleus is

$$\rho = \frac{m}{V} = \frac{Am}{\frac{4}{3}\pi\left[r_0^3A\right]} = \frac{3m}{4\pi r_0^3} = \frac{3\left(1.66\times10^{-27}\text{ kg}\right)}{4\pi\left(1.2\times10^{-15}\text{ m}\right)^3}$$

$$= \boxed{2.3\times10^{17}\text{ kg/m}^3}$$

P44.6 From $M_E = \rho_n V = \rho_n(4\pi r^3/3)$, we find

$$r = \left(\frac{3M_E}{4\pi\rho_n}\right)^{1/3} = \left[\frac{3\left(5.98\times10^{24}\text{ kg}\right)}{4\pi\left(2.30\times10^{17}\text{ kg/m}^3\right)}\right]^{1/3} = \boxed{184\text{ m}}$$

P44.7 The number of neutrons in a star of two solar masses is

$$A = \frac{2\left(1.99\times10^{30}\ \text{kg}\right)}{1.67\times10^{-27}\ \text{kg/neutron}} = 2.38\times10^{57}\ \text{neutrons}$$

Therefore,

$$\begin{aligned} r = r_0A^{1/3} &= \left(1.20\times10^{-15}\ \text{m}\right)\left(2.38\times10^{57}\right)^{1/3} \\ &= 1.6\times10^{4}\ \text{m} = \boxed{16\ \text{km}} \end{aligned}$$

P44.8 (a) The electric potential energy between two protons is

$$\begin{aligned} U &= k_e\frac{q_1q_2}{r} = k_e\frac{e^2}{r} \\ &= \left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left[\frac{\left(1.60\times10^{-19}\ \text{C}\right)^2}{4.00\times10^{-15}\ \text{m}}\right] \\ &\qquad\times\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)\left(\frac{1\ \text{MeV}}{10^{6}\ \text{eV}}\right) \\ &= \boxed{0.360\ \text{MeV}} \end{aligned}$$

(b) Figure P44.8 shows the highest point in the curve at about 4 MeV, a factor of ten higher than the value in (a).

P44.9 By energy conservation,

$$\frac{1}{2}mv^2 = q\Delta V: \qquad 2m\Delta V = qr^2B^2$$

By Newton's second law,

$$\frac{mv^2}{r} = qvB: \qquad r = \sqrt{\frac{2m\Delta V}{qB^2}}$$

Comparing radii for particles with different masses but with the same charge, we find that

$$\frac{r_2}{r_1} = \frac{\sqrt{2m_2\Delta V/qB^2}}{\sqrt{2m_1\Delta V/qB^2}} = \frac{\sqrt{m_2}}{\sqrt{m_1}}$$

For ^{12}C: $m_1 = 12$ u and $r_1 = 7.89$ cm

For ^{13}C:

$$\frac{r_2}{r_1} = \frac{r_2}{7.89\ \text{cm}} = \frac{\sqrt{m_2}}{\sqrt{m_1}} = \frac{\sqrt{13}}{\sqrt{12}} \qquad \rightarrow \qquad \boxed{8.21\ \text{cm}}$$

P44.10 By energy conservation,

$$\frac{1}{2}mv^2 = q\Delta V: \qquad 2m\Delta V = qr^2B^2$$

By Newton's second law,

$$\frac{mv^2}{r} = qvB: \qquad r = \sqrt{\frac{2m\Delta V}{qB^2}}$$

Comparing radii for particles with different masses but with the same charge, we find that

$$\frac{r_2}{r_1} = \frac{\sqrt{2m_2\Delta V/qB^2}}{\sqrt{2m_1\Delta V/qB^2}} = \frac{\sqrt{m_2}}{\sqrt{m_1}} \qquad \rightarrow \qquad \boxed{r_2 = \sqrt{\frac{m_2}{m_1}}r_1}$$

***P44.11** (a) The magnitude of the maximum Coulomb force is given by

$$\begin{aligned} F_{\text{max}} &= \frac{k_e q_1 q_2}{r_{\text{min}}^2} \\ &= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[(2)(6)(1.60\times10^{-19}\ \text{C})^2\right]}{(1.00\times10^{-14}\ \text{m})^2} \\ &= \boxed{27.6\ \text{N}} \end{aligned}$$

(b) From Newton's second law,

$$a_{\text{max}} = \frac{F_{\text{max}}}{m_\alpha} = \frac{27.6\ \text{N}}{6.64\times10^{-27}\ \text{kg}} = \boxed{4.16\times10^{27}\ \text{m/s}^2}$$

(c) The potential energy of the system at the time of the maximum force is

$$\begin{aligned} U_{\text{max}} &= \frac{k_e q_1 q_2}{r_{\text{min}}} \\ &= \left\{\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[(2)(6)(1.60\times10^{-19}\ \text{C})^2\right]}{(1.00\times10^{-14}\ \text{m})}\right\} \\ &\qquad \times\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)\left(\frac{1\ \text{MeV}}{10^6\ \text{eV}}\right) \\ &= \boxed{1.73\ \text{MeV}} \end{aligned}$$

P44.12 We obtain the alpha particle's momentum from

$$E_\alpha = 7.70 \text{ MeV} = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} \qquad \to \qquad mv = \sqrt{2mE_\alpha}$$

(a) The de Broglie wavelength of the alpha particle is (mass from Table 44.1)

$$\lambda = \frac{h}{m_\alpha v_\alpha} = \frac{h}{\sqrt{2m_\alpha E_\alpha}}$$

$$= \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2(6.64\times10^{-27}\text{ kg})(7.70\times10^{6}\text{ eV})(1.60\times10^{-19}\text{ J/eV})}}$$

$$= 5.18\times10^{-15}\text{ m} = \boxed{5.18\text{ fm}}$$

(b) Since $\boxed{\lambda \text{ is much less than the distance of closest approach,}}$ the alpha particle may be considered a particle.

P44.13 The volume of each of the golf balls is

$$V = \frac{4}{3}\pi r^4 = \frac{4}{3}\pi(0.021\,5\text{ m})^3 = 4.16\times10^{-5}\text{ m}^3$$

We take the nuclear density from Example 44.2. Then, the mass of a golf-ball sized nuclear matter is

$$m = \rho V = (2.3\times10^{17}\text{ kg/m}^3)(4.16\times10^{-5}\text{ m}^3) = 9.6\times10^{12}\text{ kg}$$

and the gravitational force between two such balls is

$$F = G\frac{m_1 m_2}{r^2} = (6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)\frac{(9.6\times10^{12}\text{ kg})^2}{(1.00\text{ m})^2}$$

$$F = \boxed{6.1\times10^{15}\text{ N toward each other.}}$$

P44.14 (a) Let *V* represent the volume of the tank. The number of molecules present is

$$N = nN_A = \frac{PV}{RT} = \frac{(1.013\times10^{5}\text{ N/m}^2)V}{(8.315\text{ J/mol}\cdot\text{K})(273\text{ K})}(6.022\times10^{23})$$

$$= (2.69\times10^{25}\text{ m}^{-3})V$$

The volume of one molecule is

$$2\left(\frac{4}{3}\pi r^3\right) = \frac{8\pi}{3}\left(\frac{1.00\times10^{-10}\text{ m}}{2}\right)^3 = 1.047\times10^{-30}\text{ m}^3$$

The volume of all the molecules is

$$\left(2.69\times10^{25}\ \text{m}^{-3}\right)V\left(1.047\times10^{-30}\ \text{m}^{3}\right)=2.82\times10^{-5}\ V$$

So the fraction of the volume occupied by the hydrogen molecules is $\boxed{2.82\times10^{-5}.}$ An atom is precisely one half of a molecule.

(b) The fraction occupied by the nucleus is found from

$$\frac{\text{nuclear volume}}{\text{atomic volume}}=\frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi(d/2)^3}=\left(\frac{r}{d/2}\right)^3$$

$$=\left(\frac{1.20\times10^{-15}\ \text{m}}{0.500\times10^{-10}\ \text{m}}\right)^3=\boxed{1.38\times10^{-14}}$$

In linear dimension, the nucleus is small inside the atom in the way a fat strawberry is small inside the width of the Grand Canyon. In terms of volume, the nucleus is *really* small.

Section 44.2 Nuclear Binding Energy

P44.15 Using Equation 44.2, the binding energy per nucleon is

$$\frac{E_b}{A}=\frac{\left[ZM(\text{H})+Nm_n-M\left({}^A_ZX\right)\right]}{A}\left(\frac{931.5\ \text{MeV}}{\text{u}}\right)$$

Using atomic masses as given in Table 44.2,

(a) For ${}^2_1\text{H}$:

$$\frac{E_b}{A}=\frac{1(1.007\ 825\ \text{u})+1(1.008\ 665\ \text{u})-2.014\ 102\ \text{u}}{2}$$

$$=\left(\frac{0.002\ 388\ \text{u}}{2}\right)\left(\frac{931.5\ \text{MeV}}{\text{u}}\right)=\boxed{1.11\ \text{MeV}}$$

(b) For ${}^4_2\text{He}$:

$$\frac{E_b}{A}=\frac{2(1.007\ 825\ \text{u})+2(1.008\ 665\ \text{u})-4.002\ 603\ \text{u}}{4}$$

$$=\left(\frac{0.030\ 377\ \text{u}}{4}\right)\left(\frac{931.5\ \text{MeV}}{\text{u}}\right)=\boxed{7.07\ \text{MeV}}$$

(c) For ${}^{56}_{26}\mathrm{Fe}$:

$$\frac{E_b}{A}=\frac{26(1.007\ 825\ \mathrm{u})+30(1.008\ 665\ \mathrm{u})-55.934\ 942\ \mathrm{u}}{56}$$
$$=\left(\frac{0.528\ 458\ \mathrm{u}}{56}\right)\left(\frac{931.5\ \mathrm{MeV}}{\mathrm{u}}\right)=\boxed{8.79\ \mathrm{MeV}}$$

(d) For ${}^{238}_{92}\mathrm{U}$:

$$\frac{E_b}{A}=\frac{92(1.007\ 825\ \mathrm{u})+146(1.008\ 665\ \mathrm{u})-238.050\ 783\ \mathrm{u}}{238}$$
$$=\left(\frac{1.934\ 207\ \mathrm{u}}{238}\right)\left(\frac{931.5\ \mathrm{MeV}}{\mathrm{u}}\right)=\boxed{7.57\ \mathrm{MeV}}$$

P44.16 We use Equation 44.2,

$$E_b(\mathrm{MeV})=\left[ZM(\mathrm{H})+Nm_n-M\left({}^{A}_{Z}\mathrm{X}\right)\right](931.494\ \mathrm{MeV/u})$$

Then, for ${}^{23}_{11}\mathrm{Na}$,

$$E_b\left({}^{23}_{11}\mathrm{Na}\right)=\left[11M(\mathrm{H})+12m_n-M\left({}^{23}_{11}\mathrm{Na}\right)\right](931.494\ \mathrm{MeV/u})$$
$$=\left[11(1.007\,825\ \mathrm{u})+12(1.008\,665\ \mathrm{u})-22.989\,769\ \mathrm{u}\right]\times(931.494\ \mathrm{MeV/u})$$
$$=186.565\ \mathrm{MeV}$$

and $$\frac{E_b}{A}=\frac{186.565\ \mathrm{MeV}}{23}=8.11\ \mathrm{MeV}$$

For ${}^{23}_{12}\mathrm{Mg}$,

$$E_b=E_b\left({}^{23}_{12}\mathrm{Mg}\right)$$
$$=\left[12M(\mathrm{H})+11m_n-M\left({}^{23}_{12}\mathrm{Mg}\right)\right](931.494\ \mathrm{MeV/u})$$
$$=\left[12(1.007\,825\ \mathrm{u})+11(1.008\,665\ \mathrm{u})-22.994\,124\ \mathrm{u}\right]\times(931.494\ \mathrm{MeV/u})$$
$$=181.726\ \mathrm{MeV}$$

and $$\frac{E_b}{A}=\frac{181.726\ \mathrm{MeV}}{23}=7.90\ \mathrm{MeV}$$

The difference is

$$\frac{\Delta E_b}{A}=\frac{E_b\left({}^{23}_{11}\mathrm{Na}\right)-E_b\left({}^{23}_{12}\mathrm{Mg}\right)}{A}$$
$$=8.11\ \mathrm{MeV}-7.90\ \mathrm{MeV}=0.210\ \mathrm{MeV}$$

The binding energy per nucleon is greater for ${}^{23}_{11}\text{Na}$ by $\boxed{0.210\text{ MeV}}$. There is less proton repulsion in ${}^{23}_{11}\text{Na}$; it is the more stable nucleus.

P44.17 From Equation 44.2, the binding energy of a nucleus is

$$E_b(\text{MeV}) = \left[ZM(\text{H}) + Nm_n - M\left({}^{A}_{Z}X\right)\right](931.494\text{ MeV/u})$$

For ${}^{15}_{8}\text{O}$:

$$E_b = \left[8(1.007\ 825\text{ u}) + 7(1.008\ 665\text{ u}) - 15.003\ 065\text{ u}\right] \times (931.494\text{ MeV/u}) = 111.96\text{ MeV}$$

For ${}^{15}_{7}\text{N}$:

$$E_b = \left[7(1.007\ 825\text{ u}) + 8(1.008\ 665\text{ u}) - 15.000\ 109\text{ u}\right] \times (931.494\text{ MeV/u}) = 115.49\text{ MeV}$$

Therefore, the binding energy is $\boxed{\text{greater for } {}^{15}_{7}\text{N by 3.54 MeV.}}$

P44.18 We find the mass difference, $\Delta M = Zm_H + Nm_n - M$, and then the binding energy per nucleon, $\frac{E_b}{A} = \frac{\Delta M(931.5)}{A}$, in units of MeV. The results are tabulated below.

Nuclei	Z	N	M in u	ΔM in u	$\frac{E_b}{A}$ in MeV
${}^{55}\text{Mn}$	25	30	54.938 050	0.517 5	8.765
${}^{56}\text{Fe}$	26	30	55.934 942	0.528 46	8.790
${}^{59}\text{Co}$	27	32	58.933 200	0.555 35	8.768

$$\boxed{\therefore\ {}^{56}\text{Fe has a greater } \frac{E_b}{A} \text{ than its neighbors.}}$$

P44.19 (a) The isobar with the highest neutron-to-proton ratio is $\boxed{{}^{139}_{55}\text{Cs}}$; the ratio is $\frac{N}{Z} = \frac{A-Z}{Z} = \frac{139-55}{55} = \frac{84}{55} = 1.53$

(b) $\boxed{{}^{139}_{57}\text{La}}$ is stable, so has the largest binding energy per nucleon (8.378 MeV).

(c) The isobars are close in Figure 44.6, the plot of binding energy per nucleon versus mass number, and there is not much detail, so we may assume they have about the same binding energy, or missing mass. However, neutrons have more mass than protons, so the isobar with more neutrons (thus, fewer protons) should be more massive: $\boxed{{}^{139}_{55}\mathrm{Cs}}$.

P44.20 (a) The radius of the ^{40}Ca nucleus is,

$$R = r_0 A^{1/3} = \left(1.20\times10^{-15}\ \mathrm{m}\right)(40)^{1/3} = 4.10\times10^{-15}\ \mathrm{m}$$

The energy required to overcome electrostatic repulsion is

$$U = \frac{3k_eQ^2}{5R} = \frac{3\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left[20\left(1.602\times10^{-19}\ \mathrm{C}\right)\right]^2}{5\left(4.10\times10^{-15}\ \mathrm{m}\right)}$$

$$= 1.35\times10^{-11}\ \mathrm{J} = \boxed{84.2\ \mathrm{MeV}}$$

(b) The binding energy of ${}^{40}_{20}\mathrm{Ca}$ ($Z = 20$, $N = A - Z = 20$) is (using Equation 44.2 and masses from Table 44.2),

$$E_b = \left[20(1.007\,825\ \mathrm{u}) + 20(1.008\,665\ \mathrm{u}) - 39.962\,591\ \mathrm{u}\right] \times (931.5\ \mathrm{MeV/u})$$

$$= \boxed{342\ \mathrm{MeV}}$$

(c) $\boxed{\text{The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.}}$

P44.21 Removal of a neutron from ${}^{43}_{20}\mathrm{Ca}$ would result in the residual nucleus, ${}^{42}_{20}\mathrm{Ca}$. If the required separation energy is ΔE_n, the overall process can be described by

$$\mathrm{mass}\left({}^{43}_{20}\mathrm{Ca}\right) + \Delta E_n = \mathrm{mass}\left({}^{42}_{20}\mathrm{Ca}\right) + \mathrm{mass}(n)$$

$$\Delta E_n = \mathrm{mass}\left({}^{42}_{20}\mathrm{Ca}\right) + \mathrm{mass}(n) - \mathrm{mass}\left({}^{43}_{20}\mathrm{Ca}\right)$$

From Table 44.2,

$$\Delta E_n = (41.958\,618\ \mathrm{u} + 1.008\,665\ \mathrm{u} - 42.958\,767\ \mathrm{u}) \times (931.5\ \mathrm{MeV/u})$$

$$= \boxed{7.93\ \mathrm{MeV}}$$

Section 44.3 Nuclear Models

P44.22 The curve of binding energy shows that a heavy nucleus of mass number A = 200 has binding energy about

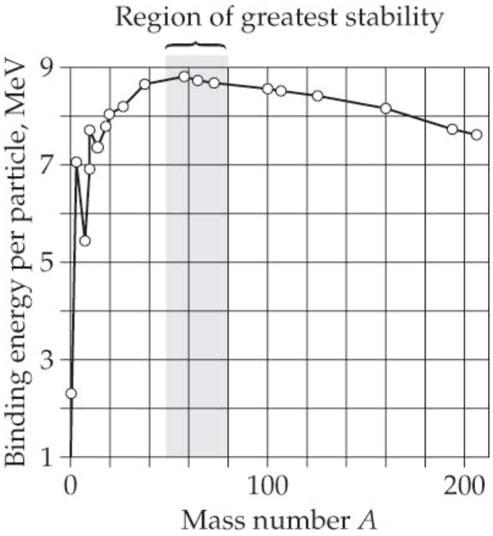
$$\left(7.8\frac{\text{MeV}}{\text{nucleon}}\right)(200\text{ nucleons}) \approx 1.56\text{ GeV}$$

Thus, it is less stable than its potential fission products, two middleweight nuclei of A = 100, together having binding energy

$$2(8.7\text{ MeV/nucleon})(100\text{ nucleons}) \approx 1.74\text{ GeV}$$

Fission then releases about

$$1.74\text{ GeV} - 1.56\text{ GeV}\ \boxed{\sim 200\text{ MeV}}$$



ANS. FIG. P44.22

P44.23 (a) In Equation 44.3, the first or "Volume" term is,

$$E_1 = C_1A = (15.7\text{ MeV})(56) = 879\text{ MeV}$$

The second, or "Surface" term is,

$$E_2 = -C_2A^{2/3} = -(17.8\text{ MeV})(56)^{2/3} = -260\text{ MeV}$$

The third, or "Coulomb" term is,

$$E_3 = -C_3\frac{Z(Z-1)}{A^{1/3}} = -(0.71\text{ MeV})\frac{(26)(25)}{(56)^{1/3}}$$
$$= -121\text{ MeV}$$

and the fourth, or "Asymmetry" term is,

$$E_4 = C_4\frac{(A-2Z)^2}{A} = -(23.6\text{ MeV})\frac{(56-52)^2}{56} = -6.74\text{ MeV}$$

The binding energy is then

$$E_b = C_1A - C_2A^{2/3} - C_3\frac{Z(Z-1)}{A^{1/3}} - C_4\frac{(A-2Z)^2}{A}$$

$$= 879\text{ MeV} - 260\text{ MeV} - 121\text{ MeV} - 6.74\text{ MeV} = \boxed{491\text{ MeV}}$$

(b) The percentages for each of the terms is as follows

$$\boxed{\text{term 1}:}\ \frac{E_1}{E_b} = \boxed{179\%};\ \boxed{\text{term 2}:}\ \frac{E_2}{E_b} = \boxed{-53.0\%};$$

$$\boxed{\text{term 3}:}\ \frac{E_3}{E_b} = \boxed{-24.6\%};\ \boxed{\text{term 4}:}\ \frac{E_4}{E_b} = \boxed{-1.37\%}$$

P44.24 (a) Nucleons on the surface have fewer neighbors with which to interact. The surface term is negative to reduce the estimate from the volume term, which assumes that all nucleons have the same number of neighbors.

(b) The volume to surface ratio for a sphere of radius *r* is

$$\frac{\text{Volume}}{\text{Area}} = \frac{(4/3)\pi r^3}{4\pi r^2} = \boxed{\frac{1}{3}r}$$

The volume to surface ratio for a cube of side length *L* is

$$\frac{\text{Volume}}{\text{Area}} = \frac{L^3}{6L^2} = \boxed{\frac{1}{6}L}$$

The sphere has a larger ratio to its characteristic length, so it would represent a larger binding energy and be more plausible for a nuclear shape.

Section 44.4 Radioactivity

***P44.25** We use Equation 44.7 for the exponential decay rate of the sample, $R = R_0e^{-\lambda t}$, where

$$\lambda = \frac{\ln 2}{26.0\text{ h}} = 0.026\,7\text{ h}^{-1}$$

Since we require a 90% decrease in activity,

$$\frac{R}{R_0} = 0.100 = e^{-\lambda t} \quad \rightarrow \quad \ln(0.100) = -\lambda t$$

then,

$$t = \frac{2.30}{0.026\,7/\text{h}} = \boxed{86.4\text{ h}}$$

P44.26 (a) From $R = R_0 e^{-\lambda t}$, the decay constant is

$$\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right) = \left(\frac{1}{4.00\text{ h}}\right)\ln\left(\frac{10.0\text{ mCi}}{8.00\text{ mCi}}\right) = 5.58\times10^{-2}\text{ h}^{-1}$$
$$= \boxed{1.55\times10^{-5}\text{ s}^{-1}}$$

(b) The half-life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{12.4\text{ h}}$$

(c) The number of original atoms can be found if we convert the initial activity from curies into becquerels (decays per second): 1 Ci $\equiv 3.70 \times 10^{10}$ Bq.

$$R_0 = 10.0\text{ mCi} = \left(10.0\times10^{-3}\text{ Ci}\right)\left(3.70\times10^{10}\text{ Bq/Ci}\right)$$
$$= 3.70\times10^{8}\text{ Bq}$$

Since $R_0 = \lambda N_0$, the original number of nuclei is

$$N_0 = \frac{R_0}{\lambda} = \frac{3.70\times10^{8}\text{ decays/s}}{1.55\times10^{-5}\text{ s}} = \boxed{2.39\times10^{13}\text{ atoms}}$$

(d) The decay rate after thirty hours is

$$R = R_0 e^{-\lambda t} = (10.0\text{ mCi})\exp\left[\left(-5.58\times10^{-2}\text{ h}^{-1}\right)(30.0\text{ h})\right]$$
$$= \boxed{1.88\text{ mCi}}$$

P44.27 The decay law is

$$dN/dt = -\lambda N$$

Then, the decay constant is

$$\lambda = -\frac{1}{N}\left(\frac{dN}{dt}\right) = -\left(\frac{1}{1.00\times10^{15}\text{ nuclei}}\right)\left(\frac{-6.00\times10^{11}\text{ nuclei}}{\text{s}}\right)$$
$$= 6.00\times10^{-4}\text{ s}^{-1}$$

and the half-life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \boxed{1.16\times10^{3}\text{ s}}$$

(This is 19.3 minutes.)

P44.28 According to Equation 44.7, the time dependence of the decay rate is $R = R_0 e^{-\lambda \Delta t}$. From this equation we can derive a relation between the change in decay rate over the time interval Δt to the decay constant. We start with $R = R_0 e^{-\lambda \Delta t}$. Then, rearranging and taking the natural log of both sides gives

$$e^{-\lambda \Delta t} = \frac{R}{R_0} \quad \rightarrow \quad \ln\left(e^{-\lambda \Delta t}\right) = \ln\left(\frac{R}{R_0}\right)$$

or

$$-\lambda \Delta t = \ln\left(\frac{R}{R_0}\right) = -\ln\left(\frac{R_0}{R}\right)$$

Solving,

$$\lambda = \frac{1}{\Delta t}\ln\left(\frac{R_0}{R}\right)$$

Now, because $\lambda = \dfrac{\ln 2}{T_{1/2}}$, we can relate the time interval Δt to the half-life:

$$\lambda = \frac{1}{\Delta t}\ln\left(\frac{R_0}{R}\right) \quad \rightarrow \quad \frac{\ln 2}{T_{1/2}} = \frac{1}{(\ln 2)\Delta t}\ln\left(\frac{R_0}{R}\right)$$

$$\frac{1}{T_{1/2}} = \frac{1}{(\ln 2)\Delta t}\ln\left(\frac{R_0}{R}\right)$$

$$T_{1/2} = \frac{(\ln 2)\Delta t}{\ln\left(R_0/R\right)}$$

P44.29 The number of nuclei that decay during the interval will be

$$\Delta N = N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$$

First we find the decay constant λ:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{64.8\ \text{h}} = 0.010\,7\ \text{h}^{-1} = 2.97 \times 10^{-6}\ \text{s}^{-1}$$

Now we find N_0:

$$N_0 = \frac{R_0}{\lambda} = \frac{(40.0\ \mu\text{Ci})}{2.97 \times 10^{-6}\ \text{s}^{-1}}\left(\frac{3.70 \times 10^{4}\ \text{s}^{-1}}{\mu\text{Ci}}\right)$$

$$= 4.98 \times 10^{11}\ \text{nuclei}$$

Substituting in these values,

$$N_1 - N_2 = \left(4.98\times10^{11}\right)\left[e^{-(\ln 2/64.8\text{ h})(10.0\text{ h})} - e^{-(\ln 2/64.8\text{ h})(12.0\text{ h})}\right]$$

$$N_1 - N_2 = \boxed{9.47\times10^{9}\text{ nuclei}}$$

P44.30 The number of nuclei that decay during the interval will be

$$N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right)$$

We wish to write this expression in terms of the half-life $T_{1/2}$ and the initial decay rate R_0. First, from the definition of λ, we have

$$\lambda = \frac{\ln 2}{T_{1/2}} \quad\rightarrow\quad e^{-\lambda t} = e^{\ln 2\left(-t/T_{1/2}\right)} = 2^{-t/T_{1/2}}$$

Now we find N_0:

$$N_0 = \frac{R_0}{\lambda} = \frac{R_0 T_{1/2}}{\ln 2}$$

Substituting in these expressions, we find that

$$N_1 - N_2 = \frac{R_0 T_{1/2}}{\ln 2}\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right) = \boxed{\frac{R_0 T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)}$$

P44.31 (a) The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{8.04\text{ d}} = \boxed{0.0862\text{ d}^{-1}}$$

$$= \frac{0.0862}{\text{d}}\left(\frac{1\text{ d}}{24\text{ h}}\right) = \boxed{3.59\times10^{-3}\text{ h}^{-1}}$$

$$= \frac{9.98\times10^{-7}}{\text{h}}\left(\frac{1\text{ h}}{3600\text{ s}}\right) = \boxed{9.98\times10^{-7}\text{ s}^{-1}}$$

(b) From $R = \lambda N$, the number of radioactive nuclei in a 6.40 mCi of ^{131}I is

$$N = \frac{R}{\lambda} = \frac{6.40\times10^{-3}\text{ Ci}}{9.98\times10^{-7}\text{ s}^{-1}}\left(\frac{3.70\times10^{10}\text{ s}^{-1}}{\text{Ci}}\right) = \boxed{2.37\times10^{14}\text{ nuclei}}$$

(c) From Equation 44.7, $R = \lambda N$, the decay rate R also undergoes exponential decay; thus, after one half-life, the rate drops from R_0 to $R_0/2$. The number of half-lives that have elapsed after 40.2 d is $n = t/T_{1/2} = 40.2\text{ d}/8.04\text{ d} = 5$, so the remaining activity of the sample is

$$R = \frac{R_0}{2^n} = \frac{R_0}{2^5} = \frac{6.40\text{ mCi}}{32} = \boxed{0.200\text{ mCi}}$$

P44.32 (a) From Equation 44.6, the fraction remaining at *t* = 5.00 yr will be

$$\frac{N}{N_0}=e^{-\lambda t}=e^{-t\ln 2/T_{1/2}}=e^{-(5.00\text{ yr})\ln 2/(12.33\text{ yr})}=\boxed{0.755}$$

(b) At *t* = 10.0 yr,

$$\frac{N}{N_0}=e^{-\lambda t}=e^{-t\ln 2/T_{1/2}}=e^{-(10.0\text{ yr})\ln 2/(12.33\text{ yr})}=\boxed{0.570}$$

(c) At *t* = 123.3 yr,

$$\frac{N}{N_0}=e^{-\lambda t}=e^{-t\ln 2/T_{1/2}}=e^{-(123.3\text{ yr})\ln 2/(12.33\text{ yr})}=e^{-10\ln 2}=\boxed{9.766\times 10^{-4}}$$

(d) No. The decay model depends on large numbers of nuclei. After some long but finite time, only one undecayed nucleus will remain. It is likely that the decay of this final nucleus will occur before infinite time.

P44.33 The number remaining after time $\frac{T_{1/2}}{2}=\frac{\ln 2}{2\lambda}$ is

$$N=N_0e^{-\lambda t}=N_0\,e^{-\lambda(\ln 2/2\lambda)}=N_0\left(e^{-\ln 2}\right)^{1/2}=N_0\left(\frac{1}{2}\right)^{1/2}=\frac{N_0}{\sqrt{2}}$$

The number decaying in this first half of the first half-life is

$$\Delta N_{\text{first half}}=N_0-\frac{N_0}{\sqrt{2}}=\left(1-\frac{1}{\sqrt{2}}\right)N_0=\left(1-\frac{\sqrt{2}}{2}\right)N_0$$

$$=\frac{\sqrt{2}}{2}\left(\sqrt{2}-1\right)N_0$$

The number remaining after time $T_{1/2}$ is $\frac{N_0}{2}$, so the number decaying in the second half of the first half-life is

$$\Delta N_{\text{second half}}=\frac{N_0}{\sqrt{2}}-\frac{N_0}{2}=\left(\frac{1}{\sqrt{2}}-\frac{1}{2}\right)N_0=\left(\frac{\sqrt{2}}{2}-\frac{1}{2}\right)N_0$$

$$=\frac{1}{2}\left(\sqrt{2}-1\right)N_0$$

The ratio required is then

$$\frac{\Delta N_{\text{first half}}=}{\Delta N_{\text{second half}}=}=\frac{\frac{\sqrt{2}}{2}\left(\sqrt{2}-1\right)N_0}{\frac{1}{2}\left(\sqrt{2}-1\right)N_0}=\sqrt{2}=\boxed{1.41}$$

P44.34 (a) $\frac{dN_2}{dt} =$ rate of change of N_2

= rate of production of N_2 – rate of decay of N_2

= rate of decay of N_1 – rate of decay of N_2

$= \lambda_1 N_1 - \lambda_2 N_2$

(b) From the trial solution,

$$N_2(t) = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(e^{-\lambda_2 t} - e^{-\lambda_1 t}\right)$$

$$\therefore \frac{dN_2}{dt} = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(-\lambda_2 e^{-\lambda_2 t} + \lambda_1 e^{-\lambda_1 t}\right) \qquad [1]$$

$$\frac{dN_2}{dt} + \lambda_2 N_2 = \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(\cancel{-\lambda_2 e^{-\lambda_2 t}} + \lambda_1 e^{-\lambda_1 t}\right) + \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}\left(\cancel{\lambda_2 e^{-\lambda_2 t}} - \lambda_2 e^{-\lambda_1 t}\right)$$

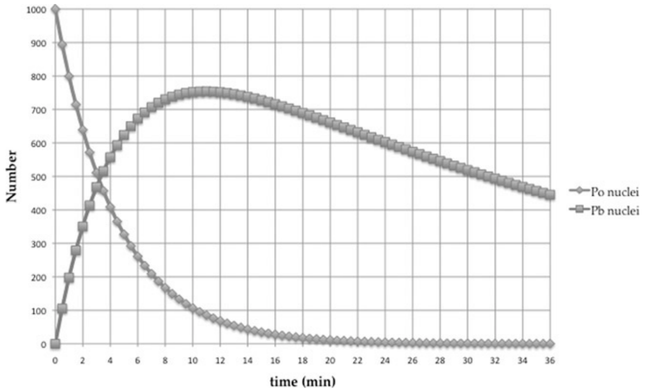
$$= \frac{N_{10}\lambda_1}{\lambda_1 - \lambda_2}(\lambda_1 - \lambda_2)e^{-\lambda_1 t} = \lambda_1 N_1$$

So $\frac{dN_2}{dt} = \lambda_1 N_1 - \lambda_2 N_2$ as required.

(c) The functions plotted in ANS. FIG. P44.34(c) are

Po nuclei: $N_1(t) = 1\,000 e^{-(\ln 2/3.10\text{ min})t}$

Pb nuclei: $N_2(t) = 1\,130.8\left[e^{-(\ln 2/26.8\text{ min})t} - e^{-(\ln 2/3.10\text{ min})t}\right]$



ANS. FIG. P44.34(c)

(d) From the graph, $t_m \approx \boxed{10.9\text{ min}}$

(e) From equation [1], $\frac{dN_2}{dt} = 0$ if

$$\lambda_2 e^{-\lambda_2 t} = \lambda_1 e^{-\lambda_1 t}$$

$$\therefore e^{(\lambda_1 - \lambda_2)t} = \frac{\lambda_1}{\lambda_2}$$

Thus, $t = \boxed{t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}}$.

(f) With $\lambda_1 = \ln 2/(3.10\text{ min})$, $\lambda_2 = \ln 2/(26.8\text{ min})$, this formula gives

$$t_m = \frac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$$

$$= \frac{\ln\left[\frac{\ln 2/(3.10\text{ min})}{\ln 2/(26.8\text{ min})}\right]}{\left(\frac{\ln 2}{3.10\text{ min}} - \frac{\ln 2}{26.8\text{ min}}\right)} = \frac{\ln\left(\frac{26.8\text{ min}}{3.10\text{ min}}\right)}{\ln 2\left(\frac{1}{3.10\text{ min}} - \frac{1}{26.8\text{ min}}\right)}$$

$$= \boxed{10.9\text{ min}}$$

This result is in agreement with the result of part (d).

Section 44.5 The Decay Processes

P44.35 Atomic masses are given in Table 44.2.

(a) For this e^+ decay,

$$Q = (M_X - M_Y - 2m_e)c^2$$
$$= [39.962\,591\text{ u} - 39.963\,999\text{ u} - 2(0.000\,549\text{ u})] \times (931.5\text{ MeV/u})$$
$$Q = -2.33\text{ MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(b) For this alpha decay,

$$Q = \left(M_X - M_Y - 2m_e\right)c^2$$
$$= \left[97.905\,287\text{ u} - 4.002\,603\text{ u} - 93.905\,088\text{ u}\right] \times \left(931.5\text{ MeV/u}\right)$$
$$Q = -2.24\text{ MeV}$$

Since $Q < 0$, the decay $\boxed{\text{cannot occur}}$ spontaneously.

(c) For this alpha decay,

$$Q = \left(M_X - M_Y - 2m_e\right)c^2$$
$$= \left[143.910\,083\text{ u} - 4.002\,603\text{ u} - 139.905\,434\text{ u}\right] \times \left(931.5\text{ MeV/u}\right)$$
$$Q = 1.91\text{ MeV}$$

Since $Q > 0$, the decay $\boxed{\text{can occur}}$ spontaneously.

P44.36 (a) The reaction is $^{3}_{1}\text{H} \rightarrow {}^{3}_{2}\text{He} + e^- + \bar{\nu}$.

Adding one electron, the reaction becomes

$$^{3}_{1}\text{H nucleus} + e^- \rightarrow {}^{3}_{2}\text{He nucleus} + 2e^- + \bar{\nu}$$

Ignoring the slight difference in ionization energies, we have

$$^{3}_{1}\text{H atom} \rightarrow {}^{3}_{2}\text{He atom} + \bar{\nu}$$

(b) The total energy released is the *Q* value:

$$Q = \left(M_{\text{H-3}} - M_{\text{He-3}}\right)c^2$$
$$Q = \left(3.016\,049\text{ u} - 3.016\,029\text{ u}\right)\left(931.5\text{ MeV/u}\right)$$
$$= 0.018\,6\text{ MeV} = \boxed{18.6\text{ keV}}$$

P44.37 From Equation 44.21, carbon-14 undergoes beta decay:

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- + \bar{\nu}$$

Adding six electrons to each side, this is the same as

$$^{14}_{6}\text{C atom} \rightarrow {}^{14}_{7}\text{N atom} + \bar{\nu}$$

The *Q* value is

$$Q = \left(M_{\text{C-14}} - M_{\text{N-14}} - m_\nu\right)c^2$$
$$= \left[14.003\,242\text{ u} - 14.003\,074\text{ u} - 0\right]\left(931.5\text{ MeV/u}\right)$$
$$= \boxed{0.156\text{ MeV}}$$

P44.38 Total Z and A are conserved.

(a) A gamma ray has zero charge and it contains no protons or neutrons. So for a gamma ray $Z = 0$ and $A = 0$. Keeping the total values of Z and A for the system conserved requires $Z = 28$ and $A = 65$ for X. With this atomic number it must be nickel, and the nucleus must be in an excited state, so X is $\boxed{{}^{65}_{28}\text{Ni}^*}$.

(b) An alpha particle, $\alpha = {}^4_2\text{He}$, has $Z = 2$ and $A = 4$. Total initial Z is 84, and total initial A is 215, so for X we require

$$Z = 84 = Z_X + 2 \quad \rightarrow \quad Z_X = 82 \quad \rightarrow \quad \text{Pb, and}$$

$$A = 215 = A_X + 4 \quad \rightarrow \quad A_X = 211, \quad \rightarrow \quad \text{X is } \boxed{{}^{211}_{82}\text{Pb}}.$$

(c) A positron, $e^+ = {}^0_1e$, has charge the same as a nucleus with $Z = 1$. A neutrino, ${}^0_0\nu$, has no charge. Neither contains any protons or neutrons. So X must have by conservation $Z = 26 + 1 + 0 = 27$; so, X is Co. And $A = 55 + 0 + 0 = 55$: X is $\boxed{{}^{55}_{27}\text{Co}}$.

P44.39 Atomic masses are given in Table 44.2. We calculate the energy released by the reaction, its Q-value, as

$$Q = \left(M_{\text{U-238}} - M_{\text{Th-234}} - M_{\text{He-4}}\right)c^2$$

$$Q = (238.050\ 783 - 234.043\ 596 - 4.002\ 603)\ \text{u}(931.5\ \text{MeV/u})$$

$$= \boxed{4.27\ \text{MeV}}$$

P44.40 (a) The decay constant is $\lambda = \ln 2/10\ \text{h} = 0.0693/\text{h}$. The number of parent nuclei is given by $N_P = N_{P,0}e^{-\lambda t} = (1.00 \times 10^6)e^{-0.0693t}$, where t is in hours.

The number of daughter nuclei is equal to the number of missing parent nuclei,

$$\boxed{N_d = N_{P,0} - N_{P,0}e^{-\lambda t} = (1.00 \times 10^6)(1 - e^{-0.0693t}), \text{ where } t \text{ is in hours.}}$$

(b) $\boxed{\text{The number of daughter nuclei starts from zero at } t = 0. \text{ The number of stable product nuclei always increases with time and asymptotically approaches } 1.00 \times 10^6 \text{ as } t \text{ increases without limit.}}$

(c) $\boxed{\text{The minimum number of daughter nuclei is zero at } t = 0. \text{ The maximum number of daughter nuclei asymptotically approaches } 1.00 \times 10^6 \text{ as } t \text{ increases without limit.}}$

(d) The rate of change is

$$\frac{dN_d}{dt} = \left(1.00\times10^{6}\right)(0+0.0693\, e^{-0.0693t}) = 6.93\times10^{4} e^{-0.0693t}$$

where $\frac{dN_d}{dt}$ is in decays per hour and *t* is in hours. $\boxed{\text{The rate of change has its maximum value, } 6.93\times10^4\ \text{h}^{-1}\text{, at } t=0\text{, after which the rate decreases more and more, approaching zero as } t \text{ increases without limit.}}$

P44.41 (a) The reaction for one particle is $\boxed{e^- + p \rightarrow n + \nu}$.

(b) For nuclei, ${}^{15}_{8}O + e^- \rightarrow {}^{15}_{7}N + \nu$.

Add seven electrons to both sides to obtain

$${}^{15}_{8}O\ \text{atom} \rightarrow {}^{15}_{7}N\ \text{atom} + \nu$$

From Table 44.2 of atomic masses,

$$Q = \left(15.003\ 065\ \text{u} - 15.000\ 109\ \text{u}\right)\left(931.5\ \text{MeV/u}\right)$$
$$= \boxed{2.75\ \text{MeV}}$$

P44.42 (a) The number of carbon atoms in the sample is

$$N_C = \left(\frac{0.021\ 0\ \text{g}}{12.0\ \text{g/mol}}\right)\left(\frac{6.02\times10^{23}\ \text{atoms}}{\text{mol}}\right) = \boxed{1.05\times10^{21}}$$

(b) 1 in 7.70 × 10^{11} carbon atoms is a ^{14}C atom. Then,

$$\left(N_0\right)_{\text{C-14}} = 1.05\times10^{21}\left(\frac{1}{7.70\times10^{11}}\right) = \boxed{1.37\times10^{9}}$$

(c) The decay constant for ^{14}C is

$$\lambda_{\text{C-14}} = \frac{\ln 2}{5\,730\ \text{yr}} = 1.21\times10^{-4}\ \text{yr}^{-1}\left(\frac{1\ \text{yr}}{3.16\times10^{7}\,\text{s}}\right)$$
$$= \boxed{3.83\times10^{-12}\ \text{s}^{-1}}$$

(d) We use $R = \lambda N = \lambda N_0 e^{-\lambda t}$. At $t = 0$,

$$R_0 = \lambda N_0 = \left(3.83\times10^{-12}\ \text{s}^{-1}\right)\left(1.37\times10^{9}\right)\left[\frac{7\left(86\ 400\ \text{s}\right)}{1\ \text{week}}\right]$$
$$= \boxed{3.17\times10^{3}\,\text{decays/week}}$$

(e) At time *t*, $R = \frac{837}{0.880} = \boxed{951\ \text{decays/week}}$.

(f) Taking logarithms,

$$\ln\frac{R}{R_0} = -\lambda t \qquad \text{so} \qquad t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right)$$

and

$$t = \frac{-1}{1.21\times10^{-4}\ \text{yr}^{-1}}\ln\left(\frac{951}{3.17\times10^{3}}\right) = \boxed{9.95\times10^{3}\ \text{yr}}$$

Section 44.6 Natural Radioactivity

P44.43 (a) The conversion is

$$4.00\ \text{pCi/L} = \left(\frac{4.00\times10^{-12}\ \text{Ci}}{1\ \text{L}}\right)\left(\frac{3.70\times10^{10}\ \text{Bq}}{1\ \text{Ci}}\right)\left(\frac{1.00\times10^{3}\ \text{L}}{1\ \text{m}^3}\right)$$
$$= \boxed{148\ \text{Bq/m}^3}$$

(b) Each cubic meter of air contains

$$N = \frac{R}{\lambda} = R\left(\frac{T_{1/2}}{\ln 2}\right) = \left(148\ \text{Bq/m}^3\right)\left(\frac{3.82\ \text{d}}{\ln 2}\right)\left(\frac{86\,400\ \text{s}}{1\ \text{d}}\right)$$
$$= \boxed{7.05\times10^{7}\ \text{atoms/m}^3}$$

(c) The density of radon in each cubic meter of air is

$$\text{density} = \left(7.05\times10^{7}\ \text{atoms/m}^3\right)\left(\frac{1\ \text{mol}}{6.02\times10^{23}\ \text{atoms}}\right)\left(\frac{222\ \text{g}}{1\ \text{mol}}\right)$$
$$= 2.60\times10^{-14}\ \text{g/m}^3$$

Since air has a density of 1.20 kg/m^3, the fraction consisting of radon is

$$\text{fraction} = \frac{2.60\times10^{-14}\ \text{g/m}^3}{1\,200\ \text{g/m}^3} = \boxed{2.17\times10^{-17}}$$

P44.44 The number of radon atoms remaining is

$$N = N_0 e^{-(\ln 2)t/T_{1/2}}$$

And the fraction remaining is

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-(\ln 2)t/T_{1/2}}$$

(a) With $T_{1/2} = 3.82$ d and $t = 7.00$ d,

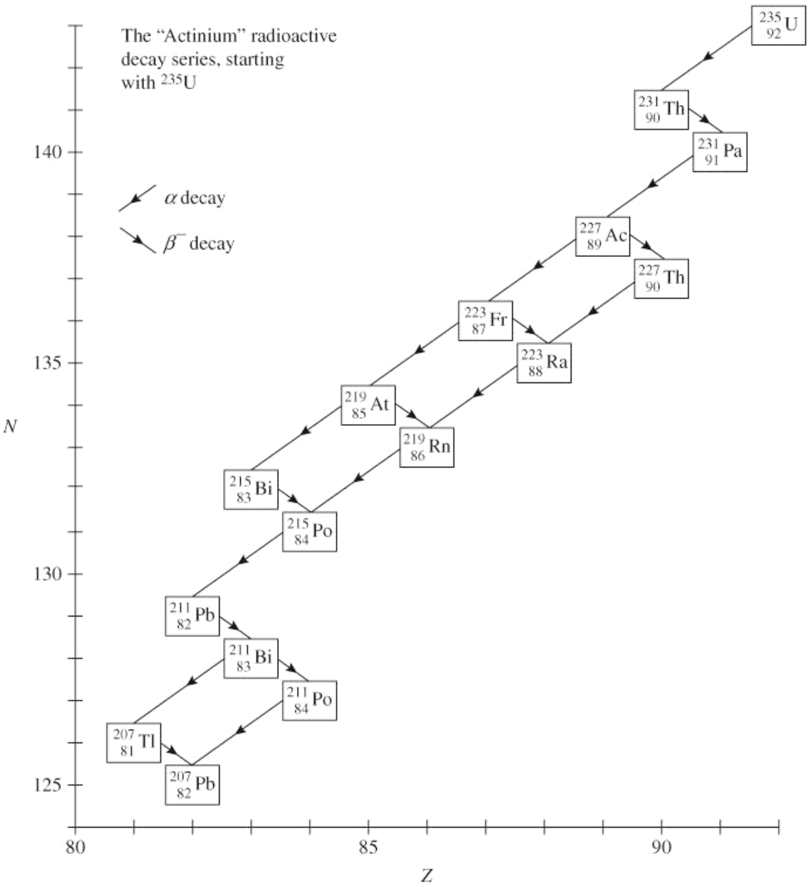
$$\frac{N}{N_0} = e^{-(\ln 2)(7.00)/(3.82)} = \boxed{0.281}$$

(b) When $t = 1.00$ yr $= 365.25$ d,

$$\frac{N}{N_0} = e^{-(\ln 2)(365.25)/(3.82)} = \boxed{1.65 \times 10^{-29}}$$

(c) $\boxed{\text{Radon is continuously created}}$ as one daughter in the series of decays starting from the long-lived isotope ^{238}U.

P44.45 We find the chemical name by looking up Z in a periodic table. The values in the shaded boxes $(^{235}\text{U}$ and $^{207}\text{Pb})$ in Figure P44.45 were given; all others have been filled in as part of the solution shown in ANS. FIG. P44.45 below.



ANS. FIG. P44.45

P44.46 (a) Let N be the number of ^{238}U nuclei and N' be ^{206}Pb nuclei.

Then $N = N_0 e^{-\lambda t}$ and $N_0 = N + N'$ so $N = (N + N')e^{-\lambda t}$ or $e^{\lambda t} = 1 + \dfrac{N'}{N}$. Taking logarithms,

$$\lambda t = \ln\left(1 + \frac{N'}{N}\right) \qquad \text{where} \qquad \lambda = \frac{\ln 2}{T_{1/2}}$$

Thus,

$$t = \left(\frac{T_{1/2}}{\ln 2}\right)\ln\left(1 + \frac{N'}{N}\right)$$

If $\dfrac{N}{N'} = 1.164$ for the $^{238}\text{U} \to {}^{206}\text{Pb}$ chain with $T_{1/2} = 4.47 \times 10^9$ yr, the age is:

$$t = \left(\frac{4.47 \times 10^9 \text{ yr}}{\ln 2}\right)\ln\left(1 + \frac{1}{1.164}\right) = \boxed{4.00 \times 10^9 \text{ yr}}$$

(b) From above, $e^{\lambda t} = 1 + \dfrac{N'}{N}$. Solving for $\dfrac{N}{N'}$ gives $\dfrac{N}{N'} = \dfrac{e^{-\lambda t}}{1 - e^{-\lambda t}}$.

With $T = 4.00 \times 10^9$ yr and $T_{1/2} = 7.04 \times 10^8$ yr for the $^{235}\text{U} \to {}^{207}\text{Pb}$ chain,

$$\lambda t = \left(\frac{\ln 2}{T_{1/2}}\right)t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{7.04 \times 10^8 \text{ yr}} = 3.938$$

and $\boxed{\dfrac{N}{N'} = 0.019\,9 \text{ for the } {}^{235}\text{U to } {}^{207}\text{Pb chain}}$.

With $T = 4.00 \times 10^9$ yr and $T_{1/2} = 1.41 \times 10^{10}$ yr for the $^{232}\text{Th} \to {}^{208}\text{Pb}$ chain,

$$\lambda t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{1.41 \times 10^{10} \text{ yr}} = 0.196\,6$$

and $\boxed{\dfrac{N}{N'} = 4.60 \text{ for the } {}^{232}\text{Th to } {}^{208}\text{Pb chain}}$.

Section 44.7 Nuclear Reactions

P44.47 Neglect recoil of product nucleus (i.e., do not require momentum conservation for the system of colliding particles). The energy balance gives $K_{\text{emerging}} = K_{\text{incident}} + Q$. To find *Q*,

$$Q = \left[\left(M_{\text{H}} + M_{\text{Al}}\right) - \left(M_{\text{Si}} + m_n\right)\right]c^2$$

$$Q = \left[(1.007\ 825 + 26.981\ 539) - (26.986\ 705 + 1.008\ 665)\right]\ \text{u} \times (931.5\ \text{MeV/u})$$

$$= -5.59\ \text{MeV}$$

Thus, $K_{\text{emerging}} = 6.61\ \text{MeV} - 5.59\ \text{MeV} = \boxed{1.02\ \text{MeV}}$.

P44.48 (a) The *Q* value of the reaction is given by

$$Q = \left[M_{^9\text{Be}} + M_{^4\text{He}} - M_{^{12}\text{C}} - m_n\right]c^2$$

$$Q = [9.012\ 182\ \text{u} + 4.002\ 603\ \text{u} - 12.000\ 000\ \text{u} - 1.008\ 665\ \text{u}] \times (931.5\ \text{MeV/u})$$

$$= \boxed{5.70\ \text{MeV}}$$

(b) For this reaction,

$$Q = \left[2M_{^2\text{H}} - M_{^3\text{He}} - m_n\right]$$

$$Q = [2(2.014\ 102\ \text{u}) - 3.016\ 029\ \text{u} - 1.008\ 665\ \text{u}] \times (931.5\ \text{MeV/u})$$

$$= \boxed{3.27\ \text{MeV}}$$

(c) The reaction in part (b) is $\boxed{\text{exothermic}}$ because the *Q* value is positive.

P44.49 Total *A* and total *Z* are conserved.

(a) For *X*, *A* = 24 + 1 – 4 = 21 and *Z* = 12 + 0 – 2 = 10, so *X* is $\boxed{^{21}_{10}\text{Ne}}$.

(b) *A* = 235 + 1 – 90 – 2 = 144 and *Z* = 92 + 0 – 38 – 0 = 54, so *X* is $\boxed{^{144}_{54}\text{Xe}}$.

(c) *A* = 2 – 2 = 0 and *Z* = 2 – 1 = +1, so *X* must be a positron.

As it is ejected, it is accompanied by a neutrino:

$$X + X' = \boxed{^0_1\text{e}^+ + \nu}$$

P44.50 (a) $\boxed{{}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{79}\text{Au}^{*} \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}}$

Note the conservation of baryon number (which you can think of as nucleon census number and call mass number in this chapter) in the superscripts: 197 + 1 = 198 + 0. Note the conservation of charge in the subscripts: 79 + 0 = 80 − 1.

(b) Consider adding 79 electrons:

$${}^{197}_{79}\text{Au atom} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{80}\text{Hg atom} + \bar{\nu} + Q$$

Then,

$$Q = \left[M_{{}^{197}\text{Au}} + m_n - M_{{}^{198}\text{Hg}}\right]c^2$$

$$Q = [196.966\ 552\ \text{u} + 1.008\ 665\ \text{u} - 197.966\ 752\ \text{u}] \times (931.5\ \text{MeV/u})$$

$$= \boxed{7.89\ \text{MeV}}$$

P44.51 We consult Table 44.2 for the masses. For the first reaction,

$${}^{9}_{4}\text{Be} + 1.665\ \text{MeV} \rightarrow {}^{8}_{4}\text{Be} + {}^{1}_{0}\text{n}$$

so $$M_{{}^{8}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} - \frac{Q}{c^2} - m_n$$

$$M_{{}^{8}_{4}\text{Be}} = 9.012\,182\ \text{u} - \frac{(-1.665\ \text{MeV})}{931.5\ \text{MeV/u}} - 1.008\,665\ \text{u}$$

$$= \boxed{8.005\,3\ \text{u}}$$

For the second reaction,

$${}^{9}_{4}\text{Be} + {}^{1}_{0}\text{n} \rightarrow {}^{10}_{4}\text{Be} + 6.812\ \text{MeV}$$

so $$M_{{}^{10}_{4}\text{Be}} = M_{{}^{9}_{4}\text{Be}} + m_n - \frac{Q}{c^2}$$

$$M_{{}^{10}_{4}\text{Be}} = 9.012\,182\ \text{u} + 1.008\,665\ \text{u} - \frac{6.812\ \text{MeV}}{931.5\ \text{MeV/u}}$$

$$= \boxed{10.013\,5\ \text{u}}$$

Section 44.8 Nuclear Magnetic Resonance and Magnetic Resonance Imaging

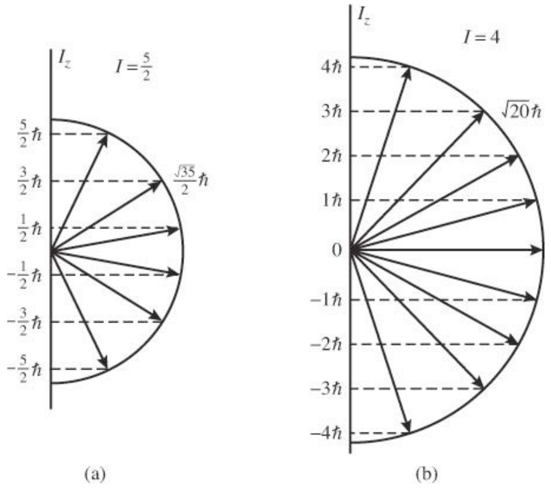
P44.52 It is the quantum particle under boundary conditions model that is behind the general rules: With angular momentum quantum number I, the magnitude of the angular momentum must be $\sqrt{I(I+1)}\,\hbar$. Whether I is an integer or a half-integer, the allowed values for one component of angular momentum being measured range from $+I\hbar$ to $+(I-1)\hbar$ to … to $-I\hbar$. Conditions that the wave function for a quantum particle must satisfy, for self-consistency under rotations in three-dimensional space, impose these requirements. We call a component being measured the z component. It can be measured more directly, as in a nuclear magnetic resonance experiment, or less directly, as from the way the angular momentum influences the intrinsic energy levels of a system and the number of available states within an energy level.

(a) With $I = 5/2$, the magnitude of the angular momentum is

$$\begin{aligned}\sqrt{I(I+1)}\,\hbar = \sqrt{\tfrac{5}{2}\left(\tfrac{5}{2}+1\right)}\,\hbar = \sqrt{35}\,\hbar/2\\ = 2.958\ 04(6.626\times10^{-34}\ \mathrm{J\cdot s})/2\pi\\ = 3.119\times10^{-34}\ \mathrm{kg\cdot m^2/s}\end{aligned}$$

The z component can take the values $+5\hbar/2$, $+3\hbar/2$, $+\hbar/2$, $-\hbar/2$, $-3\hbar/2$, and $-5\hbar/2$. These identifications are shown in ANS. FIG. P44.52(a).

(b) Similarly, with $I = 4$, the magnitude of the angular momentum of a nucleus is $\sqrt{I(I+1)}\,\hbar = \sqrt{4(4+1)}\,\hbar = \sqrt{20}\,\hbar$ and its z component must have one of the nine values $+4\hbar$, $+3\hbar$, $+2\hbar$, $+\hbar$, 0, $-\hbar$, $-2\hbar$, $-3\hbar$, $-4\hbar$, as shown in ANS. FIG. P44.52(b).



(a) (b)

ANS. FIG. P44.52

P44.53 From page 1406 and Equation 44.31, the magnetic moment is $-1.913\,5\mu_n$ for the neutron and $2.792\,8\mu_n$ for the proton, where $\mu_n = 5.05\times10^{-27}$ J/T is the nuclear magneton.

(a) The Larmor frequency of free neutrons is

$$f_n = \frac{|2\mu B|}{h} = \frac{2\left[(1.913\,5)(5.05\times10^{-27}\text{ J/T})\right](1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}}$$

$$= \boxed{29.2\text{ MHz}}$$

(b) The Larmor frequency of free protons is

$$f_p = \frac{2\left[(2.792\,8)(5.05\times10^{-27}\text{ J/T})\right](1.00\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{42.6\text{ MHz}}$$

(c) In the Earth's magnetic field,

$$f_p = \frac{2\left[(2.792\,8)(5.05\times10^{-27}\text{ J/T})\right](50.0\times10^{-6}\text{ T})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{2.13\text{ kHz}}$$

Additional Problems

***P44.54** From $R = R_0e^{-\lambda t}$ and $T_{1/2} = 5\,730$ yr for ^{14}C, the age of the sample is

$$t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right) = -T_{1/2}\left[\frac{\ln(R/R_0)}{\ln 2}\right] = -(5\,730\text{ yr})\left[\frac{\ln(0.600)}{\ln 2}\right]$$

$$= \boxed{4.22\times10^3\text{ yr}}$$

***P44.55** From $R = R_0e^{-\lambda t}$, the elapsed time is

$$t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right) = -T_{1/2}\left[\frac{\ln(R/R_0)}{\ln 2}\right] = -(14.0\text{ d})\left[\frac{\ln\left(\frac{20.0\text{ mCi}}{200\text{ mCi}}\right)}{\ln 2}\right]$$

$$= \boxed{46.5\text{ d}}$$

P44.56 The proposed reaction can be written as

$${}^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{1}_{1}\text{H} + {}^{12}_{6}\text{C}$$

While electric charge is conserved (5 + 2 = 1 + 6), the number of nucleons is not (10 + 4 ≠ 1 + 12). Therefore, this reaction cannot occur.

P44.57 (a) From Equation 44.1,

$$r = aA^{1/3} = (1.2\text{ fm})A^{1/3} = (1.2\times10^{-15}\text{ m})A^{1/3}$$

When $A = 12$, $r = 1.2\text{ fm}(12)^{1/3} = 2.7\times10^{-15}\text{ m} = \boxed{2.7\text{ fm}}$

(b) $F = \dfrac{k_e(Z-1)e^2}{r^2} = \dfrac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(Z-1)(1.60\times10^{-19}\text{ C})^2}{r^2}$

When $Z = 6$ and $r = (1.2\times10^{-15}\text{ m})(12)^{1/3}$, $F = \boxed{1.5\times10^2\text{ N}}$.

(c) $U = \dfrac{k_e q_1 q_2}{r} = \dfrac{k_e(Z-1)e^2}{r} = \dfrac{(8.99\times10^9)(Z-1)(1.6\times10^{-19})^2}{r}$

When $Z = 6$ and $r = (1.2\times10^{-15}\text{ m})(12)^{1/3}$,

$$U = 4.2\times10^{-13}\text{ J} = \boxed{2.6\text{ MeV}}$$

(d) $A = 238$, $Z = 92$, and $r = 1.2\text{ fm}(238)^{1/3} = 7.4\times10^{-15}\text{ m} = \boxed{7.4\text{ fm}}$

$$F = \boxed{3.8\times10^2\text{ N}} \quad\text{and}\quad U = 2.8\times10^{-12}\text{ J} = \boxed{18\text{ MeV}}$$

P44.58 (a) $\boxed{\text{The process cannot occur because energy input would be required}}$. Note that the mass of the proton is less than the sum of the masses of the neutron and positron (electron):

$$m_n + m_{e^+} > m_p$$
$$1.008\,665\text{ u} + 0.000\,549\text{ u}$$
$$1.009\,214\text{ u} > 1.007\,276\text{ u}$$

Therefore, the reaction $p \to n + e^+ + \nu$ would violate the law of conservation of energy.

(b) The $\boxed{\text{required energy can come from the electrostatic repulsion}}$ of protons in the parent nucleus.

(c) Add seven electrons to both sides of the reaction for nuclei ${}^{13}_{7}\text{N} \to {}^{13}_{6}\text{C} + e^+ + \nu$ to obtain the reaction for neutral atoms ${}^{13}_{7}\text{N atom} \to {}^{13}_{6}\text{C atom} + e^+ + e^- + \nu$.

$$Q = \left[m({}^{13}\text{N}) - m({}^{13}\text{C}) - m_{e^+} - m_{e^-} - m_\nu\right]c^2$$

$$Q = \left[13.005\,739\text{ u} - 13.003\,355\text{ u} - 2(5.49\times10^{-4}\text{ u}) - 0\right] \times(931.5\text{ MeV/u})$$

$$= \boxed{1.20\text{ MeV}}$$

P44.59 $E = -\vec{\mu}\cdot\vec{\mathbf{B}}$ so the energies are $E_1 = +\mu B$ and $E_2 = -\mu B$, where $B = 12.5$ T, $\mu = 2.792\,8\mu_n$, and $\mu_n = 5.05\times10^{-27}$ J/T. The energy difference is

$$\Delta E = 2\mu B = 2(2.792\,8)(5.05\times10^{-27}\ \text{J/T})(12.5\ \text{T})$$
$$= 3.53\times10^{-25}\ \text{J} = 2.20\times10^{-6}\,\text{eV} = \boxed{2.20\ \mu\text{eV}}$$

P44.60 We check the *Q* value of this reaction:

$$Q = [238.050\,788\ \text{u} - 237.051\,144\ \text{u} - 1.007\,825\ \text{u}]\times(931.5\ \text{MeV/u})$$
$$= -7.62\ \text{MeV}$$

$$\boxed{\text{The Q value of this hypothetical decay is calculated to be }-7.62\text{ MeV, which means you would have to add this much energy to the }{}^{238}\text{U nucleus to make it emit a proton.}}$$

P44.61 (a) The system of a separated proton and electron puts out energy 13.606 eV to become a hydrogen atom in its ground state. This decrease in its rest energy appears also as a decrease in mass: the mass is $\boxed{\text{smaller}}$.

(b) The mass difference is

$$|\Delta m| = \frac{|E|}{c^2} = \left[\frac{13.6\ \text{eV}}{(3.00\times10^8\ \text{m/s})^2}\right]\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)$$
$$= (2.42\times10^{-35}\ \text{kg})\left(\frac{1\ \text{u}}{1.66\times10^{-27}\ \text{kg}}\right) = \boxed{1.46\times10^{-8}\ \text{u}}$$

(c) As a percentage of the total mass,

$$\frac{1.46\times10^{-8}\ \text{u}}{1.007\,825\ \text{u}} = 1.45\times10^{-8} = \boxed{1.45\times10^{-6}\%}$$

(d) $\boxed{\text{No.}}$ The textbook table lists 1.007 825 u as the atomic mass of hydrogen. This correction of 0.000 000 01 u is on the order of 100 times too small to affect the values listed.

P44.62 We check the *Q* value of the ^{57}Co nuclei decay by e^+:

$${}^{57}_{27}\text{Co} \to {}^{57}_{26}\text{Fe} + {}^{0}_{+1}\text{e} + \bar{\nu}$$

Mass values appear in Table 44.2. For this reaction,

$$Q = [56.936\,291 - 56.935\,394 - 2(0.000\,549)]\text{u}(931.5\ \text{MeV/u})$$
$$= -0.187\ \text{MeV}$$

$$\boxed{\text{The nucleus } {}^{57}\text{Co cannot decay by e}^{+} \text{ emission because the } Q \text{ value is } -0.187 \text{ MeV.}}$$

P44.63 (a) The number of nuclei at *t* = 0 is given by

$$N_0 = \frac{\text{mass}}{\text{mass per atom}} = \frac{1.00 \text{ kg}}{(239.05 \text{ u})\left(1.66\times10^{-27} \text{ kg/u}\right)}$$

$$= \boxed{2.52\times10^{24}}$$

(b) To find the initial activity, we first compute the ecay constant:

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{\left(2.412\times10^{4} \text{ yr}\right)\left(3.156\times10^{7} \text{ s/yr}\right)} = 9.106\times10^{-13} \text{ s}^{-1}$$

Then,

$$R_0 = \lambda N_0 = \left(9.106\times10^{-13} \text{ s}^{-1}\right)\left(2.52\times10^{24}\right) = \boxed{2.29\times10^{12} \text{ Bq}}$$

(c) From $R = R_0 e^{-\lambda t}$,

$$t = \frac{-1}{\lambda}\ln\left(\frac{R}{R_0}\right) = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right)$$

$$t = \frac{1}{9.106\times10^{-13} \text{ s}^{-1}}\ln\left(\frac{2.29\times10^{12} \text{ Bq}}{0.100 \text{ Bq}}\right)$$

$$= 3.38\times10^{13} \text{ s}\left(\frac{1 \text{ yr}}{3.156\times10^{7} \text{ s}}\right) = \boxed{1.07\times10^{6} \text{yr}}$$

P44.64 (a) One liter of milk contains this many ${}^{40}\text{K}$ nuclei:

$$N = (2.00 \text{ g})\left(\frac{6.02\times10^{23} \text{ nuclei/mol}}{39.1 \text{ g/mol}}\right)\left(\frac{0.011\,7}{100}\right)$$

$$= 3.60\times10^{18} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{1.28\times10^{9} \text{ yr}}\left(\frac{1 \text{ yr}}{3.156\times10^{7} \text{ s}}\right) = 1.72\times10^{-17} \text{ s}^{-1}$$

$$R = \lambda N = \left(1.72\times10^{-17} \text{ s}^{-1}\right)\left(3.60\times10^{18}\right) = 61.8 \text{ Bq}$$

The activity is $\boxed{61.8 \text{ Bq/L}}$.

(b) For the iodine, $R = R_0 e^{-\lambda t}$, with $\lambda = \dfrac{\ln 2}{8.04 \text{ d}}$. Then,

$$t = \frac{1}{\lambda}\ln\left(\frac{R_0}{R}\right) = \frac{8.04 \text{ d}}{\ln 2}\ln\left(\frac{2\,000}{61.8}\right) = \boxed{40.3 \text{ d}}$$

P44.65 We have $N_{\text{U-235}} = N_{0,\text{ U-235}}e^{-\lambda_{\text{U-235}}t}$

and $N_{\text{U-238}} = N_{0,\text{ U-238}}e^{-\lambda_{\text{U-238}}t}$,

so $\dfrac{N_{\text{U-235}}}{N_{\text{U-238}}} = 0.007\ 25 = e^{\left(-(\ln 2)t/T_{1/2,\text{ U-235}}+(\ln 2)t/T_{1/2,\text{ U-238}}\right)}$

Taking logarithms,

$$-4.93 = \left(-\frac{\ln 2}{0.704\times 10^9\text{ yr}} + \frac{\ln 2}{4.47\times 10^9\text{ yr}}\right)t$$

$$-4.93 = \left(-\frac{1}{0.704\times 10^9\text{ yr}} + \frac{1}{4.47\times 10^9\text{ yr}}\right)(\ln 2)t$$

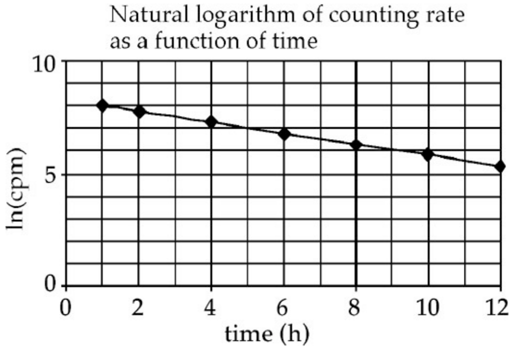
$$t = \frac{-4.93}{\left(-1.20\times 10^{-9}\text{ yr}^{-1}\right)\ln 2} = 5.94\times 10^9\text{ yr} = \boxed{5.94\text{ Gyr}}$$

P44.66 (a) See ANS. FIG. P44.66. A least-square fit to the graph yields:

$$\lambda = -\text{slope} = -\left(-0.250\text{ h}^{-1}\right) = 0.250\text{ h}^{-1}$$

and

$$\ln(\text{cpm})\big|_{t=0} = \text{intercept} = 8.30$$



ANS. FIG. P44.66

(b) From part (a),

$$\lambda = 0.250\text{ h}^{-1}\left(\frac{1\text{ h}}{60.0\text{ min}}\right) = \boxed{4.17\times 10^{-3}\text{ min}^{-1}}$$

and $T_{1/2} = \dfrac{\ln 2}{\lambda} = \dfrac{\ln 2}{4.17\times 10^{-3}\text{ min}^{-1}} = 166\text{ min} = \boxed{2.77\text{ h}}$

(c) From part (a), $\text{intercept} = \ln(\text{cpm})_0 = 8.30$.

Thus, $(\text{cpm})_0 = e^{8.30}\text{ counts/min} = \boxed{4.02\times 10^3\text{ counts/min}}$.

(d) At $t = 0$,

$$N_0 = \frac{R_0}{\lambda} = \frac{1}{\lambda}\frac{(\text{cpm})_0}{\text{Eff}} = \frac{4.02\times 10^3\ \text{counts/min}}{(4.17\times 10^{-3}\ \text{min}^{-1})(0.100)}$$

$$= \boxed{9.65\times 10^6\ \text{atoms}}$$

P44.67 (a) If ΔE is the energy difference between the excited and ground states of the nucleus of mass *M*, and *hf* is the energy of the emitted photon, conservation of energy for the nucleus-photon system gives

$$\Delta E = hf + E_r \qquad \textbf{[1]}$$

where E_r is the recoil energy of the nucleus, which can be expressed as

$$E_r = \frac{Mv^2}{2} = \frac{(Mv)^2}{2M} \qquad \textbf{[2]}$$

Since system momentum must also be conserved, we have

$$Mv = \frac{hf}{c} \qquad \textbf{[3]}$$

Hence, E_r can be expressed as $E_r = \dfrac{(hf)^2}{2Mc^2}$.

When $hf \ll Mc^2$, we can make the approximation that $hf \approx \Delta E$,

so $E_r \approx \dfrac{(\Delta E)^2}{2Mc^2}$.

(b) $E_r = \dfrac{(\Delta E)^2}{2Mc^2}$, where $\Delta E = 0.014\ 4\ \text{MeV}$

and $Mc^2 = (57\ \text{u})(931.5\ \text{MeV/u}) = 5.31\times 10^4\ \text{MeV}$.

Therefore,

$$E_r = \frac{(1.44\times 10^{-2}\ \text{MeV})^2}{2(5.31\times 10^4\ \text{MeV})} = 1.95\times 10^{-9}\ \text{MeV} = \boxed{1.95\times 10^{-3}\ \text{eV}}$$

P44.68 (a) If we assume all the ${}^{87}\text{Sr}$ came from ${}^{87}\text{Rb}$, then $N = N_0 e^{-\lambda t}$ yields

$$t = \frac{-1}{\lambda}\ln\left(\frac{N}{N_0}\right) = \frac{T_{1/2}}{\ln 2}\ln\left(\frac{N_0}{N}\right)$$

where $\quad N = N_{\text{Rb-87}}$

and $\quad N_0 = N_{\text{Sr-87}} + N_{\text{Rb-87}}.$

$$t = \frac{\left(4.75\times10^{10}\text{ yr}\right)}{\ln 2}\ln\left(\frac{1.82\times10^{10}+1.07\times10^{9}}{1.82\times10^{10}}\right) = \boxed{3.91\times10^{9}\text{ yr}}$$

(b) It could be $\boxed{\text{no older.}}$ The rock could be younger if some ^{87}Sr were originally present. We must make some assumption about the original quantity of radioactive material. In part (a) we assumed that the rock originally contained no strontium.

P44.69 The time of flight is given by $\Delta t = d/v$. Since $K = \frac{1}{2}mv^2$,

$$\Delta t = \frac{d}{\sqrt{\frac{2K}{m}}} = \frac{10.0\times10^{3}\text{ m}}{\sqrt{\frac{2(0.040\,0\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}{1.67\times10^{-27}\text{ kg}}}} = 3.61\text{ s}$$

The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(10.4\text{ min})(60\text{ s/min})} = 1.11\times10^{-3}\text{ s}^{-1}$$

Therefore we have

$$\lambda\Delta t = \left(1.11\times10^{-3}\text{ s}\right)(3.61\text{ s}) = 4.01\times10^{-3} = 0.004\,01$$

And the fraction remaining is

$$\frac{N}{N_0} = e^{-\lambda\Delta t} = e^{-0.004\,01} = 0.996\,0.$$

Hence, the fraction that has decayed in this time interval is

$$1-\frac{N}{N_0} = 0.004\,01 \quad\text{or}\quad \boxed{0.401\%}$$

P44.70 (a) For cobalt-56,

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{77.1\text{ d}}\left(\frac{365.25\text{ d}}{1\text{ yr}}\right) = 3.28\text{ yr}^{-1}$$

The elapsed time from July 1054 to July 2010 is 956 yr. Then, $R = R_0e^{-\lambda t}$ implies

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(3.28\text{ yr}^{-1}\right)(956\text{ yr})} = e^{-3\,139} = e^{-(\ln 10)1\,363} = \boxed{\sim 10^{-1\,363}}$$

(b) For carbon-14,

$$\lambda = \frac{\ln 2}{5\,730 \text{ yr}} = 1.21\times10^{-4} \text{ yr}^{-1}$$

and

$$\frac{R}{R_0} = e^{-\lambda t} = e^{-\left(1.21\times10^{-4} \text{ yr}^{-1}\right)\left(956 \text{ yr}\right)} = e^{-0.116} = \boxed{0.891}$$

P44.71 (a) For the electron capture,

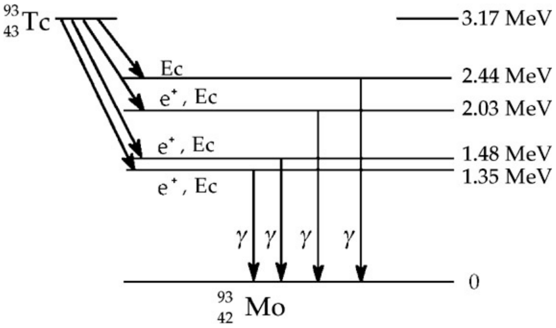
$${}^{93}_{43}\text{Tc} + {}^{0}_{-1}\text{e} \rightarrow {}^{93}_{42}\text{Mo} + \gamma$$

For positron emission,

$${}^{93}_{43}\text{Tc} \rightarrow {}^{93}_{42}\text{Mo} + {}^{0}_{+1}\text{e} + \gamma$$

The daughter nucleus in both forms of decay is $\boxed{{}^{93}_{42}\text{Mo}}$.

(b) We usually calculate the Q value under the assumption that the daughter nucleus is in its ground state, but for these decays, the Q value gives the upper limit of energy available to the daughter nucleus to be above its ground state.



ANS. FIG. P44.71

For electron capture, the disintegration energy is

$$Q = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}}\right]c^2$$

$$Q = \left[92.910\,2 \text{ u} - 92.906\,8 \text{ u}\right]\left(931.5 \text{ MeV/u}\right)$$
$$= 3.17 \text{ MeV} > 2.44 \text{ MeV}$$

so $\boxed{\text{electron capture}}$ provides enough energy for ${}^{93}_{42}\text{Mo}$ to be in $\boxed{\text{all levels}}$ above its ground state.

For e^+ emission, the disintegration energy is

$$Q' = \left[M_{^{93}\text{Tc}} - M_{^{93}\text{Mo}} - 2m_e\right]c^2.$$

$$Q' = \left[92.910\,2 \text{ u} - 92.906\,8 \text{ u} - 2(0.000\,549 \text{ u})\right](931.5 \text{ MeV/u})$$
$$= 2.14 \text{ MeV}$$

so $\boxed{e^+ \text{ emission}}$ does not supply enough energy for ${}^{93}_{42}\text{Mo}$ to be in the 4.22 MeV state, $\boxed{\text{only 1.35 MeV, 1.48 MeV, and 1.35 MeV}}$ above ground (see ANS. FIG. P44.71).

P44.72 We start with $R = R_0 e^{-\lambda t}$, and take the natural logarithm of both sides, giving $\ln R = \ln R_0 - \lambda t$, which is the equation of a straight line with $|\text{slope}| = \lambda$. The logarithmic plot shown in Figure P44.72 is fitted by

$$\ln R = 8.44 - 0.262t$$

If *t* is measured in minutes, then decay constant λ is 0.262 per minute. The half–life is

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.262/\text{min}} = \boxed{2.64 \text{ min}}$$

The reported half–life of ${}^{137}\text{Ba}$ is 2.55 min. The difference reflects experimental uncertainties.

P44.73 (a) With m_n and v_n as the mass and speed of the neutrons, Equation 9.24 for elastic collisions becomes for the two collisions, after making appropriate notational changes,

$$v_1 = \left(\frac{2m_n}{m_n + m_1}\right)v_n, \text{ and } v_2 = \left(\frac{2m_n}{m_n + m_2}\right)v_n$$

Solving,

$$(m_n + m_2)v_2 = (m_n + m_1)v_1 = 2m_n v_n$$

$$m_n(v_2 - v_1) = m_1 v_1 - m_2 v_2 \qquad \rightarrow \qquad m_n = \frac{m_1 v_1 - m_2 v_2}{v_2 - v_1}$$

(b) We obtain the neutron mass from

$$m_n = \frac{(1\text{ u})(3.30 \times 10^7 \text{ m/s}) - (14\text{ u})(4.70 \times 10^6 \text{ m/s})}{4.70 \times 10^6 \text{ m/s} - 3.30 \times 10^7 \text{ m/s}} = \boxed{1.16 \text{ u}}$$

P44.74 (a) We treat the collision of the two particles a and X as a perfectly inelastic collision: the kinetic energy that is converted into internal energy supplies the missing energy *Q*, permitting the conversion of the particles into Y and b.

Initially, the projectile M_a moves with velocity v_a while the target M_X is at rest. We have from momentum conservation for the projectile-target system:

$$M_a v_a = (M_a + M_X)v_c$$

The initial energy is

$$E_i = \frac{1}{2} M_a v_a^2$$

The final kinetic energy is:

$$E_f = \frac{1}{2}(M_a + M_X)v_c^2 = \frac{1}{2}(M_a + M_X)\left[\frac{M_a v_a}{M_a + M_X}\right]^2$$

$$= \left[\frac{M_a}{M_a + M_X}\right] E_i$$

From this, we see that E_f is always less than E_i and the change in energy, $E_f - E_i$, is given by

$$E_f - E_i = \left[\frac{M_a}{M_a + M_X} - 1\right] E_i = -\left[\frac{M_X}{M_a + M_X}\right] E_i$$

This loss of kinetic energy in the isolated system corresponds to an increase in mass-energy during the reaction. Thus, the absolute value of this kinetic energy change is equal to –*Q* (remember that *Q* is negative in an endothermic reaction). The initial kinetic energy E_i is the threshold energy E_{th}. Therefore,

$$-Q = \left[\frac{M_X}{M_a + M_X}\right] E_{th}$$

or $$E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -Q\left[1 + \frac{M_a}{M_X}\right].$$

(b) We first calculate the *Q* value for the reaction:

$$Q = \left[M_{\text{N-14}} + M_{\text{He-4}} - M_{\text{O-17}} - M_{\text{H-1}}\right]c^2$$

$$Q = [14.003\ 074\ \text{u} + 4.002\ 603\ \text{u} - 16.999\ 132\ \text{u} - 1.007\ 825\ \text{u}] \times (931.5\ \text{MeV/u})$$

$$= -1.19\ \text{MeV}$$

Then,

$$E_{th} = -Q\left[\frac{M_X + M_a}{M_X}\right] = -(-1.19\ \text{MeV})\left[1 + \frac{4.002\ 603\ \text{u}}{14.003\ 074\ \text{u}}\right]$$

$$= \boxed{1.53\ \text{MeV}}$$

P44.75 We have the following information: $N_X(0) = 2.50N_Y(0)$, $N_X(3\text{ d}) = 4.20N_Y(3\text{ d})$, and $T_{1/2\,Y} = 1.60$ d. The nuclei decay exponentially:

$$N_X(3\text{ d}) = 4.20N_Y(3\text{ d})$$

$$N_X(0)e^{-\lambda_X(3\text{ d})} = 4.20N_Y(0)e^{-\lambda_Y(3\text{ d})} = 4.20\frac{N_X(0)}{2.50}e^{-\lambda_Y(3\text{ d})}$$

$$e^{(3\text{ d})\lambda_X} = \frac{2.5}{4.2}e^{(3\text{ d})\lambda_Y}$$

Taking the natural logarithm of both sides,

$$(3\text{ d})\lambda_X = \ln\left(\frac{2.5}{4.2}\right) + (3\text{ d})\lambda_Y$$

$$(3\text{ d})\frac{0.693}{T_{1/2\,X}} = \ln\left(\frac{2.5}{4.2}\right) + (3\text{ d})\frac{0.693}{1.60\text{ d}} = 0.781$$

The half-life of X is $T_{1/2\,X} = \boxed{2.66\text{ d}}$

P44.76 We have the following information: $\dfrac{N_X(0)}{N_Y(0)} = r_1$, $\dfrac{N_X(\Delta t)}{N_Y(\Delta t)} = r_2$, and $T_{1/2\,Y} = T_Y$. The nuclei decay exponentially:

$$N_X(\Delta t) = r_2N_Y(\Delta t)$$

$$N_X(0)e^{-\lambda_X\Delta t} = r_2N_Y(0)e^{-\lambda_Y\Delta t} = \left(\frac{r_2}{r_1}\right)N_X(0)e^{-\lambda_Y\Delta t}$$

$$e^{-\Delta t\lambda_X} = \frac{r_2}{r_1}e^{-\Delta t\lambda_Y}$$

Taking the natural logarithm of both sides,

$$-\Delta t\lambda_X = \ln\left(\frac{r_2}{r_1}\right) - \Delta t\lambda_Y$$

$$\Delta t\frac{\ln 2}{T_X} = -\ln\left(\frac{r_2}{r_1}\right) + \Delta t\frac{\ln 2}{T_Y} = \ln\left(\frac{r_1}{r_2}\right) + \Delta t\frac{\ln 2}{T_Y}$$

$$\frac{1}{T_X} = \frac{\ln(r_1/r_2)}{\Delta t\ln 2} + \frac{1}{T_Y} = \frac{T_Y\ln(r_1/r_2) + \Delta t\ln 2}{T_Y\Delta t\ln 2} = \frac{\ln\left[2(r_1/r_2)^{T_Y/\Delta t}\right]}{T_Y\ln 2}$$

The half-life of X is $T_X = \boxed{\dfrac{T_Y\ln 2}{\ln\left[2(r_1/r_2)^{T_Y/\Delta t}\right]}}$

Challenge Problems

P44.77 The electric charge density in the sphere is

$$\rho = \frac{Ze}{(4/3)\pi R^3}$$

Using Gauss's Law inside the sphere,

$$E \cdot 4\pi r^2 = \left(\frac{(4/3)\pi r^3}{\in_0}\right)\frac{Ze}{(4/3)\pi R^3} :$$

or $$E = \left(\frac{1}{4\pi \in_0}\frac{Ze}{R^3}\right) r \qquad (r \le R)$$

Outside the sphere, the field is

$$E = \frac{1}{4\pi \in_0}\frac{Ze}{r^2} \qquad (r \ge R)$$

We now find the electrostatic energy

$$U = \int_{r=0}^{\infty}\left(\frac{1}{2}\in_0 E^2\right)4\pi r^2\,dr$$

$$U = \frac{1}{2}\in_0\int_0^R\left[\left(\frac{Ze}{4\pi \in_0 R^3}\right)r\right]^2 4\pi r^2\,dr + \frac{1}{2}\in_0\int_R^{\infty}\left[\frac{1}{4\pi \in_0}\frac{Ze}{r^2}\right]^2 4\pi r^2\,dr$$

$$= 2\pi \in_0\left(\frac{Ze}{4\pi \in_0}\right)^2\int_0^R\left[\frac{r^2}{R^6}\right]r^2\,dr + 2\pi \in_0\left(\frac{Ze}{4\pi \in_0}\right)^2\int_R^{\infty}\left[\frac{1}{r^4}\right]r^2dr$$

$$= \frac{Z^2e^2}{8\pi \in_0}\left[\int_0^R\frac{r^4}{R^6}dr + \int_R^{\infty}\frac{dr}{r^2}\right] = \frac{Z^2e^2}{8\pi \in_0}\left[\left(\frac{R^5}{5R^6}\right)\Big|_0^R - \left(\frac{1}{r}\right)\Big|_R^{\infty}\right]$$

$$= \frac{Z^2e^2}{8\pi \in_0}\left[\frac{R^5}{5R^6} + \frac{1}{R}\right] = \frac{3}{20}\frac{Z^2e^2}{\pi \in_0 R} = \frac{3}{5}\left(\frac{1}{4\pi \in_0}\right)\frac{Z^2e^2}{R}$$

or $$U = \boxed{\frac{3}{20}\frac{Z^2e^2}{\pi \in_0 R} = \frac{3k_eZ^2e^2}{5R}}$$

P44.78 (a) Add two electrons to both sides of the reaction to have it in neutral-atom terms:

$$4\,{}_1^1\text{H atom} \rightarrow {}_2^4\text{He atom} + Q \;\rightarrow\; Q = \Delta mc^2 = \left[4M_{{}_1^1\text{H}} - M_{{}_2^4\text{He}}\right]c^2$$

The Q value is then

$$Q = \left[4(1.007\,825\ \text{u}) - 4.002\,603\ \text{u}\right] \times (931.5\ \text{MeV/u})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)$$

$$= \boxed{4.28\times10^{-12}\ \text{J}}$$

(b) The Sun is comprised of

$$N = \frac{1.99\times10^{30}\ \text{kg}}{1.67\times10^{-27}\ \text{kg/atom}} = \boxed{1.19\times10^{57}\ \text{atoms}}$$

$$= 1.19\times10^{57}\ \text{protons}$$

(c) The energy that could be created by this many protons in this reaction is

$$\left(1.19\times10^{57}\ \text{protons}\right)\left(\frac{4.28\times10^{-12}\ \text{J}}{4\ \text{protons}}\right) = 1.27\times10^{45}$$

Then, since $P = \dfrac{E}{\Delta t}$,

$$\Delta t = \frac{E}{P} = \frac{1.27\times10^{45}\ \text{J}}{3.85\times10^{26}\ \text{W}} = 3.31\times10^{18}\ \text{s} = \boxed{105\ \text{billion years}}$$

(d) The time interval in (c) is an order of magnitude larger than the expected remaining lifetime of the Sun. Only the hydrogen in a relatively small core is available as a nuclear fuel. Only in the core are temperatures and densities high enough for the fusion reaction to be self-sustaining.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P44.2	(a) 68; (b) ${}^{68}_{30}\text{Zn}$; (c) Isotopes of other elements to the left and right of zinc in the periodic table (from manganese to bromine) may have the same mass number.
P44.4	$\sim 10^{28}$ protons; (b) 10^{28} neutrons; (c) $\sim 10^{28}$ electrons
P44.6	184 m
P44.8	(a) 0.360 MeV; (b) Figure P44.8 shows the highest point in the curve at about 4 MeV, a factor of ten higher than the value in (a).
P44.10	$r_2 = \sqrt{\dfrac{m_2}{m_1}}r_1$
P44.12	(a) 5.18 fm; (b) λ is much less than the distance of closest approach
P44.14	(a) 2.82×10^{-5}; (b) 1.38×10^{-14}
P44.16	0.210 MeV
P44.18	See P44.18 for full explanation.
P44.20	(a) 84.2 MeV; (b) 342 MeV; (c) The nuclear force is so strong that the binding energy greatly exceeds the minimum energy needed to overcome electrostatic repulsion.
P44.22	~200 MeV
P44.24	(a) Nucleons on the surface have fewer neighbors with which to interact. The surface term is negative to reduce the estimate from the volume term, which assumes that all nucleons have the same number of neighbors; (b) sphere, $\dfrac{1}{3}r$, cube, $\dfrac{1}{6}L$. The sphere has a larger ratio to its characteristic length, so it would represent a larger binding energy and be more plausible for a nuclear shape.
P44.26	(a) $1.55 \times 10^{-5}\ \text{s}^{-1}$; (b) 12.4 h; (c) 2.39×10^{13} atoms; (d) 1.88 mCi
P44.28	See P44.28 for full explanation.
P44.30	$\dfrac{R_0T_{1/2}}{\ln 2}\left(2^{-t_1/T_{1/2}} - 2^{-t_2/T_{1/2}}\right)$
P44.32	(a) 0.755; (b) 0.570; (c) 9.766×10^{-4}; (d) No. The decay model depends on large numbers of nuclei. After some long but finite time, only one undecayed nucleus will remain. It is likely that the decay of this final nucleus will occur before infinite time.

P44.34 (a) See P44.34(a) for full explanation; (b) See P44.34(b) for full explanation; (c) See ANS. FIG. P44.34(c); (d) 10.9 min; (e) $t_m = \dfrac{\ln(\lambda_1/\lambda_2)}{\lambda_1 - \lambda_2}$; (f) 10.9 min

P44.36 (a) See P44.36(a) for full explanation; (b) 18.6 keV

P44.38 (a) ${}^{65}_{28}\text{Ni}^*$; (b) ${}^{211}_{82}\text{Pb}$; (c) ${}^{55}_{27}\text{Co}$

P44.40 (a) $N_d = N_{P,0} - N_{P,0}e^{-\lambda t} = (1.00 \times 10^6)(1 - e^{-0.0693t})$, where *t* is in hours; (b) The number of daughter nuclei starts from zero at *t* = 0. The number of stable product nuclei always increases with time and asymptotically approaches 1.00×10^6 as *t* increases without limit; (c) The minimum number of daughter nuclei is zero at *t* = 0. The maximum number of daughter nuclei asymptotically approaches 1.00×10^6 as *t* increases without limit; (d) The rate of change has its maximum value, $6.93 \times 10^4\ \text{h}^{-1}$, at *t* = 0, after which the rate decreases more and more, approaching zero as *t* increases without limit.

P44.42 (a) 1.05×10^{21}; (b) 1.37×10^9; (c) $3.83 \times 10^{-12}\ \text{s}^{-1}$; (d) 3.17×10^3 decays/week; (e) 951 decays/week; (f) 9.95×10^3 yr

P44.44 (a) 0.281; (b) 1.65×10^{-29}; (c) Radon is continuously created.

P44.46 (a) 4.00×10^9 yr; (b) $\dfrac{N}{N'} = 0.019\,9$ ^{235}U to ^{207}Pb chain and $\dfrac{N}{N'} = 4.60$ for the ^{232}Th to ^{208}Pb chain

P44.48 (a) 5.70 MeV; (b) 3.27 MeV; (c) exothermic

P44.50 (a) ${}^{197}_{79}\text{Au} + {}^{1}_{0}\text{n} \rightarrow {}^{198}_{79}\text{Au}^* \rightarrow {}^{198}_{80}\text{Hg} + {}^{0}_{-1}\text{e} + \bar{\nu}$; (b) 7.89 MeV

P44.52 See ANS. FIG. P44.52(a) and (b).

P44.54 4.42×10^3 yr

P44.56 While electric charge is conserved (5 + 2 = 1 + 6), the number of nucleons is not $(10 + 4 \neq 1 + 12)$. Therefore, this reaction cannot occur.

P44.58 (a) The process cannot occur because energy input would be required; (b) Required energy can come from the electrostatic repulsion; (c) 1.20 MeV

P44.60 The *Q* value of this hypothetical decay is calculated to be –7.62 MeV, which means you would have to add this much energy to the ^{238}U nucleus to make it emit a proton.

P44.62	The nucleus ^{57}Co cannot decay by e^+ emission because the Q value is –0.187 MeV.
P44.64	(a) 61.8 Bq/L; (b) 40.3 d
P44.66	(a) See ANS. FIG. P44.66; (b) $4.17 \times 10^{-3}\ \text{min}^{-1}$, 2.77 h; (c) 4.02×10^3 counts/min ; (d) 9.65×10^6 atoms
P44.68	(a) 3.91×10^9 yr ; (b) no older
P44.70	(a) $\sim 10^{-1\,363}$; (b) 0.891
P44.72	2.64 min
P44.74	(a) See P44.74(a) for full explanation; (b) 1.53 MeV
P44.76	$\dfrac{T_Y \ln 2}{\ln\left[2\left(r_1/r_2\right)^{T_Y/\Delta t}\right]}$
P44.78	(a) 4.28×10^{-12} J; (b) 1.19×10^{57} atoms; (c) 105 billion years; (d) The time interval in (c) is an order of magnitude larger than the expected remaining lifetime of the Sun. Only the hydrogen in a relatively small core is available as a nuclear fuel. Only in the core are temperatures and densities high enough for the fusion reaction to be self-sustaining.

45

Applications of Nuclear Physics

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ45.1 Answer (c). We compute the change in mass number *A*: 235 + 1 – 137 – 96 = 3. All the protons that start out in the uranium nucleus end up in the fission product nuclei.

OQ45.2 Answer (d). The best particles to trigger a fission reaction of the uranium nuclei are slow moving neutrons. Fast moving neutrons may not stay in close proximity with a uranium nucleus long enough to have a good probability of being captured by the nucleus so that a reaction can occur. Positively charged particles, such as protons and alpha particles, have difficulty approaching the target nuclei because of Coulomb repulsion.

OQ45.3 Answer (c). The total energy released was

$$E = (17\times 10^3\ \text{ton})\left(4.2\times 10^9\ \text{J/1 ton}\right) = 7.1\times 10^{13}\ \text{J}$$

and according to the mass-energy equivalence, the mass converted was

$$m = \frac{E}{c^2} = \frac{7.1\times 10^{13}\ \text{J}}{\left(3.00\times 10^8\ \text{m/s}\right)^2} = 7.9\times 10^{-4}\ \text{kg} = 0.79\ \text{g} \sim 1\ \text{g}$$

OQ45.4 The ranking is (b) > (c) > (a) > (d). See Table 45.1 for the RBE factors. Dose (a) is 1 rem. Dose (b) is (1 rad × 10) = 10 rem. Doses (c) and (d) are (1 rad × 4 or 5) = 4 to 5 rem, but dose (d) is to the hands only (less mass has absorbed the radiation). If we assume that (a) and (b) as well as (c) were whole-body doses to many kilograms of tissue (more mass has absorbed the radiation), we find the ranking stated.

OQ45.5 Answer (c). The function of the moderator is to slow down the neutrons released by one fission so that they can efficiently cause more fissions.

OQ45.6 The ranking is $Q_1 > Q_2 > Q_3 > 0$. Because all of the reactions involve 108 nucleons, we can look just at the change in binding-energy-per-nucleon as shown on the curve of binding energy. The jump from lithium to carbon is the biggest jump (~ 5.4 → 7.7 MeV), and next the jump from $A = 27$ to $A = 54$ (~ 8.3 → 8.8 MeV), which is near the peak of the curve. The step up for fission from $A = 108$ to $A = 54$ (~ 8.7 → 8.8 MeV) is smallest. All the reactions result in an increase in binding-energy-per-nucleon, so both of the fusion reactions described and the fission reaction put out energy, so Q is positive for all.

Imagine turning the curve of binding energy upside down so that it bends down like a cross-section of a bathtub. On such a curve of total energy per nucleon versus mass number it is easy to identify the fusion of small nuclei, the fission of large nuclei, and even the alpha decay of uranium, as exoenergetic processes. The most stable nucleus is at the drain of the bathtub, with minimum energy.

OQ45.7 Answer (d). The particles lose energy by collisions with nuclei in the bubble chamber to make their speed and their cyclotron radii $r = mv/qB$ decrease.

OQ45.8 Answer (b). The cyclotron radius is given by

$$r = mv/qB = \sqrt{2\left(\tfrac{1}{2}m^2v^2\right)}\Big/qB = \sqrt{2mK}\Big/qB$$

K and B are the same for both particles, but the ratio $\sqrt{m}/q$ is smaller for the electron; therefore, the path of the electron has a smaller radius, meaning the electron is deflected more.

OQ45.9 Answer (b). The nuclei must be energetic enough to overcome the Coulomb repulsion between them so that they can get close enough to fuse, and numerous enough for many collisions to occur in a short period of time so that the reaction produces more energy than it requires to operate.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ45.1 The two factors presenting the most technical difficulties are the requirements of a high plasma density and a high plasma temperature. These two conditions must occur simultaneously.

CQ45.2 For the deuterium nuclei to fuse, they must be close enough to each other for the nuclear forces to overcome the Coulomb repulsion of the protons—this is why the ion density is a factor. The more time that the nuclei in a sample spend in close proximity, the more nuclei will fuse—hence the confinement time is a factor.

CQ45.3 The products of fusion reactors are generally not themselves unstable, while fission reactions result in a chain of reactions which almost all have some unstable products, because they have an excess of neutrons.

CQ45.4 The advantage of a fission reaction is that it can generate much more electrical energy per gram of fuel compared to fossil fuels. Also, fission reactors do not emit greenhouse gases as combustion byproducts like fossil fuels—the only necessary environmental discharge is heat. The cost involved in producing fissile material is comparable to the cost of pumping, transporting, and refining fossil fuel.

The disadvantage is that some of the products of a fission reaction are radioactive—and some of those have long half-lives. The other problem is that there will be a point at which enough fuel is spent that the fuel rods do not supply power economically and need to be replaced. The fuel rods are still radioactive after removal. Both the waste and the "spent" fuel rods present serious health and environmental hazards that can last for tens of thousands of years. Accidents and sabotage involving nuclear reactors can be very serious, as can accidents and sabotage involving fossil fuels.

CQ45.5 Fusion of light nuclei to a heavier nucleus releases energy. Fission of a heavy nucleus to lighter nuclei releases energy. Both processes are steps towards greater stability on the curve of binding energy, Figure 44.5. The energy release per nucleon is typically greater for fusion, and this process is harder to control.

CQ45.6 The excitation energy comes from the binding energy of the extra nucleon.

CQ45.7 Advantages of fusion: high energy yield, no emission of greenhouse gases, fuel very easy to obtain, reactor cannot go supercritical like a fission reactor and low amounts of radioactive waste.

Disadvantages: requires high energy input to sustain reaction, lithium and helium are scarce, and neutrons released by the reaction cause structural damage to reactor housing.

CQ45.8 For each additional dynode, a larger applied voltage is needed, and hence a larger output from a power supply—"infinite" amplification would not be practical. Nor would it be desirable: the goal is to connect the tube output to a simple counter, so a massive pulse amplitude is not needed. If you made the detector sensitive to weaker and weaker signals, you would make it more and more sensitive to background noise.

CQ45.9 The hydrogen nuclei in water molecules have mass similar to that of a neutron, so that they can efficiently rob a fast-moving neutron of kinetic energy as they scatter it. A neutron bouncing off a more massive nucleus would lose less energy, so it would continue to travel through the shield. Once the neutron is slowed down, a hydrogen nucleus can absorb it in the reaction $\mathrm{n} + {}_1^1\mathrm{H} \rightarrow {}_1^2\mathrm{H}$.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 45.1 Interactions Involving Neutrons

Section 45.2 Nuclear Fission

***P45.1** The energy consumed by a 100-W lightbulb in a 1.0-h time period is

$$E = P\Delta t = (100\ \mathrm{J/s})(1.0\ \mathrm{h})\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = 3.6\times10^{5}\ \mathrm{J}$$

The number of fission events, yielding an average of 208 MeV each, required to produce this quantity of energy is

$$n = \frac{E}{208\ \mathrm{MeV}} = \frac{3.6\times10^{5}\ \mathrm{J}}{208\ \mathrm{MeV}}\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right) = \boxed{1.1\times10^{16}}$$

P45.2 The mass of U-235 producing the same amount of energy as 1 000 kg of coal is

$$\begin{aligned} m &= (3.30\times10^{10}\ \mathrm{J})\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right) \\ &\quad\times\left(\frac{1\ \text{U-235 nucleus}}{200\ \mathrm{MeV}}\right)\left(\frac{235\ \mathrm{g}}{6.02\times10^{23}\ \mathrm{nucleus}}\right) \\ &= \boxed{0.403\ \mathrm{g}} \end{aligned}$$

P45.3 Three different fission reactions are possible:

$$ {}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{38}^{90}\text{Sr} + {}_{54}^{144}\text{Xe} + 2\,{}_0^1\text{n} \qquad \boxed{{}_{54}^{144}\text{Xe}} $$

$$ {}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{38}^{90}\text{Sr} + {}_{54}^{143}\text{Xe} + 3\,{}_0^1\text{n} \qquad \boxed{{}_{54}^{143}\text{Xe}} $$

$$ {}_0^1\text{n} + {}_{92}^{235}\text{U} \rightarrow {}_{38}^{90}\text{Sr} + {}_{54}^{142}\text{Xe} + 4\,{}_0^1\text{n} \qquad \boxed{{}_{54}^{142}\text{Xe}} $$

P45.4 If the electrical power output of 1.00 GW is 40.0% of the power derived from fission reactions, the power output of the fission process is

$$ \frac{1.00\text{ GW}}{0.400} = \left(2.50\times10^{9}\text{ J/s}\right)\left(8.64\times10^{4}\text{ s/d}\right) = 2.16\times10^{14}\text{ J/d} $$

The number of fissions per day is

$$ \left(2.16\times10^{14}\text{ J/d}\right)\left(\frac{1\text{ fission}}{200\times10^{6}\text{ eV}}\right)\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 6.75\times10^{24}\text{ d}^{-1} $$

This also is the number of ^{235}U nuclei used, so the mass of ^{235}U used per day is

$$ \left(6.75\times10^{24}\text{ nuclei/d}\right)\left(\frac{235\text{ g/mol}}{6.02\times10^{23}\text{ nuclei/mol}}\right) = 2.63\times10^{3}\text{ g/d} = \boxed{2.63\text{ kg/d}} $$

In contrast, a coal-burning steam plant producing the same electrical power uses more than 6 × 10^6 kg/d of coal.

P45.5 First, the thorium is bombarded:

$$ {}_0^1\text{n} + {}_{90}^{232}\text{Th} \rightarrow {}_{90}^{233}\text{Th} $$

Then, the thorium decays by beta emission:

$$ {}_{90}^{233}\text{Th} \rightarrow {}_{91}^{233}\text{Pa} + {}_{-1}^{0}\text{e} + \bar{\nu} $$

Protactinium-233 has more neutrons than the more stable protactinium-231, so it too decays by beta emission:

$$ {}_{91}^{233}\text{Pa} \rightarrow {}_{92}^{233}\text{U} + {}_{-1}^{0}\text{e} + \bar{\nu} $$

P45.6 (a) The energy released is equal to the *Q* value, given by

$$ Q = (\Delta m)c^2 = \left[m_n + M_{\text{U-235}} - M_{\text{Ba-141}} - M_{\text{Kr-92}} - 3m_n\right]c^2 $$

with

$$ \Delta m = [1.008\,665\text{ u} + 235.043\,923\text{ u} - 140.914\,4\text{ u} - 91.926\,2\text{ u} - 3(1.008\,665\text{ u})] = 0.185\,993\text{ u} $$

Then,

$$Q = (0.185\ 993\ \text{u})(931.5\ \text{MeV/u}) = \boxed{173\ \text{MeV}}$$

(b) The fraction of rest energy transformed is

$$f = \frac{\Delta m}{m_i} = \frac{0.185\ 993\ \text{u}}{236.05\ \text{u}} = 7.88 \times 10^{-4} = \boxed{0.078\ 8\%}$$

P45.7 The energy released in the reaction ${}^1_0\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{88}_{38}\text{Sr} + {}^{136}_{54}\text{Xe} + 12\,{}^1_0\text{n}$ is

$$\begin{aligned}Q = (\Delta m)c^2 &= \left[m_{{}^{235}_{92}\text{U}} - 11m_n - m_{{}^{88}_{38}\text{Sr}} - m_{{}^{136}_{54}\text{Xe}}\right]c^2 \\ &= \big[235.043\,923\ \text{u} - 11(1.008\,665\ \text{u}) \\ &\qquad -87.905\,614\ \text{u} - 135.907\,220\ \text{u}\big](931.5\ \text{MeV/u}) \\ &= \boxed{126\ \text{MeV}}\end{aligned}$$

P45.8 In *N* collisions, the energy is reduced from 2.00 MeV to 0.039 eV:

$$\left(2.00 \times 10^6\ \text{eV}\right)\left(\frac{1}{2}\right)^N \leq 0.039\ \text{eV}$$

$$\left(\frac{1}{2}\right)^N \leq \frac{0.039}{2.00 \times 10^6}$$

$$N \ln\left(\frac{1}{2}\right) \leq \ln\left(\frac{0.039}{2.00 \times 10^6}\right)$$

$$N \ln(2) \geq \ln\left(\frac{2.00 \times 10^6}{0.039}\right)$$

which gives

$$N \geq 25.6 \rightarrow N = \boxed{26}$$

P45.9 The mass defect is

$$\begin{aligned}\Delta m &= (m_n + M_U) - (M_{Zr} + M_{Te} + 3m_n) \\ \Delta m &= \big[1.008\,665\ \text{u} + 235.043\,923\ \text{u} \\ &\qquad - 97.912\,7\ \text{u} - 134.916\,5\ \text{u} - 3(1.008\,665\ \text{u})\big] \\ &= 0.197\ 393\ \text{u}\end{aligned}$$

The energy equivalent is

$$\Delta mc^2 = (0.197\,393\ \text{u})c^2\left(\frac{931.5\ \text{MeV}/c^2}{\text{u}}\right) = \boxed{184\ \text{MeV}}$$

P45.10 (a) At a concentration of $c = 3\ \text{mg/m}^3 = 3 \times 10^{-3}\ \text{g/m}^3$, the mass of uranium dissolved in the oceans covering two-thirds of Earth's surface to an average depth of $h_{\text{avg}} = 4$ km is

$$m_U = cV = c\left(\tfrac{2}{3}A\right)\cdot h_{\text{av}} = c\left[\tfrac{2}{3}\left(4\pi R_E^2\right)\right]\cdot h_{\text{av}}$$

or

$$m_U = \left(3\times 10^{-3}\ \frac{\text{g}}{\text{m}^3}\right)\left(\frac{2}{3}\right)4\pi\left(6.38\times 10^6\ \text{m}\right)^2\left(4\times 10^3\ \text{m}\right)$$
$$= \boxed{4\times 10^{15}\ \text{g}}$$

(b) Fissionable ^{235}U makes up 0.700% of the mass of uranium computed above. If we assume all of the ^{235}U is collected and caused to undergo fission, with the release of about 200 MeV per event, the potential energy supply is

$$E = \left(\text{number of }^{235}\text{ U atoms}\right)(200\ \text{MeV})$$
$$= \frac{0.700}{100}\left(\frac{m_U}{m_{^{235}\text{U}_{\text{atom}}}}\right)(200\ \text{MeV})$$

and at a consumption rate of $P = 1.5 \times 10^{13}$ J/s, the time interval this could supply the world's energy needs is $\Delta t = E/P$, or

$$\Delta t = \frac{0.700}{100}\left(\frac{m_U}{m_{^{235}\text{U}_{\text{atom}}}}\right)\frac{(200\ \text{MeV})}{P}$$
$$= \frac{0.700}{100}\left[\frac{4\times 10^{15}\ \text{g}}{(235\ \text{u})\left(1.66\times 10^{-27}\ \text{kg/u}\right)}\left(\frac{1\ \text{kg}}{10^3\ \text{g}}\right)\right]$$
$$\times\left[\left(\frac{200\ \text{MeV}}{1.50\times 10^{13}\ \text{J/s}}\right)\left(\frac{1.60\times 10^{-13}\ \text{J}}{1\ \text{MeV}}\right)\left(\frac{1\ \text{yr}}{3.16\times 10^7\ \text{s}}\right)\right]$$
$$= \boxed{5\times 10^3\ \text{yr}}$$

(Compare this value to that in part (b) of Problem 17, which is a more realistic estimate of the time interval for the uranium that can be extracted reasonably from the Earth.)

(c) $\boxed{\text{The uranium comes from rocks and minerals dissolved in water and carried into the ocean by rivers.}}$

(d) $\boxed{\text{No.}}$ Uranium cannot be replenished by the radioactive decay of other elements on Earth.

P45.11 One kg of enriched uranium contains 3.40% ${}^{235}_{92}\mathrm{U}$, so the mass of uranium-235 is

$$m_{235} = 0.034\,0(1\,000\text{ g}) = 34.0\text{ g}$$

In terms of number of nuclei, this is equivalent to

$$\begin{aligned} N_{235} &= (34.0\text{ g})\left(\frac{1}{235\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ atoms/mol}\right) \\ &= 8.71\times10^{22}\text{ nuclei} \end{aligned}$$

If all these nuclei fission, the energy released is equal to

$$\begin{aligned} &\left(8.71\times10^{22}\text{ nuclei}\right)\left(200\times10^{6}\text{ eV/nucleus}\right) \\ &\qquad\times\left(1.602\times10^{-19}\text{ J/eV}\right) = 2.79\times10^{12}\text{ J} \end{aligned}$$

Now, for the engine,

$$\text{efficiency} = \frac{\text{work output}}{\text{heat input}} \quad \text{or} \quad e = \frac{P\Delta r\cos\theta}{Q_h}$$

So the distance the ship can travel per kilogram of uranium fuel is

$$\Delta r = \frac{eQ_h}{P\cos(0^\circ)} = \frac{0.200\left(2.79\times10^{12}\text{ J}\right)}{1.00\times10^{5}\text{ N}} = \boxed{5.58\times10^{6}\text{ m}}$$

Section 45.3 Nuclear Reactors

***P45.12** (a) With a specific gravity of 4.00, the density of soil is $\rho = 4.00\times10^{3}\text{ kg/m}^3$. Thus, the mass of the top 1.00 m of soil is

$$\begin{aligned} m = \rho V &= \left(4.00\times10^{3}\text{ kg/m}^3\right)\left[(1.00\text{ m})\left(43\,560\text{ ft}^2\right)\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)^2\right] \\ &= 1.62\times10^{7}\text{ kg} \end{aligned}$$

At a rate of 1 part per million, the mass of uranium in this soil is

$$m_U = \frac{m}{10^6} = \frac{1.62\times10^{7}\text{ kg}}{10^6} = \boxed{16.2\text{ kg}}$$

(b) Since 0.720% of naturally occurring uranium is ${}^{235}_{92}\mathrm{U}$, the mass of ${}^{235}_{92}\mathrm{U}$ in the soil of part (a) is

$$\begin{aligned} m_{{}^{235}_{92}\mathrm{U}} &= \left(7.20\times10^{-3}\right)m_U = \left(7.20\times10^{-3}\right)(16.2\text{ kg}) \\ &= 0.117\text{ kg} = \boxed{117\text{ g}} \end{aligned}$$

P45.13 In one minute there are $N = \dfrac{60.0\text{ s}}{1.20\times10^{-3}\text{ s}} = 5.00\times10^{4}$ fissions.

So the rate increases by a factor of $(1.000\,25)^{50\,000} = \boxed{2.68\times10^{5}}$.

P45.14 (a) For a sphere: $V = \dfrac{4}{3}\pi r^3 \rightarrow r = \left(\dfrac{3V}{4\pi}\right)^{1/3}$, so

$$\frac{A}{V} = \frac{4\pi r^2}{(4/3)\pi r^3} = \frac{3}{r} = \left(\frac{36\pi}{V}\right)^{1/3} = \boxed{4.84V^{-1/3}}$$

(b) For a cube: $V = \ell^3 \rightarrow \ell = V^{1/3}$, so

$$\frac{A}{V} = \frac{6\ell^2}{\ell^3} = \frac{6}{\ell} = \boxed{6V^{-1/3}}$$

(c) For a parallelepiped: $V = 2a^3 \rightarrow a = \left(\dfrac{V}{2}\right)^{1/3}$, so

$$\frac{A}{V} = \frac{(2a^2+8a^2)}{2a^3} = \frac{5}{a} = 5\left(\frac{2}{V}\right)^{1/3} = \left(\frac{250}{V}\right)^{1/3} = \boxed{6.30V^{-1/3}}$$

(d) The answers show that the sphere has the smallest surface area for a given volume and the brick has the greatest surface area of the three. Therefore, $\boxed{\text{The sphere has minimum leakage and the parallelepiped has maximum leakage.}}$

P45.15 Recall the radius of a nucleus of mass number A is $r = aA^{1/3}$, where $a = 1.2$ fm. The center to center distance of the nuclei of helium ($A = 4$) and gold ($A = 197$) is the sum of their combined radii:

$$r = (1.2\text{ fm})(4)^{1/3} + (1.2\text{ fm})(197)^{1/3} = 8.9\text{ fm} = 8.9\times10^{-15}\text{ m}$$

The electric potential energy is

$$\begin{aligned} U = qV &= \frac{k_e q_1 q_2}{r} \\ &= \frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)\,(2)(79)(1.60\times10^{-19}\text{ C})\,e}{8.9\times10^{-15}\text{ m}} \\ &= 2.6\times10^{7}\text{ eV} = \boxed{26\text{ MeV}} \end{aligned}$$

P45.16 The power after three months is $P = 10.0\text{ MW} = 1.00\times10^{7}$ J/s. If each decay delivers $1.00\text{ MeV} = 1.60\times10^{-13}$ J, then the number of decays/s

$$= \frac{1.00\times10^{7}\text{ J/s}}{1.60\times10^{-13}\text{ J}} = \boxed{6.25\times10^{19}\text{ Bq}}$$

P45.17 (a) Do not think of the "reserve" as being held in reserve. We are depleting it as fast as we choose. The remaining current balance of irreplaceable ^{235}U is 0.7% of the whole mass of uranium:

$$(0.007\ 00)\left(4.40\times10^{6}\ \text{tons}\right)\left(\frac{10^{3}\ \text{kg}}{1\ \text{ton}}\right)\left(\frac{10^{3}\ \text{g}}{1\ \text{kg}}\right)=\boxed{3.08\times10^{10}\ \text{g}}$$

(b) The number of moles of ^{235}U in the reserve is

$$n=\frac{m}{M}=\frac{3.08\times10^{10}\ \text{g}}{235\ \text{g/mole}}=\boxed{1.31\times10^{8}\ \text{mole}}$$

(c) The number of moles found in part (b) corresponds to

$$N=nN_{\text{A}}=\left(1.31\times10^{8}\ \text{mole}\right)\left(\frac{6.02\times10^{23}\ \text{atom}}{1\ \text{mole}}\right)\left(\frac{1\ \text{nucleus}}{1\ \text{atom}}\right)$$
$$=\boxed{7.89\times10^{31}\ \text{nuclei}}$$

(d) We imagine each nucleus as fissioning, to release

$$\left(7.89\times10^{31}\ \text{fissions}\right)\left(\frac{200\ \text{MeV}}{1\ \text{fission}}\right)\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)=\boxed{2.52\times10^{21}\ \text{J}}$$

(e) The definition of power is represented by P = (energy converted)/ Δt, so we have

$$\Delta t=\frac{\text{energy}}{P}=\frac{2.52\times10^{21}\ \text{J}}{1.5\times10^{13}\ \text{J/s}}=\left(1.68\times10^{8}\ \text{s}\right)\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)$$
$$=\boxed{5.33\ \text{yr}}$$

(f) $\boxed{\text{Fission is not sufficient to supply the entire world with energy at a price of \$130 or less per kilogram of uranium.}}$

P45.18 Assuming that the impossibility is *not* that he can have this control over the process (which, as far as we know presently, *is* impossible), let's see what else might be wrong. The reaction can be written

$$^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{141}_{57}\text{La} + {}^{94}_{35}\text{Br} + n\left({}^{1}_{0}\text{n}\right)$$

where *n* is the number of neutrons released in the fission reaction. By balancing the equation for electric charge and number of nucleons, we find that *n* = 1. If one incoming neutron results in just one outgoing neutron, the possibility of a chain reaction is not there, so this nuclear reactor will not work.

***P45.19** The total energy required for one year is

$$\mathsf{E} = (2\ 000\ \text{kWh/month})(3.60\times10^{6}\ \text{J/kWh})(12.0\ \text{months})$$
$$= 8.64\times10^{10}\ \text{J}$$

The number of fission events needed will be

$$\mathsf{N} = \frac{\mathsf{E}}{\mathsf{E}_{\text{event}}} = \frac{8.64\times10^{10}\ \text{J}}{(208\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV})} = 2.60\times10^{21}$$

and the mass of this number of ^{235}U atoms is

$$\mathsf{m} = \left(\frac{\mathsf{N}}{\mathsf{N}_A}\right)\mathsf{M}_{\text{mol}} = \left(\frac{2.60\times10^{21}\ \text{atoms}}{6.02\times10^{23}\ \text{atoms/mol}}\right)(235\ \text{g/mol})$$
$$= \boxed{1.01\ \text{g}}$$

P45.20 (a) Since $K = p^2/2m$, we have

$$\mathsf{p} = \sqrt{2\mathsf{mK}} = \sqrt{2\mathsf{m}\left(\frac{3}{2}\mathsf{k}_B\mathsf{T}\right)}$$
$$= \sqrt{3(1.675\times10^{-27}\ \text{kg})(1.38\times10^{-23}\ \text{J/K})\ (300\ \text{K})}$$
$$= \boxed{4.56\times10^{-24}\ \text{kg}\cdot\text{m/s}}$$

(b) The de Broglie wavelength of the particle is

$$\lambda = \frac{\mathsf{h}}{\mathsf{p}} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{4.56\times10^{-24}\ \text{kg}\cdot\text{m/s}} = 1.45\times10^{-10}\ \text{m} = \boxed{0.145\ \text{nm}}$$

(c) This size has the same order of magnitude as an atom's outer electron cloud, and is vastly larger than a nucleus.

Section 45.4 Nuclear Fusion

P45.21 (a) Helium fusion proceeds according to

$${}^{4}_{2}\text{He} + {}^{4}_{2}\text{He} \rightarrow \boxed{{}^{8}_{4}\text{Be}} + \gamma$$

(b) The beryllium produced by helium fusion fuses with another alpha particle according to

$${}^{8}_{4}\text{Be} + {}^{4}_{2}\text{He} \rightarrow \boxed{{}^{12}_{6}\text{C}} + \gamma$$

(c) The total energy released in this pair of fusion reactions is

$$Q = (\Delta m)c^2 = \left[2m_{^4\text{He}} - m_{^8\text{B}}\right]c^2 + \left[m_{^8\text{B}} + m_{^4\text{He}} - m_{^{12}\text{C}}\right]c^2$$
$$= \left[3m_{^4\text{He}} - m_{^{12}\text{C}}\right]c^2$$
$$= \left[3(4.002\ 602\ \text{u}) - 12.000\ 000\ \text{u}\right](931.5\ \text{MeV/u})$$
$$= \boxed{7.27\ \text{MeV}}$$

P45.22 From Equation 45.2, the energy released in the reaction $^2_1\text{H} + {}^3_1\text{H} \rightarrow {}^4_2\text{He} + {}^1_0\text{n}$ is 17.59 MeV per event. The total energy required for the year is

$$E = (2\ 000\ \text{kWh/month})(12.0\ \text{months})(3.60\times10^6\ \text{J/kWh})$$
$$= 8.64\times10^{10}\ \text{J}$$

so the number of fusion events needed for the year is

$$N = \frac{E}{Q} = \frac{8.64\times10^{10}\ \text{J}}{(17.59\ \text{MeV/event})(1.602\times10^{-13}\ \text{J/MeV})}$$
$$= \boxed{3.07\times10^{22}\ \text{events}}$$

P45.23 The energy released in the reaction $^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \gamma$ is

$$Q = (\Delta m)c^2 = \left[m_{^1_1\text{H}} + m_{^2_1\text{H}} - m_{^3_2\text{He}}\right]c^2$$
$$= \left[1.007\ 825\ \text{u} + 2.014\ 102\ \text{u} - 3.016\ 029\ \text{u}\right](931.5\ \text{MeV/u})$$
$$= \boxed{5.49\ \text{MeV}}$$

P45.24 (a) We assume that the nuclei are stationary at closest approach, so that the electrostatic potential energy equals the total energy *E*. Then, from the isolated system model,

$$K_f + U_f = K_i + U_i \quad \rightarrow \quad U_f = E$$

then,

$$\frac{k_e(Z_1e)(Z_2e)}{r_{\text{min}}} = E$$

$$E = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2 Z_1Z_2}{1.00\times10^{-14}\ \text{m}}\left(\frac{1\ \text{keV}}{1.60\times10^{-16}\ \text{J}}\right)$$
$$= (144\ \text{keV})Z_1Z_2$$

or $\boxed{E = 144Z_1Z_2 \text{ where } E \text{ is in keV.}}$

(b) $\boxed{\text{The energy is proportional to each atomic number.}}$

(c) $\boxed{\text{Take } Z_1 = 1 \text{ and } Z_2 = 59 \text{ or vice versa. This choice minimizes the product } Z_1 Z_2.}$ If extra cleverness is allowed, take $Z_1 = 0$ and $Z_2 = 60$: use neutrons as the bombarding particles. A neutron is a nucleon but not an atomic nucleus.

(d) For both the D-D and the D-T reactions, $Z_1 = Z_2 = 1$. Thus, the minimum energy required in both cases is

$$E = \left(2.30\times10^{-14}\ \text{J}\right)\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right)$$

$$= \boxed{\text{144 keV for both, according to this model.}}$$

Section 45.4 in the text gives more accurate values for the critical ignition temperatures, of about 52 keV for D-D fusion and 6 keV for D-T fusion. The nuclei can fuse by tunneling. A triton moves more slowly than a deuteron at a given temperature. Then D-T collisions last longer than D-D collisions and have much greater tunneling probabilities.

P45.25 (a) The Q value for the D-T reaction is 17.59 MeV (from Equation 45.4). Specific energy content in fuel for D-T reaction (from Table 44.2, mass = 2.014 u + 3.016 u = 5.030 u):

$$\frac{(17.59\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)}{(5.030\ \text{u})\left(1.6605\times10^{-27}\ \text{kg/u}\right)} = 3.37\times10^{14}\ \text{J/kg}$$

The rate of fuel burning for the D-T reaction is then

$$r_{\text{DT}} = \frac{\left(3.00\times10^{9}\ \text{J/s}\right)(3\,600\ \text{s/hr})}{\left(3.37\times10^{14}\ \text{J/kg}\right)\left(10^{-3}\ \text{kg/g}\right)}$$

$$= \boxed{32.1\ \text{g/h burning of D and T}}$$

(b) Using energy values from Equation 45.4, the specific energy content in fuel for D-D reaction is:

$$Q = \frac{1}{2}(3.27 + 4.03) = 3.65\ \text{MeV}$$

From Table 44.2, the D-D mass is = 2(2.014 u) = 4.018 u. The specific energy content in D-D fuel is

$$\frac{(3.65\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)}{(4.028\ \text{u})\left(1.6605\times10^{-27}\ \text{kg/u}\right)} = 8.73\times10^{13}\ \text{J/kg}$$

and the rate of fuel burning for the D-D reaction is

$$r_{DD}=\frac{(3.00\times10^{9}\ \text{J/s})(3\,600\ \text{s/hr})}{(8.73\times10^{13}\ \text{J/kg})(10^{-3}\ \text{kg/g})}=\boxed{124\ \text{g/h}}$$

P45.26 (a) The radius of a nucleus with mass number A is $r = aA^{1/3}$, where $a = 1.2$ fm. The distance of closest approach is equal to the center to center distance of the two nuclei:

$$r_f=r_D+r_T=(1.20\times10^{-15}\ \text{m})\left[(2)^{1/3}+(3)^{1/3}\right]$$
$$=3.24\times10^{-15}\ \text{m}=\boxed{3.24\ \text{fm}}$$

(b) At this distance, the electric potential energy is

$$U_f=\frac{k_e e^2}{r_f}=\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{3.24\times10^{-15}\ \text{m}}$$
$$=7.10\times10^{-14}\ \text{J}=\boxed{444\ \text{keV}}$$

(c) Conserving momentum, $m_D v_i=(m_D+m_T)v_f$ or

$$v_f=\left(\frac{m_D}{m_D+m_T}\right)v_i=\boxed{\frac{2}{5}v_i}$$

(d) To find the minimum initial kinetic energy of the deuteron, we use $K_i + U_i = K_f + U_f$, where $U_i = 0$ because the deuteron starts from very far away (infinity), and with the result from part (c),

$$K_i+0=\frac{1}{2}(m_D+m_T)v_f^2+U_f$$
$$K_i=\frac{1}{2}(m_D+m_T)\left(\frac{m_D}{m_D+m_T}\right)^2 v_i^2+U_f$$

With some re-arrangement, we have

$$K_i=\left(\frac{m_D}{m_D+m_T}\right)\left(\frac{1}{2}m_D v_i^2\right)+U_f=\left(\frac{m_D}{m_D+m_T}\right)K_i+U_f$$

or

$$\left(1-\frac{m_D}{m_D+m_T}\right)K_i=U_f$$

solving for the initial kinetic energy then gives

$$K_i=U_f\left(\frac{m_D+m_T}{m_T}\right)=\frac{5}{3}(444\ \text{keV})=\boxed{740\ \text{keV}}$$

(e) The nuclei can fuse $\boxed{\text{possibly by tunneling}}$ through the potential energy barrier.

P45.27 (a) $V = \left(317\times10^6\ \text{mi}^3\right)\left(\dfrac{1\,609\ \text{m}}{1\ \text{mi}}\right)^3 = 1.32\times10^{18}\ \text{m}^3$

From the periodic table, H has atomic mass 1.007 9 and O has atomic mass 15.999 4, so water has atomic mass 18.015 2.

$$m_{\text{water}} = \rho V = \left(10^3\ \text{kg/m}^3\right)\left(1.32\times10^{18}\ \text{m}^3\right) = 1.32\times10^{21}\ \text{kg}$$

$$\begin{aligned} m_{\text{H}_2} &= \left(\frac{M_{\text{H}_2}}{M_{\text{H}_2\text{O}}}\right)m_{\text{H}_2\text{O}} = \left(\frac{2.016}{18.015}\right)\left(1.32\times10^{21}\ \text{kg}\right) \\ &= 1.48\times10^{20}\ \text{kg} \end{aligned}$$

$$\begin{aligned} m_{\text{Deuterium}} &= \left(0.030\,0\%\right)m_{\text{H}_2} = \left(0.030\,0\times10^{-2}\right)\left(1.48\times10^{20}\ \text{kg}\right) \\ &= 4.43\times10^{16}\ \text{kg} \end{aligned}$$

The number of deuterium nuclei in this mass is

$$N = \frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}} = \frac{4.43\times10^{16}\ \text{kg}}{(2.014\ \text{u})\left(1.66\times10^{-27}\ \text{kg/u}\right)} = 1.33\times10^{43}$$

Since two deuterium nuclei are used per fusion, ${}^2_1\text{H} + {}^2_1\text{H} \to {}^4_2\text{He}$, the number of events is $\dfrac{N}{2} = 6.63\times10^{42}$.

The energy released per event is

$$\begin{aligned} Q &= \left[M_{^2\text{H}} + M_{^2\text{H}} - M_{^4\text{He}}\right]c^2 \\ &= \left[2(2.014\ 102) - 4.002\ 603\right]\text{u}\left(931.5\ \text{MeV/u}\right) \\ &= 23.8\ \text{MeV} \end{aligned}$$

The total energy available is then

$$\begin{aligned} E &= \left(\frac{N}{2}\right)Q = \left(6.63\times10^{42}\right)(23.8\ \text{MeV})\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) \\ &= \boxed{2.53\times10^{31}\ \text{J}} \end{aligned}$$

(b) The time this energy could possibly meet world requirements is

$$\begin{aligned} \Delta t &= \frac{E}{P} = \frac{2.53\times10^{31}\ \text{J}}{100\left(1.50\times10^{13}\ \text{J/s}\right)} = \left(1.69\times10^{16}\ \text{s}\right)\left(\frac{1\ \text{yr}}{3.16\times10^7\ \text{s}}\right) \\ &= \boxed{5.34\times10^8\ \text{yr}} \end{aligned}$$

P45.28 (a) Including both ions and electrons, the number of particles in the plasma is $N = 2nV$, where n is the ion density and V is the volume of the container. Application of Equation 21.6 gives the total energy as

$$E = \frac{3}{2}Nk_BT = 3nVk_BT$$

$$= 3\left(2.00\times10^{13}\ \text{cm}^{-3}\right)\left[\left(50.0\ \text{m}^3\right)\left(\frac{10^6\ \text{cm}^3}{1\ \text{m}^3}\right)\right] \times\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.00\times10^8\ \text{K}\right)$$

$$E = \boxed{1.66\times10^7\ \text{J}}$$

(b) The specific heat of water is $c = 4\ 186\ \text{J/kg}\cdot{}^\circ\text{C}$, and the energy required to raise the temperature of one kilogram of water from 27.0°C to 100°C is given by Equation 20.4:

$$Q = mc\Delta T = \left(1.00\ \text{kg}\right)\left(4\ 186\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(100^\circ\text{C} - 27.0^\circ\text{C}\right)$$
$$= 3.06\times10^5\ \text{J}$$

From Table 20.2, the heat of vaporization of water is $L_v = 2.26\times10^6\ \text{J/kg}$, so that a total of

$$E_{1\ \text{kg}} = 3.06\times10^5\ \text{J} + 2.26\times10^6\ \text{J} = 2.57\times10^6\ \text{J}$$

is required to boil away each kilogram of water initially at 27.0°C. The mass of water that could be boiled away is therefore

$$m = \frac{E}{E_{1\ \text{kg}}} = \frac{1.66\times10^7\ \text{J}}{2.57\times10^6\ \text{J/kg}} = \boxed{6.45\ \text{kg}}$$

P45.29 (a) Taking $m \approx 2m_p$ for deuterons, we have

$$\frac{1}{2}m\overline{v^2} = \frac{3}{2}k_BT$$

The root-mean-square speed is

$$v_{\text{rms}} = \sqrt{\frac{3k_BT}{2m_p}} = \sqrt{\frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.00\times10^8\ \text{K}\right)}{2\left(1.67\times10^{-27}\ \text{kg}\right)}}$$
$$= \boxed{2.23\times10^6\ \text{m/s}}$$

(b) The confinement time in the absence of confinement measures is

$$\Delta t = \frac{x}{v} = \frac{0.100\ \text{m}}{2.23\times10^6\ \text{m/s}} \sim 10^{-7}\ \text{s}$$

P45.30 (a) By adding 1 + 6 = 7 and 1 + 12 = 13, we have ${}^{1}_{1}\mathrm{H} + {}^{12}_{6}\mathrm{C} \to {}^{13}_{7}\mathrm{N} + \gamma$ so nucleus *A* is $\boxed{{}^{13}\mathrm{N}.}$

(b) Now 13 – 0 = 13 and 7 – 1 = 6, so the positron decay is ${}^{13}_{7}\mathrm{N} \to {}^{13}_{6}\mathrm{C} + {}^{0}_{1}\mathrm{e} + \nu$ and nucleus *B* is $\boxed{{}^{13}\mathrm{C}.}$

(c) Similarly, we have ${}^{1}_{1}\mathrm{H} + {}^{13}_{6}\mathrm{C} \to {}^{14}_{7}\mathrm{N} + \gamma$ and nucleus *C* is $\boxed{{}^{14}\mathrm{N}.}$

(d) The hydrogen nuclei keep piling on like rugby players after a tackle. We have ${}^{1}_{1}\mathrm{H} + {}^{14}_{7}\mathrm{N} \to {}^{15}_{8}\mathrm{O} + \gamma$ and nucleus *D* is $\boxed{{}^{15}\mathrm{O}.}$

(e) Now ${}^{15}_{8}\mathrm{O} \to {}^{15}_{7}\mathrm{N} + {}^{0}_{1}\mathrm{e} + \nu,$ so nucleus *E* is $\boxed{{}^{15}\mathrm{N}.}$

(f) We calculate 15 + 1 – 4 = 12 and 7 + 1 – 2 = 6 to identify ${}^{1}_{1}\mathrm{H} + {}^{15}_{7}\mathrm{N} \to {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He}$ and nucleus *F* is $\boxed{{}^{12}\mathrm{C}.}$

(g) $\boxed{\text{The original carbon-12 nucleus is returned.}}$ One carbon nucleus can participate in the fusions of colossal numbers of hydrogen nuclei, four after four. Carbon is a catalyst.

The two positrons immediately annihilate with electrons according to ${}^{0}_{1}\mathrm{e} + {}^{0}_{-1}\mathrm{e} \to 2\gamma$. The overall reaction, obtained by adding all eight reactions, can be represented as

$$ {}^{1}_{1}\mathrm{H} + {}^{12}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H} + 2\,{}^{0}_{-1}\mathrm{e} \to {}^{4}_{2}\mathrm{He} + {}^{12}_{6}\mathrm{C} + 7\gamma + 2\nu $$

This simplifies to $\boxed{4\left({}^{1}_{1}\mathrm{H}\right) + 2\,{}^{0}_{-1}\mathrm{e} \to {}^{4}_{2}\mathrm{He} + 2\nu}$. The net reaction is identical to the net reaction in the proton–proton cycle which predominates in the Sun. In energy terms the reaction can be considered as $4\left({}^{1}_{1}\mathrm{H\ atom}\right) \to {}^{4}_{2}\mathrm{He\ atom} + 26.7\ \mathrm{MeV},$ where the *Q* value of energy output was computed in Chapter 39, Problem 67 and again in Problem 59 in this chapter.

P45.31 (a) Lawson's criterion for the D-T reaction is $n\tau \geq 10^{14}\ \mathrm{s/cm^3}$. For a confinement time of $\tau = 1.00$ s, this requires a minimum ion density of $n = \boxed{10^{14}\ \mathrm{cm^{-3}}}$.

(b) At the ignition temperature of $T = 4.5 \times 10^7$ K and the ion density found above, the plasma pressure is

$$ \begin{aligned} P &= 2nk_BT \\ &= 2\left[\left(10^{14}\ \mathrm{cm^{-3}}\right)\left(\frac{10^6\ \mathrm{cm^3}}{1\ \mathrm{m^3}}\right)\right]\left(1.38\times10^{-23}\ \mathrm{J/K}\right)\left(4.5\times10^7\ \mathrm{K}\right) \\ &= \boxed{1.2\times10^5\ \mathrm{J/m^3}} \end{aligned} $$

(c) The required magnetic energy density is then

$$u_B = \frac{B^2}{2\mu_0} \geq 10P = 10\left(1.2\times10^5\ \mathrm{J/m^3}\right) = 1.2\times10^6\ \mathrm{J/m^3}$$

which requires a magnetic field of magnitude

$$B \geq \sqrt{2\mu_0(10P)} = \sqrt{2\left(4\pi\times10^{-7}\ \mathrm{N/A^2}\right)\left(1.24\times10^6\ \mathrm{J/m^3}\right)}$$

$$= \boxed{1.8\ \mathrm{T}}$$

This is a very strong field.

Section 45.5 Radiation Damage

P45.32 (a) The number of x-ray images made per year is (assuming a 2-week vacation)

$$n = (8\ \text{x-ray/d})(5\ \text{d/wk})(50\ \text{wk/yr}) = 2.0\times10^3\ \text{x-ray/yr}$$

The average dose per photograph is

$$\frac{5.0\ \text{rem/yr}}{2.0\times10^3\ \text{x-ray/yr}} = 2.5\times10^{-3}\ \text{rem/x-ray} = \boxed{2.5\ \text{mrem/x-ray}}$$

(b) The technician receives low-level background radiation at a rate of 0.13 rem/yr. The ration dose of 5.0 rem/yr received as a result of the job to background is

$$\frac{5.0\ \text{rem/yr}}{0.13\ \text{rem/yr}} = 38$$

$\boxed{\text{The technician's occupational exposure is high compared to background radiation—it is 38 times 0.13 rem/yr.}}$

P45.33 (a) $I = I_0 e^{-\mu x}$, so $x = -\frac{1}{\mu}\ln\left(\frac{I}{I_0}\right)$, with $\mu = 1.59\ \mathrm{cm^{-1}}$.

When the intensity $I = \frac{I_0}{2}$,

$$x = -\frac{1}{1.59\ \mathrm{cm^{-1}}}\ln\left(\frac{1}{2}\right) = \boxed{0.436\ \mathrm{cm}}$$

(b) When $I = \frac{I_0}{1.00\times10^4}$,

$$x = -\frac{1}{1.59\ \mathrm{cm^{-1}}}\ln\left(\frac{1}{1.00\times10^4}\right) = \boxed{5.79\ \mathrm{cm}}$$

P45.34 (a) $I = I_0 e^{-\mu x}$, so $x = -\frac{1}{\mu}\ln\left(\frac{I}{I_0}\right)$.

When intensity $I = \frac{I_0}{2}$, $x = -\frac{1}{\mu}\ln\left(\frac{I}{I_0}\right) = -\frac{1}{\mu}\ln\left(\frac{1}{2}\right) = \boxed{\frac{\ln(2)}{\mu}}$.

(b) When intensity $I = f I_0$, $x = -\frac{1}{\mu}\ln\left(\frac{I}{I_0}\right) = -\frac{1}{\mu}\ln(f) = \boxed{-\frac{\ln f}{\mu}}$.

P45.35 The source delivers 100 mrad of 2.00-MeV γ-rays/h at a 1.00-m distance. The RBE for these γ-rays is 1.0 (from Table 45.1).

(a) From Equation 45.6,

dose in rem = dose in rad × RBE

1.00 rem = dose in rad × 1.0

or, dose in rad = 1.00 rad = $(100\times10^{-3}\ \text{rad/h})\Delta t$

which gives $\Delta t = 10.0$ h.

Thus a person would have to stand there $\boxed{\text{10.0 hours}}$ to receive 1.00 rem from a 100-mrad/h source.

(b) If the γ-radiation is emitted isotropically, the dosage rate falls off as $\frac{1}{r^2}$.

Thus a dosage 10.0 mrad/h would be received at a distance $r = \sqrt{10.0}$ m = $\boxed{3.16\ \text{m}}$.

***P45.36** For each gray (GY) or radiation, 1 J of energy is delivered to each kilogram of absorbing material. Thus, the total energy delivered in this whole body dose to a 75.0-kg person is

$$E = (0.250\ \text{Gy})\left(1\ \frac{\text{J/kg}}{\text{Gy}}\right)(75.0\ \text{kg}) = \boxed{18.8\ \text{J}}$$

P45.37 By definition, one rad increases the energy of one kilogram of the absorbing material by 1.00×10^{-2} J. The energy starts as energy carried by electromagnetic radiation, and turns entirely into internal energy. The 1 000 rad or 10.0 gray = 10.0 Gy will then put 10.0 J/kg into the body, to raise its temperature by the same amount as 10.0 J/kg of energy input by heat from a higher-temperature energy source. In $Q = mc\Delta T$ we have $Q/m = 10.0$ J/kg and

$$\Delta T = \frac{Q}{m}\frac{1}{c} = (10.0\ \text{J/kg})\left(\frac{1}{4\ 186\ \text{J/kg}\cdot{}^\circ\text{C}}\right) = \boxed{2.39\times10^{-3}\ {}^\circ\text{C}}$$

P45.38 Assume all the energy from the x-ray machine is absorbed by the water and that no energy leaves the cup of water by heat or thermal radiation. The energy input to the cup and the temperature of the water are related by

$$T_{ER} = mc\Delta T$$

Because the power input *P* is equal to $T_{ER}/\Delta T,$ we have

$$P\Delta t = mc\Delta T \quad \rightarrow \quad \Delta t = \frac{mc\Delta T}{P}$$

where we have solved for the time interval required to raise the temperature of the water. We note that the temperature of the water will increase until it is 100°C, after which the latent heat of vaporization of $L_v = 2.26 \times 10^6$ J/kg would have to be added to boil the water. For the purposes of this problem, we limit ourselves to increasing the temperature of the water to 100°C. Substituting numerical values gives

$$\Delta t = \frac{m(4\ 186\ \text{J/kg}\cdot{}^\circ\text{C})(50.0^\circ\text{C})}{(10.0\ \text{rad/s})(1 \times 10^{-2}\ \text{J/kg})m} = 2.09 \times 10^6\ \text{s} = 24.2\ \text{d}$$

Therefore, it would take over 24 days just to increase the water's temperature to 100°C, and much longer to boil it, and this technique will not work for a 20-minute coffee break!

P45.39 The number of nuclei in the original sample is

$$N_0 = \frac{\text{mass present}}{\text{mass of nucleus}} = \frac{5.00\ \text{kg}}{(89.907\ 7\ \text{u})(1.66 \times 10^{-27}\ \text{kg/u})}$$
$$= 3.35 \times 10^{25}\ \text{nuclei}$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{29.1\ \text{yr}} = 2.38 \times 10^{-2}\ \text{yr}^{-1} = 4.53 \times 10^{-8}\ \text{min}^{-1}$$

The original activity is

$$R_0 = \lambda N_0 = (4.53 \times 10^{-8}\ \text{min}^{-1})(3.35 \times 10^{25}\ \text{nuclei})$$
$$= 1.52 \times 10^{18}\ \text{decays/min}$$

The law of decay then gives us

$$\frac{R}{R_0} = \frac{10.0\ \text{decays/min}}{1.52 \times 10^{18}\ \text{decays/min}} = 6.59\ \times 10^{-18} = e^{-\lambda t}$$

and the time interval is

$$t = \frac{-\ln(R/R_0)}{\lambda} = \frac{-\ln(6.59\times10^{-18})}{2.38\times10^{-2}\ \text{yr}^{-1}} = \boxed{1.66\times10^{3}\ \text{yr}}$$

P45.40 If half of the 0.140-MeV gamma rays are absorbed by the patient, the total energy absorbed is

$$E = \frac{(0.140\ \text{MeV})}{2}\left[\left(\frac{1.00\times10^{-8}\ \text{g}}{98.9\ \text{g/mol}}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{1\ \text{mol}}\right)\right]$$

$$= (4.26\times10^{12}\ \text{MeV})(1.60\times10^{-13}\ \text{J/MeV}) = 0.682\ \text{J}$$

Thus, the dose received is Dose $= \dfrac{0.682\ \text{J}}{60.0\ \text{kg}}\left(\dfrac{1\ \text{rad}}{10^{-2}\ \text{J/kg}}\right) = \boxed{1.14\ \text{rad}}$

P45.41 The decay constant is $\lambda = \ln 2/T_{1/2} = \ln 2/17.0\ \text{d}$. The number of nuclei remaining after 30.0 days is

$$N = N_0 e^{-\lambda T} = N_0 \exp\left[\left(\frac{-\ln 2}{17.0\ \text{d}}\right)30.0\ \text{d}\right] = 0.294 N_0$$

The number decayed is $N_0 - N = N_0(1 - 0.294) = 0.706N_0$.

Then the energy release is

$$2.12\ \text{J} = (0.706N_0)(21.0\times10^{3}\ \text{eV})\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)$$

$$N_0 = \frac{2.12\ \text{J}}{2.37\times10^{-15}\ \text{J}} = 8.94\times10^{14}$$

(a) The initial activity is

$$R_0 = \lambda N_0 = \frac{\ln 2}{17.0\ \text{d}}(8.94\times10^{14})\left(\frac{1\ \text{d}}{86\ 400\ \text{s}}\right) = \boxed{4.22\times10^{8}\ \text{Bq}}$$

(b) We find the total mass contained in the seeds from

$$\text{original sample mass} = m = N_0 m_{\text{one atom}}$$

$$= 8.94\times10^{14}\left[(103\ \text{u})\left(\frac{1.66\times10^{-27}\ \text{kg}}{1\ \text{u}}\right)\right]$$

Then,

$$m = \boxed{1.53\times10^{-10}\ \text{kg}} = 1.53\times10^{-7}\ \text{g} = 153\ \text{ng}$$

P45.42 The nuclei initially absorbed are (mass from Table 44.2)

$$N_0 = \left(1.00\times10^{-9}\ \text{g}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei/mol}}{89.9\ \text{g/mol}}\right) = 6.70\times10^{12}$$

The number of decays in time *t* is

$$\Delta N = N_0 - N = N_0\left(1-e^{-\lambda t}\right) = N_0\left(1-e^{-(\ln 2)t/T_{1/2}}\right)$$

At the end of 1 year,

$$\Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left\{1-\exp\left[\left(\frac{-\ln 2}{29.1\ \text{yr}}\right)1.00\ \text{yr}\right]\right\}$$

$$= 1.58\times10^{11}$$

The energy deposited is

$$E = \left(1.58\times10^{11}\right)\left(1.10\ \text{MeV}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right) = 0.027\,7\ \text{J}$$

Thus, the dose received is

$$\text{Dose} = \left(\frac{0.027\,7\ \text{J}}{70.0\ \text{kg}}\right) = \boxed{3.96\times10^{-4}\ \text{J/kg}} = 0.039\,6\ \text{rad}$$

Section 45.6 Uses of Radiation

P45.43 (a) With $I(x) = \frac{1}{2}I_0$, $I(x) = I_0e^{-\mu x}$ becomes

$$\frac{1}{2}I_0 = I_0e^{-0.72x/\text{mm}}$$

$$2 = e^{+0.72x/\text{mm}} \rightarrow \ln 2 = 0.72x/\text{mm} \rightarrow x = \frac{(\ln 2)\ \text{mm}}{0.72}$$

$$= \boxed{0.963\ \text{mm}}$$

(b) The intensity reaching the detector through $x_1 = 0.800$ mm of steel is $I_1 = I_0e^{-\mu x_1}$. That transmitted by thickness $x_2 = 0.700$ mm is $I_2 = I_0e^{-\mu x_2}$. The fractional change is

$$\frac{I_2 - I_1}{I_1} = \frac{I_0e^{-\mu x_2} - I_0e^{-\mu x_1}}{I_0e^{-\mu x_1}} = e^{\mu(x_1-x_2)} - 1 = e^{(0.720/\text{mm})(0.100\ \text{mm})} - 1$$

$$= e^{0.0720} - 1 = +0.074\,7 = 7.47\%$$

As the thickness decreases, $\boxed{\text{the intensity increases by 7.47\%.}}$

P45.44 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \quad \text{so} \quad dN = (R - \lambda N)dt.$$

The variables are separable.

$$\int_0^N \frac{dN}{R - \lambda N} = \int_0^t dt: \quad -\frac{1}{\lambda}\ln\left(\frac{R - \lambda N}{R}\right) = t$$

so $\ln\left(\frac{R - \lambda N}{R}\right) = -\lambda t$ and $\left(\frac{R - \lambda N}{R}\right) = e^{-\lambda t}$.

Therefore $1 - \frac{\lambda}{R}N = e^{-\lambda t} \rightarrow N = \frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)$.

(b) The maximum number of radioactive nuclei would be $\boxed{\frac{R}{\lambda}}$.

P45.45 (a) The number of photons is $\frac{10^4\ \text{MeV}}{1.04\ \text{MeV}} = 9.62 \times 10^3$. Since only 50% of the photons are detected, the number of ^{65}Cu nuclei decaying is twice this value, or 1.92×10^4. In two half-lives, three-fourths of the original nuclei decay, so $\frac{3}{4}N_0 = 1.92 \times 10^4$ and $N_0 = 2.56 \times 10^4$. This is 1% of the ^{65}Cu, so the number of ^{65}Cu is $2.56 \times 10^6\ \boxed{\sim 10^6}$.

(b) Natural copper is 69.17% ^{63}Cu and 30.83% ^{65}Cu. Thus, if the sample contains N_{Cu} copper atoms, the number of atoms of each isotope is $N_{63} = 0.691\,7\ N_{\text{Cu}}$ and $N_{65} = 0.308\,3\ N_{\text{Cu}}$. Therefore,

$$\frac{N_{63}}{N_{65}} = \frac{0.691\,7}{0.308\,3}$$

or $$N_{63} = \left(\frac{0.6917}{0.3083}\right)N_{65} = \left(\frac{0.6917}{0.3083}\right)\left(2.56 \times 10^6\right) = 5.75 \times 10^6$$

The total mass of copper present is then

$$m_{\text{Cu}} = (62.93\ \text{u})N_{63} + (64.93\ \text{u})N_{65}$$

$$m_{\text{Cu}} = \left[(62.93\ \text{u})\left(5.75 \times 10^6\right) + (64.93\ \text{u})\left(2.56 \times 10^6\right)\right] \times \left(1.66 \times 10^{-24}\ \text{g/u}\right)$$

$$= 8.77 \times 10^{-16}\ \text{g}\ \boxed{\sim 10^{-15}\ \text{g}}$$

Additional Problems

P45.46 (a) The energy released by the ${}_1^1\text{H} + {}_5^{11}\text{B} \to 3\left({}_2^4\text{He}\right)$ reaction is

$$Q = \left[M_{{}_1^1\text{H}} + M_{{}_5^{11}\text{B}} - 3M_{{}_2^4\text{He}}\right]c^2$$

$$Q = \left[1.007\,825\text{ u} + 11.009\,305\text{ u} - 3\left(4.002\,603\text{ u}\right)\right] \times \left(931.5\text{ MeV/u}\right)$$

$$= \boxed{8.68\text{ MeV}}$$

(b) The particles must have enough kinetic energy to overcome their mutual electrostatic repulsion so that they can get close enough to fuse.

P45.47 From momentum conservation, we have

$$0 = m_{\text{Li}}\vec{\mathbf{v}}_{\text{Li}} + m_\alpha\vec{\mathbf{v}}_\alpha \text{ or } m_{\text{Li}}v_{\text{Li}} = m_\alpha v_\alpha$$

Thus,

$$K_{\text{Li}} = \frac{1}{2}m_{\text{Li}}v_{\text{Li}}^2 = \frac{1}{2}\frac{\left(m_{\text{Li}}v_{\text{Li}}\right)^2}{m_{\text{Li}}} = \frac{\left(m_\alpha v_\alpha\right)^2}{2m_{\text{Li}}} = \left(\frac{m_\alpha^2}{2m_{\text{Li}}}\right)v_\alpha^2$$

$$K_{\text{Li}} = \left[\frac{\left(4.002\,6\text{ u}\right)^2}{2\left(7.016\,0\text{ u}\right)}\right]\left(9.25\times10^6\text{ m/s}\right)^2$$

$$= \left(1.14\text{ u}\right)\left(1.66\times10^{-27}\text{ kg/u}\right)\left(9.25\times10^6\text{ m/s}\right)^2$$

$$K_{\text{Li}} = 1.62\times10^{-13}\text{ J} = \boxed{1.01\text{ MeV}}$$

P45.48 (a) We have $I = \frac{1}{2}\rho v\left(\omega s_{\text{max}}\right)^2$, and from Equation 17.10, $I = \frac{\left(\Delta P_{\text{max}}\right)^2}{2\rho v}$.

Substituting the second expression for *I* into the first and solving for s_{max} gives

$$s_{\text{max}} = \frac{1}{\omega}\left(\frac{2I}{\rho v}\right)^{1/2} = \frac{1}{\omega}\left[\frac{2}{\rho v}\frac{\left(\Delta P_{\text{max}}\right)^2}{2\rho v}\right]^{1/2} = \frac{\Delta P_{\text{max}}}{\omega\rho v}$$

Solving for ΔP_{max} and assuming $s_{\text{max}} \sim 2.5$ m,

$$\Delta P_{\text{max}} = \omega\rho v s_{\text{max}} = \left(1\text{ s}^{-1}\right)\left(1.20\text{ kg/m}^3\right)\left(343\text{ m/s}\right)\left(2.5\text{ m}\right)$$

$$\sim \boxed{10^3\text{ Pa}}$$

(b) The change in volume is given by

$$\Delta V = 4\pi r^2 \Delta r = 4\pi\left(14.0\times10^3\ \text{m}\right)^2(2.5\ \text{m})$$
$$= 1.23\times10^8\ \text{m}^3 \sim \boxed{6\times10^9\ \text{m}^3}$$

(c) The energy carried by the blast wave is

$$W = \left(\Delta P_{\text{max}}\right)(\Delta V) = \left(10^3\ \text{Pa}\right)\left(6\times10^9\ \text{m}^3\right) = \boxed{6\times10^{12}\ \text{J}}$$

(d) Since the blast wave carries only 10% of the bomb's energy, $6\times10^{16}\ \text{J} = \frac{1}{10}(\text{yield})$, and the bomb yield is then

$$\text{yield} = 6\times10^{13}\ \text{J}\ \boxed{\sim10^{14}\ \text{J}}$$

(e) The yield in terms of tons of TNT is

$$\frac{6\times10^{13}\ \text{J}}{4.2\times10^9\ \text{J/ton TNT}} = 1.42\times10^4\ \text{ton TNT}\boxed{\sim 10^4\ \text{ton TNT}}$$

***P45.49** The Japanese call it the *original child bomb*.

(a) Suppose each ^{235}U fission releases 208 MeV of energy. Then, the number of nuclei that must have undergone fission is

$$N = \frac{\text{total release}}{\text{energy per nuclei}} = \frac{5\times10^{13}\ \text{J}}{(208\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right)}.$$
$$= \boxed{1.5\times10^{24}\ \text{nuclei}}$$

(b) $$\text{mass} = \left(\frac{1.5\times10^{24}\ \text{nuclei}}{6.02\times10^{23}\ \text{nuclei/mol}}\right)(235\ \text{g/mol}) \approx \boxed{0.6\ \text{kg}}$$

P45.50 (a) Subtracting the background counts, the decay counts are $N_1 = 372 - 5(15) = 297$ in the first 5.00 min interval and $N_2 = 337 - 5(15) = 262$ in the second. The midpoints of the time intervals are separated by $T = 5.00$ min. We use $R = R_0 e^{-\lambda t}$, taking $t = T$ and identifying $R_0 = N_1/T = 297/5$ min and $R = N_2/T = 262/5$ min. We have then

$$\frac{N_2}{T} = \left(\frac{N_1}{T}\right)e^{-\lambda T} \quad \text{or} \quad \frac{262}{5\ \text{min}} = \left(\frac{297}{5\ \text{min}}\right)e^{-\left(\ln 2/T_{1/2}\right)(5.00\ \text{min})}$$

which gives

$$e^{-\left(\ln 2/T_{1/2}\right)T} = \frac{N_2}{N_1} \quad \text{or} \quad e^{-\left(\ln 2/T_{1/2}\right)(5.00\ \text{min})} = \frac{262}{297}$$

Solving,

$$-\frac{\ln 2}{T_{1/2}}T = \ln\left(\frac{N_2}{N_1}\right) \qquad \text{or} \qquad -\frac{\ln 2}{T_{1/2}}T = \ln\left(\frac{262}{297}\right)$$

The half-life is then

$$T_{1/2} = \frac{-\ln 2}{\ln\left(N_2/N_1\right)}T = \frac{-\ln 2}{\ln\left(262/297\right)}(5.00\text{ min}) = \boxed{27.6\text{ min}}$$

NOTE: If it seems questionable to set instantaneous decay rates equal to average decay rates, to let $R_0 = N_1/T$ and $R = N_2/T$, see the Alternate Solution to (a) below. The results are the same.

(b) The average count rate is about

$$\frac{1}{2}\left(\frac{262}{5\text{ min}} + \frac{297}{5\text{ min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) \sim 1\text{ s}^{-1}$$

but the counts are randomly spaced in time, meaning some counts near the beginning and end of each 5.00-min interval should or should not have been counted. Let's assume that the count incidence could vary by as much as 5 seconds, so we shall assume a count uncertainty of ±5. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262-5}{297+5}\right) = -\frac{\ln 2}{T_{1/2}}(5.00\text{ min}), \text{ giving } \left(T_{1/2}\right)_{\min} = 21.5\text{ min}$$

The largest credible value is found from

$$\ln\left(\frac{262+5}{297-5}\right) = -\frac{\ln 2}{T_{1/2}}(5.00\text{ min}), \text{ yielding } \left(T_{1/2}\right)_{\max} = 38.7\text{ min}$$

Thus, the half-life is about

$$T_{1/2} = \left(\frac{38.5+21.7}{2}\right) \pm \left(\frac{38.5-21.7}{2}\right)\text{ min}$$

$$= (30 \pm 8)\text{ min} = \boxed{30\text{ min} \pm 27\%}$$

<u>Alternate Solution to (a)</u> The amount of the radioactive sample at time *t* is $N = N_0e^{-\lambda t}$, where we do not know N_0. The number of decay counts between $t = 0$ and $t = T$ are

$$N_1 = N_0\left(1 - e^{-\lambda T}\right) = 297$$

and the number of decay counts between $t = 0$ and $t = 2T$ are

$$N_1 + N_2 = N_0\left(1 - e^{-\lambda 2T}\right) = 297 + 262 = 559$$

To eliminate N_0, we consider the ratio of the counts:

$$r = \frac{N_1 + N_2}{N_1} = \frac{N_0\left(1-e^{-\lambda 2T}\right)}{N_0\left(1-e^{-\lambda T}\right)} = \frac{559}{297}$$

$$r = \frac{\left(1-e^{-\lambda 2T}\right)}{\left(1-e^{-\lambda T}\right)} = \frac{\left(1-e^{-\lambda T}\right)\left(1+e^{-\lambda T}\right)}{\left(1-e^{-\lambda T}\right)} = 1 + e^{-\lambda T}$$

solving,

$$e^{-\lambda T} = r - 1 = \frac{N_1 + N_2}{N_1} - 1 = \frac{N_2}{N_1} \quad \rightarrow \quad e^{-\left(\ln 2/T_{1/2}\right)T} = \frac{N_2}{N_1}$$

which leads to the same result as above, $T_{1/2} = \dfrac{-\ln 2}{\ln\left(N_2/N_1\right)}T$.

P45.51 (a) The energy amplification is

$$\frac{E}{E_0} = \frac{\frac{1}{2}C\Delta V^2}{0.500\text{ MeV}} = \frac{\frac{1}{2}\left(5.00\times 10^{-12}\text{ F}\right)\left(1.00\times 10^{3}\text{ V}\right)^2}{\left(0.500\text{ MeV}\right)\left(1.60\times 10^{-13}\text{ J/MeV}\right)}$$

$$= \boxed{3.12\times 10^{7}}$$

(b) The number of electrons is

$$N = \frac{Q}{e} = \frac{C\Delta V}{e} = \frac{\left(5.00\times 10^{-12}\text{ F}\right)\left(1.00\times 10^{3}\text{ V}\right)}{1.60\times 10^{-19}\text{ C}}$$

$$= \boxed{3.12\times 10^{10}\text{ electrons}}$$

P45.52 (a) To conserve momentum, the two fragments must move in opposite directions with speeds v_1 and v_2 such that

$$m_1 v_1 = m_2 v_2 \quad \text{or} \quad v_2 = \left(\frac{m_1}{m_2}\right)v_1$$

The kinetic energies after the break-up are then

$$K_1 = \frac{1}{2}m_1 v_1^2 \quad \text{and} \quad K_2 = \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_2\left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right)K_1$$

The fraction of the total kinetic energy carried off by m_1 is

$$\frac{K_1}{K_{tot}} = \frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + \left(m_1/m_2\right)K_1} = \frac{m_2}{m_1 + m_2}$$

and the fraction carried off by m_2 is

$$\frac{K_2}{K_{\text{tot}}}=1-\frac{K_1}{K_{\text{tot}}}=1-\frac{m_2}{m_1+m_2}=\frac{m_1}{m_1+m_2}$$

(b) The disintegration energy is

$$Q=\left(236.045\,562\text{ u}-86.920\,711\text{ u}-148.934\,370\text{ u}\right)\times\left(931.5\text{ MeV/u}\right)$$

$$=177.4\text{ MeV}=\boxed{177\text{ MeV}}$$

(c) Immediately after fission, this *Q*-value is the total kinetic energy of the fission products. From part (a),

$$\frac{K_1}{K_{\text{tot}}}=\frac{m_2}{m_1+m_2}=\frac{K_{\text{Br}}}{Q}$$

Then,

$$K_{\text{Br}}=Q\frac{m_{\text{La}}}{m_{\text{Br}}+m_{\text{La}}}=(177.4\text{ MeV})\left(\frac{149\text{ u}}{87\text{ u}+149\text{ u}}\right)=\boxed{112.0\text{ MeV}}$$

and $K_{\text{La}}=Q-K_{\text{Br}}=177.4\text{ MeV}-112.0\text{ MeV}=\boxed{65.4\text{ MeV}}$

(d) The speed of the fragments is given by

$$v_{\text{Br}}=\sqrt{\frac{2K_{\text{Br}}}{m_{\text{Br}}}}=\sqrt{\frac{2\left(112\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{(87\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)}}$$

$$=1.58\times10^{7}\text{ m/s}=\boxed{15.8\text{ Mm/s}}$$

and

$$v_{\text{La}}=\sqrt{\frac{2K_{\text{La}}}{m_{\text{La}}}}=\sqrt{\frac{2\left(65.4\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}{(149\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)}}$$

$$=9.20\times10^{6}\text{ m/s}=\boxed{9.30\text{ Mm/s}}$$

***P45.53** (a) For each of the following six steps, the subscripts *a* - *f* of *Q* refer to the corresponding step in Problem 45.30.

For ${}^{12}\text{C}+{}^{1}\text{H}\rightarrow{}^{13}\text{N}+Q$,

$$Q_a=(12.000\,000+1.007\,825-13.005\,739)(931.5\text{ MeV})$$

$$=\boxed{1.94\text{ MeV}}$$

For the second step, add seven electrons to both sides to get:

$${}^{13}\text{N atom}\rightarrow{}^{13}\text{C atom}+e^{+}+e^{-}+Q$$

Then,

$$Q_b = [13.005\ 739 - 13.003\ 355 - 2(0.000\ 549)](931.5\text{ MeV})$$
$$= \boxed{1.20\text{ MeV}}$$
$$Q_c = [13.003\ 355 + 1.007\ 825 - 14.003\ 074](931.5\text{ MeV})$$
$$= \boxed{7.55\text{ MeV}}$$
$$Q_d = [14.003\ 074 + 1.007\ 825 - 15.003\ 065](931.5\text{ MeV})$$
$$= \boxed{7.30\text{ MeV}}$$
$$Q_e = [15.003\ 065 - 15.000\ 109 - 2(0.000\ 549)](931.5\text{ MeV})$$
$$= \boxed{1.73\text{ MeV}}$$
$$Q_f = [15.000\ 109 + 1.007\ 825 - 12 - 4.002\ 603](931.5\text{ MeV})$$
$$= \boxed{4.97\text{ MeV}}$$

(b) The energy released in the annihilations is

$$Q_3 = Q_7 = 2(0.000\ 549)(931.5\text{ MeV})$$
$$= \boxed{1.02\text{ MeV}}$$

(c) The sum is $\boxed{26.7\text{ MeV}}$, the same as for the proton-proton cycle.

(d) Not all of the energy released appears as internal energy in the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

P45.54 The original activity per area is

$$\frac{5.00\times 10^{6}\text{ Ci}}{10^{4}\text{ km}^{2}}\left(\frac{1\text{ km}}{10^{3}\text{ m}}\right)^{2} = 5.00\times 10^{-4}\text{ Ci/m}^{2}$$

The half-life is 29.1 yr. The decay law, $N = N_0 e^{-\lambda t}$, becomes the law of decrease of activity, $R = R_0 e^{-\lambda t}$. If the material is not transported, it describes the time evolution of activity per area, $R/A = R_0/A\ e^{-\lambda t}$. Solving for the time *t* gives

$$e^{\lambda t} = \frac{R_0/A}{R/A} \quad\to\quad t = \frac{1}{\lambda}\ln\left(\frac{R_0/A}{R/A}\right)$$

Substituting numerical values,

$$t = \left(\frac{29.1\text{ yr}}{\ln 2}\right)\ln\left(\frac{R_0/A}{R/A}\right) = \frac{29.1\text{ yr}}{\ln 2}\ln\left(\frac{5.00\times 10^{-4}\text{ Ci/m}^{2}}{2.00\times 10^{-6}\text{ Ci/m}^{2}}\right)$$
$$= \boxed{232\text{ yr}}$$

P45.55 The number of nuclei in 3.80 kg of ${}^{238}_{94}\text{Pu}$ is

$$N_0 = \left(\frac{3\,800\text{ g}}{238.049\,560\text{ g/mol}}\right)\left(6.022\times10^{23}\text{ nuclei/mol}\right)$$
$$= 9.61\times10^{24}\text{ nuclei}$$

The half-life of ${}^{238}_{94}\text{Pu}$ is 87.7 years, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(87.7\text{ yr})(3.155\times10^{7}\text{ s/yr})} = 2.51\times10^{-10}\text{ s}^{-1}$$

The initial activity is

$$R_0 = \lambda N_0 = \left(2.51\times10^{-10}\text{ s}^{-1}\right)\left(9.61\times10^{24}\text{ nuclei}\right) = 2.41\times10^{15}\text{ Bq}$$

The energy released in each ${}^{238}_{94}\text{Pu}\rightarrow{}^{234}_{92}\text{U}+{}^{4}_{2}\text{He}$ reaction is

$$Q = \left[M_{{}^{238}_{94}\text{Pu}} - M_{{}^{234}_{92}\text{U}} - M_{{}^{4}_{2}\text{He}}\right]c^2:$$

$$Q = [238.049\,560\text{ u} - 234.040\,952\text{ u} - 4.002\,603\text{ u}] \times (931.5\text{ MeV/u})$$
$$= 5.59\text{ MeV}$$

Thus, assuming a conversion efficiency of 3.20%, the initial power output of the battery is

$$P = (0.032\,0)R_0Q$$
$$= (0.032\,0)\left(2.41\times10^{15}\text{ decays/s}\right)(5.59\text{ MeV/decay}) \times \left(1.602\times10^{-13}\text{ J/MeV}\right)$$
$$= \boxed{69.0\text{ W}}$$

P45.56 The number of hydrogen-3 nuclei is

$$N = \left(50.0\text{ m}^3\right)\left(2.00\times10^{14}\ \frac{\text{particles}}{\text{cm}^3}\right)(100\text{ cm/m})^3$$
$$= 1.00\times10^{22}\text{ particles}$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \left(\frac{0.693}{12.3\text{ yr}}\right)\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) = 1.78\times10^{-9}\text{ s}^{-1}$$

The activity is then

$$R = \lambda N = \left(1.78\times10^{-9}\text{ s}^{-1}\right)\left(1.00\times10^{22}\text{ nuclei}\right) = 1.78\times10^{13}\text{ Bq}$$

In curies this is

$$R = \left(1.78 \times 10^{13}\ \text{Bq}\right)\left(\frac{1\ \text{Ci}}{3.70 \times 10^{10}\ \text{Bq}}\right) = 482\ \text{Ci}$$

$\boxed{\text{482 Ci, which is less than the fission inventory by on the order of a hundred million times.}}$

P45.57 The complete fissioning of 1.00 gram of ^{235}U releases

$$Q = \left(\frac{1.00\ \text{g}}{235\ \text{grams/mol}}\right)\left(\frac{6.02\times 10^{23}\ \text{atoms}}{\text{mol}}\right) \times\left(\frac{200\ \text{MeV}}{\text{fission}}\right)\left(\frac{1.60\times 10^{-13}\ \text{J}}{\text{MeV}}\right)$$

$$= 8.20\times 10^{10}\ \text{J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 20 of text for values), then

$$Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$$

$$Q = m\left[\left(4\ 186\ \text{J/kg}\ ^\circ\text{C}\right)\left(80.0\ ^\circ\text{C}\right) + 2.26\times 10^6\ \text{J/kg} + \left(2\ 010\ \text{J/kg}\ ^\circ\text{C}\right)\left(300\ ^\circ\text{C}\right)\right]$$

Therefore, $m = \dfrac{8.20\times 10^{10}\ \text{J}}{3.20\times 10^{6}\ \text{J/kg}} = \boxed{2.56\times 10^4\ \text{kg}}$

P45.58 When mass m of ^{235}U undergoes complete fission, releasing energy E per fission event, the total energy released is

$$Q = \left(\frac{m}{M_{\text{U-235}}}\right)N_A E$$

where N_A is Avogadro's number. If all this energy could be utilized to convert a mass m_w of liquid water at T_c into steam at T_h, then

$$Q = m_w\left[c_w\left(100^\circ\text{C} - T_c\right) + L_v + c_s\left(T_h - 100^\circ\text{C}\right)\right]$$

where c_w is the specific heat of liquid water, L_v is the latent heat of vaporization, and c_s is the specific heat of steam. Solving for the mass of water converted gives

$$m_w = \frac{Q}{\left[c_w\left(100^\circ\text{C} - T_c\right) + L_v + c_s\left(T_h - 100^\circ\text{C}\right)\right]}$$

$$= \boxed{\frac{mN_A E}{M_{\text{U-235}}\left[c_w\left(100 - T_c\right) + L_v + c_s\left(T_h - 100\right)\right]}}$$

P45.59 (a) $Q_{\mathrm{I}} = \left[M_A + M_B - M_C - M_E\right]c^2$, and

$$Q_{\mathrm{II}} = \left[M_C + M_D - M_F - M_G\right]c^2$$

$$Q_{\text{net}} = Q_{\mathrm{I}} + Q_{\mathrm{II}} = \left[M_A + M_B - M_C - M_E + M_C + M_D - M_F - M_G\right]c^2$$

$$Q_{\text{net}} = Q_{\mathrm{I}} + Q_{\mathrm{II}} = \left[M_A + M_B + M_D - M_E - M_F - M_G\right]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives

$$^1_1\mathrm{H} + {}^1_1\mathrm{H} + {}^{\;0}_{-1}\mathrm{e} + {}^1_1\mathrm{H} + {}^1_1\mathrm{H} + {}^{\;0}_{-1}\mathrm{e} \rightarrow {}^4_2\mathrm{He} + 2\nu$$

$$4\,{}^1_1\mathrm{H} + 2\,{}^{\;0}_{-1}\mathrm{e} \rightarrow {}^4_2\mathrm{He} + 2\nu$$

Adding two electrons to each side gives

$$4\,{}^1_1\mathrm{H}\text{ atom} \rightarrow {}^4_2\mathrm{He}\text{ atom} + 2\nu$$

Thus,

$$\begin{aligned} Q_{\text{net}} &= \left[4M_{^1_1\mathrm{H}} - M_{^4_2\mathrm{He}}\right]c^2 \\ &= \left[4(1.007\ 825\ \mathrm{u}) - 4.002\ 603\ \mathrm{u}\right](931.5\ \mathrm{MeV/u}) \\ &= \boxed{26.7\ \mathrm{MeV}} \end{aligned}$$

P45.60 (a) From the definition of the volume of a cube and the definition of mass density, we have $V = \ell^3 = \dfrac{m}{\rho}$, so

$$\ell = \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{70.0\ \mathrm{kg}}{19.1\times 10^3\ \mathrm{kg/m^3}}\right)^{1/3} = 0.154\ \mathrm{m} = \boxed{15.4\ \mathrm{cm}}$$

(b) We add 92 electrons to both sides of the given nuclear reaction. Then it becomes

$$^{238}_{\ 92}\mathrm{U}\text{ atom} \rightarrow 8\,{}^4_2\mathrm{He}\text{ atom} + {}^{206}_{\ 82}\mathrm{Pb}\text{ atom}$$

The Q value of this reaction is

$$\begin{aligned} Q &= \left[M_{^{238}_{\ 92}\mathrm{U}} - 8M_{^4_2\mathrm{He}} - M_{^{206}_{\ 82}\mathrm{Pb}}\right]c^2 \\ &= \left[238.050\,783 - 8(4.002\,603) - 205.974\,449\right](931.5\ \mathrm{MeV/u}) \\ Q &= \boxed{51.7\ \mathrm{MeV}} \end{aligned}$$

(c) $\boxed{\text{The number of decays per second is the decay rate R, and the energy released in each decay is Q. Then the energy released per unit time interval is P = QR.}}$

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00\times10^{4}\ \text{g}}{238\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{nuclei/mol}\right)$$

$$= 1.77\times10^{26}\ \text{nuclei}$$

The decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47\times10^{9}\ \text{yr}} = 1.55\times10^{-10}\ \frac{1}{\text{yr}}$$

and the rate of decays is then

$$R = \lambda N = \left(1.55\times10^{-10}\ \frac{1}{\text{yr}}\right)\left(1.77\times10^{26}\ \text{nuclei}\right)$$

$$= 2.75\times10^{16}\ \text{decays/yr}$$

so, $$P = QR = (51.7\ \text{MeV})\left(2.75\times10^{16}\ \text{yr}^{-1}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right)$$

$$= \boxed{2.27\times10^{5}\ \text{J/yr}}$$

(e) We know that

dose in rem = dose in rad × RBE

or

5.00 rem/yr = (dose in rad/yr)(1.10)

giving

(dose in rad/yr) = 4.55 rad/yr

The allowed whole-body dose is then

$$(70.0\ \text{kg})(4.55\ \text{rad/yr})\left(\frac{10^{-2}\ \text{J/kg}}{1\ \text{rad}}\right) = \boxed{3.18\ \text{J/yr}}$$

P45.61 (a) The mass of the pellet is

$$m = \rho V = \rho\frac{4\pi}{3}r^{3} = \left(0.200\ \text{g/cm}^{3}\right)\left[\frac{4\pi}{3}\left(\frac{1.50\times10^{-2}\ \text{cm}}{2}\right)^{3}\right]$$

$$= 3.53\times10^{-7}\ \text{g}$$

The pellet consists of equal numbers of ^{2}H and ^{3}H atoms, so the average molar mass is 2.50 and the total number of atoms is

$$N = \left(\frac{3.53\times 10^{-7}\ \text{g}}{2.50\ \text{g/mol}}\right)\left(6.02\times 10^{23}\ \text{atoms/mol}\right)$$

$$= 8.51\times 10^{16}\ \text{atoms}$$

When the pellet is vaporized, the plasma will consist of 2N particles (N nuclei and N electrons). The total energy delivered to the plasma is 1.00% of 200 kJ or 2.00 kJ. The temperature of the plasma is found from $E = (2N)\left(\frac{3}{2}k_BT\right)$ as

$$T = \frac{E}{3Nk_B} = \frac{2.00\times 10^3\ \text{J}}{3\left(8.51\times 10^{16}\right)\left(1.38\times 10^{-23}\ \text{J/K}\right)} = \boxed{5.68\times 10^8\ \text{K}}$$

(b) Each fusion event uses 2 nuclei, so N/2 events will occur. From Equation 45.4, the energy released by one fusion event is 17.59 MeV, so the total energy released will be

$$E = \left(\frac{N}{2}\right)Q = \left(\frac{8.51\times 10^{16}}{2}\right)(17.59\ \text{MeV})\left(1.60\times 10^{-13}\ \text{J/MeV}\right)$$

$$= 1.20\times 10^5\ \text{J} = \boxed{120\ \text{kJ}}$$

P45.62 (a) From the given equation, the ratio of the two intensities is

$$\frac{I_2}{I_1} = \frac{I_0e^{-\mu_2x}}{I_0e^{-\mu_1x}} = \boxed{e^{-(\mu_2-\mu_1)x}}$$

(b) Substituting numerical values into the equation in part (a) gives

$$\frac{I_{50}}{I_{100}} = \exp\left[-\left(5.40\ \text{cm}^{-1} - 41.0\ \text{cm}^{-1}\right)(0.100\ \text{cm})\right] = e^{3.56} = \boxed{35.2}$$

(c) Here, x = 10.0 mm = 1.00 cm, and

$$\frac{I_{50}}{I_{100}} = \exp\left[-\left(5.40\ \text{cm}^{-1} - 41.0\ \text{cm}^{-1}\right)(1.00\ \text{cm})\right] = e^{35.6}$$

$$= \boxed{2.89\times 10^{15}}$$

Thus, a 1.00-cm-thick aluminum plate has essentially removed the long-wavelength x-rays from the beam.

P45.63 The momentum of the alpha particle and that of the neutron must add to zero, so their velocities must be in opposite directions with magnitudes related by

$$m_n\vec{\mathbf{v}}_n + m_\alpha\vec{\mathbf{v}}_\alpha = 0 \quad \text{or} \quad (1.008\,7\ \text{u})v_n = (4.002\,6\ \text{u})\,v_\alpha$$

At the same time, their kinetic energies must add to 17.6 MeV:

$$E = \frac{1}{2}m_n v_n^2 + \frac{1}{2}m_\alpha v_\alpha^2 = \frac{1}{2}(1.008\ 7\ \text{u})v_n^2 + \frac{1}{2}(4.002\ 6\ \text{u})v_\alpha^2$$
$$= 17.6\ \text{MeV}$$

Substitute $v_\alpha = 0.252\ 0 v_n$ to obtain

$$E = (0.504\ 35\ \text{u})v_n^2 + (0.127\ 10\ \text{u})v_n^2$$
$$= 17.6\ \text{MeV}\left(\frac{1\ \text{u}}{931.494\ \text{MeV}/c^2}\right)$$

Solving for v_n then gives

$$v_n = \sqrt{\frac{0.018\ 9c^2}{0.631\ 45}} = 0.173c = 5.19 \times 10^7\ \text{m/s}$$

Since this speed is not too much greater than 0.1*c*, we can get a reasonable estimate of the kinetic energy of the neutron from the classical equation,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(1.008\ 7\ \text{u})(0.173c)^2\left(\frac{931.494\ \text{MeV}/c^2}{\text{u}}\right)$$
$$= \boxed{14.1\ \text{MeV}}$$

For a more accurate calculation of the kinetic energy, we should use relativistic expressions. Conservation of energy for this reaction requires that

$$E_n + E_\alpha = \left(m_n c^2 + K_n\right) + \left(m_\alpha c^2 + K_\alpha\right) = m_n c^2 + m_\alpha c^2 + K \qquad \textbf{[1]}$$

where *K* = 17.6 MeV is the total kinetic energy, and conservation of momentum for this reaction requires that

$$\vec{\mathbf{p}}_n + \vec{\mathbf{p}}_\alpha = 0 \quad \rightarrow \quad p_n = p_\alpha \qquad \textbf{[2]}$$

From the relation between total energy, mass, and momentum of a particle, we have

$$E^2 = p^2c^2 + \left(mc^2\right)^2 \quad \rightarrow \quad p^2c^2 = E^2 - \left(mc^2\right)^2 \qquad \textbf{[3]}$$

From equations [2] and [3], we may write

$$p_n^2c^2 = p_\alpha^2c^2$$
$$E_n^2 - \left(m_n c^2\right)^2 = E_\alpha^2 - \left(m_\alpha c^2\right)^2$$
$$E_n^2 - E_\alpha^2 = \left(m_n c^2\right)^2 - \left(m_\alpha c^2\right)^2$$
$$\left(E_n - E_\alpha\right)\left(E_n + E_\alpha\right) = \left(m_n c^2\right)^2 - \left(m_\alpha c^2\right)^2$$

Substituting the above expression into equation [1] gives

$$(E_n - E_\alpha)(m_n c^2 + m_\alpha c^2 + K) = (m_n c^2)^2 - (m_\alpha c^2)^2$$

$$E_n - E_\alpha = \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + K)}$$

$$E_\alpha = E_n - \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + K)}$$

Substituting this result back into equation [1] gives

$$E_n + \left[E_n - \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + K)} \right] = m_n c^2 + m_\alpha c^2 + K$$

$$2E_n = (m_n c^2 + m_\alpha c^2 + K) + \frac{(m_n c^2)^2 - (m_\alpha c^2)^2}{(m_n c^2 + m_\alpha c^2 + K)}$$

$$E_n = \frac{(m_n c^2 + m_\alpha c^2 + K)^2 + (m_n c^2)^2 - (m_\alpha c^2)^2}{2(m_n c^2 + m_\alpha c^2 + K)}$$

To find the kinetic energy of the neutron, we note that $E_n = m_n c^2 + K_n$:

$$E_n = \frac{(m_n c^2 + m_\alpha c^2 + K)^2 + (m_n c^2)^2 - (m_\alpha c^2)^2}{2(m_n c^2 + m_\alpha c^2 + K)} = m_n c^2 + K_n$$

$$K_n = \frac{(m_n c^2 + m_\alpha c^2 + K)^2 + (m_n c^2)^2 - (m_\alpha c^2)^2}{2(m_n c^2 + m_\alpha c^2 + K)} - m_n c^2$$

For $K = 17.6$ MeV,

$$m_n c^2 = (1.008\,7\text{ u})c^2(931.494\text{ MeV}/c^2\cdot\text{u}) = 939.60\text{ MeV}$$

and $$m_\alpha c^2 = (4.002\,6\text{ u})c^2(931.494\text{ MeV}/c^2\cdot\text{u}) = 3\,728.4\text{ MeV}$$

we find that $K_n = \boxed{14.0\text{ MeV}}$.

P45.64 (a) The number of Pu nuclei in 1.00 kg is

$$\frac{6.02\times10^{23}\text{ nuclei/mol}}{239.05\text{ g/mol}}(1\,000\text{ g}) = 2.52\times10^{24}\text{ nuclei}$$

The total energy is

$$(25.2\times10^{23}\text{ nuclei})\left(\frac{1\text{ fission}}{\text{nucleus}}\right)\left(\frac{200\text{ MeV}}{\text{fission}}\right) = 5.04\times10^{26}\text{ MeV}$$

$$E = \left(5.04\times10^{26}\ \text{MeV}\right)\left(4.44\times10^{-20}\ \text{kWh/MeV}\right)$$
$$= \boxed{2.24\times10^{7}\ \text{kWh}}$$

or 22 million kWh.

(b) $$E = \Delta mc^2 = \left(3.016\,049\ \text{u} + 2.014\,102\ \text{u} - 4.002\,603\ \text{u} - 1.008\,665\ \text{u}\right) \times\left(931.5\ \text{MeV/u}\right)$$

$$E = \boxed{17.6\ \text{MeV for each D-T fusion}}$$

(c) $$E_n = \left(\text{total number of D nuclei}\right)\left(17.6\ \text{MeV}\right)\left(4.44\times10^{-20}\ \text{kWh/MeV}\right)$$

$$E_n = \left(\frac{6.02\times10^{23}}{\text{mol}}\right)\left(\frac{1\,000\ \text{g}}{2.014\ \text{g/mol}}\right)\left(17.6\ \text{MeV}\right)\times\left(4.44\times10^{-20}\ \text{kWh/MeV}\right)$$
$$= \boxed{2.34\times10^{8}\ \text{kWh}}$$

(d) $$E_n = \left(\text{the number of C atoms in 1.00 kg}\right)\times\left(\frac{4.20\ \text{eV}}{\text{kg}}\right)$$

$$E_n = \left(\frac{6.02\times10^{26}}{12\ \text{g}}\right)\left(4.20\times10^{-6}\ \text{MeV}\right)\left(4.44\times10^{-20}\ \text{kWh/MeV}\right)$$
$$= \boxed{9.36\ \text{kWh}}$$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels. Burning coal in the open puts carbon dioxide into the atmosphere, worsening global warming. Plutonium is a very dangerous material to have sitting around.

P45.65 (a) We have 1.00 kg – (1.00 kg)(0.007 20) – (1.00 kg)(0.000 0500) = 0.993 kg of ^{238}U, comprising

$$N = \left(0.993\ \text{kg}\right)\left(\frac{6.02\times10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.238\ \text{kg}}\right)$$
$$= 2.51\times10^{24}\ \text{nuclei}$$

with activity

$$\begin{aligned} R = \lambda N &= \frac{\ln 2}{4.47\times 10^{9}\ \text{yr}}\left(2.51\times 10^{24}\ \text{nuclei}\right) \\ &\qquad \times\left(\frac{1\ \text{yr}}{3.16\times 10^{7}\ \text{s}}\right)\left(\frac{1\ \text{Ci}}{3.70\times 10^{10}\ \text{s}^{-1}}\right) \\ &= \boxed{3.3\times 10^{-4}\ \text{Ci}} = 330\ \mu\text{Ci} \end{aligned}$$

We have (1.00 kg)(0.007 20) = 0.007 2 kg of ^{235}U, comprising

$$\begin{aligned} N &= \left(0.007\ 2\ \text{kg}\right)\left(\frac{6.02\times 10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.235\ \text{kg}}\right) \\ &= 1.84\times 10^{22}\ \text{nuclei} \end{aligned}$$

with activity

$$\begin{aligned} R = \lambda N &= \frac{\ln 2}{7.04\times 10^{8}\ \text{yr}}\left(1.84\times 10^{22}\ \text{nuclei}\right) \\ &\qquad \times\left(\frac{1\ \text{yr}}{3.16\times 10^{7}\ \text{s}}\right)\left(\frac{1\ \text{Ci}}{3.70\times 10^{10}\ \text{s}^{-1}}\right) \\ &= 1.6\times 10^{-5}\ \text{Ci} = \boxed{16\ \mu\text{Ci}} \end{aligned}$$

We have (1.00 kg)(0.000 0500) = 5.00 × 10^{-5} kg of ^{234}U, comprising

$$\begin{aligned} N &= \left(5.00\times 10^{-5}\ \text{kg}\right)\left(\frac{6.02\times 10^{23}\ \text{nuclei}}{\text{mol}}\right)\left(\frac{1\ \text{mol}}{0.234\ \text{kg}}\right) \\ &= 1.29\times 10^{20}\ \text{nuclei} \end{aligned}$$

with activity

$$\begin{aligned} R = \lambda N &= \frac{\ln 2}{2.44\times 10^{5}\ \text{yr}}\left(1.29\times 10^{20}\ \text{nuclei}\right) \\ &\qquad \times\left(\frac{1\ \text{yr}}{3.16\times 10^{7}\ \text{s}}\right)\left(\frac{1\ \text{Ci}}{3.70\times 10^{10}\ \text{s}^{-1}}\right) \\ &= \boxed{3.1\times 10^{-4}\ \text{Ci}} = 310\ \mu\text{Ci} \end{aligned}$$

(b) The total activity is (330 + 16 + 310) μCi = 656 μCi, so the fractional contributions are, respectively, 330/656 = $\boxed{50\%}$, 16/656 = $\boxed{2.4\%}$, and 310/656 = $\boxed{47\%}$

(c) $\boxed{\text{It is dangerous, notably if the material is inhaled as a powder. With precautions to minimize human contact, however, microcurie sources are routinely used in laboratories.}}$

P45.66 (a) The number of molecules in 1.00 liter of water (mass = 1 000 g) is

$$\mathrm{N} = \left(\frac{1.00\times10^{3}\ \mathrm{g}}{18.0\ \mathrm{g/mol}}\right)\left(6.02\times10^{23}\ \text{molecules/mol}\right)$$

$$= 3.34\times10^{25}\ \text{molecules}$$

The number of deuterium nuclei contained in these molecules is

$$\mathrm{N}' = \left(3.34\times10^{25}\ \text{molecules}\right)\left(\frac{1\ \text{deuteron}}{3\ 300\ \text{molecules}}\right)$$

$$= 1.01\times10^{22}\ \text{deuterons}$$

Since 2 deuterons are consumed per fusion event, the number of events possible is $\frac{\mathrm{N}'}{2} = 5.07\times10^{21}$ reactions, and the energy released is

$$\mathrm{E}_{\text{fusion}} = \left(5.07\times10^{21}\ \text{reactions}\right)\left(3.27\ \text{MeV/reaction}\right)$$

$$= 1.66\times10^{22}\ \text{MeV}$$

$$\mathrm{E}_{\text{fusion}} = \left(1.66\times10^{22}\ \text{MeV}\right)\left(1.60\times10^{-13}\ \text{J/MeV}\right) = \boxed{2.65\times10^{9}\ \mathrm{J}}$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{\mathrm{E}_{\text{fusion}}}{\mathrm{E}_{\text{gasoline}}} = \frac{2.65\times10^{9}\ \mathrm{J}}{3.40\times10^{7}\ \mathrm{J}} = \boxed{78.0\ \text{times larger}}$$

P45.67 (a) At 6 × 10^8 K, the average kinetic energy of a carbon atom is

$$\frac{3}{2}k_{\mathrm{B}}T = (1.5)\left(8.62\times10^{-5}\ \mathrm{eV/K}\right)\left(6\times10^{8}\ \mathrm{K}\right) = \boxed{8\times10^{4}\ \mathrm{eV}}$$

Note that 6 × 10^8 K is about $6^2 = 36$ times larger than 1.5 × 10^7 K, the core temperature of the Sun. This factor corresponds to the higher potential-energy barrier to carbon fusion compared to hydrogen fusion. It could be misleading to compare it to the temperature ~ 10^8 K required for fusion in a low-density plasma in a fusion reactor.

(b) The energy released is

$$Q = \left[2M_{\mathrm{C}^{12}} - M_{\mathrm{Ne}^{20}} - M_{\mathrm{He}^{4}}\right]c^2$$

$$Q = \left[2(12.000\ 000\ \mathrm{u}) - 19.992\ 440\ \mathrm{u} - 4.002\ 603\ \mathrm{u}\right]\times\left(931.5\ \mathrm{MeV/u}\right)$$

$$= \boxed{4.62\ \mathrm{MeV}}$$

In the second reaction,

$$Q = \left[2M_{\mathrm{C}^{12}} - M_{\mathrm{Mg}^{24}}\right]c^2$$

$$Q = \left[2(12.000\ 000\ \mathrm{u}) - 23.985\ 042\ \mathrm{u}\right](931.5\ \mathrm{MeV/u})$$

$$= \boxed{13.9\ \mathrm{MeV}}$$

(c) The energy released is the energy of reaction of the number of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E = (2.00 \times 10^3\ \mathrm{g})\left(\frac{6.02 \times 10^{23}\ \mathrm{atoms/mol}}{12.0\ \mathrm{g/mol}}\right)$$

$$\times\left(\frac{4.62\ \mathrm{MeV/fusion\ event}}{2\ \mathrm{nuclei/fusion\ event}}\right)\left(\frac{1\ \mathrm{kWh}}{2.25 \times 10^{19}\ \mathrm{MeV}}\right)$$

$$\Delta E = \frac{(1.00 \times 10^{26})(4.62)}{2(2.25 \times 10^{19})}\mathrm{kWh} = \boxed{1.03 \times 10^7\ \mathrm{kWh}}$$

P45.68 From Table 44.2 of isotopic masses, the half-life of $^{32}\mathrm{P}$ is 14.26 d. Thus, the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14.26\ \mathrm{d}} = 0.048\ 6\ \mathrm{d}^{-1} = 5.63 \times 10^{-7}\ \mathrm{s}^{-1}$$

and the initial number of nuclei is

$$N_0 = \frac{R_0}{\lambda} = \frac{5.22 \times 10^6\ \mathrm{decay/s}}{5.63 \times 10^{-7}\ \mathrm{s}^{-1}} = 9.28 \times 10^{12}\ \mathrm{nuclei}$$

At $t = 10.0$ days, the number remaining is

$$N = N_0 e^{-\lambda t} = (9.28 \times 10^{12}\ \mathrm{nuclei})\exp\left[-(0.048\,6\ \mathrm{d}^{-1})(10.0\ \mathrm{d})\right]$$

$$= 5.71 \times 10^{12}\ \mathrm{nuclei}$$

so the number of decays has been $N_0 - N = 3.57 \times 10^{12}$ and the energy released is

$$E = (3.57 \times 10^{12})(700\ \mathrm{keV})(1.60 \times 10^{-16}\ \mathrm{J/keV}) = 0.400\ \mathrm{J}$$

If this energy is absorbed by 100 g of tissue, the absorbed dose is

$$\mathrm{Dose} = \left(\frac{0.400\ \mathrm{J}}{0.100\ \mathrm{kg}}\right)\left(\frac{1\ \mathrm{rad}}{10^{-2}\ \mathrm{J/kg}}\right) = \boxed{400\ \mathrm{rad}}$$

P45.69 (a) The thermal power transferred to the water is $P_w = 0.970$ (waste heat):

$$P_w = 0.970(3\ 065\ \mathrm{MW} - 1\ 000\ \mathrm{MW}) = 2.00 \times 10^9\ \mathrm{J/s}$$

r_w is the mass of water heated per hour:

$$\mathrm{r_w} = \frac{\mathrm{P_w}}{\mathrm{c}(\Delta \mathrm{T})} = \frac{(2.00\times 10^9\ \mathrm{J/s})(3600\ \mathrm{s/h})}{(4186\ \mathrm{J/kg\cdot{}^\circ C})(3.50\ {}^\circ\mathrm{C})} = \boxed{4.92\times 10^8\ \mathrm{kg/h}}$$

Then, the volume used per hour is

$$\frac{4.91\times 10^8\ \mathrm{kg/h}}{1.00\times 10^3\ \mathrm{kg/m^3}} = \boxed{4.92\times 10^5\ \mathrm{m^3/h}}$$

(b) The ^{235}U fuel is consumed at a rate

$$\mathrm{r_f} = \left(\frac{3\,065\times 10^6\ \mathrm{J/s}}{7.80\times 10^{10}\ \mathrm{J/g}}\right)\left(\frac{1\ \mathrm{kg}}{1\,000\ \mathrm{g}}\right)\left(\frac{3\,600\ \mathrm{s}}{1\ \mathrm{h}}\right) = \boxed{0.141\ \mathrm{kg/h}}$$

P45.70 We add two electrons to both sides of the given reaction.

Then, $4\ {}_1^1\mathrm{H\ atom} \to {}_2^4\mathrm{He\ atom} + 2\nu$,

where $\mathrm{Q} = (\Delta \mathrm{m})\mathrm{c}^2 = [4(1.007\,825\ \mathrm{u}) - 4.002\,603\ \mathrm{u}](931.5\ \mathrm{MeV/u})$
$= 26.7\ \mathrm{MeV}$

or $\mathrm{Q} = (26.7\ \mathrm{MeV})(1.60\times 10^{-13}\ \mathrm{J/MeV}) = 4.28\times 10^{-12}\ \mathrm{J}$

The proton fusion rate is then

$$\text{rate} = \frac{\text{power output}}{\text{energy per proton}} = \frac{3.85\times 10^{26}\ \mathrm{J/s}}{(4.28\times 10^{-12}\ \mathrm{J})/(4\ \text{protons})}$$
$$= \boxed{3.60\times 10^{38}\ \text{protons/s}}$$

Challenge Problems

P45.71 The initial specific activity of ^{59}Fe in the steel is

$$(\mathrm{R/m})_0 = \frac{20.0\ \mu\mathrm{Ci}}{0.200\ \mathrm{kg}} = \left(\frac{100\ \mu\mathrm{Ci}}{\mathrm{kg}}\right)\left(\frac{3.70\times 10^4\ \mathrm{Bq}}{1\ \mu\mathrm{Ci}}\right)$$
$$= 3.70\times 10^6\ \mathrm{Bq/kg}$$

The decay constant of ^{59}Fe is $\lambda = \dfrac{\ln 2}{45.1\ \mathrm{d}}\left(\dfrac{1\ \mathrm{d}}{24\ \mathrm{h}}\right)$.

After 1 000 h, the activity is

$$\begin{aligned}\frac{R}{m} &= \left(\frac{R}{m}\right)_0 e^{-\lambda t}\\ &= \left(3.70\times10^{6}\ \text{Bq/kg}\right)\exp\left[-\left(\frac{\ln 2}{45.1\ \text{d}}\right)\left(\frac{1\ \text{d}}{24\ \text{h}}\right)(1\ 000\ \text{h})\right]\\ &= 1.95\times10^{6}\ \text{Bq/kg}\end{aligned}$$

The activity of the oil is

$$R_{\text{oil}} = \left(\frac{800}{60.0}\ \text{Bq/liter}\right)(6.50\ \text{liters}) = 86.7\ \text{Bq}$$

Therefore,

$$m_{\text{in oil}} = \frac{R_{\text{oil}}}{(R/m)} = \frac{86.7\ \text{Bq}}{1.95\times10^{6}\ \text{Bq/kg}} = 4.44\times10^{-5}\ \text{kg}$$

So that the wear rate is $\dfrac{4.45\times10^{-5}\ \text{kg}}{1\ 000\ \text{h}} = \boxed{4.44\times10^{-8}\ \text{kg/h}}$.

P45.72 (a) The number of fissions occurring in the zeroth, first, second, …, *n*th generation is

$$N_0,\ N_0K,\ N_0K^2,\ \ldots\ ,\ N_0K^n$$

The total number of fissions that have occurred up to and including the *n*th generation is

$$N = N_0 + N_0K + N_0K^2 + \cdots + N_0K^n = N_0\left(1 + K + K^2 + \cdots + K^n\right)$$

Note that the factoring of the difference of two squares, $a^2 - 1 = (a+1)(a-1)$, can be generalized to a difference of two quantities to any power,

$$\begin{aligned}a^3 - 1 &= \left(a^2 + a + 1\right)(a - 1)\\ a^{n+1} - 1 &= \left(a^n + a^{n-1} + \cdots + a^2 + a + 1\right)(a - 1)\end{aligned}$$

Thus, $K^n + K^{n-1} + \cdots + K^2 + K + 1 = \dfrac{K^{n+1} - 1}{K - 1}$

and $\boxed{N = N_0\dfrac{K^{n+1} - 1}{K - 1}}$

(b) The number of U-235 nuclei is

$$N = (5.50\ \text{kg})\left(\frac{1\ \text{atom}}{235\ \text{u}}\right)\left(\frac{1\ \text{u}}{1.66\times10^{-27}\ \text{kg}}\right) = 1.41\times10^{25}\ \text{nuclei}$$

We solve the equation from part (a) for n, the number of generations:

$$\frac{N}{N_0}(K-1)=K^{n+1}-1$$

$$\frac{N}{N_0}(K-1)+1=K^n(K)$$

$$n\ln K=\ln\left(\frac{N(K-1)/N_0+1}{K}\right)=\ln\left(\frac{N(K-1)}{N_0}+1\right)-\ln K$$

$$n=\frac{\ln\left(N(K-1)/N_0+1\right)}{\ln K}-1=\frac{\ln\left(1.41\times10^{25}(0.1)/10^{20}+1\right)}{\ln 1.1}-1$$

$$=99.2$$

Therefore time must be allotted for 100 generations:

$$\Delta t_b = 100\left(10\times10^{-9}\ \text{s}\right)=1.00\times10^{-6}\ \text{s}=\boxed{1.00\ \mu\text{s}}$$

(c) The speed of sound in uranium is

$$v=\sqrt{\frac{B}{\rho}}=\sqrt{\frac{150\times10^{9}\ \text{N/m}^2}{18.7\times10^{3}\ \text{kg/m}^3}}=2.83\times10^{3}\ \text{m/s}=\boxed{2.83\ \text{km/s}}$$

(d) From the definitions of volume and mass density, $V=\frac{4}{3}\pi r^3=\frac{m}{\rho}$,

and

$$r=\left(\frac{3m}{4\pi\rho}\right)^{1/3}=\left(\frac{3\left(5.5\ \text{kg}\right)}{4\pi\left(18.7\times10^{3}\ \text{kg/m}^3\right)}\right)^{1/3}=4.13\times10^{-2}\ \text{m}$$

then, the time interval is given by

$$\Delta t_d=\frac{r}{v}=\frac{4.13\times10^{-2}\ \text{m}}{2.83\times10^{3}\ \text{m/s}}=1.46\times10^{-5}\ \text{s}=\boxed{14.6\ \mu\text{s}}$$

(e) 14.6 μs is greater than 1 μs, so the entire bomb can fission. The destructive energy released is

$$\left(1.41\times10^{25}\ \text{nuclei}\right)\left(\frac{200\times10^{6}\ \text{eV}}{\text{fissioning nucleus}}\right)\left(\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)$$

$$=4.51\times10^{14}\ \text{J}\left(\frac{1\ \text{ton TNT}}{4.20\times10^{9}\ \text{J}}\right)$$

$$=1.08\times10^{5}\ \text{ton TNT}$$

$$=\boxed{108\ \text{kilotons of TNT}}$$

What if? If the bomb did not have an "initiator" to inject 10^{20} neutrons at the moment when the critical mass is assembled, the number of generations would be

$$n = \frac{\ln\left(1.41\times10^{25}(0.1)/1+1\right)}{\ln 1.1} - 1 = 582.4$$

requiring $583\left(10\times10^{-9}\text{ s}\right) = 5.83\ \mu\text{s}$

This time is not very short compared with 14.6 μs, so this bomb would likely release much less energy.

P45.73 (a) E_I = 10.0 eVis the energy required to liberate an electron from a dynode. Let n_i be the number of electrons incident upon a dynode, each having gained energy $e\Delta V$ as it was accelerated to this dynode. The number of electrons that will be freed from this dynode is $N_i = n_i e\frac{\Delta V}{E_I}$.

At the <u>first</u> dynode, $n_i = 1$ and

$$N_1 = \frac{(1)e(100\text{ V})}{10.0\text{ eV}} = \boxed{10^1\text{ electrons}}$$

(b) For the <u>second</u> dynode, $n_i = N_1 = 10^1$, so

$$N_2 = \frac{(10^1)e(100\text{ V})}{10.0\text{ eV}} = 10^2$$

At the <u>third</u> dynode, $n_i = N_2 = 10^2$ and

$$N_3 = \frac{(10^2)e(100\text{ V})}{10.0\text{ eV}} = 10^3$$

Observing the developing pattern, we see that the number of electrons incident on the *n*th dynode is $n_n = N_{n-1} = 10^{n-1}$, so for the seventh and last dynode is $n_7 = N_6 = \boxed{10^6}$.

(c) The number of electrons incident on the last dynode is $n_7 = 10^6$. The total energy these electrons deliver to that dynode is given by

$$E = n_i e(\Delta V) = 10^6 e(700\text{ V} - 600\text{ V}) = \boxed{10^8\text{ eV}}$$

ANSWERS TO EVEN-NUMBERED PROBLEMS

P45.2 0.403 g

P45.4 2.63 kg/d

P45.6 (a) 173 MeV; (b) 0.078 8%

P45.8 26

P45.10 (a) 4×10^{15} g; (b) 5×10^{3} yr; (c) The uranium comes from rocks and minerals dissolved in water and carried into the ocean by rivers; (d) No.

P45.12 (a) 16.2 kg; (b) 117 g

P45.14 (a) $4.84V^{-1/3}$; (b) $6V^{-1/3}$; (c) $6.30V^{-1/3}$; (d) The sphere has minimum leakage and the parallelepiped has minimum leakage.

P45.16 6.25×10^{19} Bq

P45.18 By balancing the equation for electric charge and number of nucleons, we find that $n = 1$. If one incoming neutron results in just one outgoing neutron, the possibility of a chain reaction is not there, so this nuclear reactor will not work.

P45.20 (a) $4.56 \times 10^{-24}\ \text{kg} \cdot \text{m/s}$; (b) 0.145 nm; (c) This size has the same order of magnitude as an atom's outer electron cloud, and is vastly larger than a nucleus.

P45.22 3.07×10^{22} events

P45.24 (a) $E = 144Z_1Z_2$ where E is in keV; (b) The energy is proportional to each atomic number; (c) Take $Z_1 = 1$ and $Z_2 = 59$ or vice versa. This choice minimizes the product $Z_1 Z_2$; (d) 144 keV for both, according to this model

P45.26 (a) 3.24 fm; (b) 444 keV; (c) $\frac{2}{5}v_i$; (d) 740 keV; (e) possibly by tunneling

P45.28 (a) 1.66×10^{7} J; (b) 6.45 kg

P45.30 (a) ${}^{13}_{7}\text{N}$; (b) ${}^{13}_{6}\text{C}$; (c) ${}^{14}_{7}\text{N}$; (d) ${}^{15}_{8}\text{O}$; (e) ${}^{15}_{7}\text{N}$; (f) ${}^{12}_{6}\text{C}$; (g) The original carbon-12 nucleus is returned so the overall reaction is $4\left({}^{1}_{1}\text{H}\right) \to {}^{4}_{2}\text{He}$.

P45.32 (a) 2.5 mrem/x-ray; (b) The technician's occupational exposure is high compared to background radiation; it is 38 times 0.13 rem/yr.

P45.34	(a) $\frac{\ln(2)}{\mu}$; (b) $-\frac{\ln f}{\mu}$
P45.36	18.8 J
P45.38	It would take over 24 days to raise the temperature of the water to 100°C and even longer to boil it, so this technique will not work for a 20-minute coffee break!
P45.40	1.14 rad
P45.42	3.96×10^{-4} J/kg
P45.44	(a) See P45.44(a) for full explanation; (b) $\frac{R}{\lambda}$
P45.46	(a) 8.68 MeV; (b) The particles must have enough kinetic energy to overcome their mutual electrostatic repulsion so that they can get close enough to fuse.
P45.48	(a) 10^3 Pa; (b) 6×10^9 m^3; (c) 6×10^{12} J; (d) $\sim 10^{14}$ J; (e) $\sim 10^4$ ton TNT
P45.50	(a) 27.6 min; (b) 30 min ± 27%
P45.52	(a) See P45.52(a) for full explanation; (b) 177 MeV; (c) $K_{\text{Br}} = 112.0$ MeV, $K_{\text{La}} = 65.4$ MeV; (d) $v_{\text{Br}} = 15.8$ Mm/s, $v_{\text{La}} = 9.30$ Mm/s
P45.54	232 yr
P45.56	482 Ci, less than the fission inventory by on the order of a hundred million times.
P45.58	$\frac{mN_AE}{M_{\text{U-235}}\left[c_w\left(100 - T_c\right) + L_v + c_s\left(T_h - 100\right)\right]}$
P45.60	(a) 15.4 cm; (b) 51.7 MeV; (c) The number of decays per second is the decay rate *R*, and the energy released in each decay is *Q*. Then the energy released per unit time interval is $P = QR$; (d) 2.27×10^5 J/yr; (e) 3.18 J/yr
P45.62	(a) See P45.62(a) for full explanation; (b) 35.2; (c) 2.89×10^{15}
P45.64	(a) 2.24×10^7 kWh; (b) 17.6 MeV for each D-T fusion; (c) 2.34×10^8 kWh; (d) 9.36 kWh; (e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels. Burning coal in the open puts carbon dioxide into the atmosphere, worsening global warming. Plutonium is a very dangerous material to have sitting around.
P45.66	(a) 2.65×10^9 J; (b) 78.0 times larger

P45.68 400 rad

P45.70 3.60×10^{38} protons/s

P45.72 (a) See P45.72(a) for full explanation; (b) 1.00 μs; (c) 2.83 km/s; (d) 14.6 μs; (e) 108 kilotons of TNT

46

Particle Physics and Cosmology

CHAPTER OUTLINE

* An asterisk indicates a question or problem new to this edition.

ANSWERS TO OBJECTIVE QUESTIONS

OQ46.1 Answers (a), (b), (c), and (d). Protons feel all these forces; but within a nucleus the strong interaction predominates, followed by the electromagnetic interaction, then the weak interaction. The gravitational interaction is very small.

OQ46.2 Answer (e). Kinetic energy is transformed into internal energy: $Q = -\Delta K$. In the first experiment, momentum conservation requires the final speed be zero:

$$p_1 = mv - mv = 2mv_f \rightarrow \quad v_f = 0$$

The kinetic energy converted into internal energy is mv^2:

$$\Delta K_1 = K_f - K_i = 0 - \left(\tfrac{1}{2}mv^2 + \tfrac{1}{2}mv^2\right) = -mv^2 \quad \rightarrow \quad Q_1 = mv^2$$

In the second experiment, momentum conservation requires the final speed be half the initial speed:

$$p_2 = mv + m(0) = 2mv_f \qquad \rightarrow \qquad v_f = \frac{v}{2}$$

The kinetic energy converted into internal energy is $\dfrac{mv^2}{4}$:

$$\Delta K_2 = K_f - K_i = \frac{1}{2}(2m)\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2 = -\frac{mv^2}{4} \qquad \rightarrow \qquad Q_2 = \frac{mv^2}{4}$$

OQ46.3 Answer (b). There are $(2s+1) = \left(2\tfrac{3}{2}+1\right) = 4$ states: the *z* component of its spin angular momentum can be 3/2, 1/2, –1/2, or –3/2, in units of $\hbar$.

OQ46.4 Answer (b). According the Table 46.1, the photon mediates the electromagnetic force, the graviton the gravitational force, and the W^+ and Z bosons the weak force.

OQ46.5 Answer (c). According to Table 46.2, the muon has much more rest energy (105.7 MeV/c^2) than the electron (0.511 MeV/c^2) and the neutrinos together (< 0.3 MeV/c^2). The missing rest energy goes into kinetic energy: $m_\mu c^2 = K_{\text{total}} + m_e c^2 + m_{\bar{\nu}_e} c^2 + m_{\nu_\mu} c^2$.

OQ46.6 Answer (a). The vast gulfs not just between stars but between galaxies and especially between clusters, empty of ordinary matter, are important to bring down the average density of the Universe. We can estimate the average density defined for the Solar System as the mass of the Sun divided by the volume of a sphere of radius 2×10^{16} m:

$$\frac{2\times 10^{30}\ \text{kg}}{\frac{4}{3}\pi(2\times 10^{16}\ \text{m})^3} = 6\times 10^{-20}\ \text{kg/m}^3 = 6\times 10^{-23}\ \text{g/cm}^3$$

This is ten million times larger than the critical density $3H^2/8\pi G = 6 \times 10^{-30}$ g/cm^3.

OQ46.7 Answer (b). Momentum would not be conserved. The electron and positron together have very little momentum. A 1.02-MeV photon has a definite amount of momentum. Production of a single gamma ray could not satisfy the law of conservation of momentum, which must hold true in this—and every—interaction.

OQ46.8 The sequence is c, b, d, e, a, f, g. Refer to Figure 46.16 in the textbook. The temperature corresponding to b is on the order of 10^{13} K. That for hydrogen fusion d is on the order of 10^{7} K. A fully ionized plasma can be at 10^{4} K. Neutral atoms can exist at on the order of 3 000 K, molecules at 1 000 K, and solids at on the order of 500 K.

ANSWERS TO CONCEPTUAL QUESTIONS

CQ46.1 The electroweak theory of Glashow, Salam, and Weinberg predicted the W^+, W^-, and Z particles. Their discovery in 1983 confirmed the electroweak theory.

CQ46.2 Hadrons are massive particles with internal structure. There are two classes of hadrons: mesons (bosons) and baryons (fermions). Hadrons are composed of quarks, so they interact via the strong force. Leptons are light particles with no structure. All leptons are fermions. It is believed that leptons are fundamental particles (otherwise, there would be leptonic bosons); leptons are not composed of quarks, so they do not interact via the strong force.

CQ46.3 Before that time, the Universe was too hot for the electrons to remain bound to any nucleus. The thermal motion of both nuclei and electrons was too rapid for the Coulomb force to dominate. The Universe was so filled high energy photons that any nucleus that managed to captured an electron would immediately lose it because of Compton scattering or the photoelectric effect.

CQ46.4 Baryons are heavy hadrons; they are fermions with spin $\frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \ldots$; they are composed of three quarks. (Antibaryons are composed of three antiquarks.) Mesons are light hadrons; they are bosons with spin 0, 1, 2, …; they are composed of a quark and an antiquark.

CQ46.5 The decay is slow, relatively speaking. The decays by the weak interaction typically take 10^{-10} s or longer to occur. This is slow in particle physics. The decay does not conserve strangeness: the Ξ^0 has strangeness of -2, the Λ^0 has strangeness -1, and the π^0 has strangeness 0. (Refer to Table 46.2.)

CQ46.6 The word "color" has been adopted in *analogy* to the properties of the three primary colors (and their complements) in additive color mixing. Each flavor of quark can have colors, designated as red, green, and blue. Antiquarks are colored antired, antigreen, and antiblue. We call baryons and mesons *colorless*. A baryon consists of three quarks, each having a different color: the analogy is three primary colors combine to form no color: colorless white. A meson consists of a quark of one color and antiquark with the corresponding anticolor: the analogy is a primary color and its complementary color combine to form no color: colorless white.

CQ46.7 No. Antibaryons have baryon number –1, mesons have baryon number 0, and baryons have baryon number +1. The reaction cannot occur because it would not conserve baryon number, unless so much energy is available that a baryon-antibaryon pair is produced.

CQ46.8 The Standard Model consists of quantum chromodynamics (to describe the strong interaction) and the electroweak theory (to describe the electromagnetic and weak interactions). The Standard Model is our most comprehensive description of nature. It fails to unify the two theories it includes, and fails to include the gravitational force. It pictures matter as made of six quarks and six leptons, interacting by exchanging gluons, photons, and W and Z bosons. In 2011 and 2012, experiments at CERN produced evidence for the Higgs boson, a cornerstone of the Standard Model.

CQ46.9 (a) Baryons consist of three quarks.
(b) Antibaryons consist of three antiquarks.
(c) and (d) Mesons and antimesons consist of a quark and an antiquark.

Since quarks have spin quantum number $\frac{1}{2}$ and can be spin-up or spin-down, it follows that the baryons and antibaryons must have a half-integer spin ($\frac{1}{2}, \frac{3}{2}, \ldots$), while the mesons and antimesons must have integer spin (0, 1, 2, …).

CQ46.10 We do know that the laws of conservation of momentum and energy are a consequence of Newton’s laws of motion; however, conservation of baryon number, lepton number, and strangeness cannot be traced to Newton’s laws. Even though we do not know what electric charge *is*, we do know it is conserved, so too we do not know what baryon number, lepton number, or strangeness *are*, but we do know they are conserved—or in the case of strangeness, sometimes conserved—from observations of how elementary particles interact and decay. You can think of these conservation laws as regularities which we happen to notice, as a person who does not know the rules of chess might observe that one player’s two bishops are always on squares of opposite colors. (From the observation of the behavior of baryon number, lepton number, and strangeness in particle interactions, *gauge theories*, which are not discussed in the textbook, have been developed to describe that behavior.)

CQ46.11 The interactions and their field particles are listed in Table 46.1.

Strong Force—Mediated by gluons.
Electromagnetic Force—Mediated by photons.
Weak Force—Mediated by W^+, W^-, and Z^0 bosons.
Gravitational Force—Mediated by gravitons (not yet observed).

CQ46.12 Hubble determined experimentally that all galaxies outside the Local Group are moving away from us, with speed directly proportional to the distance of the galaxy from us, by observing that their light spectra were red shifted in direct relation to their distance from the Local Group.

CQ46.13 The baryon number of a proton or neutron is one. Since baryon number is conserved, the baryon number of the kaon must be zero. See Table 46.2.

SOLUTIONS TO END-OF-CHAPTER PROBLEMS

Section 46.1 The Fundamental Forces in Nature

Section 46.2 Positrons and Other Antiparticles

P46.1 (a) The rest energy of a total of 6.20 g of material is converted into energy of electromagnetic radiation:

$$E = mc^2 = \left(6.20\times10^{-3}\ \text{kg}\right)\left(2.998\times10^{8}\ \text{m/s}\right)^2 = \boxed{5.57\times10^{14}\ \text{J}}$$

(b)
$$5.57\times10^{14}\ \text{J} = 5.57\times10^{14}\ \text{J}\left(\frac{\$0.11}{\text{kWh}}\right)\left(\frac{\text{k}}{1\ 000}\right)\left(\frac{\text{W}}{\text{J/s}}\right)\left(\frac{1\ \text{h}}{3\ 600\ \text{s}}\right)$$
$$= \boxed{\$1.70\times10^{7}}$$

P46.2 (a) The minimum energy is released, and hence the minimum frequency photons are produced, when the proton and antiproton are at rest when they annihilate.

That is, $E = E_0$ and $K = 0$. To conserve momentum, each photon must have the same magnitude of momentum, and $p = E/c$, so each photon must carry away one-half the energy.

Thus $E_{\text{min}} = \dfrac{2E_0}{2} = E_0 = 938.3\ \text{MeV} = hf_{\text{min}}$.

$$\text{Thus,}\ f_{\text{min}} = \frac{\left(938.3\ \text{MeV}\right)\left(1.602\times10^{-13}\ \text{J/MeV}\right)}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{2.27\times10^{23}\ \text{Hz}}.$$

(b) $\lambda = \frac{c}{f_{\min}} = \frac{2.998\times10^{8}\text{ m/s}}{2.27\times10^{23}\text{ Hz}} = \boxed{1.32\times10^{-15}\text{ m}}$

P46.3 (a) Assuming that the proton and antiproton are left nearly at rest after they are produced, the energy *E* of the photon must be

$$E = 2E_0 = 2(938.3\text{ MeV}) = 1\ 876.6\text{ MeV}\left(\frac{1.602\times10^{-13}\text{ J}}{1\text{ MeV}}\right)$$
$$= 3.01\times10^{-10}\text{ J}$$

Thus, $E = hf = 3.01\times10^{-10}$ J, so

$$f = \frac{3.01\times10^{-10}\text{ J}}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{4.53\times10^{23}\text{ Hz}}$$

(b) $\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\text{ m/s}}{4.53\times10^{23}\text{ Hz}} = \boxed{6.61\times10^{-16}\text{ m}}$

P46.4 The half-life of ^{14}O is 70.6 s, so the decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{70.6\text{ s}}$.

The number of ^{14}O nuclei remaining after five minutes is

$$N = N_0 e^{-\lambda t} = \left(10^{10}\right)\exp\left[-\frac{\ln 2}{70.6\text{ s}}(300\text{ s})\right] = 5.26\times10^{8}$$

The number of these in one cubic centimeter of blood is

$$N' = N\left(\frac{1.00\text{ cm}^3}{\text{total volume of blood}}\right) = \left(5.26\times10^{8}\right)\left(\frac{1.00\text{ cm}^3}{2\ 000\text{ cm}^3}\right)$$
$$= 2.63\times10^{5}$$

and their activity is

$$R = \lambda N' = \frac{\ln 2}{70.6\text{ s}}\left(2.63\times10^{5}\right) = 2.58\times10^{3}\text{ Bq}\quad\boxed{\sim10^{3}\text{ Bq}}$$

P46.5 The total energy of each particle is the sum of its rest energy and its kinetic energy. Conservation of system energy requires that the total energy before this pair production event equal the total energy after. In $\gamma \to p^+ + p^-$, conservation of energy requires that

$$E_\gamma \to E_{p^+} + E_{p^-}$$
$$E_\gamma \to \left(m_p c^2 + K_{p^+}\right) + \left(m_p c^2 + K_{p^-}\right)$$

or $$E_\gamma = \left(E_{Rp} + K_p\right) + \left(E_{R\bar{p}} + K_{\bar{p}}\right)$$

The energy of the photon is given as

$$E_\gamma = 2.09\ \text{GeV} = 2.09\times10^3\ \text{MeV}$$

From Table 46.2 or from the problem statement, we see that the rest energy of both the proton and the antiproton is

$$E_{Rp} = E_{R\bar{p}} = m_pc^2 = 938.3\ \text{MeV}$$

If the kinetic energy of the proton is observed to be 95.0 MeV, the kinetic energy of the antiproton is

$$\begin{aligned}K_{\bar{p}} &= E_\gamma - E_{R\bar{p}} - E_{Rp} - K_p\\ &= 2.09\times10^3\ \text{MeV} - 2(938.3\ \text{MeV}) - 95.0\ \text{MeV} = \boxed{118\ \text{MeV}}\end{aligned}$$

Section 46.3 Mesons and the Beginning of Particle Physics

P46.6 The creation of a virtual Z^0 boson is an energy fluctuation $\Delta E = m_{Z^0}c^2 = 91\times10^9$ eV. By the uncertainty principle, it can last no longer than $\Delta t = \dfrac{\hbar}{2\Delta E}$ and move no farther than

$$\begin{aligned}c(\Delta t) &= \frac{hc}{4\pi\,\Delta E}\\ &= \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}{4\pi(91\times10^9\ \text{eV})}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)\\ &= 1.06\times10^{-18}\ \text{m} = \boxed{\sim10^{-18}\ \text{m}}\end{aligned}$$

P46.7 (a) The particle's rest energy is mc^2. The time interval during which a virtual particle of this mass could exist is at most Δt in $\Delta E\Delta t = \dfrac{\hbar}{2} = mc^2\Delta t$; or $\Delta t = \dfrac{\hbar}{2mc^2}$; so, the distance it could move (traveling at the speed of light) is at most

$$\begin{aligned}d \approx c\Delta t &= \frac{\hbar c}{2mc^2} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{4\pi mc^2(1.602\times10^{-19}\ \text{J/eV})}\\ &= \frac{1.240\times10^{-6}\ \text{eV}\cdot\text{m}}{4\pi mc^2}\left(\frac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = \frac{1\ 240\ \text{eV}\cdot\text{nm}}{4\pi mc^2}\\ &= \frac{98.7\ \text{eV}\cdot\text{nm}}{mc^2}\end{aligned}$$

or $d \approx \frac{98.7}{mc^2}$, where d is in nanometers and mc^2 is in electron volts.

According to Yukawa's line of reasoning, this distance is the range of a force that could be associated with the exchange of virtual particles of this mass.

(b) $\boxed{\text{The range is inversely proportional to the mass of the field particle.}}$

(c) The value of mc^2 for the proton in electron volts is 938.3×10^6. The range of the force is then

$$d \approx \frac{98.7}{mc^2} = \frac{98.7}{938.3 \times 10^6} = \left(1.05 \times 10^{-7}\text{ nm}\right)\left(\frac{10^{-9}}{1\text{ nm}}\right)$$
$$= 1.05 \times 10^{-16}\text{ m}\boxed{\sim 10^{-16}\text{ m}}$$

Section 46.4 Classification of Particles

Section 46.5 Conservation Laws

***P46.8** $\boxed{\text{Baryon number conservation allows the first and forbids the second}}$.

P46.9 The energy and momentum of a photon are related by $p_\gamma = E_\gamma/c$. By momentum conservation, because the neutral pion is at rest, the magnitudes of the momenta of the two photons are equal; thus, their energies are equal.

(a) From Table 46.2, $m_{\pi^0} = 135\text{ MeV}/c^2$. Therefore,

$$E_\gamma = \frac{m_{\pi^0}c^2}{2} = \frac{135.0\text{ MeV}}{2} = \boxed{67.5\text{ MeV}}\text{ for each photon}$$

(b) $p = \frac{E_\gamma}{c} = \boxed{67.5\text{ MeV}/c}$

(c) $f = \frac{E_\gamma}{h} = \frac{67.5\text{ MeV}}{6.626 \times 10^{-34}\text{ J}\cdot\text{s}}\left(\frac{1.602 \times 10^{-13}\text{ J}}{\text{MeV}}\right) = \boxed{1.63 \times 10^{22}\text{ Hz}}$

P46.10 The time interval for a particle traveling with the speed of light to travel a distance of 3×10^{-15} m is

$$\Delta t = \frac{d}{v} = \frac{3 \times 10^{-15}\text{ m}}{3.00 \times 10^8\text{ m/s}} = \boxed{\sim 10^{-23}\text{ s}}$$

P46.11 (a) $p+\bar{p}\to\mu^{+}+e^{-}$ $\quad L_{\mu}: 0+0\to -1+0$ and $L_{e}: 0+0\to 0+1$

$\boxed{\text{muon lepton number and electron lepton number}}$

(b) $\pi^{-}+p\to p+\pi^{+}$ $\quad \boxed{\text{charge}}: -1+1\to +1+1$

(c) $p+p\to p+p+n$ $\quad \boxed{\text{baryon number}}: 1+1\to 1+1+1$

(d) $\gamma+p\to n+\pi^{0}$ $\quad \boxed{\text{charge}}: 0+1\to 0+0$

(f) $\nu_{e}+p\to n+e^{+}$ $\quad L_{e}: 1+0\to 0-1$

$\boxed{\text{electron lepton number}}$

P46.12 (a) Baryon number and charge are conserved, with respective values of

baryon: 0 + 1 = 0 + 1

charge: 1 + 1 = 1 + 1 in both reactions (1) and (2).

(b) The strangeness values for the reactions are

(1) $S: 0+0=1-1$

(2) $S: 0+0=0-1$

$\boxed{\text{Strangeness is not conserved in the second reaction.}}$

P46.13 Check that electron, muon, and tau lepton number are conserved.

(a) $\pi^{-}\to\mu^{-}+\boxed{\bar{\nu}_{\mu}}$ $\quad L_{\mu}: 0\to 1-1$

(b) $K^{+}\to\mu^{+}+\boxed{\nu_{\mu}}$ $\quad L_{\mu}: 0\to -1+1$

(c) $\boxed{\bar{\nu}_{e}}+p^{+}\to n+e^{+}$ $\quad L_{e}: -1+0\to 0-1$

(d) $\boxed{\nu_{e}}+n\to p^{+}+e^{-}$ $\quad L_{e}: 1+0\to 0+1$

(e) $\boxed{\nu_{\mu}}+n\to p^{+}+\mu^{-}$ $\quad L_{\mu}: 1+0\to 0+1$

(f) $\mu^{-}\to e^{-}+\boxed{\bar{\nu}_{e}}+\boxed{\nu_{\mu}}$ $\quad L_{\mu}: 1\to 0+0+1$ and $L_{e}: 0\to 1-1+0$

P46.14 The relevant conservation laws are $\Delta L_{e}=0$, $\Delta L_{\mu}=0$, and $\Delta L_{\tau}=0$.

(a) $\pi^{+}\to\pi^{0}+e^{+}+?$ $\quad L_{e}: 0\to 0-1+L_{e}$ implies $L_{e}=1$, so the particle is $\boxed{\nu_{e}}$.

(b) $? \;+\mathrm{p}\rightarrow\mu^- +\mathrm{p}+\pi^+$ $\quad \mathsf{L}_\mu$: $\mathsf{L}_\mu + 0 \rightarrow +1+0+0$ implies $\mathsf{L}_\mu = 1$, so the particle is $\boxed{\nu_\mu}$.

(c) $\Lambda^0 \rightarrow \mathrm{p}+\mu^- +?$ $\quad \mathsf{L}_\mu$: $0\rightarrow 0+1+\mathsf{L}_\mu$ implies $\mathsf{L}_\mu = -1$, so the particle is $\boxed{\bar{\nu}_\mu}$.

(d) $\tau^+ \rightarrow \mu^+ + ? + ?$ $\quad \mathsf{L}_\mu$: $0\rightarrow -1+\mathsf{L}_\mu$ implies $\mathsf{L}_\mu = 1$, so one particle is $\boxed{\nu_\mu}$.

Also, L_τ: $-1\rightarrow 0+\mathsf{L}_\tau$ implies $\mathsf{L}_\tau = -1$, so the other particle is $\boxed{\bar{\nu}_\tau}$.

P46.15 (a) $\mathrm{p}^+ \rightarrow \pi^+ + \pi^0$ $\quad$ check baryon number: $1\rightarrow 0+0$

$\boxed{\text{It cannot occur because it violates baryon number conservation.}}$

(b) $\mathrm{p}^+ + \mathrm{p}^+ \rightarrow \mathrm{p}^+ + \mathrm{p}^+ + \pi^0$ $\quad \boxed{\text{It can occur.}}$

(c) $\mathrm{p}^+ + \mathrm{p}^+ \rightarrow \mathrm{p}^+ + \pi^+$ check baryon number: $1+1\rightarrow 1+0$

$\boxed{\text{It cannot occur because it violates baryon number conservation.}}$

(d) $\pi^+ \rightarrow \mu^+ + \nu_\mu$ $\quad \boxed{\text{It can occur.}}$

(e) $\mathrm{n}^0 \rightarrow \mathrm{p}^+ + \mathrm{e}^- + \bar{\nu}_\mathrm{e}$ $\quad \boxed{\text{It can occur.}}$

(f) $\pi^+ \rightarrow \mu^+ + \mathrm{n}$ $\quad$ check baryon number: $0\rightarrow 0+1$

check muon lepton number: $0\rightarrow -1+0$

check masses: $m_{\pi^+} < m_{\mu^+} + m_\mathrm{n}$

$$\boxed{\begin{array}{l}\text{It cannot occur because it violates baryon number conservation,}\\ \text{muon lepton number conservation, and energy conservation.}\end{array}}$$

P46.16 The reaction is $\mu^+ + \mathrm{e}^- \rightarrow \nu + \nu$.

muon-lepton number before reaction: (–1) + (0) = –1

electron-lepton number before reaction: (0) + (1) = 1

Therefore, after the reaction, the muon-lepton number must be –1. Thus, one of the neutrinos must be the antineutrino associated with muons, and one of the neutrinos must be the neutrino associated with electrons: $\boxed{\bar{\nu}_\mu}$ and $\boxed{\nu_\mathrm{e}}$

Thus, $\quad \mu^+ + \mathrm{e}^- \rightarrow \bar{\nu}_\mu + \nu_\mathrm{e}$.

P46.17 Momentum conservation for the decay requires the pions to have equal speeds.

The total energy of each is $\frac{497.7 \text{ MeV}}{2} = 248.8 \text{ MeV}$, so

$$E^2 = p^2c^2 + \left(mc^2\right)^2 \text{ gives}$$

$$(248.8 \text{ MeV})^2 = (pc)^2 + (139.6 \text{ MeV})^2$$

Solving, $pc = 206 \text{ MeV} = \gamma mvc = \dfrac{mc^2}{\sqrt{1-(v/c)^2}}\left(\dfrac{v}{c}\right)$:

$$\frac{pc}{mc^2} = \frac{206 \text{ MeV}}{139.6 \text{ MeV}} = \frac{1}{\sqrt{1-(v/c)^2}}\left(\frac{v}{c}\right) = 1.48$$

$$\frac{v}{c} = 1.48\sqrt{1-\left(\frac{v}{c}\right)^2}$$

and $$\left(\frac{v}{c}\right)^2 = 2.18\left[1-\left(\frac{v}{c}\right)^2\right] = 2.18 - 2.18\left(\frac{v}{c}\right)^2$$

$$3.18\left(\frac{v}{c}\right)^2 = 2.18$$

so $$\frac{v}{c} = \sqrt{\frac{2.18}{3.18}} = 0.828 \quad \text{and} \quad \boxed{v = 0.828c}.$$

P46.18 (a) In the suggested reaction $p \to e^+ + \gamma$.

From Table 46.2, we would have for baryon numbers $+1 \to 0 + 0$; thus $\Delta B \neq 0$, so baryon number conservation would be violated.

(b) From conservation of momentum for the decay: $p_e = p_\gamma$

Then, for the positron,

$$E_e^2 = (p_ec)^2 + \left(m_ec^2\right)^2$$

becomes

$$E_e^2 = \left(p_\gamma c\right)^2 + \left(m_ec^2\right)^2 = E_\gamma^2 + \left(m_ec^2\right)^2$$

From conservation of energy for the system: $m_pc^2 = E_e + E_\gamma$

or $E_e = m_pc^2 - E_\gamma$,

so $E_e^2 = (m_p c^2)^2 - 2(m_p c^2)E_\gamma + E_\gamma^2$.

Equating this to the result from above gives

$$\cancel{E_\gamma^2} + (m_e c^2)^2 = (m_p c^2)^2 - 2(m_e c^2)E_\gamma + \cancel{E_\gamma^2}$$

$$E_\gamma = \frac{(m_p c^2)^2 - (m_e c^2)^2}{2m_p c^2}$$

$$= \frac{(938.3\text{ MeV})^2 - (0.511\text{ MeV})^2}{2(938.3\text{ MeV})} = 469\text{ MeV}$$

Also, $E_e = m_p c^2 - E_\gamma = 938.3\text{ MeV} - 469\text{ MeV} = 469\text{ MeV}$,

Thus, $\boxed{E_e = E_\gamma = 469\text{ MeV}}$.

Also, $p_\gamma = \dfrac{E_\gamma}{c} = \dfrac{469\text{ MeV}}{c}$, so $\boxed{p_e = p_\gamma = 469\text{ MeV}/c}$.

(c) The total energy of the positron is $E_e = 469$ MeV,

but $E_e = \gamma m_e c^2 = \dfrac{m_e c^2}{\sqrt{1-(v/c)^2}}$,

so $\sqrt{1-\left(\dfrac{v}{c}\right)^2} = \dfrac{m_e c^2}{E_e} = \dfrac{0.511\text{ MeV}}{469\text{ MeV}} = 1.09\times 10^{-3}$,

which yields $\boxed{v = 0.000\,999\,4c}$.

P46.19 (a) To conserve charge, the decay reaction is $\Lambda^0 \to p + \pi^-$.

We look up in the table the rest energy of each particle:

$m_\Lambda c^2 = 1\,115.6\text{ MeV}$ $m_p c^2 = 938.3\text{ MeV}$

$m_\pi c^2 = 139.6\text{ MeV}$

The *Q* value of the reaction, representing the energy output, is the difference between starting rest energy and final rest energy, and is the kinetic energy of the products:

$$Q = 1\,115.6\text{ MeV} - 938.3\text{ MeV} - 139.6\text{ MeV} = \boxed{37.7\text{ MeV}}$$

(b) The original kinetic energy is zero in the process considered here, so the whole *Q* becomes the kinetic energy of the products

$$K_p + K_\pi = \boxed{37.7\text{ MeV}}$$

(c) The lambda particle is at rest. Its momentum is zero. System momentum is conserved in the decay, so the total vector momentum of the proton and the pion must be $\boxed{\text{zero.}}$

(d) The proton and the pion move in precisely opposite directions with precisely equal momentum magnitudes. Because their masses are different, their kinetic energies are not the same.

The mass of the π-meson is much less than that of the proton, so it carries much more kinetic energy. We can find the energy of each. Let p represent the magnitude of the momentum of each. Then the total energy of each particle is given by $E^2 = (pc)^2 + (mc^2)^2$ and its kinetic energy is $K = E - mc^2$. For the total kinetic energy of the two particles we have

$$\sqrt{m_p^2c^4 + p^2c^2} - m_pc^2 + \sqrt{m_\pi^2c^4 + p^2c^2} - m_\pi c^2$$
$$= Q = m_\Lambda c^2 - m_pc^2 - m_\pi c^2$$

Proceeding to solve for pc, we find

$$m_p^2c^4 + p^2c^2 = m_\Lambda^2c^4 - 2m_\Lambda c^2\sqrt{m_\pi^2c^4 + p^2c^2} + m_\pi^2c^4 + p^2c^2$$

$$\sqrt{m_\pi^2c^4 + p^2c^2} = \frac{m_\Lambda^2c^4 - m_p^2c^4 + m_\pi^2c^4}{2m_\Lambda c^2}$$
$$= \frac{1\,115.6^2 - 938.3^2 + 139.6^2}{2(1\,115.6)}\text{ MeV} = 171.9\text{ MeV}$$

$$pc = \sqrt{171.9^2 - 139.6^2}\text{ MeV} = 100.4\text{ MeV}$$

Then the kinetic energies are

$$K_p = \sqrt{938.3^2 + 100.4^2} - 938.3 = 5.35\text{ MeV}$$

and $K_\pi = \sqrt{139.6^2 + 100.4^2} - 139.6 = 32.3\text{ MeV}$

> No. The mass of the π^- meson is much less than that of the proton, so it carries much more kinetic energy. The correct analysis using relativistic energy conservation shows that the kinetic energy of the proton is 5.35 MeV, while that of the π^- meson is 32.3 Mev.

Section 46.6 Strange Particles and Strangeness

P46.20 $\boxed{\begin{array}{l}\text{The } \rho^0 \to \pi^+ + \pi^- \text{ decay must occur via the strong interaction.}\\ \text{The } K_S^0 \to \pi^+ + \pi^- \text{ decay must occur via the weak interaction.}\end{array}}$

P46.21 (a) $\pi^- + p \to 2\eta$

Baryon number: $0 + 1 \to 0$

$\boxed{\text{It is not allowed because baryon number is not conserved.}}$

(b) $K^- + n \to \Lambda^0 + \pi^-$

Baryon number: $0 + 1 \to 1 + 0$

Charge: $-1 + 0 \to 0 - 1$

Strangeness: $-1 + 0 \to -1 + 0$

Lepton number: $0 \to 0$

The interaction may occur via the $\boxed{\text{strong interaction}}$ since all are conserved.

(c) $K^- \to \pi^- + \pi^0$

Strangeness: $-1 \to 0 + 0$

Baryon number: $0 \to 0$

Lepton number: $0 \to 0$

Charge: $-1 \to -1 + 0$

Strangeness conservation is violated by one unit, but everything else is conserved. Thus, the reaction can occur via the $\boxed{\text{weak interaction}}$, but not the strong or electromagnetic interaction.

(d) $\Omega^- \to \Xi^- + \pi^0$

Baryon number: $1 \to 1 + 0$

Lepton number: $0 \to 0$

Charge: $-1 \to -1 + 0$

Strangeness: $-3 \to -2 + 0$

Strangeness conservation is violated by one unit, but everything else is conserved. The reaction may occur by the $\boxed{\text{weak interaction}}$, but not by the strong or electromagnetic interaction.

(e) $\eta \to 2\gamma$

Baryon number: $0 \to 0$

Lepton number: $0 \to 0$

Charge: $0 \to 0$

Strangeness: $0 \to 0$

No conservation laws are violated, but photons are the mediators of the electromagnetic interaction. Also, the lifetime of the η is consistent with the $\boxed{\text{electromagnetic interaction}}$.

P46.22 (a) $\mu^- \to e^- + \gamma$ $\quad L_e$: $0 \to 1 + 0$

L_μ: $1 \to 0$

$\boxed{\text{electron and muon lepton numbers}}$

(b) $n \to p + e^- + \nu_e$ $\quad L_e$: $0 \to 0 + 1 + 1$

$\boxed{\text{electron lepton number}}$

(c) $\Lambda^0 \to p + \pi^0$ $\quad$ Strangeness: $-1 \to 0 + 0$

Charge: $0 \to +1 + 0$

$\boxed{\text{charge and strangeness}}$

(d) $p \to e^+ + \pi^0$ Baryon number: $+1 \to 0 + 0$

$\boxed{\text{baryon number}}$

(e) $\Xi^0 \to n + \pi^0$ $\quad$ Strangeness: $-2 \to 0 + 0$

$\boxed{\text{strangeness}}$

P46.23 (a) $K^+ + p \to ? + p$

The strong interaction conserves everything.

Baryon number: $0 + 1 \to B + 1$ so $B = 0$

Charge: $+1 + 1 \to Q + 1$ so $Q = +1$

Lepton numbers: $0 + 0 \to L + 0$ so $L_e = L_\mu = L_\tau = 0$

Strangeness: $+1 + 0 \to S + 0$ so $S = 1$

The conclusion is that the particle must be positively charged, a non-baryon, with strangeness of +1. Of particles in Table 46.2, it can only be the $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction conserves all but strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \to ? + \pi^-$

Baryon number: $+1 \to \mathrm{B} + 0$ so $B = 1$

Charge: $-1 \to \mathrm{Q} - 1$ so $Q = 0$

Lepton numbers: $0 \to \mathrm{L} + 0$ so $\mathrm{L}_e = \mathrm{L}_\mu = \mathrm{L}_\tau = 0$

Strangeness: $-3 \to \mathrm{S} + 0$ so $\Delta S = 1$: $S = -2$

(There is no particle with $S = -4$.)

The particle must be a neutral baryon with strangeness of –2. Thus, it is the $\boxed{\Xi^0}$.

(c) $\mathrm{K}^+ \to ? + \mu^+ + \nu_\mu$

Baryon number: $0 \to \mathrm{B} + 0 + 0$ so $B = 0$

Charge: $+1 \to \mathrm{Q} + 1 + 0$ so $Q = 0$

Lepton numbers: L_e: $0 \to \mathrm{L}_e + 0 + 0$ so $L_e = 0$

L_μ: $0 \to \mathrm{L}_\mu - 1 + 1$ so $\mathrm{L}_\mu = 0$

L_τ: $0 \to \mathrm{L}_\tau + 0 + 0$ so $\mathrm{L}_\tau = 0$

Strangeness: $1 \to \mathrm{S} + 0 + 0$ so $\Delta\mathrm{S} = \pm 1$: $S = 0$

(There is no meson with $S = 2$.)

The particle must be a neutral meson with strangeness $= 0 \Rightarrow \boxed{\pi^0}$.

P46.24 (a) $\Xi^- \to \Lambda^0 + \mu^- + \nu_\mu$

Baryon number: $+1 \to +1 + 0 + 0$ Charge: $-1 \to 0 - 1 + 0$

L_e: $0 \to 0 + 0 + 0$ L_μ: $0 \to 0 + 1 + 1$

L_τ: $0 \to 0 + 0 + 0$ Strangeness: $-2 \to -1 + 0 + 0$

Conserved quantities are $\boxed{\mathrm{B},\ \text{charge},\ \mathrm{L}_e,\ \text{and}\ \mathrm{L}_\tau}$.

(b) $\mathrm{K}^0_\mathrm{S} \to 2\pi^0$

Baryon number: $0 \to 0$ Charge: $0 \to 0$

L_e: $0 \to 0$ L_μ: $0 \to 0$

L_τ: $0 \to 0$ Strangeness: $+1 \to 0$

Conserved quantities are $\boxed{\mathrm{B},\ \text{charge},\ \mathrm{L}_e,\ \mathrm{L}_\mu,\ \text{and}\ \mathrm{L}_\tau}$.

(c) $\mathrm{K}^- + \mathrm{p} \to \Sigma^0 + \mathrm{n}$

Baryon number: $0 + 1 \to 1 + 1$ Charge: $-1 + 1 \to 0 + 0$

L_e: $0 + 0 \to 0 + 0$ L_μ: $0 + 0 \to 0 + 0$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $-1 + 0 \to -1 + 0$

Conserved quantities are $\boxed{\text{S, charge, } L_e,\ L_\mu, \text{ and } L_\tau}$.

(d) $\Sigma^0 + \Lambda^0 + \gamma$

Baryon number: $+1 \to 1 + 0$ Charge: $0 \to 0$

L_e: $0 \to 0 + 0$ L_μ: $0 \to 0 + 0$

L_τ: $0 \to 0 + 0$ Strangeness: $-1 \to -1 + 0$

Conserved quantities are $\boxed{\text{B, S, charge, } L_e,\ L_\mu, \text{ and } L_\tau}$.

(e) $e^+ + e^- \to \mu^+ + \mu^-$

Baryon number: $0 + 0 \to 0 + 0$ Charge: $+1 - 1 \to +1 - 1$

L_e: $-1 + 1 \to 0 + 0$ L_μ: $0 + 0 \to +1 - 1$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $0 + 0 \to 0 + 0$

Conserved quantities are $\boxed{\text{B, S, charge, } L_e,\ L_\mu, \text{ and } L_\tau}$.

(f) $\bar{\mathrm{p}} + \mathrm{n} \to \bar{\Lambda}^0 + \Sigma^-$

Baryon number: $-1 + 1 \to -1 + 1$ Charge: $-1 + 0 \to 0 - 1$

L_e: $0 + 0 \to 0 + 0$ L_μ: $0 + 0 \to 0 + 0$

L_τ: $0 + 0 \to 0 + 0$ Strangeness: $0 + 0 \to +1 - 1$

Conserved quantities are $\boxed{\text{B, S, charge, } L_e,\ L_\mu, \text{ and } L_\tau}$.

P46.25 (a) $\Lambda^0 \to \mathrm{p} + \pi^-$ Strangeness: $-1 \to 0 + 0$, so $\Delta S = +1$

$\boxed{\text{Strangeness is not conserved.}}$

(b) $\pi^- + \mathrm{p} \to \Lambda^0 + \mathrm{K}^0$ Strangeness: $0 + 0 \to -1 + 1$, so $\Delta S = 0$

$\boxed{\text{Strangeness is conserved.}}$

(c) $\bar{\mathrm{p}} + \mathrm{p} \to \bar{\Lambda}^0 + \Lambda^0$ Strangeness: $0 + 0 \to +1 - 1$, so $\Delta S = 0$

$\boxed{\text{Strangeness is conserved.}}$

(d) $\pi^- + p \to \pi^- + \Sigma^+$ Strangeness: $0 + 0 \to 0 - 1$, so $\Delta S = -1$

$\boxed{\text{Strangeness is not conserved.}}$

(e) $\Xi^- \to \Lambda^0 + \pi^-$ Strangeness: $-2 \to -1 + 0$, so $\Delta S = +1$

$\boxed{\text{Strangeness is not conserved.}}$

(f) $\Xi^0 \to p + \pi^-$ Strangeness: $-2 \to 0 + 0$, so $\Delta S = +2$

$\boxed{\text{Strangeness is not conserved.}}$

P46.26 As a particle travels in a circle, it experiences a centripetal force, and the centripetal force can be related to the momentum of the particle.

$$\sum F = ma: \quad qvB\sin 90° = \frac{mv^2}{r} \quad \to \quad mv = p = qBr$$

(a) Using $p = qBr$ gives momentum in units of kg · m/s. To convert kg · m/s into units of MeV/c, we multiply and divide by c:

$$\left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right) = \left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)\left(\frac{c}{c}\right) = \left(\frac{\text{kg}\cdot\text{m}}{\text{s}}\right)\left(2.998\times10^8 \text{ m/s}\right)\left(\frac{1}{c}\right)$$

$$= \left(2.998\times10^8 \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2}\right)\left(\frac{1}{c}\right)$$

$$= 2.998\times10^8 \text{ J}\left(\frac{1}{c}\right)\left(\frac{1 \text{ MeV}}{1.602\times10^{-13} \text{ J}}\right)$$

$$= 1.871\times10^{21} \text{ MeV}/c$$

$$p_{\Sigma^+} = eBr_{\Sigma^+}$$

$$= \left(1.602\times10^{-19} \text{ C}\right)(1.15 \text{ T})(1.99 \text{ m})\frac{1.871\times10^{21} \text{ MeV}/c}{\text{kg}\cdot\text{m/s}}$$

$$= \boxed{686 \text{ MeV}/c}$$

$$p_{\pi^+} = eBr_{\pi^+}$$

$$= \left(1.602\times10^{-19} \text{ C}\right)(1.15 \text{ T})(0.580 \text{ m})\left(\frac{1.871\times10^{21} \text{ MeV}/c}{\text{kg}\cdot\text{m/s}}\right)$$

$$= \boxed{200 \text{ MeV}/c}$$

(b) The total momentum equals the momentum of the Σ^+ particle. The momentum of the pion makes an angle of 64.5° with respect to the original momentum of the Σ^+ particle. If we take the direction of the momentum of the Σ^+ particle as an axis of reference, and let ϕ be the angle made by the neutron's path with the path of the Σ^+

at the moment of its decay, by conservation of momentum, we have these components of momentum:

parallel to the original momentum:

$$p_{\Sigma^+} = p_n \cos\phi + p_{\pi^+} \cos 64.5°$$

thus,

$$p_n \cos\phi = p_{\Sigma^+} - p_{\pi^+} \cos 64.5°$$

$$p_n \cos\phi = 686 \text{ MeV/c} - (200 \text{ MeV/c})\cos 64.5° \qquad \textbf{[1]}$$

perpendicular to the original momentum:

$$0 = p_n \sin\phi - (200 \text{ MeV/c})\sin 64.5°$$

$$p_n \sin\phi = (200 \text{ MeV/c})\sin 64.5° \qquad \textbf{[2]}$$

From [1] and [2]:

$$p_n = \sqrt{(p_n \cos\phi)^2 + (p_n \sin\phi)^2} = \boxed{626 \text{ MeV/c}}$$

(c) $$E_{\pi^+} = \sqrt{(p_{\pi^+}c)^2 + (m_{\pi^+}c^2)^2} = \sqrt{(200 \text{ MeV})^2 + (139.6 \text{ MeV})^2}$$

$$= \boxed{244 \text{ MeV}}$$

$$E_n = \sqrt{(p_n c)^2 + (m_n c^2)^2} = \sqrt{(626 \text{ MeV})^2 + (939.6 \text{ MeV})^2}$$

$$= 1\,129 \text{ MeV} = \boxed{1.13 \text{ GeV}}$$

(d) $$E_{\Sigma^+} = E_{\pi^+} + E_n = 244 \text{ MeV} + 1129 \text{ MeV} = 1\,373 \text{ MeV} = \boxed{1.37 \text{ GeV}}$$

(e) $$m_{\Sigma^+}c^2 = \sqrt{E_{\Sigma^+}^2 - (p_{\Sigma^+}c)^2} = \sqrt{(1\,373 \text{ MeV})^2 - (686 \text{ MeV})^2} = 1\,189 \text{ MeV}$$

$$\therefore m_{\Sigma^+} = 1\,189 \text{ MeV/c}^2 = \boxed{1.19 \text{ GeV/c}^2}$$

(f) From Table 46.2, the mass of the Σ^+ particle is 1 189.4 MeV/c^2. The percentage difference is

$$\frac{\Delta m}{m} = \frac{1.19 \times 10^3 \text{ MeV/c}^2 - 1\,189.4 \text{ MeV/c}^2}{1\,189.4 \text{ MeV/c}^2} \times 100\% = 0.0504\%$$

$$\boxed{\text{The result in part (e) is within 0.05\% of the value in Table 46.2.}}$$

P46.27 The time-dilated lifetime is

$$T = \gamma T_0 = \frac{0.900 \times 10^{-10} \text{ s}}{\sqrt{1 - v^2/c^2}} = \frac{0.900 \times 10^{-10} \text{ s}}{\sqrt{1 - (0.960)^2}} = 3.214 \times 10^{-10} \text{ s}$$

During this time interval, we see the kaon travel at 0.960*c*. It travels for a distance of

$$\text{distance} = vT = 0.960\left(2.998\times10^{8}\ \text{m/s}\right)\left(3.214\times10^{-10}\ \text{s}\right)$$
$$= 9.25\times10^{-2}\ \text{m} = \boxed{9.25\ \text{cm}}$$

Section 46.7 Finding Patterns in the Particles
Section 46.8 Quarks
Section 46.9 Multicolored Quarks
Section 46.10 The Standard Model

P46.28 (a)

	K^0	d	$\bar{s}$	total
strangeness	1	0	1	1
baryon number	0	1/3	–1/3	0
charge	0	–*e*/3	*e*/3	0

(b)

	Λ^0	u	d	s	total
strangeness	–1	0	0	–1	–1
baryon number	1	1/3	1/3	1/3	1
charge	0	2*e*/3	–*e*/3	–*e*/3	0

P46.29 In the first reaction,

$$\pi^- + p \rightarrow K^0 + \Lambda^0$$

the quarks in the particles are

$$\bar{u}d + uud \rightarrow \bar{s}d + uds$$

There is a net of 1 up quark both before and after the reaction, a net of 2 down quarks both before and after, and a net of zero strange quarks both before and after. Thus, the reaction conserves the net number of each type of quark.

In the second reaction,

$$\pi^- + \text{p} \to \text{K}^0 + \text{n}$$

the quarks in the particles are

$$\bar{\text{u}}\text{d} + \text{uud} \to \bar{\text{s}}\text{d} + \text{udd}$$

In this case, there is a net of 1 up and 2 down quarks before the reaction but a net of 1 up, 3 down, and 1 anti-strange quark after the reaction. Thus, the reaction does not conserve the net number of each type of quark.

P46.30 Compare the given quark states to the entries in Tables 46.4 and 46.5:

(a) uus = $\boxed{\Sigma^+}$

(b) $\bar{\text{u}}$d = $\boxed{\pi^-}$

(c) $\bar{\text{s}}$d = $\boxed{\text{K}^0}$

(d) dss = $\boxed{\Xi^-}$

P46.31 (a)

	proton	u	u	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	e	$2e/3$	$2e/3$	$-e/3$	e

(b)

	neutron	u	d	d	total
strangeness	0	0	0	0	0
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

P46.32 (a) $\pi^+ + \mathrm{p} \to \mathrm{K}^+ + \Sigma^+$: $\quad \bar{\mathrm{d}}\mathrm{u} + \mathrm{uud} \to \bar{\mathrm{s}}\mathrm{u} + \mathrm{uus}$

up quarks: $1 + 2 \to 1 + 2$, or $\quad 3 \to 3$

down quarks: $\quad -1 + 1 \to 0 + 0$, $\quad$ or $\quad 0 \to 0$

strange quarks: $\quad 0 + 0 \to -1 + 1$, $\quad$ or $\quad 0 \to 0$

$\boxed{\text{The reaction has a net of 3 u, 0 d, and 0 s before and after.}}$

(b) $\mathrm{K}^- + \mathrm{p} \to \mathrm{K}^+ + \mathrm{K}^0 + \Omega^-$: $\quad \bar{\mathrm{u}}\mathrm{s} + \mathrm{uud} \to \bar{\mathrm{s}}\mathrm{u} + \bar{\mathrm{s}}\mathrm{d} + \mathrm{sss}$

up quarks: $-1 + 2 \to 1 + 0 + 0$, $\quad$ or $\quad 1 \to 1$

down quarks: $\quad 0 + 1 \to 0 + 1 + 0$, $\quad$ or $\quad 1 \to 1$

strange quarks: $\quad 1 + 0 \to -1 - 1 + 3$, $\quad$ or $\quad 1 \to 1$

$\boxed{\text{The reaction has a net of 1 u, 1 d, and 1 s before and after.}}$

(c) $\mathrm{p} + \mathrm{p} \to \mathrm{K}^0 + \mathrm{p} + \pi^+ + ?$: $\quad \mathrm{uud} + \mathrm{uud} \to \bar{\mathrm{s}}\mathrm{d} + \mathrm{uud} + \bar{\mathrm{d}}\mathrm{u} + ?$

The quark combination ? must be such as to balance the last equation for up, down, and strange quarks.

up quarks: $2 + 2 = 0 + 2 + 1 + ?$ $\quad$ (? has 1 u quark)

down quarks: $\quad 1 + 1 = 1 + 1 - 1 + ?$ $\quad$ (? has 1 d quark)

strange quarks: $\quad 0 + 0 = -1 + 0 + 0 + ?$ $\quad$ (? has 1 s quark)

$\boxed{\text{The reaction must net of 4 u, 2 d, and 0 s before and after.}}$

(d) quark composition $= \mathrm{uds} = \boxed{\Lambda^0 \text{ or } \Sigma^0}$

P46.33 (a) $\bar{\mathrm{u}}\bar{\mathrm{u}}\bar{\mathrm{d}}$:

$$\text{charge} = \left(-\frac{2}{3}e\right) + \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{-e}$$

(b) $\bar{\mathrm{u}}\bar{\mathrm{d}}\bar{\mathrm{d}}$:

$$\text{charge} = \left(-\frac{2}{3}e\right) + \left(\frac{1}{3}e\right) + \left(\frac{1}{3}e\right) = \boxed{0}$$

(c) $\boxed{\text{antiproton}}$; $\boxed{\text{antineutron}}$

***P46.34** The number of protons in one liter (1 000 g) of water is

$$N_p = (1\ 000\ \text{g})\left(\frac{6.02 \times 10^{23}\ \text{molecules}}{18.0\ \text{g}}\right)\left(\frac{10\ \text{protons}}{\text{molecule}}\right)$$

$$= 3.34 \times 10^{26}\ \text{protons}$$

and there are

$$N_n = (1\ 000\ \text{g})\left(\frac{6.02 \times 10^{23}\ \text{molecules}}{18.0\ \text{g}}\right)\left(\frac{8\ \text{neutrons}}{\text{molecule}}\right)$$

$$= 2.68 \times 10^{26}\ \text{neutrons}$$

So there are, for electric neutrality, $\boxed{3.34\times 10^{26}\ \text{electrons}}$.

The protonquark content is p = uud, and the neutron quark content is n = udd, so the number of up quarks is

$$2\left(3.34\times 10^{26}\right)+2.68\times 10^{26}=\boxed{9.36\times 10^{26}\ \text{up quarks}}$$

and the number of down quarks is

$$2\left(2.68\times 10^{26}\right)+3.34\times 10^{26}=\boxed{8.70\times 10^{26}\ \text{down quarks}}$$

P46.35 $\Sigma^0 + p \to \Sigma^+ + \gamma + X$

uds + uud → uus + 0 + ?

The left side has a net 3 u, 2 d, and 1 s. The right-hand side has 2 u and 1 s, leaving 2 d and 1 u missing.

$\boxed{\text{The unknown particle is a neutron, udd.}}$

Baryon and strangeness numbers are conserved.

P46.36 Quark composition of proton = uud and of neutron = udd. Thus, if we neglect binding energies, we may write

$$m_p = 2m_u + m_d \qquad \textbf{[1]}$$

and

$$m_n = m_u + 2m_d. \qquad \textbf{[2]}$$

Subtract [2] from 2 × [1]:

$$\begin{aligned} 2m_p &= 4m_u + 2m_d \\ -m_n &= -\left(m_u + 2m_d\right) \\ \hline 2m_p - m_n &= 3m_u \end{aligned}$$

We find

$$\begin{aligned} m_u &= \frac{1}{3}\left(2m_p - m_n\right) = \frac{1}{3}\left[2\left(938\ \text{MeV}/c^2\right) - 939.6\ \text{meV}/c^2\right] \\ &= \boxed{312\ \text{MeV}/c^2} \end{aligned}$$

and from either [1] or [2], $m_d = \boxed{314\ \text{MeV}/c^2}$.

Section 46.10 The Cosmic Connection

P46.37 From Equation 39.10,

$$f_{\text{observer}} = f_{\text{source}}\sqrt{\frac{1+v_a/c}{1-v_a/c}}$$

where the velocity of approach, v, is the negative of the velocity of mutual recession: $v_a = -v$.

Thus, $$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-v/c}{1+v/c}} \quad \text{and} \quad \boxed{\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}}}$$

P46.38 (a) We let r in Hubble's law represent any distance.

$$v = Hr = \left(22\times10^{-3}\ \frac{\text{m}}{\text{s}\cdot\text{ly}}\right)(1.85\ \text{m})\left(\frac{1\ \text{ly}}{c\cdot 1\ \text{yr}}\right)$$
$$\times\left(\frac{c}{3.00\times10^{8}\ \text{m/s}}\right)\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)$$
$$= \boxed{4.30\times10^{-18}\ \text{m/s}}$$

This is unobservably small.

(b) $$v = Hr = \left(22\times10^{-3}\ \frac{\text{m}}{\text{s}\cdot\text{ly}}\right)(3.84\times10^{8}\ \text{m})\left(\frac{1\ \text{ly}}{c\cdot 1\ \text{yr}}\right)$$
$$\left(\frac{c}{3.00\times10^{8}\ \text{m/s}}\right)\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right)$$
$$= 8.92\times10^{-10}\ \text{m/s} = \boxed{0.892\ \text{nm/s}}$$

Again too small to measure.

P46.39 (a) From Wien's law,

$$\lambda_{\max}T = 2.898\times10^{-3}\ \text{m}\cdot\text{K}$$

Thus,

$$\lambda_{\max} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{T} = \frac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{2.73\ \text{K}} = 1.06\times10^{-3}\ \text{m}$$
$$= \boxed{1.06\ \text{mm}}$$

(b) This is a $\boxed{\text{microwave.}}$

P46.40 (a) The volume of the sphere bounded by the Earth's orbit is

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(1.496\times10^{11}\ \text{m}\right)^3 = 1.40\times10^{34}\ \text{m}^3$$

$$m = \rho V = \left(6\times10^{-28}\ \text{kg/m}^3\right)\left(1.40\times10^{34}\ \text{m}^3\right) = \boxed{8.41\times10^{6}\ \text{kg}}$$

(b) By Gauss's law, the dark matter would create a gravitational field acting on the Earth to accelerate it toward the Sun. It would shorten the duration of the year in the same way that 8.41×10^6 kg of extra material in the Sun would. This has the fractional effect of

$$\frac{8.41\times10^{6}\ \text{kg}}{1.99\times10^{30}\ \text{kg}} = 4.23\times10^{-24}\ \text{of the mass of the Sun.}$$

$\boxed{\text{No. It is only the fraction } 4.23 \times 10^{-24} \text{ of the mass of the Sun.}}$

P46.41 (a) The energy is enough to produce a proton-antiproton pair: $k_B T \approx 2m_p c^2$, so

$$T \approx \frac{2m_p c^2}{k_B} = \frac{2(938.3\ \text{MeV})}{\left(1.38\times10^{-23}\ \text{J/K}\right)}\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)\boxed{\sim10^{13}\ \text{K}}$$

(b) The energy is enough to produce an electron-positron pair: $k_B T \approx 2m_e c^2$, so

$$T \approx \frac{2m_e c^2}{k_B} = \frac{2(0.511\ \text{MeV})}{\left(1.38\times10^{-23}\ \text{J/K}\right)}\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)\boxed{\sim10^{10}\ \text{K}}$$

P46.42 (a) The Hubble constant is defined in $v = HR$. The gap R between any two far-separated objects opens at constant speed according to $R = v\Delta t$. Then the time interval Δt since the Big Bang is found from

$$v = H\,v\Delta t \rightarrow \quad \Delta t = \frac{1}{H}$$

(b) $$\frac{1}{H} = \frac{1}{22\times10^{-3}\ \text{m/s}\cdot\text{ly}}\left[\frac{(1\ \text{yr})\cdot\left(3\times10^{8}\ \text{m/s}\right)}{1\ \text{ly}}\right] = \boxed{1.36\times10^{10}\ \text{yr}}$$

= 13.6 billion years

***P46.43** The radiation wavelength of $\lambda' = 500$ nm that is observed by observers on Earth is not the true wavelength, λ, emitted by the star because of the Doppler effect. The true wavelength is related to the observed wavelength using:

$$\frac{c}{\lambda'} = \frac{c}{\lambda}\sqrt{\frac{1-(v/c)}{1+(v/c)}}$$

Solving for the true wavelength then gives

$$\lambda = \lambda'\sqrt{\frac{1-(v/c)}{1+(v/c)}} = (500\text{ nm})\sqrt{\frac{1-(0.280)}{1+(0.280)}} = 375\text{ nm}$$

The temperature of the star is given by Wien's law,

$$\lambda_{\max}T = 2.898\times10^{-3}\text{ m}\cdot\text{K}$$

or $$T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{\lambda_{\max}} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{375\times10^{-9}} = \boxed{7.73\times10^{3}\text{ K}}.$$

P46.44 We assume that the fireball of the Big Bang is a black body. Then,

$$I = e\sigma T^4 = (1)\left(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(2.73\text{ K})^4$$
$$= \boxed{3.15\times10^{-6}\text{ W/m}^2}$$

P46.45 (a) We use primed symbols to represent observed Doppler-shifted values and unprimed symbols to represent values as they would be measured by an observer stationary relative to the source. Doppler-shift equations from Chapter 17 do not apply to electromagnetic waves, because the speed of source or observer relative to some medium cannot be defined for these waves. Instead, we use Equation 39.10, expressing it as

$$f' = \frac{c}{\lambda'} = \sqrt{\frac{1+v/c}{1-v/c}}\,f = \sqrt{\frac{1+v/c}{1-v/c}}\left(\frac{c}{\lambda}\right)$$

where v is the velocity of mutual approach. Then we have

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{1-v/c}{1+v/c}}$$

Squaring both sides, and solving,

$$\left(\frac{\lambda'}{\lambda}\right)^2 = \frac{1-v/c}{1+v/c}$$

$$\left(\frac{\lambda'}{\lambda}\right)^2 + \left(\frac{\lambda'}{\lambda}\right)^2\frac{v}{c} = 1-\frac{v}{c}$$

$$\left(\frac{\lambda'}{\lambda}\right)^2 - 1 = -\frac{v}{c}\left[\left(\frac{\lambda'}{\lambda}\right)^2 + 1\right]$$

Solving for v/c then gives

$$\frac{v}{c}=-\frac{(\lambda'/\lambda)^2-1}{(\lambda'/\lambda)^2+1}=-\frac{(510\text{ nm}/434\text{ nm})^2-1}{(510\text{ nm}/434\text{ nm})^2+1}=\frac{(1.18)^2-1}{(1.18)^2+1}$$

$$=-\frac{1.381-1}{1.381+1}=-0.160$$

The negative sign indicates that the quasar is moving away from us, or us from it. The speed of recession that the problem asks for is then

$$v=\boxed{0.160c}\text{ (or 16.0\% of the speed of light)}$$

(b) Hubble's law asserts that the universe is expanding at a constant rate so that the speeds of galaxies are proportional to their distance R from Earth, as described by $v = HR$.

So, $R=\frac{v}{H}=\frac{0.160\left(3.00\times10^{8}\text{ m/s}\right)}{2.2\times10^{-2}\text{ m/s}\cdot\text{ly}}=\boxed{2.18\times10^{9}\text{ ly}}$.

P46.46 (a) Applying the result from Problem 37, $\lambda_n'=\lambda_n\sqrt{\frac{1+v/c}{1-v/c}}$, to the definition $Z=\frac{\lambda_n'-\lambda_n}{\lambda_n}$, we have

$$Z=\frac{\lambda_n'-\lambda_n}{\lambda_n}\quad\rightarrow\quad (Z+1)\lambda_n=\lambda_n'=\lambda_n\sqrt{\frac{1+v/c}{1-v/c}}$$

$$\frac{1+v/c}{1-v/c}=(Z+1)^2$$

$$1+\frac{v}{c}=(Z+1)^2-\left(\frac{v}{c}\right)(Z+1)^2$$

$$\left(\frac{v}{c}\right)\left(Z^2+2Z+2\right)=Z^2+2Z$$

$$v=\boxed{c\left(\frac{Z^2+2Z}{Z^2+2Z+2}\right)}$$

(b) $R=\frac{v}{H}=\boxed{\frac{c}{H}\left(\frac{Z^2+2Z}{Z^2+2Z+2}\right)}$

P46.47 First, we calculate $v = HR$, using $H = 22\times 10^{-3}\ \text{m/s}\cdot\text{ly}$, and then we use the result of Problem 37, $\lambda' = \lambda\sqrt{\dfrac{1+v/c}{1-v/c}}$, and $c = 2.998\times 10^{8}$ m/s, to calculate the wavelength emitted by the galaxy.

(a) $v = \left(22\times 10^{-3}\ \text{m/s}\cdot\text{ly}\right)\left(2.00\times 10^{6}\ \text{ly}\right) = 4.4\times 10^{4}\ \text{m/s}$,

$$\lambda' = \lambda\sqrt{\frac{1+v/c}{1-v/c}} = \lambda\sqrt{\frac{1+\left(4.4\times 10^{4}\ \text{m/s}\right)/\left(2.998\times 10^{8}\ \text{m/s}\right)}{1-\left(4.4\times 10^{4}\ \text{m/s}\right)/\left(2.998\times 10^{8}\ \text{m/s}\right)}}$$

$$= (590\ \text{nm})\sqrt{\frac{1+0.000\ 146\ 8}{1-0.000\ 146\ 8}} = \boxed{590.09\ \text{nm}}$$

Similarly,

(b) $v = \left(22\times 10^{-3}\ \text{m/s}\cdot\text{ly}\right)\left(2.00\times 10^{8}\ \text{ly}\right) = 4.4\times 10^{6}\ \text{m/s}$,

$$\lambda' = (590\ \text{nm})\sqrt{\frac{1+0.014\ 68}{1-0.014\ 68}} = \boxed{599\ \text{nm}}$$

(c) $v = \left(22\times 10^{-3}\ \text{m/s}\cdot\text{ly}\right)\left(2.00\times 10^{9}\ \text{ly}\right) = 4.4\times 10^{7}\ \text{m/s}$,

$$\lambda' = (590\ \text{nm})\sqrt{\frac{1+0.146\ 8}{1-0.146\ 8}} = \boxed{684\ \text{nm}}$$

P46.48 (a) What we can see is limited by the finite age of the Universe and by the finite speed of light. We can see out only to a look-back time equal to a bit less than the age of the Universe. Every year on your birthday the Universe also gets a year older, and light now in transit arrives at Earth from still more distant objects. So the radius of the visible Universe expands at the speed of light, which is

$$\frac{dr}{dt} = c = 1\ \text{ly/yr}$$

(b) The volume of the visible section of the Universe is $\frac{4}{3}\pi r^{3}$, where r = 13.7 billion light-years. The rate of volume increase is

$$\frac{dV}{dt} = \frac{d}{dt}\left(\frac{4}{3}\pi r^{3}\right) = \frac{4}{3}\pi 3r^{2}\frac{dr}{dt} = 4\pi r^{2}c$$

$$= 4\pi\left[\left(13.7\times 10^{9}\ \text{ly}\right)\left(\frac{9.4605\times 10^{15}\ \text{m}}{1\ \text{ly}}\right)\right]^{2}\left(3.00\times 10^{8}\ \frac{\text{m}}{\text{s}}\right)$$

$$= \boxed{6.34\times 10^{61}\ \text{m}^{3}/\text{s}}$$

P46.49 The density of the Universe is

$$\rho = 1.20\rho_c = 1.20\left(\frac{3H^2}{8\pi G}\right)$$

Consider a remote galaxy at distance r. The mass interior to the sphere below it is

$$M = \rho\left(\frac{4}{3}\pi r^3\right) = 1.20\left(\frac{3H^2}{8\pi G}\right)\left(\frac{4}{3}\pi r^3\right) = \frac{0.600H^2r^3}{G}$$

both now and in the future when it has slowed to rest from its current speed $v = Hr$. The energy of this galaxy-sphere system is constant as the galaxy moves to apogee distance R:

$$\frac{1}{2}mv^2 - \frac{GmM}{r} = 0 - \frac{GmM}{R}$$

so $$\frac{1}{2}mH^2r^2 - \frac{Gm}{r}\left(\frac{0.600H^2r^3}{G}\right) = 0 - \frac{Gm}{R}\left(\frac{0.600H^2r^3}{G}\right)$$

$$-0.100 = -0.600\frac{r}{R} \quad \text{so} \quad R = 6.00r$$

The Universe will expand by a factor of $\boxed{6.00}$ from its current dimensions.

Section 46.12 Problems and Perspectives

P46.50 (a) The Planck length is

$$L = \sqrt{\frac{\hbar G}{c^3}} = \sqrt{\frac{(1.054\ 5\times10^{-34}\ \text{J}\cdot\text{s})(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}{(2.998\times10^8\ \text{m/s})^3}}$$

$$= \boxed{1.62\times10^{-35}\ \text{m}}$$

(b) The Planck time is given as

$$T = \frac{L}{c} = \frac{1.616\times10^{-35}\ \text{m}}{2.998\times10^8\ \text{m/s}} = \boxed{5.39\times10^{-44}\ \text{s}}$$

of the same order of magnitude as the ultrahot epoch.

Additional Problems

P46.51 (a) $\pi^- + \mathrm{p} \to \Sigma^+ + \pi^0$

Total charge is 0 on the left side of the equation, +1 on the right side. $\boxed{\text{Charge is not conserved.}}$

(b) $\mu^- \to \pi^- + \nu_\mathrm{e}$

The rest mass of the pion is larger than the rest mass of the muon. Muon lepton number is +1 on the left side of the equation, 0 on the right side. Electron lepton number is 0 on left side, +1 on right side. $\boxed{\text{Energy, muon lepton number, and electron lepton number are not conserved.}}$

(c) $\mathrm{p} \to \pi^+ + \pi^+ + \pi^-$

Baryon number is +1 on the left side of the equation, 0 on the right side. $\boxed{\text{Baryon number is not conserved.}}$

P46.52 In $? + \mathrm{p}^+ \to \mathrm{n} + \mu^+$, charge conservation requires the unknown particle to be neutral. Baryon number conservation requires baryon number = 0. The muon-lepton number of ? must be –1. So the unknown particle must be an muon-antineutrino $\boxed{\bar{\nu}_\mu}$.

***P46.53** The time of flight is given by $\Delta t = d/v$.

Since $K = \frac{1}{2}mv^2$,

$$\Delta t = \frac{d}{\sqrt{\frac{2K}{m}}} = \frac{10.0 \times 10^3\ \mathrm{m}}{\sqrt{\frac{2(0.040\,0\ \mathrm{eV})\left(1.60 \times 10^{-19}\ \mathrm{J/eV}\right)}{1.67 \times 10^{-27}\ \mathrm{kg}}}} = 3.61\ \mathrm{s}$$

The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{614\ \mathrm{s}} = 1.13 \times 10^{-3}\ \mathrm{s}^{-1}$.

Therefore we have

$$\lambda \Delta t = \left(1.13 \times 10^{-3}\ \mathrm{s}\right)(3.61\ \mathrm{s}) = 4.08 \times 10^{-3} = 0.004\,08$$

And the fraction remaining is

$$\frac{N}{N_0} = e^{-\lambda \Delta t} = e^{-0.004\,08} = 0.995\,9$$

Hence, the fraction that has decayed in this time interval is

$$1 - \frac{N}{N_0} = 0.004\,07 \quad \text{or} \quad \boxed{0.407\%}$$

P46.54 Let's find the minimum energy necessary for the increase in rest energy to occur.

$$\Delta E_R = (3m_e - m_e)c^2 = 2m_ec^2 = 2(0.511\text{ eV}) = 1.02\text{ eV}$$

This calculation may make it look like the reaction is possible. But there is more to the energy picture here than just the increase in rest energy. There is kinetic energy associated with the moving particles. Let's demand that energy be conserved for the isolated system:

$$E_i = E_f \rightarrow E_\gamma + m_ec^2 = 3\gamma m_ec^2 \qquad \textbf{[1]}$$

Now demand that momentum in the direction of travel of the initial photon be conserved for the isolated system:

$$p_{xi} = p_{xf} \rightarrow \frac{E_\gamma}{c} = 3\gamma m_eu \qquad \textbf{[2]}$$

Divide equation [1] by equation [2]:

$$\frac{E_\gamma + m_ec^2}{E_\gamma / c} = \frac{c^2}{u} \rightarrow \frac{E_\gamma + m_ec^2}{E_\gamma} = \frac{c}{u} = \frac{1}{\beta} \qquad \textbf{[3]}$$

where $\beta = u/c$. Multiply equation [2] by c and subtract it from equation [1]:

$$E_\gamma + m_ec^2 - E_\gamma = 3\gamma m_ec^2 - 3\gamma m_euc$$

$$\rightarrow m_ec^2 = 3\gamma m_ec^2 - 3\gamma m_euc$$

$$\rightarrow 1 = 3\gamma - 3\gamma\frac{u}{c} = 3\gamma(1-\beta)$$

Substitute for γ:

$$1 = \frac{3(1-\beta)}{\sqrt{1-u^2/c^2}} = \frac{3(1-\beta)}{\sqrt{1-\beta^2}} = 3\sqrt{\frac{1-\beta}{1+\beta}}$$

$$1+\beta = 9(1-\beta) \quad \rightarrow \quad \beta = \frac{8}{10} = 0.800$$

Substitute this value into equation [3]:

$$\frac{E_\gamma + m_ec^2}{E_\gamma} = \frac{1}{0.800}$$

$$1 + \frac{m_ec^2}{E_\gamma} = 1.25 \rightarrow \quad E_\gamma = 4m_ec^2 = 2.04\text{ MeV}$$

Therefore, the photon arriving with 1.05 MeV of energy cannot cause this reaction.

Let's check the assumptions. If the final particles have any velocity component perpendicular to the initial direction of travel of the photon, then they must be moving with a higher speed after the collision and the incoming photon energy would have to be larger. If any one of the particles had a different energy than the other two, then the only way to satisfy both energy and momentum conservation would be for at least two of the particles to have components of velocity perpendicular to the initial direction of motion of the photon, so again the incoming photon energy would have to be larger. Therefore, 2.04 MeV represents the *minimum* energy for the reaction to occur.

P46.55 We find the number *N* of neutrinos:

$$10^{46}\ \text{J} = N(6\ \text{MeV}) = N\left(6\times 1.60\times 10^{-13}\ \text{J}\right)$$
$$N = 1.0\times 10^{58}\ \text{neutrinos}$$

The intensity at our location is

$$\frac{N}{A} = \frac{N}{4\pi r^2} = \frac{1.0\times 10^{58}}{4\pi\left(1.7\times 10^{5}\ \text{ly}\right)^2}\left(\frac{1\ \text{ly}}{9.460\ 5\times 10^{15}\ \text{m}}\right)^2$$
$$= 3.1\times 10^{14}\ \text{m}^{-2}$$

The number passing through a body presenting 5 000 cm^2 = 0.50 m^2 is then $\left(3.1\times 10^{14}\ \dfrac{1}{\text{m}^2}\right)\left(0.50\ \text{m}^2\right) = 1.5\times 10^{14}$, or $\boxed{\sim 10^{14}}$.

P46.56 Since the neutrino flux from the Sun reaching the Earth is 0.400 W/m^2, the total energy emitted per second by the Sun in neutrinos in all directions is that which would irradiate the surface of a great sphere around it, with the Earth's orbit as its equator.

$$\left(0.400\ \text{W/m}^2\right)\left(4\pi r^2\right) = \left(0.400\ \text{W/m}^2\right)\left[4\pi\left(1.496\times 10^{11}\ \text{m}\right)^2\right]$$
$$= 1.12\times 10^{23}\ \text{W}$$

In a period of 10^9 yr, the Sun emits a total energy of $\Delta E = P\Delta t$.

$$E = \left(1.12\times 10^{23}\ \text{J/s}\right)\left(10^{9}\ \text{yr}\right)\left(3.156\times 10^{7}\ \text{s/yr}\right) = 3.55\times 10^{39}\ \text{J}$$

carried by neutrinos. This energy corresponds to an annihilated mass according to

$$E = m_\nu c^2 = 3.55\times 10^{39}\ \text{J} \quad \text{or} \quad m_\nu = 3.94\times 10^{22}\ \text{kg}.$$

Since the Sun has a mass of 1.989 × 10^{30} kg, this corresponds to a loss of only about $\boxed{1\ \text{part in}\ 5\times 10^{7}}$ of the Sun's mass over 10^9 yr in the form of neutrinos.

P46.57 In our frame of reference, Hubble's law is exemplified by $\vec{\mathbf{v}}_1 = \mathrm{H}\vec{\mathbf{R}}_1$ and $\vec{\mathbf{v}}_2 = \mathrm{H}\vec{\mathbf{R}}_2$.

(a) From the first equation $\vec{\mathbf{v}}_1 = \mathrm{H}\vec{\mathbf{R}}_1$ we may form the equation $-\vec{\mathbf{v}}_1 = -\mathrm{H}\vec{\mathbf{R}}_1$. This equation expresses Hubble's law as seen by the observer in the first galaxy cluster, as she looks at us to find our velocity relative to her (away from her) is $-\vec{\mathbf{v}}_1 = \mathrm{H}\left(-\vec{\mathbf{R}}_1\right)$.

(b) From both equations we may form the equation $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = \mathrm{H}\left(\vec{\mathbf{R}}_2 - \vec{\mathbf{R}}_1\right)$. This equation expresses Hubble's law as seen by the observer in the first galaxy cluster, as she looks at cluster two to find the relative velocity of cluster 2 relative to cluster 1 is $\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1 = \mathrm{H}\left(\vec{\mathbf{R}}_2 - \vec{\mathbf{R}}_1\right)$.

P46.58 $\pi^- \to \mu^- + \bar{\nu}_\mu$. By energy conservation,

$$m_\pi c^2 = E_\mu + E_{\bar{\nu}} = 139.6\text{ MeV} \qquad \textbf{[1]}$$

Because we assume the antineutrino has no mass, $E_{\bar{\nu}} = p_{\bar{\nu}}c$, and by momentum conservation, $p_\mu = p_{\bar{\nu}}$; thus, we can relate the total energies of the muon and antineutrino:

$$E_\mu^2 = \left(p_\mu c\right)^2 + \left(m_\mu c^2\right)^2 = \left(p_{\bar{\nu}}c\right)^2 + \left(m_\mu c^2\right)^2 = \left(E_{\bar{\nu}}^2\right)^2 + \left(m_\mu c^2\right)^2$$

or $$E_\mu^2 - E_{\bar{\nu}}^2 = \left(m_\mu c^2\right)^2$$

and $$\left(E_\mu + E_{\bar{\nu}}\right)\left(E_\mu - E_{\bar{\nu}}\right) = \left(m_\mu c^2\right)^2. \qquad \textbf{[2]}$$

Substituting [1] into [2], we find that

$$E_\mu - E_{\bar{\nu}} = \frac{\left(m_\mu c^2\right)^2}{\left(E_\mu + E_{\bar{\nu}}\right)} = \frac{\left(m_\mu c^2\right)^2}{m_\pi c^2} \qquad \textbf{[3]}$$

Subtracting [3] from [1],

$$\left(E_\mu + E_{\bar{\nu}}\right) - \left(E_\mu - E_{\bar{\nu}}\right) = m_\pi c^2 - \frac{\left(m_\mu c^2\right)^2}{m_\pi c^2}$$

$$2E_{\bar{\nu}} = m_\pi c^2 - \frac{\left(m_\mu c^2\right)^2}{m_\pi c^2}$$

$$E_{\bar{\nu}} = \frac{\left(m_\pi c^2\right)^2 - \left(m_\mu c^2\right)^2}{2m_\pi c^2} = \frac{(139.6\text{ MeV})^2 - (105.7\text{ MeV})^2}{2(139.6\text{ MeV})}$$

$$= \boxed{29.8\text{ MeV}}$$

P46.59 Each particle travels in a circle, so each must experience a centripetal force:

$$\sum F = ma: \quad qvB\sin 90° = \frac{mv^2}{r} \quad \rightarrow \quad mv = qBr$$

The proton and the pion have the same momentum because they have the same magnitude of charge and travel in a circle of the same radius:

$$p_p = p_\pi = p = qBr = \left(1.60\times 10^{-19}\text{ C}\right)\left(0.250\text{ T}\right)\left(1.33\text{ m}\right)$$
$$= 5.32\times 10^{-20}\text{ kg}\cdot\text{m/s}$$

so

$$pc = \left(3.00\times 10^{8}\text{ m/s}\right)\left(5.32\times 10^{-20}\text{ kg}\cdot\text{m/s}\right)\left(\frac{1\text{ MeV}}{1.60\times 10^{-13}\text{ J}}\right)$$
$$= 99.8\text{ MeV}$$

Using masses from Table 46.2, we find the total energy of the proton to be

$$E_p = \sqrt{(pc)^2 + \left(m_p c^2\right)^2} = \sqrt{(99.8\text{ MeV})^2 + (938.3\text{ MeV})^2}$$
$$= 944\text{ MeV}$$

and the total energy of the pion to be

$$E_\pi = \sqrt{(pc)^2 + \left(m_\pi c^2\right)^2} = \sqrt{(99.8\text{ MeV})^2 + (139.6\text{ MeV})^2}$$
$$= 172\text{ MeV}$$

The unknown particle was initially at rest; thus, $E_{\text{total after}} = E_{\text{total before}} =$ rest energy, and the rest energy of unknown particle is

$$mc^2 = 944\text{ MeV} + 172\text{ MeV} = 1\,116\text{ MeV}$$

$$\text{Mass} = \boxed{1.12\ \text{GeV}/c^2}$$

From Table 46.2, we see this is a Λ^0 particle.

P46.60 Each particle travels in a circle, so each must experience a centripetal force:

$$\sum F = ma: \quad qvB\sin 90° = \frac{mv^2}{r} \quad \rightarrow \quad mv = qBr$$

The particles have the same momentum because they have the same magnitude of charge and travel in a circle of the same radius:

$$p_+ = p_- = p = eBr \quad \rightarrow \quad pc = eBrc$$

We find the total energy of the positively charged particle to be

$$E_{+,\text{ total}} = \sqrt{(pc)^2 + \left(E_+\right)^2} = \sqrt{(qBrc)^2 + E_+^2}$$

and the total energy of the negatively charged particle to be

$$E_{+,\ \text{total}} = \sqrt{(pc)^2 + (E_-)^2} = \sqrt{(qBrc)^2 + E_-^2}$$

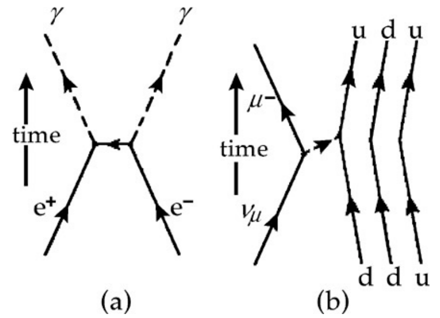
The unknown particle was initially at rest; thus, $E_{\text{total after}} = E_{\text{total before}} =$ rest energy, and the rest energy of the unknown particle is

$$mc^2 = \sqrt{(qBrc)^2 + E_+^2} + \sqrt{(qBrc)^2 + E_-^2}$$

$$\boxed{m = \frac{\sqrt{(qBrc)^2 + E_+^2} + \sqrt{(qBrc)^2 + E_-^2}}{c^2}}$$

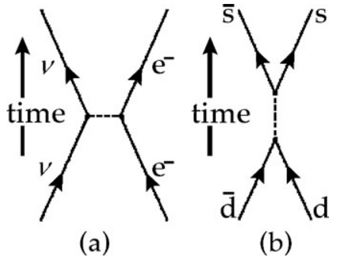
P46.61 (a) This diagram represents electron–positron annihilation. From charge and lepton-number conservation at either vertex, the exchanged particle must be an electron, $\boxed{e^-.}$

(b) A neutrino collides with a neutron, producing a proton and a muon. This is a weak interaction. The exchanged particle has charge $+e$ and is a $\boxed{W^+.}$



ANS. FIG. P46.61

P46.62 (a) The Feynman diagram in ANS. FIG. P46.62 shows a neutrino scattering off an electron, and the neutrino and electron do not exchange electric charge. The neutrino has no electric charge and interacts through the weak interaction (ignoring gravity). The mediator is a $\boxed{Z^0 \text{ boson.}}$



ANS. FIG. P46.62

(b) The Feynman diagram shows a down quark and its antiparticle annihilating each other. They can produce a particle carrying energy, momentum, and angular momentum, but zero charge, zero baryon number, and, if the quarks have opposite color charges, no color charge. In this case the mediating particle could be a $\boxed{\text{photon or } Z^0 \text{ boson.}}$

Depending on the color charges of the d and $\bar{d}$ quarks, the ephemeral particle could also be a $\boxed{\text{gluon}}$, as suggested in the discussion of Figure 46.13(b).

For conservation of both energy and momentum in the collision we would expect two mediating particles; but momentum need not be strictly conserved, according to the uncertainty principle, if the particle travels a sufficiently short distance before producing another matter-antimatter pair of particles, as shown in ANS. FIG. P46.62(b).

P46.63 The expression $e^{-E/k_BT}dE$ gives the fraction of the photons that have energy between *E* and *E* + *dE*. The fraction that have energy between *E* and infinity is

$$\frac{\int_E^\infty e^{-E/k_BT}\,dE}{\int_0^\infty e^{-E/k_BT}\,dE} = \frac{\int_E^\infty e^{-E/k_BT}\left(-dE/k_BT\right)}{\int_0^\infty e^{-E/k_BT}\left(-dE/k_BT\right)} = \frac{e^{-E/k_BT}\Big|_E^\infty}{e^{-E/k_BT}\Big|_0^\infty} = e^{-E/k_BT}$$

We require *T* when this fraction has a value of 0.010 0 (i.e., 1.00%)

and $E = 1.00 \text{ eV} = 1.60 \times 10^{-19} \text{ J}.$

Thus, $0.010\ 0 = e^{-\left(1.60\times10^{-19}\text{ J}\right)/\left(1.38\times10^{-23}\text{ J/K}\right)T}$

or $\ln(0.010\ 0) = -\dfrac{1.60\times10^{-19}\text{ J}}{\left(1.38\times10^{-23}\text{ J/K}\right)T} = -\dfrac{1.16\times10^{4}\text{ K}}{T},$

giving $T = 2.52\times10^{3}\text{ K} \sim \boxed{10^3\text{ K}}.$

P46.64 $\Sigma^0 \rightarrow \Lambda^0 + \gamma$

From Table 46.2, $m_\Sigma = 1\,192.5\ \text{MeV}/c^2$ and $m_\Lambda = 1\,115.6\ \text{MeV}/c^2$.

Conservation of energy in the decay requires

$$m_\Sigma c^2 = \left(m_\Lambda c^2 + K_\Lambda\right) + E_\gamma \quad \text{or} \quad m_\Sigma c^2 = \left(m_\Lambda c^2 + \frac{p_\Lambda^{\ 2}}{2m_\Lambda}\right) + E_\gamma$$

System momentum conservation gives $|p_\Lambda| = |p_\gamma|$, so the last result may be written as

$$m_\Sigma c^2 = \left(m_\Lambda c^2 + \frac{p_\gamma^{\ 2}}{2m_\Lambda}\right) + E_\gamma$$

or $$m_\Sigma c^2 = \left(m_\Lambda c^2 + \frac{p_\gamma^2 c^2}{2m_\Lambda c^2} \right) + E_\gamma .$$

Recognizing that $p_\gamma c = E_\gamma$, we now have

$$1\,192.5\text{ MeV} = 1\,115.6\text{ MeV} + \frac{E_\gamma^2}{2(1\,115.6\text{ MeV})} + E_\gamma$$

Solving this quadratic equation gives $E_\gamma = \boxed{74.4\text{ MeV}}$.

P46.65 $p + p \rightarrow p + \pi^+ + X$

The protons each have 70.4 MeV of kinetic energy. In accord with conservation of momentum for the collision, particle X has zero momentum and thus zero kinetic energy. Conservation of system energy then requires

$$m_p c^2 + m_\pi c^2 + m_X c^2 = \left(m_p c^2 + K_p\right) + \left(m_p c^2 + K_p\right)$$

$$\begin{aligned} m_X c^2 &= m_p c^2 + 2K_p - m_\pi c^2 \\ &= 938.3\text{ MeV} + 2(70.4\text{ MeV}) - 139.6\text{ MeV} \\ &= 939.5\text{ MeV} \end{aligned}$$

X must be a neutral baryon of rest energy 939.5 MeV. Thus, X is a $\boxed{\text{neutron.}}$

P46.66 $p + p \rightarrow p + n + \pi^+$

The total momentum is zero before the reaction. Thus, all three particles present after the reaction may be at rest and still conserve system momentum. This will be the case when the incident protons have minimum kinetic energy. Under these conditions, conservation of energy for the reaction gives

$$2\left(m_p c^2 + K_p\right) = m_p c^2 + m_n c^2 + m_\pi c^2$$

so the kinetic energy of each of the incident protons is

$$\begin{aligned} K_p &= \frac{m_n c^2 + m_\pi c^2 - m_p c^2}{2} = \frac{(939.6 + 139.6 - 938.3)\text{ MeV}}{2} \\ &= \boxed{70.4\text{ MeV}} \end{aligned}$$

Challenge Problems

P46.67 See the discussion of P46.19 in this volume for more details of the mathematical steps used in the following calculations.

From Table 46.2, $m_\Lambda c^2 = 1\ 115.6$ MeV, $m_p c^2 = 938.3$ MeV, and $m_\pi c^2 = 139.6$ MeV.

Since the Λ^0 is at rest, the difference between its rest energy and the rest energies of the proton and the pion is the sum of the kinetic energies of the proton and the pion.

$$K_p + K_\pi = 1\ 115.6 \text{ MeV} - 938.3 \text{ MeV} - 139.6 \text{ MeV} = 37.7 \text{ MeV}$$

Now, since $p_p = p_\pi = p$, applying conservation of relativistic energy to the decay process, we have

$$\left[\sqrt{(938.3 \text{ MeV})^2 + p^2c^2} - 938.3 \text{ MeV}\right] + \left[\sqrt{(139.6 \text{ MeV})^2 + p^2c^2} - 139.6 \text{ MeV}\right] = 37.7 \text{ MeV}$$

Solving yields

$$p_\pi c = p_p c = 100.4 \text{ MeV}$$

Then,

$$K_p = \sqrt{\left(m_p c^2\right)^2 + (100.4 \text{ MeV})^2} - m_p c^2 = \boxed{5.35 \text{ MeV}}$$

$$K_\pi = \sqrt{(139.6)^2 + (100.4 \text{ MeV})^2} - 139.6 = \boxed{32.3 \text{ MeV}}$$

P46.68 (a) Let E_{min} be the minimum total energy of the bombarding particle that is needed to induce the reaction. At this energy the product particles all move with the same velocity. The product particles are then equivalent to a single particle having mass equal to the total mass of the product particles, moving with the same velocity as each product particle. By conservation of energy:

$$E_{\text{min}} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + \left(p_3c\right)^2} \qquad \textbf{[1]}$$

By conservation of momentum, $p_3 = p_1$, so

$$\left(p_3c\right)^2 = \left(p_1c\right)^2 = E_{\text{min}}^2 - \left(m_1c^2\right)^2 \qquad \textbf{[2]}$$

Substitute [2] into [1]:

$$E_{\text{min}} + m_2c^2 = \sqrt{\left(m_3c^2\right)^2 + E_{\text{min}}^2 - \left(m_1c^2\right)^2}$$

Square both sides:

$$E_{\text{min}}^2 + 2E_{\text{min}} m_2 c^2 + \left(m_2 c^2\right)^2 = \left(m_3 c^2\right)^2 + E_{\text{min}}^2 - \left(m_1 c^2\right)^2$$

$$\therefore E_{\text{min}} = \frac{\left(m_3^2 - m_1^2 - m_2^2\right)c^2}{2m_2}$$

$$\therefore K_{\text{min}} = E_{\text{min}} - m_1 c^2 = \frac{\left(m_3^2 - m_1^2 - m_2^2 - 2m_1 m_2\right)c^2}{2m_2}$$

$$= \frac{\left[m_3^2 - (m_1 + m_2)^2\right]c^2}{2m_2}$$

Refer to Table 46.2 for the particle masses.

(b) $$K_{\text{min}} = \frac{[4(938.3)]^2 \text{ MeV}^2/c^2 - [2(938.3)]^2 \text{ MeV}^2/c^2}{2\left(938.3 \text{ MeV}/c^2\right)} = \boxed{5.63 \text{ GeV}}$$

(c) $$K_{\text{min}} = \frac{(497.7 + 1\,115.6)^2 \text{ MeV}^2/c^2 - (139.6 + 938.3)^2 \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2}$$

$$= \boxed{768 \text{ MeV}}$$

(d) $$K_{\text{min}} = \frac{[2(938.3) + 135]^2 \text{ MeV}^2/c^2 - [2(938.3)]^2 \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2}$$

$$= \boxed{280 \text{ MeV}}$$

(e) $$K_{\text{min}} = \frac{\left(91.2 \times 10^3\right)^2 - \left[(938.3 + 938.3)^2\right] \text{ MeV}^2/c^2}{2(938.3) \text{ MeV}/c^2} = \boxed{4.43 \text{ TeV}}$$

P46.69 (a) $\Delta E = \left(m_n - m_p - m_e\right)c^2$

From Table 44.2 of masses of isotopes,

$$\Delta E = (1.008\,665 \text{ u} - 1.007\,825 \text{ u})(931.5 \text{ MeV/u})$$

$$= \boxed{0.782 \text{ MeV}}$$

(b) Assuming the neutron at rest, momentum conservation for the decay process implies $p_p = p_e$. Relativistic energy for the system is conserved:

$$\sqrt{\left(m_p c^2\right)^2 + p_p^2 c^2} + \sqrt{\left(m_e c^2\right)^2 + p_e^2 c^2} = m_n c^2$$

Since $p_p = p_e = p$, we have

$$\sqrt{\left(m_p c^2\right)^2 + p^2 c^2} = m_n c^2 - \sqrt{\left(m_e c^2\right)^2 + p^2 c^2}$$

$$(m_pc^2)^2 + \cancel{p^2c^2} = (m_nc^2)^2 - 2m_nc^2\sqrt{(m_ec^2)^2 + p^2c^2} + (m_ec^2)^2 + \cancel{p^2c^2}$$

$$\sqrt{(m_ec^2)^2 + p^2c^2} = \frac{(m_nc^2)^2 - (m_pc^2)^2 + (m_ec^2)^2}{2m_nc^2}$$

$$p^2c^2 = \left[\frac{(m_nc^2)^2 - (m_pc^2)^2 + (m_ec^2)^2}{2m_nc^2}\right]^2 - (m_ec^2)^2$$

Refer to Table 46.2 for the particle masses.

$$p^2c^2 = \left[\frac{(939.6\text{ MeV})^2 - (938.3\text{ MeV})^2 + (0.511\text{ MeV})^2}{2(939.6\text{ MeV})}\right]^2 - (0.511\text{ MeV})^2$$

$$pc = 1.19\text{ MeV}$$

From $p_ec = \gamma m_e v_e c$, we find the speed of the electron:

$$\frac{\gamma v_e}{c} = \frac{p_ec}{m_ec^2} = \frac{1}{\sqrt{1-(v_e/c)^2}}\frac{v_e}{c}$$

$$1-\left(\frac{v_e}{c}\right)^2 = \left(\frac{v_e}{c}\right)^2\left(\frac{m_ec^2}{p_ec}\right)^2 \rightarrow \left(\frac{v_e}{c}\right)^2\left[1+\left(\frac{m_ec^2}{p_ec}\right)^2\right] = 1$$

$$\frac{v_e}{c} = \frac{1}{\sqrt{1+(m_ec^2/p_ec)^2}} = \frac{1}{\sqrt{1+(0.511\text{ MeV}/1.19\text{ MeV})^2}}$$

$$\boxed{v_e = 0.919c}$$

To find the speed of the proton, a similar derivation (basically, substituting m_p for m_e), yields

$$v_p = \frac{c}{\sqrt{1+(m_pc^2/p_ec)^2}} = \frac{2.998\times10^8\text{ m/s}}{\sqrt{1+(938.3\text{ MeV}/1.19\text{ MeV})^2}}$$

$$= 3.82\times10^5\text{ m/s} = \boxed{382\text{ km/s}}$$

(c) $\boxed{\text{The electron is relativistic; the proton is not.}}$ Our criterion for answers accurate to three significant digits is that the electron is moving at more than one-tenth the speed of light and the proton at less than one-tenth the speed of light.

P46.70 (a) At threshold, we consider a photon and a proton colliding head-on to produce a proton and a pion at rest, according to $p + \gamma \to p + \pi^0$. Energy conservation gives

$$\frac{m_p c^2}{\sqrt{1 - u^2/c^2}} + E_\gamma = m_p c^2 + m_\pi c^2$$

Momentum conservation gives $\dfrac{m_p u}{\sqrt{1 - u^2/c^2}} - \dfrac{E_\gamma}{c} = 0.$

Combining the equations, we have

$$\frac{m_p c^2}{\sqrt{1 - u^2/c^2}} + \frac{m_p c^2}{\sqrt{1 - u^2/c^2}} \frac{u}{c} = m_p c^2 + m_\pi c^2$$

$$\frac{(938.3\ \text{MeV})(1 + u/c)}{\sqrt{(1 - u/c)(1 + u/c)}} = 938.3\ \text{MeV} + 135.0\ \text{MeV}$$

so $\dfrac{u}{c} = 0.134$

and $E_\gamma = \boxed{127\ \text{MeV}}$.

(b) $\lambda_{\text{max}} T = 2.898\ \text{mm} \cdot \text{K}$

$$\lambda_{\text{max}} = \frac{2.898\ \text{mm} \cdot \text{K}}{2.73\ \text{K}} = \boxed{1.06\ \text{mm}}$$

(c) $E_\gamma = hf = \dfrac{hc}{\lambda} = \dfrac{1\,240\ \text{eV} \cdot 10^{-9}\ \text{m}}{1.06 \times 10^{-3}\ \text{m}} = \boxed{1.17 \times 10^{-3}\ \text{eV}}$

(d) In the primed reference frame, the proton is moving to the right at $\dfrac{u'}{c} = 0.134$ and the photon is moving to the left with $hf' = 1.27 \times 10^8$ eV. In the unprimed frame, $hf = 1.17 \times 10^{-3}$ eV. Using the Doppler effect equation (Equation 39.10), we have for the speed of the primed frame (suppressing units)

$$1.27 \times 10^8 = \sqrt{\frac{1 + v/c}{1 - v/c}}\, 1.17 \times 10^{-3}$$

$$\frac{v}{c} = 1 - 1.71 \times 10^{-22}$$

Then the speed of the proton is given by

$$\frac{u}{c} = \frac{u'/c + v/c}{1 + u'v/c^2} = \frac{0.134 + 1 - 1.71 \times 10^{-22}}{1 + 0.134\left(1 - 1.71 \times 10^{-22}\right)} = 1 - 1.30 \times 10^{-22}$$

And the energy of the proton is

$$\frac{m_p c^2}{\sqrt{1-u^2/c^2}} = \frac{938.3\text{ MeV}}{\sqrt{1-\left(1-1.30\times10^{-22}\right)^2}}$$

$$= 6.19\times10^{10}\times938.3\times10^{6}\text{ eV} = \boxed{5.81\times10^{19}\text{ eV}}$$

P46.71 (a) Consider a sphere around us of radius *R* large compared to the size of galaxy clusters. If the matter *M* inside the sphere has the critical density, then a galaxy of mass *m* at the surface of the sphere is moving just at escape speed *v* according to $K+U_g=0$,

or $\frac{1}{2}mv^2 - \frac{GMm}{R} = 0.$

The energy of the galaxy-sphere system is conserved, so this equation is true throughout the history of the Universe after the Big Bang, where $v=\frac{dR}{dt}$. Then,

$$\left(\frac{dR}{dt}\right)^2 = \frac{2GM}{R}$$

or $\frac{dR}{dt} = R^{-1/2}\sqrt{2GM}.$

integrating,

$$\int_0^R \sqrt{R}\,dR = \sqrt{2GM}\int_0^T dt$$

$$\left.\frac{R^{3/2}}{3/2}\right|_0^R = \sqrt{2GM}\,t\Big|_0^T \quad\text{gives}\quad \frac{2}{3}R^{3/2} = \sqrt{2GM}\,T$$

or $T = \frac{2}{3}\frac{R^{3/2}}{\sqrt{2GM}} = \frac{2}{3}\frac{R}{\sqrt{2GM/R}}.$

From above, $\sqrt{\frac{2GM}{R}} = v$

so $T = \frac{2}{3}\frac{R}{v}.$

Now Hubble's law says $v = HR$, so $T = \frac{2}{3}\frac{R}{HR} = \frac{2}{3H}.$

(b) $T = \frac{2}{3\left(22\times10^{-3}\text{ m/s}\cdot\text{ly}\right)}\left(\frac{2.998\times10^{8}\text{ m/s}}{1\text{ ly/yr}}\right) = \boxed{9.08\times10^{9}\text{ yr}}$

= 9.08 billion years

P46.72 A photon travels the distance from the Large Magellanic Cloud to us in 170 000 years. The hypothetical massive neutrino travels the same distance in 170 000 years plus 10 seconds:

$$c(170\ 000\ \text{yr}) = v(170\ 000\ \text{yr} + 10\ \text{s})$$

$$\frac{v}{c} = \frac{170\ 000\ \text{yr}}{170\ 000\ \text{yr} + 10\ \text{s}}$$

$$= \frac{1}{1 + \left\{10\ \text{s}/\left[\left(1.7\times 10^{5}\ \text{yr}\right)\left(3.156\times 10^{7}\ \text{s/yr}\right)\right]\right\}}$$

$$= \frac{1}{1 + 1.86\times 10^{-12}}$$

For the neutrino we want to evaluate mc^2 in $E = \gamma mc^2$:

$$mc^2 = \frac{E}{\gamma} = E\sqrt{1 - \frac{v^2}{c^2}} = 10\ \text{MeV}\sqrt{1 - \frac{1}{\left(1 + 1.86\times 10^{-12}\right)^2}}$$

$$= (10\ \text{MeV})\sqrt{\frac{\left(1 + 1.86\times 10^{-12}\right)^2 - 1}{\left(1 + 1.86\times 10^{-12}\right)^2}}$$

$$mc^2 \approx (10\ \text{MeV})\sqrt{\frac{2\left(1.86\times 10^{-12}\right)}{1}} = (10\ \text{MeV})\left(1.93\times 10^{-6}\right)$$

$$= 19\ \text{eV}$$

Then the upper limit on the mass is

$$m = \boxed{\frac{19\ \text{eV}}{c^2}}$$

$$m = \frac{19\ \text{eV}}{c^2}\left(\frac{\text{u}}{931.5\times 10^{6}\ \text{eV}/c^2}\right) = 2.1\times 10^{-8}\ \text{u}$$

P46.73 (a) If $2N$ particles are annihilated, the energy released is $2Nmc^2$. The resulting photon momentum is $p = \frac{E}{c} = \frac{2Nmc^2}{c} = 2Nmc$. Since the momentum of the system is conserved, the rocket will have momentum $2Nmc$ directed opposite the photon momentum.

$$\boxed{p = 2Nmc}$$

(b) Consider a particle that is annihilated and gives up its rest energy mc^2 to another particle which also has initial rest energy mc^2 (but no momentum initially).

$$E^2 = p^2c^2 + \left(mc^2\right)^2$$

Thus, $\left(2mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$.

Where *p* is the momentum the second particle acquires as a result of the annihilation of the first particle. Thus $4\left(mc^2\right)^2 = p^2c^2 + \left(mc^2\right)^2$, $p^2 = 3\left(mc^2\right)^2$. So $p = \sqrt{3}mc$.

This process is repeated *N* times (annihilate $\frac{N}{2}$ protons and $\frac{N}{2}$ antiprotons). Thus the total momentum acquired by the ejected particles is $\sqrt{3}Nmc$, and this momentum is imparted to the rocket.

$$\boxed{p = \sqrt{3}Nmc}$$

(c) Method (a) produces greater speed since $2Nmc > \sqrt{3}Nmc$.

ANSWERS TO EVEN-NUMBERED PROBLEMS

P46.2 (a) 2.27×10^{23} Hz; (b) 1.32×10^{-15} m

P46.4 $\sim10^{3}$ Bq

P46.6 $\sim10^{-18}$ m

P46.8 Baryon number conservation allows the first reaction and forbids the second.

P46.10 $\sim10^{-23}$ s

P46.12 (a) See P46.12(a) for full explanation; (b) Strangeness is not conserved in the second reaction.

P46.14 (a) ν_e; (b) ν_μ; (c) $\bar{\nu}_\mu$; (d) ν_μ, $\bar{\nu}_\tau$

P46.16 $\bar{\nu}_\mu$ and ν_e

P46.18 (a) See P46.18(a) for full explanation;
(b) $E_e = E_\gamma = 469$ MeV, $p_e = p_\gamma = 469$ MeV/c; (c) $v = 0.999\,999\,4c$

P46.20 The $\rho^0 \to \pi^+ + \pi^-$ decay must occur via the strong interaction. The $K_S^0 \to \pi^+ + \pi^-$ decay must occur via the weak interaction.

P46.22 (a) electron and muon lepton numbers; (b) electron lepton number; (c) charge and strangeness; (d) baryon number; (e) strangeness

P46.24 (a) B, charge, L_e, and L_τ; (b) B, charge, L_e, L_μ, and L_τ;
(c) S, charge, L_e, L_μ, and L_τ; (d) B, S, charge, L_e, L_μ, and L_τ;
(e) B, S, charge, L_e, L_μ, and L_τ; (f) B, S, charge, L_e, L_μ, and L_τ

P46.26 (a) $p_{\Sigma^+} = 686$ MeV/c, $p_{\pi^+} = 200$ MeV/c; (b) 626 MeV/c;
(c) $E_{\pi^+} = 244$ MeV, $E_n = 1.13$ GeV; (d) 1.37 GeV; (e) 1.19 GeV/c^2;
(f) The result in part (e) is within 0.05% of the value ion Table 46.2.

P46.28 (a) See table in P46.28(a); (b) See table in P46.28(b).

P46.30 (a) Σ^+; (b) π^-; (c) K^0; (d) Ξ^-

P46.32 (a) The reaction has a net of 3u, 0d, and 0s before and after; (b) The reaction has a net of 1u, 1d, and 1s before and after; (c) The reaction must net of 4u, 2d, and 0z before and after; (d) Λ^0 or Σ^0

P46.34 3.34×10^{26} electrons, 9.36×10^{26} up quarks, 8.70×10^{26} down quarks

P46.36 $m_u = 312$ MeV/c^2; $m_d = 314$ MeV/c^2

P46.38	(a) 4.30×10^{-18} m/s ; (b) 0.892 nm/s
P46.40	(a) 8.41×10^{6} kg ; (b) No. It is only the fraction 4.23×10^{-24} of the mass of the Sun.
P46.42	1.36×10^{10} yr
P46.44	3.15×10^{-6} W/m^2
P46.46	(a) $c\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$; (b) $\dfrac{c}{H}\left(\dfrac{Z^2+2Z}{Z^2+2Z+2}\right)$
P46.48	(a) See P46.48(a) for full explanation; (b) 6.34×10^{61} m^3/s
P46.50	(a) 1.62×10^{-35} m; (b) 5.39×10^{-44} s
P46.52	$\bar{\nu}_\mu$
P46.54	See P46.54 for full explanation.
P46.56	1 part in 5×10^{7}
P46.58	29.8 MeV
P46.60	$m=\dfrac{\sqrt{(qBrc)^2+E_+^2}+\sqrt{(qBrc)^2+E_-^2}}{c^2}$
P46.62	(a) Z^0 boson; (b) photon or Z^0 boson, gluon
P46.64	74.4 MeV
P46.66	70.4 MeV
P46.68	(a) See P46.68(a) for full explanation; (b) 5.63 GeV; (c) 768 MeV; (d) 280 MeV; (e) 4.43 TeV
P46.70	(a) 127 MeV; (b) 1.06 mm; (c) 1.17×10^{-3} eV; (d) 5.81×10^{19} eV;
P46.72	19 eV/c^2